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ABSTRACT

Following the crisis of 2008 several central banks engaged in a radical new policy experiment by setting negative policy rates. Using aggregate and bank-level data, we document a collapse in pass-through to deposit and lending rates once the policy rate turns negative. Motivated by these empirical facts, we construct a macro-model with a banking sector that links together policy rates, deposit rates and lending rates. Once the policy rates turns negative the usual transmission mechanism of monetary policy breaks down. Moreover, because a negative interest rate on reserves reduces bank profits, the total effect on aggregate output can be contractionary.

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1 Introduction

Would you give me one dollar today if I promise to give you less than one dollar back tomorrow? Conventional macroeconomic theory typically assumes that the answer is no. Rather than giving me one dollar for less in return tomorrow, you would prefer to hold on to that dollar yourself. At the heart of this argument is the role of currency as a store of nominal value: one dollar will never be worth less than one dollar in nominal terms. This logic gives rise to the so called zero lower bound on short term nominal interest rates. That is, everybody will demand at least the nominal value of what they lend out returned to them. This is a key assumption behind a vast recent literature on the zero lower bound, see e.g. Krugman (1998) and Eggertsson and Woodford (2003) for two early contributions. Recent experience in a number of countries however, put this key assumption into question. From 2012 to 2016, central banks in Switzerland, Sweden, Denmark, Japan and the Eurozone reduced their key policy rates below zero for the first time in economic history, going as far as -0.75 percent in Switzerland. The rationale behind this policy measure was to stimulate economic activity. The implementation of negative policy rates represents a new macroeconomic policy experiment. To date, there is no consensus on the effects of negative nominal interest rates - either empirically or theoretically. This paper contributes to filling this gap.

Understanding how negative nominal interest rates affect the economy is important in preparing for the next economic downturn. Interest rates have been declining for more than three decades, resulting in worries about secular stagnation (see e.g. Summers 2014, Eggertsson and Mehrotra 2014 and Caballero and Farhi 2017). In a recent working paper, Kiley and Roberts (2017) estimate that the zero lower bound on nominal interest rates will bind 30-40 percent of the time going forward. Whether setting a negative interest rate is expansionary is therefore of first order importance. Why did central banks try this untested policy? In short, they argued that there is nothing special about policy rates falling below zero. When announcing a negative policy rate, the Swedish Riksbank wrote in their monetary policy report that *"Cutting the repo rate below zero, at least if the cuts are in total not very large, is expected to have similar effects to repo-rate cuts when the repo rate is positive, as all channels in the transmission mechanism can be expected to be active"* (The Riksbank 2015). Similarly, the Swiss National Bank declared that *"the laws of economics do not change significantly when interest rates turn negative"* (Jordan, 2016). Many are skeptical however. For instance, Mark Carney of the Bank of England is *"... not a fan of negative interest rates"* and argues that *"we see the negative consequences of them through the financial system"* (Carney, 2016). Waller (2016) coins the policy as a "tax in sheep's clothing", arguing that negative interest rates act as any other tax on the banking system and thus reduces credit

growth¹.

In this paper we investigate the impact of negative central bank rates on the macroeconomy, both from an empirical and theoretical perspective². The first main contribution of the paper is to use a combination of aggregate and bank-level data to examine the pass-through of negative nominal central bank rates via the banking system. Using aggregate data, across six different economies, we show how negative policy rates have had limited pass-through to bank deposit rates, i.e. the rates customers face when they deposit their money in banks, and to lending rates, i.e. the rate at which customers borrow from banks. Making inferences about whether negative interest rates are expansionary based on aggregate data is challenging, however. We therefore proceed by using a novel, high-frequency bank level dataset on interest rates from Sweden, to explore the decoupling of lending rates from the policy rate. We document a striking decline in pass-through, and a substantial increase in heterogeneity across banks, once the policy rate becomes negative. We show that this increase in heterogeneity is linked to variation in the reliance on deposit financing: the higher the dependence on deposit financing, the smaller the effect on borrowing rates. Using a difference-in-difference approach, we show that banks with high deposit shares have significantly lower growth in loan volumes once the policy rate becomes negative, consistent with similar findings for the Euro Area (Heider, Saidi, and Schepens, 2016).

Motivated by these empirical results, the second main contribution of the paper is methodological. We construct a model, building on several papers from the existing literature, which allows us to address in the most simple setting how changes to the central bank policy rate filters through the banking system to various other interest rates, and ultimately determines aggregate output. At a minimum, such a model needs to recognize the role of money as a store of value, give a role to banks in order to allow for separate lending and borrowing rates, as well as having a well defined policy rate that may differ from the rates depositors and borrowers face. We construct a simple New Keynesian DSGE model which nests the standard one-period interest rate textbook New Keynesian model (see e.g. Woodford, 2003) and other recent variations. Our framework has four main elements. First, we explicitly introduce money along with storage costs to clarify the role of money as a store of value and how this may generate a bound on deposit rates. Second, we incorporate a banking sector and nominal frictions along the lines of Benigno, Eggertsson, and Romei (2014), which delivers well defined deposit and lending rates. Third, we incorporate demand for central

¹Other skeptics include Stiglitz (2016) and McAndrews (2015).

²Note that we do not attempt to evaluate the impact of other monetary policy measures which occurred simultaneously with negative interest rates. That is, we focus exclusively on the effect of negative interest rates, and do not attempt to address the effectiveness of asset purchase programs or programs intended to provide banks with cheap financing (such as the TLTRO program initiated by the ECB).

bank reserves as in Curdia and Woodford (2011) in order to obtain a policy rate which can potentially differ from the commercial bank deposit rate. Fourth, we allow the cost of bank intermediation to depend on banks' net worth as in Gertler and Kiyotaki (2010)³. The central bank sets the interest rate on reserves, and can choose to implement a negative policy rate as banks are willing to pay for the transaction services provided by reserves. However, due to the possibility of using money to store value, the deposit rate faced by commercial bank depositors is bounded at some level (possibly negative), in line with our empirical findings. The reason is simple: the bank's customers will choose to store their wealth in terms of paper currency if charged too much by the bank. We stipulate explicit conditions on the storage cost of money that guarantees a well-defined lower bound.

Away from the lower bound on the deposit rate, the central bank can stimulate the economy by lowering the policy rate. This reduces both the deposit rate and the rate at which households can borrow, thereby increasing demand. We show however, that once the deposit rate reaches its effective lower bound, reducing the policy rate further is no longer expansionary. As the central bank loses its ability to control the deposit rate, it cannot stimulate the demand of savers via the traditional intertemporal substitution channel. Furthermore, as banks' funding costs (via deposits) are no longer responsive to the policy rate, the bank lending channel of monetary policy breaks down. There is no stimulative effect via lower borrowing rates. Hence, as long as the deposit rate is bounded, a negative central bank rate fails to bring the economy out of a recession. We further show that if bank profits affect banks' intermediation costs, due to for instance informational asymmetries between the bank and its creditors as in Gertler and Kiyotaki (2010), negative interest rates can be contractionary through a reduction in banks' net worth.

Literature Review Jackson (2015) and Bech and Malkhozov (2016) document the limited pass-through of negative policy rates to aggregate bank rates, but do not evaluate the effects on the macroeconomy. In an empirical paper on the pass-through of negative policy rates to Euro Area banks, Heider, Saidi, and Schepens (2016) find that banks with higher deposit shares have lower lending growth in the post-zero environment. While the authors argue that this is a result of the lower bound on deposit rates, no attempt is made to formalize the mechanisms at play. Given the radical nature of the policy experiment pursued by several central banks, the theoretical literature is perhaps surprisingly silent on the expected

³Relative to the literature cited above, our model is closest to Benigno, Eggertsson, and Romei (2014). The main differences are that we explicitly incorporate cash to derive the interest rate bound rather than imposing it exogenously. Furthermore, we introduce reserves to allow the policy rate to differ from the deposit rate. Finally, we allow for the net worth of banks to have an effect on their intermediation costs.

effect of this policy within the current monetary framework⁴⁵. Two important exceptions are Brunnermeier and Koby (2016) and Rognlie (2015). The former paper defines the reversal rate as the interest rate at which further interest rate reductions become contractionary. This reversal rate, however, can in principle be either positive or negative, so it is unrelated to the observed lower bound on deposit rates resulting from money storage costs, which is central to our paper. Rognlie (2015) allows for a negative interest rate due to money storage costs. However, in that model households face only one interest rate, and the central bank can control this interest rate directly. Thus, the model does not capture the bound on deposit rates which we model here. Hence, neither of these papers capture the key mechanism in our paper, which is driven by an empirical observation which was by no means obvious ex ante: as the lower bound on the commercial bank deposit rate becomes binding, the connection between the central bank’s policy rate (which can be negative) and the rest of the interest rates in the economy breaks down.

There is a older literature however, dating at least back to the work of Silvio Gesell more than a hundred years ago (Gesell, 1916), which contemplates more radical monetary policy regime changes than we do here. This literature has been rapidly growing in recent years. In our model, the storage cost of money, and hence the lower bound, is treated as fixed. However, policy reforms can potentially alter the lower bound or even remove it completely. An example of such policies is a direct tax on paper currency, as proposed first by Gesell and discussed in detail by Goodfriend (2000) and Buiter and Panigirtzoglou (2003). This scheme directly affects the storage cost of money, and thereby the lower bound on deposits which we derive in our model. Another possibility is abolishing paper currency altogether. This policy is discussed, among others, in Agarwal and Kimball (2015), Rogoff (2017c) and Rogoff (2017a), who also suggest more elaborate policy regimes to circumvent the ZLB. An example of such a regime is creating a system in which paper currency and electronic currency trade at different exchange rates. The results presented here should not be considered as rebuffing any of these ideas. Rather, we are simply pointing out that under the current institutional framework, empirical evidence and a stylized variation of the standard New Keynesian model do not seem to support the idea that a negative interest rate policy is an effective tool to stimulate aggregate demand. This should, in fact, be read as a motivation to study further more radical proposals such as those presented by Gesell over a century ago and more recently in the work of authors such as Goodfriend (2000), Buiter and Panigirtzoglou (2003), Rogoff

⁴There is however a large literature on the effects of the zero lower bound. See for example Krugman (1998) and Eggertsson and Woodford (2006) for two early contributions.

⁵Our paper is also related to an empirical literature on the connection between interest rate levels and bank profits (Borio and Gambacorta 2017, Kerbl and Sigmund 2017), as well as a theoretical literature linking credit supply to banks net worth (Holmstrom and Tirole 1997, Gertler and Kiyotaki 2010).

(2017c) and Agarwal and Kimball (2015). In the discussion section we comment upon how our model can be extended to explore further some of these ideas, which we consider to be natural extensions.

2 Negative Interest Rates In Practice

In this section we use aggregate and bank-level data to document three facts about the pass-through of negative policy rates to bank interest rates. These facts will motivate our theoretical model presented in the next section. The aggregate data is retrieved from central banks and statistical agencies for each of the six economies we discuss. For Sweden, we also use two bank-level datasets. First, we use bank level data on monthly lending volumes from Statistics Sweden. Second, we use daily bank level data on mortgage rates for thirteen Swedish banks and credit institutions, which was provided by the price comparison site *compricer.se*⁶.

Fact 1: Limited Pass-Through to Deposit Rates In Figure (1) we plot deposit rates for six economic areas in which the policy rate is negative. Starting in the upper left corner, the Swedish central bank lowered its key policy rate below zero in February 2015. Deposit rates, which in Sweden are usually below the policy rate, did not follow the central bank rate into negative territory. Instead, deposit rates for both households and firms remain stuck at, or just above, zero. A similar picture emerges for Denmark, as illustrated in the upper right corner. The Danish central bank crossed the zero lower bound twice, first in July 2012 and then in September 2014. As was the case for Sweden, the negative policy rate has not been transmitted to deposit rates.

Consider next the Swiss and Japanese case in the middle row of Figure (1). Switzerland implemented a negative policy rate in December 2014, while the central bank in Japan lowered its key policy rate below zero in early 2016. The deposit rates in both countries were already very low, and did not follow the policy rate into negative territory. As a result, the impact on deposit rates was limited.

Finally, interest rates for the Euro Area are depicted in the bottom row of Figure (1). The ECB reduced its key policy rate below zero in June 2014. As seen from the left panel, aggregate deposit rates are high in the Euro Area and therefore have more room to fall before reaching the zero lower bound. Moreover, the deposit rate does not normally follow

⁶We have mortgage rates for different periods of fixed interest rates, ranging from three months to ten years. The fixed interest rate periods for which all financial institutions provide interest rates are 3 months, 1 year, 2 years, 3 years and 5 years. In the text we depict the results using five year periods, but we replicate our results using other interest rates in Figure (8) in the appendix.

the policy rate as closely as in the other cases we consider. One reason for this may be the underlying heterogeneity of the Euro Area⁷. Looking at Germany only, in the bottom right of the figure, we see a similar pattern emerge as in the other countries. That is, despite negative policy rates, the deposit rate appears bounded by zero. To sum up, the aggregate evidence is strongly suggestive of a lower bound on deposit rates.

