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ABSTRACT

Measuring the performance of private equity investments (buyout and venture) has historically only been possible over long horizons because the IRR on a fund is only observable following the fund's final distribution. We propose a new approach to evaluating performance using actual prices paid for limited partner shares of funds in secondary markets. We construct indices of buyout and venture capital performance using a proprietary database of secondary market prices between 2006 and 2017. These transaction-based indices exhibit significantly higher betas and volatilities, and lower alphas than NAV-based indices built from Preqin and obtained from Burgiss. There are a number of potential uses for these indices. In particular, they provide a way to track the returns of the buyout and venture capital sectors on a quarter-to-quarter basis and to value illiquid stakes in funds.

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1. Introduction

In recent decades, private equity has become an important asset class for institutional investors. A 2017 survey of institutional investors finds that 88% are invested in private equity, with nearly a third having an allocation greater than 10% (Whyte (2017)). A typical private equity investment begins with capital commitments at the fund's creation and ends with the final distribution, which is often 12 to 15 years after the initial capital commitment. The return on the fund is determined by the returns on the individual portfolio companies in which the fund invests, and is therefore only fully observable following the fund's final distribution.[1] The underlying value of these portfolio companies fluctuates with firm-specific and economy-wide news in a manner similar to that of public equities, but these fluctuations are usually not fully reflected in the valuations that funds report to their investors. Moreover, since returns are measured at such irregular, infrequent intervals, it can be quite challenging to estimate standard performance parameters such as factor alphas and betas (Axelson, Sorenson, and Strömberg (2014)).

While active markets for trading investments in private equity funds did not exist prior to 2000, in the early 2000s, a secondary market developed on which limited partners (LPs) could transact their stakes in private equity funds. In this paper, we use data obtained from a large intermediary in this market to evaluate the risk and return of private equity funds in a similar manner to the way in which investors regularly use public equity markets to understand the risk and return of publicly traded companies. Using these data we construct transaction-based indices for both buyout and venture capital funds, and use these indices to address a number of questions about the private equity market. These indices provide new insights into the performance of private equity as an asset class. In contrast to the existing literature, we find that neither buyout nor venture funds outperform public markets on a risk-adjusted basis.[2] We also

[1] Funds do report "Net Asset Values (NAVs)" to their investors, which are accounting-based valuations of the fund. These NAVs are adjusted to reflect the fund's actual value, but at any point in time, the gap between the NAV and the value of an investor's stake in a fund can be substantial.

[2] For example, current evidence suggests both buyout and venture funds outperform on a risk-adjusted basis (Cochrane (2005), Korteweg and Sorensen (2010), Higson and Stucke (2012), Harris, Jenkinson, and Kaplan (2014), and Robinson and Sensoy (2016)). It should be emphasized, however, that our study is on a more recent sample period than these other studies, so the results are not comparable and not necessarily inconsistent with the prior studies.

find that NAV-based indices, such as the *Burgiss* index, tend to significantly understate the volatility of private equity as well as its covariance with other asset classes.

The primary difficulty in constructing an index from secondary market data for private equity is accounting for the fact that not every fund trades in every period, and many funds in our sample do not trade at all. In the subsample of funds that could be matched with cash flow data from the *Preqin* database, there are 1,119 fund transactions for 630 funds (372 buyout and 258 venture) from 2006 through 2017, implying that the average fund in our data trades 1.8 times in our sample. Moreover, the funds that do trade are not random draws, and it is possible that sample selection could affect our estimates. We employ two approaches to construct our indices in light of these challenges.

First, we show that if funds transact at random, we can construct an index that tracks the price of a broad portfolio of funds, even if funds within the portfolio do not transact in our sample every period. Second, we account for the possibility that fund transactions are not random, and that the decision to transact in the secondary market could be related to fund market values or other characteristics. To account for such possible sample selection, we create a hedonic index using the approach of Heckman (1979). Using a broad universe of funds, we estimate the parameters of an econometric model using observed transaction prices and each period create an inferred price for every fund, including those that do not transact. We then use these inferred prices to construct our index.[3] We account for measurement error when estimating performance parameters by applying bias adjustments (e.g. Scholes and Williams (1977) and Blume and Stambaugh (1983)).

A striking observation about the transactions-based indices is that they are much more cyclical and exposed to market-wide risk than other indices based on reported NAVs. We estimate the beta of the transactions-based buyout index to be greater than two. As emphasized by Axelson, Sorensen and Strömberg (2014), the return on a buyout fund is essentially the return on a portfolio of highly levered firms. Even if the portfolio firms prior to the buyout have unlevered betas slightly less than 1.0, tripling

[3] Other indices based on secondary markets also use some type of interpolation to infer the prices of non-traded assets. Bond indices, for instance, often employ "matrix pricing" to infer the prices of non-traded bonds.

their leverage, as is typically done in a buyout, should lead to a portfolio beta larger than two. For comparison, we estimate the beta of a buyout index that we construct based on the NAVs reported by *Preqin* using the same set of firms that make up the transaction-based indices, and the beta of a buyout index produced by *Burgiss*. In contrast to the estimates for the transactions-based indices, the estimated betas for both of these more traditional indices are less than 0.5. These indices based on NAVs have much lower betas than the transactions-based indices because NAVs are smoothed over time and do not reflect the information contemporaneously incorporated into market prices.

An implication of the high estimated betas is that, even though the hedonic buyout index averages a 20% return over the sample period, its estimated CAPM alpha is not significantly different from zero. The relatively high average return, therefore, is just enough to compensate investors for their exposure to market-wide risk. This finding is in contrast to our positive estimates of alpha for the two NAV based indices, which mirror the results from the existing literature.

The betas of transaction-based venture capital indices are also higher than that of the corresponding NAV-based indices. We estimate betas for the transactions-based indices to be about 1.0, and for the NAV-based indices to be about 0.3. Since venture funds usually do not lever up their positions in portfolio companies, there is not a Modigliani-Miller ratcheting of betas as in buyouts, which is why venture capital betas tend to be lower than buyout betas. Nonetheless, the estimated betas of the transactions-based indices are large enough to affect inferences about performance. While NAV-based indices of venture funds have estimated alphas that are positive and statistically significant, the transactions-based indices have estimated alphas that are actually negative, although not statistically significantly different from zero.

Besides altering perceptions of private equity performance, the lack of information from secondary markets can also distort investors' portfolio decisions. For example, following the Financial Crisis of 2008 a number of investors believed that their portfolio weight in private equity had substantially increased, since NAVs of their private equity positions dropped much less in value than the market values of their stock holdings. Our analysis suggests that at the time of the Crisis, private equity funds' values had fallen by at least as much as public equities. Therefore, properly measured, the fraction of institutional portfolios made up by private equity did not increase during the Financial Crisis, as was commonly believed.

The indices we develop based on secondary market transactions allow us to evaluate the risk-adjusted, net of fee performance of broad private equity portfolios. While there are a number of papers that have estimated private equity performance, none rely on secondary market data, which is ideal for measuring the risk and returns of securities. We believe that the use of secondary market data, which is the approach commonly applied to measuring risk and return for other types of assets, is the appropriate way to measure risk and return for private equity as well.

Finally, our indices also allow us to value individual funds at any given point in time and to estimate the extent to which a NAV for a given fund differs from its value in any particular year. These market-to-book estimates could potentially be used by investors to value stakes of private equity funds in their portfolios. They suggest that the values of private equity stakes sometimes differ substantially from their NAVs. For example, in 2017 NAVs were 44% lower on average than our estimate of market values for some fund vintages. Therefore, investors using NAVs to assess their portfolios are likely to understate the value of their private equity holdings considerably. These understatements could materially affect investors' portfolio decisions as well as their spending decisions, especially if the organizations set spending levels at a fixed fraction of portfolio values.

While the indices we construct are similar to stock and bond indices that reflect prices achievable to the marginal investor who transacts in secondary markets, we note that the performance of such indices can differ from that of the typical buy-and-hold investor for two reasons. First, the duration of expected cash flows can be held relatively constant in secondary market transactions while the duration of cash flows (and hence, systematic risk) for a buy-and-hold investor dramatically shrinks over the life of the investment.[4] Second, achievable returns of all asset classes in secondary markets are influenced to some

[4] For this reason, the holding period return from buying and holding a corporate bond to maturity can differ from the return earned by engaging in secondary bond markets.

degree by transaction costs. In secondary markets for private equity, most transaction costs appear to be borne by sellers, and buyers can earn a small premium as market makers if positions are held to the end of a fund's life (Nadauld, Sensoy, Vorkink, and Weisbach, (2018), NSVW hereafter). In contrast, our index returns reflect the return from buying *and* selling at secondary market prices. To the extent that buyers earn a premium for market making, this premium is largely canceled out when the position is sold. These indices reflect the achievable benefits from participating in secondary markets similar to indices of other asset classes.

The paper proceeds as follows. Section 2 describes the other approaches that have been used to estimate the risk and return of private equity. In Section 3 we describe the secondary market data we use to construct our indices. In Section 4 we describe the empirical methods we use to construct the indices. In Section 5 we present the indices and evaluate their risk and return. In Section 6 we discuss the way that transactions-based indices can be used by limited partners to value their investments. Section 7 concludes.

2. Prior Work Measuring Private Equity Risk and Return

For most asset classes, investors rely on secondary-market transaction-based indices to measure the asset class' risk and return. Because such secondary markets did not exist for many years in private equity, risk and return have been measured using a variety of alternative approaches. In this section, we discuss each of these approaches.

Prior studies about the investment performance of private equity can be broadly classified into one of four groups, depending on the type of data used. First, a number of studies use fund-level data on cash flows paid to and received by limited partners. Second, other studies use data on private equity funds' investments in their portfolio companies and the distributions they receive from these companies. Third, some studies use venture financing rounds and exit events (IPO, acquisition, and failure), which provide intermittent estimates of market value at these times. Finally, some studies use other proxies for market value, such as NAV or the prices of publicly listed securities, especially publicly traded private equity funds.

Papers that use data on cash flows between LPs and funds have relied on the PME approach, which measures the performance of a fund relative to the public equity market at the same time.[5] Recent studies that use fund-level cash flow data find the PME for buyout funds to be in the range of 1.19-1.23 and for venture funds to be in the range of 1.06-1.36, suggesting that these two types of private equity funds beat public equity markets even after the fees that LPs pay (see Higson and Stucke (2012), Harris, Jenkinson, and Kaplan (2014), Korteweg and Nagel (2016), and Robinson and Sensoy (2016)). Other studies use fund-level cash flows to estimate CAPM betas by estimating cross-sectional regressions of fund $IRRs$ on the $IRRs$ of the public equity market measured over the life of each fund (see Ljungqvist and Richardson (2003), Kaplan and Schoar (2005), and Driessen, Lin, and Phalippou (2012)). These papers generally find betas for both private equity types to be in the range of 1.08 to 1.23. Exceptions are Kaplan and Schoar (2005) who find a buyout beta of 0.41, and Driessen Lin, and Phalippou (2012) who find a venture beta of 2.73.[6] Recent work by Ang, Chen, Goetzmann, and Phalippou (2018) uses fund-level cash flow data to extract an unobserved performance measure that allows a time series of quarterly PE returns to be decomposed into passive and active components. Their estimates suggest CAPM buyout betas of 1.25 and CAPM venture betas of 1.80, with multi-factor market betas in similar ranges.

Papers relying on cash flows between private equity funds and their portfolio firms generally estimate cross-sectional regressions of excess IRRs on the excess IRRs of factor portfolios in the cross section.[7] These papers find CAPM alphas for buyout funds to be in the range of 9.3% to 16.3% with betas in the range of 0.95 to 2.3 (see Frazoni, Nowark, and Philippou (2012) and Axelson, Sorensen, and Strömberg (2014)). In contrast to the estimates presented below, these studies estimate risk and return

[5] The PME approach was originally developed by Kaplan and Schoar (2005). $PMEs$ are calculated by discounting all cash flows of the fund at a rate equal to the total return on the S&P 500 index, and then dividing the future value of cash inflows by the future value of cash outflows. A fund with a PME above 1.0 therefore has outperformed the passive index over the evaluation period. Korteweg and Nagel (2016) provide an extension of the PME that does allow for risk adjustments.

[6] Other papers that investigate fund-level cash flows include Chen, Baierl and Kaplan (2002), Phalippou and Gottschalg (2009), and Phalippou (2012).

[7] The use of IRR is necessary since deal-level cash flows sometimes include intermediate cash flows that occur because of interim recapitalizations or equity injections.

gross of fees. Management fees, because they are a function of performance, covary positively with market returns, so will have a positive beta. Therefore, gross-of-fee betas will be larger than net-of-fee betas since gross-of-fee betas increase with both the beta of the fees and the net-of-fee betas.

