LEVERAGE

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ABSTRACT

Many stylized facts of leverage, trading, and asset prices obtain in a frictionless general equilibrium model that features agents' heterogeneity in endowments and time- varying risk preferences. Our model predicts that aggregate debt increases in expansions when asset prices are high, volatility is low, and levered households enjoy a "consumption boom." Our model is consistent with poorer households borrowing more and with intermediaries' leverage being a priced factor. In crises, levered households strongly delever by "fire selling" their risky assets as asset prices drop. Yet, as empirically observed, their debt-to-wealth ratios increase as higher discount rates make their wealth decline faster.

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1.   Introduction

This paper is concerned with the determinants of leverage, both for households and financial intermediaries. This topic has been the focus of much attention in the wake of the financial crisis of 2008. Indeed, a popular narrative on the crisis is that it was the excessive growth in both household and financial intermediary leverage that led to the crisis once it proved unsustainable, as exemplified, for instance, in the fact that household debt grew well above disposable income. There are though few models that, first, offer sharp predictions about the level and dynamics of both aggregate household debt as well as its cross sectional distribution and, second, can be calibrated to the data to obtain quantitative predictions. The purpose of this paper is to fill this void.

We construct an exchange economy populated by households that differ in both their attitudes towards risk and in their initial wealth. We posit that households' attitudes towards risk fluctuate with aggregate economic conditions and that they fluctuate more for some households than for others. These differences induce motives for risk sharing and we offer a full characterization of the efficient distribution of risk across households. Our model features the strong discount effects that the asset pricing literature has found are needed to match key aspects of the data. The model then generates at the aggregate level quantitatively plausible amounts of risk.

We decentralize the efficient allocation as follows. Because individual household endowment features idiosyncratic risk a competitive financial intermediary arises to pool and eliminate this risk from households' consumption. The alternative is to have a very large number of Arrow-Debreu markets in which each of the individual idiosyncratic risks is traded, which is potentially very costly. The existence of a financial intermediary saves on these costs. The intermediary can both grant loans and issue deposits. Households thus can replicate the efficient allocation through a dynamic strategy in which they trade the aggregate stock market and borrow and lend from and to the financial intermediary, respectively.[1] We solve for asset prices, portfolio allocations and leverage measures in closed form and investigate both the qualitative and quantitative properties of our model.

Our results connect to some stylized facts that have been the focus of the recent literature but also to some new ones. They are discussed in Section 2. Start with household leverage. The definition of leverage matters for its cyclical properties and the interpretation that we

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[1]Our model is thus quite different than Bewley (1977) and the literature that follows from it, which typically features ex-ante identical households that face uninsurable idiosyncratic shocks. These households are ex-post different depending on the realization of theses shocks and self-insure to smooth out consumption.

give to those dynamics. Data shows that household leverage is pro-cyclical when debt is normalized by income and counter-cyclical when normalized by net worth, a fact we are able to reproduce. In our model income is exogenously determined but both household debt and net worth are endogenous. Other things equal, we show that households that borrow do so proportionally to income. But there is an additional determinant of household leverage and it is the variation in the attitudes towards risk. As households' income grows they become more tolerant of risk and some of them become more tolerant than others. The more risk tolerant households are willing to take on additional risk through leverage and increase their exposure to aggregate market conditions. This second effect gives an additional "kick" to the amount of debt borrowing households take and thus debt grows faster than income.

The same mechanism, an increase in risk tolerance, lowers the discount rate households require to hold risky assets, pushing up prices and increasing net worth, which more than compensates for the growth in debt, leading to a drop in the ratio of debt to net worth. The dynamics of household leverage then are the portfolio policy dual of the traditional argument as to why habit persistence models can generate asset price volatility beyond the volatility of the underlying cash-flows: It is a product of the effect of shocks to income on the households' attitudes towards risk. Importantly, when aggregate income drops the dynamics of household leverage reverse. Borrowing households sell assets to repay their debt and delever. Debt to income drops but asset prices drop even faster as households discount risk more aggressively due to falling risk tolerance. As a result debt to net worth increases in bad times, as aggregate income falls. Ours is to our knowledge, the first model to be able to reconcile these diverging dynamics of household leverage.

A second fact is concerned with the cross sectional distribution of leverage across households. Using data from the Survey of Consumer Finances, we show that when sorting households into net worth quintiles, poorer households lever more. Our model features two sources of household heterogeneity, initial wealth and preferences. We calibrate the cross sectional distribution of these parameters to match some stylized properties of household consumption and are able to reproduce the inverse pattern linking debt to assets and net worth. The intuition is that in our model household preferences are non-homothetic so that poorer households are less risk tolerant than wealthier ones. Thus poor households that lever up must be very risk tolerant to undo the effect of wealth on risk taking, which results in high leverage. But we miss on the magnitudes: Our households are not as levered as they are in the data in periods of good economic conditions and are half as much in recessions. Thus something else needs to be brought to bear to explain the observed magnitudes.[2] Instead our

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[2]In this our model provides a benchmark that can be used to assess how much "work" other potential ingredients, such as the savings gluts hypothesis of Bernanke (2005) or the surge in credit supply associated

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model does much better in what concerns the level of aggregate household debt to income: It is about 135% of disposable income at the peak of the real estate cycle in 2007, which is just slightly less than what the model generates.

In our framework financial intermediaries provide the assets and liabilities needed to implement the efficient allocation: They grant loans to risk tolerant households that are willing to lever up to increase their exposure to aggregate risk and issue short term liabilities (deposits) that the more risk averse households use to smooth out consumption. Critically the amount of deposits they can issue is limited by the loans they can grant which is determined by the amount of leverage taken by risk tolerant households. The balance sheet of the financial intermediary thus inherits the properties of the household balance sheet. In this our model formalizes the idea that financial intermediaries' liabilities are constrained by their ability to originate assets (loans), which in turn is determined by the risk tolerant households willingness to borrow. Two important implications follow from this observation.

First, some, such as Adrian and Shin (2014) for instance, have argued that the financial intermediary leverage is driven by value-at-risk (VaR) like constraints: There is a negative relation between changes in VaR and changes in leverage. These constraints are in turn linked to the volatility of asset prices. Effectively thus, as volatility increases financial intermediaries lower their leverage. We show that this is exactly what one should expect to observe in a simple frictionless economy such as the one in this paper. In our model return volatility is countercyclical. Thus there is a negative correlation between the volatility and measures of leverage that normalize financial intermediary debt by slow moving measures such as book equity or aggregate income, but a positive one when we normalize debt by some measure of the intermediaries' net worth.

Second, a recent a literature has shown that measures of financial intermediary leverage show up as priced factors in tests of the cross section of returns (see Adrian, Etula and Muir (2014) and He, Kelly and Manela (2016)). This evidence has been put forth as proof that financial intermediaries matter for asset pricing. They might but we argue that these tests are misspecified and cannot claim to establish the impact of intermediaries on asset prices. The point is straightforward: Financial intermediaries' balance sheets reflect more fundamental forces at work, namely the gyrations of the marginal rate of substitution (MRS) of the representative consumer. It may well be the case that shocks to the capital structure of banks and other intermediaries offer a clean proxy for shocks to the MRS of the representative consumer, which is unobserved, but in isolation this evidence cannot be offered as proof that

with looser lending constraints as in Justiniano, Primiceri and Tambalotti (forthcoming), will have to do to explain the levels of household leverage that were observed in the years prior to the financial crisis of 2008.

the marginal investor is a financial institution.

In addition we clarify a debate regarding the market price of risk associated with the factor linked to the intermediaries' leverage. This price can be positive or negative depending on whether debt is normalized by some measure of income, which is slow moving, or the intermediaries equity, which incorporates the discount shocks that are key in our analysis. Our framework can reproduce the sign of the factors suggested by Adrian, Etula and Muir (2014) and He, Kelly and Manela (2016), which are different in what concerns this normalization. The intuition for this result is exactly the one advanced to understand the different dynamics of household debt depending on whether one normalizes by income or net worth.

Finally, our model features an important property of the data. In our framework households trade the stock and borrow and lend to smooth consumption and share risks. Our model can match for example the high equity premium and high volatility of returns and features frequent drops in price dividend ratios. Interestingly though, and as observed in the historical series, substantial household deleveraging only occurs in extreme realizations of the aggregate shocks, such as in the aftermath of the financial crisis of 2008. Indeed, household leverage has dropped from about 135% of disposable income in 2007Q4 to about a 100% in 2018Q2. These are magnitudes that correspond roughly to an extreme realization of the aggregate shock in our model.

Our point thus is that many stylized facts of household and financial intermediary leverage can be explained without appealing to financial frictions. But of course these frictions do matter; the question is how much. Our model can be seen as providing a benchmark to which frictions can be added to offer a quantitative assessment of their importance.

Related literature. This paper is obviously connected to the literature on optimal risk sharing, starting with Borch (1962). Much of this literature is concerned with assessing to what extent consumers are effectively insured against idiosyncratic shocks to income and wealth.[3] Our paper is closely related to Dumas (1989), Wang (1996), Bolton and Harris (2013), Longstaff and Wang (2012), and Bhamra and Uppal (2014). These papers consider two groups of agents with constant risk aversion, and trading and asset prices are generated by aggregate shocks through the variation in the wealth distribution. While similar in spirit, our model generates several novel results that do not follow from this previous work, such as procyclical debt to income ratios, countercyclical debt to wealth ratios, higher leverage amongst poorer households, consistency with asset pricing facts, and so on. Our model is more closely related to Chan and Kogan (2002), who also consider a continuum of households

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[3]See for instance Dynarski and Sheffrin (1987), Cochrane (1991), Mace (1991) and Townsend (1994).

with habit preferences and heterogeneous risk aversion. In their setting, however, households' risk aversions are constant, while in our setting they are time varying in response to business cycle variation, a crucial ingredient in our model. Moreover, Chan and Kogan (2002) do not investigate the leverage dynamics implied by their model, which is our focus.

Our framework differs from these models in another important respect. In models such as Longstaff and Wang (2012) for example, asset pricing dynamics are tightly linked to the dynamics of the cross sectional distribution of wealth. Given that these dynamics are dominated by low frequency components it is challenging to explain asset pricing empirical regularities appealing to them. Instead the asset pricing implications of our model are orthogonal to the wealth distribution: when we aggregate we obtain a representative consumer whose preferences are independent of whether wealth is held by risk tolerant or risk averse agents. Thus asset pricing dynamics can be fit independently of the wealth distribution.

Our model is related to Campbell, Grossman and Wang (1993), which explores the implications for trading volume and asset prices in a model where the motivation for trade is driven by shocks to agents' risk tolerance. More recently Alvarez and Atkenson (2017) consider a model where agents' risk tolerance is subject to uninsurable idiosyncratic shocks. In our paper instead variation in risk tolerance is driven by exposure to a business cycle factor, and the source of heterogeneity, in addition to initial endowment, is the degree of exposure to that factor. Neither Campbell, Grossman and Wang (1993) or Alvarez and Atkenson (2017) analyzes the dynamics of leverage and the distribution of leverage in the population.

Our paper is also related to the literature that links causally household debt to aggregate economic activity. For instance Eggertson and Krugman (2012) argue that a tightening of the household borrowing constraints leads to sustained depressed economic activity. Justiniano, Primiceri and Tambalotti (2015) construct a general equilibrium model that, as we do, tries to obtain reasonable levels for household leverage and find instead that the macroeconomic consequences of household leveraging and deleveraging are minor. In our model household leverage has no impact on economic activity. In fact the causality runs the other way: Bad realizations of economic activity decrease risk tolerance and lead to deleveraging cycles.

Finally, a recent literature (Barro and Mollerus (2014) and Caballero and Fahri (2014)) studies the determinants of the supply of safe assets and its connection to aggregate activity. In our model all debt is indeed risk free and the supply of safe securities is determined by the risk bearing capacity of the risk tolerant households. Our model thus also has implications for the dynamics of the supply of safe assets and their relation to aggregate variables.

2. Facts

Our model speaks to a set of empirical regularities which have been the focus of renewed interest in the aftermath of the financial crisis of 2008.

Household Leverage. Household leverage has featured prominently in policy discussions in the wake of the 2008 financial crisis. Two measures of leverage are of interest here. In the first, US households debt is normalized by their net worth, whereas is it normalized by disposable income in the second. We refer to the first measure as market leverage and the second, with some abuse of terminology, as book leverage:

$$\text{Market leverage} = \frac{\text{Household debt}}{\text{Net worth}} \quad \text{and} \quad \text{Book leverage} = \frac{\text{Household debt}}{\text{Disposable income}} \quad (1)$$

Figure 1 Panel B reproduces Figure 1 in Adrian and Shin (2010). It plots the quarterly growth in percent of total assets held by households against the quarterly growth rate in percent in market leverage. Asset growth is strongly associated with drops in market leverage: As asset values grow, mostly real estate and financial assets, net worth grows even faster than household debt and thus market leverage drops. Instead, as shown in Panel A, book leverage correlates positively, though weakly, with asset growth: As real estate and financial assets grow in value, household debt grows faster than disposable income and thus leverage increases. Given that asset growth is strongly pro-cyclical, it follows that market leverage is countercyclical and book leverage is pro-cyclical.

Figure 2, which plots market and book leverage for the last twenty years, provides a striking example of this pattern. During the years of appreciating real estate values household debt grew much faster than disposable income whereas market leverage was stable as growth in net worth compensated for the growth in household debt. When the crisis struck prices collapsed and as a result market leverage increased dramatically as households, which were deleveraging, couldn't do it fast enough to compensate for the drop in asset values, which compressed household net worth. Instead book leverage dropped as households delevered at a rate that was faster than the rate at which their disposable income fell.

There are also patterns in what concerns the cross section of household leverage. Figure 3 shows the amount of debt to total assets for households sorted on net worth quartiles in 2009 (we further split the last quartile to show the behavior of the top 10%). Data is from the Survey of Consumer Finances. The 2009 survey was an ad-hoc date in which the *same* households as in 2007 survey were surveyed again. The figure shows two regularities. First, the lower the net worth quartile to which the household belongs, the higher the ratio of debt to net worth. Second, leverage increased across all quartiles in 2009, the trough of the
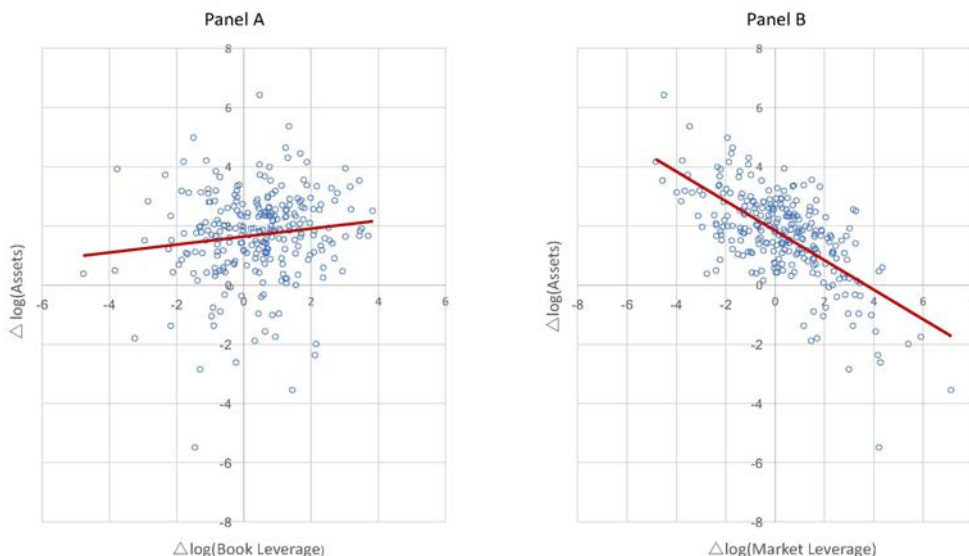
Figure 1: Total assets and the two definitions of leverage for the US household
Panel A plots the quarterly growth (in percent) in total assets held by US households against the quarterly growth (in percent) in household "market" leverage. Market leverage is defined as household debt divided by their net worth. Panel B plots the quarterly growth (in percent) in total assets held by US households against the quarterly growth (in percent) in household "book" leverage. Book leverage is defined as total household debt divided by disposable personal income. 1951Q1 to 2018Q2. Household debt is obtained by subtracting net worth from household total assets. Data source: Federal Reserve Bank of St. Louis.

financial crisis, but the increase was particularly pronounced in the lowest quartile.[4]

We show that all these patterns can be explained in a simple frictionless framework that features household heterogeneity in wealth and attitudes towards risk. The model is calibrated to match standard asset pricing regularities and thus we are able to assess leverage magnitudes in a setting with realistic risk properties, which is a novel contribution to the literature.

