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AN ANALYSIS OF THE IMPLICATIONS
FOR STOCK AND FUTURES PRICE
VOLATILITY OF PROGRAM TRADING
AND DYNAMIC HEDGING STRATEGIES

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An Analysis of the Implications for Stock Futures Price Volatility
of Program Trading and Dynamic Hedging Strategies

ABSTRACT

Recent advances in financial theory have created an understanding of the environments in which a real security can be synthesized by a dynamic trading strategy in a risk free asset and other securities. We contend that there is a crucial distinction between a synthetic security and a real security. In particular the notion that a real security is redundant when it can be synthesized by a dynamic trading strategy ignores the informational role of real securities markets. The replacement of a real security by synthetic strategies may in itself cause enough uncertainty about the price volatility of the underlying security that the real security is no longer redundant.

Portfolio insurance provides a good example of the difference between a synthetic security and a real security. One form of portfolio insurance uses a trading strategy in risk free securities ("cash") and index futures to synthesize a European put on the underlying portfolio. In the absence of a real traded put option (of the appropriate striking price and maturity), there will be less information about the future price volatility associated with current dynamic hedging strategies. There will thus be less information transmitted to those people who could make capital available to liquidity providers. It will therefore be more difficult for the market to absorb the trades implied by the dynamic hedging strategies. In effect, the stocks' future price volatility can rise because of a current lack of information about the extent to which dynamic hedging strategies are in place.

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1. INTRODUCTION

The introduction of futures and options markets in stock indexes is strongly associated with the use of programmed trading strategies. Such strategies are used for spot/futures arbitrage, market timing, and portfolio insurance. It is this last use of programmed trading strategies that raises fascinating theoretical questions, the answers to which may have practical importance for understanding the impact of such strategies on the volatility of stock and futures prices.

Recent advances in financial theory have created an understanding of the environments in which a real security can be synthesized by a dynamic trading strategy in a risk free asset and other securities.¹ The proliferation of new securities has been made possible, in part, by this theoretical work. The issuer of a new security can price the security based on its ability to synthesize the returns stream of the new security using a dynamic trading strategy in existing securities, futures and options. This use of dynamic trading strategies has been extended even further by eliminating the "new" security altogether and just selling the dynamic hedging strategy directly. Portfolio insurance is the best example of the latter phenomenon.

Herein we contend that there is a crucial distinction

¹ The seminal contribution is the Black-Scholes (1973) option pricing approach, whereby it is shown how a dynamic trading strategy in a stock and risk free asset can reproduce a European call or put option on the stock.

between a synthetic security and a real security. In particular the notion that a real security is redundant when it can be synthesized by a dynamic trading strategy ignores the informational role of real securities markets.² The prices of real securities convey important information to market participants, and this information will not be conveyed if the real security is replaced by synthetic trading strategies. In particular the replacement of a real security by synthetic strategies may in itself cause enough uncertainty about the price volatility of the underlying security that the real security is no longer redundant.

Portfolio insurance provides a good example of the difference between a synthetic security and a real security. One form of portfolio insurance uses a trading strategy in risk free securities ("cash") and index futures to synthesize a European put on the underlying portfolio. If a put was traded on a securities market, then the price of the put would reveal important information about the desire of people to sell stock consequent to adverse future price moves.³ For example, if everyone in the economy would like to get out of stocks before the price falls by more than 25%, then the price of such a put option would be very high. If only a few holders of stocks

² See Grossman (1977) for an elaboration of the informational role of securities and futures prices.

³ Throughout this paper "stock" is often used interchangeably with "stock index" to represent a portfolio of risky assets.

desired such protection then the put option's market price would be low. The put's price thus reveals information now about the fraction of people with plans to get out of (or into) stocks in the future. The put's price reveals the extent to which the strategies of people can cohere in the future. By showing people the true cost of their plan's it may discourage people from attempting to purchase too much insurance in exactly those circumstances when the dynamic hedging strategy would raise stock price volatility.⁴

All of the above informational consequences of trading in a real security are absent if the real security is replaced by dynamic hedging strategies alone. How does a purchaser of a given strategy (such as a synthetic put) know the cost of insurance? Surely the cost depends on how many other people are planning to carry out similar stock selling and purchasing plans in the future. What mechanism exists to aggregate across people the information about future trading plans which will determine the cost of the current insurance strategy?

The marketing of strategies rather than securities has far reaching implications for the volatility of the underlying stock and futures markets. There is no market force or price information which ensures that strategies can be implemented, or which informs the user of the total cost of implementation. In

⁴ The cost of the strategy is the potential upside gains that are foregone to protect against downside losses. If the stock volatility is high then this cost will be high. We will argue below that the volatility will be higher the larger is the number of investors using portfolio insurance strategies.

contrast, the purchaser of a security knows the cost of his purchase. For the economy as a whole, the price of the security reflects the cost of implementing the dynamic hedging strategy to which it may be equivalent. More importantly, the existence of a traded security will aggregate information (regarding future trading plans) which is currently dispersed among investors, and hence provide valuable information about the cost of implementing the strategy.

The current price of a traded security also reveals information to people who can currently plan to take liquidity providing positions in the future to offset the position changes implied by portfolio insurance strategies. For example, when a put option price is high, this reveals information that stock price volatility is high. Market makers, market timers, and other liquidity providers are thus informed that the future holds good opportunities for them. This leads them to make more capital available in the future to be used to take advantage of the stock price volatility. Of course, this will have the effect of reducing the actual volatility, since a lot of capital will be present to invest to take advantage of excessive price moves.

