NBER WORKING PAPER SERIES

DISASTER RISK AND ASSET RETURNS:
AN INTERNATIONAL PERSPECTIVE

Karen K. Lewis
Edith X. Liu

Working Paper 23065
http://www.nber.org/papers/w23065

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
January 2017

Disaster Risk and Asset Returns: An International Perspective
Karen K. Lewis and Edith X. Liu
NBER Working Paper No. 23065
January 2017
JEL No. F3,F4,G1

ABSTRACT

Recent studies have shown that disaster risk can generate asset return moments similar to those observed in the U.S. data. However, these studies have ignored the cross-country asset pricing implications of the disaster risk model. This paper shows that standard U.S.-based disaster risk model assumptions found in the literature lead to counterfactual international asset pricing implications. Given consumption pricing moments, disaster risk cannot explain the range of equity premia and government bill rates nor the high degree of equity return correlation found in the data. Moreover, the independence of disasters presumed in some studies generates counterfactually low cross-country correlations in equity markets. Alternatively, if disasters are all shared, the model generates correlations that are excessively high. We show that common and idiosyncratic components of disaster risk are needed to explain the pattern in consumption and equity co-movements.

Karen K. Lewis
Department of Finance, Wharton School
2300 SHDH
University of Pennsylvania
Philadelphia, PA 19104-6367
and NBER
lewisk@wharton.upenn.edu

Edith X. Liu
Federal Reserve Board of Governors
20st & Constitution Ave NW
Washington, DC 20551
edith.x.liu@frb.gov

The risk of disasters has long been proposed as an explanation for a variety of financial market anomalies. Key among these anomalies is the high equity premium in the face of relatively smooth consumption. As originally presented by Reitz (1988) and advanced by Barro (2006, 2009), a low probability of a large decline in output can sufficiently increase the variability in intertemporal marginal utility to deliver the level of equity premium seen in U.S. data. In combination with risk of government default, the potential for these disasters can also explain the level of government bill rates. Moreover, as Wachter (2013) shows, time varying disaster risk can help explain the volatility of equity returns and government bills.

Since disasters are rare in the U.S. time series, this literature uses international data to measure both the frequency and size of these events. To obtain these measures, each country is typically assumed to face the same potential decline in consumption, parameterized from observed disasters across all countries.[1] However, if true, this assumption carries important implications for the magnitude and co-movements in international asset returns. If all countries face a similar disaster risk, this risk should affect the correlation of asset returns across countries, as well.

In this paper, we study the international asset pricing moments and co-movements implied by a standard domestic-based disaster risk model. Using consumption and asset price data for seven OECD countries, we begin by evaluating each country in isolation following the standard approach in this literature. Within the constant probability of disaster framework as in Barro (2006), we ask whether differences in exposure to disaster risk can explain

---

[1] In a modification of this approach, Nakamura, Steinsson, Barro and Ursua (2013) estimate endogenous differences in timing, magnitude, and length of disasters while maintaining the assumption that the frequency and size distribution is time invariant and the same across countries. Similar to our model below, they allow for correlation in the timing of disasters. However, they use this information to match the U.S. asset pricing moments alone and do not consider the international asset pricing implications. We discuss their approach relative to ours below.

the cross-section of asset return moments for each individual country. To examine these implications, we choose model parameters that best fit the asset pricing moments using Simulated Method of Moments. For this purpose, we allow for cross-country deviations in the size of the disaster, the probability of government default, and the dividend leverage parameter. Despite allowing for these deviations, however, the model cannot match the variation in the cross-country data. We then incorporate time-varying probabilities of disasters as in Wachter (2013). Across countries, time-variation in disaster probabilities indeed improves the fit for asset return volatility and, to a lesser extent, the mean returns.

Given the best fit to individual country asset returns, we evaluate the disaster model's ability to match the international correlation of asset returns and consumption growth found in the data. For example, an empirical finding in the data is that international consumption correlations are lower than equity return correlations.[2] To determine whether the model can replicate this pattern, we analyze implied correlations under two extreme assumptions about international disasters found in the literature; that is, independent versus common disaster events.[3] Under the assumption that disaster events occur independently across countries, equity return correlations either mimic those of consumption correlations when disaster risk is constant or else are much lower than consumption when disaster risk is time-varying. By contrast, when disaster events are common, equity return correlations are near one, and are hence too high.

To address the inconsistencies posed by these two extreme cases, we posit a novel gener-

---

[2]See, for example, the discussion in Tesar (1995) and Lewis and Liu (2015).

[3]Studies that treat disasters as independent across countries include Barro (2006,2009) and Wachter (2013). In these papers, the frequency of disasters is calculated as the average number of times that output or consumption declined below a threshhold across all countries and years. Studies that treat disasters as common include Gourio, Siemens, and Verdelhan (2013) and Farhi and Gabaix (2016).

alization of the theoretical framework that incorporates both country-specific and common world disaster shocks. This generalization allows us to combine the domestic-based disaster risk model with international asset return and consumption correlations in the data to uncover country-specific versus common world disaster risk. Our evidence shows that a high degree of common disaster risk is required to explain the pattern that asset return correlations are greater than consumption growth correlations.

As this description makes clear, our objective in this paper is to highlight the international implications of existing U.S.-based disaster risk models in the tradition of Reitz (1988) and Barro (2006). For this purpose, we use a canonical disaster risk model to study its ability to fit international data moments. Therefore, we purposefully take as given the assumptions consistent with that literature and do not develop a new equilibrium model.[4] In this way, the results in our paper most directly contribute to understanding any required modifications and potential limitations of the standard model when facing international data.

Although our analysis provides a unique contribution to understanding the international dimensions of disaster risk models, a number of other papers have also addressed the impact of disasters on the macroeconomy and on asset markets. Gabaix (2008, 2012) considers disaster risk with variable severity of disasters arising from the resilience of an asset's recover rate through a "linearity generating" process. Martin (2008) solves for the welfare cost of business cycles due to disasters, but does not match to asset return data. Backus, Chernov, and Martin (2011) use U.S. equity index options to examine the implied disaster risk in consumption. Gourio (2008, 2012) evaluates the impact of disasters in a real business cycle

---

[4]By contrast, in Lewis and Liu (2017), we develop an international equilibrium model that requires a given asset and goods market structure.

model allowing for recoveries after a disaster. Nakamura, Steinsson, Barro, and Ursua (2013) also allow for recovery periods after disasters, but then estimate differing probabilities of entering disasters across countries. However, these papers do not evaluate the international asset pricing implications of disaster risk.

Two recent papers provide an exception. Gourio, Siemer and Verdelhan (2013) and Farhi and Gabaix (2016) examine the co-movements of returns and exchange rates with disasters, but they do so assuming complete markets. By contrast, our goal is to investigate the international asset market implications of existing U.S.-based empirical disaster risk models that, in turn, do not require markets to be complete. As such, we view the contribution in our paper to be complementary, but distinct from all of these papers.

The plan of the paper is as follows. Section 1 reviews the general framework used in the literature as well as the approach used in this paper. Section 2 describes the data and evaluates the model fit for countries in isolation. Section 3 describes the implications for correlations in consumption and asset returns across countries. Concluding remarks are in Section 4.

1   The Canonical Model and Framework

The disaster risk literature is grounded in a theoretical asset pricing tradition beginning with Lucas (1978), which relates returns to the intertemporal consumption optimization. Research applying this theory to data has met with mixed success. For example, as Mehra and Prescott (1986) showed in their seminal work, the risk to U.S. investors implied by historical consumption data was not sufficient to generate the observed equity premium, a

regularity often called the "equity premium puzzle." Following this observation, Reitz (1988) suggested that the risk of rare, but severe, disasters could provide a resolution to this puzzle.

The impact of rare disasters has been difficult to quantify, given the infrequency of these events in U.S. data, however. Therefore, Barro (2006) proposed using data on disasters across a large sample of countries to identify both the size and frequency of disasters in the U.S. Subsequent papers such as Barro (2009) and Wachter (2013) have also considered the implications of these disasters on various asset pricing moments such as the mean and variance of the equity returns and government bill rates. Moreover, these moments are often measured in real returns in home country prices, and presented as average asset returns (e.g., Barro (2006), Barro and Ursua (2008)).

While the consumption-based asset pricing literature on disaster risk has almost exclusively focused upon the behavior of U.S. data moments, the identifying assumption that disasters need to be measured with non-U.S. data has clear implications for the asset pricing moments of those countries as well as their cross-country co-movements. In order to evaluate these implications below, we develop a framework taken from a standard domestic-based model, modified to allow as much latitude for the model to match differing asset and consumption moments across countries. For this purpose, we incorporate country specific parameters to the framework with time-varying disaster risk developed by Wachter (2013). The Barro (2009) model with constant probability of disaster is a special case of this framework. We refer to this general framework as the "canonical model" below.

Since our contribution is to investigate this framework applied to international asset returns, we necessarily inherit both the limitations and generalities of the standard approach. Specifically, the most limited interpretation of our investigation would be that, since the

framework was developed to target domestic asset pricing moments, our analysis applies only to a world of multiple closed economies in isolation. Indeed, this narrow interpretation is consistent with the quantitative analysis in Section 2 that focuses exclusively on the analysis within each country. However, in Section 3, we show that this interpretation is likely to be overly restrictive when we examine the co-movements across countries implied by the canonical domestic-based disaster model. As demonstrated there, the domestic-based model implies positive co-movement in consumption and asset returns across countries. Therefore, to highlight the potential relationships within the standard model that may lead to these international co-movements, we review more general interpretations of the canonical model at the end of this section in Subsection 1.4.

1.1 Preferences and Consumption

To consider how a standard single-country disaster risk model may fit a cross-section of individual country asset returns, we modify the model specified in Wachter (2013) and Barro (2009) to allow all parameters other than preferences to differ by country. For consistency with this approach, we maintain the assumption from these papers that the analysis for each country is specified in units of home country consumption.

Following this framework, there is a representative consumer-investor in each country, indexed by j. These agents have identical preferences over their own aggregate consumption good defined as C_t^j at time t. To allow the model to match real consumption data that is deflated by home price indices, we assume that each country's aggregate consumption good is a composite of individual goods that will in general differ by country. This approach

ensures consistency between the theoretical framework and the data.

In particular, we assume as in Barro (2009) that preferences are recursive over time.[5] We also follow Wachter (2013) by considering the continuous time version formulated by Duffie and Epstein (1992) for the case of unitary intertemporal elasticity of substitution in consumption. This special case implies tractable, exact form solutions to our asset pricing moments below.[6] Thus, under these assumptions, utility at time t for representative consumer j, defined by V_t^j, is given by:

$$V_t^j = E_t \int_t^\infty U(C_s^j, V_s^j) ds \tag{1}$$

where

$$U(C_t^j, V_t^j) = \beta(1-\gamma)V_t^j \left[\log C_t^j - \frac{1}{1-\gamma} \log((1-\gamma)V_t^j) \right] \tag{2}$$

and where $\beta > 0$ is the rate of time preference and $\gamma > 0$ is the coefficient of relative risk aversion. Furthermore, the consumption good C_s^j in each country j is a composite of multiple heterogeneous non-durable goods each with separate prices, thereby allowing the price index to differ across countries as in the data. This approach follows a long literature in international finance that treats consumption in each country as an aggregate of individual goods.[7] Moreover, to insure a well-defined price index per country while providing the most

[5]These preferences use the form of the utility function specified in Epstein and Zin (1989) and Weil (1990). Barro (2009) argues that these preferences are needed to avoid the counterfactual implication that high price-dividend ratios predict high excess returns.

[6]This assumption allows us to adapt the closed-form solutions from Wachter (2013) to individual country asset returns, in the case where disaster intensity are time-varying. Nevertheless, when the disaster probability is constant, we could in principle allow the intertemporal elasticity of substitution to differ from one.

[7]For early examples see Adler and Dumas (1983), Cole and Obstfeld (1991), and Backus, Kehoe, and Kydland (1994). Colacito and Croce (2011) provides a more recent example.

general framework, we assume only that aggregate consumption C_s^j is isoelastic with respect to its individual goods components without specifying a particular form for the aggregation.[8] Thus, although the basic form of the utility function over aggregate consumption is the same across countries, preferences over individual goods may differ. Specifying consumption in this way provides consistency with the empirical literature that treats aggregate consumption in units that are the inverse of the price index in each country.[9] Note that this assumption implies that the value of consumption for country j in units of another country consumption will differ by a real exchange rate. We discuss the implications of variations in the real exchange rate in Subsection 1.4 as well as our quantitative analysis below.

The representative agent in each country j then chooses the sequence of C_s^j to maximize utility subject to a lifetime budget constraint of income, $Y_t^j = Y^j(\underline{\delta}_t)$, where $\underline{\delta}_t$ is a vector of state variables in the economy. In general, income is the flow of the resources available to a given country so that $\underline{\delta}_t$ reflects all of the variables influencing those resources. In a full international macroeconomic model, these variables would include variables affecting both domestic production and any net ownership of foreign production through foreign asset positions. Below we follow the asset pricing literature in taking this process as given by production side decisions in the economy and then focusing upon the asset pricing decisions conditional on income. Therefore, this income process may be considered exogenous for much of the analysis, although in Subsection 1.4 we discuss more general interpretations.

---

[8] Adler and Dumas (1983) demonstrate that when the consumption aggregator is homothetic with respect to individual goods components, then a well-defined price index holds per country, even in the absence of purchasing power parity.

[9] Technically, as described in Adler and Dumas (1983) we require the consumption aggregator to preserve the property that utility is homogeneous of degree one in the individual consumption components. This property holds for specific aggregators such as those in Colacito and Croce (2011) or Verdelhan (2010), for example. However, the condition also holds for more general aggregators as well.

Given this income process, then, the lifetime present value of these resources is the representative consumer's wealth; that is, a variable given by: $W_t^j \equiv E_t \left[\int_t^\infty \frac{\pi_s^j}{\pi_t^j} Y_s^j ds \right]$ where π_s^j is the state price density. This variable is defined as: $\pi_t^j \equiv e^{\left[\int_0^t U_V(C_s^j, V_s^j) ds \right]} U_C(C_t^j, V_t^j)$, where U_x denotes the partial derivative with respect to x. Thus, the state price density relates the value of resources to the intertemporal marginal utility of consumption. Hence, this variable must be determined in equilibrium, as described below in Subsection 1.2.

The representative agent then chooses consumption to maximize utility given in equations (1) and (2) subject to the constraint that:

$$ E_t \left[\int_t^\infty \frac{\pi_s^j}{\pi_t^j} C_s^j ds \right] \leqq W_t^j \tag{3} $$

This optimization implies a value function that gives the maximum utility as a function of the state variables, such as wealth, which we describe in more detail below.[10] Since the utility function in equation (2) is strictly increasing in aggregate consumption, the wealth constraint in equation (3) will hold with equality along any optimal path, implying an equilibrium relationship we use below.

The consumption that arises from this optimization naturally inherits a functional dependence on at least some of the variables that affect income. Defining this subset of variables as $\widetilde{\underline{\delta}}_t$, then we could rewrite consumption as: $C_t^j = C^j(\widetilde{\underline{\delta}}_t)$. While a full macroeconomic model would detail how the income process relates to consumption, much of the empirical asset pricing literature directly uses the fact noted above that in equilibrium $W_t^j = E_t \left[\int_t^\infty \frac{\pi_s^j}{\pi_t^j} C_s^j ds \right]$. As such, the behavior of consumption identified by the data is sufficient to determine the

---

[10]In discrete time, the value function would be determined using a Bellman equation. In this continuous time setting, the same is done using the Hamilton-Jacobi-Bellman equation as detailed in Appendix A.1.

behavior of wealth.

