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# THE DISCOUNT RATE FOR PUBLIC POLICY OVER THE DISTANT FUTURE

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## ABSTRACT

The choice of discount rate has a significant impact on net benefit estimates when costs today have benefits over long time horizons. Standard U.S. government practice for cost–benefit analysis is to bound such analysis using two alternative rates. These rates are meant to represent the rate of return paid by capital investment and the rate received by consumers. Previous work has shown this approach legitimately bounds the analysis—but only when future benefits accrue directly to consumers either in a two-period model or as a perpetuity. We generalize to consider arbitrary patterns of future benefits, accruing either directly to consumers or indirectly through future investment. We derive an expression for the appropriate discount rate and show that it converges to the consumption rate for benefits increasingly far into the future. More generally, the bounding rates depend on the temporal pattern of the undiscounted dollars. As an application, we estimate the appropriate discount rate for climate change damages from carbon dioxide, finding it lies in a narrow range (+/- 0.5 percent) around the consumer rate of interest.

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## 1. Introduction

Conducting cost–benefit analysis (CBA) for government policy requires a discount rate to compare costs and benefits over time in order to establish, on net, whether total benefits exceed total costs. For more than 15 years, federal guidelines for government CBA in the United States have indicated that two discount rates should be used, 7 and 3 percent (OMB 2003). When applied to government policies with costs today and benefits extending far into the future—our stylized arrangement throughout this paper[1]—the different outcomes associated with these rates can be striking. Recent government estimates of climate change benefits, for example, are six to nine times higher using 3 percent rather than 7 percent.[2]

The explicit rationale for this range of rates is based on whether the costs of regulation today fall primarily on the allocation of private capital or instead directly affect household consumption. These rates reflect the pretax return paid by private capital and the return received by consumers, respectively, with the

[1] While costs and benefits could occur in any period, we have in mind problems where costs today are being measured against benefits in future periods. Most public policy problems break down this way, as investments are made today that then yield benefits. Even when there may be operating costs in the future, these can be subtracted from the future benefit flows. In this paper, the undiscounted costs and benefits dollars are assume to be measured in their certainty equivalents.

[2] See Appendix A for details.

difference owing largely to taxes.[3] For shorthand and consistency, we refer to these two rates as the "investment rate" and "consumer rate." Ignoring risk and uncertainty, CBA using these rates provides upper and lower bounds on the estimate of net benefits when benefits directly to consumers under two alternative, specific assumptions: either the economy exists in only two periods (Harberger 1972; Sandmo and Drèze 1971) or the pattern of benefits over time is a perpetuity (Marglin 1963a, 1963b; Drèze 1974; Sjaastad and Wisecarver 1977).

The main point of this paper is to derive more general bounds on the discount rate in a multi-period model when benefits are no longer assumed to accrue directly to consumers or to be a perpetuity. In particular, we show that these bounds converge to the consumer interest rate when valuing benefits far into the future. Intuitively, the appropriate discount rate is exactly the consumption rate when all costs and benefits accrue to consumption. Impacts on investment *either* (1) have to be converted to consumption equivalents using a shadow price *or*, equivalently, (2) must be accommodated by a different discount rate. However, the effect of shadow pricing is at most a bounded multiplicative factor. Mechanically, the appropriate discount rate under approach (2) can be defined as an upward or downward adjustment to the consumption rate based on this bounded multiplicative factor. Looking far into the future, smaller and smaller

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[3] While OMB guidance clearly articulates the notion that 7 and 3 percent represent investment and consumption rates, there are several reasons to believe taxes do not entirely explain the difference. Corporate and individual marginal tax rates, perhaps averaging 40 percent each over the past few decades, do not produce such a large divergence. Moreover, not all capital is subject to corporate taxes and consumers can reduce their effective tax rate through tax-deffered savings (for retirement and education). More likely, some of this difference should be ascribed to risk.

adjustments to the consumer rate will be needed to compensate for that factor. More generally, this range depends on the pattern of future benefits over time and the shadow price of capital.

We can use the U.S. government's values of 7 and 3 percent for the investment and consumption rates to provide numeric examples.[4] Applied to the pattern of climate mitigation benefits arising in a recent National Academy of Science report (NAS 2017), the appropriate range is quite narrow. The estimated discount rate would be between 2.3 and 4.0 percent if we use the government rates to derive a bound on the shadow price. Adding additional assumptions based on a Ramsey growth model, we can derive a particular shadow price of capital and an even narrower range of 2.6–3.4 percent for the discount rate. On the other hand, the discount rate for a policy with benefits entirely in the very near term could vary from -50 to +50 percent depending on the distribution of impacts on capital versus consumption goods.

At a practical level, our results provide an argument for focusing the discussion of the social discount rate[5] on an interval centered on the consumer discount rate and based on each application's particular pattern of benefits, particularly over long horizons. This need not require any more information than the current government CBA, with selected investment and consumer rates implying a bound on the shadow price.

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[4] As noted in footnote 3, we cannot explain the difference between 7 and 3 percent based on actual tax rates, as required by the model. However, these numbers remain relevant in the policy discussion as evident by recent regulatory analyses. Alternatively, one could replace our example using different rates.

[5] We use the term "social discount rate" when we want to emphasize the use of discounting for public investments or government regulation that generates public goods.

Our focus on a policy's potential impact on investment or consumption is distinct from the important question of how uncertainty about future benefits might be correlated with market returns or consumption growth. There is a long history in the finance literature that relates this correlation to an appropriate rate of return (e.g., Fama 1977). This thinking has also been applied to public policy, particularly climate change (Gollier 2013). However, such an analysis must still contend with taxes that create a distortion between the return paid on investment and the returns received by households, regardless of correlation. This distortion creates a divergence between the shadow price of a dollar of investment and a dollar of consumption. A full welfare analysis under uncertainty must contend with *both* correlated returns *and* the potential for policy impacts on both consumption and investment. In order to focus our discussion, this paper only considers different impacts on consumption and investment, assuming the returns themselves are known.

We are not the first to suggest that the consumer rate is a more appropriate discount rate over long horizons, but we believe we are the first to provide particularly compelling arguments against the investment rate. OMB guidelines note that policies with intergenerational effects raise certain ethical concerns. They suggest that policies with such effects might consider a lower, but positive, rate as a sensitivity analysis *in addition to* calculations based on 3 and 7 percent. They do not suggest that the investment interest rate is incorrect. Government estimates of the social cost of carbon dioxide (SC-$CO_2$) under the Obama administration were based only on consumption interest rates. The underlying analysis (IWG 2010) concluded the consumer rate was appropriate based on the "economics literature," but it is hard to find such a clear declaration in the noted references. The NAS report argues against an investment interest rate because such a rate is correct only under very restrictive assumptions (NAS 2017). While this is true, the same could be said about the consumption interest rate so long as taxes create a wedge between consumer and investment rates and the impacts on capital and consumption are unclear.

There is also a different but related debate over the appropriate investment and consumption interest rates for CBA, regardless of uncertainty about which one to use or uncertainty in the future. In this paper, the 7 and 3 percent rates were chosen to keep the policy argument consistent with the federal practice. A recent government white paper, for example, attempted to distill recent evidence regarding both rates, arguing that the current consumer rate might be lower than three percent (CEA 2017). Another paper recently surveyed experts and arrived at a similar conclusion (Drupp et al. 2018). Harberger and Jenkins (2015) revisited the empirical estimates for opportunity costs of capital and suggests an upward adjustment to the investment rate.

For those concerned about CBA's application to regulating persistent environmental pollution, including carbon dioxide and other greenhouse gases, it is hard to overstate the importance of these questions about the appropriate discount rate. On October 16, 2017, the Trump administration issued a CBA of the Clean Power Plan to support the plan's repeal (U.S. EPA 2017). In contrast to the Obama administration's CBA, which used a central benefit estimate based on a three-percent discount rate (U.S. EPA 2015), the new CBA of the same regulation gave equal weight to estimates based on three and seven percent. As noted earlier and described in more detail in Appendix A, climate benefits are six to nine times higher using three rather than seven percent. By focusing on the midpoint of benefit estimates based on 3 and 7 percent, the new central benefit estimate is brought down by more than 40 percent. Whether to use the investment rate has thus emerged as a particularly salient discounting policy question.

Our paper is organized as follows. In the next section, we review the literature supporting the use of consumer and investment rates as bounding values for the social discount rate, including Marglin (1963a, 1963b), Harberger (1972), and Sjaastad and Wisecarver (1977). We also review the sharply different results obtained by Bradford (1975) when the assumptions underlying that result are relaxed. As we review these early papers, we develop a simple model to formalize ideas and define the range of social discount rates appropriate for an arbitrary pattern of benefits. We use this model to examine long-run behavior, where we show that the range of discount rates for benefits in the distant future converges to the consumption rate of interest. More generally, the range of appropriate rates depends on the temporal pattern of benefits being valued. The other key parameter that defines this range is the shadow price of capital, which we explore in more detail in section 4. We show that the ratio of investment and consumer rates is an upper bound on the shadow price under relatively weak assumptions. Based on the U.S. government rates, this would be 7/3. Using a Ramsey model along with additional assumptions about depreciation, the output-capital elasticity, and growth, we derive a shadow price of capital of roughly 1.5. This implies that the range of the social discount rate for benefits several decades in the future has already converged to roughly the consumer rate. Finally, we turn to the climate change application. Using the temporal pattern of climate change damages used in the noted NAS report (2017), we show that the appropriate rate for discounting climate damages lies between 2.6 and 3.4 percent based on a consumer rate of 3 percent and a shadow price of 1.5. Using this range, we find that estimates for the social cost of $CO_2$ differ by a factor of two. This is considerably less than estimates using 3 and 7 percent rates, where the SC-$CO_2$ varies by at least a factor of six.

