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STICKY PRICES AS
COORDINATION FAILURE

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Sticky Prices As Coordination Failure

ABSTRACT

This paper shows that nominal price rigidity can arise from a failure to coordinate price changes. If a firm's desired price is increasing in others' prices, then the gains to the firm from adjusting its price after a nominal shock are greater if others adjust. This "strategic complementarity" in price adjustment can lead to multiple equilibria in the degree of nominal rigidity. Welfare may be much higher in the equilibria with less rigidity. In addition, with multiple equilibrium degrees of rigidity, the economy may have several short-run equilibria but a unique long-run equilibrium.

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I. INTRODUCTION

Many recent attempts to provide microfoundations for Keynesian macroeconomics are based on the idea that agents in a decentralized economy are unable to coordinate their actions. Coordination problems can arise in trade (e.g., Diamond, 1982), production (e.g., Bryant, 1983), and demand (e.g., Kiyotaki, 1985).¹ As Cooper and John (1986) point out, the essential feature of coordination failure models is "strategic complementarity": a positive dependence of an agent's optimal "effort" (for example, level of production or time spent searching for trading partners) on the effort of others. Economies with strategic complementarities may possess multiple equilibria in the level of effort, with high effort equilibria Pareto superior to low effort equilibria. This formalizes the idea that an economy may be stuck in an "underemployment" equilibrium even though a superior equilibrium exists.

While recent coordination failure models capture important Keynesian ideas, they appear irrelevant to one central feature of Keynesian economics: rigidities in nominal wages and prices. Current coordination failure models contain only real variables. Indeed, many authors present such models as an alternative to older Keynesian theories that explain underemployment with nominal rigidities.

¹See also Hart (1982), Weitzman (1982), Heller (1985), Shleifer (1986), and the references in Cooper and John.

This paper shows that nominal rigidities can arise from a failure to coordinate price changes. This failure has the essential features of coordination failures involving real variables such as the level of production or trade. Flexibility of one firm's price increases the incentives for other firms to make their prices flexible. This strategic complementarity implies that there may be multiple equilibria in the degree of nominal rigidity. Equilibria with less rigidity (more "effort" devoted to price adjustment) are often Pareto superior to equilibria with more rigidity.

We demonstrate these results in a "menu cost" model similar to the ones in Mankiw (1985), Blanchard and Kiyotaki (1985), and Ball and Romer (1987a). Section II describes the model and Section III presents our main results. In the model, imperfectly competitive firms choose whether to pay a small cost of adjusting prices after a nominal shock. Previous work shows that, because of externalities from price rigidity, considerable rigidity can be an equilibrium even if the result is large, highly inefficient fluctuations in output. This paper shows that the model has additional equilibria with less rigidity. Specifically, for a range of realizations of the shock, both full adjustment of prices and complete non-adjustment are equilibria; this implies that an economy facing a distribution of shocks possesses a continuum of equilibrium degrees of rigidity. Thus the economic fluctuations that result from nominal disturbances would be greatly reduced, and welfare might greatly improve, if firms could "agree" to make their prices more flexible. The size of the continuum of equilibria is increasing in the degree of strategic complementarity

in price adjustment.²

Section IV sketches two extensions of our analysis. First, we introduce heterogeneity among price setters, which leads to equilibria in which some prices adjust to a shock and others do not. In this version of the model, there may be multiple equilibria in the proportion of prices that adjust, and hence in the size of a shock's real effects. Second, we consider a simple dynamic version of our model in which firms choose between adjusting prices every period and adjusting every two periods. There can be multiple equilibria in the frequency of price changes, and therefore in the speed with which the price level adjusts to shocks. In addition, this example illustrates a difference between our model and other coordination failure models: while the economy may possess multiple short-run equilibria, it converges to a unique long-run equilibrium (output equal to the natural rate).

Section V discusses the policy implications of our results and offers conclusions.

II. THE MODEL

The model is the same as in Ball and Romer (1987a), where we provide more details. The economy consists of N producer-consumers, or "yeoman farmers."

²The surveys of menu cost models by Blanchard (1987) and Rotemberg (1987) contain other discussions of multiple equilibria in the degree of rigidity; Rotemberg's argument is closer to ours. (Rotemberg's paper and this one were written independently and should be viewed as complementary.) Cooper (1987) presents a model of labor and commodity contracts in which both predetermined nominal prices and full indexation to nominal shocks are equilibria.

N is large. Each farmer uses his own labor to produce a differentiated product, then sells the product and purchases the products of all other farmers. Farmers take each others' prices as given.

