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ABSTRACT

Empirical evidence suggests that for many countries, retail prices of traded goods are sticky in national currencies. Movements in exchange rates then cause deviations from the law of one price, and exchange rate ëmisalignmentí, which cannot be corrected by monetary policy alone. This paper shows that a state contingent international tax policy can be combined with monetary policy to eliminate exchange rate misalignment and sustain a fully efficient welfare outcome. But this monetary-fiscal mix cannot be decentralized with non-cooperative determination of monetary and fiscal policy. Non-cooperative use of taxes and subsidies introduces strategic spillovers which opens up a fundamental conflict between the goals of output gap and inflation stabilization and those of terms of trade manipulation in an open economy. The implementation of an efficient monetary-fiscal mix requires effective cooperation in fiscal policy, while leaving monetary policy to be determined non-cooperatively. In addition, while an efficient outcome requires state contingent taxes and subsidies to eliminate exchange rate misalignment, it is still necessary to have flexible exchange rates and independent monetary policy.

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A data appendix is available at http://www.nber.org/data-appendix/w25111

1 Introduction

The benefit of flexible exchange rates represents a central pillar of open economy macroeconomics. Famously, Friedman (1953) argued for the ease of adjusting relative prices with nominal exchange rate movement as opposed to nominal prices in an environment of sticky prices. The extension of the Friedman case for flexible exchanges rate is that it allows for national monetary independence. With flexible exchange rates, individual countries can follow independent, or 'self-oriented' monetary policies (using the language of Obstfeld and Rogoff, 2002), without the need for international policy cooperation.

The more recent New Keynesian open economy literature has articulated Friedman's argument in fully specified welfare-based DSGE models. In particular Clarida, Gali and Gertler (2002) and Obstfeld and Rogoff (2002) show that an optimal monetary policy requires flexible exchange rates, and in certain environments, self-oriented monetary policy-making can achieve a global optimum, without the need for (or any benefits of) international policy cooperation. [1] Both these papers however follow the traditional assumption about traded goods pricing, notably that prices are set in producer's currency (PCP).

Recent contributions have qualified these arguments, based on the evidence that nominal exchange rate adjustment may fail to achieve efficient relative price adjustment. For instance, when traded goods prices are set in the currency of the buyer (local currency pricing, or LCP), movements in nominal exchange rates will generate deviations from the law of one price across countries. In this case, independent monetary policy and flexible exchange rates cannot fully undo the inefficiencies generated by nominal price stickiness. Engel (2011) constructs a New Keynesian open economy model with LCP, and shows that optimal monetary policy faces a trade-off between the output gap, inflation control, and exchange rate 'misalignment' due to deviations from the law of one price. In this case, even cooperative monetary policy-making cannot achieve a full global optimum.

Fujiwara and Wang (2017) extend Engel's model to the case of non-cooperative policy making. They identify a new set of strategic inefficiencies associated with non-cooperative

[1] Many other papers have analyzed non-cooperative monetary policy in open economies. See for instance Corsetti and Pesenti, 2001, and Corsetti, Dedola and Leduc, 2010. We discuss the literature more fully below.

policy-making in an environment of LCP, and show that in general there are positive gains to policy-cooperation under LCP, even in the case where no such gains would exist with PCP.

A separate and more recent literature has widened the debate on the limits to monetary policy by exploring the possibility of using targeted fiscal instruments to achieve effective relative price adjustment even when nominal prices cannot move. One strand of this literature is motivated by the constraints of a single currency area, where by definition nominal exchange rates cannot respond to country specific shocks. For instance, Fahri Gopinath and Itskhoki (2013) show that a set of taxes and subsidies can replicate an exchange rate devaluation under a fixed exchange rate regime. Other papers are motivated by the constraints on monetary policy imposed by the zero lower bound (see for instance, Correa, Fahri, Nicolini, and Teles 2013). But as we have discussed, local currency pricing also imposes limits on the ability of monetary policy to achieve fully efficient outcomes. Somewhat less attention has been paid to this case. [2] Can these constraints be undone by targeted sets of taxes and subsidies? This is the first question addressed in the current paper.

Our paper asks whether policy makers can use a set of consumption and producer taxes and subsidies to eliminate the distortions imposed by deviations from the law of one price. Do these fiscal instrument then restore the efficacy of monetary policy and the benefits of flexible exchange rates, or indeed possibly eliminate the need for exchange rate adjustment at all?

But the second more important question we address is whether the use of fiscal instruments restores the case for self-oriented policymaking (monetary independence). As noted above, Fujiwara and Wang (2017) show that there are losses from non-cooperative policy making under LCP. If policy makers have access to taxes and subsidies that can eliminate deviations from the law of one price (or exchange rate misalignment), does this imply that independent non-cooperative policy making can sustain full efficiency? Alternatively, one might conjecture that the addition of independent fiscal policy instruments into policy-

[2] A notable exception is Adao Correa and Teles 2009, who derive a set of tax policies that can achieve an efficient global outcome independent of the type of nominal price stickiness. We discuss their paper and the differences from ours in more detail below.

making introduces a new set of strategic inefficiencies in independent policymaking.

Our paper investigates these two questions. We construct a two country open economy model under LCP along the lines of Engel (2011) and Fujiwara and Wang (2017). Each country is subjected to random shocks to labour productivity. Generically, this environment is characterized by exchange rate misalignment, causing welfare losses due to deviations from the law of one price. We then identify a set of taxes and subsidies that can eliminate inefficiencies coming from these deviations from the law of one price. In particular, in response to a home country (positive) productivity shock, an optimal tax policy entails a positive home country tax on imports and a foreign country subsidy to exports, combined with the opposite response of the foreign consumption tax and the home exporter production subsidy. This set of tax-subsidy responses can perfectly replicate the relative price movement that would take place under PCP, eliminating deviations from the law of one price. Intuitively, this tax-subsidy combination will lead consumers and producers to tilt their behaviour in a manner that would occur if all prices were set in producer's currency and there existed full exchange rate pass-through in traded goods prices.

An interesting feature of this optimal policy mix is that it does not obviate the need for exchange rate adjustment. We show that, conditional on the optimal set of taxes and subsidies being in place, an optimal monetary rule will call for exchange rates to adjust to productivity shocks exactly as would occur under PCP, as in Clarida, Gali, and Gertler (2002), for instance. Thus, the use of optimal taxes and subsides fully restores Friedman's 'case for flexible exchange rates'. Translated into our model, using the welfare approximations of Engel (2011) and Fujiwara and Wang (2017), we show that a cooperative equilibrium in which the policy maker chooses both fiscal and monetary policy simultaneously will achieve the fully efficient flexible price equilibrium.

We then address the critical question of implementability. Is this equilibrium consistent with non-cooperative policy-making? Here, our results are quite striking. We find that the ability to independently choose consumption taxes and production subsidies opens up a new strategic channel in non-cooperative policy making associated with terms of trade manipulation. Acting independently, each policymaker attempts to bias the tax-subsidy instruments so as to raise its expected terms of trade. Strikingly, we show that, when the tax-subsidy choice is unrestricted, there is in fact no equilibrium to the non-cooperative game consistent with finite inflation rates and output gaps. [3]

We address this dilemma in two ways. First, we show that if we impose a particular restriction on the set of consumption taxes and production subsidies, then the equilibrium of the non-cooperative game will coincide with the cooperative equilibrium, and will attain the flexible price equilibrium. In particular this condition requires that the home consumption tax be restricted to equal the foreign production subsidy.

But this restriction may seem somewhat arbitrary, and it is unclear how it would be imposed in a non-cooperative equilibrium. To resolve this, we conclude that an efficient monetary-fiscal mix will require fiscal policy cooperation. We define a new game in which taxes and subsidies are chosen by a cooperative fiscal authority maximizing world welfare, while monetary policy is determined independently by each monetary authority. We show that the equilibrium of this game will be the same as the cooperative equilibrium in which both fiscal and monetary policy are chosen by a single benevolent authority.

The implication of these results is then clear - an optimally chosen set of fiscal responses can eliminate the restrictions on monetary policy implied by LCP, and restore the benefits of flexible exchange rates. But in order for this to be consistent with independent 'self-reliance' in monetary policy, it is necessary to have effective cooperation in fiscal policy. Alternatively, the message could be interpreted as saying that absent fiscal cooperation, or effective restrictions on fiscal responses, the use of taxes and subsidies to correct deviations from the law of one price and support independent policy-making cannot be successful.

An interesting feature of our results is that the use of optimal tax-subsidies changes the nature of optimal monetary policy under LCP. Engel (2011) argues that under LCP, even if output gaps in both countries were somehow eliminated and producer price infla-

[3] We note that the channel of terms of trade manipulation in optimal monetary policy has been explored extensively in the New Keynesian literature. See for instance Corsetti and Pesenti, 2001, and Clarida Gali and Gertler 2002 for early discussion. But our results are quite different from the conventional analysis. We show that this strategic inefficiency is generated solely by the use of state varying fiscal instruments in monetary policy. Without the use of fiscal instruments, the equilibrium under non-cooperative monetary policy is well defined, as in Fujiwara and Wang 2017.

tion were zero, consumption is misallocated if there are currency misalignments, due to a failure of efficient cross-country risk-sharing. As a consequence, he shows that with LCP, the monetary authority should target consumer price inflation instead of producer-price inflation. We find that in both cooperative and non-cooperative games, so long as fiscal instruments are set optimally, the currency misalignment is eliminated, there are no deviations from the law of one prices, and there is full cross country risk-sharing (conditional on home-bias in preferences). A consequence of this is that the monetary authority should still target producer-price inflation in the dynamic model with staggered pricing.

A later section of the paper presents a quantitative analysis of welfare gains. The welfare gains from correcting currency misalignments under a cooperative monetary policy are very small. But the welfare gains from an optimal monetary fiscal-mix, compared to a non-cooperative monetary policy equilibrium under LCP, while still small in absolute terms, are substantially larger.[4]

This paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the benchmark dynamic model with staggered price setting and explains why the introduction of a fax on the consumption of foreign goods may correct the currency misalignment. Section 4 introduces the staggered price model with state-contingent taxes and export subsidies. In section 5, we show that, with the optimal choice of these taxes and subsidies in a cooperative game, the flexible price equilibrium can be achieved. In Section 6 we explore policy-making without cooperation. We show that without restrictions on fiscal instruments, the non-cooperative equilibrium will not exist. We then introduce a three player game, where a fiscal authority chooses taxes and subsidies cooperatively, while monetary authorities follow non-cooperative policies. Section 7 presents some quantitative welfare analysis results. Section 8 discusses some other possible fiscal instruments. Section 9 concludes.

[4]This is in accord with the results of Fujiwara and Wang (2017), who quantify the welfare gains from monetary policy cooperation under LCP.

2 Related Literature

Our paper is related to the literature on optimal fiscal policy in open economies. Gali and Monacelli (2008) study how the government chooses the optimal level of public consumption in a monetary union with lump-sum taxes. They find that the choice of fiscal policy raises union welfare. Hevia and Nicolini (2013) consider a small open economy with flexible exchange rates and state-contingent assets. They find that flexible price equilibrium is implementable, but exchange rates must move across states. Benigno and Paoli (2010) also analyze optimal fiscal policy and in small open economy, but in their set-up, the optimal flexible price allocation cannot be achieved.

Our paper is quite closely related and complementary to Adao, Correia, and Teles (2009). They show that in an environment with nominal rigidities, whatever is the type of price setting (PCP or LCP), the exchange rate regime, whether flexible or fixed exchange rates, is irrelevant once fiscal policy instruments (both taxes on labor income and consumption of home and foreign goods) are taken into account. Hence fiscal policy instruments can replicate the flexible price equilibrium even under LCP. They derive these results using a Ramsey approach, which requires a benevolent social planner who can use the available policy instruments to replicate the flexible price allocation. They do not address the question of how to implement the optimal fiscal policy in a non-cooperative game, which is the main focus of our paper. We show that a Nash game of international tax policy, together with inward-looking monetary policy, can replicate the flexible price equilibrium, as long as we have fiscal policy cooperation.

Another important difference between our paper and theirs is related to the role of exchange rate flexibility. They find that the flexible price allocation can be achieved by fiscal and monetary policies that induce stable producer prices and without movements in the exchange rate. The exchange rate will not play any role, so the exchange rate regime becomes irrelevant. But in our paper, it is critical to allow for flexible exchange rates, in order to achieve complete risk sharing. Hence, the tax policies identified in our paper not only replicate the real variables under a flexible price allocation, but also replicate the efficient risk-sharing response of the exchange rate fluctuations under flexible prices. [5]

[5] Adao, Correia, and Teles 2009 also differ from us in the nature of financial markets. They allow for

Our paper also belongs to a small but fast growing literature that emphasizes the role of fiscal policy in replacing monetary policy or exchange rate policy as a macro stabilization tool. Schmitt-Grohe and Uribe (2011) show that, when there is downward wage rigidity and inelastic labor supply, a payroll subsidy alone can replicate the effect of nominal exchange rate devaluation. Farhi, Gopinath and Itskhoki (2013) show that, when the exchange rate cannot be devalued, a small set of conventional fiscal instruments can robustly replicate the real allocation attained under a nominal exchange rate devaluation in a dynamic New Keynesian open economy environment. In both these papers, the assumption is that nominal exchange rate is fixed, perhaps due to membership of a single currency area. In contrast, the exchange rate regime is flexible in our model, but exchange rate changes are inefficient due to LCP.

As mentioned in the introduction, the addition of fiscal instruments to substitute or complement monetary policy may also introduce strategic spillover channels when we extend the analysis to study non-cooperative policy-making. A key result of our paper is that absent some additional constraints on the design of corrective taxes or subsidies, in an environment where policy-makers act independently, these strategic inefficiencies may render fiscal policy ineffective in correcting macro inefficiencies.