Cash flows between private equity funds and LPs and cash flows between private equity funds and portfolio companies are not measured over uniform intervals. Axelson, Sorensen, and Strömberg (2014) argue that the irregular intervals over which returns or IRRs are measured can affect estimates of a fund's risk and return. The IRR of a fund is a function of the IRRs of the individual deals in which the fund invests. Each of these is only observable after the deal exits. The intervals over which the IRRs are calculated vary across deals and, to the extent that funds return capital at different times, across funds as well. The irregular intervals at which venture financing events occur will also cause variation in measured return horizons. Using IRRs or returns compounded over irregular intervals can result in surprisingly large biases when estimating CAPM parameters. Axelson, Sorensen, and Strömberg (2014) simulate deal-level cash flows and estimate the CAPM using cross-sectional variation in IRRs. In some specifications, the beta is underestimated relative to the true beta by 116% while in other specifications, it is overestimated by 123%.[8]

A number of papers use valuations in venture financing rounds to measure the risk-adjusted performance of venture funds. These estimates are also gross of fees, so estimates of both risk and return are inflated relative to the risk and return received by investors. Since portfolio firms that receive more rounds of financing tend to be the better performing investments, these papers have to adjust for the sample selection implicit in their reliance on data from financing rounds. Papers that estimate the parameters of sample selection models find CAPM alphas for venture firms in the range of 32% to 38% with betas of 1.9 to 2.7 (see Cochrane (2005) and Korteweg and Sorensen (2010)). Other papers use venture financing events to create hedonic and repeated sales indices that account for sample selection

[8] One common solution used to account for the irregular intervals over which IRRs are measured is to use log IRRs. This approach, however, requires specific parametric assumptions to correctly estimate alpha.

and find alphas in the range of 4% and betas in the range of 0.6 to 1.3 (see Peng (2001) and Hwang, Quigley, and Woodward (2005)).

Such indices can also be problematic, however, for three reasons. First, financing events represent prices at which some investors acquire shares in startups, but not all investors can purchase at that price and investors are not usually allowed to exit at that time. Second, in venture-backed firms, shares from different financing rounds have different rights and therefore different values. Since newly created shares in financing events often have more rights and therefore are worth more than old shares, implied valuations based on post-money valuations are likely to be upward biased (see Gornall and Strebulaev 2018). Finally, returns from venture financing events are gross of fees, leading these studies to overestimate both the risk and returns to limited partners that invest directly in venture capital funds.

Jegadeesh, Kraussl, and Pollet (2015) take another approach to estimating private equity performance based on the returns to publicly traded private equity securities. In general, these securities are publicly traded funds of funds and private equity firms. Their returns are likely to be correlated with those received by limited partners in private equity funds but are different for several reasons. Most importantly, the publicly traded securities of private equity firms tend to be of the general partners, whose claim is the present value of future fees and carried interest earned by the fund, rather than the returns earned by the limited partners in a particular fund. In addition, large, publicly traded buyout firms such as Blackrock and KKR hold a variety of investments other than private equity, including hedge funds, real estate, advisory services, etc. Finally, some of the publicly traded private equity firms are funds of funds, which are a function of the claims of limited partners. However, these funds of funds charge an extra layer of fees that sometimes vary with fund performance, making it difficult to infer the returns earned by limited partners in the underlying funds held by the publicly traded funds of funds.

Overall, evidence in the literature suggests that both buyout and venture funds tend to perform well. The average PME for both types of funds is usually estimated to be greater than 1, which implies that these funds outperform public markets. This outperformance could reflect positive alpha, the greater risk of private equity funds relative to the market, or both. While most studies of private equity performance find evidence of positive alpha, estimates of fund betas are somewhat mixed, with some studies finding betas in the range of 1.0 and other studies finding betas well above 2.0.

3. Constructing an Index Measuring Private Equity Returns

Private equity returns are a function of transaction prices, fund contributions and fund distributions. While we observe quarterly distributions and contributions for a large sample of funds, we observe market prices only for transactions intermediated by the firm that provides us with our data. In this sample, no fund transacts in every quarter and some funds never transact at all.[9] In our baseline specification we estimate a hedonic model that uses actual transaction prices to infer the prices of funds that do not trade. This approach helps avoid potential biases associated with sample selection. The inferred prices contain measurement error that is assumed to be zero and independent, similar to the setting described in Blume and Stambaugh (1983). In addition, the transactions that do occur are highly nonsynchronous. Nonsynchronous trading causes returns of the individual assets in a portfolio to be observed over different time intervals as in Scholes and Williams (1977) and Lo and MacKinlay (1990). Our observed index returns therefore contain measurement error coming from two sources: non-trading and nonsynchronous trading.[10] Both sources of measurement error can induce bias in estimated variances and covariances. We next show how we construct our indices and correct for biases in estimated moments that arise from measurement error.

3.1. Calculating Returns

Let $P_{i,t}$ denote the secondary market price of a \$10 million commitment to private equity fund i at the end of quarter t.[11] Suppose at the end of quarter $t-1$, a purchasing LP acquires a \$10 million

[9] It is possible that some funds do not transact through the intermediary providing our data, but do transact through one of their competitors.

[10] Nontrading can be modeled as extensive nonsynchronous trading as in Lo and MacKinlay (1990) if all securities trade at some point in time. In our data, however, some funds never transact.

[11] To avoid potential confusion that could arise from our use of notation and language, we note that the term *price* in private secondary markets often gets used in two different ways. Prices in the secondary market are frequently quoted as a percent-of-NAV price, notated in our equations as π_t. The term *price* could also reference the actual dollar amount an LP paid in a secondary transaction. We notate the dollar price in our equations as P_t.

commitment to each of N different private equity funds from a selling LP .[12] If the purchasing LP holds for one period and then sells all positions at the end of quarter t, the buy-and-hold return for the portfolio is given by:

$$r_t = \frac{\sum_i P_{i,t} + D_{i,t} - C_{i,t}}{\sum_i P_{i,t-1}} - 1, \tag{1}$$

where $D_{i,t}$, $C_{i,t}$ represent total distributions and calls associated with the position in fund i during quarter t. Equation (1) defines the arithmetic return on a price-weighted portfolio of N funds and can be extended to other weighting methods by appropriately scaling prices and cash flows.

While we observe quarterly distributions and calls for a broad population of funds, we observe $P_{i,t}$ only for the funds that transact. We can scale the numerator and denominator of (1) by N to obtain:

$$r_t = \frac{\bar{P}_t + \bar{D}_t - \bar{C}_t}{\bar{P}_{t-1}} - 1 \tag{2}$$

where an overline indicates an average. The N funds in the portfolio over quarter t classify a full "population" of funds. Our objective is to estimate the population average price, $\bar{P}_t$, using the sample of funds from this population that transact in quarter t to estimate the return on the portfolio of N funds. We take two approaches to estimate $\bar{P}_t$.

In the first approach, we simply use the transaction prices we have available to estimate $\bar{P}_t$. We refer to indices constructed in this manner as *naïve* indices since they ignore the possibility that fund transactions are nonrandom. In the second approach, we account for sample selection by developing a hedonic model, following Heckman (1979), in which we infer the prices of all funds in the population based on observable fund characteristics, and use all inferred prices to estimate $\bar{P}_t$. We refer to indices constructed in this manner as *hedonic* indices.

For comparison, we also construct index returns using equation (1) assuming, as is commonly done in practice, that the NAV of fund i reported at the end of quarter t equals $P_{i,t}$. Although we observe NAV for every fund in the sample, a fund's NAV is not a market-determined estimate of value. Instead,

[12] The some of the commitment (in our case $10 million) is arbitrary, in our example we only need all commitments to be of the same size. These results are generalizable to holdings of commitments of different sizes.

NAV represents an appraisal rendered by GPs who tend to smooth their NAVs over time and respond slowly to market information. (See, for example, Ewens, Jones, and Rhodes-Kropf (2013)). We also examine the moments of the *Burgiss* index, which is also constructed from funds' NAVs as a measure of value.

3.1.1 Naïve indices

To create naïve indices, we estimate $\bar{P}_t$ as the simple average across the k_t funds that transact in quarter t, $\bar{P}_{k,t}$. We first scale transaction prices by *NAV*, $\pi_{i,t}$,

$$\pi_{i,t} = \frac{P_{i,t}}{NAV_{i,t}}. \tag{3}$$

We can then express the average price as:

$$\bar{P}_{k,t} = (\bar{\pi}_t)(\overline{NAV}_t) + Cov(\pi_{i,t}, NAV_{i,t}). \tag{4}$$

since for any two random variables, X and Y:

$$\overline{XY} = (\bar{X})(\bar{Y}) + Cov(X, Y). \tag{5}$$

While we observe $\pi_{i,t}$ only for the k_t funds that transact, we do observe $NAV_{i,t}$ for the full population of N funds. An advantage of expressing average price using (4) is that it enables us to use all of the NAVs in the full population of N funds to estimate $\overline{NAV}_t$ and consequently $\bar{P}_{k,t}$, regardless of whether all funds transact in quarter t. While NAV is not the market price it is likely to contain some pricing information for the population of N funds. To compute the returns for naïve indices we use $\bar{P}_{k,t}$ as defined in (4) to estimate $\bar{P}_t$ in (2).

3.1.2 Hedonic Indices

Because some types of funds are more likely to transact than others, an estimate of $\bar{P}_t$ based on the average of observed prices could be biased because of nonrandom sample selection. We therefore develop a hedonic model as a way to obtain an estimate of $\bar{P}_t$ at a point in time conditional on the fund's characteristics and relevant economic state variables.[13] While it is common to use hedonic models to

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[13] For discussion of hedonic models and examples of their uses in other contexts, see Gatzlaff and Haurin (1998), Pakes (2003) and Hwang, Quigley, and Woodward (2005).

estimate price changes for a basket of goods with *fixed* characteristics over time (see, for example Pakes (2003)), our objective is to understand the price changes of a portfolio when some transaction prices are not observed, and fund characteristics *evolve* over time. Doing so enables us to estimate the return on the set of funds accessible to investors.

Suppose we use available data and OLS to estimate the equation:

$$\boldsymbol{\pi}_k = \boldsymbol{X_k}\widehat{\boldsymbol{\theta}}_{\boldsymbol{k}} + \boldsymbol{e_k}, \tag{5}$$

where $\boldsymbol{X_k}$ denotes a $k_t \times p$ matrix of p characteristics observable by the end of quarter t for the k funds that transact in quarter t, $\boldsymbol{\pi}_k$ denotes the $k_t \times 1$ vector of observed scaled market prices for transactions in quarter t, and $\widehat{\boldsymbol{\theta}}_{\boldsymbol{k}}$ represents a $p \times 1$ vector of estimated coefficients. In addition, assume that we observe the same p characteristics for all N funds in the population, represented as $\boldsymbol{X_N}$, an $N \times p$ matrix. Our approach uses the estimated vector of coefficients, $\widehat{\boldsymbol{\theta}}_k$, to estimate the average fitted price for the full population of N funds, $\bar{P}_k$,

$$\bar{P}_{h(k),t} = \frac{1}{N}\left[\left(\boldsymbol{X_N}\widehat{\boldsymbol{\theta}}_{\boldsymbol{k}}\right)'\boldsymbol{NAV_N}\right], \tag{6}$$

where subscript $h(k)$ refers to the hedonic approach using the k transactions, and $\boldsymbol{NAV_N}$ represents the $N \times 1$ vector of NAVs for the entire population at quarter t.

Let $\widehat{\boldsymbol{\theta}}_{\boldsymbol{N}}$ denote the parameters estimated for the equation given in (6) using the full population of N funds in quarter t (which are unobserved). If we could use $\widehat{\boldsymbol{\theta}}_{\boldsymbol{N}}$ in place of $\widehat{\boldsymbol{\theta}}_{\boldsymbol{k}}$ in (6), the resulting estimate of average fitted price, $\bar{P}_{h(N),t}$ would be identical to the population average ($\bar{P}_t$) with no measurement error, *regardless* of the explanatory variables used in $\boldsymbol{X_N}$. This highlights that our objective is not to estimate the parameters of a "true" model. Instead, our more moderate objective is to obtain an unbiased estimate of $\widehat{\boldsymbol{\theta}}_{\boldsymbol{N}}$ using *some* set of reasonable explanatory variables. When we use only the k_t funds that transact, $\bar{P}_{h(k),t}$ is an unbiased estimator of $\bar{P}_t$ provided that $\widehat{\boldsymbol{\theta}}_{\boldsymbol{k}}$ is an unbiased estimator of $\widehat{\boldsymbol{\theta}}_{\boldsymbol{N}}$. Appendix C provides a formal development of this result.

In general, the OLS estimate of $\widehat{\boldsymbol{\theta}}_{\boldsymbol{k}}$ is a biased estimator of $\widehat{\boldsymbol{\theta}}_{\boldsymbol{N}}$ unless regression residuals are independent of the selection process. To account for the possibility that regression residuals are not independent of selection, we estimate a standard Heckman (1979) sample selection model by maximum likelihood. We jointly estimate the parameters of a linear pricing equation as in (6) and a probit model of the selection outcomes as a function of fund characteristics and economic state variables.

We propose the fraction of LPs invested in a fund that are pension funds as a variable that satisfies the exclusion restriction in the selection equation.[14] Pension funds are typically buy-and-hold investors with the main investment objective of matching the duration of their liabilities. As such, pension funds are less likely to trade in the secondary market relative to other PE investors than other private equity investors, and conditional on trading, their transactions are less motivated by fund quality than other considerations, such as duration. Empirically, NSVW (2018) document that pension funds are less likely than other investors to sell their private equity stakes. Moreover, the characteristics of the LPs in general are unlikely to be correlated with innovations in transaction prices because buyers do not know the identity of the seller (or the type of seller) during the price discovery process of the transaction, since the intermediary providing our transaction data keeps the names of clients confidential. Competition among bidders should also move prices to be more independent of the seller's identity.