Farmer i 's utility function is

$$(1) \quad U_i = C_i - \frac{\varepsilon-1}{\gamma\varepsilon} L_i^\gamma - zD_i ,$$

where

$$(2) \quad C_i = N \left[\frac{1}{N} \sum_{j=1}^N C_{ij} \right]^{(\varepsilon-1)/\varepsilon} \varepsilon^{1/(\varepsilon-1)} ,$$

and where

L_i is farmer i 's labor supply;

C_i is an index of farmer i 's consumption;

C_{ij} is farmer i 's consumption of the product of farmer j ;

z is a small positive constant (the menu cost);

D_i is a dummy variable equal to one if farmer i changes his nominal price;

ε is the elasticity of substitution between any two goods ($\varepsilon > 1$);

γ measures the extent of increasing marginal disutility of labor ($\gamma > 1$).

Farmer i has a linear production function:

$$(3) \quad Y_i = L_i ,$$

where Y_i is farmer i 's output. A transactions technology determines the relation between aggregate consumption and real money balances:

$$(4) \quad C = \frac{M}{P} ,$$

where

$$(5) \quad C = \frac{1}{N} \sum_{j=1}^N C_j ;$$

$$(6) \quad P = \left[\frac{1}{N} \sum_{j=1}^N P_j^{1-\varepsilon} \right]^{1/(1-\varepsilon)} .$$

C is average consumption, P_i is the price of farmer i 's product, and P is the price index corresponding to (2).

Equations (1)-(6) determine the demand for farmer i 's product:

$$(7) \quad Y_i^D = \left(\frac{M}{P} \right) \left(\frac{P_i}{P} \right)^{-\varepsilon} .$$

Farmer i 's consumption equals his real revenues:

$$(8) \quad C_i = \frac{P_i Y_i}{P} .$$

Substituting (7) and (8) into (1) yields farmer i 's utility as a function of real balances and the farmer's relative price:

$$(9) \quad U_i = \left(\frac{M}{P} \right) \left(\frac{P_i}{P} \right)^{1-\varepsilon} - \frac{\varepsilon-1}{\gamma\varepsilon} \left(\frac{M}{P} \right)^\gamma \left(\frac{P_i}{P} \right)^{-\gamma\varepsilon} - zD_i .$$

Differentiation of (9) shows that farmer i 's utility-maximizing price neglecting menu costs is

$$(10) \quad P_i^* = P \phi M^{1-\phi} , \quad \phi = 1 - \frac{\gamma-1}{\gamma\varepsilon-\varepsilon+1} , \quad 0 < \phi < 1 .$$

ϕ is the elasticity of P_i^* with respect to the aggregate price level. (10) implies that, in the absence of menu costs, symmetric equilibrium occurs when $P_i = P = M$.

Combining (9) and (10) yields farmer i 's utility as a function of real balances, the ratio of his price to the utility-maximizing level, and the menu cost:

$$\begin{aligned}
 (11) \quad U_i &= \left(\frac{M}{P}\right)^{\gamma(1-\varepsilon+\varepsilon\phi)} \left[\left(\frac{P_i}{P_i^*}\right)^{(1-\varepsilon)} - \frac{\varepsilon-1}{\gamma\varepsilon} \left(\frac{P_i}{P_i^*}\right)^{-\gamma\varepsilon} \right] - zD_i \\
 &\equiv V\left(\frac{M}{P}, \frac{P_i}{P_i^*}\right) - zD_i.
 \end{aligned}$$

In what follows, we use (11) as our expression for utility.³

III. COORDINATION FAILURE IN PRICE ADJUSTMENT

In this section, we assume that the economy starts with $M=1$ and $P_i=P_i^*=1$ for all i . A shock occurs: M changes to $1+x$. Each farmer then decides whether to pay the menu cost and change his price to the new P_i^* . Part A of the section shows that, for a range of x , both adjustment by all farmers and non-adjustment by all are equilibria. This implies that the economy possesses a continuum of equilibrium degrees of nominal rigidity. Part B discusses the importance of strategic complementarity in price setting for this result. Part C compares welfare in the different equilibria.

The simplifying assumption that initially all prices equal one is ad hoc. Therefore, in an Appendix we follow Ball and Romer (1987a) in assuming that farmers choose initial prices optimally given a distribution of shocks with

³Our use of specific functional forms simplifies the analysis but is not important for the result below that there are multiple equilibria. Writing farmer i 's utility as a function of real money and his relative price, $W(M/P, P_i/P)$ (see (9)), all that is needed is $-W_{22} > W_{12} > 0$ at a point where $W_2(M/P, 1) = 0$ (subscripts denote partial derivatives). $W_2 = 0$ at $P_i/P = 1$ is necessary for symmetric equilibrium; $W_{22} < 0$ is the second order condition for farmer i ; $W_{12} > 0$ is necessary for the equilibrium to be stable; and $-W_{22} > W_{12}$ is required for strategic complementarity.

mean zero. Even though the mean of the ex post money supply is one, farmers set initial prices different from one -- that is, certainty equivalence fails -- because utility is not quadratic. We show, however, that the results in the text are altered only slightly.