The common feature in the disaster risk literature is that consumption is affected by infrequent but large declines in income. For example, as argued in Barro (2006), this impact on consumption can be generated by significant downturns in the macroeconomy as occurred during the Great Depression, by natural events such as earthquakes, or may be the result of wars such as the World Wars. It may also arise through large declines in productivity that affect business cycles, as articulated in Gourio (2008, 2012).

Overall, although the specific ways in which disasters affect consumption will depend upon the nature of the macroeconomy, its impact will be observed in the data. For this reason, much of the focus in the disaster risk asset pricing literature has been to analyze the consumption data directly and then use the implications to uncover the effects on wealth through the constraint in equation (3). We therefore follow this approach below by using the framework from Wachter (2013) that includes the possibility of disasters. However, we augment this process to allow the parameters to differ across countries in order to fit potential variations. Specifically, using the notation above and including the potential effects of disasters, the consumption process to be related to the data below is:

$$dC_t^j = \mu C_{t-}^j dt + \sigma^j C_{t-}^j dB_t^j + (e^{\omega^j Z_t^j} - 1)C_{t-}^j dN_t^j, \quad \forall = 1, ..., J \tag{4}$$

where C_{t-} denotes $\lim_{s \uparrow t} C_s$ and C_t is $\lim_{s \downarrow t} C_s$, dB_t^j is a standard Brownian motion that affects consumption in normal times, dN_t^j is a Poisson process that is positive when disaster events occur, and Z_t^j is a variable that determines the size of the decline in consumption conditional on a disaster occurring.

10

We follow Barro (2006, 2009) and Wachter (2013) in identifying periods when disasters occur as years in which there were declines in income or consumption below a threshold. That is, the response of consumption to these disasters is reflected in a proportional drop in level by the amount $\omega^j Z_t$, where Z_t is a random variable that reflects the size of the drop and ω^j allows for the impact of this decline to differ across countries. To capture the effect of disasters, $Z_t < 0$ and $\omega^j > 0$ so that realizations of dN_t^j reduce consumption growth. In our quantitative application below, we parameterize the distribution of Z_t with the empirical distribution of disasters using the long sample of international data from Barro and Ursua (2008).[11] This distribution is treated as time-invariant so we drop the time subscript in the remainder of the paper.

To consider time-variations in disasters, N_t^j has an intensity parameter, λ_t^j, given by:

$$d\lambda_t^j = \kappa^j \left(\overline{\lambda} - \lambda_t^j \right) dt + \sigma_\lambda^j \sqrt{\lambda_t^j} dB_{\lambda,t}^j \tag{5}$$

where $dB_{\lambda,t}^j$ is also a standard Brownian motion. Following Wacther (2013), all country-specific processes, $\left\{ dB_t^j, dB_{\lambda,t}^j, dN_t^j \right\}$ are uncorrelated with each other at a given time t within a given country j.

Since these shocks originate from income processes for each country, they are likely to be correlated across countries if, for instance, there is trade in goods or assets. Therefore, in Section 3 we consider this possibility and allow for the correlations between countries, $Corr(dB_t^i, dB_t^j)$, $Corr(dN_t^i, N_t^j)$, and $Corr(dB_{\lambda,t}^i, dB_{\lambda,t}^j)$, to be non-zero while maintaining the independence within countries as in the standard model. As a result, these variables are

---

[11]This approach has been used in a number of papers including Barro (2009), Nakamura et al (2013), and Wachter (2013).

not in general independent across countries, even though we continue to identify them with a country-specific superscript.

The consumption process in equations (4) and (5) include country-specific parameters that allow the framework to fit data across countries below. For instance, although the standard model implicitly assumes $\omega^j = 1$, we incorporate this parameter to allow for differing effects across countries. Clearly, a country with higher ω will experience a larger impact of disasters on consumption. In addition, consumption volatility in "normal times" without disasters, σ^j, and the time-varying intensity parameters, κ^j and σ^j_λ, may be country-specific. Below we also consider country differences in asset return parameters measuring leverage and government bond default rate to be detailed later.

While our specification of consumption processes in equations (4) and (5) allows some parameters to be country-specific, others are treated as common. In particular, country mean growth rates, μ, are set to be equal across countries for plausibility since our quantitative analysis will focus upon developed economies. We also assume that the long run mean of the disaster probability $\overline{\lambda}$ is common across countries in the absence of power to distinguish this parameter across countries.[12]

1.2   First-Order Condition for Intertemporal Optimization

In order to solve for implied returns using observed consumption data, we follow the literature by conditioning our analysis on the first-order condition of intertemporal utility maximization given the wealth constraint. As with other first-order conditions, it simply

---

[12]For this reason, we also treat the other parameters in the time varying intensity process, κ and σ_λ as common in most of the quantitative analysis.

provides a relationship that optimizes an objective of one agent in the economy and may not reflect the equilibrium in the presence of multiple agents. Therefore, we condition our analysis on a further identifying assumption from the standard disaster risk literature: the domestic investor's first-order condition prices the domestic equity returns and government bill rates. This condition would clearly be satisfied if financial markets were completely segmented since only domestic investors would have access to their own assets. However, this identification also holds in more general contexts as we discuss at the end of this section in Subsection 1.4.

To solve for the return on an asset that would be required by the representative investor from country j, we must derive the first-order condition that relates wealth to that asset.[13] For this purpose, we define the value function for the country j investor in terms of the state variables of wealth and the disaster probability as $H(W^j, \lambda^j)$. As noted earlier, analyzing wealth in the data is simplified since in equilibrium the budget constraint in equation (3) holds with equality; i.e., $W_t^j = E_t \int_t^\infty \frac{\pi_s^j}{\pi_t^j} C_s^j ds$. Thus, wealth can be viewed as the value of an asset that would theoretically pay a dividend mimicking realizations from the consumption process in perpetuity, an asset often called the "consumption asset." Given the equilibrium association between consumption and wealth, valuation of financial securities in representative agent frameworks often depends upon the return on this asset.[14] Indeed, for recursive preferences, Epstein and Zin (1991) and Duffie and Epstein (1992) show that each asset must satisfy a first-order condition involving its own return and the return on an asset that is a claim on future realizations of consumption.

---

[13]This approach is equivalent to the process in discrete time using the value function from the Bellman equation, thereby yielding the Euler equation.

[14]The usefulness of the equilibrium relationship between wealth and consumption is highlighted in Campbell (1993), for example.

Recognizing this relationship, we can then determine two important building blocks for valuing equity and the government bill rate used in our empirical analysis for each country j: its instantaneous risk-free rate, and the associated state-price density. We only summarize their solutions here, providing more discussion in Appendix (A.2) for the risk-free rate and in Appendix (A.3) for the state-price density. Details are in Wachter (2013), Appendix A.I and A.II.

Determining the first building block, the risk-free rate for each investor, requires solving for the value of this asset at the equilibrium level of portfolio holdings. This rate can be determined by taking the derivative of the value function $H(W^j, \lambda^j)$ with respect to the choice of the risky consumption asset at the equilibrium level implied by the wealth constraint in equation (3). Following these steps implies that the value of a risk-free rate to an investor in country j is:

$$r_t^j = \beta + \mu - \gamma(\sigma^j)^2 + \lambda_t^j E\left[e^{-\gamma \omega^j Z}(e^{\omega^j Z} - 1)\right] \tag{6}$$

where the expectation is taken over the time invariant distribution of Z.

From the perspective of investors in country j, the only source of variation in the country j risk-free rate arises from time variation in the disaster probability, λ_t^j, as shown in equation (6). Moreover, if there were no disaster risk, this rate would simply be constant at: $r_t^j = r^j = \beta + \mu - \gamma(\sigma^j)^2$. The finding that the risk-free rate is constant when consumption growth is i.i.d. is well-known.[15] By contrast, time variation in the disaster probability induces volatility in the risk free rate. Moreover, since $e^{\omega^j Z} < 1$, a higher probability of disasters, λ_t^j, implies a greater risk that a disaster event will reduce consumption. In turn, this greater

---

[15]See for example, Obstfeld (1994), Campbell and Cochrane (1999), and Lewis (2000) among others.

risk induces more demand for precautionary savings, thereby reducing the implied country j risk-free rate.[16]

Note also that since wealth of investor j is measured in consumption units of country j, this return would only be risk-free to residents of country j. In particular, since consumption in country i is measured in different consumption units, the value of consumption in country j from the perspective of country i could be written in county i units as: $\widetilde{C}_t^{i,j} \equiv Q_t^{i,j} C_s^j$ where $Q_t^{i,j}$ is the real exchange rate that values a unit of country j in country i consumption. Thus, the risk-free asset to country j's investors would be risky from the perspective of country i's investors, because it would be valued at $Q_t^{i,j} r_t^j$. Moreover, since we follow the literature in expressing all asset returns in domestic country good units below, the same real exchange rate variations will affect all relative valuations of these returns across countries.

The second key building block for valuing assets from the perspective of country j's representative investor is the intertemporal marginal utility of consumption measured through the state price density, π_t^j. Solving for this process requires using the solution for the value function $H(W^j, \lambda^j)$ and the envelope condition that $H_W = U_C(C, V)$ along the optimal path. Using the functional form for these expressions together with Ito's Lemma implies that the state price density for country j follows:

$$\frac{d\pi_t^j}{\pi_{t-}^j} = \mu_{\pi,t}^j dt - \gamma\sigma^j dB_t^j + b^j \sigma_\lambda \sqrt{\lambda_t^j} dB_{\lambda,t}^j + (e^{-\gamma\omega^j Z_t} - 1)dN_t^j, \tag{7}$$

where b^j is a positive constant that depends upon parameters of the time-varying disas-

---

[16] As noted previously, the shocks to equilibrium consumption inherit shocks from the macroeconomy. Thus, variations in the probability of disaster arise from news in the economy that alter the perceived likelihood of disasters.

ter process, κ and σ_λ; the expected size of the disaster for country j, $\omega^j Z$; and preference parameters, β and γ.[17] As noted earlier, this process is specified in units of domestic consumption, C_t^j. Therefore, the state price density will in general differ across countries, unless they are identical once converted into a common good so that $\pi_t^j = Q_t^{i,j} \pi_t^j$. We discuss this implication in Subsection 1.4 below.

Since the state-price density impacts the valuation of all risky assets in the economy, equation (7) is useful for building intuition about several asset pricing relationships we find in our quantitative analysis below. First, note that the state price in equation (7) evolves with innovations to the exogenous variables in an intuitive way. In particular, π_t^j decreases in "good times"; that is, with increases in the Brownian on normal times consumption dB_t^j according to risk aversion, γ. By contrast, the state price increases in "bad times"; that is, with innovations to the Brownian on disaster probabilities, $dB_{\lambda,t}^j$, according to the current level of the disaster probability $\sqrt{\lambda_t^j}$ and the expected size of the disaster implied through the parameter b^j. Finally, since $Z_t < 0$, disaster events generated by dN_t^j increase the state price. Note that, in the absence of time-varying probabilities, the instantaneous variance of the state-price density during normal times would be driven by the variation in normal times consumption alone. Therefore, if disaster probabilities were constant (i.e, $\sigma_\lambda = 0$), then the instantaneous volatility of the state-price in normal times would simply be $\gamma \sigma^j$, as in the standard iid Guassian model.

---

[17]Specifically, $b^j = \left(\frac{\kappa + \beta}{\sigma_\lambda^2} \right) - \sqrt{\left(\frac{\kappa + \beta}{\sigma_\lambda^2} \right)^2 - 2 \frac{E_\nu \left(e^{(1-\gamma)\omega^j Z} - 1 \right)}{\sigma_\lambda^2}}$, as described in Appendix A.1. In practice, the square-root imposes a restriction on the relationship between the expected size of disaster and the variation of the disaster probabilities, as described in Wachter (2013).

1.3 Relating Asset Prices to Observed Data

Given these building blocks, we now relate the theoretical framework to returns observed in the data. Note that the framework simply asks how a representative investor-consumer would value claims to any specific stream of future income, which potentially applies to a large number of assets. Moreover, because wealth is identified by the present value of consumption, the framework does not have anything to say about which assets actually comprise the portfolio held by the representative investors. Since our objective is to evaluate the standard disaster risk literature, therefore, we consider only the two assets typically related to the data in that literature: government bill rates and equity returns.[18] With two assets per country, this approach implies that we only focus upon $2J$ asset returns, where J is the number of countries.[19] In this section, we describe the solution of these two returns, relegating details to Appendix A.4 and Appendix A.5 for the government bill rate and the equity return, respectively.

1.3.1 Government Bill Rates

We begin by considering the government bill rates. Following Barro (2006), the return of government securities is presumed to be subject to partial default during disaster periods. This presumption is based upon the observation that crises are often associated with a decline

---

[18]As noted earlier, these are the two assets studied in the tradition of Barro (2006,2009) and Wachter (2013). However, other papers such as Backus, Chernov, and Martin (2011) and Farhi, et al (2016) analyze options. Since option analysis would require significant restructuring of the canonical framework in this paper, we leave this analysis to future research.

[19]Note, however, that since the countries have different consumption units, there will be different valuations of these returns across countries unless state price densities are equal once converted into common consumption units. Thus, in principle, we could evaluate the required returns from the perspective of each of the J representative agents. That is, if there N^j assets in country j, we could obtain $J\sum_{j}N^j$ different asset returns implied by the first-order conditions.

in the value of government securities, either through partial default or inflation. Following this literature, we define the probability of this government default for country j as q^j. Then, consider an asset that pays out government debt that is risk-free during normal times but is subject to default with probability q^j during disaster periods. In this case, a domestic investor would evaluate the asset as a combination of the risk-free rate in equation (6) and an asset that may default during disasters. Using a no-arbitrage condition for these payouts, the instantaneous required return on the j government bill rate, as measured in units of country j consumption can be shown to be:

$$r_t^{b,j} = r_t^j + \lambda_t^j q^j E\left[(e^{-\gamma\omega^j Z} - 1)(1 - e^{\omega^j Z})\right] \tag{8}$$

This solution for the government bill rate illustrates several features. First, the premium on government bills is clearly increasing in probability of default, q^j. Moreover, the volatility depends upon the variation in the probability of disasters, λ_t^j. Note that in the absence of time-varying disasters, the government bill rate, like the risk-free rate, is constant so that its variance is zero. Furthermore, a higher probability of default increases the required compensation by investors to hold government bills as indicated by the second term on the right-hand side of equation (8). Finally, as noted earlier, the solution for this government bill rate is measured in units of domestic consumption, corresponding to its treatment in the data below.

1.3.2 Equity Prices

Equity is the second asset typically studied in the disaster risk literature. Defining D_t^j as dividends paid by country j equity and F_t^j as the price of the claim to income from all future dividends using the state price of country j investors, then this equity price can be written:

$$F_t^j = E_t \left[\int_t^\infty \frac{\pi_s^j}{\pi_t^j} D_s^j ds \right]. \tag{9}$$

With this relationship, we can evaluate the behavior of the stock price over time given a process for dividends.