While the fraction-of-pension-fund-ownership variable is in our main specifications, we note that when we exclude this variable parameter estimates and standard errors change very little, suggesting that in our setting we have sufficient nonlinearity in the inverse Mills ratio and sufficient curvature in the MLE function to identify parameters with reasonable precision, even without a variable that satisfies the exclusion restriction in the selection equation.

[14] While the parameters of the Heckman model are identifiable under the normality assumption, Monte-Carlo studies indicate that the small sample properties of the MLE estimator can be improved by adding a variable associated with selection, but uncorrelated with prices to the selection equation. See Puhani (2000) for a survey. Adding such an exclusion restriction helps identify parameters when the inverse Mills ratio is a near-linear function, and variables used to model prices and selection largely overlap. Under these conditions the MLE objective function can be quite flat without exclusion restrictions, resulting in large standard errors and poor small sample performance.

3.2 Accounting for Measurement Error in Index Parameter Estimates

We use our naïve, hedonic, and NAV indices to estimate parameters such as average return, volatility, alpha, and beta. Our measured index returns are subject to nonsynchronous trading, and i.i.d. measurement error that could lead to measurement error in estimated index parameters. Appropriate adjustments can be made, however, to ensure our estimated parameters are unbiased. In deriving these adjustments, we make the following three assumptions regarding fund returns and transactions:

1) Fund returns from the end of one quarter to the next, are independent and identically distributed across time.
2) Each fund is assigned a random transaction time relative to the end of the quarter, $t - s_i(t)$, where t defines the end of the quarter, $s_i(t)$ is independent and identically distributed across time, and $0 < s_i(t) < 1$.
3) In quarter *t*, a set of $k_t < N$ funds actually transact in quarter t.

Assumption (1) is convenient for deriving the appropriate bias adjustments that arise from measurement error and non-synchronous trading. Assumption (2) specifies the nature of non-synchronous trading. Assumption (3) highlights that not all funds transact, which results in i.i.d. measurement error in our estimates of return.

We denote observed index returns as r_t^*. These are index returns with measurement error estimated using the naïve or hedonic approaches. We denote the true (unobservable) returns as r_t, the quarterly return from end-of-quarter to end-of-quarter, as defined in equation (1). We use r_t^* to obtain unbiased estimates of the parameters of r_t. In Appendix D we show that to a first-order approximation,

$$\begin{gathered} E[r_t^*] = E[r_t] \\ Var[r_t^*] + 2Cov[r_t^*, r_{t-1}^*] = Var[r_t] \\ Cov\left[r_t^*, r_{m,t}\right] + Cov\left[r_t^*, r_{m,t-1}\right] = Cov\left[r_t, r_{m,t}\right]. \end{gathered} \tag{7}$$

where $r_{m,t}$ denotes the public market return, which we assume we can observe without measurement error. The bias adjustments in (7) are valid for naïve indices only if funds transact at random. They are valid more generally for hedonic indices. Equation (7) shows that average estimated returns are unbiased. Estimated variances and covariances, however, require some adjustments to be unbiased. The moments given in (10) can also be used to derive other parameters, such as alpha and beta.[15]

4. Data on Transactions in the Private Equity Secondary Market

A large intermediary in the private equity secondary market provides us with data on all the secondary market transactions intermediated by their firm similar to data used in NSVW (2018).[16] These data identify the fund name, the vintage, the total capital committed by the seller, the amount unfunded by the seller, the purchase price, and the transaction date for funds that transacted from June of 2000 through December of 2017. Since the database contains only five transactions before 2006, we eliminate these and conduct our analysis using transactions that take place from 2006 to 2017.

We clean the data as detailed in Appendix A and pull the most recent transaction for each fund each calendar quarter. After cleaning the data, we are left with 3,404 fund transactions for 2,424 funds, which we refer to as the *transaction sample*. This transaction sample is made up of 2,066 buyout transactions and 1,338 venture transactions for 1,427 buyout funds and 997 venture funds.

LPs transact in the secondary market for many reasons, primarily, according to practitioners, for portfolio rebalancing purposes.[17] However, transactions on funds that are 10 years or older, referred to as tail-end transactions by practitioners, are essentially transactions in which the buyer is speculating about the value of the one or two particular portfolio firms left in the fund. For this reason, the prices of tail-end

[15] For our main results we do not report biased-adjusted parameters for the NAV-based indices for two reasons. First-NAV-based indices are not subject to the same kinds of measurement error as the transaction-based indices. We observe NAV for every fund in our chosen population, and NAV's are reported as of the end of each quarter. Second, we believe that bias adjustments are not standard in practice and we wish to compare the parameters of the transaction-based indices with those commonly estimated using the NAV-based indices. That being said, GPs appear to smooth their NAVs over time and respond slowly to market information, which leads to significant autocorrelation in NAVs as well as cross-autocorrelation with lagged market returns. For this reason Ewens, Jones, and Rhodes-Kropf (2013) apply bias adjustments identical to ours when estimating betas using NAV-based indices. We therefore also compute bias adjusted parameters for the NAV-based indices and discuss these results in the paper.

[16] The data used below extend the data from NSVW (2018) through the end of 2017.

[17] NSVW (2018) provide discussion and analysis of participants and motivations for secondary market trading of private equity.

transactions do not reflect the fundamental economics of private equity as an asset class, given that most of the fund is liquidated before the time of the transaction. Similarly, transactions on young funds, those 3 years or younger, primarily reflect liquidity demands of investors and so prices of young transactions do not contain much economic content other than demand for immediate liquidity.[18]

For these reasons, transactions of funds in the 4-9 year range are most likely to reflect the value of private equity as an asset class. To the extent that common factors affect all private equity funds, these transactions are likely to be informative about other funds' values as well. For this reason, we construct our indices using funds that are between 4 and 9 years old. We label the subset of the transactions sample for funds that are 4-9 years old as the *baseline-transaction sample.*

We obtain data on other fund characteristics, such as calls, distributions, NAV, fund LP type, and size, from *Preqin*, and clean these data as detailed in Appendix A. Within each calendar quarter we sum all contributions and distributions (separately) for a given fund. A reporting lag causes our data to be missing information for the last quarter of 2017, and hence, our data from *Preqin* extend from the first quarter of 2006 through the third quarter of 2017 with quarterly information on 2,287 unique funds, of which 1,251 are buyout and 1,036 are venture. We refer to these data as the *Preqin sample.*

We then merge the transactions sample with the *Preqin sample*, some of which is done by hand (see Appendix A for details). After this process, we end up with a sample on 630 funds (372 buyout and 258 venture), for which 1,119 transactions occurred through the intermediary providing our data between 2006 and 2017. We refer to these data as the *full-merged sample.* We also merge the baseline transaction sample (for funds 4-9 years old) with *Preqin* and refer to this sample as the *baseline-merged sample*.

Table 1 reports summary statistics for the various samples, with Panel A containing statistics for buyout funds and Panel B for venture funds. This table reports statistics for both the full merged sample and its complement, i.e., all quarter-fund observations in *Preqin* for which no transactions occurred. Table

[18] An important feature of these markets is that transactions are often done for portfolios of funds rather than individual funds. In Appendix B, we discuss the way in which the portfolio nature of some of our transactions affects the construction of the indices.

1 also reports statistics separately for the *baseline-merged sample* (the intersection of the set of funds in the *Preqin* universe that are four to nine years old with the transactions sample) and its complement.

The first three rows of each panel report the mean, first quartile (Q1) and third quartile (Q3) for transaction prices as a fraction of NAV, denoted throughout the paper as $\pi_{i,t}$. Consistent with prior findings, funds on average transact at a discount relative to NAV (see NSVW (2018)). Discounts are smaller for baseline transactions than for other, less typical deals. The overall average $\pi_{i,t}$ for buyout funds is between 0.82 to 0.83 but for baseline transactions is 0.89. Similarly, the overall average $\pi_{i,t}$ for venture funds is between 0.80 to 0.84 but for baseline transactions is 0.88 to 0.92. Among venture baseline transactions, the third quartile for $\pi_{i,t}$ is 1.21, highlighting the fact that many venture funds transact at a premium to NAV.

The deviation between a fund's NAV and its market price will depend on the performance of the investments that the fund has made but not yet exited. A fund that has made poorly performing investments will have a market price less than NAV, so a trade for less than NAV is not necessarily reflective of a liquidity discount. Similarly, a fund trading at a premium to NAV likely reflects good performing investments that the fund has made. The point is that the economic discount or premium at which a transaction occurs should be measured relative to the (unobservable) underlying value of the fund's assets, not the NAV.[19]

Funds that transact in the secondary market tend to be larger than the average fund in *Preqin*. The average fund size in the buyout full-merged sample is about $4.7 billion, compared to an average fund size in the buyout-compliment sample of about $1.8 billion. Similar patterns are found for venture, though venture funds in our data average about 80% to 90% smaller than buyout funds. The average fund age of transacting funds tend to be around eight to ten years in our data. In addition to being larger, buyout funds that transact display higher average $PMEs$, in the range of 1.16 to 1.18, compared to 1.12 for non-

[19] NSVW (2018) develop this issue further. They argue that a second measure of a trade's discount relative to fundamental value is the difference in returns between buyers and sellers. If transactions always occurred at fundamental values and expected returns do not change over time, then buyer and seller returns should be approximately the same. Instead, these authors find that buyers of LP stakes outperform sellers, suggesting that, on average, transaction prices tend to be lower than fundamental values.

transacting funds. However, the performance of transacting venture funds is similar to that of non-transacting funds: transacting funds have average PMEs between 0.95 and 1.04, while the average non-transacting fund PME is 1.01.[20]

Figure 1 reports the number of transactions per quarter for the baseline-transactions sample. Buyout and venture transactions are highly correlated in the time series, though the buyout market has been more active between 2009 and 2016 than the venture market. Venture transactions picked up substantially in 2014 and 2015 before tapering off slightly in 2016 and 2017.[21] Table 1 indicates the average number of transactions in the baseline transactions sample per quarter is about 19 for buyout funds and about 10 per quarter for venture funds.

5. Estimates of Secondary Market Based Private Equity Indices

5.1 Estimation Details

We estimate the return on naïve indices as described in section 3.1.1. We use the baseline-transactions sample to estimate $\bar{\pi}_t$ separately each quarter. To estimate $\overline{NAV_t}$ we combine all NAVs as reported in the full *Preqin* sample with NAVs reported by the intermediary that supplied the transactions data. For any fund in the baseline-transactions sample not in *Preqin* we use the NAV as reported by the intermediary. We estimate $\overline{NAV_t}$ separately each quarter. We use this same sample to obtain a single estimate of $Cov(\pi_{i,t}, NAV_{i,t})$ using all funds and quarters. For buyout funds, we have enough observations to estimate $\bar{\pi}_t$ in every quarter using the baseline-transactions sample. The average number of transactions per quarter for this sample is 19.1 with a first quartile of 8 and a third quartile of 27 (see Table 1).

For venture funds, we unfortunately do not have enough observations to estimate $\bar{\pi}_t$ in every quarter. Seven quarters in the baseline transactions sample have zero venture transactions, making it impossible to compute returns for the 14 quarters before and after each missing observation. (See Figure

[20] We calculate the *PME* for each fund using all cash flows up to the most recent date for which we have cash flow data in *Preqin*, using NAV as the terminal value for funds that have not liquidated.
[21] Deal volume in our sample is also influenced by fluctuations in the market share of our data provider.

1). Missing returns complicates the estimation of autocovariances and the bias adjustments outlined in section 3.2. A solution is to infer the missing transaction prices, which we do using a hedonic model. Given the lack of transactions in some quarters, however, we forgo calculating a naïve venture index.

We estimate the return on hedonic indices as described in section 3.1.2. To implement the sample selection model given in (9) we need to select the explanatory variables $\boldsymbol{x}_{i,t}$ and $\boldsymbol{z}_{i,t}$. Table 2 lists the explanatory variables we include in the model. The first 6 rows of Table 2 list the "state variables" that are constant across funds and vary only over time. The final 6 rows of Table 2 list the fund-specific variables that vary both across funds and over time.

We estimate the parameters of the sample selection model by maximum likelihood, using the full panel of data in the *Preqin* sample including both the full-merged sample and its compliment.[22] We also estimate the hedonic model by OLS using the full merged sample. In this sense we deviate slightly from Section 3.1.2 and Appendix C that advocate estimating the model separately by quarter. Although estimation of the hedonic model by quarter may be preferable, data limitations prevent us from doing so.

We estimate the standard errors of the hedonic model parameters using the quasi-maximum-likelihood approach of White (1982), which accounts for heteroscedasticity and any cross-sectional or time-series dependence, and is valid even if the true density of the residuals is not normal. We estimate standard errors for index parameters (alpha, beta, etc.) using a two-step bootstrap procedure based on the subsampling methods of Politis and Romano (1994). Details can be found in Appendix E.