A. Multiple Equilibria

We first solve for the range of shocks over which non-adjustment of all prices is an equilibrium. This is similar to computations in previous menu cost papers. Then we determine the range over which full adjustment is an equilibrium. The two ranges overlap, possibly substantially.⁴

Non-adjustment is an equilibrium when farmer i chooses not to pay the menu cost if no other farmer pays. If farmer i maintains a rigid price along with the others, then $D_i=0$. $P_i=P=1$, which implies $M/P = M$ and, using (10), $P_i/P_i^* = 1/M^{(1-\phi)}$. Thus the farmer's utility is $V(M, \frac{1}{M^{1-\phi}})$.

If farmer i pays the menu cost despite others' non-adjustment, then $D_i=1$. Adjustment of one price does not affect the aggregate price level, so $P=1$ and $M/P = M$. Adjustment allows farmer i to set $P_i=P_i^*$, so $P_i/P_i^* = 1$. Thus farmer i 's utility is $V(M, 1) - z$.

These results imply that farmer i chooses not to pay the menu cost -- and so rigidity is an equilibrium -- if

$$(12) \quad G_N < z ,$$

⁴One can show that if both full adjustment of prices and complete non-adjustment are equilibria, then there is a third equilibrium in which some prices adjust and others do not. This equilibrium is unstable.

$$G_N \equiv V(M, 1) - V\left(M, \frac{1}{M^{1-\phi}}\right).$$

G_N is the gain to a farmer from adjusting given that others do not adjust.

(12) states that rigidity is an equilibrium if G_N is less than the menu cost.

Taking a second order approximation of G_N around $M=1$ yields

$$(13) \quad G_N \approx \frac{-(1-\phi)^2}{2} V_{22} x^2,$$

where $x \equiv M-1$ and subscripts of V denote partial derivatives evaluated at $(1, 1)$

($V_{22}(1, 1) < 0$). The derivation of (13) uses the fact that $V_2(1, 1) = V_{12}(1, 1) = 0$.

(13) shows that the gain from adjusting is increasing in the size of the shock. (12) and (13) imply that the gain is less than the menu cost, and so rigidity is an equilibrium, if $|x| < x_N$, where

$$(14) \quad x_N = \sqrt{\frac{-2z}{(1-\phi)^2 V_{22}}}.$$

Price flexibility is an equilibrium when farmer i chooses to pay the menu cost if all others pay. If farmer i pays along with the others, then $D_i = 1$. $P_i = P = M$ (the equilibrium under flexible prices), which implies $M/P = 1$ and $P_i/P_i^* = 1$. Thus utility is $V(1, 1) - z$.

If farmer i does not pay the menu cost even though others do, then $D_i = 0$. $P = M$ but $P_i = 1$, which implies $M/P = 1$ and, using (10), $P_i/P_i^* = 1/M$. Farmer i 's utility is $V\left(1, \frac{1}{M}\right)$.

These results show that farmer i pays the menu cost if

$$(15) \quad G_A > z,$$

$$G_A \equiv V(1, 1) - V\left(1, \frac{1}{M}\right).$$

Flexibility is an equilibrium if G_A , the gain from adjusting given that others

adjust as well, is greater than the menu cost. A second order approximation yields

$$(16) \quad G_A \approx \frac{-1}{2} v_{22} x^2 .$$

Like G_N , G_A is increasing in the size of the shock. (15) and (16) imply that flexibility is an equilibrium if $|x| > x_A$, where

$$(17) \quad x_A = \sqrt{\frac{-2z}{v_{22}}} .$$

Combining (14) and (17) yields our central result:

$$(18) \quad \frac{x_N}{x_A} = \frac{1}{1-\phi} .$$

ϕ , the elasticity of P_i^* with respect to P , is between zero and one. Thus $x_A < x_N$. If $|x|$ is between x_A and x_N , then both rigidity and flexibility are equilibria.⁵

These results can be summarized as follows. For small monetary shocks -- $|x| < x_A$ -- each farmer refuses to pay the menu cost regardless of others' decisions, and so rigidity is the only equilibrium. For large shocks -- $|x| > x_N$ -- each farmer pays regardless of others, and so flexibility is the only equilibrium. But for shocks of intermediate size -- $x_A < |x| < x_N$ -- a farmer pays if and only if others do. To see why, consider a positive shock for concreteness and recall that a farmer's utility maximizing price, P_i^* , is $P_M^{\phi} M^{1-\phi}$. If others keep their prices fixed at one, then P_i^* rises to $M^{1-\phi}$. By