In the absence of a real traded put option (of the appropriate striking price and maturity), there will be less information about the future price volatility associated with

current dynamic hedging strategies.⁵ There will thus be less information transmitted to those people who could make capital available to liquidity providers. It will therefore be more difficult for the market to absorb the trades implied by the dynamic hedging strategies. In effect, the stocks' future price volatility can rise because of a current lack of information about the extent to which dynamic hedging strategies are in place.

These points are elaborated in this paper as follows. Section 2 presents a schematic model of the impact of portfolio insurance on the stock and futures markets. Section 3 discusses the strategies used by investors who use synthetic hedging strategies, and by market timers whose capital commitments can offset the effects of portfolio insurance. Section 4 develops a model of market equilibrium in a context where the number of users of dynamic hedging strategies is not known to all market participants. Section 5 discusses potential adaptations which may be useful to organized Exchanges in case the growth in the use of synthetic securities raises the information requirements necessary to maintain stable markets in the underlying securities.

⁵ In the Black - Scholes model, the volatility is assumed constant, so that an option of any strike price and maturity can be used to infer the volatility of the stock. Clearly, the situation considered here is one where the volatility is not constant. This is elaborated below.

2. THE ORGANIZATION OF THE MODEL

The purpose of this and the next two Sections is to provide a schematic model of the informational consequences of trading strategies which are designed to create synthetic securities. We wish to elaborate the idea that market timers must commit capital before they know the extent of usage, and the future price impact of the implementation of these strategies. This incomplete information will lessen their effectiveness in reducing the price volatility which can be caused when large portfolio insurance induced trades take place.

One purpose of the model to be developed below is to show that as the importance of portfolio insurance grows, then the price impact problem will also grow unless there is some mechanism by which the market can be informed in advance of the trades. The current market impact issues are minuscule relative to what would occur if 50% of the pension fund asset managers were to choose strategies designed to protect themselves against a loss on their stock portfolios. In order to minimize the market impact of such strategies, those who could provide substantial amounts of liquidity to the market would have to be informed substantially in advance so that they could choose not to commit their capital to other activities. In what follows the length of time between date 1 and date 2 represents the amount of time that market timers and other liquidity providers would have to avoid committing their capital to other activities so that this capital can support the purchase or sale of

securities in response to temporary price moves caused by the execution of portfolio insurance strategies.

It may help the reader to have a real example of the phenomenon under study. Such an example is the use of "sunshine trading" strategies in Stock Index Futures by the brokers for portfolio insurers (see Kidder, Peabody & Co.(1986)). A broker using the sunshine trading technique announces to the brokerage and investment community that after a fixed period of time large orders will be brought to the trading floor and auctioned off at the best price. The purpose of preannounced trading is to give the investment community time to bring "market timing capital", or to bring the orders of customers who want the other side of the trade, to the Exchange trading floor so that the execution of a large order will not cause an adverse price move.⁶

The simplest model which can bring out the distinction between real securities and synthetic securities has three trading dates:

At date 1, some fraction f of security holders choose a dynamic hedging strategy. At the same time market makers, market timers and other liquidity providers (who I will henceforth group together under the title of market timers) decide how much capital to set aside for their attempts to profit from temporary price movements.

At date 2, news arrives about the underlying worth of the stock portfolio. This triggers trades based upon the date 1 portfolio dynamic hedging strategy. The price change caused by the

⁶ This phenomenon shows that not only are portfolio insurers concerned about the price impact of their trades, but that they feel that the release of information before a trade can bring forth capital (and offsetting customer order flow) to enhance the liquidity of the market.

execution of the trades will depend on the amount of capital set aside at date 2 for market timing activity. It may be helpful to imagine that there are two possible prices at date 2, P_{2g} and P_{2b} depending on whether good or bad news about fundamentals arrives at date 2. The (market clearing) prices at which trades can be executed will depend on f as well as market timing capital (denoted by M) available at date 2. We denote this dependence by writing $P_{2g} = P_{2g}(f, M)$ and $P_{2b} = P_{2b}(f, M)$.

At date 3 the stock price returns to its normal level which reflects the underlying fundamental value of holding the stock portfolio. The normal level at date 3 depends on information about fundamentals which arrives at date 3. It may be helpful to imagine that there are two possible prices at date 3, P_{3g} and P_{3b} depending on whether good or bad news about fundamentals arrives at date 3. Of course, the news about fundamentals which arrived at date 2 will be relevant for determining the level of possible date 3 prices, and we capture this by writing $P_{3g} = P_{3g}(2g)$, if good news at date 3 was preceded by good news at date 2, or $P_{3b} = P_{3b}(2g)$ if bad news at date 3 was preceded by good news at date 2, and similarly for other combinations of date 2 and date 3 news.

Since we are focusing on the informational consequences of the substitution of synthetic securities for real securities, we assume that f is uncertain, i.e., a random variable. A summary of the resolution of uncertainty follows:

Date 1: A realization f occurs which is not public information.

Date 2: News about fundamentals arrives publicly prior to trade. The dynamic hedging strategy chosen at Date 1 is implemented.

