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**ABSTRACT**

This paper studies how interest groups allocate campaign contributions when congressmen are connected by social ties. We establish conditions for the existence of a unique Nash equilibrium in pure strategies for the contribution game and characterize the associated allocation of the interest groups' moneys. While the allocations are generally complex functions of the environment (the voting function, the legislators' preferences and the social network topology), they are simple, monotonically increasing functions of the respective legislators' Bonacich centralities when the legislators are office motivated or the number of legislators is large. Using data on the 109th-113th Congresses and on congressmen's alumni connections, we estimate the model and find evidence supporting its predictions.

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A data appendix is available at <http://www.nber.org/data-appendix/w22739>

# 1 Introduction

There is a large theoretical and empirical literature studying interest groups' influences on congressmen. This literature aims to derive and test predictions about interest groups' activities, starting with the assumption that congressmen are self-interested, individualistic utility maximizers. However, a long tradition in political science notes that treating legislators as solely self-interested individuals may be reductive, because it ignores deep connections of friendship, respect and patronage that transcend partisan or ideological divisions.<sup>1</sup> Recent work has creatively used a variety of data sources and methodologies to map legislators' social ties and show that these connections can help explain legislative success (Fowler [2006], Cho and Fowler [2010]), voting behavior (Arnold et al. [2000], Maskett [2008], Cohen and Malloy [2014]), and may provide insights on congressional power centers (Porter et al. [2005], Zhang et al. [2008]). For the most part, however, social connections among legislators have been ignored by the literature on interest groups. If interpersonal relations truly play a role in legislators' behavior, then we should expect them to play a role in how interest groups allocate resources among legislators.

In this paper, we present a new theory of campaign contributions in which legislators care about how other legislators in their social network behave. Even for realistically complex networks, our theory provides sharp predictions on how the interest groups allocate their resources based on social network topology. We then use data from the 109th-113th Congresses to estimate the model. We find robust evidence that the measures of centrality suggested by our theory have a significant influence on the spending decisions of Political Action Committees (PACs).

In our model,  $n$  legislators vote to pass or reject a policy. Legislators care about the policy outcome, but also care about the resources they can obtain from interest groups and about the behavior of other legislators to whom they are socially tied. We assume that legislators like to receive resources from interest groups (for example, because these resources increase the likelihood of being reelected);<sup>2</sup> they also like to vote for the option that they think is chosen by their friends. Social ties are represented by a network matrix whose generic element  $g_{ij}$  represents the intensity

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<sup>1</sup> See, among others, Eulau [1962], Caldeira et al. [1993], Baker [1980], Arnold et al. [2000]. Among early quantitative studies of legislators' social interactions, see Rice [1927, 1928], Rountt [1938], Patterson [1959] and Matthews and Stimpson [1975]. For historical discussions, see for example Truman [1951], Bailey and Samuel [1952] and Clapp [1963].

<sup>2</sup> While it is useful to think of the interest groups' resources as money, this does not need to be the case. An example of a non-monetary resource is information that the group can provide to the legislator.

of the influence of congressman  $j$  on  $i$ . Two interest groups compete for the legislators' votes. Interest group  $A$  aims to maximize the share of legislators who vote for a given policy; interest group  $B$  aims for the opposite result. Each interest group has a given budget and can commit to offer payments to the legislators that are contingent on the legislators' votes; the legislators cast their ballots after observing the offers. We establish the conditions for the existence of a unique pure strategy Nash equilibrium of this game and characterize the associated equilibrium allocation of resources.

Perhaps unsurprisingly, we find that the allocation of the interest groups' moneys is generally a complex function of the voting function, the legislators' preferences for the policy and the geometry of the social network. While this relationship can be characterized in closed form, in practice it may be hard to compute it exactly for large networks, creating a challenge for empirical analysis. However, we show that when legislators are office motivated or when their number is large, the relationship between network topology and allocation of resources is simple: the interest groups allocate their resources in a way that is proportional to the Bonacich measure of centrality, a well known concept of centrality in network theory (see, for example, Zenou [2015]).<sup>3</sup>

We then estimate our model and test whether the legislators' Bonacich centralities are good predictors of business PACs' contributions. To construct the social network, we use two alternative approaches.

In the first, we exploit the insight from the political science literature that congressmen become well acquainted while serving in congressional committees (see Caldeira and Patterson [1987], Masker [2008] and Bratton and Rouse [2011]). We construct social networks in which links between two congressmen are proportional to the number of shared committees. An advantage of constructing legislators' social networks with committee memberships is that committees are relatively stable over time, and thus determined long before PAC contributions are chosen.<sup>4</sup> To control for possible unobserved factors driving both committee membership and PAC contributions, we implement a two-step procedure a' la Heckman, as recommended by Blume et al. [2015].

In the second approach, we exploit the idea that educational institutions provide a basis for

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<sup>3</sup> The exact relationship between the Bonacich measure of centrality and the resources of legislators can also be characterized in closed form, but it depends on the specific assumptions on the legislator's utility function.

<sup>4</sup> For example, the chairman of the Ways and Means Committee in 2006 was Bill Thomas. He had become chairman in 2001.

social networks (see Cohen et al. [2008], Fracassi and Tate [2012], Cohen and Malloy [2014], Do et al. [2016], among others). We therefore construct social networks using the congressmen’s alumni connections: two congressmen are connected if they graduated from the same institution or if (alternatively) they graduated from the same institution in the same period. This approach gives us a network that is exogenous by construction to the political process.

Using networks constructed by committee membership and by educational institution, we obtain consistent results that support our theory. We find that standard measures of centrality like degree centrality (measuring the number of “connected” nodes), and betweenness centrality (roughly speaking measuring how well a node connects to other nodes), have no power in explaining business PAC contributions. We instead find that, as predicted by the theory, legislators’ Bonacich centralities have an highly significant effect. The relevance of the Bonacich centralities, moreover, is robust to many natural controls suggested by the previous literature on the determinants of PAC contributions: measures of members’ relative “power” inside the house (i.e., chairmanship, seniority and participation in important committees such as Appropriations or Way and Means), the per-capita income in their electoral districts, the margins of victory in the legislators’ elections (as a proxy for the competitiveness in the district), gender, party affiliation, legislators’ ideologies and Congress-specific effects (as captured by Congress fixed effects). Adding information on network topology as suggested by the theory significantly improves the fit of the model compared with alternative specifications that ignore this information.

The intuition behind the result that Bonacich centrality is a sufficient statistic to determine the allocation of resources for a sufficiently large  $n$  depends on the following simple observation: as  $n$  increases, the equilibrium probability that a legislator is pivotal for the outcome converges to zero. As the preferences of the legislator for the legislative outcome become decreasingly important, the dominant factor becomes the social network (and the interest groups’ moneys). At that point, only the Bonacich centrality matters (as opposed to other measures of centrality like degree or betweenness that focus on different dimension of the network topology). This result depends on the fact that Bonacich centrality captures the recursive nature of the legislators’ social interactions in the network, a feature that has also been highlighted in other environments (Ballester, Calvo-Armengol and Zenou [2006], Zenou [2015]).

Our work is related to three strands of literature that to date have had little overlap. First, it relates to the political science literature on social networks in Congress already mentioned above.

In addition to providing a variety of approaches to describe the legislators’ social networks, this literature has shown that legislators’ social connections explain voting behavior (Arnold et al. [2000], Porter et al. [2005], Masket [2008], Ringe et al. [2013] and Cohen and Malloy [2014]) and legislative success, as measured by successful amendments (Monsma [1966], Fowler [2006], Canen and Trebbi [2016]), or the number of bills passed (Cho and Fowler [2010]). These recent works follow an older (if less formal) tradition in political science (see Rice [1927, 1928], Routh [1938], Eulau [1962], among others).

Our work is also connected to a large theoretical and empirical literature exploring how interest groups influence Congress.<sup>5</sup> The theoretical literature has been characterized by two types of models: informative theories, in which interest groups influence legislators by providing information (Calvert [1985], Austen-Smith and Wright [1992], Austen-Smith [1995], Bennedsen and Feldmann [2002], Cotton [2012]), and campaign contribution theories, in which interest groups influence legislators by providing resources (Denzau and Munger [1986], Snyder [1991], Groseclose and Snyder [1996], Persson [1998], Diermeier and Myerson [1999], Helpman and Persson [2001], Baron [2006], Dekel et al. [2009]).<sup>6</sup> The empirical literature has studied the determinants of PACs’ allocations of campaign contributions, documenting evidence of interest groups’ strategic behavior consistent with the campaign contribution theories (Poole and Romer [1985], Snyder [1990], Grier and Munger [1991]), Stratmann [1992], Romer and Snyder [1994] and Ansolabehere and Snyder [1999]).<sup>7</sup> This literature, however, has for the most part ignored social networks in Congress and the impact that they may have on interest groups’ activities.

Finally, our work is related to the general literature on networks, which has also studied related issues of policy intervention and marketing in networks. The seminal paper studying policy intervention in networks is Ballester, Calvo-Armengol and Zenou [2006], which was among the first to propose an economic model of how the removal of a “key player” influences individual behavior. Our work differs from this because interest groups alter the agents’ payoffs by making contingent

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<sup>5</sup> See Austen-Smith [1992] and Grossman and Helpman [2001] for surveys of theoretical research and Ansolabehere et al. [2003], Stratmann [2005] and de Figueredo and Richter [2014] for surveys of empirical research.

<sup>6</sup> Related but distinct literatures are the literatures studying the influence of the choice of a single policy-maker, and the direct acquisition of citizens’ votes. For the first, see Stigler [1971], Grossman and Helpman [1994], Dixit [1996], Dixit, Grossmann and Helpman [1997], Besley and Coate [2001], among others. For the second, see Buchanan and Tullock [1962], Anderson and Tollison [1990], Piketty [1994], Dal Bo [2007], Dekel et al. [2008].

<sup>7</sup> More recent research has extended the analysis to behavior of lobbyists, uncovering evidence that they provide expertise and access (Blanes i Vidal et al. (2012), Bertrand, Bombardini and Trebbi [2014], Kang [2015], Kang and Young You [2015]).

promises, but they do not affect the network topology. The issue of marketing in networks has been studied in the computer science literature by Domingos and Richardson [2001] and Richardson and Domingos [2002], who considered the problem of a monopolist attempting to influence customers by allocating a budget of marketing resources.<sup>8</sup> The case of competitive influencers has been studied by Bharathi, Kempe and Salek [2007] who extend a contagion model by Kempe, Kleinberg and Tardos [2003] and [2005]. In these works, marketers identify nodes in a network to start a contagion process. Contagion models have been applied in the political science literature to study influence on legislators by Groenert [2010], Guzman [2010] and Groll and Prummer [2016]. These papers, however, do not provide microfoundations of the legislators’ decisions, since they assume that legislators collectively decide according to an exogenous decision function and are influenced through mechanical contagion processes that do not account for legislators’ incentives.

The remainder of this paper is organized as follows. Section 2 presents our model of legislative behavior and competitive interest groups’ activities. In Section 3, we study the equilibrium of this game and characterize the relationship between the legislators’ preferences, the voting rule, the network topology and the interest groups’ resource allocations. Section 4 brings the model to data, and Section 5 concludes.

## 2 Model

Consider a legislature with  $n$  members who choose between one of two alternatives: a new policy, denoted by  $A$ , and a status quo policy, denoted by  $B$ . All members cast a vote for either  $A$  or  $B$  and the legislature deliberates according to a  $q$ -rule with a generic  $q \in (1/2, 1)$ , such that new policy  $A$  is chosen if it achieves a share  $q$  of votes.

Two factors determine a legislator’s choice. First, each legislator cares about whether the policy is approved or not. This is described by a parameter  $v^i$ : the utility enjoyed by  $i$  if  $A$  is approved. Since  $v^i$  can be either positive or negative, we can normalize the benefit of approving  $B$  at zero.