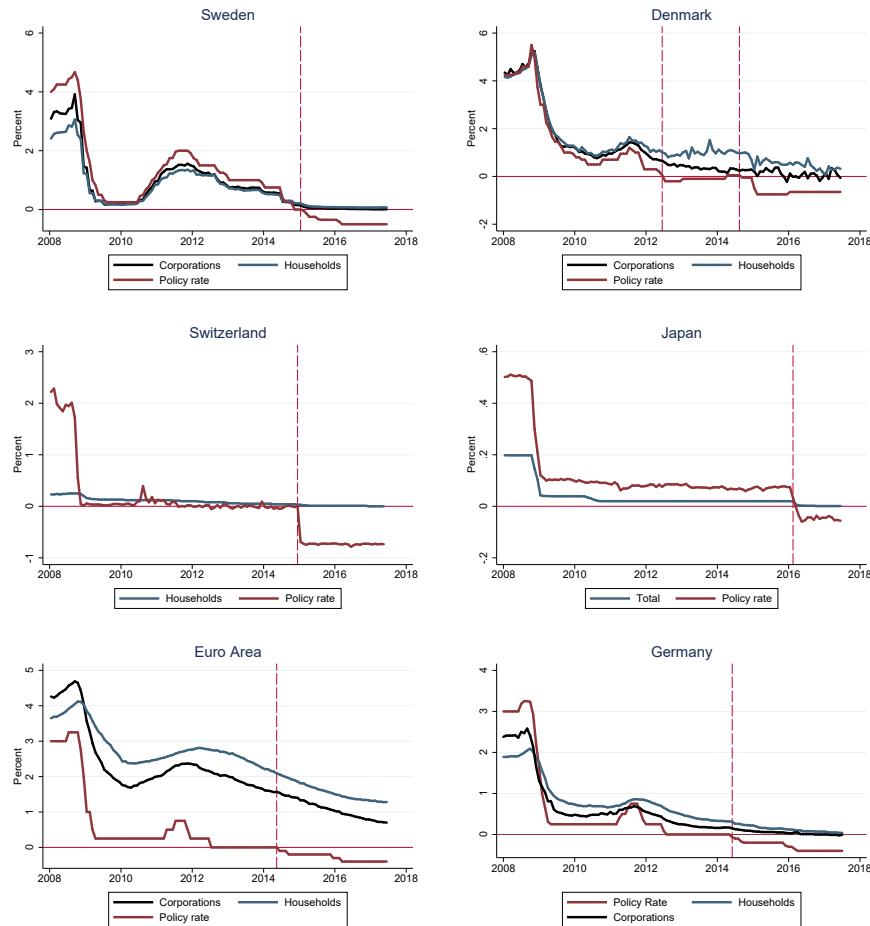


Figure 1: Aggregate Deposit Rates for Sweden, Denmark, Switzerland, Japan, the Euro Area and Germany. The policy rates are defined as the Repo Rate (Sweden), the Certificates of Deposit Rate (Denmark), SARON (Switzerland), the Uncollateralized Overnight Call Rate (Japan) and the Deposit Rate (Euro Area and Germany). The red vertical lines mark the month in which policy rates became negative. Source: The Riksbank, Statistics Sweden, the NB, the SNB, Bank of Japan, and the ECB.

⁷When considering the Euro Area it is worth noting that the negative interest rate policy was implemented together with a host of other credit easing measures, some of which implied direct lending from the ECB to commercial banks at a (potentially) negative interest rate. That policy is better characterized as a credit subsidy rather than charging interest on reserves, which the commercial banks hold in positive amounts at the central bank.

Fact 2: Limited Pass-Through to Lending Rates Although deposit rates appear bounded by zero, one might still expect negative policy rates to lower *lending* rates. As lending rates are usually above the central bank policy rate, they are all well above zero. Here we show that the pass-through of the policy rate to lending rates appears affected by the policy rate becoming negative, an empirical finding our model will replicate. In Figure (2) we plot bank lending rates for the six economic areas considered above. While lending rates usually follow the policy rate closely, there appears to be a disconnect once the policy rate breaks the zero lower bound, a feature which will become starker once we consider disaggregated bank data. Looking at the aggregate data in Figure (2), lending rates in Sweden, Denmark and Switzerland seem less sensitive to the respective policy rates once they become negative. There appears to be some reduction in Japanese lending rates at the time the policy rate went negative, but because there are no further interest rate reductions in negative territory the Japanese case is less informative⁸. Again, the Euro Area is somewhat of an outlier, as lending rates appear to have decreased. This is not surprising in light of the higher-than-zero deposit rates we documented in the previous fact. Again, for the case of Germany, in which the zero lower bound on the deposit rate *is* binding, lending rates appear less responsive.

⁸The initial reduction in Japanese lending rates could be caused by the positive part of the policy rate cut, i.e. going from a positive policy rate to a zero policy rate.

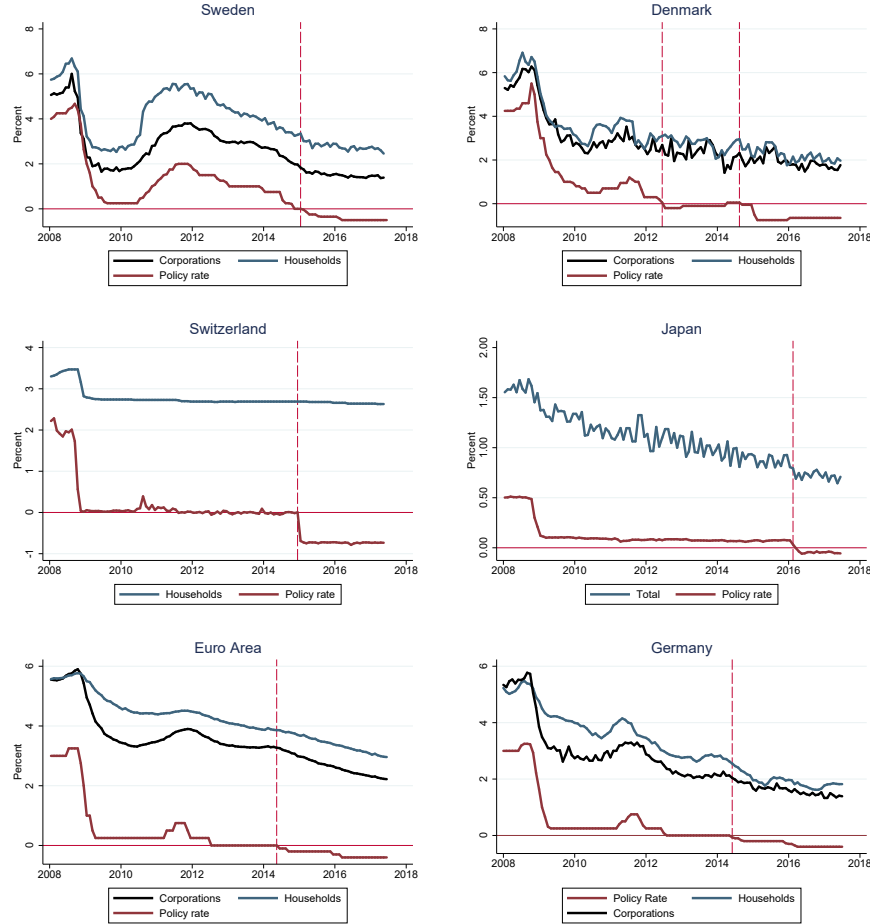


Figure 2: Aggregate Lending Rates for Sweden, Denmark, Switzerland, Japan, the Euro Area and Germany. The policy rates are defined as the Repo Rate (Sweden), the Certificates of Deposit Rate (Denmark), SARON (Switzerland), the Uncollaterized Overnight Call Rate (Japan) and the Deposit Rate (Euro Area and Germany). The red vertical lines mark the month in which policy rates became negative. Source: The Riksbank, Statistics Sweden, the NB, the SNB, Bank of Japan, and the ECB.

Perhaps the most compelling evidence on the breakdown in correlation between the policy rate and the lending rate of banks, comes from daily bank-level data from Sweden. In the left panel of Figure (3) we plot bank-level mortgage rates for thirteen banks or credit institutions. The vertical lines capture days on which the key policy rate (the repo rate) was lowered. The first two lines capture repo rate reductions in positive territory. On both of these occasions, there is an immediate and homogeneous decline in bank lending rates. The solid vertical line marks the day the repo rate turned negative for the first time, and the three proceeding lines capture repo rate reductions in negative territory. The response in bank lending rates to these interest rate cuts are strikingly different. While there is some initial reduction in lending rates, most of the rates increase again shortly thereafter. As a result, the total impact

on lending rates is limited⁹. There is also a substantial increase in dispersion, with several banks keeping their lending rate roughly unchanged despite repeated interest rate reductions below zero. In the right panel of Figure (3) we plot the minimum and maximum bank lending rate, along side the repo rate (the dashed black line). Again, the increase in dispersion after the repo rate turned negative is clearly visible. We also note that the minimum bank lending rate has stayed constant since the first quarter of 2015, despite three policy rate reductions in negative territory.

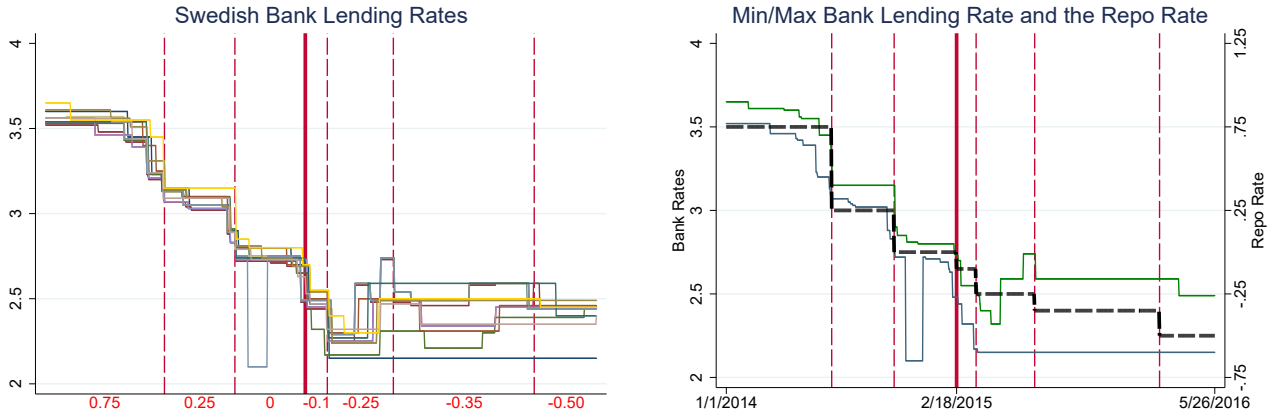


Figure 3: Bank Level Lending Rates Sweden. Interest rate on five-year mortgages. The red vertical lines mark days in which the repo rate was lowered. Left panel: lending rates by bank - the label on the x-axis shows the value of the repo rate. Right panel: the solid green (blue) line depicts the maximum (minimum) bank lending rate, while the dashed black line depicts the repo rate.

Fact 3: Increased Dispersion in Pass-Through Figure (3) showed an increase in the dispersion of bank lending rates once the policy rate fell below zero. In the left panel of Figure (4), we illustrate this explicitly by plotting the standard deviation of lending rates over time. We first note that the dispersion in bank rates appears to spike around the time when changes to the repo rate are announced. Second, and more importantly for our purpose, there is a sustained increase in dispersion after the zero lower bound is breached.

In the right panel of Figure (4) we plot the bank level correlations between the lending rates and the policy rate. The correlations captured by the blue bars are calculated for the pre-zero period, and show correlations close to one for all banks in our sample. The red bars capture correlations for the post-zero period, and show a very different picture. First, the correlations are much lower, averaging only 0.02. Second, the correlations now vary

⁹The response in aggregate deposit rates is relatively large and stable for the first four policy rate cuts in our sample. For the two last policy rate cuts, the response in the deposit rate is very limited. This lines up well with the low average response in lending rates for the final two policy rate cuts.

substantially across banks, ranging from -0.46 to 0.62. We therefore conclude that bank responses to negative interest rates are more heterogeneous than bank responses to positive interest rates.

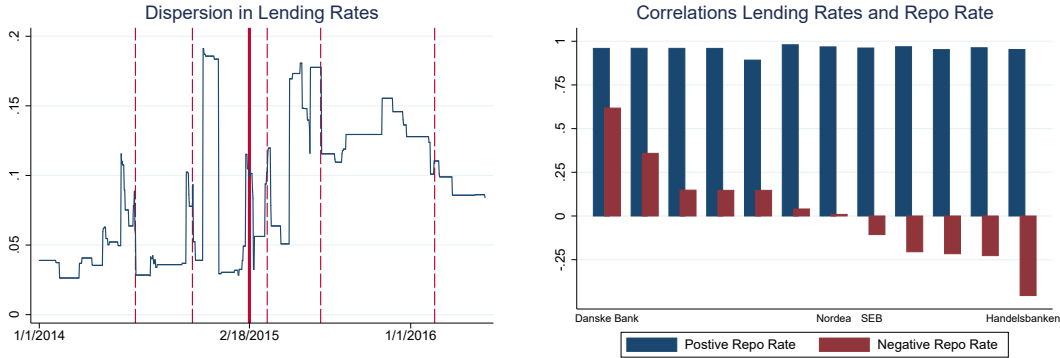


Figure 4: Left panel: Cross-sectional standard deviation in lending rates Sweden. Interest rate on five-year mortgages. Right panel: Bank-level correlations between lending rates and the repo-rate.

What is causing the dispersion in bank responses to negative interest rates? We first note that several banks initially cut interest rates when the policy rate became negative, only to raise them again shortly thereafter. This could indicate that banks are uncertain about how to set prices in the new and unfamiliar environment. However, there could also be more structural reasons why bank responses are heterogeneous. Given that there are frictions in raising different forms of financing and some sources of financing are more responsive to monetary policy changes than others, cross-sectional variation in balance-sheet components can induce cross-sectional variation in how monetary policy affects banks (Kashyap and Stein 2000). This is especially relevant in our setting. Negative interest rates have had a more limited pass-through to deposit financing relative to other sources of financing (see Figure (12) in the appendix). To investigate whether banks' funding structures affect their willingness to lower lending rates, we plot the bank level correlation between lending rates and the repo rate after the repo rate turned negative, as a function of banks' deposit shares. The result is depicted in Figure (5), and shows a correlation of -0.16. The small number of observations makes it difficult to draw any firm conclusions. However, we note that banks with high deposit shares consistently have small lending rate responses, in line with our proposed explanation. This is suggestive of the lower bound on deposit rates leading to cross-sectional differences in pass-through based on banks' share of deposit financing.

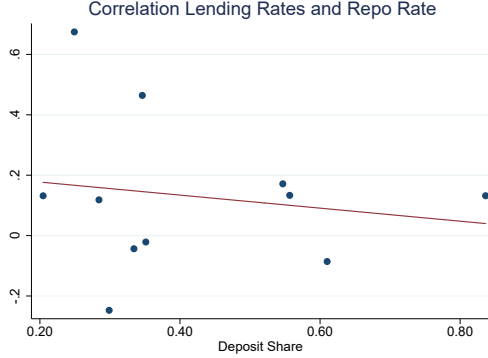


Figure 5: Correlation between lending rate and repo rate after the repo rate turned negative, as a function of the banks’ deposit share. Swedish banks. Interest rate on 5 year mortgages.