⁵As noted above, previous analyses of menu cost models (for example, Blanchard and Kiyotaki) derive the conditions under which nominal rigidity is an equilibrium. Thus they focus on what we call x_N and ignore the existence of multiple equilibria. An exception is Rotemberg (discussed in n. 2), who focuses on x_A and notes that there are multiple equilibria.

itself, this does not induce the farmer to pay the menu cost. But if others adjust their prices, then P rises to M and P_i^* rises to $M > M^{1-\phi}$. That is, if others adjust, they change their prices in the same direction as the money supply, which pushes P_i^* farther from one. Since the desired increase in P_i is larger, the incentive to adjust is larger, and so the farmer pays the menu cost. Since the farmer pays if and only if others do, there are two equilibria.⁶

These results concern equilibrium responses to a single shock. Now suppose that farmers face a distribution of shocks and choose rules for when to pay the menu cost, and consider the equilibrium rules. We restrict attention to equilibria in which all farmers pay the menu cost if $|x|$ is greater than a cutoff, x^* -- that is, if the money supply lies outside of $(1-x^*, 1+x^*)$. x^* is a natural measure of the degree of rigidity. Our results imply that any value of x^* between x_A and x_N is an equilibrium -- a farmer will adopt any value in this range as a cutoff if all others do. Thus there is a continuum of equilibrium degrees of nominal rigidity.⁷

⁶Accommodating monetary policy would be another source of multiple equilibria. Suppose the money supply rule is changed from $M=1+x$ to $M=1+c(P-1)+x$, $0 < c < 1$. Since $P=1$ if prices are rigid, x_N is not affected. But if prices are flexible, the equilibrium level of P and M is $1+[x/(1-c)]$ rather than $1+x$. As a result, $x_A = \sqrt{[-2z(1-c)^2]/V_{22}}$ and $x_N/x_A = 1/[(1-\phi)(1-c)]$. Thus accommodating monetary policy increases the range of multiple equilibria and makes multiple equilibria possible even if $\phi \leq 0$. Intuitively, accommodating policy creates an additional source of strategic complementarity: when others raise their prices, M rises, which raises P_i^* .

⁷The economy also possesses equilibria with less natural rules for when to change prices. For example, the set of realizations of M for which prices are rigid can be an asymmetric range, $(1-x_1^*, 1+x_2^*)$, or even a disconnected set, $\{(M_1^*, M_2^*), (M_3^*, M_4^*)\}$.

B. The Role of Strategic Complementarity

As the preceding discussion makes clear, multiple equilibria are possible because a farmer's incentive to adjust his price is greater if others adjust. In Cooper and John's terminology, there is "strategic complementarity" in price adjustment. Equations (12) and (15) show that this is tied to a simpler kind of strategic complementarity: the positive dependence of a farmer's utility-maximizing price neglecting menu costs on the prices of others. The degree to which G_A exceeds G_N depends on ϕ , the elasticity of P_i^* with respect to P .⁸

Equation (18) shows that the size of the range of shocks for which there are two equilibria depends on the degree of strategic complementarity. If there is little strategic complementarity -- that is, if ϕ is close to zero -- then a farmer's desired price, and hence his incentive to pay the menu cost, changes little when others adjust their prices. Thus x_N is close to x_A . In contrast, with strong strategic complementarity the range of multiple equilibria can be very large. This is illustrated by the special case of $\gamma \rightarrow 1$ (constant marginal utility of leisure). When $\gamma \rightarrow 1$, $\phi = 1$ and $P_i^* = P$ -- each farmer desires a price equal to the aggregate price level. If others do not change their prices, then farmer i has no desire to change his regardless of

⁸While the result that a farmer's utility-maximizing price increases with others' prices is clearly realistic, one can find cases in which it does not hold. ϕ can be negative -- prices can be strategic substitutes -- if farmers are risk averse in consumption (see Ball and Romer, 1987a) or if aggregate demand increases more than one-for-one with real money (as in Ball, 1987). If ϕ is negative, there is always a unique equilibrium in the fraction of farmers who adjust their prices.

the size of the shock: $G_N=0$ and $x_N \rightarrow \infty$ (see (13) and (14)). But if others adjust, the benefits of adjusting with them are positive, and so the farmer adjusts if the shock is sufficiently large. From (16) and (17), $G_A = (1/2)(\epsilon-1)x^2$ and $x_A = \sqrt{2z/(\epsilon-1)}$. Thus there are two equilibria for any $|x| > \sqrt{2z/(\epsilon-1)}$.⁹