The specific assumptions about how those dividends are identified in the data varies across studies. The most direct approach to discipline the dividend process is to use dividend data itself (e.g., Bansal and Yaron (2004), Lewis and Liu (2015)). Arguably, this approach gives the best picture of the behavior of the dividend process. However, unlike asset return data, reliable data do not exist for dividends across countries over a long history. For that reason, a typical approach in the disaster risk literature is to treat dividends as a process that mimics a more volatile version of consumption. Therefore, in this paper, we follow Wachter (2013) in assuming that the dividend process can be calculated using a process that mimics consumption multiplied by an exponential factor (e.g., Abel (1999), Wachter (2013), Gourio, Siemer and Verdelhan (2013)). That is, dividends D_t^j for country j are related to the consumption process according to: $D_t^j = \left(C_t^j \right)^{\phi^j}$ where $\phi^j > 1$ is the "leverage" parameter.[20]

Using this relationship along with the consumption process in equation (4), Ito's Lemma

---

[20]By contrast, some studies assume dividends mimic consumption itself (e.g., Mehra and Prescott (1985), Obstfeld (1994)) with no leverage parameter.

implies that the process of dividends for equity from country j is given by:

$$dD_t^j = \mu_D^j D_{t-}^j dt + \phi^j \sigma^j D_{t-}^j dB_t^j + (e^{\phi^j \omega^j Z} - 1)D_{t-}^j dN_t^j, \tag{10}$$

where $\mu_D^j = \phi^j \mu + \frac{1}{2}\phi^j (\phi^j - 1)(\sigma^j)^2$. Combining this process for dividends with the evolution of the state price density in equation (7), the diffusion for the stock price in equation (9) can be written as[21]:

$$\frac{dF_t^j}{F_{t-}^j} = \mu_{F,t}^j dt + \phi^j \sigma^j dB_t^j + g^j \sigma_\lambda \sqrt{\lambda_t^j} dB_{\lambda,t}^j + (e^{\phi^j \omega^j Z} - 1)dN_t^j, \tag{11}$$

where $\mu_{F,t}^j$ is the instantaneous mean and $g^j < 0$.[22]

The evolution of the stock price follows the essential features of the state price density in equation (7). In particular, the stock price increases with innovations in the Brownian on normal times consumption, dB_t^j, now augmented by the leverage parameter, ϕ^j. Moreover, the stock price decreases with innovations to the Brownian driving innovations to the probability of disasters, $dB_{\lambda,t}^j$, as well as disasters themselves. Also, note that in the absence of time-varying disaster probabilities, the stock price volatility in normal times would simply be that of the levered volatility of normal times consumption, $\phi^j \sigma^j$. As with all the other asset returns, the stock price evolution is in domestic consumption units. Overall, these relationships can then be used to generate the asset pricing moments in the model to compare

---

[21]Wachter (2013) in Appendix A.III derives the stock returns including the dividend payment, the solution we use to match to the equity returns. Here we provide the equity price alone for illustrative purposes only.

[22]Specifically, $g^j = G^{j\prime}(\lambda_t^j)/G^j(\lambda_t^j)$ where G^j is the price-dividend ratio for the equity of country j. This price-dividend ratio also depends upon the state price diffusion in equation (7). Ensuring that the solution of G is not imaginary restricts the relationship between not just Z and the parameters of the time-varying densities as before, but also the leverage parameter ϕ^j.

to their counterparts in the data.

1.4 Generality and Limitations of Canonical Framework

In order to consider the international implications of the literature on consumption-based asset pricing with disaster risk, we have modified the standard domestic-based model to allow for differences in the consumption and asset return data across countries.[23] Given the domestic economy focus of this literature, a narrow interpretation would be that the framework cannot do more than represent a world of multiple isolated markets with exogenously specified consumption. However, the literature on consumption-based asset pricing has demonstrated over the past few decades that this interpretation may be unduly restrictive. A more generous interpretation would be that the model reflects a world in which consumption is an endogenous outcome of a larger production process, potentially generated by international trade in goods and financial assets. In order to consider this possibility, then, we next review these alternative, more general interpretations of the literature as well as the limitations imposed by its basic identifying assumptions.

1.4.1 Exogenous versus Endogenous Consumption

The framework above is conditioned on a particular consumption process as given by equation (4) and therefore a narrow interpretation would presume that consumption is exogenous. There are other interpretations, however. Barro (2009) describes how a similar process for consumption obtains when output is an endowment process. Furthermore, other papers

---

[23]In particular, removing all the j superscripts and setting $\omega^j = 1$ reduces all the equations above to the Wachter (2013) model when the disaster probability λ is time-varying and to a continuous time version of the Barro (2009) model when it is constant.

have studied a richer production side of the analysis. For example, Gourio (2012) develops a

production-based model with capital and labor that endogenously generates a consumption

process with disaster shocks. Similarly, Gourio, Siemer, and Verdelhan (2013) analyze a pro-

duction economy in a two-country model with open financial markets, implying a consump-

tion process with infrequent, but large declines. The international setting then imposes an

additional world resource constraint to the framework above. That is, for each time period,

world consumption equals world income; that is, $\sum_j Q_t^{i,j} C_t^j = \sum_j Q_t^{i,j} Y_t^j$, for $t \in \{0, ..., \infty\}$.

Overall, a key feature common to disaster risk models is that income to the economy

experiences large declines that, in turn, dramatically reduces consumption. Moreover, since

observed consumption is the outcome of decisions made by individuals operating in the

true economy, it reflects the optimal process given the constraints faced by agents.[24] The

approach has also been used to examine risk-sharing in Cochrane (1991a), Lewis (1996), and

Lewis and Liu (2015).

1.4.2 Complete versus Incomplete Asset Markets

The domestic-based model above uses the first-order condition of domestic investors to price

domestic assets, without specifying whether foreign investors also hold these assets. Implic-

itly, then much of the domestic-based asset pricing literature assumes that domestic agents

are the marginal investors who determine the price of domestic assets. To see why, consider

---

[24]Treating consumption as measured in the data as an endogenous outcome also has a long tradition in macro-finance, dating to Lucas (1978,1982).On the connection between consumption and asset pricing data, see Hansen and Singleton (1983), Cochrane (1991a), Campbell (1993), Campbell and Cochrane (1999) and Bansal and Yaron (2004), among others. Kocherlakota (1996) provides a very useful review of the behavior of consumption and asset prices required to generate asset returns. In some cases, fuller production-based models are separately specified to demonstrate how an observed consumption process can be generated. For example, Kaltenbrunner and Lochstoer (2010) show how a persistent autoregressive component to consumption can arise endogenously even though the technological process in production is only subject to i.i.d. shocks.

again the canonical model above. In this setting, the source of income to the domestic investor, Y_t^j, affects the state price density, π_t^j, that is used in turn to value the two domestic assets measured in home country consumption: equity and government bills. As noted earlier, a narrow interpretation that clearly delivers this result is that each country is completely segmented in its financial market.[25] More generally, however, the analysis is also consistent with at least two other interpretations: either markets are complete or they are incomplete in a particular way described below.

The first alternative interpretation is that markets could be complete. In this case, the countries would share the same state price density as measured in the same consumption units. Therefore, if purchasing power parity does not hold complete markets would require that $Q_t^{i,j} = \left(\pi_t^i / \pi_t^j \right)$ or if it does hold $\pi_t^j = \pi_t^i$, for all i, j. Given our goal of analyzing the standard model and allowing for the most general treatment in our analysis below, we do not impose these restrictions a priori but instead allow the data to reflect any such relationship.

A second alternative interpretation is that markets are incomplete in a way such that the domestic investors are the marginal investors that price domestic assets in the given data sample. This interpretation is based upon the idea that the pricing impact of some investors in the market may not be apparent during periods when those investors are inframarginal. This notion is consistent with some studies of incomplete markets. For example, Telmer (1993) describes a model in which agents can only trade a risk-free bond, implying multiple equilibria over time. In this equilibrium, one agent will often be at a corner solution.[26]

---

[25]Note that countries need not be segmented in goods markets for the asset return equations to hold, however. If countries are engaged in international trade, then income processes, Y_t^j, will in general be correlated across countries, correspondingly implying that consumption processes, C_t^j, will be correlated internationally as we find in Section 3.

[26]When markets are incomplete, the Euler equation holds for each agent but is not unique in aggregate. See Telmer (1993), Bakshi, Cerrato, and Crosby (2015) and Lustig and Verdelhan (2016), for example.

Further, Heaton and Lucas (1996) consider domestic economies in which agents only have access to a risk-free bond but markets are incomplete while Baxter and Crucini (1995) consider a similar financial market in the international setting. Although the implications of these studies are specific to their market structure, they raise the possibility that the first-order condition of some investors may be more important in pricing in any particular period.

Overall, then, the standard disaster risk model that conditions on the domestic household's valuation of domestic assets may be interpreted consistently with the international data in three ways. Financial markets are either (a) completely segmented; (b) completely integrated; or (c) incomplete in a manner such that the domestic representative household is the marginal investor during the data sample.

1.4.3 Purchasing Power Parity versus Differing Goods Prices Across Countries

The consumption-based asset pricing literature typically converts asset returns and consumption into real growth rates using the domestic price index. This approach has also been maintained in the disaster risk literature beginning with Reitz (1988) as well as the more recent literature starting with Barro (2006) and calculated in international data by Barro and Ursua (2008). Clearly, this practice implicitly converts these data and their moments into domestic consumption units that differ by country. For this reason, the aggregate consumption indices in the utility function (1) as well as the state price densities for each country and their corresponding equity prices and government bill rates are all specified in home consumption units in the framework above.

Therefore, we measure these returns in a manner that is consistent with consumption

in each country and do not take a stand on whether the prices are equal across countries or not. Again, this treatment admits a range of possibilities from the empirical analysis. If purchasing power parity holds, then current real payouts will be valued equivalently by investors across countries. However, if purchasing power parity does not hold, investors will view foreign payouts differently because of variation in the real exchange rate. That is, dividends of equity from country j from the perspective of country i would be: $Q_t^{ij} D_t^j$. Intuitively, it would be like the value in real U.S. terms of a dividend paid in real German consumption units. We follow this treatment so that our structural framework will match the empirical analysis in the literature that converts all units into domestic real units.

2   Single Country Implications of Disaster Risk

Above we described how to modify a standard disaster risk model to allow for potential cross-country differences. In this section, we consider the implications of this framework for non-U.S. countries, focusing on the data of each country individually as in the U.S.-based approach. In Section 3, we will examine their cross-country implications.

2.1   Data by Country

Following much of the disaster risk literature, we base our empirical analysis on the long time series sample of consumption and asset return moments across countries reported in Barro and Ursua (2008). For the 21 OECD countries in the sample, this data set provides consumption beginning in the range of 1800 to 1913, depending upon the country. These data are constructed by deflating with their respective country consumer price indices. Since

consumer prices are known to differ across countries due to Purchasing Power Parity (PPP) deviations, the measured consumption identified with C_t^j will presumably not be in the same units of goods across countries. However, individual country consumption are in the same units as their own asset returns payouts as developed in Section 1.

Available asset return data generally begin later than consumption, precluding a long history analysis of all 21 countries in the Barro-Ursua (2008) set. For this reason, we focus upon seven countries with asset pricing data that begin relatively early: Australia, Canada, France, Germany, Japan, the United Kingdom, and the United States. Among these countries, the United Kingdom's stock return data starts the earliest in 1791, while Canada begins the latest in 1934. In all cases, the bond return data are available either earlier or at the same time as the stock return data.

Table 1 Panel A reports the means and standard deviations for the equity return, the government bill rate, and consumption for these seven countries.[27] As the table shows, the mean consumption growth rates of the countries are similar across countries, ranging between 1.47% and 2.48%. By contrast, the asset pricing estimates vary widely across countries. For example, Australia has the highest mean stock return at 10.27% while the lowest is France at 5.43%. Also, the standard deviations of equities are relatively similar for Australia, Canada, France, the U.K., and the U.S., but are higher for Germany and Japan at around 30%. A similar pattern may be seen in the standard deviation of the bill rates, as those estimates exhibit substantially higher volatility in Germany and Japan. A wide range of mean bill rates is also apparent, and those levels are even negative for Germany and France. In our

---

[27]For Canada, the bill rate is unavailable from Barro and Ursua (2008). We therefore use the bond rate for this country.

quantitative analysis below, we refer to moments calculated over this full sample of data as "Unconditional," following Wachter (2013).[28]

One reason for the range of moments may be that disasters affect countries heterogeneously in the sample. As such, it may be informative to condition the moments on years when disasters are absent. Therefore, following Wachter (2013), we also examine post war data as a subset of our full sample, representing a data sample that excludes large disasters. We refer to these moments as "Conditional."

Again using the Barro and Ursua (2008) data, we recompute consumption growth and asset pricing moments for the period after 1947. Table 1, Panel B shows the range of annual data moments across the seven countries for this postwar sample. The range of mean equity returns is smaller, but the range of standard deviations across countries remain large. Not surprisingly, the standard deviations of consumption growth and government bill rates are uniformly lower in the postwar period without disasters than in the full sample.

2.2 Matching the moments: Constant Disaster Probability

Given the solutions for the asset pricing returns and the consumption processes in Section 1, we now ask how well the model can fit each individual country's consumption and asset return data. For this purpose, we use Simulated Method of Moments. We describe results from this analysis next, relegating details of the simulation to Appendix C.

We begin by considering the constant disaster risk model assuming the baseline parameters from Barro (2006). In particular, these parameters are a relative risk aversion of $\gamma = 4$, a

---

[28]Given that Germany and France have negative average bill rates over the sample, we constrain their targeted bill rates at the lower bound of zero for all analysis below based upon this period.

rate of time preference of $\beta = 0.03$, a "normal times" consumption growth rate of $\mu = 0.025$, and a probability of disaster event of $\lambda_t^j = \bar{\lambda} = 1.7\%$ for all countries j, where the latter is calculated as the proportion of years when GDP dropped by 15% or greater. We also assume that the distribution of the sizes of consumption declines due to disasters, Z, is given by the historical sample in Barro and Ursua (2008).

Table 2 reports measures of the model fit for both the Unconditional Model Moments including disasters, targeting the data moments in Table 1 Panel A, and the Conditional Model Moments during "normal times," targeting the post-war data in Table 1 Panel B. For each version, we use Simulated Method of Moments (SMM) to provide the best fit of our model parameters for each country's data moments. In order to fit these parameters, we target the following seven data moments for each of the countries: (a) the consumption growth standard deviation, (b) the mean equity premium, (c) the standard deviation of the equity return, (d) the Sharpe ratio, (e) the mean government bill rate, and (f) the standard deviation of the government bill rate. Using these target moments, we estimate the following model parameters: (a) the probability of government bond default, q^j; (b) the dividend-consumption leverage parameters, ϕ^j; (c) the proportion of disaster state consumption decline, ω^j, relative to the standard model; and (d) the volatility of consumption in normal times, σ^j. We calibrate the mean consumption growth rate μ to be equal to the average mean growth rate across countries.