Another way to investigate whether the degree of deposit financing affects bank behavior is to look at lending volumes, rather than interest rates. This is done in Heider, Saidi, and Schepens (2016) for Euro Area banks. Consistent with our proposed explanation, they find that banks with higher deposit shares had lower lending growth after the policy rate turned negative. Here we show that their results also hold for Swedish banks. Following Heider, Saidi, and Schepens (2016) we use the difference in difference framework specified in equation (1). I_t^{post} is an indicator variable equal to one after the policy rate became negative, while $DepositShare_i$ is the deposit share of bank i in year 2013. We include bank fixed effects δ_i and month-year fixed effects δ_t . Standard errors are clustered at the bank level. We restrict our sample to start in 2014, following Heider, Saidi, and Schepens (2016) in choosing a relatively short time period around the event date. The coefficient of interest is the interaction coefficient β . If banks with high deposit shares have lower credit growth than banks with low deposit shares after the policy rate breaches the zero lower bound, we expect to find $\hat{\beta} < 0$.

$$\Delta \log(Lending_{it}) = \alpha + \beta I_t^{post} \times DepositShare_i + \delta_i + \delta_t + \epsilon_{it} \quad (1)$$

The regression results are reported in Table (1). The interaction coefficient is negative as expected, and significant at the ten percent level. Hence, the results for European banks from Heider, Saidi, and Schepens (2016) seem to hold also for Swedish banks¹⁰: credit growth in the post-zero environment is lower for banks which rely heavily on deposit financing. We thus conclude that the bound on the deposit rate is limiting the reaction in banks’ lending rates. This observation is important for our theoretical evaluation in the next section, where we make the assumption that banks are entirely funded via deposits. The bound on the deposit

¹⁰Our estimate of β is larger in absolute value than the one found in Heider, Saidi, and Schepens (2016), but we lack the statistical power to establish whether the coefficients are in fact significantly different.

rate then creates a bound on the banks' financing costs, creating a link between deposit and lending rates as seen in the data.

	(1)
	$\Delta \log(Lending)$
$I^{post} \times DepositShare$	-0.0297* (-1.72)
Constant	0.00962 (1.46)
Clusters	31
Bank FE	yes
Month-Year FE	yes
Observations	1,046

t statistics in parentheses, Std. err. clustered at bank level
* $p < .10$, ** $p < .05$, *** $p < .01$

Table 1: Regression results from estimating equation (1). Monthly bank level data from Sweden.

To summarize, we conclude that the repeated reductions in central bank rates below zero have not led to negative deposit rates. In fact, in most countries deposit rates appear stuck at zero, and do not react to further interest rate reductions. Further, lending rates appear elevated as well, causing the spread between deposit rates and lending rates to remain fairly constant - or even increase for some banks. Finally, the variation in bank responses to negative interest rates is consistent with the theory that the pass-through to lending rates is lower for banks which largely finance themselves with deposits, due to the apparent lower bound on deposit rates. This is important, for deposits will be the principal way in which banks finance themselves in our theoretical model. Motivated by these empirical facts, we now develop a formal framework for understanding the impact of negative central bank policy rates. As in the data, the deposit rate will be subject to a lower bound, resulting from storage costs associated with holding money. This lower bound on the deposit rate will affect banks' willingness to lower lending rates. Although the policy rate in our model can be negative, what matters for the effectiveness of monetary policy is to what degree negative policy rates are transmitted to other interest rates in the economy.

3 Model

3.1 Households

We consider a closed economy, populated by a unit-measure continuum of households. Households are of two types, either patient (indexed by superscript s) or impatient (indexed by superscript b). Patient households have a higher discount factor than impatient agents, i.e. $\beta^s > \beta^b$. The total mass of patient households is $1 - \chi$, while the total mass of impatient households is χ . In equilibrium, impatient households will borrow from patient households via the banking system, which we specify below. We therefore refer to the impatient households as “borrowers” and the patient households as “savers”.

Households consume, supply labor, borrow/save and hold real money balances. At any time t , the optimal choice of consumption, labor, borrowing/saving and money holdings for a household $j \in \{s, b\}$ maximizes the present value of the sum of utilities

$$\mathcal{U}_t^j = \mathbb{E}_t \sum_{T=t}^{\infty} (\beta^j)^{T-t} \left[U(C_T^j) + \Omega \left(\frac{M_T^j}{P_T} \right) - V(N_T^j) \right] \zeta_t \quad (2)$$

where ζ_t is a random variable following some stochastic process and acts as a preference shock¹¹. C_t^j and N_t^j denote consumption and labor for type j respectively, and the utility function satisfies standard assumptions clarified below.

Households consume a bundle of consumption goods. Specifically, there is a continuum of goods indexed by i , and each household j has preferences over the consumption index

$$C_t^j = \left(\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \quad (3)$$

where $\theta > 1$ measures the elasticity of substitution between goods.

Agents maximize lifetime utility (equation (2)) subject to the following flow budget constraint:

$$M_t^j + B_{t-1}^j (1 + i_{t-1}^j) = W_t^j N_t^j + B_t^j + M_{t-1}^j - P_t C_t^j - S(M_{t-1}^j) + \Psi_t^j + \psi_t^j - T_t^j \quad (4)$$

B_t^j denotes one period risk-free debt of type j ($B_t^s < 0$ and $B_t^b > 0$). For the saver, B_t^s consists of bank deposits and government bonds, both remunerated at the same interest

¹¹We introduce the preference shock as a parsimonious way of engineering a recession.

rate i_t^s by arbitrage. Borrower households borrow from the bank sector only, at the banks' lending rate i_t^b . $S(M_{t-1}^j)$ denotes the storage cost of holding money. Ψ_t^j is type j 's share of firm profits, and ψ_t^j is type j 's share of bank profits. Let Z_t^{firm} denote firm profits, and Z_t denote bank profits. We assume that firm profits are distributed to both household types based on their population shares, i.e. $\Psi_t^b = \chi Z_t^{firm}$ and $\Psi_t^s = (1 - \chi) Z_t^{firm}$. Bank profits on the other hand are only distributed to savers, which own the deposits by which banks finance themselves¹². Hence, we have that $\psi_t^b = 0$ and $\psi_t^s = Z_t$.

The optimal consumption path for an individual of type j has to satisfy the standard Euler-equation

$$U'(C_t^j) \zeta_t = \beta^j (1 + i_t^j) \mathbb{E}_t (\Pi_{t+1}^{-1} U'(C_{t+1}^j) \zeta_{t+1}) \quad (5)$$

Optimal labor supply has to satisfy the intratemporal trade-off between consumption and labor¹³

$$\frac{V'(N_t^j)}{U'(C_t^j)} = \frac{W_t^j}{P_t} \quad (6)$$

Finally, optimal holdings of money have to satisfy¹⁴

$$\frac{\Omega' \left(\frac{M_t^j}{P_t} \right)}{U'(C_t^j)} = \frac{i_t^j + S'(M_t^j)}{1 + i_t^j} \quad (7)$$

The lower bound on the deposit rate \underline{i}^s is typically defined as the lowest value of i_t^s satisfying equation (7). The lower bound therefore depends crucially on the marginal storage cost. With the existence of a satiation point in real money balances, zero (or constant) storage costs imply $S'(M_t^s) = 0$ and $\underline{i}^s = 0$. That is, the deposit rate is bounded at exactly zero. With a non-zero marginal storage cost however, this is no longer the case. If storage cost are convex, for instance, the marginal storage cost is increasing in M_t^s . In this case, there is no lower bound. Based on the data from section (2), a reasonable assumption is that the deposit rate is bounded at a value close to zero. This is consistent with a proportional storage

¹²Distributing bank profits to both household types would make negative interest rates even more contractionary. The reduction in bank profits would reduce the transfer income of borrower households, causing them to reduce consumption. We believe this effect to be of second order significance, and so we abstract from it here.

¹³We assume that the function V is increasing in N and convex with well defined first and second derivatives.

¹⁴We assume a satiation point for money. That is, at some level \bar{m}^j households become satiated in real money balances, and so $\Omega'(\bar{m}^j) = 0$.

cost $S(M_t^s) = \gamma M_t^s$, with a small $\gamma > 0$. We therefore assume proportional storage costs for the rest of the paper, in which the lower bound on deposit rates is given by $\underline{i}^s = -\gamma$ ¹⁵.

We assume that households have exponential preferences over consumption, i.e. $U(C_t^j) = 1 - \exp\{-qC_t^j\}$ for some $q > 0$. The assumption of exponential utility is made for simplicity, as it facilitates aggregation across agents. Under these assumptions, the labor-consumption trade-off can easily be aggregated into an economy-wide labor market condition¹⁶

$$\frac{V'(N_t)}{U'(C_t)} = \frac{W_t}{P_t} \quad (8)$$

Letting G_t denote government spending¹⁷, aggregate demand is given by

$$Y_t = \chi C_t^b + (1 - \chi) C_t^s + G_t \quad (9)$$

3.2 Firms

Each good i is produced by a firm i . Production is linear in labor, i.e.

$$Y_t(i) = N_t(i) \quad (10)$$

where $N_t(i)$ is a Cobb-Douglas composite of labor from borrowers and savers respectively, i.e. $N_t(i) = (N_t^b(i))^\chi (N_t^s(i))^{1-\chi}$, as in Benigno, Eggertsson, and Romei (2014). This ensures that each type of labor receives a total compensation equal to a fixed share of total labor expenses. That is,

$$W_t^b N_t^b = \chi W_t N_t \quad (11)$$

$$W_t^s N_t^s = (1 - \chi) W_t N_t \quad (12)$$

where $W_t = (W_t^b)^\chi (W_t^s)^{1-\chi}$ and $N_t = \int_0^1 N_t(i) di$.

Given preferences, firms face a downward-sloping demand function

¹⁵This nests the case of no storage costs, in which case $\gamma = 0$.

¹⁶To see this, just take the weighted average of equation (6) using the population shares χ and $1 - \chi$ as the respective weights.

¹⁷Government policies are explained below.

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t \quad (13)$$

We introduce nominal rigidities by assuming Calvo-pricing. That is, in each period, a fraction α of firms are not able to reset their price. Thus, the likelihood that a price set in period t applies in period $T > t$ is α^{T-t} . Prices are assumed to be indexed to the inflation target Π . The firm problem is standard and is solved in Appendix D.

3.3 Banks

Our banking sector is made up of identical, perfectly competitive banks. Bank assets consist of one-period real loans l_t . In addition to loans, banks hold real reserves $R_t \geq 0$ and real money balances $m_t = \frac{M_t}{P_t} \geq 0$, both issued by the central bank¹⁸. Bank liabilities consist of real deposits d_t . Reserves are remunerated at the interest rate i_t^r , which is set by the central bank. Loans earn a return i_t^l . The cost of funds, i.e. the deposit rate, is denoted i_t^s . Banks take all of these interest-rates as given.

Financial intermediation takes up real resources. Therefore, in equilibrium there is a spread between the deposit rate i_t^s and the lending rate i_t^l . We assume that banks' intermediation costs are given by a function $\Gamma\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t\right)$, in which $z_t = \frac{Z_t}{P_t}$ is real bank profit. In order to allow for the intermediation cost to be time-varying for a given set of bank characteristics, we include a stochastic cost-shifter \bar{l}_t . This cost-shifter may capture time-variation in borrowers default probabilities, changes in borrower households borrowing capacity, bank regulation etc. (Benigno, Eggertsson, and Romei 2014).

We assume that the intermediation costs are increasing and convex in the amount of real loans provided. That is, $\Gamma_l > 0$ and $\Gamma_{ll} \geq 0$. Central bank currency plays a key role in reducing intermediation costs¹⁹. The marginal cost reductions from holding reserves and money are captured by $\Gamma_R \leq 0$ and $\Gamma_m \leq 0$ respectively. We assume that the bank becomes satiated in reserves for some level \bar{R} . That is, $\Gamma_R = 0$ for $R \geq \bar{R}$. Similarly, banks become satiated in money at some level \bar{m} , so that $\Gamma_m = 0$ for $m \geq \bar{m}$. Banks can thus reduce their intermediation costs by holding reserves and/or cash, but the opportunity for cost reduction can be exhausted. Finally, we assume that higher profits (weakly) reduce the marginal cost of lending. That is, we assume $\Gamma_{lz} \leq 0$. We discuss this assumption below.

¹⁸Because we treat the bank problem as static - as outlined below - we can express the maximization problem in real terms.

¹⁹For example, we can think about this as capturing in a reduced form way the liquidity risk that banks face. When banks provide loans, they take on costly liquidity risk because the deposits created when the loans are made have a stochastic point of withdrawal. More reserves helps reduce this expected cost.

Following Curdia and Woodford (2011) and Benigno, Eggertsson, and Romei (2014) we assume that any real profits from the bank's asset holdings are distributed to their owners in period t and that the bank holds exactly enough assets at the end of the period to pay off the depositors in period $t + 1$ ²⁰. Furthermore, we assume that storage costs of money are proportional and given by $S(M) = \gamma M$. Under these assumptions, real bank profits can be implicitly expressed as:

$$z_t = \frac{i_t^b - i_t^s}{1 + i_t^s} l_t - \frac{i_t^s - i_t^r}{1 + i_t^s} R_t - \frac{i_t^s + \gamma}{1 + i_t^s} m_t - \Gamma \left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t \right) \quad (14)$$

Any interior l_t , R_t and m_t have to satisfy the respective first-order conditions from the bank's optimization problem²¹

$$l_t : \frac{i_t^b - i_t^s}{1 + i_t^s} = \frac{1}{\bar{l}_t} \Gamma_l \left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t \right) \quad (15)$$

$$R_t : -\Gamma_R \left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t \right) = \frac{i_t^s - i_t^r}{1 + i_t^s} \quad (16)$$

$$m_t : -\Gamma_m \left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t \right) = \frac{i_t^s + \gamma}{1 + i_t^s} \quad (17)$$

The first-order condition for real loans says that the banks trade off the marginal profits from lending with the marginal increase in intermediation costs. The next two first-order conditions describe banks demand for reserves and cash. We assume that reserves and money are not perfect substitutes, and so minimizing the intermediation cost implies holding both reserves and money. This is not important for our main result²².

The first-order condition for loans pins down the equilibrium credit spread ω_t defined as

$$\omega_t \equiv \frac{1 + i_t^b}{1 + i_t^s} - 1 = \frac{i_t^b - i_t^s}{1 + i_t^s} \quad (18)$$

Specifically, it says that

²⁰The latter is equivalent to assuming that $(1 + i_t^b) l_t + (1 + i_t^r) R_t + m_t - S(m_t) = (1 + i_t^s) d_t$.

