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Ricardo J. Caballero
Alp Simsek

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ABSTRACT

We develop a model of gross capital flows that addresses the tension between their fickleness during foreign crises and retrenchment during local crises. In a symmetric environment with domestic crises, capital flows mitigate fire sales since fickle inflows exit the crisis location at weak prices whereas past outflows provide liquidity at higher valuations. However, due to the public good aspect of flows' liquidity services, local policymakers with financial stability concerns may restrict flows. Greater scarcity of safe assets and lower correlation between crises increase gross flows. With asymmetric locations, the model features reach-for-safety and reach-for-yield flows that can be destabilizing.

Ricardo J. Caballero
Department of Economics, E52-528
MIT
77 Massachusetts Avenue
Cambridge, MA 02139
and NBER
caball@mit.edu

Alp Simsek
Department of Economics, E52-552
MIT
77 Massachusetts Avenue
Cambridge, Ma. 02139
and NBER
asimsek@mit.edu

A Dynamic link to the most recent version of the paper is available at
https://www.dropbox.com/s/zf48a2dd5cfga64/fickleFlowsMostRecent_Public.pdf?dl=0

1. Introduction

Capital inflows are large, often exceeding 20 percent of GDP per year for advanced economies and half of that for emerging market economies (see, e.g., Lane and Milesi-Ferretti (2007); Lane (2013)). But they are also *fickle*; that is, foreign investors have a tendency to exit when the country is in distress. The combination of their large size and fickleness has made capital flows a perennial source of headaches for policymakers around the world. This concern has spawned an academic and policy literature that attempts to identify the fire-sale externalities of fickleness and the need to regulate them (see, e.g., IMF (2012) for a recent survey).

Less noticed than fickleness, but as prevalent, is *retrenchment.* That is, local investors reduce their foreign investments during local crises and use their global liquidity at home (see, e.g., Forbes and Warnock (2012) and Broner et al. (2013a,b)). In the presence of retrenchment, it is no longer a foregone conclusion that gross flows reduce financial stability even in the presence of financially destabilizing fickleness. Our main goal in this paper is to address this tension and its implications. To this end, we develop a stylized model of capital flows that takes as given an extreme form of fickleness and asks whether capital flows can still be a useful source of liquidity in a global economy exposed to domestic fire sales.

Our model features several locations, each of which is associated with a risky asset. The asset always pays a fixed amount but the timing of the payoff is uncertain. Specifically, with some probability, each location experiences a "liquidity shock" in which case its asset payoff is delayed to a future period. During a liquidity shock, the asset is traded in a financial market at an endogenous price. There is a group of agents ("distressed sellers") that sell their endowed assets to raise capital, and another group of agents ("banks") that use their liquid resources to purchase risky assets. We make parametric assumptions so that the asset's price during a liquidity shock is below its fundamental value (the price that would obtain with abundant liquidity) and is determined by banks' available liquidity. We refer to this situation as a *fire sale*, and analyze how capital flows interact with fire sales.[1]

We model capital flows by considering an ex-ante period in which banks make a decision whether to invest in the risky asset of their own location or the foreign location (or to consume). The key assumption is that banks are home biased in the sense that they are extremely fickle in foreign locations: specifically, if the foreign location experiences a liquidity shock, then they sell their risky asset holdings in that location regardless of the price. This assumption captures a variety of factors that could handicap foreigners during local distress: asymmetric information or Knightian uncertainty, deterioration of property rights, asymmetric regulation, and so on. We remain agnostic about the source of fickleness and view it as a simple modeling device to

[1] In practice, crises and fire sales are often associated with banks' distress. We find it convenient to model "distressed sellers" and "banks" as separate agents but our analysis does not rely on this abstraction. Our main results also hold in an alternative version of the model in which liquidity shocks are events in which banks experience losses and are forced to sell their risky assets to "secondary buyers," which convert these assets to an alternative use with lower payoff (see Remark 2 and Appendix A.5).

capture the asymmetric behavior of locals and foreigners during crises. Specifically, while foreign banks in our model sell local assets, local banks simultaneously retrench and use their global liquidity to purchase local assets at fire-sale prices. Hence, our model naturally generates the fickleness and retrenchment patterns that we observe in the data.

Within this environment, our first finding is that foreign investment happens despite the fickleness element (which occasionally forces banks to sell at fire-sale prices) as long as there are domestic fire sales. The reason is that foreign assets tend to retain their value and provide valuable liquidity during a domestic liquidity shock.

We then analyze how these fickle capital flows affect global financial stability. Our main result is that, in a symmetric environment, capital flows increase fire-sale asset prices during local liquidity shocks despite their fickleness. The intuition is that fickle foreign banks sell local assets at fire-sale prices, but local banks obtain liquidity from their diversified foreign assets at relatively high valuations. In a symmetric environment, every pre-crisis inflow is matched by a pre-crisis outflow of equal size. During a crisis, the value of previous outflows exceeds the value of liquidated inflows, and thus symmetric capital flows provide liquidity and increase fire-sale prices. That is, once we focus on certain types of flows that could be subject to fire sales (such as equity, long-term debt, or unsecured short-term debt), our model suggests that they *create global liquidity despite their fickleness.*

Our model further reveals that regulating capital flows is subject to a coordination problem. Specifically, even though in global equilibrium capital flows increase fire-sale prices, they might be restricted by local policymakers whose objective is to stabilize domestic financial markets. The reason is that there is a public good aspect to the global liquidity generated via fickle capital flows. Every capital inflow into a location is an outflow from the perspective of some other locations. Local regulators take into account the fickleness cost of inflows, but not the retrenchment benefit of inflows for those other locations, which leads to excessive restrictions on capital flows relative to a coordinated outcome.

We also investigate the determinants of gross capital flows in our setting. We find that greater scarcity of safe assets naturally increases gross capital flows—a situation reminiscent of the period before the global financial crisis. The reason is that flows (imperfectly) substitute for safe assets by creating liquidity during local liquidity shocks. We also find that an increase in the perceived correlation among liquidity shocks reduces gross capital flows—a situation reminiscent of the period after the financial crisis. When banks believe liquidity shocks might be global in scope, they perceive that gross flows create less liquidity. Moreover, the resulting reduction in gross flows reduces liquidity and fire-sale prices even if the global shock is ultimately not realized.

We envision the symmetric case of our model as roughly capturing the gross flows among developed markets. Our results are qualified when there are substantial asymmetries in liquidity or investment returns across different regions of the world. We identify two potentially destabilizing mechanisms—reach for safety and reach for yield—that apply when developed markets with substantial liquidity but relatively low returns trade capital flows with emerging markets with smaller liquidity but relatively high returns.

The reach-for-safety mechanism is driven by cross-location differences in liquidity (captured by the availability of local safe assets in our model). The greater liquidity in a developed market location makes its assets relatively attractive for the banks in other locations. Other things equal, this induces the developed market location to experience greater inflows relative to its outflows (or run current account deficits). Moreover, when there are global liquidity shocks, the inflows into the developed market location are relatively safe whereas the outflows are relatively risky. That is, the banks in the developed market sell liquidity insurance (at a premium) to the emerging markets. These types of reach-for-safety flows exacerbate the financial crises in the developed market while mitigating the crises in emerging markets. (See Gourinchas and Rey (2007); Gourinchas et al. (2010, 2012) for evidence on the venture capitalist and insurer role played by the U.S. in the global system).

The reach-for-yield mechanism is driven by cross-location differences in investment returns. If the return in an emerging market location is greater than in other markets, then foreign banks invest in this location not only to mitigate their domestic fire sales but also to chase after high returns. This process stops only when fickle inflows are sufficiently large that the emerging market location experiences deeper fire sales compared to other locations (thereby reducing its appeal for foreign banks). Thus, we find that fickle flows that are driven by the pursuit of higher returns are destabilizing for the emerging markets at the receiving end.[2]

Related literature. At the core of our mechanism is international diversification. There is an extensive literature that attempts to understand capital flows using frictionless models of international risk sharing (see e.g., Grubel (1968); Cole and Obstfeld (1991); Van Wincoop (1994); Lewis (2000); Coeurdacier and Rey (2013)). The main reason for diversification in our model is different from the ones highlighted in this literature, as in our model international liquidity is used to fund the comparative advantage of domestic banks during fire sales.

Our paper is part of a literature that focuses on gross positions held by sophisticated financial intermediaries, and emphasizes the role of these flows in allocating liquidity where it is most needed (see, for instance, Brunnermeier et al. (2012); Bruno and Shin (2013); Miranda-Agrippino and Rey (2015); Gabaix and Maggiori (2015); Fostel et al. (2015)). A related literature emphasizes that the fickleness of inflows exacerbates domestic fire-sale externalities, so that while potentially useful for capital allocation purposes, flows can *increase* crisis risks and should be subject to macroprudential regulation (see, for instance, Caballero and Krishnamurthy (2004); Jeanne and Korinek (2010); Ostry et al. (2010); Caballero and Lorenzoni (2014); Calvo (2016); Korinek and Sandri (2016)). We identify similar issues but explore the global equilibrium implications of fickleness and the policy coordination issues that arise in this global context.

[2]The concern with destabilization was a central theme of the post World War II meetings at Bretton Woods (e.g., Forbes (2016)), and it has reemerged in earnest in the post subprime crisis era, mostly in response to the spillovers of developed markets' expansionary monetary policies onto emerging market economies (see, e.g., IMF (2012)) but also onto other developed market economies (see, e.g., Klein (2012)).

We take fickleness as given. In this methodological sense we also relate to Scott and Uhlig (1999), who take as given the fickleness of financial investors and study the impact of this feature on economic growth. The all-or-none attitude of fickle foreign banks is extreme in our model, but it is intended to capture a variety of reasons for why foreign investors are likely to exit during turmoil (see Remark 3), including the attitude of Knightian agents facing an unfamiliar situation. As such, it relates to Dow and da Costa Werlang (1992); Caballero and Krishnamurthy (2008); Caballero and Simsek (2013); Haldane (2013). We develop this Knightian uncertainty interpretation in Appendix A.1.

The core reason for capital flows in our environment is the scarcity of locally safe assets that provide liquidity during domestic fire sales. In this sense, our work is related to the literature on limited availability of safe assets and its macroeconomic consequences (e.g., Caballero (2006); Caballero et al. (2008, 2016); Bernanke et al. (2011); Gorton et al. (2012); Krishnamurthy and Vissing-Jorgensen (2012); Gorton (2016)).

Our two central ingredients are endogenous liquidity creation and fire sales. As such, our paper relates to Allen and Gale (1994) who endogenize market size and volatility in a closed economy with entry costs. In our model liquidity is created in a manner akin to Holmström and Tirole (1998), although our context and mechanism are different. Our model also shares elements of the limits-to-arbitrage and fire sales literature. In particular, the liquidity pricing of local assets is similar to, e.g., Allen and Gale (1994); Shleifer and Vishny (1997); Gabaix et al. (2007); Lorenzoni (2008); Krishnamurthy (2010); Gromb and Vayanos (2016); Holmström and Tirole (2001). In addition to these mechanisms, we highlight the benefit of gross flows as a stabilization channel.

The rest of the paper is organized as follows. Section 2 reviews the empirical literature documenting the prevalence of simultaneous fickleness and retrenchment of capital flows. Section 3 presents the model and defines the equilibrium. Section 4 characterizes the equilibrium and illustrates how symmetric capital flows help to create liquidity and mitigate crises. It also characterizes the asset prices and shows that foreign investment is associated with a risk premium. Section 5 concerns the optimal regulation of capital flows in our environment. It shows that the equilibrium allocation (typically) features too little gross flows due to pecuniary externalities, and that local policymakers that attempt to improve liquidity in their own location might end up further reducing gross flows. Section 6 develops a special case of the model ("the beta model"), and uses it to analyze the determinants of gross capital flows and to characterize a global liquidity cycle in capital flows and asset prices. Section 7 considers a variant of the model in which an (infinitesimal) country has different return and liquidity parameters than the remaining countries, and uses it to analyze asymmetric flows driven by reach for safety and yield. Section 8 concludes and is followed by an (online) appendix that contains various extensions of the model as well as the proofs of the propositions.

2. Fickleness and Retrenchment: Some Facts

Our model is built on the observation that capital inflows are fickle during crises and that capital outflows retrench during those episodes. In this section we review evidence that supports the widespread nature of these patterns.

To start with a policy perspective, Obstfeld (2012) documents the fickleness and retrenchment that occurred in the U.S. at the peak of the subprime financial crisis, and argues that retrenchment helped to mitigate the crisis. Specifically, he writes:

> Figure ... illustrates the example of the United States over the two quarters of intensive global deleveraging following the Lehman Brothers collapse in September 2008.... Gross capital inflows, which in previous years had been sufficient to more than cover even a 2006 net current account deficit of 6 percent of GDP, went into reverse, as foreigners liquidated $198.5 billion in U.S. assets. In addition, the U.S. financed a current account shortfall of $231.1 billion (down sharply from the current account deficit of $371.4 billion over the previous two quarters). Where did the total of nearly $430 billion in external finance come from? It came from U.S. sales of $428.4 billion of assets held abroad....

More systematic analysis of capital flows typically relies on International Monetary Fund's (IMF) Balance of Payments Statistics. Using this data, Broner et al. (2013a,b) document that capital inflows and outflows are both procyclical, meaning that fickleness and retrenchment are empirical regularities that apply beyond the U.S. They write: "during contractions foreigners reduce their investments in domestic assets and domestic agents reduce their investments abroad. This retrenchment toward home financial markets is particularly acute during crises." Figure 1 from their work, which we reproduce here, documents these patterns for a wide range of countries.[3] As they also note, these patterns are difficult to reconcile with standard macroeconomic models without frictions, because the shocks (e.g., to domestic productivity) in those models typically affect domestic agents' and foreigners' domestic investments in the same direction. Rather, the evidence is more easily reconciled with models in which crises affect domestic agents and foreigners asymmetrically, which motivates our fickleness assumption.

More recently, Avdjiev et al. (2017) analyze international capital flows by the sectors that send or receive them (banks, corporates, or sovereigns) and find that the fickleness and retrenchment patterns in the aggregate data are largely accounted for by global banks. These banks seem to be especially important to understand the retrenchment of outflows in developed markets, whereas sovereigns (that increase their borrowing or draw down their reserves during

[3] Similarly, Forbes and Warnock (2012) document that retrenchment is a widespread phenomenon that applies for the outflows of countries as diverse as the U.S. and Chile. And Bluedorn et al. (2013) document that capital flows are fickle for all countries, developed and emerging, although the former experience less volatility of total net inflows despite greater volatility of each component.

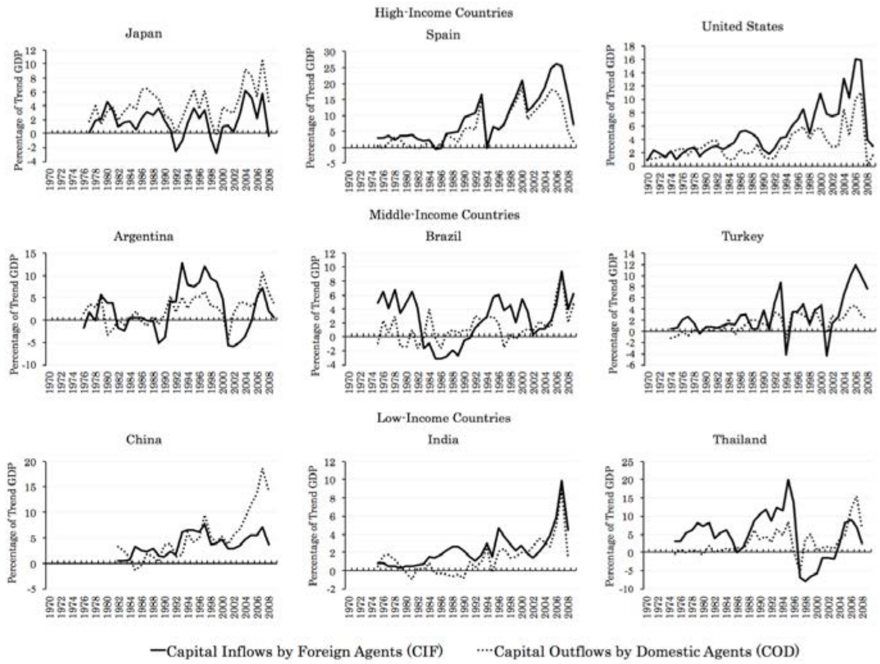

Figure 1: This figure from Broner et al. (2013a,b) shows the capital inflows and outflows for a sample of countries based on IMF's Balance of Payments statistics. CIF is equal to the net purchases of domestic assets by non-residents, and COD is equal to the net purchases of foreign assets by domestic agents (including international reserves). Reprinted with permission.

crises) seem to account for some retrenchment in emerging markets. This motivates our emphasis on "banks" as the main empirical counterpart to the agents in our model, as well as our interpretation that our baseline symmetric model applies most naturally to developed markets.

These patterns are further corroborated by Jeanne and Sandri (2017), who document a high positive correlation between outflows and inflows, which rises with financial development. Moreover, they note that less developed economies address the reduced correlation between outflows and inflows by accumulating international reserves. Relatedly, Alberola et al. (2016) document that the retrenchment of outflows is stronger in countries that have higher international reserve ratios.[4] While we do not explicitly model emerging market central banks that accumulate international reserves, the prevalence of retrenchment in those economies suggests that many of the mechanisms that we emphasize are likely to be relevant in those contexts (with "central banks" often taking the functional role of "banks" in our model, either directly or indirectly through their implicit support to local banks).

[4] They conclude that ".... While (international reserves) do not prevent a reduction of inflows by foreign investors... they facilitate financial retrenchment by resident investors... The economic significance of this (retrenchment) effect during periods of financial stress is substantial. Domestic outflows might contract by up to 6 percentage points of GDP for an average country..."

3. The Model

The model features three periods, $t \in \{0, 1, 2\}$, and a single consumption good in each period. There is a continuum of mass one of locations denoted by superscript $j \in J$. In period 1, an aggregate state $s \in S = \{1, ... |S|\}$ is drawn with probability $\gamma_s > 0$, where $\sum_{s \in S} \gamma_s = 1$. The aggregate state determines the probability of a liquidity shock in a location, which is the same across locations and denoted by $\pi_s \in [0, 1]$. Specifically, a random variable ω^j is drawn for each location j and i.i.d. across j, with $\pi_s = \Pr\left(\omega^j = b\right)$ and $1 - \pi_s = \Pr\left(\omega^j = g\right)$. We say that a location with $\omega^j = b$ experiences a liquidity shock. We assume $\sum_S \gamma_s \pi_s \in (0, 1)$ so the probability of a liquidity shock is positive but less than one. We also assume π_s is strictly increasing in s so that aggregate states with greater s are associated with greater likelihood of liquidity shocks.

There are three types of assets. First, in each location, there is a linear technology in period 0: investing one unit of the consumption good produces one unit of a location-specific risky asset. If $\omega^j = g$, then each unit of the asset pays R units in period 1 and 0 units in period 2. If instead $\omega^j = b$, so that the location experiences a liquidity shock, then each unit of the asset pays 0 units in period 1 and R units in period 2. In this case, the asset is traded in period 1 at an endogenous price p_s^j. We concentrate our attention on symmetric equilibria in which the price when $\omega^j = b$ is the same for all locations, that is, $p_s^j \equiv p_s$ for each j.

Second, there is also a risk-free asset that pays 1 unit of the consumption good in period 1 (and 0 units in period 2). The risk-free asset is in fixed supply: specifically, there are η units in each location (endowed to the local banks that will be described below). In period 0, the risk-free asset is traded at an endogenous price q_f.

Third, there are also Arrow-Debreu financial securities that facilitate the sharing of aggregate risk. Specifically, for each aggregate state $s \in S$, there is an Arrow-Debreu security that pays 1 unit of the consumption good in period 1 if state s is realized (and 0 units in all other states or in period 2). In period 0, the Arrow-Debreu security for state s is traded at an endogenous price q_s. The Arrow-Debreu securities are in zero net supply.[5]

There are two types of agents. First, in each location, there is a mass of agents that we refer to as "distressed sellers." These agents have preferences given by $E\left[\tilde{c}_{2,s}\right]$, where $\tilde{c}_{2,s}$ denotes their consumption in period 2 conditional on aggregate state s. These agents have access to a linear technology that converts 1 unit of the consumption good in period 1 into λ units of the consumption good in period 2. The payoff from this technology cannot be pledged to other agents, so when λ is sufficiently large (which will be the case we will focus on) the distressed sellers face balance sheet constraints.

In period 1, the distressed sellers are endowed with e units of the risky asset of their own location. If $\omega^j = g$ is realized in their location, then they receive Re units of the consumption

[5] Note that agents cannot trade financial contracts whose payoffs are contingent on the realizations of the local liquidity shocks, $\left\{\omega^j\right\}_j$. We make this assumption primarily for simplicity. In Appendix A.2, we develop a version of the model with complete markets and show that our main results continue to apply in this setting.

good from their endowment. They invest this in the linear technology and consume the output in period 2, that is, $\tilde{c}_{2,s}\left(\omega^j = g\right) = \lambda Re$. If instead $\omega^j = b$ is realized, then they decide whether to keep their endowment or to sell it to reinvest in the project. We let $\tilde{\chi}^j_s \in [0, e]$ denote their holdings of the asset at the end of period 1, and note that their consumption in period 2 is,

$$\tilde{c}_{2,s}\left(\omega^j = b\right) = \tilde{\chi}^j_s R + \lambda\left(e - \tilde{\chi}^j_s\right) p_s. \tag{1}$$

As long as $\lambda p_s > R$ for each s, distressed sellers optimally choose $\tilde{\chi}^j_s = 0$ and sell all of their endowments regardless of the state. We will make parametric assumptions so that this will be the case along the equilibrium path for most of our analysis (see Remark 2 below for the role that distressed sellers play in our analysis).

In each location, there is also a second group of agents with mass one, which we refer to as "banks." These are the main agents in our model and their preferences are,

$$E\left[u\left(c_0\right) + c_{1,s} + c_{2,s}\right] \tag{2}$$

where the utility function, $u(\cdot)$, satisfies $u'(c) > 0, u''(c) < 0$ for each $c > 0$ as well as the Inada-type conditions, $u'(0) = \infty$ and $u'(1) < R$. Note that these preferences also imply that, if $\omega^j = b$ is realized, then (local) banks would be indifferent to hold the asset if and only if $p_s = R$. We will make parametric assumptions so that the equilibrium asset price will be below this level, $p_s < R$, which we refer to as *fire sales*.[6]

Banks in each location j are endowed with one unit of the consumption good in period 1, as well as η units (all of the fixed supply) of the risk-free asset. In period 0, they choose an investment strategy, $x^{j',j}$, in risky assets across locations, j'. We impose that $x^{j,j}$ is point mass, and $x^{j',j}$ for $j' \neq j$ is a density with respect to the Lebesgue measure. Banks also choose how many consumption units to invest in the risk-free asset, y, or in Arrow-Debreu securities, $(z_s)_s$. Banks' budget constraint in period 0 is,

$$c_0 + x^{j,j} + x^{out,j} + yq_f + \sum_s z_s q_s = 1 + \eta q_f, \text{ where } x^{out,j} = \int_{j' \neq j} x^{j',j} dj'. \tag{3}$$

Here, $x^{out,j}$ denotes the outflows: the aggregate amount of investment made by banks in location j in other locations. Banks are not allowed to short-sell risky assets, $x^{j',j} \geq 0$ for each j', but they are allowed to take unrestricted positions on the risk-free asset or the Arrow-Debreu securities subject to obtaining nonnegative consumption in all periods and states.[7]

[6] Recall that we assume the assets in locations without a liquidity shock ($\omega^j = g$) pay early in period 1. This simplifies the exposition by ensuring that we do not need to worry about asset prices or fire sales in these locations (the ex-dividend price would always be zero). Equivalently, we could assume the risky asset always pays later (in period 2), but make parametric assumptions (e.g., on the asset endowment of distressed sellers) such that the locations without liquidity shocks are not subject to fire sales ($p_s\left(\omega^j = g\right) = R$).

[7] The binding constraint here will be the short-selling of the local asset, $j' = j$. In our model, the price of the local asset is correlated with local liquidity shocks, and thus, short selling the local asset could provide some insurance with respect to liquidity shocks. See Appendix A.2 for the complete markets case.

In period 1, if $\omega^{j'} = g$ and $j' \neq j$, then banks receive R units of the consumption good from their risky asset investments in location j'. By an exact law of large numbers, these locations generate $x^{out,j}(1-\pi_s)R$ units of the consumption good (see Uhlig (1996) for details).[8] If instead $\omega^{j'} = b$ and $j' \neq j$, then banks are required to sell their risky asset holdings in this location, which captures our main fickleness assumption (see Remark 3 below for various interpretations). By the same law of large numbers, these sales generate $x^{out,j}\pi_s p_s$ units of the consumption good. Hence, the amount of resources banks receive from investments in other locations is given by $x^{out,j}\overline{R}_s$, where

$$\overline{R}_s = (1-\pi_s)R + \pi_s p_s, \tag{4}$$

denotes the expected one-period payoff from a unit of foreign investment conditional on the aggregate state s. In addition, banks receive $y + z_s$ units of the consumption good from their investments in the risk-free asset and the Arrow-Debreu securities.