Table 2, Panel A reports the parameter estimates targeting the moments over the full "Unconditional" Barro-Ursua sample, including years when disasters were observed. Despite the range of data moments across countries, the estimates for the Unconditional Model Moments provide a relatively tight range of country-specific parameters. Indeed, for most

of the parameters, the estimates correspond to those found in the literature for the U.S. For example, the probability of government default is in the range given by $q = [0.42, 0.45]$, close to the assumption of $q = 0.4$ in Barro (2006). Moreover, the fitted "normal times" consumption volatility, σ, estimates are around 2.3% and therefore near standard estimates. The range of the estimate for the leverage parameter ϕ are between 2.6 and 2.8, near the Abel (1999) assumption of 3. Overall, these estimates are all relatively in line with values required to fit asset pricing moments in the U.S., even though the target data moments are for non-U.S. countries that often have quite different values. By contrast, the range of estimates of ω are all lower than one, $\omega^j = [0.84, 0.88]$, indicating that the consumption loss in the event of a disaster is somewhat lower than that assumed in the standard U.S.-targeted model.

This tight range of fitted parameters creates difficulties in matching the wide range of asset pricing moments, however. As Table 2, Panel A reports, the annualized asset pricing moments from the model are much closer across countries than their data counterparts in Table 1, Panel A. For example, in the model, the mean and standard deviation of government bill rates are all relatively uniform and do not differ across countries by more than 0.5%. In the data, by contrast, the means vary by almost 4% and the standard deviation by over 10%. Similar discrepancies can be found in the equity returns. Even the more modest range of the consumption volatilities across countries cannot be generated by the model.

The last row of Table 2 Panel A emphasizes the poor fit by reporting the sum of squared difference ("Sum of Sq Difference") between targeted data moments and the average model-derived moments across parameter estimates. As the numbers indicate, the model fits particularly poorly for Australia and Japan.

29

A potential problem with these results is that, depending upon the occurrence and severity of disasters, the data estimates may not accurately report the population data moments. For this reason, Panel B of Table 2 reports the results of an alternative SMM analysis targeting the model under "normal" times excluding disasters, given by the postwar data moments in Table 1, Panel B. As before, to get an aggregate measure of how the model fits across all the data moments, we report the Sum of Squared Difference.

Despite the differences in the samples, the fitted parameters targeted to the Conditional data moments provide a qualitatively similar pattern as before and, once again, are similar to the estimates found for the U.S. There are some differences, however. The values for the probability of government default q are now somewhat higher, between 0.432 and 0.521. Furthermore, the leverage parameter ϕ estimates range between 2.731 and 3.005, and thus are all close to 3. By contrast, the implied loss in consumption, captured by ω, is lower than standard disaster studies of the U.S. that assume $\omega = 1$. These estimates are very similar across countries at about 81% to 87% of the size generated by the distribution from the Barro data.

Given the similarities in parameters across countries, the model again shows little variation in implied moments across countries. The government bill rates are mostly between 3% - 4%. Moreover, as noted earlier, during normal times, the model implies that the bill rate is constant so that the bill rate volatility is zero. The equity premium and the volatility of equity returns are fairly uniform around 5% and 6.5%, respectively. Thus, the model cannot explain the range of equity premia from 6.35% to 11.90% reported in Table 1, Panel B. Furthermore, in most cases, the model-implied volatility of equity is much lower than the 15% to 33% found in the data. The only notable exception is Japan which, given its large

consumption growth volatility, generates a lower bill rate, a higher equity premium, and a higher equity standard deviation.

2.3   The Size of Disasters With Constant Disaster Probabilities

In light of these difficulties with matching moments for individual countries, we next study how variations in key parameters affect the implied asset prices. For this purpose, we simulate the model based upon varying levels of, alternatively, ω, ϕ, and q. These results are shown in Table 3.

The table begins by reporting the effects of a range of values of the severity of disasters through ω. In particular, the numbers in the first three columns labeled "Baseline" show how asset pricing moments vary when the proportionate size of the disaster ω to the standard model ranges from $\omega = 0.8$ to 1, holding constant the other parameters. For example, as ω increases towards one, the government bill return declines. As noted in Section 1, a more severe disaster increases savings, thereby reducing the risk-free rate. As a consequence, the government bond rate declines with higher ω as well, although this effect is muted by the increased default risk premium. In the absence of time variation in disaster probability, higher ω has a large affect on the mean risk-free rate but not the volatility of the risk-free rate. For consumption growth, by contrast, the increase in ω has a larger effect on volatility than on the mean.

Under the next three columns labeled "High ϕ", Table 3 shows the effects of a higher leverage parameter. Here we assume the leverage parameter ϕ to be 3.0 rather than 2.8, and then re-examine the results from varying ω. Comparing the results to the first three

columns of Table 3 shows that a higher leverage ratio increases the equity premium. It also increases the standard deviation of the market return, albeit much more modestly. On the other hand, comparing the model moments in the "Baseline" model to the "High ϕ" model shows that these effects from higher leverage become muted when there is less sensitivity to disaster risk; that is, when ω is lower. For example, comparing the Mean Equity Premium, the higher leverage ratio increases the equity premium 24 bps from 7.88% to 8.12% when ω is one, but only 17 bps when ω is 0.8. The government bond rate is unaffected by any changes since, as noted earlier, it is independent of the equity dividend leverage parameter ϕ.

The last three columns of Table 3 labeled "Low q" report the implications for the model moments when the probability of government bond default conditional on a disaster q declines to 25% rather than the 40% given in the Baseline Model. Comparing these results to those in the first three columns makes clear that a decrease in the probability of government default decreases the government bill rate. The intuition is clear. A lower probability of loss reduces the implied default risk premium as shown in equation (8). Moreover, this lower government bill rate correspondingly increases the equity premium. Reducing the likelihood of the default loss in disasters also reduces the volatility of the government bill rate.

Table 3 also reports the model-implied moments for each set of parameter values. For the Conditional Model Moments computed during normal times alone, the increase in leverage ratio, ϕ, again produces a noticeable increase on the mean equity premium, and a slight increase on the equity return volatility. Similarly, the increase in q decreases the return and volatility of the government bill rate. Finally, given the low volatility of the equity return for the conditional moments, the model suggests implausibly large conditional Sharpe ratios

that range between 1.11 and 1.46, for the cases when ω is one.

In summary, the model with constant probability of disaster fails to explain the volatility of asset returns. The volatility of equity returns are significantly lower than in the data, while the volatility of government bills during periods without crises is counterfactually zero. In the next section, therefore, we evaluate the effects of incorporating time-varying disaster risk on the cross-country variation in asset pricing moments.

2.4 Matching the moments: Time-Varying Disaster Intensity

We now allow for time variation in the probability of disasters following Wachter (2013) and ask whether the model can generate better fitting cross-country differences in asset returns. The time-varying disaster intensity process in equation (5) introduces two new parameters: the volatility of the probability, σ_λ, and its mean reversion, κ. We therefore began by conducting SMM to target these two new parameters in addition to the four parameters formerly fitted. However, including these two new parameters generally meant that the SMM optimization would not converge because σ_λ and κ tended to exceed conditions required for the distribution of λ to be well-defined, even when we restricted these parameters to be the same across countries.[29] For this reason, in the results reported below, we instead constrain some of the parameters and conduct SMM to obtain the best fit to the data for the other parameters.

We take as a base case for the time-varying disaster probability model the parameters

---

[29]The intensity process in equation (5) has a stationary Gamma distribution only for $\frac{1}{2}\sigma_\lambda^2 < \kappa\overline{\lambda}$. Thus, for given assumptions about the mean of disaster probabilities, $\overline{\lambda}$, the volatility of probabilities, σ_λ, cannot be too high and the degree of mean reversion, κ, cannot be too low. Otherwise, the distribution of probabilities becomes degenerate.

from Wachter (2013). In particular, we assume for time preference that $\beta = 0.12$, for risk aversion that $\gamma = 3$, and for the average probability of disaster that $\overline{\lambda} = 3.55\%$.[30] Given these assumptions, we then analyze the model in two ways. First, we maintain the same disaster risk parameters (κ, σ_λ) as in Wachter (2013), and use SMM to fit, respectively, country-specific government default, consumption volatility, leverage ratio, and size of disaster as in: $\{q, \sigma, \phi, \omega\}$. This case is most comparable to the constant disaster probability model results in Section 2.1. Second, we hold fixed those same parameters, and fit κ and σ_λ to match data moments. For parsimony, we report in the text only the analysis targeting the Unconditional data moments, although the results targeting Conditional data moments are qualitatively similar.

Panel A of Table 4 shows the results when we choose the leverage parameter ϕ, probability of government default q, the volatility of normal times consumption σ, and the proportional impact of disasters ω, to match the target data moments, while fixing the volatility of disaster intensity σ_λ and its persistence parameter κ. In particular, normal times consumption volatility is calibrated to the standard deviation of Post-War consumption growth in the data as before. As the table shows, there is now a wider range of parameters. For example, Australia has a lower leverage parameter and probability of government default than the other countries. The Sum of Squared Difference is substantially lower across all countries compared to those in the constant probability model for Unconditional Moments in Table 2, Panel A. Furthermore, the differences in parameters now generate a wider range in asset return moments across countries. Indeed, the mean of government bill rates becomes too low for Australia. The model also generates equity volatility closer to the data.

---

[30]This probability is based on treating disasters as a decline in consumption of 10% or more.

Table 4, Panel B reports the estimates based upon reversing this process. Specifically, we now fix the probability of government default q, the leverage parameter ϕ, and the size of disasters ω, to those in Wachter (2013), and instead use SMM to fit parameters in the distribution process for the intensity λ_t to best match the targeted Unconditional Model moments. As the table shows, the parameters vary in a narrow range. The degree of mean reversion κ is essentially unchanged across countries, while σ_λ varies only between 0.066 to 0.087. In practice, this narrow range is dictated by the condition that the distribution of λ be stationary. But the resulting implications for the government bill rates and equity premium are again that asset pricing moments cannot vary much across countries. This finding is highlighted by the larger Sum of Squared Difference in Panel B relative to Panel A.

Overall, therefore, the best fit is the model with time-varying probabilities of disasters in Table 2, Panel A, allowing for differences across countries in government default recovery rates, consumption volatility, equity leverage, and effects of disaster shocks.

2.5 The Size of Disasters With Varying Disaster Probabilities

As the above results show, the means of asset returns are similar to those of the U.S. market, although there is a higher variation in standard deviations of returns. Therefore, we now ask how differences in the impact of disasters can affect the moments.

The results are given in Table 5. For the base case model, reported in column 1, we first report the implications for Unconditional and Conditional Model Moments when ω is constrained to be 1, as in Wachter (2013). By contrast, the second and third columns report the results when $\omega = 0.85$ and $\omega = 0.95$, respectively.[31] As the results show, reducing ω from

---

[31]We also considered the cases when $\omega > 1$, but these violated the condition that is required to give a

1 to 0.85 increases the bill rate since the disaster has less of an impact. It correspondingly reduces the size of the unconditional equity premium from 7.6% to 4.6%. At the same time, the unconditional equity volatility also decreases as the impact on consumption is lessened, declining from 20% to 16.6%.

In the following three columns, labeled "High ϕ", we consider the impact from increasing the leverage parameter ϕ to 3. The results are quite similar for the bill rate and volatility. However, the unconditional equity premium and volatility are higher as a direct implication of the increased leverage. Moreover, the range of potential sizes of declines is narrower since the model generates imaginary solutions for $\omega = 0.85$.

Finally, the last three columns, labeled "Low q", report the same analysis as the base case, but now setting the probability of government default conditional on a disaster at $q = 0.25$, down from 0.40. The lower risk of default increases desired precautionary savings at the benchmark disaster size, $\omega = 1$, so that the unconditional mean bill rate is an implausibly low value of 0.6%. As the size of disaster declines to $\omega = 0.95$ and $\omega = 0.85$, the precautionary motive is offset somewhat and the bill rate increases while the equity premium correspondingly shrinks.

Similar patterns hold for the Conditional Model Moments. In general, the precautionary motive for holding bonds decreases with lower ω and q so that implied government bill rates increase. Generally, the equity premium moves inversely with these relationships.

Overall, the time-varying model does provide an improved match for the volatility of asset returns. This improvement, coupled with the fact that there is less variability across

---

non-imaginary solution of b in the state price density given in Equation (7). In other words, the restriction that $\left(\frac{\kappa+\beta}{\sigma_\lambda^2}\right)^2 > 2\frac{E_\nu\left(e^{(1-\gamma)\omega Z_t}-1\right)}{\sigma_\lambda^2}$ was violated. See footnote 29.

countries in the asset return variances, allows the model, even within a narrow range of parameters, to fit these moments better.

3   International Correlation Implications of Disaster Risk

The analysis in Section 2 above examined in isolation the country effects of the canonical disaster risk framework. In this section, we begin to examine the international implications of this framework. To do so, we ask how well the model can generate the international co-movements of asset returns and consumption that are observed in the data. In particular, a common empirical finding is that the correlations of consumption are lower than the correlations of equity returns (see Tesar (1995) and Lewis and Liu (2015)).

In the canonical disaster framework considered here, these moments are also affected by real exchange rate variations since consumption and asset returns are measured using a price index specific to each country. To see this relationship, recall that the value of consumption in country j in units of country i is $C_t^{ij} \equiv Q_t^{i,j} C_t^j$. Then the correlation of consumption growth measured in home consumption units as in the Barro and Ursua (2008) data set will include the effects of the correlation of the real exchange rates when viewed from the perspective of a common numeraire. That is, the correlation of consumption between country i and country j measured in country i price indices is: $Corr\left(\frac{dC_t^i}{C_t^i}, \frac{dC_t^j}{C_t^j}\right) = Corr\left(\frac{dC_t^i}{C_t^i} \frac{d(C_t^{ij}/Q_t^{i,j})}{C_t^{ij}/Q_t^{i,j}},\right)$ and similarly for asset returns. Clearly, these correlations differ from the perspective of a given home representative agent depending upon the real exchange rate. Therefore, in this section, we also demonstrate that the standard correlation patterns between consumption and asset returns hold in the data when measured in this way.

37

In order to consider these international co-movements in the model, we must discipline the international correlation for each of the random variables, dB, dB_λ, and dN. The data counterpart for the first of these variables is straightforward since during "normal times," $Corr(dB_t^i, dB_t^j)$ is equal to the correlation of consumption growth between the two countries. Therefore, we calibrate this number to the data correlation in the Conditional data sample. By contrast, there are insufficient observations of disasters to identify the correlation of the disaster events and their probabilities in the data. For this reason, we turn to the assumptions in the literature.

There are typically two extreme assumptions in the literature about the co-occurence of disasters across countries: (a) they are independent; or (b) they are common. As an example of a study using the first assumption, the Barro (2006) model calculates the frequency of disaster events as independent across countries. Moreover, the realization of the event itself in the form of the Poisson jump N_t^j only affects one country. Thus, there are no feedback effects from disasters in foreign countries that would affect the home country disaster process. Therefore, we start our analysis of disasters with this assumption in Section 3.1. As an example of a study using the second assumption, Gouirio, Siemer and Verdelhan (2011) assume that disasters are common across countries. In Section 3.2 below, we examine the model under this assumption.

