TARIFFS AND SAVING IN A
MODEL WITH NEW FAMILIES

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ABSTRACT

The paper explores how a tariff may affect saving through intergenerational redistribution of income that is caused by changes in factor prices and by the distribution of tariff revenue. The model is a Blanchard-type overlapping generations model. Two types of revenue distribution schemes are examined -- lump-sum distribution of current revenues to currently living individuals, and distribution as a subsidy to holders of physical wealth. (There is no fiscal policy in this paper -- the government budget is continuously balanced). We draw some general conclusions about the non-neutralities that arise in this type of model as opposed to single-generation models, or models in which perfect bequest motives exist.

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1. Introduction

In policy discussions, it is often suggested that increased tariffs will improve a country's current account. To the economic theorist, it is not immediately obvious how a distortionary tax change should affect the incentives to save and invest -- whose difference comprises a current account imbalance. Here we take a look at one aspect of the effect of tariffs on saving in a neoclassical model.

This paper analyzes the effects of tariffs on saving in a small open economy using the uncertain lifetimes version of the overlapping generations model, developed by Yaari (1965) and Blanchard (1984,1985). Several authors[1] have used this model to examine the role of public sector budget deficits because it fails to display Ricardian debt-neutrality, so that the intertemporal pattern of net lump-sum transfers to individuals has real effects. We examine the intertemporal effects of a permanent tariff change, abstracting from other aspects of fiscal policy. The distribution of the incidence of the tariff across different factors, and the method of distribution of the tariff revenue, have important consequences for aggregate per capita saving and, therefore, the current account. The intersectoral and intergenerational effects of the tariff have intertemporal impacts for the

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[1] See Blanchard (1984,1985). In the international context, see Buiter (1986a,b,c), Frenkel and Razin (1986), Kouri (1986), van Wijnbergen (1985), Eaton (1987), Smith (1987) and our earlier paper, Engel and Kletzer (1986). See also Weil (1985).

same reason that debt-neutrality fails; however we constrain the public sector budget to be in balance continuously.[2]

After laying out the model in section 2, we proceed by examining first a special case of the model in which the import good is not produced domestically. Tariff revenue is assumed to be redistributed lump-sum to living individuals. We find that under this distribution scheme, the change in the tariff has consequences for aggregate saving. The tariff is essentially an equal tax on both physical wealth and non-tangible wealth, while the lump-sum redistribution is a subsidy only to non-tangible wealth. When the incidence of the tariff cum subsidy scheme is not neutral across generations, total expenditure in this economy is affected because of the imperfect claim of currently living individuals on income from non-tangible assets in the future.

We next take up models in which the import good is produced. Here, a change in the tariff has additional effects on expenditure through its power to change the factoral distribution of income.

It is important to note that these effects are different than those that appear in other models of the current account in which no new generations are born. (In fact, both of these effects are present even when the tariff would have no effect on saving in a model with a single generation.)[3] As we will show, the fact that new generations are born with an imperfect bequest motive

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[2] Our analysis of the distributional impact of taxes bears some resemblance to that of Chamley and Wright (1987).

[3] This general feature of the uncertain lifespans model has also been noted by Buiter (1986b).

means that even a small tariff will alter saving. These effects occur even in the absence of any first-order distortion, or presence of a "pure substitution effect".[4]

In section 4, we consider an alternative scheme for redistribution of tariff revenue. If the economy has positive holdings of tangible assets (foreign currency bonds and land), the revenue is redistributed as a subsidy to tangible assets. If there are net negative holdings of tangible assets, the revenue is redistributed as a subsidy to net tangible debt. We show that for any given level of the tariff, the government has a choice of how to redistribute revenue. If they choose to subsidize steady-state tangible assets, the steady-state tangible asset position will be positive. If they choose to have a negative subsidy rate to steady-state tangible assets -- hence, a positive subsidy to steady-state tangible debt -- the economy will have a negative position in tangible assets in steady state. Thus, by choosing how to set the subsidy rate for any given tariff rate, the government can determine the net position in tangible assets in steady state for the economy. We then show how changes in the tariff rate affect saving.

Section 5 concludes.

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[4] See for example Razin and Svensson (1983), Edwards (1987) and the endogenous discount rate model in our earlier paper, Engel and Kletzer (1986). There are first-order effects in Razin and Svensson because the consumers' price indices are allowed to change from period to period. We rule this out in our model. Edwards also introduces non-traded goods.

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2.   The Model

We study a small country that takes as given the world interest rate, r, and the world price of good 2 in terms of good 1, which we set equal to one. Both goods are traded and consumed. We consider the effects of increasing a tariff on good 2.

Goods are produced using standard neoclassical production processes. There are at least two factors of production, so factor returns and output levels are determined exactly. All factor supplies are constant (there are no intermediate goods, and all non-labor factors can be considered to be types of land) and are normalized to one. With unchanging factor supplies and relative price of commodities, factor returns and output levels are constant over time. A permanent change in the tariff may lead to a once and for all shift in factor prices and production levels. The production side of the economy can be left in this general form for the dynamic analysis, although we will compare the effect of a permanent tariff change for three special cases: only the export good is produced; both goods are produced in the Heckscher-Ohlin model; and, both goods are produced in the specific-factors model.

Household consumption behavior is derived using the uncertain lifetimes version of the overlapping generations model, developed by Yaari (1965) and Blanchard (1984,1985). We adopt a continuous-time version in which each individual faces a constant (age and time independent) instantaneous probability of death, π, less than unity, and there is no bequest motive. At each instant, a new cohort of size π+n is born, where n is the constant proportionate rate of population growth. The dynamics of per-capita saving are identical for all values of π+n that exceed zero (see Buiter (1986c)).

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Weil (1985) shows that an overlapping generations model results when π is zero and n is positive. In a model with infinitely-lived dynastic families in which each individual possesses a perfect bequest motive, if there is birth of new dynasties, then the model will lead to the same saving dynamics as in Weil, because currently living families do not care about the consumption of future dynasties. We use Blanchard's version in which π is positive and n equals zero, because labor force growth is unessential to our examination of the savings effects of tariff changes. Therefore, the population is constant with size equal to one.

Because consumers have uncertain lifetimes, their effective subjective discount rate is $\delta+\pi$, where δ is the positive pure rate of time preference.