Finally, topic wise, this paper is also closely related to the literature on optimal monetary policy in open economy under LCP. This question is treated in Devereux and Engel (2003). As discussed above, Engel (2011) shows that even with policy coordination, optimal monetary policy cannot replicate the flexible price equilibrium, due to the currency misalignment induced by LCP. Fujiwara and Wang (2017) focus on a non-cooperative game under LCP in a two-country dynamic general equilibrium model. They find that in this case, policy makers face extra trade-offs regarding stabilizing real marginal cost induced by the deviations from law of one price. Both papers therefore emphasize the im-

two state-contingent domestic bonds, but in their model there are no state-contingent international bonds. So there is not risk-sharing across countries in their baseline model. When state-contingent bonds can be traded across countries, they show that an extra fiscal instrument, a state-contingent government bond, is needed to produce the irrelevance result. Our framework by contrast is based on the assumption of complete markets for cross-country risk-sharing. Section 8 below allows for a labor income tax and consumption tax in our model, and shows that these instruments cannot replicate the flexible price equilibrium in a non-cooperative Nash game.

portance of currency misalignment in the optimal monetary policy. Our paper identifies the a monetary-fiscal mix that can correct this misalignment, but also the restrictions on independent policy-making required to implement such a policy.

3 Baseline Model and the Consumption Tax

In this section, we derive the loss function under the cooperative game in the standard LCP model as in Engel (2011) and Fujiwara and Wang (2017). From the loss function, we will explore the possible fiscal instruments that can be used to correct the welfare loss due to deviations from law of one price under local currency pricing.

Our baseline model is similar to Engel (2011) and Fujiwara and Wang (2017). There are two countries of equal size, Home and Foreign, each populated with a continuum of households with population size normalized to unity. Households in each country consume both home and foreign goods with a symmetric home bias. They supply labor to firms in a competitive labor market. Firms are monopolisticly competitive and produce differentiated goods. They set prices in a staggered manner (Calvo 1983) and in local currency. In the model, the government levy a lump-sum tax on households and subsidize firms to eliminate steady-state distortions from monopolistic pricing. To avoid complexity, we assume a complete financial market.

3.1 Household

The representative household is assumed to maximize lifetime utility defined as,

$$U = E_{t=0}^{\infty}\beta^t(\frac{C_t^{1-\rho}}{1-\rho} - \eta L_t)$$

where $C_t = C_{ht}^{\frac{v}{2}} C_{ft}^{1-\frac{v}{2}}$. When $v > 1$, the household exhibits home-bias in preference. The budget constraint under complete markets is given by

$$P_tC_t + B_{ht+1} + \sum_{\zeta^{t+1} \in Z_{t+1}} B(\zeta^{t+1}|\zeta^t)D(\zeta^{t+1}) = W_tL_t + R_{t-1}B_{h,t} + \Pi_t + T_t + D(\zeta^t), \quad (3.1)$$

where $D(\zeta^t)$ represents the household's payoff on state-contingent claims at state ζ^t. $B(\zeta^{t+1}|\zeta^t)$ is the price of a claim that pays one dollar in state ζ^{t+1}, conditional on state ζ^t occurring at time t. B_h is household's holding of domestic bonds, and R_t is the domestic gross interest rate between period t and $t+1$. The consumption-based price index is

$$P_t = \Theta^{-1} P_{hh,t}^{\frac{v}{2}} P_{fh,t}^{1-\frac{v}{2}} \tag{3.2}$$

where $\Theta = (\frac{v}{2})^{\frac{v}{2}}(1-\frac{v}{2})^{1-\frac{v}{2}}$. P_{hht} and P_{fht} represent the price set by firms for home produced goods and foreign produced goods sold in the home market, respectively. Trade in state-contingent nominal assets across countries will lead to the standard risk-sharing condition:

$$\frac{C_t^{-\rho}}{P_t} = \Gamma \frac{C_t^{*-\rho}}{S_t P_t^*}, \tag{3.3}$$

where S_t is the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency, and $P_t^* = \Theta^{-1}(P_{hf,t}^*)^{1-\frac{v}{2}} P_{ff,t}^{*\frac{v}{2}}$ is the foreign price level. [6] Γ is the state-invariant weight, in a symmetric model, $\Gamma = 1$.

Similarly, foreign households maximize

$$U^* = E_{t=0}^{\infty} \beta^t \left(\frac{C_t^{*1-\rho}}{1-\rho} - \eta L_t^*\right)$$

3.2 Production

Each firm i in the home economy has the following production technology.

$$Y_t(i) = Z_t L_t(i) \tag{3.4}$$

where $Z_t = exp(\theta_t)$ is a country-specific shock, and θ_t is distributed with mean zero and variance σ_θ^2. We assume a Dixit-Stiglitz consumption structure. Each firm produces a differentiated good facing a downward-sloping individual demand curve and chooses its optimal price along the demand curve. Export prices are assumed to be set in local

[6] Equation (\ref{eq:risk-sharing}) implies that one dollar can get the same marginal utility of consumption across countries, or the ratio of marginal utilities of consumption is equal to the real exchange rate.

currency (LCP). As in Clarida, Gali, and Gertler (2002), firm adjust prices following the standard Calvo mechanism.

In the home country, a firm may reset its prices with probability $1-\kappa$ each period. When the firm resets prices, it will be able to reset two prices, $P^o_{hh,t}(i)$ in home currency for sales in the home market, and $P^{o*}_{hf,t}(i)$ in foreign currency for sales in the foreign market. The firm maximizes the following objective function,

$$E_t \sum_{j=0}^{\infty} \kappa^j \beta_{t,t+j}(1+\tau)[(P^o_{hht}(i) - MC_{t+j}(i))C_{h,t+j}(i) + (S_{t+j}P^{o*}_{hf,t}(i) - MC_{t+j}(i))C^*_{h,t+j}(i)]$$

where $C_{ht}(i)$ and $C^*_{ht}(i)$ are the demand for the home good i from the home and foreign markets, respectively, and $\beta_{t,t+j} = \beta^j(\frac{C_{t+j}}{C_t})^{-\rho}(\frac{P_t}{P_{t+j}})$ is the stochastic discount factor. $\tau = \frac{1}{\lambda-1}$ is a subsidy imposed by the government to eliminate the steady state monopolistic distortion.

The firm in the foreign country faces an analogous problem, and chooses $P^{*o}_{ff,t}$, in terms of foreign currency and $P^o_{fh,t}$, in terms of home currency.

3.3 Market Clearing Conditions and Equilibrium

The goods market clearing conditions for home and foreign goods are thus given by,

$$Y_t = \frac{v}{2}\frac{P_tC_t}{P_{hh,t}}\Delta_{hh,t} + (1-\frac{v}{2})\frac{P^*_tC^*_t}{P^*_{hf,t}}\Delta^*_{hf,t} \tag{3.5}$$

$$Y^*_t = (1-\frac{v}{2})\frac{P_tC_t}{P_{fh,t}}\Delta_{fh,t} + \frac{v}{2}\frac{P^*_tC^*_t}{P^*_{ff,t}}\Delta^*_{ff,t} \tag{3.6}$$

where Δ are price dispersion terms

$$\Delta_{hh,t} = \int_0^1 (\frac{P_{hh,t}(i)}{P_{hh,t}})^{-\lambda}di,\ \Delta^*_{hf,t} = \int_0^1 (\frac{P^*_{hf,t}(i)}{P^*_{hf,t}})^{-\lambda}di \tag{3.7}$$

$$\Delta^*_{ff,t} = \int_0^1 (\frac{P^*_{ff,t}(i)}{P^*_{ff,t}})^{-\lambda}di,\ \Delta_{fh,t} = \int_0^1 (\frac{P_{fh,t}(i)}{P_{fh,t}})^{-\lambda}di \tag{3.8}$$

We also have the two domestic bond markets clearing condition ($B_h = 0$ and $B^*_f = 0$) and the state-contingent bond market clearing condition. We define the real exchange rate, the terms of trade and deviation from law of one price as below, respectively.

$$e_t = \frac{S_t P_t^*}{P_t}, Q_t = \frac{P_{fht}}{S_t P_{hft}^*} \tag{3.9}$$

$$d_t = \frac{S_t P_{hft}^*}{P_{hht}}, d_t^* = \frac{P_{fht}}{S_t P_{fft}^*} \tag{3.10}$$

The equilibrium is characterized by the household labor supply, price index, firms' optimal pricing strategy and labor demand, as well as the market clearing conditions and risk–sharing condition as listed in Table 1.

3.4 Linear-quadratic Loss Function and Consumption Tax

We first define the loss function under the cooperative case and check if it is possible to use a fiscal instrument to correct the currency misalignment. Also, as discussed in Clarida, Gali and Gertler (2002), the terms of trade effect and risk-sharing effect cancel out with each other when $\rho = 1$, which greatly simplifies the expression for the loss function. So for simplicity, we first assume $\rho = 1$. Since countries are of equal size, the global welfare loss in the cooperative game is $L_0 = L_{h,0} + L_{h,0}^*$, where $L_{h,0}$ and $L_{h,0}^*$ are the loss functions for the home and foreign countries, respectively.

In the following analysis, $\widehat{X_t} = \log(X_t) - \log(\overline{X})$ indicates the log-deviation of a variable from the respective steady state. All the nominal variables are normalized by the CPI. For example, the real price $p_{x,t} = \frac{P_{x,t}}{P_t}$ is the price level relative to CPI index; $mc_t = \frac{MC_t}{P_t}$ is the real marginal cost at Home. The global loss function[7] when $\rho = 1$ is given by

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\begin{array}{c} \frac{1}{2}(\widehat{Y}_t - \theta_t)^2 + \frac{1}{2}(\widehat{Y}_t^* - \theta_t^*)^2 \\ + \frac{\lambda v}{4\delta}\pi_{hh,t}^2 + \frac{\lambda(2-v)}{4\delta}\pi_{fh,t}^2 + \frac{\lambda v}{4\delta}(\pi_{ff,t}^*)^2 + \frac{\lambda(2-v)}{4\delta}(\pi_{hf,t}^*)^2 \\ + \frac{v(2-v)}{8}(\widehat{d}_t^*)^2 + \frac{v(2-v)}{8}(\widehat{d}_t)^2 \end{array} \right]$$

The first two terms are quadratic deviations from steady state output. The following four terms represent squared inflation rates of local as well as imported products, which captures the costs of price dispersion. For example, $\pi_{hh,t} = \pi_t + \widehat{p}_{hh,t} - \widehat{p}_{hh,t-1}$ is the

[7] Detailed derivation of the loss function under the cooperative game is described in Engel (2011) and Fujiwara and Wang (2017).

Table 1: Equations of the Model

Home	Foreign
(a) Optimal conditions for consumers	
$\frac{W_t}{P_t}C_t^{-\rho}=\eta$ $\frac{1}{R_t}=\beta E_t\frac{C_t^{\rho}}{C_{t+1}^{\rho}\pi_{t+1}}$	$\frac{W_t^*}{P_t^*}C_t^{*-\rho}=\eta$ $\frac{1}{R_t^*}=\beta E_t\frac{C_t^{*\rho}}{C_{t+1}^{\rho}\pi_{t+1}^*}$
(b) Production and Optimal pricing for firms	
$Y_t=Z_tL_t$ $P_{hh,t}^o=\frac{E_t\sum_{j=0}^{\infty}\kappa^j\beta_{t,t+j}P_{hh,t+j}^{\lambda}MC_{t+j}C_{h,t+j}}{E_t\sum_{j=0}^{\infty}\kappa^j\beta_{t,t+j}P_{hh,t+j}^{\lambda}C_{h,t+j}}$ $P_{fh,t}^o=\frac{E_t\sum_{j=0}^{\infty}\kappa^j\beta_{t,t+j}^*P_{fh,t+j}^{\lambda}MC_{t+j}^*C_{f,t+j}}{E_t\sum_{j=0}^{\infty}\kappa^jS_{t+j}^{-1}\beta_{t,t+j}^*P_{fh,t+j}^{\lambda}C_{f,t+j}}$	$Y_t^*(i)=Z_tL_t$ $P_{hf,t}^{o*}=\frac{E_t\sum_{j=0}^{\infty}\kappa^j\beta_{t,t+j}(P_{hf,t+j}^*)^{\lambda}MC_{t+j}C_{h,t+j}^*}{E_t\sum_{j=0}^{\infty}\kappa^jS_{t+j}\beta_{t,t+j}(P_{hf,t+j}^*)^{\lambda}C_{h,t+j}^*}$ $P_{ff,t}^{o*}=\frac{E_t\sum_{j=0}^{\infty}\kappa^j\beta_{t,t+j}^*(P_{ff,t+j}^*)^{\lambda}MC_{t+j}^*C_{f,t+j}^*}{E_t\sum_{j=0}^{\infty}\kappa^j\beta_{t,t+j}^*(P_{ff,t+j}^*)^{\lambda}C_{f,t+j}^*}$
(c) Price Index	
$P_t=\Theta^{-1}P_{hh,t}^{\frac{v}{2}}P_{fh,t}^{1-\frac{v}{2}}$ $P_{hh,t}=[\kappa P_{hh,t-1}^{1-\lambda}+(1-\kappa)(P_{hh,t}^o)^{1-\lambda}]^{\frac{1}{1-\lambda}}$ $P_{fh,t}=[\kappa P_{fh,t-1}^{1-\lambda}+(1-\kappa)(P_{fh,t}^o)^{1-\lambda}]^{\frac{1}{1-\lambda}}$	$P_t^*=\Theta^{-1}(P_{hf,t}^*)^{1-\frac{v}{2}}P_{ff,t}^{*\,\frac{v}{2}}$ $P_{hf,t}^*=[\kappa P_{hf,t-1}^{*1-\lambda}+(1-\kappa)(P_{hf,t}^{*o})^{1-\lambda}]^{\frac{1}{1-\lambda}}$ $P_{ff,t}^*=[\kappa P_{ff,t-1}^{*1-\lambda}+(1-\kappa)(P_{ff,t}^{*o})^{1-\lambda}]^{\frac{1}{1-\lambda}}$
(d) Market Clearing Conditions	
$Y_t=\frac{v}{2}\frac{P_tC_t}{P_{hh,t}}\Delta_{hh,t}+(1-\frac{v}{2})\frac{P_t^*C_t^*}{P_{hf,t}^*}\Delta_{hf,t}^*$ $\Delta_{hh,t}=\int_0^1(\frac{P_{hh,t}(i)}{P_{hh,t}})^{-\lambda}di$ $\Delta_{hf,t}^*=\int_0^1(\frac{P_{hf,t}^*(i)}{P_{hf,t}^*})^{-\lambda}di$	$Y_t^*=(1-\frac{v}{2})\frac{P_tC_t}{P_{fh,t}}\Delta_{fh,t}+\frac{v}{2}\frac{P_t^*C_t^*}{P_{ff,t}^*}\Delta_{ff,t}^*$ $\Delta_{ff,t}^*=\int_0^1(\frac{P_{ff,t}^*(i)}{P_{ff,t}^*})^{-\lambda}di$ $\Delta_{fh,t}=\int_0^1(\frac{P_{fh,t}(i)}{P_{fh,t}})^{-\lambda}di$
Risk sharding condition:	
$C_t^{-\rho}=\Gamma\frac{(C_t^*)^{-\rho}}{e_t}$	

where $\Theta=(\frac{v}{2})^{\frac{v}{2}}(1-\frac{v}{2})^{1-\frac{v}{2}}$ is a constant term; $\Gamma=1$.

deviation of inflation of home goods sold in home from the steady state ($\bar{\pi_{hh}} = 0$), and the other terms are defined analogously. As in Clarida, Gali and Gertler (2002), the terms of trade $\widehat{q}_t = \widehat{p}_{fh,t} - \widehat{e}_t - \widehat{p}^*_{hf,t}$ will not appear in the loss function, given $\rho = 1$. The last two terms $\widehat{d}_t = \widehat{p}^*_{hf,t} + \widehat{e}_t - \widehat{p}_{hh,t}$ and $\widehat{d}^*_t = \widehat{p}_{fh,t} - \widehat{e}_t - \widehat{p}^*_{ff,t}$ are related to the welfare loss due to deviations from the law of one price (LOOP) . This is the currency misalignment problem emphasized in Engel(2011)[8].