## 2. History of discounting at consumption and investment interest rates

Discussions of discounting in CBA for public policy have recognized two main approaches to identifying appropriate discount rates. A *prescriptive* approach examines the ethical basis for discounting and focuses on some notion of how society ought to value future consequences. A *descriptive* approach instead focuses on the observed behavior of, and rates of interest faced by, households and firms (Arrow et al., 1996).

The United Kingdom and France have largely taken a prescriptive approach. The U.K. Green Book requires a 3.5 percent social time preference rate (in real terms) for government appraisal of costs and benefits over 30 years (HM Treasury 2018). That rate decreases to 1 percent for evaluation beyond 30 years. Similarly, the French guideline (France Stratégie 2017) recommends a 2.5 percent risk-free discount rate to the year 2070, gradually declining to 1.5 percent in the more distant future. These rates are based on an underlying ethical notion of discounting based on pure time preference and growth.

Government policy in the United States and Canada has largely followed the descriptive approach. Circular A-94 (BOB 1969) established 10 percent as the official discount rate for U.S. government CBAs. This rate was based on the "average rate of return on private investment, before taxes and after inflation." It was later revised to 7 percent (OMB 1992), and the after-tax rate of 3 percent was added even more recently (Lew 2000; OMB 2003). A similar guideline was adopted by Canada which recommends using a real rate of 8 percent for public cost-benefit analysis when funds are primarily extracted from capital market (TBS 2007). This rate is estimated to be a weighted average of costs of funds from three sources – return of capital, post-tax return on savings, and marginal cost of foreign borrowing. When consumer consumption is involved, a rate of 3 percent is suggested.

### 2.1 The descriptive approach and challenges

This paper speaks to the descriptive approach. This idea that public investments ought to provide the same return as observed private investments has a long history in economic thought (Harberger 1972; Lind 1982, 1990). Put simply, why invest in public projects that provide a lower return than private alternatives? Moreover, if the public project is displacing private investment, the notion is even more compelling as the opportunity cost of the forgone investment. Arguably, most public projects or government regulation involve up-front capital investment via either government spending or private dollars.

One problem for the descriptive approach arises because, when thinking about private investment alternatives, observed returns differ across different types of private investments. Uncertainty about the return to various investments, coupled with correlation between a specific investment's returns and the broader market, has long been recognized as an explanation for such differences. The capital asset pricing model (CAPM) has been a central feature in the finance literature, positing a risk premium based on an investment returns' volatility and correlation with the market (Fama 1977; Dixit and Williamson 1989; van Ewijk and Tang 2003; Hultkrantz, Krüger, and Mantalos 2014). This approach has in turn been applied to public investments (Gollier 2013). However, the U.S. government has largely ignored uncertainty and correlation among investment returns, focusing on "best" estimates of costs and benefits and a single, average estimate of the rate of return to private investment.[6] A notable exception is the

[6] An interesting question for future work would be to consider how and when such correlation could be practically incorporated in government CBA.

social cost of carbon dioxide estimates, that posit alternative discount rates based on uncertain outcomes and possible correlation with market returns (IWG 2010).

Distinct from the range of private sector returns associated with different risk profiles, a second complication arises when a public project is viewed through the eyes of consumers. Economists generally look to households and revealed preference as the ultimate arbiters of welfare value. Household preferences should be revealed by the intertemporal prices that they face, so the relevant question should be the rate of interest available for household savings. In a competitive capital market without tax distortions, a household's return to savings and the return on capital investment are both equal to the same rate.

In practice, however, taxes on the income from capital drive a wedge between investment returns and the returns experienced by consumers (Baumol 1968). This can lead us to ask, what kind of return does a household require to be better off? A consumer perspective would be particularly compelling for public projects that take away household consumption in one period and pay it back in another.

This second challenge for the descriptive approach is to decide, which is correct, the perspective of households or of firms? Recognizing the lack of a clear answer, government policy changed in 2000 to provide equal prominence to both a consumer and investment rate—3 and 7 percent, respectively (Lew 2000; OMB 2003).

Without knowing which perspective is correct, it is appealing to imagine that the correct CBA lies between those computed using these two discount rates (or, alternatively, between some other pair of defined discount rates). As it turns out, that is precisely the case. The theoretical underpinnings of this practice date back to Marglin (1963a, 1963b), Harberger (1972), and Sjaastad and Wisecarver (1977), who all argue that the correct discount rate lies between the consumer and investment rates of interest. Here, we briefly review the Harberger and Sjaastad and Wisecarver approaches, which provide two different framings that yield the same result. As we will see, however, these cases require quite strong assumptions and, weakening the assumptions, the bounding rates become much larger—unless we begin to examine longer time horizons.

### *2.2 Opportunity cost versus shadow price of capital approach*

Among the different approaches, Harberger's 1972 intuition and the opportunity cost approach perhaps has had the largest influence on public policy guidelines. The Harberger approach derives a weighted average discount rate with a simple partial equilibrium, in which the distortion between consumption and investment interest rates is attributed to taxes on capital. A government project or policy drives up demand for investment goods thus leading to displaced private investment and reduced consumption (i.e. by the same amount of increased savings). In the partial equilibrium model, the opportunity cost of capital for public projects $\rho_h$ can be derived graphically[7].

$$\rho_h = \frac{\Delta C}{\Delta C + \Delta I} r_c + \frac{\Delta I}{\Delta C + \Delta I} r_i, \tag{1}$$

where $\Delta C$ and $\Delta I$ are the share of reduced consumption and private investment from the public investment. Equation (1) indicates that the required return on the public project is the weighted average of the consumer rate $r_c$ and investment rate $r_i$ – the opportunity cost.

[7] See Appendix B for details.

Harberger's partial equilibrium analysis has been derived in a general equilibrium context by Sandmo and Drèze (1971) and Drèze (1974) for both the case of a two-period economy and the case of an infinite horizon where investments (public and private) yield a perpetual return. This approach views the rates of return as the relevant prices associated with investment and consumption displaced by a public investment. Implicit in this approach is that public and private investment have similar patterns of future returns over time (and, more specifically, as either a perpetuity or entirely in the next, final period).

An alternative approach is to focus on the implied consumption impact of any action and then to discount at the consumer rate of interest. That is, any investment effects are first translated into future consumption impacts, then discounted back to the period of the investment effect using the consumer rate. This places consumer valuation at the center of the analysis (Marglin 1963a, 1963b; Dasgupta, Marglin, and Sen 1972; Bradford 1975). This generally leads one dollar of investment in any period to be valued more than one dollar of consumption in that period when there is a tax distortion. This ratio is referred to as the shadow price of capital. More generally, shadow prices are appropriate for either unpriced goods that affect household utility or, as in the case of investment, priced goods where the price is distorted by taxes, regulation, or market failures. The shadow price of such a good in period $t$ reflects the equivalent dollar change in period $t$ consumption from a dollar change in that good at the established equilibrium. Once we have consumption-equivalent changes in each period, the overall net present value (NPV) is computed using a consumption rate of interest.

### 2.3 A general model to reconcile the two approaches

We would expect the opportunity cost approach and shadow price of capital approach to deliver the same result when assumptions about the pattern of investment returns are the same. Sjaastad and Wisecarver (1977) show precisely this result for both aforementioned cases related to the opportunity cost approach: a two-period economy and a perpetual stream of constant benefits. We now develop those cases on our way to the most general possible model.

The first case essentially follows Harberger. Assuming the world exists in two periods, we can write the NPV of a project that yields future benefits $B$ for each dollar invested today as

$$NPV = -\theta_0 + \frac{B}{(1 + r_c)}, \tag{2}$$

where $r_c$ is the consumption rate of interest and $\theta_0$ is the social opportunity cost per government dollar spent today in terms of household consumption:

$$\theta_0 = \alpha \cdot v + (1 - \alpha) \cdot 1. \tag{3}$$

In this definition, $\alpha$ is the share of today's cost displacing capital and $(1 - \alpha)$ is the share of displacing consumption. The shadow price of capital is denoted by $v$. The parameter $\theta_0$ is therefore bounded by 1 and $v$. Without a future beyond the second period, it is easy to figure out the shadow price of a capital investment with return $r_i$ as (4) in a two-period model,

$$v_{2p} = \frac{1 + r_i}{1 + r_c}. \tag{4}$$

That is, the value of the investment today is the gross return, including return of principal, $1 + r_i$ valued in terms of today's consumption based on the consumer discount rate $r_c$. Here, we use the subscript "*2p*" to remind us this is the shadow price in a two-period model. If we rewrite the *NPV* in terms of "project dollars" by dividing by $\theta_0$, we have

$$\begin{aligned} NPV' &= -1 + \frac{B}{\theta_0(1 + r_c)} = -1 + \frac{B}{1 + (\alpha r_i + (1 - \alpha) r_c)} \\ &= -1 + \frac{B}{1 + \rho_h}. \end{aligned} \tag{5}$$

This yields the Harberger result that the correct social discount rate, once we roll shadow price effects into the discount rate, is a weighted average of the consumer and investment rates.

