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LENDING TO AN
INSECURE SOVEREIGN

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Lending to an Insecure Sovereign

ABSTRACT

This paper analyzes a reputational equilibrium for sovereign debt in a model in which the sovereign borrows to finance spending for defense against threats to its survival in power. In this model, the amount of sovereign debt and defense spending, the resulting survival probability, and the sovereign's implied discount rate for future consumption are determined simultaneously. The optimal amount of debt and defense spending equates the marginal cost of defense spending in reducing the level of consumption to the marginal benefit of defense spending in increasing the probability of surviving to enjoy future consumption. In the reputational equilibrium, however, the amount of debt and the associated discount rate must be small enough that the short-run gains from debt repudiation are not larger than the long-run costs from the loss of a trustworthy reputation.

The analysis shows that the interest rate on the sovereign's debt and the discount rate for the sovereign that results from optimal borrowing and defense spending can be small enough that optimal borrowing and defense spending satisfy the condition for a reputational equilibrium. In this case, the sovereign's inability to make an irrevocable commitment not to repudiate its debts does not hinder its ability to finance its defense against threats to its survival. This result is more likely to obtain the smaller is the expected rate of return that lenders require, the larger is the amount of servicing that a potential successor sovereign would rationally provide for debts incurred by the current sovereign, and the closer is the relation between the current sovereign's discount rate and its probability of surviving in power.

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A distinguishing feature of sovereignty is the power to abrogate commitments without having to answer to a higher legal authority. In particular, the debts of a sovereign, unlike private debts, are not subject to laws regarding bankruptcy and enforcement of collateral. These observations suggest that a sovereign's decision not to repudiate its current debts depends primarily on the sovereign's concern about its reputation--that is, about the effect of this decision on lender expectations about future repudiation, which in turn determine the sovereign's continued access to loans.

In recent papers, Grossman & Van Huyck (January 1987, August 1987), we have analyzed reputational equilibria for sovereign debts. In a reputational equilibrium, the short-run benefits from repudiation are smaller than the long-run costs from the loss of a trustworthy reputation. This equilibrium condition can allow positive sovereign debt, but also can imply a limitation on sovereign borrowing that is smaller than the amount that the sovereign would borrow if it could make an irrevocable commitment not to repudiate.

A key factor in determining the amount of debt that reputational considerations can support is the rate at which the sovereign discounts the benefits of future borrowing. If the sovereign discounts the future heavily, then it has little concern for its reputation and lenders could not lend even a small amount to the sovereign without inducing repudiation. At the other extreme, if the sovereign puts a high enough value on its ability to borrow in the future, then reputational considerations can substitute fully for the sovereign's inability to make irrevocable commitments.

The sovereign's discount rate presumably depends mainly, if not solely, on its probability of surviving in power. Previous

analyses of reputational equilibria have treated the discount rate, and by implication the sovereign's survival probability, as exogenous. This assumption abstracts from the effect, which may well be important, that the sovereign's reputation and its consequent ability to borrow (or to engage in other actions, such as the collection of seigniorage, that depend on its reputation) have on its survival probability.

The present paper analyzes a reputational equilibrium for a model in which the sovereign borrows in order to finance its defense against threats to its survival. In this equilibrium, in contrast to previous analyses, the sovereign's ability to borrow and its discount rate are determined simultaneously.

1. Analytical Framework

The sovereign's objective in issuing debt in period τ is to maximize U_τ , which is the expected sum of future consumption over an horizon of h periods--that is,

$$(1) \quad U_\tau = E_\tau \sum_{t=\tau+1}^{\tau+h} c_t,$$

where c_t is consumption in period t . One interpretation of this objective is that the sovereign is Pigovian and that c_t measures the total consumption of the sovereign's subjects. An alternative interpretation is that the sovereign acts as a proprietor and that c_t measures the consumption of the sovereign's court, which in modern states would include the entire political establishment or ruling group that extracts the rents associated with the existing sovereignty. The analysis is robust to these alternative interpretations of the sovereign's objective. In any event, these two measures of consumption are likely to be highly correlated. The assumptions implicit in

equation (1) that utility is linear in consumption and that the sovereign does not discount future consumption that occurs either before or in period $t+h$ are convenient simplifications.