How can a fiscal instrument correct the welfare loss due to $\widehat{d}_t$ and $\widehat{d}^*_t$? Exploring why they show up in the welfare loss may help us to answer this question. Technically, $\widehat{d}^*_t$ and $\widehat{d}_t$ appear in the loss function through the second order approximations of market clearing condition (3.5) and (3.6).

$$\begin{aligned} & \theta_t + \widehat{L}_t - \widehat{C}_t + \frac{2-v}{2}(\widehat{C}_t - \widehat{C}^*_t) + \frac{v}{2}\widehat{p}_{hh,t} - \frac{2-v}{2}\widehat{p}^*_{hf,t} \\ = & \frac{v(2-v)}{8}(\widehat{d}_t)^2 + \frac{v\lambda}{4\delta}\pi^2_{hh,t} + \frac{(2-v)\lambda}{4\delta}\pi^{*2}_{hf,t} \end{aligned} \tag{3.11}$$

$$\begin{aligned} & \theta^*_t + \widehat{L}^*_t - \widehat{C}^*_t - \frac{2-v}{2}(\widehat{C}_t - \widehat{C}^*_t) + \frac{v}{2}\widehat{p}^*_{ff,t} - \frac{2-v}{2}\widehat{p}_{fh,t} \\ = & \frac{v(2-v)}{8}(\widehat{d}^*_t)^2 + \frac{(2-v)\lambda}{4\delta}\pi^2_{fh,t} + \frac{v\lambda}{4\delta}\pi^{*2}_{ff,t} \end{aligned} \tag{3.12}$$

The deviation from LOOP affects the consumption demand for home produced goods sold in foreign and foreign produced goods sold in home country. Intuitively, in face of a positive technology shock in the home country, the local currency pricing strategy results in a different response of price adjustment in different markets. This term associated with $\widehat{d}_t$ in the loss function measures the efficiency loss of the home country due to LCP, since the foreign country cannot allocate their consumption expenditure as efficiently as under the PCP case (where LOOP holds).

A simple way to correct the currency misalignment is to introduce a consumption tax on imported goods which can respond to foreign technology shocks so that after-tax, the deviation from the law of one price is zero. In other words, we can introduce a tax $t_{c,t}$ on home imported goods and a tax $t^*_{c,t}$ on foreign imported goods such that the consumption demand for imported goods will respond efficiently, as under the PCP case. Specifically,

[8] In Engel, currency misalignment is defined as $m_t = \frac{1}{2}(\widehat{d}_t + \widehat{d}^*_t)$.

the price of home imported goods faced by consumers will be $P_{fh,t}(1+t_{c,t})$, while the price of foreign imported goods faced by consumers is $P^*_{hf,t}(1+t^*_{c,t})$. Then, hopefully in log terms,

$$\widehat{d}_t + \widehat{t}_{c,t} = \widehat{p}^*_{hf,t} + \widehat{e}_t - \widehat{p}_{hh,t} + \widehat{t}_{c,t} = 0 \tag{3.13}$$

$$\widehat{d}^*_t + \widehat{t}^*_{c,t} = \widehat{p}_{fh,t} - \widehat{e}_t - \widehat{p}^*_{ff,t} + \widehat{t}^*_{c,t} = 0 \tag{3.14}$$

In the following section, we show that this consumption tax on imported goods can correct the deviations from LOOP.

4 The Model with Fiscal Instruments

In this section, we introduce a consumption tax into the benchmark model and explore the implications for welfare. We find that the introduction of a consumption tax will lead to production distortions, so we also need to introduce a production subsidy to correct this inefficiency.

4.1 The System with Consumption Taxes

As discussed in Section 3, if we introduce a tax $t_{c,t}$ on home imported goods and a tax $t^*_{c,t}$ on foreign imported goods, the demand functions for imported varieties at Home and at Foreign are

$$C_{f,t}(i) = (1-\frac{v}{2})\frac{P_t C_t}{P_{fh,t}(1+t_{c,t})}(\frac{P_{fh,t}(i)}{P_{fh,t}})^{-\lambda} \tag{4.15}$$

$$C^*_{h,t}(i) = (1-\frac{v}{2})\frac{P^*_t C^*_t}{P^*_{hf,t}(1+t^*_{c,t})}(\frac{P^*_{hf,t}(i)}{P^*_{hf,t}})^{-\lambda} \tag{4.16}$$

Given the proposed consumption taxes, the equilibrium conditions are identical to the benchmark except the consumer price index price index (3.2), and the market clearing condition (3.5) and (3.6). The consumer price indices are

$$P_t = \Theta^{-1} P_{hh,t}^{\frac{v}{2}}((1+t_{c,t})P_{fh,t})^{1-\frac{v}{2}} \tag{4.17}$$

$$P^*_t = \Theta^{-1}(P^*_{hf,t}(1+t^*_{c,t}))^{1-\frac{v}{2}} {P^*_{ff,t}}^{\frac{v}{2}} \tag{4.18}$$

Note that, $1+t_{ct}$ is the state-contingent consumption tax (tax, if $t_c > 0$; subsidy, if $t_c < 0$) on imported foreign goods. The market clearing conditions are

$$Y_t = \frac{v}{2}\frac{P_t C_t}{P_{hh,t}}\Delta_{hh,t} + (1-\frac{v}{2})\frac{P_t^* C_t^*}{P_{hf,t}^*(1+t_{c,t}^*)}\Delta_{hf,t}^* \tag{4.19}$$

$$Y_t^* = (1-\frac{v}{2})\frac{P_t C_t}{P_{fh,t}(1+t_{c,t})}\Delta_{fh,t} + \frac{v}{2}\frac{P_t^* C_t^*}{P_{ff,t}^*}\Delta_{ff,t}^* \tag{4.20}$$

4.2 First Order Log-linearization

The market clearing conditions (4.19) and (4.20) are approximated as

$$\widehat{Y}_t = \widehat{C}_t - \frac{v}{2}\widehat{p}_{hh,t} - \frac{2-v}{2}(\widehat{p}_{hf,t}^* + \widehat{t}_{c,t}^*) - \frac{2-v}{2}\frac{1}{\rho}\widehat{e}_t \tag{4.21}$$

$$\widehat{Y}_t^* = \widehat{C}_t^* - \frac{v}{2}\widehat{p}_{ff,t}^* - \frac{2-v}{2}(\widehat{p}_{fh,t} + \widehat{t}_{c,t}) + \frac{2-v}{2}\frac{1}{\rho}\widehat{e}_t \tag{4.22}$$

The terms $\widehat{p}_{hf,t}^* + \widehat{t}_{c,t}^*$ and $\widehat{p}_{fh,t} + \widehat{t}_{c,t}$ reflect the real cost faced by consumers when they choose to consume imported goods. The New Keynesian Phillips curves are

$$\pi_{hh,t} = \delta\left[\widehat{mc}_t - \widehat{p}_{hh,t}\right] + \beta E_t \pi_{hh,t+1} \tag{4.23}$$

$$\pi_{ff,t}^* = \delta\left[\widehat{mc}_t^* - \widehat{p}_{ff,t}^*\right] + \beta E_t \pi_{ff,t+1}^* \tag{4.24}$$

$$\pi_{hf,t}^* = \delta\left[\widehat{mc}_t - \widehat{p}_{hf,t}^* - \widehat{e}_t\right] + \beta E_t \pi_{hf,t+1}^* \tag{4.25}$$

$$\pi_{fh,t} = \delta\left[\widehat{mc}_t^* - \widehat{p}_{fh,t} + \widehat{e}_t\right] + \beta E_t \pi_{fh,t+1} \tag{4.26}$$

Equations (4.23)-(4.24) are Phillips curves for domestically produced goods. Equations (4.25)-(4.26) are Phillips curves for imported goods under local currency pricing. The log-linearized real marginal costs (relative to real producer price index) are:

$$\begin{aligned}\widehat{mc}_t - \widehat{p}_{hh,t} &= \rho\widehat{Y}_t - \theta_t + \frac{2-v}{2}(1-\rho)(\widehat{q}_t + \widehat{e}_t) + \frac{2-v}{2}(\widehat{d}_t + \widehat{t}^*_{c,t}) \\ &\quad + \frac{(2-v)(1-\rho)}{2}(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) \qquad (4.27)\end{aligned}$$

$$\begin{aligned}\widehat{mc}^*_t - \widehat{p}^*_{ff,t} &= \rho\widehat{Y}^*_t - \theta^*_t + \frac{2-v}{2}(1-\rho)(\widehat{q}^*_t - \widehat{e}_t) + \frac{2-v}{2}(\widehat{d}^*_t + \widehat{t}_{c,t}) \\ &\quad + \frac{(2-v)(1-\rho)}{2}(\widehat{t}^*_{c,t} - \widehat{t}_{c,t}) \qquad (4.28)\end{aligned}$$

$$\begin{aligned}\widehat{mc}^*_t - \widehat{p}_{fh,t} + \widehat{e}_t &= \rho\widehat{Y}^*_t - \theta^*_t + \frac{(2-v)}{2}(1-\rho)(\widehat{q}^*_t - \widehat{e}_t) \\ &\quad - \frac{v}{2}(\widehat{d}^*_t + \widehat{t}_{c,t}) + \frac{2-v}{2}(\rho-1)(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) + \widehat{t}_{c,t} \qquad (4.29)\end{aligned}$$

$$\begin{aligned}\widehat{mc}_t - \widehat{p}^*_{hf,t} - \widehat{e}_t &= \rho\widehat{Y}_t - \theta_t + \frac{2-v}{2}(1-\rho)(\widehat{q}_t + \widehat{e}_t) \\ &\quad - \frac{v}{2}(\widehat{d}_t + \widehat{t}^*_{c,t}) - \frac{(2-v)}{2}(\rho-1)(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) + \widehat{t}^*_{c,t} \qquad (4.30)\end{aligned}$$

The log deviation of marginal cost terms are similar to those in Fujiwara and Wang (2017), but now they involve the consumption tax terms, $\widehat{t}^*_{c,t}$ and $\widehat{t}_{c,t}$. The intuition is straight forward. The tax will distort the consumption choice and has an impact on wages, which is eventually reflected in marginal cost. The first two terms represent the effect of output and the productivity shock on production while the third term is related to the effect of terms of trade on marginal cost (relative to the producer price index). The fourth term is the (after-tax) deviation from the law of one price. Note that compared to Engel (2011), it now includes a log tax term due to the presence of the consumption tax. The fifth term is the difference between the consumption tax across countries. Comparing the third and the fifth term, we can see that the role of the tax rate difference on marginal cost is identical to that of the term of trade. This is intuitive. Also, these two terms will only affect the marginal cost when $\rho > 1$.

Finally, there exists a sixth term, the consumption tax, in Equations (4.29) and (4.30), implying the tax will affect marginal cost relative to the foreign producer price index directly. This is because when prices facing consumers are affected by the consumption tax (subsidy), the export firms' price setting will be distorted. In other word, the consumption taxes (subsidies), although correcting the deviations from law of one price, distorts the production decision, which in turn affects the relative real marginal cost of imported goods. To eliminate this distortion, we need to impose a producer subsidy (tax) on export firms.

