NBER WORKING PAPER SERIES

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Working Paper 22811
http://www.nber.org/papers/w22811

NATIONAL BUREAU OF ECONOMIC RESEARCH
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November 2016

The authors served as consultants to AT&T in its merger with DIRECTV. Early versions of the results presented here were discussed in filings with the U.S. Department of Justice and Federal Communications Commission on behalf of AT&T. We thank Joshua Gans for helpful comments and Wayne Gao for research assistance. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 22811
November 2016
JEL No. L13,L4

ABSTRACT

The distinction between complements, substitutes, and independent goods is important in many contexts. It is well known that when consumers' conditional indirect utilities for two goods are superadditive, the goods are gross complements. Generalizing insights in Gans and King (2006) and Gentzkow (2007), we show that superadditivity between one pair of goods can also introduce complementarity between competing pairs of goods. One implication is that lower prices can result from a merger between producers of goods that themselves offer no superadditivity.

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1 Introduction

Whether two good are complements is often of interest. For example, if two goods are complements, a merger of their producers tends to reduce prices because the merged firm will internalize the benefits that lowering the price of one good has on demand for the other.[1]

A sufficient condition for two goods to be complements is that they have superadditive conditional indirect utilities. Sources of such superadditivity include direct consumption synergies, bundle discounts, convex loyalty rewards, price club memberships, convex shipping discounts (e.g., Amazon Prime), and benefits of one-stop shopping. We show that, whatever the source, superadditivity between one pair of goods creates complementarity, not only between that pair but also between pairs of competing goods. An important implication is that a merger between producers of goods that exhibit no superadditivity can lead to lower prices.

To suggest why, consider two categories of goods, apples and beer, with no intrinsic complementarities in consumption: once purchased, the value a consumer places on consuming a good from each category is the sum of the values she places on consuming each good alone. Apples are offered by a greengrocer; beer by a liquor store. A supermarket offers both apples and beer. A consumer experiences travel cost t from each visit to one of these sellers. Consider a consumer who buys apples from the greengrocer and beer from the liquor store but, at current prices, finds this only slightly preferable to her second-best option of buying both from the supermarket. If the liquor store raises its beer price, this consumer will switch to purchasing *both* goods from the supermarket, because doing so saves t in travel cost. This is the only response to the price change affecting the greengrocer's apple sales. Thus, an increase in the price of beer at the liquor store reduces demand for the greengrocer's apples. Symmetrically, an increase in the greengrocer's apple price reduces demand for the liquor store's beer. Therefore, these two goods are gross complements.

Our result generalizes an observation in Gans and King (2006), who studied pricing in a stylized "linear city" model with no outside good, assuming that all consumers purchase one good from each of two product categories. They note that a key force in their analysis is the fact that bundle discounts offered for one pair of goods create negative cross-price elasticities between the goods of competing firms. We focus on this phenomenon itself and show that it extends to a much more general random utility model, with no restriction on the form of competition or the source of superadditivity in consumers' preferences.

2 Baseline: Two Goods

We begin with the two-good random utility model of Gentzkow (2007), dropping his functional form and distributional assumptions. Consider a market with two goods, A and B, and a continuum of consumers with mass normalized to one. Each consumer desires at most one unit of each good. A given consumer i obtains conditional indirect utility ("utility")

[1] See Cournot (1838) and Economides and Salop (1992)).

a^i when purchasing A alone, and b^i from purchasing B alone. Each utility accounts for the good's price; thus, for example, an increase in the price of A corresponds to a reduction in a^i for every consumer. Throughout we normalize the utility of the outside good to zero. Consumer i's utility from purchasing both goods is

$$a^i + b^i + \Delta$$

where $\Delta \geq 0$ and, for simplicity, Δ is constant across consumers. When $\Delta > 0$, consumers have superadditive utilities.

To ease notation, we henceforth drop the superscript i from utilities. Let F denote the joint distribution of (a, b) across consumers. For expositional convenience, we assume F has support $\mathbb{R}^2$ and is absolutely continuous with respect to the Lebesgue measure.

Consumers in different regions of $\mathbb{R}^2$ will make different choices. As in Gentzkow (2007), an examination of how price changes alter the measure of consumers in each choice region demonstrates that superadditivity leads to complementarity.