The horizon h corresponds to the prospective longevity of the sovereign's survival in power. The analysis assumes that h is a random variable defined over the non-negative integers. The probability that the sovereignty, having survived to period t , will not survive to period $t+1$ is $1-\gamma_{t+1}$, where $0 < \gamma_{t+1} < 1$. In other words, the realization of h is derived from a stochastic process with the property

$$\Pr (h = t+1 | h \geq t) = 1 - \gamma_{t+1}.$$

Given this stochastic process, and given that h is the only stochastic element in the model, calculation of the expectation in equation (1) yields a discounted sum of consumption over an infinite horizon--namely,

$$(2) \quad U_t = \gamma_{t+1}c_{t+1} + \gamma_{t+1}\gamma_{t+2}c_{t+2} + \gamma_{t+1}\gamma_{t+2}\gamma_{t+3}c_{t+3} + \dots$$

Accordingly to equation (2), the contribution of consumption in any future period to U_t is larger the larger is the probability that the sovereignty will survive to that period.

The essential innovation in the present analysis is that it treats the probability γ_{t+1} that the current sovereignty will survive from period t to period $t+1$ as endogenous. Specifically, this probability depends on the amount that the sovereign spends in period t on defense against threats to its survival. These threats can involve the possibility of conquest by external aggressors or the possibility of insurrection and overthrow by internal rivals.

Spending on defense, denoted by b_t , is measured as a fraction of gross income per period available to the sovereign. The analysis assumes that a positive survival probability requires positive defense spending and that γ_{t+1} increases with b_t at a decreasing rate--that is,

$$(3) \quad \gamma_{t+1} = f(b_t), \text{ with } f(0) = 0, f(1) = 1, f' > 0, f'' < 0.$$

Equation (3) implies a fixed relation between γ_{t+1} and b_t . Also, to focus on the role of sovereign debt in determining the sovereign's probability of surviving, the analysis assumes that all defense spending must be financed by borrowing. [Useful extensions of the analysis would allow the intensity of threats to survival and the resulting relation between the survival probability and defense spending to vary over time and also would allow the sovereign to choose a combination of borrowing and self finance for defense spending.] In addition, the present analysis abstracts from other motivations for borrowing emphasized in previous studies--namely, productive investment and risk shifting.

If the sovereign survives from period t to period $t+1$, its consumption in period $t+1$ equals its gross income, y , minus s_{t+1} , the amount it spends in period $t+1$ to service debts issued to finance previous defense spending--that is,

$$(4) \quad c_{t+1} = y - s_{t+1} \text{ for } t = \tau+1, \dots, \tau+h.$$

In this context, corresponding to the alternative interpretations of c_t , y can represent either the gross income of the sovereign's subjects or the gross rents extracted by the ruling group. The simplifying assumption that y is constant is consistent with the simplifying assumption that the sovereign

borrowers only to finance defense spending, and not to finance productive investments or to shift risk.

The final simplifying assumptions are that the sovereign's debts all mature in one period, that the sovereign's lenders, are atomistic, and that market clearing implies that the expected rate of return on sovereign debt equals a fixed required expected rate of return, ρ . Let $S^e(b_t)$ be the amount of debt servicing that lenders in period t expect to receive in period $t+1$ from the current sovereign if the current sovereign survives to period $t+1$ and let $N^e(b_t)$ be the amount of debt servicing that lenders expect to receive in period $t+1$ from a new sovereign if the current sovereign does not survive. Then the market-clearing condition for sovereign debt implies that b_t , $S^e(b_t)$, and $N^e(b_t)$ satisfy

$$(5) \quad \gamma_{t+1} S^e(b_t) + (1-\gamma_{t+1})N^e(b_t) = (1+\rho)b_t y.$$

The next step is to consider the determination of $S^e(b_t)$ and $N^e(b_t)$. In general, it seems reasonable to assume that

$$(6) \quad N^e(b_t) < S^e(b_t).$$

Condition (6) says that lenders never expect to receive more servicing of the current sovereign's debts from a successor sovereign than they expect to receive from the current sovereign. This assumption rules out the paradoxical possibility that lenders to the current sovereign would prefer that the current sovereign not survive.

