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ABSTRACT

The paper uses bank- and instrument-level data on asset holdings and liabilities to identify and estimate a general equilibrium model of trade in financial instruments. Bilateral ties are formed as each bank selects the size and the diversification of its assets and liabilities. Shocks propagate due to the response, rather than the size, of bilateral ties to such shocks. This general equilibrium propagation of shocks reveals a financial network where the strength of a tie is determined by the sensitivity of an instrument's return to other instruments' returns. General equilibrium analysis predicts the propagation of real, financial and policy shocks. The network's shape adjusts endogenously in response to shocks, to either amplify or mitigate partial equilibrium shocks. The network exhibits key theoretical properties: (i) more connected networks lead to less amplification of partial equilibrium shocks, (ii) the influence of a bank's equity is independent of the size of its holdings; (ii) more risk-averse banks are more diversified, lowering their own volatility but increasing their influence on other banks. The general equilibrium based network model is structurally estimated on disaggregated data for the universe of French banks. We used the estimated network to assess the effects of ECB quantitative easing policy on asset prices, balance-sheets, individual bank distress risk, and networks systemicness.

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"It's not a bank large enough to cause systemic crisis," said Lorenzo Codogno, a former chief economist at Italy's treasury who operates LC Macro Advisors, a consultancy in London. *"But,"* he added, *"we have seen that even small banks can cause huge problems."* – E.C.B. Takes Reins of Italian Bank to Prevent Wider Crisis, New York Times, January 2, 2019.

1 Introduction

Does financial network structure matter for financial stability? A seminal theoretical literature (Acemoglu, Ozdaglar & Tahbaz-Salehi 2015, Elliott, Golub & Jackson 2014, Cabrales, Gottardi & Vega-Redondo 2017) suggests that contractual links between banks' balance sheets shape a network structure that affects aggregate stability above and beyond what is captured by standard aggregate and bank-level prudential ratios. Elliott et al. (2014) shows that a moderate increase in diversification increases network connectivity and *amplifies* the propagation of defaults, while more complete diversification makes financial networks more resilient to bi-lateral default transmission. On the empirical side, a key challenge is the development of identifiable and micro-founded endogenous network formation models that (i) allow for the endogenous emergence *and* transformation of financial ties, (ii) match the large heterogeneity of institution- and security-level holdings, (iii) test the empirical relevance of theoretical network mechanisms.

This paper addresses these three challenges. First, it puts forward a model of heterogeneous banks and securities where financial linkages form endogenously, shaping a network of interlinked banks' balance-sheets through trade in securities. The network is shaped by bank-specific beliefs about future payoffs, combined with their risk-aversion and cost of equity; as well as by the cost of trading financial instruments. Individual banks' decisions to form bilateral ties makes a network of banks emerge out of a structural general equilibrium model of trade in financial assets and liabilities. Second, beliefs about payoffs, risk aversions, and costs of equity are structurally estimated[1] using disaggregated balance sheet information on assets and liabilities across a number of financial instruments for the entire universe of the 303 French banks. Third, estimated parameters enable

[1]In that sense, the paper follows the literature sparked by Ross (2015), which aims at identifying the market's subjective probabilities of future events from prices. Here we back out each institution's forecast from the time variation of demands.

a comparative statics prediction of the network response to financial shocks such as the ECB's quantitative easing programs. In particular, this large policy shock affects network topology, with consequences for asset prices, banks' distance to insolvency and their influence on key parts of the economy.

The microfounded model is one of a competitive equilibrium in incomplete markets and incomplete information, where heterogeneous banks agree to disagree about the future payoff and price and cannot fully learn other institutions' beliefs from the observation of equilibrium prices. Under such market and information structure, trade occurs in contrast with the typical no-trade result (Merton 1987, Milgrom & Stokey 1982). At general equilibrium, banks form bilateral ties with each other by either buying assets or supplying liabilities. Trading occurs, and ties form, for two reasons: first, because banks are heterogenous in their beliefs, and second, for risk-sharing purposes as they differ in risk aversion.

The network emerges from its ability to turn partial equilibrium shocks into general equilibrium impacts. Indeed, partial equilibrium shocks propagate through ties by causing shifts in the diversification of assets and liabilities. Hence, the propagation of shocks across banks in general equilibrium provides the micro-foundation of a weighted network of banks whose edges weights represent the partial equilibrium adjustment in returns for any pair of equity, and whose adjacent matrix represents the general equilibrium response of the partial equilibrium change in equity return on the entire network. The combination of banks' rebalancing of their assets and liabilities for all financial instruments, and each market's adjustment of prices to achieve partial equilibrium, implies a sequence of propagation of shocks that brings markets back to a general equilibrium. This general equilibrium framework yields a network, which is based on the final impact of a change in the equity price of a bank on the equity prices of the other banks.

Such network is amenable to typical measures of bank influence and bank sensitivity to network shocks. We derive measures of bank's influence and sensitivity which capture the chain sequence of balance sheet rebalancing; such complex propagation mechanism can be measured with a simple transform of the array of general equilibrium equity response. Any parameter change or shock alters the structure of the network itself, resulting in time-varying shifts in influence and sensitivity, and thus shifts in the propagation of future shocks.

The influence of a bank may not be driven by its size: the strength of the financial network links crucially depends on the economy-wide elasticity of *substitution* between financial instruments rather than on the size of holdings, or the size of the balance sheets. Indeed, we highlight the presence of small banks (in terms of equity or total assets) that have a sizable influence on other banks. The elasticity of substitution is central in workhorse models of the industrial organization literature (Berry, Levinsohn & Pakes 1995), yet has been so far absent in both the theoretical financial network literature which focuses primarily on the number of links in incomplete networks, and in policy discussion which focuses on the size of the banks and that of asset holdings to detect sources of systemic risk. Micro-foundations of the network reveal an important tension between the bank's balance-sheet size, bank's decision to diversify and the effects of such diversification on the system-wide fragility. In particular, we show while the more risk-averse banks reduce the variance of their returns by diversifying their portfolio, they increase their influence in the network. An increase in a bank's risk-aversion increases both the influence and the sensitivity of its equity. This novel finding which derives from the endogeneity of the network to individual asset demand and supply decisions runs in vast contrast with two common "rules of thumb" approach: first, the largest banks may not be the most systemic ones; and second, portfolio diversification may increase influence and systemicness rather than reduce it.

The key topological characteristics of the financial network are pinned down by the trade costs, risk aversion and beliefs of the underlying banks. This approach enables us to understand the evolution of the financial network over time and to derive out-of-sample predictions for asset prices, net asset demand, banks balance-sheet, for any shock or policy shift. Such counterfactual analysis captures both network propagation and network transformation and derives implications for measures of systemicness and financial fragility of individual institutions. This paper is to the best of our knowledge the first to provide a direct structural link between net financial asset trade in general equilibrium, network structure, and network effects.

The model is identified under the framework that each bank is using its own factor model of return forecasting. Such factors also explain institutions' asset and liability diversification. Under rather standard assumptions, a key result is that the mapping is one-to-one: from the structural parameters (beliefs, risk aversion, cost of equity) into the optimal portfolio of assets and liabilities. Hence the model is identified by mapping available time-varying security-level holding information from banks' longitudinal balance-sheet into structural parameters. Identifying the factor model of net demands yields the identification of the factor model used to predict returns, as well as each institution's risk aversion and cost of equity. As each set of factors has a specific correlation structure, this yields the first two moments and comoments, i.e. banks' beliefs about returns and correlations. This process then identifies the financial network implied by the partial and general equilibrium response to any change in the model's parameters.

The model is estimated on disaggregated information for the universe of French banks. The banking sector is disaggregated at the individual bank level for 303 banks using balance-sheet data that includes cross-holdings of each bank's equity and detailed instrument holdings. Ties of banks with other sectors are observed using a quarterly dataset of whom-to-whom sector-level and instrument-level information for seven sectors of the French economy and the rest of the world. Sectoral balance sheets provide the bilateral ties of 20 financial instruments, which includes traded securities, non-traded financial assets (e.g. loans), and real assets.

The model's estimation starts by estimating each bank's factor model. A small number of factors is able to explain the bulk of the variance in net demands. The factors exhibit a close correspondence with drivers of the *Global cycle* and the *Eurozone cycle* that correlate with US and Euro policy rates, global GDP growth and global trade growth, the VIX measure of implied global risk aversion, or interest rates on government securities for the countries that were subject to the sovereign crisis of 2011 (Greece, Ireland, Italy, Portugal and Spain). Return beliefs are consistent with realized returns: they explain up to 39 percent of the variance of ex-post returns depending on the sector and instrument. The factor structure on return beliefs is rather similar across sectors, up to a few changes in the ordering of the factors (e.g banking being more sensitive to short-run factors and insurance to long run factors). Return beliefs are generally in line with ex-post returns. They tend to be correlated across sectors and especially so for the banking, mutual funds, and corporate sectors. The belief model fails, however, to predict returns in one key instance: during the 2007-2008 crisis when some institutional sectors made counter-cyclical net asset purchases, i.e. buying assets whose returns are declining. Such misperception right before the financial crisis is akin to Gennaioli & Shleifer (2018). Such crisis led to a broad reassessment of beliefs, yet not all banks reacted in the same way: more risk-averse banks (but not larger and more leveraged banks) revised their beliefs more aggressively following these forecasting errors.

The paper simulates the propagation of the ECB's quantitative easing policy: how such propagation affects asset prices, banks' balance sheets, and the shape of the network itself. One quarter of quantitative easing in the Euro area leads to a reduction of the French long term government yield by 44 bps along with a flattening of the yield curve and an increase in the equity premium. Banks rebalanced their balance sheet holdings from debt to equity, and, in parallel, increased (decreased) their equity (bond) liabilities, and increased their distance to insolvency (according to the first order variation of Atkeson, Eisfeldt & Weill (2017) in general equilibrium). Effects are strongly heterogeneous across banks. Less risk averse and more influential banks reshuffle their their balance sheets less. The financial network becomes less fragile and systemic after the policy shock: mean influence and mean sensitivity goes down. Such reduction is larger for more influent and/or more sensitive banks but, importantly, is uncorrelated with their asset size.

The paper contributes to the theoretical and the empirical literatures. First, the paper follows the theoretical research program laid out by Elliott et al. (2014), who stated that "a fully endogenous study of the network of cross-holdings and of asset holdings is a natural next step." The paper endogenizes the network structure by using a general equilibrium model of asset trade. Second, the paper estimates the network structure using disaggregated data on bank's balance-sheets. This builds a bridge between the empirical literature on asset demand among heterogeneous institutions (Koijen & Yogo 2019, Miranda-Agrippino & Rey 2015), extending it to endogenous asset *supply* and asset demand, and the theoretical network literature.

The paper contributes to the theory literature by providing a model that sits on a continuum between non-tradable contracts and costlessly tradable securities. The higher the trade costs, the more persistent the bilateral ties formed between institutions. The financial network forms endogenously as linkages are constrained by financial instruments' trade costs, yet securities can be traded on the secondary market. Highly tradable instruments (e.g. listed equity, with lower trade costs) lead to more volatile links and more price-elasticities. A periphery of fairly static, hard-to-trade instruments, forms around a set of highly tradable instruments. As such our network differs with networks with non-tradable bilateral contracts in which ties remain as initially formed until a default occurs, as in Acemoglu et al. (2015). The two types of networks (trade-based v. contract-based) differ both in terms of network formation and in terms of network propagation. Attempts to endogenize the second type of network relies on network formation games as in Cabrales et al. (2017) and in Farboodi (2014), while endogenizing the first type of network relies on a general equilibrium model of asset trade as in this paper. Propagation in bi-lateral networks occurs when a negative shock to a debtor party makes him unable to honor a contract with a creditor, which in turn lack the funds needed to deliver on its on its obligations to third parties. By contrast, propagation in our network results from prices changes and balance sheet reshuffling and can occur even across seemingly unrelated assets and across seemingly unrelated institutions. Our paper provides micro-level foundations to the propagation mechanism among vulnerable banks analyzed by Greenwood, Landier & Thesmar (2015).

Our market-implied endogenous weighted network provides a novel perspective on insolvency risk due to the propagation of shocks through the financial network structure (Acemoglu et al. 2015, Elliott et al. 2014, Farboodi 2014, Cabrales et al. 2017, Greenwood et al. 2015) . Whether a shock is amplified or mitigated through network propagation can be simply measured by the ratio between the general equilibrium and the partial equilibrium response to the shock. The measure of conductance proposed by Cheeger (1969) suggests that a higher conductance, i.e. a more tightly connected weighted network, exhibits less amplification. This is reminiscent of Elliott et al. (2014): beyond some threshold, a more diversified network reduces default contagion. Furthermore, by estimating the response of banks' entire balance-sheets to shocks, we can measure how the financial network amplifies or mitigates default risk – as measured by distance to insolvency -- even in absence of a realized default. Such a measure is especially relevant for regulators that would like to assess the risk of capital shortfalls and the potential need for recapitalization or forced deleveraging in a bank. In that dimension, our paper proposes both micro-foundations and empirical estimation.

By using a general equilibrium model of asset trade among heterogenous agents, this paper follows a finance tradition that goes back to Merton (1987) and includes seminal contributions such as Berrada (2006), Gandhi & Serrano-Padial (2015), Coimbra & Rey (2017). The contribution of the paper to this literature is to show how to use observable net demands for financial instruments to structurally estimate a model with a large dimension of heterogeneity (risk-aversion, beliefs, cost of equity, trading costs, regulatory constraints). By backing out beliefs in a framework with heterogeneous agents, the model follows the precedent set by Ross (2015) in the case of a representative agent. The paper shares with Koijen & Yogo (2019) the objective of a structural model that simultaneously matches asset demands and imposes market clearing but our approach differs in two dimensions. First, we simultaneously model the demand and supply of assets, which is key to derive the network of linkages between bank's balance sheets. Second, rather than estimating a discrete choice model to understand portfolio shares, we are using the observed structure and *size* of net demands for financial instruments to reveal the factor structure of beliefs about asset returns and explain institution size and leverage.

The paper contributes to the literature on the evaluation of the effects of shocks on both individual banks and the stability of the financial system by providing an empirical framework that is (i) immune from the Lucas' critique, (ii) jointly takes into account the propagation of shocks in the network and the transformation of the network, (iii) yields bank-level prediction of balance-sheets, asset prices, and network-based measures of financial fragility. The paper addresses these 3 challenges by bridging the gap between the theoretical literature on the effect of monetary policy on risk-taking by heterogenous banks (Coimbra & Rey 2017), and the empirical literature. Our structural approach complements the reduced-form empirical approaches of Dell'Ariccia, Laeven & Suarez (2017) for the US, and of Koijen & Yogo (2019) for the Euro Area, on the risk-exposure and asset prices consequences of large-scale asset purchases. It does so by micro founding the demand and price responses to shocks affecting bank's balance-sheets and by using the tools of network analysis to characterize the evolution of financial fragility.

The paper proceeds as follows. Section 2 describes the endogenous diversification of assets and liabilities of each bank at market equilibrium. The Section makes the financial network emerge endogenously from such institution-level arbitrage. Section 3 introduces the identification and estimation of return beliefs using the time varying cross-correlation of security holdings. Section 5 takes the model to the data and estimates beliefs at the security and sector levels. Section 6 uses such structurally estimated beliefs and risk aversions to simulate the impact of quantitative easing interventions by the European Central Bank on key asset prices, balance-sheets, and network properties.

2 The Financial Network

This Section presents the model of endogenous network formation between firms who trade in both liquid and illiquid assets. The relative illiquidity of a subset of financial instruments leads to a persistent yet evolving network in which firms' balance-sheet are connected by both liquid assets and by costly-to-trade assets. Firms form ties between firms by either purchasing assets or supplying liabilities. Trading occurs, and ties form, for two reasons: first, as firms are heterogeneous in their beliefs, and second, for risk-sharing purposes. Yet, trading costs, due to transaction costs, lack of market clearing mechanisms (over-the-counter securities), or other drivers, limit the ability of firms to engage in the full set of profitable trades. This leads to a network structure whose ability to respond to shocks is affected by the magnitude of trading costs.

This Section presents such constrained general equilibrium framework, which provides the micro-level foundations for the network of banks. The network of banks measures the response of firms' equities to partial equilibrium shocks. Such partial equilibrium shocks propagate as they cause a re-diversification of assets and liabilities. Key measures summarize respectively each firm's *influence* and *exposure*, which respectively characterize a firm's systemicness and financial fragility.

2.1 Balance-Sheet Diversification and Size

Firms $i = 1, 2, \ldots, N$ trade $j = 1, \ldots, J$ financial instruments over time periods indexed by t. In such period t, financial instrument j delivers an observable payoff π_{jt} and has a market price p_{jt}.

In period t, firm i's belief about the *payoff* of financial instrument j in $t+1$ is represented by a firm- and instrument-specific random variable $\widetilde{\pi}_{ijt+1}$. While the beliefs about end-of-period returns are firm-specific (each firm estimates a forecast of returns), the asset price p_{jt} is public information. In that sense, the law of one price implies that the uncertainty lies in the projection of future dividends and future security values.

$$\widetilde{r_{ijt}} = \frac{\widetilde{\pi}_{ijt}}{p_{jt}} = \frac{\mathbb{E}\left[\widetilde{p_{jt+1}} + \widetilde{d_{jt+1}} | \Omega_{it}\right]}{p_{jt}} \tag{1}$$

where $\widetilde{p_{jt+1}}$ is the price in period $t+1$, $\widetilde{d_{jt+1}}$ the dividend in period $t+1$ and Ω_{it} the information set of firm i in period t. Firm i's beliefs about returns are represented in vector form as $\tilde{\mathbf{r}}_{it} = \tilde{\boldsymbol{\pi}}_{it}/\mathbf{p}_t$.

Each firm i forms such beliefs based on (i) on a firm-specific information set and (ii) a firm-specific forecasting model.[2] Model and information are not directly observable by the econometrician. The firm's joint distribution of instrument-level returns $\tilde{\mathbf{r}}$ is a continuous function of $\mathbb{R}^J$ noted $f_i(\tilde{\mathbf{r}})$.

[2] Both the information set and the model are parameterized and estimated in Section 3.3.

Firm i seeks to maximize the return on its equity E_{it} by raising funds on the market, through the emission of liabilities, and investing the total sum of the capital and these raised funds in financial instruments. The firm's trading activity is represented by a level of gross demand D_{ijt} and gross supply S_{ijt} (in value) that is both security- and firm-specific. The demand in value is the product of its price and its quantity: $D_{ijt} = p_{jt} \cdot z^D_{ijt}$; and similarly $S_{ijt} = p_{jt} \cdot z^S_{ijt}$.

Each firm's equity is part of the set of instruments. We write $e(i) \in \{1, 2, \ldots, J\}$ the function that maps firm i's equity into the set of instruments. Hence $E_{it} = p_{e(i)t} \cdot z^S_{ie(i)}$. Firm i's number of shares $z^S_{ie(i)}$ is fixed, while its price $p_{e(i)t}$ is free to adjust to market demand and supply. Each additional unit of total demand $\mathbf{D}'_{it}\mathbf{1}_J = \sum_j D_{ijt}$ for assets beyond initial capital E_{it} requires raising a corresponding additional unit of liability $\mathbf{S}'_{it}\mathbf{1}_J = \sum_j S_{ijt}$, with $\mathbf{1}_J$ the J-column vector of ones. The firm's balance sheet satisfies the usual equality of assets and liabilities:

$$\mathbf{D}'_{it}\mathbf{1}_J = \mathbf{S}'_{it}\mathbf{1}_J + E_{it} \tag{2}$$

which simplifies to $\boldsymbol{\Delta}'_{it}\mathbf{1}_J = E_{it}$ when defining the net demand vector as $\boldsymbol{\Delta}_{it} = \mathbf{D}_{it} - \mathbf{S}_{it}$. Hence the net demand in period t, together with firm i's current capital E_{it}, makes up period t's balance sheet and asset-liability diversification; the firm achieves such diversification by trading $\boldsymbol{\Delta}_{it} - \boldsymbol{\Delta}_{it-1} \in \mathbb{R}^J$.

Firm i faces a trading cost $\gamma_{ij} \geq 0$ per quantity traded for adjusting its net demand Δ_{ijt} for security j from Δ_{ijt-1} to Δ_{ijt}. When the trading cost $\gamma_{ij} = 0$, the firm's holdings of security j are not constrained by its past holdings.[3] When $\gamma_{ij} \to \infty$ the firm's current holdings of such illiquid assets are entirely pinned down by its previous holdings.

The firm's risk appetite is represented by a utility function $u(\cdot)$, a function of its net income. Noting $\boldsymbol{\Delta}_{it}$ firm i's stacked vector of net demands in period t, the firm maximizes:

$$\begin{aligned} \max_{\boldsymbol{\Delta}_{it} \in \mathbb{R}^J} & \int u_i\left(\boldsymbol{\Delta}'_{it}\tilde{\mathbf{r}}\right) f_i(\tilde{\mathbf{r}}) d\tilde{\mathbf{r}} - \left\| \boldsymbol{\gamma}_i / \mathbf{p} \cdot (\boldsymbol{\Delta}_{it} - \boldsymbol{\Delta}_{it-1}) \right\|^2 \\ \text{s.t.} \quad & \boldsymbol{\Delta}'_{it}\mathbf{1}_J = E_{it}, \end{aligned} \tag{3}$$

where the vector of trading costs $\boldsymbol{\gamma}_i/\mathbf{p}$ is per unit of security traded. Such optimization program yields a vector $\boldsymbol{\Delta}_{it}(\mathbf{p}_t)$ of net demands as a function of the price vector $\mathbf{p}_t$ given the joint distribution

[3] Such trading costs are structurally identified in Section 5.

of its beliefs about future payoffs, its initial equity, and its initial balance sheet diversification $\mathbf{\Delta}_{it-1}$. Utility functions u_i are increasing, continuous, and concave. The cost of equity λ_i is the Lagrange multiplier of the funding constraint $\mathbf{\Delta}'_{it}\mathbf{1}_J = E_{it}$.