All forms of physical wealth are perfect substitutes, so that they earn the same rate of return, r, as an internationally traded bond. We assume that consumers have access to a perfect annuities market. Each consumer can contract with an insurance company to receive an additional rate of return π on tangible assets while she lives. In exchange, the company receives her net wealth if she dies. Conversely, if a consumer has negative net holdings of tangible assets, then she agrees to pay a premium π per unit of debt on the condition that the insurance company assumes her debt upon death.

Two types of wealth are assumed not transferable to the insurer for an annuity. The consumer's human wealth (the discounted value of labor income) has no value upon death, so that the company is unwilling to pay anything for the privilege of owning this asset after the person's death. Also, since tariff revenue is distributed only to living persons, the individual has no claim to tariff revenue after death to transfer to the insurer. We refer to the sum of these two types of wealth as non-tangible assets.

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In the Yaari-Blanchard model, an individual born at time i will maximize
the expectation of the discounted stream of felicity of current consumption.
The objective function for an individual born at time i is given by:

$$(1) \qquad V_i(t) = \int_t^\infty u(c_{1i}(s), c_{2i}(s)) e^{-(\delta+\pi)(s-t)} ds$$

where $c_{1i}(s)$, $c_{2i}(s)$ are individual i's consumption at time s of goods 1 and
2, respectively. The individual's budget constraint at time t is

$$(2) \qquad \dot{w}_{it} = (r+\pi)w_{it} + \omega_{it} + R_{it} - I_{it}.$$

w_{it} is tangible wealth.[5] Income from non-tangible wealth is given by the sum
of labor income, ω_{it}, and net transfers, R_{it}. Expenditure at domestic prices
on consumables is denoted by I_{it}, which equals the sum $c_{1i}(t) + pc_{2i}(t)$, where
p is the domestic (cum tariff) price of good 2. The details of the derivation
of individual and aggregate consumption dynamics are given in the Appendix.
We make the assumptions that the felicity function, $u(c_1, c_2)$, is homothetic
and displays constant relative risk aversion to allow linear aggregation of
individuals' consumption plans.

An important feature of the Yaari-Blanchard model is that the pure
subjective rate of discount need not equal the world rate of interest to
assure convergence of aggregate per capital wealth and consumption to steady
state values under individual intertemporal optimization. Because individuals
face a positive probability of death at each instant, aggregate per capita
wealth can converge to a finite level when r exceeds δ, even though each
individual plans to accumulate unbounded wealth over an infinite horizon (and

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[5] The "·" above a letter refers to its time derivative.

analogously, when δ exceeds r). Individuals born at any given time comprise an exponentially decreasing fraction of the population as they age (in Weil (1985), this happens through population growth alone). The appendix restates Blanchard's condition for existence and stability of the steady state.

Output of the two goods is given by y_1 and y_2. Aggregate consumption is represented by c_1 and c_2. Total expenditure at domestic prices is given by

$$I = c_1 + pc_2.$$

Total expenditure at world prices is

$$z = c_1 + c_2.$$

Tariff revenue in the aggregate is given by

$$R_t = (p-1)(c_{2t} - y_{2t}).$$

The aggregate lump-sum transfer to consumers at time t, R_t, equals the actual tariff revenue collected at time t. We assume a continuously balanced public sector budget. Because felicity is homothetic, the age distribution of total revenue has no consequences if the transfer is lump-sum and received only by those currently alive.

The aggregate value of non-tangible wealth (aggregating as in Blanchard) is given by:

$$N_t = [\omega/(r+\pi)] + \int_t^\infty R_s e^{-(r+\pi)(s-t)} ds.$$

(The wage rate is age independent so that ω depends only on r and p for the small country, and r and p do not change -- except for the one time permanent change in p from the tariff.)

Aggregate tangible wealth, w_t, is defined by

$$w_t = a_t + b_t.$$

b_t is aggregate net claims on foreigners. a_t is the value of land. Under the constant returns to scale production assumption,

$$(3) \qquad a_t = (y_1 + py_2 - \omega)/r.$$

Therefore, a_t depends only on the paths of p and r.

Aggregate consumption at any time t is given by the simple linear relationships (see the Appendix):

$$(4) \qquad I_t = \Delta(w_t + N_t)$$

$$c_{1t} = (1 - \eta(p))I_t, \text{ and}$$

$$pc_{2t} = \eta(p)I_t,$$

where

$$\Delta = r + \pi + (\delta - r)/\sigma,$$

and $0 \le \eta \le 1$; $\eta'(p) \gtrless 0$.

The coefficient of relative risk aversion is given by σ.

Aggregating as in Blanchard yields equations for accumulation of tangible and non-tangible assets:

$$(5) \qquad \dot{w} = rw_t + (\omega + R_t) - I_t,$$

and,

$$(6) \qquad \dot{N}_t = (r+\pi)N_t - (\omega + R_t).$$

Note that tariff revenues may be expressed as

$$R_t = \alpha I_t - (p-1)y_{2t},$$

where

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$$\alpha(p) = [1 - \frac{1}{p}]\eta(p).$$

In what follows, we will generally assume $\alpha' > 0$. This would hold, for example, with Cobb-Douglas utility ($\eta'(p) = 0$). It could be violated if the demand elasticity of substitution between goods is sufficiently high and initial tariff levels are sufficiently greater than zero.

The tangible wealth accumulation equation can be rewritten as

(7) $$\dot{w}_t = rw_t + \omega - (p-1)y_{2t} + \alpha I_t - I_t.$$

Since a_t is constant over time, $\dot{b}_t = \dot{w}_t$. Also note that

(8) $$z_t = (1-\alpha)I_t.$$

Equations (3), (7) and (8) may be used to derive

(9) $$\dot{b}_t = rb_t + y_1 + y_2 - z_t.$$

Equations (4), (5), (7) and (8) give the dynamics of expenditure at world prices:

(10) $$\dot{z}_t = (r+\pi-\Delta)z_t - (1-\alpha)\pi\Delta(a_t + b_t).$$

(Remember, a_t is constant.)

Equations (9) and (10) constitute a second order dynamic system that expresses the motion of the economy.

The steady-state levels of z and b can be obtained by setting $\dot{b} = 0$ in equation (9) and $\dot{z} = 0$ in equation (10). We get[6,7]

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[6] A - over a variable represents its steady-state value.