Finally, note that the degree of strategic complementarity necessary for multiple equilibria is much weaker here than in most models of coordination failure. In previous work, an agent's choice of "effort" is a continuous variable. In this case, a necessary condition for multiple equilibria is that over some range an agent's effort increase more than one-for-one with others' effort: a reaction function must somewhere have slope greater than one to cross the 45 degree line more than once. In our model, the choice of whether to pay the menu cost is discrete. As a result, there are multiple equilibria as long as a farmer's desired price is simply increasing in others' prices. For some values of the shock, this weak strategic complementarity is enough for adjustment by others to push the gain from adjusting above the menu

⁹ ϕ also approaches one as $\epsilon \rightarrow \infty$ -- that is, as the product market approaches perfect competition. In this case, however, $G_N \rightarrow \infty$ and $x_N \rightarrow 0$ (x_N/x_A still approaches infinity because x_A approaches zero more quickly than x_N). When markets are competitive, a farmer's desired price change is small if others' prices are rigid, but the cost of forgoing even a small change is large. Formally, $G_N \rightarrow \infty$ because V_{22} grows more quickly than $(1-\phi)^2$ shrinks (see (13)).

cost.¹⁰

C. Welfare

Many coordination failure models possess multiple equilibria that can be Pareto ranked. In particular, high "effort" equilibria (for example, those with high levels of production) are often superior to low effort equilibria. It is natural to ask whether this is the case in the current model. When there are multiple equilibria in the degree of price rigidity, is less rigidity (more effort expended on price adjustment) better?

To study welfare, we assume as above that farmers face a distribution for the monetary shock, x , and pay the menu cost if $|x|$ exceeds a cutoff, x^* . For a symmetric distribution with mean zero, we derive the socially optimal value of x^* -- the one that maximizes farmers' expected utility. To determine the welfare properties of equilibrium rigidity, we compare the optimal x^* to x_A and x_N , the endpoints of the range of equilibria. We continue to assume that farmers initially set their prices to one, the equilibrium value in the absence of shocks; the Appendix studies the case in which initial prices are

¹⁰The importance of whether choice variables are continuous or discrete carries over to other settings. For example, multiple equilibria in the length of labor contracts requires that a firm's optimal contract length be increasing at least one-for-one in the length of other contracts (see the related discussion in Section IVB). In contrast, multiple equilibria in the fraction of firms that renegotiate contracts after a shock requires only that the gains from renegotiating be increasing in the fraction that renegotiate.

chosen optimally given the distribution of shocks.¹¹

Recall that a farmer's utility is $V(1,1) - z$ if all farmers pay the menu cost and $V(M, 1/M^{1-\phi})$ if none pays. Thus, since all pay if $|x| > x^*$, expected utility is

$$(19) \quad E[U_i] = [1 - (F(1+x^*) - F(1-x^*))][V(1,1) - z] \\ + \int_{M=1-x^*}^{1+x^*} V(M, \frac{1}{M^{1-\phi}}) f(M) dM ,$$

where $F(\cdot)$ is the cumulative distribution function for M and $f(\cdot)$ is the density function. The first order condition for the socially optimal x^* , denoted x_S , is

$$(20) \quad -2[V(1,1) - z] + V(1+x_S, \frac{1}{(1+x_S)^{1-\phi}}) \\ + V(1-x_S, \frac{1}{(1-x_S)^{1-\phi}}) = 0 ,$$

where we use the fact that $f(1+x) = f(1-x)$ by our assumption that $f(\cdot)$ is symmetric around one. A second order approximation leads to

$$(21) \quad x_S = \sqrt{\frac{-2z}{V_{11} + (1-\phi)^2 V_{22}}} .$$

Our central welfare result follows from substituting the appropriate derivatives of $V(\cdot)$ into (21) and the expressions for x_N and x_A :

$$(22) \quad x_A < x_S < x_N .$$

¹¹We study average welfare given a distribution of shocks because the welfare effect of rigidity after an individual shock depends on the sign of the shock (Mankiw, 1985; Ball and Romer, 1987a). Non-adjustment to a fall in the money supply reduces output and welfare. But non-adjustment to a positive shock increases output. This raises welfare because, under imperfect competition, the no-shock level of output is too low.

Since $x_S < x_N$, there is a range of equilibrium values of x^* ($x_S < x^* < x_N$) with too much rigidity -- in these equilibria, all farmers would be better off if the cutoff were lowered. Since $x_S > x_A$, there is a range of equilibria with too much flexibility. Finally, the social optimum ($x^* = x_S$) is always an equilibrium.