The approach accounts for the two-step estimation procedure used to estimate the index parameters. For the buyout naïve index, the first step involves estimating parameters such as $\bar{\pi}_t$, $\overline{NAV}_t$, and $Cov(\pi_{i,t}, NAV_{i,t})$. For the hedonic indices the first step involves estimating the parameters of the hedonic model. After constructing the indices using these estimated parameters, the second step

[22] Monte Carlo experiments indicate that MLE is often more efficient than the two-step approach originally proposed by Heckman (1979) (see Puhani (2000)). In addition, MLE allows for straightforward computation of robust asymptotic standard errors, is convenient for conducting standard model diagnostics, and imposes the natural restriction that $|\rho| \leq 1$ where ρ represents the correlation between $\epsilon_{i,t}$ and $\upsilon_{i,t}$. Regardless, our results are virtually unchanged using either approach to estimate the parameters of the model.

involves estimating the index parameters themselves. Our two-step bootstrap approach also accounts for any cross-sectional and time-series dependence in the data.[23]

5.1. Pricing Parameters

Table 3 presents our estimates of the parameters for the sample selection model for both buyout and venture funds. The selection model involves two stages, a selection equation and a pricing equation. Panel A reports estimates of the selection equation, while Panel B reports estimates of the pricing equation. Panel B contains the parameters we use to infer the prices of funds that do not transact, using both OLS and the Heckman sample selection model.

The estimates presented in Panel A suggest that a number of variables are associated with fund selection. Among the economy-level variables, the estimates indicate that funds are more likely to transact when public equity markets are valued highly (the aggregate market-to-book ratio for equities (MTB_t)), and when private equity has been performing well, (the average fund's PME). The coefficient on MTB_t for buyout funds is 0.20 (t-statistic = 1.7) and the coefficient on the average PME_t is 1.05 (t-statistic = 1.8). Both coefficients are higher and statistically significant for venture funds. Both fund types are also more likely to transact when the valuation confidence index is high, and when the crash confidence index is low. These represent times when institutional investors believe the public market is not overvalued, but the likelihood of a crash is relatively higher.

Among the fund-specific variables, fund size strongly affects the likelihood that a stake in the fund is traded on the secondary market for buyouts and venture. Not surprisingly, larger funds are much more likely to transact than smaller funds. In addition, fund age also affects the likelihood of a transaction. The coefficients on both $AGE < 4$ (dummy equal to one if fund is less than 4 years old) and $4 \leq AGE \leq 9$ (dummy equal to one if fund is 4 to 9 years old) are negative and statistically significant. Funds older than 9 years (the group excluded in the age-dummy-variable classification) are the most

[23] Alternatively, we estimate standard errors for index parameters by GMM assuming we observe index returns without estimation error. Our reported bootstrapped *t*-statistics are about 40% lower on average than the GMM *t*-statistics that ignore first-stage estimation error.

likely type of fund to be traded on the secondary market (although there are fewer such funds, so the most common transaction is for funds between 4 and 9 years old, which we refer to as "baseline transactions").

Finally, and importantly for our econometric design, stakes in private equity funds held by pension funds are less likely to transact than stakes held by other types of investors for both buyout and venture. The coefficient on $Frac.Pension_i$ (the fraction of LPs that are pension funds) is -0.44 for buyout and -0.23 for venture, both of which are statistically significantly different from zero. Consistent with the views of practitioners communicated to us through private conversations, pension funds appear to have less pressure to change portfolio strategies than other investors, so are less likely to sell stakes in the secondary market. This negative relation between $Frac.Pension_i$ and the likelihood of transacting suggests that this variable satisfies the exclusion restriction, since the identity of the investors affects the likelihood of a trade. However, there is no reason why it should affect the pricing of deals, since the prices are determined by a competitive bidding process in which the identity of sellers is withheld from buyers in the bidding process.

The bottom rows in Panel A test the null hypothesis that there are omitted variables correlated with both fund selection and fund premiums, $H_0: \rho = 0$. The Heckman (1979) sample selection model helps ensure that our parameter estimates are unbiased even if $\rho \neq 0$, while if $\rho = 0$ then OLS estimates of the pricing equation are also unbiased. The low t-statistic on ρ for buyout funds (-0.3) as well as the Wald, likelihood ratio, and Lagrange multiplier test fail to reject the hypothesis that ρ equals zero. In contrast, the high t-statistic on ρ for venture funds (-7.0) and the low p-values on the Wald, and likelihood ratio test statistics indicate that we can reject the null hypothesis that $\rho = 0$ for venture funds. These findings are consistent with the OLS estimates of the pricing equation in Panel B. For buyout funds, the Heckman and OLS test statistics are virtually identical, suggesting that both approaches provide consistent estimates. In contrast, the estimated parameters are somewhat different for the Heckman and OLS models using venture funds, which, combined with the nonzero estimated ρ, suggests that the OLS estimates for venture funds potentially suffer from selection bias.

The results in Table 3 Panel B for the pricing equation indicate that scaled market prices tend to be higher for both buyout and venture funds when the crash confidence index is high. These are times when a crash in the public market appears less likely from the view of institutional investors.

Turning to results for the fund-specific variables, scaled prices tend to be higher for funds with higher valuations as measured by $NAV_{i,t}$, and for transactions aged between 4 and 9 years. We observe higher valuations for funds that are 4 to 9 years old, in part, because such funds are likely to have more capital invested than younger or older funds. Since portfolio firms increase in value on average and NAVs reflect historical cost, NAVs of funds with more invested capital tend to have relatively high deviations from the underlying value of their portfolio firms.

The bottom row of Panel B reports the R-square of the pricing regression. Our hedonic pricing model is able to explain about 34% of the variation in scaled prices for buyout funds, and about 30% of the variation in scaled prices for venture funds.

5.2. Private Equity Indices over Time

Figure 2 graphs the size weighted hedonic indices for both buyout and venture over the 2006-2017 sample period. For comparison we also present the performance of the public equity market, as reported on Ken French's website, the index based on *Preqin* reported NAVs we describe above, and the *Burgiss* Index that is used by practitioners.

Figure 2 illustrates that the transactions-based indices are much more volatile than the two NAV based indices (*Preqin* and *Burgiss*). NAVs adjust slowly to changes in the value of the private equity funds' assets. The NAV-based indices do not immediately adjust to information likely to affect future cash flows in the manner of public equity markets. Consequently, the indices based on NAVs tend to be "smoother" than the transactions based private equity indices or public equity markets.

For this reason, NAV based indices do not reflect changes in information to the degree that our transactions-based index does. For example, the only sharp decline in equity markets during our sample period occurred during the 2008 Financial Crisis. During this period, the NAV indices declined somewhat because assets were written down, but not nearly as much as overall public equity indices. In contrast, the transactions-based indices decline at least as much as public equity markets: during 2008 the public equity market index declined 37% while the transactions-based buyout index declined 51% and the transactions-based venture index declined 33%. Even though private equity positions were not written down completely during the 2008 crisis, their value appears to have declined by at least as much as the public equity markets at that time.

5.3. Risk and Return of Private Equity Indices

5.3.1. Buyout Indices

Table 4 reports estimates of risk and return for the buyout transaction-based indices we create and for alternative buyout indices as well. Panel A reports estimates for hedonic indices that are size-weighted and price-weighted, as well as the naïve index that is price-weighted. Panel B reports moment estimates for size-weighted *Preqin* NAV-based indices and the differences between these and the Hedonic ones.

Consistent with Figure 2, the estimates in Panel A of Table 4 indicate that buyout funds have performed very well over our sample period (2006 to 2017). The two hedonic indices each have an expected annual return for a baseline fund between 18-20% and the naïve index has an expected return of 30%. These values are relatively high because our sample period was a good one for the buyout industry.

In addition to the expected returns, the estimated betas of the transactions-based indices are also large. The estimated betas of the two hedonic buyout indices are over 2 (2.43 and 2.25) and for the naïve buyout index is 1.43. These estimates are substantially larger than those reported in the prior literature (an exception is Axelson, Sorensen, and Strömberg (2014)). However, betas of this magnitude *are* consistent with theory. Private equity funds are portfolios of equity positions in leveraged buyouts. Since buyouts tend to be much more highly levered than public firms, Modigliani-Miller Proposition 2 implies that buyouts should have substantially higher betas than public firms. For example, Axelson, Jenkinson, Strömberg, and Weisbach (2013) report a mean debt to total capital ratio of 70% in their sample of 1,157 LBOs. In contrast, typical large publicly traded firms have approximately a 20-25% debt to total capital ratio. Even if the firms experiencing buyouts are less risky than average so have asset betas somewhat smaller than 1, the equity portion of the LBO should nonetheless have a beta greater than 2 given these leverage ratios (see Axelson, Sorensen, and Strömberg (2014) for more discussion).

A consequence of these relatively high estimates of beta is that even though the expected returns of the transactions-based buyout indices are high (at least 18%), the estimated alphas are not statistically significantly different from zero. The high returns earned by buyout funds appear to reflect the "beta risk" coming from the leverage in the buyouts rather than an abnormal return beyond what is justified by their risk.

Panel B of Table 4 compares the estimates from the size-weighted hedonic transactions-based index to the more commonly used NAV based indices, both the one we construct from the *Preqin* database and the one distributed by *Burgiss*. Both these indices have lower expected returns than the transactions-based indices, with expected returns ranging from 10 to 13 percent. In addition, there is a large difference in volatility between the transactions-based indices and the NAV-based ones, with the standard deviation of the NAV-based indices less than that of the public equity market (9% for both *Preqin* and *Burgiss* compared to 16% for the public equity market), and the standard deviation for the transactions-based index equal to 40%.

The estimated betas for these indices are less than 0.5, equal to 0.45 for both *Preqin* and *Burgiss.* It is hard to reconcile betas this low for buyout funds with theory; buyout funds take companies whose assets presumably have betas of close to 1, and add substantial leverage to them. Modigliani-Miller Proposition 2 implies that the leverage should substantially increase beta, not decrease it.[24]

Regardless of the reason for the low estimated beta for the NAV indices, they have an important implication. Because of the low betas, standard estimates of alpha are positive (5% for *Burgiss* and 8% for *Preqin*) and statistically significantly different from zero for the *Preqin* index. These positive alphas

[24] One explanation for the low betas of NAV-based indices is that NAVs do not tend to adjust at the time of market movements because the information in these movements is not incorporated into the NAVs at the time it becomes public. The bias adjusted beta for the *Preqin* index that should account for cross-autocorrelation between the index and lagged market returns, however, is only 0.73, and for the *Burgiss* index is only 0.67. The bias adjusted volatility that accounts for autocorrelation is only 0.13 for both the *Preqin* and *Burgiss* indices.

differ from the zero alphas estimated for the transactions-based indices. Even though the transactions-based indices have higher average returns than the NAV based ones, the transactions-based buyout indices are associated with an alpha close to zero because the estimated betas are so much higher. In contrast, the NAV-based buyout indices are associated with a positive alpha.

5.3.2. Venture Capital Indices

Table 5 reports estimates of risk and return for venture funds in the same format as Table 4. As discussed in section 5.1, we do not calculate a naïve venture index given the lack of transactions in some quarters. As has been documented elsewhere and is evident from Figure 2, venture did not perform well as an asset class during our sample period, 2006-2017. The expected returns on the venture indices is 3% to 4%, both of which trail the 9.43% average return on the public equity market. The estimated beta is around 1 (1.04 for the size weighted and 0.99 for the price weighted). These estimated betas are similar to the CAPM beta of small-cap tech firms over our sample period, which we estimate to be 1.06.[25] The estimated alphas for the transactions-based indices are negative, but are not statistically significantly different from zero.

As with buyouts, these estimates markedly contrast with estimates using the NAV-based indices. For both the index we construct from *Preqin* NAVs and the one create using *Burgiss*, the expected returns are higher than the transactions based indices (8% and 11%), and the estimated betas are lower than the transactions based indices (0.31). For this reason, and the estimated alphas of the NAV-based indices are positive (5% and 7%) and in the case of *Burgiss*, statistically significantly different from zero. These alphas are also significantly higher than those of the transaction indices as reported at the bottom of Panel B. The smoothing coming from the way that NAVs are constructed tends to lower the estimated beta because the effect of market-wide factors are incorporated with only a long lag. Similar to buyout funds, the standard approach to measuring private equity performance tends to understate the systematic risk and therefore to overstate the alpha of venture funds. The bias adjusted betas for the *Preqin* and *Burgiss*

[25] We pull firms with a Fama-French 49-Industry classification code of 35, 36, or 37. We sort these firms into quintiles at the beginning of each month and create value-weighted portfolios. The estimated beta of the bottom quintile from 2006 through 2017 is 1.06.

venture NAV-based indices are only 0.47 and 0.50, while the bias adjusted volatilities are only 0.09 and 0.11, respectively.

5.4. The Impact of the 2008 Financial Crisis

An important issue in interpreting the results is the role of the 2008 Financial Crisis. It is often thought (hoped) that the Financial Crisis was a one-time event that will not be repeated, so many studies exclude this period in the hope of having a sample that will be more representative of the future. On the other hand, a fund's beta is relevant to an investor because it measures the sensitivity of fund returns to market declines, and the Financial Crisis was the only major market-wide decline in equity markets during our sample period. In addition, because the cash flows produced by private equity funds for investors come through the highly pro-cyclical exits of portfolio companies, investors in private equity funds such as endowments who rely on cash flows to fund operations should be particularly concerned about the way their value changes during market downturns.

For these reasons, we focus our discussion on the estimates from Tables 4 and 5 based on the entire sample, but for completeness, present estimates of private equity risk and return omitting the Financial Crisis in Table 6. To calculate the hedonic indices without the Financial Crisis, we use the same parameter estimates for the pricing equation as in Tables 4 and 5, and then omit years 2008 and 2009 when estimating index returns.