Proposition 1. *Goods A and B are strict gross complements if $\Delta > 0$, and independent goods if $\Delta = 0$.*

Proof. A is purchased (alone or with B) whenever $a > 0$. When $a \leq 0$, A is purchased if and only if both $-\Delta \leq a \leq 0$ and $a + b + \Delta \geq 0$.[2] Total demand for A is therefore

$$1 - F_A(0) + \int_{-\Delta}^{0} \Pr\left(a + b + \Delta \geq 0 | a\right) dF_A(a), \tag{1}$$

where F_A denotes the marginal distribution of a. An in increase in the price of good B implies a reduction of b in terms of first-order stochastic dominance; if $\Delta > 0$, this means that demand for good A falls. If $\Delta = 0$, however, (1) does not depend on b. □

Figure 1 illustrates the product choice regions with $\Delta > 0$.[3] The thick diagonal line segment shows the set of consumer utilities producing indifference between the AB "bundle" and the outside good. For consumers whose utilities are bounded away from this line segment, a small change in b has no effect on the decision to purchase A. However, when the price of good B increases, a mass of consumer utilities just above this locus of indifference will move downward across the line segment, leading the associated consumers to switch from AB to the outside good.

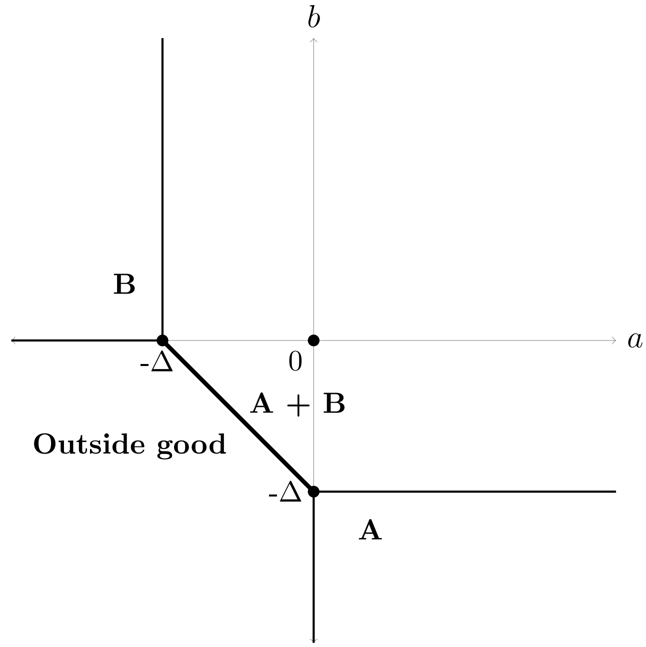
3 Competing Varieties

The equivalence between complementarity and superadditivity of the conditional indirect utilities breaks down when consumers can choose from multiple varieties within each product

[2]We ignore ties since these occur with probability zero and therefore have no effect on demand.

[3]This figure is also found as the "$\Delta > 0$" panel of Figure 1 in Gentzkow (2007).

Figure 1: Two-Good Case



category. Let goods $(A_1, \ldots, A_J)$ denote different varieties in category A, with $(B_1, \ldots, B_K)$ denoting varieties in category B. Each consumer again values at most one unit from each category.[4] For example, a consumer might wish to have both home television service (category A) and home internet service (category B), but does not benefit sufficiently from a second variety of either category of service to offset its price.

Each consumer has (conditional indirect) utilities for the standalone goods given by a_j and b_k. As before, utilities are net of prices and vary randomly across consumers. Let H denote the joint distribution of $(a_1, \ldots, a_J, b_1, \ldots, b_K)$. We assume H has support $\mathbb{R}^J \times \mathbb{R}^K$ and is absolutely continuous. As a notational convention, we introduce $a_0 \equiv 0$ and $b_0 \equiv 0$.

The utility from purchasing A_1 and B_1 is superadditive, equal to

$$a_1 + b_1 + \Delta \tag{2}$$

where Δ is nonnegative and, for simplicity, constant across consumers.[5] For the remaining combinations of goods, utilities are additive.[6] Thus, by Proposition 1, for $j \neq 1$ and $k \neq 1$,

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[4]Formally this is a restriction on the utilities of consumer choices involving more than one unit of A or B. This places no restriction on the joint distribution H defined below.

[5]Our result extends to the case of more than one pair of goods with superadditive utilities by letting $a_1 + b_1 + \Delta$ represent a consumer's maximum utility among all superadditive pairs and, in the proof, considering j and k from the goods not part of any superadditive pair.