Leverage and the risk of financial intermediaries. Household leverage has a clear counterpart in the leverage of financial institutions. The top two panels of Figure 4 reproduce Figure 3 in Adrian and Shin (2014). These authors plot the change in assets against changes in book (left panel) and enterprise leverage (right panel). Given that asset growth is procyclical, the plots show that market leverage is strongly countercyclical whereas book leverage is procyclical, as it was the case with households.

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[4]Some care needs to be exerted when interpreting Figure 3. Poor households' main assets are housing while rich households' main assets are investment in public or private equity, which are levered securities. In the internet appendix we adjust the data to take into account the implicit leverage in equity and private businesses and obtain similar results.
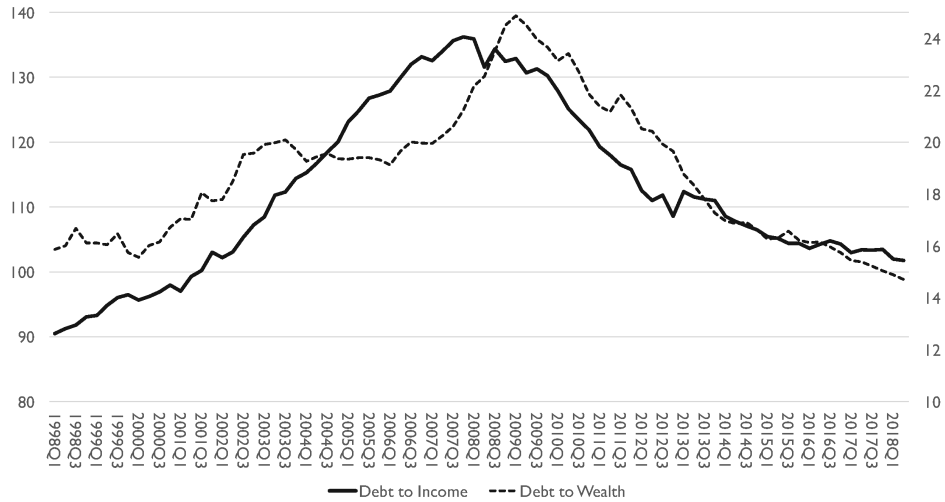
Figure 2: US Household leverage during the real estate bubble and burst

The figure shows the time series of market (dashed line; right axis) and debt-to-income (solid line; left axis) for US households between 1998Q1 to 2018Q2. Market leverage is defined as total assets of US households divided by their net worth. Book leverage is defined as total assets divided by disposable personal income. Data source: Federal Reserve Bank of St. Louis.
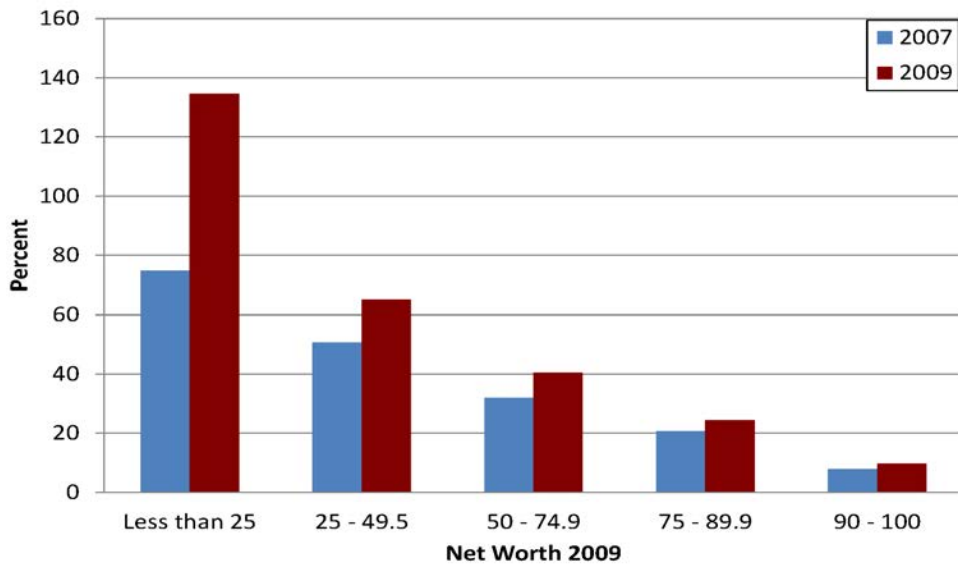
Figure 3: The cross section of household leverage

This figure plots the distribution of debt-to-asset ratios from the Survey of Consumer Finances in 2007 and in 2009, which was conducted on the same sample of households. The sample is restricted to households with debt.

Adrian and Shin (2010,2014) argue that in good times intermediaries increase the size of their balance sheet by issuing debt, which grows relative to book equity. Instead during bad times, risk measures such as VaR increase, which constrains financial intermediaries' capital, forcing them to liquidate assets and reduce debt. The bottom panel of Figure 4 shows that indeed an increase in Value-of-Risk of financial institutions – that is, the volatility of their risky assets – generates a decline in their (book) leverage because of active develeraging by part of financial intermediaries. Others have suggested a slightly different story for the procyclicality of book leverage. For instance, Geanakoplos (2009) and Gorton and Metrick (2012) argue that procyclical leverage is the mirror image of increased collateral requirement during downturns (increased "haircuts"). Geanakoplos terms such dynamics "the leverage cycle," arguing that leverage is high when volatility is low and prices are high. Finally, He and Krishnamurthy (2013) provide a theoretical model of procyclical market leverage also based on the constraints of financial institutions.

All these authors emphasize some form of friction to explain these regularities and none of them can simultaneously reconcile the pro- and countercylicality of book and market leverage, respectively. We argue instead that these regularities result from the fact that households intermediate through financial institutions and thus household portfolio choices determine to a large extent intermediaries' leverage. In our framework households deposit with financial intermediaries as well as borrow from them. The reason financial intermediaries exist is because households are exposed to idiosyncratic risk. Financial intermediaries are able to diversify this risk and save on markets as a result. The balance sheet of the financial intermediaries, which are assumed to be competitive, is thus a reflection of the household balance sheet and follows to a large extent their active leveraging and deleveraging.

Financial intermediary leverage as a risk factor. A recent literature studies the impact of financial intermediaries' leverage on asset prices. For instance, Adrian, Etula, and Muir (AEM, 2014) show that a book leverage factor prices the cross-section of equity returns. He, Kelly, and Manela (HKM, 2017) show that a market leverage factor prices also prices equity returns as well as other asset classes. All this is taken as evidence of the role of financial institution in pricing risky assets. This is motivated by models such as He and Krishnamurthy (2013) in which households are barred from investing directly in risky assets and financial institutions become the "marginal pricer". Instead in our model the pricing ability of factors linked to financial intermediaries' balance sheets is simply a reflection of the risk factors affecting households.
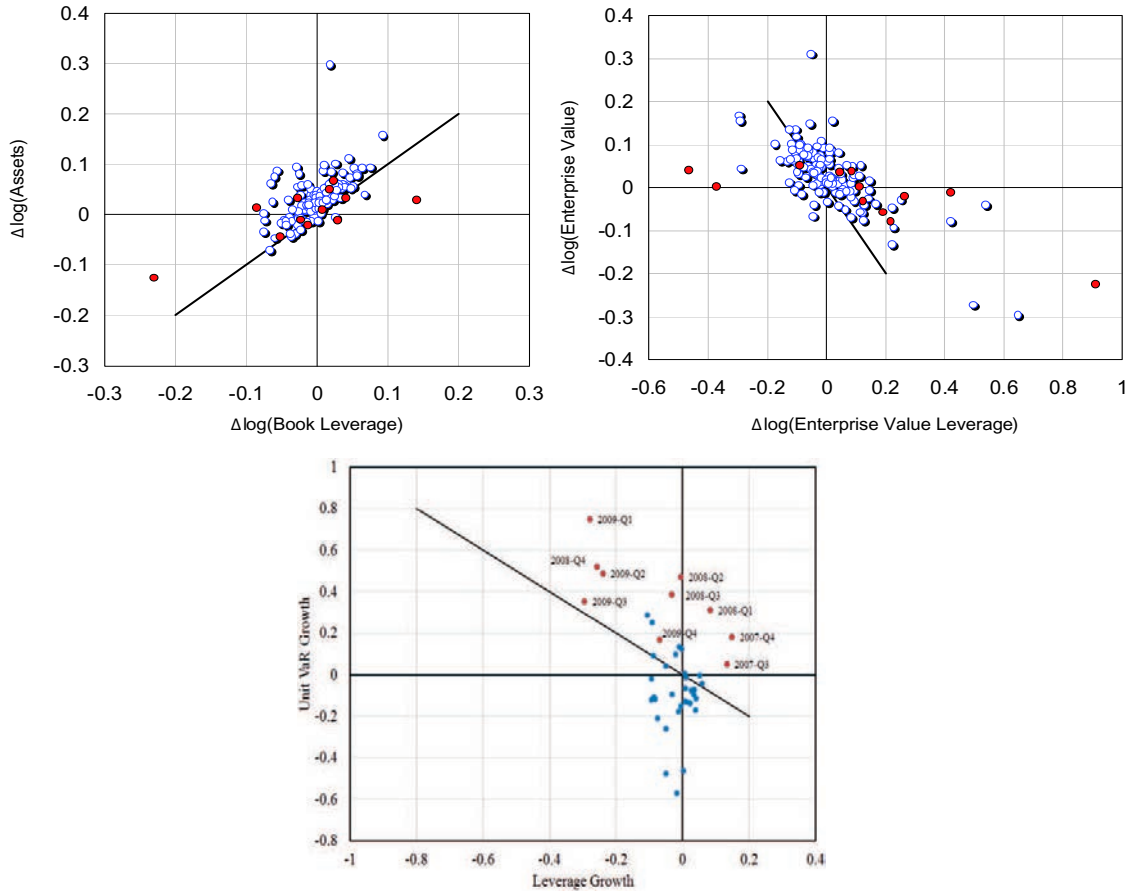
Figure 4: Leverage and Risk (Adrian and Shin, 2014)

The top two panels reproduce Figure 3 in Adrian Shin (2014). The left panel shows the scatter chart of the asset-weighted growth in book leverage and total assets for the eight largest U.S. broker dealers and banks. The right panel is the scatter for the asset-weighted growth in enterprise value leverage and enterprise value. The dark dots correspond to the period 2007-2009. The bottom panel reproduces Figure 6 in Adrian and Shin (2014) and it plots the annual growth rate in unit VaR against the annual growth rate in leverage.

In addition, the market price of risk of this leverage factor depends on whether book or market leverage is used. AEM's book leverage factor has a negative market price of risk, while HKM's market leverage factor has positive market price of risk. The two different signs in the market price of risk have generated some controversy in the literature about the proper definition of intermediary leverage (see Section 4 in HKM). The different signs of the market price of risk have a natural explanation in our framework. It is simply the result of the different cyclical behavior of household leverage depending on whether one normalizes by net worth, which depends on prices, or income.

Panel A of Table 1 reports results using the same data in AEM and HKM to price the

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Table 1: The Market Price of Leverage Risk. Panel A reports Fama-MacBeth regressions where the set of test portfolios are the standard 25 Fama-French portfolios sorted on size and book to market. Column I reports the standard CAPM regression. Column II adds to the market, the market leverage, defined as Debt/Equity $= 1/$(capital ratio)$ - 1$, which is a transformation of the capital ratio= Equity/(Debt + Equity) introduced in He, Kelly and Manela (2017). Column III reports the same regression where instead of using market leverage we use book leverage, defined as in Adrien, Etula, and Muir (2014). Panel B reports time-series predictability regressions of future excess returns on book leverage, while Panel C reports time-series predictability regression on market leverage. The sample period is 1970-2012. t-statistics are in parenthesis.

Panel A. Cross-sectional Regressions

	I	II	III
α	3.19	0.76	1.07
	(3.05)	(0.62)	(0.97)
Market Return	-0.89	0.97	0.82
	(-0.72)	(0.69)	(0.61)
Market Leverage		-0.22	
		(-2.13)	
Book Leverage			0.63
			(3.07)
R^2 (%)	6.54	50.77	53.35

Panel B. Time-Series Predictability with Book Leverage

	1 year	2 year	3 year	4 year	5 year
Coef ($\times 100$)	-1.78	-1.79	-2.17	-3.13	-9.89
	(-0.83)	(-0.72)	(-0.89)	(-1.03)	(-3.29)
R^2	0.01	0.01	0.01	0.01	0.07

Panel C. Time-Series Predictability with Market Leverage

	1 year	2 year	3 year	4 year	5 year
Coef ($\times 100$)	3.66	6.21	8.56	10.03	13.06
	(1.57)	(1.50)	(2.18)	(2.51)	(3.84)
R^2	0.04	0.07	0.10	0.12	0.19

Fama French 25 value and size sorted portfolios.[5]

The first column reports the CAPM regression, in which the aggregate market portfolio is the main risk factor. As is well known, the CAPM fails to price these portfolios. The R^2 is a puny 6.5%, the alpha is strongly positive, and the average market return is negative. The second column shows that market leverage is able to explain a large fraction of the variation of the portfolios. The market return becomes positive (but not statistically significant), the

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[5]Data on the AER and HKM factors are available on the HKM web site. We transform HKM capital ratio = Equity/(Debt + Equity) into a debt-to-equity ratio Debt/Equity $= 1/$(capital ratio)$ - 1$. We then normalize both factors to have zero mean and variance one. The sample is 1970 through 2012.

alpha is zero, and the market price of risk is negative, and significant. Finally, column III shows the same results for book leverage, and obtains similar results, but now with a positive market price of risk.

Panels B and C of Table 1 provide additional support. They report the results of predictive regression of future excess returns using book and market leverage, respectively. Book leverage has only mild predictive power of future returns, and with a negative sign. In contrast, market leverage displays stronger predictability of future returns, with a positive sign: High aggregate market leverage predicts higher future returns.

In contrast to the recent literature, our model offers a different explanation for the role of factors linked to intermediaries' leverage in asset pricing tests and rationalizes the sign of the market price of risk associated with that factor depending on the specific definition of leverage adopted. Our explanation does not rely on financial frictions but our point is not that frictions are not important. Rather, it is that financial intermediaries' leverage is driven, at least to some extent, by the marginal rate of substitution of the representative consumer and that in fact may provide a cleaner proxy to other ones suggested in the literature. Thus its apparent pricing ability.