We show that neither of these two assumptions of disasters can explain the data patterns. Therefore, in Section 3.4, we introduce a new hybrid version, finding that both common and country-specific disasters are required for the canonical model to match the correlation of asset returns and consumption across countries.[32]

---

[32]Since the objective of the paper is to examine the implications of the standard disaster risk model, we

3.1 International Co-movements Assuming Independent Disasters

To examine international co-movements, we begin by considering the pattern of consumption co-movements implied by equation (4), repeated here for convenience:

$$\frac{dC_t^j}{C_{t-}^j} = \mu dt + \sigma^j dB_t^j + (e^{\omega^j Z_t} - 1)dN_t^j, \quad \forall = 1, ..., J.$$

If indeed, the Poisson process generating the disaster is independent across countries, then clearly the instantaneous consumption correlations are just given by the correlation of the normal times Brownians, as observed in the data:

$$Corr\left(\frac{dC_t^i}{C_{t-}^i}, \frac{dC_t^j}{C_{t-}^j}\right) = Corr(dB_t^i, dB_t^j) \equiv \rho^{ij}. \tag{12}$$

Similarly, asset price correlations are also only affected by the correlation of Brownians, implying that there are no effects on the correlation across countries due to disaster risk. To see why, consider the state price processes across countries from equation (7) under the assumptions that the dN_t^j realizations and the Brownians on their intensities $dB_{\lambda,t}^j$ are independent across countries. In this case, the correlation of the state price processes are also the correlation of normal times consumption; that is, $Corr\left(\frac{d\pi_t^i}{\pi_t^i}, \frac{d\pi_t^j}{\pi_t^j}\right) = \rho^{ij}$. The state price processes are only correlated across countries due to their normal times consumption correlations since agents view the impact of disasters on consumption as uncorrelated. As

---

simply generate the model correlation without taking a stand on the source of that correlation. This approach leaves open the question of how much international market integration is implied by the effects of disaster risk on asset return comovements. More generally, a large literature has studied the degree of financial market integration in international markets. See, for example, Dumas and Solnik (1995), Pukthuanthong and Roll (2009), Bekaert, Harvey, Lundblad, and Siegel (2011), and Carrieri, Chaieb, and Errunza (2013), among others. In Lewis and Liu (2015,2017), we focus upon the potential degree of integration in a long run risk model and disaster risk model, respectively.

a result, the instantaneous correlation in stock prices across countries using the process for

equities in equation (11) and the assumptions of independent disasters is also:

$$Corr\left(\frac{dF_t^i}{F_{t-}^i}, \frac{dF_t^j}{F_{t-}^j}\right) = \rho^{ij}. \tag{13}$$

Furthermore, the independence of disasters implies that the government bill rates will be

independent. Simple inspection of the government bill rate in equation (8) makes clear why.

As noted earlier, these rates only vary due to the changes in the probability of disasters,

according to λ_t^j. When λ_t^j and λ_t^i are uncorrelated for all i, j, then government bill rates

will be as well. Therefore, the instantaneous correlation of government bill rates under

independent disasters is:

$$Corr\left(\frac{dr_t^{b,i}}{r_t^{b,i}}, \frac{dr_t^{b,j}}{r_t^{b,j}}\right) = 0. \tag{14}$$

Note also that if disasters are independent, the correlation of consumption and asset

pricing moments will be the same whether using a full sample including disasters or a sub-

sample excluding those disasters.

3.2 International Co-movements Assuming Common Disasters

By contrast to the assumption of independence, most of the disasters identified in the data

by Barro (2006) and Barro and Ursua (2008) occur at roughly the same time for the OECD

countries. For example, during the periods of the Great Depression and the World Wars,

most of the countries were in disaster states, whether measured by declines in GDP of at

least 15% as measured by Barro (2006), or by declines of consumption of at least 10% as

classified by Wachter (2013). Nakamura, Steinsson, Barro, and Ursua (2013) also estimate a world disaster event during these periods.

Thus, an alternative assumption may be that all countries share the same disaster risk process so that $dN_t^j = dN_t^w, \forall j$ where dN_t^w is a Poisson world disaster event shock that has an intensity λ_t^w. In this case, consumption for country j follows:

$$\frac{dC_t^j}{C_{t-}^j} = \mu dt + \sigma^j dB_t^j + (e^{\omega^j Z} - 1)dN_t^w, \quad \forall = 1, ..., J \tag{15}$$

where dN_t^w is a Poisson jump process and the intensity process follows:

$$d\lambda_t^w = \kappa \left(\overline{\lambda}^w - \lambda_t^w \right) dt + \sigma_\lambda \sqrt{\lambda_t^w} dB_{\lambda,t}^w. \tag{16}$$

To highlight the effects of the common disaster risk, we assume that the mean reversion parameter, κ, and the volatility of the probability, σ_λ, are the same as defined earlier, and are equal across countries. However, the impact of the disaster may affect country consumption growth rates differently through the size of ω^j. Note that even if $\omega^j = \omega, \forall j$ so that all growth rates decline by the same proportion during a disaster, the impact on levels is unique to each country since consumption goods units potentially differ across countries due to real exchange rate effects.

If the disaster event is common, the correlation of consumption and asset returns will also depend upon the correlation of the disaster components. To see why, consider the effect on consumption correlations from these common disasters assuming for simplicity that the intensity on the world disaster shock is constant so that $\lambda_t^w = \overline{\lambda}^w$. Defining the size of the

decline in the consumption growth during disasters as $K^j \equiv (e^{\omega^j Z} - 1)$ and using moment properties of Poisson processes, the correlation of consumption across countries is given by:

$$Corr\left(\frac{dC_t^i}{C_{t-}^i}, \frac{dC_t^j}{C_{t-}^j}\right) = \frac{\sigma^i \sigma^j \rho^{ij} + K^i K^j \lambda^w}{\sqrt{(\sigma^i)^2 + (K^i)^2 \lambda^w}\sqrt{((\sigma^j)^2 + (K^j)^2 \lambda^w}} \tag{17}$$

Or, in the case where the effect of disasters is the same so that $\omega^i = \omega^j \; \forall j$, and $K^j = K^i = K$, the instantaneous correlation is:

$$Corr\left(\frac{dC_t^i}{C_{t-}^i}, \frac{dC_t^j}{C_{t-}^j}\right) = \frac{\sigma^i \sigma^j \rho^{ij} + (K)^2 \lambda^w}{\sqrt{(\sigma^i)^2 + K^2 \lambda^w}\sqrt{((\sigma^j)^2 + K^2 \lambda^w}} \tag{18}$$

Using the relevant parameter values, it can be shown that the correlation of consumption growth in equation (18) is greater than the correlation of normal times consumption, ρ^{ij}. Thus, in this case of shared disaster risk, consumption is more correlated.

Now consider the effects on asset prices. The correlation in the state prices in equation (7) will include a common disaster shock, dN_t^w, thereby increasing the correlation in state prices. In particular, since disaster risk is shared, both by a common occurrence of the disaster event, dN_t^w, and by common changes in its probability, $dB_{\lambda,t}^w$, then intertemporal marginal utility increases at the same time across countries. As a result, equity prices will share this same higher correlation due to the common disaster shock. To see this relationship, consider the stock price process in equation (11) under constant disaster risk so that $dB_{\lambda,t}^w = 0$. Then the stock price evolution becomes:

$$\frac{dF_t^j}{F_{t-}^j} = \mu_{F,t}^j dt + \phi^j \sigma^j dB_t^j + (e^{\phi^j \omega^j Z} - 1)dN_t^w,$$

42

As this equation shows, when disaster intensities are constant, instantaneous variations in equity returns are generated exclusively through the Brownian on "normal times" consumption, dB_t, and the disaster event shock, dN_t^w, as in consumption. However, these effects on equity prices are magnified relative to consumption growth by the leverage parameter ϕ. Thus, the correlations in equity prices are the same as those of consumption in equation (17) except that now $K^j \equiv (e^{\phi^j \omega^j Z} - 1)$ and σ^j are replaced by $\sigma^j \phi^j$.

When disaster intensities are time-varying, the correlation of asset prices can be higher than consumption, however. Rewriting the stock price in equation (11) to include the common disaster event and common time-varying probabilities, the process becomes:

$$\frac{dF_t^j}{F_{t-}^j} = \mu_{F,t}^j dt + \phi^j \sigma^j dB_t^j + g\sigma_\lambda \sqrt{\lambda_t^w} dB_{\lambda,t}^w + (e^{\phi^j \omega^j Z} - 1) dN_t^w.$$

In this case, stock price changes have a higher correlation due to the perfect correlation in $dB_{\lambda,t}^w$; that is, through innovations to the probability of a common world disaster.

We next evaluate the quantitative implications for both common and independent disasters on the correlations across countries.

3.3 Matching the Moments: Correlations

In order to understand the degree of co-movement between returns, we focus upon two countries, the U.S. and the U.K. Table 6 reports under "Data Correlation" the cross-country correlations of consumption growth, equity returns and government bill rates for these two countries using the Barro and Ursua (2008) data. In the rows labeled, "Unconditional," the table gives the correlations calculated over the entire history of common consumption

and asset return data from 1870 onwards, while the rows labeled "Conditional" provide the same statistics calculated in the post-War data. Both the Unconditional and Conditional correlations show the pattern typically found in the literature that consumption correlations are lower than asset return correlations.

Table 6 also gives these same correlations calculated from the model simulations assuming the Wachter (2013) parameter values for two identical countries with correlation in normal times consumption give by the post-War data correlation between the U.S. and the U.K. These simulations are conducted for two cases. Panel A reports the correlations of consumption, equity returns, and government bills under the assumption that the disaster probability is constant and that $\lambda_t^j = \lambda_t = \overline{\lambda} = 3.5\%$ for $j = 1, 2$. The table reports both the "Unconditional" correlations over all realizations as well as the "Conditional" correlations excluding the disaster events. The second column gives the results when the effects upon consumption due to disasters are assumed to be uncorrelated across countries as in equation (12). As described above, the correlation of equity during normal times, consistent with the "Conditional" results, is determined by normal times consumption. Over the whole sample including disasters reported in the Unconditional results, however, consumption and equity return correlations are driven down to implausibly low numbers by independent realizations of dN_t^j.

Alternatively, the last column reports the results assuming a common world disaster as in equation (15). Since countries are affected by a common world disaster, the disasters across countries are not only perfectly correlated, but also have the same magnitude of realizations of Z. Once again, the consumption and equity return correlations are similar during normal times, as the "Conditional" results show. However, now the periods of disasters are shared

across countries, both for the timing and size of disasters. As a result, the periods of disasters generate a strong common component during those periods, driving the consumption correlations very high. Indeed, the correlation of consumption is higher than that of equity returns, inconsistent with the data.

Panel B of Table 6 provides the correlation when the probability of disasters is time varying as in equation (17). In this case, an important component of the correlation of equity returns is determined by the co-movement of time-variation in disaster probabilities, λ_t. The column labeled "No Disaster Correlation" shows that when these disaster probabilities are uncorrelated, even the normal times Conditional correlations in equities are lower than consumption at 0.056. As in the static probability case, the independence of disasters renders the Unconditional correlations in both consumption and equity to be implausibly low at about 5%. Also, as described in the previous subsection, the correlation of the bill rates is driven entirely by the correlation in the disaster probabilities. Thus, when the probabilities of disasters are independent across countries, the correlation of bill rates becomes zero.

By contrast, when disaster events occur at the same time across countries, the correlations in asset returns are much higher. For this case, reported in the last column of Table 6 Panel B, equity return correlations at 0.946 are significantly higher than those of consumption. Moreover, this high correlation is maintained in the full sample at 0.958. At the same time, both conditional and unconditional government bill rates carry a correlation of one, since they are driven by the same common disaster probability.

In summary, the investigation highlights problems with both versions of assumptions on disaster risk. When disasters are independent, correlations of asset returns are too low, indeed lower than consumption correlations. On the other hand, when disasters are shared,

asset return correlations are too high and near one.

3.4   Country and World Disasters

As the results above demonstrate, standard assumptions about disaster risk across countries imply counterfactual implications for the normal times correlation in equity returns. In the absence of time-varying disaster intensities, the variation in equity returns is too low and the equity return correlation is driven entirely by the correlation in consumption. In the presence of time-variation in disaster intensities, however, the equity return correlation is too high if the disaster events are common and too low if disaster events are independent.

This observation suggests that a more plausible assumption is that some disasters are shared while others are country-specific. To allow for this possibility, we specify the disaster event in the consumption process as a mixture of two Poisson jump processes. Moreover, note that there are two potential dimensions in which disaster risk can be correlated: the disaster event, N_t, and the size of consumption decline conditional on disaster, Z. To focus upon the role of disaster events, we assume that disasters are only correlated through the Poisson process that guides the timing of the disaster, and allow the size of disasters Z^j to be independent for each country j.[33] Then the consumption process in this case is:

$$\frac{dC_t^j}{C_{t-}^j} = \mu dt + \sigma^j dB_t^j + (e^{\omega^j Z^j} - 1)(dN_t^j + dN_t^w) \tag{19}$$

where N_t^j has disaster intensity λ_t^j and N_t^w has disaster intensity λ_t^w. In other words, the

---

[33]Note that the assumption is only made to connect with the data on the prior section. In principle, we could consider the case where the world disaster is big and the country-specific disaster is smaller, or vice versa.

probability of a disaster in each country can be generated by a world disaster shock, dN_t^w, or a country-specific shock, dN_t^j. In turn, each of these shocks are driven by their own time-varying probability processes as in equations (5) and (16).

As such, the correlation of consumption is a mixture of the independent and common jump processes. For example, in the special case when the probability of disasters is constant so that $\lambda_t^i + \lambda_t^w = \lambda^i + \lambda^w$, $\forall t$, the consumption correlation is:

$$Corr\left(\frac{dC_t^i}{C_{t-}^i}, \frac{dC_t^j}{C_{t-}^j}\right) = \frac{\sigma^i \sigma^j \rho^{ij} + K^i K^j \lambda^w}{\sqrt{(\sigma^i)^2 + (K^i)^2 \lambda^w}\sqrt{((\sigma^j)^2 + (K^j)^2 \lambda^w}} \tag{20}$$

where $K^j \equiv (e^{\omega^j Z^j} - 1)$ as before. However, note that the probability of world disasters λ^w must now be lower since the total probability of a disaster is given by the sum of the probabilities, $\lambda_t^i + \lambda_t^w$. In order to match the data, we therefore impose the condition that the means of the two intensity processes equals that of the data; or $\tau^i \overline{\lambda}^w + (1 - \tau^i)\, \overline{\lambda}^i = \widetilde{\lambda}^i$ where $\widetilde{\lambda}^i$ is the weighted mean of the joint Poisson process and $0 < \tau^i < 1$ is the share of disaster risk of country i that is due to country-specific disasters.

In this case, the consumption correlations will be lower than in the case of common world disasters due to the presence of uncorrelated country-specific disasters, thereby lowering λ^w. The realizations of these common and country-specific disaster events therefore affect equity correlations as well. The probability of a disaster for country i given that country j is in a disaster will lie between the extreme cases of 0 and 1.

Identifying the share of country-specific versus world disaster probabilities poses a difficulty with empirically evaluating the differing disaster risks across countries. Since there are not enough disaster events in a given country, there are insufficient observations to detect

common versus country-specific disaster events. However, Lewis and Liu (2015) propose an identification approach that is useful in this context. Given the pattern of consumption correlation such as in equation (20) along with the implied correlation patterns of asset returns in the model, we can recover the relevant patterns for identification such as λ^i, λ^w. Using the framework above, we can vary the share of country-specific disaster risk through τ^i to match the observed correlation patterns.

Table 7 demonstrates the relationship implied by varying τ^i between 0 and 1 for the Static Disaster case in Panel A and the Time-Varying Disaster case in Panel B. To be consistent with our prior analysis, we set $\tilde{\lambda}^i = 3.55\%$. We continue to maintain the assumption that the variance of the probability of country-specific and world disasters are the same, as for the case of the U.S. and U.K. example. As the weight on the world disaster increases, the correlation of asset returns increases. For example, for the Time-Varying case under normal times (signified by the Conditional rows), the correlation of equity returns increases from 0.055 when disasters are uncorrelated to 0.946 when they are perfectly correlated. A similar pattern holds for the bill rate.

