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THE CASE OF THE NEGATIVE NOMINAL INTEREST RATES: NEW ESTIMATES OF THE TERM STRUCTURE OF INTEREST RATES DURING THE GREAT DEPRESSION

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ABSTRACT

During the 1930s and early 1940s U.S. Treasury bonds and notes had negative nominal yields as they approached maturity. But since an investor can always hold cash, this is impossible. Any bond must have a positive nominal yield. This paper poses a resolution to this puzzle: in addition to making coupon payments, Treasury securities were options that gave the owner the right to buy a new security on a future date. The paper proposes a method for valuing this 'exchange privilege' and computing the yield to the coupon bearing component of these composite bond/options. The case of the negative nominal interest rates demonstrates that the construction of accurate data requires close examination of the institutional environment, even when studying financial markets.

The corrected bond and note yields are used to calculate new estimates of the term structure of interest rates from 1929 to 1949. These new data allow one to follow changes in the both the level and the shape of the yield curve during the Great Depression.

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1 Introduction

On December 31, 1932 the New York Times listed the yield on a $3\frac{1}{2}\%$ United States Liberty Bond as -1.74%. This seems impossible. An investor can always hold cash rather than an interest bearing security, so any bond should have a positive nominal yield. It is well known that during the Great Depression the prices of Treasury Bills at auction occasionally exceeded par. But the negative yields were extremely small, on the order of -.05%.¹ Yields of this small a magnitude can be explained by both the fact that Treasury Bills were exempt from personal property taxes in some states [See Homer (1976) pg. 355.] and that Treasury securities were required as collateral for a bank to hold U.S. Government deposits.² Negative nominal yields on the order of -2% are an entirely different story. In fact, from mid-1932 through mid-1942, the vast majority of coupon bearing U.S. Government securities bore negative nominal yields as they neared maturity.³

Since negative nominal yields are impossible in a world where one can always hold cash, these securities must have had other attributes that were being valued. During the 1930s, the standard practice of the U. S. Treasury was to issue new bonds with coupon rates that implied market prices above par, but sell them at par. Holders of maturing bonds and notes were given preferential treatment in the distribution of the new issue. Coupon bearing Treasury securities had what was called an 'exchange privilege'. At maturity, they could be exchanged at par for a new issue. Government bonds and notes were not just coupon securities; they were options as well. The option had value that was included in the quoted price. As a bond approached maturity, this premium caused the price to rise high enough that the computed yield was negative.

The solution to the first puzzle, that of the negative nominal interest rates, has given way to a second one: Why did the Treasury sell new issues at prices below those prevailing in the market? The answer to this question can be found by studying the institutional environment of the 1930s. Legal

¹Bids in excess of par were received throughout 1939, 1940 and 1941. The highest recorded was 100.018 on January 8, 1941. See the Annual Report of the Secretary of the Treasury, 1941 pg. 301.

²I have also heard the claim that banks substituted Treasury Bills for smaller denomination currency in making interbank transfers, and so the negative yield reflected convenience. Unfortunately, this could not be substantiated.

³The plots in both Durand (1942) and the U.S. Treasury Bulletin for 1939 imply negative nominal yields for maturities below two years. Childs (1947, pg. 259) also notes the existence of negative nominal yields in the 1930's but provides no explanation.

constraints forced the Treasury to sell new securities at par. To insure that an offering actually sold, the coupon rate had to be set above the current market interest rate. Initial purchasers were paid to place the new issue. This was the method of underwriting.

The purpose of this paper is to describe the conditions that led to the apparent negative nominal interest rates and then use this information to construct accurate data on the returns to holding U.S. Government securities during the 1930s and 1940s. Proper computation of the term structure during the 1930s requires careful examination of the institutions of the bond market and Treasury debt management. In what follows, a method for valuing the exchange privilege is described and used to correct the measurement of the yields of traded securities. These are used to construct term structure estimates from 1929 to 1949 that are consistent with those currently in use. These new data replace the sketchy data contained in the Federal Reserve Board's Banking and Monetary Statistics of the United States, and for the first time allow one to follow changes in the shape of the term structure during the Great Depression. The interest rate data can be added to new data on three and six month time loans in Mankiw and Miron (1985) and the new output, production and unemployment data in Romer (1986a, 1986b, 1986c and 1987).

The motivation for constructing this new data set is twofold. First, empirical research in macroeconomics often relies on the use of lengthy time series data.⁴ While Salomon Brothers publishes estimates of yields at 3 months, 1, 2, 3, 4, 5, 10, and 20 years to maturity beginning in 1950,⁵ data on the term structure of interest rates prior to 1950 are noticeably missing. Second, the resurgence of interest in the economics of the Great Depression⁶ makes it all the more important to exploit new data sources.

The remainder of this paper is divided into five sections. Section II describes the raw data collected and used in the study. The following section provides a detailed account of the Treasury practices that caused nominal interest rates to be negative. The rationale for the Treasury's behavior is also examined. A method for valuing the exchange provision is then pro-

⁴This is true of the original work on business cycle dating summarized in Moore and Zarnowitz (1986) and the more recent studies of the effects of money by Friedman and Schwartz (1982) and investment by Gordon and Veitch (1986).

⁵Recently, McCulloch (1987) has estimated coupon corrected yield curves for December 1946 to February 1987 that will likely replace the Salomon data in future research.

⁶Papers by Bernanke (1983, 1986), Bernanke and Powell (1986), Field (1984), Hamilton (1987) and the essays in Brunner (1981) are examples.

posed and used to compute the yield to the coupon bearing component of the composite bond/option. To allow the complete use of the information in the data, it is necessary to study the tax status of existing securities. This is the task of Section 4. The following section uses this tax information, together with the corrected yields from Section 3, to construct estimates of the term structure using a technique derived by Nelson and Seigel (1985). The concluding section provides a comparison of the new interest rate series with those previously available and finds that there are substantial differences. The adjustments for the exchange privilege lead to systematically higher estimates of yields at maturities below five years.

2 Data

Existing data on nominal interest rates prior to World War II are both limited in scope and imprecise. The Federal Reserve Board's *Banking and Monetary Statistics of the United States* contains several series for interest rates during the inter-war period, but it is difficult to tell exactly how the numbers were constructed and to what securities they actually refer. For example, Table No. 122 on page 460 of the *Banking and Monetary Statistics* includes monthly series for '3- to 5-year tax exempt Treasury notes', while Table No. 128 on page 468 reports longer term bond yields under the simple heading 'U.S. Government'. The second of these refers to the unweighted average of the yield on all outstanding bonds with at least twelve years to maturity. Clearly, there is motivation for collecting a new and more complete set of interest rate data.

Construction of a new data set on the term structure requires information on the prices of outstanding Treasury issues. These raw data were collected from the *New York Times* financial column entitled 'Bond Sales on the New York Stock Exchange.' Quotes on the *prices* of all U.S. Treasury Bonds, Notes and Certificates of Indebtedness were collected from the *New York Times* for the final trading day of each month from January 1929 to December 1949. The data set is complete in that it contains a yield for every bond, note or certificate for every month during which it was in existence. It is composed of all 152 coupon bearing securities either in existence in January 1929 or issued during the twenty-one year period examined. Of this total, 56 are bonds, 54 are notes, and 42 are certificates of indebtedness.

In addition to coupon securities, beginning in mid-1931 data were collected on the *yield* of Treasury Bills with three months to maturity — prices are not reported.⁷ As is currently the case, Treasury Bills were pure discount securities. Other Treasury Bills of shorter maturity were excluded since the major objective is to study yields at longer maturities.⁸

As is nearly always the case in research on financial markets, the data refer to dealer price quotes. There is no guarantee that actual transactions occurred at these prices. This problem is minimized by computing yields based on the mean of the bid/ask spread. But it is impossible to know how large an error comes from systematic differences between dealer quotes and transactions prices.

It is possible, however, to insure that trading occurred. The New York Times does report volume. For example, on January 30, 1932, volume in the $3\frac{3}{8}\%$ Treasury Bonds of 1940-43 amounted to \$130,000. While this is a very small fraction of the nearly \$360 million of the issue outstanding, it is important that there was some trading. To make the data set complete, in several isolated cases it was necessary to use price quotes that did not reflect trading on the New York Stock Exchange. These quotes were found in the New York Times under the heading 'U.S. Bond Quotations — Closing quotations for issues not traded in on [sic] the Stock Exchange yesterday.'

Since the majority of U.S. Treasury bonds issued during this period contained call provisions, there is a problem in computing the yield to maturity. Fortunately, except for several very special cases, all bonds were called on the first allowable date. As such, all yields were computed to the call date.⁹

The raw data consists of 9070 observations over 252 months, or just under 36 observations per month, on average. These raw data are available from the author on standard diskettes. As one would expect, the number of observations is small during the first few years, increasing substantially with the debt issues of the middle 1930s and again with the issues during World

⁷Childs (1947, pg. 432) describes early Treasury Bill issues. While the first Treasury Bills were issued in 1929, it was not until 1931 that a series can be constructed that is composed solely of issues with three months to maturity. During 1929 and 1930, bills were issued at irregular intervals and matured in three, six, nine or twelve months. Three month Treasury Bill rates were found for February, April and May 1931, as well as every month beginning with July 1931.