Second, each legislator cares directly about the vote he casts. This reflects two facts: first, interest groups observe a legislator’s actions and may choose to reward votes with monetary contributions; and second, a legislator is influenced by other legislators and derives utility from

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<sup>8</sup> In their model, the key determinant of the monopolist’s allocation is the degree centrality of a node, a measure that is not relevant in our theory and does not appear significant in our empirical analysis.

voting that depends on how his peers behave. We write legislator  $i$ 's direct utility of voting for policy  $p \in \{A, B\}$  as:

$$U^i(p) = \omega(s^i(p)) + \phi \sum_j g_{i,j} x_j(p) + \varepsilon_p^i. \quad (1)$$

The first term in (1) is the utility of the interest groups' contributions:  $s^i(p)$  is the sum of contributions pledged to  $i$  in exchange for a vote for  $p$  and  $\omega(s)$  is the utility that legislator  $i$ 's receives from contribution  $s$ . We assume  $\omega(\cdot)$  is an increasing, concave, differentiable function with  $\lim_{s \rightarrow 0} \omega'(s) = \infty$ ,  $\lim_{s \rightarrow \infty} \omega'(s) = 0$ . The second term describes the social interaction effects. As in Ballester, Calvo-Armengol and Zenou [2006], the social network is described by a  $n \times n$  matrix  $G$  with generic element  $g_{i,j} > 0$ :  $x_j(p)$  is an indicator function equal to one if legislator  $j$  votes for  $p$  and zero otherwise and  $g_{i,j}$  measures the strength of the social influence of legislator  $j$  on legislator  $i$ . Without loss of generality, we normalize the social weights so that for any  $i$ ,  $\sum_j g_{i,j} = 1$  and we assume that  $\sum_i g_{i,j} \leq \bar{g}$  for all  $j$  and some bounded  $\bar{g} < 1$ . The final term in (1) represents other exogenous factors that may affect  $i$ 's preference for or aversion to voting for  $p$ . We can set  $\varepsilon_A^i = \varepsilon^i$ , where  $\varepsilon^i$  can be positive or negative, and normalize  $\varepsilon_B^i$  at zero.

For future reference, we say that a legislator is *office motivated* if he does not care about the policy outcome (so  $v^i = 0$ ); we say that a legislator is *policy motivated* if he does care about the policy outcome (so  $v^i > 0$  or  $v^i < 0$ ).

The key assumption in (1) is that legislators like to conform to the behavior of the members of their social circle. Apart from the general evidence on social influence in Congress mentioned in the introduction, this assumption is well supported, both empirically and theoretically. On the empirical front, conformism is a phenomenon that has been well documented in the psychology literature (e.g., Asch [1951], Deutsch and Gerard [1955], Ross, Bierbrauer and Hoffman [1976] and Jones [1984]). More specifically, Cohen and Malloy [2014] have recently shown that personal connections amongst U.S. politicians have significant impacts on Senate voting behavior, even after controlling for political ideology. Canen and Trebbi [2016] have formulated and structurally estimated a model of legislative behavior in which voting depends on legislators' social ties. On the theoretical front, various authors have proposed microfoundations of conformist preferences as in (1), rationalizing them as implications of the agents' quests for social status (see Akerlof [1980], Jones [1984] and Bernheim [1994]).



Two interest groups, also denoted  $A$  and  $B$ , attempt to influence the policy outcome. Interest group  $A$  is interested in persuading as many legislators as possible to choose policy  $A$ ; interest group  $B$ , instead, is interested in persuading the legislators to choose policy  $B$ . Each interest group is endowed with a budget  $W$  and promises a contingent payment to each legislator who follows its recommendation. Specifically, interest group  $A$  promises a vector of payments  $\mathbf{s}_A = (s_A^1, \dots, s_A^n)$  to the legislators where  $s_A^i$  is the payment received by legislator  $i$  if he chooses  $A$ ; similarly, interest group  $B$  promises a vector of payments  $\mathbf{s}_B = (s_B^1, \dots, s_B^n)$  to the legislators where  $s_B^i$  is the payment received by legislator  $i$  if he votes for  $B$ .<sup>9</sup>

We assume that the interest groups do not know with certainty the legislators' preferences, and so are unable to perfectly forecast how payments affect their voting behavior. Specifically, we assume  $\varepsilon^i$  is an independent, uniformly distributed variable with mean zero and density  $\Psi > 0$ , whose realization is observed only by  $i$ . Let  $\varphi_i$  be the probability that  $i$  votes for  $A$  and  $\varphi = (\varphi_i)_{i=1}^n$  be the associated vector of probabilities. Let moreover  $q^i(\varphi)$  be legislator  $i$ 's pivot probability, that is the probability that a vote by  $i$  for  $A$  changes the outcome from  $B$  to  $A$  given  $\varphi$ . Legislator  $i$  is willing to vote for  $A$  if and only if:

$$E [U^i(B) - U^i(A)] \leq v^i q^i(\varphi). \quad (2)$$

The right hand side of (2) is the expected benefit of helping policy  $A$  win: the utility of the policy  $v^i$  times the probability that the vote is actually decisive in determining the outcome. The left hand side is the implicit cost of voting for  $A$  in terms of loss of monetary contributions, personal aversion and "social" pressure.<sup>10</sup> Naturally, we must have  $\varphi_i = E(x_i(A))$ , so (2) can be re-written as a condition on  $s_A^i, s_B^i, \varphi$  and  $\varepsilon^i$  only:

$$\varepsilon^i \leq \omega(s_A^i) - \omega(s_B^i) + v^i q^i(\varphi) + \phi \sum_j g_{i,j} (2\varphi_j - 1), \quad (3)$$

In the following, we focus on environments in which for any feasible  $s_A, s_B$  there is sufficient uncertainty that the probability of (3) is interior and so no interest group can be sure about a

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<sup>9</sup> In Section 5 we extend this basic model in various directions: we allow for more than two interest groups (Section 5.2); we consider alternative objective functions for the interest groups (Section 5.3); and we consider the case in which the legislators vote on multiple policies and interest groups have heterogeneous preferences on the policies (Section 5.4).

<sup>10</sup> From (1), we can see that  $U^i(p)$  is a function of the actions of the other legislators,  $x_j(p)$  for  $j \neq i$ . Since the agent does not know them, they are evaluated at their expected values: this is the reason we have an expectation in (2). Note moreover that  $\varepsilon^i$  is known to the agent, so it enters (2) only as a parameter.

legislator’s decision. Let  $\bar{v}$  be the highest valuation in absolute value:  $\bar{v} = \max_i |v^i|$ . A sufficient condition for this to be true, which we will maintain throughout the paper, is the following:

**Assumption 1.**  $\Psi(\bar{v} + \phi + \omega(2W)) < 1/2$ .

The important observation is that this condition is satisfied if  $\Psi$  is sufficiently small, i.e. if there is sufficient uncertainty on the legislators’ preferences.

A strategy for interest group  $l$  is a probability distribution over the set of feasible transfers  $S$ , that is:

$$S = \{s : \sum_i s^i \leq W, s^i \geq 0 \text{ for } i = 1, \dots, n\}.$$

A pair of strategies constitute a Nash equilibrium if they are mutually optimal: the strategy of interest group  $A$  maximizes the expected number of legislators who adopt  $A$  given  $\varphi$  and interest groups  $B$ ’s strategy; and the strategy of interest group  $B$  minimizes the expected number of legislators who adopt  $A$  given  $\varphi$  and interest group  $A$ ’s strategy. In the remainder of the paper we focus on equilibria in pure strategies, that is on pairs of vectors  $\mathbf{s}_A, \mathbf{s}_B$  in  $S \times S$  that are mutually optimal. Proposition 1 and 2 guarantee that a pure strategy equilibrium exists and is unique.

In the following pages we consider very complex networks that cannot be easily visualized.<sup>11</sup>

In these cases it is useful to define simple statistics that describe the position of an agent in the network. A standard measure in the theory of networks that will play an important role in the analysis below is Bonacich Centrality (Bonacich [1987], Ballester, Calvo-Armengol and Zenou [2006]). For a given network matrix  $\widehat{G}$ , the vector of Bonacich centralities, if it exists, is defined as:

$$\mathbf{b}(\delta, \widehat{G}) = (I - \delta\widehat{G})^{-1} \cdot \mathbf{1}, \tag{4}$$

where  $\delta < 1$  is a positive parameter that controls the rate of decay of the influence in indirect links,  $I$  is the identity matrix and  $\mathbf{1}$  is a column vector of ones. The Bonacich centrality of legislator  $i$  with respect to  $\widehat{G}$  and  $\delta$  is the  $i$ th entry of  $\mathbf{b}(\delta, \widehat{G})$ . Bonacich centralities may not exist because the matrix  $I - \delta\widehat{G}$  may fail to be invertible. Invertibility is guaranteed for any  $\widehat{G}$  if  $\delta$  is sufficiently small. For the remainder of the paper, a condition that guarantees that the relevant Bonacich centralities exist in our environment is the following:

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<sup>11</sup> In Section 4, we apply the model to the U.S. Congress. In this case, the network has over 400 nodes (the congressmen) and thousands of links.

**Assumption 2.** *The matrix  $I - 2\phi\Psi G$  is invertible and positive.*

Note that as for Assumption 1, this condition is satisfied if  $\Psi$  and/or  $\phi$  is sufficiently small.<sup>12</sup>

In general, it is difficult to compare the Bonacich centralities in networks with different  $n$  because an increase in the number of agents may completely change the topology of the network. However, the comparison is straightforward when the agents in the networks can be classified into a finite number of types, each comprising a given fraction of population. We say that two legislators  $i$  and  $j$  have the same type if they have the same preferences,  $v^i = v^j$ , and if they interact in the same way with the other legislators, so  $g_{i,k} = g_{j,k}$  and  $g_{k,i} = g_{k,j}$  for all  $k = 1, \dots, n$ . As we formally prove in Lemma 3.1, presented in the online appendix, in this case each agent of the same type has the same centrality and, more importantly, the centralities depend only on the share of the population of each type. In the following analysis we assume that there is at most a finite number  $m$  of types of legislators.<sup>13</sup>

### 3 Equilibrium contributions

The game described in the previous section has two stages. In the first stage, the *influence stage*, the interest groups simultaneously promise monetary contributions to the legislators contingent on their votes. In the second stage, the *voting stage*, the legislators simultaneously choose how to vote given the interest groups' promises. We can solve this game by backward induction: first, we solve the voting stage, taking as given the allocation of transfers; second, we solve the influence stage, given the continuation value for the voting stage.

#### 3.1 The voting stage

Each legislator chooses his ballot on the basis of his preferences, the monetary promises and his expectations of the other legislators' behavior. Because of this, the voting probabilities must be jointly determined in equilibrium and no legislator can be treated in isolation. From (3) we have

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<sup>12</sup> See, for example, Theorem 1 in Ballester, Calvo-Armengol and Zenou [2006].

<sup>13</sup> Naturally this assumption is without loss of generality if  $n$  is finite and it will play a role only when we consider sequences of economies as  $n \rightarrow \infty$ .

that the legislators' probabilities of choosing  $A$ ,  $\varphi$ , are characterized by the nonlinear system:

$$\begin{pmatrix} \varphi_1 \\ \dots \\ \varphi_n \end{pmatrix} = \begin{pmatrix} 1/2 + \Psi \left( \omega(s_A^1) - \omega(s_B^1) + v^1 q^1(\varphi) + \phi \sum_j g_{1,j} (2\varphi_j - 1) \right) \\ \dots \\ 1/2 + \Psi \left( \omega(s_A^n) - \omega(s_B^n) + v^n q^n(\varphi) + \phi \sum_j g_{n,j} (2\varphi_j - 1) \right) \end{pmatrix}. \quad (5)$$

For any  $\mathbf{s} = \mathbf{s}_A, \mathbf{s}_B$ , the system of equations (5) defines a function  $T(\mathbf{s}, \varphi)$  that maps the vector of probabilities  $\varphi$  to itself. A voting equilibrium is a fixed point  $\varphi(\mathbf{s}) = T(\mathbf{s}, \varphi(\mathbf{s}))$  of this correspondence. Since  $T$  is continuous in  $\varphi$  from  $[0, 1]^n$  to itself, Brouwer's fixed point theorem implies that an equilibrium exists for any pair  $\mathbf{s}_A, \mathbf{s}_B$  of transfers by the interest groups.