[7] The stability condition implies $(\Delta-r)(r+\pi) - \alpha\Delta\pi > 0$, and $\Delta > r > 0$. So, $\bar{z} > 0$. These facts are demonstrated in the Appendix.

(11)
$$\bar{z} = \frac{\Delta \pi (1-\alpha)}{(\Delta - r)(r+\pi) - \alpha \Delta \pi} (\omega + (1-p)y_2),$$

and

(12)
$$\bar{b} = \frac{\bar{z} - y_1 - y_2}{r}.$$

The appendix shows the conditions under which the dynamic system is saddle stable. The accumulation of bonds over time is given by

(13)
$$\dot{b}_t = \theta (b_t - \bar{b}_t),$$

where $\theta < 0$ is the stable root of the system.

3. Effects of Tariff Changes

Here we examine the effects of increasing the tariff permanently at some time. We are particularly interested in the response of saving and the current account. At the moment the tariff is imposed, the country's claims on foreigners, b_t, cannot jump. So, from equation (13), the effect of an increase in tariffs on saving and the current account, starting from a position of steady state is given by (recall the assumption that $\alpha' > 0$)

(14)
$$\dot{db}_t/d\alpha = -\theta (d\bar{b}/d\alpha)$$

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which has the same sign as $d\bar{b}/d\alpha$.[8]

a. Specialization in Production of the Export Good

In the case in which the import good (good 2) is not produced, the wage rate, ω, and the value of land, a, are unaffected by changes in the tariff. Output of good 2, y_2, is zero, and output of good 1 will not respond to tariff movements.

From equation (12)

(15) $$d\bar{b}/d\alpha = (1/r)(d\bar{z}/d\alpha).$$

From (11),

(16) $$d\bar{z}/d\alpha = \frac{r\Delta\pi(r-\delta)\omega}{\sigma[(\Delta-r)(r+\pi) - \alpha\Delta\pi]^2} \gtrless 0 \text{ as } r \gtrless \delta.$$

Hence, from (14), (15) and (16) it follows that an increase in tariffs will improve the current account (increase saving) when the personal discount rate is less than the world interest rate, but will worsen the current account (lower saving) when the discount rate exceeds the world interest rate.

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[8] If we are initially away from steady state, $db/d\alpha = -\theta(d\bar{b}/d\alpha) + (b-\bar{b})d\theta/d\alpha$. From the expression for θ in the appendix, $d\theta/d\alpha = \Delta\pi((\Delta+\pi)^2 - 4\alpha\Delta\pi)^{-1/2} > 0$. If initially the current account is in deficit, so $(b-\bar{b}) > 0$, then the effect of a tariff increase on the current account is more positive relative to a starting position of current account balance, and vice-versa for a current account initially in surplus.

It is useful to pursue this from a different tack to develop intuition.
From equation (9)

$$db_t/d\alpha = - dz_t/d\alpha.$$

Hence

$$dz_t/d\alpha = (\theta/r)(d\bar{z}/d\alpha).$$

When long-run expenditure $\bar{z}$ rises, current expenditure, z_t, falls. An
increase in the tariff will cause z_t to rise when $\delta > r$ and fall when $r > \delta$.

From equation (4)

$$dI_t/d\alpha = \Delta \ dN_t/d\alpha.$$

N_t will change when the tariff rises because the discounted value of tariff
revenue will increase. This value depends on the expected amount of change in
expenditure currently and in the future. The appendix demonstrates that
starting from steady state

$$dN_t/d\alpha = \frac{\theta(\theta-r)}{(1-\alpha)\Delta[(\Delta-r)(r+\pi) - \alpha\Delta\pi]} \quad \bar{I} > 0.$$

So, $dI_t/d\alpha > 0$. Expenditure measured in domestic goods prices necessarily
increases as the tariff rises.

Now $z_t = I_t - R_t$, under complete specialization. Clearly if $r > \delta$, the
increase in tariff revenue exceeds the increase in I_t (so z_t falls), and when
$\delta > r$, the increase in I_t exceeds the increase in tariff revenue.

It is very helpful to consider the special case of free trade initially.
Then,

$$dR_t/d\alpha = \bar{I}.$$

We also have

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$$dN_t/d\alpha = \bar{I}/(r + \pi),$$

which simply equals the discounted value of a permanent increase in tariff revenue equal to today's increase. We can write

$$I_t = (r + \pi + (\delta-r)/\sigma)(w_t + N_t).$$

So

$$dI_t/d\alpha = (r + \pi + \frac{\delta - r}{\sigma}) \, dN_t/d\alpha = (1 + \frac{\delta-r}{\sigma(r+\pi)}) \, dR_t/d\alpha.$$

When $\delta > r$ $(r > \delta)$ the marginal propensity to consume out of permanent income is greater than (less than) one, and

$$dz_t/d\alpha = dI_t/d\alpha - dR_t/d\alpha = \frac{\delta-r}{\sigma(r+\pi)} \, \bar{I}.$$

In the complete specialization model, a tariff increase leads to an increase in spending in terms of domestic prices. The tariff revenue generates future income (in terms of domestic prices) and, therefore, increases the value of non-tangible wealth. If the increase in spending falls short of the increase in current tariff revenue $(\delta < r)$, saving and the current account increase, but if the increase in spending exceeds the increase in current revenue $(\delta > r)$, saving and the current account decline.

In models in which no new families are born and there is a perfect bequest motive, if there were no distortions in the economy (such as existing tariffs) a small increase in tariffs would have no effect on expenditure (except possibly through a "pure substitution effect" which is ruled out here

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by our assumptions on preferences).[9] It is important to note that in this model even when the initial tariff is zero, a small increase in tariffs has a first order effect on expenditure.

Consider for a moment a scheme for redistributing tariff revenue that makes the imposition of a tariff neutral. Since tariff revenue is proportional to expenditure measured in terms of the domestic good, I, a subsidy to expenditure clearly would neutralize the effect of the tariff. In this case we know

$$z_t = \Delta(w_t + N_t).$$

But, then using equation (8), we would have

$$I_t = (1/(1-\alpha))\Delta(w_t + N_t).$$

Tariff revenue would be given by

$$R_t = (\alpha/(1-\alpha))\Delta(w_t + N_t).$$

Notice that in this case the tariff is effectively a proportional tax on total wealth $(w_t + N_t)$ at the rate $(\alpha/(1-\alpha))\Delta$. The tariff is neutral when the

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[9]   See Engel and Kletzer (1986) for a demonstration of this in a model with a representative consumer who has an infinite horizon and an endogenous rate of time preference. Razin and Svensson (1983) discuss a "pure substitution effect" that is ruled out by assumption in this model. Because the felicity function is identical in all periods, and prices are constant, the exact price index does not change over time in our set-up.