Date 3: News about fundamentals arrives publicly prior to trade. Price is determined fully by fundamentals⁷.

For expositional simplicity, at this stage in the analysis we ignore transaction costs and the distinction between futures and spot transactions in the underlying stock portfolio. The

⁷ Date 3 is a theoretical device to tie down the equilibrium. The time between date 2 and date 3 is the length of time necessary for the temporary price impact of the date 2 trades to disappear. That is, the expected return from holding stocks from date 3 forward would be uncorrelated with the date 2 news event.

purpose of the model is to show how incomplete information at date 1 about the fraction f of portfolio managers using synthetic option strategies can leave market timers unprepared at date 2 to offset the trades of portfolio hedgers, and that this causes the date 2 stock price to be more volatile than it would have been had real put options been traded at date 1.⁸ We begin by explaining the behavior of each of the types of traders, and then analyze how the behavior determines market clearing prices.

⁸ It should be emphasized that there may be incomplete information about more than just the fraction of investor capital managed with the use of portfolio insurance strategies. There may also be incomplete information about the type of strategy used, e.g. there can be incomplete information about the horizon, and/or strike price of the implicit put options being used. We focus on incomplete information about f , for expositional simplicity alone. The basic principle would be unaffected by more complex types of incomplete information.

3. TRADING STRATEGIES UTILIZED

3A. MARKET MAKERS, MARKET TIMERS, AND OTHER LIQUIDITY SUPPLIERS

At date 1 members of this group must decide how much capital to commit to activities which would leave their capital unavailable for market timing at date 2. That is, at date 1 capital can be committed or invested in activities for which it would be very costly or impossible to withdraw the funds and use it to capitalize market making transactions at date 2. For example if a pension fund invests some of its capital in mortgages, then it will be very costly for it to sell these mortgages at short notice to use its capital to take advantage of a market timing opportunity. Similarly, an investment bank may commit its capital to financing various activities other than market timing. These date 1 commitments of capital to activities for which there is a large cost of withdrawal at date 2 will lessen the funds available for date 2 market timing activities.

How much capital will firms make available for market timing activities? Clearly, this depends on the date 1 expected reward from taking market timing positions at date 2. We now argue that this date 1 expected reward will be higher the larger is the volatility of date 2 expected stock returns around the normal expected return. For example, if market timers at date 1 knew for certain that the expected return at date 2 for holding the stock from date 2 to date 3 would equal the normal return for holding the risk associated with the stock fundamentals,

then they would have no particular incentive at date 1 to commit capital to date 2 market timing activities.

The above point can be clarified by reference to the situation where there is either good or bad news about fundamentals. Suppose that at date 2 the two possible prices in the absence of parties using portfolio insurance would be P_{2g}^* and P_{2b}^* . These numbers have the property that portfolio owners would be willing to hold their existing stock levels anticipating a random return of P_3/P_{2b}^* computed from the bad news of date 2 to date 3, and P_3/P_{2g}^* computed from the good news at date 2 to date 3.⁹ Suppose that the implementation of dynamic hedging strategies will cause the date 2 price to be lower than P_{2b}^* in the bad news state, and higher than P_{2g}^* in the good news state. Since by assumption, date 3 is a point where prices are driven by fundamentals, this implies that the expected return as of the date 2 good news state (for holding the stock until date 3) will be lower than the normal expected return, and the expected return in the date 2 bad news state will be higher than the normal expected return. In the date 2 bad news state the market timers will make a net expected reward by increasing their stock holding, and in the date 2 good news state they will make an expected reward by decreasing their holdings (possibly taking a short position).

The above argument shows that the market timing rewards

⁹ Throughout the paper, a price with a state subscript absent represents a random variable.

will be high the larger is P_{2g} above P_{2g}^* , and the smaller is P_{2b} below P_{2b}^* . This is precisely the statement that the larger is the excess volatility in the date 2 prices, the larger will be the expectation as of date 1 that rewards can be made from market timing activity at date 2. Thus given that there is a real opportunity cost of committing funds for market timing activities, a higher date 2 excess price volatility will bring forth more market timing capital. This supply curve for market timing capital will be denoted by $M(V)$, where V is the excess volatility of date 2 prices, as anticipated at date 1. Note that by definition M includes the possibility of leverage. That is, M gives the absolute value of the dollar size of the position that the market timer can take at date 2.

It is now possible to describe the trading activity of market timers at date 2. The fraction of $M(V)$ which is invested will depend on P_2/P_2^* . When that ratio is small (and less than 1) a larger proportion of $M(V)$ will be invested, but by definition never more than 100%. Similarly, when P_2/P_2^* is large (and larger than 1) up to $M(V)$ dollars worth of the stocks will be sold.¹⁰

When market clearing prices at date 2 are generated, it will be shown that the market timers trading strategy serves a stabilizing function. If at date 1 the market timers know that

¹⁰ This is a crude description of the extent to which date 1 commitments enable date 2 trades. Our argument requires only that the size of market timers' trades at date 2 is an increasing function of the volatility they anticipated as of date 1.

there is going to be date 2 volatility, then they will commit capital to be used at date 2 to buy stocks when the price is lower than its normal level, and to sell stocks when the price is above its normal level. This argument relies crucially on the hypothesis that market timers know the date 2 volatility at date 1. We will see that if volatility is generated by the use of synthetic securities, then this volatility will be larger the larger is the fraction of portfolio managers (f) at date 1 who commit to a dynamic hedging strategy. To the extent that market timers do not know f at the time they choose M , they will find it difficult to forecast volatility.