In general, (5) may admit multiple solutions and the solution may not be well behaved in the monetary transfers (as, for example, multiplicity may induce  $\varphi$  to be discontinuous in  $\mathbf{s}_A, \mathbf{s}_B$ ). The following result shows that, indeed, (5) admits a unique, well behaved solution when the legislators are office motivated, or when they are policy motivated and there is sufficiently high uncertainty on the legislators' types.

**Lemma 1.** *With office motivated legislators, there is a unique vector of equilibrium probabilities  $\varphi(\mathbf{s}) = \{\varphi_1(s), \dots, \varphi_n(s)\}$  solving (5). Moreover, the sum of the equilibrium probabilities  $\sum_i \varphi_i(s)$  is increasing, differentiable in  $s_A^i$  (respectively decreasing and differentiable in  $s_B^i$ ) for all  $i$ , and concave in  $\mathbf{s}_A$  (respectively convex in  $\mathbf{s}_B$ ). With policy motivated legislators, there is a  $\Psi^*$  such that the same properties are true for  $\Psi \leq \Psi^*$ .*

To see the intuition of this result, consider first the case in which legislators are office motivated (i.e.  $v^j = 0$  for all  $j$ ). In this case, (5) is a linear system with a unique solution  $\varphi^*$ . Consider now the marginal effect of an increase in  $s_A^i$ . Differentiating (5), we obtain:

$$\partial \varphi_j^* / \partial s_A^i = \Psi \left[ \omega'(s_A^i) \cdot 1_{j,i} + 2\phi \sum_l g_{j,l} \cdot \partial \varphi_l^* / \partial s_A^i \right], \quad (6)$$

where  $1_{j,i}$  is an indicator function equal to 1 when  $j = i$  and 0 otherwise. The first term in the square parenthesis is the direct effect of an increase in  $s_A^i$ : it induces a marginal change in legislator  $j$ 's utility of  $\omega'(s_A^i)$  if  $j = i$ , and zero otherwise. The second term is the indirect network effect: the change in  $i$ 's behavior induces a change in legislator  $l$ 's behavior  $\partial \varphi_l^* / \partial s_A^i$ , which in turn affect  $j$ 's behavior in a recursive fashion. The system of equations (6) can be rewritten in matrix form as:  $D\varphi = \Psi [D\omega + 2\phi G \cdot D\varphi]$ . We therefore have:

$$D\varphi = \Psi [I - 2\Psi\phi \cdot G]^{-1} D\omega, \quad (7)$$

where  $D\varphi$  and  $D\omega$  are the Jacobians of, respectively,  $\varphi$  and  $\omega$ ; and  $I - 2\Psi\phi G$  exists and is positive by Assumption 2. Since  $D\omega = (0, \dots, 0, \partial\omega(s_A^i)/\partial s_A^i, \dots, 0)^T$ , we have that  $\partial\varphi_j^*/\partial s_A^i = m_{j,i}\omega'(s_A^i) > 0$  and  $\partial^2\varphi_j^*(\mathbf{s})/\partial^2 s_A^i = m_{j,i}\omega''(s_A^i) < 0$  where  $m_{j,i}$  is the  $j$ th element of  $(I - 2\Psi\phi G)^{-1}$ . Voting probabilities are therefore unique, increasing and concave in  $s_A^j$ . A similar argument establishes that they also are decreasing and convex in  $s_B^i$ .

With policy motivated legislators, the analysis is a little more complicated because we need to take into account the pivot probabilities, which are nonlinear functions in  $\varphi$ . Lemma 1 shows that when there is sufficiently high uncertainty on the legislators' preferences, these nonlinearities are not problematic because the pivot probabilities are sufficiently insensitive to changes in the monetary allocations.

In the following, we will maintain the assumption that legislators are not policy motivated or, if they are policy motivated,  $\Psi$  is sufficiently small that the properties described in Lemma 1 are satisfied:

**Assumption 3.** *There is sufficient uncertainty on the legislators' preferences so that  $\sum_i \varphi_i(s)$  is increasing, differentiable in  $s_A^i$  (respectively decreasing and differentiable in  $s_B^i$ ) for all  $i$ , and concave in  $\mathbf{s}_A$  (respectively convex in  $\mathbf{s}_B$ ).*

Figure 1 illustrates the system (5) in a simple “star” network example in which there is a central legislator, say legislator 0, who is connected to all other legislators and  $n - 1$  peripheral legislators  $j = 1, \dots, 4$ , who in turn are connected only to the central legislator.<sup>14</sup> The symmetric structure implies that the probabilities of  $j = 1, \dots, 4$  are equal and so (5) collapses to two equations in two unknowns,  $\varphi_0$  and  $\varphi_j = \varphi_{-0}$  for all  $j = 1, \dots, 4$ . Assuming that legislators have the same logarithmic utility  $\omega_i(s) = \log(s)$ , the voting probabilities are characterized by:

$$\begin{aligned}\varphi_0 &= \Psi \cdot (\log(s_A^0/s_B^0) + 4\phi(2\varphi_{-0} - 1) + 6\varphi_{-0}(1 - \varphi_{-0})^2 - b), \\ \varphi_{-0} &= \Psi \cdot (\log(s_A^{-0}/s_B^{-0}) + \phi(2\varphi_0 - 1) + 3\varphi_0\varphi_{-0}(1 - \varphi_{-0})^2 + 3(1 - \varphi_0)(1 - \varphi_{-0})\varphi_{-0}^2 - b)\end{aligned}\tag{8}$$

where  $s_j^0$  (respectively,  $s_j^{-0}$ ) is the transfer by interest group  $j$  to legislator 0 (respectively,  $-0$ ). The intersection of the thick lines in Figure 1 illustrates the solution of (8) and a voting equilibrium in the case in which the interest group allocates  $W = 10$  evenly.<sup>15</sup> Given  $B$ 's promise  $\mathbf{s}_B$ , interest

<sup>14</sup> Formally,  $g_{0,j} = g_{j,0} = 1$  for all  $j$  and  $g_{i,j} = 0$  if neither  $i$  nor  $j$  are equal to zero.

<sup>15</sup> Specifically, in the example of Figure 2 we assume  $\phi = 0.25$ ,  $\Psi = 1$ ,  $b = 0$ ,  $q = 1/2$  and  $v^i = 1$  for all  $i$ .

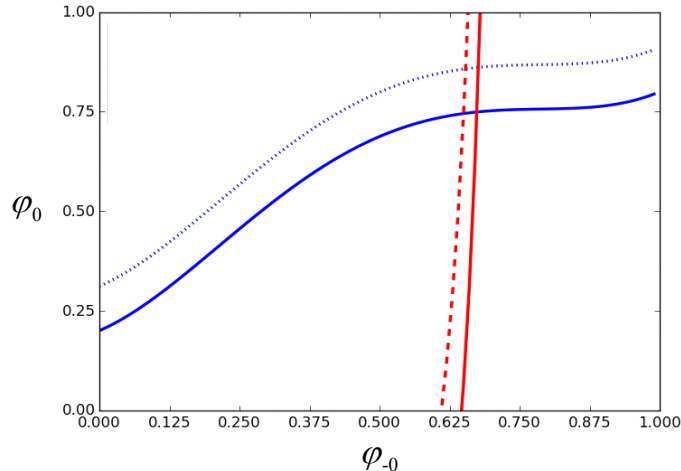


Figure 1: The flatter blue lines represent the reaction function of agent 0 to  $\varphi_{-0}$  (i.e. the first equation in (8)). The steeper lines are the reaction functions of all the other agents agents to  $\varphi_0$  (i.e. the second equation in (8)). The intersections of the reaction functions correspond to voting equilibria for different allocations of the campaign contributions.

group  $A$  can control the equilibrium probabilities by changing  $\mathbf{s}_A$ . The dashed lines in Figure 1 illustrate the effect of a redistribution by  $A$  of money on  $\varphi = (\varphi_0, \varphi_{-0})$  from the initial even distribution ( $s^i = 2$  for all  $i$ ) to a distribution that favors  $i = 0$ :  $s_A^0 = 4$ ,  $s_A^j = s_A^{-0} = 3/2$  for  $j = 1, \dots, 4$ . Despite the fact that each legislator does not directly care about the transfers sent to the other players, his behavior is indirectly affected by the transfers to the other legislators since these transfers affect behavior in his social network.

### 3.2 The influence stage

We can now turn to the interest groups' problems in the first stage. Interest group  $A$  solves:

$$\max_{\mathbf{s}_A \in S} \left\{ \sum_i [\varphi_i(\mathbf{s}_A, \mathbf{s}_B)] \right\} \quad (9)$$

taking  $\mathbf{s}_B$  as given. Interest group  $B$ 's problem is the mirror image of  $A$ 's problem, as it attempts to minimize the objective function of (9) taking  $s_A$  as given.

Under the conditions of Lemma 1, (9) is a standard maximization program. This implies that  $A$ 's optimal choice is uniquely defined and a continuous function in  $s_B$  (and symmetrically  $B$ 's reaction function is a continuous function of  $s_A$ ). The Brouwer's fixed-point theorem implies

that a Nash equilibrium in pure strategies exists for sufficiently low  $\Psi$ . The equilibrium solution, moreover, must satisfy the first order condition:

$$\sum_j \partial \varphi_j(\mathbf{s}_A, \mathbf{s}_B) / \partial s_l^i = \lambda_l \text{ and } \sum_{j=1}^n s_l^j = W \text{ for } i = 1, \dots, n, l = A, B \quad (10)$$

where  $\lambda_l$  is the Lagrangian multiplier associated with the budget constraints  $\sum_i s_l^i \leq W$  in interest group  $l$ 's problem. As formally proven in Propositions 1 and 2, moreover,  $A$ 's and  $B$ 's problems have the same Lagrangian multipliers  $\lambda_A = \lambda_B = \lambda_*$ , since they are symmetric. To discuss the implications of (10) intuitively, we will first consider the case in which legislators are office motivated. We then generalize the results to the case of legislators that are policy motivated.

### 3.2.1 Office motivated legislators

We can rewrite the necessary and sufficient condition with respect to  $s_A^i$  (10) in matrix form as

$$D\varphi^T \cdot \mathbf{1} = \lambda_*$$

where  $D\varphi^T = (\partial \varphi_1^* / \partial s_A^1, \dots, \partial \varphi_n^* / \partial s_A^1)$  and  $\mathbf{1}$  is a  $n$ -dimensional column vector of ones. Using (7), we have:

$$\begin{aligned} D\varphi^T \cdot \mathbf{1} &= \Psi \cdot D\omega^T \cdot (I - \phi^* \cdot G^T)^{-1} \cdot \mathbf{1} = \lambda_* \\ \Rightarrow D\omega^T \cdot \mathbf{b}(\phi^*, G^T) &= \lambda_* / \Psi \end{aligned} \quad (11)$$

where  $\phi^* = 2\Psi\phi$  and for the last equality we used the definition of the vector of Bonacich centralities (4). Recall that  $D\omega$  is a vector of zeros except for its  $i$ th element that is equal to  $\omega'(s_*^i)$ . We can therefore write our necessary and sufficient condition (10) as:

$$b_i(\phi^*, G^T) \cdot \omega'(s_*^i) = \lambda_* \text{ for } i = 1, \dots, n \quad (12)$$

where, without loss in generality, we have incorporated the constant  $\Psi$  in the Lagrangian multiplier  $\lambda_*$ .

The necessary and sufficient condition (12) shows the determinants of the interest group's monetary allocation. The interest group chooses  $s_*^i$  to equalize the marginal cost of resources and their marginal benefit. The marginal cost is measured by the Lagrangian multiplier  $\lambda_*$  of (9). The marginal benefit is measured by the increase in expected votes for  $A$ . Equation (12) makes clear that, because of network effects, the direct benefit of making a transfer to  $i$  is magnified by

a factor that is exactly equal to  $b_i(\phi^*, G^T)$ , the Bonacich centrality of  $i$  in  $G^T$  with a constant  $\phi^*$ .