The term $\|\boldsymbol{\gamma}_i/\mathbf{p}\cdot(\mathbf{\Delta}_{it}-\mathbf{\Delta}_{it-1})\|^2$ captures the firm's balance sheet inertia: although the firm aims at changing demand and supply according to shifts in the distribution of return beliefs f_i, it might not be able to achieve a full adjustment of its balance-sheet from one quarter to the next. Firms are highly specialized in trading and managing certain classes of financial instruments, and therefore vary in their ability to respond to a change in beliefs about returns of a particular instrument. Here $\cdot$ (resp., $/$) is the term by term vector product (resp., term by term division) and $\|\cdot\|^2$ is a norm on trading activity in $\mathbb{R}^J$ that measures the magnitude of trading activity.

2.2 Market Equilibrium with Trade Costs

Given each firm's net demand for each financial instrument i as a function of (i) the price vector, (ii) return beliefs, (iii) initial equity, (iv) initial balance sheet, this subsection establishes the existence and properties of the market equilibrium in t with trading costs.

Two key ingredients enable trade in contrast with Milgrom & Stokey (1982): first, markets are not complete as the set of instruments does not include the full set of firm-specific contingent claims. Second, each firm's model, that maps private information into forecasts of returns, is not common knowledge, which mirrors the framework of Lintner's (1969). Hence observing the firm's demand for assets does not fully reveal its information. Firms trade (i) for risk-sharing purposes as their risk preferences u_i differ, and (ii) as they 'agree to disagree' each holding beliefs $\tilde{\mathbf{r}}_i$ about stochastic returns.

Market equilibrium in t is defined as follows. For any instrument j that is also the equity of a firm i, $j = e(i)$, the net demand of instrument j is $\sum_{i'=1}^{N}\Delta_{i'e(i)t} - E_{it}$; otherwise it is simply $\sum_{i'=1}^{N}\Delta_{i'jt}$. Denote the stacked J-vector of firms' equity by $\mathbf{E}_t$, equal to E_{it} when $j = e(i)$ and 0 otherwise.

Definition 1. ***(Equilibrium)*** *An equilibrium in period t is a J-vector of prices for each financial* *instrument $\mathbf{p}_t^*$ that clears each of the J instrument markets:*

$$\sum_{i=1}^{N} \boldsymbol{\Delta}_{it}\left(p_{1t}^*, p_{2t}^*, \ldots, p_{Jt}^*\right) = \mathbf{E}_t\left(p_{1t}^*, p_{2t}^*, \ldots, p_{Jt}^*\right), \tag{4}$$

where $\boldsymbol{\Delta}_{it}\left(\mathbf{p}_t^*\right)$ and $\mathbf{E}_t\left(\mathbf{p}_t^*\right)$ are J-column vectors.

Proposition 1. ***(Existence and Local Uniqueness of Equilibrium)*** *There exists an equilibrium price vector $\mathbf{p}_t^* \in \mathbb{R}^J$. Such equilibrium is locally unique almost surely.*

Proof. See Appendix A.1 for existence based on standard arguments. Finite trade costs do not affect the existence proof as the trade cost term $\|\boldsymbol{\gamma}_i/\mathbf{p} \cdot (\boldsymbol{\Delta}_{it} - \boldsymbol{\Delta}_{it-1})\|^2$ is homogeneous of degree zero. Intuitively, such trade costs simply lower the price elasticity of demand. Local uniqueness follows from a result by Sard (1942) and used in Debreu et al. (1970). As demand is a continuous function of beliefs, utility functions, and equity, the set of critical points of the Jacobian has measure zero. □

2.3 Shock Propagation Through the Financial Network

The financial network predicts how partial equilibrium shocks to a limited set of equity or non-equity instruments affect other banks' value by propagating through balance sheets whose shifts in diversification is potentially limited by trade costs. The framework will suggest that with large trade costs $\boldsymbol{\gamma} \to \infty$, the network is akin to a fixed network of equity cross-holdings and of holdings of primitive instruments, including debt. With finite trade costs, the network ties' sensitivity to prices is a key mechanism that explains the amplification or the mitigation of shocks.

The model is flexible and can accommodate a large variety of shocks such as either (i) a shock to one firm's equity value, which affects the equity value of all other firms in the economy, (ii) monetary policy interventions that shift the demand for debt instruments or (iii) shifts in the market beliefs about a subset of financial instruments, e.g. driven by external factors such as market volatility or forecasts of fundamentals. In each case, the financial network multiplies the initial, partial equilibrium shock, and yields a predicted general equilibrium impact for each of the financial securities.

Each of the three types (i)–(iii) of partial equilibrium shocks can be represented by a shock $d\theta$

to a parameter θ that affects net demands. If, for instance, $d\theta$ is a shock to the market value of the equity of firm k, then $d\theta = dE_k$ and the partial equilibrium shift in net demand is $\partial \Delta_k / \partial \theta \cdot d\theta$.

Firm i's general equilibrium equity value shift is driven by shifts in its assets and liabilities' market values, written compactly as:

$$\frac{dE_i}{d\theta} = \mathbf{1}' \frac{d}{d\theta} \boldsymbol{\Delta}_i \tag{5}$$

as a consequence of the funding constraint $E_i = \mathbf{1}' \boldsymbol{\Delta}_i$ (Section 2.1).

The total impact of the shock $d\theta$ on the vector $\boldsymbol{\Delta}_i \in \mathbb{R}^J$ of net demands of each firm $i = 1, 2, \ldots, N$ is the sum of a partial equilibrium term and a general equilibrium term. The total derivative of net demand w.r.t. the shock expands according to the chain rule as:

$$\frac{d\boldsymbol{\Delta}_i}{d\theta} = \frac{\partial \boldsymbol{\Delta}_i}{\partial \log \mathbf{p}} \frac{d \log \mathbf{p}}{d\theta} + \frac{\partial \boldsymbol{\Delta}_i}{\partial \theta}, \tag{6}$$

where $\frac{\partial \boldsymbol{\Delta}_i}{\partial \theta} \in \mathbb{R}^J$ is the partial equilibrium rebalancing of assets and liabilities at constant prices; $\frac{\partial \boldsymbol{\Delta}_i}{\partial \log \mathbf{p}} \in M_{J,J}(\mathbb{R})$ is the sensitivity of net demand to the prices of financial instruments, a consequence of banks' rebalancing of their balance sheet in response to price changes. $\frac{d \log \mathbf{p}}{d\theta} \in \mathbb{R}^J$ is the market-wide shift in the price vector, common to all institutions as the law of one price holds.

The set of instruments is split into *primitive* instruments and *equity* instruments, $\mathbf{p}' = ((\mathbf{p}^p)' \, (\mathbf{p}^e)')$ and $\boldsymbol{\Delta}_i' = ((\boldsymbol{\Delta}_i^p)' \, (\boldsymbol{\Delta}_i^e)')$. The equity instruments play a special role as they pin down the market value of the firm. Thus:

$$\frac{dE_i}{d\theta} = \mathbf{1}' \frac{\partial \boldsymbol{\Delta}_i}{\partial \theta} + \mathbf{1}' \begin{pmatrix} \frac{\partial \boldsymbol{\Delta}_i^p}{\partial \log \mathbf{p}^p} & \frac{\partial \boldsymbol{\Delta}_i^p}{\partial \log \mathbf{p}^e} \\ \frac{\partial \boldsymbol{\Delta}_i^e}{\partial \log \mathbf{p}^p} & \frac{\partial \boldsymbol{\Delta}_i^e}{\partial \log \mathbf{p}^e} \end{pmatrix} \cdot \begin{pmatrix} \frac{d \log \mathbf{p}^p}{d\theta} \\ \frac{d \log \mathbf{p}^e}{d\theta} \end{pmatrix}, \tag{7}$$

and $\mathbf{1}' \frac{\partial \Delta_i}{\partial \theta}$ is the partial equilibrium shift in firm i's value, simply noted $\mathbf{pe}$. With a perfectly inelastic supply of equity shares,[4] the fluctuation in the market value of equity is driven solely by the change in the market price of equity. This implies that $\frac{d \log \mathbf{p}^e}{d\theta} = (\text{diag}\mathbf{E})^{-1} \frac{d\mathbf{E}}{d\theta}$. In this expression, the presence of the equity values E_i and $\mathbf{E}$ on both sides will provide us with the equilibrium response of firms' values to the shock $d\theta$. Such response will depend on equity cross-holdings as $\mathbf{1}' \boldsymbol{\Delta}_i^e$ is the row vector of bank i's holdings of other banks' equity.

[4] While this allows for stock splits, an extension accommodates the case of public offerings and stock buy-backs.

Such cross-holdings of bank equity by firm i respond to the vector of equity prices $\log \mathbf{p}^e$. The impact of log prices on the value of cross-holdings is driven by two terms. The first term is the mechanical impact of prices on the value of the holdings, keeping the number of shares held constant. The second term captures the endogenous response of the number of bank shares held to price changes.

$$\mathbf{1}_e' \frac{\partial \mathbf{\Delta}_i^e}{\partial \log \mathbf{p}^e} = \mathbf{1}_e' \frac{\partial \left(\mathbf{p}^e \cdot \mathbf{z}_i^e\right)}{\partial \log \mathbf{p}^e} = \underbrace{\mathbf{1}_e' \mathrm{diag} \mathbf{\Delta}_i^e}_{Equity\ Holdings\ H_i} + \underbrace{\mathbf{1}_e' \left[\left(\mathbf{p}^e \mathbf{1}_e'\right) \cdot \frac{\partial \mathbf{z}_i^e}{\partial \log \mathbf{p}^e} \right]}_{Response\ of\ equity\ holdings\ \partial H_i / \partial \log \mathbf{p}^e} \tag{8}$$

where $\cdot$ is the term by term vector product. This is simply noted, for the sake of clarity $H_i + \partial H_i / \partial \log \mathbf{p}^e$. The first term H_i is the row vector of holdings of other banks' equity (in value) as in Elliott et al. (2014). The second term, noted $\partial H_i / \partial \log \mathbf{p}^e$ is due to bank i's response of holdings to the price change.

The response of each firm i's market value in equation 7 can now be stacked into an equity vector $\mathbf{E} = \{E_i\}_i$.[5] This provides a familiar formula. This formula expresses the response of firms' value to a shock $d\theta$ in terms of a network of holdings H and a partial equilibrium shock $\mathbf{pe}$. Stacking the scalars $\frac{dE_i}{d\theta}$ into a column vector,

$$\frac{d\mathbf{E}}{d\theta} = \left[\mathbb{I} - \left(H + \frac{\partial H}{\partial \log \mathbf{p}^e} + \left\{ \frac{\partial (\mathbf{1}_p' \mathbf{\Delta}_i^p)}{\partial \log \mathbf{p}^e} \right\}_i \right) (\mathrm{diag} \mathbf{E})^{-1} \right]^{-1} \left[\frac{\partial \mathbf{B}}{\partial \theta} + \left\{ \frac{\partial \mathbf{1}_p' \mathbf{\Delta}_i^p}{\partial \log \mathbf{p}^p} + \frac{\partial \mathbf{1}_e' \mathbf{\Delta}_i^e}{\partial \log \mathbf{p}^p} \right\}_i \frac{d \log \mathbf{p}^p}{d\theta} \right] \tag{9}$$

where the first term can be simply written $\left[\mathbb{I} - \mathcal{A}\right]^{-1} \mathbf{pe}$. In this expression, $\mathcal{A}$ is the adjacency of this paper's financial network. Such adjacency matrix is equal to Elliott et al.'s (2014) adjacency matrix of cross holdings $H\,(\mathrm{diag}\mathbf{E})^{-1}$ in the specific case where (i) the number of bank shares held by any other bank is kept constant, (ii) there is no interbank linkage through other instruments than equity. Then,

Definition 2. ***(Endogenous Financial Network)*** *The financial network is a weighted and directed graph $\mathcal{G} = (V, E)$ whose vertices V are the banks $\{1, 2, \dots, N\}$. There is an edge $(i, i', a_{ii'}) \in E$ from bank i to bank i' if a partial equilibrium shock in the value (assets–liabilities) of bank i affects the market price of the equity of bank i'. The weight $a_{ii'}$ of the edge (i, i') measures the magnitude*

[5] Throughout the paper, the notation $\{X_i\}_i = X$ means that a series of row vectors $i = 1, 2, \dots, N$ is stacked into a squared matrix of size N.

and the sign of the impact. The adjacency matrix of such financial network is $\mathcal{A}$. The general equilibrium impact of a partial equilibrium shock on banks' values is:

$$\frac{d\mathbf{E}}{d\theta} = [\mathbb{I} - \mathcal{A}]^{-1} \left[\mathbf{pe} + \left\{ \frac{\partial \mathbf{1}_p' \mathbf{\Delta}_i^p}{\partial \log \mathbf{p}^p} + \frac{\partial \mathbf{1}_e' \mathbf{\Delta}_i^e}{\partial \log \mathbf{p}^p} \right\}_i \frac{d \log \mathbf{p}^p}{d\theta} \right], \tag{10}$$

When the number of equity and non-equity instruments held by each bank is fixed, and when the price of non-equity instruments is fixed, the network is simply the network of cross-holdings, $\mathcal{A} = C$, where $C = H\,(diag\mathbf{E})^{-1}$ is the matrix of cross-holdings as a proportion of each bank's market capitalization. In general, Equation (9) shows that the propagation of shocks is affected by two terms in addition to C:

$$\mathcal{A} \equiv C + \frac{\partial H}{\partial \log \mathbf{p}^e} (\mathrm{diag}\mathbf{E})^{-1} + \left\{ \mathbf{1}_p' \frac{\partial \mathbf{\Delta}_i^p}{\partial \log \mathbf{p}^e} \right\}_i (\mathrm{diag}\mathbf{E})^{-1}, \tag{11}$$

where $\frac{\partial H}{\partial \log \mathbf{p}^e}\,(diag\mathbf{E})^{-1}$ is the network of cross-holdings' response to equity price changes; and $\left\{ \mathbf{1}_p' \frac{\partial \mathbf{\Delta}_i^p}{\partial \log \mathbf{p}^e} \right\}_i$ is the set of links between banks that are not equity links.

2.4 Network Structure: Amplification or Mitigation

Whether a partial equilibrium shock is amplified or mitigated depends on the structure of the financial network. Noting $\|\mathbf{pe}\|$ the magnitude of the partial equilibrium shock,[6] the ratio of the magnitude of the general equilibrium shock $\left\|\frac{d\mathbf{E}}{d\theta}\right\|$ to the partial equilibrium shock $\|\mathbf{pe}\|$ is a measure of the magnification of the initial partial equilibrium shock. When $\left\|\frac{d\mathbf{E}}{d\theta}\right\| / \|\mathbf{pe}\| > 1$ the partial equilibrium shock is amplified, while in the opposite case the shock is mitigated. The maximum of such ratio is noted α:

$$\alpha = \max \frac{\left\|\frac{d\mathbf{E}}{d\theta}\right\|}{\|\mathbf{pe}\|}, \tag{12}$$

where the maximum is taken over all possible partial equilibrium shocks $\mathbf{pe}$ in $\mathbb{R}^N$. It is equal to the norm of the inverse of the Laplacian $(1 - \mathcal{A})^{-1}$ of the graph $\mathcal{G}$. A result by Cheeger (1969) implies that α has an upper bound equal to a measure of the network's connectedness. Hence network shock amplification is related to the graph's topology.

[6] Results of propositions 2 and 3 are independent of the specific norm. Such results apply for any norm on the Euclidean space $\mathbb{R}^N$ with dimension N equal to the number of institutions.

We proceed as follows. Given a partition of the financial network $\mathcal{G}$ into two subgraphs G and $\overline{G}$, the *conductance* of the cut $(G, \overline{G})$ measures the connection between subgraph G and subgraph $\overline{G}$:

$$\varphi(G) = \frac{\sum_{j \in G, j' \in \overline{G}} a_{jj'}}{\min(a(G), a(\overline{G}))} \tag{13}$$

where $a(G) = \sum_{j \in G} \sum_{j' \in V} a_{jj'}$. The Cheeger constant h_G is the minimum of the conductance $\varphi(G)$ over all subgraphs G of $\mathcal{G}$.

$$h_G = \min \varphi(G) \tag{14}$$

Then, Cheeger (1969) shows that the second smallest eigenvalue of the Laplacian is bounded below by $h_G^2/2$. Given that the maximum of α over all potential partial equilibrium shocks is the norm of the inverse of the Laplacian, we obtain the following proposition:

Proposition 2. ***(Network Conductance and Maximum Propagation of Shocks)*** *In a financial network with a higher conductance (more 'tightly' connected) shocks are less magnified. Precisely,*

$$\frac{1}{2h_G} \leq \alpha \leq \frac{2}{h_G^2}. \tag{15}$$

In such inequality, $\frac{1}{2h_G} \leq \frac{2}{h_G^2}$ *as the degree of each vertex of the network is* 1*, which implies that* $\varphi(G) \leq 1$*, and* $h_G \leq 1$*.*

Proof. This result is proven by Cheeger (1969). □

The estimation of the Cheeger constant is challenging as the minimum is taken over all possible subgraphs of $\mathcal{G}$, of which there are $2^J - 1$. A common approach to this issue is to use a spectral cut (Spielman 2007): given the eigenvector corresponding to the second smallest eigenvalue of the laplacian, securities are sorted into each of the subgraph according to the sign of their value in the eigenvector. Spielman (2007) shows that this leads to a cut corresponding to the minimum 14.

2.5 Measuring the Systemicness of Institutions

While the previous analysis focuses on the economy-wide propagation of partial equilibrium shocks, we focus here on individual securities. This will allow us to measure the position of a firm's equity in the network of financial instruments.

Two simple measures characterize (i) the *influence* and (ii) the *sensitivity* of a bank to a partial equilibrium shock. Influence is a measure of the systemicness bank, sensitivity measures its fragility. Influence typically differs from sensitivity: institutions at the fringes of the financial network exhibit high sensitivity but negligible influence.

Definition 3. ***(Influence and Sensitivity)*** *The vector of influences of each bank is*[7]

$$\mathbf{Influence}_t' = \mathbf{1}' \left[I - \mathcal{A}_t\right]^{-1}, \tag{16}$$

measuring the general equilibrium change in equity value of the total financial system caused by a unit increase of a bank's partial equilibrium shock. Influence is akin to the centrality measure presented in Bonacich (1987). The sensitivity vector is

$$\mathbf{Sensitivity}_t = \left[I - \mathcal{A}_t\right]^{-1} \mathbf{1}, \tag{17}$$

measuring the impact on each bank of a unit increase in value of all bank equities.

The structure of the financial network provides an upper bound on the average influence and sensitivities. The following proposition suggests that a "more connected" network, in the sense of Cheeger's conductance, will also have smaller influences and sensitivities.

Proposition 3. ***(Magnification)*** *An upper bound for the sum of the magnitudes of influences (resp., sensitivities) is the number α that measures the amplification or mitigation of shocks:*

$$\sum_{i=1}^{N} |Influence_{it}| \leq \alpha \leq \frac{2}{h_G^2}, \tag{18}$$

with the same inequality for the sum $\sum_{i=1}^{N} |Sensitivity_{it}|$.

Proof. Endow the space $\mathbb{R}^N$ with the L^1 norm $\|(x_1, \ldots, x_N)\| = \sum_{i=1}^{N} |x_i|$. Then: $\|\mathbf{Influence}_t\| \leq \frac{1}{N} \left\|(I - \mathcal{A}_t)^{-1}\right\| \|\mathbf{e}\| = \alpha$. The inequality $\alpha \leq \frac{2}{h_G^2}$ comes from proposition 2. Given that the spectrum of a matrix is identical to the spectrum of its transpose, the same inequality holds for the sensitivities. $\square$

[7] One potential issue is that the influence vector includes the security itself. Thus, an alternative definition excludes each instrument from the influence vector. This yields similar results in the empirical sections of this paper. A similar comment applies to the sensitivity vector.

Influence and sensitivity measures are typically uncorrelated with the size of holdings. In particular, a bank's holdings can be small but exhibit a strong elasticity with respect to the returns of other securities. Appendix Figure 9 plots network influence against the size of holdings using estimates provided in Section 4, and finds no systematic correlation between the two metrics.

Figure 2 illustrates an equilibrium tension between the firm-level need for diversification, and the financial connections that such diversification implies. In particular, while the more risk averse firms *reduce* the variance of their returns by diversifying their portfolio, they *increase* their influence in the network. This is illustrated by two simulations of market equilibrium, one with a uniform distribution of risk aversions, over $[1, 10]$; and one with a uniform distribution of risk aversions over $[1, 15]$. While the distribution of risk aversions is uniform, the distribution of balance sheet sizes is such that the least risk-averse firms have balance sheet sizes about 2.5 times the balance sheet size of firms with a risk aversion above 4 (Subfigure (b)). Subfigures (a) and (c) illustrate the trade-off between firm-level diversification and network-level influence: firms with the lowest risk aversion have the equity with the greatest influence (as defined in equation (16)), and the greatest impact of their equity on their immediate connections (Subfigure (c)).