²¹Assuming that $\Gamma \left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t \right)$ is such that there exists a unique z solving equation (14).

²²The assumption that banks always wants to hold some reserves is however important for the effect of negative interest rates on bank profitability. If we instead assume that the sum of money holdings and reserves enters the banks cost function as one argument, the bank would hold only money once $i^r < -\gamma$. Hence, reducing the interest rate on reserves further would not affect bank profits. However, such a collapse in central bank reserves is not consistent with data, suggesting that banks want to hold some (excess) reserves.

$$\omega_t = \frac{1}{\chi \bar{b}_t} \Gamma_l \left(\frac{b_t^b}{\bar{b}_t}, R_t, m_t, z_t \right) \quad (19)$$

where we have used the market clearing condition in equation (20) to express the spread as a function of the borrowers real debt holdings b_t^b ²³.

$$l_t = \chi b_t^b \quad (20)$$

That is, the difference between the borrowing rate and the deposit rate is an increasing function of the aggregate relative debt level, and a decreasing function of banks' net worth.

Why do bank profits affect intermediation costs? We have assumed that the marginal cost of extending loans (weakly) decreases with bank profits. That is, $\Gamma_{lz} \leq 0$. This assumption captures, in a reduced form manner, the established link between banks' net worth and their operational costs. We do not make an attempt to microfound this assumption here, which is explicitly done in among others Holmstrom and Tirole (1997) and Gertler and Kiyotaki (2010)²⁴.

In Gertler and Kiyotaki (2010) bank managers may divert funds, which means that banks must satisfy an incentive compatibility constraint in order to obtain external financing. This constraint limits the amount of outside funding the bank can obtain based on the banks net worth. Because credit supply is determined by the total amount of internal and external funding, this means that bank lending depends on bank profits. In an early contribution, Holmstrom and Tirole (1997) achieve a similar link between credit supply and bank net worth by giving banks the opportunity to engage (or not engage) in costly monitoring of its non-financial borrowers. For recent empirical evidence on the relevance of bank net worth in explaining credit supply, see for example Jiménez and Ongena (2012).

Importantly, our main result is that negative interest rates are *not expansionary*. This does not depend on profits affecting intermediation costs. However, the link between profits and the intermediation cost is the driving force behind negative interest rates being *contractionary*. If we turn off this mechanism, negative interest rates still reduce bank profits, but this does not feed back into aggregate demand²⁵.

²³Following equation (20) we also assume that $\bar{l}_t = \chi \bar{b}_t$.

²⁴Another way to interpret the implied link between bank profits and credit supply is to include a capital requirement. In Gerali, Neri, Sessa, and Signoretti (2010) a reduction in bank profits reduces the banks' capital ratio. In order to recapitalize the bank lowers credit supply.

²⁵Alternatively, we could assume that bank profits do not affect intermediation costs, but that bank profits are distributed to all households. A reduction in bank profits would then reduce aggregate demand through the borrowers budget constraint.

3.4 Policy

The consolidated government budget constraint is given by

$$B_t^g + M_t^{tot} + P_t R_t = (1 + i_{t-1}^g)B_{t-1}^g + M_{t-1}^{tot} + (1 + i_{t-1}^r)P_t R_{t-1} + G_t - T_t \quad (21)$$

where B_t^g is one period government debt, $M_t^{tot} = M_t + M_t^s + M_t^b$ is total money supply - which is the sum of money held by banks and each household type, i_t^g is the one period risk-free rate on government debt, G_t is government spending, and $T_t = \chi T_t^b + (1 - \chi) T_t^s$ is the weighted sum of taxes on the two household types.

The conventional way of defining monetary and fiscal policy, abstracting from reserves and the banking sector (see e.g. Woodford 2003), is to say that fiscal policy is the determination of end of period government liabilities, i.e. $B_t^g + M_t^{tot}$, via the fiscal policy choice of G_t and T_t . Monetary policy on the other hand, determines the *split* of end of period government liabilities B_t^g and M_t^{tot} , via open market operations. This in turn determines the risk-free nominal interest rate i_t^g through the money demand equations of the agents in the economy. The traditional assumption then, is that the one period risk-free rate on government debt corresponds to the policy rate which the monetary authority controls via the supply of money through the money demand equation.

We define monetary and fiscal policy in a similar way here. Fiscal policy is the choice of fiscal spending G_t and taxes T_t . This choice determines total government liabilities at the end of period t - the left hand side of equation (21). Total government liabilities are now composed of public debt and the money holdings of each agent, as well as reserves held at the central bank. Again, monetary policy is defined by how total government liabilities is split between government bonds B_t^g , and the overall supply of central bank issuance. In addition, we assume that the central bank sets the interest rate on reserves i_t^r . The supply of central bank currency is then given by

$$S_t = P_t R_t + M_t + M_t^s + M_t^b \quad (22)$$

Given these assumptions, the financial sector itself determines the allocation between reserves and money. That is, the split between the money holdings of different agents and reserves held by banks is an endogenous market outcome determined by the first order conditions of banks and households.

In order to clarify the discussion, it is helpful to review two policy regimes observed in the US at different times. Consider first the institutional arrangement in the US prior to the crisis, when the Federal Reserve paid no interest on reserves, so that $i_t^r = 0$. As seen from equation (16), this implies that banks were not satiated in reserves. The policy maker

then chose S_t so as to ensure that the risk-free rate was equal to its target. In this more general model, the policy rate is simply the risk-free nominal interest rate, which is equal to the deposit rate and, assuming that depositors can also hold government bonds, the interest rate paid on one period government bonds, i.e. $i_t^s = i_t^g$.

Consider now an alternative institutional arrangement, in which paying interest on reserves is a policy tool. Such a regime seems like a good description of the post-crisis monetary policy operations, both in the US and elsewhere. The central bank now sets the interest rate on reserves equal to the risk-free rate, i.e. $i_t^r = i_t^s = i_t^g$, and chooses S_t to implement its desired target. From the first order condition for reserves (16), we see that $i_t^s = i_t^r$ implies that $\Gamma_R = 0$. Hence, as long as banks are satiated in reserves, the central bank implicitly controls i_t^s via i_t^r . A key point, however, is that $\Gamma_R = 0$ is not always feasible due to the lower bound on the deposit rate. If the deposit rate is bounded at $\underline{i}^s = -\gamma$, and the central bank lowers i_t^r below $-\gamma$, then $i_t^s > i_t^r$. The first order condition then implies $\Gamma_R > 0$. Intuitively, it is not possible to keep banks satiated in reserves when they are being charged for their reserve holdings. More explicitly, we assume that the interest rate on reserves follows a Taylor rule given by equation (23). Because of the reserve management policy outlined above, the deposit rate in equilibrium is either equal to the reserve rate or to the lower bound, as specified in equation (24). We can now ask a well defined question: what happens if the interest rate on reserves is lowered below the lower bound on the deposit rate?

$$i_t^r = r_t^n \Pi_t^{\phi_\pi} Y_t^{\phi_Y} \quad (23)$$

$$i_t^s = \max \{ \underline{i}^s, i_t^r \} \quad (24)$$

Before closing this section it is worth pointing out that it seems exceedingly likely that there also exists a lower bound on the reserve rate. Reserves are useful for banks because they are used to settle cash-balances between banks at the end of each day. However, banks could in principle settle these balances outside of the central bank, for example by ferrying currency from one bank to another (or more realistically trade with a privately owned clearing house where the commercial banks can store cash balances). Hence, because banks have the option to exchange their reserves for cash, there is a limit to how negative i_t^r can become. We do not model this bound here, as it does not appear to have been breached (yet) in practice. Instead we focus on the bound on deposit rates - which is observable in the data.

3.5 Equilibrium Conditions: A Summary

After having introduced central bank policy in the previous section we are now ready to define an equilibrium in our model²⁶. For a given initial price dispersion Δ_0 and debt level b_0^b , and a sequence of shocks $\{\zeta_t, \bar{l}_t\}_{t=0}^\infty$, an equilibrium is a process for the 17 endogenous variables $\{C_t^b, C_t^s, b_t^b, m_t^b, m_t^s, \tau_t^s, s_t, Y_t, \Pi_t, F_t, K_t, \Delta_t, \lambda_t, l_t, R_t, m_t, z_t\}_{t=0}^\infty$ and the 3 endogenous interest rates $\{i_t^b, i_t^s, i_t^r\}_{t=0}^\infty$ such that the following equilibrium conditions hold²⁷. First, from the household problem we have the borrowers budget constraint (4) with $j = b$ ²⁸, the Euler equations and money demand (5) and (7) with $j = \{s, b\}$. Further we have the aggregate demand condition outlined in equation (9), leaving us with six equations from the demand side of the economy.

From the firm side we have the variables $\Pi_t, F_t, K_t, \Delta_t$ and λ_t . Π_t denotes inflation, while F_t and K_t are defined in appendix D. Δ_t and λ_t denote price dispersion and the weighted marginal utility of consumption respectively. The five equilibrium conditions from the firm side are listed in appendix D as equations (35), (36), (37), (38), and (39).

From the bank side we have four equilibrium conditions. These are bank profits given by equation (14), the first order condition for lending given by equation (15), the first order condition for reserves given by equation (16), and the first order condition for money given by equation (17). In addition, market clearing in the credit market requires that equation (20) is satisfied.

Government policy is summarized by the consolidated budget constraint in equation (21), which pins down the overall tax level, and the supply of base money in equation (22).

The final two equilibrium conditions are the policy equations from the previous section. Specifically, the interest rate on reserves follows the Taylor rule in equation (23) and the deposit rate is determined by equation (24). That leaves us with 20 equations to solve for the 20 endogenous variables listed above. These non-linear equilibrium conditions are summarized in appendix E.

²⁶We make a few assumptions that simplifies the model. First, we set $G_t = B_t^g = 0$. Transfers then only represent central bank profits/losses, which we assume accrue to the saver along with bank profits. If we instead assume that bank and central bank profits are distributed to both household types, negative interest are in fact even more contractionary.

²⁷Inflation is defined as $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, and we define $\tau_t^s \equiv \frac{T_t^s}{P_t}$.

²⁸Technically, we also need to solve for firm profits and labor income which enter into the borrowers budget constraint. When we list the non-linear equilibrium equations in Appendix E we have inserted for firm profits, and then labor income cancels. Hence we do not need to explicitly solve for these two variables.

3.6 Generalization of the Standard New Keynesian Model

We take a log-linear approximation of the equilibrium conditions around the steady state. The steady state equations, as well as the log-linearized equilibrium conditions are listed in appendix E²⁹. Here we reproduce the key equations in order to make the following observation: in the absence of interest rate bounds, and any shocks that create a trade-off between inflation and output, our model reduces to the standard New Keynesian model. The central bank can replicate a zero inflation target and keep output at its natural level at all times. Our log-linearized model is therefore a natural generalization of the textbook New Keynesian model with an endogenous natural rate of interest.

We first note that the supply side collapses to the standard case. That is, the supply side can be summarized by the generic Phillips curve³⁰

$$\hat{\pi}_t = \kappa \hat{y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} \quad (25)$$

The demand side is governed by the IS-curve in equation (26), which is derived by combining the aggregate resource constraint and the Euler equations of the savers and borrowers, where $\sigma \equiv \frac{1}{qY}$ ³¹.

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \sigma \left(\hat{i}_t^s - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^n \right) \quad (26)$$

In the standard model, the natural rate of interest r_t^n is exogenous. In our case the natural rate of interest is endogenous, and depends on the shocks to the economy and the agents' decisions. Specifically, the natural rate of interest takes the following form

$$\hat{r}_t^n = \hat{\zeta}_t - \mathbb{E}_t \hat{\zeta}_{t+1} - \chi \hat{\omega}_t \quad (27)$$

The natural rate of interest depends on the preference shock and the spread between the deposit rate and the borrowing rate, $\hat{\omega}_t \equiv \hat{i}_t^b - \hat{i}_t^s$. Given the assumed functional form for the banks intermediation cost Γ outlined in Appendix E, the feedback effect from bank profits

²⁹The log-linearized equilibrium conditions listed in the appendix are the IS-curve in equation (68), the expression for the real interest rate in equation (69), the borrowers Euler equation in equation (70), the savers Euler equation in (71), the borrowers budget constraint in equation (72), the borrowers money demand in equation (73), the savers money demand in equation (74), total central bank currency in equation (75), the governments consolidated budget constraint in equation (76), the Phillips curve in equation (77), the definition of the interest rate spread in equation (78), the value of the interest rate spread in equation (79), bank profits in equation (80), banks' reserve demand in equation (81), banks' money demand in equation (82), the Taylor rule in equation (83), and the lower bound on the deposit rate in equation (84).

³⁰We define $\kappa = \frac{1-\alpha}{\alpha}(1-\alpha\beta)(\eta+\sigma^{-1})$, in which $\beta = \chi\beta^b + (1-\chi)\beta^s$.

³¹The log-linearized resource constraint is given by $\hat{y}_t = \frac{\chi c^b}{y} \hat{c}_t^b + \frac{(1-\chi)c^s}{y} \hat{c}_t^s$, in which we have used the assumption that $G_t = 0$.

to the marginal intermediation cost is captured by the parameter $\iota \geq 0$. A higher ι means higher feedback from profits to intermediation costs. The credit spread is then pinned down by the relative amount of debt and bank profits.

$$\hat{\omega}_t = \frac{i^b - i^s}{1 + i^b} \left((\nu - 1) \hat{b}_t^b - \nu \hat{b}_t - \iota \hat{z}_t \right) \quad (28)$$

More private debt increases the interest rate spread, thereby reducing the natural rate of interest. In addition, the interest rate spread now depends on bank profits. The reason is that higher profits reduces intermediation costs, thereby lowering the banks' required interest rate margin. Both the preference shock and the shock to the economy's debt capacity are exogenous. However, the total debt level and bank profits are endogenous. In order to solve for these two variables, the entire system of equations outlined in the appendix needs to be solved.