An immediate implication of (12) is the following result:

**Proposition 1.** *With office motivated legislators, there is a unique equilibrium in which the interest groups choose the same vector of transfers  $s_*$ . The vector  $s_*$  solves the problem:*

$$\max_{s \in S} \left\{ \sum_j b_j(\phi^*, G^T) \cdot \omega_j(s^j) \right\} \quad (13)$$

where  $b_j(\phi^*, G^T)$  is the Bonacich centrality measure of  $i$  in  $G^T$  with coefficient  $\phi^* = 2\Psi\phi$ .

If we assume that the utility from money is logarithmic, then the transfer promised to legislator  $i$  is exactly proportional to his Bonacich centrality, with a factor of proportionality that depends on the inverse of the shadow cost of resources  $\lambda^*$ . In general, (13) shows that money is chosen in order to maximize a weighted sum of the legislators' monetary utilities, where the weights are exactly equal to the respective Bonacich centrality measures.

### 3.2.2 Policy motivated legislators

When legislators are not purely office motivated, the analysis is complicated by the fact that a marginal increase in a payment  $s_A^i$  has an additional effect on voting probabilities that does not exist with exclusively office motivated legislators. By affecting the voting probabilities of all players, an increase in  $s_A^i$  changes the pivot probabilities  $q(\varphi) = (q^i(\varphi))_{i=1}^n$ . This effect is irrelevant with office motivated legislators because they do not care about the policy outcome.

Taking this into account, the analysis proceeds in the same way as above assuming  $n$  sufficiently large so that the objective function of (9) is concave. Concavity and the symmetry of the two groups' problems imply that the equilibrium is unique and symmetric with  $s_A = s_B = s_*$  (a formal proof is presented in the proof of Proposition 2 in the appendix). Given this, (5) becomes the system:

$$\begin{pmatrix} \varphi_1^* \\ \dots \\ \varphi_n^* \end{pmatrix} = \begin{pmatrix} 1/2 + \Psi \left( v^1 q^1(\varphi) + \phi \sum_j g_{1,j} (2\varphi_j^* - 1) \right) \\ \dots \\ 1/2 + \Psi \left( v^n q^n(\varphi) + \phi \sum_j g_{n,j} (2\varphi_j^* - 1) \right) \end{pmatrix}. \quad (14)$$

This system admits a solution that depends *only* on exogenous variables  $\phi$ ,  $G$  and  $(v^i)_{i=1}^n$ . The equilibrium vector  $\varphi^* = (\varphi_1^*, \dots, \varphi_n^*)$  can therefore be taken as a function of only the primitives of the model.



Let  $D\mathbf{q}_*$  be the Jacobian of  $\mathbf{q}(\boldsymbol{\varphi}) = (q^1(\boldsymbol{\varphi}), \dots, q^n(\boldsymbol{\varphi}))^T$  evaluated at  $\boldsymbol{\varphi}^*$ . Moreover, let  $V$  be the diagonal matrix with  $i$ th diagonal term equal to  $v^i$ . Given this we can define the following *Modified Bonacich* centrality measure in  $V$ ,  $G^T$  and coefficients  $\Psi$  and  $\phi^*$ :

$$\mathbf{b}^{\mathcal{M}}(\phi^*, \mathbf{V}, \mathbf{G}^T) = [I - (\phi^* G^T + \Psi D\mathbf{q}_*^T \cdot V)]^{-1} \cdot \mathbf{1}. \quad (15)$$

This formula augments the standard Bonacich formula by incorporating information on the legislators' preferences and equilibrium pivot probabilities. It is easy to see that when  $v^i = 0$  for all  $i$ , it coincides with (4) with  $\delta = \phi^*$  and  $\widehat{G} = G^T$ .

Following the same steps as in the previous section, we can now characterize the equilibrium allocation solely in terms of the modified Bonacichs. We have:

**Proposition 2.** *With policy motivated legislators, there is a unique equilibrium in which the interest groups choose the same vector of transfers  $s_{**}$ . The vector  $s_{**}$  solves the problem:*

$$\max_{\mathbf{s} \in S} \left\{ \sum_j b_j^{\mathcal{M}}(\phi^*, V, G^T) \cdot \omega_j(s^j) \right\} \quad (16)$$

where  $b_j^{\mathcal{M}}(\phi^*, V, G^T)$  is the *Modified Bonacich centrality* of  $j$  in  $V$ ,  $G^T$  with coefficient  $\phi^* = 2\Psi\phi$ .

It should be stressed that  $\mathbf{b}^{\mathcal{M}}(\phi^*, \mathbf{V}, \mathbf{G}^T)$  can be constructed exclusively using the exogenous fundamentals of the problem  $q$ ,  $\phi$ ,  $V$ ,  $G$  and  $\Psi$ , so it can itself be taken as a primitive of the model. Indeed  $\mathbf{b}^{\mathcal{M}}(\phi^*, \mathbf{V}, \mathbf{G}^T)$  and the solution  $\mathbf{s}_{**}$  can be found following simple steps:

- Solve (14) to find  $\boldsymbol{\varphi}^*$  as function of the primitives (that is  $q$ ,  $\phi$ ,  $V$ ,  $G$  and  $\Psi$ ).
- Find  $D\mathbf{q}_*$  exclusively as function of  $\boldsymbol{\varphi}^*$ .
- Compute  $\mathbf{b}^{\mathcal{M}}(\phi^*, \mathbf{V}, \mathbf{G}^T)$  using (15) and solve (16) for  $\mathbf{s}_{**}$ .

A problem with Proposition 2 is that it may be laborious to compute the vector of weights  $\mathbf{b}^{\mathcal{M}}(\phi^*, \mathbf{V}, \mathbf{G}^T)$  for large networks since the construction of the pivot probabilities is quite complicated in the presence of many heterogeneous legislators with different voting probabilities. The weights  $\mathbf{b}^{\mathcal{M}}(\phi^*, \mathbf{V}, \mathbf{G}^T)$ , moreover, do not have an immediate interpretation in terms of the standard measures of network centrality because they do not depend only on the network topology  $G$ , but on preferences and the voting rule as well.

There are two cases in which we should expect the formulas in (15) to be simple. The first is when the legislators have weak preferences for the policy outcome, so  $v_i$  is small in absolute value

for all  $i$ . This is a simple implication of the fact that (15) is continuous in  $v_i$ , so the modified Bonacichs converge to the originals as  $v_i \rightarrow 0$ . Recalling that  $\bar{v} = \max_i |v_i|$ , we have:

**Corollary 1.** *The equilibrium allocation with policy motivated legislators converges to the allocation with office motivated legislators as  $\bar{v} \rightarrow 0$ .*

The second case is when the number of legislators is large. Intuitively, we should expect pivot probabilities to be quite low and irrelevant in all cases except when  $n$  is very small. In situations with a sufficiently large  $n$  we should expect the social factors described by the simple Bonacich centralities to be dominant. To formalize this point, consider a sequence of networks  $G_n$  with  $n$  legislators of  $m$  types  $j = 1, \dots, m$  with associated sequences of equilibria with office motivated legislators,  $\mathbf{s}_*^n = (s_*^{n,1}, \dots, s_*^{n,n})$ , and policy motivated legislators,  $\mathbf{s}_{**}^n = (s_{**}^{n,1}, \dots, s_{**}^{n,n})$ . In the case with policy motivated legislators, the legislators' preferences are described by some vector  $\mathbf{v} = (v_1, \dots, v_m)$ , where  $v_l$  is the preferences of a legislator of type  $l = 1, \dots, m$ . We have:

**Proposition 3.** *The equilibrium allocation with policy motivated legislators converges to the allocation with office motivated legislators as  $n \rightarrow \infty$ .*

Proposition 3 make clear that when  $n$  is large, the main determinant of the allocation of money is effectively the centrality of the legislator as measured by the standard Bonacichs  $b_j(\phi^*, G^T)$ . Therefore, when studying the U.S. Congress (which has hundreds of legislators), it is essentially without loss of generality to use simple Bonacich centralities to predict how interest groups allocate resources.

## 4 Evidence from the U.S. Congress

### 4.1 Empirical model

To make the empirical predictions of the model precise, let us assume we observe data from  $\bar{r}$  congresses ( $r = \{1, \dots, \bar{r}\}$ ), each comprised of  $n$  congressmen, characterized by a network  $G_r = \{g_{ij,r}\}$  and by a budget for an interest group's activities  $W_r$ . In equilibrium, each congressman  $j$  receives an offer  $s_{r,A}^j$  from  $A$  and an offer  $s_{r,B}^j$  from  $B$ , both equal to a common value  $s_r^j$ . Since the congressmen all vote either for  $A$  or  $B$ , the model predicts that all congressmen receive a contribution  $s_r^j$  with probability one.

Propositions 1-3 show that, in equilibrium, the contributions either solve (13) or are close to this solution. From the first order necessary and sufficient condition of this problem we have

$b_j(\phi^*, G_r^T) \cdot w'_j(s_r^j) = \lambda_r$ , where  $b_j(\phi^*, G_r^T)$  is the Bonacich centrality of  $j$  in Congress  $r$  and  $\lambda_r$  is the Lagrangian multiplier in Congress  $r$  associated with a budget  $W_r$ . We now assume that the utility is a logarithmic function  $\omega_j(s) = \log(s)$ . The first order condition can then be written as:

$$s_r^j = (1/\lambda_r) \cdot b_j(\phi^*, G_r^T). \quad (17)$$

Using the definition of the Bonacich centrality, this relation can be re-written in matrix form as:

$$\mathbf{s}_r = (1/\lambda_r) \cdot (I - \phi^* G_r^T)^{-1} \cdot \mathbf{1}, \quad (18)$$

where  $\mathbf{s}_r = (s_{1,r}, \dots, s_{n,r})^T$  and  $\mathbf{1}$  is a vector of ones.

Before bringing (18) to the data, it is useful to note that there is evidence supporting the assumption that interest groups may have direct preferences on the characteristics of the legislators whose votes they buy. For example, women in congress receive smaller campaign contributions from PACs than men, a fact that is probably better explained by interest groups' biases than by other factors influencing, say, women's preferences for contributions. To allow for these potentially relevant factors, it is useful to consider a slightly more general model in which interest groups maximize a weighted sum of the voting probabilities, where the weights capture their preferences for the legislators. Interest group  $A$ 's problem becomes:

$$\max_{\mathbf{s}_A \in \mathcal{S}} \left\{ \sum_i [\theta_i \cdot \varphi_i(\mathbf{s}_A, \mathbf{s}_B)] \right\}. \quad (19)$$

Following similar steps as in the derivation of (11), we can see that condition (18) becomes:<sup>16</sup>

$$\mathbf{s}_r = (I - \phi^* G_r^T)^{-1} \cdot \boldsymbol{\theta}, \quad (20)$$

where  $\boldsymbol{\theta}_r^* = (\theta_{1,r}, \dots, \theta_{n,r})^T$  and  $\theta_{i,r} = \theta_i/\lambda_r$ . Condition (20) says that transfers are proportional to a weighted Bonacich centrality measure with weights  $\theta$ .<sup>17</sup> This generalization provides us additional flexibility to control for factors influencing interest groups' preferences and lets the data speak about the relative importance of these factors.

To bring (20) to the data, we assume that  $\theta_{i,r}$  is a linear function of a  $\kappa$ -dimensional vector of congressman  $i$ 's characteristics in congress  $r$ ,  $X_{j,r}$ , with coefficients  $\beta = (\beta_1, \dots, \beta_\kappa)^T$ :

$$\boldsymbol{\theta}_r = \alpha \cdot \mathbf{1} + X_r \beta + \boldsymbol{\epsilon}_r, \quad (21)$$

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<sup>16</sup> The formal steps for this equation are presented in the online appendix.