3 Structural Estimation

Structural estimation of the model in a parametric context requires (i) a specification of banks' utility functions u_i, i.e. risk preferences, (ii) a specification of institutions' belief formation model and data. The first point is addressed by using CARA utility with trade costs. The CARA setup is equivalent to maximizing expected returns under a value-at-risk constraint. In this case the importance of the value-at-risk constraint is measured by the risk-aversion parameter ρ_i (Coimbra & Rey 2017). Point (ii) is addressed by considering that each institution builds a specific factor model of returns.

3.1 Parameterization: Demand, Trade Costs, Beliefs

Each firm trades off the expected return and the expected variance of such stochastic net income. The relative importance of such variance for firm i is noted ρ_i.[8] Hence firm i chooses its net demand

[8]This mean-variance goal for a firm formally corresponds to the concept of absolute risk aversion in the context of household choice under Gaussian return uncertainty with Constant Absolute Risk Aversion (CARA).

of instruments by maximizing

$$\begin{aligned} &\text{argmax}_{\boldsymbol{\Delta}_{it}}\, Q(\boldsymbol{\Delta}_{it}) \equiv \boldsymbol{\Delta}_{it}'\boldsymbol{\mu}_{it} - \frac{1}{2}\rho_i\boldsymbol{\Delta}_{it}'\boldsymbol{\Sigma}_{it}\boldsymbol{\Delta}_{it} - \left\|\boldsymbol{\gamma}_i/\mathbf{p}\cdot(\boldsymbol{\Delta}_{it}-\boldsymbol{\Delta}_{it-1})\right\|^2 \\ &\text{s.t. } \boldsymbol{\Delta}_{it}'\mathbf{1}_J = E_{it} \end{aligned} \tag{19}$$

where $\boldsymbol{\mu}_{it} \equiv \mathbb{E}\left[\widetilde{\boldsymbol{r}_{it}}\right] \equiv \mathbb{E}\left[\widetilde{\boldsymbol{r}}_t|\Omega_{it}\right]$ is the J-vector of mean return beliefs and $\boldsymbol{\Sigma}_{it} \equiv \text{Var}\left[\widetilde{\boldsymbol{r}_{it}}\right] \equiv \text{Var}\left[\widetilde{\boldsymbol{r}}_t|\Omega_{it}\right]$ is the J-square matrix of the variance-covariance of return beliefs. Stochastic beliefs about returns are not multi-collinear and have strictly positive variances, in other words the variance-covariance of beliefs $\text{Var}(\tilde{\mathbf{r}}_{it})$ is symmetric, positive-definite.

The following proposition provides a closed-form expression for net demands with firm-specific beliefs and trade costs:

Proposition 4. (CARA Firm Demand with Trade Costs) *Each firm i's net demand for instruments depends on the first two moments of its return beliefs, its risk aversion, its cost of capital, and its adjustment costs. Formally, the J-vector of net demands is*

$$\boldsymbol{\Delta}_{it} = \left(\mathit{\Gamma}_i + \rho_i\Sigma_{it}\right)^{-1}\left(\boldsymbol{\mu}_{it} - \eta_{it}\mathbf{1} + \mathit{\Gamma}_i\boldsymbol{\Delta}_{it-1}\right), \tag{20}$$

where η_{it} is the cost of equity, i.e. the impact of a marginal relaxation of the equity constraint on the mean-variance objective,

$$\eta_{it} = \frac{\mathbf{1}'\left(\mathit{\Gamma}_i + \rho_i\Sigma_{it}\right)^{-1}\boldsymbol{\mu}_{it} - E_{it} + \mathbf{1}'\mathit{\Gamma}_i\boldsymbol{\Delta}_{it-1}}{\mathbf{1}'\left(\mathit{\Gamma}_i + \rho_i\Sigma_{it}\right)^{-1}\mathbf{1}}. \tag{21}$$

The Lagrange multiplier η_{it} is the marginal value of the funding constraint $\boldsymbol{\Delta}_{it}'\mathbf{1}_J = E_{it}$.

Proof. See Appendix A.1. □

Mean-Variance Frontier at Market Equilibrium

Equilibrium prices clear the market of each security j and reflect the underlying trade-off between risk and return of each individual institution i. Thus the equilibrium market price p_i should reflect the distribution of beliefs $\widetilde{\pi}_{ijt}$ about payoffs.

This is what Figure 1 illustrates in a market simulation with $J = 100$ instruments and $N = 3$

banks. When institutions have common beliefs, figure 1 shows that the risk-return frontier describes a typical mean-variance frontier (Cochrane 2009) with no convex hull. Equilibrium prices are obtained by a contraction mapping derived from the general equilibrium fixed point in prices. Each point of these simulated graphs is a security. Different shades and point types are for each of the 3 banks. With common beliefs, the mean return and risk converge to a parabola, where securities with an average positive (resp., negative) covariance with other securities' payoffs lie in the upper part (resp., lower part) of the parabola.

When institutions disagree about the mean and variance of payoffs, or when institutions face constraints on their demand or supply of securities, securities typically lie inside the convex hull of the mean-variance frontier. The slope of the fitted line of the upper part of the parabola is an increasing function of the bank's risk aversion.

3.2 Model Identification: Intuitions

Knowing firms' beliefs, risk aversion, and equity levels is key to the estimation of the sensitivity of network ties to shocks (network adjacency matrix of equation (11)).

Indeed, bilateral ties emerge endogenously from banks' beliefs about stochastic returns (means, variances, and correlations), and from their risk aversion and equity level. In turn, the *observation* of time-varying financial ties and equity levels identifies the time-variation of banks' stochastic beliefs and their risk aversion.

For instance, in a simple case with only two assets, a negative correlation between the demand Δ_{ijt} for listed equities and the demand $\Delta_{ij't}$ of long term debt reveals the belief that the return $\widetilde{r}_{ijt}$ on listed equities and the return $\widetilde{r}_{ij't}$ on long term debt are negatively correlated. The relationship between beliefs about correlations $\mathrm{Corr}(\widetilde{r}_{ijt}, \widetilde{r}_{ij't})$ and the magnitude of time-variation in holdings identifies the firm's risk aversion ρ.

Each firm is characterized by (i) beliefs about $J(J-1)/2$ correlations and J means in each time period t, (ii) a constant risk aversion parameter ρ, and (iii) an equity level E_t. For each firm i, we observe (a) assets and liabilities Δ_{ijt} for each of the J instruments in each time period t and (b) their equity level E_t. Identifying beliefs and risk aversion requires a set of assumptions on the time-varying dynamics of first- and second-order moments. The literature has provided a range of approaches to model time-varying correlation structures (Engle & Kelly 2012, Harvey 2010, Harvey

& Thiele 2016). In this paper, each bank forecasts returns using a firm-specific factor model, which provides the variance-covariance structure of the forecast returns.

The following sections show that (i) assuming a firm-specific factor structure in stochastic returns implies a firm-specific factor structure in net demands, and (ii) the relationship from the factor structure of returns to the factor structure of net demands can be *inverted* to identify each firm's beliefs about returns' means, variances, and correlations. Both factors and loadings are specific to each bank i, reflecting the fact that firms have heterogeneous beliefs about both the factors that price assets *and* about the comovement of prices with factors.

Trade costs are identified by estimating the impact of the previous period's holdings on current holdings. We lighten the exposition by presenting identification with no trade cost, and present identification with such costs in Appendix Proposition 7.

3.3 From Return Beliefs to Net-Demands: A Dynamic Factor Model

Recent empirical asset pricing literature (Miranda-Agrippino & Rey 2015, Koijen & Yogo 2019) suggests that unobservable and persistent factors drive asset returns. Similarly, in this paper, each firm uses a factor model turning public and private information into a forecast of the joint distribution of returns.

Data used in this paper includes up to 820 financial instruments (Section 4). Given such level of disaggregation, financial instruments are grouped into natural classes, e.g. listed and unlisted equities, long-term and short-term debt. Within each instrument class, the firm forms beliefs about the joint distribution of financial instruments' returns; the firm also forms beliefs about the joint distribution of returns across instrument classes. For the sake of simplicity, we present here an approach without grouping by instrument class.

Stochastic return beliefs about instruments follow a factor structure:

$$\widetilde{\mathbf{r}}_{it} = \boldsymbol{\varphi}_i + \boldsymbol{\Lambda}_i \mathbf{f}_{it+1} + \boldsymbol{\varepsilon}_{it}, \quad \Sigma_{\varepsilon i} \equiv Var\left(\boldsymbol{\varepsilon}_{it}\right), \tag{22}$$

where there are two sources of uncertainty: first, the variance of the forecast factor $\mathbf{f}_{it+1}|\mathbf{f}_{it}$ measures the uncertainty coming from firm i's factor structure; second, $\boldsymbol{\varepsilon}_{it}$ measures firm i's idiosyncratic uncertainty about returns. Each factor $\mathbf{f}_{it+1}$ follows an autoregressive process:

$$\mathbf{f}_{it+1} = \boldsymbol{\phi}_i + \Phi_i \mathbf{f}_{it} + \mathbf{u}_{it+1},\ \Sigma_{ui} \equiv Var(\mathbf{u}_{it+1}), \tag{23}$$

where $\boldsymbol{\phi}_i$ is a $K \times 1$ vector of factor constants, Φ_i is the K-diagonal matrix of autoregressive coefficients. $\mathbf{u}_{it+1}$ are the innovations to factors.

Overall, firms form beliefs in two steps. First, firms forecast the value of the factors $\mathbf{f}_{it+1}$; second, firms use the loadings $(\boldsymbol{\varphi}_i, \boldsymbol{\Lambda}_i, \Sigma_{\varepsilon i})$ to forecast the mean and the variance-covariance of returns.

Proposition 5 below shows that when firms use a dynamic factor model to forecast return beliefs, then their net demands also follow a dynamic factor model. Indeed,

$$\boldsymbol{\Delta}_{it} = \mathbf{c}_i + \mathbf{L}_i \mathbf{f}_{it+1} - \mathbf{h}_{it}. \tag{24}$$

The vector of constants $\mathbf{c}_i$, the net-demand loadings $\mathbf{L}_i$, and the time-varying term $\mathbf{h}_{it}$ are a function of the risk aversion ρ, the constant $\boldsymbol{\varphi}_i$ and the loadings $\boldsymbol{\Lambda}_i$:

$$\mathbf{c}_i = \frac{1}{\rho_i}\left[\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i}\right]^{-1} (\boldsymbol{\varphi}_i + \Lambda_i \boldsymbol{\phi}_i)\,, \quad L_i = \frac{1}{\rho_i}\left[\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i}\right]^{-1} \Lambda_i \Phi_i, \tag{25}$$

and $\mathbf{h}_{it} = \frac{1}{\rho_i}\left[\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i}\right]^{-1} \eta_{it} \mathbf{1}$.

The factors $\mathbf{f}_{it}$ evolve according to the same autoregressive model as the autoregressive model for the factors of the returns (Equation 23). This result is summed up in the following proposition:

Proposition 5. ***(Implied Net-Demand Dynamic Factor Model)*** *There exists a mapping from the factor structure of return beliefs to the factor structure of net demands, noted δ:*

$$\delta:\ (\rho_i, \boldsymbol{\varphi}_i, \Lambda_i, \Sigma_{\varepsilon i}) \longmapsto (\mathbf{c}_i, L_i, \mathbf{h}_{it})\,, \tag{26}$$

The mapping is from $\mathbb{R} \times \mathbb{R}^J \times \mathcal{L} \times M_{J,J}$ to $\mathbb{R}^J \times M_{J,K} \times M_{J,J}$. The set $\mathcal{L}$ is the set of loadings $\left\{\Lambda_i \in M_{J,K} \middle| \mathbf{1}' (\Lambda_i \odot \Lambda_i)\, \mathbf{1} = K\right\}$ that sum to K. In this K is the number of factors, J the number of instruments, and $M_{p,q}$ the set of real matrices of dimension $p \times q$.

Proof. See Appendix A.1. □

3.4 From Net-Demands to Return Beliefs: Identification

The estimation of the dynamic factor model of net-demands and of the autoregressive process of the factors yields for each firm i: (i) the autoregressive dynamic of factors $(\boldsymbol{\phi}_i, \Phi_i, \Sigma_{ui})$, (ii) the constant and loadings $(\mathbf{c}_i, L_i)$ of net-demands, and (iii) the transformed cost of equity $\mathbf{h}_{it}$. The model is identified by finding the factor structure of return beliefs $(\boldsymbol{\varphi}_i, \Lambda_i, \Sigma_{\epsilon i})$ that matches these reduced-form estimates. This is equivalent to the inversion of the mapping δ.

The autoregressive dynamics $(\boldsymbol{\phi}_i, \Phi_i, \Sigma_{ui})$ of the factors of return beliefs is identical to the autoregressive dynamic of the factors for net demands. For the constants and loadings of return beliefs, cost of equity, and risk aversion the proof is more elaborate. The following proposition shows that δ is invertible:[9]

Proposition 6. ***(Identification of Return Beliefs and Risk Aversion)*** *The mapping δ from the factor structure of return beliefs to the factor structure of net demands can be inverted. $(\rho_i, \boldsymbol{\varphi}_i, \Lambda_i, \Sigma_{\varepsilon i}) = \delta^{-1}(c_i, L_i, \Sigma_{\varepsilon i})$ identifies (i) risk aversion ρ_i , (ii) the constant and the loadings of the return belief factor structure, (iii) the variance-covariance matrix of idiosyncratic risk.*

Proof. The closed-form expression for the inverse δ^{-1} is presented in Appendix A.1. The intuition of the proof is that the loadings on return beliefs are scaled equivalents of the loadings on net demands. The scaling is a function of risk aversion and the variance-covariance of return loadings. □

This key proposition allows us to conclude the identification of the model without trade costs. Indeed, firm i's ex-ante beliefs about returns are identified as:

$$E(\widetilde{\mathbf{r}}_{it}) = \boldsymbol{\varphi}_i + \Lambda_i \cdot E(\mathbf{f}_{it+1}|\mathbf{f}_{it}), \quad \mathrm{Var}(\widetilde{\mathbf{r}}_{it}) = \mathrm{Var}(\Lambda_i \cdot \mathbf{f}_{it+1}) + \Sigma_{\varepsilon i} \tag{27}$$

which provides each bank's demand vector $\boldsymbol{\Delta}_{it}$ at any price vector $\mathbf{p}_t \in \mathbb{R}^J_{+*}$.

Appendix Proposition 7 provides the identification procedure for the case with finite trade costs, where the source of identifying variation is the correlation between past and current holdings.

3.5 Estimation Procedure

The model's estimation proceeds as follows and yields the paper's financial network in step 6:

[9] In the presence of adjustment costs, both propositions 5 and 6 still apply.

1. The factor model for net demand is estimated for each bank. This yields estimated factors $\hat{\mathbf{f}}_{it}$ in each period, loadings $\hat{\mathbf{L}}_i$, the constants $\hat{\mathbf{c}}_i$, the residual $\hat{\mathbf{h}}_{it}$.

2. The autoregressive process for the unobserved factors $\hat{\mathbf{f}}_{it+1}|\hat{\mathbf{f}}_{it}$ is estimated. In this paper we use a first-order autoregressive process. This yields the constant $\boldsymbol{\phi}_i$, the autoregressive coefficient $\boldsymbol{\Phi}_i$, and the variance-covariance matrix Σ_{ui} of innovations.

3. Using the inverse of the mapping δ from reduced form factor structure $(\mathbf{c}_i, \mathbf{L}_i, \mathbf{h}_{it})$ to the structural parameters we estimate the risk aversion, as well as the constant, loadings, and variance-covariance of the factor structure of return beliefs $(\rho_i, \boldsymbol{\varphi}_i, \Lambda_i, \Sigma_{\varepsilon i})$.

4. The mean return $E(\widetilde{\mathbf{r}}_{it})$, the variance-covariance matrix $\mathrm{Var}(\widetilde{\mathbf{r}}_{it})$ of the returns, and the cost of equity η_{it} follow from equation 27.

5. These parameter pin down the firm's net demand $\boldsymbol{\Delta}_{it}$ for any price vector $\mathbf{p}_t$ according to proposition 4.

6. The financial network is the matrix $\mathcal{A}_t$ obtained by summing the network of holdings (observed), the response of equity holdings to equity prices (estimated), and the response of non-equity holdings to equity prices (Definition 2).

The next section estimates beliefs, risk aversions, and the financial network using a comprehensive longitudinal balance-sheet data with assets and liabilities of all French banks.

4 Data

Building this paper's network of banks leads to the following data challenges: first, measuring institution-level equity cross-holdings across bank and non-bank shareholders, in the longitudinal dimension; second, by measuring banks' balance sheets and the non-equity links between them, again in the longitudinal dimension; third, by measuring the response of the financial instruments (loans, debt) of other sectors (household sector, non-financial corporate sector, rest of the world) to shocks occurring within the banking sector (e.g. ECB interventions) and outside of it. These three challenges are addressed by merging the financial instruments of individual banks' balance sheets with (i) other banks' balance sheets, and (ii) with the balance sheets of other sectors than the financial sector.

Equity Cross Ownership

For banks' balance sheets and cross-equity ownership at the bank level, we rely on the Bureau Van Dijk's collection of detailed data for the period 2013–2018 on cross-equity ownerships, built from annual reports. The 2013-2018 is particularly relevant as it covers the period of the ECB's Public Sector Purchase Program. For France, the Bureau Van Dijk bank data are a longitudinal panel of 294 commercial banks, followed over 24 quarters, where balance sheets are reported at a disaggregated level (so-called C1, C2, and U1, U2 consolidation levels) that shows both time-varying parent-subsidiary relationships as well as time-varying ownership of equity across groups. For instance, HSBC France fully owns HSBC Real Estate Leasing France as well as HSBC Factoring France and HSBC SFH. The latter subsidiary is specialized in mortgage lending activities. Such parent-subsidiary relationships evolve over time: while Edmond de Rothschild France is fully owned by Edmond de Rothschild in March 2013, the 100% share ownership declines to 86% in March 2016. The longitudinal equity panel also reports holdings of shares across groups, such as the 35% ownership of La Banque Postale Financement (a subsidiary specialized in consumer credit) by Société Générale. This enables an estimation of the net demand $\boldsymbol{\Delta}_i^e$ of banks for equity instruments. Hence such longitudinal holdings data enables the construction of the $C_t = H_t(\text{diag}\boldsymbol{E}_t)^{-1}$ network adjacency matrix in each quarter from $t = 2013Q1$ to $t = 2018Q4$.

Figure 7 plots the network implied by cross holdings of equities for the 4th quarter of 2013. The figure makes apparent the partition of the French banking sector into a number of banking groups formed by a holding company surrounded by a myriad of subsidiaries. Note that there is almost no linkages between banking groups. As we shall discuss in Section 6 such network structure will change rather dramatically when the other sources of network links, i.e. the terms from equation (11), will be factored in.

Whom-to-Whom Sectoral Data

Yet, this paper's general equilibrium modeling suggests that such a network of cross holdings may not take into account links across banks due to other instruments (e.g. loans, debt, deposits), due to connections through other sectors (e.g. the non-financial corporate). This requires building a data set of sectors, where banks' balance sheets sum up to the total assets and liabilities of the financial sector. Each bank balance sheet data obtained through the Bureau Van Dijk provides itemized holdings of securities, where securities are sorted into holdings within the financial sector and holdings outside the sector. For instance, such balance sheets include the EUR amount of deposits from the banking sector, and from other sectors such as the household sector. This enables an estimation of the net demand $\mathbf{\Delta}_i^p$ of the net demand of banks i for 20 primitive non-equity instruments: one real asset and *19* categories of financial instruments including currency, deposits, securities (stocks, debt, fund shares), loans (short-term, long-term), entitlements (insurance, pension), and derivatives.

For other sectors than the financial sector, we rely on the detailed security-holdings statistics collected by the *Banque de France* at quarterly frequency through the *PROTIDE* survey. One key feature of this data set is that it provides whom-to-whom holdings, in contrast with U.S. Flow of Funds data. The six sectors are 1) banking, which includes the central bank, 2) insurance, 3) mutual funds, 4) the corporate sector, 5) household, including non-profit institutions, and 6) the public sector.

We map the full asset- and liability-side of banks' balance-sheet by accounting category into the same instrument categories as available in the sectoral accounts (European System of Accounts 2010), in particular debt securities (short-term F.31 and long-term F.32), equity (listed F.511 and unlisted F.512), and fund shares (money market F.521 and non-money market F.522).

Three facts assess the validity of the longitudinal whom-to-whom-sector-bank matched data set. First, we check that the total assets of all banks, minus the interbank cross-equity holdings, match the total assets of the financial sector. Such check reveals that cumulative assets under management of all banks included in the final dataset amount on average (over time) to approximately 80% of the aggregate banking sector balance-sheet as observed in the sectoral accounts data (ESA 2010). The remaining 20% are largely explained by the fact that the banking sector definition applied here also includes money-market funds, owing to the fact that liabilities of money-market funds exhibit similar characteristics as deposits offered by monetary banks. Assets under management display a typical 'superstar' distribution, as predicted by the model's simulation displayed in Figure 2. The model's simulations indeed predict the endogenous emergence of a set of large banks. A similar pattern is observed in the data, the $90th$ percentile holds on average 77%, and the 99% percentile on average 28% of total banking assets. Second, we check that the total sum of bank holdings of specific securities outside the financial sector matches the total EUR holdings of the financial sector of this security. Third, banks' demand for assets $\boldsymbol{\Delta}_{it} = \boldsymbol{\Delta}_{it}\left(p_{1t}^{*}, p_{2t}^{*}, \ldots, p_{Jt}^{*}; \boldsymbol{\theta}\right)$ aggregates up with similar properties as a sector-level CARA demand where the sector-level risk aversion is the harmonic mean of individual banks' risk aversions.