For the second case, consider a public project or policy yielding a perpetual stream of constant benefits, $B$, every period in the future. We can again define $NPV$ as

$$NPV = -\theta_0 + \sum_{t=1}^{\infty} \frac{B}{(1+r_c)^t}. \tag{6}$$

And we can again rewrite in terms of project dollars by slightly rearranging,

$$\begin{aligned} NPV = -\theta_0 + \sum_{t=1}^{\infty} \frac{B}{(1+r_c)^t} = -\theta_0 + \frac{B}{r_C} = \theta_0 \left(-1 + \frac{B}{r_c\theta_0}\right) \\ = \theta_0 \cdot NPV', \end{aligned} \tag{7}$$

where

$$NPV' = -1 + \sum_{t=1}^{\infty} \frac{B}{(1+r_c\theta_0)^t}. \tag{8}$$

As in the two-period case, a key parameter is $\theta_0$ and, in turn, the shadow price of capital $v$ that defines $\theta_0$. If we further assume each dollar of capital investment has a perpetual return of $r_i$ dollars of household consumption in each future period, as first proposed by Marglin, it is straightforward to compute the shadow price:

$$v_m = \sum_{t=1}^{\infty} \frac{r_i}{(1+r_c)^t} = \frac{r_i}{r_c}. \tag{9}$$

Here, we use the subscript "*m*" to denote Marglin's shadow price (we will return to the more general question of shadow prices in section 4). We can substitute this shadow price of capital $v_m$ into $\theta_0$, and $\theta_0$ into (8), to yield

$$NPV' = -1 + \sum_{t=1}^{\infty} \frac{B}{\left(1 + (\alpha r_i + (1-\alpha) r_c)\right)^t}$$
$$= -1 + \sum_{t=1}^{\infty} \frac{B}{(1+\rho_h)^t}. \tag{10}$$

where $\rho_h$ is the Harberger discount rate defined in (1). Again, we have the effective social discount rate equal to a weighted average of the investment and consumer rates.

This result, and the discussion in Sjaastad and Wisecarver more generally, emphasizes how the opportunity cost of capital and shadow price of capital approaches can yield the same result. Both can support the idea that the social discount rate equals a weighted average of the investment and consumption rates of interest. With the assumption of costs up front and benefits in the future, already implicit in these derivations, a CBA based on the investment and consumer rates provides bounding values.

We note that this result supports the recommended approach followed by the U.S. government since 2000. It also highlights the rather strong assumptions that are required. In particular, benefits either are repaid in a second, final period[8] or are constant and perpetual. Our next step is relax these assumptions, beginning with work by Bradford (1975).

Here, we also briefly note a second, subtle but equally important, assumption lurking in Equations (2) and (6). While much attention focuses on whether costs in the first period fall on either consumption or investment, benefits

[8] As shown in Bradford (1975), the key assumption is that the distortion between the consumer and investment rates vanishes in the second period and the shadow price of capital equals one.

are assumed to accrue entirely in the form of consumption. The two-period model has no investment in the second, final period, and the assumption is unavoidable. This is more evident as an assumption in the perpetuity models developed by Sjaastad and Wisecarver, Marglin, and Drèze. If we relax the assumption that benefits accrue entirely to consumers,[9] the range of discount rates becomes centered on the consumer interest rate, rather than having that rate as a lower bound. Bradford takes this approach, allowing both costs and benefits to count as consumption or investment.

### *2.4 Shadow price of capital, generalization over two periods*

Following the idea in Bradford (1975), let $\beta$ be the share of benefits accruing to private investment in a second, but not final, period. The parameter $\theta_1 = \beta v + (1 - \beta)$ is the consumption equivalent of one dollar of benefits, averaged across the share accruing to consumption and the share accruing to capital. Here, we assume $v$ is also the shadow price of capital in the second period.[10] For a dollar of investment yielding total benefits $B$ in the next period, we now generalize (2)

$$NPV = -\theta_0 + \frac{\theta_1 B}{(1 + r_c)}. \tag{11}$$

As before, we can rewrite this NPV in terms of project dollars,

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[9] In the case of climate change, it is easy to imagine that many mitigation benefits will accure to investment—damages from storms and flooding, increased electricity capacity for cooling, etc.

[10] Unlike Bradford, we assume the same shadow price in both periods. This is unimportant for our ultimate goal, which is to consider the consequences of extreme (largest and smallest) values of the ratio $\theta_0/\theta_1$, equalling $v$ and $1/v$, respectively, given $v \geq 1$, and determined by the largest value of $v$ in both cases.

$$NPV' = \frac{NPV}{\theta_0} = -1 + \frac{\left(\frac{\theta_1}{\theta_0}\right) B}{(1 + r_c)} = -1 + \frac{B}{1 + \left((1 + r_c)\frac{\theta_0}{\theta_1} - 1\right)} \quad (12)$$

$$= -1 + \frac{B}{1 + \rho_b},$$

where $\rho_b$ is Bradford's discount rate,

$$\rho_b = (1 + r_c)\frac{\theta_0}{\theta_1} - 1 \quad (13)$$

Given both $\theta_0$ and $\theta_1$ are bounded by the shadow price $v$, e.g., $\theta \in [1, v]$, the range of potential values is quite large as illustrated in Figure 1 for $r_c = 3$ percent. When $\theta_0/\theta_1$ ranges from 2/3 to 3/2, e.g., a bounding shadow price of 1.5, the social discount rate varies from –30 to +60 percent. This bounding shadow price reflects our discussion in section 4 based on a long-run growth equilibrium.

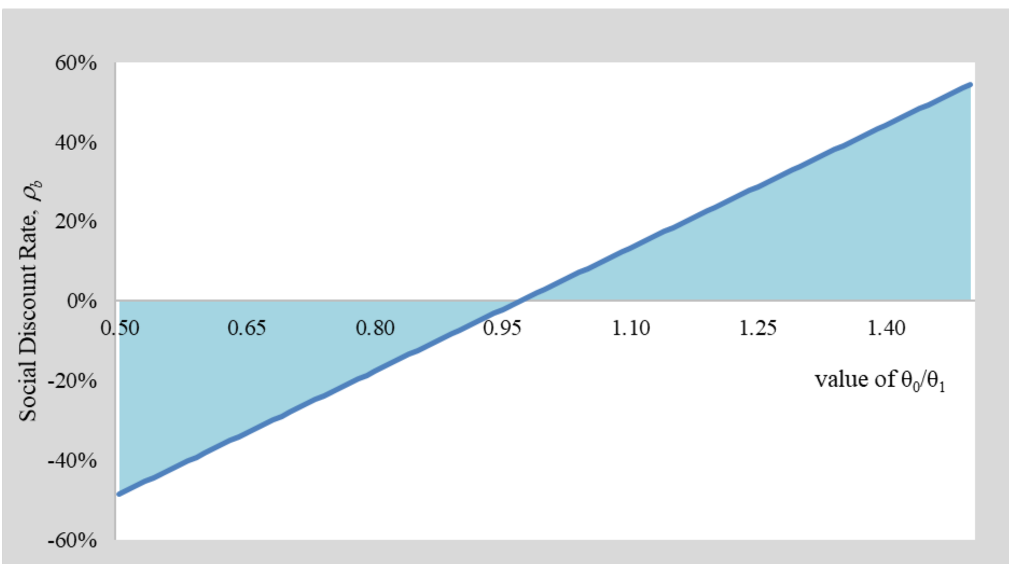


Figure 1 Values of social discount rate in Bradford's two-period model versus the ratio cost and benefit prices, $\theta_0/\theta_1$, when $r_c = 3$ percent

Bradford himself concludes that quite a wide range of social discount rates are possible for benefits over a one-year horizon.

The possibility of such wide ranging discount rates might appear somewhat hopeless as a practical matter. However, this is the relevant range for benefits just one year in the future embedded in a much longer (perhaps infinite) horizon model. Over such a short horizon, the differential welfare effects caused by displacing investment versus consumption dwarf ordinary discounting effects. The same is not true when we consider benefits arising further in the future, a topic to which we now turn.

## 3. A multiperiod model for the social discount rate

Consider a public investment project or regulatory program where one dollar of cost is expended at the beginning of the initial period, but now an arbitrary time series of benefits follows in future periods, $\{B_t, t = 1,2, \ldots, T\}$. The multiperiod cost–benefit flow is summarized in Table 1.

Note that this generalized problem nests the previous examples where $B_t = B$ and either $T = 1$ or $T \rightarrow \infty$. Following the previous examples, we are looking for the social discount rate that converts future benefits into equivalent dollars of current costs, allowing the share of impacts on investment and consumption to vary. As before, we begin by considering the $NPV$ in terms of household

Table 1. Generalized Benefit Stream from a Dollar of Project Cost

| Period | t = 0 | t = 1 | t = 2 | … | t = T |
|---|---|---|---|---|---|
| Project costs vs. (net) benefits in terms of undiscounted project dollars | $-1$ | $B_1$ | $B_2$ | … | $B_T$ |

consumption of a single project dollar spent, a modified version of (2), (6), and (11).

$$NPV = -\theta_0 + \sum_{t=1}^{T} \frac{\theta_1 B_t}{(1+r_c)^t} \tag{14}$$

As before, $\theta_0 = \alpha v + (1-\alpha)$ is the consumption-equivalent cost of an initial project dollar spent based on a weighted average of the shadow price of capital and the (numeraire) price of consumption, and $\theta_1 = \beta v + (1-\beta)$ is the same conversion for future benefits. Here, we assume without loss of generality that all benefits accrue to household consumption and capital in the same way over time.[11]

We can then construct the alternative $NPV'$ in terms of dollars of initial project cost:

$$\begin{aligned} NPV' = \frac{NPV}{\theta_0} &= -1 + \sum_{t=1}^{T} \frac{\left(\frac{\theta_1}{\theta_0}\right) B_t}{(1+r_c)^t} \\ &= -1 + \sum_{t=1}^{T} \frac{B_t}{\left(1 + \left((1+r_c)\left(\frac{\theta_0}{\theta_1}\right)^{\frac{1}{t}} - 1\right)\right)^t}. \end{aligned} \tag{15}$$

So, the implied social discount rate $\rho_t$ at any point in time is given by

$$\rho_t = (1+r_c)\left(\frac{\theta_0}{\theta_1}\right)^{\frac{1}{t}} - 1. \tag{16}$$

[11] As noted in footnote 10, our ultimate focus is on the range of social discount rates; this will be determined by extreme assumptions about how costs versus benefits accrue. The most extreme assumptions will be that costs accrue to capital and benefits to household consumption, and vice versa.