3. The model

Preferences and endowments. We posit a continuous time single good exchange economy populated by a continuum of households indexed by i. These households have preferences for period $t \in [0, \infty)$ over consumption C_{it} given by

$$u\left(C_{it}; \psi_{it}, Y_t, t\right) = e^{-\rho t} \log\left(C_{it} - \psi_{it} Y_t\right). \tag{2}$$

Utility is then derived from the distance between individual consumption and aggregate output Y_t, scaled by the process ψ_{it}. Y_t follows

$$\frac{dY_t}{Y_t} = \mu_Y \ dt \ + \ \sigma_Y(I_t) \ dZ_t. \tag{3}$$

where Z_t is a Brownian motion, μ_Y is a constant,[6] but the volatility $\sigma_Y(I_t)$, which we refer to as *economic uncertainty*, depends on a state variable I_t that summarizes the state of the economy. I_t follows

$$dI_t = k\left(\overline{I} - I_t\right) dt - v \ I_t \ \left[\frac{dY_t}{Y_t} - \mu_Y dt\right] \tag{4}$$

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[6]The main results of the paper carry through with a richer specification of the drift μ_Y.

That is, I_t increases after bad aggregate shocks, $\frac{dY_t}{Y_t} < \mu_Y dt$, and it hovers around its central tendency $\overline{I}$. It is useful to interpret I_t as a *recession indicator*: During good times I_t is low and during bad times I_t is high.

Finally, ψ_{it} in (2) is a function on I_t, $\psi_{it} \equiv \psi_i(I_t)$. To obtain closed form solutions for prices and quantities we assume a specific functional form for $\psi_i(\cdot)$:

$$\psi_i(I_t) = \gamma_i \left(1 - I_t^{-1}\right), \tag{5}$$

where γ_i are positive constants normalized so that $\int \gamma_i di = 1$ and we assume throughout that $I_t > 1$ so that $\psi_i(I_t) > 0$. This restriction is achieved by assuming that $\sigma_Y(I_t) \to 0$ as $I_t \to 1$.[7]

Intuitively, ψ_{it} regulates the local curvature of the utility function, with higher ψ_{it} implying a higher curvature. Indeed, for given consumption C_{it} and output Y_t, the local risk aversion (LRA) is

$$\mathrm{LRA}_{it} \equiv -\frac{u_{CC}\, C_{it}}{u_C} = 1 + \frac{\psi_{it}}{C_{it}/Y_t - \psi_{it}} \tag{6}$$

As we will show below (see equation (10)), $C_{it}/Y_t > \psi_{it}$ and thus households have a LRA strictly greater than that of log utility. Attitudes towards risk are thus determined by both ψ_{it} and the consumption share C_{it}/Y_t. For a given consumption share, a higher ψ_{it} implies a higher LRA. From (5), ψ_{it} is monotonically increasing in γ_i and the recession indicator I_t. Therefore, our preference specification implies that households with higher γ_i have higher LRA and, in addition, all households' LRAs increase in recessions, when I_t is higher, albeit heterogeneously depending on γ_i. Our model thus allows us to introduce heterogeneous variation in households' risk preferences during the business cycle in a simple way.

To conclude the model, each agent is born at time 0 endowed with a tree that produces Y_{it}. We do not need to make assumptions on Y_{it} except that its aggregate $Y_t = \int Y_{it} di$ follows the dynamics in (3). The time-0 values of households' stochastic endowments are heterogeneous and denoted by ω_i. We normalize prices so that $\int \omega_i di = 1$.

Financial intermediary and financial markets. Households are born at time 0 endowed with individual trees whose output bear idiosyncratic risk. As we show below, holding idiosyncratic risk is suboptimal for households and therefore they want to share risks with other households. It is costly to share risks by trading with each other and thus they form an intermediary to pool their idiosyncratic risks.

Specifically, at time 0 households form a financial intermediary by pledging their endowment in exchange for a portfolio that consists of shares of the intermediary and either

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[7]We use the simple notation $\int di$ to indicate the integration over agents' density $f(\gamma_i, \omega_i)$.
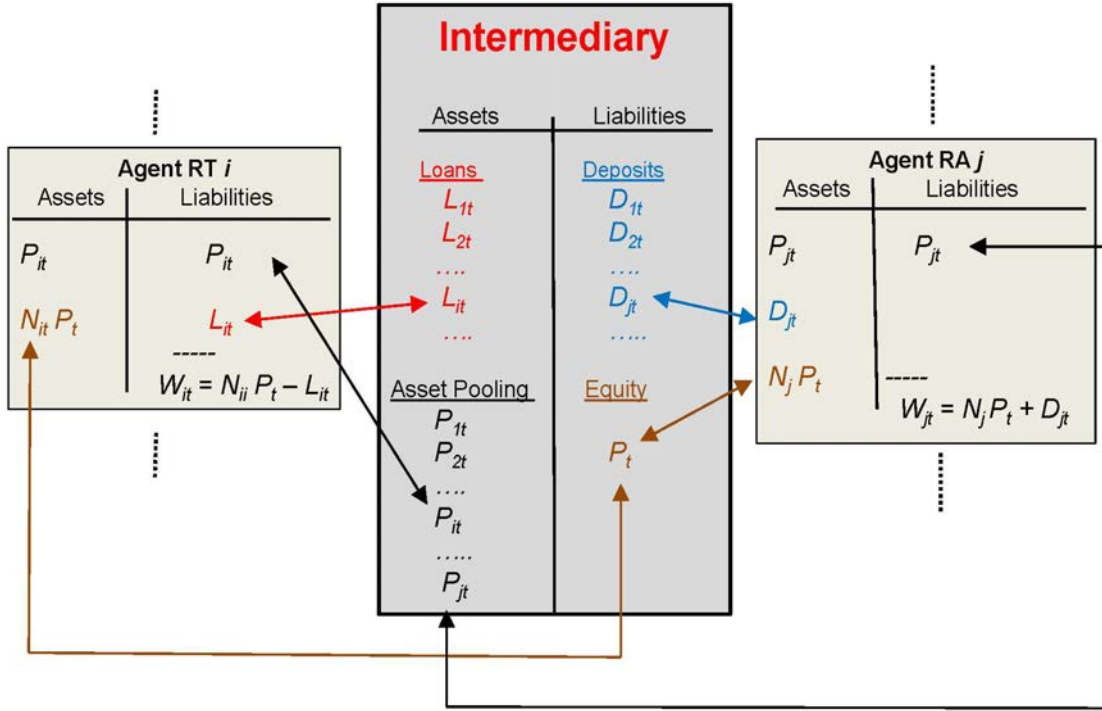
Figure 5: Balance sheets of households and the financial intermediary

loans or deposits. The intermediary issues a unit amount of shares to households, who then trade them competitively. Because the intermediary pools the idiosyncratic risks of the trees of individual households, the intermediary's stock price only depends on aggregate shocks, which is the desired exposure of all households. Moreover, depending on risk aversion, each household may decide to invest some funds in overnight safe demand deposits, or borrow to lever up their investment in the intermediary risky stock. We assume a single intermediary for simplicity, but we could have any number of them.

Figure 5 shows the balance sheets of the intermediary as well as of the two types of households that, as we will show, arise in equilibrium, risk-tolerant (RT) households and risk-averse (RA) households. The assets of the intermediary are comprised of the set of individual trees pledged by households, whose values are P_{it}, and the loans it issues to the RT households, denoted by L_{it}. The liabilities of the intermediary are comprised of demand deposits, denoted by D_{jt}, and by the equity shares issued to the households, whose value is P_t. Let r_t denote the equilibrium risk-free rate at which both borrowing and lending occurs.

The balance sheet of each household i has its own tree P_{it} both as an asset, as that's its endowment, and as a liability, as it pledges its cash flows to the intermediary. In addition,

a RA household j holds N_{jt} shares of the intermediary's stock and D_{jt} in demand deposits, while a RT household i borrow L_{it} from the intermediary and purchases N_{it} shares of the intermediary's. The net worth $W_{i,t}$ of each household i at time $t \geq 0$ is then

$$W_{i,t} = \begin{cases} N_{i,t}P_t - L_{i,t} & \text{for the Risk Tolerant (RT) household} \\ N_{i,t}P_t + D_{i,t} & \text{for the Risk Averse (RA) household} \end{cases} \tag{7}$$

Given ω_i is the value of household's initial endowment, it has to be that $\omega_i = W_{i,0}$.

Discussion. Our preference specification (2) possesses numerous appealing properties, which we now discuss. As shown in (6), our model implies heterogeneity across agents and across time in the local curvature of the utility function, determined by the interaction of the parameter γ_i and the recession indicator I_t. First, there is substantial evidence of cross sectional dispersion in attitudes towards risk in the population (see, for instance, Barsky, Juster, Kimball and Shapiro (1997), Guiso and Paiella (2008) and Chiappori and Paillea (2011)). Second, as for the time series variation, Guiso, Sapienza and Zingales (2018), using a large sample of clients of an Italian bank, find that measures of risk aversion increased after the 2008 financial crisis. They find that the increase in risk aversion is more pronounced for those experiencing large losses in wealth, though the increase in risk aversion occurs even for those agents who did not experience any loss. In our model all variables are perfectly correlated and thus we cannot produce this "pure" discount effect.

In addition, for a given ψ_{it}, households who are richer consume more of aggregate output and thus from (6) result in a lower curvature. That is, our preference specification imply non-homotheticity at the individual level. There is strong evidence in favor of this property in the data; roughly, richer households are less risk averse. Households with higher endowment thus increase the share of wealth invested in the risky asset, an empirical regularity found in surveys of household finances even when restricted to those who participate in the stock market (Wachter and Yogo, 2010). More generally, the portfolio allocation predictions of our model are consistent with the empirical evidence of Calvet and Sodini (2014).

Households trade with an intermediary for risk sharing: First, the intermediary allows households to shed the idiosyncratic risk implied by their initial endowment, as they sell it in exchange of claims to the pooled aggregate endowment. Second, the intermediary allows households to reach their desired dynamic investment mix by borrowing or lending. This second risk-sharing channel arises out of households' differences in attitudes towards risk (differences in γ_i) and thus also play a role in the equilibrium distribution of risk in the population. To illustrate the role of each source of cross sectional variation we also investigate the two polar cases of homogeneous preferences ($\gamma_i = 1$ for all i) and/or homogeneous

endowments ($\omega_i = 1$ for all i).[8]

In our model, the role of the intermediary is to complete financial markets even in the absence of Arrow-Debreu securities. By pooling idiosyncratic risks and selling shares that are effectively claims on the the aggregate endowment, the intermediary can offer a security that only depends on aggregate shocks without assuming any additional risks because its balance sheet is perfectly hedged from the law of large numbers. In this respect, however, we make the additional assumption that households do not default on their promises. That is, the sale of their tree to the intermediary at time 0 in exchange of shares of the latter is final. We emphasize that the intermediary is passive: it only allows households to costlessly trade with each other. The no-profit condition may be the result of perfect competition in the market for intermediation services.

Finally we need assumptions on the relevant household parameters to guarantee that their marginal utility remains positive in all possible states. The following is a sufficient condition and is assumed throughout

$$\omega_i > \gamma_i \left(1 - \overline{I}^{-1}\right) \qquad \text{for all} \quad i. \tag{A1}$$

In sum, risk sharing in our model operates through the balance sheet of the financial intermediary: households dynamically trade the intermediary's stock and borrow from and lend to it to obtain the desired consumption path. The amount of demand deposits that the financial intermediary can issue is limited by the amount of loans that is able to grant. The balance sheet of the intermediary thus endogenously expands and contracts over the business cycle with the variation in the households' attitudes towards risk.

4. Equilibrium

The portfolio problem. Given prices $\{P_t, r_t\}$ households choose consumption C_{it}, the amount of shares of the intermediary stock N_{it}, and the amount of deposits D_{it} or loans L_{it} to maximize their expected utilities

$$\max_{\{C_{it}, N_{it}, D_{it}, L_{it},\}} E_0 \left[\int_0^\infty e^{-\rho t} \log\left(C_{it} - \psi_{it} Y_t\right) dt \right]$$

subject to the dynamic budget constraint

$$dW_{it} = N_{it}(dP_t + Y_t dt) + (D_{it} - L_{it})r_t dt - C_{it} dt \qquad \text{with} \qquad W_{i,0} = \omega_i.$$

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[8]We are obviously not the first two focus on these two sources of cross sectional differentiation; see for instance Longstaff and Wang (2012) and Bolton and Harris (2013). Empirically these sources of variation have been investigated by, for example, Chiappori and Paeilla (2011) and Calvet and Sodini (2014).

The optimal allocation only depends on the net position $(D_{it} - L_{it})$. We break the indeterminacy by assuming that only either D_{it} or L_{it} can be positive at any given time t.

Definition of a competitive equilibrium. A competitive equilibrium is a series of stochastic processes for prices $\{P_t, r_t\}$ and allocations $\{C_{it}, N_{it}, D_{it}, L_{it}\}_{i \in \mathcal{I}}$ such that households maximize their intertemporal utility and markets clear $\int C_{it} di = Y_t$, $\int N_{it} di = 1$, and the intermediary balance sheet clears $\int D_{it} di = \int L_{it} di$. The economy starts at time 0 in its stochastic steady state $I_0 = \overline{I}$. Without loss of generality, we normalize the initial output $Y_0 = \rho$ for notational convenience.

For later reference, it is useful to illustrate some steps of the derivation of the competitive equilibrium. Details are contained in the Internet Appendix. Because markets are dynamically complete, each agent solves the static problem

$$\max_{\{C_{it}\}} E_0 \left[\int_0^\infty e^{-\rho t} \log \left(C_{it} - \psi_{it} Y_t \right) dt \right] \quad \text{subject to} \quad E_0 \left[\int_0^\infty M_t\, C_{it} dt \right] \leq w_i\, M_0 \quad (8)$$

where M_t is the state price density. The first order condition of the corresponding Lagrangean implies that

$$u_C(C_{it}; \psi_{it}, Y_t, t) = \frac{e^{-\rho t}}{C_{it} - \psi_{it} Y_t} = \frac{1}{\phi_i} M_t \qquad \text{for all } i, \quad (9)$$

where ϕ_i is the inverse of the Lagrange multiplier of the static budget constraint in (8), normalized such that $\int \phi_i di = 1$. It is easy to show that[9]

$$M_t = e^{-\rho t}\, Y_t^{-1}\, I_t \qquad \text{and} \qquad C_{it} = \left[\psi_{it} + \phi_i I_t^{-1} \right] Y_t. \quad (10)$$

Consumption for all households is increasing in aggregate output but it increases more for those households for whom ψ_{it} is larger as their marginal utility increases more with increases in aggregate output.[10]

Proposition 1 *(Efficient allocation).* *Let the economy be at its stochastic steady state at time 0, $I_0 = \overline{I}$, and normalize $Y_0 = \rho$. Then (a) the (inverse of) Lagrange multipliers are*

$$\phi_i = \gamma_i + (\omega_i - \gamma_i) \overline{I} \quad (11)$$

(b) The optimal consumption path for household i is given by

$$C_{it} = s_i\left(I_t\right) Y_t \qquad with \qquad s_i\left(I_t\right) = \gamma_i + (w_i - \gamma_i)\frac{\overline{I}}{I_t} \in (0,1) \quad (12)$$

---

[9]It is enough to solve for C_{it} in (9), integrate across households, and use the resource constraint $\int C_{it} di = Y_t$ to yield M_t. Plugging this expression in (9) yields C_{it}.