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revenue is rebated as a proportional subsidy to total wealth.[10]

In contrast, under the lump-sum redistribution to living persons considered in this section, the tariff is still a proportional tax on total wealth:

$$R_t = a\Delta(w_t + N_t),$$

but the revenue is returned purely as a subsidy to non-tangible wealth. The tariff changes consumption because the redistribution scheme has first-order effects on expenditure.

When there is a permanent increase in the tariff, total wealth is taxed at a greater rate both now and in the future. The tax on tangible wealth is a fully-capitalized loss to living individuals (because of the perfect annuities market). The losses from the tax on future non-tangible wealth are only partially capitalized by living individuals. A neutral redistribution scheme would be to return the revenue in an equal subsidy to tangible and non-tangible wealth. Any other scheme has consequences for total expenditure measured at world prices. For example, the lump-sum redistribution considered in this section takes revenue from taxes on tangible and non-tangible wealth and redistributes it purely as a subsidy to non-tangible assets. In section 4 we consider another non-neutral scheme in which the revenue is redistributed

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[10] Under the "neutral" scheme, the level of c_1 and c_2 will change (because the tariff is a tax on c_2, but all expenditure is subsidized). However, $c_1 + c_2$ (=z) will not be affected. Of course, expenditure in domestic prices changes as $(p-1)c_2$ is altered, but this is exactly the change in tariff revenue.

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as a subsidy to tangible assets.[11]

In this section, both forms of wealth are being taxed by the tariff but the revenue is all coming back as a lump-sum transfer. In the future, that revenue (which will be generated partially by a tax on physical assets and partially by a tax on non-tangible assets) will be redistributed to all individuals who are alive at that time -- some of whom are not yet born. Thus, living individuals are not fully compensated for the burden of the tax they bear. The only neutral scheme (_   ´ ·   _   ´   ) would give as a lump-sum redistribution to individuals living at any time only that share of the revenue collected that is effectively a tax on non-tangible wealth. With lump-sum redistribution of revenues, the burden of the tax is not spread across generations in the same way as the redistribution of the revenue -- which causes the pattern of saving to change across generations.

b.   Both Goods Produced

In addition to the effect on saving generated by redistribution of tariff revenue, there is an effect on total expenditure caused by changes in the factor composition of income. In a model where both the export and import good are produced domestically, and there are at least two factors of production, the change in the domestic relative price of the goods has implications for spending levels. In particular, if the tariff adjusts the

[11]   Eaton (1987) considers a similar model, but one in which there are monopoly firms that have a claim on tariff revenue (yet another non-neutral redistribution scheme).

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size of income derived from tangible versus non-tangible forms of wealth, aggregate saving may be altered.

This effect is separate from any impact the tariff may have on saving by decreasing the total value of output at world prices from the distortionary effects of non-lump-sum taxes. To make this point most forcefully, we will first consider a small tariff starting from a point of free trade, so that distortions are second-order small. Thus, this effect is not present in those models with no new families and perfect bequest motives.

It is useful to note from equation (3) above that the value of land, a_t, can be expressed as

$$(17) \qquad a_t = (1/r)[y_1 + y_2 - (\omega - (p-1)y_2)].$$

The value of land equals the value of output at world prices less the value of the output of labor and the value of the tariff distortion of output.

Also, note that non-tangible wealth can be expressed as

$$(18) \qquad N_t = \frac{\omega}{r+\pi} - \frac{(p-1)y_2}{r+\pi} + \int_t^\infty \alpha I_s e^{-(r+\pi)(s-t)} ds.$$

From equation (4), expenditure at domestic prices, I_t, is proportional to the sum of tangible and non-tangible wealth. Examination of equations (17) and (18) reveal how a change in tariffs will affect I_t. In the previous section we saw the effects of a permanent tariff increase on the sum of the discounted values of future αI_s. But here there is an additional effect that comes from changes in $\omega - (p-1)y_2$. For example, if the tariff raises the wage rate (in terms of the exportable), the value of non-tangible wealth increases by $(1/(r+\pi))$ times the change in the wage. However, the value of land falls by $(1/r)$ times the change in the wage. The total effect of a given increase

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in wages on wealth and spending is negative, because the social discount rate that values the flow of income from tangible assets, r, is less than the corresponding interest rate for non-tangible assets, $r+\pi$. The future changes in the product of land are fully capitalized into the current value of land (because of the perfect annuities market), but future changes in wage income are not (because in the future the labor force will consist only partly of those living now, and partly of some who are not currently alive). Unlike models where agents have infinite lives, a change in the source of factor income has implications for the total value of wealth.

A simple expression can be derived for the change in I_t when tariffs increase, starting from initially free trade. Note, first, in this case

$$d[(p-1)y_2]/dp = y_2.$$

We then have

$$dI_t/dp = [\Delta\alpha'/(r+\pi)]\bar{I} - [\pi\Delta/r(r+\pi)](d\omega/dp - y_2).$$

The first term in this expression is identical to the one discussed at length in the previous section, and the second term corresponds to the effect explained in the preceding paragraph. (Note that there is no change in $y_1 + y_2$ if we start at free trade and have an infinitesimal increase in the tariff rate.) The change in expenditure at world prices, z_t, which in this case equals the negative of the change in saving and the current account, is given by:

$$dz_t/dp = [(\delta-r)\alpha'/\sigma(r+\pi)]\bar{I} - [\pi\Delta/r(r+\pi)](d\omega/dp - y_2).$$

The change in expenditure depends on how wages in terms of the export good change, but the size and direction of this movement depends upon the

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production structure. In a Heckscher-Ohlin set-up, in which both goods are produced with intersectorally mobile land and labor, the rate will rise if the protected sector is labor-intensive and fall if that sector is land-intensive. The value of land will rise if the protected sector is land-intensive, and conversely if the protected sector is labor-intensive. The size of these effects also depends upon the exact production function. Thus, taking into account the effects of tariffs on factor prices makes the response of saving to tariffs ambiguous.