<sup>17</sup> The concept of weighted Bonacich centrality measure is introduced by Ballester, Calvo-Armengol and Zenou [2006].

where  $\epsilon_r = (\epsilon_1, \dots, \epsilon_n)^T$  is a vector of random variables uncorrelated with  $X_r$  and with mean zero, describing unobserved heterogeneity in interest group preferences for the various congressmen.

Premultiplying both sides of (20) by  $(I - \phi^* G_r^T)$  and using (21), our first order necessary and sufficient condition generates the following model:

$$\mathbf{s}_r = \alpha \cdot \mathbf{1} + \phi^* G_r^T \mathbf{s}_r + X_r \boldsymbol{\beta} + \boldsymbol{\epsilon}_r. \quad (22)$$

For a sample with  $\bar{r}$  networks, stack up the data by defining  $\mathbf{s} = (s'_1, \dots, s'_{\bar{r}})^T$ ,  $\boldsymbol{\epsilon} = (\epsilon'_1, \dots, \epsilon'_{\bar{r}})^T$ ,  $X = \text{diag}\{X_r\}_{r=1}^{\bar{r}}$ ,  $G^T = \text{diag}\{G_r^T\}_{r=1}^{\bar{r}}$ . For the entire sample, the model is:

$$\mathbf{s} = \alpha \cdot \mathbf{1} + \phi^* G^T \mathbf{s} + X \boldsymbol{\beta} + \boldsymbol{\epsilon}. \quad (23)$$

Once we specify the social networks per Congress  $G_r^T$  and the relevant vector  $X_{j,r}$  of variables affecting  $j$ 's utility in congress  $r$ , we can estimate  $\alpha$ ,  $\phi^*$  and  $\boldsymbol{\beta}$ . Model (23) is a *spatial autoregressive model* (SAR), the parameters of which can be jointly obtained using Maximum Likelihood (see, e.g. Anselin, 1988).<sup>18</sup>

This model allows us to obtain an estimate of the impact of a congressman's social ties on the allocation of PACs' campaign contributions. Recall that  $\phi^* = \Psi \phi$ , where  $\Psi$  is the density of the unobserved preference parameter  $\varepsilon^i$  (see (1)) and  $\phi$  is the parameter describing the network externality (again see (1)). Since  $\Psi > 0$ , we the social network matters in the allocation of political contributions if and only if  $\phi^* > 0$ . The key hypothesis to be tested is therefore whether  $\phi^* > 0$ .

In Section 4.2, we describe the construction of the networks  $G_r^T$ , the control variables  $X_{j,r}$  and the data on PAC contributions used for  $s$ . In Section 4.3, we present the empirical results.

## 4.2 Data description

### 4.2.1 Congressional networks

Naturally, the most accurate way to map a congressman's social ties is to directly observe his social behavior and habits, or use surveys and direct interviews. This type of data is unfortunately available only for a few state assemblies and limited to a few years,<sup>19</sup> but insights from this

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<sup>18</sup> An OLS estimation of this system would not be consistent because of the simultaneity which is endemic in spatial autoregressive models (see, e.g., Anselin, 1988).

<sup>19</sup> Routh [1938] presents a quantitative analysis on the social interactions of the members of the floor of the Illinois Senate in 1937. Masket [2008] uses data on the seating assignments in the 1949 California Assembly. Caldeira and Patterson [1987] analyze survey data from the 1965 Iowa legislature. Arnold et al. [2000] present evidence from a survey of the Ohio legislature in 1993.

literature can be used to take advantage of richer and more widely available datasets. In the following, we adopt two alternative but complementary approaches. In the first, we construct social networks using membership in congressional committees: we postulate that a higher number of shared committees between two congressmen implies a stronger social connection between them. This approach is motivated by the fact that, as we will discuss more extensively below, works studying direct surveys of legislator social networks have identified committee memberships as a key factor in the formation of social links (Caldeira and Patterson [1987], Caldeira et al. [1993] and Arnold et al. [2000]). In the second approach, we construct the network using congressmen’s alumni connections: two congressmen are connected if they graduated from the same educational institution, using academic institutions attended for both undergraduate and graduate degrees.<sup>20</sup> This approach is motivated by studies showing long-lasting effects of shared educational networks. In particular, Cohen and Malloy (2014) have shown that alumni connections can help explain voting behavior in the Senate. The two approaches are complementary: in the first, social connections are assumed to be generated by shared work experience in Congress; in the second, by shared educational experiences before being elected. In the remainder of this subsection, we describe these approaches in greater detail.

**Committee membership network** Studying social ties in a state legislature for which a detailed survey is available, Caldeira et al. [1993] find that representatives who share committee assignments are more likely to identify one another as a “friend” or “respected legislator,” and that the probability of social bonds increases with the number of shared assignments. As noted by Caldeira et al. [1993], “the business of the legislature largely happens in its committees and subcommittees, where legislators become familiar with and take a measure of colleagues in a task-oriented environment. Legislators on the same committees or subcommittees share substantive interests and common workloads, so they have good reasons for establishing a relationship” (p. 12).<sup>21</sup>

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<sup>20</sup> In our baseline analysis of Section 4.3, we do not include information on the period of graduation. In an extension presented in Section 5.1, we show that the results remain qualitatively unchanged if we add information on the graduation period. Specifically, if we establish a link between two legislators if they graduate in the same institution within four years and within two years.

<sup>21</sup> These findings are confirmed using data from different legislatures and years. Arnold et al. [2000] shows that membership in the same congressional committee is among the most significant predictors of friendship, even after accounting for factors such as gender, race, party affiliation and distance between districts. Masket [2008] shows that the number of common committees is significant factor determining agreement in voting behavior. Bratton

Following this insight, we construct a legislative network using data on congressional committee assignments published by the *Clerk’s Office of the House of Representatives*. We set a link between two congressmen to be equal to the number of committees in which they both sit.<sup>22</sup> We use information on the last five election cycles, i.e. from the 109th Congress (election cycle 2004) to the 113th Congress (election cycle 2012). Each network includes roughly 440 Representatives (including midterm replacements) and about 20 standing committees.

Naturally, legislators’ unobservable characteristics may affect both the amount of contributions received and committee assignments. If this is the case, the network structure is (at least in part) endogenous. To control for network endogeneity, we implement an Heckman correction. The idea is to estimate an extended version of our model in which we explicitly account for a possible correlation between unobserved factors driving network formation and outcomes. Qu and Lee (2015) implement a control function approach for the estimation of a spatial autoregressive model with an endogenous spatial matrix in a geographic context. The strategy is to model proximity between areas as a function of observed characteristics at a first stage and then add a function of the first stage residuals to the outcome equation. We apply this framework to the case of a network model: while Qu and Lee (2015) model links between areas, we model links between politicians. We consider a standard dyadic model of link formation, used previously in the literature (see, e.g., Fafchamps and Gubert [2007], Mayer and Puller [2008], Lai and Reiter [2000], Apicella, Marlowe, Fowler and Christakis [2012] and Attanasio, et al. [2012]). When used in our context, the probability that two politicians  $i$  and  $j$  are assigned to the same committee is explained by distance between them in terms of characteristics:

$$g_{ij,r} = \delta_0 + \sum_l \delta_l |x_{i,r}^l - x_{j,r}^l| + \eta_{ij,r}, \quad (24)$$

where  $x_{i,r}^l$  for  $l = 1, \dots, L$  are  $i$ ’s characteristics. Let us assume that  $E(\epsilon_{i,r}^2) = \sigma_\epsilon^2$ , that  $E(\epsilon_{i,r}\eta_{ij,r}) = \sigma_{\epsilon\eta}$  for all  $i \neq j$  and that  $E(\eta_{ij,r}\eta_{ik,r}) = \sigma_\eta^2 \forall j = k$  and  $E(\eta_{ij,r}\eta_{ik,r}) = 0 \forall j \neq k$ .<sup>23</sup> Under such assumptions, the expected value of the error term conditional on the link formation is

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and Rouse [2011] show that sharing a committee is a significant factor determining cosponsorship between two representatives. Interestingly, these works suggest that committee affiliation appears to provide a milieu in which friendship and respect may unfold across party lines.

<sup>22</sup> We have also considered alternative ways to constrict the network by weighting links on the basis of party affiliation. Results are robust to these alternative specifications. We discuss these extensions in Section 5.1.

<sup>23</sup> These assumptions imply that the selection effect is the same for all politicians (i.e., the correlation between unobservable characteristics determining link formation and unobservable characteristics driving outcome is the same for everyone).

$E(\epsilon_{i,r}|\eta_{i1,r}, \dots, \eta_{in-1,r}) = \psi\xi_{i,r}$ , where  $\psi = \sigma_{\epsilon\eta}/\sigma_{\eta}^2$  and  $\xi_{i,r} = \sum_{j \neq i} \eta_{ij,r}$ . If  $\psi = 0$ , the links between individuals can be treated as exogenous. It is possible, however, that selection on unobservables can generate a positive  $\psi$ . In this case, equation (22) can be rewritten as:

$$\mathbf{s}_r = \alpha \cdot \mathbf{1} + \phi^* G_r^T \mathbf{s}_r + X_r \boldsymbol{\beta} + \psi \boldsymbol{\xi}_r + \epsilon_r, \quad (25)$$

where  $\boldsymbol{\xi}_r = (\xi_{i,r}, \dots, \xi_{n,r})'$  and the term  $\psi \boldsymbol{\xi}_r$  captures the selectivity bias.<sup>24</sup> Following Qu and Lee (2015), we can now estimate equation (25) after replacing  $\boldsymbol{\xi}_r$  with its estimated counterpart  $\widehat{\boldsymbol{\xi}}_r$  from the first stage OLS regression of (24).<sup>25</sup>

**Alumni network** Following Cohen and Malloy (2014), we extract information on the universities attended by the congressmen using the *Biographical Directory of the United States Congress* available online (<http://bioguide.Congress.gov/biosearch/biosearch.asp>) and construct a membership network based on educational experience.<sup>26</sup> Specifically, we match politicians to their colleges and universities. A tie between two congressmen exists if they graduated from the same institution.<sup>27</sup>

Relative to the committee membership network described above, this approach gives us a network that is exogenous to the political process. To prove that alumni networks are still relevant even many years after the congressmen attended school, in Table 1 we have estimated a dyadic regression model (similar to (24)) where links between legislator  $i$  and  $j$  in the alumni networks,  $g_{ij,A}$  are used as explanatory variables for cosponsorship activities in congress, controlling for similarities in terms of party, gender, state, number of shared committees and Congress fixed effects. Cosponsorship activity is measured by directional links  $g_{ij,L}$  equal to the number of bills

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<sup>24</sup> The extended model (24)-(25) is identified even if the  $x_{i,r}^l$  variables used in the link formation and in the outcome equation completely overlap. The dyad-specific variables in the link formation equation (24) (i.e. nonlinear functions of  $x_{i,r}^l, s$ ) are naturally excluded from the outcome equation (25). See also Hsieh and Lee [2016].

<sup>25</sup> It should be noted that we are not directly interested in estimating choice probabilities, but only the degree of correlation between  $\epsilon_i$  and  $\eta_{ij}$ . Therefore, similarly to Qu and Lee [2015], we use a linear probability model for (24). Inference is complicated because the selectivity term is a generated regressor from a previous estimation and no closed form solution is available for the ML adjusted standard errors estimates in a network context. We use bootstrapped standard errors with 1000 replications.

<sup>26</sup> We use academic institutions attended for both undergraduate and graduate degrees. In dealing with multiple campuses, we match each satellite campus as a separate university (e.g., University of California at Los Angeles, San Diego, and Berkeley are treated as separate universities). We match specialized school to the university. We drop observations where a specialized school name could match multiple universities (e.g., School of Management).