Banks' total resources in period 1, as well as what they do with these resources, also depends on the shock in their own location. If $\omega^j = g$, then banks' risky asset investment in own location pays $Rx^{j,j}$ units in period 1. Moreover, banks do not have a remaining investment opportunity so they consume all of their available resources in period 1. Then, banks' budget constraints in state $\omega^j = g$ can be written as,

$$\begin{aligned} c_{1,s}\left(\omega^j = g\right) &= x^{j,j}R + x^{out,j}\overline{R}_s + y + z_s \\ \text{and } c_{2,s}\left(\omega^j = g\right) &= 0. \end{aligned} \tag{5}$$

If instead $\omega^j = b$, then banks' risky asset investment in own location pays zero units in period 1. However, banks are not required to sell their holdings in their own location. We let $\chi_s^j \geq 0$ denote banks' position in the local risky assets in period 1 when $\omega^j = b$ and the aggregate state is s. Then, banks' budget constraints in state $\omega^j = b$ can be written as,

$$\begin{aligned} c_{1,s}\left(\omega^j = b\right) + \chi_s^j p_s &= x^{j,j}p_s + x^{out,j}\overline{R}_s + y + z_s, \\ c_{2,s}\left(\omega^j = g\right) &= \chi_s^j R. \end{aligned} \tag{6}$$

Putting everything together, banks in each location j make an investment plan, $\left[x^{j',j} \geq 0\right]_{j'}, y^j, \left(z_s^j, \chi_s^j \geq 0\right)_s$, to maximize their expected utility in (2), where c_0^j is determined by Eq. (3); $c_{1,s}^j\left(\omega^j = g\right)$ and $c_{2,s}^j\left(\omega^j = g\right)$ are determined by Eq. (5), and $c_{1,s}^j\left(\omega^j = b\right)$ and $c_{2,s}^j\left(\omega^j = b\right)$ are determined by Eq. (6); and consumption in all periods and states are nonnegative, $c_0^j \geq 0, c_{1,s}^j \geq 0, c_{2,s}^j \geq 0$.

[8] More precisely, conditional on the aggregate state s, the return from these locations corresponds to an integral over random variables, $\int_{j' \in [0,1]} Rx^{j',j}\mathbf{1}\left[\omega^{j'} = g\right]dj'$ (where $\mathbf{1}\left[\omega^{j'} = g\right]$ denotes the indicator variable). We obtain the law of large numbers by interpreting this as a *Pettis integral,* which is a generalization of the Lebesgue integral to vector-valued functions. We then use a slight extension of Theorem 3 in Uhlig (1996) to evaluate the integral as equal to, $\int_{j' \in [0,1]} Rx^{j',j}(1-\pi_s)dj' = R(1-\pi_s)x^{out,j}$ with probability one.

The equilibrium with symmetric prices is a collection of optimal allocations for distressed sellers and banks, together with prices, $(p_s)_s, q_f, (q_s)_s$, that ensure market clearing. The market clearing condition for the risky asset in a location j with $\omega^j = b$ in period 1 is given by,

$$e + x^{in,j} + x^{j,j} = \tilde{\chi}_s^j + \chi_s^j \text{ where } x^{in,j} = \int_{j' \neq j} x^{j,j'} dj'. \tag{7}$$

Here, $x^{in,j}$ denotes the inflows: the aggregate amount of investment in a location made by banks in other locations. The left side captures the supply of risky assets, which comes from the distressed sellers' endowment, the ex-ante inflows, and the ex-ante local investments. The right side captures the demand, which comes only from the distressed sellers and the local banks, because foreign banks sell all of their asset holdings when $\omega^j = b$. The market clearing condition for the risk-free asset in period 0 is given by,

$$\int_j y^j dj = \eta. \tag{8}$$

Finally, the market clearing condition for the Arrow-Debreu security for state s is given by,

$$\int_j z_s^j dj = 0. \tag{9}$$

Remark 1 (Interpreting Risky and Safe Assets)**.** *We view the risky assets in our model as corresponding to securities that are typically held by banks and that can be subject to fire sales. Some examples are equity, long-term debt (bank loans as well as portfolio debt), and unsecured short-term debt that is subject to default risk (as the threat of default during an aggregate liquidity event effectively turns it into an illiquid investment at such times). In contrast, safe assets are those that are not subject to fire sales and that yield a relatively high payoff during distress events, e.g., short-term debt that is highly collateralized or issued by entities with negligible default risk. We assume safe assets are scarce, which is consistent with a growing empirical literature (see, for instance, Gorton and Laarits (2018)).*

Remark 2 (An Alternative Model with Distressed Banks)**.** *In Appendix A.5, we build an alternative model in which there are no separate distressed sellers, and liquidity shocks are events in which banks experience losses (so they are the distressed agents). When this happens, banks are forced to sell risky assets to another group of agents, "secondary buyers" (that reside in the same location), which convert these assets to an alternative use that generates lower payoff. We show that our main results continue to apply in this relatively more standard setting (see, for instance, Kiyotaki and Moore (1997)). Hence, the role of "distressed sellers" is to introduce the standard balance sheet channel into our model while simplifying the analysis. By endowing these agents with risky assets and a technology with high return (large λ), we mechanically generate liquidity-driven asset sales and the misallocation of capital (from high to low-marginal-utility agents) that results from these sales.*

Remark 3 (Interpreting Fickleness)**.** *Our fickleness assumption captures a variety of factors which, during a local distress event, reduces foreign banks' valuation of local risky asset relative to that of locals (local banks in the main model and secondary buyers in the expanded model of Appendix A.5). One interpretation is asymmetric information or uncertainty: that is, foreign banks have an information disadvantage that becomes acute when the local market is distressed (see Appendix A.1 for a formalization based on Knightian uncertainty). This interpretation is broadly consistent with a large literature that studies portfolio home bias (see, for instance, Gehrig (1993); Brennan and Cao (1997); Van Nieuwerburgh and Veldkamp (2009)). Other interpretations are asymmetric property rights that make foreign banks more likely to be expropriated or defaulted upon in distressed markets compared to locals (see Broner et al. (2014)) for a formalization in the context of the European sovereign debt crisis); or asymmetric regulation that increases foreign banks' cost of investment in distressed markets relative to their local counterparts (see Uhlig (2014) for a model along these lines in the context of European crisis).*

4. Gross Flows and Global Liquidity Creation

In this section we characterize the equilibrium. We show that, despite the fickleness element, gross flows exist, contribute to global liquidity creation, and mitigate fire sales. We also characterize the equilibrium asset prices and returns in period 0 and show that foreign investment is associated with a risk premium even though banks have linear utility in period 1 (thus the standard source of the risk premium is absent). Throughout the rest of the paper, we focus on the following parametric condition.

Assumption 1. $eR/\lambda < \eta < eR$.

The right side of the inequality ensures the equilibrium features fire sales, $p_s < R$. The left side ensures $\lambda p_s > R$, so that distressed sellers always sell their assets, $\tilde{\chi}_s^j = 0$ [cf. Eq. (1)].

4.1. Equilibrium and liquidity creation

Under Assumption 1, we conjecture an equilibrium with symmetric prices that satisfy $p_s \in (R/\lambda, R)$ for each s. We also conjecture symmetric equilibrium allocations in which each location invests the same amount in the risky assets of each other location, $x^{j',j} = x^{out,j}$ for each $j' \neq j$; and all locations choose identical allocations. We denote these symmetric allocations by dropping the superscript j, that is,

$$x^{out,j} \equiv x^{out}, y^j \equiv y, z_s^j \equiv z_s \text{ for each } j.$$

Note that these assumptions also imply that the inflows into a location are equal to the outflows, $x^{in,j} \equiv x^{in} = x^{out}$ [cf. (3) and (7)]. When it is clear from the context, we also drop the superscript "in" or "out" and denote these symmetric gross flows with simply x.

Since banks have linear utility between periods 1 and 2, the presence of fire sales ($p_s < R$) implies that banks in locations with state $\omega^j = b$ invest all of their resources in period 1 in the risky asset, that is, $c^j_{1,s}\left(\omega^j = b\right) = 0$ and their position, χ^j_s, is determined by Eq. (6). In addition, since locations have symmetric allocations, the market clearing conditions (8) and (9) imply $y = \eta$ and $z_s = 0$. Combining these observations with the budget constraints $(3), (5), (6)$, we obtain,

$$\begin{aligned} c_0 + x^{j,j} + x^{out} &= 1, \\ c_{1,s}\left(\omega^j = g\right) &= x^{j,j}R + x^{out}\overline{R}_s + \eta, \\ c_{2,s}\left(\omega^j = b\right) &= \left(x^{j,j}p_s + x^{out}\overline{R}_s + \eta\right)\frac{R}{p_s}. \end{aligned}$$

Substituting these expressions into the objective function in (2), and rearranging terms, the representative banks' problem can be written as,

$$\max_{x^{j,j},x^{out}} u\left(1 - x^{j,j} - x^{out}\right) + x^{j,j}R + \sum_s \gamma_s \left(x^{out}\overline{R}_s + \eta\right) M_s, \tag{10}$$

$$\text{where } M_s \equiv 1 - \pi_s + \pi_s \frac{R}{p_s}. \tag{11}$$

M_s denotes the expected period-1 marginal utility conditional on the aggregate state s. When $\omega^j = g$, local banks do not have an investment opportunity in period 1 and consume their available resources. When $\omega^j = b$, local banks take advantage of local fire sales in period 1 to invest their available resources so as to obtain greater marginal utility, $R/p_s > 1$. The expression for expected marginal utility, M_s, combines these two cases.

To solve problem (10), first note that the ex-ante marginal utility from investing in the local risky asset is simply equal to its payoff, R. In contrast, the ex-ante marginal utility from investing in foreign risky assets is given by, $\sum_s \gamma_s \mu_s(p_s)$, where

$$\mu_s(p_s) \equiv \overline{R}_s M_s = \left((1-\pi_s)R + \pi_s p_s\right)\left(1 - \pi_s + \pi_s \frac{R}{p_s}\right).$$

The function, $\mu_s(p_s)$, captures the ex-ante marginal utility conditional on the aggregate state s and given the price level p_s. For banks, one-period payoff from foreign investment is relatively low, $\overline{R}_s < R$, because they are fickle and sell their risky assets when there is a liquidity shock in the foreign location. On the other hand, expected period-1 marginal utility is relatively high, $M_s > 1$, because they retrench and use the liquidity from foreign assets to arbitrage fire sales during a local liquidity shock. The ex-ante marginal utility, $\mu_s(p_s) = \overline{R}_s M_s$, combines banks' costs and benefits from foreign investment. Our next result characterizes this expression and shows that it always exceeds the ex-ante marginal utility from local investment.

Lemma 1. *For each aggregate state s with $\pi_s \in (0,1)$, the ex-ante marginal utility from foreign investment, $\mu_s(p_s)$, is strictly decreasing in p_s over the range $p_s \in (0, R]$, and it satisfies $\mu_s(R) =$*

R. In particular, when $p_s \in (0, R)$, we have $\mu_s(p_s) > R$ and investing in foreign risky assets dominates investing in local risky assets, that is, $x^{j,j} = 0$.

Intuitively, the presence of local fire sales induces banks to obtain liquidity insurance by investing in foreign risky assets. Consistent with this intuition, a decline in the fire-sale price, p_s, increases the marginal benefit from foreign investment, $\mu_s(p_s)$.

Combining Lemma 1 with problem (10), we also characterize the equilibrium level of foreign investment as the solution to,

$$u'\left(1 - x^{out}\right) = E\left[\overline{R}_s M_s\right] = \sum_s \gamma_s \mu_s(p_s). \tag{12}$$

Banks buy foreign risky assets up to the point at which the ex-ante marginal utility from investment is equal to their current marginal utility from consumption. Note also that a reduction in the fire-sale price p_s (in any aggregate state with $\pi_s > 0$) increases x^{out}: a lower price level increases the value of liquidity insurance and the latter is achieved by increasing foreign investment.

Next consider the determination of the fire-sale asset prices, p_s. Recall that $c_{1,s}^j\left(\omega^j = b\right) = 0$ and χ_s^j is determined by Eq. (6) after substituting $y = \eta$ and $z_s = 0$. Substituting this expression as well as $\tilde{\chi}_s^j = 0$ into the market clearing condition (7), we obtain an expression for the fire-sale price,

$$p_s = \frac{\eta + x^{out}\overline{R}_s}{e + x^{in}}. \tag{13}$$

The denominator of this term captures the total amount of sales, which come from liquidity-driven sales (e) and the past inflows, all of which are liquidated in a crisis in view of the fickleness assumption. The numerator corresponds to the local banks' wealth, which comes from their safe assets and their foreign asset positions that are determined by the past outflows. Eq. (13) says that (when there are fire sales) the asset price is determined by the cash-in-the-market per asset. This expression illustrates the key tension captured by our model: while past inflows tend to reduce the fire-sale price, past outflows provide liquidity to retrenching local banks and help to stabilize fire sale prices.

Recall also that, in a symmetric equilibrium, inflows are equal to outflows, $x^{in} = x^{out} = x$. After substituting this into Eq. (13) and rearranging terms, the fire-sale asset prices can also be written as,

$$p_s = P_s^{mc}(x) \equiv \frac{\eta + x(1-\pi_s)R}{e + x(1-\pi_s)}. \tag{14}$$

The last equality defines the market clearing relation, $p_s = P_s^{mc}(x)$, which describes the price level in state s as a function of the gross flows. The following lemma resolves the tension between inflows and outflows, and shows that retrenchment dominates fickleness.

Lemma 2. *Under Assumption 1, for each aggregate state s with $\pi_s < 1$, the market clearing price level, $P_s^{mc}(x)$, is strictly increasing in symmetric gross flows, x.*

The intuition for why retrenchment dominates fickleness can be understood by inspecting Eq. (13). Note that past inflows (x in the denominator) are liquidated at the fire-sale return, p_s. However, past outflows (x in the numerator) provide liquidity to retrenching local banks at a higher return, $\overline{R}_s$. When $p_s < R$ and $\pi_s < 1$, the fire-sale return is lower than the return from foreign investment, $p_s < \overline{R}_s = (1 - \pi_s) R + \pi_s p_s$. It follows that the symmetric flows increase liquidity and fire-sale prices. Despite their fickleness, gross flows help to bring the excess liquidity in foreign financial markets that do not experience liquidity shocks into the local market that has a liquidity shock.

The equilibrium level of gross flows and prices, $x, (p_s)_s$, are characterized by solving Eq. (12) together with Eq. (14) for each aggregate state s. We next state our main result, which establishes the existence of a unique symmetric equilibrium that features $x \in (0,1)$, and $p_s \in (R/\lambda, R)$ (verifying our conjecture). The result also compares the equilibrium prices with those that would obtain in the autarky allocation in which all foreign investment is banned. In autarky, banks solve the same portfolio problem as before with the additional restriction that $x^{j,j'} = 0$ for any $j' \neq j$. It is then easy to check that banks hold some local risky assets, $x^{j,j} = \underline{x} > 0$, where $\underline{x}$ is the solution to, $u'(1 - \underline{x}) = R$. However, as illustrated by Eqs. (6) and (7), these local investments do not generate any additional liquidity when there is a local liquidity shock in period 1. Therefore, the fire-sale price is still characterized by Eq. (14) after substituting zero capital flows, $p^{aut} = \eta/e$. By Lemma 2, this is lower than the equilibrium price.

Proposition 1 (Equilibrium Capital Flows and Global Liquidity Creation)**.** *Consider the model with Assumption 1. There exists a unique symmetric equilibrium allocation,* $x^{j,j}, \left(x^{out} = x^{in} = x\right), y, (z_s, \chi_s)_s$*, with symmetric prices,* $(p_s)_s, q_f, (q_s)_s$*. The equilibrium allocation satisfies* $x^{j,j} = 0, y = \eta, z_s = 0$*. The tuple* $(x, (p_s)_s)$ *is characterized by Eqs.* (12) *and* (14)*, and satisfies* $x \in (0,1)$ *and* $p_s \in (R/\lambda, R)$ *for each* s*. Capital flows create liquidity in the sense that the fire-sale price is greater than the price that would obtain in the autarky allocation in which all foreign investment is banned, that is,* $p_s \geq p^{aut} = \eta/e$ *for each* s *with strict inequality if* $\pi_s < 1$*.*

We postpone the characterization of financial asset prices, $q_f, (q_s)_s$, to the end of this section. Next, we illustrate Proposition 1 for the special case with a single aggregate state (that is, S is a singleton). We let $\pi \equiv \pi_s \in (0,1)$ denote the probability of a liquidity shock, $p \equiv p_s$ denote the fire-sale asset price, and $\mu(p)$ and $P^{mc}(x)$ denote the functions characterized in Lemmas 1 and 2. Figure 2 visualizes the resulting equilibrium. The declining curve corresponds to the optimality condition, $u'(1-x) = \mu(p)$. The increasing curve corresponds to the market clearing relation, $p = P^{mc}(x)$. The equilibrium corresponds to the intersection. Note also that the equilibrium price is strictly greater than the autarky price, which illustrates that gross flows help to create liquidity and mitigate fire sales despite their fickleness.

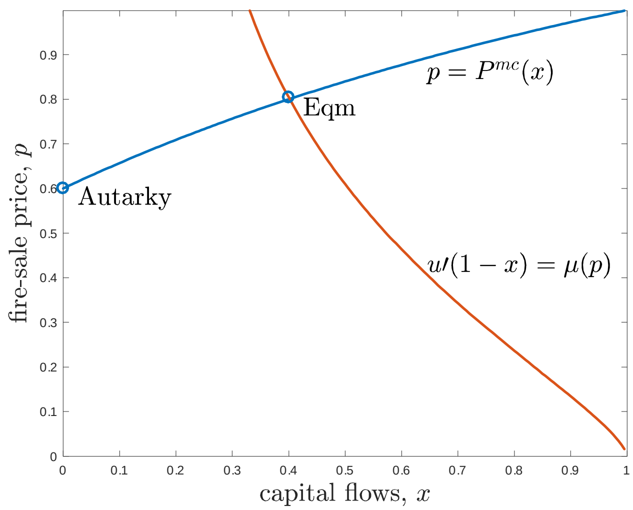

Figure 2: Equilibrium when there is a single aggregate state.

4.2. Asset prices and returns

Let us go back to the case with multiple aggregate states and complete the characterization of equilibrium that we started in Proposition 1. Recall our convention that π_s is strictly increasing in s so that states with greater s are associated with greater likelihood of liquidity shocks. Combining this with Eq. (14) illustrates that p_s is strictly decreasing in s: that is, states with a greater likelihood of liquidity shock are associated with strictly lower equilibrium prices. Intuitively, these states feature less global liquidity since more locations are simultaneously hit by the liquidity shock. This also implies that the payoff from foreign investment, $\overline{R}_s = (1 - \pi_s) R + \pi_s p_s$, is strictly decreasing in s; whereas the marginal utility in period 1, $M_s = 1 - \pi_s + \pi_s \frac{R}{p_s}$ is strictly increasing in s.

Next consider the asset prices in period 0, $q_f, (q_s)_s$. Recall that the equilibrium features $y = \eta$ and $z_s = 0$ for each s. Suppose banks in a location consider changing these allocations, y^j, z_s^j. Following similar steps as above (and using $x^{j,j} = 0$) the optimal allocations solve,

$$\max_{y^j, \left(z_s^j\right)_s} u\left(c_0\right) + \sum_s \gamma_s \left(x^{out} \overline{R}_s + y^j + z_s^j\right) M_s, \tag{15}$$
$$\text{where } c_0 = 1 - x^{out} - q_f \left(\eta^j - y^j\right) - \sum_s q_s z_s^j.$$

Using the optimality condition for z_s^j, we obtain an expression for Arrow-Debreu prices,

$$\frac{q_s}{\gamma_s} = \frac{M_s}{u'\left(1-x\right)}. \tag{16}$$

As usual, the stochastic discount factor (SDF), q_s/γ_s, is determined by the expected marginal utility in the corresponding state divided by the marginal utility in period 0. Note also that q_s/γ_s is strictly increasing in s (because M_s is strictly increasing). As expected, states with greater probability of liquidity shocks feature more expensive state prices. For future reference, note also that substituting Eq. (16) into (12) implies $1 = \sum_s \overline{R}_s q_s$: that is, the cost of foreign diversified investment is equal to the value of a replicating portfolio.

Using the optimality condition for η^j, we also solve the risk-free asset price as,

$$q_f = \frac{E\left[M_s\right]}{u'\left(1-x\right)}. \tag{17}$$

We define the risk-free return as the inverse of this price, $R_f \equiv 1/q_f$. Using Eq. (12) to substitute for $u'\left(1-x\right)$ in the previous expression, we further obtain,

$$R_f = \frac{E\left[\overline{R}_s M_s\right]}{E\left[M_s\right]} = E\left[\overline{R}_s\right] + \frac{cov\left(\overline{R}_s, M_s\right)}{E\left[M_s\right]}. \tag{18}$$

All else equal, the risk-free rate is lower when the expected return from risky assets is lower and when these risky returns covary negatively with marginal utility.

In our setting, the covariance is negative because $\overline{R}_s$ is strictly decreasing in s and M_s is strictly increasing in s. Combining this observation with Eq. (18), we also find that the risk premium on foreign assets is positive,

$$E\left[\overline{R}_s\right] - R_f = -\frac{cov\left(M_s, \overline{R}_s\right)}{E\left[M_s\right]} \geq 0, \tag{19}$$

with strict inequality as long as there are multiple states. Intuitively, the value of the foreign investment is reduced by the fact that they pay relatively less when aggregate liquidity is relatively scarce. The following result summarizes the characterization of asset prices.

Proposition 2 (Equilibrium Asset Prices and Risk Premia)**.** *Consider the symmetric equilibrium characterized in Proposition 1. The fire-sale price, p_s, is strictly decreasing in s (which captures the likelihood of the liquidity shock). The state price, q_s/γ_s, is characterized by Eq.* (16) *and is strictly increasing in s. The price and the return of the risk-free asset are characterized by Eqs.* $(17-18)$. *The risk premium on foreign investment is characterized by Eq.* (19), *and is strictly positive as long as there are multiple aggregate states.*

5. Regulating Gross Flows

In this section, we analyze the desirability of policies that regulate capital flows. We first characterize the constrained optimal allocation and show that the equilibrium is constrained inefficient due to pecuniary externalities. We then show that local policymakers that are motivated by addressing these externalities (in their own location) can fail to do so because there is a public good aspect to liquidity creation that generates a need for policy coordination.

Throughout, we focus on the special case with a single aggregate state so we drop the subscript s from all variables. We also assume the policymakers are utilitarian with identical welfare weights on all agents: the social welfare in each location j is the sum of (local) banks' and (local) distressed sellers' expected utilities, $W^j = u\left(c_0^j\right) + E\left[c_1^j + c_2^j\right] + E\left[\tilde{c}_2^j\right]$.

5.1. Constrained optimal allocation and externalities

Consider a constrained social planner that can dictate (symmetric) period 0 local and foreign investment in each location but otherwise cannot interfere with the equilibrium allocations. We denote the local investment with $x^{j,j}$, foreign investment with x, and the resulting equilibrium price with p. In view of Assumption 1, we conjecture that the resulting price satisfies $p \in (R/\lambda, R)$ for any choice, $\left(x^{j,j}, x\right)$.

Following similar steps as in Section 3, it can be checked that the market clearing condition (14) still applies. Moreover, the social welfare that results from this allocation is given by,

$$W^j = u\left(1 - x - x^{j,j}\right) + \left(x^{j,j} + x + e\right) R + \eta + (\lambda - 1)\, e\overline{R}, \tag{20}$$

where $\overline{R} = (1-\pi)\, R + \pi p$. This expression can be understood by considering the net production in periods 1 and 2. Conditional on risky investment $x + x^{j,j}$ in period 0, the risky assets produce a total of $\left(x + x^{j,j} + e\right) R$ units of the consumption good in either periods 1 or 2. Safe assets produce an additional η units in period 1. Finally, the investment activity by distressed sellers uses (in expectation) $e\overline{R}$ units of the consumption good in period 1 and delivers $\lambda e\overline{R}$ units in period 2, for an expected net production of $(\lambda - 1)\, e\overline{R}$. All of these resources are consumed by either banks or distressed sellers in periods 1 or 2. Since these agents have linear utility over these periods, the utilitarian social welfare is given by (20) (see Appendix A.6 for details).

The constrained social planner chooses $x, x^{j,j} \geq 0$ to maximize (20) subject to the market clearing condition (14). Since p is strictly increasing in x (cf. Lemma 1) but does not depend on $x^{j,j}$, the optimum features $x^{j,j} = 0$. That is, local investment is dominated not only in equilibrium but also in the constrained optimum.