2. An Irrevocable Servicing Commitment

Suppose, hypothetically, that in period t the sovereign could irrevocably commit itself, as well as its potential successors, to service its debts in full -- that is, to follow in period $t+1$ the debt servicing policy given by

$$(7) \quad s_{t+1} = (1+R_t)b_t y,$$

where R_t is the contractual interest rate on the sovereign's debt. This irrevocable servicing commitment would determine the lenders' expectation about debt servicing to be

$$(8) \quad S^e(b_t) = N^e(b_t) = (1+R_t)b_t y.$$

For equation (8) to be consistent with equation (5) -- that is, for the expected return to lenders to equal the required expected rate of return ρ -- the contractual interest rate R_t must equal ρ .

If it could make such an irrevocable servicing commitment, the sovereign could issue any amount of debt that it chooses as long as it offers this contractual interest rate. Accordingly, the sovereign would choose b_t to maximize U_t as given by equation (2), subject to the constraints given by equations (3), (4), (7), and $R_t = \rho$. The sovereign would also know that, if it survives, it would face the identical problem in choosing b_{t+1} , b_{t+2} , etc. Thus, the sovereign's problem is equivalent to the problem of choosing a constant amount of defense expenditures and debt, denoted by b , to maximize

$$(9) \quad U = \frac{Yc}{1-\gamma}, \quad \text{subject to}$$

$$(10) \quad \gamma = f(b), \quad c = y - s, \quad \text{and} \quad s = (1+\rho)by.$$

The critical value for this problem, denoted by \hat{b} , satisfies the first-order condition $dU/db = 0$, which implies

$$(11) \quad f(\hat{b}) \frac{dc}{db} + \frac{f'(\hat{b})\hat{c}}{1-f(\hat{b})} = 0,$$

where $dc/db = -(1+\rho)$ and $\hat{c} = y - (1+\rho)\hat{b}y$.

The relevant second-order condition, $d^2U/db^2 < 0$, is unambiguously satisfied. Equation (11) says that the optimal amount of debt and defense spending, \hat{b} , equates the marginal cost of defense spending in reducing the level of consumption to the marginal benefit of defense spending in increasing the probability of surviving to enjoy future consumption. The implied values $\hat{\gamma} = f(\hat{b})$ and \hat{c} are the optimal survival probability and the optimal level of consumption.

3. A Reputational Equilibrium

In reality, a sovereign, not being subject to higher legal authority, cannot irrevocably commit itself, or its potential successors, not to repudiate its debts. Consequently, lenders must limit the amount of sovereign debt such that the sovereign, taking account of the benefits and costs associated with repudiation, would not find repudiation to be a desirable policy. Because, for any positive amount of debt, repudiation has the benefit of increasing current consumption, an equilibrium with a positive amount of debt requires that repudiation would imply an offsetting cost in the form of a reduction in expected future consumption. Specifically, an equilibrium with a positive amount of debt requires that, given the interest rate on this debt, the expected value of the sovereign's consumption is at least as large if the sovereign services its debts in full as it would be if the sovereign were to repudiate its debts.

In a reputational model of sovereign debt, the sovereign's current and past debt servicing decisions and its future consumption opportunities are linked through its reputation for trustworthiness. Specifically, lenders base their expectations about the sovereign's future debt servicing on the sovereign's current and past record of debt servicing. Given this linkage, a rational sovereign would consider how its current debt servicing is likely to affect its reputation and how its reputation affects its ability to issue debt now and in the future. [Assuming that the process by which it appoints and removes individual policymakers permits the sovereign to translate its objectives into policy, reputation resides with the sovereign and not with individual policymakers.]