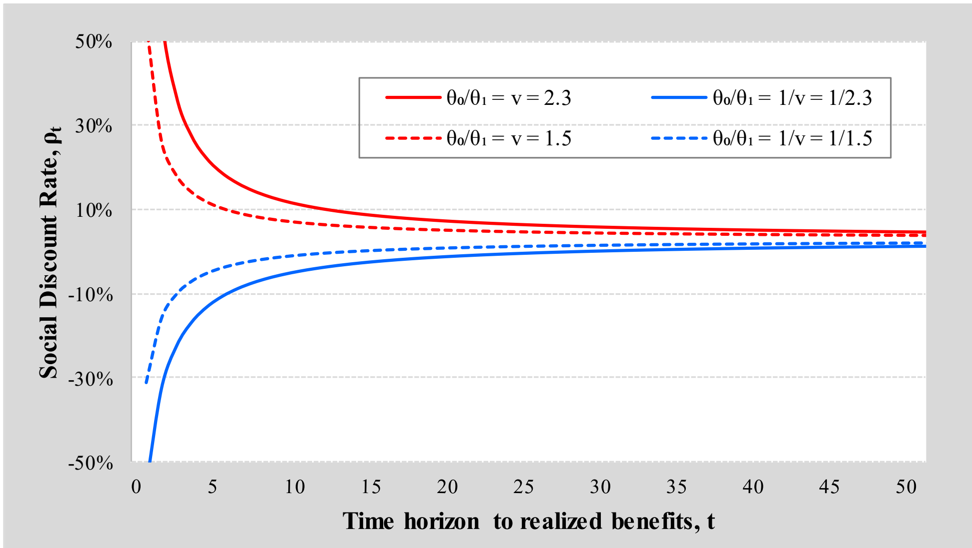


Figure 2 Range of possible social discount rates, versus time horizon, based on a consumer rate of 3 percent, an investment rate of 7 percent, and alternate values of the shadow price of capital ($v$ = 2.3, 1.5).

This definition (16) for social discount rate as a function of horizon $t$ is one of our two main results. We can make three immediate observations about (16). First, using the shadow-price approach, the appropriate social discount rate depends on how costs and benefits accrue with respect to capital and consumption, captured by $\theta_0/\theta_1$, as well as the underlying shadow price $v$ used to define $\theta_0$ and $\theta_1$. It has a form similar to the Bradford rate $\rho_b$ in (13), except the expression $\theta_0/\theta_1$ is raised to the power $1/t$. Second, following from this observation, the social discount rate varies over time but it matches $\rho_b$ when $t = 1$.

Third, and most importantly, as $t \to \infty$, the social discount rate converges to the consumption interest rate. [12] Figure 2 shows how the range of possible social discount rates arising from different values of $(\theta_0/\theta_1)^{-t}$ varies based on $t$ and two underlying shadow price $v$. For example, after 50 years and assuming $v$ = 2.3, the range of social discount rates is 1.3 to 4.8 percent. This result is important because it provides a strong argument against the idea that it is appropriate to use a rate as high as 7 percent as we discount benefits further in the future. This is true even when costs displace investment and even over horizons as short as a few decades.

If we want to define a single social discount rate that generates the same $NPV'$ for the entire pattern of indicated benefits and a particular ratio $\theta_0/\theta_1$, we can find $\rho^*$ implicitly defined by the following equation,

$$\sum_{t=1}^{T} \frac{B_t}{(1+\rho^*)^t} = \sum_{t=1}^{T} \frac{B_t}{(1+\rho_t)^t} = \sum_{t=1}^{T} \frac{B_t}{\left(1+\left((1+r_c)\left(\frac{\theta_0}{\theta_1}\right)^{\frac{1}{t}}-1\right)\right)^t}. \tag{17}$$

[12] This result appears similar to a result in Little and Mirrlees (p. 283, 1974) and Squire and van der Tak (p. 142, 1975). However, these authors explicitly ignore the (our) developed country context where capital income is taxes (footnote 1, page 285, Little and Mirrlees 1974). They have in mind a developing country where capital markets fail to equate investment demand and savings supply through the interest rate. Instead, the return to investment exceeds the consumer rate of interest. By construction, the shadow price of investment (in terms of consumption) must fall over time. This does not necessarily lead the interest rates to converge. Rather, it will prioritize projects that pass a CBA even when the shadow price of capital is high; other projects will wait. If the rates converge in their model, it is because capital markets begin to function (and absent taxes on capital income).

Equation (17) is our second key result and provides two more observations. First, the appropriate constant social discount rate generally depends on the pattern of benefits, $\{B_t, t = 1, \ldots, T\}$. Intuitively, for a given $\theta_0/\theta_1$, $\rho^*$ will lie somewhere in the range $(\rho_1, \rho_T)$, depending in part on the temporal distribution of benefits. In the special case of constant perpetuity of benefits (i.e. $B_t = B, T \to \infty$), we can show $\rho^* = (\theta_0/\theta_1)r_c$. This agrees with the previous result, e.g., when we use the Marglin shadow price $v_m = r_i/r_c$ and assume benefits accrue as consumption so $\theta_1 = 1$, we have $\rho^* = \alpha r_c + (1-\alpha)r_i$.

Second, regardless of the benefit pattern, the highest appropriate rate, based on uncertainty about $\theta_0/\theta_1$, will arise when we fix $\theta_0/\theta_1$ at its maximum (e.g., the greatest emphasis on current period costs), and the lowest appropriate rate will arise when we fix $\theta_0/\theta_1$ at its minimum. These values are given by $v$ and $1/v$, respectively.[13]

As noted, the previous literature focused on more restrictive assumptions. Results showing the social discount rate to be a weighted average of the investment and consumer rates hinged on a two-period model or a perpetuity. Later, Bradford (1975) assumed a one-time benefit payment in the second period of an otherwise multiperiod model. We can now see these as three special cases in our framework. Moreover, we consider our model applied to the case of a single benefit at a distant horizon as a fourth special case. These are summarized in Table 2.

[13] That $\theta_0/\theta_1 = v$ implies the case where 100 percent of the social costs today displaces private capital and the entire future benefits are allocated to household consumption; the exact opposite is true for $\theta_0/\theta_1 = 1/v$.

Table 2. Discount Rate Defined in Four Special Cases of Equation (17)

| Assumptions | **Case I: Two-period model** (Harberger; Sandmo and Drèze; Sjaastad and Wisecarver; Bradford) | **Case II: Benefits paid as perpetuity** (Drèze; Sjaastad and Wisecarver) | **Case III: Benefits paid at the beginning** (Bradford) | **Case IV: Benefits paid in the distant future** |
|---|---|---|---|---|
| **Benefits stream** | $T = 1, B_1 = B$ | $B_t = B,$ $\forall t = 1, \dots, \infty$ | $B_1 = B$ $B_t = 0, \forall t =$ $2, \dots, T$ | $B_t = 0,$ $\forall t = 1, \dots, T-1$ $B_T = B$ |
| **Constraint on ratio of cost–benefit shadow price** | Implied by two-period model; $v = \dfrac{1 + r_i}{1 + r_c}$ and $\theta_1 = 1$ | Yes, $v = r_i / r_c$ and $\theta_1 = 1$ | No, $\dfrac{1}{v} < \dfrac{\theta_0}{\theta_1} < v$ | No, $\dfrac{1}{v} < \dfrac{\theta_0}{\theta_1} < v$ |
| **Social discount rate** | $\rho^* = \rho_h$ $= (1 + r_c)\theta_0 - 1$ $= \alpha r_i + (1 - \alpha) r_c$ | $\rho^* = \rho_h$ $= r_c \theta_0$ $= \alpha r_i + (1 - \alpha) r_c$ | $\rho^* = \rho_b$ $= (1 + r_c)\left(\dfrac{\theta_0}{\theta_1}\right) - 1$ | $\rho^* = \rho_T$ $= (1 + r_c)\left(\dfrac{\theta_0}{\theta_1}\right)^{\frac{1}{T}} - 1$ $\rho^* \xrightarrow{T \to \infty} r_c$ |

Table 2 highlights that we can obtain all the previous results regarding the social discount rate from equation (17). The simple result from either a two-period or perpetuity model of benefits, the complex results from Bradford, and the convergent results in our paper—these are all special cases of that equation. They arise by making a specific assumption about the pattern of benefits and/or that benefits accrue only to consumption (and not to investment).