Estimating Realized Returns

A key test of the model's estimates of beliefs is to compare banks' beliefs about returns to their realizations, for each instrument. In order to obtain the first and second moment of returns, we construct time series of returns (i) due to changes in prices and (ii) due to payoffs, e.g. dividend income.

Returns due to valuation changes can be derived from information on the amount in Euro of stocks outstanding and valuation changes by financial instrument and sector. While the data does not include changes in the number of instruments, a key insight is that, while different banks and sectors change their holdings differently, they experience the same price shifts. Appendix B.1 shows how a panel regression with institution and instrument fixed effects can identify returns using the variation of balance-sheet positions and instruments' income.

Returns due to payoffs on financial instruments are constructed from information recorded in the income accounts on different types of income received and paid by sector. Types of income are dividends, interest payments, investment income attributable to mutual fund shareholders, insurance policy holders, and investment income payable on pension entitlements.

5 Structural Parameters

The procedure described in section 3 enables to estimate the structural parameters of the model: beliefs about future returns, risk-aversion and trading costs. The first step of the procedure is the estimation of the dynamic factor model.

5.1 Return Beliefs

Dynamic Factors and Macroeconomic Variables The estimation of the dynamic factor model used to construct return beliefs enables both the factors and the factor loadings to vary across institutional sectors. The estimation results reveal however that the factors are rather similar across sectors with some variation in their order of importance for explaining the variance of net demand.

Figure 3, left panel, plots the time series of the first three factors for the banking sector with shaded area corresponding to the Global Financial Crises (2007-2008) and the European Debt Crisis (2011-2013). These first three factors explain 41% of the variance in net assets demand. Figure 3, left panel, right panel, plot the same factors alongside with macroeconomic variables which exhibit the strongest co-movement with each of the factor. The first and second factor captures remarkably well the Global Financial Crisis and the European Debt Crisis. The first factor co-moves with World GDP growth and captures the global cycle and the global crisis. The second factor co-moves tightly with the average spread with the average interest rate of the GIIPS countries (Greece, Ireland, Italy, Portugal, Spain). Given the exposure of the French banking sector the Euro Sovereign Debt crisis in Southern Europe, it is very reassuring to see it captured by the second factor. The third factor co-moves with the Euro Area GDP growth. Overall the first three factors used to explain net asset demand by the banking sector display a time series profile which captures well either the 2007-2008 financial crisis or the 2011-2013 Eurozone sovereign crisis and correlate well with variables capturing the global cycle, the Euro cycle, and sovereign stress in the GIIPS countries. The net demands of the other sectors are roughly explained by the same set of factors but their importance tends to change. The insurance and corporate sector net asset demands areas explained more by the Global and Euro growth cycles and less so my sovereign stress in the GIIPS countries.

Return Beliefs and Ex-post Returns The estimated factor model for return beliefs, described in Section 3.4, yields for each sector and each instrument, a one-quarter-ahead return forecast. Figure 4 and Figure 5 plot alongside the time series of realized ex-post returns the time series of corresponding return beliefs, the 95th percent confidence forecast interval band, for the financial sectors and the real sectors, and for each of the following financial instruments: Bonds (Short-Term and Long-Term), Stocks (Listed and Unlisted), Mutual Fund Shares (Money Market and

Non-Money Market).

The R-square of an OLS regression of ex-post returns on ex-ante return beliefs and a constant is reported on the right-hand upper corner of each plot. In many cases, the ex-ante return beliefs predict well ex-post returns. The corporate sector ex-ante return beliefs explain 31 percent of the variance of ex-post stock returns. The household sector ex-ante return beliefs explain 35 percent of the variance of the Non Money Market Mutual Funds ex-post returns. In both cases, the time series of return belief tracks very well the asset crash of 2007-2008, and the subsequent rebound. Other good predicting performance include the prediction of unlisted stock returns by the banking sector and the household sector, the prediction of short-term bonds return by the insurance sector, the prediction of the return to mutual funds by the corporate sector, the household sector, and the mutual fund sector itself. In several instances however, the model either does not predict ex-post return or more puzzlingly its predictions negatively correlate with ex-post returns. We shall notice that this feature is mostly driven by the 2007-2008 crisis. A potential explanation is that several institutional sectors during that period had to increase their purchases of assets even if their returns were declining. Since beliefs are implicitly derived from net demands, those counter-cyclical purchases can drive the negative correlation between return beliefs and ex-post returns. This is the case for the banking sector which hoarded short-term liquid assets during the crisis, for the public sector who bought large stock shares to recapitalize the banking sector and the automobile sector during the crisis, and for the insurance sector which increased considerably its asset holdings during the crisis.[10]

Comparing beliefs across sector reveal that while there is substantial disagreement in beliefs at each point in time, the return beliefs of the banking sector and the mutual fund sectors display a strong comovement. The correlation between the return beliefs of the banking sector and those of the mutual fund sectors are high for most financial instruments: listed stocks (0.54), unlisted stocks (0.46), non money-market mutual fund share (0.34), short-term debt (0.40). There are however episodes in which the return beliefs differ substantially. For example, the mutual fund sector exhibited much more pessimistic beliefs during the 2007-2008 crisis about the return to stocks and to debt securities, while the banking sector became more pessimist on the returns to

[10] The increasing size the insurance sector balance sheet is document in HRV. Its counter-cyclical purchases during the crisis can be, in part, explained by government moral suasion as a way to facilitate the banking sector deleveraging.

stocks and mutual fund shares during the sovereign crisis of 2011-2012. One interpretation is that the bank bailout of 2007-2008 avoided the need for banks to engage in massive fire-sales (with deep discount prices), while the mutual fund sector faced large withdrawals from customers and had to engage in such fire-sales. On the opposite, banks were suffering from significant liquidity or solvency stress during the sovereign debt crisis, due to their exposure to GIIPS debt, and therefore faced a pressure to sell-off rapidly other assets that the mutual fund sector did not experience then. The comparison of beliefs among the three real sectors reveal a sharp contrast between the belief returns of the corporate and household that typically comove, and that of the public sector that often displays counter-cyclical beliefs, and especially so during the 2007-2008 crisis. This is consistent with the role played by government in providing bailout and in the debt financing of large public sector deficits during the crisis.

Bank-Specific Return Beliefs In order to obtain bank's specific return beliefs, we regress net-demands of each bank for each financial instrument on the net-demand factors estimated for the banking sector computed using sectoral data. The time-variation in net-demands for financial instruments and factors identifies the net-demands loadings. The underlying assumption is that each bank forms belief using the same return-belief factors as the banking sector as a whole but with bank-specific loadings. From that point, onwards, we can apply the procedure described in and obtain all the bank-specific structural parameters according to the procedure described here 3.5.

Belief Updating and Bank's heterogeneity.

We can assess the heterogeneity of the return beliefs' model across banks by looking at how they differentially incorporate information from realized returns in updating their return belief. We do so by regressing the difference between realized return and bank's return belief on the same variable lagged – by itself and interacted with bank's characteristics – controlling for time, instrument, and bank fixed effects.

Table 1, column 1, shows that the differential between realized returns and return belief is mean reverting suggesting that, on average, banks do update their belief in order to minimize their forecast error about realized returns. There is, however, considerable heterogeneity in this correction mechanism. Using our model estimates of risk-aversion, we show that more risk-averse banks are *more aggressively* updating their belief than less risk-averse banks (Column 2). Risk-aversion is therefore associated with banks being more Bayesian in using the realized information, and implicitly less contrarian, that is less likely to withstand beliefs that are not (yet) being validated by realized returns. By contrast, banks that are more leveraged, or larger, are *less aggressively* correcting their beliefs. This result, which stands when controlling for risk aversion, suggests a possible overconfidence bias of larger and more leveraged banks in the long run validity of their return belief model.

In short it is highly reassuring to observe that the pattern of belief updating observed in the data is largely consistent with priors on which type of banks are more or less likely to follow a Bayesian updating behavior.

5.2 Risk-aversion

Section 3.4 suggests that the mapping between the factor structure of net demand and that of the return beliefs identifies beliefs about all instruments' first and second-order moments (variances and covariances), up to a global constant of overall volatility σ_{ε}^2. Specifically the model identifies the product $\rho\sigma_{\varepsilon}^2$, that is the degree of risk-aversion multiplied by the residual variance of the belief return model. We recover the degree of risk-aversion as follows. We match the overall residual variance of the return belief model to the overall residual variance of ex-post returns, that is the fraction of the variance of the ex-post returns that is not explained by the factors.[11] Table A, upper panel, report risk-aversion estimates, one by sector, obtained by matching variances. As in typical in the equity risk premium literature (Ang 2014, Ait Sahalia & Lo 2000, Ross 2015), this approach generates estimates ranging from 8.9 to 21.7, depending on the institutional sector, and which are within the range of estimates of the literature. [12]

Sector-level risk aversion estimates are also ranked in a natural way: a higher estimated risk-

[11] Note that here were are only scaling the *residual variance* of the belief model. Therefore the residual variance could be of the same order of magnitude of that of ex-post return and yet the variance of beliefs conditional on factors could be sensibly smaller than that of ex-post returns.

[12] While imposing the matching of belief returns to ex-post returns is a way to discipline belief formation, one cannot rule out that beliefs are substantially more volatile than ex-post returns as papers have suggested, e.g. Shleifer & Summers (1990). We thus test the robustness of our results to letting the variance of return beliefs to be a multiple of the variance of ex-post returns (Table A, bottom three panels); in this case estimated risk-aversion parameters are substantially smaller.

aversion in financial sectors subject to capital requirement (banks, insurance) than for the mutual funds sector or the corporate sector.

Bank-level estimates display substantial heterogeneity, and estimates that the disaggregation of the banking sector is key in lowering risk aversion estimates. The mean equal to 4.25, while the median is 0.25, with a standard deviation of 7.7. The large degree of heterogeneity in risk-aversion is expected given differences in size at given return-belief and cost of equity. These differences in risk aversion generate substantial heterogeneity in the response to monetary policy shocks (Section 6.1).

5.3 Trading costs

Trading costs are essential ingredients to the estimation of the model. They indeed directly impact the relative persistence of linkages in the financial network. The structural estimation of trading costs does not impose *a priori* restriction on the relative liquidity of different asset classes. Figure 6 plots the deviation of median trade costs from the overall median trade cost by major asset classes. Reassuringly long term loans is the less liquid asset class, and equity the most liquid ones, short-term loans and debt displaying an intermediate degree of liquidity.

6 The Network in General Equilibrium: Estimating the Impact of ECB Quantitative Easing

Section 2.3 suggests that the network structure of interlinkages between banks turns partial equilibrium shocks to banks' value into general equilibrium shocks that affect the market pricing of equity. As such, shocks to the demand of a specific financial instrument, e.g. debt, have two impacts: first, they affect the diversification of constrained banks connected through their balance sheet interlinkages, the phenomenon described in equation (10). The propagation of the shock is in part due to the sensitivity of holdings, constrained by their trade cost; such trade cost is lowest for traded equity, and highest for non-traded loans. Empirically estimated trade costs reflect the homogeneity of securities and their ability to be traded on a market-clearing exchange. In order to understand the propagation channels, we must look at the structure of the network beyond the predetermined network of cross-holdings. Figure 8 plots the sensitivity of the network of cross-holdings to changes in equity prices and Figure A the linkages between banks that are not equity links. In sharp contrast with Figure 7, the network structure that emerges from these two additional components includes many important links *between* banking groups, and thus contributes to the propagation of shocks throughout the banking sector. Note also that Societe Generale appears now at the center of the *between-groups* links but did not display any special feature when only the network of cross-holdings was considered.

Second, the shock changes the structure of the network itself: the position of banks in the network (their influence and sensitivity, defined in Section (2.5)) shifts in a way that either mitigates or amplifies the shock (Section (2.4)). These features are analyzed empirically in the case of the ECB's quantitative easing below.

6.1 Structural Policy Evaluation: the case of ECB Quantitative Easing.

In this section, we use our structural model to assess the effects on bank balance-sheet and on the financial network of a large-scale quantitative easing policy through which a central bank expands its balance-sheet by purchasing securities while increasing its monetary base. Precisely, we simulate a demand shock for government bonds which corresponds to the cumulative net purchases over a quarter during the first phase of the Extended Asset Program (March 2015 to March 2016) of the European Central Bank. The quarterly demand shock (Euro 180 bn.) corresponds to three times the monthly purchases (Euro 60 Bn.) For simplicity, we choose for the shock to replicate the Public Sector Purchase Program (PSPP) and thus ignore the purchase of non-government bonds. We choose to simulate the shock one year before the actual shock occur so that the estimated return beliefs used in the simulation are not contaminated by the actual shock.

Table 2, Panel (a) summarizes the calibration of the shock which follows the design of actual ECB policy: (i) asset purchases are made in proportion to country's share in the capital of the ECB, which implies that 20 percent (80 percent) of bonds purchased were French Government bonds (Other Euro Area Government bonds); (ii) the proportion of short and long-term government bonds are made such so as to keep constant the outstanding share of each type of debt. The calibration of the shock thus implies a initial impact 4 times bigger for the rest of the Euro Area government bond market than for the French market.

Impact on yields and spreads Table 2, Panel (b) reports the effects of the QE shock on asset prices in partial and in general equilibrium on French government bonds, corporate bonds (spread) and equity (premium). In partial equilibrium, the yield on French government debt decreases to clear the additional demand by the Eurosystem. The yield on long-term French debt decreases substantially more (-7.4 bps) than the one on short-term French debt (-1.0 bps) implying a flattening of the yield curve (decrease in the term premium). Since in partial equilibrium only the prices of government debt change, the equity premium increases. The general equilibrium effect is much stronger than the partial equilibrium one with a decline in long-term (short-term) bond yield of -43.9 bps (-37.4).[13] The price of the corporate bonds and that of equity increase sharply in general equilibrium so that the term premium and equity premium remain the same. In short, the calibrated QE shock, despite a modest partial equilibrium impact, has a strong impact on government bond yields in general equilibrium and a very large pass-through into the price of other assets (corporate bonds, equity).

Even if we do not use any information on quantities, prices or beliefs during the period of the policy shock, we obtain structural estimates for the effects on yield which are reassuringly of the same order magnitude to those obtained from reduced-form regressions using actual data for the period of the shock. Andrade, Breckenfelder, De Fiore, Karadi & Tristani (2016) report a median impact of 43bps on 10-year government bonds based on a meta-study of 24 studies. Their own estimates based on 2-days event windows around the program announcement and around the program implementation suggests an effect of 13 basis points after the announcement and an additional 14 basis points after the implementation. Koijen, Koulischer, Nguyen & Yogo (2018) using an IV approach, exploiting the predictability of purchases found an effect ranging from 2bps to 60 bps depending on country and maturity.

Impact on banks' balance sheets Figure 3 summarizes the effects of the ECB shock on various balance sheet measures for individual banks. Figure 3 plots the distributions of the log-change in these various measures between the 25th and the 75th percentiles and report the mean and various quantile of the distribution of log changes below each distributional plot. The median log-change in the size of banks' balance sheets is close to zero but this hides considerable heterogeneity, with

[13] Network structure matters for general equilibrium price responses as the comparative statics on prices $d \log \mathbf{p}/d\theta$ depend on the adjacency matrix of the network graph.

a subset of banks experiencing a substantial reduction in balance-sheet size (at the 25th percentile, the reduction is −2.1 percent). On the asset side, the most critical change regards debt holdings, which is not surprising given the nature of the shock. The median change in debt holdings is –0.77 percent but there is substantial heterogeneity: at the 25th percentile, banks reduce their holding by -6.0 percent. The equity holdings change very little except at tails of the distribution resulting in an increase in the mean equity equity holdings of +10.6 percent. On the liability side, there is little change in the equity position of banks except at the upper tail (95th quantile) where banks increase their equity position by 9.2 percent. There is however a substantial decrease in debt liabilities resulting in a mean reduction of −7.4 percent, with 25 banks experiencing a reduction of more than 3.9 percent. Altogether the ECB shock results in a reduction in balance sheet size through a simultaneous reduction in both debt holdings and debt liabilities for a sizable fraction of the banks.

The bottom panel plots asset volatility and distance to insolvency (Atkeson et al. 2017). The distance to insolvency increases for almost all banks even if the magnitude of the change is small. The majority of banks experience a reduction of asset volatility – the median bank experiences a decline in volatility of −0.2 percentage point, but there is substantial heterogeneity across banks with the 25th percentile experiencing a reduction of −1.1 percent and 95th percentile an increase of 1.4 percent.

Table 4 explores how banks differ in their response to the ECB quantitative easing shocks, that is how the rebalancing of their portfolio of bonds and equities varies with their initial portfolio, their risk-aversion, as well as in their degree of influence and sensitivity in the network. There is a considerable literature on the heterogeneity of the response of banks to monetary policy shocks as a function of their exposure to interest rate risk and other shocks (Flannery & James 1984, Landier, Sraer & Thesmar 2013, Dell'Ariccia et al. 2017). A key unsolved question we address here is whether the *position* of banks in the financial network, as captured here by their influence and sensitivity, is a key source of heterogeneity.

The ECB shock increases the equity-premium (+7.23 bps) which should lead to a rebalancing of the portfolio towards equity and away from bonds. Indeed the constant terms on Table 4 reveals a reduction in debt holdings and an increase in equity holdings. There is however substantial heterogeneity in the magnitude of such response. First, banks with initially large debt equity holdings reduce their holdings by less, and those with initially large equity holdings increase their holdings by less. Second, rebalancing is more muted for more risk-averse banks: there is both a lower increase in their equity holdings and a lower decline of bond holdings than other banks. Third, network-based heterogeneity does significantly matter: banks that are more influent also rebalance significantly less between bonds and equities. By contrast, the degree of sensitivity does not correlate significantly with changes in bonds and equity holdings.

Impact on network systemicness The effects of the ECB policy shocks on the network and its systemicness can be measured through changes in influence and sensitivity for each of the 306 banks. In order to compute such changes, we recompute those network measures in the new market equilibrium following the sovereign asset purchases by the ECB, and compare them to the initial estimates. Summary statistics for changes in influence and sensitivity suggest substantial shifts in banks' network positions. The ECB shock causes a mean reduction of influence and sensitivity of 0.45 and 0.27 percentage points respectively following the quarterly ECB sovereign purchases (Euro 180 bn). Since the later shock represents only about 8.2 percent of the total ECB program (Euro 2200 bn.), we shall expect for the overall program to have a sizable reduction in systemicness. Furthermore, the effects are extremely skewed with large changes happening on the left tail of the distribution. 10 (5) percent of banks experience of reduction of their influence of more than 1.74 ppt (3.87 ppt). Figure 10 shows a very strong negative correlation between the changes of the influence and sensitivity measure and their initial level, that is, the banks that were more sensitive and more influent show a larger reduction in their systemicness than the others. If the objective of the ECB program was to reduce the systemic risk stemming from banks that were either a potential source of fragility to the network (because of their influence) or at great risk of being subject to risk (because of their sensitivity), our results show that it is actually what does happen after the policy intervention. By contrast, Appendix Figure C displays no relationship between changes in influences and sensitivity and the size of the banks.

7 Conclusion

This paper shows how the structural estimation of general equilibrium securities trade, both on the asset and on the liability sides, is a micro-foundation for a network of of banks. Shocks propagate through this network and cause an endogenous transformation of the structure of the network in the response to the shock. While only using net demands as an input, the estimated network can be estimated and used to simulate the effect of a large policy shocks such as ECB quantitative easing. Such shock reduces government bonds yield, increases distance to insolvency for banks, and reduce the systemicness and fragility of the financial network.