[10]Marginal utilities remain positive, $(C_{it} - \psi_{it} Y_t)^{-1} = \frac{I_t}{\phi_i Y_t} > 0$, and thus households' utilities are well defined. The marginal utility is lower the higher the aggregate output and the lower the recession indicator.

The inverse of the Lagrange multipliers ϕ_i in (11) are increasing in the initial aggregate endowment ω_i and decreasing in γ_i (as $\overline{I} > 1$). Higher initial endowment loosens the financial constraint and thus reduces the Lagrange multiplier. Similarly, higher γ_i increases the marginal utility of consumption and thus the desire to increase consumption, making the financial constraint tighter. The inverse of the Lagrange multipliers ϕ_i can also be interpreted as Pareto weights in a planner problem, and we refer to them as such at times.

Equation (12) shows the equilibrium sharing rule. Households with high endowment w_i or low γ_i enjoy a high consumption share $s_i(I_t) = C_{it}/Y_t$ during good times, that is, when the recession indicator I_t is low, and vice versa. This is intuitive given the discussion of local risk aversion in (6). Indeed, substituting now in LRA the equilibrium consumption we find

$$\text{LRA}_{it} = -\frac{C_{it}u_{cc}(C_{it},\psi_{it},Y_t,t)}{u_c(C_{it},\psi_{it},Y_t,t)} = \frac{I_t + (\omega_i/\gamma_i - 1)\overline{I}}{1 + (\omega_i/\gamma_i - 1)\overline{I}} \tag{13}$$

Given that $I_t > 1$, in equilibrium, each agent i's LRA_{it} is decreasing in ω_i/γ_i: Households with low initial endowment ω_i relative to γ_i are more risk averse in equilibrium than those with high endowment relative to γ_i. Efficiency calls for households with $\omega_i > \gamma_i$ to consume a bigger share of aggregate output in good times in exchange for a lower share in bad times thus insuring households with $\omega_i < \gamma_i$. This effect is standard in the risk sharing literature that features households with CRRA preferences (see Longstaff and Wang (2014) and Veronesi (2018)). In our model though there are two additional effects relative to that literature. First, in our framework the amount of risk sharing also depends on the recession index because it generates systematic heterogeneous variation in household' risk aversion over the business cycle. For instance, households with $\omega_i > \gamma_i$ are relatively more risk tolerant in the peak of the cycle than in the trough which expands risk sharing possibilities. Second, and unlike in the CRRA case, our preferences are non-homothetic and thus initial endowment affects households' risk aversion: Even if all agents had identical preferences, $\gamma_i = 1$ for all i, agents with higher ω_i still consume more in good times and less in bad times.

4.1. Asset Prices

Proposition 2 *(Competitive equilibrium). The equilibrium stock price and interest rate are*

$$P_t = \left(\frac{\rho + k\overline{I}I_t^{-1}}{\rho(\rho+k)}\right)Y_t \tag{14}$$

$$r_t = \rho + \mu_Y - (1+v)\sigma_Y^2(I_t) + k\left(1 - \overline{I}I_t^{-1}\right) \tag{15}$$

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The stock price in Proposition 2 is identical to the one found in Menzly, Santos and Veronesi (MSV, 2004) once we define $S_t = I_t^{-1}$. This should not be surprising as the state price density in (10) is similar to the one obtained in that paper. An important benefit of this result is that we are able to calibrate the economy to yield reasonable asset pricing quantities. Indeed, given the state price density in (10) we obtain the following proposition:

Proposition 3 *(Stochastic discount factor). Given the risk-free rate r_t in (15), the stochastic discount factor follows*

$$\frac{dM_t}{M_t} = -r_t dt - \sigma_{M,t} dZ_t \qquad with \qquad \sigma_{M,t} = (1+v)\sigma_Y(I_t), \qquad (16)$$

Intuitions for the asset pricing implications are then well understood. Start with the risk free rate r_t. The terms $\rho + \mu_Y - \sigma_Y^2(I_t)$ in (15) are the standard log-utility terms: time discount, expected aggregate consumption growth, and precautionary savings. The two additional terms $k(1 - \overline{I}I_t^{-1})$ and $v\ \sigma_Y(I_t)$, are additional intertemporal substitution and precautionary savings terms, respectively, associated with the external habit features of the equivalent representative agent model (see MSV for details).

As for (14), a negative aggregate shock $dZ_t < 0$ decreases the price directly through its impact on Y_t, but it also increases households risk aversion through I_t. As a result of this, households require a higher discount to hold risky securities which produces an additional drop in prices. Thus, our model with time-varying risk preferences yields higher volatility of returns when compared with a model with log preferences, for example:

$$\sigma_P(I_t) = \sigma_Y(I_t)\left(1 + \frac{vk\overline{I}}{\rho I_t + k\overline{I}}\right). \qquad (17)$$

In addition, returns are predictable both because the market price of risk is time varying (see (16)) and there is variation in aggregate consumption volatility ($\sigma_Y(I_t)$). This generates the predictability of stock returns. Indeed,

$$E_t[dR_P - r_t dt] = \sigma_M(I_t)\sigma_P(I_t)dt \qquad where \qquad dR_P = (dP_t + Y_t dt)/dt \qquad (18)$$

All these effects combine to generate a higher equity premium.

Formulas (14) and (15) of P_t and r_t in Proposition 2 also imply the following result:

Corollary 4 *Asset prices are independent of the endowment distribution across households as well as the distribution of preferences. In particular the model has identical asset pricing implications even if all households are identical, i.e. $\gamma_i = 1$ and $\omega_i = 1$ for all i.*

In our framework standard Gorman aggregation results hold and thus there is a representative household that one can use for pricing purposes. Corollary 4 simply emphasizes that the preferences of this representative household are independent of the distribution of endowments and preferences of the underlying households. Thus P_t in equation (14) and r_t in (15) are independent of the distribution of either current consumption or wealth in the population. This property distinguishes our model from the existing literature, such as Longstaff and Wang (2012) or Chan and Kogan (2002). In these papers, the variation in risk premia is driven by endogenous changes in the cross-sectional distribution of wealth. Roughly more risk-tolerant households hold a higher proportion of their wealth in stocks. A drop in stock prices reduces the fraction of aggregate wealth controlled by such households and hence their contribution to the aggregate risk aversion. The conditional properties of returns thus rely on strong fluctuations in the cross sectional distribution of wealth. Instead, in our model households' risk aversions change, which in turn induces additional variation in premia and puts less pressure on the changes in the distribution of wealth to produce quantitatively plausible conditional properties for returns. Indeed, Corollary 4 shows that the asset pricing implications are identical even when households are homogeneous and thus there is no variation in cross-sectional distribution of wealth. Our model then features a clean separation between its asset pricing implications and its implications for trading, leverage and risk sharing. In particular, the corollary clarifies that equilibrium prices and quantities do not need to be causally related to each other, but rather comove with each other because of fundamental state variables, such as I_t in our model.

5. Leverage

We characterize now both household and financial intermediary leverage. The main results are presented in the same order we discussed the main empirical regularities in Section 2.

5.1. Household leverage

5.1.1. Households' Investments and Saving Decisions

Proposition 5 *(Optimal Allocations). In equilibrium:*

a) *Households with $\gamma_i > \omega_i$ are risk averse (RA) and save in risk-free deposits:*

$$D_{it} = v \left(\gamma_i - \omega_i \right) H \left(I_t \right) Y_t > 0 \tag{19}$$

b) *Household with $\omega_i > \gamma_i$ are risk tolerant (RT) and borrow from the intermediary:*

$$L_{it} = v\left(\omega_i - \gamma_i\right) H\left(I_t\right) Y_t > 0 \tag{20}$$

c) *All households buy N_{it} shares in the intermediary stock:*

$$N_{it} \;=\; \gamma_i + (\rho + k)(1 + v)\left(\omega_i - \gamma_i\right) H\left(I_t\right) \tag{21}$$

where

$$H\left(I_t\right) \;=\; \frac{\overline{I}}{\rho I_t + k(1+v)\overline{I}} > 0 \tag{22}$$

Implementation of the efficient allocation described in Proposition 1 requires that households with $\omega_i < \gamma_i$ save and households with $\omega_i > \gamma_i$ borrow. Thus, the terminology introduced in (7): Households for whom $\omega_i < \gamma_i$ are the RA households and households with $\omega_i > \gamma_i$ are the RT households. We use these expressions throughout. Notice as well the dependence of D_{it} and L_{it} on the recession indicator I_t, which reflects the dependence of household portfolios on the fluctuations of their attitudes towards risk. We explore this dependence in the next section.

The next corollary shows that RT households borrow to achieve a position in stocks that is higher than 100% of their wealth.

Corollary 6 *(Household positions in stocks). The investment in stock of household i in proportion to wealth is*

$$\frac{N_{it} P_t}{W_{it}} = \frac{1 + v\left(1 - \frac{\rho I_t}{\rho I_t + \overline{I}[k + (\rho + k)(\omega_i/\gamma_i - 1)]}\right)}{1 + v\left(1 - \frac{\rho I_t}{\rho I_t + \overline{I}k}\right)} > 1 \quad \text{if and only if} \quad \omega_i > \gamma_i. \tag{23}$$

Expression (23) shows that RT households invest comparatively more in stocks. In particular, because households' preferences are non-homothetic in wealth, for given preference parameter γ_i there is a positive relation between wealth and the share of the portfolio held in risky assets, a result with strong empirical support (see, for instance, by Wachter and Yogo, 2010, section 2.2). Obviously, nonhomotheticity can obtain in a variety of settings.[11] But expression (23) has some specific implications that have been taken to the data by Calvet and Sodini (2014). Indeed we show in the Internet Appendix that (23) can be written as

$$\frac{N_{it} P_t}{W_{it}} = \frac{\mathsf{SR}\left(I_t\right)}{\sigma_P\left(I_t\right)}\left(1 - \frac{\theta_i Y_t}{W_{it}}\right), \tag{24}$$

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[11]Wachter and Yogo (2010) for instance write a model in which nonhomotheticity obtains because the households have non-separable preferences over two kinds of goods, a basic good and a luxury one.

where $\mathsf{SR}\,(I_t) = (1+v)\,\sigma_P\,(I_t)$ is the Sharpe ratio of the risky asset and θ_i is a household specific constant. Equation (24) is a version of equation (2) in Calvet and Sodini (2014, page 876).[12] These authors test a variety of implications of (24) in a large panel of Swedish twins (which serves to control for differences in risk preferences) and find strong support for them.

5.1.2. Households' debt-to-income and debt-to-wealth ratios

We now show that our model is consistent with the business cycle dynamics of household leverage highlighted in Section 2. The RT households who lever up at time t borrow L_{it} from the intermediary (see Figure 5). We characterize the two definitions of leverage introduced there, namely, debt-to-income and debt-to-wealth ratios (see equation (1)).

To characterize debt-to-income ratios we need a specification of household income, Y_{it}. For simplicity, we assume that $Y_{it} = \omega_i Y_t \varepsilon_{it}$, where ε_{it} is a stationary idiosyncratic risk with $\varepsilon_{it} > 0$ and $E_t[\varepsilon_{it}] = 1$.

Proposition 7 *(Household leverage) Let $\omega_i > \gamma_i$. Then*

a) *Debt-to-income and debt-to-wealth ratios are given by:*

$$\frac{L_{it}}{Y_{it}} = v\,(1 - \gamma_i/\omega_i)\,H\,(I_t)\,\varepsilon_{it}^{-1} \quad and \quad \frac{L_{it}}{W_{it}} = \frac{v\rho(\rho + k)\,(\omega_i - \gamma_i)\,H\,(I_t)}{\gamma_i\rho + ((\rho + k)\,\omega_i - \gamma_i\rho)\,\overline{I}I_t^{-1}}, \quad (25)$$

respectively, where $H\,(I_t)$ is given by (22).

b) *The higher the ratio γ_i/ω_i and the lower the leverage whether measured relative to income or wealth.*

c) *Debt-to-income ratios, L_{it}/Y_{it}, are procyclical on average. Debt-to-wealth ratios, L_{it}/W_{it}, are countercyclical if $I_t < I^{**}$ where I^{**} is the threshold given by (IA.17) in the Internet Appendix.*

As seen in (a) and (b), leverage is fully characterized by the time series behavior I_t and the cross section of γ_i/ω_i.

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[12]Equation (2) in Calvet and Sodini (2014) is $\phi_{it} = \frac{\mathsf{SR}}{\gamma\sigma_P}\,(1 - \theta_i X_{it}/W_{it})$, where X_{it} is a subsistence or habit level in consumption. This equation obtains in a variety of habit setups (see Section II of the Internet Appendix of Calvet and Sodini (2014)). In expression (2) of our model, aggregate output, Y_t, takes the place of "habit" in traditional models.

Point(c) is the critical point of proposition, as it shows that whether one normalizes by the "income" of the household, Y_{it}, or her wealth W_{it} matters for the business cycle properties of the particular measure of leverage. As the economy improves and I_t decreases risk-tolerant households borrow more relative to their income. The reason is that as the economy improves the local curvature of the utility function decreases (see expression (13)). Wealthy households (high ω_i) or households with low exposure to aggregate shocks (low γ_i) are willing to take on more risk by borrowing more and investing more in the financial intermediary's stock.

The implications for leverage when normalizing by wealth, W_{it}, are instead different. As the economy improves the stock price increases and it does so more than the amount of debt issued by the borrowing households. In effect, the difference in the time series behavior of both measures of leverage is driven by the fact that in debt-to-wealth ratios, L_{it}/W_{it}, discount effects are present in the denominator whereas they are not in debt-to-income ratios, L_{it}/Y_{it}; whether one normalizes by income or wealth matters for the cyclical properties of leverage.[13] Put it in another way, the variation on leverage depends on the business cycle variable I_t, which affects also the stock return volatility and risk premium. As I_t increases, all agents' risk aversions increase, which induce them to decreases their debt-to-income L_{it}/Y_{it}, but at the same time the risk premium increases, which induces them to increase the amount of debt in percentage of wealth to take advantage of the improved investment opportunities.

The results in Proposition 7 hold also in aggregate once we integrate across all risk-tolerant households. Therefore, the dynamics of aggregate debt-to-income and debt-to-wealth ratios are thus in line with the evidence in Figure 2, which shows the increase in debt-to-income ratio of households before the 2008 crisis but the increase in debt-to-wealth ratio after the crisis kicked in, in 2009.

In addition, the results of Proposition 7 are also consistent with Figure 3, which shows that households with lower wealth are those with higher debt-to-wealth ratios. Our model can easily match this pattern by assuming that γ_i/ω_i is positively correlated with ω_i, a relation that is also automatically imposed by the wealth constraint ($A1$). Moreover, under such assumptions, point (c) of the proposition implies indeed that debt-to-wealth ratio of poorer agents increases the most during bad times, as shown in Figure 3. These results thus show that our model is able to match qualitatively important patterns in the data. In Section 6. we investigate the model's quantitative performance and show in simulation that the same pattern occurs even when γ_i and ω_i are independent of each other provided that the distribution of γ_i is sufficiently wide.

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[13]The threshold I^{**} in point (c) of Proposition 7 is very high and rarely reached in simulations.

5.1.3. Leverage and consumption

Our framework has tight implications for the relation between leverage and current and future consumption at the individual household level.