In a reputational equilibrium, the amount of debt and the interest rate are such that only a sovereign that irrationally ignored its reputation would behave opportunistically and repudiate its debts. Using their knowledge and of how repudiation would affect a sovereign's reputation, lenders are able to calculate the maximum amount of debt servicing that the current sovereign and potential successor sovereigns rationally would choose to provide. Given the stationary structure of the model, these amounts, denoted \bar{s} and \bar{n} , are time invariant. Together with the market-clearing condition for sovereign debt, \bar{s} and \bar{n} imply a time-invariant maximum amount of debt, denoted \bar{b} , that the current sovereign can issue if it has a trustworthy reputation. Specifically, from equations (3) and (5), the sovereign faces the constraint

$$(12) \quad b_t < \bar{b},$$

where \bar{b} satisfies $f(\bar{b})\bar{s} + [1-f(\bar{b})]\bar{n} = (1+\rho)\bar{b}y$.

To analyze the determination of the current sovereign's reputation, assume that all sovereigns always behave rationally, except for an infinitesimal fraction, ϵ , of sovereigns who inexplicably lose the rational ability to resist the temptation to behave opportunistically. A loss of rational restraint could result either from idiosyncratic irrationality or from a breakdown in the process by which the individuals who compose the sovereignty reach their decisions. Either infirmity, however uncommonly it occurs, is intrinsic and irreversible.

Knowing this pattern of sovereign behavior, lenders, when dealing with a specific sovereign, attach probability $1-\epsilon$, which equals approximately unity, to rational and, hence, trustworthy behavior as long as this sovereign has not behaved opportunistically in the past. For the current sovereign, rational behavior implies full servicing of debts that it has incurred or that its predecessor sovereigns incurred up to the maximum amount of servicing \bar{s} . With a trustworthy reputation, the current sovereign can issue debt up to the amount \bar{b} .

If, alternatively, a sovereign ever has failed to exercise rational restraint, lenders would expect such opportunistic behavior by this sovereign in the future. In this case, $S^e(b_t)$ would equal zero and by condition (6) $N^e(b_t)$ would also equal zero. Thus, the market-clearing condition would imply that this sovereign would be unable to issue any debt. Note that this outcome would depend only on the expectations of atomistic lenders and would not require or involve collusive strategic behavior by lenders. [This analysis assumes for simplicity that lenders never forget a past repudiation and, hence, that a sovereign could never recover a trustworthy reputation once it had been lost. See Grossman & Van Huyck (January 1987, May 1987) for a more general model of the lenders' memory process.]

To summarize, if lenders in period τ expect the current sovereign to behave rationally -- that is, if in all periods from the inception of its sovereignty through τ the current sovereign has serviced existing debts, whether incurred by itself in its predecessors, in full up to the amount \bar{s} -- and if lenders expect that a successor sovereign would behave rationally, then lenders' expectations are

$$(13) \quad \text{for } t = \tau, S^e(b_t) = (1+R_t)b_t y \leq \bar{s}$$

$$\text{and } N^e(b_t) = \min[(1+R_t)b_t y, \bar{n}] \quad \text{and}$$

$$\text{for } \tau+h > t > \tau, \text{ either } S^e(b_t) = (1+R_t)b_t y$$

$$\text{and } N^e(b_t) = \min[(1+R_t)b_t y, \bar{n}]$$

$$\text{if } s_{t-j} = S^e(b_{t-j-1}) \text{ for all } j = 0, \dots, t-\tau-1,$$

$$\text{or } S^e(b_t) = N^e(b_t) = 0 \text{ otherwise.}$$

In addition, if the sovereign has a trustworthy reputation and, hence, is able to issue a positive amount of debt, then the contractual interest rate, R_t , must imply an expected return to lenders equal to the required expected rate of return ρ . In other words, condition (13) must be consistent with equation (5), which implies that R_t must satisfy

$$(14) \quad 1+R_t = \frac{1+\rho}{\gamma_{t+1} + (1-\gamma_{t+1}) \min[1, \bar{n}/(1+R_t)b_t y]},$$

where, from equation (3), $\gamma_{t+1} = f(b_t)$.

Equation (14) implies that, with γ_{t+1} less than unity, R_t exceeds ρ if and only if \bar{n} is less than $(1+R_t)b_t y$ -- that is, if and only if a new sovereign, who has a positive probability of replacing the current sovereign, would not rationally service the current sovereign's debts in full.