The table results suggest combinations of world and country-specific disasters that may match the pattern of correlations given that consumption correlations are equal to their data counterparts by construction. Using again the example of the Conditional period Time-Varying Disaster case, when $\tau = 0.6$, the correlation of equity returns is 0.59 while that of bill returns in 0.61, both close to their data counterparts of 0.58 and 0.63, respectively. The fit to the full sample period reported as the Unconditional results are not as tight, in part because of the low data correlation of 0.12 over the full period since 1870. This tendency may reflect an increase in consumption correlations over time as the world has become more

integrated. Overall, the evidence suggests a higher degree of world disaster risk than country disaster risk is needed to explain the degree of co-movement between asset returns.

4   Concluding Remarks

A growing literature examines the impact of disaster risk on the macroeconomy and asset returns. Indeed, the relevance of this risk has become more evident since the recent financial crises. A standard approach in this literature is to use international data to make inferences about the frequency and size of these disasters. Nevertheless, the focus of these studies has largely targeted U.S. consumption and asset return behavior.

In this paper, we began to ask what this literature means from an international perspective. For this purpose, we evaluated international asset returns through the lens of a canonical disaster risk model as articulated in Barro (2006, 2009) and allowing for time-varying disasters as in Wachter (2013). Our analysis led to three main findings. First, while the disaster risk model does well in explaining U.S. asset returns, it is less successful in matching the range of asset return behavior observed internationally. Second, the degree to which the model can explain international asset return co-movements hinges largely on the importance of a common disaster risk across countries. Specifically, if the frequency and size of disasters is independent across countries, the correlation of asset returns is implausibly low. By contrast, if all disasters are common, these correlations are near one and, hence, unrealistically large. Third, these findings suggest that international correlations of asset returns and consumption can provide an identification of the importance of world versus country-specific disasters. Calibrating the model to the correlations between the U.S. and

the U.K. implies that 60% of the disaster risk is common between the two countries. Overall, this paper shows that the international dimensions of standard disaster risk models carry important implications for their saliency.

REFERENCES

Abel, Andrew, 1999, "Risk Premia and Term Premia in General Equilibrium," Journal of Monetary Economics, 43, 3-33.

Adler, Michael; Dumas, Bernard, 1983, "International Portfolio Choice and Corporation Finance: A Synthesis," Journal of Finance, 38.3, 925-984.

Backus, David; Chernov, Mikhail; Martin, Ian, 2011, "Disasters Implied by Equity Index Options," Journal of Finance, 66: 6, 1969-2012.

Backus, David K.; Kehoe, Patrick J.; Kydland, Finn E., 1994, "Dynamics of the Trade Balance and the Terms of Trade: The J-curve?" American Economic Review, 84:1, 84-103.

Bakshi, Gurdip; Cerrato, Mario; Crosby, John, 2015, "Risk Sharing in International Economies and Market Incompleteness," University of Maryland Working Paper, December.

Bansal, Ravi; Yaron, Amir. 2004 "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," Journal of Finance, 59.4, 1481-1509.

Barro, Robert J., 2006, "Rare Disasters and Asset Markets in the Twentieth Century," Quarterly Journal of Economics, 121, 823-866.

Barro, Robert J., 2009, "Rare Disasters, Asset Prices, and Welfare Costs," American Economic Review, 99, 243-264.

Barro, Robert J.; Ursua, Jose F., 2008, "Macroeconomic Crises Since 1870," Brookings Papers on Economic Activity, 1, 255-350.

Baxter, Marianne; Crucini, Mario J. 1995, "Business Cycles and the Asset Structure of Foreign Trade." International Economic Review, November, Vol. 36(4), 821-854.

Bekaert, Geert; Harvey, Campbell R; Lundblad, Christian T; Siegel, Stephan, 2011, "What Segments Equity Markets?" Review of Financial Studies, 24.12, 3841-3890.

Campbell, John Y. 1993, "Intertemporal Asset Pricing without Consumption Data," American Economic Review, 83.3: 487-512.

Campbell, John Y.; Cochrane, John H. 1999, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," Journal of Political Economy, 107.2, 205-251.

Carrieri, F.; Chaieb, I.; Errunza, V. 2013, "Do Implicit Barriers Matter for Globalization?" Review of Financial Studies, 26:7, 1694-1739

Cochrane, John H., 1991, "A simple test of consumption insurance," The Journal of Political Economy, 99(5), pp. 957-976 "

Colacito, Riccardo; Croce, Mariano M.; 2011, "Risks for the Long Run and the Real Exchange Rate," Journal of Political Economy, 119(1), pp. 153-181

Cole, Harold L.; Obstfeld, Maurice, 1991, "Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?" Journal of Monetary Economics, 28, 3-24.

Duffie, Darrell, 2010. Dynamic asset pricing theory. Princeton University Press.

Duffie, Darrell; Larry G. Epstein, 1992, "Asset pricing with stochastic differential utility", Review of Financial Studies 5, 411–436.

Duffie, Darrell, and Costis Skiadas, 1994, "Continuous-time asset pricing: A utility gradient approach," Journal of Mathematical Economics 23, 107–132.

Dumas, Bernard; Solnik, Bruno, 1995, "The World Price of Foreign Exchange Risk," Journal of Finance, 50:2, 445-79.

Epstein, Larry; Zin, Stanley, 1989, "Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework," Econometrica 57, 937–969.

Epstein, Larry; Zin, Stanley; 1991, "Substitution, risk aversion and the temporal behavior of consumption and asset returns: An empirical analysis," Journal of Political Economy 99(2), 263-286.

Farhi, Emmanuel; Gabaix, Xavier, 2016, "Rare Disasters and Exchange Rates," The Quarterly Journal of Economics 131, 1-52.

Gabaix, Xavier, 2008, "Variable Rare Disasters: A Tractable Theory of Ten Puzzles in Macro-Finance," American Economic Review 98, 64-67.

Gabaix, Xavier, 2012, "Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance," Quarterly Journal of Economics 127, 645–700.

Gourio, Francois, 2008, "Disasters and recoveries," American Economic Review Papers and Proceedings, 98, 68–73.

Gourio, Francois, 2012, "Disaster Risk and Business Cycles," American Economic Review, 102, 2734-2766.

Gourio, François; Siemer, Michael; Verdelhan, Adrien, 2013, "International Risk Cycles," Journal of International Economics, 89:2, 471–484.

Hansen, Lars P.; Singleton, Kenneth J., 1983, "Stochastic consumption, risk aversion, and the temporal behavior of asset returns," The Journal of Political Economy, pp. 249-265.

Heaton, John; Lucas, Deborah J., 1996, "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing," The Journal of Political Economy, 104 (3), pp. 443-487.

Kaltenbrunner, Geog; Lochstoer, Lars. 2010, "Long-Run Risk through Consumption Smoothing," Review of Financial Studies 23, pp. 3141 - 3189.

Kocherlakota, Narayana R. 1996, "The Equity Premium: It's Still a Puzzle," Journal of Economic Literature 34.1, pp.42-71.

Lewis, Karen K. 1996, "What Can Explain the Apparent Lack of International Consumption Risk Sharing?" Journal of Political Economy 104.2, 267-297.

Lewis, Karen K. 2000, "Why Do Stocks and Consumption Imply Such Different Gains from International Risk Sharing?" Journal of International Economics 52.1: 1-35.

Lewis, Karen K.; Liu, Edith X., 2015, "Evaluating International Consumption Risk Sharing Gains: An Asset Return View," Journal of Monetary Economics, 71, 84 - 98.

Lewis, Karen K.; Liu, Edith X., 2016, "Disaster Risk and International Asset Markets," In Progress.

Lucas, Robert E., Jr. 1978, "Asset Prices in an Exchange Economy," Econometrica, 46.6: 1429-1445.

Lucas, Robert E, Jr. 1982, "Interest Rates and Currency Prices in a Two-Country World," Journal of Monetary Economics,10.3: 335-359.

Lustig, Hanno; Verdelhan, Adrien, 2016, "Does Incomplete Spanning in International Financial Markets Help to Explain Exchange Rates?" Stanford University Working paper, March.

Martin, Ian W., 2008, "Disasters and the welfare cost of uncertainty," The American Economic Review, 98(2), 74 - 78.

Mehra, Rajnish; Prescott, Edward, 1985, "The Equity Premium: A Puzzle", Journal of Monetary Economics, 15, 145-161.

Nakamura, Emi; Steinsson, Jon; Barro, Robert; Ursua, Jose, 2013, "Crises and Recoveries in an Empirical Model of Consumption Disasters," American Economic Journal: Macroeconomics, 5.3, 35-74.

Obstfeld, Maurice, 1994. "Risk-Taking, Global Diversification, and Growth," The American Economic Review, 84(5), pp.1310-1329.

Pukthuanthong, Kuntara; Roll, Richard, 2009, "Global Market Integration: An Alternative Measure and Its Application," Journal of Financial Economics 94,2, 214-232.

Reitz, Thomas A. 1988, "The Equity Risk Premium: A Solution," Journal of Monetary Economics, 22, 117-131.

Tesar, Linda L., 1995, "Evaluating the gains from international risksharing," Carnegie-Rochester Conference Series on Public Policy, 42, 95-143.

Telmer, Christopher, 1993, "Asset Pricing Puzzles and Incomplete Markets," Journal of Finance, 48(5), pp. 1803-1832.

Verdelhan, Adrien, 2010, "A Habit-Based Explanation of the Exchange Rate Risk Premium," Journal of Finance, Journal of Finance, 65:1, 123-145.

Weil, Philippe, 1990, "Nonexpected utility in macroeconomics," Quarterly Journal of Economics 105, 29–42.

Wachter, Jessica A., 2013. "Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?" The Journal of Finance, 68(3), pp.987-1035.

Table 1: Data Moments (Annual %)

Panel A: Unconditional (Full Sample)	AUS	CAN	FRA	GER	JPN	UK	US
Mean Cons Growth	1.54	1.92	1.62	1.89	2.48	1.47	1.85
Std Dev Cons Growth	5.06	4.74	6.74	5.70	6.89	2.83	3.60
Mean Equity Premium	9.01	3.89	6.04	9.11	8.85	4.62	6.28
Mean Equity Return	10.27	7.81	5.43	7.58	9.28	6.41	8.27
Std Dev Equity Return	16.16	17.54	20.78	29.76	30.17	17.65	18.66
Sharpe Ratio	0.56	0.22	0.29	0.31	0.29	0.26	0.34
Mean Govt Bill	1.26	3.92	-0.61	-1.53	0.43	1.79	1.99
Std Dev Govt Bill	5.66	11.99	9.96	17.88	14.75	6.24	4.82
Panel B: Conditional (Post-War Sample)	AUS	CAN	FRA	GER	JPN	UK	US
Mean Cons Growth	2.45	2.22	3.34	3.56	5.47	2.38	2.40
Std Dev Cons Growth	3.19	2.24	3.70	3.36	6.19	2.21	2.07
Mean Equity Premium	8.49	6.35	8.15	10.63	11.90	7.77	8.16
Mean Equity Return	9.04	7.82	7.31	10.74	9.32	8.88	8.86
Std Dev Equity Return	20.09	15.82	25.42	33.66	33.73	22.51	17.44
Sharpe Ratio	0.42	0.40	0.32	0.32	0.35	0.35	0.47
Mean Govt Bill	0.55	1.46	-0.84	0.11	-2.57	1.11	0.70
Std Dev Govt Bill	5.14	3.81	9.86	12.05	15.67	3.67	3.19

[†] Data from Barro and Ursua (2008)

Table 2: Constant Disaster Model Fit - SMM

Panel A: Target Unconditional Data Moments

	AUS	CAN	FRA	GER	JPN	UK	US
Fitted Parameters							
q	0.421	0.442	0.431	0.429	0.447	0.445	0.436
ϕ	2.664	2.795	2.630	2.805	2.755	2.749	2.741
ω	0.884	0.840	0.854	0.843	0.844	0.843	0.846
σ (in %)	2.17	2.26	2.28	2.31	2.29	2.26	2.24
Unconditional Model Moments (Annual %)							
Std Dev Cons Growth	2.12	2.34	2.31	2.43	2.17	2.23	2.34
Mean Equity Premium	5.36	4.74	4.85	4.80	4.68	4.76	4.73
Std Dev Equity Return	9.95	10.22	9.94	10.67	10.36	10.19	10.18
Sharpe Ratio	0.54	0.46	0.49	0.45	0.45	0.47	0.46
Mean Govt Bill	2.86	3.31	3.18	3.20	3.30	3.30	3.24
Std Dev Govt Bill	2.35	2.29	2.11	2.42	2.22	2.08	2.27
Mean Sq Diff	0.286	0.052	0.079	0.093	0.286	0.067	0.105

Panel B: Target Conditional Data Moments

	AUS	CAN	FRA	GER	JPN	UK	US
Fitted Parameters							
q	0.488	0.521	0.462	0.457	0.432	0.502	0.497
ϕ	2.945	3.005	2.768	2.840	2.731	2.865	2.969
ω	0.839	0.817	0.843	0.841	0.873	0.814	0.808
σ (in %)	3.19	2.24	3.85	3.57	6.25	2.45	2.37
Conditional Model Moments (Annual %)							
Std Dev Cons Growth	3.19	2.25	3.85	3.60	6.25	2.44	2.35
Mean Equity Premium	6.26	5.12	6.61	6.48	9.97	5.03	5.07
Std Dev Equity Return	10.32	7.37	11.73	11.29	19.29	7.63	7.61
Sharpe Ratio	0.61	0.70	0.57	0.59	0.52	0.67	0.67
Mean Govt Bill	3.44	3.90	3.17	3.24	1.96	3.84	3.88
Std Dev Govt Bill	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Mean Sq Diff	0.050	0.099	0.093	0.141	0.076	0.128	0.053

[†] For the constant disaster risk model, we follow Barro (2006) in calibrating disaster frequency ($\bar{\lambda} = 1.7\%$), and follow Panel C of Table 5 of Wachter (WP 2009) in calibrating consumption growth parameter ($\mu = 2.52\%$), preference parameters ($\beta = 0.012$, $\gamma = 3$), and average disaster risk probability or intensity ($\bar{\lambda} = 3.55\%$).