However, the level of foreign investment in a constrained optimum can be different than in equilibrium. Specifically, the optimality condition for foreign investment implies,

$$u'\left(1 - x\right) = R + (\lambda - 1)\, e\pi \frac{dp}{dx} \quad \text{where} \quad \frac{dp}{dx} = \frac{(1-\pi)}{e + x\,(1-\pi)}\,(R - p)\,. \tag{21}$$

The constrained optimum is found by solving this expression together with the market clearing condition (14). Under Assumption 1, there exists a unique intersection that satisfies $x \in (0, 1)$ and $p \in (R/\lambda, R)$. For comparison, recall that the equilibrium is characterized by solving a different optimality condition (12) together with the same market clearing condition (14) (see Figure 2). Hence, the equilibrium is typically constrained inefficient. The following proposition characterizes the direction of the inefficiency.

Proposition 3 (Constrained Optimal Allocation)**.** *Consider the model with Assumption 1 and a single aggregate state. The constrained optimal allocation, (x, p), is characterized as the unique solution to Eqs.* (21) *and* (14)*. Compared to the equilibrium allocation, denoted by (x^{eq}, p^{eq}), the constrained planner chooses greater x (which also leads to greater p) if and only if,*

$$\frac{e\lambda + x^{eq}(1-\pi) + x^{eq}\pi(R/p^{eq})}{e + x^{eq}} > \frac{R}{p^{eq}}. \tag{22}$$

To understand this result, note that investing in foreign assets creates liquidity and increases the fire-sale price p (via Eq. (14)). The increase in the fire-sale price increases the wealth of the sellers and reduces the wealth of the buyers. These effects represent pecuniary externalities that are ignored by banks but are taken into account by the planner. Condition (22) says that foreign investment is associated with net positive pecuniary externalities if and only if it increases sellers' average marginal utility more than it decreases buyers' marginal utility. Note that during a liquidity shock $e + x^{eq}$ units of the asset are sold (at the fire-sale price) from a mix of agents to local banks. The right side of the expression describes the period-1 marginal utility of the buyers (local banks), which is equal to R/p^{eq}. The left side describes the weighted-average period-1 marginal utility of the sellers, where the weights are proportional to the number of units that they sell. The distressed sellers have weight e and marginal utility λ, foreign investors in locations without a liquidity shock have weight $x^{eq}(1-\pi)$ and marginal utility 1, and foreign investors in locations with a liquidity shock have weight $x^{eq}\pi$ and marginal utility R/p^{eq}.

Next consider the limit as $\lambda \to \infty$. In this limit, condition (22) holds and thus the constrained optimal allocation features greater x and p. In fact, it can also be checked that the foreign investment approaches one (its maximum feasible level) in the constrained optimum, whereas it is strictly below one in the competitive equilibrium. Intuitively, the sellers' average marginal utility is dominated by distressed sellers' marginal utility, which is large and exceeds buyers' marginal utility. This in turn leads to positive pecuniary externalities from foreign investment.

Now consider the case with lower levels of λ. In this case, condition (22) can be violated because some of the sellers are fickle foreign banks that have lower marginal utility than buyers (local banks). We verify that this can happen under some configuration of parameters. In this case, constrained optimum features smaller x and p than the equilibrium. Since raising fire-sale asset prices benefits not only the distressed sellers but also the fickle foreigners with low marginal utility, if the effect through the distressed sellers is weak, then foreign investment generates negative pecuniary externalities. The planner opts for lower foreign investment and lower fire-sale prices to transfer wealth from fickle foreign banks to local banks.[9]

As we describe in Remark 2, we view the distressed sellers in our setting as a modeling device to capture liquidity-driven sales that transfer risky assets from high to low-marginal

[9] Note also that this possibility emerges because of market incompleteness that enable local and foreign banks to have different marginal utility during a domestic liquidity shock. As emphasized by Geanakoplos and Polemarchakis (1986), pecuniary externalities partially substitute for the missing insurance market.

utility agents. In view of this interpretation, we take the case with high λ and positive pecuniary externalities from foreign investment as the more natural benchmark for welfare analysis.

5.2. Public good aspects of liquidity creation

We next investigate whether local policymakers that act in isolation can achieve globally optimal outcomes without coordination. Specifically, suppose each location is associated with a local policymaker that maximizes the utilitarian social welfare in its own location. For the baseline scenario, we also focus on the limit, $\lambda \to \infty$, while we relegate the discussion of the case with lower λ to the end of this section. As $\lambda \to \infty$, maximizing the utilitarian social welfare becomes equivalent to maximizing p^j (see Appendix A.3). That is, similar to the social planner in the previous subsection, local policymakers' objective is to increase the fire-sale price. The difference is that they exclusively care about the price in their own location.

To simplify the exposition, we equip local policymakers with a single and binary policy instrument, $b^j \in \{0, 1\}$, which they choose at the beginning of period 0 (before any other decision is made). If the policymaker sets $b^j = 0$, then foreigners are allowed to invest in location j as in our baseline model. If instead $b^j = 1$, then foreign investment is banned in location j. In period 0, banks choose their portfolio, $\left[x^{j',j} \geq 0\right]_{j'}$, subject to the additional constraint that $x^{j',j} = 0$ for each $j' \neq j$ and $b^{j'} = 1$. The remaining ingredients are the same as in Section 3.

If all policymakers choose $b^j = 0$, then we recover the equilibrium allocation with free flows. If instead all policymakers choose $b^j = 1$, then all foreign investment is banned, and we recover the autarky allocation. Recall that the price with free flows is strictly greater than in autarky (see Figure 2). Thus, a global policymaker that prescribes symmetric policies (with the objective of maximizing the symmetric fire-sale prices, p) would allow capital flows in all locations. Note that this is consistent with our analysis in Section 5.1: the global policymaker creates as much liquidity as possible given the instruments she has access to.

We next characterize the Nash equilibrium outcome, and contrast with the coordinated solution. To this end, first consider the equilibrium for a given configuration of policy choices. Suppose the sets of locations with $b^j = 1$ ("banned locations") and $b^j = 0$ ("free locations") are Lebesgue measurable, respectively with measures $B \in [0, 1)$ and $1 - B$. As before, we focus on a symmetric equilibrium in which each banned location chooses identical and fully diversified foreign investment in each free location, denoted by $x^{ban} \geq 0$, and experiences identical fire-sale prices, $p^{ban} \in (0, R)$. Likewise, each free location chooses identical and fully diversified foreign investment in each free location, denoted by $x^{free} \geq 0$, and experiences identical fire-sale prices, $p^{free} \in (0, R)$. Following similar steps as above, the fire-sale price levels satisfy,

$$
\begin{aligned}
p^{ban} &= \frac{\eta + (1 - B)\, x^{ban} \overline{R}^{free}}{e}, \\
p^{free} &= \frac{\eta + (1 - B)\, x^{free} \overline{R}^{free}}{e + B x^{ban} + (1 - B)\, x^{free}},
\end{aligned}
$$

where $\overline{R}^{free} = (1-\pi) R + \pi p^{free}$. The equilibrium tuple, $\left(p^{ban}, p^{free}, x^{ban}, x^{free}\right)$, is characterized by solving these equations jointly with the optimality conditions (see Appendix A.3). These expressions illustrate that the banned locations experience smaller inflows than their outflows, whereas the free locations experience the opposite. These net imbalances raise the fire-sale price in banned locations while lowering the price in free locations. In equilibrium, the level of outflows also react to these changes (in fact, the banned locations can also feature some local investment due to their reduced need for outside liquidity), but these induced effects do not overturn the initial effect. Specifically, $p^{ban} > p^{free}$ in any symmetric equilibrium with $B \in [0,1)$.

Now consider the Nash equilibrium among the policymakers. Since $p^{ban} > p^{free}$ for any $B \in [0,1)$, the only candidate for equilibrium is the autarky allocation in which all policymakers ban capital inflows. In Appendix A.3, we verify that this is indeed an equilibrium. As Figure 2 illustrates, the Nash equilibrium outcome is sharply different than the coordinated solution, and it features strictly lower fire-sale prices and lower welfare in every location.

The reason for this discrepancy is that global liquidity is a public good: that is, policymakers that make locally optimal policy choices ignore their impact on global liquidity. Every inflow into a location corresponds to outflows that provide liquidity and raise fire-sale asset prices in the sending locations. In the limit $\lambda \to \infty$, greater asset prices improve welfare by mitigating fire-sale externalities. Local planners ignore the beneficial effects of inflows at sending locations while fully internalizing the fickleness costs, as those are felt at the local level, which leads to too little capital flows and insufficient liquidity creation.

Next consider the alternative scenario in which λ is lower. As our analysis in Section 5.1 suggests, the global policymaker might prefer to reduce foreign investment and fire-sale prices. Nonetheless, coordination improves welfare also in this less standard case. To illustrate this, consider the extreme version of this case in which the policymakers have the objective function $-p^j$: that is, they would like to exacerbate the severity of liquidity crises in their own location. It can then be checked that a coordinated solution would ban capital flows whereas the Nash equilibrium would feature free capital flows. In this scenario, liquidity is a public "bad", which leads to a coordination problem in the opposite direction.

The general point is that, when liquidity has a first-order effect on welfare, there is a need for coordinating capital flows as they contribute to global liquidity despite their fickleness. Moreover, fickleness exacerbates the coordination problem because it lowers local liquidity and induces local policymakers to take different actions than what a global policymaker would prescribe.

6. Determinants of Gross Flows

So far we have illustrated how capital flows contribute to global liquidity and how there is a public good aspect to liquidity creation. In this section we analyze the determinants of gross flows in our setting. We also illustrate how our model can generate a global cycle in capital flows and asset prices driven by the perceived likelihood of global liquidity shocks.

6.1. The beta model

We show these results using a special case of the model ("the beta model") that leads to closed form solutions. In this model, liquidity shocks are either completely uncorrelated or fully correlated across regions. Specifically, suppose there are three aggregate states, $s \in \{1, 2, 3\}$, that feature,

$$\pi_1 = 0, \qquad \pi_2 = \pi, \qquad \pi_3 = 1, \tag{23}$$

for some $\pi \in (0, 1)$. In particular, state $s = 2$ corresponds to the state in which the liquidity shocks are i.i.d. across the regions. States $\{1, 3\}$ together can be thought of as a "correlated shock" state in which the liquidity shocks are perfectly correlated across the locations. Specifically, either all locations are hit (state 3) or no location is hit (state 1). We also assume the state probabilities are given by,

$$\gamma_1 = \beta(1 - \pi), \qquad \gamma_2 = 1 - \beta, \qquad \gamma_3 = \beta\pi. \tag{24}$$

Here, the parameter $\beta \in (0, 1)$ captures the extent to which the shocks are correlated. The limit, $\beta \to 0$, corresponds to the single-state case with uncorrelated shocks (see Figure 2), whereas the other limit, $\beta \to 1$, corresponds to perfectly correlated shocks.

Note that the expected returns and marginal utilities are given by,

$$\begin{aligned} \overline{R}_1 &= R, \qquad \overline{R}_2 = (1 - \pi) R + \pi p_2, \qquad \overline{R}_3 = p_3, & (25)\\ M_1 &= 1, \qquad M_2 = 1 - \pi + \pi R / p_2, \qquad M_3 = R / p_3. & (26) \end{aligned}$$

Next note that $\mu_1 (p_1) = \overline{R}_1 M_1 = R$ and $\mu_3 (p_3) = \overline{R}_3 M_3 = R$. Thus, Eq. (12) becomes,

$$\begin{aligned} & u' (1 - x) = E \left[\overline{R}_s M_s \right] = \beta R + (1 - \beta) \mu_2 (p_2), & (27)\\ \text{where} \quad & \mu_2 (p_2) = \overline{R}_2 M_2 = ((1 - \pi) R + \pi p_2) (1 - \pi + \pi R / p_2). \end{aligned}$$

The market clearing condition (14) implies,

$$p_2 = P_2^{mc} (x) = \frac{\eta + x (1 - \pi) R}{e + x (1 - \pi)}. \tag{28}$$

Eqs. (27) and (28) determine the pair, (x, p_2). Figure 3, which is a generalized version of Figure 2, provides a pictorial illustration of the equilibrium pair, (x, p_2).

Using the market clearing condition (14), we also calculate the price in state 3 (with $\pi_3 = 1$) as (p_1 plays no role as there are no liquidity shocks in state 1),

$$p_3 = \frac{\eta}{e}. \tag{29}$$

This is also the average fire-sale price conditional on a liquidity shock in the correlated shock state, $\{1,3\}$ (since $\pi_1 = 0$ and $\pi_3 = 1$). Note that we have, $p_3 < p_2$: that is, the correlated state features lower fire-sale prices on average than the uncorrelated state.

Next consider the prices and returns for financial assets. Using Eq. (25), the expected return on foreign assets is given by,

$$E\left[\overline{R}_s\right] = (1-\pi) R + \pi \left(\beta p_3 + (1-\beta) p_2\right). \tag{30}$$

In particular, asset returns depend on the weighted average fire-sale prices across the correlated and uncorrelated states. State prices are given by $q_s/\gamma_s = M_s/u'(1-x)$ where M_s are characterized in (26).

Finally, combining Eq. (18) with Eqs. (26) and (27), the risk-free rate can be calculated as,

$$R_f = \frac{E\left[\overline{R}_s M_s\right]}{E\left[M_s\right]} = \frac{\beta R + (1-\beta)\mu(p_2)}{\beta(1-\pi+\pi R/p_3) + (1-\beta)(1-\pi+\pi R/p_2)}, \tag{31}$$

The risk premium can be obtained from Eqs. (30) and (31). Thus, Eqs. $(27-31)$ provide a closed-form characterization of the equilibrium in the beta model. We next use this model to establish a number of comparative statics results.

6.2. Safe-asset scarcity

Consider a scarcity of liquidity as captured by a reduction in the supply of safe assets, η. Recall that the equilibrium pair, (x, p_2), is characterized by the optimality condition (27) and the market clearing relation (28). The left panel of Figure 3 illustrates that a decline in η shifts the market clearing equation downwards, without affecting the optimality condition. This increases the capital flows, x, while also reducing the fire-sale price in the uncorrelated state, p_2. Intuitively, the fire-sale price declines because banks have less liquidity to arbitrage asset fire sales (cf. (28)). The anticipation of these more severe fire sales induce greater ex-ante investment in foreign risky assets so as to obtain liquidity insurance (cf. (27)). When there is greater scarcity of liquidity, there is greater need for global liquidity creation, and gross capital flows increase to satisfy this need.

Next note that, by Eq. (29), the scarcity of liquidity also reduces the fire-sale price in the correlated shock state, p_3. In fact, the price declines more in this state than in the uncorrelated state because gross flows do not provide liquidity when there is a correlated shock. That is, a reduction in η reduces p_3/p_2.

Finally, consider the impact on asset prices in period 0. By Eq. (30), the expected return on risky foreign assets also declines due to lower fire-sale prices. In Appendix A.6, we show that the risk-free return characterized by Eq. (31) also declines. Finally, consider the risk premium on foreign financial assets, $E\left[\overline{R}_s\right] - R_f$. It is easy to check that the risk premium becomes zero as $\eta \to eR$ (as this limit features $p_s \to R$ and $M_s \to 1$ for each s) whereas it is strictly positive for any $\eta < eR$ (see Eq. (19)). Thus, the risk premium also increases in the neighborhood of

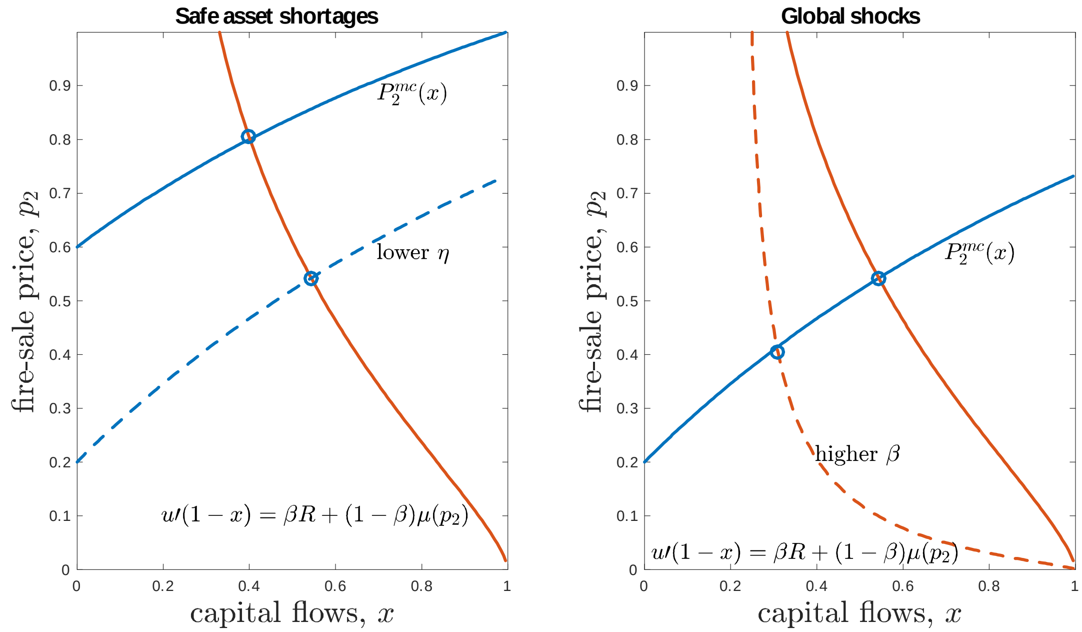

Figure 3: Comparative statics of equilibrium with respect to changes in the supply of safe assets (left panel) and the probability of correlated liquidity shocks (right panel).

abundant safe assets, $\eta = eR$. The following result summarizes this discussion.

Proposition 4. *Consider the beta model described in this section. A reduction in η (that exacerbates the safe-asset scarcity) increases gross flows, x, reduces fire-sale prices in both states, p_2, p_3, as well as the relative fire-sale price in the correlated state, p_3/p_2. It reduces expected risky asset returns, $E\left[\overline{R}_s\right]$, as well as the risk-free return, R_f. In the (lower) neighborhood of $\eta = eR$, it also increases the risk premium on foreign assets, $E\left[\overline{R}_s\right] - R_f$.*

This result provides one explanation for the worldwide increase in gross capital flows in the run-up to the Global Financial Crisis (see Bluedorn et al. (2013)). From the lens of our model, the gross flows increased at least in part as a response to the global asset scarcity that developed in early 2000s (see e.g., Caballero (2006)).

6.3. Global shocks and the global financial cycle

Now consider an increase in the (perceived or real) probability of the correlated shock state, β. The right panel of Figure 3 illustrates that this shifts the optimality curve downward without affecting the market clearing equation. Hence, it lowers the gross flows, x, as well as the fire-sale price in the uncorrelated state, p_2. Correlated shocks reduce the value of liquidity insurance which translates into lower ex-ante investment, x, and lower ex-post liquidity. The latter lowers the price even in the uncorrelated state. That is, the presence of correlated shocks affects asset prices even if those shocks are ultimately unrealized.

Next consider the impact on asset prices in period 0. By Eq. (30), the expected return on risky foreign assets $E\left[\overline{R}_s\right]$ declines due to lower fire-sale prices. In Appendix A.6, we show that the risk-free return R_f characterized by Eq. (31) also declines. As the liquidity shocks become more correlated, the risk-free asset becomes more valuable as it provides liquidity when there is a global liquidity shock. Finally, consider the risk premium on foreign financial assets, $E\left[\overline{R}_s\right] - R_f$. It is easy to check that the risk premium is zero in the limit as $\beta \to 0$ (as this limit is equal to a single aggregate state, in which case there is no aggregate risk) whereas it is strictly positive for any $\beta > 0$ (see Eq. (19)). Thus, the risk premium is increasing in the neighborhood of no global shocks, $\beta = 0$. The following result summarizes this discussion.

Proposition 5. *Consider the beta model described in this section (with Assumption 1). An increase in β (that makes the liquidity shocks more correlated) reduces the capital flows, x, and reduces fire-sale asset price in the uncorrelated state, p_2. It reduces the expected return on foreign financial assets, $E\left[\overline{R}_s\right]$, as well as the risk-free interest rate, R_f. In the neighborhood of $\beta = 0$, it also increases the risk premium on foreign assets, $E\left[\overline{R}_s\right] - R_f$.*

An increase in β in this model can be thought of as capturing a "risk-off" environment in which the banks retrench into their home markets (even at date 0, before crises are realized). This reduces the capital flows and liquidity creation, while also reducing the risk-free rate and increasing the risk premia. This result is consistent with the large decline in gross capital flows in the aftermath of the Global Financial Crisis (see Bluedorn et al. (2013) and Lane and Milesi-Ferretti (2012); Milesi-Ferretti and Tille (2011)). From the lens of our model, the global crisis increased the (real or perceived) correlations of financial crises, which in turn reduced the usefulness and the magnitude of gross capital flows.

7. Reach for Safety and Yield

In our baseline model gross capital flows are entirely driven by the liquidity insurance motive. In this section, we consider a slight variant of the baseline model to illustrate two additional mechanisms that might drive capital flows and investigate how they affect fire sales.

7.1. The model with a special location

Suppose all but one of the locations are "regular" and have the parameters as described in the previous section. The remaining location is "special" and has potentially different parameters, (η^*, R^*): its supply of safe assets is given by η^*, and the return on its risky assets is given by R^*. As before, banks in the special location are endowed with all of the safe assets in this location as well as one unit of the consumption good in period 0. The rest of the model is unchanged. To simplify the exposition, we assume $\pi_1 > 0$ so liquidity shocks happen with strictly positive probability in all aggregate states. Appendix A.4 extends the analysis to cases with $\pi_1 = 0$ (which includes as a special case the beta model from Section 6).

Since the special location has Lebesgue measure zero, the equilibrium allocations and prices in the regular locations are the same as the symmetric equilibrium characterized in the previous sections, denoted by $x, (p_s)_s, q_f, (q_s)_s$. Our goal in this section is to characterize the equilibrium allocations and prices in the special location, which we denote by $x^{*,*}, x^{in,*}, x^{out,*}, y^*, (z_s^*)_s, (\tilde{\chi}_s^*)_s, (p_s^*)_s$.[10] Throughout, we assume the parameters in the special location satisfy the following.

Assumption 2. $R^* - R \in \left[0, \frac{\eta}{e} \frac{\sum_s q_s \pi_s}{\sum_s q_s (1-\pi_s)}\right], \eta^* - \eta \geq -\frac{\sum_s q_s p_s x}{q_f}$.

In particular, the special location has weakly greater return, $R^* \geq R$, which suffices to cover the cases of interest, and its parameters are not too different from those in regular locations, which helps to obtain an interior solution.

First consider the investments made by regular locations' banks in the special location, $x^{in,*}$. In view of Assumption 2, we conjecture an equilibrium with strictly positive inflows, $x^{in,*} > 0$. We assume this equilibrium satisfies the no-arbitrage condition,

$$1 = \sum_s \overline{R}_s^* q_s = \sum_s \overline{R}_s q_s, \text{ where } \overline{R}_s^* = (1 - \pi_s) R^* + \pi_s p_s^*. \tag{32}$$

This condition says that (when there is positive investment into the special location) the cost of investment (one unit) must be equal to the value of a replicating portfolio of Arrow-Debreu securities.[11] The second equality (which we established in Section 4.2) says that the same condition also holds for investment into regular locations. Hence, Eq. (32) can also be viewed as saying that the (foreign) banks are at the margin indifferent between investing in the special location and regular locations.

Next consider the investments made by the special location's banks, $x^{out,*}, y^*, (z_s^*)_s$. These are not uniquely determined, because there are multiple equivalent ways of obtaining the same payoff vector. We therefore define the location's liquidity purchase (or sale) in each state as,

$$l_s^* = x^{out,*} \overline{R}_s + y^* + z_s^* - \eta^*.$$

Note that l_s^* captures the additions to the location's liquidity starting with its endowment, η^*. We conjecture that the equilibrium features fire sales also in the special location, $p_s^* < R$ for

[10] As before, we focus on symmetric equilibria for regular countries so that $x^{in,*}$ (resp. $x^{out,*}$) denotes the risky investment inflows into (resp. outflows from) the special country from (resp. into) each regular country.

[11] We technically state this as an assumption because banks in regular locations are actually indifferent to take any nonnegative position in the special location, as this location has measure zero. In a version of the model in which the special countries have strictly positive but small mass $\Delta > 0$, condition (32) would always hold in equilibrium. Hence, the equilibrium we analyze can be viewed as the limit of these equilibria as $\Delta \to 0$.

each s. Then, banks in the special location solve the following analogue of problem (10),

$$\max_{x^{*,*}\geq 0,(l_s^*\geq -\eta^*)_s} u\left(c_0^*\right)+x^{*,*}R^*+\sum_s \gamma_s l_s^* M_s^*,$$
$$\text{where } c_0^*+x^{*,*}+\sum_s q_s l_s^*=1. \tag{33}$$

In Appendix A.6, we show that $x^{*,*}=0$, that is, local investment is dominated by foreign investment also for the special location. The remaining optimality conditions (for an interior solution) can be written as,

$$\frac{q_s}{\gamma_s}=\frac{M_s^*}{u'\left(c_0^*\right)}=\frac{M_s}{u'\left(c_0\right)}, \text{ where } M_s^*=1-\pi_s+\pi_s\frac{R^*}{p_s^*}. \tag{34}$$

Hence, the relative SDF is equal to the relative marginal utility of banks in the special location, as well as those in the regular locations (as we established in Section 4.2).