Taking account of reputation, the rational sovereign's problem in period τ is to choose a program $(b_t)_{t=\tau}^{\infty}$ to maximize U_{τ} , as given by equation (2), subject to equations (3), (4), (6), and (14) and conditions (12) and (13). The solution to this problem describes a reputational equilibrium. Given the stationary structure of the model, this program, denoted by b^* , is time invariant.

To derive b^* , define $V_{\tau+1}$ to be the sum, given that the sovereign has survived to period $\tau+1$, of consumption in period $\tau+1$ and the expectation formed in period $\tau+1$ of total consumption in periods $\tau+2$ through $\tau+h$ -- that is,

$$(15) \quad V_{\tau+1} = c_{\tau+1} + E_{\tau+1} \sum_{t=\tau+2}^{\tau+h} c_t.$$

Given the stochastic process generating h , equation (15) implies

$$(16) \quad V_{\tau+1} = c_{\tau+1} + \gamma_{\tau+2} c_{\tau+2} + \gamma_{\tau+2} \gamma_{\tau+3} c_{\tau+3} + \dots$$

Condition (12) implies that b^* is less than or equal to \bar{b} and, hence, is a member of the set of amounts of debt that satisfy

$$(17) \quad V_{\tau+1}^* > V_{\tau+1}^0,$$

where $V_{\tau+1}^*$ is the value of $V_{\tau+1}$ that results from borrowing b^* and servicing this debt in full in every period from $\tau+1$ through $\tau+h$ and $V_{\tau+1}^0$ is the value of $V_{\tau+1}$ that would result from borrowing b^* in period τ and repudiating this debt in period $\tau+1$. Condition (17) says that b^* is such that a plan that for all $t > \tau+1$ involves servicing this debt in full would generate in period $\tau+1$ at least as high expected utility for a rational sovereign as would a decision to repudiate. If b^* equals \bar{b} , then condition (17) is satisfied as an equality. If b^* is less than \bar{b} , then condition (17) is satisfied as an inequality.

Given that, if the sovereign services its debt in full, lenders do not change their expectations, any value of b^* that satisfies condition (17) also satisfies the analogous condition for period $\tau+1$ and, by extension, for every subsequent period. Therefore, the sovereign's plan to keep its trustworthy reputation in the future is time consistent.

Equation (16) implies that

$$(18) \quad V_{\tau+1}^* = \frac{c^*}{1-\gamma^*},$$

where, from equations (3), (4), (7), and (14), we have

$$\gamma^* = f(b^*), \quad c^* = y - s^*, \quad s^* = (1+R^*)b^*y,$$

$$\text{and } 1+R^* = \frac{1+\rho}{\gamma^* + (1-\gamma^*) \min(1, \bar{n}/s^*)}.$$

To calculate $V_{\tau+1}^O$, observe that by repudiating its debts in period $\tau+1$ the sovereign would be able to consume all of y in that period, but would be unable to borrow again and, hence, would have no chance to survive beyond period $\tau+1$. Thus, we have

$$(19) \quad V_{\tau+1}^O = y.$$

4. Does Reputation Support Optimal Defense Spending?

The analysis in Section 2 derived the optimal amount of debt and defense spending, \hat{b} , which would equate the marginal cost and marginal benefit of defense spending and would yield the highest value of U_{τ} subject to the constraint of full debt servicing. Thus, if condition (17) is satisfied for b^* equal to \hat{b} -- that is, if \hat{b} is not larger than \bar{b} -- then the sovereign would borrow \hat{b} , and \hat{b} is the reputational equilibrium. If, alternatively, condition (17) is not satisfied for b^* equal to \hat{b} , lenders would not permit the sovereign to borrow \hat{b} . Such a constraint would prevent the sovereign from achieving both optimal defense spending and an optimal survival probability.