Table 3: Constant Disaster - Varying ω

ω	1.0	0.9	0.8	1.0	0.9	0.8	1.0	0.9	0.8
Parameters									
		Baseline			High ϕ			Low q	
q	0.4	0.4	0.4	0.4	0.4	0.4	**0.25**	**0.25**	**0.25**
ϕ	2.8	2.8	2.8	**3.0**	**3.0**	**3.0**	2.8	2.8	2.8
σ (in %)	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Unconditional Model Moments (Annual %)									
Mean Cons Growth	1.86	1.92	1.99	1.86	1.92	1.99	1.86	1.92	1.99
Std Dev Cons Growth	5.99	5.46	4.94	5.99	5.46	4.94	5.99	5.46	4.94
Mean Equity Premium	7.88	5.56	3.91	8.12	5.76	4.08	8.73	6.17	4.34
Std Dev Equity Return	10.74	10.27	9.79	11.27	10.79	10.31	10.74	10.27	9.79
Sharpe Ratio	0.60	0.27	0.02	0.59	0.28	0.04	0.76	0.39	0.11
Mean Govt Bill	1.44	2.78	3.69	1.44	2.78	3.69	0.59	2.18	3.26
Std Dev Govt Bill	2.77	2.81	2.83	2.77	2.81	2.83	2.07	2.10	2.12
Conditional Model Moments (Annual %)									
Mean Cons Growth	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50
Std Dev Cons Growth	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Mean Equity Premium	8.79	6.40	4.67	9.07	6.64	4.87	9.73	7.09	5.19
Std Dev Equity Return	6.19	6.13	6.08	6.65	6.58	6.53	6.19	6.13	6.08
Sharpe Ratio	1.15	0.55	0.13	1.11	0.55	0.15	1.46	0.78	0.30
Mean Govt Bill	1.66	3.00	3.91	1.66	3.00	3.91	0.72	2.30	3.39
Std Dev Govt Bill	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

[†] For the constant disaster risk model, we follow Barro (2006) in calibrating disaster frequency ($\bar{\lambda} = 1.7\%$), and follow Panel C of Table 5 of Wachter (WP 2009) in calibrating consumption growth parameter ($\mu = 2.52\%$), preference parameters ($\beta = 0.012$, $\gamma = 3$), and average disaster risk probability or intensity ($\bar{\lambda} = 3.55\%$).

Table 4: Time Varying Disaster Model Fit - SMM

Panel A: Identical Disaster Parameters							
	AUS	CAN	FRA	GER	JPN	UK	US
Fitted Parameters							
q	0.208	0.597	0.597	0.588	0.592	0.598	0.598
ϕ	2.011	2.874	2.960	3.490	3.493	2.722	2.585
ω	0.977	0.863	0.845	0.823	0.824	0.874	0.889
σ (in %)	3.73	1.60	1.61	1.98	2.12	1.58	1.56
Identical Parameters							
κ	0.08	0.08	0.08	0.08	0.08	0.08	0.08
σ_λ	0.067	0.067	0.067	0.067	0.067	0.067	0.067
Unconditional Model Moments (Annual %)							
Std Dev Cons Growth	7.41	6.59	6.51	6.66	6.75	6.51	6.55
Mean Equity Premium	7.41	7.52	7.71	9.20	9.26	7.18	6.84
Std Dev Equity Return	16.87	22.21	22.93	28.69	28.86	20.93	19.74
Sharpe Ratio	0.44	0.35	0.35	0.33	0.33	0.36	0.36
Mean Govt Bill	0.09	1.62	1.63	1.55	1.55	1.63	1.63
Std Dev Govt Bill	3.53	4.12	4.08	4.1	4.1	4.08	4.13
Mean Sq Diff	0.015	0.028	0.008	0.020	0.013	0.013	0.002
Panel B: Country Specific Disaster Parameters							
	AUS	CAN	FRA	GER	JPN	UK	US
Fitted Parameters							
κ	0.200	0.200	0.199	0.199	0.199	0.198	0.199
σ_λ	0.066	0.078	0.079	0.079	0.085	0.087	0.085
Identical Parameters							
q	0.4	0.4	0.4	0.4	0.4	0.4	0.4
ϕ	2.6	2.6	2.6	2.6	2.6	2.6	2.6
ω	1.0	1.0	1.0	1.0	1.0	1.0	1.0
σ (in %)	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Unconditional Model Moments (Annual %)							
Std Dev Cons Growth	6.45	6.6	6.57	6.64	6.67	6.61	6.69
Mean Equity Premium	4.66	4.79	4.85	4.78	4.91	4.95	4.93
Std Dev Equity Return	12.20	12.78	12.90	12.99	13.29	13.40	13.25
Sharpe Ratio	0.42	0.41	0.41	0.40	0.40	0.40	0.41
Mean Govt Bill	0.95	0.94	0.93	0.95	0.92	0.96	0.98
Std Dev Govt Bill	3.34	3.51	3.49	3.54	3.59	3.59	3.55
Mean Sq Diff	0.022	0.047	0.025	0.060	0.055	0.024	0.009

[†] For the time-varying disaster risk model, we follow Wachter (2013) for "Identical Parameters", mean consumption growth parameter ($\mu = 2.52\%$), preference parameters ($\beta = 0.012$, $\gamma = 3$), and average disaster risk probability or intensity ($\bar{\lambda} = 3.55\%$).

Table 5: Time Varying Disaster - Varying ω

ω	1.0	0.95	0.85	1.0	0.95	0.85	1.0	0.95	0.85
Parameters									
		Baseline			**High ϕ**			**Low q**	
q	0.4	0.4	0.4	0.4	0.4	0.4	**0.25**	**0.25**	**0.25**
ϕ	2.6	2.6	2.6	**3.0**	**3.0**	**3.0**	2.6	2.6	2.6
κ	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
σ_λ	0.067	0.067	0.067	0.067	0.067	0.067	0.067	0.067	0.067
σ **(in %)**	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Panel A: Unconditional Moments (Annual %)									
Mean Cons Growth	1.63	1.68	1.76	1.63	1.68	N/A	1.63	1.68	1.76
Std Dev Cons Growth	6.36	6.07	5.50	6.36	6.07	N/A	6.36	6.07	5.50
Mean Equity Premium	7.62	6.34	4.61	8.56	7.18	N/A	8.07	6.73	4.88
Std Dev Equity Return	19.95	18.79	16.62	23.39	22.36	N/A	19.95	18.79	16.62
Sharpe Ratio	0.39	0.35	0.29	0.37	0.33	N/A	0.41	0.36	0.30
Mean Govt Bill	1.01	1.33	1.86	1.01	1.33	N/A	0.56	0.95	1.59
Std Dev Govt Bill	3.79	3.55	3.11	3.79	3.55	N/A	3.55	3.26	2.76
Panel B: Conditional Moments (Annual %)									
Mean Cons Growth	2.52	2.52	2.52	2.52	2.52	N/A	2.52	2.52	2.52
Std Dev Cons Growth	1.99	1.99	1.99	1.99	1.99	N/A	1.99	1.99	1.99
Mean Equity Premium	8.87	7.55	5.73	9.97	8.55	N/A	9.43	8.03	6.10
Std Dev Equity Return	17.73	16.57	14.41	21.19	20.20	N/A	17.73	16.57	14.41
Sharpe Ratio	0.49	0.49	0.49	0.49	0.49	N/A	0.49	0.49	0.49
Mean Govt Bill	1.38	1.69	2.18	1.38	1.69	N/A	0.82	1.20	1.81
Std Dev Govt Bill	2.00	1.74	1.32	2.00	1.74	N/A	2.48	2.16	1.64

[†] We also follow Wachter (2013) for additional parameters: consumption growth ($\mu = 2.52\%$), and preference parameters ($\beta = 0.012$, $\gamma = 3$).

Table 6: Model Implied Correlation

Panel A: Static Disaster

	Data Correlation	No Disaster Correlation	Perfect Disaster Correlation
Unconditional:			
Corr Cons Growth	0.119	0.034	0.951
Corr Equity Return	0.473	0.117	0.873
Corr Bill Rate	0.564	0.000	1.0
Conditional:			
Corr Cons Growth	0.496	0.495	0.495
Corr Equity Return	0.576	0.495	0.495
Corr Bill Rate	0.628	N/A	N/A

Panel B: Time Varying Disaster

	Data Correlation	No Disaster Correlation	Perfect Disaster Correlation
Unconditional:			
Corr Cons Growth	0.119	0.046	0.950
Corr Equity Return	0.473	0.045	0.958
Corr Bill Rate	0.564	0.000	1.000
Conditional:			
Corr Cons Growth	0.496	0.490	0.489
Corr Equity Return	0.576	0.056	0.946
Corr Bill Rate	0.628	0.000	1.000

Table 7: Model Implied Correlation - Varying World and Individual Country Disaster

Panel A: Static Disaster							
	Data	$\tau = 0.0$	$\tau = 0.2$	$\tau = 0.4$	$\tau = 0.6$	$\tau = 0.8$	$\tau = 1.0$
Unconditional:							
Corr Cons Growth	0.119	0.048	0.144	0.270	0.373	0.491	0.614
Corr Equity Return	0.473	0.122	0.242	0.367	0.495	0.634	0.766
Corr Bill Rate	0.564	0.000	0.053	0.114	0.200	0.279	0.304
Conditional:							
Corr Cons Growth	0.496	0.495	0.494	0.494	0.495	0.495	0.494
Corr Equity Return	0.576	0.494	0.493	0.493	0.494	0.494	0.494
Corr Bill Rate	0.628	N/A	N/A	N/A	N/A	N/A	N/A
Panel B: Time Varying Disaster							
	Data	$\tau = 0.0$	$\tau = 0.2$	$\tau = 0.4$	$\tau = 0.6$	$\tau = 0.8$	$\tau = 1.0$
Unconditional:							
Corr Cons Growth	0.119	0.042	0.169	0.281	0.387	0.482	0.613
Corr Equity Return	0.473	0.037	0.236	0.396	0.565	0.722	0.920
Corr Bill Rate	0.564	0.002	0.117	0.230	0.357	0.434	0.561
Conditional:							
Corr Cons Growth	0.496	0.492	0.495	0.494	0.492	0.495	0.494
Corr Equity Return	0.576	0.055	0.257	0.416	0.586	0.748	0.946
Corr Bill Rate	0.628	0.007	0.225	0.416	0.612	0.793	1.000

A   Appendix: Asset Return Solutions

In the text, we use solutions for asset returns for each country that follow from straightforward extensions of the analysis in Wachter (2013) Appendix A. While we summarize these solutions here, interested readers may wish to consult Wachter (2013) for detailed derivations.

A.1   The Value Function

The asset returns are determined through the intertemporal first order condition maximizing consumption and portfolio allocations given income. Since wealth is related to equilibrium consumption through the budget constraint in equation (3), these returns depend upon their riskiness relative to a claim on realizations of consumption in all future periods (Epstein and Zin (1989,1991), Duffie and Epstein (1992)) and a risk-free rate. Using this relationship allows us to solve for the value function and, as described in below, the state price density and risk-free rate as we describe below. Given these solutions, the price of any other asset can then be determined through standard arbitrage arguments as described in Wachter (2013).

Defining the price of a claim on C_t^j for the representative agent in country j as S_t^j and assuming that the consumption process has the form given in equation (4), the price of this claim follows[34]:

$$dS_t^j = \mu S_{t-}^j dt + \sigma^j S_{t-}^j dB_t^j + (e^{\omega^j Z_t} - 1)S_{t-}^j dN_t^j \tag{21}$$

Next, defining α_t^j as the fraction of wealth W_t^j that the representative agent allocates to this risky asset, the price-dividend ratio for country j as $\ell^j \equiv (S_t^j/C_t^j)$ and $(1 - \alpha_t^j)$ fraction of

---

[34]Since the purpose of this appendix is to consider possible differences across countries, we signify differences across countries with j superscripts. Note, however, that we do not impose any assumption about the degree of integration or comovement in assets in these derivations. We return to these issues in Appendix B below.

their wealth to the risk free asset, then the wealth process for country j follows:

$$dW_t^j = W_t^j \alpha_t^j dS_t^j + W_t^j (1 - \alpha_t^j) r_t^j dt - C_t^j dt \tag{22}$$

$$= (\alpha_t^j W_{t-}^j (\mu - r_t^j + (\ell^j)^{-1}) + W_{t-}^j r_t^j - C_t^j) dt + \alpha_t^j W_{t-}^j \sigma^j dB_t^j + \alpha_t^j W_{t-}^j (e^{\omega^j Z_t} - 1) dN_t^j$$

Thus, α_t^j is investor j's portfolio share in the consumption asset while $(1 - \alpha_t^j)$ is investor j's portfolio share in a country j risk-free asset. Note that in this formulation of wealth, the price-dividend ratio for this consumption claim $\ell^j = (C_t^j / W_t^j)$ is a constant because the intertemporal elasticity of consumption is one (see Weil (1990)).

Defining the value function as $H(W_t^j, \lambda_t^j)$, the Hamilton-Jacobi-Bellman equation relates the instantaneous expected change in the value function plus flow utility to zero at the optimum, similar to the role of the Bellman equation in discrete time. Given the wealth process in equation (22), the risky asset process in equations (21), the felicity function in equation (2), and solving for this HJB as a direct application of Ito's lemma with jumps (see Duffie(2010)) implies that this equation is:

$$\sup_{\alpha_t^j, C_c^j} \{ H_W(\alpha_t^j W_t^j (\mu - r_t^j + (l_t^j)^{-1}) + W_t^j r_t^j - C_t^j) + H_\lambda \kappa(\bar{\lambda} - \lambda_t^j) + \tfrac{1}{2} H_{WW}(\alpha_t^j W_t^j \sigma^j)^2$$

$$+ \tfrac{1}{2} H_{\lambda\lambda} \sigma_\lambda^2 \lambda_t^j + \lambda_t^j E_\nu [H(W_t(1 + \alpha_t^j (e^{\omega^j Z} - 1)), \lambda_t^j) - H(W_t^j, \lambda_t^j)] + f(C_t^j, J) \} = 0 \tag{23}$$

where H_i and H_{iq} are the first and second derivatives of H with respect to i and to i and q, respectively, and E_ν is the expectation taken over the time invariant distribution ν of Z. Solving this equation for the value function $H(W_t^j, \lambda_t^j)$ then provides the utility from lifetime consumption as a function of state variables W_t^j and λ_t^j. In equilibrium, W_t^j is given by the

consumption asset so that, $\alpha_t^j = 1$. Using this fact along with the consumption process in equation (4), we solve for this value function by guess-and-verify following the conjecture form in Wachter (2013) :

$$H(W_t^j, \lambda_t^j) = I^j(\lambda_t^j)^{1-\gamma} \frac{\left(W_t^j\right)^{1-\gamma}}{1-\gamma}. \tag{24}$$

Using this form of the value function and the envelope condition that $H_W = U_C(C, V)$, it follows that: $\ell^j = \ell = \beta^{-1}$. Thus, with identical preferences, all countries have the same wealth-consumption ratio, even though they are potentially priced in completely segmented financial markets with different consumption units. Further conjecturing the explicit form for the proportionality term in the value function as

$$I^j(\lambda_t^j) = e^{a^j + b^j \lambda_t^j} \tag{25}$$

and following the same steps as Wachter (2013), it can be shown that for the benchmark case considered in the text when $\sigma_\lambda^j = \sigma_\lambda; \kappa^j = \kappa, \forall j = 1, ...J$:

$$b^j = \left(\frac{\kappa + \beta}{\sigma_\lambda^2}\right) - \sqrt{\left(\frac{\kappa + \beta}{\sigma_\lambda^2}\right)^2 - 2\frac{E_\nu \left(e^{(1-\gamma)\omega^j Z_t} - 1\right)}{\sigma_\lambda^2}} \tag{26}$$

$$a^j = \frac{1-\gamma}{\beta}\left(\mu - \frac{1}{2}\gamma(\sigma^j)^2\right) + (1-\gamma)\log(\beta) + b^j \frac{\kappa \overline{\lambda}}{\beta}. \tag{27}$$

Note that the effects on the value function from time-variation in the disaster intensities, λ_t, as captured by b^j in equation (26) only differ across countries according to how much consumption declines when a disaster occurs, as measured by ω^j. However, the constant

effect as captured by a^j in equation (27) also varies by countries according to the volatility of normal times consumption, σ^j, reflecting cross-country heterogeneity across countries from the standard certainty equivalent consumption measure.