We view Case IV in Table 2 as a powerful result; namely, the social discount rate converges over long horizons to the consumption rate of interest. However, the speed of convergence will be determined by the magnitude of the shadow price and the range of possible values for $\theta_0/\theta_1$. For a shadow price of capital close to 1, the range of the social discount rate quickly narrows down to the rate of return on consumption, although the process is slower with a larger shadow-price value. As illustrated by Figure 2, when the shadow price of capital is 1.5, the upper bound of social discount rate falls below seven percent after a decade and the possible range shrinks to 4 to 2 percent after three decades. For the high shadow price case (i.e. $v = 2.3$), on the other hand, it takes twice the number of years. In the next section, we consider two approaches to specifying the shadow price of capital and, in turn, estimating the range of possible social discount rates at each horizon.

## 4. The shadow price of capital

We consider two approaches to estimate the shadow price of capital in our calculations. Both approaches start with consideration of a small change in capital today, trace the impacts over time, and then compute the NPV of the resulting changes in consumption at the consumer interest rate. The key parameter in the resulting expression is the savings rate. In one approach, we consider a reasonable way to bound the savings rate. In the other, we calculate the savings rate assuming an underlying Ramsey growth model.

We have already noted the general approach in Marglin (1963a, 1963b) and Sjaastad and Wisecarver (1977). They assumed any change in capital generated a perpetuity of flows at rate $r_i$, all accruing to consumption. In turn, they showed that $v = r_i/r_c$. However, a more careful consideration requires thinking about whether flows accrue to consumption or investment, what to assume about tax revenue, and the role of depreciation (Bradford 1975; Mendelsohn 1981, 1983; Lind 1982; Lyon 1990).

Following Lyon (1990), we specify a constant savings rate $s$ from incremental gross capital returns, equal to net returns $r_i$ plus depreciation $\mu$. We then think about stepping through the sequence of events after an incremental change in capital. A change in private investment $\Delta K_t$ in period $t$ produces a return equal to the gross rate of return from capital before depreciation of $\Delta Y_{t+1} = (r_i + \mu)\Delta K_t$ in period $t + 1$. This return will be divided between reinvestment, $\Delta Z_{t+1} = s(r_i + \mu)\Delta K_t$, taxes on the net capital return, $\tau r_i \Delta K_t$, and direct consumption, $\Delta Y_{t+1} - \Delta Z_{t+1} - \tau r_i \Delta K_t$, in period $t + 1$. Capital stock in the next period will increase by the amount reinvested net of depreciation.

As a rule of thumb in welfare analysis, the change in taxes is assumed to generate current-period government spending that increases current-period consumption dollar for dollar.[14] Thus the total flow to household consumption is $\Delta Y_{t+1} - \Delta Z_{t+1}$.

The dynamics of $\Delta K_t$-induced flows are illustrated in Figure 3 for the first period after an incremental change in the capital stock. The flow to consumption (including the benefit from government spending) is given by $\Delta C_{t+1} = (1-s)(r_i+\mu)\Delta K_t$. Tracing out the next period based on the new $\Delta K_{t+1} = \big(s(r_i+\mu) + (1-\mu)\big)\Delta K_t$ yields

$$\begin{aligned} \Delta C_{t+2} &= (1-s)(r_i+\mu)\Delta K_{t+1} \\ &= (1-s)(r_i+\mu)\big(s(r_i+\mu)+(1-\mu)\big)\Delta K_t. \end{aligned} \tag{18}$$

Repeating this every period and constructing the discounted sum of consumption changes at the consumption rate of interest yields (see Appendix C for additional details)

$$v = \frac{(1-s)(r_i+\mu)}{r_c+\mu-s(r_i+\mu)}. \tag{19}$$

One way to understand (19) is to consider the numerator and denominator. For a given capital stock deviation in any period, the numerator gives the portion flowing to household welfare. That is, it equals the gross return $r_i+\mu$, times the nonreinvested fraction $1-s$. The denominator then reflects the adjustments

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[14] One could assume tax revenues flow at least partly into government projects/investment that then provide consumer benefits in future periods, that government funds exceed the value of consumption, or both. This is easily accommodated in the current framework by considering that each dollar of such government investment will have an equivalent current-period consumption value greater than one—what is referred to as the "marginal value of public funds." Allowing this shadow price to be $v_G$, the numerator in (19) would be replaced by $(1-s)(r_i+\mu)+(v_G-1)\tau r_i$. That is, each period has a "bonus" from tax revenue being diverted into more valuable public projects. Based on recent estimates of the marginal value of public funds (Hendren 2014), this does not significantly alter our results, raising our preferred estimate of $v$ from 1.5 to 1.7.

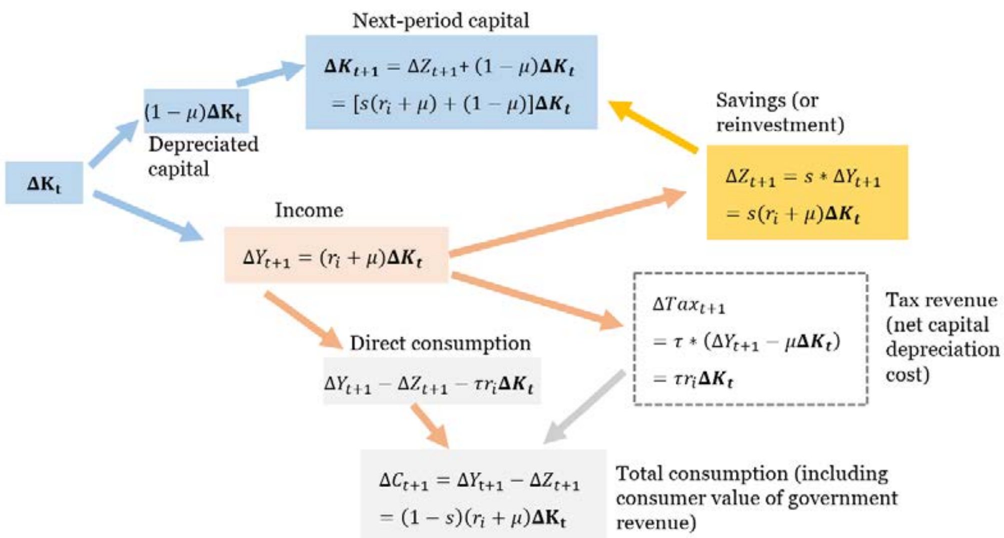


Figure 3 Dynamic flows of an incremental change in capital stock in period $t$

necessary to value an initial capital stock deviation over time. The consumption discount rate governs the household valuation, lowering the value over time. The term $s(r_i + \mu) - \mu$ governs the change in the actual capital stock deviation each period. The deviation increases by the reinvestment each period $s(r_i + \mu)$ and decreases by the amount of depreciation μ.

Note that if $r_i = r_c$ and there is no distortion between the value of capital and consumption, Expression (19) for $v$ simplifies to $v = 1$.

Expression (19) replaces the problem of identifying the shadow price of capital with the problem of identifying values for the savings rate and capital depreciation (in addition to the consumption and investment interest rates). One approach is to think about possible bounding values. In both Mendelsohn (1983) and Lyon (1990), they consider the condition necessary for the shadow price of capital to be finite. This amounts to a condition that consumption grows more slowly than the consumption interest rate.[15]

However, we would argue that a more reasonable condition for a steady-state economy is to have a nonexplosive capital stock (i.e., $\Delta K_{t+1} \leq \Delta K_t$). That is, if one adds or subtracts a little capital from the economy, it goes back to its original equilibrium or path. This implies that reinvestment $s(r_i + \mu)\Delta K_t$ should be less than depreciation, $\mu \Delta K_t$, or

$$s \leq \frac{\mu}{r_i + \mu}. \tag{20}$$

Using this expression we can bound the numerator in (19), $(1 - s)(r_i + \mu) \leq r_i$, and the denominator, $r_c + \mu - s(r_i + \mu) \geq r_c$. So, the shadow price of capital at steady state is less than the ratio of the two interest rates. We also know this ratio equals 1 if no distortion from taxes is imposed to the market ($r_i = r_c$). Putting these together,

$$1 \leq v \leq \frac{r_i}{r_c}. \tag{21}$$

That is, we have replicated Marglin's (and others) shadow price of capital as a maximum shadow price under what we believe are more general/appropriate conditions.

If we focus on the government's values of 7 and 3 percent for $r_i$ and $r_c$, respectively, we have the upper bound for $v$ being 7/3. It is this value of the shadow price that motivated the solid blue and red lines in Figure 2. Based on

[15] This condition is specified in (C8) of Appendix C.

those values, the range of the social discount rate becomes 1.2–4.8 percent after 50 years. It narrows to 2.5–3.5 percent after 175 years.

Instead of looking for bounding values, our second approach turns to a structural model to see how the shadow price relates the savings rate to underlying technology or preference parameters and then to specify those parameters. In Appendix D, we use the Ramsey growth model with a Cobb-Douglas production function to derive the steady-state savings rate ($s^*$) with a tax distortion τ on the labor and capital income (such that $r_c = (1 - \tau)r_i$). That yields

$$s^* = \frac{(\mu + g + n)a}{\mu + r_i}. \tag{22}$$

Here, $g$ is the growth rate of labor-augmenting productivity, $n$ is the population growth rate, and $a$ is the capital-output elasticity (capital share) in the production function. As before, μ is the depreciation rate of capital and $r_i = r_c/(1 - \tau)$ is the investment rate of interest, equal to the grossed-up consumption interest rate. In the Ramsey model, this consumption rate of interest is, in turn, related to pure time preference, utility curvature, and productivity growth, which we have subsumed into $r_c$. Choosing parameters from the literature (see Table D-1), along with $r_i$ = 7 percent and $r_c$ = 3 percent, yields $s \approx$ 23 percent from (22) and $v \approx 1.5$ from (19). The range of social discount rates at different future horizons based on this value of $v$ is indicated by the dashed line in Figure 2. In particular, the range of social discount rates narrows to 1.6–4.4 percent after 30 years and 2.1–3.8 percent after 50 years. After 100 years, the range is 2.6–3.4 percent.