To determine whether \hat{b} is the reputational equilibrium, it is necessary to evaluate condition (17) for b^* equal to \hat{b} . To facilitate the analysis, assume that $f(b) = b^{\alpha}$, $0 < \alpha < 1$. With this assumption, condition (17) and equations (18) and (19) imply that b^* satisfies the condition

$$(20) \quad b^* \leq (1+R^*)^{-\frac{1}{1-\alpha}},$$

where $1+R^* = \frac{1+\rho}{\gamma^* + (1-\gamma^*) \min(1, \bar{n}/s^*)}$, $\gamma^* = (b^*)^\alpha$,

and $s^* = (1+R^*)b^*y$.

Moreover, the derivative of U with respect to b , calculated from equations (9) and (10), becomes

$$(21) \quad \frac{dU}{db} = \frac{b^\alpha y}{1-b^\alpha} \left\{ -(1+\rho) + \frac{\alpha[1-(1+\rho)b]}{b(1-b^\alpha)} \right\}.$$

Now suppose that s^* is not greater than \bar{n} , which means that lenders calculate that it would be rational for a successor sovereign to service fully the debts incurred by the current sovereign. In this case, condition (20) becomes

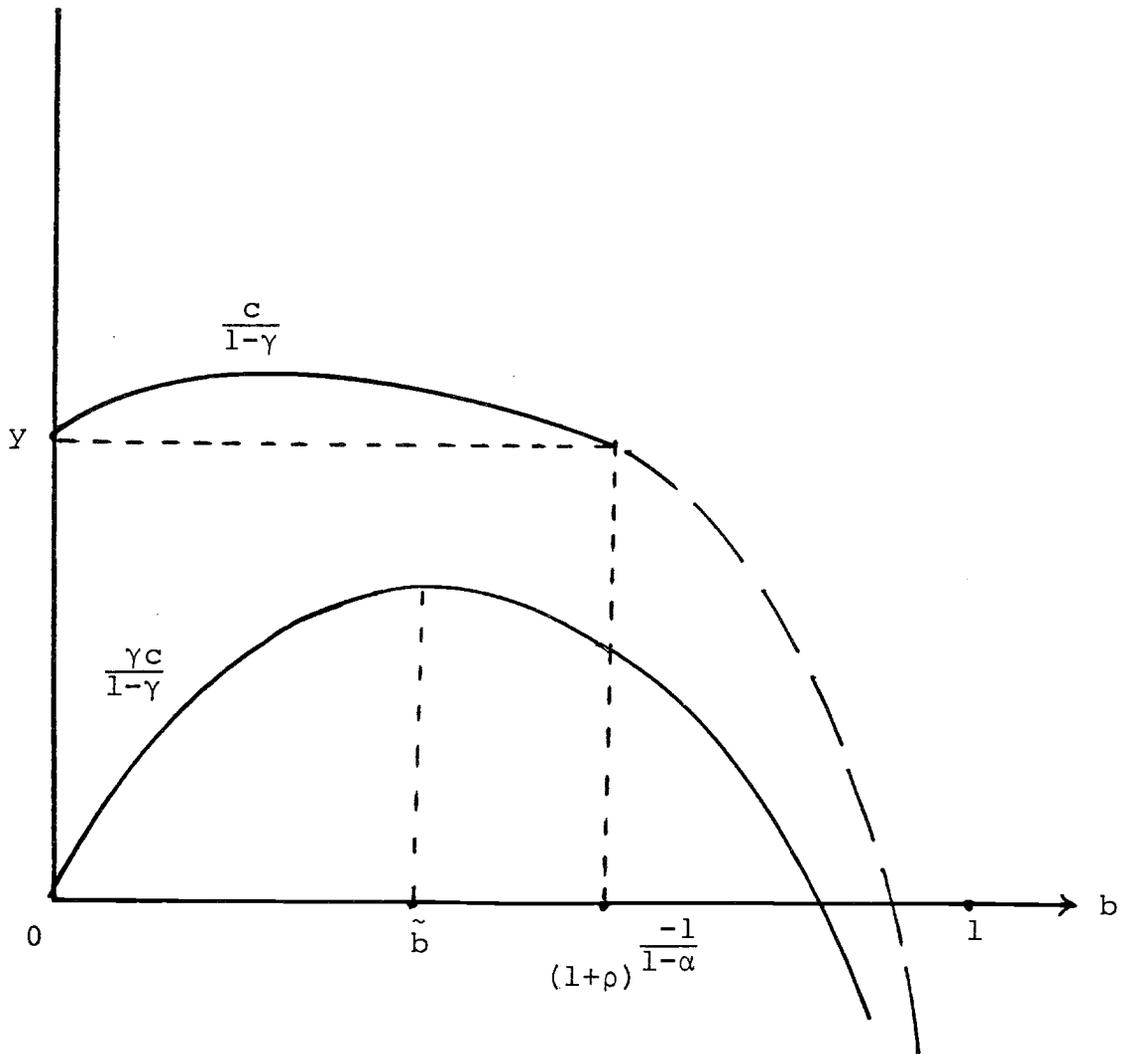
$$b^* \leq (1+\rho)^{-\frac{1}{1-\alpha}} = \bar{b}.$$