Finally, consider the equilibrium value of fire-sale prices in the special location, $(p_s^*)_s$. Using similar steps as before, these prices are determined by the following analogue of Eq. (13),

$$p_s^*=\frac{\eta^*+l_s^*}{e-\tilde{\chi}_s^*+x^{in,*}} \tag{35}$$
$$\text{where} \qquad \tilde{\chi}_s^*=0 \text{ if } \lambda p_s^*\geq R^* \text{ and } \tilde{\chi}_s^*=e \text{ if } \lambda p_s^*<R^*. \tag{36}$$

Here, $\tilde{\chi}_s^*$ denotes the amount of assets the distressed sellers in the special location optimally retain [cf. Eq. (1)], which is not necessarily zero unless we strengthen Assumption 1.[12] The following result establishes the existence of equilibrium.

Proposition 6. *Consider the model with Assumption 1 and $\pi_1>0$, together with a special location that satisfies Assumption 2. There exists an equilibrium in which the allocations and prices for regular locations are characterized by Propositions 1-2. In the special location, there is no local investment, $x^{*,*}=0$. The remaining allocations, $x^{in,*}, c_0, (l_s^*)_s, (p_s^*)_s, (\tilde{\chi}_s^*)_s$, are characterized as the unique solution to the system of equations* $(32-36)$.

We next use this model to illustrate capital flows driven by reach for safety and yield. Unlike regular locations, the special location trades safe and contingent assets in equilibrium so its inflows and outflows are not necessarily determined by investments in risky assets. We adopt the convention that total inflows are equal to the inflows into its risky assets, $\overline{x}^{in,*}\equiv x^{in,*}$;

[12] Specifically, we could ensure this by considering the limit, $\lambda\to\infty$. However, this is not necessary since the prices in the special location are not affected by the level of λ. If the parameters are such that the distressed sellers prefer to retain some assets, $\tilde{\chi}_s^*>0$, then banks in regular locations send greater inflows to neutralize the price impact of the additional demand (therefore leaving p_s^* unchanged). This also generates a source of innocuous indeterminacy for the knife-edge case, $\lambda p_s^*=R^*$, in which distressed sellers are indifferent to retain or sell assets. In (36), we resolve this indeterminacy by assuming that they sell when indifferent.

whereas total outflows account for the net trade of safe and contingent assets,[13]

$$\begin{aligned} \overline{x}^{out,*} &\equiv x^{out,*} + q_f \left(y^* - \eta^*\right) + \sum_s q_s z_s^* \\ &= \sum_s q_s l_s^* = 1 - c_0. \end{aligned} \tag{37}$$

Here, the second line uses the definition of l_s^* (together with $\sum_s \overline{R}_s q_s = 1$) as well as the budget constraint (33). Hence, total outflows are equal to the value of banks' liquidity purchases in period 0. Since there is no local investment in period 0, this is also equal to the banks' endowment in period 0 (one) net of their consumption.

7.2. Reach for safety and global imbalances

We first abstract away from return differences, $R^* = R$, and focus on the effect of asymmetries in the liquidity supply, η^*. A developed location with deep financial markets and a large supply of safe assets—such as the U.S.—can be thought of as featuring $\eta^* > \eta$. Conversely, an emerging market location is captured by $\eta^* < \eta$.

As a benchmark, consider the autarky allocation in which the location does not trade flows with regular locations. It is easy to check that the location's autarky price is characterized by $p_s^* = \min\left(R, \frac{\eta^*}{e}\right)$ for each s (see Proposition 1 for a similar result for regular locations). In particular, a developed location with sufficiently high liquidity, $\eta^* > eR$, would completely avoid fire sales whereas regular locations would experience fire sales, that is, $p_s^* = R > p_s$ for each s. In contrast, an emerging market location with $\eta^* < \eta$ would experience more severe fire sales than other locations, $p_s^* = \frac{\eta^*}{e} < p_s$ for each s.

We next analyze the equilibrium with free capital flows. Using Proposition 6, it is easy to verify that the equilibrium in the special location obtains when, $c_0^* = c_0, \tilde{\chi}_s^* = 0$, and,

$$\begin{aligned} p_s^* &= p_s < R \\ x^{in,*} &= x + (e + x)(\Lambda - 1), \\ \overline{x}^{out,*} &= x, \\ l_s^* &= \left(x\overline{R}_s + \eta\right)\Lambda - \eta^*, \end{aligned} \tag{38}$$

where we refer to Λ as *the leverage ratio of outflows*, and define it as,

$$\Lambda = \frac{x + q_f \eta^*}{x + q_f \eta}. \tag{39}$$

[13] With trade in safe and contingent assets, there is an indeterminacy in gross flows since the special location can always sell a financial asset to regular locations and purchase exactly the same asset from those locations. This would increase inflows as well outflows without any additional effects. Our definition excludes these types of spurious flows and focuses on the lowest level of gross flows that could emerge in our setting.

The first line in (38) says that the asset prices in the special location are the same as those in regular locations. In particular, even though a developed location with $\eta^* > eR$ would avoid fire sales in autarky, it cannot escape fire sales in equilibrium with free capital flows. Conversely, an emerging market location with $\eta^* < \eta$ obtains higher fire-sale prices with free capital flows than what it would obtain in autarky.

To understand these results, consider the case of a developed location with $\eta^* > \eta$ (the case with $\eta^* < \eta$ is symmetric). All else equal, greater liquidity in this location, increases the fire-sale prices, which increases the expected return, $\sum_s \overline{R}_s^* q_s$, above the level obtained in regular locations, $\sum_s \overline{R}_s q_s$. This temporarily violates the indifference condition (32) and makes the location's assets attractive to foreign banks. This translates into greater inflows, $x^{\text{in},*}$, which worsens fire-sale prices as illustrated by Eq. (35). This process stops only when the special location also experiences severe fire sales that equate its expected return with those in regular locations. In fact, the second and the third lines of Eq. (38) illustrate that the developed location receives more inflows than outflows (it has a current account deficit). These net inflows neutralize the location's initial liquidity advantage and induce fire sales in equilibrium.

While this intuition explains why the developed location experiences fire sales as severe as in the regular locations *on average*, it does not explain why the asset prices are equated *state-by-state*. In fact, from the earlier market clearing condition (14), one could expect a developed location to have relatively high prices in states with high π_s in which the global liquidity is low—because its greater domestic liquidity supply could provide some cushion. This does not happen in our model because banks in the developed location do not necessarily retain their initial endowments of liquidity. Rather, as captured by Eq. (34), they trade financial assets so as to move their liquidity across aggregate states.

The last line in (38) characterizes the equilibrium outcome from these trades. Banks in the developed location can be thought of as selling some of their safe asset endowments, $\eta^* - \eta\Lambda$ (which is positive when $\eta^* > \eta$), to make a leveraged investment in foreign diversified portfolio. We refer to Λ as the leverage ratio of outflows, because it captures the value of foreign risky asset investments divided by the value of outflows, $\sum_s q_s x \overline{R}_s \Lambda / x$. For a developed location, the leverage ratio is greater than one, $\Lambda > 1$, meaning that the location's outflows are riskier than in other locations. Intuitively, banks in the developed location are selling some of their excess liquidity to take advantage of the positive risk premium on foreign assets [cf. Eq. (19)]. In our model, this effect is strong and ensures that the location has the same (fire-sale) asset price as regular locations in every state.

Conversely, an emerging market location with $\eta^* < \eta$ has more outflows relative to its inflows ($\overline{x}^{out,*} = x > x^{in,*}$), and its outflows are also safer than those in regular locations, $\Lambda < 1$. Intuitively, the scarcity of liquidity in this location reduces the inflows, while also inducing the location to purchase safe assets from abroad to obtain additional liquidity (by paying the risk premium). These forces improve the fire-sale prices relative to what the location would obtain in autarky. The following result summarizes this discussion.

Proposition 7. *Consider the setup in Proposition 6 with $R^* = R$ and $\eta^* \neq \eta$. With free financial flows, the equilibrium allocations in the special location are given by* (38)*. Regardless of its liquidity supply, the location experiences fire sales with prices that are equal to those in regular locations, $p_s^* = p_s < R$ for each s. When $\eta^* > \eta$, the location receives more inflows than its outflows, $x^{in,*} > \overline{x}^{out,*} = x$, and has riskier (more leveraged) outflows than regular locations, $\Lambda > 1$. When $\eta^* < \eta$, the location has more outflows than its inflows, $\overline{x}^{out,*} = x > x^{in,*}$, and safer (less leveraged) outflows than regular locations, $\Lambda < 1$.*

These results suggest that the *reach-for-safety* flows have potentially destabilizing effects for developed markets with $\eta^* > \eta$ but stabilizing effects for emerging markets with $\eta^* < \eta$.

The results are also consistent with the empirical work by Gourinchas and Rey (2007); Gourinchas et al. (2010), who document that the outflows of the U.S. are riskier than its inflows. They show that the U.S. earns a risk premium on capital flows in normal times, but it also transferred resources and provided insurance to the rest of the world during the Global Financial Crisis. Our model suggests these transfers are likely to have exacerbated the severity of the GFC in the U.S., while mitigating its impact in the locations that held the (relatively) safe U.S. assets.

A natural question is how the global liquidity cycle we described earlier affects flows driven by reach for safety. We address this question in Appendix A.4, where we extend the analysis to the beta model we analyzed in Section 6. We show that an increase in the correlation parameter, β, increases the absolute value of the location's imbalances as a fraction of outflows, $\left|x^{\text{in},*} - x\right| / x$, as well the absolute value of its relative leverage ratio, $|\Lambda - 1|$. In particular, a developed location with $\eta^* > \eta$ increases its (proportional) current account deficit as well as the riskiness of its outflows. Conversely, an emerging market location with $\eta^* < \eta$ increases its (proportional) current account surplus and the safety of its outflows. These results suggest that the "risk-off" induced by the increase in β strengthens the flows driven by reach for safety.

7.3. Reach for yield

We next abstract away from differences in liquidity supply, $\eta^* = \eta$, and investigate the effect of asymmetries in return, R^*. We focus on the more interesting case with $R^* > R$, so that the special location can be thought of as a rapidly growing or high yielding emerging market country. These types of countries appear to have relatively attractive fundamental returns, especially in recent years in which the asset returns in developed markets have been unusually low.

Note that Eqs. (32) and (34) can be combined to obtain,

$$\sum_{s \in S} q_s \left(\left(1 - \pi_s\right) R^* + \pi_s p_s^* \right) = \sum_{s \in S} q_s \left(\left(1 - \pi_s\right) R + \pi_s p_s \right), \tag{40}$$

$$\frac{1 - \pi_s + \pi_s \frac{R^*}{p_s^*}}{1 - \pi_s + \pi_s \frac{R}{p_s}} = \frac{u'\left(c_0^*\right)}{u'\left(c_0\right)} \text{ for each } s \in S. \tag{41}$$

This represents a system of $|S|+1$ equations in $|S|+1$ unknowns, $\left(p_s^*\right)_s, c_0^*$. In Appendix A.6, we show that there is a unique solution to this system (see Lemma 4). The remaining equilibrium allocations are characterized by solving the remaining equations listed in Proposition 6.

We also show that when $R^* > R$ the solution satisfies, $p_s^* < p_s$ for each s, that is, the fire-sale prices in the special location are lower than in regular locations. To understand this result, suppose the locations had identical fire-sale prices, $p_s^* = p_s$. This would violate the indifference condition (40), because foreign banks would strictly prefer to invest in the special location. This would increase the inflows into the special location $x^{in,*}$, and lower the fire-sale prices according to (35). This process stops only when the special location's fire-sale prices are lower and the indifference condition is reestablished. Using more subtle arguments, it can further be seen that the special location obtains a lower fire-sale price in every aggregate state s.

Furthermore, we show that p_s^*/p_s is strictly increasing in s: that is, fire-sale prices in the special location are relatively higher in aggregate states with greater likelihood of liquidity shocks. Intuitively, as illustrated by Eq. (41), local banks in the special location distribute their liquidity across states so as to equate their expected marginal utilities. Since crises are more frequent in states with greater s, they purchase relatively more liquidity insurance for these states. This helps to mitigate somewhat the fire sales caused by the reach-for-yield inflows in states with greater s (e.g., the global financial crisis), at the expense of deepening the fire sales in states with smaller s (more localized crises).

Relatedly, the solution satisfies, $c_0^* < c_0$, which implies $\overline{x}^{out,*} > x$ [cf. Eq. (37)]. Thus, the special location has greater outflows than the regular locations. This reflects that the banks in the special location take precaution not only by purchasing more insurance (as in the previous result) but also by holding more foreign assets. Nonetheless, with free capital flows, these attempts at obtaining greater insurance make the location attractive to foreigners and ultimately translate into greater inflows. Formally, we show that $x^{in,*} > \overline{x}^{out,*} > x$: the reach for yield increases the location's inflows more than its outflows. The following result summarizes this discussion.

Proposition 8. *Consider the setup in Proposition 6 with $\eta^* = \eta$ and $R^* > R$. With free financial flows, the special location experiences deeper fire sales than the regular locations in all states but less so in more distressed states, that is, $p_s^*/p_s < 1$ for each s, and p_s^*/p_s is strictly increasing in s. The special location's inflows exceeds its outflows, $x^{in,*} > \overline{x}^{out,*}$, which in turn exceeds the gross flows in (otherwise comparable) regular locations, $\overline{x}^{out,*} > x$.*

These results are consistent with a growing empirical literature on the impact of depressed interest rates in developed markets on the surge of capital inflows to emerging markets (see, for instance, Shin (2014); Tillmann (2016)).

In Appendix A.4, we extend this analysis to the beta model from Section 6 so as to investigate how the global return and risk conditions affect the reach for yield. We show that, all else equal, a decline in investment return in regular locations, R, as well as the correlation parameter, β, reduces a weighted-average fire-sale price in the special location relative to its counterpart in regular locations. The intuition regarding R is that lower returns in other locations make investing in high-yielders more attractive, but this is ultimately countered by more severe fire-sale prices. The intuition regarding β is that investing in high-yielders makes losses during local crises, and these losses are less costly when the local crises are less correlated with aggregate distress states. Thus, a reduction in correlations also makes investing in high-yielders more attractive and lowers their fire-sale prices.

8. Final Remarks

We developed a global equilibrium model of capital flows that addresses the tension between their fickleness during foreign crises and retrenchment during local crises. Our main finding is that gross capital flows create liquidity and stabilize domestic fire sales. Moreover, the destabilizing forces in the reach-for-safety and reach-for-yield scenarios we considered stem not from the associated gross flows, but from their imbalance (net flows).

In the model the distinction is stark: symmetric flows are stabilizing while imbalances are not for the net recipient country. However, the practical message is broader as ultimately what matters for stabilization is not whether a country has a current account deficit or not, but whether its fickle inflows exposed to domestic fire sales are comparable in magnitude to retrenchable outflows. When viewed in this light, it is apparent that, e.g., (non-fickle) foreign direct investment flows are stabilizing while fully collateralized short term debt denominated in foreign currency is not (it is fickle but not exposed to domestic fire sales).

Finally, while one could loosely connect the net imbalances in our model with exchange rates, we leave a proper analysis of this important dimension for future work. In practice, exchange rate movements are dominated by forces that drive asset markets and as such the forces behind flows in our model should also affect exchange rates, at least over short horizons. In turn, movements in exchange rates generate important valuation effects, which in all likelihood interact with the insurance mechanisms we highlight in this paper.

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Online Appendix

This appendix is organized as follows. Appendices A.1-A.5 analyze various extensions of the model in the main text. Appendix A.6 contains the proofs omitted from the main text or the appendices.

A.1. Endogenizing fickleness with Knightian uncertainty

In the main text, we assumed that the banks are fickle in the sense that, if there is a liquidity shock in a foreign location, then they sell their asset holdings in that location regardless of the price. We view this assumption as a modeling device to capture the heterogeneous behavior of foreigners and locals during local crises that we systematically observe in the data (see Section 2). We also view it as capturing various factors that could handicap foreigners during local crises, such as asymmetric information or Knightian uncertainty, deterioration of property rights, asymmetric regulation, and so on. In this section, we explicitly incorporate one such factor, Knightian uncertainty, and illustrate that it can endogenously generate fickleness.

The model is the same as in the main text with the difference that foreign banks do not have to sell the assets of a location that experiences a liquidity shock in period 1. Instead, banks face uncertainty about the asset's payoff in period 2, and choose actions that are robust with respect to this uncertainty.

Formally, consider a location j' that experiences a liquidity shock in period 1, that is, $\omega^{j'} = b$. In the main text, we assumed the asset pays R units in period 2 with certainty. We now suppose the banks in location j believe these assets pay R units in period 2 with probability $\phi^{j',j} \in [0,1]$, and zero units in period 2 with probability $1-\phi^{j',j}$. Moreover, the banks face uncertainty about the parameter $\phi^{j',j}$, in the sense that they consider a range of probabilities possible. We let $\left[\underline{\phi}^{j',j}, \overline{\phi}^{j',j}\right]$ (with $\underline{\phi}^{j',j} \leq \overline{\phi}^{j',j}$) denote the range of possibilities the banks find possible regarding the assets in location j'.

Following Gilboa and Schmeidler (1989)'s Maximin expected utility representation, we assume the banks maximize the following objective function (in period 1 as well as period 0),

$$\min_{\left\{\phi^{j',j} \in \left[\underline{\phi}^{j',j}, \overline{\phi}^{j',j}\right]\right\}_{j' \in J}} E\left[u\left(c_0^j\right) + c_{1,s}^j + c_{2,s}^j\right]. \tag{A.42}$$

Thus, the banks act according to the worst case scenario within the range of probabilities that they find possible.[14] We capture fickleness by assuming that foreigners face greater parameter uncertainty than locals, that is,

$$\underline{\phi}^{j,j} = \overline{\phi}^{j,j} = 1 \text{ for each } j, \text{ and } \underline{\phi}^{j',j} \equiv \underline{\phi} < \underline{\phi}^{j',j} = 1 \text{ for each } j, j'. \tag{A.43}$$

In particular, foreigners' worst case probability, captured by $\underline{\phi}$, is worse than locals' worse case probability, which we assume is equal to one. The latter assumption also implies that locals do not face any parameter uncertainty regarding the risky assets in their own location. This does not play an important role beyond simplifying the analysis.

We let $\chi_s^{j',j}$ denote the position that the banks in location j take in period 1 in the risky assets of location j' (conditional on that location experiencing a liquidity shock, $\omega^{j'} = b$). In the main text, we assumed $\chi_s^{j',j} = 0$ for each $j' \neq j$, and we used the shorthand notation $\chi_s^j \equiv \chi_s^{j,j}$ to capture local investment. Here, the banks make an investment plan for all locations, $\left[\chi_s^{j',j} \geq 0\right]_{j'}$, where we impose

[14] This behavior is arguably reasonable for situations in which economic agents face ambiguity as opposed to quantifiable uncertainty. We believe crises are typically associated with this type of uncertainty (see Caballero and Krishnamurthy (2008); Caballero and Simsek (2013) for further discussion).

$\chi_s^{j,j}$ to be a point mass, and $\chi_s^{j',j}$ for $j' \neq j$ to be a density with respect to the Lebesgue measure. As before, the banks also choose the period 0 allocations, $\left[x^{j',j} \geq 0\right]_{j'}, y^j, \left(z_s^j\right)_s$, subject to the restrictions described in Section 3.

The equilibrium with symmetric prices is a collection of allocations, $\left[x^{j',j} \geq 0\right]_{j'}, y^j, \left(z_s^j\right)_s, \left[\chi_s^{j',j}\right]_{j'}$ that maximize the objective function in $(A.42)$ for each j, and prices $(p_s)_s, q_f, (q_s)_s$ that ensure market clearing. The market clearing conditions for the safe asset and the Arrow-Debreu securities are the same as conditions (8) and (9) in the main text, but the market clearing condition for risky assets of location j in period 1 is slightly different and given by [cf. condition (7)],

$$e + x^{in,j} + x^{j,j} = \tilde{\chi}_s^j + \chi_s^{j,j} + \int_{j'} \chi_s^{j,j'} dj'. \tag{A.44}$$

In particular, we now also take into account the possible demand for the risky asset that comes from other locations' banks, $\int_{j'} \chi_s^{j,j'} dj'$.

We characterize the equilibrium for parameters that satisfy Assumption 1 as well as the following.

Assumption K. $\left(1 - \underline{\phi}\right) R < \eta/e$.

With these parameters, we conjecture that the equilibrium characterized in Section 4 remains an equilibrium also in this context. In particular, we claim that it is optimal for banks to choose $\chi_s^{j',j} = 0$ for each $j' \neq j$. To verify this, note that Eq. (14) implies the equilibrium price satisfies $p_s \geq \eta/e$ for each s. Combining this with Assumption K implies $p_s > \left(1 - \underline{\phi}\right) R$ for each s. Thus, foreign banks believe that the asset's price exceeds its payoff under the worst case probability. In view of the preferences in $(A.42)$, it is then easy to check that choosing $\chi_s^{j',j} = 0$ for each $j' \neq j$ is an optimal strategy.

Next note that, after substituting $\chi_s^{j',j} = 0$ for each $j' \neq j$ into the objective function in $(A.42)$ and observing that investors do not face any Knightian uncertainty in their own location, we recover the same objective function as in the main text [cf. Eq. (2)]. Likewise, after substituting $\chi_s^{j',j} = 0$ for each $j' \neq j$ into the market clearing condition $(A.44)$, we recover the same market clearing condition as in the main text [cf. Eq. (7)]. It follows that the equilibrium that we characterized in Section 4 remains an equilibrium also in this context.

A.2. Complete markets with respect to local liquidity shocks

In the main text we assumed the banks cannot trade financial contracts contingent on the realizations of local liquidity shocks. In this section we relax this assumption. We construct a version of the model in which markets are complete with respect to local liquidity shocks, and we establish that the equivalent of our main result (Proposition 1) also applies in this case. For simplicity, we also focus attention on the case in which S is a singleton so there is no aggregate risk (the results can be generalized). This implies that the Arrow-Debreu securities for aggregate states are redundant so we drop them from the notation (equivalently, we set $z_s^j = 0$ for each j). Throughout, we also drop the subscript s from the notation.

The model with complete markets. There are now location-specific Arrow-Debreu securities. The security for location j pays one unit of the consumption good if $\omega^j = b$ is realized in period 1 (and zero units if $\omega^j = g$ is realized or in period 2). In period 0, the Arrow-Debreu security for location j is traded at an endogenous price q_{loc}^j. We concentrate our attention on symmetric equilibria in which the price is the same for all locations, that is, $q_{loc}^j \equiv q_{loc}$ for each j. Location-specific Arrow-Debreu securities are in zero net supply.

In period 0, banks choose an investment strategy on these Arrow-Debreu securities denoted by $z_{loc}^{j',j}$. We impose that $z_{loc}^{j,j}$ is point mass, and $z_{loc}^{j',j}$ for $j' \neq j$ is a density with respect to the Lebesgue measure. As before, banks also choose the investment strategy on risky assets, $x^{j',j}$, and their position on the risk-free asset, y. Their budget constraint in period 0 is the following analogue of Eq. (3),

$$c_0 + x^{j,j} + x^{out,j} + yq_f + \left(z_{loc}^{j,j} + z_{loc}^{out,j}\right) q_{loc} = 1 + \eta q_f, \tag{A.45}$$
$$\text{where } x^{out,j} = \int_{j' \neq j} x^{j',j} dj' \text{ and } z_{loc}^{out,j} = \int_{j' \neq j} z_{loc}^{j',j} dj'.$$

As before, banks are not allowed to short-sell risky assets, $x^{j',j} \geq 0$ for each j', but they are allowed to take unrestricted positions on the risk-free asset or location-specific Arrow-Debreu securities subject to obtaining nonnegative consumption in all periods and states.