In a specific-factors model in which labor is free to move between sectors, but other factors cannot, the increase in the tariff will raise the wage in terms of the export good. The value of land in the export sector will decline, and the value of land in the import sector will rise. Again, the total effect of the tariff on saving is ambiguous.

The general expression for the change in saving starting from a position in which a tariff was already in place is given by:

$$\frac{db}{dp} = \frac{-\theta\Delta\pi(r-\delta)a'[\omega - (p-1)y_2]}{\sigma[(\Delta-r)(r+\pi) - a\Delta\pi]^2} - \frac{\theta(1-a)\Delta\pi}{r[(\Delta-r)(r+\pi) - a\Delta\pi]}[-y_2 + \frac{d\omega}{dp} - (p-1)\frac{dy_2}{dp}]$$

$$- \frac{\theta}{r}(p-1)\frac{dy_2}{dp}.$$

In general, the sign of this derivative is indeterminate.

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4.   Alternate Redistribution Scheme

In the previous section, all tariff revenue was redistributed as lump-sum transfers to the currently alive. This scheme has the effect of increasing the value of non-tangible wealth (for the "usual" case in which α rises with the tariff rate). An interesting alternative is the redistribution of tariff revenue in the form of a subsidy to tangible assets. In this section, we consider the remittance of all current tariff revenue through a linear subsidy to holdings of tangible wealth. This scheme is identical to a reduction of the tax on non-wage income (interest and rents) financed by the tariff increase in a model with a more complex fiscal policy in place.

To isolate the effect of the change in the redistribution plan, we assume that the country is completely specialized in production of the exportable. The tariff revenue is redistributed in proportion to each living individual's tangible wealth, so that the aggregate transfer is $\beta_t w_t$, where β_t is the proportionate rate. The effective market return on these assets becomes $r + \pi + \beta_t > 0$.

Total tariff revenue is given by αI_t, where α is as previously defined. The balanced budget requirement implies that

$$(19) \qquad\qquad \alpha I_t = \beta_t w_t,$$

at all times. While α is a constant for a fixed tariff rate, β will vary with I and w. Therefore, the model is now non-linear.

In the Blanchard model, net holdings of tangible assets, w_t, can assume

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negative values. This can happen if total foreign indebtedness exceeds the total value of land. In order to satisfy equation (19), clearly β_t must be negative in these cases, since α and I_t are always positive. Hence, $\beta_t < 0 \leftrightarrow w_t < 0$, and $\beta_t > 0 \leftrightarrow w_t > 0$.

The dynamics of aggregate tangible wealth and consumption expenditure valued in world prices are given by (see Appendix):

$$(20) \qquad \dot{w}_t = (r+\beta_t)w_t + \omega - I_t$$

$$(21) \qquad \dot{z}_t = ((r+\beta_t-\delta)/\sigma)z_t - (1-\alpha)\pi_t \Delta_t w_t,$$

where Δ_t is defined by

$$\Delta_t^{-1} = \int_t^\infty e^{\int_t^s [(r-\delta)/\sigma - (r+\pi)] + ((1-\sigma)/\sigma)\beta(u)du} \, ds.$$

Using (19) and recalling that $z_t = (1-\alpha)I_t$, equation (20) becomes

$$(22) \qquad \dot{w}_t = rw_t + \omega - z_t.$$

Equations (21) and (22) are a dynamic system in two variables. The appendix demonstrates the conditions under which this system is saddle stable. An equation for the accumulation of foreign bonds near steady state is given by

$$\dot{b} = \lambda(b_t - \bar{b}).$$

As discussed at the beginning of section 3, the change in saving and the current account in response to a tariff increase, starting from steady state, has the same sign as the change in $\bar{b}$, the long run position in international bonds.

Setting $\dot{z} = 0$ and $\dot{w} = 0$, steady-state tangible wealth is given by:

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$$
(23) \qquad \bar{w} = \frac{(r+\bar{\beta}-\delta)/\sigma}{(r+\pi+\bar{\beta})(\pi-(r+\bar{\beta}-\delta)/\sigma)}\, \omega.
$$

The stability conditions imply $(\pi-(r+\bar{\beta}-\delta)/\sigma) > 0$, so $\bar{w} > 0 \leftrightarrow r+\bar{\beta}-\delta > 0$ and $\bar{w} <$ $0 \leftrightarrow r+\bar{\beta}-\delta < 0$. From the discussion above, this implies $\bar{\beta} > 0 \leftrightarrow r+\bar{\beta}-\delta > 0$ and $\bar{\beta}$ $< 0 \leftrightarrow r+\bar{\beta}-\delta < 0$. More will be said about this presently.

Because only the export good is produced, the tariff will not change the value of land, which implies $d\bar{w}/dp = d\bar{b}/dp$. From (23) we have

$$
(24) \qquad \frac{d\bar{b}}{d\bar{\beta}} = \frac{\bar{w}}{(r+\bar{\beta}-\delta)} \left[\frac{(\delta+\pi)\pi + (r+\bar{\beta}-\delta)^2/\sigma}{(r+\pi+\bar{\beta})(\pi - (r+\bar{\beta}-\delta)/\sigma)} \right] > 0.
$$

This result is entirely plausible -- an increase in the subsidy to tangible wealth increases the steady-state holdings of that type of wealth in the form of foreign bonds. We need to investigate how $\bar{\beta}$ changes when the tariff increases to understand the effects of tariffs on the current account. In those cases in which an increase in p causes $\bar{\beta}$ to rise, saving and the current account will rise, and when an increase in p leads to a decrease in $\bar{\beta}$, saving and the current account decline.

Solving for relation (19) in steady state yields a quadratic relationship between α and $\bar{\beta}$:

$$
(25) \qquad \alpha\pi\bar{\Delta} = \bar{\beta}((r+\bar{\beta} - \delta)/\sigma),
$$

where,

$$
\bar{\Delta} = (r + \pi + \bar{\beta} - (r + \bar{\beta} - \delta)/\sigma).
$$

This implies that the constraint (19) does not determine $\bar{\beta}$ uniquely for any tariff rate. For any given α, there are two choices for $\bar{\beta}$ that satisfy (19).