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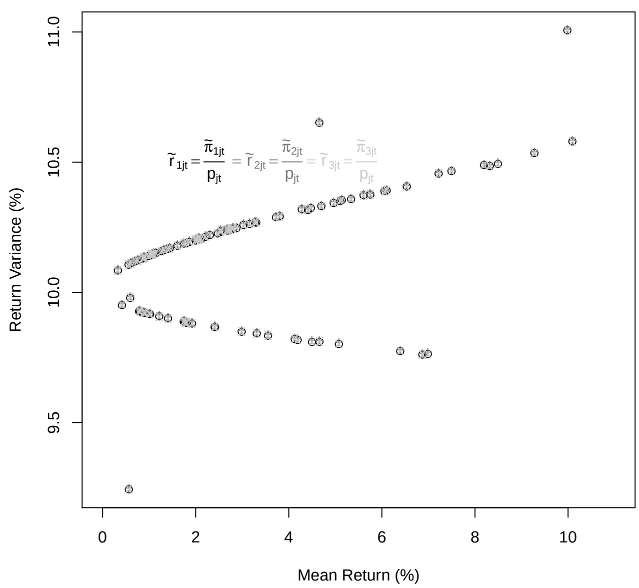
Figure 1: The Equilibrium Mean-Variance Frontier with Disagreement About Stochastic Payoffs

Prices converge to an equilibrium vector price vector $\mathbf{p}^*$ *(*$\boldsymbol{\Delta}(\mathbf{p}^*) = 0$*) that arbitrages return and risk. With all firms agreeing about the distribution of stochastic payoffs, expectations and variances of returns* $\tilde{\mathbf{r}}_i = \tilde{\boldsymbol{\pi}}_i/\mathbf{p}$ *describe a mean-variance frontier. For securities that are positively correlated with the average portfolio, equilibrium means and variances of returns sit on an upward sloping 1-dimensional curve. Securities that are negatively correlated with the average portfolio sit on the lower part of the mean-variance parabola. When firms disagree about payoffs, the mean-variance arbitrage curve has points in its convex hull. There are also such points whenever institutions face constraints (see Appendix Section 1.2).*

(a) Agreement About Returns



(b) Disagreement About Returns

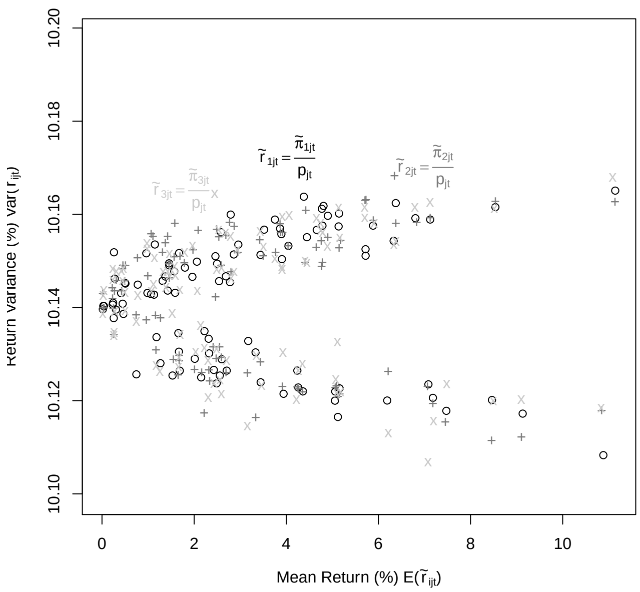
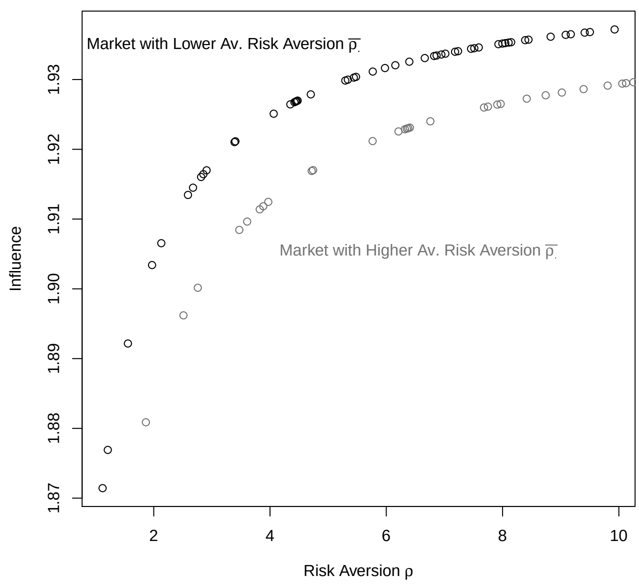


Figure 2: The Equilibrium Relationship between a bank's Influence and its Risk Aversion

These figures present the equilibrium relationship (i) between each bank's risk aversion ρ_i *and its influence in the financial network, defined in Section 2.5; (ii) between each bank's risk aversion* ρ_i *and its balance sheet size. The first market simulation (black points) has a lower average risk aversion: risk aversions are drawn from a uniform distribution over* $[1, 10]$. *The second market simulation (grey points) has a higher average risk aversion: they are drawn from a uniform distribution over* $[1, 15]$

(a) Influence of a Firm's Equity and Its Risk Aversion



(b) Balance Sheet Size and Risk Aversion

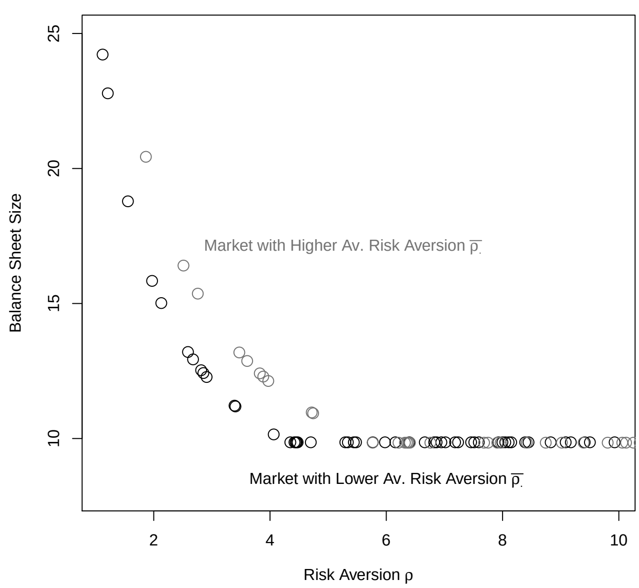


Figure 3: Banks' Estimated Return-Beliefs Factors and Macro-Variables

This figure shows the first three factors of return-beliefs and net-demands of the banking sector. Shaded areas correspond to the Global Financial Crisis (2007Q3-2008Q4) and the European Debt Crisis (2011Q4-2013Q1), respectively. Subgraphs (b), (d), and (f) relate standardized factors to standardized observable macro-variables.

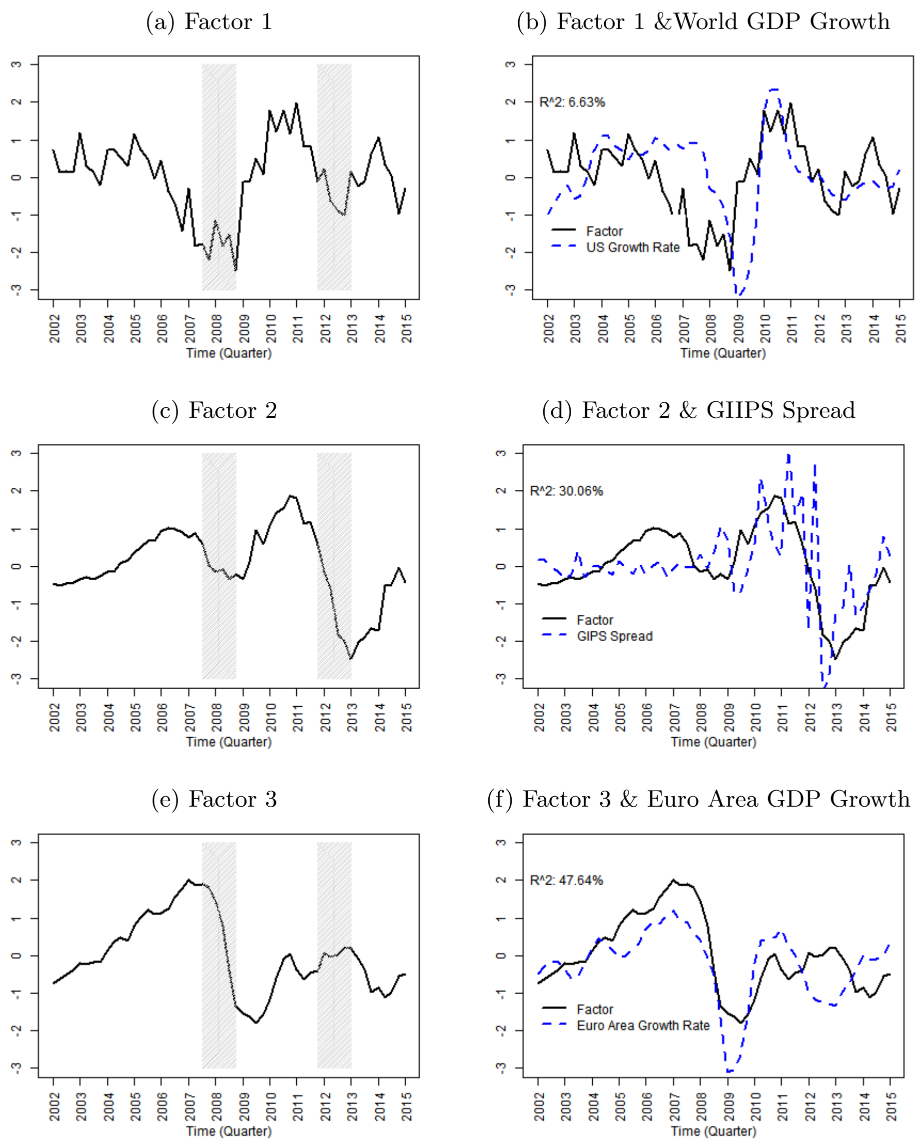


Figure 4: Comparing Identified Beliefs to Actual Returns

These plots compare average sector-level beliefs about returns with actual returns.

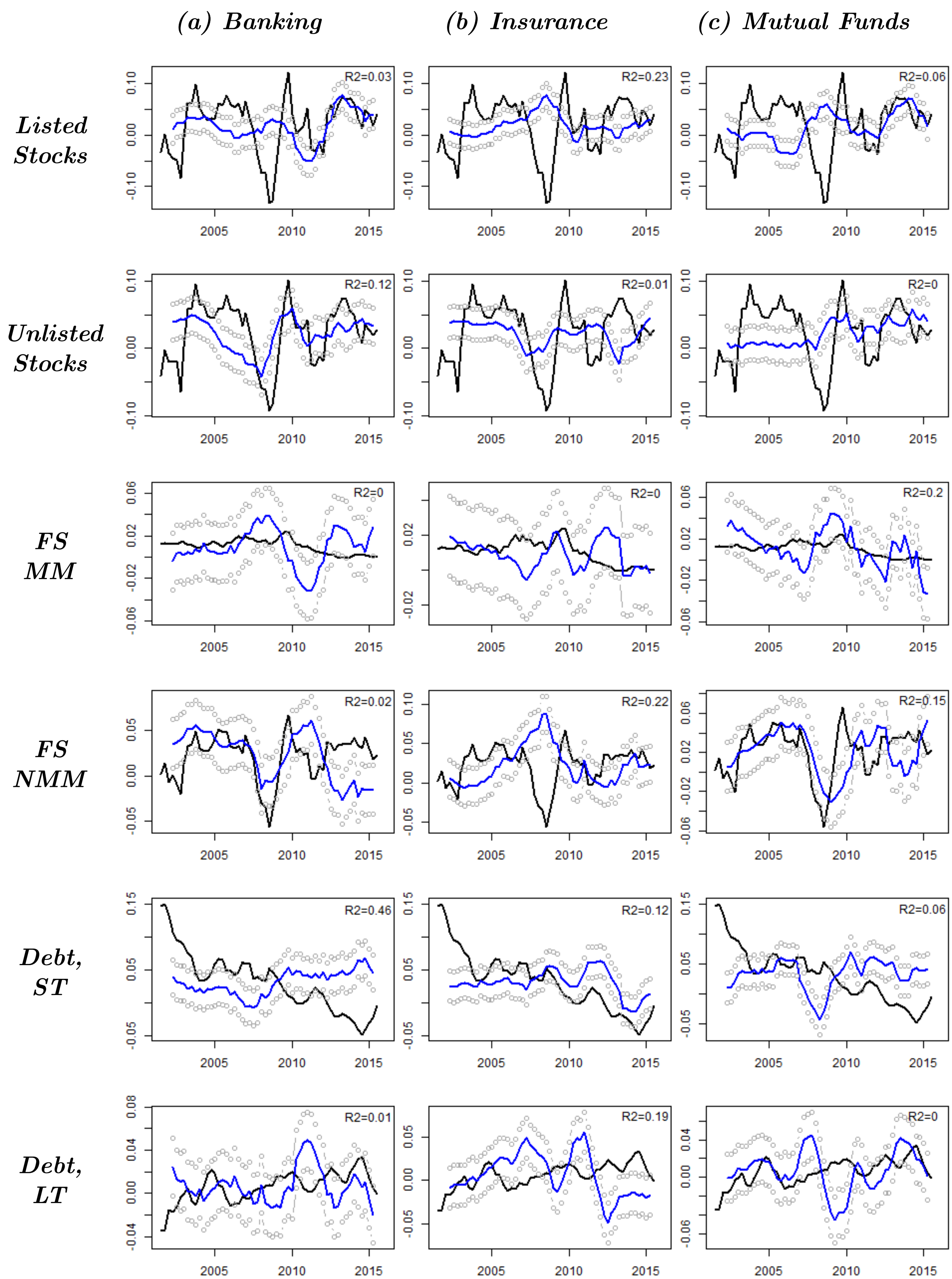


Figure 5: Comparing Identified Beliefs to Actual Returns (Non-Financial Sector)

The expected return belief is drawn in blue with 95% confidence bands based on the sectors return uncertainty. Ex-post returns are drawn in black.

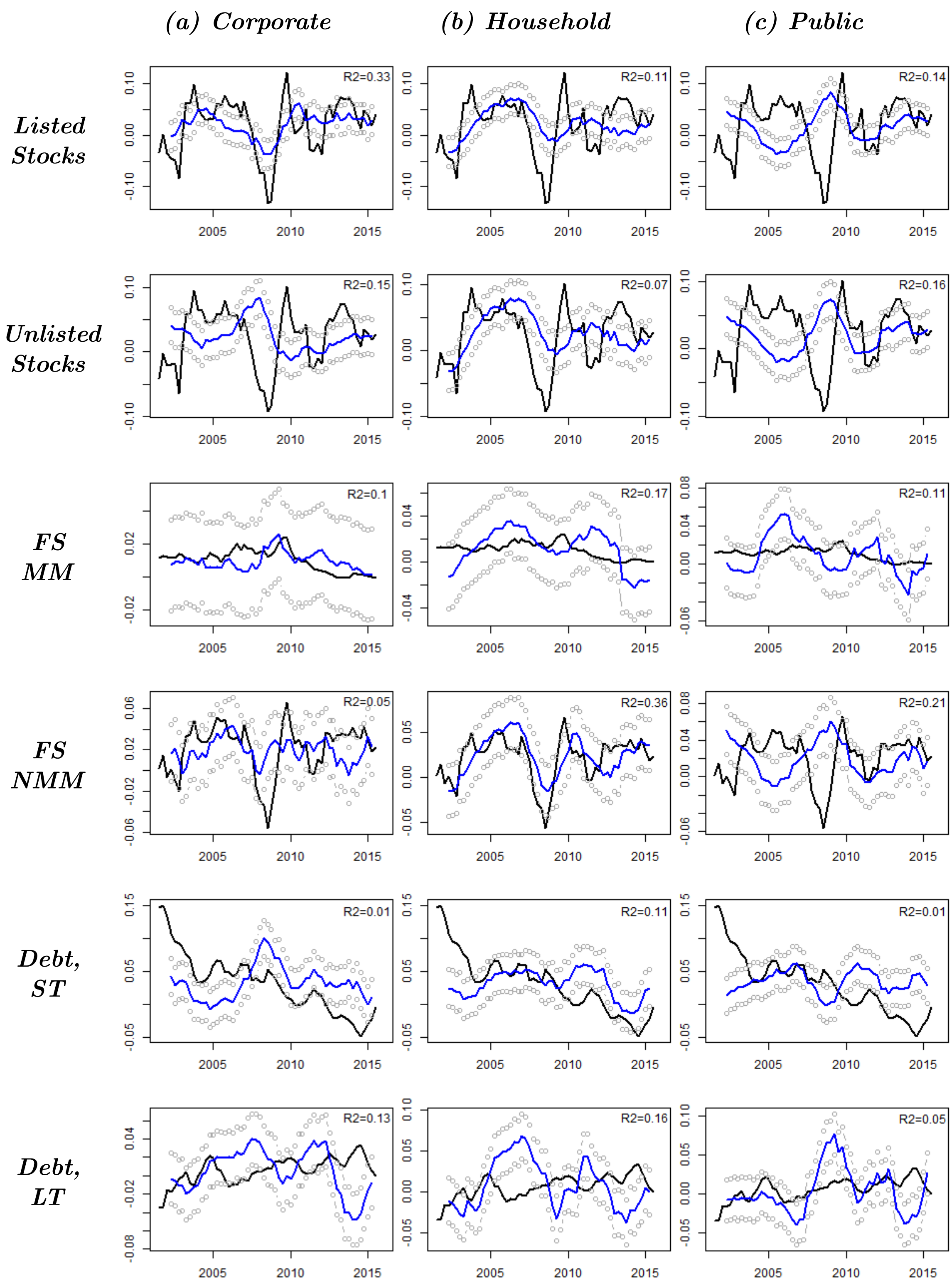


Figure 6: Estimated Trade Costs by Financial Instrument Type

This figure shows the median of estimated trade costs for a selection of financial instruments, i.e. Equity, Debt, Short-term Loans, and Long-Term Loans relative to the overall median of trade costs.

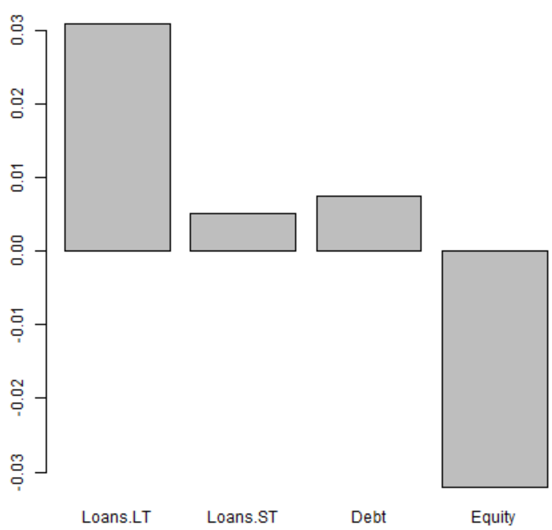
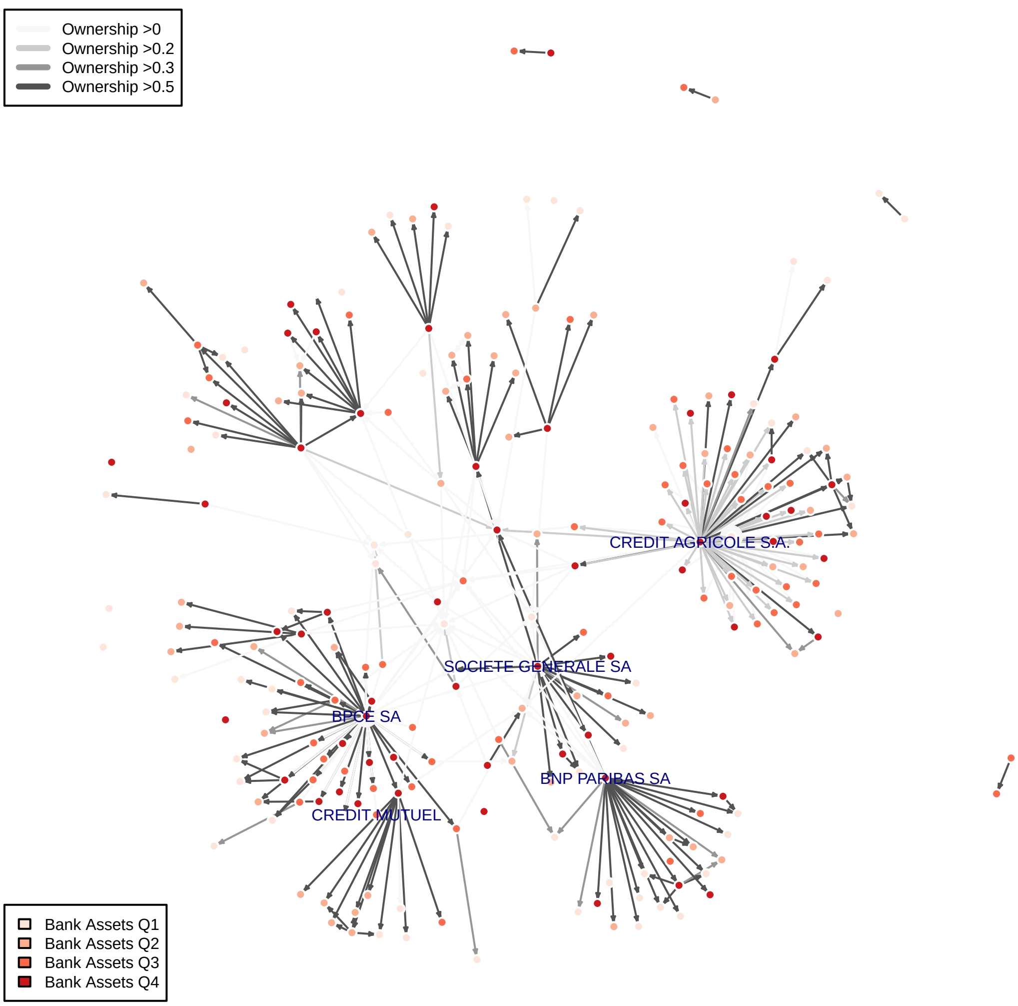


Figure 7: The Network of Cross Holdings

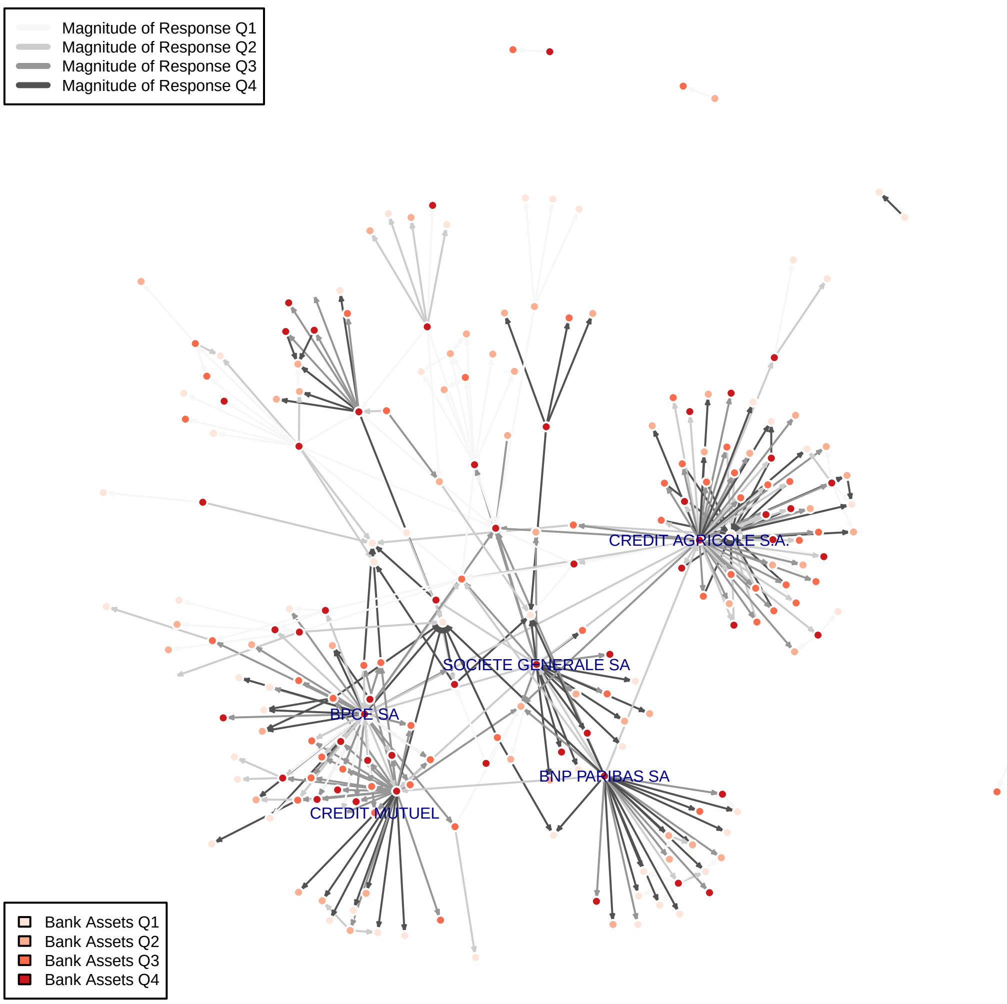
This figure depicts the network of cross holdings C (Equation 11 of Definition 2) for the 4th quarter of 2013. This network does not incorporate the response $\frac{\partial H}{\partial \log \mathbf{p}^e} (diag\mathbf{E})^{-1}$ *of the holdings to equity prices and the interbank links going through primitive instruments* $\left\{ \mathbf{1}_p' \frac{\partial \mathbf{\Delta}_i^p}{\partial \log \mathbf{p}^e} \right\}_i (diag\mathbf{E})^{-1}$*. As such it represents the network of cross holdings introduced by Elliott et al. (2014).*



The directed edges indicate ownership. Darker edges correspond to larger share ownership. The vertices are colored according to the total assets of each bank.