4.3 Introducing Export Subsidies

Export subsidies will be proportional to the export prices, since export goods will be taxed by the importing consumers government. Specifically, when home firms export to the foreign country, they will receive a subsidy $s_{e,t}$ proportional to their sales revenue. Given the proposed export subsidies, the objective functions of firms in the home and foreign countries will change accordingly.

$$E_t \sum_{j=0}^{\infty} \kappa^j \beta_{t,t+j}(1+\tau)[(P^o_{hht}(i) - MC_{t+j}(i))C_{h,t+j}(i) + (S_{t+j}(1+s_{e,t+j})P^{o*}_{hf,t}(i) - MC_{t+j}(i))C^*_{h,t+j}(i)]$$

$$E_t \sum_{j=0}^{\infty} \kappa^j \beta^*_{j,t+j}(1+\tau)[(\frac{P^o_{fh,t}(k)}{S_{t+j}}(1+s^*_{e,t+j}) - MC^*_{t+j}(k))C_{f,t+j}(k) + (P^{*o}_{ff,t}(k) - MC^*_{t+j}(k))C^*_{f,t+j}(k)]$$

The subsidies only enter the the pricing setting of exported goods while goods sold in the domestic market are unaffected.

$$P^{o*}_{hf,t}(i) = \frac{E_t \sum_{j=0}^{\infty} \kappa^j \beta_{t,t+j}(P^*_{hf,t+j})^{\lambda} MC_{t+j} C^*_{h,t+j}}{E_t \sum_{j=0}^{\infty} \kappa^j S_{t+j}(1+s_{e,t+j})\beta_{t,t+j}(P^*_{hf,t+j})^{\lambda} C^*_{h,t+j}} \tag{4.31}$$

$$P^o_{fh,t}(k) = \frac{E_t \sum_{j=0}^{\infty} \kappa^j \beta^*_{t,t+j} P^{\lambda}_{fh,t+j} MC^*_{t+j} C_{f,t+j}}{E_t \sum_{j=0}^{\infty} \kappa^j S^{-1}_{t+j}(1+s^*_{e,t+j})\beta^*_{t,t+j} P^{\lambda}_{fh,t+j} C_{f,t+j}} \tag{4.32}$$

When subsidies are introduced, import price inflation rates depend not only on the real marginal cost (relative to the real producer price index) but also on the subsidies received by exporters.

$$\pi^*_{hf,t} = \delta\left[\widehat{mc}_t - \widehat{p}^*_{hf,t} - \widehat{e}_t - \widehat{s}_{e,t}\right] + \beta E_t \pi^*_{hf,t+1} \tag{4.33}$$

$$\pi_{fh,t} = \delta\left[\widehat{mc}^*_t - \widehat{p}_{fh,t} + \widehat{e}_t - \widehat{s}^*_{e,t}\right] + \beta E_t \pi_{fh,t+1} \tag{4.34}$$

In the following analysis, we will first derive the global loss function with tax and subsidy for the cooperative game. We check if these fiscal instruments can correct the welfare loss due to deviation from LOOP and help to achieve the flexible price equilibrium.

Note that in the dynamic model, the most desirable level to which monetary policy can deliver is the flexible price equilibrium level, which will be used as a target for international tax policies. Since we remove the monopolistic competition by a constant tax, it is also the efficient equilibrium, as that in Fujiwara and Wang (2017).

5 The Cooperative Equilibrium

Since the effect of currency misalignment on marginal cost does not depend on whether ρ is greater, smaller, or equal to 1, we will look at the $\rho = 1$ first to simplify our analysis. This also helps to compare with the loss function in Section 1. The case with $\rho > 1$ will be discussed later.

5.1 The Cooperative Game when $\rho = 1$

We first look at the case of cooperative policy when $\rho = 1$. In this case, it is assumed that there exists an authority who will choose monetary and fiscal instruments to maximize the joint (global) welfare of the home and foreign country. As shown in the Technical Appendix, the second-order approximation of global welfare $L_0 = L_{h,0} + L^*_{h,0}$, after the introduction of a consumption tax and export subsidy is:

$$P(1) \quad :$$

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\begin{array}{c} \frac{1}{2}(\widehat{Y}_t - \theta_t)^2 + \frac{1}{2}(\widehat{Y}^*_t - \theta^*_t)^2 \\ +\frac{\lambda v}{4\delta}\pi^2_{hh,t} + \frac{\lambda(2-v)}{4\delta}\pi^2_{fh,t} + \frac{\lambda v}{4\delta}(\pi^*_{ff,t})^2 + \frac{\lambda(2-v)}{4\delta}(\pi^*_{hf,t})^2 \\ +\frac{v(2-v)}{8}(\widehat{d}^*_t + \widehat{t}_{c,t})^2 + \frac{v(2-v)}{8}(\widehat{d}_t + \widehat{t}^*_{c,t})^2 \end{array} \right]$$

These terms are identical to the loss function derived from the standard model except for the last two terms on $\widehat{d}^*_t$ and $\widehat{d}_t$. Comparing to that in Section 3.4 (or that in Engel, 2011 and Fujiwara and Wang, 2017), in our model with fiscal instruments, they show up in the loss function together with the consumption tax.

The authority will maximize global welfare subject to the following five constraints.

$$\pi_{hh,t} = \delta\left[\widehat{Y}_t - \theta_t + \frac{2-v}{2}\widehat{d}_t + \frac{2-v}{2}\widehat{t}^*_{c,t}\right] + \beta E_t \pi_{hh,t+1} \tag{5.35}$$

$$\pi^*_{hf,t} = \delta\left[\widehat{Y}_t - \theta_t - \frac{v}{2}\widehat{d}_t + \frac{2-v}{2}\widehat{t}^*_{c,t} - \widehat{s}_{e,t}\right] + \beta E_t \pi^*_{hf,t+1} \tag{5.36}$$

$$\pi^*_{ff,t} = \delta\left[\widehat{Y}^*_t - \theta^*_t + \frac{2-v}{2}\widehat{d}^*_t + \frac{2-v}{2}\widehat{t}_{c,t}\right] + \beta E_t \pi^*_{ff,t+1} \tag{5.37}$$

$$\pi_{fh,t} = \delta\left[\widehat{Y}^*_t - \theta^*_t - \frac{v}{2}\widehat{d}^*_t + \frac{2-v}{2}\widehat{t}_{c,t} - \widehat{s}^*_{e,t}\right] + \beta E_t \pi_{fh,t+1} \tag{5.38}$$

$$\widehat{Y}_t - \widehat{Y}^*_t = \frac{v}{2}\widehat{d}_t - \frac{v}{2}\widehat{d}^*_t + \widehat{q}_t + \frac{2-v}{2}(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) \tag{5.39}$$

where $\delta = \frac{(1-\beta\kappa)(1-\kappa)}{\kappa}$. The first four constraints are new Keynesian Phillips curve (NKPC) when $\rho = 1$, and the last one comes from the goods market clearing conditions. The choice variables for the cooperative game are $\{\widehat{Y}_t - \theta_t, \widehat{Y}^*_t - \theta^*_t, \pi_{hh,t}, \pi^*_{hf,t}, \pi^*_{ff,t}, \pi_{fh,t}, \widehat{d}^*_t,$ $\widehat{d}_t, \widehat{t}_{c,t}, \widehat{t}^*_{c,t}, \widehat{s}_{e,t}, \widehat{s}^*_{e,t}\}$. Solving the optimization problem gives us the following proposition. The domestic output gap is defined as deviation from the natural level of output (the output level in the flexible price equilibrium) $\widetilde{Y}_t = \widehat{Y}_t - \widehat{Y}^f_t$, where notations with superscript "f" denote the variables in flexible price equilibrium.

Proposition 1 *In the cooperative game with $\rho = 1$, consumption tax rates are chosen to correct deviations from law of one price $\widehat{d}^*_t$ and $\widehat{d}_t$, whereas export subsidy rates are used to offset the corresponding externality of the consumption tax. The optimal monetary policy requires monetary authorities to target zero PPI inflation and a zero output gap.*

$$\widetilde{Y}_t = \widetilde{Y}_t^* = 0$$
$$\widehat{Y}_t = \widehat{Y}^f_t = \theta_t, \ \widehat{Y}^*_t = \widehat{Y}^{*f}_t = \theta^*_t$$
$$\widehat{d}_t + \widehat{t}^*_{c,t} = 0, \ \widehat{d}^*_t + \widehat{t}_{c,t} = 0$$
$$\widehat{t}^*_{c,t} = \widehat{s}_{e,t}, \ \widehat{t}_{c,t} = \widehat{s}^*_{e,t}$$
$$\pi_{hh,t} = 0, \ \pi^*_{ff,t} = 0$$
$$\pi^*_{hf,t} = 0, \ \pi_{fh,t} = 0$$

The proof is given in the Technical Appendix. The global welfare loss comes from the output gap, inflation instability and the deviation from LOOP. Engel (2011) provides a solution for this cooperative game without tax and subsidy. He finds that the optimal solution is to target zero CPI inflation instead of PPI inflation, since policymakers cannot eliminate "currency misalignment" due to LCP. Thus, even with policy coordination the optimal monetary policy cannot replicate the flexible price equilibrium. In our model, however, once the international tax and subsidy scheme are introduced to correct the distortion induced by the deviation from LOOP, the social planner can achieve an efficient equilibrium, which is identical to that under flexible prices or PCP (under optimal monetary policy).

Given the above optimal policies, what exactly are the deviations of LOOP and the fiscal policies in this dynamic model? Zero PPI inflation, or $\pi_{hh,t} = \pi^*_{hf,t} = 0$, implies $\widehat{d}_t = \widehat{S}_t$.[9] Similarly, $\widehat{d}^*_t = -\widehat{S}_t$. Substituting this relation, solution to the cooperative game, and the expression for $\widehat{q}_t$ into Equation (5.39) gives $\widehat{d}_t = -\widehat{d}^*_t = \theta_t - \theta^*_t$ and tax schemes $\widehat{t}_{c,t} = -\widehat{t}^*_{c,t} = \theta_t - \theta^*_t$.[10] In other words, consumption tax rates are exactly the gap between home and foreign productivity shocks. The Phillips curves (5.36) and (5.38) indicate that the inflation dynamics of imported goods are directly affected by consumption tax rates. So to offset this externality, export subsidies $\widehat{s}_{e,t}$ and $\widehat{s}^*_{e,t}$ eliminate the impact of consumption taxes on inflation of imported goods. Finally, $\widehat{S}_t = \theta_t - \theta^*_t$ implies that the exchange rate is flexible and responds to productivity shocks in the same manner as that under PCP.

5.2 Cooperative Game when $\rho > 1$

As discussed above, when $\rho > 1$, the terms of trade effect and the risk-sharing effect will enter the loss function. There will also be some spillover effect of international productivity. In Section 1 of the Appendix, we present the loss function and the constraints faced by the social planner for the cooperative game in the $\rho > 1$ case. The solutions to the cooperative

[9] Substituting $\widehat{e}_t = \widehat{P}^*_t + \widehat{S}_t - \widehat{P}_t$, $\widehat{p}^*_{hf,t} = \widehat{P}_{hf,t} - \widehat{P}^*_t$, and $\widehat{p}_{hh,t} = \widehat{P}_{hh,t} - \widehat{P}_t$ into $\widehat{d}_t = \widehat{p}^*_{hf,t} + \widehat{e}_t - \widehat{p}_{hh,t}$ gives $\widehat{d}_t = \widehat{S}_t$.

[10] Tax rates are zero in steady state, so $\widehat{t}_{c,t} = t_{c,t}$ and $\widehat{t}^*_{c,t} = t^*_{c,t}$.

game can be described as:

$$
\begin{aligned}
\widetilde{Y}_t &= \widetilde{Y}_t^* = 0 \\
\widehat{Y}_t &= \widehat{Y}_t^f = \theta_t + \frac{(1-\rho)}{\rho}\left(\theta_t - \frac{(2-v)v}{2}(\theta_t - \theta_t^*)\right) \\
\widehat{Y}_t^* &= \widehat{Y}_t^{*f} = \theta_t^* + \frac{(1-\rho)}{\rho}\left(\theta_t^* + \frac{(2-v)v}{2}(\theta_t - \theta_t^*)\right) \\
\widehat{d}_t + \widehat{t}_{c,t}^* &= 0,\ \widehat{d}_t^* + \widehat{t}_{c,t} = 0 \\
\widehat{t}_{c,t}^* &= \widehat{s}_{e,t},\ \widehat{t}_{c,t} = \widehat{s}_{e,t}^* \\
\pi_{hh,t} &= 0,\ \pi_{ff,t}^* = 0 \\
\pi_{hf,t}^* &= 0,\ \pi_{fh,t} = 0
\end{aligned}
$$

When $\rho > 1$, although there exists an international productivity spillover in the game, the consumption tax will still correct the deviation from LOOP and the export subsidy will still offset the distortion of consumption tax on allocation. Given that the currency misalignment induced by LCP is corrected by the tax, the optimal monetary policy will still target zero PPI inflation. Due to the presence of international spillover, a country's equilibrium output does not reflect exactly the its corresponding productivity shock, but the output gap, properly defined, is still zero. Therefore, even when $\rho > 1$, we can still have the following result. The combination of international tax policy and an inward-looking monetary policy targeting PPI inflation can correct the currency misalignment and replicate the flexible price equilibrium. [11]

Having shown that these fiscal instruments can correct the currency misalignment due to LCP and deliver the flexible price equilibrium under the cooperative game, the next step is to explore the implementation of these international fiscal policies.

[11] If there is no home bias ($v = 1$), we can show that the optimal solutions are identical to the responses under flexible price equilibrium in the static model of Devereux and Engel (2003). The optimal output deviations from steady state values are $\widehat{Y}_t - \theta_t = \widehat{Y}_t^* - \theta_t^* = \frac{(1-\rho)}{\rho}(\theta_t + \theta_t^*)$ and consumption deviations are $\widehat{C}_t = \widehat{C}_t^* = \frac{1}{2\rho}(\theta_t + \theta_t^*)$. The global welfare loss is $L^W = E_t \sum_{t=0}^{\infty} \beta^t \frac{\rho-1}{4\rho}(\theta_t + \theta_t^*)^2$.

6 Non-cooperative Game and Implementability

We first ask if the solution to the international tax non-cooperative game exists. If not, then we ask what kind of condition on the fiscal instruments is needed to ensure the existence of a Nash international monetary game.