<sup>27</sup> As noted in footnote 21, in Section 5.1 we extend the analysis considering two variations of link definition in which two legislators are linked if they attended the same institution in overlapping periods.

by  $j$  that  $i$  has cosponsored.<sup>28</sup> We thus run the following OLS regression:

$$g_{ij,L} = \gamma_0 + \gamma_1 g_{ij,A} + \sum_l \gamma_l |x_{i,r}^l - x_{j,r}^l| + e_{ij,r}. \quad (26)$$

Panel (a) of Table 1 shows that two politicians who attended the same college or university are more likely to cosponsor the same piece of legislation than two politicians who attended different universities, keeping constant similarities in terms of observed characteristics. The results are robust to the inclusion of both legislator  $i$  and legislator  $j$ 's total number of connections. This check addresses the concern that two politicians may happen to endorse the same bill simply because they are connected to many politicians.<sup>29</sup>

It should be highlighted that the alumni network and the committee assignment network capture two alternative channels through which social connections in Congress are formed: as said before, the first through a shared educational experience; the second through a shared work experience in Congress. This can be seen from Panel (b) of Table 1, showing the OLS results of model (26) where the alumni connections are used as explanatory variables for committee membership, keeping unchanged the structure of the control variables. In this model specification, the dependent variable,  $g_{ij,C}$ , takes value one if the two politician sit in the same committee and zero otherwise. We find only a mild association between the two networks in this case. The regression explains less than 1% of the committee formation process versus about 11% of the legislative endorsement process. These results are consistent with the idea that the allocation of politicians into committees is largely beyond the choice of the single politician.<sup>30</sup>

By using alma mater connections, we are able to link more than fifty percent of congressmen. As shown in the online appendix (Table A.1), these congressmen do not significantly differ from the entire sample in terms of characteristics. We only oversample legislators who graduated from top 10 universities, since they are relatively more likely to be in the Congress. In the following analysis, we control for attendance at a top-10 university with a dummy variable.

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<sup>28</sup> To construct the cosponsorship networks we collected all pieces of legislation proposed in the U.S. House from the 109th-113th Congresses from the Library of Congress data information system, THOMAS (<http://thomas.loc.gov>).

<sup>29</sup> This finding is in line with Cohen and Malloy's [2014] results showing that alumni networks help explain voting patterns of Senators from the 101st to the 110th Congresses.

<sup>30</sup> We do not use patterns of cosponsorship to measure network centrality in our analysis precisely because cosponsorship is determined simultaneously with monetary contributions and is entirely determined by endogenous choices of the congressmen.



## 4.2.2 Other variables and controls

**Control variables** The vector of variables  $X_{i,r}$  (and the associated matrix  $X$ ) measures the susceptibility of a congressman to PAC contributions. The classic variables used to explain campaign contributions to legislators in the literature are the degree of electoral competition, the per capita income in the electoral district, measures of members' relative "power" inside the house and indicators of a congressman's ideology, political party, gender and seniority in his current committee.<sup>31</sup>

Information on politicians' characteristics including gender and party of affiliation is provided by *GovTrack*.<sup>32</sup> Charles Stewart and Jonathon Woon's website is used to obtain information on committee appointments, seniority and chairmanship.<sup>33</sup> One-year estimates of per capita income by congressional district are provided by the American Community Survey (ACS). For each congressman, electoral competition is measured by the margin of victory.<sup>34</sup> Each candidate's margin of victory is derived from the FEC's Federal Elections publications. These publications provide statistics on candidates' vote shares. Since the publications often omit special election results, we supplement the FEC reports with information from individual state agencies. The ideologies of the congressmen are measured using the first dimension of the dw-nominate score (McCarty et al. [1997]).<sup>35</sup> The "power" of the congressman is measured by three variables. First, we have a dummy variable indicating whether the member is a committee chair.<sup>36</sup> Secondly, we have a dummy variable indicating that the member is on one of the powerful committees (Ways and Means, Energy and Commerce, Appropriations, Rules or Financial services), in which

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<sup>31</sup> For electoral competitiveness, the idea is that a close race increases an incumbent's demand for PAC contributions, producing an exogenous shift in contributions via an increase in the propensity to "sell" services, including roll call votes. For the "power" of a member, the argument is that groups give more to powerful members because their support is especially valuable. The political district income is used to capture price differences in most campaign inputs, such as labor and advertising prices, between districts. The inclusion of the politicians' ideologies captures the fact that congressmen with more extreme ideologies are more difficult to persuade.

<sup>32</sup> Seniority has been manually adjusted for the few cases in which a congressman changed commission during the term.

<sup>33</sup> See [http://web.mit.edu/17.251/www/data\\_page.html#2](http://web.mit.edu/17.251/www/data_page.html#2). This website does not contain information for the 113th Congress. We extract the House of Representative committee roster for the 113th Congress from the website <http://media.cq.com/pub/committees/index.php>.

<sup>34</sup> Margin of victory as a measure of electoral competition is used by Poole, Romer and Rosenthal [1987], Grier and Munger [1991] and Romer and Snyder [1994], among others.

<sup>35</sup> To isolate this index for one Congress at a time, we used the modified DW-Nominate coordinates developed by Nokken and Poole [2004]. Data are available at <http://voteview.com>.

<sup>36</sup> A dummy variable for committee leadership is used in Romer and Snyder [1994].

an individual is likely to receive greater PAC contributions (Grier and Munger [1991], [1993] and Romer and Snyder [1994]). Finally, we include a dummy variable indicating whether the politician is on one of the committees that is joint with the Senate (Economic, Taxation, Library or Printing). To control for electoral cycle fixed effects, we include in our analysis four election cycle dummies, Y06-07, Y08-09, Y10-11, Y12-13. These are intended to control for changes in the number of PACs over time and changes in nominal and real PAC budgets, as well as for year-specific factors affecting PAC contributions. We also use the information on the college attended by each politician to control for unobserved ability. Politicians who graduated from a top university may be particularly able individuals, and such an ability may also attract campaign contributions. As mentioned before, we add in the regression a dummy variable which is equal to one if the politician attended a top-10 university and zero otherwise. In our sample about 6% of the congressmen attended a top-10 university.<sup>37</sup> Table A.1 contains a detailed description of our data, as well as summary statistics for our sample.

**Campaign contributions data.** Campaign contributions data from the Federal Election Commission (FEC) files are collected and aggregated by the Center for Responsive Politics (CRP). The CRP provides details on the date, type, industry to which the PAC is associated and recipient of each contribution. We consider the total amount of contributions from PACs and reduce the effect of possible outliers by trimming the distribution at the 1st and 99th percentiles.<sup>38</sup> In our data, the money spent by PACs for a given candidate range from \$9519 to \$7,178,406, whereas total spending ranges from \$310 million for the 110th Congress to \$453 million for the 112th Congress.

### 4.3 Empirical findings

Column (1) of Table 2 presents the Maximum Likelihood estimates of our model (equation (23)) using the *committee membership network*.<sup>39</sup> The estimates reveal a positive and statistically significant estimate of  $\phi^*$ , which confirms the presence of externalities as predicted by our theory.

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<sup>37</sup> US university ranking is taken by U.S News and World Report available online at <http://www.usnews.com/rankings>. The top 10 universities include Princeton University, Harvard University, Yale University, Columbia University, Stanford University, University of Chicago, Duke University, University of Pennsylvania, John Hopkins University and Dartmouth College. We report the results that use the most recent ranking (year 2014). Results using the top 10 dummy based on ranking from different years remain qualitatively unchanged.

<sup>38</sup> This data has been extensively used in the literature on economics and politics, following Poole and Rosenthal [1997].

<sup>39</sup> We report here the estimates with the more extensive set of controls. In the online appendix, we show the robustness of the results for alternative sets of controls (see Tables A.2 and A.3).

In column (2), we show the estimation results when controlling for network endogeneity in the committee membership network (model (25)). Here too we find a statistically significant estimate of  $\phi^*$ . It is interesting to note that the estimate of the selection correction term is negative. This is consistent with the presence of politicians' unobservable characteristics that are correlated positively with the contributions received and negatively with the probability of having links. A politician's expertise on a specific topic could be an example of such an omitted factor. Indeed, highly specialized politicians are likely to sit on fewer committees, and politician expertise is likely to be positively correlated with the contributions received (at least from the interest groups focused in that area). The last column of Table 2 (column (3)) reports the results when social connections in Congress are measured using the *alumni network*. In this case too, the evidence remains highly supportive of network effects: the estimate of our target parameter  $\phi^*$  is statistically significant and positive.

Perhaps unsurprisingly, we find that the effects of the *Margin of Victory*, *Chair*, *Relevant Committee* and *Party* are all significant and with the expected sign. A positive effect of *Chair* and *Relevant Committee* confirms the fact that congressmen in positions of leadership and members of important committees receive more attention from interest groups. The estimated effect of *Joint Committee* is also positive, though statistically different from zero only when using the Committee networks. Since the variable *Party* is equal to 1 when the legislator is Republican, our results shows that Republicans receive more contributions than Democrats. Among other reasons, this can be explained by the fact that the Republicans had the majority in all Congresses we consider except for the first two. A negative effect of the *Margin of Victory* coefficient suggests that congressmen who face tight elections have higher needs for campaign finance, are more susceptible to interest groups' influence, and therefore receive more money. We also find a positive and significant effect of *Per Capita Income*, indicating that politicians facing higher local prices in campaign inputs need more money. Being female is associated with receiving lower contributions, but this effect is not statistically significant when using the alumni network. The negative effect of *Seniority* is consistent with the results in Grier and Munger [1986]. A negative and statistically significant effect of *DW\_ideology* indicates that politicians with more extreme ideologies receive less money, in line with the idea that they are more difficult to persuade. Perhaps unsurprisingly, when using the committee network, we find that legislators who studied in top universities receive more contributions *ceteris paribus*.

The findings discussed above should be contrasted with two benchmarks: the OLS estimates ignoring the network effects; and estimates using other standard measures of centrality that do not have a theoretical foundation. With respect to the first benchmark, Table 3 column (1) reports the OLS estimates of the traditional model where campaign contributions are explained using legislators’ characteristics and Congress fixed effects, ignoring that congressmen are connected. In column (3), we report the OLS results for the model with no network effects for the restricted sample that we use for the alumni network. The important observation is that for both the committee network and the alumni network, the inclusion of network effects significantly improves the fit of the model. The relative goodness of fit of the different models is measured estimating both models by maximum likelihood and using the Akaike information criterion (AIC).<sup>40</sup> It is reported in the bottom panel of Table 3. Given a set of candidate models for the data, the preferred model is the one with the minimum AIC value (see, e.g., Burnham and Anderson [2002]). Table 3 shows that the model with network effects (columns (2) and (4)) outperforms the model with no network effects (columns (1) and (3)), irrespective of the network definition. We formally test the model fit increase of the spatial autoregressive model versus the traditional linear regression (i.e.  $\phi^* = 0$ ) using a likelihood ratio test.<sup>41</sup> In both cases, the likelihood comparison clearly rejects the hypothesis that  $\phi^*$  can be set to 0 (p-value equal to 0.000).

In comparing the estimates of the covariates in the models with and without network effects, we should note that the interpretation of the coefficients of the control variables in the OLS and in the ML models are different. When  $\phi^* > 0$ , the marginal effect of the  $k$ -th covariate in model (22) is not just  $\beta_k$ , but  $\Sigma = (I_{n_r} - \phi^* G_r^T)^{-1} (I_{n_r} \beta_k)$ , which is an  $n_r \times n_r$  matrix with its  $(i, j)$ -th element representing the effect of a change in  $x_{jk,r}$  on  $y_{i,r}$ . Thus, while the OLS model produces homogeneous estimates for the effects of covariates, the model with network effects displays marginal effects that are necessarily heterogeneous across individuals.

The second set of benchmarks that we consider are the predictions obtained using other standard measures of network centrality (which are not supported by a theoretical analysis). Table 4 presents OLS estimates of the relationship between PAC electoral contributions and *Degree*,

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<sup>40</sup> The AIC is a measure of the relative quality of statistical models for a given set of data. Let  $L$  be the maximum value of the likelihood function for the model; let  $k$  be the number of estimated parameters in the model. Then the AIC value of the model is  $2k - 2 \ln L$  (Akaike [1974]).