A.2 Risk Free Rate

Substituting the solutions for b^j and a^j in equations (26) and (27) into equation (25) and the result into equation (24) gives the explicit solution for the value function. Further substituting this value function solution into the HJB equation (23) and taking the derivative with respect to α_t^j implies the following first order condition:

$$(\mu - r_t^j + \ell^{-1}) - \gamma\alpha_t^j(\sigma^j)^2 + \lambda_t^j E_v[(1 + \alpha_t^j(e^{\omega^j Z} - 1))^{-\gamma}(e^{\omega^j Z} - 1)] = 0. \tag{28}$$

Using the facts noted earlier that $\ell^{-1} = \beta$ and that in equilibrium $\alpha_t^j = 1$ and rearranging equation (28) to solve for r_t^j verifies the risk-free rate equation (6) given in the paper repeated here:

$$r_t^j = \beta + \mu - \gamma(\sigma^j)^2 + \lambda_t^j E\left[e^{-\gamma\omega^j Z}(e^{\omega^j Z} - 1)\right]$$

A.3 The State Price Density

The state price density measures the agent's value of holding risky assets according to the intertemporal marginal utility of consumption. Following Duffie and Skiadas (1994), this state price is:

$$\pi_t^j = \exp\left\{\int_0^t U_V(C_s^j, V_s^j)ds\right\} U_C(C_s^j, V_s^j). \tag{29}$$

Using the solution for the value function in equations (24) and (25), the envelope condition that $H_W = f_C(C, V)$ implies that the marginal untility of consumption is:

$$U_C(C_t^j, V_t^j) = \beta^\gamma (C_t^j)^{-\gamma} I^j(\lambda_t^j).$$

Applying Ito's Lemma to the state price equation (29) and using the solutions for the terms in the value function in equations (26), and (27), this form for marginal utility gives the solution for the state price in equation (7) of the text rewritten here:

$$\frac{d\pi_t^j}{\pi_{t-}^j} = \mu_{\pi,t}^j dt - \gamma \sigma^j dB_t^j + b^j \sigma_\lambda \sqrt{\lambda_t^j} dB_{\lambda,t}^j + (e^{-\gamma \omega^j Z_t} - 1) dN_t^j \tag{30}$$

where $\mu_{\pi,t}^j = -r_t^j - \lambda_t^j E(e^{-\gamma \omega^j Z_t} - 1)$.

A.4   Government Bill Rate

Modifying Wachter (2013) to be country-specific, the price of government debt is:

$$\frac{dL_t^j}{L_{t-}^j} = r_t^{b,j} dt + (e^{\omega^j Z_{L,t}} - 1) dN_t^j \tag{31}$$

where $Z_{L,t} = Z_t$ with probability q^j; $Z_{L,t} = 0$ with probability $(1-q^j)$. Using the relationship between the state-price density in equation (30) and the price of government debt in equation (31), the drift term $r_t^{b,j}$ can be solved as:

$$r_t^{b,j} = r_t^j + \lambda_t^j q^j E\left[(e^{-\gamma \omega^j Z} - 1)(1 - e^{\omega^j Z})\right],$$

obtaining equation (8) in the text.

A.5 Equity Price

The price of equity is the value of a perpetual claim on dividends, defined as D_t. Then

letting F_t^j denote a claim on the dividends in country j, this price has the form given by

equation (9) in the text given here:

$$F_t^j = E_t(\int_t^\infty D_s^j \frac{\pi_s^j}{\pi_t^j} ds)$$

Following Abel (1999) as well as Wachter (2013) and Gouirio, Siemer and Verdelhan (2013),

among others, we identify these dividends in the data as a levered claim on consumption,

$D_t^j = (C_t^j)^{\phi^j}$, where ϕ^j is the leverage parameter. Using Ito's Lemma, the dividend process

is:

$$dD_t^j = \phi^j(\mu + \frac{1}{2}(\sigma^j)^2(\phi^j - 1))D_{t-}^j dt + \sigma^j \phi^j D_{t-}^j dB_t^j + [(e^{\omega^j Z})^{\phi^j} - 1]D_{t-}^j dN_t^j.$$

Using this relationship and applying Ito's Lemma to the equity price equation (9) verifies

the equity price diffusion given in equation (11) in the text repeated here:

$$\frac{dF_t^j}{F_{t-}^j} = \mu_{F,t}^j dt + \phi^j \sigma^j dB_t^j + g^j \sigma_\lambda \sqrt{\lambda_t^j} dB_{\lambda,t}^j + (e^{\phi^j \omega^j Z} - 1)dN_t^j.$$

where $g^j = G^{j\prime}(\lambda_t^j)/G^{j\prime}(\lambda_t^j)$ where $G^{j\prime}(\lambda_t^j)$ is the price-dividend ratio for the equity of country

j.

We use this price process in combination with the dividend payout process to generate

equity returns in the quantitative analysis. Calculating these returns requires valuing the

dividend payout strips at each future date as described in Wachter (2013), Appendix A.III.

B   Appendix: International Correlations

This appendix describes the cross-country correlations solutions reported in Section 3 in the text given various assumptions about disaster events. The consumption processes in equation (4) are repeated here:

$$dC_t^j = \mu C_{t-}^j dt + \sigma^j C_{t-}^j dB_t^j + (e^{\omega^j Z_t} - 1)C_{t-}^j dN_t^j, \quad \forall = 1,...,J$$

where N_t^j is a Poisson process with time-varying intensity parameter, λ_t^j, given by equation (5) as:

$$d\lambda_t^j = \kappa \left(\overline{\lambda} - \lambda_t^j \right) dt + \sigma_\lambda \sqrt{\lambda_t^j} dB_{\lambda,t}^j$$

Similarly, the stock prices are in equation (11) and the risk-free rate is in equation (6).

Independent Country Poisson Processes: When dN_t^j are independent Poisson processes and $dB_{\lambda,t}^j$ are uncorrelated across countries, then inspection of equations (4) and (11) demonstrates that international correlations of consumption and equity prices only arise through the "normal times" Brownians. Thus, $Corr\left(\frac{dC_t^i}{C_{t-}^i}, \frac{dC_t^j}{C_{t-}^j}\right) = Corr\left(\frac{dF_t^i}{F_{t-}^i}, \frac{dF_t^j}{F_{t-}^j}\right) = Corr(dB_t^i, dB_t^j) \equiv \rho^{ij}$. Moreover, since risk-free rates given in equation (6) only vary due to the Brownian on the intensity, $dB_{\lambda,t}^j$, and these are independent across countries, then $Corr\left(r_t^i, r_t^j\right) = 0$.

Common World Poisson Process: When disasters are driven by a common world Poisson process, the dN_t^j are replaced by a common world process dN_t^w with its own in-

tensity process, $d\lambda_t^w$. In this case, the correlation between the disaster components across countries is one. Also, since the variance of a Poisson process is equal to its intensity, $Corr\left((e^{\omega^i Z} - 1)dN_t^w, (e^{\omega^j Z} - 1)dN_t^w\right) = (e^{\omega^i Z} - 1)(e^{\omega^j Z} - 1)\lambda_t^w$ Since the normal times Brownians dB_t^j and the disaster risk components, dN_t^w and $dB_{\lambda,t}^w$, are all independent of each other, the correlation of consumption becomes:

$$Corr\left(\frac{dC_t^i}{C_{t-}^i}, \frac{dC_t^j}{C_{t-}^j}\right) = \frac{\sigma^i \sigma^j \rho^{ij} + (e^{\omega^i Z} - 1)(e^{\omega^j Z} - 1)\lambda^w}{\sqrt{(\sigma^i)^2 + (e^{\omega^i Z} - 1)^2 \lambda^w}\sqrt{(\sigma^j)^2 + (e^{\omega^j Z} - 1)^2 \lambda^w}}$$

when the intensity process is constant over time, thus verifying equation (17). Following similar steps, the correlation in stock price changes with constant intensity is:

$$Corr\left(\frac{dF_t^i}{F_{t-}^i}, \frac{dF_t^j}{F_{t-}^j}\right) = \frac{\phi^i \phi^j \sigma^i \sigma^j \rho^{ij} + (e^{\phi^i \omega^i Z} - 1)(e^{\phi^j \omega^j Z} - 1)\lambda^w}{\sqrt{(\phi^i \sigma^i)^2 + (e^{\phi^i \omega^i Z} - 1)^2 \lambda^w}\sqrt{(\phi^j \sigma^j)^2 + (e^{\phi^j \omega^j Z} - 1)^2 \lambda^w}}.$$

Mixed Independent and Common World Poisson Processes: When disasters are driven by both processes, the consumption process is as in equation (19) in the text:

$$\frac{dC_t^j}{C_{t-}^j} = \mu dt + \sigma^j dB_t^j + (e^{\omega^j Z} - 1)(dN_t^j + dN_t^w)$$

Again, the country disasters dN_t^j are independent and therefore add no correlation to international consumption. However, the world disaster is perfectly correlated as above. Therefore following the same steps as for the Common World Poisson Process implies the solution in equation (19).

C Simulations

Using the closed-form asset return solutions outlined in Appendices A and B, this section describes the simulation methods used to compute model moments, estimate individual country model parameters using Simulated Method of Moments, and compute asset return and consumption growth comovements.

C.1 Model-Implied Moments

To compute model moments, we first simulate a long time series (50,000 years) of shocks to consumption growth (dB_t), shocks to disaster intensity $(dB_{\lambda,t})$, and consumption declines in disaster (Z_t). In the time varying case, we compute the time series of λ_t using equation (5), the shock to the disaster intensity $(dB_{\lambda,t})$, and parameters (κ, σ_λ). Then with the generated series of disaster probabilities, λ_t, we simulate the Poisson process (dN_t), which determines whether there is a disaster in each period, and combine it with Z_t, which is the change to consumption growth in the event of a disaster, to form a disaster series. Finally, we combine the disaster series with simulated direct consumption growth shocks (dB_t) to compute the consumption process described in equation (4) over 50,000 years. Then applying the above closed-form solutions with a given preference parameters, we compute the model implied asset returns and model moments.

C.2 Single Country Simulated Method of Moments

In Section 2, we use Simulated Method of Moments (SMM) to find the parameters that best match the data moments of consumption and asset return moments for each country.

As noted in Section 2, we find the best fit for four parameters: the probability of government bond default, q^j; the dividend-consumption leverage parameters, ϕ^j; the proportion of disaster state consumption decline, ω^j; and the volatility of normal times consumption, σ^j. All of these parameter fits are further conditioned on the mean consumption growth rate parameter μ that is set equal to the average mean growth rate across countries. Using SMM, we find the best fitting parameters of the model to target seven data moments: (a) the mean government bill rate, (b) the standard deviation of the government bill rate, (c) the mean equity premium, (d) the standard deviation of the equity return, (e) the Sharpe ratio, and (f) the consumption growth standard deviation.

For this purpose, we simulate the model to obtain the seven data moments and iterate on values of q^j, σ^j, ϕ^j, and ω^j until the sum of squared difference between the data and model moments are minimized. Specifically, we begin with an initial guess for the parameters (q^j, σ^j, ϕ^j, and ω^j) anchored by the parameters values in Wachter (2013). Then drawing a long time series of shocks (dB_t, $dB_{\lambda,t}$, Z_t), we simulate the consumption process described in equation (4) over 50,000 years. For the parameters not being estimated, such as preference parameters (β, γ) and mean consumption growth (μ), we use values advocated by Barro (2006) for the constant disaster risk case and Wachter (2013) for the time varying disaster risk case. Given these shocks and parameters, we compute model-implied moments described above. This set of model moments is then compared to the data to arrive at a Sum of Squared Difference. To find the set of parameters that achieve the lowest sum of squared difference, this process is repeated over a ranges of parameters, with each iteration computing the model implied moments using the same set of shocks but different parameter values.[35]

---

[35]In general, less than 100 iterations are need to find the combination of parameter values that minimize

Although our long history run of $50,000$ Brownians and Poisson draws should provide a fairly accurate measure of the distribution of the parameters, we take the simulation a step further in order to ensure stability of the estimates. For this purpose, we repeat the entire procedure described above multiple times (50 times for the time-invariant case, 25 times for the time-varying case). Tables 2 and 4 report the average of the parameter estimates across these simulation runs. We find that our estimates are little changed after multiple runs, suggesting that the estimates are indeed stable.

C.3 International Country Correlation

In Section 3, we report implied model consumption and asset return correlations with various assumptions about the comovement in disasters across countries. In order to make sure our results are not driven by asymmetries across countries, we consider two identical countries with parameters calibrated as in Wachter (2013) with $\mu = 0.025$, $\sigma = 0.02$, $q = 0.4$, $\kappa = 0.08$, $\sigma_\lambda = 0.067$. All cases treat the "normal times" consumption as a common drift μ and standard deviation σ as before. However, in this case, the cross-country correlation of the "normal times" consumption Brownians ρ^{12} are measured from the correlation between the U.S. and the U.K. in the post-War period without disasters as given in Tables 6 and 7 in the "Data" column.

In Table 6, we further calculate the disaster events in two polar ways. The "No Disaster Correlation" case generates the simulated disaster effects by drawing two independent Poisson processes $\{dN_t^1, dN_t^2\}$ and, moreover, two individual draws of the size of the disasters,

the sum of squared difference. However, this depends somewhat on the range on each parameter. Given model boundary condition constraints, we typically bound our parameter estimates to be $\omega \in [0.7, 1.0]$, $\phi \in [1.5, 3.5]$, $q \in [0.2, 0.6]$.

Z_t^j.[36] Furthermore, for the "Time Varying Disaster" case reported in Panel B, the intensities of disasters, λ_t^i, are generated from draws of the $\left\{dB_{\lambda,t}^1, dB_{\lambda,t}^2\right\}$ processes as independent zero normal variables and hence these intensities are uncorrelated. Table 6 also reports the opposite case as "Perfect Disaster Correlation." In this case, a common world Poisson process dN_t^w is generated with a common disaster effect Z_t^w.

Table 7 reports correlations of simulated data from a hybrid case with both independent and common disasters. Generating the disaster effects for this case follows the same steps as above, but generates the Poisson process as $dN_t^j + dN_t^w$ where the mean of the probability of this joint Poisson is $\tau^i \overline{\lambda}^w + (1 - \tau^i) \overline{\lambda}^i = \widetilde{\lambda}^i$ where $\widetilde{\lambda}^i = 3.55\%$, the frequency of disaster imposed before.[37] Since we use a single draw to generate the Poisson processes, we further assume that the size of the disaster Z is the same whether it originates from the world disaster through dN_t^w, or originates from the individual country through dN_t^j. As before, the distribution of the intensity processes are the same.

---

[36]Barro (2006) and Wachter (2013) assume the disaster process is uniform. Therefore, we checked our simulations based upon the Poisson process by comparing to the Uniform distribution case, finding similar results. We base our simulations on the Poisson assumption to ensure that our cross-country correlations accurately correspond with the model.

[37]We approximate this process in our simulation as a single Poisson process calculated as a mixture of two Poisson processes. This process in turn is used to generate asset returns as described in Appendix A as a mixture of Poissons in place of the given Poisson jump process dN. As this approximation imposes restrictions on the joint process of returns, we intend to relax these restrictions in future work.