6.1 Non-existence of Nash Game on Tax Policies

Define the non-cooperative game as one where the policy-maker in each country independently chooses both monetary variables and fiscal variables. Again, first focus on the case $\rho = 1$. In this section, following Benigno and Woodford (2005) and Fujiwara and Wang (2017), we use a second-order approximation methods to derive individual country loss functions and characterize the conditions for a non-cooperative equilibrium. Here we just sketch the main steps for the derivation and leave the details to the Technical Appendix. The home household's lifetime expected utility can be approximated up to the second order as

$$\begin{aligned} W_{h,0} &= E_t \sum_{t=0}^{\infty} (\beta)^t \left[\frac{C_t^{1-\rho}}{1-\rho} - \eta L_t \right] \\ &\approx E_t \sum_{t=0}^{\infty} (\beta)^t \overline{C}^{1-\rho} \left[\widehat{C}_t - \widehat{L}_t + \frac{1-\rho}{2} \widehat{C}_t^2 - \frac{1}{2} \widehat{L}_t^2 \right] \end{aligned}$$

The third term in the bracket represents the log consumption volatility and the fourth term is just the output gap. Both can be derived from first-order approximation of the system. However, to get $\widehat{C}_t - \widehat{L}_t$ we need to use a second order approximation. So we first log-linearize the system around the zero inflation, zero tax/subsidy steady state without monopolistic distortions. Then using this log-linearized system, we can get the second-order approximation of the AS equations (the four price indices) and the market clearing conditions, which will allow us to express $\widehat{C}_t - \widehat{L}_t$ in terms of policy parameters and inflation.

As shown in the Technical Appendix, in the non-cooperative game, when $\rho = 1$, the loss functions for home $L_{h,0}$ and foreign $L_{h,0}^*$ are given by, respectively

$$
\begin{aligned}
P(2) &\quad : \\
L_{h,0} &= E_t \sum_{t=0}^{\infty} (\beta)^t \left\{ \begin{array}{c} \frac{1}{2}(\widehat{Y}_t - \theta_t)^2 + \frac{v(2-v)}{8}(\widehat{d}_t + \widehat{t}_{c,t}^*)^2 + \frac{\lambda}{2\delta}\left[\frac{v}{2}\pi_{hh,t}^2 + \frac{(2-v)}{2}\pi_{fh,t}^2\right] \\ -\frac{2-v}{4}\left(\widehat{t}_{c,t} - \widehat{s}_{e,t}^*\right)^2 + \frac{2-v}{4}\left(\widehat{t}_{c,t}^* - \widehat{s}_{e,t}\right)^2 + \frac{2-v}{2}[(\widehat{t}_{c,t} - \widehat{s}_{e,t}^*) - (\widehat{t}_{c,t}^* - \widehat{s}_{e,t})] \\ +\frac{2-v}{4}\left((\widehat{Y}_t^* - \theta_t^*) - \frac{v}{2}(\widehat{d}_t^* + \widehat{t}_{c,t})\right)^2 - \frac{2-v}{4}\left((\widehat{Y}_t - \theta_t) - \frac{v}{2}(\widehat{d}_t + \widehat{t}_{c,t}^*)\right)^2 \end{array} \right\} \\
L_{h,0}^* &= E_t \sum_{t=0}^{\infty} (\beta)^t \left\{ \begin{array}{c} \frac{1}{2}(\widehat{Y}_t^* - \theta_t^*)^2 + \frac{v(2-v)}{8}(\widehat{d}_t^* + \widehat{t}_{c,t})^2 + \frac{\lambda}{2\delta}\left[\frac{v}{2}\pi_{ff,t}^{*2} + \frac{(2-v)}{2}\pi_{hf,t}^{*2}\right] \\ +\frac{2-v}{4}\left(\widehat{t}_{c,t} - \widehat{s}_{e,t}^*\right)^2 - \frac{2-v}{4}\left(\widehat{t}_{c,t}^* - \widehat{s}_{e,t}\right)^2 - \frac{2-v}{2}[(\widehat{t}_{c,t} - \widehat{s}_{e,t}^*) - (\widehat{t}_{c,t}^* - \widehat{s}_{e,t})] \\ -\frac{2-v}{4}\left((\widehat{Y}_t^* - \theta_t^*) - \frac{v}{2}(\widehat{d}_t^* + \widehat{t}_{c,t})\right)^2 + \frac{2-v}{4}\left((\widehat{Y}_t - \theta_t) - \frac{v}{2}(\widehat{d}_t + \widehat{t}_{c,t}^*)\right)^2 \end{array} \right\}
\end{aligned}
$$

The first three terms represent the welfare cost due to output gaps, deviation from the law of one price and price dispersion, respectively. The following four terms are welfare losses due to spillovers from non-cooperative behaviour. Note that $(\widehat{t}_{c,t} - \widehat{s}_{e,t}^*)^2, (\widehat{t}_{c,t}^* - \widehat{s}_{e,t})^2, (\widehat{t}_{c,t} - \widehat{s}_{e,t}^*)$ and $(\widehat{t}_{c,t}^* - \widehat{s}_{e,t})$ only appear in the non-cooperative loss but not in co-operative case. The last two terms represent an inefficiency from fluctuations in marginal cost, which represent the additional objectives of policy makers in the non-cooperative game, as emphasized in Fujiwara and Wang (2017).

The home policy maker is subject to the following constraints,

$$
\begin{aligned}
\pi_{hh,t} &= \delta\left[\widehat{Y}_t - \theta_t + \frac{2-v}{2}\widehat{d}_t + \frac{2-v}{2}\widehat{t}_{c,t}^*\right] + \beta E_t \pi_{hh,t+1} && (6.40) \\
\pi_{fh,t} &= \delta\left[\widehat{Y}_t^* - \theta_t^* - \frac{v}{2}\widehat{d}_t^* + \frac{2-v}{2}\widehat{t}_{c,t} - \widehat{s}_{e,t}^*\right] + \beta E_t \pi_{fh,t+1} && (6.41) \\
\widehat{Y}_t - \widehat{Y}_t^* &= \frac{v}{2}\widehat{d}_t - \frac{v}{2}\widehat{d}_t^* + \widehat{q}_t + \frac{2-v}{2}(\widehat{t}_{c,t} - \widehat{t}_{c,t}^*) && (6.42)
\end{aligned}
$$

Similarly, the constraints for the foreign country are

$$
\begin{aligned}
\pi_{ff,t}^* &= \delta\left[\widehat{Y}_t^* - \theta_t^* + \frac{2-v}{2}\widehat{d}_t^* + \frac{2-v}{2}\widehat{t}_{c,t}\right] + \beta E_t \pi_{ff,t+1}^* && (6.43) \\
\pi_{hf,t}^* &= \delta\left[\widehat{Y}_t - \theta_t - \frac{v}{2}\widehat{d}_t + \frac{2-v}{2}\widehat{t}_{c,t} - \widehat{s}_{e,t}\right] + \beta E_t \pi_{hf,t+1}^* && (6.44) \\
\widehat{Y}_t - \widehat{Y}_t^* &= \frac{v}{2}\widehat{d}_t - \frac{v}{2}\widehat{d}_t^* + \widehat{q}_t + \frac{2-v}{2}(\widehat{t}_{c,t} - \widehat{t}_{c,t}^*) && (6.45)
\end{aligned}
$$

In the Nash game without restriction, the home country chooses $\{\widehat{Y}_t - \theta_t, \pi_{hh,t}, \pi_{fh,t}, \widehat{d}_t, \widehat{t}_{c,t}, \widehat{s}_{e,t}, \widehat{q}_t\}$, while foreign country chooses $\left\{\widehat{Y}_t^* - \theta_t^*, \pi_{hf,t}^*, \pi_{ff,t}^*, \widehat{d}_t^*, \widehat{t}_{c,t}^*, \widehat{s}_{e,t}^*, \widehat{q}_t^*\right\}$. First order conditions are reported in the Technical Appendix. We find that Nash game equilibrium does not exist.

Proposition 2 *The equilibrium of non-cooperative international tax Nash game does not exist.*

The proof is given in the Technical Appendix. When deriving the loss function of each country, the linear terms $\widehat{C}_t - \widehat{L}_t$ are replaced by not only the price dispersion terms $\widehat{\Delta}_t$ but also the terms of trade $-\widehat{q}_t$ which is absent in the closed economy. Thus, each central bank in an open economy has incentive to strategically manipulate the terms of trade in its favour. This represents the term of trade externality as analyzed in Corsetti and Pesenti (2001), Benigno (2002) and Benigno and Benigno (2006). In our framework with fiscal instruments, we have the expression of $\widehat{C}_t - \widehat{L}_t$ in the following way:

$$\begin{aligned}\widehat{L}_t - \widehat{C}_t \quad = \quad & \frac{v(2-v)}{8}(\widehat{d}_t + \widehat{t}^*_{c,t})^2 + \frac{\lambda(1+\alpha)}{4\delta}\left[\frac{v}{2}\pi^2_{hh,t} + \frac{(2-v)}{2}\pi^2_{fh,t}\right] \\ & -\frac{(2-v)}{4}\left(\widehat{t}_{c,t} - \widehat{s}^*_{e,t}\right)^2 + \frac{(2-v)}{4}\left(\widehat{t}^*_{c,t} - \widehat{s}_{e,t}\right)^2 + \frac{(2-v)}{2}\left((\widehat{t}_{c,t} - \widehat{s}^*_{e,t}) - (\widehat{t}^*_{c,t} - \widehat{s}_{e,t})\right) \\ & +\frac{(2-v)}{4}\left((\widehat{Y}^*_t - \theta^*_t) - \frac{v}{2}(\widehat{d}^*_t + \widehat{t}_{c,t})\right)^2 - \frac{(2-v)}{4}\left((\widehat{Y}_t - \theta_t) - \frac{v}{2}(\widehat{d}_t + \widehat{t}^*_{c,t})\right)^2\end{aligned} \tag{6.46}$$

Therefore, linear terms $(\widehat{t}_{c,t} - \widehat{s}^*_{e,t})$ and are part of term of trade externality. Each country has the incentive to manipulate this term and increase domestic households' welfare. From the home loss function P(2), we see that at the initial point where $(\widehat{t}^*_{c,t} - \widehat{s}_{e,t}) = 0$, the home country would have an incentive to increase $E(\widehat{t}^*_{c,t} - \widehat{s}_{e,t})$, which would reduce its expected loss, since this would raise its expected terms of trade. This is reflected in the first order condition under noncooperative behaviour when the government chooses the optimal subsidy $\widehat{s}_{e,t}$.

$$-\frac{(2-v)}{2}\left(\widehat{t}^*_{c,t} - \widehat{s}_{e,t}\right) + \frac{(2-v)}{2} = 0 \tag{6.47}$$

So the home government would wish to have $E(\widehat{t}^*_{c,t} - \widehat{s}_{e,t}) > 0$. But from (6.41), at $\widehat{Y}^*_t - \theta^*_t = 0$ and $\widehat{d}^*_t + \widehat{t}_{c,t} = 0$, this would imply that $\pi_{fh,t}$ cannot converge to zero, even in the absence of productivity shocks. In fact, as shown in the technical appendix, any solution to the non-cooperative game must have $\pi_{fh,t}$ diverging to infinity. Therefore, there is no equilibrium with finite inflation rates. As a result there is a fundamental inconsistency between the policymakers goal of inflation control and the desire to exploit its terms of trade advantage in the non-cooperative game where each policymaker has responsibility for both monetary policy and fiscal instruments.

6.2 Implementability in a Three-player Nash Game

Is there any restriction that could help to regain the Nash equilibrium? Obviously, from the analysis above, one natural restriction would be $\widehat{s}_{e,t} = \widehat{t}^*_{c,t}$ and $\widehat{s}^*_{e,t} = \widehat{t}_{c,t}$. By artificially imposing a restriction on tax policy such that the home consumption tax exactly equals the export subsidy for foreign exporters, the terms of trade externality due to international tax competition externality can be avoided. After this restriction is imposed, there will be a solution to the restricted Nash game and it replicates the flexible price equilibrium.

Proposition 3 *With the restriction that $\widehat{t}^*_{c,t} = \widehat{s}_{e,t}$ and $\widehat{t}_{c,t} = \widehat{s}^*_{e,t}$, the non-cooperative equilibrium exists. It also replicates the flexible equilibrium. That is, the solutions to the restricted Nash game are the same as those in the cooperative game.*

Proofs are given the Technical Appendix. Thus, we conclude that even under the non-cooperative game, the international tax policy combined by the consumption tax and export subsidy can be used to replicate the flexible price equilibrium and improve global welfare.

How can this condition be rationalized? We now define a mixed environment where there is cooperation in fiscal policy but non-cooperative monetary policy.

To achieve this goal, we distinguish between fiscal authorities and monetary authorities. Instead of assuming that the monetary authorities choose the level of tax or subsidies, we assume that they take the tax and subsidies policy rules as given, but choose inflation and output gaps independently (non-cooperatively). Meanwhile, the fiscal authorities choose the optimal responses of tax and subsidies. But to avoid the terms of trade externality in the choice of fiscal instruments, fiscal policies must be chosen cooperatively. Hence, we define a 3 player Nash game, where there are two monetary authorities and a global fiscal alliance.

The objective functions and constraints of the home and foreign monetary authorities are identical to the problem characterized by P(2), the only difference is that $\left\{\widehat{t}_{c,t}, \widehat{s}_{e,t}\right\}$ and $\left\{\widehat{t}^{*}_{c,t}, \widehat{s}^{*}_{e,t}\right\}$ are not their choice variables. The objective function of global fiscal authorities, as well as the constraint, is identical to the problem characterized by P(1) but they can only choose the tax and subsidies instruments.

Proposition 4 *The solution to the 3 player Nash game is the same as those in the co-operative game. In other words, effective fiscal policy coordination in tax policies among governments is necessary for non-cooperative (or self-oriented) monetary policy to sustain an efficient outcome.*

Proofs are given the Technical Appendix. Intuitively, fiscal authorities can use the consumption tax to correct the welfare costs due to currency misalignment and ensure that this consumption tax will not affect the inflation rate, by using export subsidies. The governments can implement this fiscal scheme without coordinating with the monetary authorities. In effect, coordination between fiscal authorities is more desirable than coordination between monetary authorities, from the perspective of correcting currency misalignment.[12]

Clarida, Gali and Gertler (2002) show that when $\rho = 1$, there is no gain from policy coordination under PCP framework. In our model, once the deviation of law of one price due to LCP is corrected by the international tax coordination there is no extra gain from monetary policy cooperation. This implication differs from Fujiwara and Wang (2017), where they find that there is some policy coordination gain from monetary policy coordination under LCP. This is because, in our model, once the currency misalignment is corrected, the optimal monetary policy can again target producer price stability, as in the case of PCP. Therefore, the optimal monetary and fiscal policy mix delivers the same results under both the cooperative game and non-cooperative game, which is the flexible price equilibrium under PCP. In our model, the key policy coordination gain is from the

[12]We note again that fiscal policy to deal with the welfare loss due to local currency pricing is also proposed by Adao, Correia, and Teles (2009). The key difference here however it that optimal tax policy is derived from three-player Nash game among coordinated fiscal alliance, home and foreign money authorities.

fiscal policy coordination. We discuss this in Section 7, where we define the welfare gain as the difference between welfare under the LCP case without fiscal policy and the flexible price equilibrium.