<sup>41</sup> Let  $lik_1$  define the log-likelihood of the unrestricted model (column (2)) and  $lik_2$  the log-likelihood of the restricted model (column (1)), the likelihood ratio test statistic  $LR = 2(lik_1 - lik_2)$  is asymptotically distributed as a chi-squared with 1 degree of freedom.

*Betweenness* and *Closeness* centralities using both network definitions.<sup>42</sup> We find that the effects of Degree and Closeness centralities are not significantly different from zero. The effect of Betweenness is statistically significant in sign for the committee networks, but insignificant in magnitude and negative. All the control variables have the expected signs, the same as in the estimates of Table 2. The important observation is that as it is shown by the  $r$ -squared in the bottom part of Table 4, the performance of these models with traditional network centralities is not different from the performance of a regression model where PAC contributions are explained with no reference to the network topology (columns 1 and 5 in Table 4).

## 5 Discussions and extensions

### 5.1 Robustness checks: alternative network definitions

Table 5 collects the maximum likelihood estimation results of (23) when we adopt network definitions enriched with additional information. In the first two panels, the committee membership data is enriched with additional information on party affiliation or the role of congressmen in the legislature. The first column of each panel shows the baseline MLE estimates, while the second column reports the MLE estimates with the control function correction. In the network presented in Section 4, congressmen are linked if they belong to the same committee; the intensity of the link is a count variable representing the number of shared committees, and does not incorporate information on the party affiliations of the linked congressmen. In the first two columns of Table 5, we adopt a *Partisanship Weighted Network* (PWN) that reflects the fact that two congressmen from the same party have more opportunities to form a social bond and influence each other. Specifically, in the PWN, the intensity is doubled when legislators are affiliated with the same party. Using this network, the results of the estimate of (23) are qualitatively the same as in the previous analysis, though the estimate of  $\phi^*$  is now larger.

Politicians connected with committee chairs may be more influential than those who are not. To reflect this fact, in the second panel of Table 5 we adopt a *Chairmanship Weighted Network* (CWN) in which the intensity of the  $i, j$  link (i.e., the link describing the influence of  $j$  on  $i$ ) is doubled when  $i$  and  $j$  are in the same committee and  $j$  is its chairman. The resulting network

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<sup>42</sup> Degree centrality counts the total number of direct connections. Closeness centrality measures the length of the average shortest path passing between a node and all the others. The measure is normalized by the degree. Betweenness is equal to the number of shortest paths from all nodes to all others that pass through that node. See Jackson [2008] for an introduction and detailed description of these measures.

is directional as it reflects asymmetric influences between members of the same committee when one of the two has a position of leadership. Table 5 shows that our results remain qualitatively unchanged irrespective of the definition of Congressional network adopted.

The last panel of Table 5 enriches the alumni network by using information on graduation time. In this definition, two congressmen are connected if they attended the same academic institution at the same time. We use four-year and two-year windows. Again, the results of the analysis remain unchanged.

## 5.2 Multiple interest groups

In the preceding analysis, we maintained the assumption of two interest groups, one for  $A$  and one for  $B$ . It is natural to extend the results to the case in which we have  $K$  interest groups for  $A$  and  $K$  for  $B$ , each endowed with a budget  $W$ . Let  $s_{j,A}^i$  be the contribution promised by the  $j$ th interest group for  $A$  to the  $i$ th legislator with  $\mathbf{s}_{j,A} = (s_{j,A}^1, \dots, s_{j,A}^n)$  and  $\mathbf{s}_A = (\mathbf{s}_{1,A}, \dots, \mathbf{s}_{K,A})$ . The problem faced by an interest group  $j$  of the  $A$  type is similar to (9), with the only difference being that now both  $\mathbf{s}_{-j,A}$ , the choice of all other  $K - 1$  interest groups supporting  $A$ , and  $\mathbf{s}_B$ , the choice of all  $K$  interest groups supporting  $B$ , are taken as given.

Following the same steps as above, we can show that, if legislators are office motivated or if they are policy motivated and there is sufficient uncertainty on their preferences, there is a unique equilibrium in which all interest groups commit to the same transfer  $s_{j,A}^i = s_{k,B}^i = s_*^i$  for any  $j$ ,  $k$  and  $i$ . This implies that the voting probabilities are derived exactly as in Section 3.2. The analysis is unaffected by the size of  $K$  because the marginal effect of a contribution on the voting probabilities is independent of the contributions of other interest groups.<sup>43</sup> Assuming, as in Section 4.1, that the monetary utility is  $\omega_j(s) = \log(s)$  and the interest group's objective is (19), we have that the total contribution received for voting  $A$  in congress  $r$  is just  $K$  times the formula in (20):  $\mathbf{s}_r = K \cdot (I - \phi^* G_r^T)^{-1} \cdot \boldsymbol{\theta}_r$ . Given (21), we obtain the same spatial autoregressive model (23) discussed in Section 4.1. Since these values differ from the previous analysis only by a factor of proportionality, there is no qualitative change in the result and its implications for the empirical analysis.

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<sup>43</sup> This can be seen from (6) with office motivated and (27) with office and policy motivated.

### 5.3 Alternative objective functions

In the analysis presented above, we assume that interest groups maximize the expected number of supporters. This objective function is typically assumed in probabilistic models of electoral competition (see Lindbeck and Weibull [1987]). There are, however, environments in which interest groups care about legislators' votes only to the extent that it allows them to reach a given threshold of support (such as a majority). The analysis presented above easily extends to these cases.

To extend the analysis, let us now assume that the interest groups' preferences are represented by a sequence of thresholds  $(z_j, u_j)_{j=0}^J$  for some finite  $J$  with  $z_0 = 0$  and  $u_0 > 0$  and  $z_j < z_{j+1}$  and  $u_j < u_{j+1}$  for all  $j = 0, \dots, J-1$ , such that  $A$ 's utility can be written as a step function:  $u_A(\sum_i x_i(P)) = u_j$  if  $\sum_i x_i(P) \in (z_j, z_{j+1}]$  for  $j \leq J-1$  and  $u_J$  for  $\sum_i x_i(P) > z_J$ . A special example of these preferences is when interest groups care only about obtaining a majority. In this case, the utility is characterized by just one threshold and  $z_1 = \frac{n-1}{2}$  for  $n$  odd or  $z_1 = \frac{n}{2}$  for  $n$  even and utility level  $u_1 > u_0$ .

Following the same steps as above, it is straightforward to verify that, when legislators are office motivated or when they are policy motivated and there is sufficient uncertainty on their preferences, we have a unique equilibrium in which interest groups offer the same monetary contributions  $\mathbf{s}_A = \mathbf{s}_B = \mathbf{s}_{**}$ . Also, as before,  $\mathbf{s}_{**}$  is characterized as the maximization of a weighted sum of the monetary utilities:

$$\max_{\mathbf{s} \in S} \left\{ \sum_j b_j^{\mathbf{z}, \mathbf{u}}(\phi^*, V, G^T) \cdot \omega_j(s^j) \right\},$$

where  $\mathbf{b}^{\mathbf{z}, \mathbf{u}}(\phi^*, \mathbf{V}, \mathbf{G}^T) = (b_j^{\mathbf{z}, \mathbf{u}}(\phi^*, V, G^T))_{j=1}^n$  are weights that depend on  $\phi^*, V, G^T$  and on the thresholds  $\mathbf{z}, \mathbf{u} = (z_j, u_j)_{j=0}^J$  (a formal derivation of these weights is presented in Section 5 in the online appendix). The key observation is that the importance of the thresholds vanishes as  $n \rightarrow \infty$ . Indeed, as we formally prove in Section 5 in the online appendix, for any  $\mathbf{z}, \mathbf{u}$  we have  $b_j^{\mathbf{z}, \mathbf{u}}(\phi^*, V, G^T) \rightarrow \mathbf{b}(\phi^*, \mathbf{G}^T)$ . In this case too, therefore, the equilibrium allocation of transfers depends only on the Bonacich centralities for large  $n$ .

### 5.4 Heterogeneous policies

Another assumption we made in the previous analysis is that legislators vote only on one policy. In reality, legislators vote on many policies that could be very different and attract the attention

of different sets of interest groups (defense, agriculture, trade, etc.). In these cases, we might have a set  $H = \{1, \dots, h\}$  of different votes, with policy  $j = 1, \dots, h$  associated with  $N_j$  interest groups in favor and  $N_j$  against, and a per interest group budget  $W_j$ .

Once again, the analysis is quite similar to the analysis presented above. Assuming logarithmic utility, it is easy to see that in this environment each interest group interested in policy  $j \in H$  makes a transfer  $\mathbf{s}^j = (1/\lambda_{r,j}) \cdot (I - \phi^* G_r^T)^{-1} \cdot \boldsymbol{\theta}$  and so the total vector of contributions is  $\mathbf{S}^r = \sum_j \mathbf{s}^j = \left[ \sum_j N_j / \lambda_{r,j} \right] \cdot (I - \phi^* G_r^T)^{-1} \cdot \boldsymbol{\theta}$ , that is proportional to  $(I - \phi^* G_r^T)^{-1} \cdot \boldsymbol{\theta}$  as in Section 4.1.

## 6 Conclusions

In this paper, we present a new theory of competitive vote-buying to study campaign contributions when legislators care about the behavior of other legislators to whom they are socially connected. The theory predicts that campaign contributions are increasing in the legislators' Bonacich centralities, a standard measure of centrality in networks.

As a first attempt to bring these predictions to the data, we estimate the model with data on PAC contributions in the last five Congresses (the 109th-113th). To measure the legislators' social network and control for endogeneity we propose two approaches. In the first, we exploit the insight from the political science literature that congressmen become well acquainted while serving in congressional committees. We therefore construct social networks in which links between two congressmen are proportional to the number of committees in which they both sit, controlling for possible unobserved factors driving both committee membership and PAC contributions by including an Heckman correction term. In the second approach, we exploit the insight that educational institutions provide a basis for social networks. We therefore construct the social network using the congressmen's alumni connections: two congressmen are connected if they graduated from the same institution or if (alternatively) they graduated from the same institution in the same period. This approach provides a network that is exogenous by construction to interest groups' activities.

With both approaches, we obtain results supporting our theory. As predicted by the theory, legislators' Bonacich centralities significantly impact campaign contributions. The results are robust to the inclusion of established determinants of PAC contributions used in previous literature. Adding information on the topology of the legislators social network significantly improves the fit



of the model compared with alternative specifications that ignore this information.

We believe there is significant room for further analysis on the impact of legislators' social networks on interest groups' campaign contributions and other influence activities. While our analysis has focused on monetary contributions, it would be interesting to extend the basic theory to situations in which interest groups offer other types of valuable resources, including expertise and contacts with other legislators. It would be particularly interesting to allow the interest groups to affect the network topology by establishing links between legislators, blending our analysis with Ballester, Calvo-Armengol and Zenou [2006]'s analysis of key players. This would improve understanding of the extent to which legislators' social networks affect the activities of lobbyists, who provide campaign contributions, services, and networking resources in the U.S. Congress.

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## 7 Appendix

### 7.1 Proof of Lemma 1

The proof for the case with office motivated legislators is presented in Section 3.1. For the case with policy motivated legislators, see the online appendix.