In period 1, if $\omega^{j'} = g$ and $j' \neq j$, then banks receive zero units from the Arrow-Debreu security for their own location. By an exact law of large numbers, they also receive $\pi z_{loc}^{out,j}$ units from the Arrow-Debreu securities on other locations. Combining this with the analysis in Section 3, the banks' budget constraint in state $\omega^j = g$ is the following analogue of Eq. (5),

$$\begin{aligned} c_1\left(\omega^j = g\right) &= x^{j,j}R + x^{out,j}\overline{R} + y + \pi z_{loc}^{out,j} \\ \text{and } c_2\left(\omega^j = g\right) &= 0. \end{aligned} \tag{A.46}$$

If instead $\omega^j = b$, then banks receive $z_{loc}^{j,j}$ units from the Arrow-Debreu security of their own location, and still $\pi z_{loc}^{out,j}$ units from the Arrow-Debreu securities on other locations. Their budget constraint in state $\omega^j = b$ is then the following analogue of Eq. (6),

$$\begin{aligned} c_1\left(\omega^j = b\right) + \chi^{j,j}p &= x^{j,j}p + x^{out,j}\overline{R} + y + z_{loc}^{j,j} + \pi z_{loc}^{out,j}, \\ c_2\left(\omega^j = b\right) &= \chi^{j,j}R. \end{aligned} \tag{A.47}$$

Putting everything together, banks in each location j make an investment plan, $\left[x^{j',j} \geq 0\right]_{j'}, y^j, \left[z_{loc}^{j',j} \geq 0\right]_{j'}, \chi^j$, to maximize their expected utility, $E\left[u\left(c_0\right) + c_1 + c_2\right]$, where where c_0 is determined by Eq. (3); $c_1^j\left(\omega^j = g\right)$ and $c_2^j\left(\omega^j = g\right)$ are determined by Eq. $(A.46)$, and $c_1^j\left(\omega^j = b\right)$ and $c_2^j\left(\omega^j = b\right)$ are determined by Eq. $(A.47)$; and consumption in all periods and states are nonnegative, $c_0^j \geq 0, c_1^j \geq 0, c_2^j \geq 0$.

The equilibrium with symmetric prices is a collection of optimal allocations for distressed sellers and banks, together with prices, p, q_f, q_{loc}, that ensure market clearing. The market clearing conditions for risky and safe assets are the same as Eqs. (7) and (8) in the main text. For each location j, there is also a market clearing condition for the corresponding Arrow-Debreu security, which can be written as,

$$z_{loc}^{j,j} + z_{loc}^{in,j} = 0 \text{ for each } j, \text{ where } z_{loc}^{in,j} = \int_{j' \neq j} z_{loc}^{j,j'} dj'. \tag{A.48}$$

Liquidity creation with complete markets. We characterize the equilibrium under the following strengthening of Assumption 1.

Assumption 1$^{\text{C}}$. $eR/\lambda < \eta < eR\pi - \underline{x}\left(1-\pi\right)R$, where $\underline{x} \in (0,1)$ is the solution to $u'\left(1-\underline{x}\right) = R$.

For any $\pi \in (0,1)$, the required upper bound on η is smaller than its counterpart in Assumption 1, eR.

However, it approaches that upper bound as $\pi \to 1$. Hence, for any η that satisfies Assumption 1, this assumption also holds for sufficiently large levels of the probability of the liquidity shock, π.

Under Assumption 1^{C}, we conjecture an equilibrium with symmetric prices that satisfy $p \in (R/\lambda, R)$. As before, distressed sellers sell all of their endowments, $\tilde{\chi}^j = 0$. For banks, we conjecture symmetric equilibrium allocations that satisfy, $x^{out,j} = x^{out}, y^j \equiv y$, as well as,

$$z_{loc}^{out,j} \equiv -z \text{ and } z_{loc}^{j,j} \equiv z.$$

Hence, banks choose symmetric allocations on foreign Arrow-Debreu securities, which we denote by $-z$. They also take the opposite position in the Arrow-Debreu security of their own location, denoted by z, so that the market clearing condition $(A.48)$ is satisfied.

Following similar steps as in Section 4, the budget constraints $(A.45), (A.46), (A.47)$ imply,

$$\begin{aligned} c_0 + x^{j,j} + x^{out} &= 1, \\ c_1\left(\omega^j = g\right) &= x^{j,j} R + x^{out} \overline{R} + \eta - \pi z, \\ c_2\left(\omega^j = b\right) &= \left(x^{j,j} p + x^{out} \overline{R} + \eta + z - \pi z\right) \frac{R}{p}. \end{aligned}$$

Substituting these expressions into the objective function and rearranging terms, the representative investor solves the following analogue of problem (10),

$$\begin{aligned} &\max_{x^{j,j}, x^{out}, z} u\left(1 - x^{j,j} - x^{out}\right) + x^{j,j} R + \left[\begin{array}{c} (1-\pi)\left(x^{out}\overline{R} + \eta - \pi z\right) \\ +\pi\left(x^{out}\overline{R} + \eta + (1-\pi) z\right) \frac{R}{p} \end{array} \right], \qquad \text{(A.49)} \\ \text{s.t.} \quad & x^{out}\overline{R} + \eta - \pi z \geq 0 \text{ and } x^{out}\overline{R} + \eta + (1-\pi) z \geq 0. \end{aligned}$$

Note that banks' marginal utility from increasing z is given by, $\pi(1-\pi)(-1+R/p)$, which is strictly positive because $p < R$. Thus, banks set z as large as possible subject to the nonnegative consumption constraints, which implies,

$$z = \frac{x^{out}\overline{R} + \eta}{\pi}. \qquad \text{(A.50)}$$

Intuitively, increasing z is similar to purchasing insurance with respect to local liquidity shocks: it transfers wealth from the state without a liquidity shock to the state with a liquidity shock. Banks purchase as much insurance as possible because their marginal utility is greater when there is a local liquidity shock.

After substituting $(A.50)$, banks' problem $(A.49)$ can be written as,

$$\max_{x^{j,j}, x^{out}} u\left(1 - x^{j,j} - x^{out}\right) + x^{j,j} R + \left(x^{out}\overline{R} + \eta\right) \frac{R}{p}.$$

Since $\overline{R}R/p > R$, we have $x^{j,j} = 0$, that is, local investment is dominated also when markets are complete. Foreign investment is determined by the following analogue of the optimality condition (12),

$$u'(1-x) = \left((1-\pi) R + \pi \frac{R}{p}\right) \frac{R}{p}, \qquad \text{(A.51)}$$

where we used $x = x^{out} = x^{in}$ to denote the gross flows and substituted $\overline{R} = (1-\pi) R + \pi \frac{R}{p}$.

Following similar steps as in Section 4, the market clearing condition for risky assets can be written as the following analogue of Eq. (13),

$$p = \frac{\eta + x^{out}\overline{R} + (1-\pi)\, z}{e + x^{in}}. \tag{A.52}$$

After substituting Eq. $(A.50)$ and rearranging terms, we also obtain the following analogue of Eq. (14),

$$p = \frac{\eta + x\,(1-\pi)\, R}{e\pi}. \tag{A.53}$$

To understand this condition, note that the denominator, $e\pi$, captures the aggregate amount of liquidity need from distressed sellers in period 1. The numerator captures the aggregate amount of liquidity available in period 1, which comes from the supply of the safe asset, η, as well as the aggregate payoff from investments in locations that do not experience a liquidity shock, $x\,(1-\pi)\,R$. Thanks to complete markets, all aggregate liquidity is pooled together and used to finance distressed sellers' projects in locations that experience a liquidity shock.

The equilibrium is characterized as the pair, (x, p), that solves Eqs. $(A.51)$ and $(A.53)$. Under Assumption 1^{C}, it can be checked that there is a unique solution that satisfies $x \in (\underline{x}, 1)$ and $p \in (R/\lambda, R)$, verifying our conjecture. Banks' positions in Arrow-Debreu securities is characterized by Eq. $(A.50)$ given the equilibrium levels of x and p.

It follows that, as long as aggregate liquidity is sufficiently scarce, the equilibrium features fire sales also in this case. Complete markets help to distribute all available liquidity to locations that experience liquidity shocks, thereby making it harder to obtain fire sales in equilibrium, but they do not necessarily prevent fire sales. As this intuition would suggest, the fire-sale price in this case is always strictly greater than its counterpart in the baseline model. Specifically, comparing Eqs. $(A.51 - A.53)$ with $(12 - 14)$ in the baseline setting, note that both curves (in the $x - p$ space) are shifted upwards in this setting. This leads to a greater fire-sale price, p. The upward shift of the market clearing curve reflects the more efficient allocation of liquidity. The upward shift of the optimality curve means that, for a given level of fire-sale price p, banks undertake greater foreign investment (because liquidity from foreign investment is allocated more efficiently). Both effects increase the fire-sale price, p. On the other hand, the equilibrium level of x might be lower than before, because a high fire-sale price lowers the need for obtaining liquidity via foreign investment.

Our main result regarding the liquidity-creation role of gross flows continues to apply in this setting (cf. Proposition 1). Specifically, Eq. $(A.53)$ illustrates that the fire-sale price is increasing in x. For comparison, consider also the autarky outcome in which all foreign risky investment is banned, $x^{j',j} = 0$ for each $j' \neq j$ (but investment in Arrow-Debreu securities for foreign locations are allowed). It can be checked that the equilibrium price in autarky is given by $p^{aut} = \eta / (e\pi)$, which is strictly lower than the equilibrium price with free capital flows. Thus, foreign investment increases the fire-sale prices also in this setting.

Our result regarding the public good aspect of liquidity creation also applies in this setting (cf. Section 5.2). Specifically, Eq. $(A.52)$ illustrates that outflows and inflows have opposing effects on the fire-sale price. It can then be checked that policymakers with the objective of increasing fire-sale prices would ban inflows in a Nash equilibrium, whereas a global policymaker would allow them.

A.3. Equilibrium with bans on capital flows

This section completes the analysis of equilibrium with policymakers that choose whether to ban capital inflows that we described in Section 5.2. Recall also that we focus on the limit, $\lambda \to \infty$. We first illustrate that in this limit, policymakers can be equivalently thought of as maximizing the local fire-sale price, p^j. Note that distressed sellers' expected consumption during a liquidity shock is given by [cf. Eq. (1)],

$$\tilde{c}_2^j \left(\omega^j = b\right) = \max_{\tilde{\chi}^j} \left(\tilde{\chi}^j R + \lambda \left(e - \tilde{\chi}^j\right) p^j\right).$$

Thus, as long as p^j is bounded from below (which is the case in the scenarios we will consider), $\lim_{\lambda \to \infty} \tilde{c}_2^j \left(\omega^j = b\right) / \lambda = ep^j$. Next note that utilitarian social welfare function satisfies,

$$\frac{W^j}{\lambda} = \frac{u\left(c_0^j\right) + E\left[c_1^j + c_2^j\right] + (1-\pi)\, \tilde{c}_2^j \left(\omega^j = g\right)}{\lambda} + \pi \frac{\tilde{c}_2^j \left(\omega^j = b\right)}{\lambda}.$$

Taking the limit as $\lambda \to \infty$, and observing that all terms except for the last one approach zero, we obtain, $\lim_{\lambda \to \infty} W^j / \lambda = \pi e p^j$. Since $\pi, e > 0$, maximizing this expression is the same as maximizing the local fire-sale price, p^j. Intuitively, in this limit, the balance sheet channel is very strong, and the planners focus on increasing the investment of distressed sellers.

Next recall that the banning decision is denoted by $b^j \in \{0,1\}$. Banks choose their portfolio, $\left[x^{j',j} \geq 0\right]_{j'}$, subject to the additional constraint that $x^{j',j} = 0$ for each $j' \neq j$ and $b^{j'} = 1$. The remaining ingredients are the same as in Section 3. We first characterize the equilibrium for a given configuration of banning decisions. We then consider the Nash equilibrium in which the banning decisions are optimal.

The extreme cases in which $b^j = 0$ for each j (free capital flows), or $b^j = 1$ for each j (autarky) are characterized in Section 4.1. Consider the intermediate case in which capital flows are banned in some locations. Specifically, suppose the sets of locations with $b^j = 1$ ("banned locations") and $b^j = 0$ ("free locations") are Lebesgue measurable, respectively with measures $B \in [0,1)$ and $1 - B > 0$. In this case, we consider a symmetric equilibrium in which each location invests the same amount in the safe asset and contingent securities, $y^j = \eta$ and $z = 0$; each ban location makes a symmetric investment in each free location, denoted by x^{ban}, and obtains a price denoted by p^{ban}; and each free location makes a symmetric investment in each free location (other than itself), denoted by x^{free}, and obtains a price denoted by p^{free}. Under Assumption 1, we also conjecture that a symmetric equilibrium features fire sales and strictly positive outflows in both banned and free locations, $p^{ban}, p^{free} \in (0, R)$ and $x^{ban}, x^{free} > 0$. We will also find it convenient to work with the modified variables, $X^{ban} > (1-B)\, x^{ban}, X^{free} = (1-B)\, x^{free}$ that describe the total amount of foreign investment made by respectively each banned and free location.

First consider the free locations. Following similar steps as in Section 4, banks solve the following analogue of problem (10),

$$\max_{x^{j,j} \geq 0, X^{free} \geq 0} u\left(1 - x^{j,j} - X^{free}\right) + x^{j,j} R + \left(X^{free} \overline{R}^{free} + \eta\right) M^{free},$$

where $\overline{R}^{free} = (1-\pi)\, R + \pi p^{free}$ and $M^{free} = 1 - \pi + \pi R / p^{free}$. By Lemma 1, we have $M^{free} \overline{R}^{free} = \mu\left(p^{free}\right) > R$. Thus, in free locations, local investment is dominated, $x^{j,j} = 0$, and foreign investment is characterized by the optimality condition,

$$u'\left(1-X^{free}\right)=\mu\left(p^{free}\right). \tag{A.54}$$

Note also that each free location's inflows are given by,

$$x^{in,j}=\int_{j'}x^{j',j}dj'=(1-B)\,x^{free}+Bx^{ban}=X^{free}+\frac{BX^{ban}}{1-B}.$$

Using Eq. (6) (and symmetry), its banks' asset demand in period 1 is given by, $\chi^j = \left(\eta+X^{free}\overline{R}^{free}\right)/p^j$. Note also that, since we consider the limit in which $\lambda\to\infty$ and the fire-sale price is positive, distressed sellers sell all of their endowments, $\tilde{\chi}^j=0$ [cf. Eq. (1)]. Substituting these expressions into Eq. (7), we obtain the market clearing condition,

$$p^{free}=\frac{\eta+X^{free}\overline{R}^{free}}{e+X^{free}+BX^{ban}/\left(1-B\right)}. \tag{A.55}$$

Next consider the banned locations. Banks solve a similar problem,

$$\max_{x^{j,j}\geq 0,X^{ban}\geq 0} u\left(1-x^{j,j}-X^{ban}\right)+x^{j,j}R+\left(X^{ban}\overline{R}^{free}+\eta\right)M^{ban},$$

where $M^{ban}=1-\pi+\pi R/p^{ban}$. In this case, $\overline{R}^{free}M^{ban}$ is not necessarily strictly greater than R, and there might be some local investment. Using our conjecture, $X^{ban}>0$, the optimality conditions can be written as,

$$\begin{aligned} u'\left(1-x^{j,j}-X^{ban}\right) &= M^{ban}\overline{R}^{free} \\ u'\left(1-x^{j,j}-X^{ban}\right) &\geq R \text{ with strict inequality only if } x^{j,j}=0. \end{aligned}$$

To simplify these conditions, we let $\overline{p}^{ban}\in(0,R)$ denote the unique solution to the equation,

$$\left(1-\pi+\pi R/\overline{p}^{ban}\right)\left(\left(1-\pi\right)R+\pi p^{free}\right)=R. \tag{A.56}$$

We then combine the two optimality conditions to obtain the following,

$$\begin{cases} u'\left(1-X^{ban}\right)=\left(1-\pi+\pi R/p^{ban}\right)\overline{R}^{free}, & \text{if } p^{ban}<\overline{p}^{ban} \\ X^{ban}\in[0,\underline{x}] & \text{if } p^{ban}=\overline{p}^{ban} \end{cases}, \tag{A.57}$$

where $\underline{x}$ denotes the solution to $u'\left(1-\underline{x}\right)=R$. Hence, if the fire-sale price is equal to the upper bound, $\overline{p}^{ban}$, then banks in banned locations are indifferent between local and foreign investment. If the price is below this level, they make only foreign investment.

Finally, note also that a banned location's inflows are zero, $x^{in,j}=0$. Following similar steps as in free locations, we obtain the market clearing condition,

$$p^{ban}=\frac{\eta+X^{ban}\overline{R}^{free}}{e}. \tag{A.58}$$

The equilibrium is characterized by the tuple, $\left(X^{free},p^{free},X^{ban},p^{ban}\right)$, that solve Eqs.

$(A.54), (A.55), (A.57)$, and $(A.58)$. The following lemma, the proof of which is relegated to the end of this section, verifies that there exists an equilibrium that satisfies, $p^{free}, p^{ban} \in (0, R), X^{free}, X^{ban} \in (0, 1)$. It also shows that, in every such equilibrium, fire-sale prices in banned locations are greater than in free locations.

Lemma 3. *For each $B \in [0, 1)$, there exists a tuple, $p^{free}, p^{ban} \in (0, R), X^{free}, X^{ban} \in (0, 1)$, that solves Eqs. $(A.54), (A.55), (A.57)$, and $(A.58)$. Moreover, every solution satisfies $p^{ban} > p^{free}$.*

Next consider a Nash equilibrium in which policymakers choose $b^j \in \{0, 1\}$ to maximize p^j. Since every symmetric equilibrium with $B \in [0, 1)$ features $p^{ban} > p^{free}$, the only candidate for a Nash equilibrium is the autarky allocation in which all policymakers ban inflows, $b^j = 1$ for each j and $B = 1$.

We finally check that, under no arbitrage, the autarky allocation corresponds to a Nash equilibrium. Recall that in autarky every location (which is a banned location) invests a positive amount in the local risky asset, $\underline{x} > 0$, and features features the price, $p^{ban} = \eta/e$. Suppose the policymaker in an (infinitesimal) location j switches to $b^j = 0$ and allows capital flows. We let p^{free} denote the equilibrium fire-sale price that would obtain in this location. Under no arbitrage, this price satisfies $p^{free} < p^{ban} = \eta/e$. To see this, suppose $p^{free} \geq p^{ban}$. This would imply $M^{ban}\overline{R}^{free} \geq M^{free}\overline{R}^{free} > R$ (cf. Lemma 1), which in turn would violate no arbitrage because in equilibrium banned locations invest a positive amount in local risky assets. This proves $p^{free} < p^{ban}$, which in turn proves that the autarky allocation is a Nash equilibrium.

Proof of Lemma 3. First note that, given any $X^{ban} \in (0, 1)$, there exists a unique pair, $p^{free} \in (0, R)$ and $X^{free} \in (0, 1)$, that solves Eqs. $(A.54)$ and $(A.55)$. Moreover, increasing X^{ban} weakly decreases P^{free} and weakly increases X^{free}. We therefore denote the solution with $\mathbf{p}^{free}\left(X^{ban}\right)$ and $\mathbf{X}^{free}\left(X^{ban}\right)$, and note that $\mathbf{p}^{free}(\cdot)$ is a weakly decreasing function and $\mathbf{X}^{free}(\cdot)$ is a weakly increasing function. Note also that $\mathbf{p}^{free}(0) = p$ and $\lim_{X^{ban} \to 1} \mathbf{p}^{free}\left(X^{ban}\right) = 0$, where p denotes the equilibrium price in the baseline model with free flows.

Next note that Eq. $(A.57)$ describes p^{ban} as a function of X^{ban} and p^{free}, which we denote with $\mathbf{p}_1^{ban}\left(X^{ban}, p^{free}\right)$. It can be checked that $\mathbf{p}_1^{ban}(\cdot)$ is weakly decreasing in X^{ban} and strictly increasing in p^{free}. Hence, after substituting the weakly decreasing function, $\mathbf{p}^{free}\left(X^{ban}\right)$, we obtain the weakly decreasing function, $f\left(X^{ban}\right) \equiv \mathbf{p}_1^{ban}\left(X^{ban}, \mathbf{p}^{free}\left(X^{ban}\right)\right)$. Note also that $\lim_{X^{ban} \to 1} f\left(X^{ban}\right) = 0$, and

$$f(0) = \mathbf{p}_1^{ban}(0, p) = \overline{p}^{ban} > p,$$

where the last inequality follows from applying Eq. $(A.56)$ with $p^{free} = p$ and observing that $\mu(p) > R$.

Likewise, note that Eq. $(A.58)$ describes p^{ban} as another function of X^{ban} and p^{free}, which we denote with $\mathbf{p}_2^{ban}\left(X^{ban}, p^{free}\right)$. After substituting $\mathbf{p}^{free}\left(X^{ban}\right)$, we obtain the continuous function, $g\left(X^{ban}\right) \equiv \mathbf{p}_2^{ban}\left(X^{ban}, \mathbf{p}^{free}\left(X^{ban}\right)\right)$. Note also that $\lim_{X^{ban} \to 1} g\left(X^{ban}\right) > 0$ and $g(0) = \eta/e < p$ (since p denotes the equilibrium in the baseline model).

Using the continuity of functions $f(\cdot)$ and $g(\cdot)$, it follows that there exists $X^{ban} \in (0, 1)$ such that $f\left(X^{ban}\right) = g\left(X^{ban}\right)$. By definitions of $f(\cdot)$ and $g(\cdot)$, X^{ban} corresponds to a solution. The remaining variables are found from, $p^{free} = \mathbf{p}^{free}\left(X^{ban}\right), X^{free} = \mathbf{X}^{free}\left(X^{ban}\right)$, and $p^{ban} = f\left(X^{ban}\right)$. It can also be checked that the solution is interior and satisfies $p^{free}, p^{ban} \in (0, R), X^{free}, X^{ban} \in (0, 1)$.

We next show that, in every solution, the banned locations feature greater fire-sale prices, $p^{ban} > p^{free}$. Suppose, to reach a contradiction, that $p^{ban} \leq p^{free}$. Eq. $(A.56)$ implies that $\overline{p}^{ban} > p^{free}$. Thus, we have $p^{ban} \leq p^{free} < \overline{p}^{ban}$. Eq. $(A.57)$ then implies that $x^{j,j} = 0$ and outflows are determined by

$$u'\left(1 - X^{ban}\right) = \left(1 - \pi + \pi R/p^{ban}\right) \overline{R}^{free} \geq \mu\left(p^{free}\right),$$

where the last inequality follows since $p^{ban} \leq p^{free}$. Combining this with Eq. $(A.54)$ implies that $X^{ban} \geq X^{free}$. Combining this inequality with Eqs. $(A.55)$ and $(A.58)$ then implies $p^{ban} > p^{free}$, which yields a contradiction and completes the proof. □

A.4. Reach for yield and safety in the beta model

In Section 7, we assumed $\pi_1 > 0$ (liquidity shocks have positive probability in all aggregate states) so as to simplify the exposition. In this section, we characterize the equilibrium when $\pi_1 = 0$. A special case is the beta model we introduced in Section 6. After we characterize the equilibrium, we use the beta model to establish additional comparative statics of the reach for safety and yield.

As in Section 7, we consider a special location that has potentially different parameters, (η^*, R^*). As before, the parameters satisfy Assumption 2. The new assumption is that $\pi_1 = 0$, that is, there exists an aggregate state in which the conditional probability of liquidity shocks is zero. As before, we denote the equilibrium allocations in the special location with $x^{*,*}, x^{in,*}, (l_s^*)_s, (\tilde{\chi}_s^*)_s, (p_s^*)_s$, where recall that $l_s^* = x^{out,*}\overline{R}_s + y^* + z_s^* - \eta^*$ denotes the banks' liquidity purchase (and the individual components, $x^{out,*}, y^*, z_s^*$, are not uniquely determined).

In Proposition 6, we show that $x^{*,*} = 0$, and that the remaining equilibrium allocations in the special location are characterized as the solution to Eqs. $(32 - 36)$. Most of these equations also continue to apply in this setting with the exception of the optimality condition for outflows, Eq. (34). In this case, since liquidity shocks happen with zero probability in state $s = 1$, banks might prefer to bring zero net liquidity to this state. Thus, we need to allow for the possibility of a corner solution for this state. To this end, we replace the optimality condition (34) with the following conditions,

$$\frac{M_s^*}{u'(c_0^*)} = \frac{q_s}{\gamma_s} = \frac{M_s}{u'(c_0)} \text{ for each } s > 1, \tag{A.59}$$

$$\text{and } c_0^* < c_0, l_1 = -\eta^*; \text{ or } c_0^* = c_0, l_1 > -\eta^*, p_1^* = p_1. \tag{A.60}$$

Here, Eq. $(A.59)$ says that the earlier optimality conditions continue to apply for states $s > 1$ (that feature $\pi_s > 0$). Eq. $(A.60)$ is the optimality condition for state $s = 1$. To understand this condition, note that $\pi_1 = 0$ implies $M_1^* = M_1 = 1$. For regular locations, we have an interior solution also for this state (in a symmetric equilibrium), which implies that banks are indifferent to invest in this state, $M_1/u'(c_0) = q_1/\gamma_1$. Eq. $(A.60)$ states the optimality condition for the banks in the special location depending on whether $c_0^* < c_0$ or $c_0^* = c_0$ (the remaining case, $c_0^* > c_0$, can be ruled out). If $c_0^* < c_0$, then the special location features, $M_1^*/u'(c_0^*) < q_1/\gamma_1$, which implies that banks strictly prefer to reduce their investment in this state and there is a corner solution, $l_1 = -\eta^*$. If $c_0^* = c_0$, the special location features, $M_1^*/u'(c_0^*) = q_1/\gamma_1$, which implies the banks are indifferent to invest in this state. In this case, p_1^* is not uniquely determined.[15] But this indeterminacy is innocuous since $\pi_1 = 0$ and liquidity shocks happen with zero probability in this state. We resolve this indeterminacy by assuming, $p_1^* = p_1$.