While the standard New Keynesian model only has one interest rate, our model has three distinct interest rates. We have an interest rate on borrowing i_t^b , as well as an interest rate on savings i_t^s . In addition we also have an interest rate on reserves i_t^r . The log-linearized reserve rate is set by the central bank according to the standard Taylor rule

$$\hat{i}_t^r = \hat{r}_t^n + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t \quad (29)$$

Because the central bank keeps the bank sector satiated in reserves whenever feasible, the deposit rate is equal to the reserve rate when the lower bound is not binding, and is equal to the lower bound otherwise

$$\hat{i}_t^s = \max \left\{ \hat{\underline{i}}^s, \hat{i}_t^r \right\} \quad (30)$$

where $\hat{\underline{i}}^s = \frac{\beta^s}{\pi} (1 - \gamma) - 1$ is the lower bound on the deposit rate expressed in deviation from steady state. The current characterization of the model, given by equations (25) - (30) is incomplete, but we relegate to the appendix the full set of equations needed to describe the dynamics of private debt and bank profits in order to conserve space. This partial representation, however, is sufficient to make several observations. If there is no constraint on the policy rate and we set $i_t^s = i_t^r$, then the central bank can fully offset any variations in r_t^n via the policy rate. In this case, as in the standard model, it is easy to confirm that there is an equilibrium in which $\hat{i}_t^s = \hat{i}_t^r = \hat{r}_t^n$ and $\hat{\pi}_t = \hat{y}_t = 0$, and the model reduces to the standard model. Recall that we have abstracted from any shocks that trigger a trade-off between inflation and output, making this outcome feasible.

More generally, the characterization above clarifies how our model provides additional

details on the transmission of monetary policy. If the central bank lowers the reserve rate, this lowers the deposit rate through equation (30). The reduction in the deposit rate stimulates the consumption of saver households. In addition, lowering the deposit rate reduces the banks financing costs. This increases their willingness to lend, putting downwards pressure on the borrowing rate and thereby stimulating the consumption of borrower households. Hence, the reduction in the reserve rate leads to a reduction in the other interest rates in the economy, thereby stimulating aggregate demand.

Most macro models either implicitly or explicitly impose a zero lower bound on the interest rate controlled by the central bank, an assumption recent experience has called into question. Despite negative *policy rates* however, the deposit rate seems stuck at an apparent lower bound. As evident from equation (30), lowering the reserve rate below this bound and into negative territory has no effect on the deposit rate. Further, because the deposit rate stays unchanged, there is no stimulative effect on banks' financing costs and so no increase in their willingness to lend. As a result, there is no longer a boost to aggregate demand. Moreover, because charging a negative interest rate on reserves reduces bank profits, the interest rate spread in equation (28) increases. This implies an increase in the borrowing rate, and so aggregate demand falls. Hence, when the deposit rate is stuck at the lower bound, further reductions in the reserve rate have a *contractionary* effect on the economy. Note that this result is due to the feedback effect from bank profits to the interest rate spread. If we shut down this channel by setting $\iota = 0$ in equation (28), negative interest rates are neither expansionary nor contractionary. We now highlight these results with two numerical examples.

4 The Effects of Monetary Policy in Positive and Negative Territory

In this section, we compare our baseline model (which we refer to as the negative reserve rates model) to two other models. The first is the standard lower bound NK model. In this case, there is an identical effective lower bound on both the deposit rate and the central banks policy rate. The second is the frictionless model, in which both the deposit rate and the central bank policy rate can fall below zero.

We consider two different shocks. First, a temporary preference shock, which effectively makes agents more patient and so delays consumption. Second, a debt deleveraging shock in which the economy faces a permanent reduction in the borrowing capacity given by \bar{l}_t . We then evaluate to which degree the central bank can stimulate aggregate demand by lowering

the reserve rate.

4.1 Calibration

We pick the size of the two shocks to generate an approximately 4.5 percent drop in output on impact. This reduction in output is chosen to roughly mimic the average reduction in real GDP in Sweden, Denmark, Switzerland and the Euro Area in the aftermath of the financial crisis, as illustrated in figure (9) in the Appendix³². The drop in output in the US was of similar order. The persistence of the preference shock is set to generate a duration of the lower bound of approximately 12 quarters. We choose parameters from the existing literature whenever possible. We target a real borrowing rate of 4 %³³ and a real deposit rate of 1.5 %, yielding a steady state credit spread of 2.5 %. The preference parameter q is set to 0.75, which generates an intertemporal elasticity of substitution of approximately 2.75, in line with Curdia and Woodford (2011). We set the proportional storage cost to 0.01, yielding an effective lower bound of - 0.01 %. This is consistent with the deposit rate being bounded at zero for most types of deposits, with the exception of slightly negative rates on corporate deposits in some countries. We set $\bar{R} = 0.07$, which yields steady-state reserve holdings in line with average excess reserves relative to total assets for commercial banks from January 2010 and until April 2017³⁴. We set $\bar{m} = 0.01$, implying that currency held by banks in steady state accounts for approximately 1.5 percent of total assets. This currency amount corresponds to the difference between total cash assets reported at US banks and total excess reserves from January 2010 until April 2017.

The parameter ν measures the sensitivity of the credit spread to private debt. We set ν so that a 1 % increase in private debt increases the credit spread by 0.12 %, as in Benigno, Eggertsson, and Romei (2014). Given the steady-state credit spread, \bar{l} pins down the steady-state level of private debt. We choose \bar{l} to target a steady state private debt-to-GDP ratio of approximately 95 percent, roughly in line with private debt in the period 2005 - 2015 (Benigno, Eggertsson, and Romei, 2014). The final parameter is ι . In our baseline scenario

³²Detrended real GDP fell sharply from 2008 to 2009, before partially recovering in 2010 and 2011. The partial recovery was sufficiently strong to induce an interest rate increase. We focus on the second period of falling real GDP (which occurred after 2011), as negative interest rates were not implemented until 2014-2015. Targeting a reduction in real GDP of 4.5 percent is especially appropriate for the Euro Area and Sweden. Real GDP fell by somewhat less in Denmark, and considerably less in Switzerland. This is consistent with the central banks in the Euro Area and Sweden implementing negative rates because of weak economic activity, and the central banks in Denmark and Switzerland implementing negative rates to stabilize their exchange rates.

³³This is consistent with the average fixed-rate mortgage rate from 2010-2017. Series MORTGAGE30US in the St.Louis Fed's FRED database.

³⁴We use series EXCSRESNS for excess reserves and TLAACBW027SBOG for total assets from commercial banks, both in the St.Louis Fed's FRED database.

we set $\iota = 0.2$. While ι is not important for our main result that negative interest rates are not expansionary, it is important for determining the feedback effect from bank profits to aggregate demand. In Table 3 in the next section we show how the potentially contractionary effect of negative interest rates depends quantitatively on ι .

All parameter values are summarized in Table (2). Due to the occasionally binding constraint on i_t^s , we solve the model using OccBin (Guerrieri and Iacoviello, 2015) for the preference shock. For simplicity, we consider a cashless limit for the household's problem ³⁵.

4.2 Preference Shock

We start by investigating how the economy responds to a shock to agents marginal utility of consumption, a standard ZLB shock dating at least back to Eggertsson and Woodford (2003).

The results of the exercise are depicted in Figure (6). We start by considering the completely frictionless case, referred to as the *No bound* case. In this scenario it is assumed that both the policy rate and the deposit rate can turn negative, as illustrated by the dashed black lines in Figure (6). The preference shock reduces aggregate demand and inflation, triggering an immediate response from the central bank. The policy rate is lowered well below zero. The central bank keeps banks satiated in reserves whenever it can. As long as the deposit rate is not bounded, this policy is feasible. A reduction in the reserve rate is then accompanied by an identical reduction in the deposit rate. The reduction in deposit rates reduces banks' financing costs. As a result, banks supply more credit and the borrowing rate declines. In the frictionless case, the aggressive reduction in the policy rate means that the central bank is able to perfectly counteract the negative shock. Hence, all of the reaction comes through interest rates, with no reduction in aggregate demand.

³⁵There are no additional insights provided by allowing households to hold money in the numerical experiments, even if this feature of the model was essential in deriving the bound on deposits.

Parameter	Value	Source/Target
Inverse of Frisch elasticity of labor supply	$\eta = 1$	Justiniano et.al (2015)
Preference parameter	$q = 0.75$	Yields IES of 2.75(Curdia and Woodford, 2011)
Share of borrowers	$\chi = 0.61$	Justiniano et.al (2015)
Steady-state gross inflation rate	$\Pi = 1.005$	Match annual inflation target of 2%
Discount factor, saver	$\beta^s = 0.9901$	Annual real savings rate of 1.5 %
Discount factor, borrower	$\beta^b = 0.9963$	Annual real borrowing rate of 4 %
Probability of resetting price	$\alpha = 2/3$	Gali (2008)
Taylor coefficient on inflation gap	$\phi_\Pi = 1.5$	Gali (2008)
Taylor coefficient on output gap	$\phi_Y = 0.5/4$	Gali (2008)
Elasticity of substitution among varieties of goods	$\theta = 7.88$	Rotemberg and Woodford (1997)
Proportional storage cost of cash	$\gamma = 0.01$ %	Effective lower bound $i_t^s = -0.01$ %
Reserve satiation point	$\bar{R} = 0.07$	Target steady-state reserves/total assets of 13 %
Money satiation points	$\bar{m} = 0.01$	Target steady-state cash/total assets of 1.5 %
Marginal intermediation cost parameters	$\nu = 6$	Benigno, Eggertsson, and Romei (2014)
Level of safe debt	$\bar{l} = 1.3$	Target debt/GDP ratio of 95 %
Link between profits and intermediation costs	$\iota = 0.2$	1 % increase in profits \approx 0.01 % reduction in credit spread
Shock	Value	Source/Target
Preference shock	2.5 % temporary decrease in ζ_t	Generate a 4.5 % drop in output on impact
Persistence of preference shock	$\rho = 0.9$	Duration of lower bound of 12 quarters
Debt deleveraging shock	50 % permanent reduction in \bar{l}	Generate a 4.5 % drop in output on impact

Table 2: Parameter values

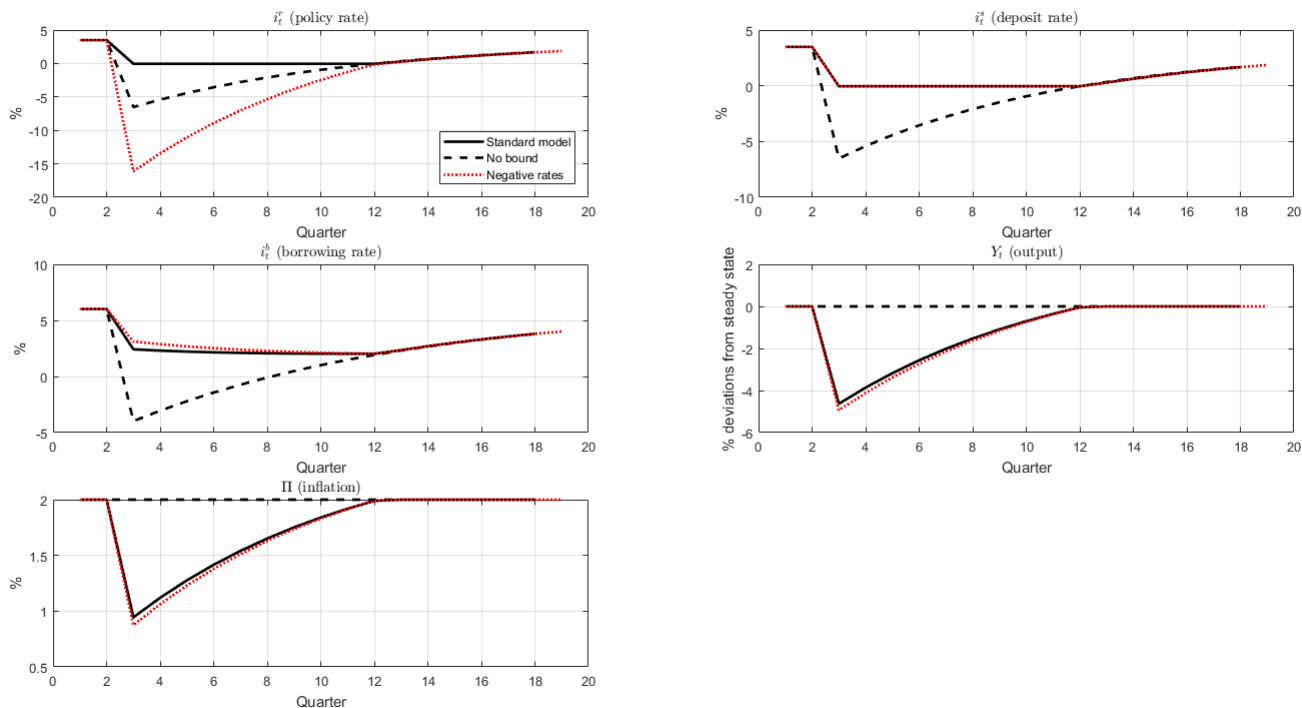


Figure 6: Impulse response functions following an exogenous decrease in the marginal utility of consumption (ζ_t), under three different models. *Standard model* refers to the case where there is an effective lower bound on both deposit rates and the central bank’s policy rate. *No bound* refers to the case where there is no effective lower bound on any interest-rate. *Negative rates* refers to the model outlined above, where there is an effective lower bound on the deposit rate but no lower bound on the policy rate.

Contrast the frictionless case to the standard case, in which both the policy rate and the deposit rate are bounded. In this case, the central bank is not able to offset the shock, and output is below its steady state value for the full duration of the shock. This scenario is outlined by the solid black line in Figure (6). As before, the central bank reacts to the shock by lowering the policy rate. However, the policy rate soon reaches the lower bound, and cannot be lowered further. As a result, both the policy rate and the deposit rate are stuck at the lower bound for the duration of the shock. This transmits into the borrowing rate, which falls less than in the previous case, due to the limited reduction in financing costs. Because of the inability of interest rates to fully adjust to the shock, aggregate output and inflation remain below their steady state values for the full duration of the shock.

Finally, we consider the case deemed to be most relevant compared to what we see in the data. While the policy rate is not bounded, there exists an effective lower bound on the deposit rate. This case is illustrated by the red dashed lines in Figure (6). The central bank

reacts to the shock by aggressively reducing the policy rate³⁶. However, the deposit rate only responds until it reaches its lower bound, at which point it is stuck. As a result, the borrowing rate does not fall as much as in the frictionless case, and the central bank is once again unable to mitigate the negative effects of the shock on aggregate demand and inflation. Hence, the central bank cannot stimulate aggregate demand by lowering its policy rate below zero.