⁸In addition, all interest bearing government debt not issued directly by the U.S. Treasury, such as securities issued by the Federal Home Loan Bank Board or the Reconstruction Finance Corporation, is omitted.

⁹An alternative is to compute the yield to the call date when the price of the security exceeds par, and the yield to the final maturity date when the price is below par. Use of this rule would have virtually no effect on the results since bonds nearly always sold at prices in excess of par.

War II. In 1929, 1930 and 1931 there are an average of only 14 data points per month. By 1933, the average is over 20, rising steady to 40 in 1939, to 54 in 1945 and falling to 38 in 1949. The implication is that the estimated yield curves will be less accurate for the earlier period simply because of the paucity of data.

3 Negative Nominal Yields and the Exchange Privilege

Consider the following exercise. Take the data described in Section 2 for a representative month and compute the yield to maturity for all the coupon bearing securities based on the mean of the bid/ask spread. The results for February 1935 are plotted in Figure 1.¹⁰ In the figure, N's refer to fully tax exempt securities and P's refer to partially tax exempt securities. This distinction is discussed further in Section 4. The solid line is an estimate of a term structure using the techniques described in Section 5. (Following the standard convention, all interest rates are in bond yield equivalents — two times the six month rate.)

Figure 1 has several striking features. First, except for the single N representing the 3 month Treasury Bill yield of .15%, the yield curve is smoothly upward sloping. If one were to neglect the vertical scale, the picture would not seem odd. The problem is that the lowest point is a Treasury note with 5 months to expiration and a yield of -1.25%. If this result were obtained for an isolated month, one would be inclined to check the raw price data for errors. But negative yields arise consistently from 1932 through 1942.

Discussions of the period note the existence of negative nominal yields. They point out that during the 1930s the standard practice of the U.S. Treasury was to issue new bonds above par and give holders of maturing bonds, notes and certificates preferential treatment in distributing the new issue. Maturing securities had an 'exchange privilege' which gave them added value.

The remainder of this section is divided into two parts. The first provides a discussion of the institutional environment that led to the apparent negative nominal yields and discusses the reason for the Treasury to issue

¹⁰A similar diagram was constructed for every month of the data set. From 1934 to 1941 all of the figures had the same general features as Figure 1.



securities in the way that it did. This is followed by a detailed description of how to correct the data for the existence of the exchange privilege.

3.1 The Exchange Privilege

Each year, the Annual Report of the Secretary of the Treasury describes the offerings of securities during that year. In the 1930s, new offerings were announced from one to two weeks prior to the date of issue. The announcement stipulated the method of payment. The purchaser was either required to pay cash, required to exchange an existing security (valued at par), or given a choice of the two. Of the 86 new and additional offerings of bonds, notes and certificates of indebtedness from 1932 to 1940, 15 required cash payment, 31 could be obtained only by exchange, and the remaining 40 gave the purchaser a choice.¹¹

For reasons that will be discussed below, the Treasury's regular practice was to fix the coupon rate on a new issue above the current interest rate for a bond of equivalent maturity, causing the initial price of the new bond in the securities market to exceed par. Exchange allowed the holder of a maturing security to reap the benefit of this, giving value to the exchange privilege.¹² Of the 57 coupon bearing securities that matured between 1932 and 1940, 54 could be exchanged at maturity for new issues that initially sold in excess of par.

Cash payment was by subscription. Prospective purchasers made application for a certain amount of the issue and sent either 5% or 10% (depending on the issue) of the face value as a deposit. Subscription was guaranteed up to some level, usually \$5,000 or \$10,000. Individuals' requests in excess of the minimum were filled as a percentage of the total of all applications. For example, subcribers to the $3\frac{1}{8}\%$ 1949-52 bond, whose issue was announced on December 3, 1934, were alloted 18% of the amount they requested, but not less than \$10,000. Between 1932 and 1940, cash subscribers, on average, were alloted 15.4% of their requests, but not less than \$5263.

Once the allotment was determined, a cash subscriber could take delivery by paying the remaining balance. For example, a request for \$100,000

¹¹The total of 86 issues exceeds the actual number of new securities by 19 because of the practice of making additional offerings of alreading existing securities.

¹²Durand (1942) mentions the exchange privilege, but implies that its value is derived from the saving in brokerage fees that comes from rolling over an investment. It is difficult to see why someone wishing a long term security would buy a maturing one simply for the benefit of having it roll over. Theis (1985) in replicating the work of Durand also notes the existence of the exchange privilege and correctly points out the source of its value.

might require a \$10,000 deposit. If the final allotment were 18%, then upon delivery the subscriber must pay the balance of \$8,000. Because the bonds were issued above par, a cash subscriber could make a profit by selling them immediately. In the case of the $3\frac{1}{8}$ % 1949-52 bond, the bid price on December 15, 1934 was $101\frac{13}{32}$, implying a profit of $1\frac{13}{32}$. Alternatively, since the offering announcement guaranteed a minimum allotment, in this case \$10,000, a subscriber could sell the securities to a dealer on a when-issued basis. In this second case, the investor would take delivery of the bonds and immediately hand them over to the dealer, retaining the difference between par and the when-issued price that was previously agreed upon.¹³

Neither of the strategies associated with cash subscription was without risk. Since the market price of the bond on the issue date was uncertain at the time of subscription, there is clear risk in actually taking delivery and then selling the bonds on the open market. Since the allotment was not guaranteed, an investor had no way of knowing the quantity that would be delivered and could not safely sell more than the guaranteed amount on a when-issued basis. Exchange, on the other hand, was less risky since the amount of the new issue received was always guaranteed.

At this point, it is useful to compute the realized values of both the exchange privilege and the profit from cash subscription. The profit from cash subscription is easily determined by collecting data on the first quoted price of a new issue and taking the difference from par. In order to value the exchange privilege, information in the Treasury's offering notices, reprinted in the Annual Report of the Secretary of the Treasury, was used to match each maturing note and bond, beginning with the 2% Treasury note maturing on March 15, 1932, with the new issues for which it could be exchanged. Then the value of each new security on its issue date was determined by using the closing quotation from the New York Times on that date.¹⁴ The realized value of the exchange privilege is the difference between the first bid price of the new issue and par. When a security could be exchanged for more than one new one, the value was assumed to be that of the most lucrative trade

¹³Porter (1938,1939) calls this a 'free-ride' and describes in detail how to make a quick profit in the week preceding the new issue. She suggested subscribing, and selling the guaranteed amount, then only \$1000, on a when-issued basis. According to a Bell's account in the December 11, 1938 New York Times, Porter's article in the December 1938 issue of Scribner's Magazine set off a rush of subscriptions during that month and caused the Treasury to reduce the guaranteed amount.

¹⁴In several cases, no quote was found in the newspaper. For bonds, the first available quotation reported by Childs was substituted. For notes, the first available quote was located in the New York Times.

available. Obviously, the realized value was always nonnegative.

To illustrate the procedure, take an example. The $2\frac{1}{2}\%$ Note issued on January 29, 1934 and maturing on March 15, 1935 could be exchanged for a $1\frac{5}{8}\%$ Note maturing on March 15, 1940. The March 16, 1935 New York Times reported the first bid on the new issue as $101\frac{5}{32}$, 1.16% above par.

Between 1932 and 1940, the average value of the exchange privilege realized by holder of maturing coupon secruities was 1.1% with a standard deviation of .67%. Cash subscribers realized an average profit of .68% with a standard deviation of .51%.

It appears that the mechanism used to issue and refund Treasury debt involved giving away substantial amounts money. But closer examination of both the legal and economic environment of the 1930s leads to an explananation of the Treasury's behavior. From the end of 1929 to the end of 1939 the interest bearing debt of the U.S. Government more than doubled, rising from \$16 billion to \$41 billion. Prior to the Depression, major buildups of government debt had only occured during wartime and the severe Depressions of the 19th century. As such, the Treasury had no real mechanism for issuing debt. The network of dealers and banks that serve to distribute newly issued securities today was not yet in place.

Current law also constrained Treasury actions. The Second Liberty Bond Act, which gave authority for the issuance of Treasury debt, required that new Treasury Bonds and Certificates of Indebtedness be issued at par, and new Notes issued at not less than par.¹⁵ Given this statute, the only way to guarantee that a new issue would be sold (or maturing securities presented for exchange), was to set the coupon rate on the new bond or note above the current market interest rate on a comparable security.¹⁶

As is mentioned above, participation in either subscription or exchange entailed risks, and so some sort of compensation was in order. With a potential exchange, there was no way of knowing what the value would be until the full transaction was complete. The characteristics of the new security were announced only a few weeks prior to maturity of the existing bond or note. For subscribers, there was the uncertainty about the size of the allotment and the movement of interest rates over the week prior to the physical delivery of the securities. The compensation for this risk is analogous to the fee paid to underwriters of corporate securities who commit

¹⁵See U.S. Department of Treasury (1938).

¹⁶ Perhaps surprisingly, auctions of coupon securities by the U.S. Treasury did not begin until 1970. Treasury Bills, on the other had, have been auctioned since their inception in 1929.

themselves to selling a fixed quantity of a stock or bond at a given price on a future date, thereby assuming the risk inherent in price fluctuations.