[15] The reason for this indeterminacy is that banks in the special location can transfer liquidity from state 1 to other states (therefore lowering p_1^*), and banks in regular locations can send greater inflows into the special location to neutralize the price impact of these liquidity transfers (therefore leaving p_s^* unchanged for $s > 1$).

The following result, which is the analogue of Proposition 6 for this setting, summarizes this discussion and establishes the existence of an equilibrium. The proof is relegated to Appendix A.6.

Proposition 9. *Consider the model with Assumption 1 and $\pi_1 = 0$, together with a special location that satisfies Assumption 2. There exists an equilibrium in which the allocations and prices for regular locations are characterized by Propositions 1-2. In the special location, there is no local investment, $x^{*,*} = 0$. The remaining allocations, $x^{in,*}, c_0, (l_s^*)_s, (p_s^*)_s, (\tilde{\chi}_s^*)_s$, are characterized as the unique solution to the system of equations $(32-36)$ after replacing Eq. (34) with Eqs. $(A.59-A.60)$.*

As before, we also adopt the convention that total inflows are equal to the inflows into its risky assets, $\overline{x}^{in,*} \equiv x^{in,*}$; whereas total outflows account for the net trade of safe and contingent assets and are given by Eq. (37).

Next consider the beta model we analyzed in Section 6. Note that Proposition 9 also applies for this model. We next use this special case to analyze the determinants of reach for safety and yield.

Reach for safety in the beta model. First suppose $R^* = R$ and consider the effect of asymmetries in the liquidity supply, η^*. It is easy to verify that the the closed-form solution described in Section 7.2 also applies in this setting. Thus, the results in Proposition 7 apply. In particular, fire-sale prices are independent of η^*. A location with $\eta^* > \eta$ features more inflows relative to its outflows, $x^{\text{in},*} > \overline{x}^{out,*} = x$, and it also has riskier (more leveraged) outflows relative to its inflows, $\Lambda > 1$. A location with $\eta^* < \eta$ features more outflows than its inflows, $\overline{x}^{out,*} = x > x^{in,*}$, and safer (less leveraged) outflows than regular locations, $\Lambda < 1$.

We next analyze the comparative statics of equilibrium with respect to the global risk conditions, which we capture with β (see Section 6). Suppose β increases so that the liquidity shocks become more correlated. As captured by Proposition 5, this decreases the symmetric flows, x, as well as the risk-free rate, R_f. Consider the effect on the net imbalances (as a fraction of outflows), $\left|x^{\text{in},*} - x\right|/x$, as well the absolute value of its relative leverage ratio, $|\Lambda - 1|$. After rearranging Eqs. (38), we obtain closed-form expressions,

$$\begin{aligned}
\frac{\left|x^{\text{in},*} - x\right|}{x} &= \left(\frac{e}{x} + 1\right) |\Lambda - 1| \\
|\Lambda - 1| &= \frac{|\eta^* - \eta|}{xR_f + \eta}.
\end{aligned}$$

Since an increase in β reduces x and R_f, it also increases both $|\Lambda - 1|$ and $\frac{\left|x^{\text{in},*} - x\right|}{x}$. In particular, for a developed location with $\eta^* > \eta$, it increases the (proportional) current account deficit and makes the outflows riskier. Conversely, for an emerging market with $\eta^* < \eta$, it increases the (proportional) current account surplus and makes the outflows safer. The following result, which is the analogue of Proposition 7 for this setting, summarizes this discussion.

Proposition 10. *Consider the setup in Proposition 6 with parameters $R^* = R$ and $\eta^* \neq \eta$ and for the beta model described in Section 6. All of the results stated in Proposition 7 continue to apply in this setting. In addition, an increase in β (that makes the liquidity shocks more correlated) increases $\left|x^{in,*} - x\right|/x$ and $|\Lambda - 1|$.*

Reach for yield in the beta model. Next suppose $\eta^* = 0$ and consider the effect of asymmetries in return, R^*. As in the main text, we consider the case with $R^* > R$. Using Proposition 9, and Eqs.

(23) and (24), the foreigners' optimality (or indifference) condition (32) can be rewritten as,

$$q_1 R^* + q_2 \left((1-\pi) R^* + \pi p_2^* \right) + q_3 p_3^* = q_1 R + q_2 \left((1-\pi) R + \pi p_2 \right) + q_3 p_3. \tag{A.61}$$

Likewise, local banks' optimality conditions $(A.59)$ for states 2 and 3 can be rearranged as,

$$\frac{1-\pi+\pi\frac{R^*}{p_2^*}}{1-\pi+\pi\frac{R}{p_2}} = \frac{\frac{R^*}{p_3^*}}{\frac{R}{p_3}} = \frac{u'\left(c_0^*\right)}{u'\left(c_0\right)}. \tag{A.62}$$

These two equations determine (p_2^*, p_3^*), as well as c_0^*. Lemma 4 in Appendix A.6 establishes that there is a unique solution that also satisfies $\frac{u'(c_0^*)}{u'(c_0)} > 1$. By $(A.60)$, this implies $l_1^* = -\eta^*$ (there is a corner solution for state 1). The remaining allocations, $x^{in,*}, p_1^*, (l_s^*)_{s\in\{2,3\}}, (\tilde{\chi}_s)_s$, are characterized by solving Eqs. $(32-36)$ with the exception of Eq. (34). The price in state 1 is zero, $p_1^* = 0$, since banks have zero net liquidity in this state, $l_1^* + \eta^* = 0$.

Lemma 4 further shows that the solution satisfies $p_2^* < p_2, p_3 < p_3^*$: that is, the location experiences more severe fire sales in both distress states. Moreover, the relative depth of fire sales is greater in the idiosyncratic shock state than in the aggregate shock state, $p_2^*/p_2 < p_3^*/p_3$. It can also be seen that $x^{in,*} > \overline{x}^{out,*} > x$: that is, the reach for yield increases the location's outflows, but it also increases its inflows more than its outflows. Hence, all of the results in Proposition 8 also apply in this case as long as we focus on the prices for aggregate states $s > 1$ (with $\pi_s > 0$). The remaining price is equal to zero, $p_1^* = 0$.

We next investigate the comparative statics of equilibrium with respect to the global return and risk conditions, which we capture with R and β. To this end, we substitute $q_s = \gamma_s \frac{M_s}{u'(c_0)}$ [cf. Eq. (34)] into Eq. $(A.61)$ and rearrange terms to obtain,

$$\begin{aligned} &\left(R^* - R\right)(1-\pi)\left(M_1\beta + M_2(1-\beta)\right) = \left(\overline{p} - \overline{p}^*\right)\pi\left(M_2(1-\beta) + M_3\beta\right), \\ &\text{where } \overline{p} = \frac{p_2 M_2(1-\beta) + p_3 M_3 \beta}{M_2(1-\beta) + M_3\beta} \text{ and } \overline{p}^* = \frac{p_2^* M_2(1-\beta) + p_3^* M_3\beta}{M_2(1-\beta) + M_3\beta}. \end{aligned} \tag{A.63}$$

Here, the second line defines the probability and price weighted average fire-sale prices. Eq. $(A.63)$ captures the trade-off from investing in the special location relative to other locations. Banks collect net positive returns if there is no crisis (captured by the left side), but they make net negative returns if there is a crisis (captured by the right side). Net gains are multiplied by the probability of no crisis $(1-\pi)$ and the average marginal utility conditional on no crisis. Net losses are calculated in similar fashion. The indifference condition obtains when the weighted net gains and the net losses are equated.

Eq. $(A.63)$ shows that, all else equal, a decline in investment returns in other locations, R, makes investing in the special location more attractive. In equilibrium, this tends to decrease the relative fire-sale price in the special location—so as to counter the greater net gains with greater net losses conditional on a crisis. However, the result does not immediately follow since the marginal utilities are also endogenous and depend on R. In Appendix A.6, we formally establish that a decrease in R decreases $\overline{p}^* - \overline{p}$.

Eq. $(A.63)$ also shows that, all else equal, a decline in the correlation parameter, β, makes investing in the special location relatively more attractive: it decreases the weighting term on the right (loss) side while increasing the weighting term on the left (gain) side since $M_1 < M_2 < M_3$. In equilibrium, this tends to reduce the relative fire-sale price in the special location. In Appendix A.6, we formally show a decrease in β decreases $\overline{p}^* - \overline{p}$. The following proposition, which is the analogue of Proposition 8 for this setting, summaries this discussion.

Proposition 11. *Consider the setup in Proposition 6 with parameters $\eta^* = \eta$ and $R^* > R$ and for the special case of the "beta model" described in Section 6. All of the results stated in Proposition 8 continue to apply in this setting for states $s \in \{2, 3\}$ with $\pi_s > 0$. The remaining state $s = 1$ (with $\pi_1 = 0$) features $l_1^* = -\eta^*$ and $p_1^* = 0$. In addition, a decrease in R as well as a decrease in β reduces the special location's relative weighted average fire-sale price, $\overline{p}^* - \overline{p}$ [defined in (A.63)].*

A.5. An alternative model with distressed banks

In the main text, we built a model in which liquidity shocks are events such that a group of agents ("distressed sellers") sell financial assets at fire-sale prices to invest in a profitable project, and another group of agents ("banks") arbitrage these fire sales. In practice, crises and fire-sales are often associated with losses to financial institutions, that we view as corresponding to "banks" in our model. In this appendix, we build a model in which there are no distressed sellers, and liquidity shocks are events such that banks experience losses (so they are the distressed agents). When this happens, banks are forced to sell their assets to another group of agents, "secondary buyers," that reside in the same location as banks. We show that the equivalent of our main result (Proposition 1) continues to apply in this case.

We then characterize the constrained optimal allocation in this environment and establish the equivalent of our main welfare result (Proposition 3). In this setting, fire sales are costly because they tighten banks' financial constraints and generate a misallocation of productive resources to secondary buyers (similar to Kiyotaki and Moore (1997)). As long as banks are net sellers of risky assets (during a liquidity shock), the constrained optimum features greater ex-ante foreign investment and greater ex-post fire-sale prices compared to the competitive equilibrium, because the planner internalizes that greater prices relax banks' financial constraints.

Taken together, our results in this appendix highlight that the "distressed sellers" in our main model are a modeling device that introduces the standard balance sheet channel into the model while simplifying the analysis. These agents generate the liquidity demand that triggers fire sales; and when the return from their projects (λ) is high, they also capture the social cost of fire sales.

An alternative model. As before, there are three periods, $t \in \{0, 1, 2\}$, and a single consumption good in each period. There is a continuum of mass one of locations denoted by superscript $j \in J$. A random variable ω^j is drawn for each location j and i.i.d. across j, with $\pi = \Pr\left(\omega^j = b\right)$ and $1 - \pi = \Pr\left(\omega^j = g\right)$. We say that a location with $\omega^j = b$ experiences a liquidity shock.

There are two types of assets. First, in each location, there is a linear technology in period 0: investing one unit of the consumption good produces one unit of a location-specific risky asset. If $\omega^j = g$, then the asset generates $R + G$ units of the consumption good in period 1 (and 0 units in period 2), where R denotes the baseline payoff and G denotes an additional gain realized in the good state. If $\omega^j = b$, then the asset generates R units of the consumption good in period 2. But it also generates a loss of L units of consumption good in period 1 (that is, the owner of the asset is obliged to pay L units of consumption good). To simplify the exposition, we also assume $G(1 - \pi) = \pi L$, so that the asset's expected payoff after accounting for the gains and losses is still R.

When $\omega^j = b$, the asset is traded at an endogenous price. As before, we concentrate attention on symmetric equilibria in which the price (after the payment of L) is the same across all locations, denoted by p.

As before, there is also a risk-free asset that pays 1 unit of the consumption good in period 1 (and 0 units in period 2). The risk-free asset is in fixed supply: specifically, there are η units in each location (endowed to the local banks that will be described below). In period 0, the risk-free asset is traded at an endogenous price q_f. (We do not introduce the Arrow-Debreu securities since there is only a single aggregate state).

In each location, there are two types of agents which we refer to as "secondary buyers" and "banks." The secondary buyers have preferences $E[\tilde{c}_1]$. When the local state is $\omega^j = b$, they have access to a technology that converts the asset in their location into consumption goods in period 1. If they purchase $\tilde{\chi}^j \geq 0$ units of the asset, then they produce $\psi R \log\left(\tilde{\chi}^j/\psi + 1\right)$ units of the consumption good (for some $\psi > 0$). Thus, their consumption is given by

$$\tilde{c}_1 = \psi R \log\left(\tilde{\chi}^j/\psi + 1\right) - p\tilde{\chi}^j.$$

We chose the functional form for their production to obtain a simple expression for demand. Specifically, the optimality condition implies the secondary buyers' demand for the asset is given by,

$$\tilde{\chi}^j = \psi\left(R/p - 1\right) \text{ for each } p \leq R. \tag{A.64}$$

The role of these buyers is to generate misallocation and welfare losses from fire sales.

Banks have the same preferences as in the main text,

$$E\left[u\left(c_0\right) + c_1 + c_2\right].$$

They are endowed with e units of the asset in their location, which they are not allowed to sell in period 0. (We could endogenize this by introducing some local expertise with diminishing returns. The endowment e, represents the comparative advantage of local banks to lend in the local market.) In addition, they are endowed with 1 unit of the consumption good in period 0 as well as η units of the risk-free asset.

As in the main text, banks in location j choose an investment strategy, $x^{j',j}$, in risky assets across locations, j'. They also choose how many consumption units to invest the risk-free asset, y. Banks' budget constraint in period 0 is,

$$c_0 + x^{j,j} + x^{out,j} + yq_f = 1 + \eta q_f, \text{ where } x^{out,j} = \int_{j' \neq j} x^{j',j} dj'.$$

Here, $x^{out,j}$ denotes the outflows: the aggregate amount of investment made by banks in location j in other locations. Banks are not allowed to short-sell risky assets, $x^{j',j} \geq 0$ for each j', but they are allowed to take unrestricted positions on the risk-free asset subject to obtaining nonnegative consumption in all periods and states.

In period 1, if $\omega^{j'} = g$ and $j' \neq j$, then banks receive $x^{out,j}\overline{R}$ units of consumption good from their foreign positions, where we define $\overline{R} = (1-\pi) R + \pi p$ similar to the main text. Note that banks' losses and gains on average cancel (by assumption) and we are left with the same expression as in the main text. Banks also receive y units of the consumption good from their investments in the safe asset.

If $\omega^j = g$, they also receive $\left(e + x^{j,j}\right)(R + G)$ units from the local asset. Moreover, banks do not have a remaining investment opportunity so they consume all of their available resources in period 1, that is:

$$\begin{aligned}
c_1\left(\omega^j = g\right) &= \left(e + x^{j,j}\right)(R+G) + x^{out,j}\overline{R} + y \\
\text{and } c_2\left(\omega^j = g\right) &= 0.
\end{aligned}$$

If instead $\omega^j = b$, then banks lose $\left(e + x^{j,j}\right) L$ units from the local asset. They invest all of their available resources to purchase local assets (since $p < R$). Their budget constraint can be written as,

$$\begin{aligned}
c_1\left(\omega^j = b\right) + \chi^{j,j} p &= \left(e + x^{j,j}\right)(p - L) + x^{out,j}\overline{R} + y, \\
c_2\left(\omega^j = g\right) &= \chi^{j,j} R.
\end{aligned}$$

As before, banks choose $\left[x^{j',j} \geq 0\right]_{j'}, y^j, \left(z^j, \chi^j \geq 0\right)$, to maximize their expected utility subject to nonnegative consumption requirements, $c_0^j \geq 0, c_1^j \geq 0, c_2^j \geq 0$.

The equilibrium is a collection of optimal allocations and market clearing conditions. The market clearing condition for the risky asset in a location j with $\omega^j = b$ in period 1 can be written as,

$$e + x^{in,j} + x^{j,j} = \tilde{\chi}^j + \chi^j \text{ where } x^{in,j} = \int_{j' \neq j} x^{j,j'} dj'. \tag{A.65}$$

The market clearing condition for the risk-free asset in period 0 can be written as,

$$\int_j y^j dj = \eta.$$

Equilibrium in the alternative model. We assume the parameters satisfy.

Assumption 1^{A}. $eL - \psi R < \eta < eL$.

The right side of the inequality ensures the equilibrium features fire sales, $p < R$. The left side ensures the fire-sale price is strictly positive, $p > 0$. Specifically, we conjecture an equilibrium with symmetric fire-sale prices, $p \in (0, R)$. We also conjecture symmetric equilibrium allocations, denoted by x^{out}, y. Later, we will strengthen this assumption to facilitate the welfare analysis. As before, the symmetry implies that $y = \eta$, and that outflows and inflows are the same, $x^{in} = x^{out}$. When it is clear from the context, we denote this symmetric flows with x.

Since banks have linear utility between periods 1 and 2, the presence of fire sales ($p < R$) implies that banks in locations with state $\omega^j = b$ invest all of their resources in period 1 in the risky asset, that is, $c_1^j\left(\omega^j = b\right) = 0$ and

$$\chi^{j,j} = \frac{\left(e + x^{j,j}\right)(p - L) + x^{out,j}\overline{R} + \eta}{p}. \tag{A.66}$$

In addition, since locations are symmetric, the market clearing condition implies $y = \eta$. Combining these observations with the budget constraints,

$$\begin{aligned}
c_0 + x^{j,j} + x^{out} &= 1, \\
c_{1,s}\left(\omega^j = g\right) &= \left(e + x^{j,j}\right)(R + G) + x^{out}\overline{R} + \eta, \\
c_{2,s}\left(\omega^j = b\right) &= \frac{\left(e + x^{j,j}\right)(p - L) + x^{out,j}\overline{R} + \eta}{p} R.
\end{aligned}$$

Substituting these expressions into the objective function, and rearranging terms, the banks' problem can be written as,

$$\max_{x^{j,j},x^{out}} u\left(1-x^{j,j}-x^{out}\right)+\left(e+x^{j,j}\right)R^{loc}+\left(x^{out}\overline{R}+\eta\right)M,$$

where $M=1-\pi+\pi\frac{R}{p}$ as in the main text and

$$R^{loc}=(1-\pi)(R+G)+\pi\frac{p-L}{p}R,$$

denotes the marginal utility from investing in the local asset. It is easy to check that $R^{loc}<R<\overline{R}M$ (the last inequality follows from Lemma 1), which implies $x^{j,j}=0$. As before, local investment in period 0 is dominated by foreign investment.

Banks' foreign investment is then characterized by solving,

$$u'(1-x)=\mu(p)\equiv\overline{R}M, \tag{A.67}$$

where $\overline{R}=(1-\pi)R+\pi p$ and $M=1-\pi+\pi\frac{R}{p}$ as in the main text. As before, this provides a decreasing relationship between x and p.

Next consider the determination of the fire-sale asset price, p. Substituting for $\tilde{\chi}^j,\chi^j$ from Eqs. $(A.64)$ and $(A.66)$ into the market clearing condition $(A.69)$ (and using $x^{j,j}=0$ and $y=\eta$) we obtain,

$$p=\frac{\psi R+e(p-L)+x^{out}\overline{R}+\eta}{\psi+e+x^{in}}. \tag{A.68}$$

After substituting $x^{in}=x^{out}=x$ and rearranging terms, we further obtain,

$$p=\frac{\psi R+\eta-eL+x\overline{R}}{\psi+x}.$$

Substituting the expression for $\overline{R}$ and rearranging, we further obtain,

$$p=\frac{\psi R+\eta-eL+x(1-\pi)R}{\psi+x(1-\pi)}. \tag{A.69}$$

As before, this provides an increasing relation between x and p.

The equilibrium is characterized by Eqs. $(A.67)$ and $(A.69)$. Under Assumption 1^{A}, there exist a unique solution, (x,p), which also satisfies $p\in(0,R)$ and $x\in(0,1)$.

Global liquidity creation in the alternative model. We next illustrate that our main result regarding the liquidity-creation role of gross flows (Proposition 1) applies in this setting. To this end, we characterize the autarky allocation in which foreign flows are banned. In particular, banks solve the problem described above with the additional restriction, $x^{out,j}=0$. In this case, there might be some local investment. Specifically, we have,

$$u'\left(1-x^{j,j}\right)\geq(1-\pi)(R+G)+\pi\frac{p^{aut}-L}{p^{aut}}R, \tag{A.70}$$

with strict inequality only if $x^{j,j} = 0$. Note that this describes a (weakly) increasing relation between p^{aut} and $x^{j,j}$. The interpretation is that lower fire-sale prices make the losses from risky assets costlier and reduce the attractiveness of risky investment. Following similar steps as above, the market clearing condition becomes,

$$p^{aut} = \frac{\psi R + \eta - \left(e + x^{j,j}\right) L}{\psi}. \tag{A.71}$$

This describes a decreasing relation between p^{aut} and $x^{j,j}$. The interpretation is that greater risky investment leads to greater bank losses, lower liquidity, and lower fire-sale prices during liquidity shocks. The equilibrium is characterized by solving by Eqs. $(A.70)$ and $(A.71)$. Under Assumption 1^{A}, there exists a unique solution, $\left(p^{aut}, x^{j,j}\right)$, which satisfies $p^{aut} \in (0, R)$ and $x^{j,j} \in [0,\underline{x})$ (where recall that $\underline{x}$ denotes the solution to $u'(1-\underline{x}) = R$).

Comparing Eqs. $(A.69)$ and $(A.71)$, we have,

$$p > \frac{\psi R + \eta - eL}{\psi} \geq \frac{R\psi + \eta - \left(e + x^{j,j}\right) L}{\psi} = p^{aut}.$$

Here, the first inequality follows since $x > 0$ and the second inequality follows since $x^{j,j} \geq 0$. It follows that the equilibrium fire-sale price exceeds the autarky level, $p > p^{aut}$, also in the alternative model. Intuitively, as illustrated by Eq. $(A.68)$, gross flows create liquidity also in the alternative model because inflows are liquidated at the fire-sale price p, whereas outflows provide liquidity at a higher return $\overline{R}$.

Public good aspect of liquidity creation in the alternative model. Eq. $(A.68)$ also illustrates that outflows help to increase local fire-sale prices whereas inflows tend to reduce them. This implies that the coordination problem between local policymakers that we highlighted in the main text also applies in this setting. In particular, suppose local policymakers have the objective to raise local fire-sale prices, p^j, and they choose whether or not to ban capital flows, $b^j \in \{0, 1\}$ (as in Section 5.2). It is then easy to check that the global policymaker that could coordinate policy across locations would prescribe $b^j = 0$ for each j, whereas the Nash equilibrium features $b^j = 1$ for each j and results in the autarky allocation. Hence, absent coordination, local policymakers set excessive restrictions on capital flows, which leads to lower liquidity creation and lower fire-sale prices compared to a coordinated outcome.

Constrained optimal allocation and externalities in the alternative model. We next characterize the constrained optimal allocation in this environment and illustrate the externalities. As in Section 5.1, suppose the policymakers have the utilitarian social welfare function,

$$W^j = u\left(c_0^j\right) + E\left[c_1^j + c_2^j\right] + E\left[\tilde{c}_2^j\right].$$

Consider a global planner that can dictate (symmetric) period 0 local and foreign investment in each location but otherwise cannot interfere with the equilibrium allocations. We denote the local investment with $x^{j,j}$, foreign investment with x, and the resulting equilibrium price with p. Using the functional form for secondary buyers' demand [cf. Eq. $(A.64)$], and following similar steps as in Section 5.1, we calculate the planner's objective function as,

$$\begin{aligned} W^j &= u\left(1 - x - x^{j,j}\right) + \left(x^{j,j} + x + e\right) R + \eta + \pi f\left(\tilde{\chi}\right), \qquad \text{(A.72)} \\ \text{where } \tilde{\chi} &= \psi\left(R/p - 1\right) \text{ and } f\left(\tilde{\chi}\right) = -\tilde{\chi} R + \psi R \log\left(\tilde{\chi}/\psi + 1\right). \end{aligned}$$

This expression is the analogue of Eq. (20) for this setting. The term, $f(\tilde{\chi})$, captures the net production that results from the transfer of resources from banks to secondary buyers. It is easy to check that $f(0) = 0$ and $f'(\tilde{\chi}) < 0$ for each $\tilde{\chi} \geq 0$. Thus, net production is negative (and increasingly so for greater levels of $\tilde{\chi}$). This illustrates that the transfer of assets to secondary buyers reduces the utilitarian social welfare.