In the absence of perfect information about volatility, market timers will choose an M that is optimal for some average level of volatility, denoted by M_a . In situations where the volatility V is high, M_a will be less than $M(V)$. In situations where V is low, M_a will be higher than $M(V)$. Therefore the stabilizing role of market timers will be impeded by imperfect information about the determinants of price volatility.

3B. BUY AND HOLD PORTFOLIO MANAGERS

These parties do not follow dynamic hedging strategies. In particular, their risk preferences are such that, at prices P_{2g}^* and P_{2b}^* , they would keep their portfolio unchanged at date 2 in response to the date 2 news about fundamentals. In particular when $f = 0$, the whole market is composed of people with these risk preferences and P_2^* gives the price at which the expected

returns from holding stock are such that a buy and hold strategy (from date 1 to date 3) is optimal.

We are making a slightly artificial distinction between market timers and those following passive investment strategies. In general, if P_2 is less than P_2^* , then investors who planned to have a passive strategy as of date 1, may find a high expected return to increasing their investment in risky assets. Thus, this group may also serve a market timing function. However, it is our assumption that their response to temporary price moves is much smaller than market timers. This is because, their portfolio objective specifies a particular fraction of portfolio value to be invested in the risky asset, and a fall in price gives them a lower portfolio value. Thus even in the face of higher expected returns per unit risk, these investors need not increase significantly their holdings of risky assets due to the fall in their portfolio value when P_2 is less than P_2^* .

3C. USERS OF SYNTHETIC SECURITIES, PORTFOLIO INSURERS

An investor uses a dynamic trading strategy in market contexts where the securities which would generate his desired pattern of returns across states of nature are unavailable.¹¹ This is a statement about the risk preferences and information of the investor, the risk preferences and information of the

¹¹ See Leland (1980), and Benninga and Blume (1985) for an analysis of the sources of demand for portfolio insurance.

other market participants, as well about the number of explicit securities marketed. Trivially, if all investors were identical, then they would all choose buy and hold strategies in the market index portfolio. On the other hand if investors are sufficiently diverse, the only situation in which market equilibrium will involve all traders choosing buy and hold strategies at date 1 in real securities is if the market is explicitly complete, i.e., for every state s_3 , there exists a portfolio of securities which when held to date 3 gives \$1 if and only if state s_3 occurs (or equivalently, if European options at all possible striking prices are marketed, where some of the striking prices may have to depend on the history which leads up to the final payoff if investors desire path dependent final payoffs).

Our securities and futures markets allow an investor to achieve a middle ground between the above extremes. Investors may well be sufficiently diverse that a buy and hold strategy in a stock index is not optimal for everyone, however markets are not sufficiently complete that a buy and hold strategy in a risk free security and an option with the investor's desired striking price is marketed. The investor may still be able to achieve the same outcome (or close to it), by using a dynamic trading strategy to synthesize the desired security.¹²

Consider the following very simple example. Suppose that

¹² See Cox and Rubinstein (1985) for an exposition of dynamic trading strategies which synthesize options and other contingent claims.

the stock price is \$10 at date 1, and at date 2 it either rises or falls from its date 1 level by 10%. Suppose further that the at date 3 the stock price can either rise or fall from its date 2 level by 10%. Thus there are three possible date 3 prices : \$8.1, \$9.9, and \$12.1. Let the investor start with 100 shares, and assume that the risk free interest rate is 0%. Suppose that the investor's preferences are such that he wants to get the highest expected date 3 wealth subject to the constraints that: (a) his date 3 wealth is no lower than \$900, and (b) he is allowed to invest no more than his total wealth in the risky asset. If the expected return on the stock is higher than that of the risk free asset, then it can be shown that the optimal trading strategy for the investor is to (i) invest all of his date 1 wealth in the risky asset (i.e., buy 100 shares at date 1); (ii) if the price at date 2 is \$9, then he sells all 100 shares and invests in the risk free asset; (iii) if the price at date 2 is \$11, then he simply holds on to his 100 shares.

Notice that the above strategy makes the holdings of the risky asset very volatile. A high expected return is achieved (subject to the constraint that the portfolio have a terminal value no lower than \$900) by a high initial investment in risky assets supported by a plan to sell off all stocks at date 2 if the price falls. This is an extreme form of portfolio insurance. A plan which did not involve the sale of all stocks at date 2 in the event of a price fall would require a smaller initial investment in the risky asset, and have lower expected

returns.

The strategy also has the property that the final payoff to the strategy is path dependent (i.e., the strategy has a different payoff when the price reaches \$9.9 at date 3 by first reaching \$9 at date 2, than would be the case if \$9.9 is reached from \$11 at date 2). For some reason many portfolio insurers avoid the use of such path dependent strategies even though they yield a higher expected return for the same level of insurance.¹³

A strategy used by many portfolio insurers is a path independent one where a dynamic trading strategy is chosen which replicates the payoff which would derive from investing an amount of \$S in the stock and buying a put with a striking price of \$900.¹⁴ The value for S is found by noting that the cost of the put plus the investment in the stock S must equal the date 1 value of the portfolio which in the above example is \$1000. This strategy has the same qualitative property as the one given above: the risky asset is sold if the date 2 price is lower than the date 1 price. However the path of holdings in the stock is somewhat different: all of the portfolio is not invested in the risky asset at date 1, and all of the risky asset is not sold

¹³ The replication of such strategies in a complete market would require trading in real securities for which a buy and hold strategy would yield a path dependent payoff.