Two pieces of evidence support the view that the exchange privilege and the profit to cash subscription were underwriting spreads. First, the magnitude of the differential is appropriate. Cohan (1961), in his study of the cost of floating private debt in the 1930s, concludes that gross underwriting spreads for offerings of Aaa public utility bonds between 1935 and 1940 ranged from 1.65% to 2.01%. The discrepancy between this and the approprixmately 1% compensation for underwriting Treasury issues is easily explained by differences in risk.¹⁷

Additional evidence comes from looking at the identity of the initial purchasers of the Treasury's new offerings. During the 1930s, individuals in the Second Federal Reserve District, New York, were alloted over 50% of all new securities (either on subscription or exchange). It is natural to conclude that the banks and dealers in New York City, who dominate this Federal Reserve District, were being paid a fee to insure placement of the bonds.¹⁸

The impact of the legal constraints is also easy to demonstrate. Again, take the example of the $1\frac{5}{8}\%$ Note issued on March 15, 1935 and maturing 5 years later. As has already been noted, on March 16, 1935 the first bid for the new issue was $101\frac{5}{32}$. This implies a yield to maturity of 1.38%. During this period, there seems to have been a convention that all coupon rates were quoted in even $\frac{1}{8}$ ths.¹⁹ While the Treasury could have set the coupon rate at $1\frac{1}{2}\%$ and still sold the issue — the initial price would have been approximately $100\frac{19}{32}$ — this may not have been viewed as sufficient compensation for potential underwriters (brokers or individuals) to be willing to accept the risk associated with subscribing to this new issue.

Contemporary beliefs, as expressed in January 2, 1939 issue of *Barron's*, support this view. An article entitled 'Valuing of "Rights" in Treasury Notes' states, in part:

¹⁷The fact that issues were heavily oversubscribed suggests that the payment offered by the government exceeded the market clearing underwriting fee. But since everyone knew how the subscription procedure worked, there must have been substantial gaming involved in determining the subscription amounts. In fact, if every subscriber was indifferent about being alloted an additional bonds, the ratio of requests to allotments could have become arbitrarily large.

¹⁸This is similar to the underwriting mechanism of the 1950s described in Bloch (1963). Then, banks were allowed to buy new issues by simply crediting the Treasury's tax and loan account at that bank.

¹⁹The first coupon security that did not have a coupon rate that was a multiple of $\frac{1}{8}$ was a 0.90% Note issued on December 1, 1944.

At the present time, the Treasury is faced with the prospect of having to borrow substantial amounts of new money for some time to come. In addition, there is a large volume of short-term Treasury obligations that must be refunded during the next few years. Under these circumstances, [Treasury] Secretary [Henry] Morgenthau has apparently concluded that it is wise to make new United States Treasury issues unusually attractive to investors.

3.2 Computing the Corrected Yields

An estimate of the market value of the exchange privilege substantially prior to the maturity of a security is needed to correct the data for the value of the exchange privilege.²⁰ The effect of the exchange privilege is to raise the price of a bond above what it otherwise would be. An interpretation of this is that securities were trading as if their face value exceeded 100 by a 'bonus' representing the value of the exchange privilege. Once the bonus is estimated, the yield to the coupon bearing component of the composite security can be computed.

The realized value of the exchange privilege — computed by assuming that an investor holds a bond to maturity, makes the exchange and sells the new security on the day of issue — is of no use. As is clear from the previous discussion, the realized value is a biased estimate of the market's expectation, since it includes an underwriting spread. Fortunately, an arbitrage condition can be used to value the exchange privilege and correct the yield estimates.

All coupon bearing securities in the sample made payments at six month intervals. This means that all notes, bonds and certificates with less than six months to maturity were pure discount securities.²¹ Beginning in June of 1931, the Government regularly issued three month Treasury Bills. Arbitrage implies that the yield on a note with less than six months to maturity and a bill maturing on the same day must be the same. This provides a simple way of calculating the market (or implied) value of the exchange privilege. Three months or less prior to maturity, each coupon bearing security can be matched with a Treasury Bill maturing on the same day. The implied value of the exchange privilege is the difference between the traded

²⁰The same article in *Barron's* quoted above contains subjective estimates of the value of the exchange privilege that differ by small amounts from those computed here.

²¹The fact that interest on coupon bearing securities accrues linearly introduces a small error that is imperceptible at low interest rates.

price of the security and the price implied by the Treasury Bill rate, appropriately discounted.²²

To see how the computation is done, define P as the price quoted in the newspaper for a bond nearing maturity. An individual purchasing the bond must pay this price, plus accrued interest. Interest on government securities accrues linearly between coupon payments. Assume the bond pays a coupon C per year, or $\frac{1}{2}C$ every six months, and has m years to maturity. Since m is less than $\frac{1}{2}$ year (the bond has less than six months to maturity), the last coupon payment was $(\frac{1}{2} - m)$ years ago, and the accrued interest is $\frac{C}{2}(\frac{1}{2} - m)$. The price with accrued interest is just $P' = P + \frac{C}{2}(\frac{1}{2} - m)$. Arbitrage requires that the yield to holding this security equal the yield to holding a Treasury Bill maturity in m years, call this r. The implied value of the exchange privilege (Pr^e) , is calculated from the arbitrage relationship:²³

$$P' = \frac{100 + \frac{1}{2}C + Pr^e}{(1+r)^m} .$$
 (1)

The computation is very simple. Take the example of the $2\frac{1}{2}\%$ note maturing on March 15, 1935. On December 30, 1934, with two and one-half months to maturity, the closing quotation for the mean of the bid/ask spread was 101.19, so the actual price with accrued interest was $101.19 + 2.5(\frac{3.5}{12}) =$ 101.92. If calculated naively, this implies a nominal yield to maturity of -3.28% at an annual rate. On the same date, the Treasury Bills maturing on both March 7 and March 21, 1934 yielded 0.20% bid, but no ask is reported. Assuming a bid/ask spread of $\frac{2}{32}$'s indicates a mean bid/ask spread yield of 0.05%. The implied value of the exchange privilege is calculated as the face value that is consistent with a price of 101.92 and a yield of 0.05%:

$$101.92 = \frac{100 + \frac{2.5}{2} + Pr^e}{(1 + 0.0005)^{\frac{2.5}{12}}} .$$
 (2)

For this case, the value of Pr^e is 0.68. The bond is trading as if its face value were 100.68. As noted above, the bond could have been traded in for a new security selling for 101.16. So, while the realized value was 1.16, the implied or market expected value was only 0.68.

²²This ignores the tax distortions in the Treasury Bill data mentioned in the introduction, which are clearly small relative to the problem caused by the exchange privilege.

²³ Tax considerations do not effect this calculation, since both interest and capital gains on government securities were tax-exempt prior to 1941. See Section 4.1 for a partial discussion.

This procedure was employed for all coupon securities maturing between March 1932 and December 1944.²⁴ All of the estimates are based on the mean of the bid and ask price and maturity dates that match within three days. When an ask price was not available, one was computed from the bid assuming a bid/ask spread of $\frac{2}{32}$.²⁵ (It is worth noting that marking the coupon security to the Treasury Bill rate makes the information in the note or bond yield redundant. As such, the yield curves estimated in Section 5 do not utilize the coupon security yields at short maturities.)

A simple univariate regression can be used to summarize the relationship between the realized and the market expected value of the exchange premium. Assuming that the realized premium (Pr) equals the expected premium (Pr^e) plus an orthogonal error, the appropriate regression is

$$\begin{array}{rcl} Pr(i) &=& .461 &+& 1.032 & Pr^{e}(i) \\ && & (.161) && (.216) \end{array} \tag{3}$$

(Numbers in parentheses are standard errors) Mean of Pr = 1.036 Number of Observations = 65 \overline{R}^2 = .25.

As anticipated, the market implied premium is correlated with the realized value, but systematically underestimates it.²⁶ The evidence supports the hypothesis that the value of the exchange privilege was related to its function as an underwriting fee.

Once the implied market value of the exchange privilege is determined for every relevant coupon security, the yields can be recomputed. For each security, $Pr^{e}(i)$ is assumed to be an increment to the face value. The yield is recomputed for the *entire* lifetime of the note or bond assuming that the face value is $[100 + Pr^{e}(i)]$, not the usual 100. For example, in the case of

²⁴While the practice of allowing payment by exchange continued beyond 1944, the terms were no longer as favorable. Allotment was not guarantee, and so the value of the 'privilege' disappeared.

²⁵The results are not sensitive to the use of either the bid or ask in place of the midpoint of the spread — the estimated values of the exchange privilege change by less than 0.0001.