Suppose also that ρ and α are not too large. Specifically, assume that

$$\alpha(1+\rho)^{\frac{\alpha}{1-\alpha}} \leq 1.$$

In this case, equation (21) implies that, if b were equal to \bar{b} , dU/db would be negative. But, condition (11), which determines \hat{b} , sets dU/db equal to zero. Thus, the critical value, \hat{b} , is less than \bar{b} . Accordingly, in this case, b^* equals \hat{b} .

In the figure, the locus labelled, $\frac{\gamma c}{1-\gamma}$ depicts the problem given by equations (9) and (10), whose solution is \hat{b} , whereas the locus labelled $\frac{c}{1-\gamma}$ depicts the borrowing constraints in the reputational equilibrium. The solid segment of this locus indicates the set of values of b that satisfy condition (17). The figure illustrates that this set includes \hat{b} .



The assumption that s^* is not greater than \bar{n} , of course, seems unduly restrictive. In general, we want to consider values of \bar{n} less than s^* and even the possibility that \bar{n} equals zero, which would mean that lenders expect that a new sovereign would repudiate the debts of the current sovereign. A value of \bar{n} less than s^* would imply that any given value of b^* would require a higher contractual interest rate, R^* . This change, in turn, would imply lower value for \bar{b} . As \bar{n} approaches zero, the condition $\hat{b} < \bar{b}$ would be satisfied only for lower and lower values of ρ and α .

It would also be interesting to consider the possibility that the current sovereign's discount rate does not depend only on its probability of surviving in power. Specifically, suppose that the current sovereign exhibits pure time preference and, hence, would discount the future even if it were certain to survive in power forever. Pure time preference would imply lower values for both \hat{b} and \bar{b} . Whether the condition $\hat{b} < \bar{b}$ would be satisfied for high enough values of pure time preference does not seem to be clear.

5. Summary

This paper analyzes a reputational equilibrium for sovereign debt in a model in which the sovereign borrows to finance spending for defense against threats to its survival in power. In this model, the amount of sovereign debt and defense spending, the resulting survival probability, and the sovereign's implied discount rate for future consumption are determined simultaneously. The optimal amount of debt and defense spending equates the marginal cost of defense spending in reducing the level of consumption to the marginal benefit of defense spending in increasing the probability of surviving to enjoy future

consumption. In the reputational equilibrium, however, the amount of debt and the associated discount rate must be small enough that the short-run gains from debt repudiation are not larger than the long-run costs from the loss of a trustworthy reputation.

The analysis shows that the interest rate on the sovereign's debt and the discount rate for the sovereign that results from optimal borrowing and defense spending can be small enough that optimal borrowing and defense spending satisfy the condition for a reputational equilibrium. In this case, the sovereign's inability to make an irrevocable commitment not to repudiate its debts does not hinder its ability to finance its defense against threats to its survival. This result is more likely to obtain the smaller is the expected rate of return that lenders require, the larger is the amount of servicing that a potential successor sovereign would rationally provide for debts incurred by the current sovereign, and the closer is the relation between the current sovereign's discount rate and its probability of surviving in power.

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