The estimated betas are much lower when the crisis years are excluded, the betas for both buyout and venture indices are both less than 1.0. Excluding the financial crisis, the alphas for buyout are higher. The differences in the transactions-based index when the Financial Crisis is excluded emphasizes the fact that private equity returns during this period were poor, and by excluding them, we potentially overstate the performance of private equity funds.

6. An Application: "Matrix Pricing" of Private Equity Funds

Unlike some securities, private equity funds rarely trade, so for most funds, there is not a recent price from which one can value the fund. For this reason, most funds are valued by limited partners at the

NAV, which can deviate substantially from the best available estimate of the fund's underlying value. These valuations are used for a number of purposes by investors in funds, including portfolio allocation decisions across asset classes, and spending decisions, which are usually made by universities and foundations as a fixed percentage of a portfolio's assessed value. More accurate pricing of limited partner stakes in private equity funds could have substantial consequences for investors since it could lead to portfolio allocation decisions and spending rules corresponding to better estimates of the underlying values of an institution's private equity investments.

One possible approach to valuing stakes in private equity funds more accurately would be to follow the practice of "matrix pricing" commonly used to price bonds. Matrix pricing consists of pricing bonds that do not trade based on the prices of bonds that do trade. The idea is that the same fundamentals affect similar bonds in the same manner, so prices of bonds that do not trade likely move approximately the same amount as prices of similar bonds that do trade. Since private equity funds that invest in one type of asset are likely affected by a number of the same shocks to their fundamentals, they can be priced using comparable methods with transactions-based indices.

For any fund, the fund's history of quarterly cash inflows and outflows can be combined with the quarterly returns of the hedonic indexes to calculate market values in the following way. Beginning with the first contribution in the amount C which occurs in quarter q_0, represented as $C(q_0)$, subtract any distributions D that occur in quarter q_0, represented as $D(q_0)$. At the end of the initial quarter q_0, the book value, and absent extreme circumstances, the market value of the investment in the fund, V, should be $V_0 = C(q_0) - D(q_0)$. Incorporating subsequent changes in market values via the index, the end of the next quarter market value is calculated as $V_1 = V_0(1+r) + C(q_1) – D(q_1)$, where r is the hedonic-predicted return for a given fund type (either buyout or venture). At the end of quarter 2, the market value equals $V_2 = V_1(1+r) + C(q_2) - D(q_2)$. This process is repeated for each quarter to obtain a market value for any quarter between the origination date and the fund's liquidation date. In circumstances where the full history of calls and distributions is not available, the beginning value is set equal to the first available

NAV at time t. In this circumstance, value at time 1 is calculated as $V_1 = NAV(q_0)(1+r) + C(q_1) - D(q_1)$. The resulting market values at time t, represented as V_t, are compared to NAVs at time t, denoted NAV_t, by taking the ratio of the two, V_t/NAV_t. The resultant market-to-book ratio measures the deviation between market and book values in each quarter.

We perform this calculation for each fund in our sample for every quarter in our sample period (2006 to 2017), and report the estimated market to book ratios for the 4th quarter of each year in Table 7. Because our hedonic indexes are estimated with transactions from funds that are between 4 and 9 years old, we only calculate market values for 4-9 year old funds. Following the procedure described above, beginning in Q1 of the fifth year for each fund, we use the fund-specific predicted quarterly return and fund-specific calls and distributions to calculate a market value for each fund. For the purposes of reporting quarterly average market-to-book ratios, we sum market values for each fund in a quarter and divide by the sum of fund NAVs in the quarter. Because each fund in our sample is a different age in calendar time, we report results for vintage year between 2006 and 2017.

Panel A of Table 7 reports the Q4 average market-to-book ratio for buyout funds. Averages for each of the vintages are reported across the bottom of the table. Average market-to-book ratios range from a low of 0.91 for the 2003 vintage of funds to high of 1.39 for 2005 vintage buyout funds. Market-to-book ratios are considerably lower during the Financial Crisis, ranging between 0.65 and 0.77 in 2008 for the 3 vintages that were old enough for our hedonic estimation. Funds that invested during the 2007 and 2008 vintages did so at lower valuations, and therefore have high average market-to-book ratios in subsequent years when markets recovered. For example, these estimates indicate that by the end of 2017, a 2008 buyout fund has a NAV that is understated by 44% relative to its market value.

Panel B reports market-to-book ratios for venture funds. These ratios are substantially lower than the market-to-book ratios reported for buyout funds. This difference reflects the fact that returns were lower for venture than buyout during our sample period (see Figure 2 and Tables 4 and 5). The market-to-book ratios for venture funds were less impacted by the Financial Crisis; ratios reach 0.8, 0.8, and 0.92 for the three age-eligible vintages during 2008. The 2004 vintage reports the highest average market-to-book ratios, with 2009 vintage funds reporting the lowest.

Individual funds could mark their values to market using the hedonic approach in one of two ways. The most accurate approach would be to generate fund-specific market values using the hedonic-estimated coefficients applied to the fund's attributes. A simpler approach, but one that nonetheless be a substantial improvement over using NAVs, is to multiply the NAV of each of LP's investments by the average market-to-book ratio of the industry. For example, an LP would multiply the NAV of each 4-9 year old fund in his or her portfolio by the appropriate ratio from Table 8. For younger funds, the deviation between NAVs and market value is likely to be smaller but could be estimated using the coefficients from the hedonic regressions. Tail-end funds will have only a few portfolio companies left and their values will vary depending on the fortunes of these particular investments, so matrix pricing is likely to be less useful for valuing these funds.

7. Conclusion

Measuring the performance of private equity investments (buyout and venture) has historically only been possible over long horizons because a fund's return is only observable following the fund's final distribution. However, in recent years, a secondary market has developed in which investors in private equity funds can trade their stakes. Prices from this market provide a source of data useful for measuring the risk and return of private equity funds in a manner to that commonly used to measure returns for other securities.

We construct indices of buyout and venture capital performance using a proprietary database of secondary market prices of private equity stakes between 2006 and 2017. These indices provide a new approach to measuring the risk and return of private equity investments. In addition, they can be used by investors for a number of purposes, including using the estimates of private equity risk and return to make better portfolio decisions, and valuing investors' private equity investments by a method similar to the "matrix pricing" approach commonly applied to thinly traded bonds.

Our estimates indicate that the transactions-based indices of buyout and venture capital track the public equity market much more closely than the two NAV-based indices we consider, one constructed from *Preqin* data and the one produced by *Burgiss* that is used by practitioners. The hedonic buyout indices we construct have betas greater than 2, consistent with the notion that buyouts that increase leverage also increase beta. The hedonic venture indices have betas slightly higher than 1. Both of these findings are in marked contrast to the NAV-based indices, whose betas are estimated to be less than 0.5. Since NAVs adjust with a long lag, estimated betas from NAV-based indices likely understate the extent to which private equity returns depend on market-wide factors.

The high beta on the transactions-based indices has important implications for our understanding of private equity performance. Because of the high beta, the buyout indices have an estimated alpha of close to zero, even though the indices have an average return of over 20% during our sample period. The high beta estimates raise the required return to a level where even this high return level is not abnormally large, so our estimates of alpha are not significantly different from zero. This finding is counter to our estimates of alpha for NAV-based indices as well as estimates of alpha in the literature, which are consistently positive and statistically significantly different from zero.

For the transactions-based venture capital indices, we estimate betas around 1, which are also substantially larger than the betas for the NAV-based indices (around .3). These high betas imply that the transactions-based venture capital indices have a statistically insignificant (but negative) alpha, in contrast to the positive estimated alpha for the NAV-based indices. This negative alpha is likely reflective of our sample period being a poor one for the venture capital industry rather than a statement about the underlying economics of venture capital. If there had been a secondary market during the booming 1990s and one had constructed a transactions-based index, it most likely would have had a positive alpha. The point, however, is that regardless of when one uses an index for which prices adjust to market-wide information in real time, one estimates much higher betas and consequently lower alphas.

These indices also have a number of potential uses for investors. Better estimates of private equity risk and return should affect the optimal portfolio decisions of investors when deciding on the allocation to private equity in their portfolios. In addition, the index can be used to provide more accurate valuations of stakes in private equity funds that investors hold in a manner similar to the "matrix pricing" approach commonly used to price illiquid bonds. Our estimates suggest that the use of NAV for valuation as is done by most LPs can be misleading, and NAVs often substantially misstate the value of an investor's private equity holdings. Improving these valuations is likely to affect investors' decisions about both the portfolio allocations and the amount they spend from their invested assets.

Undoubtedly, there are uses for the index we have not discussed in this paper. For example, one could design derivative contracts based on an index of private equity returns. These derivatives could potentially be useful to investors or General Partners who wish to hedge risks in their portfolios, or to speculate on the performance of the buyout or venture capital sectors. Better indices of private equity performance such as the ones presented here clearly have much to offer the private equity community.

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Appendix A: Data Details

We first classify funds in the transactions data as either buyout or venture funds based on the specified fund type. Specifically, we classify funds as buyout funds that are labeled as “Buyout,” “Small Buyout,” “Mid-Cap Buyouts,” “Mid-Cap Buyout,” “Buyout/Growth,” “Mega Buyouts,” “Large-Cap Buyouts,” and “All Private Equity”. We classify funds as venture funds that are labeled “Venture,” “Venture (General),” “Early Stage,” “Early Stage VC,” “Early Stage: Seed,” “Early Stage: Start-up,” “Expansion/Late Stage,” and “Growth, and Mezzanine”. After classifying funds as either buyout or venture and eliminating all other transactions we have 5,214 fund transactions from 2006 through 2017, of which 3,277 are for buyout funds and 1,937 are for venture funds.

We hand check similar fund names to develop a list of unique fund names. We then clean the transactions data as follows:

1) Eliminate transactions with a price less than zero.
2) Eliminate transactions with a NAV less than zero.
3) Eliminate transactions that have the same price for every fund in the portfolio transaction.
4) Eliminate two transactions that appear to be obvious data errors.
5) Eliminate transactions for which the total amount committed by the seller minus the unfunded commitment is less than zero.
6) Eliminate transactions for which the total capital committed is less than or equal to zero.
7) Eliminate transactions for which the fund name is missing.

9) If multiple transactions occur on the most recent transaction date for a given fund/quarter, use only the transaction based on the highest total commitment.
10) If multiple transaction records exist with the same fund name, vintage, and commitment on the most recent transaction date for a given fund/quarter, take the mean transaction price (as a percent of NAV) as the price for this fund/quarter.

12) Eliminate all remaining transactions for which the price, as a percent of NAV, is greater than 3 standard deviations away from the mean price across funds for a given quarter.

After cleaning the data as described above, we are left with 3,404 fund transactions of which 2066 are for buyout funds and 1,338 are for venture funds. These transactions are for 2,424 unique funds, 1,427 of which are buyout and 997 of which are venture. These 3,404 transactions are the "Transactions Sample" highlighted in the first column of Table 1.

We also classify funds in the Preqin data as buyout or venture based on the fund type as specified by Preqin . Specifically, we classify funds as buyout funds that are labeled "Buyout." We classify funds as venture funds that are labeled "Venture (General)," "Venture Debt," "Growth," "Early Stage," "Early Stage: Seed," "Early Stage: Start-up," and "Expansion / Late Stage."

Using the Preqin data, we compute the realized NAV-based return for each fund, i, in every quarter, t, as $(NAV_{i,t} - C_{i,t} + D_{i,t})/NAV_{i,t-1} - 1$, where $C_{i,t}$, $D_{i,t}$, denote the calls and distributions made by fund i during quarter t, and $NAV_{i,t}$ is the end-of quarter NAV. For each fund-quarter we also compute the internal rate of return, $IRR_{i,t}$ from fund initiation through the end of the quarter, treating $NAV_{i,t}$ as a terminal cash flow in the final quarter, t. We then clean the Preqin data as follows:

1) Eliminate any fund-quarters for which $NAV_{i,t-1} = 0$, or the NAV-based return is otherwise missing. Note that we retain records for which $NAV_{i,t} = 0$. Once NAV hits zero, however, we no longer include the fund in the sample.
2) Eliminate stale NAVs, those with a report date prior to the end of each quarter.
3) Identify fund-quarters for which the NAV-based return is in the 1% tails across all fund-quarters. For these observations, eliminate the fund-quarter record only if IRR_{t-1} or IRR_t is more than 5000 basis points away from the IRR self-reported by the fund after and nearest to t in calendar time. We obtain year-end self-reported IRRs from Preqin.

We then merge our Preqin data with the explanatory variables described in Table 3. We find that a few fund-quarters are missing data on either fund size or the fraction of LPs that are pension funds. After eliminating these observations, our filtered Preqin dataset represents a full panel that includes data on fund cash flows and explanatory variables. This panel contains 41,464 fund-quarter observations from 2006 through 2017. The observations are for 2,287 unique funds, 1,251 of which are buyout, and 1,036 of which are venture. These 41,464 observations are the Preqin Universe highlighted in the second and third columns of Table 1.

To merge the transaction and cash-flow data, we first identify funds with identical fund names in the two databases and designate these as a match. We then identify fund names in the transaction and Preqin data that are "similar" and that also have the same vintage. Fund names A and B are considered similar if fund name A contains the first 10 characters of fund name B anywhere in the fund name string. We then hand check this list to determine which funds match. After merging we have transactions and Preqin cash-flow data in the merged sample for 630 funds, 372 of which are buyout and 258 of which are venture. These 630 funds account for 1,119 transactions, 716 of which are buyout, and 403 of which are venture. These 1,119 transactions are the merged sample highlighted in the second column of Table 1.