At first glance, the model with negative interest rates looks similar to the standard model. Interestingly, there is an important difference between imposing negative interest rates and not doing so - which in effect makes negative interest rates contractionary in our exercise. Output declines more if the central bank chooses to lower its policy rate below zero. The reason is the negative effect on bank profits resulting from the negative interest rate on reserves. Banks hold reserves in order to reduce their intermediation cost, but when the policy rate is negative they are being charged for doing so. At the same time, their financing costs are unresponsive due to the lower bound on the deposit rate. Hence, bank profits are lower when the policy rate is negative³⁷. This decline in bank profits feeds back into aggregate demand through the effect of bank net worth on the marginal lending cost. Lower net worth increases the cost of financial intermediation, which reduces credit supply and dampens the pass-through of the policy rate to banks' lending rates.

The importance of profits for banks intermediation costs are parametrized by ι . In Table 3 we report the effect on output and the borrowing rate for different assumptions about ι . In the case in which there is no feedback from bank profits to intermediation costs ($\iota = 0$), the output drop under negative rates corresponds to the output drop under the standard model. The same holds for the borrowing rate. As ι increases, the reduction in the borrowing rate is muted due to the increase in intermediation costs. As a result, output drops by more. For ι sufficiently high, the borrowing rate actually *increases* when negative policy rates are introduced. This is consistent with the bank-level data on daily interest rates from Sweden, where some banks in fact *increased* their lending rate following the introduction of negative interest rates. Bech and Malkhozov (2016) report a similar increase in lending rates in Switzerland.

³⁶This reaction is exaggerated by our assumption that the central bank literally follows the Taylor rule in setting the interest rate on reserves, while in practice central banks only experimented with modestly negative rates.

³⁷The fall in bank profits is large in our case, and profits become negative in the negative reserve rate model. The reason is that the central bank follows the Taylor rule and sets the reserve rate equal to - 17 % in an unsuccessful effort to mitigate the shock. This is an extreme policy compared to the recent experience with negative interest rates, in which interest rates are only modestly below zero.

Model	Output, % deviation from SS	Reduction in borrowing rate, percentage points
Standard	4.5	3.5
$\iota = 0$	4.5	3.5
$\iota = 0.1$	4.7	3.3
$\iota = 0.15$	4.8	3.1
$\iota = 0.2$	4.9	2.9
$\iota = 0.25$	5.0	2.7
$\iota = 0.5$	5.7	1.1
$\iota = 0.7$	6.9	-1.6

Table 3: The effect of a preference shock on output and the borrowing rate (on impact) with negative policy rates for different values of ι .

To summarize, negative interest rates are not expansionary relative to the standard case in which the policy rate is not set below zero. In fact, when $\iota > 0$ there is an additional dampening effect on aggregate demand, making negative interest rates contractionary.

4.3 Debt deleveraging shock

We next consider the debt deleveraging shock in Figure (7). Specifically, we consider a permanent reduction in the debt limit \bar{l}_t ³⁸, a shock often referred to as a “Minsky Moment” (Eggertsson and Krugman, 2012). This directly increases the interest rate spread, causing the borrowing rate to increase. The initial increase in the borrowing rate is substantial, due to the shock’s impact on bank profits and the feedback effect via ι . In the frictionless case, the central bank can perfectly counteract this by reducing the reserve rate below zero. Given the bound on the deposit rate however, the central bank loses its ability to bring the economy out of a recession. Any attempt at doing so, by reducing the reserve rate below zero, only lowers bank profits and aggregate demand further.

In some respects this shock – with the associated rise in the borrowing rate – resembles more the onset of the financial crisis, when borrowing rates (in some countries) increased. The preference shock is more consistent with the situation further into the crisis, when both deposit and lending rates were at historical low levels (perhaps reflecting slower moving factors such as those associated with secular stagnation, see Eggertsson and Mehrotra (2014)). From the point of view of this paper however, it makes no difference which shock is considered in terms of the prediction it has for the effect of negative central bank rates. In both cases, the policy is neutral when there is no feedback from bank profits, and contractionary when there is such a feedback.

³⁸A temporary shock to banks’ intermediation costs yield qualitatively similar results.

Debt deleveraging shock

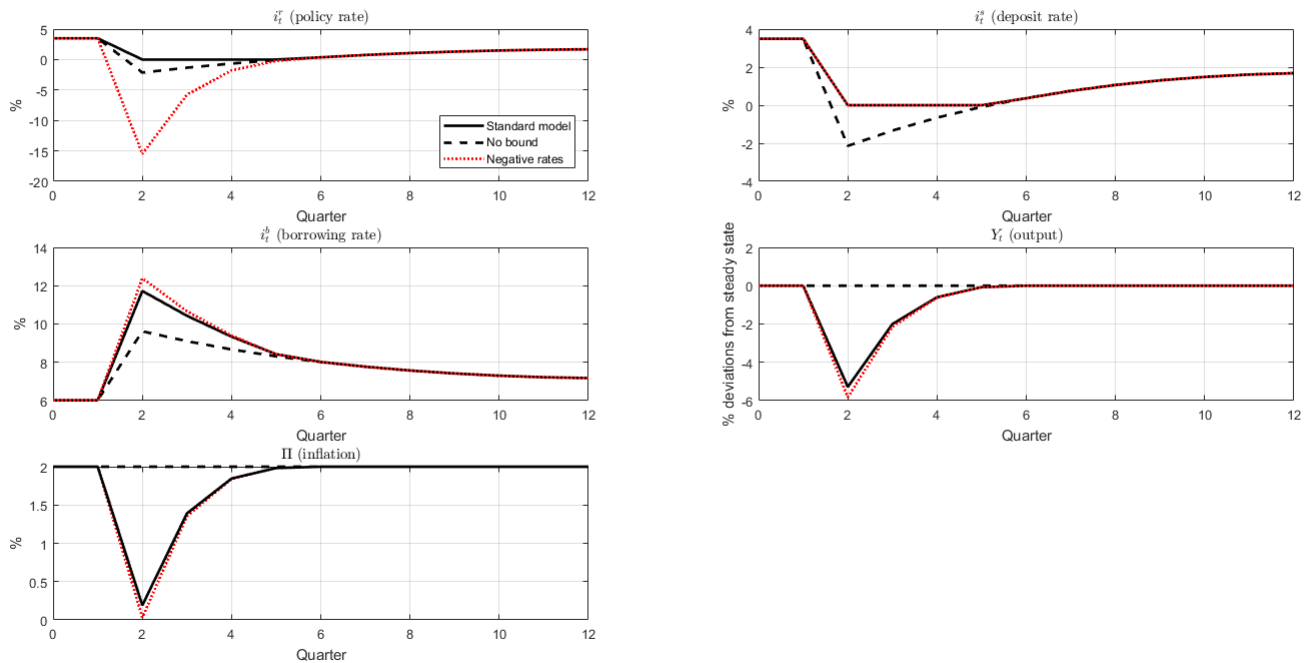


Figure 7: Impulse response functions following a debt deleveraging shock (a permanent reduction in \bar{l}_t), under three different models. *Standard model* refers to the case where there is an effective lower bound on both deposit rates and the central bank’s policy rate. *No bound* refers to the case where there is no effective lower bound on any interest-rate. *Negative rates* refers to the model outlined above, where there is an effective lower bound on the deposit rate but no lower bound on the policy rate.

5 Discussion

Our main theoretical result is that negative central bank rates are not expansionary. This result relies crucially on deposit rates being bounded, as observed in the data. The intuition is straightforward. When deposit rates are kept from falling, banks’ funding costs are constant. Hence, banks are unwilling to lower lending rates, as this would reduce the spread between deposit rates and borrowing rates - thereby reducing bank profits. This link between the deposit rate and the borrowing rate means that the bound on the deposit rate transmits into a bound on the lending rate, consistent with the empirical evidence from Sweden. Note that the result that negative interest rates are non-expansionary does not rely on the effect of bank profits on the real economy. That is, as long as the deposit rate is bounded, negative interest rates are always non-expansionary in our model.

The second result we want to highlight is the negative impact on bank profits. Charging

banks to hold reserves at the central bank lowers their profits. The impact of lower bank profits on aggregate demand depends on the model specifications. There is at least two ways in which lower bank profits may reduce economic activity. First, as in the main model specification, lower bank profits could increase banks financing costs, thereby reducing credit supply. This reduces the consumption of borrower households, and thereby aggregate demand. Second, we could distribute bank profits to all household types, thereby creating a negative impact on output through the borrowers budget constraint. We prefer the former channel, as this is the mechanism which has been extensively discussed in popular accounts, and seems of more quantitative relevance. Under either one of these two assumptions, negative interest rates are not only non-expansionary, but are contractionary.

In our model exercise, the storage cost of money was held fix. However, one could allow the storage cost of money - and therefore the lower bound on the deposit rate - to depend on policy. One way of making negative negative rates be expansionary, which is consistent with our account, is if the government takes actions to increase the cost of holding paper money. There are several ways in which this can be done. The oldest example is a tax on currency, as outlined by Gesell (1916). Gesell's idea would show up as a direct reduction in the bound on the deposit rate in our model, thus giving the central bank more room to lower the interest rate on reserves - and the funding costs of banks. Another possibility is to ban higher denomination bills, a proposal discussed in among others Rogoff (2017c). To the extent that this would increase the storage cost of money, this too, should reduce the bound on the banks deposit rate. An even more radical idea, which would require some extensions to our model, is to let the reserve currency and the paper currency trade at different values. This proposal would imply an exchange rate between electronic money and paper money, and is discussed in Agarwal and Kimball (2015), Rogoff (2017a) and Rogoff (2017b). A key pillar of the proposal - but perhaps also a challenge to implementability - is that it is the reserve currency which is the economy wide unit of account by which taxes are paid, and accordingly what matters for firm price setting. If such an institutional arrangement is achieved, then there is nothing that prevents a negative interest rate on the reserve currency while cash in circulation would be traded at a different price, given by an arbitrage condition. We do not attempt to incorporate this extension to our model, but note that it seems relatively straightforward, and has the potential of solving the ZLB problem. Indeed, the take-away from the paper should not be that negative nominal rate are always non-expansionary, simply that they are predicted to be so under the current institutional arrangement. This gives all the more reason to contemplate departures from the current framework, such as those mentioned briefly here and discussed in detail by the given authors.

We conclude this paper by discussing some arguments put forward by proponents of

negative interest rates. Addressing all of these arguments formally would require expanding our model substantially. Instead, we elaborate informally on whether we believe any of these changes would alter our main conclusions

Lower Lending Rates While the lower bound on deposit rates generally seems to have been accepted as an empirical fact, the lower bound on lending rates is not as widely recognized. Some proponents of negative interest rates argue that regardless of deposit rates, negative policy rates should transmit into lending rates as usual. The deputy governor of monetary policy at the Bank of England highlighted such a link between the reserve rate and the lending rate: *“But any attempt by banks to substitute out of reserves into other assets, including loans, would lead to downward pressure on the interest rates on those assets. Eventually, the whole constellation of interest rates would shift down, such that banks were content to hold the existing quantity of reserves. This is exactly the mechanism that operates when the Bank Rate is reduced in normal times; there is nothing special about going into negative territory.”* (Bean 2013). Similar explanations for the expansionary effects of negative interest rates have been provided by other central banks (The Riksbank 2016, Jordan 2016). This argument is potentially problematic for two reasons. First, in our model, banks are reluctant to cut lending rates when the deposit rate is stuck at its lower bound. Doing so would reduce their profits, as their funding costs are not responding to the negative policy rate. This connection between the deposit rate and the lending rate seems consistent with data. Looking at aggregate data in Figure (2) suggested that the bound on deposit rates was transmitting into a bound on lending rates as well. The bank level data from Sweden in Figure (4) further confirmed the collapse in pass-through from policy rates to lending rates once the policy rate turned negative. Another source of empirical evidence comes from the ECBs lending survey. As illustrated in Figure (13) in the appendix, 80-90 percent of banks in the Euro Area say that the negative policy rate has not contributed to increased lending volumes. There have also been reports of some banks *increasing* their lending rates in response to negative policy rates. This contractionary response seems particularly well documented in the Swiss case (Jordan 2016, Bech and Malkhozov 2016). Second, even if lending rates responded to negative policy rates as usual, this would imply a reduction in bank profits. The reduction in lending rates would squeeze banks’ profit margins, potentially reducing credit supply and thereby economic activity. This effect would only be in place when the policy rate was sufficiently low to make the lower bound on deposit rates binding, implying that negative interest rates *are* in fact special.

Additional Funding Sources In our model deposits constitute the sole financing source for banks. In reality banks have access to several funding sources, with potentially different sensitivity to negative policy rates. Figure (12) in the appendix illustrates some interest rates relevant for banks' financing costs in Sweden. In general, negative interest rates seem to have passed through to interbank rates (although these are typically banks charging other banks for transaction services, so the fact that interbank rates turn negative has no effect on the aggregate funding costs of the banking system). Also the interest rates on some short-term asset backed securities considered "safe", in some cases issued by banks, have gone negative. This implies that the negative policy rate may be reducing banks financing costs somewhat, even though deposit rates are bounded. Accordingly, we might expect the negative policy rate to have some effect on lending rates. In Sweden, deposits account for more than 40 percent of total liabilities (see Figure (10) in the appendix), and this share is typically even higher in the Euro Area. Hence, the deposit rate is quantitatively the most important interest rate for evaluating banks' financing costs. If the policy rate no longer affects the deposit rate, the central banks' ability to influence banks' funding costs is substantially reduced. However, one could imagine that banks respond to negative policy rates by shifting away from deposits to alternative sources of financing, in response to negative policy rates. This could increase the effectiveness of the monetary policy transmission. As illustrated in Figure (10) in the appendix, this does not seem to be the case. Swedish banks in aggregate actually increased their deposit share after the central bank rate turned negative in early 2015.

Bank Fees While banks have been unwilling to lower their deposit rates below zero, there has been some discussion surrounding their ability to make up for this by increasing fees and commission income. In our model there are no fixed costs involved with opening a deposit account, but allowing for this would imply that the interest rate on deposits could exceed the effective return on deposits. If banks respond to negative policy rates by increasing their fees, this could in principle reduce the effective deposit rate, and thereby lower banks' funding costs. However, as illustrated in Figure (11), the commission income of Swedish banks as a share of total assets actually fell after the policy rate turned negative. Hence, the data does not support the claim that the effective deposit rate is in fact falling. Given that depositors understand that higher fees reduce the effective return on their savings, this is perhaps not surprising. In any event, the ability to store money will ultimately put a bound on the banks ability to impose fees.