²⁸The comparison assumes that an individual cashes in the new security on the day it is issued. Prior to December of 1940, the capital gain from the sale of a note or certificate of deposit was nontaxable. If, however, the premium were taxable as a short term capital gain, this would provide another explanation for the difference between the realized and implied values in equation (3). For reasons that are described in the Section 4, it is only beginning in 1941 that the tax effects could have been significant. Examination of the data shows that the relationship between the realized and implied premium has no systematic difference over the two periods.

the $2\frac{1}{2}\%$ Note described above, the yield for every month from January 1934 to February 1935 was recomputed assuming the face value was 100.68.²⁷

Adopting this procedure entails making a very strong, but unavoidable assumption. For the entire lifetime of a bond, market participants are assumed to perfectly anticipate what the value of the exchange privilege will be when the security reaches three months to maturity. Since the prices of all government securities, except for Treasury Bills, were subject to the distortions of the exchange privilege, there is no other way of determining the implicit value of the coupon bearing component of a bond or note at any time other than when its maturity is less than three months. Since no other data are available, there is no other way to proceed.²⁸

Figure 2 plots the estimated yields corrected for the value of the exchange privilege for February of 1935. Again, P and N denote to differences in tax treatment and the solid line is a term structure using the techniques described in Section 5. For longer maturities, in excess of seven years or so, the data are nearly identical to the uncorrected yields plotted in Figure 1. But for the shorter maturities, below five years, the yields are now strictly positive and smoothly upward sloping. Futhermore, the three month Treasury Bill yield that is so much higher than the remainder of the yield curve in Figure 1 no longer stands out.

There is obviously more noise in the corrected data than in the raw data in Figure 1. Any plot of the yield to maturity against the time to maturity for coupon securities will only produce a smooth pattern, even in theory, if all the coupon rates are the same. This explains why, even at longer maturities, the figure reveals small vertical displacements. Matters are obviously worse at the shorter maturities. While this may be due to inaccuracies in the quoted prices,²⁹ some of the errors, are too large to be accounted for by anything but mismeasurement of the value of the exchange privilege. For

²⁷The fact that both the realized and the market expected value of exchange privilege were always strictly greater than zero, suggests that the option was always in the money. If this was the case, an options pricing model is not need to compute the decay in the value of the exchange privilege going back in time. The market implied value must decay at the rate of interest implying that the method used is correct.

²⁸ It is possible to examine the fluctuation in the market value of the exchange privilege associated with a given security. This is done by recomputing the value of Pr^e for the observations when the bond or note had less than three months to maturity. The results of this exercise show only small movements.

²⁹ Prices quotes are for the end of the day, but reflect information and revision at different times of the day. Because of the nonlinearity of the yield computation, price quotation errors cause larger errors in yields at at shorter maturities.





example, on December 31, 1938 the corrected yield to maturity for the $2\frac{1}{8}\%$ Note maturing on June 15, 1939 is estimated to be 1.025% (the uncorrected yield is -2.13%). At the same time, the T-Bill maturing on March 29, 1934 yielded 0.05%. A data point like this one is clearly visible on a scatter plot. The error is much to large to be the result of an error in a price quote that is incorrect by several 32nd's. Mismeasurement of the value of the exchange privilege is the clear source. It is important to keep in mind that the shorter the time to maturity, the larger these errors become. As such, in the following analysis *all* coupon bearing securities with maturity of less than six months are omitted.

Errors and all, the corrections for the exchange privilege are extremely important. They completely eliminate the existence of apparent negative nominal interest rates on coupon bearing securities and allow computation of yield curves for the 1930s.

4 Tax Considerations

Throughout the 1930s and 1940s, securities with different tax status coexisted. It is important, therefore, to take account of the tax treatment of interest payments in determining how to use the information contained in all of the data.

Changes in both individual and corporate taxes allow division of the 1929 to 1949 period into two distinct subperiods. The dividing point is January 1941. Prior to 1941, the interest income on nearly all U.S. Government securities was either partially or wholly tax exempt.³⁰ Beginning on March 1, 1941, all newly issued securities bore interest that was fully taxable. The interest on Treasury notes issued in December 1940 and January 1941 was fully taxable as well.

The remainder of this section describes the details of the tax treatment of interest payments to individuals and corporations during these two period. The conclusion is that for the 1941 to 1949 period it is possible to conform to the precedent set by the Salomon data and quote nominal interest rates of fully taxable securities on a before tax basis. From 1929 to 1940 all the interest on government debt was essentially exempt from taxation, and so the yields measure after tax returns.

³⁰Several issues prior to June of 1930 bore interest that was fully taxable to individuals, but wholly tax exempt for corporations. These will not be dealt with separately.

4.1 1929 to 1940

For the majority of the first twelve years of the sample, interest on Treasury Bills, Notes and Certificates of Indebtness was completely exempt from both personal and corporate taxes. Interest on Treasury bonds, however, was partially tax exempt. The meaning of the partial tax exemption depended on whether the bond was owned by an individual or a corporation. It is important to consider both cases.

For individuals, the interest on the first \$5000 face value of bonds held was exempt from income tax. Any additional interest was taxable at the 'surtax' rate. During this period, the individual income tax was composed of a 'normal' tax, which had at most two steps and a surtax that was graduated with as many as 55 steps. Today, the personal income tax simply sums the two. As has been the case in recent history, tax rates changed frequently. For the bulk of the period, the normal tax rate was 4 percent. The surtax rate ranged from 1% to 75% depending on income.³¹

For corporations, interest on all government obligations was fully tax exempt prior to 1934. From 1934 to 1940, the interest on the face value in excess of \$5000 was taxable at the 'declared value excess profits' tax rate, but was exempt from the normal corporate income tax. Income in excess of a certain percentage of the book value of a corporation's capital stock was subject to this tax. In 1934-35, the rate was 5% on profits in excess of 10% of book value. From 1934 to 1940, this form of the excess profits tax rose to 6% on profits between 10% and 15% of the book value of the capital stock, and equaled 12% of profits in excess of 15% of book value.

It is difficult to believe that the partial tax exemption had any value to individuals holding Treasury Bonds since the majority of the owners of the bonds almost certainly held amounts in excess of \$5000. Interest they received was, to a first approximation, fully taxable. For corporations, however, the tax exemption was very important.

The implication is that a proper comparison of partially and wholly tax exempt bonds requires some inference about the identity of the owner of the marginal bond. While it is impossible to determine the marginal participant in the securities market, information on the average participant can be gleaned from data on the ownership of the bonds and the interest income on which taxes were paid.

Data from tax returns show that the vast majority of the securities were owned by corporations. Individuals were required to report ownership of

³¹See Statistics of Income For 1950, Part 1, pg. 319 ff. for data on the surtax.

government debt. Tabulations of these returns in various issues of the *Statistics of Income* show that only 10% of the outstanding debt was owned individuals. Corporations, on the other hand, reported holding in excess of half of the debt.³²

Data from corporate income tax returns allow examination of the tax liability faced by corporations receiving interest payments from the U.S. Treasury. These data show that the declared value excess profits tax paid on interest was negligible. The marginal tax rate applicable to interest income faced by corporations was on the order of $\frac{1}{2}$ %.

The conclusion is that, between 1929 and 1940 partially tax exempt securities can be treated as if they were wholly tax exempt. Corporations owned the bulk of the securities and faced tax rates that were negligible. This classification allows construction of estimates of the term structure of nominal interest rates on tax exempt securities prior to World War II.

4.2 1941 to 1949

Beginning in 1941, the interest on all newly issued Treasury securities was treated as regular income to individuals and normal profits to corporations. For individuals, this did not represent a very large change, since the partial tax exemption of the previous years had very little impact. But for corporations, the tax status of these new issues made their yields substantially different. Not only was the interest on the taxable securities subject to the normal corporate profits tax and the declared value excess profits tax, but it was also subject to a new excess profits tax and numerous surtaxes during World War II.

To see the difference this made to a corporation, take 1941 as an example. Interest paid on partially tax exempt bonds was subject to the declared value added excess profits tax with a maximum rate that had risen to 13.2%, plus a surtax with a maximum rate of 7%. By contrast, interest from fully taxable securities was subject to both the normal corporate profits tax and an additional excess profits tax. The normal profits tax rate was 24% for corporations with net income in excess of \$38,500 and an excess profits tax. The new excess profits tax rates were very high. They began at 20% for income less than \$20,000 and increased to a maximum of 50% for income in excess of \$500,000. The highest tax rate on fully taxable interest income

³²The remaining 40% is are likely owned by entities that are neither domestic corporations nor individuals, i.e. foreigners, trusts and the government.

was nearly 95%.³³ This is substantially higher than the maximum rate of 20.2% on the interest from partially tax exempt bonds.

Throughout WWII, corporate and personal taxes remained very high. When the war ended, tax rates were revised. Individual taxes fell slightly in 1946, reflecting a change in the level of the normal tax from 6% to 3%. Corporate taxes, on the other hand, fell substantially when the excess profits tax was eliminated. As a result, the maximum corporate profits tax on income in excess of \$50,000 went down to 38%.

It is clear that, with the changes in tax law and the introduction of fully taxable U.S. Government securities, any comparison of yields must take tax status into account. Unlike the earlier period, securities that appeared to be different, now were. It is impossible, however, to determine what the effective after tax yields were given that the law changed frequently and that the data on the composition of ownership is not sufficiently detailed.

The conclusion is that data exist for estimation of a nontaxable term structure from 1929 to 1940 and a taxable term structure from 1941 to 1949. Unfortunately, any attempt to derive a consistent term structure, either taxable or tax exempt, for the entire period would require *ad hoc* adjustments to one period or the other.