Corollary 8 *Household i's consumption growth satisfies*

$$\frac{dC_{it}}{C_{it}} = \mu_{C,it}dt + \sigma_{C,it}dZ_t \tag{26}$$

where

$$\mu_{C,it} = \mu_Y + \frac{(\omega_i - \gamma_i)\overline{I}}{\gamma_i I_t + (\omega_i - \gamma_i)\overline{I}}F(I_t) \tag{27}$$

$$\sigma_{C,it} = \left(1 + \frac{v(\omega_i - \gamma_i)\overline{I}}{\gamma_i I_t + (\omega_i - \gamma_i)\overline{I}}\right)\sigma_Y(I_t) \tag{28}$$

with

$$F(I_t) = k(1 - \overline{I}I_t^{-1}) + (1+v)v\ \sigma_Y^2(I_t) \tag{29}$$

If $\sigma_Y(I_t)$ is increasing in I_t with $\sigma_Y(1) = 0$, then there exists a unique solution I^* to $F(I^*) = 0$ such that for all i and j with $\gamma_i/\omega_i < 1 < \gamma_j/\omega_j$ we have

$$E\left[\frac{dC_{it}}{C_{it}}\right] < \mu_Y < E\left[\frac{dC_{jt}}{C_{jt}}\right] \qquad for \qquad I_t > I^* \tag{30}$$

$$E\left[\frac{dC_{it}}{C_{it}}\right] > \mu_Y > E\left[\frac{dC_{jt}}{C_{jt}}\right] \qquad for \qquad I_t < I^* \tag{31}$$

This corollary shows that cross-sectionally, RT households, those with $\gamma_i < \omega_i$, have lower expected growth rate of consumption than RA households when I_t is low, and viceversa. We know that these are also times when such households are heavily in debt. It follows then that households who are heavily leveraged enjoy both a high consumption boom in good times, but a lower *future* expected consumption growth.[14]

Corollary 9 *Highly leveraged households enjoy high consumption shares in good times but have lower expected consumption going forward.*

To reiterate, leverage and consumption patterns are not casually related. They are both driven by changes in the attitudes towards risk: After a sequence of good economic

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[14]Parker and Vissing-Jørgensen (2009) use the Consumer Expenditure (CEX) Survey to show that the consumption growth of high-consumption households is significantly more exposed to aggregate fluctuations than that of the typical household.

shocks aggregate risk aversion declines. Thus, RT households borrow more and experience a consumption "boom". The increase in consumption is due to the higher investment in stocks that have higher payoffs in good times. Good times mean lower individual (and aggregate) risk aversion and thus these same households take on more leverage. Hence, our model predicts a positive comovement of leverage and consumption at the household level. Finally, mean reversion in I_t also implies that RT households are also those that suffer a bigger drop in consumption growth once I_t increases.

This implication of our model speaks to some of the recent debates regarding the low consumption growth experienced by levered households following the Great Recession. Some have argued that the observed drop in consumption growth was purely due to a wealth effect, as levered households tend to live in counties that experienced big drops in housing values, whereas others have emphasized the critical role of debt in explaining this drop.[15] Clearly these effects are important but our contribution is to show that because leverage is an endogenous variable, high leverage followed by low consumption is precisely the prediction of our model even without causal effect of leverage on consumption.

5.2. Financial intermediary leverage

The definition of leverage for the financial intermediary is similar to that of the households. The total amount of debt of the intermediary is

$$\mathcal{D}_t = \int_{i:\gamma_i < \omega_i} D_{it} di. \tag{32}$$

Again, we define two measures of intermediary leverage: The first measure, $\mathcal{D}_t/Y_t$ normalizes the amount of debt on the liability side of the intermediary's balance sheet by the aggregate output at time t, the income flowing from the assets held. The second, $\mathcal{D}_t/P_t$, instead normalizes by the market value of intermediary equity, P_t. We are interested in the cyclical properties of these measures and whether they can serve as factors priced in the cross section.

5.2.1. Time series properties of financial intermediary leverage

Proposition 10 *(Financial intermediary leverage)*

a) *The amount of debt issued by the financial intermediary, $\mathcal{D}_t$, is given by*

$$\mathcal{D}_t = v K_1 H(I_t) Y_t \qquad where \qquad K_1 \equiv \int_{i:\omega_i > \gamma_i} (\omega_i - \gamma_i) di > 0 \tag{33}$$

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[15]See for instance Mian and Sufi (2014, in particular pages 39-45) for a summary of these differing views.

and $H(I_t)$ is given in expression (22).

b) *Debt-to-output ratio and debt-to-equity ratio are given by, respectively:*

$$\mathcal{D}_t/Y_t = vK_1H(I_t) \qquad and \qquad \mathcal{D}_t/P_t = \frac{v\rho(\rho+k)K_1H(I_t)}{\rho + k\overline{I}I_t^{-1}}. \qquad (34)$$

c) *Intermediary's debt-to-output ratio, $\mathcal{D}_t/Y_t$, is procyclical. The intermediary debt-to-equity ratio, $\mathcal{D}_t/P_t$, it is countercyclical provided $I_t < I^{**}$, where I^{**} is given in equation (IA.17) in the Internet Appendix.*

The implications for the leverage of the financial intermediary follow immediately from the results on household leverage. After all the financial intermediary's leverage is directly linked to the short debt issued to saving households who use it to hedge their exposure against aggregate shocks. In turn the amount of short debt issued by the financial intermediary is backed by the loans granted to borrowing households.

Financial intermediary debt, $\mathcal{D}_t$, increases as the economy improves (as I_t decreases) on account in turn of the increase of the risk bearing capacity of RT households who take on leverage. This allows the financial intermediary to grant more loans to those household which in turn allows it to issue more short term debt to the RA households. Conversely, the risk bearing capacity of borrowing households diminishes as the economy deteriorates and with it the supply of safe assets, precisely when it is most needed. Our model thus provides a theory of the supply of safe assets that is determined by the risk bearing capacity of borrowing households.[16]

Whether we normalize financial intermediary debt, $\mathcal{D}_t$, by aggregate output or by equity matters for whether aggregate leverage is pro- or countercyclical. Proposition 10 establishes that as the economy improves debt grows more than aggregate output, Y_t, and thus the procyclicality of $\mathcal{D}_t/Y_t$. The countercyclicality of intermediary's debt-to-equity ratio $\mathcal{D}_t/P_t$ instead has to do with the discount effects that characterize our model. A large realization of Y_t increases aggregate wealth on account of both the direct effect but also the additional increase in valuations as the households' risk aversion drops (see (14)). This results in a countercyclical leverage measure.

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[16]This is an issue studied by other papers. See for instance Barro and Mollerus (2014), who propose a model based on Epstein-Zin preferences to offer predictions about the ratio of safe assets to output in the economy. Gorton, Lewellen and Metrick (2012) and Krishnamurthy and Vissing-Jorgensen (2012) provide empirical evidence regarding the demand for safe assets. In all these papers the presence of "outside debt" in the form of government debt plays a critical role in driving the variation of the supply of safe assets by the private sector, a mechanism that is absent in this paper.

The time-series properties of intermediary leverage described in Proposition 10 are consistent with the empirical regularities discussed in Section 2. As displayed in the top panels of Figure 4 book leverage decreases when firms' asset value decrease, while market leverage increases when the intermediaries' valuations decrease. While in this literature this dynamics is interpreted as the active deleveraging of intermediaries, our results show that similar results obtain in general equilibrium due to the demand and supply of credit from households.

Because the dynamics of leverage depends on I_t which also affects the volatility of the risky asset (see (17)), we obtain the following:

Corollary 11 *On average, the intermediary debt-to-output ratio, $\mathcal{D}_t/Y_t$, is high when aggregate volatility $\sigma_P(I_t)$ is low and the price of risky assets P_t is high.*

In our model, both return volatility and debt-to-output ratio of the intermediary depend on the single state variable I_t which is then the source of the comovement. This result is consistent with the evidence in the bottom panel of Figure 4, which shows that VaR measures correlate negatively with leverage, as VaR is in turn positively related to measures of asset return volatility.

5.2.2. Intermediary asset pricing and leverage risk price

Our model also sheds light on recent empirical findings in the "intermediary asset pricing" literature as discussed in Section 2. In our one-factor model the conditional CAPM holds. If we could easily measure I_t in the data, we could compute expected returns off the conditional CAPM. However, suppose that economy's aggregate risk aversion, which is proportional to I_t, is not observable. The leverage ratio of intermediaries is instead observable and it can be used as a proxy, thus explaining the empirical evidence.

Formally, let $\mathcal{L}_t$ denote either of the two measures of financial intermediary leverage that we have considered. Then if $\mathcal{L}_t$ is monotonic in I_t there exists a function $q(\cdot)$ such that[17] $I_t = q(\mathcal{L}_t)$. The state price density is then

$$M_t = e^{-\rho t} Y_t^{-1} I_t = e^{-\rho t} Y_t^{-1} q(\mathcal{L}_t).$$

The volatility of the SDF is thus $\sigma_{M,t} = \sigma_{Y,t} - \frac{q'(\mathcal{L}_t)}{q(\mathcal{L}_t)} \sigma_{\mathcal{L},t}$ where $\sigma_{\mathcal{L},t}$ is the volatility of leverage. The risk premium for *any* asset with return dR_{it} can then be written as

$$E_t[dR_{it} - r_t dt] = Cov_t\left(\frac{dY_t}{Y_t}, dR_{it}\right) + \lambda_t^{\mathcal{L}} Cov_t(d\mathcal{L}_t, dR_{it}) \tag{35}$$

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[17]For this heuristic argument, we restrict $I_t < I^{**}$ so that $\mathcal{D}_t/P_t$ is monotonic. See Proposition 10.

where

$$\lambda_t^{\mathcal{L}} = -\frac{q'(\mathcal{L}_t)}{q(\mathcal{L}_t)} \tag{36}$$

The first term of (35) corresponds to the usual log-utility, consumption-CAPM term, while the second term corresponds to the additional risk premium due to shocks to $\mathcal{L}_t$. $\lambda_t^{\mathcal{L}}$ is the market price of leverage risk.[18] We then obtain

Corollary 12 *(Price of leverage risk)*

a) *The price of leverage risk is positive, $\lambda_t^{\mathcal{D}/Y} > 0$, when leverage is defined as $\mathcal{L}_t = \mathcal{D}_t/Y_t$.*

b) *The price of leverage risk is negative, $\lambda_t^{\mathcal{D}/P} < 0$, when leverage is defined as $\mathcal{L}_t = \mathcal{D}_t/P_t$.*

As shown in Proposition 10, $\mathcal{D}_t/Y_t$ is procyclical. That is, leverage is high when on average, marginal utilities are low. Thus the market price of risk associated with this measure of leverage is positive. Instead $\mathcal{D}_t/P_t$ is countercyclical and thus it is high when on average marginal utilities are high and thus the market price of risk is in this case negative.

To link these results to the empirical evidence in AEM and HKM (see Table 1 in Section 2.) we can equate $\mathcal{D}_t/Y_t$ to the "book leverage" of financial intermediaries and $\mathcal{D}_t/P_t$ to their "market leverage". Indeed, like our measure of debt-to-output $\mathcal{D}_t/Y_t$, book leverage does not depend on market prices and thus it is procyclical. In contrast, market leverage depends on asset prices, and thus the discount effect that renders it countercyclical.

Finally, notice that good time, when I_t is low, are also periods when expected excess returns are low and so is typically aggregate uncertainty $\sigma_Y(I_t)$.[19] Because $\mathcal{D}_t/Y_t$ is procyclical and $\mathcal{D}_t/P_t$ is countercyclical the following corollary obtains immediately,

Corollary 13 *(Predictability of Future Excess Returns) Let the risk risk premium $E[dR_{it} - r_t dt] = \sigma_M(I_t)\sigma_P(I_t)$ be countercyclical. Then a high intermediary debt-to-output ratio $\mathcal{D}_t/Y_t$ predicts lower future excess returns, while a high intermediary debt-to-equity ratio $\mathcal{D}_t/P_t$ predicts high future excess returns. The regression coefficient is negative for intermediary debt-to-output ratio and positive for intermediary debt-to-equity ratio.*

These theoretical results are consistent with Panel B and C of Table 1 that show that high book leverage weakly predicts future lower excess returns, while high market leverage

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[18]This decomposition is for illustrative purposes only. All shocks are perfectly correlated in our model and so there is only one priced of risk factor.

[19]Note that we have not made any assumptions yet on $\sigma_Y(I_t)$, except that it vanishes for $I_t \to 1$.

predicts higher future returns. Our simulations below also show that interestingly our model is also consistent with market leverage being better in predicting returns than book leverage, as the price at denominator induces far more variation of market leverage than book leverage.

5.3. Discussion

Risk sharing and leverage are in our model two related but distinct concepts. Efficient risk sharing requires marginal utilities (scaled by the Pareto weights, ϕ_i; see equation (11)) to be equated across households (see equation (9)). How the competitive equilibrium implements the efficient allocation described in Proposition 1 depends on the specific financial market structure assumed and thus so do the leverage implications of our model. With this in mind, it is useful to consider how the portfolio allocations in Proposition 2 implement the efficient allocation described in Proposition 1 through a standard replication argument. Let W_{it} be the value of the contingent claim that at each point in time and state delivers as a dividend the consumption of agent i, C_{it}, associated with the efficient allocation (see equation (12)). We show in the Internet Appendix that the value of this contingent claim would be:

$$W_{it} = E_t \left[\int_t^\infty \frac{M_\tau}{M_t} C_{i\tau} d\tau \right] = \frac{\rho \gamma_i + (\rho(\omega_i - \gamma_i) + k\omega_i)\overline{I} I_t}{\rho(\rho + k)} Y_t. \tag{37}$$

Clearly a financial structure that features these contingent claims can equally implement the efficient allocation: Each agent would buy his corresponding contingent claim at date 0 and consume the dividends C_{it} throughout. Following Cox and Huang (1989) the stock investment and borrowing/lending decision in Proposition 5 simply replicates the cash-flows of this contingent claim

$$N_{it} P_t + B_{it} = W_{it}. \tag{38}$$

where B_{it} is a position in risk-free bonds. In our model, we have that demand for deposits is $D_{it} = B_{it}$ if $B_{it} > 0$ and demand for loans from the intermediary is $L_{it} = -B_{it}$ if $B_{it} < 0$. For this to be satisfied for every t (and pay C_{it} as dividend), it must be the case that the portfolio and the security have the same sensitivity to shocks dZ_t. Denoting by $\sigma_{Wi}(I_t)$ the volatility of $\log(W_{it})$, the portfolio allocation N_{it} and B_{it} must then satisfy

$$N_{it} = \frac{W_{it} \, \sigma_{Wi}(I_t)}{P_t \, \sigma_P(I_t)} \qquad \text{and} \qquad B_{it} = W_{it} - N_{it} P_t = W_{it} \left(1 - \frac{\sigma_{Wi}(I_t)}{\sigma_P(I_t)} \right). \tag{39}$$

The bond position, B_{it}, depends on the ratio of volatilities $\frac{\sigma_{Wi}(I_t)}{\sigma_P(I_t)}$: If this ratio is greater than one, the agent is leveraging his investment in the stock market. The volatility of the contingent claim is

$$\sigma_{Wi}(I_t) = \sigma_Y(I_t) \left(1 + \frac{v \, (k + (\rho + k)(\omega_i/\gamma_i - 1)) \, \overline{I}}{\rho I_t + (k + (\rho + k)(\omega_i/\gamma_i - 1)) \overline{I}} \right). \tag{40}$$

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Comparing this expression with $\sigma_P(I_t)$ in (17), we see that $\sigma_{Wi}(I_t) > \sigma_P(I_t)$ if and only if $\omega_i > \gamma_i$. That is, RT households ($\omega_i > \gamma_i$) borrow to leverage their portfolio. Intuitively, from the optimal risk sharing rule (12), RT households have a high consumption share in good times, when I_t is low, and a low consumption share in bad times, when I_t is high. This particular consumption profile implies that the value of the contingent claim W_{it} is more sensitive to discount rate shocks than the stock price P_t. As a result the "replicating" portfolio requires some leverage to match such sensitivity.