The reason that too much rigidity is possible is similar to the one in Ball and Romer (1987a). Suppose that all farmers start with an arbitrary x^* . If one farmer lowers his cutoff while the others do not, the farmer's only benefit is that he sets $P_i = P_i^*$ more frequently. But if others reduce x^* as well, there is an additional benefit: stabilization of real aggregate demand. Because the incentive for an individual to reduce x^* is smaller than the gain if all do, values of x^* above x_S can be equilibria.

Values of x^* below x_S can be equilibria -- there can be too much flexibility -- because a farmer's gain from raising x^* is also smaller if he does so by himself than if all do. If the others do not join the farmer in raising x^* , then for some shocks he does not adjust his price but others do. Others' adjustment increases movements in P_i^* , which raises the farmer's loss from non-adjustment. (Others' adjustment still benefits the farmer by stabilizing real aggregate demand, but this effect is smaller.)

While both excessive rigidity and excessive flexibility are possible, the magnitudes of the potential losses are very different. Neglecting the menu cost, full flexibility is always optimal; thus the net loss from too much flexibility is bounded by the menu cost. In actual economies, menu costs appear small. In contrast, Ball and Romer (1987a) show that the loss from too

much rigidity can be arbitrarily large, because the externality from increased fluctuations in real aggregate demand can be large compared to the private cost of rigidity. Thus excessive price flexibility is not likely to be a serious economic problem, while excessive rigidity may be.

IV. EXTENSIONS

A. Heterogeneous Agents

In the model of Section II, multiple equilibria arise when each farmer chooses to adjust his price if and only if others do. The desire to make the same decision as others is crucial. A natural question is whether multiple equilibria are possible if heterogeneity leads some agents to adjust while others do not. This section shows that models with heterogeneity can possess multiple equilibria in the proportion of prices that adjust, and therefore in the size of the real effects of a nominal shock. We focus on heterogeneity in the size of menu costs, which is the simplest case. Strategic complementarity is necessary for multiple equilibria; the sufficient condition depends on the distribution of the menu cost. We also briefly discuss the case of heterogeneous productivity shocks.

Assume that the menu cost, z , varies across farmers with a cumulative distribution function $H(z)$. After a shock, farmers with z below some critical level adjust their prices and the others do not. Let k be the proportion that adjust. We derive an equilibrium condition for k .

Let $P_A(x,k)$ be the price set by those who adjust and let $P(x,k)$ be the aggregate price level. Note that $P_A = P_1^* = P^\phi(1+x)^{1-\phi}$ and (approximating

(6) $P \approx kP_A + (1-k)$. These relations imply

$$(23) \quad P_A(x,k) \approx 1 + \frac{1-\phi}{1-\phi k} x .$$

By reasoning similar to that of Section III, the gain from adjusting is

$$(24) \quad G(x,k) = V\left(\frac{1+x}{P(x,k)}, 1\right) - V\left(\frac{1+x}{P(x,k)}, \frac{1}{P(x,k)^\phi (1+x)^{1-\phi}}\right) .$$

Using (23) and (24), one can show that

$$(25) \quad G(x,k) \approx \frac{-1}{2} \left(\frac{1-\phi}{1-\phi k}\right)^2 V_{22} x^2 .$$

The crucial result is

$$(26) \quad \frac{\partial G(x,k)}{\partial k} > 0 .$$

The gain from adjusting is increasing in the proportion of firms that adjust. This is a generalization of the earlier result that the gains are greater when all adjust than when none adjusts. Again, adjustment by others moves the price level in the same direction as the money supply, which increases the deviation of P_1^* from one.

A farmer pays his menu cost if it is less than $G(x,k)$. Thus the proportion that pay is $H(G(x,k))$, and an equilibrium k is one that satisfies $k=H(G(x,k))$. A necessary condition for multiple equilibria is $\frac{\partial H(G(x,k))}{\partial k} > 0$ over some range. Since $\frac{\partial H(G(x,k))}{\partial k} = \frac{dH}{dG} \cdot \frac{\partial G}{\partial k}$ and $H(\cdot)$ is increasing over some range, the condition reduces to (26), which holds because of strategic complementarity. The sufficient condition depends on the size of x and the shape of $H(\cdot)$; it is easy to find examples both of multiple equilibria and of unique equilibria.

Heterogeneity in real shocks leads to similar results. Suppose that the

production function, (3), is replaced by

$$(27) \quad Y_i = \theta_i L_i .$$

Assume that θ_i varies across farmers, and that a shock to θ_i occurs at the same time as the monetary shock. In this version of the model, a farmer chooses to pay the menu cost if θ_i is above an upper cutoff or below a lower cutoff; both critical values depend on x and k . Again, one can show that multiple equilibria are possible and that strategic complementarity is a necessary condition.