Following similar steps as above, the market clearing condition is given by the following analogue of Eq. $(A.69)$,

$$p = \frac{\psi R + \eta - \left(e + x^{j,j}\right) L + x(1-\pi) R}{\psi + x(1-\pi)}. \tag{A.73}$$

The constrained social planner chooses $\left(x, x^{j,j}, p\right)$ to maximize the expression in $(A.72)$ subject to this condition. It is easy to check that the marginal utility from foreign investment strictly exceeds R whereas the marginal utility from local investment is strictly less than R. Hence, the local investment is dominated, $x^{j,j} = 0$ (as in the main text). Intuitively, the planner prefers foreign investment because this increases the fire-sale price and reduces misallocation, whereas local investment reduces the fire-sale price further and exacerbates misallocation.

After setting $x^{j,j} = 0$, the market clearing condition $(A.73)$ becomes equivalent to its counterpart in equilibrium, condition $(A.69)$. Thus, the planner maximizes $(A.72)$ subject to this condition. Taking the first order conditions, we obtain the following analogue of Eq. (21) in Section 5.1,

$$\begin{aligned}
u'(1-x) &= R + \pi f'(\tilde{\chi}) \frac{d\tilde{\chi}}{dp} \frac{dp}{dx}, \\
\text{where } f'(\tilde{\chi}) &= -R + \frac{R}{1 + \tilde{\chi}/\psi} = -(R-p), \\
\frac{d\tilde{\chi}}{dp} &= -\frac{\psi R}{p^2}, \\
\frac{dp}{dx} &= \frac{(1-\pi)}{\psi + x(1-\pi)} (R-p).
\end{aligned}$$

Here, the second line uses the definition of $f(\cdot)$ in $(A.72)$, the third line uses the definition of $\tilde{\chi}$ in $(A.64)$, and the last line uses the expression for p in $(A.69)$ to evaluate the corresponding derivatives.

Combining these expressions, the planner's optimality condition becomes,

$$u'(1-x) = R + R\pi \left(\frac{R}{p} - 1\right)^2 \frac{\psi(1-\pi)}{\psi + x(1-\pi)}. \tag{A.74}$$

As before, this represents a decreasing relationship between (x, p). The equilibrium is characterized by solving this expression together with Eq. $(A.74)$. Under Assumption 1^{A}, there exists a solution that satisfies $p \in (0, R)$ and $x \in (0, 1)$.

As in Section 5.1, the equilibrium and the constrained optimum share the market clearing condition $(A.69)$ but they differ because they are associated with different optimality conditions $(A.74)$ and $(A.67)$. After comparing the right-hand-terms of conditions $(A.74)$ and $(A.67)$, and rearranging terms, it is easy to check that the constrained optimal allocation features greater x (and greater p) if and only if the following inequality holds,

$$\frac{\psi}{\psi + x^{eq}(1-\pi)} > \frac{p^{eq}}{R} = \frac{\psi + \frac{\eta - eL}{R} + x^{eq}(1-\pi)}{\psi + x^{eq}(1-\pi)}.$$

Here, (x^{eq}, p^{eq}) denote the competitive equilibrium allocation and the equality follows from Eq. $(A.69)$. After rearranging terms and combining with Eq. $(A.66)$, the previous inequality is equivalent to,

$$eL > \eta + Rx^{eq}(1-\pi). \tag{A.75}$$

When this inequality is satisfied, the constrained optimum features greater (x, p) compared to the equilibrium. Otherwise, it features smaller (x, p). Hence, the analogue of Proposition 3 in Section 5.1 also applies in this setting.

To understand condition $(A.75)$, suppose the parameters are such that banks' demand for assets in equilibrium satisfies [cf. $(A.66)$],

$$\chi^{j,j} = \frac{e(p^{eq} - L) + x^{eq}\overline{R}^{eq} + \eta}{p^{eq}} < e. \tag{A.76}$$

This says that local banks are net sellers of the asset in the sense that their demand $\chi^{j,j}$ is below their endowment e. This condition implies Eq. $(A.75)$: that is, if local banks are net sellers in equilibrium, then the constrained optimum features greater (x, p). The intuition is that increasing fire-sale prices via greater foreign investment reduces the wealth of secondary buyers but increases the wealth of local banks (when they are net sellers) as well as fickle foreign banks. Since the secondary buyers have lower marginal utility than both local and foreign banks, foreign investment is associated with positive externalities, and the equilibrium features too little liquidity creation and too low prices.[16]

Next consider the following strengthening of Assumption 1^{A}.

Assumption $1^{\mathrm{A'}}$. $eL - \psi R < \eta < eL - R$.

The right side of this inequality implies condition $(A.76)$, which in turn implies condition $(A.75)$. Thus, as long as banks' losses during liquidity shocks are sufficiently large, so that they are net sellers of risky assets, then the equilibrium features too little foreign investment and too low fire-sale prices compared to the constrained optimum.

A.6. Omitted proofs

Proof of Lemma 1. We have,

$$\begin{aligned}\mu_s'(p_s) &= \pi_s\left(1 - \pi_s + \frac{R}{p_s}\pi_s\right) - \pi_s\frac{R}{p_s^2}\left((1-\pi_s)R + \pi_s p_s\right)\\ &= \pi_s(1-\pi_s)\left(1 - \frac{R^2}{p_s^2}\right).\end{aligned}$$

Hence, $\mu_s(p_s)$ is strictly decreasing over the range $p_s \in (0, R)$. Using $\mu_s(R) = R$, we also obtain $\mu_s(p_s) > R$, which in turn implies $x^{j,j} = 0$. □

[16] Note that conditions $(A.75)$ and $(A.76)$ are similar but not identical. In particular, there might be parameters in which the latter is violated, so that banks are net buyers, but the former is satisfied so that the constrained optimum features greater (x, p) than the equilibrium. Intuitively, when local banks are net buyers, increasing the price reduces the wealth of secondary buyers as well as local banks while it raises the wealth of fickle foreign banks. Since secondary buyers have lower marginal utility than foreign banks but local banks have higher marginal utility than foreign banks, the effect of this wealth transfer is in general ambiguous.

Proof of Lemma 2. Taking the derivative of Eq. (14), we obtain,

$$\frac{d}{dx}\left(\frac{\eta + x\left(1-\pi_s\right)R}{e + x\left(1-\pi_s\right)}\right) = \frac{\left(1-\pi_s\right)}{e + x\left(1-\pi_s\right)}\left(R - P_s^{mc}\left(x\right)\right) > 0.$$

Here, the inequality follows since $\eta < eR$ implies $P_s^{mc}\left(x\right) = \frac{\eta + x(1-\pi_s)R}{e+x(1-\pi_s)} < R$, which completes the proof. □

Proof of Proposition 1. Most of the characterization of equilibrium is provided in the main text. It remains to check there exists a unique solution to Eqs. (12) and (14). To this end, define the function,

$$F\left(X\right) = u'\left(1-X\right) - \sum_s \gamma_s \mu_s\left(p_s\right), \text{ where } p_s = \frac{\eta + X\left(1-\pi_s\right)R}{e + X\left(1-\pi_s\right)} \text{ for each } s.$$

By Lemma 2, $F\left(X\right)$ is strictly increasing over the range, $X \in (0,1)$. Note also that,

$$\begin{aligned} F\left(0\right) &= u'\left(1\right) - \sum_s \gamma_s \mu_s\left(\frac{\eta}{e}\right) \\ &< R - \sum_s \gamma_s \mu_s\left(\frac{\eta}{e}\right) < 0, \end{aligned}$$

where the first inequality follows since we assume $u'\left(1\right) < R$ and the second one follows since $\mu_s\left(\frac{\eta}{e}\right) > R$ by Lemma 1. Finally, note that $F\left(1\right) = \infty$ since we assume $u'\left(0\right) = \infty$. By continuity, there exists a unique solution to the equation, $F\left(x\right) = 0$, over the range, $(0,1)$. This proves the existence and the characterization of equilibrium. Since $\eta < eR$ implies $P_s^{mc}\left(x\right) = \frac{\eta+x(1-\pi_s)R}{e+x(1-\pi_s)} < R$, the equilibrium also satisfies $p_s < R$ for each s. Likewise, $\eta > eR/\lambda$ implies $P_s^{mc}\left(x\right) > P_s^{mc}\left(0\right) = \eta/e > R/\lambda$.

We next characterize the autarky equilibrium in which all banks are required to make zero foreign investment, $x^{j',j} = 0$ for each $j' \neq j$. In this case, banks solve problem (10) with the additional constraint, $x^{out,j} = 0$. It follows that there is some local investment in equilibrium, and the level of local investment is given by $x^{j,j} = \underline{x} > 0$, where $\underline{x}$ denotes the solution to $u'\left(1-\underline{x}\right) = R$. Using Eq. (6) (and symmetry), we have $\chi^j = \left(x^{j,j} + \eta\right)/p^j$. Substituting this into the market clearing condition (7), we obtain $p^{aut} = \eta/e$. This is equal to $P^{mc}\left(0\right)$: that is, the equilibrium price is the same as in the baseline model after setting foreign investment equal to zero. Using Lemma 2 and $x > 0$, this implies $p_s > p^{aut}$ for each s with $\pi_s < 1$. We also have $p_s = p^{aut}$ for a state with $\pi_s = 1$. This completes the proof. □

Proof of Proposition 2. Provided in the main text. □

Proof of Proposition 3. The analysis in the main text characterizes the allocation that obtains when the planner chooses $x, x^{j,j}$. In particular, Eq. (14) applies conditional on the planner's choice of x. Under Assumption 1, the resulting price satisfies $p \in (R/\lambda, R)$.

Next consider the social welfare that results from this allocation. Note that distressed sellers sell all of their endowments to invest in new projects with return λ. Thus, their expected equilibrium consumption is given by

$$E\left[\tilde{c}_2^j\right] = \lambda e \overline{R},$$

where $\overline{R} = (1-\pi) R + \pi p$. For banks, following similar steps as in Section 4, we have,

$$\begin{aligned} c_0^j &= 1 - x - x^{j,j}, \\ c_1^j \left(\omega^j = g\right) &= \eta + x\overline{R} + x^{j,j} R \text{ and } c_2^j \left(\omega^j = g\right) = 0, \\ \text{and } c_2 \left(\omega^j = b\right) &= \left(\eta + x\overline{R} + x^{j,j} p\right) \frac{R}{p} \text{ and } c_1^j \left(\omega^j = b\right) = 0, \end{aligned}$$

where (x, p) are characterized as the unique solution to Eqs. (12) and (14).

Substituting these expressions into the utilitarian social welfare function, $u\left(c_0^j\right) + E\left[c_1^j + c_2^j\right] + \lambda E\left[\tilde{c}_2^j\right]$, we obtain,

$$\begin{aligned} W^j &= u\left(1 - x - x^{j,j}\right) + (1-\pi)\left(\eta + x\overline{R} + x^{j,j} R\right) + \pi\left(\eta + x\overline{R} + x^{j,j} p\right)\frac{R}{p} + \lambda e\overline{R} \\ &= u\left(1 - x - x^{j,j}\right) + \eta + x^{j,j} R + x\overline{R} + \pi\left(\eta + x\overline{R}\right)\left(\frac{R}{p} - 1\right) + \lambda e\overline{R} \\ &= u\left(1 - x - x^{j,j}\right) + \eta + x^{j,j} R + xR + \pi (R - p)\left(\frac{\eta + x\overline{R}}{p} - x\right) + \lambda e\overline{R} \\ &= u\left(1 - x - x^{j,j}\right) + \eta + x^{j,j} R + + xR + \pi (R - p)\, e + \lambda e\overline{R}. \end{aligned}$$

Here, the second line groups terms together, the third line substitutes $\overline{R} = R - (R-p)\pi$, and the last line substitutes $p = \frac{\eta + x\overline{R}}{e+x}$ [cf. Eq. (13)].

Substituting $(R-p)\pi = R - \overline{R}$ into the previously displayed equation, we finally obtain Eq. (20) in the main text, which we replicate here,

$$W^j = u\left(1 - x - x^{j,j}\right) + \left(x^{j,j} + x + e\right) R + \eta + (\lambda - 1)\, e\overline{R},$$

where $\overline{R} = (1-\pi) R + \pi p$. The social planner chooses $x, x^{j,j}$ to maximize this expression subject to the market clearing condition (14). The constrained optimum features $x^{j,j}$ because $\frac{dW^j}{dx^j} = R < \frac{dW^j}{dx} = R + (\lambda - 1)\, e\pi \frac{dp}{dx}$.

The optimality condition for foreign investment results in Eq. (21) in the main text, which we replicate here,

$$\begin{aligned} u'(1-x) &= R + (\lambda - 1)\, e\pi \frac{dp}{dx} \\ \text{where } \frac{dp}{dx} &= \frac{(1-\pi)}{e + x(1-\pi)} (R - p)\,. \end{aligned}$$

Note that this describes a strictly decreasing relation between p and x. Eq. (14) describes a strictly increasing relation. Under Assumption 1, there exists a unique intersection that satisfies $x \in (0,1)$ and $p \in (R/\lambda, R)$.

We next analyze how the constrained optimum compares with the equilibrium. Note that the constrained optimum features greater x (and p) than the equilibrium if and only if the right side of Eq. (21) exceeds the right side of Eq. (12),

$$R + (\lambda - 1)\, e\pi \frac{dp}{dx} > M\overline{R},$$

when both expressions are evaluated in the equilibrium allocation (x^{eq}, p^{eq}). After substituting M and

$\overline{R}$ as well as $\frac{dp}{dx}$ from Eq. (21), and rearranging terms, this inequality becomes,

$$1 + (\lambda - 1) e \frac{\pi (1-\pi)}{e + x^{eq} (1-\pi)} \frac{R - p^{eq}}{R} > \left(1 - \pi + \pi \frac{R}{p^{eq}}\right) \left(1 - \pi + \pi \frac{p^{eq}}{R}\right).$$

After expanding the terms on the right hand side and canceling terms from both sides, this inequality becomes,

$$\frac{\lambda e + x^{eq} (1-\pi)}{e + x^{eq} (1-\pi)} > \frac{R}{p^{eq}}.$$

This is equivalent to condition (22), which completes the proof. $\square$

Proof of Proposition 4. The analysis in the main text shows that $dx/d\eta < 0$ and $dp_2/d\eta > 0$: that is, reducing η increases x and reduces p_2 (see Figure 3). Using (29), p_3 also declines. To show that p_3/p_2 declines, note that

$$\begin{aligned}
\frac{d(p_3/p_2)/d\eta}{p_3/p_2} &= \frac{dp_3/d\eta}{p_3} - \frac{dp_2/d\eta}{p_2} \\
&= \frac{dp_3/d\eta}{p_3} - \frac{\partial P_2(x;\eta)/\partial\eta + (\partial P_2(x;\eta)/\partial x)(dx/d\eta)}{p_2} \\
&> \frac{dp_3/d\eta}{p_3} - \frac{\partial P_2(x;\eta)/\partial \eta}{p_2} \\
&= \frac{1}{\eta} - \frac{1}{\eta + x(1-\pi)} > 0.
\end{aligned}$$

Here, the third line follows since $dx/d\eta < 0$ and $\partial P_2(x;\eta)/\partial x > 0$ (by Lemma 2) and the last line follows by evaluating the derivatives from Eqs. (28) and (29) and using $x \in (0,1)$. Hence, reducing η also reduces p_3/p_2.

Combining these results with Eq. (30), $E\left[\overline{R}_s\right]$ also declines. To show the effect on R_f, note that Eq. (31) can be rewritten as,

$$\begin{aligned}
R_f &= \frac{E\left[\overline{R}_s M_s\right]/M_2}{E[M_s]/M_2} && \text{(A.77)} \\
\text{where } \frac{E\left[\overline{R}_s M_s\right]}{M_2} &= \beta \frac{1}{(1-\pi)/R + \pi/p_2} + (1-\beta)((1-\pi)R + \pi p_2) \\
\text{and } \frac{E[M_s]}{M_2} &= \beta \frac{1 - \pi + \pi R/p_3}{1 - \pi + \pi R/p_2} + 1 - \beta.
\end{aligned}$$

Note that the term in the numerator, $E\left[\overline{R}_s M_s\right]/M_2$, is an average of the arithmetic and the harmonic averages of R and p_2. Since $dp_2/d\eta > 0$, we also have $\frac{d\left(E\left[\overline{R}_s M_s\right]/M_2\right)}{d\eta} > 0$. For the term in the denominator, note that,

$$\frac{dE[M_s]/M_2}{d\eta} = \frac{\beta\pi(1 - \pi + \pi R/p_3)}{1 - \pi + \pi R/p_2} \left(-\frac{dp_3/d\eta}{p_3} \frac{1}{(1-\pi)p_3 + \pi R} + \frac{dp_2/d\eta}{p_2} \frac{1}{(1-\pi)p_2 + \pi R}\right).$$

This is strictly negative since $\frac{dp_3/d\eta}{p_3} > \frac{dp_2/d\eta}{p_2}$ and $\frac{1}{(1-\pi)p_3 + \pi R} > \frac{1}{(1-\pi)p_2 + \pi R}$. Combining the effects on the numerator and the denominator, we obtain $\frac{dR_f}{d\eta} > 0$. In particular, decreasing η decreases R_f.

Finally, to show that reducing η increases the risk premium in a neighborhood of $\eta = eR$, we let $x(\eta), [p_s(\eta)]_s$ denote the equilibrium as a function of η. The analysis in the proof of Proposition 1 implies that $x(\cdot), [p_s(\cdot)]_s$ are continuous functions for $\eta < eR$. Taking the limit of Eq. (14), we obtain, $\lim_{\eta \to eR} p_s = R$. Using Eqs. (4) and (11), we further obtain $\lim_{\eta \to eR} \overline{R}_s = R$ and $\lim_{\eta \to eR} M_s = 1$ for each s. Using Eq. (18), we obtain $\lim_{\eta \to eR} R_f = R$. Thus, the risk premium in the limit is zero, $\lim_{\eta \to eR} E\left[\overline{R}_s\right] - R_f = 0$. However, by Eq. (19), $E\left[\overline{R}_s\right] - R_f > 0$ for each $\eta < eR$. It follows that reducing η increases the risk premium in a neighborhood of $\eta = eR$, completing the proof. □

Proof of Proposition 5. The analysis in the main text shows that $dx/d\beta < 0$ and $dp_2/d\beta < 0$: that is, increasing β reduces x and p_2 (see Figure 3). Combining these results with Eq. (30), $E\left[\overline{R}_s\right]$ also declines.

Next consider the effect on R_f. Combining Eqs. (31) and (27), we obtain,

$$\begin{aligned} R_f &= \frac{u'(1-x)}{E[M_s]} \\ \text{where } E[M_s] &= \beta(1-\pi+\pi R/p_3) + (1-\beta)(1-\pi+\pi R/p_2). \end{aligned}$$

First consider the effect on the numerator. Since $dx/d\beta < 0$, we have $d(u'(1-x))/d\beta < 0$. Next consider the effect on the denominator, which can be evaluated as,

$$\frac{dE[M_s]}{d\beta} = \pi\left(\frac{R}{p_3} - \frac{R}{p_2}\right) - (1-\beta)\pi\frac{R}{(p_2)^2}\frac{dp_2}{d\beta} > 0.$$

Here, the inequality follows since $p_3 < p_2$ and $dp_2/d\beta < 0$. Combining the two effects shows $dR_f/d\beta < 0$, that is, increasing β reduces R_f.

Finally, consider the effect on the risk premium, $E\left[\overline{R}_s\right] - R_f$. Following similar steps as in the proof of Proposition 4, we have $\lim_{\beta \to 0} E\left[\overline{R}_s\right] - R_f = 0$, and $E\left[\overline{R}_s\right] - R_f > 0$ for any $\beta > 0$. This shows that increasing β also increases the risk premium, $E\left[\overline{R}_s\right] - R_f$, in a neighborhood of $\beta = 0$, completing the proof. □

The following lemma is used in the proofs of Propositions 6, 9, and 8 analyzed in Section 7 and Appendix A.4. The lemma considers the system of equations (40) and (41) in the main text, which we reproduce here to facilitate the exposition,

$$\sum_{s \in S} q_s\left((1-\pi_s)R^* + \pi_s p_s^*\right) = \sum_{s \in S} q_s\left((1-\pi_s)R + \pi_s p_s\right), \tag{A.78}$$

$$\frac{1-\pi_s+\pi_s\frac{R^*}{p_s^*}}{1-\pi_s+\pi_s\frac{R}{p_s}} = \frac{u'(c_0^*)}{u'(c_0)} \text{ for each } s \in \overline{S} = \{s \in S \quad | \pi_s > 0\}. \tag{A.79}$$

Here, $\left(c_0, (p_s)_{s \in S}\right)$ correspond to the equilibrium variables in regular locations characterized in Proposition 1. The set $\overline{S} \subset S$ includes the aggregate states in which liquidity shocks happen with strictly positive probability. In Section 7, we assume $\pi_1 > 0$ which also implies this corresponds to all states, that is, $\overline{S} = S$. In Appendix A.4, we assume $\pi_1 = 0$ which implies that it corresponds to all states except for state 1, that is, $\overline{S} = S \setminus \{1\}$.

Lemma 4. *Under Assumption 2, there exists a unique solution to the system of equations* $(40 - A.79)$, *denoted by* $\left(c_0^*, (p_s^*)_{s \in \overline{S}}\right)$. *When* $R^* = R$, *the solution satisfies* $c_0^* = c_0$ *and* $p_s^* = p_s$ *for each* $s \in \overline{S}$. *When* $R^* > R$, *the solution satisfies* $\frac{u'(c_0^*)}{u'(c_0)} > \frac{R^*}{R} > 1$, *and* $p_s^*/p_s < 1$ *for each* $s \in \overline{S}$. *It also satisfies,* $p_s^*/p_s < p_{s'}^*/p_{s'}$ *for each* $s, s' \in \overline{S}$ *with* $s < s'$ *(that satisfies* $\pi_s < \pi_{s'}$*).*

Proof. We first show that there exists a unique solution. Note that, for each $s \in \overline{S}$ and $C_0 \in (0, c_0]$, Eq. $(A.79)$ has a unique solution, p_s^*. We denote the solution with the function, $P_s^*(C_0)$ defined over the range $C_0 \in (0, c_0]$. Note also that $P_s^*(C_0)$ is strictly increasing in C_0 and it satisfies $\lim_{C_0 \to 0} P_s^*(C_0) = 0$ and $P_s^*(C_0) = \frac{R^*}{R} p_s \geq p_s$.

We next substitute $P_s^*(c_0^*)$ into Eq. $(A.78)$ to observe that the equilibrium consumption is determined by, $F(c_0^*) \equiv 0$, where we define $F(\cdot)$ as the function,

$$F(C_0) = \sum_s q_s \left((1 - \pi_s) R^* + \pi_s P_s^*(C_0) \right) - \sum_s q_s \left((1 - \pi_s) R + \pi_s p_s \right).$$

Note that $F(C_0)$ is strictly increasing over $(0, c_0]$, and it satisfies $F(C_0) \geq 0$ since $R^* \geq R$ and $P_s^*(C_0) \geq p_s$. We also have

$$\begin{aligned} \lim_{C_0 \to 0} F(C_0) &= \sum_s q_s (1 - \pi_s) R^* - \sum_s q_s \left((1 - \pi_s) R + \pi_s p_s \right) \\ &\leq \sum_s q_s (1 - \pi_s)(R^* - R) - \sum_s q_s \pi_s \eta / e \\ &< 0. \end{aligned}$$

Here, the first line uses $\lim_{\tilde{c}_0 \to 0} P_s^*(C_0) = 0$, the second line follows from $p_s \geq \frac{\eta}{e}$ (cf. Eq. (14)), and the last line follows from Assumption 2. It follows that there exists a unique level of consumption, $c_0^* \in (0, c_0]$, that satisfies $F(c_0^*) = 0$. This in turn implies that the induced level of asset prices, $(P_s^*(c_0^*))_{s \in \overline{S}}$, solve the system $(A.78 - A.79)$.

Next suppose $R^* = R$. In this case, it is easy to check (by guess and verify) that the solution features $c_0^* = c_0$ and $p_s^* = p_s$ for each $s \in \overline{S}$.

Next suppose $R^* > R$. In this case, we first show that $\frac{u'(c_0^*)}{u'(c_0)} > \frac{R^*}{R}$. To prove this, let $\overline{c}_0$ denote the level of consumption that satisfies $\frac{u'(\overline{c}_0)}{u'(c_0)} = \frac{R^*}{R}$. From our earlier analysis, it suffices to show that $F(\overline{c}_0) > 0$. To this end, we substitute $\frac{u'(\overline{c}_0)}{u'(c_0)} = \frac{R^*}{R}$ into Eq. $(A.79)$ to observe that $P_s(\overline{c}_0)$ satisfies,

$$(1 - \pi_s) \left(\frac{1}{R^*} - \frac{1}{R} \right) + \pi_s \left(\frac{1}{P_s(\overline{c}_0)} - \frac{1}{p_s} \right) = 0. \tag{A.80}$$

Since $R^* > R$, this equation implies $P_s(\overline{c}_0) \leq p_s$, with strict equality if $\pi_s < 1$. Note also that we have $p_s < R < R^*$ (see Proposition 1). Using these expressions, we obtain,

$$\begin{aligned} (1 - \pi_s)(R^* - R) + \pi_s (P_s(\overline{c}_0) - p_s) &= (1 - \pi_s) \frac{(R^* - R)}{R^* R} + \pi_s \frac{(P_s(\overline{c}_0) - p_s)}{R^* R} \\ &\geq (1 - \pi_s) \frac{(R^* - R)}{R^* R} + \pi_s \frac{(P_s(\overline{c}_0) - p_s)}{P_s(\overline{c}_0) p_s} \\ &= 0, \end{aligned}$$

with strict inequality if $\pi_s < 1$. Here, the inequality follows because $P_s(\overline{c}_0) - p_s \leq 0$ and $P_s(\overline{c}_0) p_s < R^* R$, and the last line follows from Eq. $(A.80)$. After multiplying the previously displayed inequality with q_s and summing over s (and observing that $\pi_s < 1$ for at least one state), we obtain $F(\overline{c}_0) > 0$. This proves that $c_0^* < \overline{c}_0$ and thus $\frac{u'(c_0^*)}{u'(c_0)} > \frac{R^*}{R}$.