[6]The asymmetry in superadditivity modeled here arises naturally when Δ reflects, e.g., one-stop shopping,

A_j and B_k would be independent goods in a market without competing varieties. However, in the multi-variety setting, superadditive utilities for A_1 and B_1 induce complementarity between these A_j and B_k as well.[7]

Proposition 2. *For all $j \neq 1$ and $k \neq 1$, A_j and B_k are strict gross complements iff $\Delta > 0$.*

Proof. Take $j \neq 1$ and $k \neq 1$ and consider the effect of a change in the price of B_k on demand for A_j (a symmetric argument applies to the effect of A_j's price on demand for B_k). A_j is purchased by a consumer if and only if both

$$a_j \geq \max_{j'} a_{j'} \tag{3}$$

and

$$a_j + \max_{k'} b_{k'} \geq a_1 + b_1 + \Delta. \tag{4}$$

If $\Delta = 0$, (3) implies (4), so that the values of b_k have no effect on demand for A_j. Now suppose $\Delta > 0$. If the price of B_k rises by $\delta \in (0, \Delta)$, all consumers for whom $a_j = \max_{j'} a_{j'}$, $b_k = \max_{k'} b_{k'}$, and

$$a_j \in (a_1 + b_1 + \Delta - b_k, a_1 + b_1 + \Delta - b_k + \delta)$$

will switch from purchasing A_j (along with B_k) to purchasing A_1 (along with B_1). Our support assumption guarantees that there is a positive measure of such consumers. No other consumers will alter their choices with respect to A_j. $\square$

Figure 2 illustrates in the case $J = K = 2$. The figure shows choice regions in the space of (a_2, b_2), with (a_1, b_1) held fixed at particular values, $(\bar{a}_1, \bar{b}_1)$. In general different values of (a_1, b_1) create different graphs and product choices.[8] For Figure 2 we have selected $(\bar{a}_1, \bar{b}_1)$ such that the pair (A_1, B_1) is preferred to the outside good and either A_1 or B_1 alone. Note that A_2 is never purchased when $a_2 < 0$; but when $a_2 > \bar{a}_1 + \Delta$, A_2 is purchased regardless of b_2. Consumers in the lower left region choose (A_1, B_1) ; those in the upper right region choose (A_2, B_2). The thick diagonal line segment represents the consumers who are indifferent between the two pairs. Substitution between these pairs creates the strict complementary between A_2 and B_2.

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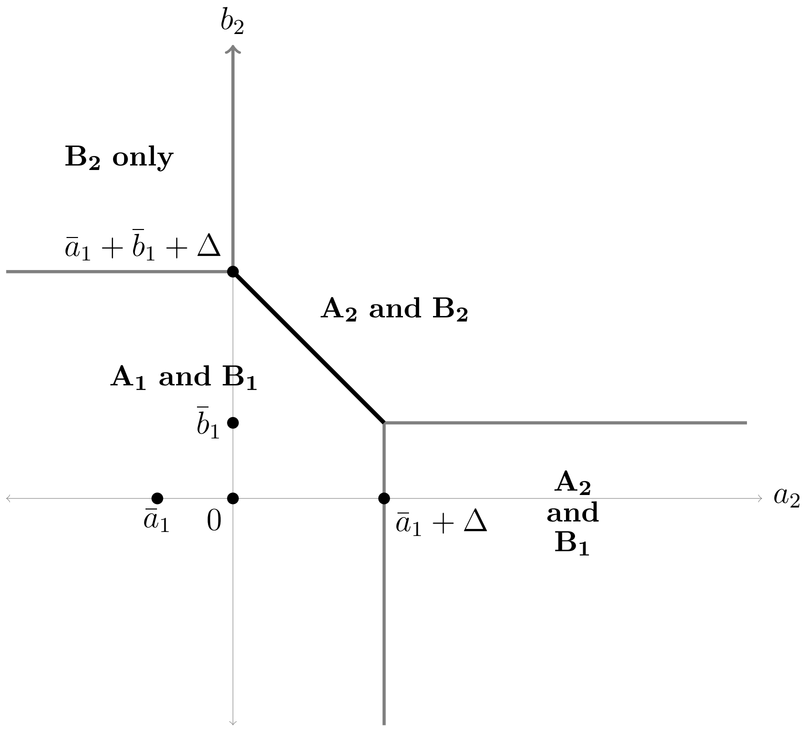
bundle discounts, convex shipping discounts, or loyalty rewards. Less extreme asymmetry may sometimes arise in the case of intrinsic consumption synergies between different varieties of A and B, e.g., due to varying aesthetic or technological compatibility.

[7] To reconcile these observations, consider the case $J = K = 2$ and note that one cannot construct the total demand for good A_2 from the demand for A_2, B_2, A_2B_2 and outside option "all other goods." For example, the option A_2B_1 would be then be part of the outside option.

[8] Although Figure 2 has a form similar to Figure 2 in Gans and King (2006), there are important differences. Because Gans and King (2006) assume that every consumer purchases either A_1 or A_2 and either B_1 or B_2, a consumer chooses among only four options rather than 9 (more generally, $(J + S) \times (K + S)$, where S denotes the number of pairs exhibiting superadditivity). And in their two-dimensional linear city model, a single two-dimensional diagram fully characterizes the map from consumer types to the four choice probabilities.

Figure 2: Four-Good Case.



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