B. Long-Run and Short-Run Effects of Nominal Shocks

This section describes a difference between our model and previous work on coordination failures. In earlier models, which include only real variables, there is never any reason for an economy in an "underemployment" equilibrium to leave it. For example, if each agent in the Diamond model does not search because others do not search, this situation does not improve over time. In contrast, our model of nominal rigidities suggests differences between short-run and long-run equilibria. It is plausible to suppose that eventually menus wear out or enough shocks accumulate to cause all firms to adjust their prices. If this is the case, then our model implies that there is a unique long-run response to a shock -- prices adjust fully and output converges to the natural rate -- but that there may be multiple equilibrium

transition paths.¹²

To formalize this idea, we add simple dynamics to our model. Assume that the money supply follows a random walk. A farmer sets his price for a fixed length of time and chooses the length to maximize utility. Specifically, he chooses between setting his price every period after observing the current money supply and setting it for two periods at every even period. The farmer pays a fixed menu cost for every adjustment.¹³

This version of the model can possess multiple equilibria in the frequency of price changes. The proof is a straightforward extension of the one for the static model. Intuitively, more frequent price adjustment implies that the aggregate price level responds more quickly to shocks. By making the price level more volatile, frequent adjustment by others increases the fluctuations in a farmer's desired nominal price, and thus raises his incentive to adjust frequently. For some parameter values, a farmer's incentive to change his price every period exceeds the added menu costs if and only if others adjust every period.

¹²Of course it is not clear whether a unique long-run equilibrium for output is a desirable feature of our model. While many modern Keynesians believe that output converges to a natural rate, many older Keynesians emphasize permanent underemployment equilibria. In addition, Campbell and Mankiw (1986) and Blanchard and Summers (1986) provide empirical evidence that shocks to output and employment have permanent effects.

¹³This is a version of the "contract length" model introduced by Gray (1978). Note that we assume synchronized timing of price changes -- if farmers set prices for two periods, they all set them in the same (even) periods. Since the model contains only aggregate shocks, this is the equilibrium timing (see Fethke and Policano, 1984, and Ball and Romer, 1987b). Our results would not change if we assumed asynchronized timing.

Since all prices adjust eventually, money is neutral in the long run. But there may be two possible short-run responses to a monetary shock: full price adjustment and neutrality, or price rigidity and real effects that last for a period.

We believe that similar results arise in more general models, such as continuous time models in which firms can choose any frequency of price changes. These models are likely to possess richer sets of equilibria (for example, several equilibrium speeds at which output returns to the natural rate following a shock). We suspect that, as in models with heterogeneity, sufficient conditions for multiple equilibria depend on assumptions about functional forms.¹⁴

V. CONCLUSION

This paper shows that nominal price rigidity can arise from a failure of firms to coordinate price changes. Increases in price flexibility by different firms are strategic complements -- greater flexibility of one firm's price raises the incentives for other firms to make their prices more flexible. Strategic complementarity leads to multiple equilibria in the degree of nominal rigidity, and welfare may be much higher in the low rigidity equilibria. Thus the inefficient economic fluctuations resulting from nominal

¹⁴In this case, the specification of the cost of changing prices is likely to be important. In Ball's (1987) model of labor contracts, the cost of writing a contract is fixed and the equilibrium contract length is unique. One can show, however, that if shorter contracts are less expensive than longer contracts, the model may possess multiple equilibria.

shocks might be greatly reduced if agents could "agree" to move to a superior equilibrium.

The essential element of our model -- strategic complementarity leading to multiple equilibria -- is also central to models of coordination failure in trade, production, and demand. But in contrast to previous work, our model possesses a unique long-run equilibrium despite the multiplicity of short-run equilibria.

Multiple equilibria that are Pareto ranked imply a role for government policy: moving the economy to a superior equilibrium. In actual economies, for example, incentives for firms to sign shorter labor contracts or to adopt greater indexation might lead to an equilibrium with less nominal wage rigidity. Government intervention could be temporary -- after a policy led firms to increase flexibility, each firm would have sufficient private incentives to maintain flexibility.

As Cooper and John point out, strategic complementarity implies that policy has a multiplier.¹⁵ For example, if intervention led some firms to shorten their labor contracts, this would increase the incentives for other wage and price setters to reduce rigidity. Thus a policy aimed at part of the labor market could reduce rigidity throughout the economy.