¹⁴ See Rubinstein (1985), and Brennan and Schwartz (1987) for a discussion of these strategies.

off at date 2 if the price falls.¹⁵

Another form of portfolio insurance, called constant proportion portfolio insurance (CPPI) moves the investment in the risky asset linearly with how much higher the value of the portfolio is than the insurance level (\$900 in the above example).¹⁶ This trading strategy also has the property that a fall in the stock price will lead the investor to reduce his holdings of risky assets.

In summary, the users of portfolio insurance will tend to have demands for stocks that are more price sensitive than those investors utilizing buy and hold strategies.

¹⁵ 68.97 shares of the stock are held at date 1, and if the price falls then 34.48 shares are held at date 2, while if the price rises then 97.18 shares are held at date 2.

¹⁶ See Black and Jones (1986), and Perold (1986) for a discussion of this strategy.

4. MARKET EQUILIBRIUM

In this Section we tie the strategies of various investors together for the purpose of analyzing market equilibrium. We will analyze 3 cases. In the first case market timers at date 1 know the extent to which dynamic hedging strategies are being used. In the second case, market timers do not know the extent to which such strategies are being used but real put options are traded at date 1. In the third case, market timers do not know the extent to which dynamic hedging strategies are being used and there are insufficient real index options markets to convey this information.

4A. EXTENT OF ADOPTION OF DYNAMIC HEDGING STRATEGIES IS KNOWN

For expositional simplicity, we focus on the case where there are two possible public news announcements about fundamentals at both date 2 and date 3. Hence a model of market clearing involves finding a date 1 price P_1 , and a date 2 price for each announcement P_{2g} , P_{2b} such that the securities market clears at each date and state. It is clear that if the fraction f of investors using dynamic hedging strategies is known (and the types of strategies being used are also known), then it will be possible for all parties to forecast the volatility of date 2 prices, and hence there will be prices P_1 , P_{2g} , and P_{2b} such that if all traders anticipate these prices, and if the dynamic hedging strategies are indeed feasible at these prices, then the stock market will clear at those prices.

Market Clearing at Date 2

The above remarks may be slightly clarified by the use of the following notation:

Let $X(P_2/P_2^*, M; N)$ be the demand function of market timers at date 2, thought of as a function of the price P_2 relative to its normal level, the capital which they can commit M , and the public news about fundamentals N . As we noted earlier, if $P_2 = P_2^*$, then they demand no shares. As P_2 falls relative to the P_2^* which is appropriate for the information N , they increase their holdings.

The demand function of the buy and hold investors is also a function $Y(P_2/P_2^*; N)$ of the price relative to its normal level. However unlike the market timers, for the reasons given above, this demand will not be very sensitive to price changes. In the extreme case of a buy and hold investor, it will be totally insensitive. We assume that if $P_2 = P_2^*$, then $Y = 100\%$. That is, if the market was composed only of buy and hold investors, then these investors would demand 100% of the outstanding shares of stock.

The desired holdings of those investors who are using a dynamic hedging strategy is $Z(P_2; N)$. Their desired holdings of shares will fall as P_2 falls. There may even be a critical level beyond which they desire to hold no shares.

Given the news N , a market clearing price P_2 will satisfy:

$$(1) \quad X(P_2/P_2^*, M; N) + (1-f)Y(P_2/P_2^*; N) + (f)Z(P_2; N) = 100\%$$

This is the statement that P_2 will adjust until 100% of the outstanding stock is held by those people who, at price P_2 , no longer desire to trade. We write the market clearing price as a function of f , and N : $P_2 = P_2(f, N)$. Note that if $f=0$, so that no dynamic hedgers are present, then the market clearing price will be P_2^* . This means that if good news arrives then $P_2 = P_{2g}^*$, and if bad news arrives then $P_2 = P_{2b}^*$. The difference between P_{2g}^* and P_{2b}^* gives a measure of the normal level of volatility in the market.

Now consider the case where $f > 0$, so dynamic hedgers are

present. In such a situation when bad news arrives, the market will no longer clear at P_{2b}^* . This is because the demand of the hedgers is lower than the demand of the buy and hold investors at P_{2b}^* . Market clearing will require a price lower than P_{2b}^* . How much lower depends on the impact of market timers. If market timers have a very large presence in the market, i.e., M is very large, then even a very small deviation of P_2 from P_2^* will cause large trades by market timers. On the other hand if M is small then it will take a large deviation of P_2 from P_2^* to clear the market when f is large.

In summary if V denotes the volatility of date 2 prices in response to news about fundamentals, then V will depend on f and M , which we write as $V(f, M)$. Volatility will rise with f and fall with M .

Market Clearing at Date 1

The above analysis of the market at date 2 can be used to analyze the behavior of market participants at date 1. Under the assumption that f is known at date 1, the market timers will be able to infer the volatility of date 2 prices, and hence their potential benefits from committing resources, M to market timing activities. In particular, the function $V(f, M)$ generates an aggregate demand for market timing services (which we denote by $M^d(V; f)$), since it implies a particular return to date 1 investments in obtaining capital commitments for the purpose of date 2 market timing activities. As we noted earlier there are

costs of obtaining market timing capital. These costs generate a supply curve for market timing capital M , denoted by $M^S(V)$. The intersection of these two curves (i.e., the M such that $M = M_D(V;f) = M_S(V)$) will generate an M and a V which depend on f , denoted respectively by $M(f)$ and $V(f)$.