## 5. Social discount rate for climate change

In section 3, we have shown that for public investment projects or regulatory analyses with long-term cost–benefit consequences, the social discount rate calculation contains less uncertainty when future benefits are mostly paid in

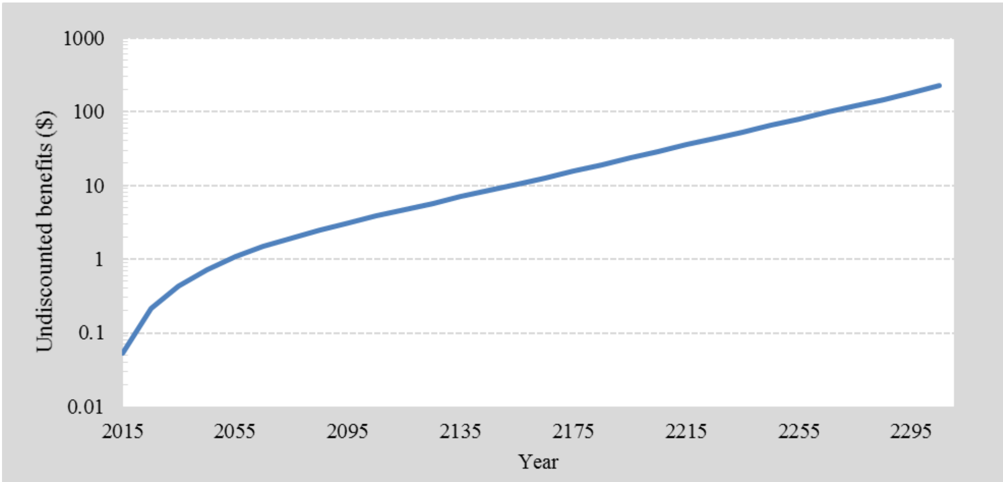

Figure 4 Undiscounted climate damages from one incremental metric ton of CO2 emitted in 2015 under a 2.2-percent economic growth scenario

the distant future. In the extreme (Case IV in Table 2), the effective discount rate converges to the consumption rate of return. This result is quite powerful, but the precise question about the appropriate range hinges on the shadow price of capital and the actual pattern of future benefits over time. Having discussed the shadow price of capital in section 4, we now turn to potential patterns of future benefits. In particular, we focus on climate change mitigation benefits.

Figure 4 illustrates an increasing sequence of undiscounted future climate damages from an incremental ton of $CO_2$ emitted in 2015 under a 2.2-percent economic growth scenario (NAS 2017). The benefits of avoiding these damages each year, discounted to the base year, 2015, would then be used to construct a benefit estimate for tons reduced in 2015, referred to as the social cost of carbon dioxide (SC-CO2).[16]

As noted earlier, each year in the future will generally have a different range of discount rates given by (16). However, can we plug this pattern of benefits, $\{B_t\}$, into Expression (17) to define a range of rates $\rho^*$ appropriate for the entire pattern of benefits over time. The only uncertainty defining those rates is the ratio $\theta_0/\theta_1$. As discussed in section 3, this ratio ranges from $1/v$ to $v$, depending on the distribution of costs and benefits over capital and consumption. In section 4, assuming the consumption and investment interest rates are 3 and 7 percent, we argued that the range of values for $v$ is between 1 and 7/3 ($\approx$ 2.33). For each value in this interval, we can solve for the upper and lower bound of $\rho^*$ using Equation (17). Figure 5 plots those bounding social discount rates over this range for the shadow price of capital $v$. At the extreme $v = 2.3$, the range is 2.3–4.0 percent. Based on a Ramsey model calculation, we suggested a preferred shadow price close to 1.5. Again, from Figure 5, we can see that this indicates a narrower range for the social discount rate of 2.6–3.4 percent when applied to climate change.

[16] The calculation would need to be repeated for each year of avoided emission in the CBA. That is, for each year *t* where emissions reductions occur, compute a sequence of undiscounted future climate damages from incremental emission in year *t* and then discount those damages back to year *t* to value emission reductions in that year.

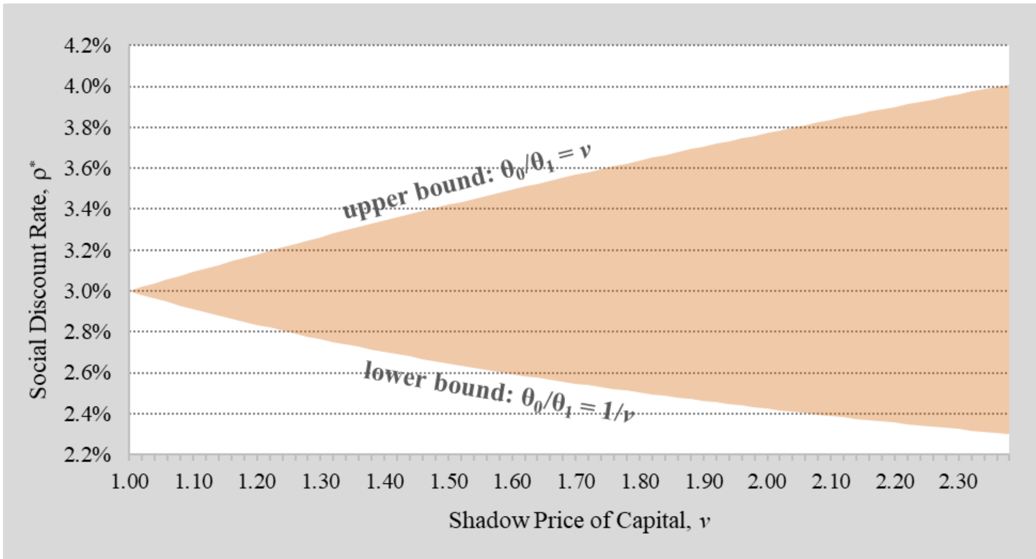


Figure 5 The social discount rate for climate damages is bounded around a narrow range around an assumed 3-percent consumption rate of interest. The value of $\rho^*$ falls approximately between 2.6 and 3.4 percent when $v = 1.5$ and between 2.3 and 4.0 percent when $v = 2.33$

Of course, finding the social discount rate for climate change mitigation benefits is not an end unto itself. The goal is to estimate the per ton benefit (and ultimately to conduct a complete CBA). Using the pattern of benefits in Figure 4 and a particular discount rate, we can compute the NPV of mitigation benefits from one ton of reduced emission. When we do this with the original three- and seven-percent rates, we find a social cost of $CO_2$ of $49 and $5.9 (in 2015$ per metric ton $CO_2$), respectively. The ratio, roughly 8:1, was noted in the introduction. If we instead use the preferred shadow price of capital and implied range of 2.6–3.4 percent for the social discount rate, we estimate the social cost of $CO_2$ as $77 and $34, respectively, or a ratio of about 2:1. This is still a rather wide range of potential benefits. However, we see that correct attention to the uncertain impacts on consumption and investment, and consequences for the social discount rate, does eliminate the low end of estimates suggested by recent government estimates (U.S. EPA 2017). Moreover, the estimates are now more centered around the three-percent estimate of $49.

## 6. Conclusions

For policy questions with significant consequences in the future, the choice of discount rate can be one of the most influential parameters in any cost-benefit analysis (CBA). As shown in the CBAs for the Clean Power Plan (CPP), the magnitude of forgone climate benefits is reduced by a factor of between six and nine when the discount rate is increased from three percent to seven percent (U.S. EPA 2017). The underlying basis for these discount rates has been divergent views about whether to use a consumption or investment rate of interest. With significant taxes on income, particularly capital income, there will be divergence between the rate of return on investment and the after-tax return on household savings.

The economic literature has tended to advocate using a consumption rate in conjunction with shadow prices to convert impacts on investment into consumption equivalents. However, absent clear guidance on how to implement this in practice, the government has resorted to using alternative consumption and investment rates as bounding cases, generally giving equal weight to both values. This approach follows early results by Marglin (1963a, 1963b), Harberger (1972), and Sjaastad and Wisecarver (1977), using a particular shadow price and allowing the share of costs accruing to investment to vary between 0 and 1. This result is correct when future benefits follow a particular pattern: they exist either in a simple two-period model or as a constant perpetuity. This result also assumes that benefits accrue entirely to consumption.

In this paper, we have demonstrated that the correct range of rates based on the shadow-price approach depends on the shadow price of capital *and* the temporal pattern of benefits. For example, if the shadow price of capital is our preferred value of 1.5, and 3 percent is the consumption rate of interest, then the range of appropriate social discount rates is 2.1–3.8 percent after 50 years. Over longer time horizons, this range converges to the consumer rate. More generally, the appropriate range of social discount rates will depend on the precise temporal pattern of benefits.

Our main result (Equation (17)) provides the formula relating the social discount rate to the pattern of benefits over time, the shadow price of capital, and consumption rate of interest. Applied to the NAS pattern of benefits from climate change and our preferred shadow price of capital, the appropriate discount rate is 2.6–3.4 percent. This, in turn, leads to estimates of the social cost of $CO_2$ of $77 and $34, respectively. This is much narrower than the range indicated by discount rates of three and seven percent.