Equation (39) also highlights the reason why the aggregate debt-to-output ratio, which is equal to $\mathcal{D}_t/Y_t$, increases in good times. This is due to a "level effect": from (40) and (17) the ratio of volatilities actually declines as I_t decreases. This is intuitive as the hypothetical contingent claim pays out more in good times and hence becomes less sensitive to discount rate shocks then. However, from (37) the value of the hypothetical contingent claim W_{it} increases in good times because the discount rate declines and more than overcomes the decline in the ratio of volatilities. As a result, aggregate debt increases in good times.

While a procyclical aggregate debt-to-output ratio may seem intuitive, it is not normally implied by, for instance, standard CRRA models with differences in risk aversion. In such models, less risk averse households borrow from more risk averse agents, who want to hold riskless bonds rather than risky assets. As aggregate wealth becomes more concentrated in the hands of less risk-averse agents, the need of borrowing and lending declines, which in turn decreases aggregate debt.[20] Moreover, a decline in aggregate uncertainty – which normally occur in good times – actually decreases leverage in such models, as it reduces the risk-sharing motives of trade (see Veronesi (2018)). In our model, in contrast, the decrease in aggregate risk aversion in good times make households with high-risk bearing capacity even more willing to take on risk and hence increase their supply of risk-free assets to those who have a lower risk bearing capacity.

[20]Longstaff and Wang (2012), Figure 5, shows the standard inverse-U shape relation between market leverage and the share of consumption of the least risk-averse agent s. Market leverage is minimized at the extremes for $s = 0$ and $s = 1$. While Longstaff and Wang do not report a similar plot for the debt-to-output ratio, it is straightforward verify that the same inverse-U shape holds also for the debt-to-output ratio.

6. Quantitative implications

6.1. Parameters and simulations

We now provide a quantitative assessment of the effects discussed in previous sections. There are two sets of parameters to consider, those that pertain to the aggregate time series properties of the model and those that relate to the cross sectional dispersion in households' attitudes towards risk and wealth.

In what concerns the time series parameters we follow MSV closely as our model aggregates to a representative consumer household which is identical to the one in that paper. The only exception is that in the present model the aggregate endowment process is heteroskedastic. Our theoretical results do not depend on the functional form of $\sigma_Y(I_t)$ but obviously to simulate the model we need to specify one. We assume that

$$\sigma_Y(I_t) = \sigma^{max}\left(1 - I_t^{-1}\right) \tag{41}$$

Assumption (41) implies that output volatility increases when the recession index increases, but it is also bounded between $[0, \sigma^{max}]$.[21] This is consistent with existing evidence that aggregate uncertainty increases in bad times (see e.g. Jurado, Ludvigson, and Ng (2015)), it satisfies the technical condition $\sigma_Y(I_t) \to 0$ as $I_t \to 1$, and it also allows us to compare our results with previous literature, as we obtain

$$dI_t = k(\overline{I} - I_t)dt - (I_t - 1)\overline{v}dZ_t$$

with $\overline{v} = v\sigma^{max}$ which is similar to the one in MSV.

σ^{max} is chosen to match the average consumption volatility $E[\sigma_Y(S_t)] = std[\Delta \log(C_t^{data})]$, where the expectation can be computed from the stationary density of I_t.[22] The rest of the parameters are similar but not identical to MSV and are reported in Panel A of Table A.1 in Appendix A1.1. Panels B and C of that table shows that, similarly to MSV, the model is able to match the main properties of stock returns, both conditionally and unconditionally. Figure A.1, which reproduces Figure 1 in MSV.[23]

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[21]The alternative of assuming e.g. $\sigma_Y(I_t)$ as linear in I_t would result in $\sigma_Y(I_t)$ potentially diverging to infinity as I_t increases. We also assume that $\sigma_Y(I)$ is multiplied by a "killing function" $k(I^{-1})$ such that $k(x) \to 0$ when $x \to 0$ to ensure that integrability conditions are satisfied (see Ceriditto and Gabaix (2008)). We do not make such function explicit for notational convenience.

[22]See the Appendix in MSV. In addition, note that in MSV, $\alpha = \overline{v}/\sigma$ and therefore we compute $\overline{v} = \alpha\sigma$. Finally, MSV has I_t bounded below by a parameter $\lambda > 1$ while in our model I_t is bounded below by 1.

[23]Like Campbell and Cochrane (1999) and Menzly, Santos, and Veronesi (2004), our model also implies a positive relation between stock return volatility and equity risk premium, for which though there is little evidence in the data.

6.2. The cross-section of household leverage

We now proceed to specify the distribution of initial endowments w_i and of preferences γ_i. A full micro-founded "calibration" is clearly problematic in our setting, given the types of preference specification. We resort to illustrate the model's predictions through a reasonable numerical illustration which yields sensible quantities for some observables, such as households consumption volatility and debt levels. We assume that the risk aversion parameters γ_i are uniformly distributed $\gamma_i \sim U[1 - \gamma, 1 + \gamma]$, so as $\int \gamma_i di = 1$. The uniform distribution is a reasonable starting point, as it bounds above the parameter γ_i.

Endowments ω_i must meet Assumption $A1$, that is, $\omega_i > \gamma_i \left(1 - \overline{I}^{-1}\right)$. To obtain draws of ω_i that are independent of γ_i we thus have to restrict the distribution of ω_i so that the support is bounded from below by $\underline{\omega} = \max(\gamma_i)(1 - \overline{I}^{-1}) = (1 + \gamma)(1 - \overline{I}^{-1})$. To ensure a positively skewed distribution of wealth, we thus assume that

$$\omega_i = \underline{\omega} + e^{\mu_w + \sigma_w \varepsilon_i}$$

with $\varepsilon_i \sim N(0, 1)$ and $\mu_w = \log\left[1 - \underline{\omega}\right] - \sigma_w^2/2$ to ensure $E[\omega_i] = 1$.[24] The only free parameters of the cross-sectional distribution of agents are thus γ and σ_w.

We choose γ and σ_w with an eye on relevant moments of individual households' consumption growth, such as average household consumption growth (arithmetic or log), its mean and median total and systematic volatility, and the cross-sectional dispersion of both. One important stumbling block to estimate the total and systematic volatility of household consumption growth is the lack of reliable panel data on households consumption, which has limited the empirical work on the time-series properties of individual households' consumption. However, the Internet Appendix IA2. describes a novel methodology to estimate households' total and systematic consumption volatility from cross-sectional consumption data, and its application to the Survey of Consumer Expenditure (CEX). For our estimation, we use the dataset compiled by Kocherlakota and Pistaferri (2009) which spans the period 1980–2005.

Panel A of Table 2 reports the results. The average quarterly (arithmetic) growth rate is about 6%, which is large but mostly driven by the large cross-sectional heterogeneity in quarterly growth rates. Indeed, the median is slightly negative and the cross-sectional standard deviation is 40%, in line with estimates by e.g. Constantinides and Ghosh (2017). The log-growth indeed shows a slightly negative mean, which is close to the median, highlighting the positive skewness of the consumption data.

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[24]Indeed $E[\omega_i] = \underline{\omega} + E[e^{\mu_w + \sigma_w \varepsilon}] = \underline{\omega} + e^{\mu_w + \sigma_w^2/2} = 1$.

The total quarterly volatility is also large, at 36.5%, and it displays a strong positive skewness, as its median is much lower at 27.1%, and its dispersion (standard deviation) is at 42.4%. Clearly, much of this quarterly consumption volatility is due to idiosyncratic shocks and residual seasonalities.[25] As one would expect, quarterly systematic volatility is lower than the total volatility: the average is almost 8.9%, and the median is just 6.6%. The dispersion is still large, but reasonable, at 10.4%.

Panel B of Table 2 contains the same moments as Panel A but from the simulated model. We consider various combinations γ for the uniform $U[1 - \gamma, 1 + \gamma]$ and the dispersion σ_w of the lognormal distribution of endowments. We focus on the case $U[0, 2]$ and $\sigma_w = 0.75$ and discuss the other parameter configurations in the Internet Appendix. The simulated moments for consumption growth and systematic volatility are reasonable and close to the data, with the important exception that our model is not able to generate the large cross-sectional dispersion in quarterly consumption growth. This is to be expected, as the cross-sectional dispersion in the data quarter by quarter is likely due to idiosyncratic shocks, which are absent in our model.[26] More specifically, with those parameters, the model generates a mean growth rate of 0.9%, with the median at 0.5% and a cross-sectional dispersion of 6.3%. There is positive skewness, but not at the levels observed in the data, as this is "systematic skewness". Indeed, the mean consumption volatility is at 8.7%, with the median at 5.9% and dispersion at 11.3%. These values are similar to the corresponding values in Panel A for systematic volatility. This calibration also generate positive skewness in systematic volatility, as observed in the data.

Panel A of Figure 6 shows the distribution of endowment in a simulation of 200,000 agents when $\sigma_w = 0.75$. The distribution is strongly positively skewed. Because of the restriction $\int \omega_i di = 1$, the distribution shows a large mass of households with $\omega_i < 1$ to allow for some households with a very large endowment. Panel B shows the relation between endowment ω_i and leverage, namely, $\omega_i - \gamma_i$. Indeed, recall that only households with $\omega_i - \gamma_i > 0$ lever up (see Proposition 5). The model is such that agents with a wide range of endowments both borrow and lend, depending on their risk aversion. However, agents with very large endowment are borrowers only, as their risk aversion is low.

Our assumption on the joint distribution of preferences γ_i and endowments ω_i in the

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[25]As explained in the Internet Appendix IA2., for each household i we mitigate the influence of seasonality by computing the average $\widehat{\sigma}_{it}^2$ over the three quarters of available variance observations.

[26]More precisely, household's endowments feature idiosyncratic shocks in our model but diversification through the intermediary's balance sheet eliminates them from the households' equilibrium consumption processes. Likewise, our model is not rich enough to be able to generate a wealth distribution that is close to the one in the data.

Table 2: Cross-Sectional Parameters and Household Consumption Moments.

Panel A. Households Quarterly Consumption Moments. Data

	Growth Rate (%)				Volatility (%)		
	Mean	Median	Std. Dev.		Mean	Median	Std. Dev.
Arithmetic	6.04	-0.63	40.13	Total	36.53	27.10	42.35
Logarithmic	-0.59	-0.66	35.78	Systematic	8.94	6.61	10.42

Panel B. Households Quarterly Consumption Moments. Model

$U[\underline{\gamma},\overline{\gamma}]$	σ_w	Arithmetic Growth Rate (%)				Volatility (%)		
		Mean	Median	Std. Dev.		Mean	Median	Std. Dev.
$U[0,2]$	1.00	0.99	0.53	7.09		9.63	6.21	12.64
$U[0,2]$	0.75	0.91	0.53	6.35		8.77	5.86	11.30
$U[0,2]$	0.50	0.84	0.53	5.60		7.91	5.54	9.69
$U[0,2]$	0.25	0.80	0.53	5.14		7.47	5.41	8.52
$U[0,2]$	0.00	0.82	0.51	5.33		7.87	5.77	8.81
$U[1,1]$	1.00	0.57	0.53	2.12		3.17	2.26	3.79
$U[1,1]$	0.75	0.55	0.51	1.74		2.66	1.72	3.50
$U[1,1]$	0.50	0.54	0.53	1.24		2.05	1.35	2.43
$U[1,1]$	0.25	0.52	0.52	0.65		1.51	1.21	1.59
$U[1,1]$	0.00	0.53	0.53	0.00		1.15	1.15	0.00

previous section also yields a cross section of debt-to-wealth ratio, L_{it}/W_{it}, that matches well its empirical counterpart around the financial crisis in 2007 - 2009. Recall that Figure 3 shows two patterns of leverage across households. First, households with lower net worth have higher debt-to-wealth ratios. Second, the debt-to-wealth ratio of poorer households increased dramatically between 2007 and 2009, while the same is not true for richer households.

Figure 7 plots L_{it}/W_{it} of the borrowing RT households by wealth percentile in simulations, which aims to replicate Figure 3.[27] It does for booms (S_t high), recessions (S_t low), and crisis (S_t very low). First, in general, households with lower net worth (W_{it}) take on more debt as a fraction of assets. To understand this result first recall that Figure 7 shows the leverage of borrowing households, what we have termed throughout the RT households, and that our preferences are non-homothetic: given γ, low net worth households are more risk averse than richer households. The intuition is clear: conditional on borrowing a poor household must have a much higher risk tolerance than an average rich household and for this reason they take on more leverage. Notice that this result obtains even when the distribution of ω_i

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[27]As discussed in footnote 4, in the data there are also cross-sectional differences in terms of the assets held by different households, such as housing or equity. Ours is clearly a simplification but an extension to multiple assets with housing would likely produce similar results.
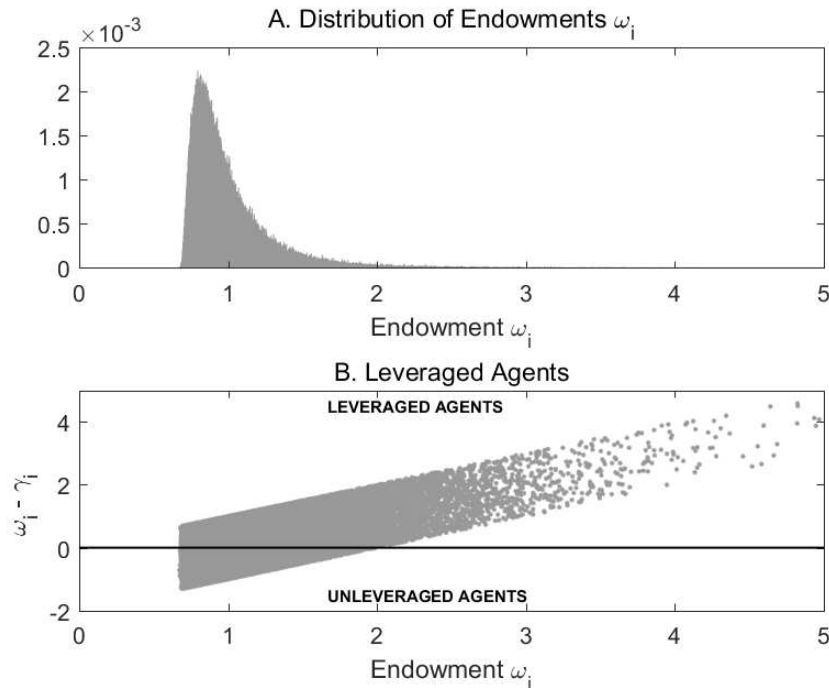
Figure 6: Preference and Endowment Distribution. Panel A plots the distribution of endowments $w_i = \underline{w} + e^{\mu_w + \sigma_w \varepsilon_i}$ where $\underline{w} = 2(1 - \overline{I}^{-1})$, $\mu_w = log[1 - \underline{w}] - \frac{1}{2}\sigma_w^2$, $\sigma_w = 0.75$ and $\varepsilon \sim N(0, 1)$. Panel B shows the relation between endowments ω_i and $\omega_i - \gamma_i$; recall that households with $w_i - \gamma_i > 0$ take on debt.

and γ_i are independent from each other. This revealed preference argument has important implications regarding the inferences on risk preferences one should draw from household portfolio decisions. On observing the patterns observed in the data two hypothesis are equally consistent: that the initial wealth and risk tolerance correlate negatively in the population or that, as in our case, preferences are non-homothetic and selection is inducing the particular pattern in Figure 3.