The less costly it is to commit capital to market timing activities, the larger will be M for a given level of f . That is, the more it will be the case that the demands of market timers offset the demands of investors using dynamic hedging strategies. Thus, date 2 price volatility $V = V(f) = V(f, M(f))$ will be low if it is not costly to commit capital to market timing activities at date 1.

The Feasibility of Portfolio Insurance

Finally, M determines the feasibility of certain types of portfolio insurance. Recall that M determines the level of the date 2 price in the presence of bad news about fundamentals, P_{2b} . An insurer would not be able to offer a dynamic strategy which assured a price higher than P_{2b} . In the event that bad news arrives at date 2 the joint execution of all the portfolio insurance strategies will force the price down to P_{2b} , so insurers would not be able to execute stop loss orders at a price higher than P_{2b} . Of course, if it is known at date 1 that market timer presence will be large at date 2, then it will be possible to offer insurance at levels almost as high as P_{2b}^* , since P_{2b} will be almost as high as P_{2b}^* .

4B. EXTENT OF ADOPTION OF DYNAMIC HEDGING STRATEGIES IS UNKNOWN, BUT REAL PUT OPTIONS ARE TRADED AT DATE 1.

The previous analysis was predicated on the notion that the degree of date 2 price volatility was known at date 1. If this is not known then it will be difficult, if not impossible for market makers to know the benefits of their date 1 capital commitments, and for date 1 insurers to know that their date 2 trading strategy can be implemented. This is exactly the type of situation where a real put options market may have a very important role.

If portfolio insurers implement their strategies via the purchase of put options at date 1, then the price of put options will reveal the fraction of investors who are using portfolio insurance strategies. Since the price of the put is a function of the anticipated volatility of the stock, the price will equivalently reveal the volatility of the stock.

To understand the ability of prices to aggregate information, imagine that a fraction f of investors decides to use portfolio insurance, and in a market where this is known, a volatility $V(f)$ would be implied. This in turn would imply a particular date 1 price for the put, say $Q(V(f)) = Q(f)$. Now suppose that traders do not know what the volatility will be because they do not know f . Suppose, for example that this leads to a put price below $Q(f)$. Could this really represent a market equilibrium? It could not, because the users of dynamic hedging strategies would find it cheaper to use real puts to execute their trades than synthetic strategies, and this would

drive up the put price. A similar argument obtains on the downside when the put price is higher than $Q(f)$. It would then be optimal for some portfolio insurer to sell puts and cover this sale with a dynamic hedging strategy.¹⁷ In the terminology of Grossman (1976), the put price is a sufficient statistic for the one dimensional variable V .

After all investors have learned the information about the stock's volatility from observing the option price, the option can indeed be a redundant security (in the sense that its date 2 and date 3 value can be replicated using a dynamic hedging strategy in the risk free asset and the stock). However, since the option is not informationally redundant, the volatility of stock prices can be substantially lower in an economy where real options are traded than it would be in an economy in which

¹⁷ The above argument is true for situations where a given investor knows that his decision to use a portfolio insurance strategy is correlated with the decision of others, and therefore each investor has a little information about the overall fraction of users. The reader may wonder what would happen if each user of portfolio insurance did not know how many others are using it, and thus he would not know the volatility. This is irrelevant in a situation where the only variable which affects the put price is the volatility. In order for the put price to be below $Q(V)$, a substantial portion of the market must expect a volatility lower than V . A price below $Q(V)$, say Q_1 would be consistent with a lower volatility than V , say V_1 . Each investor desiring portfolio insurance who is certain that the volatility is V_1 , will be indifferent between the appropriate dynamic hedging strategy and holding the option at a price of Q_1 . However, if the investor is even slightly uncertain about his ability to execute the appropriate dynamic hedging strategy then he will prefer the option. The number of people who prefer the option will be proportional to the number of investors who desire portfolio insurance strategies. This will cause the price of the option to reveal the intensity of investor desire for portfolio insurance.

market timers have no way to forecast the extent to which their capital is in demand. This is because the option price will inform market timers about the profitability of committing their capital to volatility reducing trades at date 2. A high option price is suggestive of high date 2 price volatility, which is suggestive of a high expected return from committing capital to market making activity at date 2.

4C. EXTENT OF ADOPTION OF DYNAMIC HEDGING STRATEGIES IS UNKNOWN,
NO REAL PUT OPTIONS ARE TRADED AT DATE 1.

This is a situation where price signals about the extent of adoption of portfolio insurance strategies are absent, or arrive too late.¹⁸ At date 1, market timers must make capital commitments based upon incomplete information, and investors choose dynamic hedging strategies under incomplete information. As a consequence, a put option will no longer be a redundant security, i.e., it can be impossible to replicate its payoff using a dynamic hedging strategy because the stock volatility is unknown.