Alternative Transmission Mechanisms Our focus is on the effect of monetary policy on bank interest rates. When the policy rate is positive, lowering it reduces the interest rates

charged by banks, which increases credit supply and thereby economic activity. We have shown that this mechanism is no longer active when the policy rate is negative. However, it is still possible that negative interest rates stimulate aggregate demand through other channels. Both the Swiss and Danish central banks motivated their decision to implement negative central bank rates by a need to stabilize the exchange rate. This open economy dimension of monetary policy is absent from our analysis. The point we want to make is that the pass-through to bank interest rates - traditionally the most important channel of monetary policy - is not robust to introducing negative policy rates. The impact on the exchange rate would depend on which interest rates are most important for explaining movements in the exchange rate, which in turn can depend on several institutional details.

Even if lending volumes do not respond to negative policy rates, there could potentially be an effect on the composition of borrowers. It has been suggested that banks may respond to negative interest rates by increasing risk taking. This could potentially increase lending rates, resulting in upward pressure on the interest rate margin. Heider, Saidi, and Schepens (2016) find support for increased risk taking in the Euro Area, using volatility in the return-to-asset ratio as a proxy for risk taking. According to their results, banks in the Euro Area responded to the negative policy rate by increasing return volatility. This is certainly not the traditional transmission mechanism of monetary policy, and it is unclear whether such an outcome is desirable.

Other policies Our model exercise focuses exclusively on the impact of negative policy rates³⁹. Other monetary policy measures which occurred over the same time period are not taken into account. This is perhaps especially important to note in the case of the ECB, which implemented its targeted longer-term refinancing operations (TLTROs) simultaneously with lowering the policy rate below zero. Under the TLTRO program, banks can borrow from the ECB at attractive conditions. Both the loan amount and the interest rate are tied to the banks' loan provision to households and firms. The borrowing rate can potentially be as low as the interest rate on the deposit facility, which is currently -0.40 percent⁴⁰. Such a subsidy to bank lending is likely to affect both bank interest rates and bank profits, and could potentially explain why lending rates in the Euro Area have fallen more than in other

³⁹It is worth mentioning that in our model all reserves earn the same interest rate. In reality, most central banks have implemented a tiered remuneration scheme, in which case the marginal and average reserve rates differ. For example, some amount of reserves may pay a zero interest rate, while reserves in excess of this level earn a negative rate. We outline the policy schemes in the different countries in appendix C. Allowing for more than one interest rate on reserves would not qualitatively alter our results, but may be relevant for a more detailed quantitative assessment.

⁴⁰In our model we only consider a negative interest rate on bank assets, as we impose $R \geq 0$. The TLTRO program implies a negative interest rate on a bank liability.

places once the policy rate turned negative.

6 Concluding remarks

Since 2014, several countries have experimented with negative nominal central bank interest rates. In this paper, we have documented that negative central bank rates have not been transmitted to aggregate deposit rates, which remain stuck at levels close to zero. As a result of this, aggregate lending rates remain elevated as well. Using bank level data from Sweden, we documented a disconnect between the policy rate and lending rates, once the policy rate fell below zero. Motivated by our empirical findings, we developed a New Keynesian framework with savers, borrowers, and a bank sector. By including money storage costs and central bank reserves, we captured the disconnect between the policy rate and the deposit rate at the lower bound. In this framework we showed that negative policy rates were at best irrelevant, but could potentially be contractionary due to a negative effect on bank profits.

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Appendix A: Additional Figures

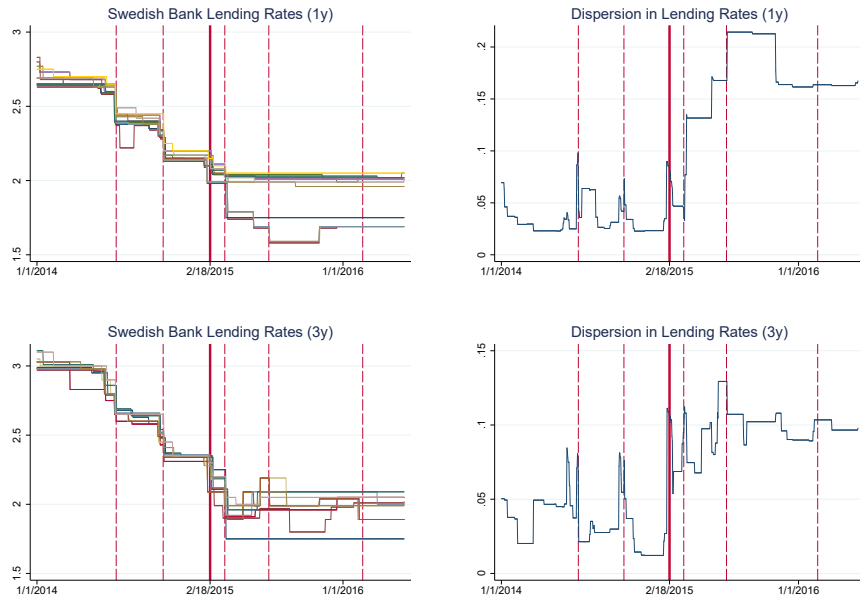


Figure 8: Bank Level Lending Rates Sweden. Interest rate on one-year and three-year mortgages.

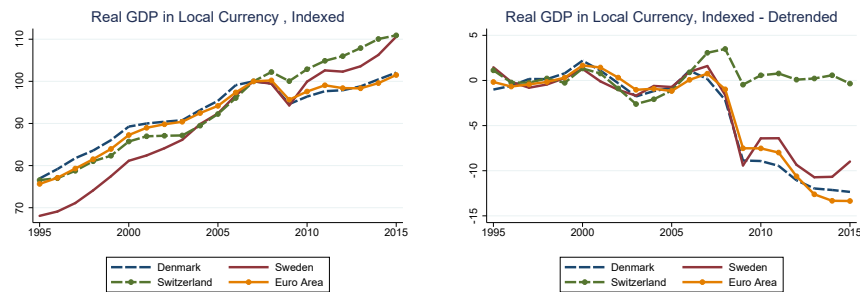


Figure 9: Gross Domestic Product in Constant Prices. Local Currency. Indexed so that $GDP^{2008}=100$. The right panel shows the detrended series using a linear time trend based on the 1995-2007 period.

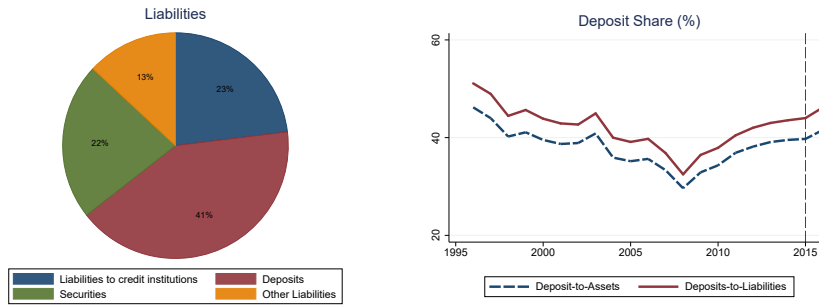


Figure 10: Decomposition of Liabilities (average 1996-2016) and Deposit Share for Swedish Banks.

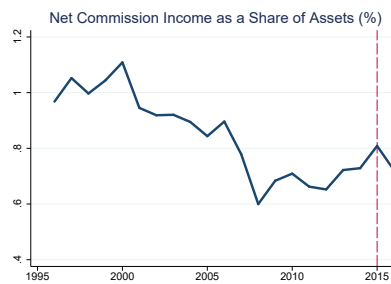


Figure 11: Net Commission Income as a Share of Total Assets for Swedish Banks. Source: Statistics Sweden.

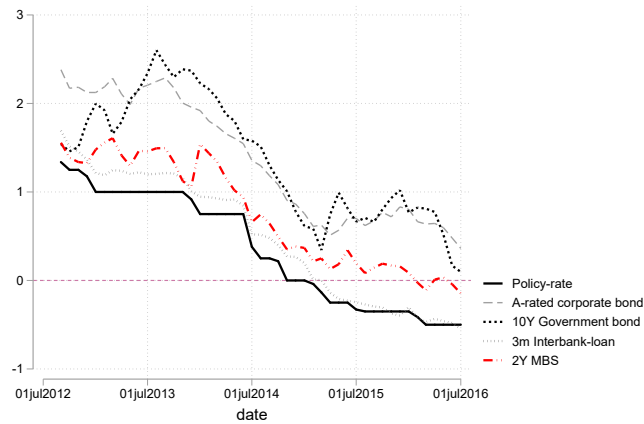


Figure 12: Other interest rates in Sweden. Source: Statistics Sweden.

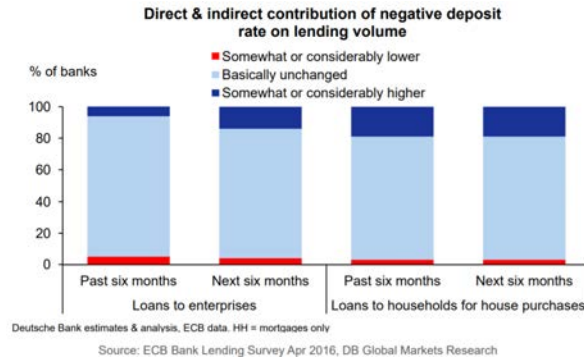


Figure 13: Share of banks answering that the negative ECB deposit rate has a negative, neutral or positive effect on their lending volume. Source: Deutsche Bank.

Appendix C: Marginal and Average Rate on Reserves

In our model, central bank reserves earn a single interest rate i^r . In reality, central banks can adopt exemption thresholds and tiered remuneration schemes so that not all reserves earn the same interest rate. Hence, even though the key policy rate is negative, not all central bank reserves necessarily earn a negative interest rate. Here we provide a short overview of the different remuneration schedules implemented in the Euro Area, Denmark, Japan, Sweden and Switzerland. For a more detailed analysis see Bech and Malkhozov (2016).

In the Euro Area, required reserves earn the main financing operations rate - currently set at 0.00 percent. Excess reserves on the other hand, earn the central bank deposit rate - currently set at -0.40 percent. Hence, only reserves in excess of the required level earn a negative interest rate. A similar remuneration scheme is in place in Denmark. Banks can deposit funds at the Danish central bank at the current account rate of 0.00 percent. However, there are (bank-specific) limits on the amount of funds that banks can deposit at the current account rate. Funds in excess of these limits earn the interest rate on one-week certificates of deposits - currently set at -0.65 percent.

The Riksbank issues one-week debt certificates, which currently earn an interest rate of -0.50 percent. While there is no reserve requirement, the Swedish central bank undertakes fine-tuning operations to drain the bank sector of remaining reserves each day. These fine-tuning operations earn an interest rate of -0.60 percent⁴¹. The Swiss central bank has the lowest key policy rate at -0.75 percent. However, due to high exemption thresholds the majority of reserves earn a zero interest rate. The Bank of Japan adopted a three-tiered remuneration schedule when the key policy rate turned negative. As a result, central bank reserves earn an interest rate of either 0.10, 0.00 or -0.10 percent.

⁴¹Any residual reserves earn the deposit rate of -1.25 percent.

Due to the tiered remuneration system, there is a gap between the average and the marginal reserve rate. Bech and Malkhozov (2016) calculate this gap as of February 2016, as illustrated in figure (14).

Central bank remuneration schedules (mid-February 2016)				Table 1
	European Central Bank	Sveriges Riksbank	Swiss National Bank	Danmarks Nationalbank
Exemption threshold	Minimum reserve requirement	.	Individual exemption	Current account limit
Aggregate amounts	Local currency, in billions			
Overnight deposits (reserves)				
Below threshold	113	-50 ¹	303	29
Above threshold	650	.	170	. ³
Term (one-week)	.	187	.	119
Policy rates	Basis points			
Overnight deposits (reserves)				
Below threshold	5	-60 ²	0	0
Above threshold	-30	.	-75	. ³
Term (one-week)	.	-50	.	-65
Weighted average rate	-25	-52	-27	-52
Marginal minus average rate⁴	-5	-8	-48	-13

¹ Amount of fine-tuning operations. In addition, overnight deposits with central bank represent SEK 0.01 billion. ² Rate applied to fine-tuning operations. Overnight deposits with central bank earn -125 basis points. ³ Amounts above the aggregate current account limit are converted into one-week certificates of deposit (Box 2). ⁴ Marginal rate is the rate on overnight deposits with central bank above exemption threshold.

Sources: Central banks; authors' calculations.

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Figure 14: Reserve Rates - Source: Bech and Malkhozov (2016).

Appendix D: Firm Problem

A firm that is allowed to reset their price in period t sets the price to maximize the present value of discounted profits in the event that the price remains fixed. That is, each firm i choose $P_t(i)$ to maximize

$$\mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left[\Pi^{T-t} \frac{P_t(i)}{P_T} Y_T(i) - \frac{W_T}{P_T} Y_T(i) \right] \quad (31)$$

where $\lambda_T \equiv q(\chi \exp\{-qC_T^b\} + (1-\chi) \exp\{-qC_T^s\})$, which is the weighted marginal utility of consumption and $\beta \equiv \chi\beta^b + (1-\chi)\beta^s$ ⁴².

Denoting the markup as $\mu \equiv \frac{\theta}{\theta-1}$, firms set the price as a markup over the average of expected marginal costs during the periods the price is expected to remain in place. That is,

⁴²Recall that the firm is owned by both types of households according to their respective population shares.

the first-order condition for the optimal price $P(i)_t^*$ for firm i is

$$\frac{P(i)_t^*}{P_t} = \mu \frac{\mathbb{E}_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \frac{1}{\Pi^{T-t}} \right)^\theta \frac{W_T}{P_T} Y_T \right\}}{\mathbb{E}_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \frac{1}{\Pi^{T-t}} \right)^{\theta-1} \frac{W_T}{P_T} Y_T \right\}} \quad (32)$$

This implies a law of motion for the aggregate price level

$$P_t^{1-\theta} = (1-\alpha) P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta} \Pi^{1-\theta} \quad (33)$$

where P_t^* is the optimal price from equation (32), taking into account that in equilibrium $P_t^*(i)$ is identical for all i . We denote this price P_t^* .