Importantly, we believe this paper provides a strong caution against the investment interest rate as a benchmark for discounting in government CBAs for projects with long horizons, when benefits, like costs, can fall on consumption, investment, or both. Over long horizons, we have shown that the appropriate rate converges to the consumption rate, regardless of the shadow price and the incidence of consumption or investment. If uncertainty exists about whether benefits, as well as costs, accrue to investment or consumption, the appropriate social discount rate should be roughly centered on the consumption rate of interest.

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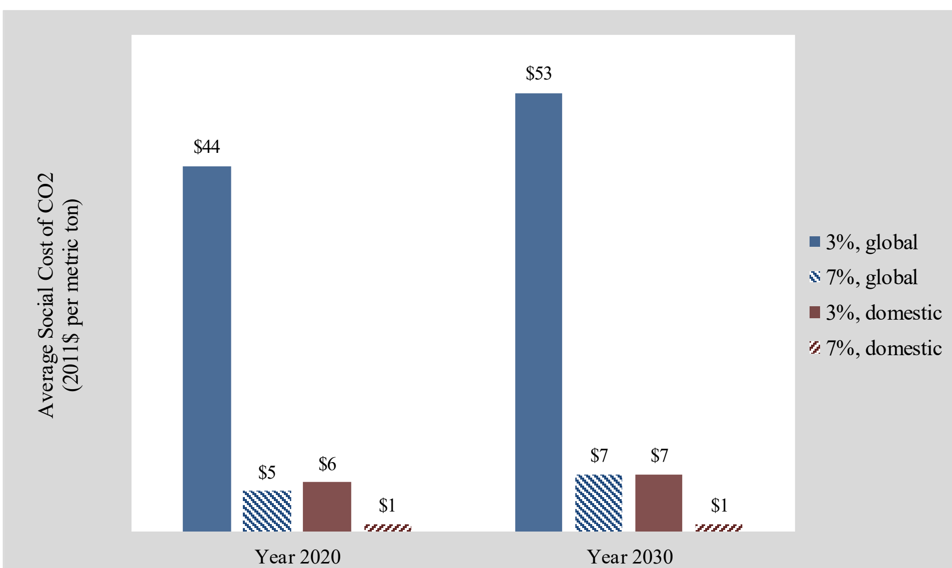
# For Online Publication

## Appendices

### A. Impact of discount rate choice on the cost–benefit analysis for repealing the Clean Power Plan

In October 2017, the Trump administration released a revised CBA of the Clean Power Plan as part of a regulatory impact analysis associated with the plan's proposed repeal. This 2017 CBA changed two key assumptions compared to the 2015 CBA. First, the revised CBA calculates climate damages (or forgone climate benefits) based on a U.S. domestic social cost of carbon dioxide (SC-$CO_2$) estimate rather than a global SC-$CO_2$. The domestic SC-$CO_2$ is approximated as




Source: Table 3-7, Appendix C.3 (U.S. EPA 2017), and Table 4-2 (U.S. EPA 2015). Numbers are rounded to the nearest integer.

Figure A-1. Interim SC-$CO_2$ estimates for 2020 and 2030 (in 2011 dollars per metric ton $CO_2$). Values are rounded to the nearest integer.

10–20 percent of the global values used in the 2015 analysis. As illustrated in Figure A-1, this brings down the average SC-$CO_2$ value by a factor of seven (U.S. EPA 2017). Second, although the 2015 CBA uses three percent as the central discount rate value, the 2017 CBA presents results for discount rates of three and seven percent without a central value. The SC-$CO_2$ estimates are about six to nine times higher based on three rather than seven percent (U.S. EPA 2017). Equally weighting these two rates, the estimate of the average global SC-$CO_2$ in year 2020 drops from $44 (based on three percent) to $24.5 (midpoint of estimates based on three and seven percent). The average domestic SC-$CO_2$ drops from $6 to $3. That is, the new discount rate approach lowers the effective SC-$CO_2$—and estimated climate benefits in the CBA—by 44 percent.

## B. Opportunity Cost of capital for public projects

The Harberger approach begins with a simple partial equilibrium that explains the distortion between the consumption and investment interest rates. That is, there is a supply schedule of savings $S(r_c)$ net of taxes on capital income and a demand for investment (public and private) gross of taxes on capital return $I(r_i) = I_{private}(r_i) + I_{public}$. Here, we assume public investment is fixed. The investment and consumption interest rates are related by the tax rate $\tau$; that is, $r_c = r_i(1 - \tau)$. An initial equilibrium is given by the intersection of the solid blue (investment) and red (savings) lines with $I = S = I_0$ in Figure B - 1.

We now consider how a government project or policy that increases demand for investment goods by a marginal amount, say $1, affects private investment and savings (with increases in savings reducing consumption by an equal amount)[17]. The outcome can be depicted graphically in Figure B - 1 by the equilibrium $I' = S = I_1$. Savings rise (and consumption declines) by $\Delta C$. Meanwhile, private investment declines by $\Delta I$, remembering that public investment demand (including the new additional dollar) is fixed. The opportunity cost of postponing consumption by one period is given by the light-shaded area $r_c \Delta C$, and the opportunity cost of postponing private investment is the dark-shaded area $r_i \Delta I$—the forgone surplus in the figure.[18] So long as the additional dollar of government-caused demand repays that amount (plus the $1 principal) in the next period, the economy maintains its current level of welfare. This leads to a weighted cost of capital,[19]

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[17] In this paper, dollars spent today on public investment are not narrowly defined as costs of a project generating financial returns but broadly refer to any public policy or regulations that lead to government spending, alter private investment decisions, or both.

[18] This is ignoring the slight difference in prices at the new equilibrium.

[19] See page 99 of Harberger (1972).

$$\rho_h = \frac{\Delta C}{\Delta C + \Delta I} r_c + \frac{\Delta I}{\Delta C + \Delta I} r_i,$$

Here, we let $\rho_h$ reflect the social discount rate defined by Harberger. That is, the required return on the public project is the weighted average of the consumer and investment interest rates that makes the economy whole—hence the notion of opportunity cost. Recalling our assumption that costs are up front and benefits flow in the future, the alternative use of the investment and consumer rates provides bounding values for the CBA without knowing exactly how consumption and investment are affected. The investment rate will maximally disfavor future benefits, and the consumer rate will maximally favor them.

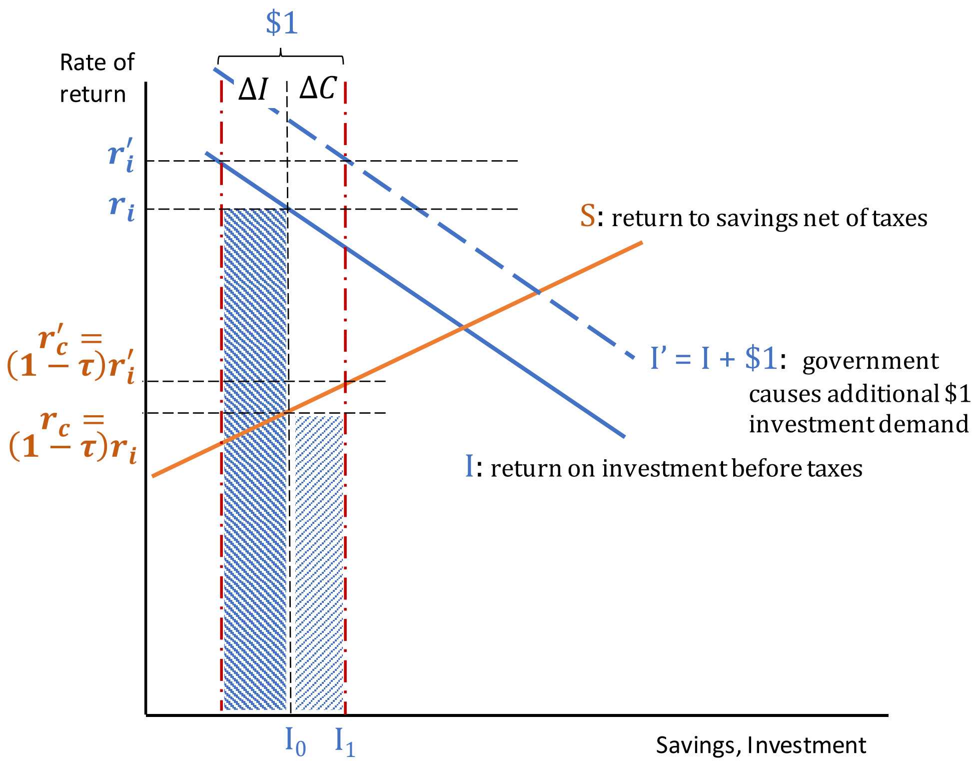


Figure B - 1 Partial equilibrium model of opportunity cost of capital for public projects

## C. Expression of the shadow price of capital

An incremental change in capital at the beginning of period $t$, $\Delta K_t$, will transfer to a stream of future capital and consumption. Figure 3 illustrates the dynamics. A change in capital $\Delta K_t$ in period $t$ will result in changes in both gross income $Y_{t+1}$ and savings $Z_{t+1}$ in the next period.

$$\Delta Y_{t+1} = (r_i + \mu)\Delta K_t \tag{C1}$$

$$\Delta Z_{t+1} = s \cdot \Delta Y_{t+1} = s(r_i + \mu)\Delta K_t \tag{C2}$$

The incremental change in next-period capital $K_{t+1}$ is the sum of direct changes in postdepreciation capital and indirect changes in savings.