¹⁵There is a multiplier even if the equilibrium degree of rigidity is unique (for example, in a dynamic model). The size of the multiplier is increasing in the degree of strategic complementarity.

APPENDIX

This Appendix relaxes the assumption that all prices equal one before the monetary shock occurs. We assume instead that farmers choose initial prices optimally and show how this affects our results. The analysis draws heavily on Ball and Romer (1987a). As in that paper, we assume that the distribution of the monetary shock is symmetric around zero, single-peaked, and continuous.

The price that a farmer sets before observing the money supply depends on others' initial prices and on the value of the cutoff x^* . In symmetric equilibrium, each farmer's initial price is

$$(A1) \quad P_0(x^*) \approx 1 + \frac{\gamma \hat{\sigma}_M^2}{2} (x^*)^2,$$

where $\hat{\sigma}_M^2(x^*)$ is the variance of M conditional on $1-x^* < M < 1+x^*$ (see our earlier paper). Given our use of second order approximations, assuming initial prices equal to P_0 rather than one does not affect our results about equilibrium rigidity: the expressions for x_N and x_A in the text remain valid (our earlier paper shows this for x_N). We now show, however, that the socially optimal degree of rigidity changes slightly.

If initial prices are P_0 and prices are rigid, then $M/P = M/P_0$ and $P_i/P_i^* = P_0^{1-\phi}/M^{1-\phi}$. Thus when initial prices are set optimally, a farmer's expected utility, (19), becomes

$$(A2) \quad E[U_i] = [1 - (F(1+x^*) - F(1-x^*))][V(1,1) - z] \\ + \int_{M=1-x^*}^{1+x^*} V\left(\frac{M}{P_0(x^*)}, \frac{[P_0(x^*)]^{1-\phi}}{M^{1-\phi}}\right) f(M) dM.$$

The first order condition for x_S is

$$\begin{aligned}
(A3) \quad & -[f(1-x_S)+f(1+x_S)][V(1,1)-z] \\
& + f(1-x_S)V\left(\frac{1-x_S}{P_0(x_S)}, \frac{[P_0(x_S)]^{1-\phi}}{(1-x_S)^{1-\phi}}\right) \\
& + f(1+x_S)V\left(\frac{1+x_S}{P_0(x_S)}, \frac{[P_0(x_S)]^{1-\phi}}{(1+x_S)^{1-\phi}}\right) \\
& + \left\{ \int_{M=1-x_S}^{1+x_S} \left[V_1\left(\frac{M}{P_0(x_S)}, \frac{[P_0(x_S)]^{1-\phi}}{M^{1-\phi}}\right) \left(\frac{-M}{[P_0(x_S)]^2}\right) \right. \right. \\
& \left. \left. + V_2\left(\frac{M}{P_0(x_S)}, \frac{[P_0(x_S)]^{1-\phi}}{M^{1-\phi}}\right) \left(\frac{(1-\phi)[P_0(x_S)]^{-\phi}}{M^{1-\phi}}\right) \right] f(M)dM \right\} P_0'(x_S) = 0,
\end{aligned}$$

where $P_0' \equiv (\partial P_0 / \partial x^*)$. Taking a second order approximation and substituting (A1) for P_0 yields

$$\begin{aligned}
(A4) \quad & -[f(1-x_S)+f(1+x_S)][V(1,1)-z] \\
& + f(1-x_S)\{V(1,1) + V_1[-x_S - (\gamma/2)\hat{\sigma}_M^2] + (1/2)V_{11}x_S^2 + (1/2)V_{22}(1-\phi)^2x_S^2\} \\
& + f(1+x_S)\{V(1,1) + V_1[x_S - (\gamma/2)\hat{\sigma}_M^2] + (1/2)V_{11}x_S^2 + (1/2)V_{22}(1-\phi)^2x_S^2\} \\
& - (\gamma/2)V_1(f(1+x_S)+f(1-x_S))(x_S^2 - \hat{\sigma}_M^2) = 0.
\end{aligned}$$

Finally, using the fact that $f(1+x)=f(1-x)$, the solution for x_S is

$$(A5) \quad x_S \approx \sqrt{\frac{-2z}{V_{11}+(1-\phi)^2V_{22}-\gamma V_1}}.$$

Substituting the derivatives of $V(\cdot)$ into (A5) establishes that $x_S < x_N$ and that x_S can be either greater or less than x_A . The possibility of $x_S < x_A$ -- which implies that all equilibria possess too much rigidity -- is the main departure of these results from the ones in the text. The explanation is that P_0 , the price level under rigidity, is greater than one, its level in the

text. Thus real aggregate demand under rigidity is lower than in the text, which makes it more likely that reducing rigidity would increase welfare.

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