If investors continue to use portfolio insurance strategies

¹⁸ I assume that the date 1 price of the stock index does not reveal the intensity of date 1 adoptions. Date 1 is supposed to be the date at which market timers must make capital commitments to attempt to profit from date 2 price volatility caused by date 1 adoptions. If the price is already varying at date 1 from its normal level because of adoptions, then the dates should be relabelled and we should start the analysis at an earlier date. In general as described in Grossman and Stiglitz (1976), there will be "noise" in the stock price which will prevent it from completely revealing such information.

in the presence of uncertain volatility, then not only will volatility be uncertain but it will be larger than it would otherwise have been. Recall that the capital commitments of market timers serve to reduce volatility. In particular if option price or other information reveals the extent of portfolio insurance usage f , then in times of high usage, they commit more capital. That is, capital commitments can be tailored to the anticipated volatility caused by adoption of portfolio insurance strategies. The inability to tailor capital commitments will reduce the average gain from such market timing investments. Therefore average volatility can rise, and be accompanied by a fall in market timing capital commitments.

The above remarks can be better understood by the following example. Suppose 50% of the time no investors pursue portfolio insurance strategies at date 1, while the other 50% of the time most of the investors use portfolio insurance. (This is just a method for describing the uncertainty market timers have about adoption.) At date 1, the market timers do not know which type of situation they are facing. If no investors are using portfolio insurance strategies, then date 2 price volatility will be very low, and the benefits from date 1 capital commitment to market timing will be low. The situation is reversed if many investors are using portfolio insurance strategies. If market timers knew which situation they were in, then they would commit capital where appropriate, and actual volatility will be low. Lacking information about adoption,

however, they commit an "average" amount of capital which is correct for the "average" situation. As a consequence when adoption is low, their capital is unnecessary, and when adoption is high their capital is inadequate to prevent excessive date 2 price volatility.

It should be emphasized that in a continuous time version of this model, market participants will discover information about the intensity of portfolio insurance usage by observing realized stock price volatility. If stock price volatility is variable only because insurance adoption is variable, then realized volatility will be a very good signal for adoption intensity. Further, if adoption intensity changes slowly relative to the rate at which new information about fundamentals arrives, then market timers will be able to commit capital in response to observed changes in realized volatility in such a way that "excessive" volatility is reduced.

5. CONCLUSIONS AND RECOMMENDATIONS

The theoretical perspectives developed herein show that a synthetic security puts quite different informational burdens on market participants than a real security. If an investor chooses a dynamic trading strategy to synthesize a European put option, then he should be very concerned with the number of other investors who have chosen similar strategies. He may very well find his own strategy infeasible if a substantial number of other traders are using the same strategy. Even if his strategy is feasible, it may cost far more than anticipated. On the other hand if an investor could buy a real European option with the desired strike price and expiration day, then the price of the option would reveal the cost of the trading strategy. He would not have to know what other traders are doing in order to know whether his strategy is feasible.

The above informational role of prices occurs in many contexts. Hayek (1945) wrote:

"We must look at the price system as ... a mechanism for communicating information if we want to understand its real function ... The most significant fact about this system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action ... by a kind of symbol, only the most essential information is passed on ..."

I have shown elsewhere how this view of prices helps illuminate the informational role of securities markets.¹⁹ Focusing on the informational role of markets seems especially appropriate in

¹⁹ See Grossman (1976).

attempting to forecast the consequences of substituting real securities for synthetic securities.

I have argued that market timers and other liquidity providers will find it difficult to engage in stabilizing trades when they have poor information about the desire for their services. In the absence of a real options market it will be difficult to forecast price volatility, and hence difficult to forecast the effective demand for commitments of capital to market timing activities. Equally important, portfolio insurers will not know the cost of their strategies when they do not know the intensity of which other investors are using similar strategies. If a substantial number of investors suddenly decide to use insurance strategies predicated on historical levels of stock volatility, then this will raise stock and stock index futures price volatility.

The above theoretical perspective should not be construed as suggesting that (a) portfolio insurance, or dynamic hedging strategies are bad, or (b) that the increased use of such strategies has caused an increase in stock and futures price volatility. First, dynamic hedging strategies clearly play an important and useful role in increasing the feasible set of payoffs available to investors. It is costly (both privately and socially) to have liquid, real markets in every imaginable security. Dynamic hedging strategies permit us to economize on the number of active markets. Second, even if dynamic hedging strategies have contributed (or will contribute as their

importance grows) to stock price volatility, it does not follow that this is, in net, socially harmful, or worthy of regulation. To say that the use of a strategy imposes costs, hardly implies that these costs outweigh their benefits.

A more relevant question is: how can the Exchanges reduce the costs imposed if volatility increases with an increase in the adoption of dynamic hedging strategies? To answer this question, recall that the source of the problem is that market participants lack current information about the future trading plans of other participants. If many investors today adopt portfolio insurance strategies, then this implies that many will be sellers in the future when prices fall. This creates a current opportunity for market timers to commit resources, if only they were aware of the existing plans of other traders.

It hardly seems practical to solve this problem by suggesting that the Exchanges require all members to publicize their plans and the plans of their customers. Aside from the obvious enforcement difficulties, it would have the effect of forcing those people who may not be "informationless" portfolio hedgers to reveal their strategies. How could the Exchanges distinguish those investors who invest resources in the collection of market timing information, from those traders who are simply pursuing "informationless" trading for the purpose of synthesizing a put option? It surely will not help the informational efficiency of markets if the Exchanges force individuals to reveal (legally obtained) information which they

want to keep secret and which they have expended real resources to acquire. This would only reduce the amount of information collected in the first place, and thus inhibit market timing activities which are volatility reducing.