This analysis also implies that $p_s^*/p_s < 1$ for each $s \in \overline{S}$, because

$$p_s^* = P_s\left(c_0^*\right) < P_s\left(\bar{c}_0\right) \leq p_s.$$

Here, the first inequality follows since the function $P_s(\cdot)$ is strictly increasing and the second inequality follows from Eq. $(A.80)$.

We next show that p_s^*/p_s is strictly increasing in s. To reach a contradiction, suppose there exists $s, s' \in \overline{S}$ with $s < s'$ (and thus, $0 < \pi_s < \pi_{s'}$) such that,

$$p_s^*/p_s \geq p_{s'}^*/p_{s'}. \tag{A.81}$$

Note that this implies $p_{s'}^* \leq p_s^* \frac{p_{s'}}{p_s} < p_s^*$ (since $p_{s'} < p_s$ fore regular locations). Using the inequalities, $p_{s'}^* < p_s^*$ and $p_{s'} < p_s$, together with $R^* > R, p_s^* < p_s, p_{s'}^* < p_{s'}$, we further obtain,

$$\frac{R^*}{p_{s'}^*} > \frac{R^*}{p_s^*} ... \frac{R}{p_{s'}} > \frac{R}{p_s}. \tag{A.82}$$

Here, the notation with three dots means that we cannot compare the terms, $\frac{R^*}{p_s^*}$ and $\frac{R}{p_{s'}}$, but all the other inequalities hold. Next note that $(A.81)$ implies the inequality,

$$\frac{R^*}{p_{s'}^*}\frac{R}{p_s} \geq \frac{R^*}{p_s^*}\frac{R}{p_{s'}}. \tag{A.83}$$

Combining this with the ordering in $(A.82)$, we also obtain the inequality,[17]

$$\frac{R^*}{p_{s'}^*} + \frac{R}{p_s} \geq \frac{R^*}{p_s^*} + \frac{R}{p_{s'}}. \tag{A.84}$$

Using these inequalities, we next obtain,

$$\begin{aligned}
\frac{u'\left(c_0^*\right)}{u'\left(c_0\right)} &= \frac{1-\pi_s+\pi_s\frac{R^*}{p_s^*}}{1-\pi_s+\pi_s\frac{R}{p_s}} \\
&< \frac{1-\pi_{s'}+\pi_{s'}\frac{R^*}{p_s^*}}{1-\pi_{s'}+\pi_{s'}\frac{R}{p_s}} \\
&\leq \frac{1-\pi_{s'}+\pi_{s'}\frac{R^*}{p_{s'}^*}}{1-\pi_{s'}+\pi_{s'}\frac{R}{p_{s'}}} = \frac{u'\left(c_0^*\right)}{u'\left(c_0\right)},
\end{aligned}$$

which yields a contradiction. Here, the first and the last equalities follow from Eq. $(A.79)$. The inequality in the second line follows since $\frac{R^*}{p_s^*} > \frac{R}{p_s}$ and $\pi_{s'} > \pi_s$. The inequality in the third line follows from the inequalities in $(A.83)$ and $(A.84)$.

This proves by contradiction that the opposite of $(A.81)$ must hold, that is, $p_s^*/p_s < p_{s'}^*/p_{s'}$ for each $s, s' \in \overline{S}$ with $s < s'$. This completes the proof of the lemma. $\square$

[17] To prove this, consider four numbers ordered according to, $a > b...c > d$, and that satisfy $ad \geq bc$. Without loss of generality, suppose also $b > c$ (the other case is symmetric). We claim that $a + d \geq b + c$. To reach a contradiction, suppose $a + d < b + c$. This implies, $d < c - (a - b)$. Multiplying this with $a - (a - b) = b$, we obtain $ad - (a - b)\,d < bc - (a - b)\,b$. Since $ad \geq bc$ and $-d > -b$ (and $a - b > 0$), this yields a contradiction and proves that $a + d \geq b + c$.

Proof of Proposition 6. First consider banks' decisions in period 0. Following similar steps as in Section 4, they solve the following problem,

$$\max_{x^{*,*},x^{out,*},y^*,(z^*_s)_s} u\left(c^*_0\right) + x^{*,*}R^* + \sum_s \gamma_s \left(x^{out,*}\overline{R}_s + y^* + z^*_s\right) M^*_s,$$
$$\text{where } c^*_0 + x^{*,*} + x^{out,*} + q_f y^* + \sum_s q_s z^*_s = 1 + q_f \eta^*$$
$$\text{and } M^*_s \equiv 1 - \pi_s + \pi_s \frac{R^*}{p^*_s}.$$

Substituting $l^*_s = x^{out,*}\overline{R}_s + y^* + z^*_s - \eta^*$ and using the pricing relations $\sum_s q_s \overline{R}_s = 1$ and $\sum_s q_s = q_f$, the problem reduces to solving the following problem (which we also state in the main text),

$$\max_{x^{*,*}\geq 0,(l^*_s\geq -\eta^*)_s} u\left(c^*_0\right) + x^{*,*}R^* + \sum_s \gamma_s l^*_s M^*_s, \tag{A.85}$$
$$\text{where } c^*_0 + x^{*,*} + \sum_s q_s l^*_s = 1.$$

Here, the constraint, $l^*_s \geq -\eta^*$, follows since banks are required to have nonnegative consumption.

We next show that $x^{*,*} = 0$, that is, banks in the special location also strictly prefer foreign risky assets (in period 0) to local risky assets. By Eq. (32), we have $E\left[\overline{R}^*_s \frac{q_s}{\gamma_s}\right] = 1$. This implies that one of the two must hold: (i) there exists an aggregate state $\tilde{s}$ such that $\overline{R}^*_{\tilde{s}} \frac{q_{\tilde{s}}}{\gamma_{\tilde{s}}} < 1$, or (ii) $\overline{R}^*_s \frac{q_s}{\gamma_s} = 1$ for each s.

First suppose (i) holds so that there exists an aggregate state $\tilde{s}$ such that $\overline{R}^*_{\tilde{s}} \frac{q_{\tilde{s}}}{\gamma_{\tilde{s}}} < 1$. From problem $(A.85)$, the marginal return from investing in state $\tilde{s}$ is given by $\frac{\gamma_{\tilde{s}}}{q_{\tilde{s}}} M^*_{\tilde{s}}$. Combining this with the inequality, $\overline{R}^*_{\tilde{s}} \frac{q_{\tilde{s}}}{\gamma_{\tilde{s}}} < 1$, we obtain,

$$\frac{\gamma_{\tilde{s}}}{q_{\tilde{s}}} M^*_{\tilde{s}} > M^*_{\tilde{s}} \overline{R}^*_{\tilde{s}} \geq R^*.$$

Here, the second inequality follows since Lemma 1 shows $M^*_{\tilde{s}} \overline{R}^*_{\tilde{s}} > R^*$ if $\pi_{\tilde{s}} \in (0,1)$, and it can be checked that $M^*_{\tilde{s}} \overline{R}^*_{\tilde{s}} = R^*$ if $\pi_{\tilde{s}} = 0$ or $\pi_{\tilde{s}} = 1$. Thus, in this case, investing in the aggregate state $\tilde{s}$ strictly dominates investing in the local risky asset.

Now suppose (ii) holds so that $\overline{R}^*_s \frac{q_s}{\gamma_s} = 1$ for each s. Let $\overline{s}$ be a state with $\pi_{\overline{s}} \in (0,1)$. Then, the marginal return from investing in state $\overline{s}$ is given by,

$$\frac{\gamma_{\overline{s}}}{q_{\overline{s}}} M^*_{\overline{s}} = M^*_{\overline{s}} \overline{R}^*_{\overline{s}} > R^*,$$

where the second inequality follows from Lemma 1. Thus, in this case, investing in the aggregate state $\overline{s}$ strictly dominates investing in the local risky asset.

Combining the two cases, we conclude that $x^{*,*} = 0$. Next consider the optimal liquidity holdings, l^*_s. Assuming there is an interior solution, the optimality conditions for problem $(A.85)$ result in Eqs. (34) listed in the main text.

Next consider banks' and distressed sellers' decisions in period 1. Banks spend all of their available liquidity in period 1 to purchase risky assets, which implies $\chi^*_s = l^*_s / p^*_s$. Distressed sellers' optimal demand, $\tilde{\chi}^*_s$, is determined by Eq. (36). Substituting these expressions (as well as $x^{*,*} = 0$) into Eq. (7), we also obtain the market clearing condition (35).

It remains to show that (under Assumption 2) there exists a unique solution, $x^{in,*}, c_0, (l^*_s)_s, (p^*_s)_s, (\tilde{\chi}^*_s)_s$, to the equations listed. Note that Eqs. (32) and (34) correspond to the system of equations analyzed in Lemma 4. Using the lemma, there exists a unique solution to these equations that satisfy $c_0^* \leq c_0$ and $p_s^* \leq p_s$ for each $s \in S$ (note that $S = \overline{S}$ since $\pi_1 > 0$). Given prices, Eq. (36) uniquely determines distressed sellers' demand levels, $\tilde{\chi}_s^*$.

To characterize the remaining allocations, we multiply the market clearing conditions (35) with q_s and aggregate over all states to obtain,

$$\sum_s q_s p_s^* \left(e - \tilde{\chi}_s^* + x^{in,*}\right) = \sum_s q_s \left(\eta^* + l_s^*\right) = 1 + q_f \eta^* - c_0^*. \tag{A.86}$$

Here, the second equality follows from the budget constraint (33). Hence, the inflows solve the equation, $G\left(x^{in,*}\right) = 0$, where we define the function,

$$G\left(X^{in}\right) = \sum_s q_s p_s^* \left(e - \tilde{\chi}_s^* + X^{in}\right) + c_0^* - \left(1 + q_f \eta^*\right). \tag{A.87}$$

Note that $G\left(X^{in}\right)$ is increasing in X^{in} with $\lim_{\tilde{x}^{in} \to \infty} G\left(X^{in}\right) = \infty$. Note also that

$$\begin{aligned} G(0) &\leq \sum q_s p_s e + c_0 - \left(1 + q_f \eta^*\right) \\ &= 1 + q_f \eta - \sum_s q_s p_s x - \left(1 + q_f \eta^*\right) \\ &= q_f \left(\eta - \eta^*\right) - \sum_s q_s p_s x < 0. \end{aligned}$$

Here, the first line follows since $p_s^* \leq p_s, c_0^* \leq c_0, \tilde{\chi}_s^* \geq 0$, the second line follows since the analogue of Eq. (A.86) also holds for regular locations, and the last line follows from Assumption 2. It follows there exists a unique solution, $x^{in,*} > 0$, to the equation, $G\left(x^{in,*}\right) = 0$, which determines the equilibrium level of inflows.

Finally, using Eq. (35), we also obtain the equilibrium liquidity holdings,

$$l_s^* = p_s^* \left(e - \tilde{\chi}_s^* + x^{in,*}\right) - \eta^* \text{ for each } s.$$

These liquidity holdings satisfy the budget constraint (33) since $x^{in,*}$ satisfies Eq. (A.86). This characterizes the equilibrium and completes the proof of Proposition 6. □

Proof of Proposition 9. Most of the proof parallels the proof of the proof of Proposition 6. Specifically, the same steps in that proof imply that $x^{*,*} = 0$ also in this case. It remains to show that (under Assumption 2) there exists a unique solution, $x^{in,*}, c_0, \left(l_s^*\right)_s, \left(p_s^*\right)_s, \left(\tilde{\chi}_s^*\right)_s$, to the equations listed.

First consider the consumption level, c_0^*, as well as the fire-sale prices, p_s, for states $s > 1$ (that feature $\pi_s > 0$). Note that Eqs. (32) and (A.59) correspond to the system of equations analyzed in Lemma 4. Using the lemma, there exists a unique solution to these equations that satisfy $c_0^* \leq c_0$ and $p_s^* \leq p_s$ for each s with $\pi_s > 0$.

Next consider state $s = 1$ (that features $\pi_1 = 0$). For this state, the price is characterized by Lemma 4 and Eq. (A.60). When $R^* = R$, we have an interior solution with $c_0^* = c_0$ and $p_1 = p_1^*$. When $R^* > R$, we have a corner solution with $c_0^* < c_0, l_1^* = -\eta^*$. This also implies $p_1 = 0$ in view of Eq. (35).

It follows that the equilibrium consumption level and prices, $\left(c_0^*, \left(p_s^*\right)_{s \in S}\right)$, are uniquely characterized and they satisfy $c_0^* \leq c_0$ and $p_s^* \leq p_s$ for each s. The rest of the proof follows the steps in the proof of Proposition 6. In particular, distressed sellers' demand levels, $\tilde{\chi}_s^*$, are determined by Eq. (36) given the prices. The inflows are characterized as the unique solution to $G\left(x^{in,*}\right) = 0$ (where $G(\cdot)$ denotes the function defined in $(A.87)$). There exists a unique and strictly positive solution to this equation because $p_s^* \leq p_s, c_0^* \leq c_0, \tilde{\chi}_s^* \geq 0$ and the parameters in the special location satisfy Assumption 2. The equilibrium liquidity purchases are determined by, $l_s^* = p_s^*\left(e - \tilde{\chi}_s^* + x^{in,*}\right) - \eta^*$, given the remaining allocations. $\square$

Proof of Proposition 7. It can be checked that Eqs. (38) solve the equation system $(32-36)$. By Proposition 6, this solution correspond to the equilibrium in the special location. The remaining statements in the proposition follow by inspecting the closed-form solution. $\square$

Proof of Proposition 10. It can be checked that Eqs. (38) solve the equation system $(32-36)$ after replacing Eq. (34) with Eqs. $(A.59-A.60)$. By Proposition 9, this solution corresponds to the equilibrium in the special location. The remaining statements in Proposition 7 follow from inspecting the closed-form solution. The proof for the comparative static results with respect to β is provided in Appendix A.4. $\square$

Proof of Proposition 8. The results regarding prices directly follow from Lemma 4. Note that the lemma also implies $c_0^* < c_0$, which in turn implies $\overline{x}^{out,*} = 1 - c_0^* > x = 1 - c_0$. It remains to show that $x^{in,*} > \overline{x}^{out,*} = 1 - c_0^*$. To this end, consider the function $G(\cdot)$ defined by Eq. $(A.87)$ in the proof of Proposition 6. Recall that the inflows are defined as the solution to $G\left(x^{in,*}\right) = 0$. Next note that,

$$\begin{aligned}
G\left(\overline{x}^{out,*}\right) &= \sum_s q_s\left(e - \tilde{\chi}_s^* + \overline{x}^{out,*}\right)p_s^* + c_0^* - \left(1 + q_f\eta^*\right) \\
&< \sum_s q_s\left(e + \overline{x}^{out,*}\right)p_s + c_0^* - \left(1 + q_f\eta\right) \\
&= c_0^* - c_0 + \left(\overline{x}^{out,*} - x\right)\sum_s q_s p_s \\
&= \left(c_0^* - c_0\right)\left(1 - \sum_s q_s p_s\right).
\end{aligned}$$

Here, the second line uses $p_s^* < p_s, \tilde{\chi}_s^* \geq 0, \eta^* = \eta$, the third line follows since the analogue of Eq. $(A.86)$ also holds for the regular locations, and the last line substitutes $\overline{x}^{out,*} = 1 - c_0^*$ and $x = 1 - c_0$. Next note that $\sum_s q_s \overline{R}_s = 1$ [cf. Eq. (32)]. Since $p_s \leq \overline{R}_s$, with strict inequality for states with $\pi_s > 0$, this implies $\sum_s q_s p_s < 1$. Combining this with $c_0^* < c_0$, we have $G\left(\overline{x}^{out,*}\right) < 0$. Since $G(\cdot)$ is an increasing function, it follows that the solution to the equation, $G\left(x^{in,*}\right) = 0$, satisfies $x^{in,*} > \overline{x}^{out,*}$, completing the proof. $\square$

Proof of Proposition 11. Note that Eqs. (40) and $(A.62)$ are a special case of the equation system analyzed in Lemma 4 when $S = \{1,2,3\}$ and $\pi_1 = 0$ (so $\overline{S} = \{2,3\}$). Applying the lemma, there exists a unique solution, (c_0^*, p_2^*, p_3^*), that features $c_0^* < c_0$ and $p_2^*/p_2 < p_3^*/p_3 < 1$. Since $c_0^* < c_0$, by Eq. $(A.60)$, we also have $l_1^* = -\eta^*$. By Eq. (35), this also implies $p_1^* = 0$. The inequalities, $x^{in,*} > \overline{x}^{out,*}$ and $\overline{x}^{out,*} > x$, follow from the same argument as in the proof of Proposition 8.

It remains to establish the comparative statics with respect to R and β. First consider a decrease in R. Let $\tilde{p}_2 = p_2/R$ denote the price-to-return ratio in state 2. Then, equations (27) and (28) (that characterize the equilibrium in regular locations) can be written in terms of $(\tilde{p}_2, x)$ as,

$$u'(1-x) = R\left(\beta + (1-\beta)(1-\pi+\pi\tilde{p}_2)\left(1-\pi+\pi\frac{1}{\tilde{p}_2}\right)\right),$$
$$\text{and } \tilde{p}_2 = \min\left(1, \frac{\eta/R + x(1-\pi)}{e + x(1-\pi)}\right).$$

As before, the first equation describes $\tilde{p}_2$ as a decreasing function of x, the second equation describes $\tilde{p}_2$ as an increasing function of x, and the equilibrium corresponds to the intersection. Moreover, decreasing R strictly decreases the first curve for each x, and (under Assumption 1) strictly increases the second curve for each x. It follows that decreasing R decreases the equilibrium level of foreign investment, x. Thus, decreasing R also decreases the price level, $p_2 = \min\left(R, \frac{\eta + Rx(1-\pi)}{e + x(1-\pi)}\right)$, while leaving $p_3 = \min\left(R, \frac{\eta}{e}\right)$ unchanged.

Next note that Eq. $(A.63)$ implies,

$$\frac{\overline{p} - \overline{p}^*}{R^* - R} = \frac{1-\pi}{\pi}\frac{M_1\beta + M_2(1-\beta)}{M_2(1-\beta) + M_3\beta}. \tag{A.88}$$

This implies that $\frac{\overline{p}-\overline{p}^*}{R^*-R} < \frac{1-\pi}{\pi}$ since $M_1 < M_2 < M_3$. After substituting for $M_s = 1 - \pi_s + \pi_s \frac{R}{p_s}$, the equation can also be written as,

$$\frac{\overline{p} - \overline{p}^*}{R^* - R}\frac{\pi}{1-\pi} = \frac{\beta\frac{1}{R} + (1-\beta)\xi(R)}{(1-\beta)\zeta(R) + \beta\frac{1}{p_3}}, \text{ where } \xi(R) = (1-\pi)\frac{1}{R} + \pi\frac{1}{p_2}.$$

It can be checked that increasing $\xi(R)$ increases the right hand side (since it is less than one). Thus, decreasing R increases $\frac{\overline{p}-\overline{p}^*}{R^*-R}$, both directly via the $1/R$ term in the numerator, and indirectly by increasing $\xi(R) = (1-\pi)\frac{1}{R} + \pi\frac{1}{p_2}$. It follows that decreasing R decreases $\overline{p}^* - \overline{p}$.

Next consider a decrease in β. By Proposition 5, this increases x, which in turn increases p_2 and leaves p_3 unchanged. Thus, it also decreases M_2 and leaves M_1 and M_3 unchanged.

Inspecting Eq. $(A.88)$ illustrates that decreasing β tends to increase $\frac{\overline{p}-\overline{p}^*}{R^*-R}$ by decreasing the weight on the smaller marginal utility (M_1) in the numerator as well as the weight on the larger marginal utility (M_3) in the denominator. However, decreasing β also generates an indirect effect since it also decreases M_2. As it turns out, the indirect effect tends to decrease $\frac{\overline{p}-\overline{p}^*}{R^*-R}$, counteracting the direct effect. We conjecture that the indirect effect does not overturn the direct effect, that is, $\frac{d}{d\beta}\left(\frac{\overline{p}-\overline{p}^*}{R^*-R}\right) < 0$, which in turn implies that decreasing β decreases $\overline{p} - \overline{p}^*$ (equivalently, increases $\overline{p}^* - \overline{p}$).

To prove this conjecture, we differentiate Eq. $(A.88)$ with respect to β, which implies that $\frac{d}{d\beta}\left(\frac{\overline{p}-\overline{p}^*}{R^*-R}\right) < 0$ if and only if,

$$\frac{M_1\beta + M_2(1-\beta)}{M_2(1-\beta) + M_3\beta} > \frac{M_1 + \frac{d}{d\beta}(M_2(1-\beta))}{M_3 + \frac{d}{d\beta}(M_2(1-\beta))}.$$

We make a second conjecture that $\frac{d}{d\beta}(M_2(1-\beta)) < 0$. Under this conjecture, the above inequality holds because,

$$\frac{M_1\beta + M_2(1-\beta)}{M_2(1-\beta) + M_3\beta} > \frac{M_1}{M_3} > \frac{M_1 + \frac{d}{d\beta}(M_2(1-\beta))}{M_3 + \frac{d}{d\beta}(M_2(1-\beta))}.$$

Here, the first equality follows from $M_1 < M_2 < M_3$, and the second inequality uses $M_1 < M_3$ together with $\frac{d}{d\beta}\left(M_2\left(1-\beta\right)\right) < 0$.

Hence, it remains to prove the second conjecture, $\frac{d}{d\beta}\left(M_2\left(1-\beta\right)\right) < 0$. To this end, note that Eq. (27) in Section 6 implies,

$$u'\left(1-x\right) = R\beta + \left(\left(1-\pi\right)R + \pi p_2\right)\left(1-\beta\right)M_2.$$

Taking the derivative with respect to β, and using $\frac{du'(1-x)}{d\beta} < 0$ (since increasing β decreases x), we obtain,

$$R + \pi\frac{dp_2}{d\beta}\left(1-\beta\right)M_2 + \left(\left(1-\pi\right)R + \pi p_2\right)\frac{d}{d\beta}\left(M_2\left(1-\beta\right)\right) < 0.$$

From here, note that $R + \pi\frac{dp_2}{d\beta}\left(1-\beta\right)M_2 > 0$ implies that $\frac{d}{d\beta}\left(M_2\left(1-\beta\right)\right) < 0$. That is, our second conjecture follows from a third conjecture,

$$\left(1-\beta\right)\pi\left(\frac{-dp_2}{d\beta}\right)M_2 < R. \tag{A.89}$$

To prove the third conjecture, note that Eq. (27) can also be written as,

$$\frac{u'\left(1-x\right)}{R} = \beta + \left(1-\beta\right)\left(1-\pi+\pi\frac{p_2}{R}\right)\left(1-\pi+\pi\frac{R}{p_2}\right).$$

Taking the derivative with respect to β, and using $\frac{du'(1-x)/R}{d\beta} < 0$, we obtain,

$$\left(1-\beta\right)\pi\left(\frac{-dp_2}{d\beta}\right)\frac{M_2}{R} < \frac{\left(1-\pi+\pi\frac{p_2}{R}\right)\left(1-\pi+\pi\frac{R}{p_2}\right)-1}{\frac{1-\pi+\pi\frac{p_2}{R}}{\left(1-\pi\right)\frac{1}{R}+\pi\frac{1}{p_2}}\frac{R}{p_2^2}-1} < 1.$$

Hence, the last inequality follows since it is equivalent to, $\left(1-\pi+\pi\frac{R}{p_2}\right)\left(\left(1-\pi\right)\frac{1}{R}+\pi\frac{1}{p_2}\right) < \frac{R}{p_2^2}$, which in turn holds since $1-\pi+\pi\frac{R}{p_2} < \frac{R}{p_2}$ and $\left(1-\pi\right)\frac{1}{R}+\pi\frac{1}{p_2} < \frac{1}{p_2}$. This establishes the third conjecture in $(A.89)$, which in turn implies $\frac{d}{d\beta}\left(\frac{\overline{p}-\overline{p}^*}{R^*-R}\right) < 0$. This completes the proof. $\square$