The second important pattern in Figure 3 is that debt-to-wealth ratios L_{it}/W_{it} increase markedly during crises, that is, those rare times in which S_t is on the left-hand-side of its distribution (see Panel A of Figure A.1). This is an important channel in our model: While households who borrow deleverage when I_t increases, and hence reduce their amount of debt, the debt-to-asset ratio actually increases. The reason is that the value of assets declines by even more (see Proposition 7). That is, households engage in active debt repayment but household leverage when debt is normalized by wealth, L_{it}/W_{it}, increases nonetheless. This pattern is particularly evident during the financial crisis of 2008 (see Figure 2).[28]

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[28]The Internet Appendix documents that a similar plot obtains in the case of Spain which also has a comprehensive household survey (the "Encuesta Financiera de las Familias" or EFF). We thank Olympia Bover of the Bank of Spain for pointing out this to us.
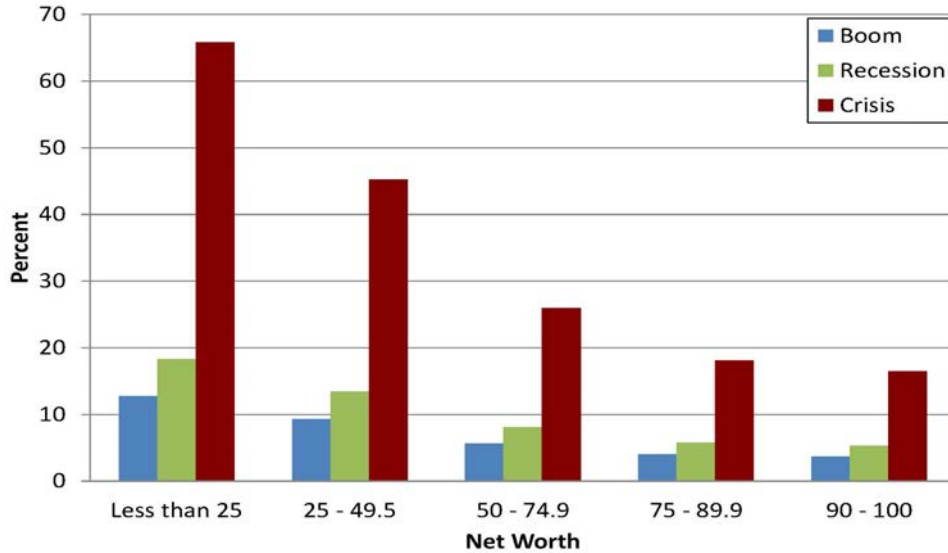
Figure 7: Debt-to-assets ratios across the wealth distribution in the model . This figure plots the distribution of debt-to-asset ratios of households who take on debt in simulations during three types of periods: Booms (I_t low), recessions (I_t high) and crisis (I_t very high).

Notice though that we miss on the magnitudes. For instance, as shown in Figure 3 the leverage of low net worth households at the trough of the crisis, in 2009, is above 130%, whereas it is about half of that in our simulations. The disparity in magnitudes is even more pronounced in good times. Thus additional ingredients need to be brought to bear in order to explain why low net worth households levered as much as they did during the years leading up to the financial crisis of 2008.

In sum, our model is able to capture an important fact in the cross section, that the less wealthy lever more. This stands in contrast with most models with heterogeneous agents, such as, for example, Dumas (1982) and Longstaff and Wang (2012). There, less risk averse households lever up, invest in risky stocks, and become richer as a result. These models thus imply counterfactually that leverage is more pronounced amongst richer agents and are unable to explain the patterns in Figure 3. In contrast, in our model the two different sources of heterogeneity, combined with the implicit assumption that households with low endowment have lower habit loading γ_i, imply that poor households lever up more, consistently with the data. Of course, there is an important difference between our model and the data: in our model agents poorer households lever to purchase equity issued by the financial intermediary, while in the data they lever to purchase durable goods and housing.
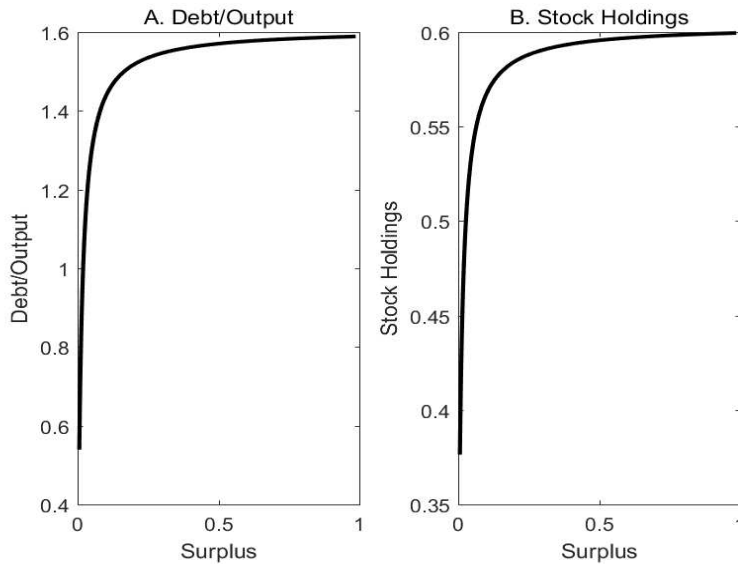
36

Figure 8: Aggregate leverage and Stock Holdings of Levered households Panel A plots $\mathcal{D}_t/Y_t$, the aggregate debt to output ratio in the economy (see expression (34)) as a function of the surplus consumption ratio S_t. Panel B reports the aggregate holdings of stocks for the RT households.

6.3. Aggregate leverage, panic deleveraging and stock prices

Our model has implications for the dynamics of the aggregate household leverage, which is also the leverage of the financial intermediary in our model. Panel A of Figure 8 shows the aggregate debt-to-output ratio as a function of S_t for our choice of parameter values. Panel B shows the aggregate stock holdings of RT households. As can be seen in expression (34) the behavior of $\mathcal{D}_t/Y_t$ with respect to S_t depends on the shape of the function $H(I_t)$. Recall that this function is decreasing and convex in I_t, and thus increasing and concave in S_t. In particular, our parametric choices imply that aggregate deleveraging accelerates as bad times morph into severe distress as househoulds' risk aversions skyrocket.[29] Instead for high values of S_t the function is relatively flat and variation of the surplus ratio in that domain do not result in big swings in either aggregate stock holdings or the aggregate debt-to-output ratio. Because the state variable does not visit that range of values very often (see the stationary density of S_t in Panel A of Figure A.1) it follows then that the extreme periods of deleveraging do not happen often.

Figure 9 further emphasizes the point. It shows the time series behavior of several quantities of interest over a 100 years of artificial quarterly data. Panel A shows the realization of the surplus consumption ratio $S_t = I_t^{-1}$, while panel B reports the corresponding economic

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[29]It is important to emphasize that these results do not depend on the specific assumptions made on the functional form for $\sigma_Y(I_t)$ as the function $H(I_t)$ does not depend on it.
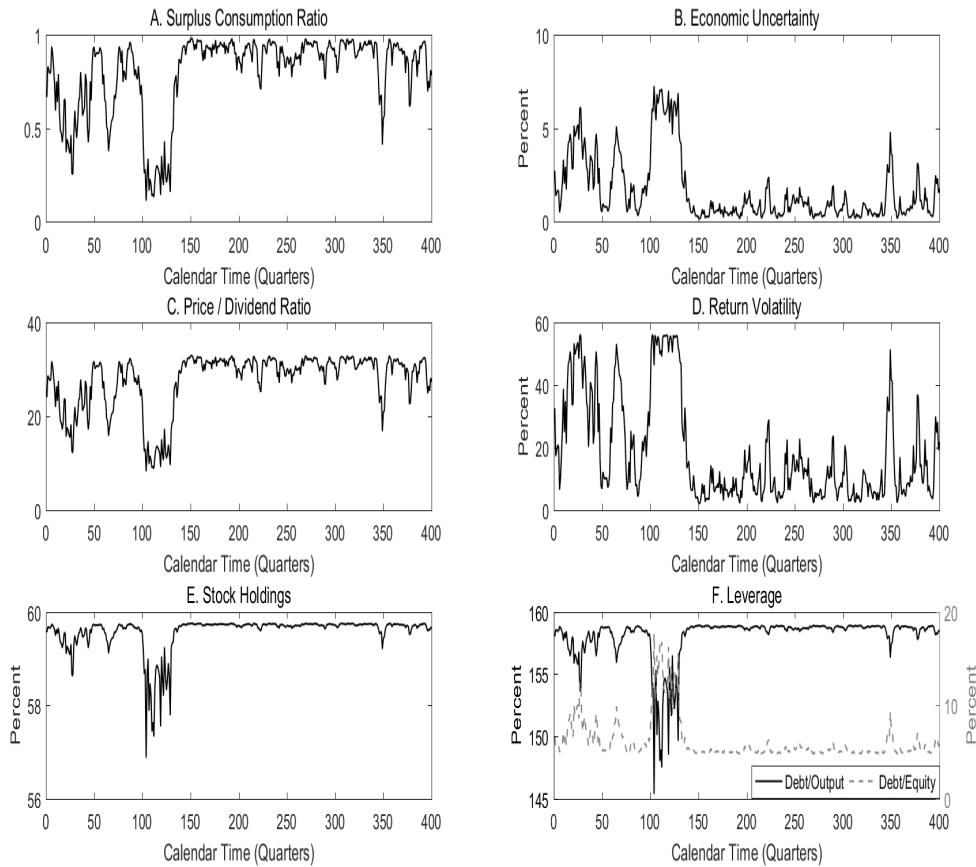
Figure 9: "Fire Sales" in a Simulation Run . This figure plots the time series of several quantities in 100 years of quarterly artificial data. Panel A reports the "surplus consumption ratio" $S_t = I_t^{-1}$. Panel B reports the consumption volatility $\sigma_Y(I_t)$. Panel C and D report the price-dividend ratio and the stock return volatility, respectively. Panel E reports the aggregate position in risky stock of levered households (grey dashed line, right axis). Panel F reports the aggregate debt-to-wealth ratio of levered agents (dashed red line; right axis) and the aggregate debt-to-output ratio (solid blue line, left axis).

uncertainty $\sigma_Y(I_t)$. Economic uncertainty increases in bad times but not unreasonably so as the conditional volatility is only slightly above 6% when the economy is in deep distress. Panel C shows the variation in the price-dividend ratio due to variation in the surplus consumption ratio, with a visible drop from the mid 30s to about 15 early on in the sample and again dips to almost 10 between quarters 100 and 140, when the recession is more sustained. This is the standard behavior of asset prices in external habit economies in the presence of negative shocks in consumption growth. Finally Panel D shows the stock return volatility, which increases dramatically during bad times, to almost 60% during periods of deep distress. Panel E shows the behavior of the aggregate stock holdings of the RT households. Finally Panel F reports the intermediary's leverage, both when we normalize by output, $\mathcal{D}_t/Y_t$, as well as with equity, $\mathcal{D}_t/P_t$.

Panels E and F illustrates the impact of the variation of the surplus consumption ratio on the stock position of leveraged households and the leverage ratios. The variation in both quantities is rather limited most of the time, except during extreme bad events. It is thus in these occasions, as the surplus consumption ratio drops and economic uncertainty increases, that levered households decrease their indebtness and liquidate their positions in risky assets. Comparing Panels E and F with Panel C, we see that during such times prices drop substantially and leveraged households delever as well by selling stocks. A possible interpretation of the comovemet of these time series is that the "price pressure" generated by the stock selling shown in Panel E is causing the price decline in Panel C. This interpretation is incorrect. As shown in Corollary 4 the asset pricing implication of our model are identical to those that obtain in a representative household framework and the same sequence of aggregate shocks would have led to the same path for asset prices.

Our model does not produce large swings in household leverage, except in situations of deep distress but the speed of adjustment is much faster, as should be expected from a frictionless model. For example, as shown in Figure 2, aggregate household debt-to-income peaked at about 135% in early 2007 and dropped to about 100% in the years following the crisis. Our model delivers slightly higher magnitudes. In simulations, debt-to-income is close to 160% and drops to about 145% when the surplus consumption ratio suffers a strong drop (between quarter 100 and 140), but it does it so much faster than in the data.

In our framework the balance sheet of the financial intermediary responds passively to the households' portfolio decisions. As shown in Panel F $\mathcal{D}_t/Y_t$ drops when stock prices drop but this deleveraging is simply a reflection of the fact that as the economy deteriorates RT households become more risk averse and decrease the amount they borrow. Because the asset side of the intermediary's balance sheet contracts so does the liability side and thus
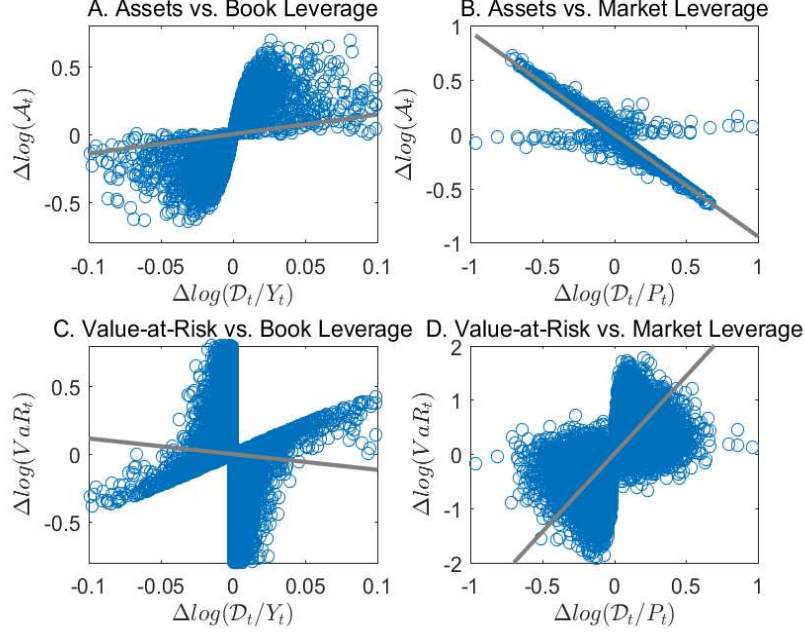
Figure 10: Simulations: Panel A and B plot log changes in total assets, defined as $\mathcal{A}_t \equiv \int P_{it} di + \int_{i:\omega_i > \gamma_i} L_{it} di$, against two measures of financial intermediary leverage, our proxy for book leverage $\mathcal{D}_t/Y_t$, and market leverage, $\mathcal{D}_t/P_t$. Panels C and D show the VaR_t, defined as $VaR_t = 2.325 \times \sigma_{\mathcal{A},t}$ against the corresponding measures of leverage.

the reduction in the amount of deposits that the intermediary can issue to RA households. Notice then that the dynamics of the balance sheet of the financial intermediary are delinked, for example, from any form VaR constraints such as in Adrian and Shin (2014), as illustrated in Figure 4 in Section 2. Our point is not that frictions do not matter but rather that there are more fundamental forces at work driving the low frequency dynamics of balance sheets and that frictions are likely to be an amplification factor rather than the primal cause of fluctuations.