5 Estimating the Term Structure

To estimate the term structure, it is necessary to fit a curve through the scatter of points similar to Figure 2 for each month of the sample. There is a large literature on estimating the term structure of interest rates.³⁴ What is needed here is a technique that provides a sufficiently broad set of alternative shapes, but is parsimonious in its parameterization. Considering that the early months have fewer than 15 data points apiece, it is important to use a method that requires estimation of the fewest parameters possible.

Nelson and Seigel (1985) derive a four parameter model that allows for humped, monotonic and S-shaped yield curves. Their specification, derived

³³ From 1941 to the end of 1945 both corporate and individual taxes were confiscatory. The highest surtax rate for individual rose from 75% in 1940 to 91% in 1945, while the highest marginal tax rate for corporations exceeded 95%.

³⁴Durand (1942, 1958) and Durand and Winn (1947) pioneered the field by drawing freehand curves through scatter diagrams. McCulloch (1971) and Shea (1985) provide examinations of analytical curve fitting techniques which use cubic splines and exponentials. Brown and Dybvig (1986) estimate yield curves using the model derived by Cox, Ingersoll and Ross (1985).

as the solution to a differential equation relating the forward rate to the time to maturity, is

$$R(m) = a + b \frac{m[1 - e^{-\frac{m}{\tau}}]}{\tau} + c e^{-\frac{m}{\tau}} , \qquad (4)$$

where R(m) is the yield to maturity m, and a, b, c and τ are parameters. While Nelson and Seigel apply (4) to data on pure discount securities, here it is used as an approximation for securities regardless of whether they are coupon bearing or not.

It would be preferable to use a technique that accounts for the fact that securities with different coupon rates and the same maturity date are expected to have different yields. But methods such as those in McCulloch (1971) or Brown and Dybvig (1986), require large amounts of data. The errors that are introduced by ignoring the differences in coupon rates are clearly small relative to the corrections made for the exchange privilege.³⁵

Equation (4) was modified to take into account the tax differences described in Section 4. For the 1941 to 1949 period a multiplicative constant was estimated to allow yields on securities with different tax status to differ systematically. The following specification was used:

$$R(m) = (1 + \alpha_p D_p + \alpha_t D_t) f(m, \theta) , \qquad (5)$$

where

 $D_p = 1$ if the security is partially tax exempt and 0 otherwise, $D_t = 1$ if the security is fully taxable and 0 otherwise,

 $f(m, \theta)$ is the Nelson and Seigel function in (4), and the α 's are parameters that measure the difference between either partially tax exempt or fully taxable securities and nontaxable ones.³⁶

For each month from January 1929 to December 1949, estimation of equation (5) proceeded as follows. First, because of the inaccuracies in the

³⁵A comparison of Figures 1 and 2 shows that coupon rate differences cause errors on the order of 0.1 percentage points, while adjustment for the exchange privilege increases measured yields by amounts in excess of a full percentage point.

³⁶An important problem in more recent yield curve estimation does not arise here. McCulloch (1975) discusses how, for fully taxable securities selling below par, the differential tax treatment of the principle appreciation and the coupon payment can produce misleading results. But for the period under study, bonds sold almost exclusively above par.

procedure for valuing the exchange privilege, all coupon securities with less than six months to maturity were omitted.³⁷

From 1934 to 1945, the Treasury Bill rate was usually below 0.25%. As such, the yield curves came very close to zero at short maturities. If left unconstrained, estimates of the yields at three months to maturity were occasionally negative. The solution is to force the estimates to go exactly through the Treasury Bill rate at a maturity of three months. The constraint is imposed by restricting the value of the constant term a in equation (4).

Finally, as suggested by Nelson and Seigel, estimation was conditional on the parameter τ . Plots of the data show that the yield curve becomes flat at longer maturities, suggesting that τ should not be in a range above 200, and so a search was done over a grid from 10 to 250 in increments of 10. The final estimate minimized the sum of squared residuals over this range.³⁸

The results are yield curve estimates for each month. For February 1935, the fitted values are plotted as the solid line in Figure 2. As can be seen from the figure, the line fits fairly well for maturities contained in the data set. In fact, the fitted values account for 90% of the variation in the data in over 200 of the 252 months. Extrapolation to maturities longer than existing securities can be misleading, however. The fitted values turn down at longer maturities, while the scatter plot shows no signs of a downward slope. This implies that the estimates are likely to be unreliable at maturities past twenty years, and have only limited accuracy past fifteen years.

The appendix reports estimates of nominal interest rates at maturities from three months to twenty years, monthly from January 1929 to December 1949. The data are for the last trading day of each month. The full data set is available from the author on standard diskettes.

³⁷Both because of their call provisions and their tax status, the $3\frac{1}{2}\%$ Treasury Notes of March, September and December of 1930-32 were also omitted. These notes were fully taxable and tended to fall on the yield curve when the maturity date was assumed to be the final redemption date. Since they were actually called during 1930 and 1931, it was unclear how to differentiate between the value of the call provision and the value of the coupon payments. In addition, after December 1930, all Liberty Bonds were omitted. These were issues used to finance the World War I that contained provisions that allowed them to be called beginning in 1932. They are the only bonds in the sample that were not called on the first date allowed.

³⁸It was not possible to estimate τ by simple nonlinear least squares. For a number of months, the estimate of τ grew too large. As τ grows, $e^{-\frac{m}{\tau}}$ goes to zero and c in (4) cannot be estimated.

6 Concluding Remarks

The mystery of negative nominal interest rates has been solved. The legal and economic environment of the 1930s restricted the method in which the Treasury issued and refunded coupon bearing securities. The Treasury was required by law to issue new bonds at par, and to insure that an offering sold, coupon rates were set so that initial market prices exceeded par. In this way individuals and brokers were paid an underwriting fee to place the new securities. Since holders of maturing securities were given preference in the distribution of a new issue, the quoted prices reflected the value of an exchange privilege — the option to hold the bond or note to maturity and roll it over into a new security. The increase in the price was large enough that the yield, computed in the standard way, appeared negative. Adjustment for this distortion in the price allows recomputation of the yield to the coupon bearing component of the composite bond/option.

Taking account of the value of the exchange privilege is obviously important. Any comparison of nominal interest rates with and without the adjustment shows systematic differences. Figure 3, for example, plots the Federal Reserve Board's series entitled '3- to 5-year tax exempt Treasury notes', from *Banking and Monetary Statistics* Table 122 against the new estimates for tax exempt yields on U.S. Government securities with four years to maturity. Figure 4 compares the new ten year to maturity estimates with the FRB series for 'U.S. Government' bonds.

Both plots show striking differences. As one would expect, the FRB medium term series is systematically too low, since it fails to account for the value of the exchange privilege. The new four year estimates are on average 0.27 percentage points, or 30%, higher. This represents a revision in the level of not only the nominal interest rate for this period, but an increase in the estimate of the real interest rate as well.

Differences are also apparent in comparing the old and new series for longer term yields. This time the FRB series is higher than the new ten year series from 1935 through 1940. Over the entire period, the average level of the FRB series is 0.16 percentage points higher than the ten year series. In addition, the old series is too stable, with a standard deviation 0.22 percentage points below the new ten year estimates. Examination of new series at longer maturities shows that the FRB bond data is close to the new estimates at fifteen years to maturity.

The usefulness of these new data is without question. Much of the argument over the casues of the length and depth of the Great Depression turns

FIGURE 3 Comparison of FRB 3–5's and New 4 Year Estimates, 1933 to 1940



Dash: New 4 year estimates, tax-exempt Solid: 3-5 year tax-exempt FIGURE 4 Comparison of FRB Government Bond and New 10 Year Estimates, 1929 to 1940



Dash: New 10 year estimates, tax-exempt Solid: FRB U.S. Government Bonds on attempts to interpret movements in interest rates. The new data will allow detailed study of a type that could not have previously been undertaken. In particular, they can be used to examine movements in the slope of the term structure and shifts in the spread between corporate and U.S. Government bond yields in the crucial period from 1929 to 1933. Hopefully, this will allow investigators to differentiate among the various theories for the causes of the most severe economic downturn of the twentieth century.

Appendix

The following table contains the constant maturity nominal term structure estimated using the Nelson and Seigel specification described in Section 5. From 1929 to 1940, the estimates are for nominal yields on wholly tax exempt securities. From 1941 to 1949 the estimates are for nominal yields on fully taxable securities. All data refer to the last trading day of the month.