$$\Delta K_{t+1} = \Delta Z_{t+1} + (1 - \mu)\Delta K_t = [s(r_i + \mu) + 1 - \mu]\Delta K_t \tag{C3}$$

The proportion of income that remains after savings is the dollars for next-period consumption.

$$\Delta C_{t+1} = \Delta \mathrm{Y}_{\mathrm{t+1}} - \Delta Z_{t+1} = (1 - s)(\mathrm{r_i} + \mu)\Delta K_t \tag{C4}$$

The shadow price of capital, $v_t$, is the present value of all future consumption changes after period $t$ discounted with the consumption rate of interest $r_c$. Hence, we can derive a difference equation for equilibrium state shadow price and savings:

$$v_t = \frac{(1 - s)(r_i + \mu)}{1 + r_c} + \frac{s(r_i + \mu) + (1 - \mu)}{1 + r_c} * v_{t+1} \tag{C5}$$

or equivalently the sum of consumption streams as a geometric sequence:

$$v_t = \frac{(1 - s)(\mu + r_i)}{1 - r_c} * [1 + \gamma + \gamma^2 + \gamma^3 + \cdots], \tag{C6}$$

where $\gamma = \frac{s(\mu+r_i)+1-\mu}{1+r_c}$. Therefore, at a steady state of the economy, the shadow price of capital ($v$) has the following relationship with savings, consumption rate of return, and pretax marginal rate of return to investment:

$$v = \frac{(1-s)(r_i+\mu)}{r_c+\mu-s(r_i+\mu)}. \tag{C7}$$

We can place some bounds on this expression by considering practical restrictions. For example, both Mendelsohn (1983) and Lyon (1990) focus on a condition for a less-than-infinity $v_t$; namely, the need to have a nonexplosive consumption flow. This requires the propensity of savings to be bounded by a depreciation-adjusted ratio of the two interest rates:

$$\gamma < 1 \rightarrow s < \frac{r_c+\mu}{r_i+\mu} \tag{C8}$$

However, we think a more reasonable condition for a steady-state economy is to have $\Delta K_{t+1} < \Delta K_t$ in (C3). This avoids a situation where a small perturbation leads to a permanent (or explosive) shift. Intuitively, savings out of gross income must be less than capital depreciation.

$$s \leq \frac{\mu}{r_i+\mu} \tag{C9}$$

This condition (C9) leads the shadow price of capital to be constrained by the ratio of interest rates.

$$1 \leq v \leq \frac{r_i}{r_c} \tag{C10}$$

## D. Steady-state shadow price in the Ramsey growth model

### *C-1. Ramsey model setup*

The Ramsey model setup follows the framework in Cass (1965). Aggregate output, $Y_t$, is produced with two inputs: labor $L_t$ and capital $K_t$. Let $A_t$ denote labor-augmenting productivity. The production function, $Y_t = F(K_t, A_t L_t)$, is assumed to have declining marginal product of capital and is homogeneous of degree 1 in capital and labor[20]. Both capital and labor are essential inputs for production.[21]

Productivity $A_t$ and population $L_t$ are assumed to grow exponentially at an exogenous rate, $g$ $(g > 0)$ and $n$ $(n > 0)$, respectively.

$$A_t = A_0 e^{gt} \tag{D1}$$

$$L_t = L_0 e^{nt} \tag{D2}$$

Variables are redefined by standardizing them with respect to $A_t L_t$. Standardized production output, $y_t$, can thus be written as a function of standardized capital, $k_t \overset{\text{def}}{=} \frac{K_t}{A_t L_t}$.

$$y_t = \frac{Y_t}{A_t L_t} = F\left(\frac{K_t}{A_t L_t}, 1\right) = F(k_t, 1) \overset{\text{def}}{=} f(k_t) \tag{D3}$$

Total production output $Y_t$ is allocated between consumption $C_t$, savings $Z_t$, and tax payment (with tax rate $\tau$). We treat depreciation ($\mu$) as a tax-deductible expense.

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[20] $F(\theta K_t, \theta A_t L_t) = \theta F(K_t, A_t L_t), \forall \theta > 0$

[21] Given $A_t > 0$, then $F(0, A_t L_t) = F(K_t, A_t * 0) = 0$.

$$Y_t = \tau Y_t - \tau\mu K_t + C_t + Z_t \tag{D4}$$

Similarly, consumption is also standardized by $A_t L_t$: $c_t \stackrel{\text{def}}{=} C_t/(A_t L_t)$ .

$$Z_t = A_t L_t \cdot [(1-\tau)f(k_t) + \tau\mu k_t - c_t] \tag{D5}$$

With capital depreciation ($\mu > 0$), gross investment involves net investment $\dot{K}$ and replacement investment $\mu K$. Equation (D6) holds because a net increase in gross capital comes from positive savings net out capital depreciation.

$$\dot{K} = Z_t - \mu K_t = A_t L_t \cdot [(1-\tau)(f(k_t) - \mu k_t) - c_t] \tag{D6}$$

Since

$$\dot{K} = \frac{d(A_t L_t k_t)}{dt} = \dot{A} L_t k_t + A_t \dot{L} k_t + A_t L_t \dot{k}\ , \tag{D7}$$

and, by assumption in (D1) and (D2), $\dot{A} = A_t g$, and $\dot{L} = L_t n$, we can derive the differential equation to describe how the standardized capital varies over time by combining these two equations. Namely,

$$\dot{k} = (1-\tau)(f(k_t) - \mu k_t) - c_t - (g+n)k_t. \tag{D8}$$

Turning to household preferences, we assume a time-invariant instantaneous utility function $U(.)$ that depends on per capita consumption $A_t c_t$. More specifically, assume $U$ is an isoelastic utility function.

$$U(x) = \begin{cases} \dfrac{x^{1-\eta}}{1-\eta}, & \eta > 0 \ and \ \eta \neq 1 \\ \ln(x), & \eta = 1 \end{cases} \tag{D9}$$

So, the marginal utility function is

$$U'(x) = x^{-\eta}. \tag{D10}$$

To find the market equilibrium under perfect competition, we solve the social planner's problem (D11)[22] to find the optimal growth path $\{(c_t, k_t): t \geq 0\}$ that maximizes social welfare. Social welfare is the aggregated utility of all consumers, where the discount rate of pure time preference is $\delta$ $(\delta > n)$.

$$\max_{c \geq 0} \int_0^{\infty} e^{-(\delta - n)t} U(A_t c_t)\, dt$$
$$\text{s.t. } \dot{k} = (1-\tau)(f(k_t) - \mu k_t) - c_t - (g+n)k_t \tag{D11}$$

*C-2. Estimate the steady-state shadow price*

We assume a solution $(c_t^*, k_t^*)$ to the necessary conditions for the social planner's problem (D11) that converges to a finite positive steady state $(c^*, k^*)$. The equilibrium conditions can be found by letting $\dot{c} = \dot{k} = 0$.

$$\dot{k} = 0 \rightarrow\ c^* = (1-\tau)(f(k^*) - \mu k^*) - (g+n)k^* \tag{D12}$$

$$\dot{c} = 0 \rightarrow\ \delta + g\eta = (1-\tau)(f'(k^*) - \mu) \tag{D13}$$

[22] Tax rate is fixed at $\tau$. Initial population stock is assumed to be $L_0 = 1$.

Table D-1. Parameters in the Ramsey Model for Numeric Estimation of the Shadow Price of Capital

| Parameter | Value in the numeric example | Source |
|---|---|---|
| Growth rate of productivity, $g$ | 2% | NAS (2017) |
| Growth rate of labor, $n$ | 1% | NAS (2017) |
| Capital depreciation rate, $\mu$ | 10% | Nordhaus (2017) |
| Output elasticity of capital, $a$ | 0.3 | Nordhaus (2017) |

Assuming Cobb-Douglas production, so $y_t = k_t^a$, these two equations can be solved for steady-state capital and consumption.

$$k^* = \left[\frac{\mu}{a} + \frac{\delta + g\eta}{a(1-\tau)}\right]^{\frac{1}{a-1}} = \left[\frac{\mu + r_i}{a}\right]^{\frac{1}{a-1}} \quad \text{(D14)}$$

$$c^* = (1-\tau)((k^*)^a - \mu k^*) - (g+n)k^* \quad \text{(D15)}$$

Here, we have simplified the expression for $k^*$ by recognizing that $\delta + g\eta$ is what we have been calling the consumer interest rate $r_c$. It is reflects the consumer's willingness to trade consumptions across periods. And $(\delta + g\eta)/(1-\tau) = f'(k^*) - \mu$ is what we have been calling the investment interest rate $r_i$. It reflects the pretax (and postdepreciation) return to investment.

From (D5), the equilibrium propensity of savings $z^*/y^*$ can be written as a function of the parameters.

$$s^* = 1 - \tau - \frac{\tau\mu k^* - c^*}{f(k^*)} = [\mu + g + n] \cdot (k^*)^{1-a}$$
$$= \frac{(\mu + g + n)a}{\mu + r_i} \qquad \text{(D16)}$$

We parameterize the Ramsey model in Table D-1 to determine the savings rate $s^*$ and shadow price of capital in (19). In particular, we take $r_c$ = three percent and $r_i$ = seven percent (based on OMB guidelines). This leaves $\mu$, $g$, $n$, and $a$. We assume depreciation $\mu$ = 10 percent, economic growth $g$ = 2 percent, population growth $n$ = 1 percent, and the capital-output elasticity $a$ = 0.3. This yields a steady-state savings rate $s^*$ = 23 percent, and the value of the shadow price $v$ is about 1.5.