Appendix B. Portfolio Transactions in the Secondary Market

A unique feature of the PE secondary market is the fact that individual funds are frequently sold as part of a larger portfolio transaction. For example, a seller might offer to sell their ownership stake in five unique funds, hoping to sell a portfolio of holdings in one large transaction. In a portfolio transaction, the buyer submits an offer price for the entire portfolio of funds, and the buyer and seller enter into a contract to eventually transfer ownership based on the portfolio offer price. Given that the construction of our price index relies on the market prices paid for individual funds, rather than one price for a portfolio of funds, it is important to consider the economics that govern how prices get assigned to individual funds in a portfolio transaction to determine whether and how portfolio transactions could influence the index.

Once a buyer and seller are in contract to transfer ownership of the portfolio of funds, the process moves to a second phase. During the diligence process, buyers assign prices to each of the individual funds in the portfolio, subject to the constraint that the size-weighted average of the individual prices equals the winning offer price. The price allocation process can be nuanced because although the buyer bids on the full portfolio, they may in reality only have strong demand for certain funds in the offered portfolio. The buyer's assignment of prices to individual funds will reflect their demand for those funds. High prices are assigned to funds the buyer most demands and lower prices are assigned to funds they demand less, again, subject to the constraint that size-weighted average prices equal the full portfolio bid. Conversations with industry experts indicate that there are times when prices allocated to individual funds result in certain of the funds being excluded from the final transaction. Thus, the prices assigned to individual funds are a reflection of demand for the funds, albeit filtered through the portfolio purchasing process.

While the assignment of prices to individual funds is ultimately a reflection of demand, the concern is whether prices determined through a portfolio process are somehow systematically different from single-fund sale prices. Any bias in our index stemming from portfolio transactions would have to display time series properties given that our object of interest from the index is quarter-over-quarter returns. The fraction of portfolio transactions each quarter is volatile, but shows no consistent trend quarter-over-quarter.

Appendix C. Unbiased Estimator of Average Price

Let $\boldsymbol{\pi_N}$ denote the $N \times 1$ vector of scaled prices using all N funds, and let $\boldsymbol{e_N}$ be the $N \times 1$ vector of population residuals. Then if $E\left[\widehat{\boldsymbol{\theta}}_{\boldsymbol{k}}\right] = \widehat{\boldsymbol{\theta}}_{\boldsymbol{N}}$,

$$\begin{aligned} E[\overline{\boldsymbol{P}}_{\boldsymbol{k}} \mid \boldsymbol{X_N}, \boldsymbol{NAV_N}] &= \frac{1}{N}\left[\left(\boldsymbol{X_N}\boldsymbol{E}[\widehat{\boldsymbol{\theta}}_{\boldsymbol{k}}]\right)' \boldsymbol{NAV_N}\right] \\ &= \frac{1}{N}\left[\left(\boldsymbol{X_N}\widehat{\boldsymbol{\theta}}_{\boldsymbol{N}}\right)' \boldsymbol{NAV_N}\right] \\ &= \frac{1}{N}(\boldsymbol{\pi_N} - \boldsymbol{e_N})' \boldsymbol{NAV_N} \\ &= \bar{P}_t - \frac{1}{N}\boldsymbol{e}'_{\boldsymbol{N}}\boldsymbol{NAV_N}. \end{aligned} \tag{8}$$

It follows that $E[\overline{\boldsymbol{P}}_{\boldsymbol{k}} \mid \boldsymbol{X_N}, \boldsymbol{NAV_N}] = \bar{P}_t$ provided that $\boldsymbol{e}'_{\boldsymbol{N}}\boldsymbol{NAV_N} = \boldsymbol{0}$, which we can ensure by including NAV_i as one of the elements in $\boldsymbol{X_N}$**.** In our model we include $\log(NAV_i)$ as an explanatory variable in the interest of aspiring towards stationarity and normality. Using NAV_i instead results in very little change in results.

Appendix D. Bias Adjustments

In this appendix we derive the bias adjustments as given in (7). If all funds transact at some point in time every quarter, the observed return for the portfolio of N funds with non-synchronous trading, r_t^n, can be defined as

$$\begin{aligned} r_t^n &= \frac{\sum_i P_{i,t}^n + D_{i,t}^n - C_{i,t}^n}{\sum_i P_{i,t}^n} - 1 \\ &= \frac{\bar{P}_t^n + \bar{D}_t^n - \bar{C}_t^n}{\bar{P}_{t-1}^n} - 1. \end{aligned} \tag{9}$$

where $P_{i,t}^n$ denotes the price of fund i at time $t - s_i(t)$, $D_{i,t}^n$, $C_{i,t}^n$ denote the appropriate distributions received and calls paid, and an overline indicates average. In addition, let $\tilde{r}_t$ denote the return from buying every fund at their assigned transaction times, $t - s_i(t)$, and selling them at time t. To a first order approximation $r_t \approx \log(1 + r_t)$ and

$$\log(1 + r_t^n) = \log(1 + r_t) - \Delta\log\left(1 + \tilde{r}_{i,t}\right). \quad (10)$$

We don't observe r_t^n since all funds do not transact. Instead we estimate r_t^n using the hedonic and naïve approaches. Consider first the naïve indices, where we estimate $\bar{P}_t^n$ as the average of available transaction prices. The observed transaction price for any fund can always be written as the product of the population average across all N funds, $\bar{P}_t^n$, and a fund-specific scaling constant, $\left(1 + \delta_{i,t}\right)$,

$$P_{i,t}^n = \bar{P}_t^n\left(1 + \delta_{i,t}\right), \quad (11)$$

where $\delta_{i,t} > -1$ and the population average of $\delta_{i,t}$ across all N funds is identically equal to zero, $\bar{\delta}_t = 0$. If we let $\bar{\delta}_{k,t}$ denote the average value of $\delta_{i,t}$ across the k_t funds that transact, then the observed estimate of the average price using these k_t funds, $\bar{P}_{k,t}^n$, is given by

$$\bar{P}_{k,t}^n = \bar{P}_t^n(1 + \bar{\delta}_{k,t}), \quad (12)$$

where $\bar{\delta}_{k,t}$ is the average value of $\delta_{i,t}$ across the k_t funds that transact. If funds transact with independent uniform probability, then $\bar{\delta}_{k,t}$ is independent across time and mean zero. It follows that for observed naïve index return, r_t^*,

$$r_t^* \approx \log(1 + r_t^*) = \log(1 + r_t^n) + \Delta\bar{\delta}_{k,t} + \phi \quad (13)$$

where ϕ is a small correction term to account for cash flows,[26] and we use the first-order approximation $\log\left(1 + \bar{\delta}_{k,t}\right) = \bar{\delta}_{k,t}$.

Consider now the hedonic indices for which we estimate $\bar{P}_{h(k),t}$ as

$$\bar{P}_{h(k),t} = \frac{1}{N}\left[\left(\boldsymbol{X_N}\boldsymbol{\hat{\theta}_k}\right)'\boldsymbol{NAV_N}\right], \quad (14)$$

where $\bar{P}_{h(k),t}$ represents the estimate of $\bar{P}_t^n$ using the hedonic approach, and $\boldsymbol{\hat{\theta}_k}$ is estimated using either OLS or the Heckman (1979) sample selection model. If $\boldsymbol{\theta}$ could be estimated using the full population of N funds, then $\bar{P}_{h(k),t}$ would be identical to the full population average, $\bar{P}_t^*$. (See Appendix C.) Since we estimate $\boldsymbol{\theta}$ using only funds that transact, the regression parameters will be estimated with error,

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[26]The correction term is given by $\phi = \log[(1 + \delta_t)\bar{P}_t^n + \bar{D}_t^n - \bar{C}_t^n] - \log[(1 + \delta_t)(\bar{P}_t^n + \bar{D}_t^n - \bar{C}_t^n)]$.

$$\widehat{\boldsymbol{\theta}}_k = \boldsymbol{\theta}_{\boldsymbol{N}} + \boldsymbol{e}_{\boldsymbol{k}}, \tag{15}$$

where $\boldsymbol{\theta}_{\boldsymbol{N}}$ represents the full population estimate, and $\boldsymbol{e}_{\boldsymbol{k}}$ is a mean zero $p \times 1$ vector that is independent of other variables used in the estimation of average price. Equation (15) implies that we can again write the estimated average price as

$$\bar{P}_{h(k),t} = \bar{P}_t^n\left(1 + \bar{\delta}_t\right), \tag{16}$$

where for the hedonic indices,

$$\bar{\delta}_t = \frac{1}{N}\left[\frac{(\boldsymbol{X}_{\boldsymbol{N}}\boldsymbol{e}_{\boldsymbol{k}})'\boldsymbol{NAV}_{\boldsymbol{N}}}{\bar{P}_t^n}\right] \tag{17}$$

Since $\bar{P}_t^n$ is the deterministic average (nonsynchronous) price using the full population of funds, it is independent of $\boldsymbol{e}_{\boldsymbol{k}}$, and $\bar{\delta}_t$ again represents a zero-mean independent random variable. In this case, the observed index return can again, be expressed as in (13).

Combining (10) and (13) it is straight forward to show that

$$\begin{aligned} E[r_t^*] &= E[r_t] \\ Var[r_t^*] &= Var[r_t] + 2Var\left[\,\bar{\delta}_{k,t}\right] + 2Var\left[\tilde{r}_{i,t}\right] - 2Cov(r_t, \tilde{r}_t) \\ Cov\left[r_t^*, r_{m,t}\right] &= Cov\left[r_t, r_{m,t}\right] - Cov\left[\tilde{r}_t, r_{m,t}\right] \\ Cov[r_t^*, r_{t-1}^*] &= -Var\left[\bar{\delta}_{k,t}\right] - Var\left[\tilde{r}_{i,t}\right] + Cov\left(r_t, \tilde{r}_{i,t}\right) \\ Cov\left[r_t^*, r_{m,t-1}\right] &= Cov\left[\tilde{r}_t, r_{m,t}\right]. \end{aligned} \tag{18}$$

where $r_{m,t}$ is the observed public equity market return with no measurement error. The first line of (18) indicates that measured portfolio returns are unbiased. The second line of (18) indicates that the observed portfolio variance may be either over- or understated (see discussion in Scholes and Williams (1979) and Lo and MacKinlay (1990)). The third line of (18) indicates the covariance of the observed portfolio return with the market is understated due to the reduced contemporaneous overlap in these measured returns from non-synchronous trading. The fourth and fifth lines of (21) provide results for the observed autocovariance and cross-autocovariance with the market. Finally, by adding the moments defined in (18) the bias adjustments defined in (7) can be derived.

Appendix E. Standard Errors for Index Parameters

We first subsample consecutive overlapping blocks of data used to estimate the hedonic model, where a block is defined as all data in the cross section for dates within a given date range. The number of dates that define a block (block size), is determined using the "minimum volatility method". (See Politis, Romano, and Wolf (1999), pp. 197-200.) We determine optimal block sizes separately for each empirical specification, e.g., hedonic buyout size-weighted, hedonic buyout price-weighted, etc. Optimal block sizes range from 25 to 33. By sampling blocks we preserve the time-series dependence structure.

For a given block we estimate the parameters of the hedonic (or naïve) model which are then used to create an index. We next sample from the created index. For this sampling procedure we estimate an AR(1) and sample from the residuals with uniform probability and with replacement. We jointly select contemporaneous market returns and risk-free returns needed to estimate alpha and beta. We then use the estimated parameters from the AR(1) and selected residuals to create a new index of identical size. Using the newly created index, the selected market returns and risk-free returns, we estimate index parameters (mean, alpha, beta, volatility, etc). For each block we repeat the second-stage sampling procedure 100 times. The standard error for an index parameter is the standard deviation of the parameter across all blocks and sampling procedures.

Figure 1. Transactions per Period, Baseline Transactions Sample, Funds 4-9 Years Old

This figure illustrates the number of transactions we observe per period in the baseline transactions sample for funds that are four to nine years old, as documented in Table 1. This sample contains 900 buyout transactions and 461 venture transactions.