TABLE A.1

Constant Maturity Yield Curve, 1929 to 1949

Time to Maturity										
Month	3mon	1yr	2yr	3yr	4yr	5yr	10yr	15yr	20yr	
				N	ontaxal	ole				
129	4.72	4.55	4.37	4.22	4.09	3.99	3.69	3.57	3.51	
229	4.75	4.62	4.46	4.32	4.20	4.10	3.77	3.66	3.70	
329	5.03	4.89	4.73	4.58	4.45	4.33	3.92	3.76	3.79	
429	5.09	4.86	4.59	4.38	4.21	4.07	3.69	3.59	3.58	
529	5.19	5.04	4.86	4.70	4.55	4.42	3.95	3.73	3.71	
629	4.86	4.76	4.64	4.53	4.42	4.33	3.94	3.69	3.54	
729	4.94	4.83	4.69	4.56	4.44	4.33	3.87	3.55	3.35	
829	4.69	4.61	4.51	4.42	4.33	4.25	3.93	3.72	3.60	
929	4.89	4.80	4.69	4.58	4.48	4.39	4.00	3.71	3.50	
1029	4.00	4.37	4.47	4.44	4.36	4.28	3.84	3.40	2.96	
1129	3.01	3.59	3.83	3.87	3.84	3.80	3.57	3.34	3.10	
1229	2.82	3.43	3.69	3.75	3.74	3.72	3.57	3.41	3.26	

				\mathbf{Time}	to Ma	aturity			
Month	3mon	1 yr	2yr	3yr	4yr	5yr	10 yr	15yr	20yr
				_					
130	3.84	3.90	3.90	3.87	3.84	3.81	3.64	3.47	3.31
230	3.04	3.52	3.71	3.75	3.74	3.71	3.55	3.39	3.23
330	2.71	3.22	3.45	3.51	3.51	3.50	3.41	3.33	3.24
430	2.96	3.35	3.53	3.57	3.57	3.56	3.48	3.40	3.31
530	2.07	2.94	3.34	3.45	3.46	3.46	3.36	3.27	3.17
630	1.78	2.43	2.93	3.19	3.32	3.38	3.35	3.23	3.11
730	1.54	2.53	3.01	3.16	3.20	3.22	3.24	3.25	3.26
830	2.09	2.49	2.85	3.07	3.21	3.30	3.35	3.25	3.12
930	1.42	2.36	2.83	2.98	3.03	3.06	3.14	3.21	3.28
1030	1.76	2.28	2.70	2.93	3.05	3.12	3.19	3.19	3.19
1130	1.53	1.97	2.39	2.67	2.85	2.98	3.22	3.27	3.30
1230	1.25	1.93	2.48	2.79	2.96	3.05	3.20	3.24	3.28
131	1.10	1.59	2.06	2.39	2.63	2.79	3.18	3.36	3.50
231	1.50	1.72	2.00	2.24	2.45	2.64	3.24	3.38	3.19
331	1.33	2.01	2.55	2.85	3.01	3.11	3.24	3.28	3.31
431	1.20	1.87	2.40	2.71	2.88	2.98	3.16	3.23	3.30
531	0.65	1.17	1.70	2.09	2.38	2.60	3.06	3.16	3.17
631	0.60	1.05	1.53	1.92	2.23	2.47	3.07	3.15	3.06
731	0.35	0.78	1.28	1.70	2.04	2.32	3.07	3.18	2.98
831	0.40	1.31	2.04	2.45	2.69	2.83	3.08	3.19	3.29
931	1.37	1.67	2.02	2.32	2.57	2.79	3.40	3.45	3.16
1031	2.50	2.68	2.90	3.09	3.25	3.40	3.81	3.83	3.56
1131	2.40	2.58	2.79	2.98	3.14	3.28	3.72	3.78	3.57
1231	2.75	3.80	4.28	4.41	4.43	4.41	4.28	4.15	4.01
132	2.30	3.91	4.64	4.83	4.85	4.82	4.60	4.37	4.14
232	2.37	3.65	4.23	4.38	4.39	4.36	4.16	3.95	3.73
332	1.75	3.35	4.08	4.28	4.31	4.29	4.13	3.95	3.77
432	0.55	1.76	2.70	3.20	3.47	3.60	3.67	3.57	3.46
532	0.25	1.34	2.35	3.01	3.44	3.71	4.06	3.95	3.78
632	0.25	1.30	2.27	2.90	3.30	3.55	3.84	3.69	3.49
732	0.25	1.42	2.35	2.85	3.13	3.29	3.46	3.47	3.46
832	0.20	1.39	2.33	2.85	3.13	3.28	3.44	3.43	3.42
932	0.10	1.03	1.90	2.47	2.85	3.09	3.43	3.38	3.28
1032	0.10	0.89	1.69	2.27	2.68	2.97	3.49	3.45	3.29
1132	0.05	0.86	1.68	2.26	2.68	2.97	3.48	3.41	3.22
1232	0.05	0.80	1.55	2.10	2.49	2.77	3.29	3.28	3.15

Time to Maturity									
Month	3mon	1yr	2yr	3yr	4yr	5yr	10yr	15 yr	20yr
133	0.05	0.73	1.43	1.97	2.37	2.67	3.26	3.20	2.96
233	0.63	1.74	2.62	3.09	3.34	3.47	3.56	3.49	3.42
333	1.00	1.91	2.62	3.01	3.22	3.33	3.42	3.39	3.34
433	0.35	1.28	2.14	2.70	3.06	3.28	3.54	3.40	3.22
533	0.20	1.06	1.85	2.38	2.72	2.95	3.28	3.25	3.17
633	0.15	0.91	1.67	2.21	2.59	2.86	3.29	3.18	2.95
733	0.25	1.09	1.87	2.38	2.72	2.94	3.26	3.22	3.14
833	0.12	0.86	1.60	2.14	2.52	2.78	3.25	3.19	3.01
933	0.05	0.83	1.61	2.17	2.57	2.85	3.31	3.22	3.01
1033	0.15	1.04	1.87	2.42	2.77	3.00	3.31	3.25	3.13
1133	0.35	1.61	2.60	3.12	3.39	3.53	3.57	3.43	3.29
1233	0.50	2.26	3.11	3.37	3.45	3.48	3.52	3.55	3.58
134	0.60	2.14	2.89	3.13	3.22	3.26	3.35	3.44	3.53
234	0.15	1.34	2.28	2.79	3.07	3.21	3.34	3.29	3.24
334	0.15	0.90	1.65	2.18	2.56	2.82	3.24	3.13	2.91
434	0.15	0.80	1.47	1.95	2.30	2.56	3.08	3.12	3.07
534	0.15	0.67	1.24	1.69	2.06	2.34	3.00	2.99	2.72
634	0.15	0.60	1.12	1.54	1.90	2.18	2.91	2.95	2.67
734	0.15	0.60	1.12	1.55	1.91	2.20	2.94	2.94	2.58
834	0.20	0.76	1.36	1.83	2.19	2.47	3.07	3.04	2.78
934	0.25	1.08	1.85	2.37	2.70	2.92	3.25	3.23	3.16
1034	0.25	0.90	1.55	2.03	2.37	2.61	3.07	3.06	2.95
1134	0.25	0.90	1.55	2.02	2.36	2.60	3.05	3.04	2.92
1234	0.20	0.71	1.26	1.71	2.07	2.35	2.98	2.96	2.68
135	0.20	0.63	1.12	1.53	1.87	2.15	2.84	2.83	2.48
235	0.15	0.51	0.94	1.32	1.64	1.93	2.73	2.76	2.24
335	0.15	0.48	0.87	1.22	1.53	1.80	2.65	2.87	2.67
435	0.15	0.47	0.85	1.19	1.50	1.77	2.64	2.88	2.66
535	0.15	0.49	0.89	1.23	1.53	1.79	2.55	2.74	2.59
635	0.15	0.45	0.81	1.13	1.42	1.67	2.51	2.78	2.62
735	0.15	0.44	0.79	1.11	1.39	1.65	2.50	2.80	2.69
835	0.20	0.55	0.96	1.32	1.64	1.91	2.73	2.84	2.48
935	0.20	0.61	1.08	1.47	1.79	2.06	2.77	2.92	2.81
1035	0.20	0.56	0.98	1.34	1.65	1.91	2.70	2.90	2.77
1135	0.15	0.52	0.96	1.33	1.64	1.91	2.71	2.90	2.79
1235	0.15	0.48	0.87	1.22	1.52	1.79	2.65	2.89	2.74