Since prices are sticky, there exists price dispersion which we denote by

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di \quad (34)$$

with the law of motion

$$\Delta_t = \alpha \left(\frac{\Pi_t}{\Pi} \right)^\theta \Delta_{t-1} + (1-\alpha) \left(\frac{1 - \alpha \left(\frac{\Pi_t}{\Pi} \right)^{\theta-1}}{1-\alpha} \right)^{\frac{\theta}{\theta-1}} \quad (35)$$

We assume that the disutility of labor takes the form $V(N_t^j) = \frac{(N_t^j)^{1+\eta}}{1+\eta}$. We can then combine equations (32) - (35), together with the aggregate labor-consumption trade-off (equation (8)) to get an aggregate Phillips curve of the following form:

$$\left(\frac{1 - \alpha \left(\frac{\Pi_t}{\Pi} \right)^{\theta-1}}{1-\alpha} \right)^{\frac{1}{\theta-1}} = \frac{F_t}{K_t} \quad (36)$$

where

$$F_t = \lambda_t Y_t + \alpha\beta \mathbb{E}_t \left\{ F_{t+1} \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\theta-1} \right\} \quad (37)$$

and

$$K_t = \mu \frac{\lambda_t \Delta_t^\eta Y_t^{1+\eta}}{z \exp\{-zY_t\}} + \alpha\beta \mathbb{E}_t \left\{ K_{t+1} \left(\frac{\Pi_{t+1}}{\Pi} \right)^\theta \right\} \quad (38)$$

and

$$\lambda_T \equiv z (\chi \exp \{-qC_T^b\} + (1 - \chi) \exp \{-qC_T^s\}) \quad (39)$$

Since every firm faces demand $Y(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$ and $Y_t(i) = N_t(i)$, we can integrate over all firms to get that

$$N_t = \Delta_t Y_t \quad (40)$$

Appendix E: Equilibrium

Non-linear Equilibrium Conditions

For given initial conditions Δ_0, b_0^b and a sequence of shocks $\{\zeta_t, \bar{l}_t\}_{t=0}^{\infty}$, an equilibrium in our model is a sequence of endogenous prices $\{i_t^s, i_t^b, i_t^r\}_{t=0}^{\infty}$ and endogenous variables $\{C_t^b, C_t^s, b_t^b, m_t^b, m_t^s, \tau_t^s, s_t, Y_t, \Pi_t, F_t, K_t, \Delta_t, \lambda_t, l_t, R_t, m_t, z_t\}_{t=0}^{\infty}$ such that the 20 equations listed below are satisfied.

$$\exp \left\{ -qC_t^b \right\} \zeta_t = \beta^b \left(1 + i_t^b \right) \mathbb{E}_t \left(\Pi_{t+1}^{-1} \exp \left\{ -qC_{t+1}^b \right\} \right) \zeta_{t+1} \quad (41)$$

$$\exp \left\{ -qC_t^s \right\} \zeta_t = \beta^s \left(1 + i_t^s \right) \mathbb{E}_t \left(\Pi_{t+1}^{-1} \exp \left\{ -qC_{t+1}^s \right\} \right) \zeta_{t+1} \quad (42)$$

$$C_t^b + m_t^b + \frac{1 + i_{t-1}^b}{\Pi_t} b_{t-1}^b = \chi Y_t + \frac{1 - \gamma}{\Pi_t} m_{t-1}^b + b_t^b \quad (43)$$

$$\frac{\Omega' \left(m_t^b \right)}{U' \left(C_t^b \right)} = \frac{i_t^b + \gamma}{1 + i_t^b} \quad (44)$$

$$\frac{\Omega' \left(m_t^s \right)}{U' \left(C_t^s \right)} = \frac{i_t^s + \gamma}{1 + i_t^s} \quad (45)$$

$$\Pi_t s_t = s_{t-1} + i_{t-1}^r R_{t-1} - \Pi_t \tau_t^s \quad (46)$$

$$s_t = R_t + m_t + m_t^s + m_t^b \quad (47)$$

$$Y_t = \chi C_t^b + (1 - \chi) C_t^s \quad (48)$$

$$\left(\frac{1 - \alpha \left(\frac{\Pi_t}{\Pi} \right)^{\theta-1}}{1 - \alpha} \right)^{\frac{1}{\theta-1}} = \frac{F_t}{K_t} \quad (49)$$

$$F_t = \lambda_t Y_t + \alpha \beta \mathbb{E}_t \left\{ F_{t+1} \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\theta-1} \right\} \quad (50)$$

$$K_t = \mu \frac{\lambda_t \Delta_t^\eta Y_t^{1+\eta}}{q \exp \left\{ -qY_t \right\}} + \alpha \beta \mathbb{E}_t \left\{ K_{t+1} \left(\frac{\Pi_{t+1}}{\Pi} \right)^\theta \right\} \quad (51)$$

$$\lambda_t = q \left(\chi \exp \left\{ -qC_t^b \right\} + (1 - \chi) \exp \left\{ -qC_t^s \right\} \right) \quad (52)$$

$$\Delta_t = \alpha \left(\frac{\Pi_t}{\Pi} \right)^\theta \Delta_{t-1} + (1 - \alpha) \left(\frac{1 - \alpha \left(\frac{\Pi_t}{\Pi} \right)^{\theta-1}}{1 - \alpha} \right)^{\frac{\theta}{\theta-1}} \quad (53)$$

$$z_t = \frac{i_t^b - i_t^s}{1 + i_t^s} l_t - \frac{i_t^s - i_t^r}{1 + i_t^s} R_t - \frac{i_t^s + \gamma}{1 + i_t^s} m_t - \Gamma \left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t \right) \quad (54)$$

$$\frac{i_t^b - i_t^s}{1 + i_t^s} = \frac{1}{\bar{l}_t} \Gamma_l \left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t \right) \quad (55)$$

$$-\Gamma_R \left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t \right) = \frac{i_t^s - i_t^r}{1 + i_t^s} \quad (56)$$

$$-\Gamma_m \left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t \right) = \frac{i_t^s + \gamma}{1 + i_t^s} \quad (57)$$

$$i_t^r = r_t^n \Pi_t^{\phi_\pi} Y_t^{\phi_Y} \quad (58)$$

$$i_t^s = \max \left\{ -\gamma, i_t^r \right\} \quad (59)$$

$$l_t = \chi b_t^b \quad (60)$$

Steady state

We denote the steady-state value of a variable X_t as X .

First, observe that in steady-state inflation is at the inflation target Π . As a result, there is no price dispersion ($\Delta = 1$).

Combining this with the Phillips curve, we have that steady-state output is pinned down by the following equation

$$\mu \frac{Y^\eta}{q \exp\{-qY\}} = 1 \quad (61)$$

From the Euler equation of a household of type j we have that

$$1 + i^j = \frac{\Pi}{\beta^j} \quad (62)$$

Using the steady-state interest rates, we can jointly solve for all bank-variables. Notice that in steady-state banks are satiated in reserves, and so $R = \bar{R}$ by assumption. Furthermore, if the intermediation cost function is additive between money and the other arguments (which we assume, see below), the steady-state level of money holdings for banks is independent of other bank variables. Therefore, only bank profits and bank lending have to be solved jointly.

Given total debt and interest rates, the borrowers budget constraint and money demand can be solved for steady state consumption and money holdings:

$$C^b = \chi Y + \frac{\Pi - 1 - i^b}{\Pi} b^b - \frac{\Pi - 1 + \gamma}{\Pi} m^b \quad (63)$$

$$\Omega'(m^b) = \frac{i^b + \gamma}{1 + i^b} U'(C^b) \quad (64)$$

Then, using the aggregate resource constraint we have that

$$C^s = \frac{1 - \chi^2}{1 - \chi} Y + \frac{\chi}{1 - \chi} \left(\frac{\Pi - 1 - i^b}{\Pi} b^b - \frac{\Pi - 1 + \gamma}{\Pi} m^b \right) \quad (65)$$

The savers money demand follows from

$$\Omega'(m^s) = \frac{i^s + \gamma}{1 + i^s} U'(C^s) \quad (66)$$

Finally, given the steady-state holdings of reserves and real money balances we can use the total money supply equation and the consolidated government budget constraint to solve for the remaining variables.

Log-linearized equilibrium conditions

We log linearize the non-linear equilibrium conditions around steady state, and define $\hat{X} \equiv \frac{X_t - \bar{X}}{\bar{X}}$. For the intermediation cost function we assume the following functional form

$$\Gamma \left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t \right) = \begin{cases} \left(\frac{l_t}{\bar{l}_t} \right)^\nu (z_t)^{-\iota} + \frac{1}{2} (R_t - \bar{R})^2 + \frac{1}{2} (m_t - \bar{m})^2 & \text{if } R_t < \bar{R} \text{ and } m_t < \bar{m} \\ \left(\frac{l_t}{\bar{l}_t} \right)^\nu (z_t)^{-\iota} & \text{if } R_t \geq \bar{R} \text{ and } m_t \geq \bar{m} \end{cases} \quad (67)$$

By combining the two Euler equations and the aggregate demand equation we derive the IS curve in equation (68), in which we define $\sigma = \frac{1}{qY}$. By combining the five supply side equations we derive the Phillips curve in equation (77). We define the real interest rate \hat{r}_t^n in equation (69), and the interest rate spread in equation (78). We also use the market clearing condition to substitute \hat{l}_t for \hat{b}_t^b . Hence, an equilibrium of the log-linearized model is a process for the 17 endogenous variables $\left\{ \hat{c}_t^b, \hat{c}_t^s, \hat{b}_t^b, \hat{m}_t^b, \hat{m}_t^s, \hat{y}_t, \hat{r}_t^n, \hat{\pi}_t, \hat{\tau}^s, \hat{s}_t, \hat{m}_t, \hat{R}_t, \hat{z}_t, \hat{\omega}_t, \hat{i}_t^b, \hat{i}_t^s, \hat{i}_t^r \right\}_{t=0}^\infty$ such that the 17 equations listed below are satisfied. Note that the expressions for \hat{R}_t and \hat{m}_t , in equations (81) and (82) respectively, only hold when the bank is not satiated.

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \sigma \left(\hat{i}_t^s - \mathbb{E}_t \pi_{t+1} - \hat{r}_t^n \right) \quad (68)$$

$$\hat{r}_t^n = \hat{\zeta}_t - \mathbb{E}_t \hat{\zeta}_{t+1} - \chi \hat{\omega}_t \quad (69)$$

$$\hat{c}_t^b = \hat{c}_{t+1}^b - \frac{1}{qc^b} \left(\hat{i}_t^b - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{\zeta} + \mathbb{E}_t \zeta_{t+1} \right) \quad (70)$$

$$\hat{c}_t^s = \hat{c}_{t+1}^s - \frac{1}{qc^s} \left(\hat{i}_t^s - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{\zeta} + \mathbb{E}_t \zeta_{t+1} \right) \quad (71)$$

$$\begin{aligned} c^b \hat{\pi}_t + c^b \hat{c}_t^b &= \hat{\pi}_t (\chi y + b^b) + \chi y \hat{y}_t + b^b \hat{b}_t^b - \frac{b^b}{\pi} \hat{i}_{t-1}^b - (1 + i^b) \frac{b^b}{\pi} \hat{b}_{t-1}^b \\ &\quad - m^b (\hat{m}_t^b + \hat{\pi}_t) + (1 - \gamma) \frac{m^b}{\pi} \hat{m}_{t-1}^b \end{aligned} \quad (72)$$

$$\frac{\Omega''(m^b) m^b}{\Omega'(m^b)} \hat{m}_t^b = -qc^b \hat{c}_t^b - \frac{i^b + \gamma - 1}{i^b + \gamma} \hat{i}_t^b \quad (73)$$

$$\frac{\Omega''(m^s) m^s}{\Omega'(m^s)} \hat{m}_t^s = -qc^s \hat{c}_t^s - \frac{i^s + \gamma - 1}{i^s + \gamma} \hat{i}_t^s \quad (74)$$

$$\hat{s}_t = \frac{R}{s} \hat{R}_t + \frac{m}{s} \hat{m}_t + \frac{m^b}{s} \hat{m}_t^b + \frac{m^s}{s} \hat{m}_t^s \quad (75)$$

$$\hat{\pi}_t + \hat{s}_t = \frac{1}{\pi} \hat{s}_{t-1} + \frac{R}{s\pi} \left(\hat{i}_{t-1}^r + i^r \hat{R}_t \right) - \frac{\tau^s}{s} (\hat{\tau}_t^s + \hat{\pi}_t) \quad (76)$$

$$\hat{\pi}_t = \kappa \hat{y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} \quad (77)$$

$$\hat{i}_t^b = \hat{i}_t^s + \hat{\omega}_t \quad (78)$$

$$\hat{\omega}_t = \frac{i^b - i^s}{1 + i^b} \left((\nu - 1) \hat{b}_t^b - \nu \hat{b} - \iota \hat{z}_t \right) \quad (79)$$

$$\begin{aligned} \hat{i}_t^s + \hat{z}_t &= \frac{\chi b^b}{(1 + i^s) z} \left(\hat{i}_t^b - \hat{i}_t^s + (i^b - i^s) \hat{b}_t^b \right) - \frac{R}{(1 + i^s) z} \left(\hat{i}_t^s - \hat{i}_t^r \right) \\ &\quad - \frac{m}{(1 + i^s) z} \left(\hat{i}_t^s + (i^s + \gamma) \hat{m}_t \right) - \frac{\Gamma}{(1 + i^s) z} \hat{i}_t^s \\ &\quad - \frac{\Gamma z}{\iota} \left(\nu \left(\hat{b}_t^b - \hat{b} \right) + \iota \hat{z}_t \right) - \frac{m(m - \bar{m})}{z} \hat{m}_t \end{aligned} \quad (80)$$

$$\hat{R}_t = \frac{1}{(1 + i^s) R} \left((\bar{R} - 1) \hat{i}_t^s + \hat{i}_t^r \right) - \hat{i}_t^s \quad (81)$$

$$\hat{m}_t = \frac{m - \bar{m}}{m} \frac{1 - i^s - \gamma}{i^s + \gamma} \hat{i}_t^s \quad (82)$$

$$\hat{i}_t^r = \hat{r}_t^n + \phi_\Pi \hat{\pi}_t + \phi_Y \hat{y}_t \quad (83)$$

$$\hat{i}_t^s = \max \left\{ -\gamma \beta^s - (1 - \beta^s), \hat{i}_t^r \right\} \quad (84)$$