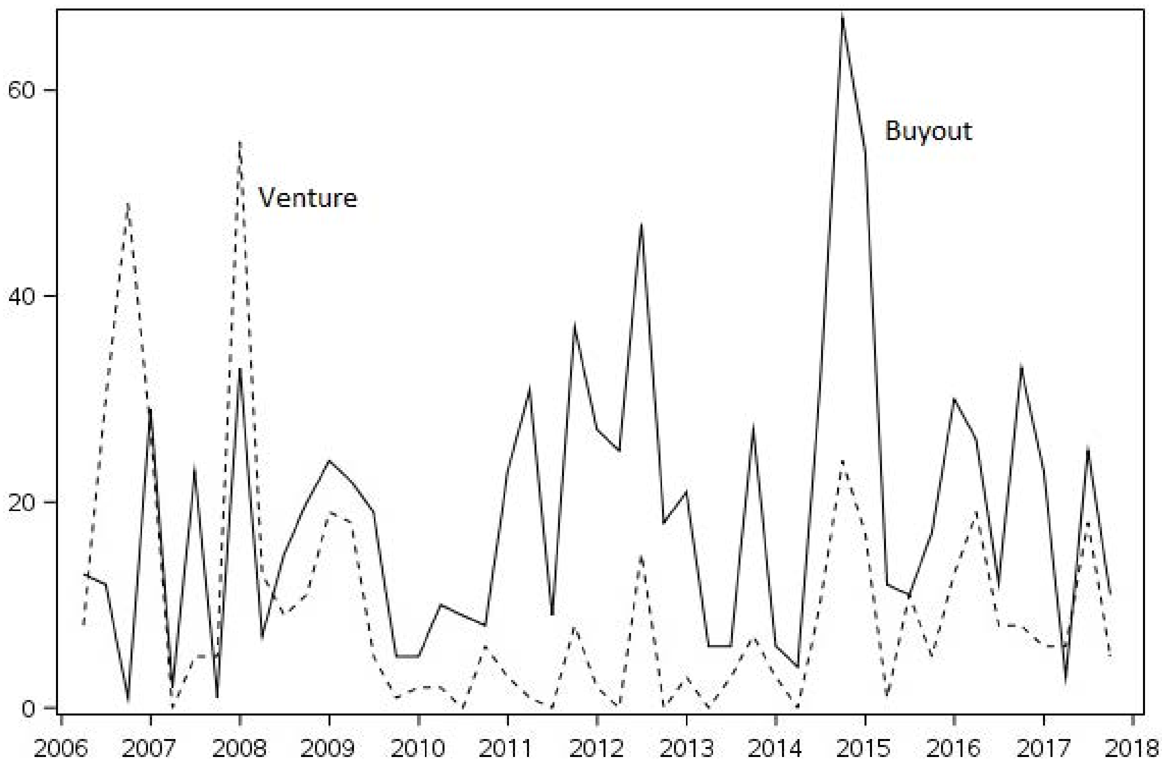
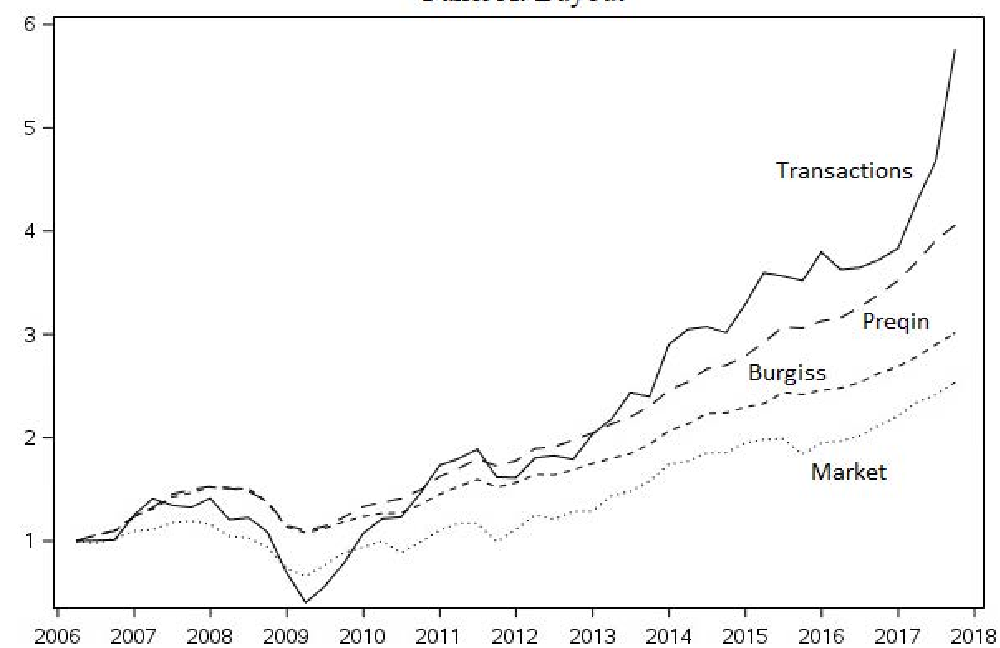


Figure 2. Buyout and Venture Indices Over Time

This figure illustrates the value of investing $1 in an index at the beginning of 2006 in each index as labeled. Panel A. is for buyout funds and Panel B. is for venture funds. The market index is based on the public market return as posted on Ken French's website. We build the Preqin indices based on NAVs reported in Preqin using funds in the Preqin Universe that are 4-9 years old. The transactions indices are the size-weighted hedonic indices.

Panel A. Buyout



Panel B. Venture

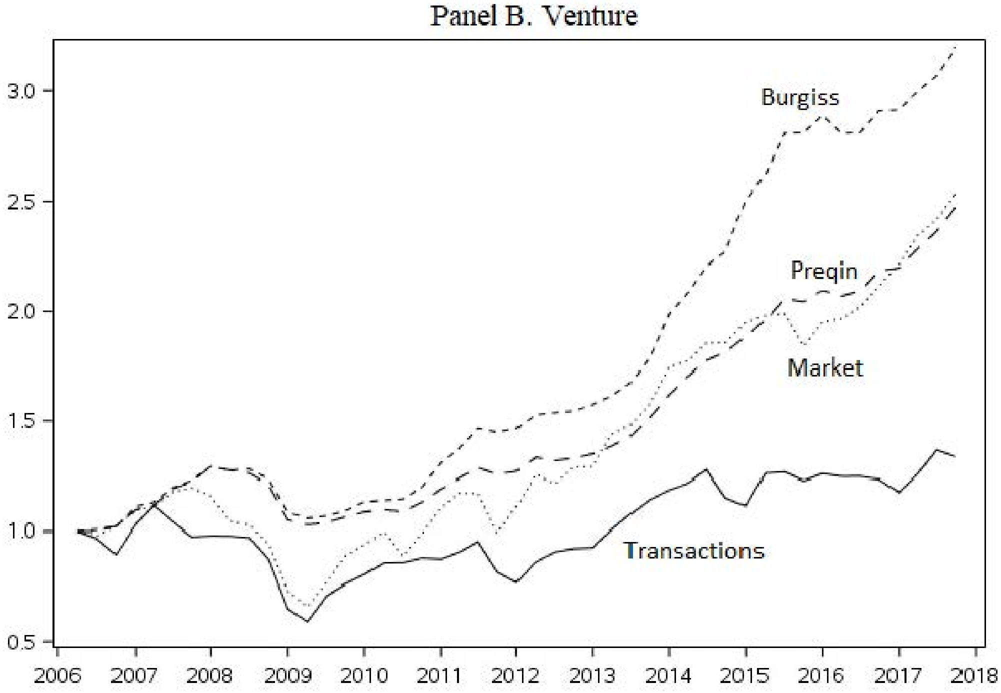


Table 1. Summary Statistics

This table reports summary statistics for the data samples we use. Panel A is for buyout funds while Panel B is for Venture funds. The "transactions full sample" is the cleaned sample of all transactions as described in the text. The "transactions baseline sample" is the subset of transactions in the transactions sample that is for funds 4-9 years old. The "*Preqin* Universe" is the cleaned sample of cash flow data and other characteristics from *Preqin* as described in the text. We create the "full merged sample" by merging the transactions full sample with the *Preqin* universe. The "compliment" sample in the *Preqin* universe represent those fund-quarter records in *Preqin* for funds that do not transact. We also report summary statistics for the *Preqin* universe for funds that are four to nine years old. We create the "baseline merged" sample by merging funds in the *Preqin* universe with the baseline sample. The "compliment" sample for the *Preqin* universe for funds that are four to nine years old represent the fund-quarter records in *Preqin* for funds that are four to nine years old that do not transact. $\pi_{i,t}$ is the fund price as a fraction of NAV. $Size(\$MM)$ represents total commitments in \$US millions. Age is fund age in years. $Trans\ per\ Qtr$ is the number of transactions per quarter. PME is the Kaplan Schoar (2005) PME using NAV as terminal value for funds that have not yet liquidated. N is the total number of observations in the sample. "Mean" is the average across funds and across time, "Q1" is the 25th percentile, and "Q3" is the 75th percentile.

Panel A. Buyout

		Transactions		*Preqin Universe*		*Preqin Universe*	
			4-9 Yrs Old	*Full Sample*		*4-9 Yrs Old*	
		Full Sample	*Baseline*	*Full Merged*	*Compliment*	*Baseline Merged*	*Compliment*
$\pi_{i,t}$	Mean	0.83	0.89	0.82		0.89	
	Q1	0.70	0.80	0.69		0.80	
	Q3	1.00	1.03	1.00		1.02	
Size ($MM)	Mean	4699.2	5403.4	4715.8	1844.5	5416.4	1921.4
	Q1	1396.0	1800.0	1398.0	400.0	1800.0	410.0
	Q3	6000.0	7626.8	6000.0	2100.0	7626.8	2175.5
Age (years)	Mean	9.4	6.8	8.0	6.4	6.50	6.37
	Q1	6.0	5.0	5.0	3.0	5.00	5.00
	Q3	12.0	8.0	11.0	9.0	8.00	8.00
Trans per Qtr	Mean	44.0	19.1	15.2		7.7	
	Q1	14.0	8.0	6.0		4.0	
	Q3	56.0	27.0	25.0		12.0	
PME	Mean	1.19	1.16	1.18	1.12	1.16	1.18
	Q1	0.94	0.96	0.93	0.89	0.94	0.94
	Q3	1.40	1.31	1.38	1.30	1.32	1.34
N		2,066	900	716	20,346	364	9,138

Table 1. *Continued*

Panel B. Venture							
		Transactions		*Preqin Universe Full Sample*		*Preqin Universe 4-9 Yrs Old*	
		Full Sample	*4-9 Yrs Old Baseline*	*Full Merged*	*Compliment*	*Baseline Merged*	*Compliment*
$\pi_{i,t}$	Mean	0.80	0.88	0.84		0.92	
	Q1	0.60	0.70	0.63		0.75	
	Q3	0.99	1.08	1.01		1.21	
Size ($MM)	Mean	650.1	711.8	645.7	415.3	711.8	407.4
	Q1	286.5	376.4	285.0	165.0	376.4	171.0
	Q3	830.0	865.4	830.0	500.0	865.4	500.0
Age (years)	Mean	10.8	7.0	10.0	7.3	6.70	6.54
	Q1	7.0	6.0	6.0	4.0	6.00	5.00
	Q3	14.0	8.0	14.0	11.0	8.00	8.00
Trans per Qtr	Mean	28.5	9.8	8.6		3.2	
	Q1	6.0	2.0	2.0		0.0	
	Q3	40.0	13.0	14.0		4.0	
PME	Mean	1.03	0.95	1.04	1.01	0.95	0.98
	Q1	0.56	0.60	0.64	0.64	0.67	0.61
	Q3	1.23	1.14	1.21	1.16	1.16	1.17
N		1,338	461	403	19,999	152	8,938

Table 2. Explanatory Variable Descriptions

This table describes the explanatory variables used in our hedonic models. The first six variables are “state variables” that are the same across all funds and vary only across time. The last six variables are “fund specific variables” that vary across funds and also (potentially) across time.

State Variables	MTB	The average end-of-month market-to-book ratio over the transaction quarter, calculated using all stocks with share code 10 or 11 in CRSP.
	Volatility	The annualized standard deviation of the value-weighted portfolio of all stocks in CRSP with share code 10 or 11 using daily data over the transaction quarter.
	Value Confidence Ind.	The average end-of-month value of the Valuation Confidence Index over the transaction quarter from the International Center for Finance at Yale. Institutional Investors are asked to report their assement of stock market value relative to fundamental value. The Valuation Confindence Index is one minus the percentage of respondants who think that the market is overvalued. We scale this variable by 100.
	Crash Confidence Ind.	The average end-of-month value of the Crash Confidence Index over the transaction quarter from the International Center for Finance at Yale. Institutional Investors are asked to report the probability of a catostrophic market crash in the next six months. The Crash Confidence Index is the percentage of respondants who think that the probability is less than 10%. We scale this variable by 100.
	Avg. Market NAV	The average log NAV reported at the end of the transaction quarter across funds in our Preqin Universe.
	Average Market PME	The average Kaplan Schoar (2005) PME at the end of the transaction quarter across funds.
Fund SpecificVriables	Log Size	The log size of the transacting fund (total commitments of all limited partners).
	NAV	The log NAV for the transacting fund reported as of the end of the transaction quarter.
	PME	The Kaplan Schoar (2005) PME of the transacting fund as of the end of the transaction quarter, calculated using data from Preqin.
	AGE< 4	Age dummy that equals 1 if the year of transaction quarter minus the vintage year for the transacting fund is less than 4.
	4<=AGE<=9	Age dummy that equals 1 if the year of the transaction quarter minus the vintage year for the transacting fund is greater than or equal to 4 and less than or equal to 9.
	Frac. Pension	The fraction of the transacting fund's limited partners that are pension funds.

Table 3. Sample Selection Model Parameters

This table reports the estimates of the Heckman (1979) sample selection model. Panel A reports estimates of $\boldsymbol{\gamma}$ for the selection equation while Panel B reports estimates of $\boldsymbol{\theta}$ for the "pricing equation", while. "Heckman" refers to the sample selection model, while "OLS" refers to the model estimated by simple OLS with no selection equation. Variables are described in Table 2. We estimate the standard errors of model parameters using the quasi-maximum-likelihood approach of White (1982). Significance at the 1%, 5%, and 10% levels is indicated, respectively, by "***", "**", and "*".