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	Time to Maturity								
Month	3mon	1yr	2yr	3yr	4yr	5yr	10yr	15yr	20yr
		-							
136	0.20	0.52	0.90	1.23	1.53	1.79	2.63	2.88	2.72
236	0.20	0.49	0.83	1.15	1.43	1.67	2.51	2.81	2.71
336	0.20	0.48	0.82	1.13	1.40	1.64	2.46	2.74	2.65
436	0.20	0.48	0.82	1.12	1.39	1.63	2.45	2.73	2.62
536	0.20	0.49	0.83	1.13	1.40	1.64	2.40	2.67	2.63
636	0.15	0.48	0.86	1.19	1.46	1.70	2.40	2.67	2.73
736	0.15	0.49	0.88	1.20	1.48	1.70	2.38	2.64	2.73
836	0.20	0.50	0.84	1.14	1.40	1.62	2.31	2.58	2.66
936	0.18	0.51	0.89	1.20	1.46	1.68	2.31	2.58	2.71
1036	0.10	0.50	0.93	1.26	1.52	1.72	2.28	2.56	2.76
1136	0.11	0.44	0.80	1.08	1.31	1.50	2.08	2.41	2.69
1236	0.20	0.66	1.06	1.33	1.51	1.64	2.06	2.40	2.74
107	0.00	0 70	1 1 7	1 45	1 40	1 = 0	0.14	a 4 a	0 70
137	0.22	0.73	1.17	1.45	1.63	1.76	2.14	2.43	2.72
237	0.25	0.78	1.23	1.51	1.69	1.81	2.16	2.42	2.67
337	0.60	1.10	1.64	1.93	2.11	2.24	2.56	2.80	3.03
437	0.00	1.07	1.52	1.84	2.06	2.23	2.62	2.80	2.96
537 697	0.45	1.20	1.63	1.81	1.92	2.01	2.39	2.77	3.15
037 797	0.39	1.20	1.73	1.92	2.03	2.11	2.40	2.80	3.14
(3)	0.32	1.14	1.00	1.79	1.89	1.98	2.34	2.69	3.05
837 027	0.42	1.33	1.82	2.01	2.11	2.18	2.49	2.79	3.08
937	0.20	1.10	1.04	1.80	1.90	2.05	2.43	2.80	3.17
1037	0.23	1.11	1.59	1.80	1.92	2.01	2.40	2.80	3.19
1137	0.12	1.03	1.53	1.(4	1.80	1.95	2.34	2.73	3.12
1237	0.12	0.92	1.37	1.58	1.(1	1.81	2.27	2.72	3.17
138	0.14	0.90	1.34	1.54	1.66	1.76	2.22	2.67	3.12
238	0.12	0.82	1.23	1.43	1.57	1.67	2.17	2.67	3.16
338	0.12	0.88	1.32	1.53	1.66	1.77	2.26	2.75	3.24
438	0. 08	0.67	1.05	1.24	1.38	1.50	2.05	2.59	3.14
538	0.08	0.37	0.71	0.99	1.24	1.45	2.13	2.50	2.75
638	0.08	0.31	0.60	0.87	1.11	1.33	2.12	2.52	2.63
738	0.08	0.35	0.67	0.95	1.18	1.39	2.08	2.45	2.68
838	0. 08	0.61	0.96	1.14	1.28	1.39	1.94	2.49	3.03
938	0.12	0.41	0.74	1.02	1.26	1.46	2.13	2.48	2.71
1038	0.05	0.32	0.63	0.91	1.14	1.35	2.06	2.43	2.64
1138	0.05	0.32	0.64	0.92	1.16	1.38	2.11	2.48	2.68
1238	0.05	0.28	0.56	0.81	1.04	1.26	2.04	2.45	2.59

Time to Maturity									
Month	3mon	1 yr	2yr	3yr	4yr	5yr	10yr	15yr	20yr
139	0.01	0.24	0.52	0.78	1.01	1.22	2.01	2.42	2.56
239	0.05	0.26	0.51	0.75	0.96	1.16	1.92	2.35	2.55
339	0.05	0.24	0.48	0.69	0.90	1.08	1.81	2.26	2.50
439	0.05	0.23	0.46	0.67	0.87	1.05	1.76	2.20	2.44
539	0.05	0.21	0.42	0.61	0.79	0.95	1.62	2.05	2.32
439	0.05	0.42	0.68	0.84	0.95	1.06	1.55	2.04	2.54
739	0.05	0.40	0.65	0.79	0.91	1.01	1.49	1.96	2.44
839	0.10	0.64	0.96	1.12	1.23	1.32	1.73	2.14	2.55
939	0.10	0.49	0.90	1.23	1.48	1.69	2.27	2.58	2.81
1039	0.05	0.29	0.58	0.84	1.07	1.28	2.00	2.38	2.56
1139	0.05	0.25	0.50	0.72	0.93	1.13	1.85	2.27	2.45
1239	0.05	0.22	0.43	0.63	0.82	0.99	1.70	2.18	2.49
140	0.05	0.22	0.44	0.65	0.84	1.01	1.73	2.21	2.51
240	0.05	0.22	0.43	0.63	0.82	0.99	1.71	2.21	2.54
340	0.05	0.20	0.39	0.57	0.74	0.90	1.57	2.07	2.43
440	0.05	0.20	0.40	0.59	0.76	0.92	1.60	2.10	2.45
540	0.08	0.65	1.00	1.17	1.29	1.40	1.87	2.34	2.82
640	0.08	0.43	0.69	0.85	0.97	1.08	1.62	2.15	2.68
740	0.06	0.43	0.69	0.85	0.97	1.08	1.61	2.13	2.65
840	0.06	0.23	0.44	0.63	0.82	0.99	1.69	2.17	2.49
940	0.06	0.21	0.40	0.58	0.75	0.92	1.59	2.10	2.46
1040	0.06	0.20	0.38	0.55	0.71	0.86	1.53	2.07	2.49
1140	0.06	0.16	0.30	0.44	0.58	0.72	1.37	1.94	2.46
1240	0.06	0.16	0.31	0.45	0.58	0.72	1.35	1.90	2.39

Time to Maturity

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$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$					Time	to Ma	aturity	,		
Taxable	Month	3mon	1yr	2yr	3yr	4yr	5yr	10yr	15yr	20yr
						Taxabl	e			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	141	0.09	0.26	0.48	0 69	0.88	1.06	1.82	2 36	2 73
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	241	0.08	0.24	0.44	0.62	0.00	0.95	1.56	1 03	2.10
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	341	0.15	0.34	0.57	0.78	0.98	1 17	1.89	2.35	2.10
5410.130.310.540.750.941.131.872.382.706410.130.290.500.690.881.051.762.262.607410.130.280.470.650.820.991.672.172.548410.130.290.490.690.861.031.722.212.549410.130.300.510.710.901.071.752.192.4610410.130.320.560.770.961.131.742.072.2511410.350.500.690.861.021.161.732.082.2712410.350.570.821.031.221.371.892.172.361420.250.480.761.001.211.391.992.282.402420.300.510.760.991.191.372.002.322.434420.350.570.831.071.271.452.062.352.453420.300.510.760.991.191.372.002.322.434420.350.650.931.191.372.002.322.434420.350.620.931.191.401.582.102.332.444220.350.660.941.201.411.592.122.34	441	0.13	0.31	0.53	0.74	0.94	1.12	1.86	2.38	2.00
641 0.13 0.29 0.50 0.69 0.88 1.05 1.76 2.26 2.60 741 0.13 0.28 0.47 0.65 0.82 0.99 1.67 2.17 2.54 841 0.13 0.29 0.49 0.69 0.86 1.03 1.72 2.21 2.54 941 0.13 0.30 0.51 0.71 0.90 1.07 1.75 2.19 2.46 1041 0.13 0.32 0.56 0.77 0.96 1.13 1.74 2.07 2.25 1141 0.35 0.50 0.69 0.86 1.02 1.16 1.73 2.08 2.27 1241 0.35 0.57 0.82 1.03 1.22 1.37 1.89 2.17 2.36 142 0.25 0.48 0.76 1.00 1.21 1.39 1.99 2.28 2.40 242 0.30 0.53 0.80 1.04 1.25 1.44 2.06 2.35 2.45 342 0.30 0.51 0.76 0.99 1.19 1.37 2.00 2.32 2.43 442 0.35 0.57 0.83 1.07 1.27 1.45 2.06 2.35 2.46 542 0.38 0.61 0.88 1.11 1.31 1.48 2.03 2.28 2.40 642 0.34 0.60 0.90 1.14 1.34 1.52 2.04 2.27	541	0.13	0.31	0.54	0.75	0.94	1.13	1.87	2.38	2 70
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	641	0.13	0.29	0.50	0.69	0.88	1.05	1.76	2.26	2.60
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	741	0.13	0.28	0.47	0.65	0.82	0.99	1.67	2.17	2.54
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	841	0.13	0.29	0.49	0.69	0.86	1.03	1.72	2.21	2.54
$ \begin{array}{ccccccccccccccccccccccccccccccc$	941	0.13	0.30	0.51	0.71	0.90	1.07	1.75	2.19	2.46
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1041	0.13	0.32	0.56	0.77	0.96	1.13	1.74	2.07	2.25
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1141	0.35	0.50	0.69	0.86	1.02	1.16	1.73	2.08	2.27
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	1241	0.35	0.57	0.82	1.03	1.22	1.37	1.89	2.17	2.36
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	142	0.25	0.48	0.76	1.00	1.21	1.39	1.99	2.28	2.40
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	242	0.30	0.53	0.80	1.04	1.25	1.44	2.06	2.35	2.45
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	342	0.30	0.51	0.76	0.99	1.19	1.37	2.00	2.32	2.43
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	442	0.35	0.57	0.83	1.07	1.27	1.45	2.06	2.35	2.46
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	542	0.38	0.61	0.88	1.11	1.31	1.48	2.03	2.28	2.40
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	642	0.34	0.60	0.90	1.14	1.34	1.52	2.04	2.27	2.40
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	742	0.35	0.62	0.93	1.19	1.40	1.58	2.10	2.33	2.44
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	842	0.35	0.63	0.94	1.20	1.41	1.59	2.12	2.34	2.44
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	942	0.35	0.65	0.98	1.24	1.46	1.63	2.12	2.34	2.46
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1042	0.35	0.63	0.94	1.20	1.42	1.60	2.13	2.35	2.46
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1142	0.35	0.64	0.96	1.23	1.45	1.63	2.16	2.38	2.49
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	1242	0.32	0.63	0.97	1.24	1.46	1.63	2.14	2.35	2.47
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	143	0.32	0.59	0.89	1.15	1.37	1.55	2.12	2.36	2.46
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	243	0.32	0.59	0.90	1.16	1.38	1.57	2.13	2.37	2.47
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	343	0.35	0.64	0.97	1.23	1.44	1.62	2.11	2.34	2.47
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	443	0.35	0. 65	0.98	1.24	1.45	1.63	2.12	2.33	2.45
$ \begin{array}{ccccccccccccccccccccccccc$	543	0.35	0. 63	0.94	1.19	1.40	1.57	2.06	2.28	2.43
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	643	0.35	0.61	0.90	1.15	1.36	1.53	2.06	2.30	2.44
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	743	0.35	0.61	0.91	1.16	1.37	1.55	2.08	2.32	2.45
9430.350.660.981.241.441.602.052.282.4510430.350.661.001.261.461.622.072.302.4711430.350.630.951.221.441.622.162.382.4912430.350.650.971.231.451.622.122.352.48	843	0.35	0.63	0.95	1.21	1.41	1.58	2.08	2.31	2.46
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	943	0.35	0.66	0.98	1.24	1.44	1.60	2.05	2.28	2.45
11430.350.630.951.221.441.622.162.382.4912430.350.650.971.231.451.622.122.352.48	1043	0.35	0.66	1.00	1.26	1.46	1.62	2.07	2.30	2.47
1243 0.35 0.65 0.97 1.23 1.45 1.62 2.12 2.35 2.48	1143	0.35	0.63	0.95	1.22	1.44	1.62	2.16	2.38	2.49
	1243	0.35	0.65	0.97	1.23	1.45	1.62	2.12	2.35	2.48