Figure 8: The Sensitivity of Interbank Cross Holdings to Price Changes

This figure depicts the sensitivity of the network of cross holdings (Equation 11 of Definition 2) for the 4th quarter of 2013. This network represents the response term $\frac{\partial H}{\partial \log \mathbf{p}^e} (diag\mathbf{E})^{-1}$ *of the holdings to equity prices.*

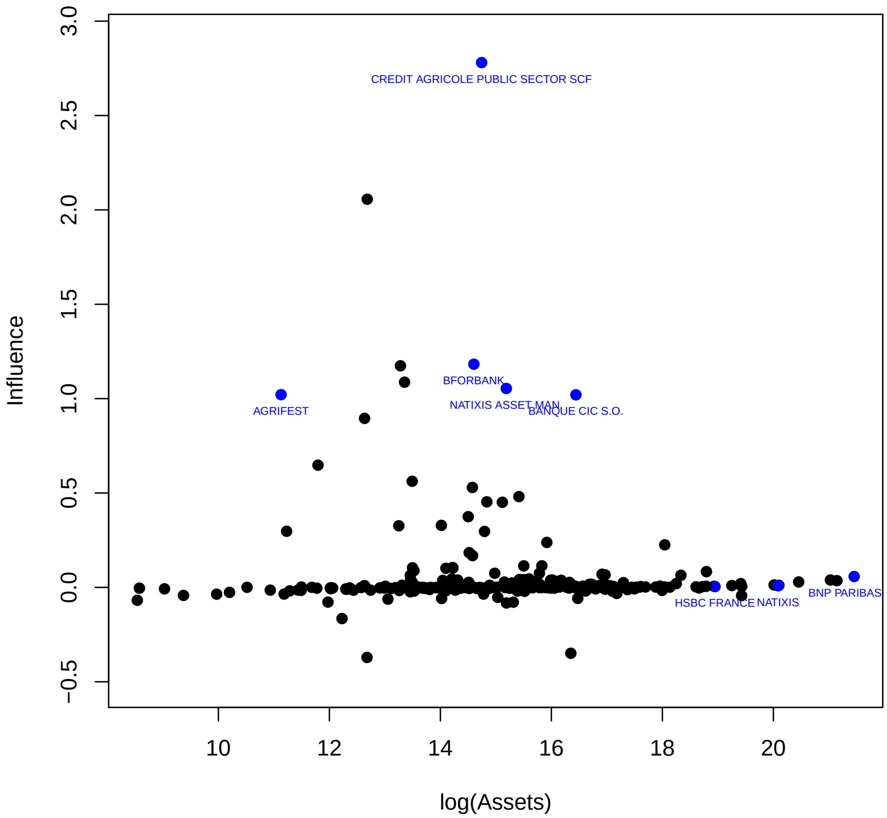


The directed edges indicate ownership. Darker edges correspond to larger share ownership. The vertices are colored according to the total assets of each bank.

Figure 9: The Correlation of Total Assets with Network Influence and Sensitivity

The top (resp., bottom) figure relates network influence (resp., sensitivity) to bank size. See Section 2 for the definition of these network measures.

(a) Influence and Size



(b) Sensitivity and Size

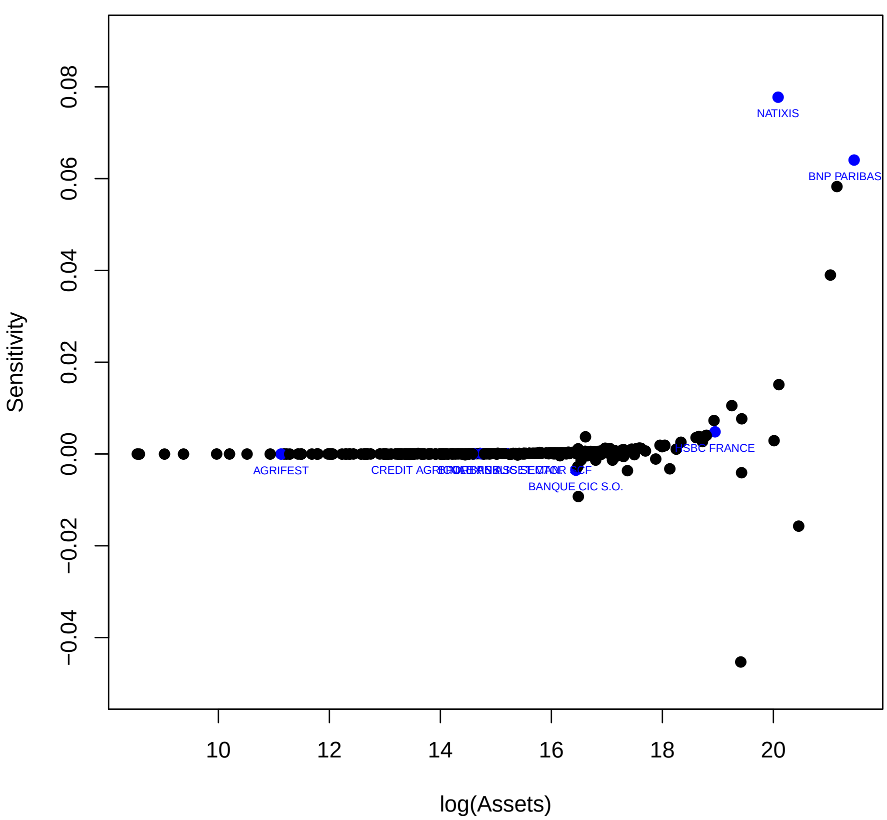
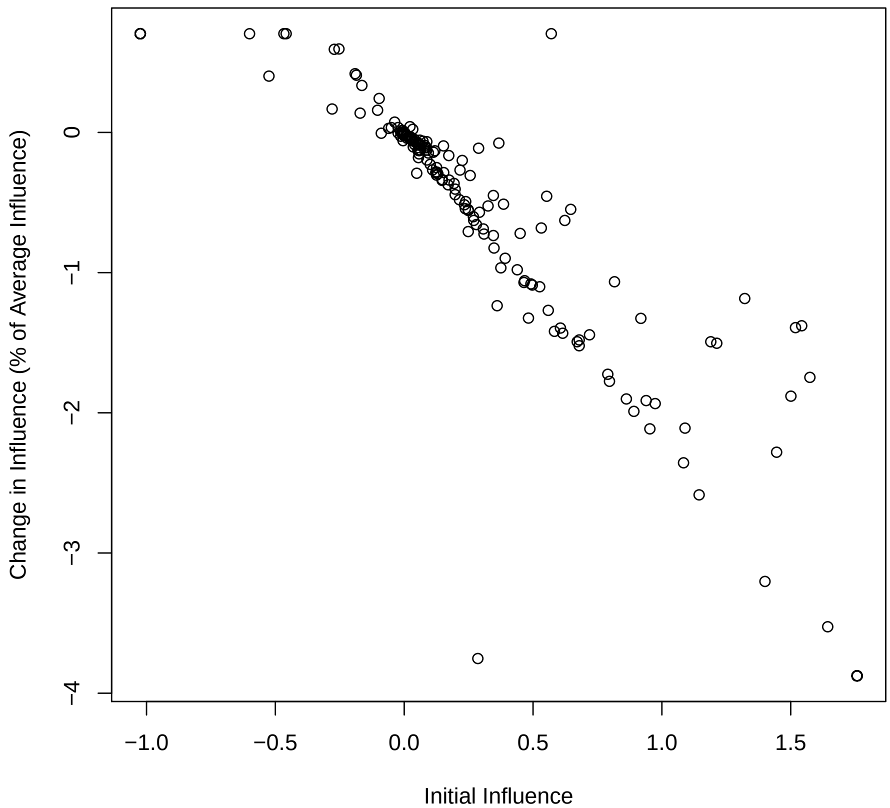


Figure 10: ECB Shock and Network Structure: Impact of Monetary Policy on Influence and Sensitivity

These two figures present the impact of the ECB shock on the influence and the sensitivity of banks. The impact is a general equilibrium change, i.e. after the series of balance sheet re-diversification leading to the equilibrium price vector.

(a) Impact on Firm Influence in General Equilibrium



(b) Impact on Firm Sensitivity in General Equilibrium

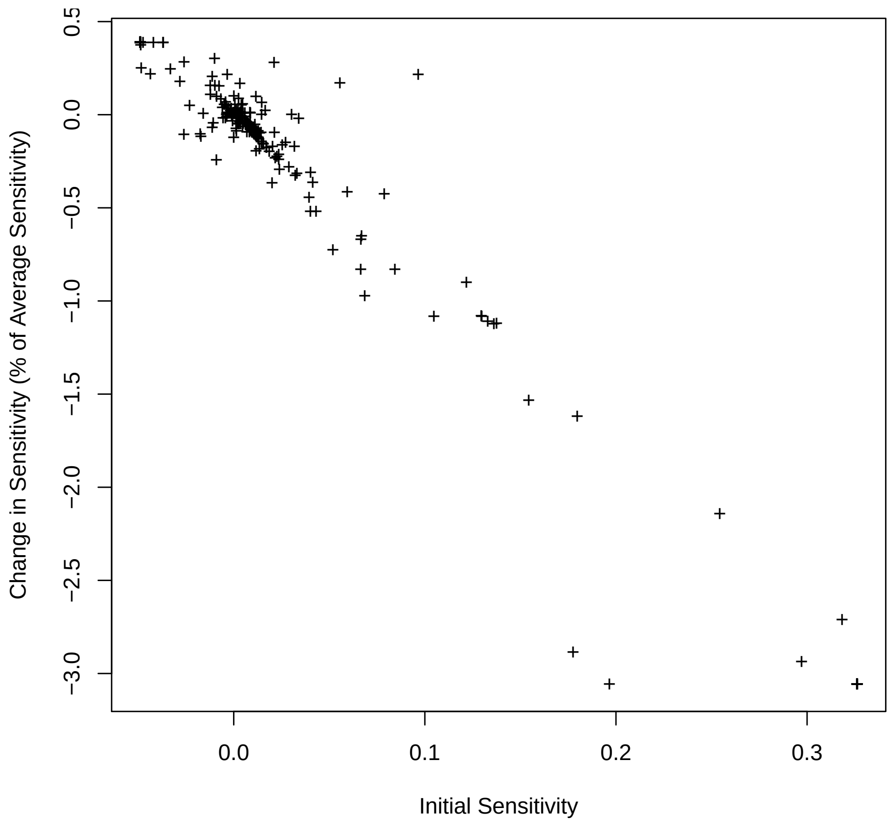


Table 1: How do Banks Form their Beliefs? How Beliefs about Returns Respond to Realizations

This table shows results of panel regressions of the difference between realized returns and beliefs about mean returns on its lag and banks' characteristics.

	Dependent variable: Realized Return *minus* Bank's Belief in t			
	(1)	(2)	(3)	(4)
Realized Return *minus* Bank's Belief in $t-1$	0.846***	0.906***	0.323***	0.908***
	(0.004)	(0.014)	(0.083)	(0.011)
$\times$ Log Risk Aversion		-0.022***		
		(0.005)		
$\times$ Leverage			0.554***	
			(0.088)	
$\times$ Log Size				0.015***
				(0.002)
Fixed Effects				
Time	Yes	Yes	Yes	Yes
Bank	Yes	Yes	Yes	Yes
Instrument	Yes	Yes	Yes	Yes
Adjusted R^2	0.757	0.757	0.758	0.757
F Statistic	404.715	402.474	403.06	403.083
Observations	18,802	18,802	18,802	18,802

Table 2: Simulation of a PSPP Shock, Bond Yields, Corporate Spread, and the Equity Premium

This table shows the calibration (Panel (a)) of an increase in the ECB's Public Sector Purchase Program (PSPP) by 180 bil. Eur in 2014Q1 and the response of selected asset prices (Panel (b))

(a) Calibration of the ECB's PSPP Purchase Shock

		ECB Capital Key† (%)	Share Outstanding (%)	Net Demand Shock (bil. Eur)
French Government Debt	Long Term	20.00	88.05	31.70
	Short Term	20.00	11.95	4.30
Other Euro Area Government Debt	Long Term	80.00	86.82	13.18
	Short Term	80.00	125.02	18.98
Deposits				-180.00

(b) Response of Selected Asset Prices

		Effect (in bps)	
		Partial Equilibrium‡	General Equilibrium
French Government Bond Yield	Long Term	-7.43	-43.90
	Short Term	-1.00	-37.43
French Corporate Spread	Long Term	7.43	7.56
	Short Term	1.00	1.08
Term Premium		-6.42	-6.46
Equity Premium		7.43	7.23

†*: The ECB capital is paid in by the national central banks of all EU Member States. Each country's share is calculated using a key based on its population and gross domestic product.*
‡*: In partial equilibrium asset prices only change in markets that are directly impacted by the PSPP shock. For example, since the equity premium is calculated as the difference between the return on equity and the yield on long-term government debt, it moves in the same magnitude as the latter.*

Table 3: Banks' Heterogeneous Response to the ECB's PSPP Shock

This table shows the general equilibrium change of banks' balance-sheets to an increase in the ECB's Public Sector Purchase Program (PSPP) by 180 bil. Eur in 2014Q1. Density plots depict the kernel density estimation of the empirical distribution of the general equilibrium change between the first and third quartile. Distance to insolvency (DI) from Atkeson et al. (2017) is calculated as the the drop in asset value that would render the firm insolvent, measured in units of the firm's asset standard deviation.

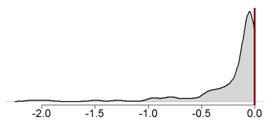
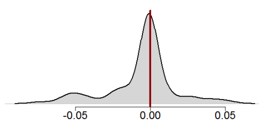
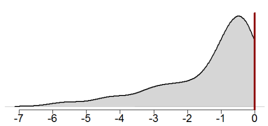
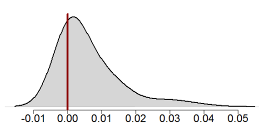
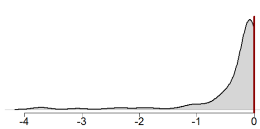
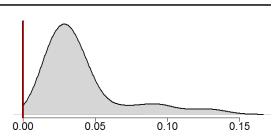
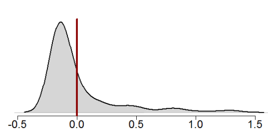
			Q75-	Quantile				
GE-Initial in:		Mean	Q25	5%	25%	50%	75%	95%
Size (%)		−6.25	2.11	−35.76	−2.12	−0.08	−0.01	−0.00
Assets								
Equity (%)		10.67	0.14	−16.61	−0.08	−0.00	0.07	32.30
Debt (%)		−23.45	5.89	−131.92	−6.02	−0.77	−0.13	−0.01
Liabilities								
Equity (%)		0.66	0.04	−1.50	0.00	0.00	0.04	9.17
Debt (%)		−7.43	3.85	−42.14	−3.86	−0.09	−0.02	−0.01
DI (×100)		0.55	0.13	0.00	0.01	0.03	0.14	1.75
Asset Vol. (ppts)		5.09	1.60	−1.10	−0.19	−0.12	1.41	36.36

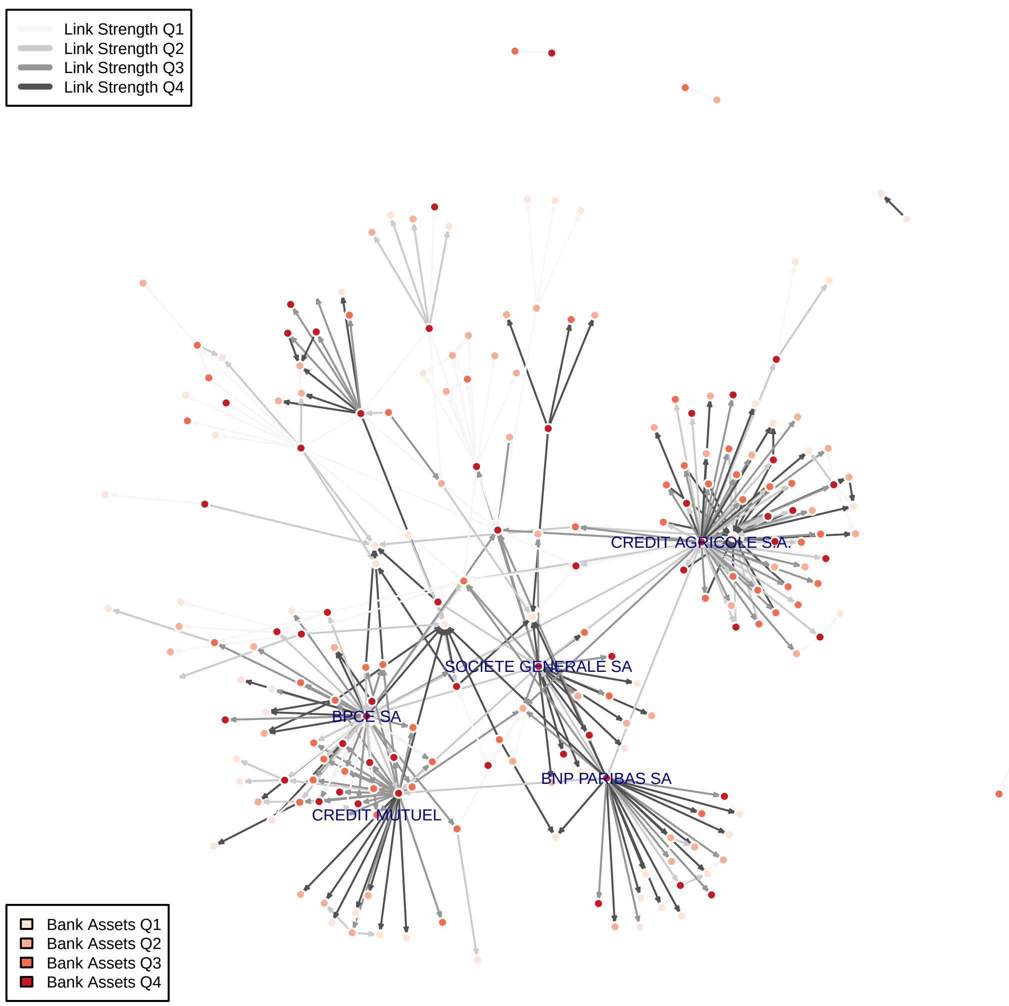
Table 4: Banks' Heterogeneous Response to the ECB's PSPP Shock – Initial Characteristics

This table shows results from an OLS regression of banks' balance-sheet responses to an increase in the ECB's Public Sector Purchase Program (PSPP) by 180 bil. Eur in 2014Q1 on banks' risk aversion (columns (1) and (4)), cost of equity (columns (2) and (5)), and the initial balance-sheet (columns (5) and (6)). Estimated log-risk-aversion has a sample average of 0.6*, a standard deviation of* 3.3*, and an interquartile range of* 7.1*.*

	(1) Δ log Bonds	(2) Δ log Bonds	(3) Δ log Bonds	(4) Δ log Equity	(5) Δ log Equity	(6) Δ log Equity
Initial Influence		0.087**			−0.011*	
		(0.039)			(0.006)	
log Risk Aversion	0.122***	0.121***	0.118***	−0.008***	−0.008***	−0.009***
	(0.023)	(0.022)	(0.020)	(0.003)	(0.002)	(0.003)
log Bonds	0.075***	0.074***	0.079***			
	(0.022)	(0.022)	(0.024)			
log Size	−0.013	−0.004	−0.014	0.001	0.001	0.001
	(0.021)	(0.021)	(0.021)	(0.002)	(0.002)	(0.002)
Initial Sensitivity			−0.545			−0.045
			(0.607)			(0.034)
log Equity				−0.008***	−0.009***	−0.007***
				(0.0026)	(0.003)	(0.002)
Constant	−0.228***	−0.265***	−0.197***	0.014*	0.015*	0.017**
	(0.068)	(0.073)	(0.061)	(0.007)	(0.008)	(0.009)
Observations	155	155	155	154	154	154
R−squared	0.418	0.425	0.429	0.277	0.29	0.287

Appendix Figure A: Linkages through Primitive Instruments

This figure depicts the interbank linkages due to primitive instruments (Equation 11) for the 4th quarter of 2015. This network represents the term $\left\{\frac{\partial(\mathbf{1}_p'\boldsymbol{\Delta}_i^p)}{\partial \log \mathbf{p}^e}\right\}_i$ *in the expression of the network definition 2.*



The directed edges indicate ownership. Darker edges correspond to larger share ownership. The vertices are colored according to the total assets of each bank.