6.3 $\rho > 1$ case

We also check if the results discussed above hold when $\rho > 1$. Section 2 and 3 of the Appendix give the loss function and constraints faced by players for the restricted Nash game and for the three-player Nash game. We show that both Nash games deliver exactly the same solution as that in the cooperative game. The proof is given in the Technical Appendix. This implies that for a more general setup, we can still show that the combination of monetary policy and state-contingent taxes and subsidies can be used to eliminate the inefficiency caused by LCP and replicate the flexible price equilibrium.

In summary, we show that even under the non-cooperative Nash game, a state-contingent tax policy combination of consumption tax (subsidy) and export subsidy (tax) can fully correct the currency misalignment due to LCP. But critically, this requires cooperation in fiscal policy. The independent determination of taxes and subsidies, while allowing policymakers to eliminate currency misalignment, opens up a new strategic channel through the terms of trade externality which, as we have shown, is inconsistent with the desire to eliminate output gaps and achieve zero inflation.

7 Quantitative Welfare Gain

In this section, we give a quantitative evaluation of the gains to employing state contingent fiscal policies to correct currency misalignment. We can define the welfare gain as the welfare achieved under the fully efficient allocation (which can be supported under the 3 player Nash game), relative to the welfare under a non-cooperative monetary policy equilibrium, without the use of fiscal policies. This latter equilibrium is that analyzed by Fujiwara and Wang (2017).

As shown above, when optimal, state-contingent fiscal policies are used, the economy can achieve the fully efficient allocations. Hence, from one perspective, the welfare loss

Table 2: Parameter values (Baseline)

Parameter	Description	Value
β	discount factor	0.99
η	preference weight on labor	2
ρ	inverse of elasticity of intertemporal substitution	2
κ	degree of price stickiness	0.75
λ	elasticity of substitution across individual goods	11
v	home country size and weight on home produced	1.5
ρ_θ	persistence of productivity shock	0.95
σ	productivity shock size	0.01

of currency misalignment in Engel (2011), which cannot be eliminated by the optimal monetary policy (CPI targeting), can be taken as the welfare improvement of our policy mix, relative to the optimal cooperative monetary policy derived in Engel (2011). To obtain a quantitative welfare improvement, we conduct a simple quantitative exercise following Engel (2011). All parameters are documented in Table 2 and identical to that in Engel (2011). We compare the welfare (in terms of steady state consumption) under optimal monetary policy in Engel (2011) and that in flexible price equilibrium. The welfare loss under the efficient, flexible price equilibrium is -0.0168% while the loss under CPI targeting under optimal cooperative monetary policy is -0.0442%. The difference is due to the presence of currency misalignment, which implies that even under the cooperative game the households have to give up 0.0274% of their steady state consumption compared with the welfare under the flexible price allocation. [13] Moreoever, if we compare our result with that in Fujiwara and Wang (2017), the non-cooperative monetary game under LCP, the welfare gain from fiscal policy, or more specifically fiscal coordination, is actually even higher, equal to around 0.0468% of consumer's steady state consumption.

Figure 1 and Figure 2 illustrate the gains to optimal taxes and subsidies to correct

[13] When calculating the optimal monetary policy under both cooperative and non-cooperative monetary policy, we limit our policy instrument to the PPI inflation and the output gap in home and foreign countries.

misalignment under LCP. Figure 1 indicates that the welfare gains by correcting the "currency misalignment" are larger when the frequency to change price is lower. Figure 2 documents that the welfare gains relative to cooperative monetary policy are largest when there is no home bias ($v = 1$). As shown in the cooperative game, the welfare loss due to currency misalignment is simply given by $\frac{v(2-v)}{8}[(\widehat{d}_t)^2 + (\widehat{d}_t^*)^2]$. Therefore, when $v = 1$, the welfare loss reaches the maximum. Also, from this equation, when the economy is closed (at $v = 0$ or $v = 2$), and only consumes one good, the welfare loss is zero, since there is no distortion from "currency misalignment".

The relationship between home bias and the welfare gain relative to the non-cooperative monetary policy [14] contrasts with that for the cooperative monetary policy case and is non-monotonic in the parameter v.

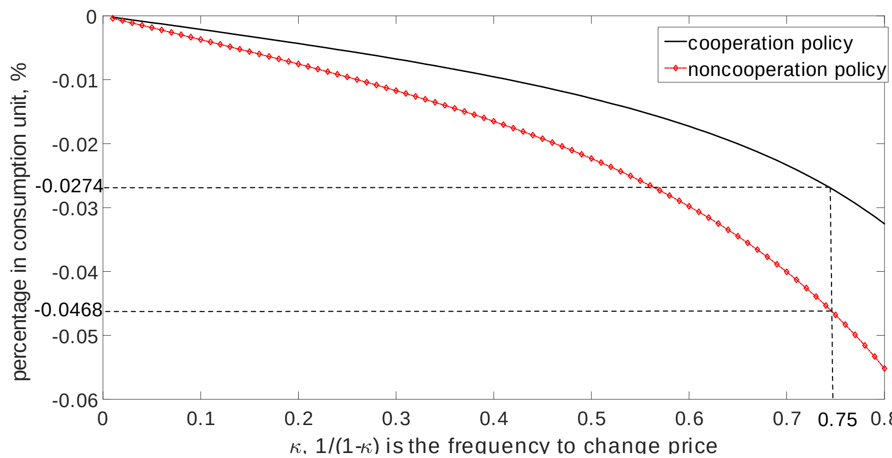

Figure 1: Welfare Gain from Fiscal Policy Coordination as Functions of κ

[14] It should be noted that for some values of ν, the non-cooperative monetary game with PPI inflation and output gap cannot be evaluated as the solution exhibits indeterminacy. Here we only report results for those values of ν under which the solution to the non-cooperative monetary game is unique.

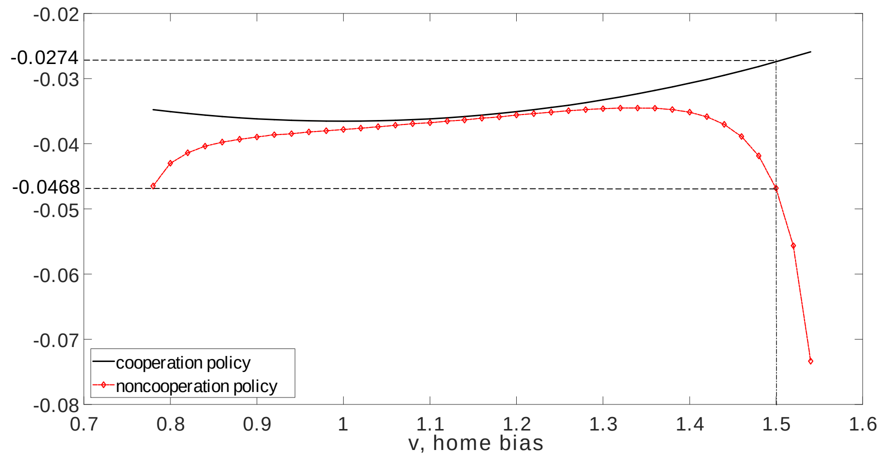

Figure 2: Welfare Gain from Fiscal Policy Coordination as Functions of v

8 Discussion

8.1 Other Tax Instruments

In this subsection, we introduce some other tax instruments discussed in the literature, particularly that used in Adao, Correia, and Teles (2009). They show that a labor income tax and consumption taxes on both home goods and foreign goods can be used to replicate the flexible prices equilibrium. And given these taxes, the exchange rate regime and the currency of pricing is irrelevant. It should be noted that since we assume complete risk sharing, our setting differs from theirs in terms of asset market completeness. But it is still important to ascertain whether these particular fiscal instruments can be used in our dynamic pricing setting model to replicate the flexible price equilibrium. The model specification is similar to the benchmark dynamic model, so we only list a few key equations which change when various tax instruments are considered.

Household budget constraint:

$$\begin{aligned} & (1+t_{h,t})P_{hht}C_{h,t} + (1+t_{f,t})P_{fht}C_{f,t} + M_{t+1} + B_{ht+1} + \sum_{\zeta^{t+1}\in Z_{t+1}} B(\zeta^{t+1}|\zeta^t)D(\zeta^{t+1}) \\ = \;& (1-\tau_t)W_tL_t + R_{t-1}B_{ht} + \Pi_t + M_t + T_t + D(\zeta^t) \qquad (8.48) \end{aligned}$$

where $t_{h,t}$ and $t_{f,t}$ are the consumption tax on home and foreign goods, respectively; and τ_t is the labor income tax. Labor supply curve becomes:

$$(1-\tau_t)W_t\frac{C_t^{-\rho}}{P_t}=\eta \tag{8.49}$$

Demand for home and foreign good are now given by:

$$C_{h,t} = \frac{v}{2}\frac{P_tC_t}{P_{hht}(1+t_{h,t})} \tag{8.50}$$

$$C_{f,t} = (1-\frac{v}{2})\frac{P_tC_t}{P_{fht}(1+t_{f,t})} \tag{8.51}$$

Note that firms' pricing equations are now the same as those in the standard literature since export subsidies are not considered.

Finally, the home market clearing condition is

$$Y_t=\frac{v}{2}\frac{P_tC_t}{P_{hht}(1+t_{h,t})}\Delta_{hh,t}+(1-\frac{v}{2})\frac{P_t^*C_t^*}{P_{hft}^*(1+t_{h,t}^*)}\Delta_{hf,t}^* \tag{8.52}$$

The foreign counterparts are defined analogously. We then check if these instruments can replicate the flexible price equilibrium. For simplicity and the purpose of comparison, we focus on the case where $\rho=1$. We first look at the cooperative game. As shown in the Technical Appendix, the solution to the cooperative game is as follows:

$$\begin{gathered}
\pi_{hh,t}=\pi_{hf,t}^*=\pi_{ff,t}^*=\pi_{fh,t}=0\\
\widehat{Y}_t-\theta_t=0;\widehat{Y}_t^*-\theta_t^*=0\\
\widehat{t}_{h,t}=\widehat{t}_{h,t}^*=-\widehat{\tau}_t\\
\widehat{t}_{f,t}=\widehat{t}_{f,t}^*=-\widehat{\tau}_t^*\\
\widehat{d}_t^*=\widehat{d}_t=\widehat{S}_t=0
\end{gathered}$$

This solution indeed establishes that the flexible price equilibrium can be achieved using these instruments under a cooperative game. This is consistent with Adao, Correa and Teles (2009), although we have a different international financial market structure in our model. However, two things should be noted. First, the level of the home and foreign labor income tax is indeterminate. The need to eliminate currency misalignment requires for instance that $d_t + \widehat{t}^*_{h,t} - \widehat{t}_{h,t} = 0$ and $\widehat{t}_{h,t} + \widehat{\tau}_t = 0$, but we cannot uniquely pin down the individual tax rates. Both home and foreign consumers are subject to the same tax responses on home goods, and similarly for the tax responses on foreign goods. The labor income tax is used to offset the supply effects of the tax rates, similarly to the role of the export subsidy in our previous model. But what matters in terms of adjusting relative demand efficiently is the difference between the tax on home goods and that on foreign goods, not the individual tax rates.

Second, the exchange rate has to be fixed. Once the responses of taxes on home and foreign goods have been put in place, nominal prices do not move, and since the taxes respond identically for both home and foreign consumers, any movement in the exchange rate would in fact generate a deviation from the law of one price. To eliminate this, we require the exchange rate to be fixed.

We then explore if the flexible price equilibrium can be supported in the non-cooperative Nash game. As discussed in the Technical appendix, it cannot. In fact, we arrive at a non-existence condition exactly as in our main model, because individual authorities would wish to have a non-zero expected labor tax, leading to no finite equilibrium level of inflation. Hence, as in our main model, these instruments cannot be used to replicate the flexible price equilibrium in a non-cooperative fiscal policy game. Some further restrictions or international cooperation would be needed.

9 Conclusion

This paper shows that international tax policy can be used to correct the inefficiency problem due to nominal rigidities and local currency pricing in the open economy. We find that a state-contingent tax policy combination of consumption tax (subsidy) and export subsidy (tax) can fully eliminate the problem of currency misalignment (deviations from the law of one price), and combined with a monetary policy that leads the exchange rate to respond to productivity shocks, achieves the flexible price allocation. But this monetary fiscal mix cannot be decentralized in an environment of fully 'self-reliant' monetary and fiscal policy. The reason is that the non-cooperative use of taxes and subsidies to offset the inefficiencies of currency misalignment opens up a strategic inefficiency in a non-cooperative setting which leads to a fundamental conflict between the goals of output gap and inflation stabilization and terms of trade manipulation. The implementation of an efficient monetary-fiscal mix requires effective cooperation in fiscal policy, while leaving monetary policy to be determined non-cooperatively.

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A Appendix

In this appendix, we describe the loss function and the constraint for the cooperative game and non-cooperative game when $\rho > 1$.