Time to Maturity									
Month	3mon	1 yr	2yr	3yr	4yr	5yr	10yr	15 yr	20yr
						·			
144	0.20	0.65	0.00	1.96	1 47	1.64	2 10	0 20	9 40
144	0.32	0.00	0.99	1.20	1.47	1.04	2.10	2.02	2.43
244	0.32	0.04	0.99	1.20	1.40	1.05	2.03	2.02	2.40
044 444	0.00	0.04	0.97	1.20	1.44	1.01	2.12	2.00	2.43
444 544	0.30	0.02	0.93	1.19	1.40	1.00	2.10	2.40	2.00
044 644	0.30	0.02	0.93	1.20	1.42	1.00	2.17	2.40	2.50
044	0.30	0.00	0.90	1.24	1.40	1.00	2.14	2.30	2.00
(44	0.30	0.00	0.90	1.21 1 17	1.40	1.01	2.10	2.00	2.00
844	0.30	0.01	0.91	1.17	1.09	1.07	2.10	2.40	2.49
944	0.30	0.02	0.93	1.19	1.41	1.00	2.10	2.39 9.26	2.49
1044	0.30	0.00	0.90	1.24	1.40	1.00	2.14	2.30	2.00
1144	0.35	0.07	1.02	1.20	1.49	1.00	2.11	4.00 0.20	2.00
1244	0.35	0.67	1.02	1.28	1.49	1.05	2.11	2.32	2.40
145	0.35	0.60	0.90	1 14	1.35	1.53	2.07	2.31	2.42
945	0.35	0.00	0.00	1 13	1 33	1.49	1.98	2.23	2.40
240	0.35	0.01	0.80	1 13	1.32	1.48	1.98	2.24	2.41
115	0.00	0.01	0.00	1 10	1.02	1 40	1.83	2.13	2.41
545	0.35	0.01	1 00	1 22	1.20	1 39	1.75	2.10	2.45
645	0.00	0.01	1.05	1 18	1.02	1.35	1 70	2.05	2.41
745	0.35	0.10	1.00	1.10	1 21	1.38	1.73	2.00	2.41
245 945	0.35	0.81	1.00	1.22	1 22	1 40	1 74	2.01	2.41
045	0.30	0.02	1.10	1.24	1 21	1 30	1 73	2.00	2.11
940	0.30	0.01	1.03	1 10	1.01	1.05	1 70	2.01	2.12
1040	0.00	0.79	1.00	1.15	1.20	1.00	1.66	2.01	2.00
1945	0.00	0.74	0.00	1.10	1.24	1.02	1.00	1 08	2.00
1240	0.55	0.74	0.99	1.12	1.20	1.20	1.00	1.50	2.02
146	0.35	0.69	0.91	1.03	1.11	1.19	1.53	1.87	2.22
246	0.35	0.65	0.85	0.96	1.05	1.12	1.47	1.82	2.16
346	0.35	0.71	0.93	1.04	1.12	1.19	1.50	1.81	2.12
446	0.35	0.79	1.05	1.18	1.26	1.32	1.63	1.92	2.22
546	0.35	0.80	1.06	1.18	1.27	1.33	1.64	1.94	2.24
646	0.35	0.75	1.00	1.12	1.21	1.28	1.60	1.91	2.23
746	0.35	0.78	1.04	1.17	1.25	1.32	1.65	1.97	2.30
846	0.35	0.81	1.09	1.22	1.30	1.37	1.69	2.01	2.33
946	0.35	0.86	1.16	1.29	1.38	1.44	1.75	2.06	2.37
1046	0.35	0.85	1.13	1.27	1.35	1.42	1.72	2.03	2.33
1146	0.35	0.90	1.21	1.35	1.43	1.50	1.79	2.08	2.37
1246	0.35	0.86	1.15	1.28	1.36	1.43	1.72	2.02	2.31

Time to Maturity									
Month	3mon	1 yr	2yr	3yr	4yr	5yr	10yr	15yr	20yr
147	0.35	0.84	1.12	1.25	1.33	1.40	1.71	2.01	2 31
247	0.35	0.84	1.12	1.25	1.34	1.40	1.70	2.00	2.30
347	0.35	0.81	1.09	1.21	1.30	1.37	1.68	1.99	2.30
447	0.35	0.86	1.15	1.28	1.37	1.43	1.72	2.01	2.30
547	0.35	0.86	1.15	1.28	1.36	1.43	1.72	2.01	2.29
647	0.35	0.87	1.17	1.31	1.39	1.46	1.76	2.05	2.35
747	0.70	1.00	1.20	1.30	1.38	1.44	1.75	2.05	2.36
847	0.73	0.98	1.15	1.25	1.33	1.40	1.71	2.03	2.34
947	0.78	1.02	1.18	1.28	1.35	1.42	1.74	2.05	2.37
1047	0.85	1.13	1.31	1.41	1.49	1.55	1.85	2.14	2.44
1147	0.91	1.12	1.32	1.46	1.57	1.65	1.96	2.22	2.49
1247	0.92	1.10	1.30	1.48	1.64	1.77	2.20	2.40	2.47
148	0.94	1.11	1.31	1.49	1.64	1.77	2.20	2.40	2 48
248	0.96	1.12	1.30	1.47	1.62	1.75	2.19	2.40	2.10 2.47
348	0.99	1.14	1.31	1.47	1.61	1.73	2.16	2.38	2.48
448	0.96	1.12	1.31	1.47	1.61	1.73	2.14	2.36	2.50
548	0.96	1.08	1.23	1.37	1.49	1.61	2.05	2.31	2.44
648	0.97	1.13	1.32	1.48	1.62	1.74	2.15	2.37	2.50
748	0.96	1.14	1.34	1.51	1.66	1.79	2.20	2.39	2.50
848	1.04	1.20	1.39	1.55	1.70	1.82	2.22	2.41	2.48
948	1.08	1.24	1.43	1.59	1.73	1.85	2.23	2.41	2.49
1048	1.08	1.27	1.47	1.64	1.78	1.89	2.23	2.40	2.51
1148	1.08	1.24	1.43	1.59	1.72	1.83	2.20	2.39	2.51
1248	1.10	1.23	1.39	1.53	1.65	1.76	2.16	2.38	2.48
149	1.11	1.22	1.36	1.49	1.60	1.71	2.12	2 35	2 46
249	1.11	1.22	1.36	1.49	1.60	1.71	2.11	2.33	2.43
349	1.11	1.21	1.34	1.46	1.57	1.67	2.07	2.31	2 45
449	1.10	1.20	1.33	1.44	1.55	1.65	2.06	2.32	2.47
549	1.10	1.19	1.31	1.43	1.53	1.63	2.03	2.31	2.50
649	1.02	1.11	1.22	1.33	1.43	1.53	1.93	2.23	2.45
749	0.98	1.07	1.18	1.29	1.39	1.49	1.90	2.20	2.42
849	1.01	1.07	1.16	1.25	1.33	1.42	1.81	2.15	2.45
949	1.00	1.08	1.18	1.28	1.37	1.46	1.84	2.15	2.39
1049	1.02	1.10	1.20	1.29	1.38	1.47	1.85	2.15	2.38
1149	1.06	1.13	1.22	1.31	1.40	1.48	1.84	2.14	2.38
1249	1.05	1.11	1.19	1.27	1.35	1.43	1.79	2.11	2.39

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