This is perhaps easiest to understand in the case in which there is no tariff. Clearly $\bar{\beta} = 0$ satisfies the government budget constraint. But it is

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also true that $\bar{\beta} = \delta - r$ will ensure a balanced budget in steady-state. Such a choice will lead steady-state wealth to be zero, so total subsidies will also be zero.

We can derive an expression for local derivatives of $\bar{\beta}$ with respect to α:

(26) $\qquad d\bar{\beta}/d\alpha = \pi\bar{\Delta}^2/[(\bar{\Delta}-\bar{\beta})(r+\bar{\beta}-\delta)/\sigma + \bar{\beta}(r+\pi+\bar{\beta})/\sigma].$

By the stability condition, $\bar{\Delta}-\bar{\beta} > 0$. Recalling that when $\bar{\beta}$ is positive when $r + \bar{\beta} - \delta$ is positive , then the derivative is positive if $\bar{\beta}$ is positive and conversely.

Figure 1 shows the relation between α and β when $r > \delta$. This country would have positive steady-state holdings of tangible assets in the absence of any subsidy to wealth or debt. When α is zero (p = 1), $\bar{\beta}$ is either zero or is negative (= $\delta - r$). For positive values of α, there is always a positive $\bar{\beta}$ that satisfies the government budget constraint (the top half of the graph). If this $\bar{\beta}$ is chosen, then clearly $r + \bar{\beta} - \delta$ is greater than zero, and steady state w is positive. But it is also true for all positive values of α there is a negative value of $\bar{\beta} < \delta$ -r which satisfies equation (19). In this case, $r + \bar{\beta} - \delta < 0$, and steady-state foreign debt exceeds the value of land ($\bar{w}$ is negative). Here the tariff revenue is rebated as a subsidy to negative holdings of tangible wealth.

Figure 2 takes up the case in which in the absence of subsidies the country would be long-run debtors in tangible wealth -- that is, the case in which $\delta > r$. If α is zero, $\bar{\beta}$ is either zero or $\delta - r > 0$. Again, for any positive value of α there is a positive value of $\bar{\beta}$ that satisfies the balanced budget requirement. In this case $\bar{\beta} > \delta - r$, which implies that $r + \bar{\beta} - \delta > 0$, and $\bar{w}$ is positive. It is also the case that there is a negative value of $\bar{\beta}$
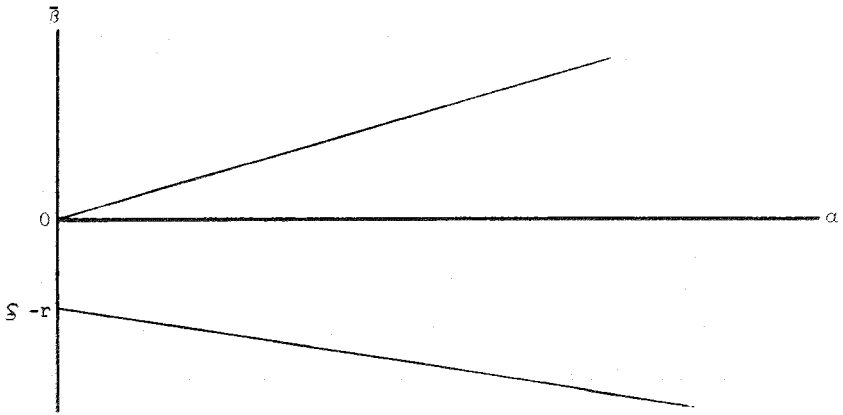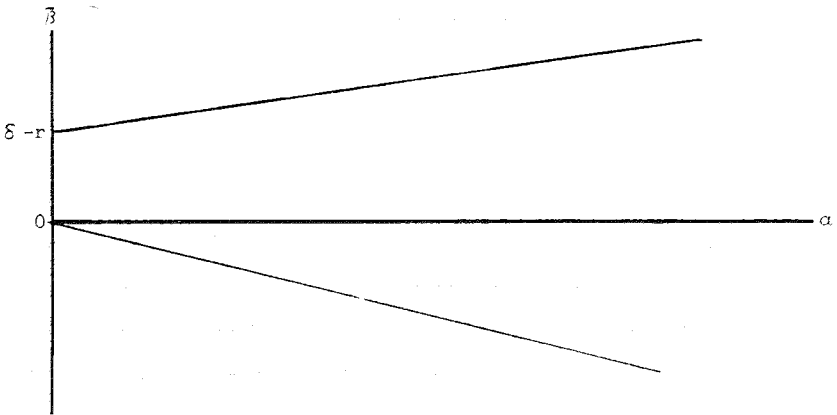
Figure 1

Figure 2

that sets total subsidies equal to total tariff revenue. For these choices of $\bar{\beta}$, $r + \bar{\beta} - \delta < 0$, and $\bar{w} < 0$.

The government can always choose a value of $\bar{\beta}$ to ensure that long-run foreign debt is less than the value of land ($\bar{w} > 0$) if it so chooses (and vice-versa if it wants $\bar{w} < 0$). It can do so by altering the rate of return on tangible assets available to residents. (This ultimately means changing the country's international debt position, since in the aggregate the value of land holdings cannot be altered.) Perhaps the surprising thing is that it can always choose such a subsidy rate and keep the budget balanced irrespective of the relation of δ to r.

Using equations (24) and (26) we can see how the current account must change as tariffs increase. Not surprisingly, when $\bar{\beta}$ is positive, so $\bar{w}$ is positive, an increase in the tariff will increase the subsidy to tangible wealth and therefore increase current saving and the current account. Likewise, when $\bar{\beta}$ is negative, so $\bar{w}$ is negative, as the tariff rises the subsidy to tangible debt goes up, and present saving and the current account decline.[12]

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[12] This analysis assumes that when the tariff changes infinitesimally, the subsidy rate does not jump discretely.

4. Conclusion

In models with only one generation of consumers, tariffs influence saving through changes in wealth caused by the tariff distortion. That channel of influence is present in our overlapping generations model with uncertain lifespans.[13]

However, we emphasize other channels which are special in models in which new families are born. The tariff can change total wealth through redistributing income between tangible and non-tangible assets. This happens in the first place when tariff revenue is redistributed lump-sum and takes on the characteristics of labor income. It also occurs because tariffs change factor prices, which in turn alter the distribution of wealth between land and human wealth.

We also explore a mechanism by which the proceeds from tariffs can be rebated in a way to affect the incentives to hold tangible assets. We show that government has some scope to significantly affect the net holdings of international bonds while still maintaining budget balance.