Appendix Table A: Risk Aversion and Return Belief Uncertainty

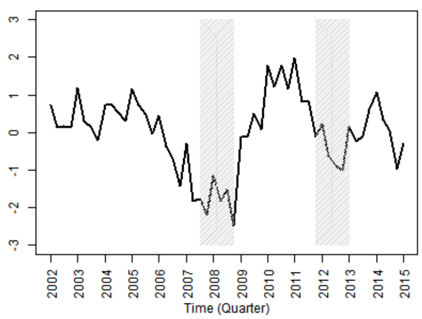
This table presents the estimates of risk aversion and the standard deviation of return beliefs for different values of the calibration ratio δ of economy-wide return beliefs over economy-wide ex-post returns.

		Banking	***Insurance***	***Mutual Funds***	***Corporate***	***Household***	***Public***	***RoW***
$\delta = 1$	ρ	21.69	23.97	8.94	12.02	13.94	9.053	12.046
	σ_ϵ	0.01	0.008	0.009	0.0111	0.0118	0.0104	0.009
$\delta = 2$	ρ	14.18	16.96	7.56	8.42	12.12	6.36	9.22
	σ_ϵ	0.0183	0.0156	0.0130	0.0227	0.0160	0.0211	0.0539
$\delta = 4$	ρ	8.73	11.99	5.52	6.01	8.66	4.76	8.74
	σ_ϵ	0.0357	0.0312	0.0272	0.0446	0.0326	0.0376	0.0604
$\delta = 6$	ρ	4.67	7.58	3.45	3.86	5.57	2.99	6.78
	σ_ϵ	0.0898	0.0779	0.0721	0.1082	0.0817	0.1055	0.1024

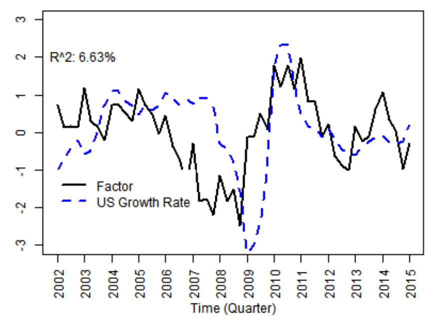
Appendix Figure B: Banks' Estimated Return-Beliefs Factors and Macro-Variables

This figure shows the first three factors of return-beliefs and net-demands of the banking sector. Shaded areas correspond to the Global Financial Crisis (2007Q3-2008Q4) and the European Debt Crisis (2011Q4-2013Q1), respectively. Subgraphs (b), (d), and (f) relate standardized factors to standardized observable macro-variables.

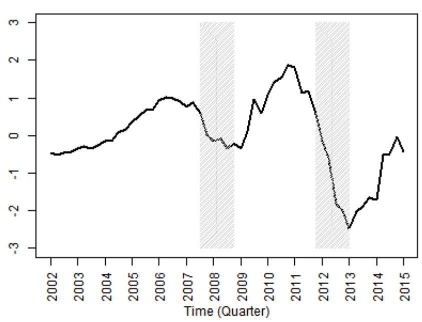
(a) Factor 1



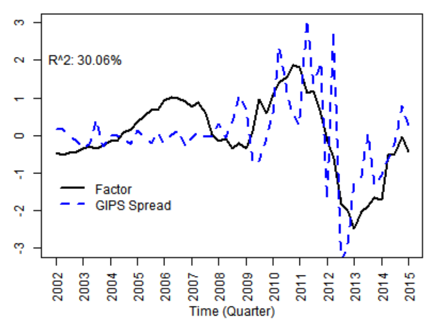
(b) Factor 1 &World GDP Growth



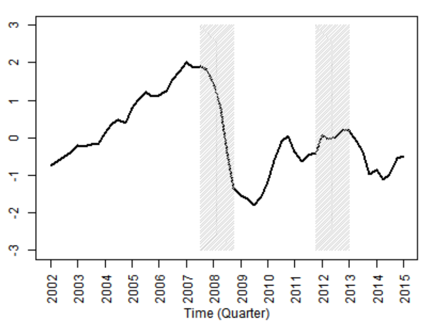
(c) Factor 2



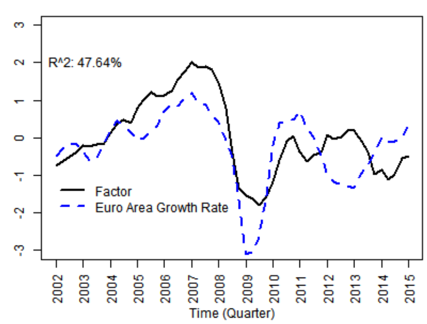
(d) Factor 2 & GIIPS Spread



(e) Factor 3



(f) Factor 3 & Euro Area GDP Growth

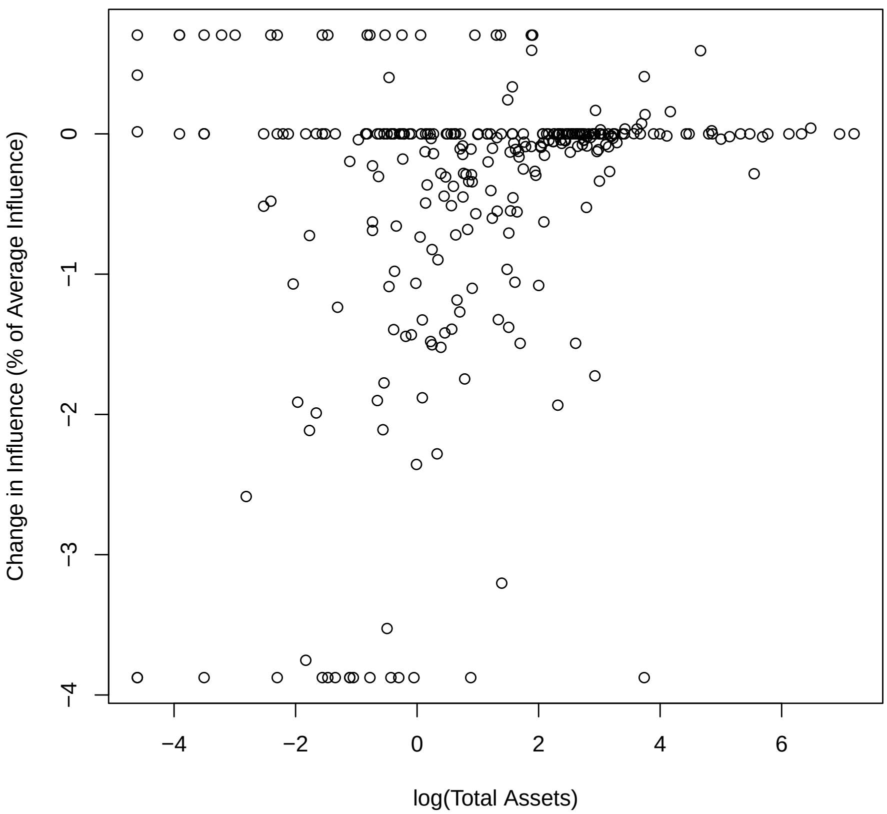


Source of Macro-Variables: IMF WEO database and IFS.

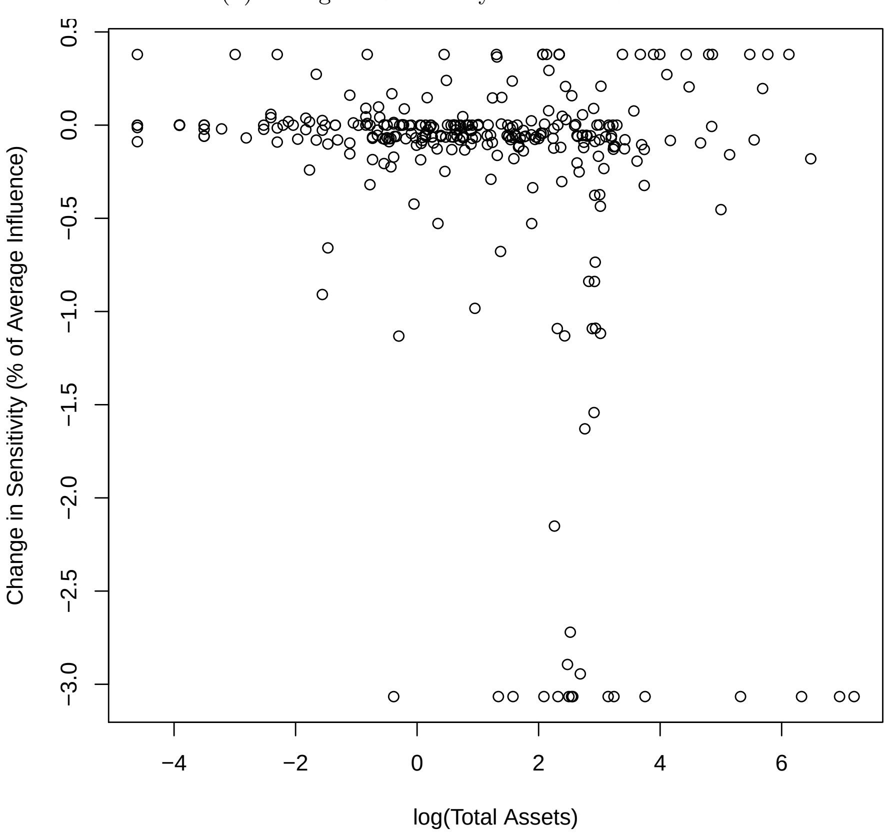
Appendix Figure C: ECB Shock and Bank Size

These two figures present the impact of the ECB shock on the influence and the sensitivity of banks, against their size (total assets).

(a) Change in Influence and Bank Size



(b) Change in Sensitivity and Bank Size



A Theory

A.1 Proofs: General Equilibrium Model

Proposition 1. (Existence of Equilibrium)

Proof. For clarity of exposition and without loss of generality, we omit the subscript t in the notations for this proof. Write $\mathbf{\Delta}(\mathbf{p}) = \sum_{i=1}^{N} \mathbf{\Delta}_i(p_1, p_2, \ldots, p_J) - \mathbf{E}(p_1, p_2, \ldots, p_J)$. Throughout the paper, $\mathbf{\Delta}(\mathbf{p})$ is the value of net demand. We thus write $\mathbf{z}(\mathbf{p})$ the net demand in units of financial instruments, and thus $\mathbf{\Delta}(\mathbf{p}) = \mathbf{p} \odot \mathbf{z}(\mathbf{p})$ is the term by term product of the price vector and the net demand vector in units.

An equilibrium price vector is a J-vector $\mathbf{p}$ such that $\mathbf{z}(\mathbf{p}) = 0$.

Following Mas-Colell, Whinston, Green et al. (1995), we need to show that the aggregate net demand curve $\mathbf{p} \in \mathbb{R}^J_{+*} \mapsto \mathbf{z}(\mathbf{p}) \in \mathbb{R}^J$ satisfies the following properties:

1. $\mathbf{z}(\cdot)$ is continuous.
2. $\mathbf{z}(\cdot)$ is homogenous of degree zero.
3. $\mathbf{p} \cdot \mathbf{z}(\mathbf{p}) = 0$ for all $\mathbf{p}$, i.e. Walras law is satisfied.
4. There is an $s > 0$ such that $z_j(\mathbf{p}) > -s$ for all financial instruments $j = 1, 2, \ldots, J$ and for all price vectors $\mathbf{p}$.
5. If $\mathbf{p}^n \to \mathbf{p}$, where $\mathbf{p} \neq 0$ and $p_j = 0$ for some j, then

$$\max\left\{z_1(\mathbf{p}^n), \ldots, z_J(\mathbf{p}^n)\right\} \to \infty$$

The continuity of $\mathbf{z}(\cdot)$ over $\mathbb{R}^J_{+*}$ is established as both the joint distribution of returns f_i and the utility function u_i of each firm i are continuous functions of $\mathbf{p}$ at each point of $\mathbb{R}^{+*J}$.

The homogeneity of $\mathbf{z}(\cdot)$ is established by writing the first-order condition of program (3) with $\|\cdot\|$ the Euclidean norm:

$$\int u_i'\left(\mathbf{\Delta}_{it}'\tilde{\mathbf{r}}\right) p_{jt}\tilde{r}_{ijt} f_i(\tilde{\mathbf{r}}) d\tilde{\mathbf{r}} - 2\left(\gamma_{ij}/p_j\right)^2 p_{jt} \cdot \left(z_{ijt} - z_{ijt-1}\right) + \lambda_i \Delta_{ijt} = 0,$$

where λ_i is the Lagrange multiplier of the funding constraint, which is homogeneous of degree 0. The first two terms are also homogeneous of degree 0, which establishes the homogeneity of degree 0 of z.

Walras law follows from the sum of the funding constraints, as

$$\mathbf{p} \cdot \mathbf{z}(\mathbf{p}) = \sum_{j=1}^{J} \sum_{i=1}^{N} (\Delta_{ij}(\mathbf{p}) - \mathbf{1}(j(i) = j) E_i(\mathbf{p})) = 0. \tag{28}$$

On point 4.: the existence of a lower bound for net demands $\mathbf{z}(\mathbf{p})$ follows from the funding constraint (2). Indeed, assume that there is no s such that $z_j(\mathbf{p}) > -s$ for all financial instruments $j = 1, 2, \ldots, J$ and for all price vectors $\mathbf{p}$. Then we can build a sequence $(s^k, \mathbf{p}^k)$ such that $s^k \to \infty$ as $k \to \infty$, and for each k, there is a $j'(k)$ in $\{1, 2, \ldots, J\}$ for which $-s^k > z_{j'(k)}(\mathbf{p}^k)$. Given the funding constraint, this implies that there will be a similar sequence for which demand will go to infinity. Formally, there is a sequence $(\sigma^k, \boldsymbol{\pi}^k)$ such that $\sigma^k \to \infty$ as $k \to \infty$, and for each k, there is a $j''(k)$ in $\{1, 2, \ldots, J\}$ for which $z_{j''(k)}(\boldsymbol{\pi}^k) > \sigma^k$. This, however, implies that the variance of the firm's portfolio diverges to infinity as $k \to \infty$, which cannot be a solution to the optimization program (19).The last point follows from the fact that if asset j's return diverges to infinity, $\mu_j \to \infty$, one of the components of net demand will diverge to infinity. Given that properties 1–5 are satisfied, an equilibrium price vector $\mathbf{p}^*$ exists with $\mathbf{z}(\mathbf{p}^*) = 0$. □

A.2 Proofs of Identification

Proposition 5. (Implied Net Demand Dynamic Factor Model)

Proof. From the expression for net-demand,

$$\hat{\boldsymbol{\Delta}}_{it} = \frac{1}{\rho_i} \Sigma_{it}^{-1} (\boldsymbol{\mu}_{it} - \eta_{it} \mathbf{1}_J), \tag{29}$$

and, using the factor structure of return beliefs (22), the moments of stochastic return beliefs are parametrized as

$$\boldsymbol{\mu}_{it} = (\boldsymbol{\varphi}_i + \Lambda_i \boldsymbol{\phi}_i + \Lambda_i \Phi_i \mathbf{f}_{it-1}) \tag{30}$$

$$\Sigma_{it} = \left(\Lambda_i \Sigma_{ui} \Lambda_i'\right) + \sigma_{\epsilon i}^2 Id + \Sigma_{ei}. \tag{31}$$

Noting $\Sigma_{\epsilon i} \equiv \sigma_{\epsilon i}^2 Id + \Sigma_{ei}$, realize that the variance covariance of stochastic return beliefs can be written as a product, such that net demands are:

$$\hat{\mathbf{\Delta}}_{it} = \frac{1}{\rho_i} \underbrace{\left(I + \Sigma_{\epsilon i}^{-1}\left(\Lambda_i \Sigma_{ui} \Lambda_i'\right)\right)^{-1} \Sigma_{\epsilon i}^{-1}}_{X} \left[\boldsymbol{\varphi}_i + \Lambda_i \mathbf{f}_{it} - \mathbf{1}_B \eta_{it}\right]. \tag{32}$$

Notice that the inverse of the variance covariance of return beliefs, Σ_{it}^{-1}, can be written as an infinite sum as

$$\begin{aligned}
\left[I - \left(-\Sigma_{\epsilon i}^{-1}\left(\Lambda_i \Sigma_{ui} \Lambda_i'\right)\right)\right] &= I + \left(-\Sigma_{\epsilon i}^{-1}\left(\Lambda_i \Sigma_{ui} \Lambda_i'\right)\right) \\
&+ \left(-\Sigma_{\epsilon i}^{-1}\left(\dots\right)\right)^2 + ...,
\end{aligned}$$

and that therefore the term noted X can be written as

$$\begin{aligned}
X &= \Sigma_{\epsilon i}^{-1}\left[I - \left(\Lambda_i \Sigma_{ui} \Lambda_i'\right)\Sigma_{\epsilon i}^{-1} + \left(-\left(\Lambda_i \Sigma_{ui} \Lambda_i'\right)\Sigma_{\epsilon i}^{-1}\right)^2 + ...\right] \\
&= \Sigma_{\epsilon i}^{-1}\left[I + \left(\Lambda_i \Sigma_{ui} \Lambda_i'\right)\Sigma_{\epsilon i}^{-1}\right]^{-1} \\
&= \left[\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\epsilon i}\right]^{-1}.
\end{aligned}$$

The matrix $\Sigma_{\epsilon i}$ is a measure of the *unexplained variance* of stochastic return beliefs. Then net demand has the factor structure:

$$\hat{\mathbf{\Delta}}_{it} = \mathbf{c}_i + L_i \mathbf{f}_{it-1} + \mathbf{h}_{it}, \tag{33}$$

where constants, loadings and transformed cost of equity of the net demand factor model are given by:

$$\begin{aligned}
\mathbf{c}_i &= \frac{1}{\rho_i}\left[\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\epsilon i}\right]^{-1}\left(\boldsymbol{\varphi}_i + \Lambda_i \boldsymbol{\phi}_i\right) \\
L_i &= \frac{1}{\rho_i}\left[\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\epsilon i}\right]^{-1}\Lambda_i \Phi_i \\
\mathbf{h}_{it} &= \frac{1}{\rho_i}\left[\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\epsilon i}\right]^{-1}\mathbf{1}\eta_{it}.
\end{aligned}$$

The introduction of trade costs Γ_i at the instrument-level adds an additional term Γ_i^{-1} inside the inversion and identification can proceed as before. $\square$

Proposition 7. (Identification of Return Beliefs and Risk Aversion)

Proof. Because all three objects $\Sigma_{\mathcal{E}i}$,Σ_{ui}, Φ_i are known parameters in this mapping, we introduce scaled return belief loadings $\widetilde{\Lambda_i} \equiv \Sigma_{\mathcal{E}i}^{-1/2}\Lambda_i\Sigma_{ui}^{1/2}$, and scaled net-demand loadings $\widetilde{L_i} \equiv \Sigma_{\mathcal{E}i}^{-1/2}L_i\Phi_i^{-1}\Sigma_u^{1/2}$ to simplify notations. Then, scaled belief loadings and risk aversion need to satisfy (dropping the firm index):

$$\frac{1}{\rho}\left(I+\widetilde{\Lambda}\widetilde{\Lambda}'\right)^{-1}\widetilde{\Lambda}=\widetilde{L}. \tag{34}$$

The invertibility of the mapping g is proven in two steps: *First*, we show that for a given risk aversion ρ, $\widetilde{\Lambda}$ can be obtained by applying the method of undetermined coefficients and solving a quadratic matrix equation. *Second*, ρ is determined using the fact that Λ must be in the domain of the mapping g, i.e. a matrix of factor loadings satisfying the property that the sum of its squared elements is equal to K.