Panel A: Selection Equation

		Buyout		Venture	
		estimate	(t-stat)	estimate	(t-stat)
	Intercept	-5.18	-(1.2)	-31.83	-(6.4)***
State Variables	MTB	0.20	(1.7)*	1.61	(9.5)***
State Variables	Volatility	0.24	(0.8)	-1.08	-(2.3)**
State Variables	Value Confidence Ind.	0.46	(1.8)*	2.89	(8.3)***
State Variables	Crash Confidence Ind.	-1.36	-(3.0)***	-3.85	-(6.4)***
State Variables	Avg. Market NAV	-0.07	-(0.3)	1.25	(4.3)***
State Variables	Average Market PME	1.05	(1.8)*	5.65	(7.0)***
Fund Specific	Log Size	0.42	(24.9)***	0.39	(15.2)***
Fund Specific	NAV	0.03	(1.9)*	-0.01	-(0.5)
Fund Specific	PME	-0.11	-(2.2)**	0.01	(0.5)
Fund Specific	AGE < 4	-0.86	-(14.0)***	-0.94	-(11.0)***
Fund Specific	4<=AGE<=9	-0.37	-(7.0)***	-0.50	-(8.7)***
Fund Specific	Frac. Pension	-0.44	-(5.1)***	-0.23	-(2.6)**
	ρ	-0.02	-0.15		
		$\chi^2(1)$	p-value	$\chi^2(1)$	p-value
	Wald	0.02	(0.88)	26.21	(0.00)
	Likelihood Ratio	0.01	(0.94)	4.17	(0.04)
	Lagrange Multiplier	0.00	(0.97)	0.15	(0.70)

Table 3. *Continued*

Panel B: Pricing Equation

		Buyout				Venture			
		OLS		Heckman		OLS		Heckman	
		estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
	Intercept	-14.33	-(7.7)***	-14.29	-(7.6)***	-2.80	-(1.0)	7.36	(1.5)
State Variables	MTB	0.18	(2.3)**	0.18	(2.2)**	0.31	(3.1)***	-0.17	-(0.8)
State Variables	Volatility	0.41	(3.3)***	0.41	(3.2)***	-0.25	-(0.8)	0.17	(0.5)
State Variables	Value Confidence Ind.	-0.33	-(2.2)**	-0.33	-(2.1)**	0.21	(0.9)	-0.61	-(1.6)
State Variables	Crash Confidence Ind.	0.71	(3.0)***	0.72	(2.9)***	1.06	(2.5)**	2.24	(3.9)***
State Variables	Avg. Market NAV	0.97	(8.0)***	0.97	(8.0)***	0.09	(0.5)	-0.30	-(1.3)
State Variables	Average Market PME	-0.47	-(1.7)*	-0.47	-(1.7)*	0.75	(1.6)	-1.11	-(1.3)
Fund Specific	Log Size	0.01	(1.1)	0.01	(0.5)	0.00	(0.0)	-0.12	-(2.6)***
Fund Specific	NAV	0.03	(3.6)***	0.03	(3.5)***	0.03	(2.5)**	0.04	(2.4)**
Fund Specific	PME	0.10	(3.3)***	0.10	(3.3)***	-0.02	-(0.7)	-0.02	-(0.6)
Fund Specific	AGE< 4	0.04	(0.9)	0.04	(0.8)	0.00	(0.0)	0.29	(2.2)**
Fund Specific	4<= AGE<=9	0.10	(3.4)***	0.10	(3.2)***	0.02	(0.5)	0.17	(2.3)**
	R-square	34%		34%		22%		30%	

Table 4. Secondary Market Based Buyout Indices: 2006-2017

This table reports moment estimates for buyout indices using data from 2006-2017. Panel A is for our transactions-based indices, Panel B is for Preqin NAV-based indices. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the baseline merged samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1 except for the autocorrelation. We estimate standard errors for index parameters (alpha, beta, etc.) using a two-step bootstrap procedure based the subsampling methods of Politis and Romano (1994). Significance at the 1%, 5%, and 10% levels is indicated, respectively, by "***", "**", and "*".

Panel A: Transactions-Based Indices

	Hedonic Size Weighted		Hedonic Price Weighted		Naïve Price Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.20	(1.4)	0.18	(1.3)	0.30	(1.9)*
β	2.43	(3.8)***	2.25	(3.6)***	1.43	(2.2)**
α	-0.02	-(0.1)	-0.02	-(0.2)	0.16	(0.9)
σ	0.40	(2.5)**	0.38	(2.4)**	0.33	(2.4)**
Sharpe	0.49	(0.5)	0.48	(0.5)	0.88	(0.6)
Corr Mkt	0.95	(3.0)***	0.94	(2.6)***	0.69	(2.0)**
Autocorr	0.37	(1.7)*	0.35	(1.5)	-0.19	-(1.0)

Panel B: Size-Weighted NAV Indices

	Preqin		Burgiss	
	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.13	(3.3)***	0.10	(2.7)***
β	0.45	(4.1)***	0.45	(4.3)***
α	0.08	(2.1)**	0.05	(1.4)
σ	0.09	(2.7)***	0.09	(2.8)***
Sharpe	1.26	(0.8)	0.97	(0.8)
Corr Mkt	0.78	(4.5)***	0.77	(4.8)***
Autocorr	0.55	(2.0)**	0.50	(1.9)*

Differences

Relative to Hedonic Size-Weighted Index

	Preqin		Burgiss	
	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.08	(0.6)	0.10	(0.8)
β	1.97	(3.3)***	1.98	(3.3)***
α	-0.10	-(0.8)	-0.07	-(0.6)
σ	0.31	(2.3)**	0.31	(2.2)**
Sharpe	-0.77	-(0.7)	-0.48	-(0.5)
Corr Mkt	0.17	(0.5)	0.17	(0.5)
Autocorr	-0.18	-(0.6)	-0.14	-(0.5)

Table 5. Secondary Market Based Venture Indices: 2006-2017

This table reports moment estimates for buyout indices using data from 2006-2017. Panel A is for our transactions-based indices, Panel B is for Preqin NAV-based indices. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the baseline merged samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1 except for the autocorrelation. We estimate standard errors for index parameters using a two-step bootstrap procedure based the subsampling methods of Politis and Romano (1994). Significance at the 1%, 5%, and 10% levels is indicated, respectively, by “***”, “**”, and “*”.

Panel A: Transactions-Based Indices				
	Hedonic Size Weighted		Hedonic Price Weighted	
	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.04	(0.5)	0.03	(0.4)
β	1.04	(2.7)***	0.99	(3.0)***
α	-0.06	-(0.8)	-0.07	-(1.0)
σ	0.19	(2.6)**	0.18	(2.8)***
Sharpe	0.17	(0.2)	0.12	(0.2)
Corr Mkt	0.84	(2.4)**	0.84	(2.7)***
Autocorr	0.23	(1.1)	0.23	(1.2)

Panel B: Size-Weighted NAV Indices				
	Prequin		Burgiss	
	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.08	(2.7)***	0.11	(2.9)***
β	0.31	(4.0)***	0.31	(3.5)***
α	0.05	(1.6)	0.07	(2.0)**
σ	0.07	(3.8)***	0.08	(5.3)***
Sharpe	1.08	(1.1)	1.24	(1.6)
Corr Mkt	0.70	(4.3)***	0.61	(3.6)***
Autocorr	0.43	(1.8)*	0.40	(1.8)*
Differences Relative to Hedonic Size-Weighted Index				
	Prequin		Burgiss	
	estimate	(t-stat)	estimate	(t-stat)
E[r]	-0.04	-(0.8)	-0.07	-(1.2)
β	0.73	(2.0)**	0.73	(1.9)*
α	-0.11	-(1.7)*	-0.13	-(2.0)**
σ	0.12	(1.9)*	0.11	(1.7)*
Sharpe	-0.91	-(1.2)	-1.07	-(1.7)*
Corr Mkt	0.14	(0.4)	0.23	(0.7)
Autocorr	-0.21	-(0.8)	-0.17	-(0.7)

Table 6. Secondary Market Based Indices 2006-2017 Excluding the Financial Crisis

This table reports moment estimates for buyout indices using data from 2006-2017 excluding the years 2008 and 2009. Panel A is for our transactions-based indices, Panel B is for Preqin NAV-based indices, and Panel C is for the difference. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the baseline merged samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1 except for the autocorrelation. For the naïve index, β and α are also bias adjusted. We estimate standard errors for index parameters using a two-step bootstrap procedure based the subsampling methods of Politis and Romano (1994). Significance at the 1%, 5%, and 10% levels is indicated, respectively, by "***", "**", and "*".

Panel A: Transactions-Based Indices

	Buyout				Venture	
	Hedonic Size Weighted		Naïve Price Weighted		Hedonic Size Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.23	(2.4)**	0.29	(2.7)***	0.06	(1.0)
β	0.92	(2.0)**	0.42	(0.7)	0.72	(2.2)**
α	0.11	(1.0)	0.23	(1.5)	-0.04	-(0.5)
σ	0.18	(2.6)***	0.09	(1.1)	0.13	(2.7)***

Panel B: Size-Weighted NAV Based Indices

	Buyout				Venture			
	Prequin		Burgiss		Prequin		Burgiss	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.17	(6.0)***	0.14	(4.7)***	0.12	(5.9)***	0.14	(5.3)***
β	0.29	(3.6)***	0.30	(4.0)***	0.22	(3.9)***	0.23	(2.7)***
α	0.12	(4.9)***	0.09	(3.8)***	0.08	(4.8)***	0.11	(4.2)***
σ	0.06	(3.9)***	0.06	(4.1)***	0.05	(6.4)***	0.06	(6.1)***

Differences Relative to Hedonic Size-weighted Indices

	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.07	(0.8)	0.09	(1.1)	-0.06	-(1.0)	-0.08	-(1.4)
β	0.64	(1.5)	0.62	(1.4)	0.50	(1.6)	0.49	(1.5)
α	-0.01	-(0.1)	0.02	(0.2)	-0.12	-(1.6)	-0.14	-(1.9)*
σ	0.12	(1.9)*	0.12	(1.9)*	0.09	(1.8)*	0.07	(1.4)

Table 7. Market-to-Book Ratios of Private Equity Investments

This table reports year-end average market-to-book ratios. Market values for each fund are calculated using the following procedure. We begin by assuming that the market value of the fund is equal to NAV in years one through four of the fund's life. We then calculate the market value each quarter from years 5-9 for fund *i* using the following formula:

$$market\ value_{i,t} = market\ value_{i,t-1} * (1 + r_t) + Calls_t - Distributions_t \ .$$

For the first quarter in year five, we use NAV as the preceding quarter's market value. The aggregate market-to-book ratio reported in this table is calculated as the sum of the individual fund's market value within each quarter divided by the sum of the individual fund's NAV in each quarter. We report the resultant market-to-book ratio for Q4 of each year, with the exception of 2017, where we report values as of Q2 due to data limitations. Panel A reports results for buyout funds. Panel B reports results for venture funds.

Panel A. Buyout

	Vintage Year											
	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
2006	1.14	--	--	--	--	--	--	--	--	--	--	--
2007	1.04	1.02	--	--	--	--	--	--	--	--	--	--
2008	0.77	0.65	0.76	--	--	--	--	--	--	--	--	--
2009	0.97	0.86	1.00	1.04	--	--	--	--	--	--	--	--
2010	1.24	1.03	1.23	1.28	1.05	--	--	--	--	--	--	--
2011	1.20	0.92	1.12	1.16	0.91	0.87	--	--	--	--	--	--
2012	--	0.98	1.25	1.28	0.97	1.01	1.05	--	--	--	--	--
2013	--	--	1.80	1.71	1.19	1.23	1.25	1.13	--	--	--	--
2014	--	--	--	1.86	1.14	1.17	1.17	0.99	1.02	--	--	--
2015	--	--	--	--	1.28	1.28	1.31	1.05	1.02	1.03	--	--
2016	--	--	--	--	--	1.22	1.28	0.91	0.91	0.93	1.00	--
2017	--	--	--	--	--	--	1.44	1.01	1.07	1.10	1.17	1.13
Average	1.06	0.91	1.19	1.39	1.09	1.13	1.25	1.02	1.00	1.02	1.08	1.13

Table 7. *Continued*

Panel B. Venture												
	Vintage Year											
	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
2006	1.03	--	--	--	--	--	--	--	--	--	--	--
2007	0.88	0.87	--	--	--	--	--	--	--	--	--	--
2008	0.80	0.80	0.92	--	--	--	--	--	--	--	--	--
2009	0.98	0.99	1.14	1.01	--	--	--	--	--	--	--	--
2010	0.89	1.02	1.09	0.99	0.92	--	--	--	--	--	--	--
2011	0.72	0.83	0.98	0.80	0.83	0.90	--	--	--	--	--	--
2012	--	0.86	0.99	0.86	0.90	0.99	1.01	--	--	--	--	--
2013	--	--	0.97	0.90	0.94	1.07	1.09	0.97	--	--	--	--
2014	--	--	--	0.72	0.70	0.85	0.91	0.79	0.90	--	--	--
2015	--	--	--	--	0.63	0.78	0.87	0.82	0.94	0.97	--	--
2016	--	--	--	--	--	0.72	0.86	0.68	0.88	0.92	0.96	--
2017	--	--	--	--	--	--	0.95	0.74	0.96	1.02	1.06	0.90
Average	0.88	0.90	1.02	0.88	0.82	0.89	0.95	0.80	0.92	0.97	1.01	0.90