The analysis in this paper is purely positive. Conclusions about the welfare effects of the tariffs are not drawn, and would in general depend upon

---

[13] In the absence of distortions tariffs can change saving through the substitution effect discussed in Razin and Svensson (1983). The effect is non-zero when price indi es change over time. That is ruled out here by the assumptions of identical felicity functions over time and constant prices.

the weights given to the utility of the differe t generations.[14] We are not able to contribute to the issue of whether tarif 's should be used to alter the current account.

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[14] Calvo and Obstfeld (1985) is a general examination of welfare issues in this type model.

Appendix

The purpose of this appendix is to fill in some of the steps in the derivations discussed in the text.

Models with Lump-Sum Subsidies

Individuals maximize utility given by (1) subject to the budget constraint (2). We assume constant relative risk aversion and homothetic preferences, so the indirect felicity function, v, for individual i can be written as

$$v_i(I,p) = [I_i^{1-\sigma}/(1-\sigma)]\nu(p).$$

The Hamiltonian for person i's optimization problem is given by

$$H = [I_i^{1-\sigma}/(1-\sigma)]\nu(p) + q[(r+\pi)w_i + \omega + R_i - I_i].$$

The first-order conditions yield

$$I_{it}^{-\sigma}\,\nu(p) = q_t,$$

$$\dot{q}_t/q_t = \delta - r.$$

These imply

$$\sigma\dot{I}_{it}/I_{it} = r - \delta,$$

or,

$$I_{it} = I_{is}e^{[(r-\delta)/\sigma](t-s)}.$$

We use the transversality condition

$$\lim_{t \to \infty} w_{it} e^{-(r+\pi)t} = 0.$$

Using the transversality condition, we integrate the dynamic budget constraint (2) to get

$$\int_t^\infty I_{is} e^{-(r+\pi)(s-t)} ds = w_{it} + N_{it},$$

where N_{it} is defined by

$$N_{it} = [\omega/(r+\pi)] + \int_t^\infty R_{is} e^{-(r+\pi)(s-t)} ds,$$

noting that all individuals are paid ω for their labor.

Using our expression for I_{it} we get

$$I_{it} = \Delta(w_{it} + N_{it}).$$

Aggregation to derive expressions for I_t, N_t and $\dot{w}_t$ follow directly as in Blanchard (1985, pp. 228-229). Note that we are able to aggregate for a general constant relative risk aversion utility function because r is constant.

The steady-state values $\bar{z}$ and $\bar{b}$ come directly from equations (9) and (10), setting $\dot{z}$ and $\dot{b}$ to zero and using the definition of a_t given in equation (3).

Note that under this revenue transfer scheme, the model is linear.

The eigenvalues of the dynamic system are given by the solution to

$$(r + \pi - \Delta - \theta)(r - \theta) = (1 - \alpha)\Delta\pi$$

which yields the negative root

$$\theta = (1/2)[2r + \pi - \Delta - ((\Delta + \pi)^2 - 4\alpha\Delta\pi)^{1/2}].$$

The system is saddle stable when θ is negative.

We must prove two propositions -- that saddle stability ($\theta < 0$) implies $\Delta > r > 0$, and $(\Delta-r)(r+\pi) - a\Delta\pi > 0$.

Note that $(\Delta+\pi)^2 - 4a\Delta\pi = (\Delta-\pi)^2 + 4(1-a)\Delta\pi$. So, as long as $0 < a < 1$, $((\Delta+\pi)^2 - 4a\Delta\pi)^{1/2}$ must be a real number. Also note that in the special case of free trade when $a = 0$, $\theta = r - \Delta$ and both propositions follow immediately from $\theta < 0$.

In general, first take the case in which $2r + \pi - \Delta > 0$. Note that this implies that θ equals one-half of $2r + \pi - \Delta$ minus the <u>positive</u> square root of $(\Delta + \pi)^2 - 4a\Delta\pi$.

First, we will show in this case $\Delta > r > 0$. Suppose $\Delta < 0$. Then the smallest that θ can be is when $a = 1$, so that

$$\theta = (1/2)(2r+\pi-\Delta-((\Delta+\pi)^2-4a\Delta\pi)^{1/2}) = r > 0,$$

hence a contradiction, so $\Delta > 0$.

Since $\Delta > 0$, it follows immediately from comparing the $a = 0$ root (which equals $r - \Delta$), that $r - \Delta < \theta < 0$, so $\Delta > r$.

Now, to show in this case that $(\Delta-r)(r+\pi) - a\Delta\pi > 0$, note that we have

$$(2r + \pi - \Delta)^2 < (\pi + \Delta)^2 - 4a\Delta\pi.$$

Multiplying out and cancelling directly yields our result.

The second case is when $2r + \pi - \Delta < 0$. Note first in this case that $\Delta > r$ directly.

We also have

$$
\begin{aligned}
(\delta-r)(r+\pi) - a\Delta\pi &> (\Delta-r)(r+\pi) - \Delta\pi \\
&= -r\pi + r(\Delta-r) \\
&> -r\pi + r(r+\pi) \quad \text{(because } 2r+\pi-\Delta > 0) \\
&= r > 0. \quad \blacksquare
\end{aligned}
$$

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The derivations of section 3 are straightforward until $dN_t/d\alpha$. This expression can be derived directly by calculating the expression for N_t from its definition and using the fact that $\dot{I}_t = \theta(I_t - \bar{I})$. However, an easier way to get it is by the back door. Note that

$$dN_t/d\alpha = (1/\Delta)\ dI_t/d\alpha$$

$$= [1/(1-\alpha)\Delta][\bar{I} + dz_t/d\alpha].$$

But,

$$dz_t/d\alpha = [(r+\pi-\Delta)/(\theta+\Delta-\pi-2r)]\bar{I},$$

where we have used $\bar{I} = (1/(1-\alpha))\bar{z}$, used the definition of $\bar{z}$ from equation (11) (with $y_2 = 0$), used the expression for $d\bar{z}/d\alpha$ (equation (16)), used the fact that $dz_t/d\alpha = (\theta/r)(d\bar{z}/d\alpha)$ and made the handy substitutions

$$(r-\delta)/\sigma = r + \pi - \Delta$$

and,

$$\theta(\theta+\Delta-\pi-2r) = (\Delta-r)(r+\pi) - \alpha\Delta\pi.$$

A bit more manipulation then yields the expression for $dN_t/d\alpha$ in the text.