A.1 Cooperative Game: $\rho > 1$ Case

As shown in the Technical Appendix, the loss function in the cooperative game could be expressed as

$$L_0 = E_t \sum_{t=0}^{\infty} (\beta)^t \left\{ \begin{array}{c} \frac{1}{2}(\widehat{Y}_t - \theta_t)^2 + \frac{1}{2}(\widehat{Y}_t^* - \theta_t^*)^2 \\ +\frac{v(2-v)}{8}(\widehat{d}_t + \widehat{t}_{c,t}^* + \frac{1}{\rho}\widehat{e}_t - \widehat{e}_t)^2 \\ +\frac{v(2-v)}{8}(\widehat{d}_t^* + \widehat{t}_{c,t} - \frac{1}{\rho}\widehat{e}_t + \widehat{e}_t)^2 \\ -\frac{1-\rho}{2}\left[\widehat{Y}_t - \frac{2-v}{2}\widehat{q}_t + \frac{2-v}{2}(\widehat{t}_{c,t}^* - \widehat{t}_{c,t}) + \frac{2-v}{2}(\frac{1}{\rho} - 1)\widehat{e}_t\right]^2 \\ -\frac{1-\rho}{2}\left[\widehat{Y}_t^* + \frac{2-v}{2}\widehat{q}_t + \frac{2-v}{2}(\widehat{t}_{c,t} - \widehat{t}_{c,t}^*) - \frac{2-v}{2}(\frac{1}{\rho} - 1)\widehat{e}_t\right]^2 \\ +\frac{\lambda}{2\delta}\left[\frac{v}{2}\pi_{hh,t}^2 + \frac{(2-v)}{2}\pi_{fh,t}^2\right] + \frac{\lambda}{2\delta}\left[\frac{v}{2}\pi_{ff,t}^{*2} + \frac{(2-v)}{2}\pi_{hf,t}^{*2}\right] \end{array} \right\}$$

The constraints are

$$\pi_{hh,t} = \delta \left[\begin{array}{c} \rho\widehat{Y}_t - \theta_t + \frac{2-v}{2}(\widehat{d}_t + \widehat{t}_{c,t}^*) + \frac{2-v}{2}(1-\rho)(\widehat{q}_t + \widehat{e}_t) \\ +\frac{(2-v)(1-\rho)}{2}(\widehat{t}_{c,t} - \widehat{t}_{c,t}^*) \end{array} \right] + \beta E_t \pi_{hh,t+1} \quad \text{(A.1)}$$

$$\pi_{hf,t}^* = \delta \left[\begin{array}{c} \rho\widehat{Y}_t - \theta_t - \frac{v}{2}(\widehat{d}_t + \widehat{t}_{c,t}^*) + \frac{2-v}{2}(1-\rho)(\widehat{q}_t + \widehat{e}_t) \\ -\frac{(2-v)}{2}(\rho - 1)(\widehat{t}_{c,t} - \widehat{t}_{c,t}^*) + \widehat{t}_{c,t}^* - \widehat{s}_{e,t} \end{array} \right] + \beta E_t \pi_{hf,t+1}^* \quad \text{(A.2)}$$

$$\pi_{ff,t}^* = \delta \left[\begin{array}{c} \rho\widehat{Y}_t^* - \theta_t^* + \frac{2-v}{2}(\widehat{d}_t^* + \widehat{t}_{c,t}) + \frac{2-v}{2}(1-\rho)(\widehat{q}_t^* - \widehat{e}_t) \\ +\frac{(2-v)(1-\rho)}{2}(\widehat{t}_{c,t}^* - \widehat{t}_t^c) \end{array} \right] + \beta E_t \pi_{ff,t+1}^* \quad \text{(A.3)}$$

$$\pi_{fh,t} = \delta \left[\begin{array}{c} \rho\widehat{Y}_t^* - \theta_t^* - \frac{v}{2}(\widehat{d}_t^* + \widehat{t}_{c,t}) + \frac{(2-v)}{2}(1-\rho)(\widehat{q}_t^* - \widehat{e}_t) \\ +\frac{2-v}{2}(\rho - 1)(\widehat{t}_{c,t} - \widehat{t}_{c,t}^*) + \widehat{t}_{c,t} - \widehat{s}_{e,t}^* \end{array} \right] + \beta E_t \pi_{fh,t+1} \quad \text{(A.4)}$$

$$\widehat{Y}_t - \widehat{Y}_t^* = \frac{v}{2}\widehat{d}_t - \frac{v}{2}\widehat{d}_t^* + \widehat{q}_t + \frac{2-v}{2}(\widehat{t}_{c,t} - \widehat{t}_{c,t}^*) + (v-1)(\frac{1}{\rho} - 1)\widehat{e}_t \quad \text{(A.5)}$$

In this more general case, the risk sharing effect will affect the welfare, so the variation of consumption enters into welfare loss function. Further, term of trade, as well as tax difference $\widehat{t}_{c,t} - \widehat{t}_{c,t}^*$, has an impact on inflation dynamics via marginal costs.

For the social planner, he chooses $\{\widehat{Y}_t-\theta_t, \widehat{Y}_t^*-\theta_t^*, \pi_{hh,t}, \pi_{hf,t}^*, \pi_{ff,t}^*, \pi_{fh,t}, \widehat{d}_t, \widehat{d}_t^*, \widehat{t}_{c,t}, \widehat{t}_{c,t}^*, \widehat{s}_{e,t}^*, \widehat{s}_{e,t}, \widehat{q}_t, \widehat{e}_t\}$. to maximize the joint/global welfare.

As shown in the Technical Appendix, the solution to the corporative game is listed in the corresponding section.

A.2 Non-cooperative Game with Restriction: $\rho > 1$ Case

After imposing the restriction $\widehat{t}_{c,t}^* = \widehat{s}_{e,t}$, $\widehat{t}_{c,t} = \widehat{s}_{e,t}^*$,, the loss function for home and foreign countries becomes[15]

Home loss function:

$$L_{h,0} = E_t \sum_{t=0}^{\infty} (\beta)^t \left\{ \begin{array}{c} \frac{1}{2}(\widehat{Y}_t-\theta_t)^2 \\ +\frac{v(2-v)}{8}(\widehat{p}_{hft}^* + \frac{1}{\rho}\widehat{e}_t - \widehat{p}_{hh,t} + \widehat{t}_{c,t}^*)^2 \\ +\frac{\lambda(1+\frac{\alpha}{\rho})}{4\delta}\left[\frac{v}{2}\pi_{hh,t}^2 + \frac{(2-v)}{2}\pi_{fh,t}^2\right] \\ +\frac{\lambda(1-\frac{\alpha}{\rho})}{4\delta}\left[\frac{v}{2}\pi_{ff,t}^{*2} + \frac{(2-v)}{2}\pi_{hf,t}^{*2}\right] \\ -\frac{1-\rho}{2}\widehat{C}_t^2 + \frac{(2-v)}{4\rho}(1-\rho)^2\left((\widehat{C}_t^*)^2 - \widehat{C}_t^2\right) \\ +\frac{v(2-v)}{8}\frac{1-\rho}{\rho}\left((\widehat{Y}_t-\theta_t) + \frac{2-v}{2}(\widehat{p}_{hft}^* + \frac{1}{\rho}\widehat{e}_t - \widehat{p}_{hh,t} + \widehat{t}_{c,t}^*)\right)^2 \\ +\frac{v(2-v)}{8}\frac{\rho-1+\frac{2}{v}}{\rho}\left((\widehat{Y}_t^*-\theta_t^*) - \frac{v}{2}(\widehat{p}_{fht} - \frac{1}{\rho}\widehat{e}_t - \widehat{p}_{ff,t}^* + \widehat{t}_{c,t})\right)^2 \\ -\frac{v(2-v)}{8}\frac{1-\rho}{\rho}\left((\widehat{Y}_t^*-\theta_t^*) + \frac{2-v}{2}(\widehat{p}_{fht} - \frac{1}{\rho}\widehat{e}_t - \widehat{p}_{ff,t}^* + \widehat{t}_{c,t})\right)^2 \\ -\frac{v(2-v)}{8}\frac{\rho-1+\frac{2}{v}}{\rho}\left((\widehat{Y}_t-\theta_t) - \frac{v}{2}(\widehat{p}_{hft}^* + \frac{1}{\rho}\widehat{e}_t - \widehat{p}_{hh,t} + \widehat{t}_{c,t}^*)\right)^2 \end{array} \right\}$$

The home government minimizes the loss function by choosing $\left\{\widehat{Y}_t-\theta_t, \pi_{hh,t}, \pi_{fh,t}, \widehat{d}_t, \widehat{t}_{c,t}, \widehat{q}_t, \widehat{e}_t\right\}$, taking $\{\widehat{Y}_t^*-\theta_t^*, \pi_{hf,t}^*, \pi_{ff,t}^*, \widehat{t}_{c,t}^*, \widehat{d}_t^*\}$ as given, subject to following constraints,

[15] The detailed derivation is given in the Technical Appendix.

$$\pi_{hh,t} = \delta \left[\begin{array}{c} \rho\widehat{Y}_t - \theta_t + \frac{2-v}{2}(\widehat{d}_t + \widehat{t}^*_{c,t}) + \frac{2-v}{2}(1-\rho)(\widehat{q}_t + \widehat{e}_t) \\ +\frac{(2-v)(1-\rho)}{2}(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) \end{array} \right] + \beta E_t \pi_{hh,t+1} \tag{A.6}$$

$$\pi_{fh,t} = \delta \left[\begin{array}{c} \rho\widehat{Y}^*_t - \theta^*_t - \frac{v}{2}(\widehat{d}^*_t + \widehat{t}_{c,t}) + \frac{(2-v)}{2}(1-\rho)(\widehat{q}^*_t - \widehat{e}_t) \\ +\frac{2-v}{2}(\rho-1)(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) \end{array} \right] + \beta E_t \pi_{fh,t+1} \tag{A.7}$$

$$\widehat{Y}_t - \widehat{Y}^*_t = \frac{v}{2}\widehat{d}_t - \frac{v}{2}\widehat{d}^*_t + \widehat{q}_t + \frac{2-v}{2}(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) + (v-1)(\frac{1}{\rho} - 1)\widehat{e}_t \tag{A.8}$$

Foreign loss function is given by:

$$L^*_{h,0} = E_t \sum_{t=0}^{\infty} (\beta)^t \left\{ \begin{array}{c} \frac{1}{2}(\widehat{Y}^*_t - \theta^*_t)^2 \\ +\frac{v(2-v)}{8}(\widehat{p}_{fht} - \frac{1}{\rho}\widehat{e}_t - \widehat{p}^*_{ff,t} + \widehat{t}_{c,t})^2 \\ +\frac{\lambda(1-\frac{\alpha}{\rho})}{4\delta}\left[\frac{v}{2}\pi^2_{hh,t} + \frac{(2-v)}{2}\pi^2_{fh,t}\right] + \frac{\lambda(1+\frac{\alpha}{\rho})}{4\delta}\left[\frac{v}{2}\pi^{*2}_{ff,t} + \frac{(2-v)}{2}\pi^{*2}_{hf,t}\right] \\ -\frac{1-\rho}{2}\widehat{C}^{*2}_t + \frac{(2-v)}{4\rho}(1-\rho)^2\left(\widehat{C}^2_t - (\widehat{C}^*_t)^2\right) \\ -\frac{v(2-v)}{8}\frac{1-\rho}{\rho}\left((\widehat{Y}_t - \theta_t) + \frac{2-v}{2}(\widehat{p}^*_{hft} + \frac{1}{\rho}\widehat{e}_t - \widehat{p}_{hh,t} + \widehat{t}^*_{c,t})\right)^2 \\ -\frac{v(2-v)}{8}\frac{\rho-1+\frac{2}{v}}{\rho}\left((\widehat{Y}^*_t - \theta^*_t) - \frac{v}{2}(\widehat{p}_{fht} - \frac{1}{\rho}\widehat{e}_t - \widehat{p}^*_{ff,t} + \widehat{t}_{c,t})\right)^2 \\ +\frac{v(2-v)}{8}\frac{1-\rho}{\rho}\left((\widehat{Y}^*_t - \theta^*_t) + \frac{2-v}{2}(\widehat{p}_{fht} - \frac{1}{\rho}\widehat{e}_t - \widehat{p}^*_{ff,t} + \widehat{t}_{c,t})\right)^2 \\ +\frac{v(2-v)}{8}\frac{\rho-1+\frac{2}{v}}{\rho}\left((\widehat{Y}_t - \theta_t) - \frac{v}{2}(\widehat{p}^*_{hft} + \frac{1}{\rho}\widehat{e}_t - \widehat{p}_{hh,t} + \widehat{t}^*_{c,t})\right)^2 \end{array} \right\}$$

The foreign government minimizes the loss function by choosing $\left\{\widehat{Y}^*_t - \theta^*_t, \pi^*_{hf,t}, \pi^*_{ff,t}, \widehat{t}^*_{c,t}, \widehat{d}^*_t, \widehat{q}^*_t, \widehat{e}_t\right\}$ given $\left\{\widehat{Y}_t - \theta_t, \pi_{hh,t}, \pi_{fh,t}, \widehat{d}_t, \widehat{t}_{c,t}\right\}$, subject to following constraints,

$$\pi^*_{hf,t} = \delta \left[\begin{array}{c} \rho\widehat{Y}_t - \theta_t - \frac{v}{2}(\widehat{d}_t + \widehat{t}^*_{c,t}) + \frac{2-v}{2}(1-\rho)(\widehat{q}_t + \widehat{e}_t) \\ -\frac{(2-v)}{2}(\rho-1)(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) \end{array} \right] + \beta E_t \pi^*_{hf,t+1} \tag{A.9}$$

$$\pi^*_{ff,t} = \delta \left[\begin{array}{c} \rho\widehat{Y}^*_t - \theta^*_t + \frac{2-v}{2}(\widehat{d}^*_t + \widehat{t}_{c,t}) + \frac{2-v}{2}(1-\rho)(\widehat{q}^*_t - \widehat{e}_t) \\ +\frac{(2-v)(1-\rho)}{2}(\widehat{t}^*_{c,t} - \widehat{t}^c_t) \end{array} \right] + \beta E_t \pi^*_{ff,t+1} \tag{A.10}$$

$$\widehat{Y}_t - \widehat{Y}^*_t = \frac{v}{2}\widehat{d}_t - \frac{v}{2}\widehat{d}^*_t + \widehat{q}_t + \frac{2-v}{2}(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) + (v-1)(\frac{1}{\rho} - 1)\widehat{e}_t \tag{A.11}$$

As shown in the Technical Appendix, the restricted Nash game will deliver exactly the same solution as that in the cooperative game.