Figure 10 reproduces Figure 4 in Section 2. Panels A and B shows the relation between asset growth (on the y-axis) and leverage growth (on x-axis), where leverage is "book leverage" (Panel A) and market leverage (Panel B) in simulations. The intermediary's assets, $\mathcal{A}_t$, are equal to the sum of the values of the individual trees plus the total value of the loans L_{it} granted, that is $\mathcal{A}_t \equiv \int P_{it} di + \int_{i:\omega_i > \gamma_i} L_{it} di$. There is negative relation in Panel A and positive relation in Panel B, exactly as in Panels A and B of Figure 4, respectively. Panels C and D plot the relation between the change in the intermediary value-at-risk and leverage growth in the two cases, respectively. We approximate the intermediary value-at-risk as $VaR_t = 2.325 \times \sigma_{\mathcal{A},t}$, that is, assuming assets are (approximately) normally distributed. The volatility of assets is given by $\sigma_{\mathcal{A},t} = \frac{P_t}{\mathcal{A}_t} \times \sigma_{P,t}$ as the volatility of loans L_{it} is zero.

As can be seen from Panel C, book leverage growth is negatively related on average with change in value at risk, consistently with the evidence put forth by Adrian and Shin (2014) (see Panel C of Figure 4). But the relation is not causal: Both leverage ratios and asset volatility are driven by I_t. As I_t increases, intermediaries delever and asset volatility increases. Panel D reports the same simulation results but with market leverage on the x-axis, in which case a clear positive relation appears. Unfortunately, we do not have an empirical counterpart to compare this plot to. Still, the message of Panels C and D is that passive deleveraging and market price variation can as well generate the type of empirical predictions that are usually argued as evidence of active balance sheet management by financial intermediaries.[30]

6.4. Intermediary Asset Pricing

We showed in subsection 5.2.2. that financial intermediary leverage should be expected to be a predictor of the cross section of stock returns and that the sign of the market price of risk depends on the specific definition of leverage used. To check that this result obtains in simulations we perform standard Fama-MacBeth cross-sectional regressions and use both measures of leverage as risk factors, with the different signs depending on definitions (see Table 1 in Section 2.)

For convenience, we consider as test assets the contingent claim securities W_{it} in expression (37) that pay the dividend C_{it} over time. Recall we use the standard Fama-French 25 portfolios sorted by size and book-to-market as test assets in the empirical data in Table 1. We normalize, both in simulations and in the empirical data, the leverage factors to have mean zero and variance one to facilitate the comparison between the coefficients obtained in the regressions run with simulated and empirical.

Panel A of Table 3 shows the results of Fama-MacBeth cross-sectional regressions with simulated data and should be compared with Panel A of Table 1. In our model the conditional CAPM holds: the first column of Panel shows a strong quarterly coefficient of 1.5 and the R^2 (not reported) is 100%, which is unsurprising as our model has only one shock and thus all returns are perfectly correlated. Similarly, we do not report t-statistics, as they are all very large given the large number of artificial data (except for the alpha's, which are close to zero). Column II shows that the estimated market price of risk of market leverage is

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[30]In this conclusion, our paper echoes Welch (2004), who argues that the determinants of corporate leverage are not active decisions by management related to market timing, taxes or other considerations put forth by the capital structure literature, but rather stock return dynamics, which account for about 40% of debt ratios fluctuations.

Table 3: The Market Price of Leverage Risk in Simulations. Panel A reports Fama-MacBeth regressions in a sample of simulated data from our model. The set of test portfolios are the contingent claims that pay the efficient allocation C_{it} for each household i (see (12)) and returns are calculated using prices P_{it} (see expression (37)). Panels B and C report time series regressions of market returns on book and market leverage, respectively lagged one to five years. $t-$statistics are in parenthesis.

Panel A. Cross-Sectional Regressions			
	I	II	III
α	0.01	-0.02	0.04
Market Return	1.51	1.54	1.48
Market Leverage		-0.05	
Book Leverage			0.02

Panel B. Predictability with Book Leverage					
	1 year	2 year	3 year	4 year	5 year
β	-0.79	-1.49	-2.04	-2.44	-2.76
$t(\beta)$	-4.11	-3.34	-3.09	-3.04	-3.06
R^2	2.66%	5.45%	7.80%	9.51%	10.99%

Panel C. Predictability with Market Leverage					
	1 year	2 year	3 year	4 year	5 year
β	4.24	7.27	9.38	10.83	11.88
$t(\beta)$	25.80	26.72	27.02	27.67	28.57
R^2	14.39%	24.58%	31.18%	35.36%	38.31%

negative, while column III shows that the estimated market price of risk of book leverage is positive, consistently with Panel A and with the results in Corollary 12. The magnitudes though are smaller which is unsurprising as the conditional CAPM holds in our framework. Moreover, the model doesn't offer a counterpart to value- and size-sorted portfolios and hence the test asset average returns don't display as large a spread as in the data.[31] Still, the simulation results highlight that endogenous leverage ratios – which only proxy for shocks to risk aversion – show up in cross-sectional regressions as risk factors and with different signs depending on their definitions as found in empirical work.

An additional prediction of our model is at the time-series level (see Corollary 13), which is consistent with Panels B and C of Table 1 in Section 2. Panels B and C of Table 3 provides evidence from artificial data. In this case, book leverage (Panel B) is always significant, but

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[31]The spread in annual average returns across portfolios is 12% in the data while only 4.7% in the model.

we note both a lower R^2 and $t-$statistics compared to market leverage (Panel C). That is, our model is consistent with the empirical finding that market leverage should be a better predictor of future stock returns. Indeed, while book leverage and market leverage are clearly related to each other, they are not perfectly correlated. In our simulated data, market leverage and book leverage have a correlation of -83% in levels, and -75% in first differences. In the data, they have a correlation of -39% in levels and -31% in first difference. The imperfect correlation in simulation is due to the non-linearities implicit in the model.

In sum, measures of financial intermediary leverage show up as risk factors in tests of the cross section of stock returns. This evidence has been interpreted as evidence that financial intermediaries act as marginal investors in many markets. Our contribution is to show that this is not necessarily the case. In our model fluctuations in the intermediary's balance sheet are driven in turn by fluctuations in the households' attitudes towards risk. Thus it might be the case that the predictive success of measures of financial intermediaries' leverage is simply due to the fact that it proxies for these changes in the attitudes towards risk and is unrelated to leverage ratio constraints.

7.   Conclusions

We propose a general equilibrium exchange economy populated with heterogenous households. Households differ in their attitudes towards risk and also in their initial endowment. In addition attitudes towards risk fluctuate with aggregate economic conditions, but by more for some households than for others. In particular, during bad times some agents become more risk averse than others which induces motives for risk sharing and trading. We posit the existence of a financial intermediary that can issue deposits and grant loans and show how households can achieve their optimal allocation through a dynamic trading strategy that combines aggregate stock market positions and either borrowing from or lending to the intermediary. The model aggregates to a representative household that features also time changing attitudes towards risk. Our framework is thus able to generate the strong discount effects that have been shown to be key in addressing well known asset pricing regularities in the data. Because it generates endogenously a reasonable amount of risk it serves as a useful framework to evaluate household portfolio decisions.

Our model is consistent with many stylized facts in the data, such as the procyclicality of households' debt-to-income ratios and countercyclicality of debt-to-net worth ratios. In addition, we are able to match stylized patterns of the cross sectional distribution of leverage as a function of net worth. In particular poorer households lever more than wealthier

households and debt-to-net worth increases in bad times for all households, independently of their net worth. The intermediaries' balance sheets reflect the economy's aggregate risk aversion and they expand and contract as households' demand for loans and deposits change over the business cycle. Because the intermediaries' balance sheet reflects the state of the economy and are easier to measure than households risk preferences, intermediaries' leverage ratios can serve as proxies for the potentially poorly measured marginal rates of substitution of the representative household. We are able to qualitatively replicate standard tests in the financial intermediation and asset pricing literature which have been put forth as evidence of the existence of the asset pricing role of frictions and capital constraints. We argue that these tests offer no such proof as our results obtain in a frictionless complete markets framework.

Our model is simple, however, in that it only has one state variable, all quantities move in lock-step and thus there is an unrealistic perfect (positive or negative) correlation between leverage, prices, volatility, expected return, consumption, and so on. It is this assumption which allows for closed form solutions in quantities and prices and thus obtain a better understanding of the various economic forces at work. Future research should focus on generalizing our simple setting to obtain more realistic dynamics. Two avenues of future research seem particularly fruitful given the simplicity of our setting: First, it would be useful to explicitly model housing as a second risky asset that also provides housing services to households. In our calibration, poor agents borrow more to buy the risky asset, which would match well the data if the class of risky assets were to also comprise housing. We conjecture that our main results would remain unscathed by this generalization but it would be an interesting extension nonetheless. A second extension is to consider idiosyncratic preference shocks, as in Alvarez and Atkenson (2017), and solve for the incomplete market version of the model. Indeed, Alvarez and Atkenson (2017) show in a three-period model but with more general recursive utilities that preference shocks impact asset prices and trading, and that it is possible to solve for the equilibrium even when market are incomplete. The extension to a dynamic economy such our ours may bring about additional dynamics and possibly allow the model to better match the distribution of consumption growth and its total volatility. Such model should also perform better in matching basic properties of the wealth distribution, which our current one-factor model is unable to fully explain.

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Table A.1: Parameters and Moments. Panel A reports the parameters for our calibration of the time series properties of the model. σ^{max} which is chosen to match the average volatility of consumption, which is the new parameter relative to MSV. Panel B reports a set of moments for the aggregate stock market and interest rates, as well as consumption growth, and compares with the same moments in artificial data obtained from a 10,000-year Monte Carlo simulation of the model. Panel C similarly reports the R^2 of predictability regressions in the model and in the data, using the price-dividend ratio as predictor.

Panel A. Parameter Estimates

ρ	k	$\overline{I}$	$\overline{v}$	μ	σ^{max}
0.0416	0.1567	1.5	1.1353	0.0218	0.0819

Panel B. Moments (1952 − 2014)

	$E[R]$	$Std(R)$	$E[r_f]$	$Std(r_f)$	$E[P/D]$	$Std[P/D]$	SR	$E[\sigma_t]$	$Std(\sigma_t)$
Data	7.13%	16.55%	1.00%	1.00%	38	15	43%	1.41%	0.52%
Model	6.06%	21.83%	2.09%	2.87 %	28.45	4.92	27.78%	1.46%	1.41%

Panel C. P/D Predictability R^2

	1 year	2 year	3 year	4 year	5 year
Data	5.12%	8.25%	9.22%	9.59%	12.45%
Model	14.29%	24.03%	30.84%	35.30%	38.61%

A1.   Appendix

A1.1.   Quantitative implications of the model: Asset pricing

Figure A.1 reports the conditional moments implied by the model as a function of the surplus-consumption ratio S_t. As in MSV Figure 1, Panel A reports the stationary distribution of the surplus-consumption ratio S_t and shows that most of the probability mass is around $\overline{S} = 0.667$, although S_t drops considerably below occasionally. The price-dividend ratio is increasing in S_t (panel B), while volatility, risk premium and interest rates decline with S_t (panel C) for the area with positive mass. Note that our choice of parameters is such to give near zero mass to the area in which $\sigma_P(I_t)$ and expected return $E_t[dR_t - r_t dt]$ are increasing in $S_t = I_t^{-1}$.[32] Finally, the Sharpe ratio is also strongly time varying, and it is higher in bad times (low S_t) and lower in good times (high S_t). This figure is very similar to Figure 1 in MSV.

Given the parameters in Panel A of Table A.1, we simulate 10,000 years of quarterly

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[32]Figures 4 and 5 of Campbell and Cochrane (1999) also display expected excess return and return volatility that are monotonically decreasing in the surplus consumption ratio S_t.
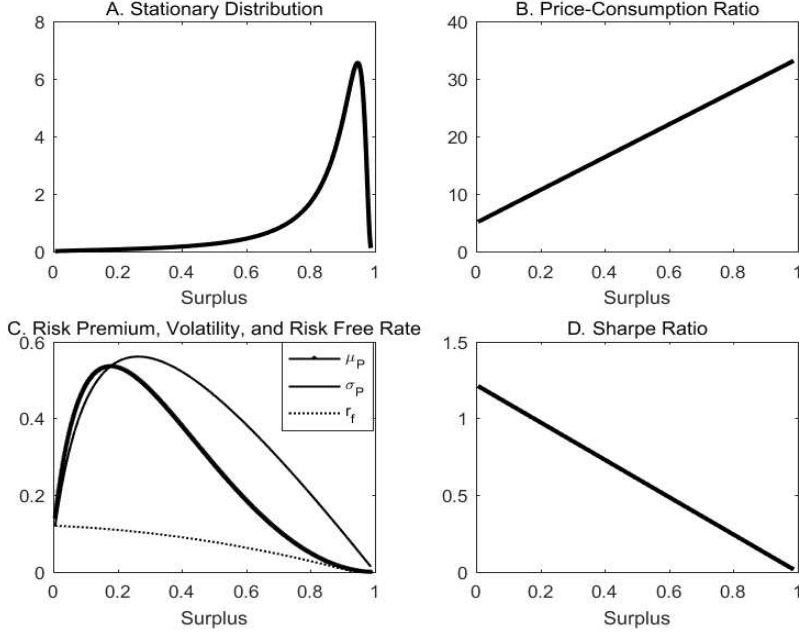
Figure A.1: Conditional Moments. Panel A shows the stationary probability density function of the surplus consumption ratio S_t. Panel B shows the P/D ratio as a function of S_t. Panel C plot the expected excess return $E_t[dR_P - r_t dt]$, the return volatility $\sigma_P(S_t)$ and the interest rate $r(S_t)$ as functions of S_t. Finally, Panel D shows the Sharpe ratio $E_t[dR_P - r_t dt]/\sigma_P(S_t)$ against S_t.

data and report the aggregate moments in Panel B. As in MSV, Table 1, the model fits well the asset pricing data, though both the volatilities of stock returns and of the risk free rate are higher than their empirical counterparts.[33] Still, the model yields a respectable Sharpe ratio of 32.64%. Finally, the simulated model generates an average consumption volatility of 1.43% with a standard deviation of 1.18%. This latter variation is a bit higher than the variation of consumption volatility in the data (0.52%), where the latter is computed fitting a GARCH(1,1) model to quarterly consumption data, and then taking the standard deviation of the annualized GARCH volatility. Our calibrated number is however lower than the standard deviation of dividend growth' volatility, which is instead around 1.50%.

The calibrated model also generates a strong predictability of stock returns (Panel C), with R^2 ranging between 14.18% at one year to 35.92% at 5 year. This predictability is stronger than the one generated in MSV and also the one in the data. This is due to the combined effect of time varying economic uncertainty (*i.e.* the quantity of risk) and time varying risk aversion (*i.e.* the market price of risk), which move in the same direction.

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[33]The volatility of the risk free rate can be substantially reduced by making the natural assumption that expected dividend growth μ_Y decreases in bad times, i.e. when the recession indicator I_t is high. Indeed, in the extreme, by assuming $\mu_Y(I_t) = \overline{\mu}_Y + (1 - v)\sigma_Y(I_t)^2 - k(1 - \overline{I}I_t^{-1})$, which is decreasing in I_t, we would obtain a constant interest rates $r = \rho + \overline{\mu}_Y$. No other result in the paper depend on $\mu_Y(I_t)$ and thus all the other results would remain unaltered by the change.