The subsequent expressions in section 3a for the cases of initially free trade all follow directly by setting $\alpha = 0$ in the more general expressions.

Equation (17) for non-tangible wealth follows directly from the definitions of N_t and R_t.

The expression for dI_t/dp when $p = 1$ initially can be derived from differentiating the expressions for N_t and w_t noting that $I_t = \Delta(a_t + b_t + N_t)$. If ω were unchanged and $y_2 = 0$, as in section 3a, the derivative would be exactly the one in that section. That is, we would have for the case of $\alpha = 0$

initially

$$dI_t/dp = [\Delta a'/(r+\pi)]\bar{I}.$$

The additional term, $-[\pi\Delta/r(r+\pi)](d\omega/dp-y_2)$ comes from the changes in ω and $(p-1)y_2$ in the expressions for a_t and N_t.

The general expression for $d\dot{b}/dp$ at the end of section 3 is derived by noting that

$$d\dot{b}/dp = -\theta \ d\bar{b}/dp,$$

and differentiating expression (12) using

$$d(y_1+y_2)/dp = (1-p)dy_2/dp.$$

Model with Subsidies to Tangible Assets

Under this redistribution scheme, the effective discount rate includes a term, β_t, which depends upon time along an equilibrium path. We assume perfect foresight. We also assume $r + \pi + \beta_t > 0$.

Setting up the problem in a way analogous to the previous section, we get

$$\sigma\dot{I}_{it}/I_{it} = r - \delta + \beta_t$$

yielding

$$I_{it} = I_{is}e^{(1/\sigma)\int_s^t(r + \beta_u - \delta)du}.$$

Imposing the transversality condition and integrating gives

$$I_{it} = \Delta_t(\omega_{it} + N_{it})$$

where Δ_t is defined in the text, and

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$$N_{it} = \int_t^\infty \omega \, e^{-\int_t^s (r + \beta_u - \pi)du} ds.$$

Aggregation yields

$$I_t = \Delta_t(w_t + N_t)$$

with $w_t = a_t + b_t$ and $N_t = N_{it}$.

Differentiating the expression for N_t with respect to time gives

$$\dot{N}_t = (r + \pi + \beta_t)N_t - \omega.$$

Using the aggregation techniques of Blanchard (1985) we get

$$\dot{w}_t = (r+\beta_t)w_t + \omega - I_t.$$

We also have

$$\dot{I}_t = \dot{\Delta}_t(w_t + N_t) + \Delta_t(\dot{w}_t + \dot{N}_t)$$

$$= \dot{\Delta}_t \Delta_t^{-1} I_t + (r+\pi+\beta_t)\Delta_t(w_t + N_t) - \pi\Delta_t w_t - \Delta_t I_t$$

$$= -\Delta_t(\dot{\Delta_t^{-1}})I_t + (r+\pi+\beta_t)I_t - \pi\Delta_t w_t - \Delta_t I_t.$$

But

$$(\dot{\Delta_t^{-1}}) = -1 + [r+\pi+\beta_t - (r+\beta_t-\delta)/\sigma]\Delta_t^{-1},$$

so, after some cancellations,

$$\dot{I}_t = [(r+\beta_t-\delta)/\sigma]I_t - \pi\Delta_t w_t.$$

The expression for $\dot{z}_t$ in the text is obtained by using $z_t = (1-\alpha)I_t$ and $\dot{z}_t = (1-\alpha)\dot{I}_t$. The equation for $\dot{w}_t$ comes from these facts and $\alpha I_t = \beta_t w_t$.

The expression for $\bar{w}$ comes from setting $\dot{w}_t = 0$ and $\dot{z}_t = 0$ in equations (20) and (21). We use the fact that

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$$\pi\bar{\Delta}(1-\alpha) - r((r+\bar{\beta}-\delta)/\sigma) = (r+\pi+\bar{\beta})[\pi - (r+\bar{\beta}-\delta)/\sigma]$$

which comes from setting $\alpha\bar{I} = \bar{\beta}\bar{w}$.

The dynamic system is non-linear, but can be linearized near steady state as

$$\dot{w}_t = r(w_t-\bar{w}) - (z_t-\bar{z})$$

and

$$\dot{z}_t = (r+\pi+\bar{\beta}-\bar{\Delta})(z_t-\bar{z}) - (1-\alpha)\pi\bar{\Delta}(w_t-\bar{w}).$$

The stable root is given by

$$\lambda = (1/2)[2r + \bar{\beta} + \pi -\bar{\Delta} - ((\bar{\beta}+\pi-\bar{\Delta})^2 + 4(1-\alpha)\bar{\Delta}\pi)^{1/2}].$$

The system is saddle stable when λ is negative.

We need to prove that saddle stability ($\lambda < 0$) implies

$$\pi - (r+\bar{\beta}-\delta)/\sigma = \bar{\Delta} - r - \bar{\beta} = \bar{\Delta}\pi(1-\alpha) - r(r+\bar{\beta}-\delta)/\sigma > 0.$$

First take the case in which $2r + \bar{\beta} + \pi - \bar{\Delta} > 0$.

Then we must subtract the <u>positive</u> square root of $(\bar{\beta}+\pi-\bar{\Delta})^2 + 4(1-\alpha)\bar{\Delta}\pi$ from $2r + \bar{\beta} + \pi - \bar{\Delta}$ to get 2λ. So, it follows that

$$(2r + \bar{\beta} + \pi -\bar{\Delta})^2 > (\bar{\beta} + \pi - \bar{\Delta})^2 + 4(1-\alpha)\bar{\Delta}\pi.$$

Cancellation yields

$$\bar{\Delta}\pi(1-\alpha) - r(r+\bar{\beta}-\delta)/\sigma > 0.$$

The second case is when $2r + \bar{\beta} + \pi - \bar{\Delta} < 0$. This implies directly

$$\bar{\Delta} - r - \bar{\beta} > r + \pi > 0. \qquad\qquad \blacksquare$$

Note, we have also implicitly shown in both cases $\bar{\Delta} - \bar{\beta} > 0$.

The expression for $d\bar{\beta}/d\alpha$ in equation (26) comes from equation (25).

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