The maintenance of privacy for those investors who desire privacy is not a problem for our purposes if the disclosure is voluntary. An investor whose trades are for the purpose of synthesizing an option will have no need for secrecy in his trading plans. This is seen clearly in Kidder Peabody's "sunshine" trades for the portfolio insurance strategy firm of Leland, O'Brien Rubinstein Associates. They use preannounced trading to reveal themselves to be "informationless" and to enhance the number of investors willing to take the other side of their trades. It is interesting to note that Kidder Peabody was unable to fully preannounce their trades because of the possible conflict with Exchange rules against prearranged trading.

I think that the Exchanges could avoid the problem of prearranged trading, and also create a system conducive to voluntary disclosure by the following system. Each Exchange could set up a system where stop loss and other limited orders would be sent to a central computer where they are aggregated and the results made public continuously. For the New York Stock Exchange a special system is feasible.²⁰ With many

²⁰ The NYSE recognizes the need for providing advance information about future order flow. Its experiments with: (a) disclosing "market on close" orders prior to the close of

specialists already using an electronic "book", it is feasible to link the books across stocks and publicly display the size of the limit orders for various indexes in which there are futures or options. For example, the aggregate of buy orders could be computed under the hypothesis that each component in the S&P 500 falls in price by 1%. This can be done by looking at the "book" for each component stock, finding the number of shares to be bought on the specialists book if the price for that stock falls by 1%, and computing the weighted sum across the stocks in the S&P index. A similar calculation could be performed for a range of percentage up moves and down moves of the stocks in the index. The final result would be a chart indicating the total buy orders in the specialist books for the index at various relative price moves in the index. A similar chart could be constructed for sell orders. Finally, a chart could be constructed for the net buy (buy minus sell) orders for the index.

The transmittal of information about size of net buy orders at prices for the index which are away from the current price will allow investors to gauge the depth of the market. If net sells are very high (due to stop loss orders of portfolio insurers) at a price just below the current price, then market timers know that there will be opportunities for advantageous trades. They will have time to raise the capital (or contact

trading, and (b) disclosing order imbalances prior to the opening of trade, are examples of the type of mechanism I am proposing, and its motivation is similar to mine.

their own brokerage customers), which will lessen the impact of the execution of the stop loss orders.

There is another adaptation for the NYSE which would interact with the above system, and also enhance the execution of index trades. The Exchange could set up a system by which there is a limit order (and stop loss order) electronic book in various stock indexes. For example, members could enter such orders on the electronic book for the S&P 500 which would specify that if the index hits a particular level, then for example, 20 units of the index should be sold. When the index hit that level the computer would send sell orders for the components of the index to the specialists' posts for execution. The aggregate positions in the electronic book would be made public, so that the public would know how many index trades can be expected to be executed at various index prices. Again, the dissemination of such information would enhance the effectiveness of market timing activities, and reduce volatility.

It is somewhat more difficult to effect similar changes on futures Exchanges. However, it is crucial to realize that the index futures and options markets do not exist in a vacuum. If futures contracts are sold by a portfolio insurer, as a low (transactions cost) alternative to selling the portfolio's common stock holdings, then this must impact on the cash (i.e., stock) market. Index arbitrage will cause the cash prices to (roughly) stay in line with the futures prices. If the cash

market is illiquid, then the futures market will show an increase in volatility.

The relevant adaptation for futures markets would be a system by which brokers could commit themselves to execute orders at various prices, and the Exchange would aggregate these commitments and display them on a screen to various interested parties.²¹ To avoid liquidity reducing prearranged trades, the screen need not even identify the source of the orders. The Exchange would have to find some method of assuring that brokers carried out their commitments. It should be noted that the physical arrangement of most trading pits, and the hectic pace of activity may make it difficult for a broker to carry out his commitment. For example, a broker could always claim that he tried to carry out the commitment, but trading was too hectic. Some problems of implementation may be alleviated if a particular part of the trading pit is designated as the place where brokers with preannounced trades must stand. Of course, this may create prearrangement abuses. I don't believe that these problems are insurmountable, however the creation of an

²¹ This suggestion goes somewhat beyond current proposals to enable "sunshine" trading. Current proposals are concerned with transmittal of information to market participants regarding a broker's commitment to execute a trade at some time in the near future. The purpose of such a proposal is to lower the market impact to a portfolio insurer for a trade that he has just decided to make. My proposal is to show the market the whole schedule of trades, at prices away from the current price (aggregated over all customers who desire to participate). A floor trader could look at such a schedule and see that if the index price falls then there will be heavy selling. This alerts the floor to the need for more liquidity before heavy selling drives the price down.

electronic display of stop loss and limit orders clearly creates special problems for futures markets.²²

Finally, it should be emphasized that the suggested adaptations are for a "problem" that may not now exist, and may never exist. The implementation of any proposal contained herein should await careful measurement of the market impact of synthetic hedging strategies. The purpose of this Section is to illustrate the potential application of a theoretical perspective which emphasizes the informational role of markets; it should not be construed as a practical guide for regulation or for the modification of Exchange rules.

²² See Miller and Grossman (1986) for a discussion of some of the differences between futures markets and the NYSE.

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