A.3 Three Players Nash Game with Restriction: $\rho > 1$ Case

- The problem of Home monetary authority:[16]

loss function:

$$L_{h,0} = E_t \sum_{t=0}^{\infty} (\beta)^t \left\{ \begin{array}{c} \frac{1}{2}(\widehat{Y}_t - \theta_t)^2 \\ +\frac{v(2-v)}{8}(\widehat{p}^*_{hft} + \frac{1}{\rho}\widehat{e}_t - \widehat{p}_{hh,t} + \widehat{t}^*_{c,t})^2 \\ +\frac{\lambda(1+\frac{\alpha}{\rho})}{4\delta}\left[\frac{v}{2}\pi^2_{hh,t} + \frac{(2-v)}{2}\pi^2_{fh,t}\right] \\ +\frac{\lambda(1-\frac{\alpha}{\rho})}{4\delta}\left[\frac{v}{2}\pi^{*2}_{ff,t} + \frac{(2-v)}{2}\pi^{*2}_{hf,t}\right] \\ -\frac{v(2-v)}{8}\frac{\rho-1+\frac{2}{v}}{\rho}\left((1-\rho)\widehat{C}_t + \widehat{t}_{c,t} - \widehat{s}^*_{e,t}\right)^2 - \frac{1-\rho}{2}\widehat{C}_t^2 \\ +\frac{v(2-v)}{8}\frac{\rho-1+\frac{2}{v}}{\rho}\left((1-\rho)\widehat{C}^*_t + \widehat{t}^*_{c,t} - \widehat{s}_{e,t}\right)^2 \\ +\frac{v(2-v)}{8}\frac{\rho-1}{\rho}(1-\rho)^2(\widehat{C}_t^2 - \widehat{C}_t^{*2}) \\ +\frac{v(2-v)}{4}\frac{\rho-1+\frac{2}{v}}{\rho}\left((\widehat{t}_{c,t} - \widehat{s}^*_{e,t}) - (\widehat{t}^*_{c,t} - \widehat{s}_{e,t})\right) \\ +\frac{v(2-v)}{8}\frac{1-\rho}{\rho}\left((\widehat{Y}_t - \theta_t) + \frac{2-v}{2}(\widehat{p}^*_{hft} + \frac{1}{\rho}\widehat{e}_t - \widehat{p}_{hh,t} + \widehat{t}^*_{c,t})\right)^2 \\ +\frac{v(2-v)}{8}\frac{\rho-1+\frac{2}{v}}{\rho}\left((\widehat{Y}^*_t - \theta^*_t) - \frac{v}{2}(\widehat{p}_{fht} - \frac{1}{\rho}\widehat{e}_t - \widehat{p}^*_{ff,t} + \widehat{t}_{c,t})\right)^2 \\ -\frac{v(2-v)}{8}\frac{1-\rho}{\rho}\left((\widehat{Y}^*_t - \theta^*_t) + \frac{2-v}{2}(\widehat{p}_{fht} - \frac{1}{\rho}\widehat{e}_t - \widehat{p}^*_{ff,t} + \widehat{t}_{c,t})\right)^2 \\ -\frac{v(2-v)}{8}\frac{\rho-1+\frac{2}{v}}{\rho}\left((\widehat{Y}_t - \theta_t) - \frac{v}{2}(\widehat{p}^*_{hft} + \frac{1}{\rho}\widehat{e}_t - \widehat{p}_{hh,t} + \widehat{t}^*_{c,t})\right)^2 \end{array} \right\} \quad \text{(A.12)}$$

The home government minimizes the loss function by choosing $\left\{\widehat{Y}_t - \theta_t, \pi_{hh,t}, \pi_{fh,t}, \widehat{d}_t, \widehat{q}_t, \widehat{e}_t\right\}$, taking $\{\widehat{Y}^*_t - \theta^*_t, \pi^*_{hf,t}, \pi^*_{ff,t}, \widehat{d}^*_t, \widehat{t}_{c,t}, \widehat{t}^*_{c,t}, \widehat{s}_{e,t}, \widehat{s}^*_{e,t}\}$ as given, subject to following constraints, shadow prices associated with these constraints are $\{\varpi_{1,t}, \varpi_{2,t}, \varpi_{3,t}, , \varpi_{4,t}\}$:

$$\pi_{hh,t} = \delta \left[\begin{array}{c} \rho\widehat{Y}_t - \theta_t + \frac{2-v}{2}(\widehat{d}_t + \widehat{t}^*_{c,t}) + \frac{2-v}{2}(1-\rho)(\widehat{q}_t + \widehat{e}_t) \\ +\frac{(2-v)(1-\rho)}{2}(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) \end{array} \right] + \beta E_t \pi_{hh,t+1} \quad \text{(A.13)}$$

$$\pi_{fh,t} = \delta \left[\begin{array}{c} \rho\widehat{Y}^*_t - \theta^*_t - \frac{v}{2}(\widehat{d}^*_t + \widehat{t}_{c,t}) + \frac{(2-v)}{2}(1-\rho)(\widehat{q}^*_t - \widehat{e}_t) \\ +\frac{2-v}{2}(\rho-1)(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) + \widehat{t}_{c,t} - \widehat{s}^*_{e,t} \end{array} \right] + \beta E_t \pi_{fh,t+1} \quad \text{(A.14)}$$

$$\widehat{Y}_t - \widehat{Y}^*_t = \frac{v}{2}\widehat{d}_t - \frac{v}{2}\widehat{d}^*_t + \widehat{q}_t + \frac{2-v}{2}(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) + (v-1)(\frac{1}{\rho} - 1)\widehat{e}_t \quad \text{(A.15)}$$

$$\widehat{e}_t = \frac{v}{2}\widehat{d}_t - \frac{v}{2}\widehat{d}^*_t - (1-v)\widehat{q}_t - \frac{2-v}{2}(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) \quad \text{(A.16)}$$

[16] The detailed derivation is given in the Technical Appendix.

- The problem of Foreign monetary authority:

loss function:

$$L^*_{h,0} = E_t \sum_{t=0}^{\infty} (\beta)^t \left\{ \begin{array}{c} \frac{1}{2}(\widehat{Y}^*_t - \theta^*_t)^2 \\ +\frac{v(2-v)}{8}(\widehat{p}_{fht} - \frac{1}{\rho}\widehat{e}_t - \widehat{p}^*_{ff,t} + \widehat{t}_{c,t})^2 \\ +\frac{\lambda(1-\frac{\alpha}{\rho})}{4\delta}\left[\frac{v}{2}\pi^2_{hh,t} + \frac{(2-v)}{2}\pi^2_{fh,t}\right] + \frac{\lambda(1+\frac{\alpha}{\rho})}{4\delta}\left[\frac{v}{2}\pi^{*2}_{ff,t} + \frac{(2-v)}{2}\pi^{*2}_{hf,t}\right] \\ +\frac{v(2-v)}{8}\frac{\rho-1+\frac{2}{v}}{\rho}\left((1-\rho)\widehat{C}_t + \widehat{t}_{c,t} - \widehat{s}^*_{e,t}\right)^2 - \frac{1-\rho}{2}\widehat{C}^{*2}_t \\ -\frac{v(2-v)}{8}\frac{\rho-1+\frac{2}{v}}{\rho}\left((1-\rho)\widehat{C}^*_t + \widehat{t}^*_{c,t} - \widehat{s}_{e,t}\right)^2 \\ -\frac{v(2-v)}{8}\frac{\rho-1}{\rho}(1-\rho)^2(\widehat{C}^2_t - \widehat{C}^{*2}_t) \\ -\frac{v(2-v)}{4}\frac{\rho-1+\frac{2}{v}}{\rho}\left((\widehat{t}_{c,t} - \widehat{s}^*_{e,t}) - (\widehat{t}^*_{c,t} - \widehat{s}_{e,t})\right) \\ -\frac{v(2-v)}{8}\frac{1-\rho}{\rho}\left((\widehat{Y}_t - \theta_t) + \frac{2-v}{2}(\widehat{p}^*_{hft} + \frac{1}{\rho}\widehat{e}_t - \widehat{p}_{hh,t} + \widehat{t}^*_{c,t})\right)^2 \\ -\frac{v(2-v)}{8}\frac{\rho-1+\frac{2}{v}}{\rho}\left((\widehat{Y}^*_t - \theta^*_t) - \frac{v}{2}(\widehat{p}_{fht} - \frac{1}{\rho}\widehat{e}_t - \widehat{p}^*_{ff,t} + \widehat{t}_{c,t})\right)^2 \\ +\frac{v(2-v)}{8}\frac{1-\rho}{\rho}\left((\widehat{Y}^*_t - \theta^*_t) + \frac{2-v}{2}(\widehat{p}_{fht} - \frac{1}{\rho}\widehat{e}_t - \widehat{p}^*_{ff,t} + \widehat{t}_{c,t})\right)^2 \\ +\frac{v(2-v)}{8}\frac{\rho-1+\frac{2}{v}}{\rho}\left((\widehat{Y}_t - \theta_t) - \frac{v}{2}(\widehat{p}^*_{hft} + \frac{1}{\rho}\widehat{e}_t - \widehat{p}_{hh,t} + \widehat{t}^*_{c,t})\right)^2 \end{array} \right\} \tag{A.17}$$

The foreign monetary authority minimize loss function by choosing $\left\{\widehat{Y}^*_t - \theta^*_t, \pi^*_{hf,t}, \pi^*_{ff,t}, \widehat{d}^*_t, \widehat{q}^*_t, \widehat{e}^*_t\right\}$ given $\left\{\widehat{Y}_t - \theta_t, \pi_{hh,t}, \pi_{fh,t}, \widehat{d}_t, \widehat{t}_{c,t}, \widehat{t}^*_{c,t}, \widehat{s}_{e,t}, \widehat{s}^*_{e,t}\right\}$, subject to following constraint, shadow prices are $\{\varpi^*_{1,t}, \varpi^*_{2,t}, \varpi^*_{3,t}, \varpi^*_{4,t}\}$:

$$\pi^*_{hf,t} = \delta \left[\begin{array}{c} \rho\widehat{Y}_t - \theta_t - \frac{v}{2}(\widehat{d}_t + \widehat{t}^*_{c,t}) + \frac{2-v}{2}(1-\rho)(\widehat{q}_t + \widehat{e}_t) \\ -\frac{(2-v)}{2}(\rho-1)(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) + \widehat{t}^*_{c,t} - \widehat{s}_{e,t} \end{array} \right] + \beta E_t \pi^*_{hf,t+1} \tag{A.18}$$

$$\pi^*_{ff,t} = \delta \left[\begin{array}{c} \rho\widehat{Y}^*_t - \theta^*_t + \frac{2-v}{2}(\widehat{d}^*_t + \widehat{t}_{c,t}) + \frac{2-v}{2}(1-\rho)(\widehat{q}^*_t - \widehat{e}_t) \\ +\frac{(2-v)(1-\rho)}{2}(\widehat{t}^*_{c,t} - \widehat{t}^c_t) \end{array} \right] + \beta E_t \pi^*_{ff,t+1} \tag{A.19}$$

$$\widehat{Y}_t - \widehat{Y}^*_t = \frac{v}{2}\widehat{d}_t - \frac{v}{2}\widehat{d}^*_t + \widehat{q}_t + \frac{2-v}{2}(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) + (v-1)(\frac{1}{\rho} - 1)\widehat{e}_t \tag{A.20}$$

$$\widehat{e}_t = \frac{v}{2}\widehat{d}_t - \frac{v}{2}\widehat{d}^*_t - (1-v)\widehat{q}_t - \frac{2-v}{2}(\widehat{t}_{c,t} - \widehat{t}^*_{c,t}) \tag{A.21}$$

- The problem of global fiscal alliance:

loss function:

$$
\begin{aligned}
L_0 &= L_{h,0} + L_{h,0}^* \\
&= E_t \sum_{t=0}^{\infty} (\beta)^t \left\{ \begin{array}{c}
\frac{1}{2}(\widehat{Y}_t - \theta_t)^2 + \frac{1}{2}(\widehat{Y}_t^* - \theta_t^*)^2 \\
+\frac{v(2-v)}{8}(\widehat{d}_t + \widehat{t}_{c,t}^* + \frac{1}{\rho}\widehat{e}_t - \widehat{e}_t)^2 \\
+\frac{v(2-v)}{8}(\widehat{d}_t^* + \widehat{t}_{c,t} - \frac{1}{\rho}\widehat{e}_t + \widehat{e}_t)^2 \\
-\frac{1-\rho}{2}\left[\widehat{Y}_t - \frac{2-v}{2}\widehat{q}_t + \frac{2-v}{2}(\widehat{t}_{c,t}^* - \widehat{t}_{c,t}) + \frac{2-v}{2}(\frac{1}{\rho} - 1)\widehat{e}_t\right]^2 \\
-\frac{1-\rho}{2}\left[\widehat{Y}_t^* + \frac{2-v}{2}\widehat{q}_t + \frac{2-v}{2}(\widehat{t}_{c,t} - \widehat{t}_{c,t}^*) - \frac{2-v}{2}(\frac{1}{\rho} - 1)\widehat{e}_t\right]^2 \\
+\frac{\lambda}{2\delta}\left[\frac{v}{2}\pi_{hh,t}^2 + \frac{(2-v)}{2}\pi_{fh,t}^2\right] + \frac{\lambda}{2\delta}\left[\frac{v}{2}\pi_{ff,t}^{*2} + \frac{(2-v)}{2}\pi_{hf,t}^{*2}\right]
\end{array} \right\}
\end{aligned}
$$

Optimal choices are $\left\{\widehat{t}_{c,t}, \widehat{t}_{c,t}^*, \widehat{s}_{e,t}^*, \widehat{s}_{e,t}\right\}$. The constraints are (A.13)-(A.16) and (A.18)-(A.19). $\xi_{5,t}$ and $\xi_{6,t}$ are the lagrangian multipliers of corresponding constraints of market clearing condition and real exchange rate.

As shown in the Technical Appendix, this Nash game will deliver exactly the same solution as that in the cooperative game.