We start by writing scaled return belief loadings, $\widetilde{\Lambda}$, as a matrix product of scaled net-demand loadings and an unknown square-matrix,

$$\widetilde{\Lambda}=\widetilde{L}\cdot\left(\rho^{-1}I-\widetilde{\Lambda}'\widetilde{L}\right)^{-1}. \tag{35}$$

Equation (35) leads us to conjecture that the solution for $\widetilde{\Lambda}$ is of the form,

$$\widetilde{\Lambda}=\widetilde{L}\cdot X, \tag{36}$$

where X is an undetermined square-matrix of coefficients. From equations (35) and 36 it follows that

$$X=\left(\rho^{-1}I-X'\widetilde{L}'\widetilde{L}\right)^{-1}. \tag{37}$$

Because equation (37) is symmetric, it must be that $X=X'$ and equation (37) can be written as the quadratic matrix equation

$$\underbrace{I}_{\mathcal{A}}\cdot X^{-2}\underbrace{-\rho^{-1}I}_{\mathcal{B}}\cdot X^{-1}\underbrace{+\widetilde{L}'\widetilde{L}}_{\mathcal{C}} \quad = \quad 0, \tag{38}$$

which has, if *(i)* $\mathcal{A} = I$, *(ii)* $\mathcal{BC} = \mathcal{CB}$, and *(iii)* $\mathcal{B}^2 - 4\mathcal{AC}$ has a square-root, the solution

$$X^*(\rho) = \left(\frac{1}{2} \left[\rho^{-1} I + \left(\rho^{-2} I - 4\widetilde{L}'\widetilde{L} \right)^{1/2} \right] \right)^{-1}. \tag{39}$$

Conditions *(i)* and *(ii)* are easily checked to be satisfied. Condition *(iii)* is satisfied as long as ρ is small enough for $\mathcal{B}^2 - 4\mathcal{AC}$ to be positive semi-definite. A positive semi-definite matrix has a unique positive semi-definite square root. While it is irrelevant for the variance of return beliefs, which square root is chosen, the natural solution is the positive semi-definite square root. Indeed, the positive semi-definite square root will preserve the direction of factors as identified by the factor model on net-demands. Choosing another square root would make it necessary to change the direction of factors.

With $X^*(\rho)$, risk aversion ρ follows from the predicate of the domain of the mapping g. The sum of squared elements of Λ must be equal to the number of factors, K.

$$\mathbf{1}' \left(\Lambda \circ \Lambda \right) \mathbf{1} = \mathbf{1}' \left(\left(\Sigma_{\mathcal{E}}^{1/2} \widetilde{L} X^*(\rho) \Sigma_u^{-1/2} \right) \odot \left(\Sigma_{\mathcal{E}}^{1/2} \widetilde{L} X^*(\rho) \Sigma_u^{-1/2} \right) \right) \mathbf{1} = K. \tag{40}$$

□

Proposition 7. (Identification of Beliefs with Trade Costs) *The optimal net demand schedule of firm i in the presence of adjustment costs can be written as*

$$\boldsymbol{\Delta}_{it} = \boldsymbol{c}_i + L_i \mathbf{f}_{it-1} + \boldsymbol{h}_{it} + G_i \boldsymbol{\Delta}_{it-1}, \tag{41}$$

where the constant vector C_i, the loadings L_i, the transformed cost of equity $\overline{\eta_{it}}$, and G_i the transformed adjustment costs are functions of the unobserved factor structure of return beliefs, the coefficients of the auto-regressive processes of underlying factors, the adjustment costs and risk preference:

$$\begin{cases} \boldsymbol{c}_i &= \left(I + \Gamma_i/(\rho_i \sigma_{\epsilon i}^2) + \frac{\sigma_{ui}^2}{\sigma_{\varepsilon i}^2} \Lambda_i \Lambda_i' \right)^{-1} \Lambda_i \left(\boldsymbol{\varphi}_i + \boldsymbol{\phi_i} \right) / (\rho_i \sigma_{\epsilon i}^2) \\ L_i &= \left(I + \Gamma_i/(\rho_i \sigma_{\epsilon i}^2) + \frac{\sigma_{ui}^2}{\sigma_{\varepsilon i}^2} \Lambda_i \Lambda_i' \right)^{-1} \Lambda_i \Phi_i / (\rho_i \sigma_{\epsilon i}^2) \\ \boldsymbol{h}_{it} &= - \left(I + \Gamma_i/(\rho_i \sigma_{\epsilon i}^2) + \frac{\sigma_{ui}^2}{\sigma_{\varepsilon i}^2} \Lambda_i \Lambda_i' \right)^{-1} \mathbf{1} \eta_{it} / (\rho_i \sigma_{\epsilon i}^2) \\ G_i &= \left(I + \Gamma_i/(\rho_i \sigma_{\epsilon i}^2) + \frac{\sigma_{ui}^2}{\sigma_{\varepsilon i}^2} \Lambda_i \Lambda_i' \right)^{-1} \Gamma_i / (\rho_i \sigma_{\epsilon i}^2) \end{cases} \tag{42}$$

The identification of structural parameters in the presence of adjustment costs is very similar to the identification without adjustment costs. Indeed, the additional observation G_i is used to express the ratio $\frac{\Gamma_i}{\rho_i \sigma_{\varepsilon i}^2} = \left(I + \frac{\sigma_{ui}^2}{\sigma_{\epsilon i}^2}\Lambda_i\Lambda_i'\right)\left(G_i\left(I - G_i\right)^{-1}\right)$ in terms of structural parameters and observations. This can be plugged into the expression of net demand factor loadings, L_i, to obtain

$$\begin{aligned}
L_i &= \left(I + \left(I + \frac{\sigma_{ui}^2}{\sigma_{\epsilon i}^2}\Lambda_i\Lambda_i'\right)\left[G_i\left(I - G_i\right)^{-1}\right] + \frac{\sigma_{ui}^2}{\sigma_{\varepsilon i}^2}\Lambda_i\Lambda_i'\right)^{-1}\Lambda_i\Phi_i/(\rho_i\sigma_{\epsilon i}^2) \\
L_i &= \left(\left(I + \frac{\sigma_{ui}^2}{\sigma_{\epsilon i}^2}\Lambda_i\Lambda_i'\right)\left[I + G_i\left(I - G_i\right)^{-1}\right]\right)^{-1}\Lambda_i\Phi_i/(\rho_i\sigma_{\epsilon i}^2) \\
\left(I - G_i\right)^{-1}L_i &= \left(I + \frac{\sigma_{ui}^2}{\sigma_{\epsilon i}^2}\Lambda_i\Lambda_i'\right)^{-1}\Lambda_i\Phi_i/(\rho_i\sigma_{\epsilon i}^2)
\end{aligned}$$

and the mapping g in the text (Equation 26) is modified to

$$g: \quad \left(\frac{1}{\rho_i\sigma_{\varepsilon i}^2}, \Lambda_i\right) \longmapsto \left(\frac{\sigma_{ui}^2}{\sigma_{\varepsilon i}^2}, \underbrace{\frac{1}{\rho_i\sigma_{\epsilon i}^2}\left(I + \frac{\sigma_{ui}^2}{\sigma_{\varepsilon i}^2}\Lambda_i\Lambda_i'\right)^{-1}\Lambda_i\Phi_i}_{(I - G_i)^{-1}L_i}\right). \tag{43}$$

The identification of the remaining parameters proceeds in the same way as in the model without adjustment costs using the relations from Proposition 7.

A.3 Firms' Demand with Exogenous Constraints

This section introduces occasionally binding linear constraints. Note $C_{\Delta i}$ and C_{Ei} two matrices of size $K \times J$ that collect coefficients on net demands$\mathbf{\Delta}_{it}$ and initial capital $\mathbf{E}_{it}$, respectively, and $\mathbf{c}_i$ a K-column vector of scalars, then K linear constraints can be expressed as:

$$C_{\Delta i}\mathbf{\Delta}_{it} + C_{Ei}\mathbf{E}_{it} \leq \mathbf{c}_i \tag{44}$$

The set of constraints of the above form comprises the following regulatory and internal constraints:

- *Funding constraint:*

$$\mathbf{1}'\mathbf{\Delta}_{it} - \mathbf{1}'\mathbf{E}_{it} \leq 0$$

- *Short-selling constraint*: firm i cannot demand (resp. supply) instruments j in a subset

$\mathcal{J} \subset \{1, 2, \ldots, J\}$. These constraints effectively define the *line of business* of a firm. Banks may for instance be prevented from selling insurance policies.

$$\boldsymbol{\Delta}_{it}(j) \geq 0 \, for \, j \in \mathcal{J} \quad (resp., \geq 0 \, for \, j \in \mathcal{J})$$

- *Portfolio value constraints* that fix a threshold under which the market value of a given portfolio of instruments should not fall (relative to equity or in absolute terms). These may include *liquidity constraints* (such as the Liquidity Coverage Ratio as defined in Basel III stipulating that High Quality Liquid Assets have to be hold in sufficient quantity to cover refinancing risks) or a *leverage ratio* setting a minimum level of equity relative to total assets.

$$\frac{\mathbf{a}'\boldsymbol{\Delta}_{it}}{\mathbf{b}'\boldsymbol{\Delta}_{it} + \mathbf{d}'\mathbf{E}_{it}} \geq z_i$$

Demands for financial instruments then take a simple form based on the marginal cost of the constraint and the multivariate Sharpe ratio $\Sigma_{it}^{-1}\boldsymbol{\mu}_{it}$.

Proposition 8. (Firm Demand with Constraints) *In the presence of occasionally binding linear constraints of the form as in equation 44, firm i's net demand for instruments depend on the first two moments of its return beliefs, its risk aversion, and a vector of Lagrange multipliers,* $\boldsymbol{\eta}_{it}$ *associated to the occasionally binding constraints:*

$$\boldsymbol{\Delta}_{it} = \frac{1}{\rho_i}\Sigma_{it}^{-1}\left(\boldsymbol{\mu}_{it} - C'_{\Delta i}\boldsymbol{\eta}_{it}\right). \tag{45}$$

The Lagrange multipliers depend on a k-column vector of latent and unobservable slack-variables $\mathbf{s}_{it}$*,*

$$\boldsymbol{\eta}_{it} = \left(C_{\Delta i}\Sigma_{it}^{-1}C'_{\Delta i}\right)^{-1}\left[C_{\Delta i}\Sigma_{it}^{-1}\boldsymbol{\mu}_{it} - \rho_i\left(\mathbf{c}_i - C_{Ei}\mathbf{E}_{it} + \mathbf{s}_{it}\right)\right] \tag{46}$$

where the vector of slack variables has the following truncated form:

$$\mathbf{s}_{it} \equiv \max\left\{-\frac{1}{\rho_i}C_{\Delta i}\Sigma_{it}^{-1}\boldsymbol{\mu}_{it} + \mathbf{c}_i - C_{Ei}\mathbf{E}_{it}, \mathbf{0}\right\}. \tag{47}$$

Since $\mathbf{s}_{it}$ *is a function of prices, the presence of occasionally binding constraints thus effectively* *introduces a discontinuity in firm i's net demand for financial instruments. In contrast to the baseline case of only a funding constraint, now discontinuities in net-demands arise endogenously as a function of (i) the moments of the return belief distribution* $(\boldsymbol{\mu}_{it}, \Sigma_{it})$ *and initial equity* $\mathbf{E}_{it}$*.*

Proof. The set of K occasionally binding constraints $C_{\Delta i}\boldsymbol{\Delta}_{it} + C_{Ei}\mathbf{E}_{it} \leq \mathbf{c}_i$ can be transformed into a set of binding constraints, by introducing a vector of slack variables $\mathbf{z}_{it}$, defined by

$$\mathbf{z}_{it}^2 \equiv \mathbf{c}_i - C_{\Delta i}\boldsymbol{\Delta}_{it} - C_{Ei}\mathbf{E}_{it}. \tag{48}$$

The firm then not only chooses a vector of net demands $\boldsymbol{\Delta}_{it}$, but also the vector of slack variables $\mathbf{z}_{it}$ in order to maximize the mean variance objective

$$\boldsymbol{\Delta}_{it}'\boldsymbol{\mu}_{it} - \frac{1}{2}\rho_i\boldsymbol{\Delta}_{it}'\Sigma_{it}\boldsymbol{\Delta}_{it} - \boldsymbol{\eta}_{it}'\left[\mathbf{c}_i - C_{\Delta i}\boldsymbol{\Delta}_{it} - C_{Ei}\mathbf{E}_{it} - \mathbf{z}_{it}^2\right] \tag{49}$$

The F.O.C.s of this problem then are

$$\begin{aligned}
\boldsymbol{\Delta}_{it} &= \frac{1}{\rho_i}\Sigma_{it}^{-1}\left(\boldsymbol{\mu}_{it} - C_{\Delta i}'\boldsymbol{\eta}_{it}\right)\\
\mathbf{0} &= \operatorname{diag}\left(\mathbf{z}_{it}^2\right)\boldsymbol{\eta}_{it}\\
\boldsymbol{\eta}_{it} &= \left(C_{\Delta i}\Sigma_{it}^{-1}C_{\Delta i}'\right)^{-1}\left[C_{\Delta i}\Sigma_{it}^{-1}\boldsymbol{\mu}_{it} - \rho_i\left(\mathbf{c}_i - C_{Ei}\mathbf{E}_{it} - \mathbf{z}_{it}^2\right)\right],
\end{aligned}$$

to which a solution in closed form exists. Indeed, define $\mathbf{s}_{it} \equiv \mathbf{z}_{it}^2$, then plugging in the expression for the Lagrange multipliers into the F.O.C. of the slack variables yields a system of independent (one for each slack variable $\mathbf{s}_{it}(k)$) quadratic equations:

$$\operatorname{diag}\left(\mathbf{s}_{it}\right)\cdot\left(C_{\Delta i}\Sigma_{it}^{-1}C_{\Delta i}'\right)^{-1}\left[\frac{1}{\rho_i}C_{\Delta i}\Sigma_{it}^{-1}\boldsymbol{\mu}_{it} - \mathbf{c}_i + C_{Ei}\mathbf{E}_{it} + \mathbf{s}_{it}\right] = \mathbf{0}. \tag{50}$$

Therefore there are potentially two solutions for each slack variable $\mathbf{s}_{it}(k)$, indeed

$$\mathbf{s}_{it} = \mathbf{0} \wedge \mathbf{s}_{it} = -\frac{1}{\rho_i}C_{\Delta i}\Sigma_{it}^{-1}\boldsymbol{\mu}_{it} + \mathbf{c}_i - C_{Ei}\mathbf{E}_{it} \tag{51}$$

However, since $\mathbf{s}_{it} \equiv \mathbf{z}_{it}^2 \geq \mathbf{0}$, values with $\mathbf{s}_{it}(k) < 0$ can be excluded. Furthermore, we impose that if there exists a solution $\mathbf{s}_{it}(k) > 0$, we choose this solution, which corresponds to the case where constraint k is not binding. Note that this is no loss of generality. Indeed, if the objective is quasi-convex in net-demands, there is one global maximum of the mean-variance objective, which can be one of the two: within the set of constraints or outside the set of constraints. If it is within, then the constraints are not binding, and we find the global maximum by choosing the solution for which $\mathbf{s}_{it}(k) > 0$ (i.e. constraint k does not bind). If the global maximum is outside the set of constraints, $\mathbf{s}_{it}(k) < 0$, and we thus go to the boundary of the set of constraints at which $\mathbf{s}_{it}(k) = 0$. Therefore the slack variable are:

$$\mathbf{s}_{it} = \max\left\{-\frac{1}{\rho_i} C_{\Delta i} \Sigma_{it}^{-1} \boldsymbol{\mu}_{it} + \mathbf{c}_i - C_{Ei}\mathbf{E}_{it}, \mathbf{0}\right\} \tag{52}$$

□

B Data

B.1 Estimation of Returns

There are two challenges for deriving returns on financial instrument categories from sectoral account data.

1. Returns are specific to the sectoral investment position. The sectoral accounts provide information on aggregates of financial instruments of the same type. No information exists on sectoral portfolio allocations across financial instruments of the same type. Returns realized on financial instruments of the same type therefore vary across sector, due to differences in inter-type allocations.

2. Pay-offs are recorded only at the sectoral level as income received or payed in the income accounts. Therefore dividends, interest payments and investment income cannot be directly attributed to financial instruments.

We use the variation of investment positions across sectors and financial instruments, the variation of valuation changes across sectors and financial instruments and the variation of income received or payed across sectors to estimate subsequently: (i) the return due to valuation changes and (ii) the return due to payoffs. Financial instruments are indexed by $j = 1, ..., J$ and sectors are indexed by $i = 1, ..., I$. Furthermore, $\boldsymbol{P}_t^D$ and $\boldsymbol{P}_t^S$ are matrices of size $I \times J$ that collect the demand and supply by I sectors for J financial instruments, respectively. The rate of return due to valuation changes is denoted by g_{jt+1} and the rate of return due to payoffs is denoted by π_{jt+1}.

Assumption 1. (Separate Estimation of Returns and Shifts in Demand) *The return on asset j demanded or supplied by sector i have an asset-specific component and a component specific to the financial position $f \in \{D, S\}$ of sector i.*

$$\pi_{ijt+1}^f = \pi_{jt+1} + u_{it+1}^f, \quad f \in \{D, S\} \tag{53}$$

$$g_{ijt+1}^f = g_{jt+1} + v_{it+1}^f, \quad f \in \{D, S\} \tag{54}$$

where D denotes the demand/asset side of the balance-sheet and S the supply/liability side of the balance-sheet. We assume that the component specific to the financial position $f \in \{D, S\}$ of sector i is iid distributed with mean zero.

Collecting the pay-offs and valuation gains received or payed on J assets by I sectors in the matrices $\boldsymbol{\Pi}_t^f$ and $\boldsymbol{G}_t^f$ of dimension $I \times J$, respectively, Assumption 1 implies that the return in amount of currency units realized can be written in matrix notation as

$$\boldsymbol{\Pi}_t^f = \boldsymbol{P}_t^f diag\left(\boldsymbol{\pi}_{t+1}\right) + diag\left(u_{t+1}^f\right)\boldsymbol{P}_t^f, \quad f \in \{D, S\} \tag{55}$$

$$\boldsymbol{G}_t^f = \boldsymbol{P}_t^f diag\left(\boldsymbol{g}_{t+1}\right) + diag\left(v_{t+1}^f\right)\boldsymbol{P}_t^f, \quad f \in \{D, S\} \tag{56}$$

As argued above, not all returns realized by sectors on financial instruments are fully observable. For example, only the sum of interest payments received by sector i is observable, but not interest payments received on loans and coupon payments received on debt securities separately. Indeed, there are matrices $\boldsymbol{O}^\Pi$ and $\boldsymbol{O}^G$ of dimension $J \times K$ that map unobservable return components to observable functions of returns, such that we can transform equations 5556 to

$$\boldsymbol{\Pi}_t^f \boldsymbol{O}^\Pi = \left[\boldsymbol{P}_t^f diag\left(\boldsymbol{\pi}_{t+1}\right) + diag\left(u_{t+1}^f\right)\boldsymbol{P}_t^f\right]\boldsymbol{O}^\Pi, \quad f \in \{D, S\} \tag{57}$$

$$\boldsymbol{G}_t^f \boldsymbol{O}^G = \left[\boldsymbol{P}_t^f diag\left(\boldsymbol{g}_{t+1}\right) + diag\left(v_{t+1}^f\right)\boldsymbol{P}_t^f\right]\boldsymbol{O}^G, \quad f \in \{D, S\} \tag{58}$$

Note that if all returns were observable, $\boldsymbol{O}^{\Pi} = \boldsymbol{O}^{G} = \boldsymbol{I}$. Finally, we estimate g_{jt+1}, π_{jt+1} such as to minimize the sum of squared residuals. This method simplifies to weighted Ordinary Least Squares when we assume additionally that the sector and financial position specific components of returns are uncorrelated.

B.2 Imputation of the Real Asset

The estimation of the model requires information on complete balance-sheets, i.e. stock and change (valuation + flows + other changes) for both financial and non-financial assets. These data are available in the following form:

1. Financial asset stocks and changes: at quarterly frequency from quarterly sectoral accounts from EUROSTAT. The datasets are: nasq_10_f_bs, nasq_10_f_gl, nasq_10_f_tr, nasq_10_f_oc.

2. Non-financial asset stocks: at annual frequency from both EUROSTAT (nasa_10_nfa) and INSEE.

3. Non-financial asset changes to due flows: at quarterly frequency from EUROSTAT (nasq_10_nf_tr), but strangely not from INSEE.

4. Non-financial asset changes due to valuation, flows and other changes: at annual frequency from INSEE, but not from EUROSTAT.

Quarterly stocks and changes of the financial part of sectoral balance-sheet are directly obtained from the sectoral financial accounts from EUROSTAT. The non-financial part of the balance-sheet are obtained in three steps:

1. We aggregate quarterly flows from EUROSTAT to annual flows and calculate the share of the annual flow that takes place in each quarter. For example we will find that in 2008.1 30% of the 2008 annual flow took place, etc.

2. We take annual changes (valuation + flows + other changes) from INSEE and apply the inter-annual breakdown for flows obtained in Step 1 to all types of changes.

3. We take the annual stocks from INSEE and build up the quarterly stocks from the quarterly series of changes obtained in Step 2.