### 7.2 Proof of Propositions 1-2

We prove the result for general  $v = (v^1, \dots, v^n)$ . This allows us to prove Proposition 2 and then Proposition 1 as a special case of Proposition 2. Following the same steps as in Section 3.2.1, we can derive:

$$D\varphi = \Psi(I - \Psi(V \cdot D\mathbf{q}_* + 2\phi G))^{-1} \cdot D\omega, \quad (27)$$

where  $V$  is the  $n$ -dimensional diagonal matrix with  $i$ th diagonal entry equal to  $v^i$ ,  $D\mathbf{q}_*$  is the  $n$ -dimensional matrix with generic  $i, j$  element equal to  $q_j^i$  as defined in Section 3.2.2. The first order necessary and sufficient condition of the problem solved by interest group  $l$  can be written in matrix form as  $D\varphi^T \cdot \mathbf{1} = \lambda_l$ , where  $\lambda_l$  is the Lagrangian multiplier of interest group  $l$ 's program. Using (27), we have:

$$\begin{aligned} D\varphi^T \cdot \mathbf{1} &= [\Psi(I - \Psi(V \cdot D\mathbf{q} + 2\phi G))^{-1} \cdot D\omega]^T \cdot \mathbf{1} \\ &= \Psi \cdot D\omega^T (I - (\phi^* G^T + \Psi D\mathbf{q}_*^T V))^{-1} \cdot \mathbf{1} = \lambda_l \\ &\Rightarrow D\omega^T \cdot \mathbf{b}^{\mathcal{M}}(\phi^*, \mathbf{V}, \mathbf{G}^T) = \lambda_l / \Psi \end{aligned} \quad (28)$$

for  $l = A, B$ , where for the last equality we used (4) and  $\phi^* = 2\phi\Psi$ . Note that  $D\omega$  is a vector of zeros except for its  $i$ th element that is equal to  $\omega'(s_*^i)$ . We can therefore write our necessary and sufficient conditions (10) as:

$$b_i^{\mathcal{M}}(\phi^*, V, G^T) \cdot \omega'(s_A^i) = \lambda_A \quad (29)$$

$$b_i^{\mathcal{M}}(\phi^*, V, G^T) \cdot \omega'(s_B^i) = \lambda_B \quad (30)$$

where, without loss of generality, we have incorporated the constant  $\Psi$  in the Lagrangian multipliers. Assume by contradiction that  $\lambda_A > \lambda_B$  (respectively,  $\lambda_A < \lambda_B$ ), we then must have  $\omega'(s_A^i) > \omega'(s_B^i)$  (resp.,  $\omega'(s_A^i) < \omega'(s_B^i)$ ) for any  $i$ , implying  $\sum s_B^i > \sum s_A^i = W$  (resp.,  $\sum s_A^i > \sum s_B^i = W$ ), a contradiction. We conclude that there is a unique solution  $(\lambda_*, \mathbf{s}_*)$  such that  $\lambda_i = \lambda_*$  and  $\mathbf{s}_i = \mathbf{s}_*$  for  $i = A, B$ . ■



### 7.3 Proof of Proposition 3

Let  $\iota(\cdot)$  be a function that maps agents to their respective groups and let  $H$  be the  $m \times m$  matrix describing the relationships between the types, so that  $g_{ij} = h_{\iota(i), \iota(j)}$ . We start from two preliminary results. The first result shows that, when we have a finite number of types, the Bonacich centralities are well-defined functions of only the shares of the types  $\alpha$  and of the matrix  $H$  describing the relationships between the types. We have:

**Lemma 3.1.** *For any  $i = 1, \dots, n$ ,  $b_i(\phi^*, G^T)$  is equal to  $\bar{b}_{\iota(i)}(H, \alpha)$  defined by:*

$$\bar{\mathbf{b}}(H, \alpha) = \left[ I + \phi^* \tilde{H}^T \right]^{-1} \cdot \mathbf{1}, \quad (31)$$

where  $\bar{\mathbf{b}}(H, \alpha) = (\bar{b}_1(H, \alpha), \dots, \bar{b}_m(H, \alpha))^T$  and  $\tilde{H}$  is the  $m \times m$  matrix with element  $i, j$  equal to  $\tilde{h}_{i,j} = \alpha_j h_{i,j} / (\sum_l h_{i,l})$ .

**Proof.** See the online appendix. ■

Note that  $\tilde{H}$  is a  $m \times m$  matrix with bounded elements since  $\tilde{h}_{i,j} \leq \sum_i \tilde{h}_{i,j} \leq \sum_i g_{i,j} \leq \bar{g}$ .

The second preliminary result shows that as  $n \rightarrow \infty$ , the equilibrium pivot probabilities and the sum of their derivatives converges to zero. For a sequence of equilibria  $(\varphi_l^n)_l$ , let  $q_n^i$  be the associated pivot probability of legislator  $i$ , and  $q_{n,j}^i$  be the derivative of  $q_n^i$  with respect to  $\varphi_j^n$ . We have:

**Lemma 3.2.**  $\lim_{n \rightarrow \infty} q_n^i = 0$ ,  $\lim_{n \rightarrow \infty} \sum_{j=1}^n |q_{n,j}^i| = 0$ , for any  $i, j$ .

**Proof.** See the online appendix. ■

To complete the proof, consider a sequence of populations of size  $n \rightarrow \infty$  in which the network is  $G_n$  and the share of type  $j$  is  $\alpha_n^j \rightarrow \alpha^j$ . We need to show that  $b_i^{\mathcal{M}}(\phi^*, V, G_n^T) \rightarrow \bar{b}_{\iota(i)}(H, \alpha)$  for all  $i$  as  $n \rightarrow \infty$ . To keep the notation simple, let  $\tilde{b}_j^n$  be the Modified Bonacich of an agent of type  $j$ . We can write:

$$\begin{aligned} \tilde{b}_{\iota(i)}^n &= 1 + \phi \sum_{l=1}^m n_l h_{l, \iota(i)}^n \tilde{b}_l^n + v_{\iota(i)} \sum_{j=1}^n q_{n,j}^i b_j^n \\ &= 1 + \phi \sum_{l=1}^m \tilde{h}_{l, \iota(i)}^n \tilde{b}_l^n + v_{\iota(i)} \sum_{l=1}^m n_l \tilde{q}_{n,l}^{\iota(i)} \tilde{b}_l^n \end{aligned} \quad (32)$$

where  $\tilde{h}_{i,j} = n_i h_{i,j}$  and  $\tilde{q}_{n,l}^{\iota(i)}$  is the derivative of the pivot probability of an agent of type  $\iota(i)$  with respect to the voting probability of a type  $l$ . Note that  $\sum_{l=1}^m |n_l \tilde{q}_{n,l}^{\iota(i)}| = \sum_{j=1}^n |q_{n,j}^i|$  and, by Lemma 3.2,  $\sum_{j=1}^n |q_{n,j}^i| \rightarrow 0$  as  $n \rightarrow \infty$ . It follows that we can write  $\tilde{\mathbf{b}}^n = \Psi \cdot$

$\left[ I + \phi^* \left[ \tilde{H}^n \right]^T + O(n) \right]^{-1} \cdot \mathbf{1}$ , where  $\tilde{\mathbf{b}}^n = (\tilde{b}_1^n, \dots, \tilde{b}_m^n)^T$ ,  $\tilde{H}^n$  is the  $m \times m$  matrix with element  $i, j$  equal to  $\tilde{h}_{i,j} = n_i h_{i,j}$  and  $O(n)$  is a  $m \times m$  matrix with all terms converging to zero as  $n \rightarrow \infty$ . Note that  $\tilde{h}_{i,j} \leq \sum_{i=1}^m n_i h_{i,j} = \sum_{l=1}^n g_{l,j} \leq \bar{g}$ , so  $\tilde{H}^n$  converges to a positive and bounded  $m \times m$  matrix  $\tilde{H}$ . Taking the limit as  $n \rightarrow \infty$ , we obtain:  $\lim_{n \rightarrow \infty} \tilde{\mathbf{b}}^n = \Psi \left[ I + \phi^* \tilde{H}^T \right]^{-1} \cdot \mathbf{1}$ . It follows that  $b_i^{\mathcal{M}}(\phi^*, V, G_n^T) \rightarrow \bar{b}_{\iota(i)}(H, \boldsymbol{\alpha})$  for all  $i$  as  $n \rightarrow \infty$  as requested. ■

**TABLE 1. Predictive power of alumni network for cosponsorship and committee networks**

	Dep. Var.: Link in cosponsorship network ( $g_{ij,L=1}$ )			Dep. Var.: Link in committee network ( $g_{ij,C=1}$ )		
	OLS	OLS	OLS	OLS	OLS	OLS
	(1)	(2)	(3)	(1)	(2)	(3)
Link in alumni network ( $g_{ij,A=1}$ )	0.180*** (0.008)	0.069*** (0.007)	0.069*** (0.007)	0.006 (0.006)	0.012* (0.007)	0.012* (0.007)
Same party (1=yes)		0.284*** (0.002)	0.285*** (0.002)		-0.007*** (0.001)	-0.007*** (0.001)
Same gender (1=yes)		-0.0001 (0.002)	-0.0001 (0.002)		-0.006*** (0.001)	-0.006*** (0.002)
Same state (1=yes)		0.255*** (0.004)	0.254*** (0.004)		-0.024*** (0.003)	-0.025*** (0.003)
N. of shared committees		0.083*** (0.002)	0.087*** (0.002)			
Intercept	0.362*** (0.002)	0.188*** (0.003)	0.182*** (0.002)	0.204*** (0.002)	0.213*** (0.002)	0.213*** (0.002)
Legislator <i>i</i> connections	No	No	Yes	No	No	Yes
Legislator <i>j</i> connections	No	No	Yes	No	No	Yes
Time dummies	Yes	Yes	Yes	Yes	Yes	Yes
R2	0.008	0.11	0.11	0.002	0.003	0.003
N.obs.	244,519	244,519	244,519	242,735	242,735	242,735

Notes: OLS estimated coefficients and standard errors (in parentheses) are reported. \*, \*\*, \*\*\* indicate statistical significance at the 10, 5 and 1 percent levels.

**TABLE 2. Main estimation results**

	Dep. Var.: PAC contributions (\$mil)		
	Committee network		Alumni network
	MLE	MLE-corrected	MLE
	(1)	(2)	(3)
$\phi$	0.2088 *** (0.0697)	0.2165*** (0.0703)	0.0837*** (0.0262)
Party (1=Republican)	0.1443** (0.0573)	0.1473*** (0.0011)	0.2212*** (0.0801)
Gender (1=Female)	-0.0950* (0.0535)	-0.09472*** (0.001)	-0.0685 (0.0761)
Chair (1=Yes)	0.4006*** (0.0967)	0.3959*** (0.0020)	0.4759*** (0.1321)
Seniority	-0.0154*** (0.0034)	-0.0153*** (0.00001)	-0.0169*** (0.0047)
Margin of Victory	-0.8972*** (0.0885)	-0.8959*** (0.0019)	-0.7793*** (0.1202)
Per capita Income	0.0061** (0.0025)	0.0062*** (0.00004)	0.0067* (0.0035)
DW_ideology	-1.0774*** (0.1241)	-1.0817*** (0.0031)	-1.1171*** (0.1670)
Relevant Committee (1=Yes)	0.1037** (0.0413)	0.0998*** (0.0007)	0.1135** (0.0575)
Joint Committee (1=Yes)	0.1694** (0.0861)	0.1669*** (0.0016)	0.0810 (0.1128)
Top 10 university (1=Yes)	0.0581 (0.0809)	0.0579*** (0.0011)	0.0790 (0.0900)
Unobservables ( $\psi$ )		-0.1132*** (0.0016)	
Intercept	1.3019*** (0.1072)	1.2949*** (0.0629)	1.2895*** (0.1330)
Time dummies	Yes	Yes	Yes
N. obs.	2,128	2,128	1,166

Notes: ML estimated coefficients and standard errors (in parentheses) are reported. In column (2) standard errors are bootstrapped with 1000 replications. A precise definition of control variables can be found in Table A.1. \*, \*\*, \*\*\* indicate statistical significance at the 10, 5 and 1 percent levels.

**TABLE 3. Model comparisons**

	No network effects		Committee network	
	OLS	MLE	OLS	Alumni network MLE
	(1)	(2)	(3)	(4)
$\phi$		0.2165*** (0.0703)		0.0837*** (0.0262)
Party (1=Republican)	0.1568*** (0.0576)	0.1473*** (0.0011)	0.2157*** (0.081)	0.2212*** (0.0801)
Gender (1=Female)	-0.0944* (0.054)	-0.09472*** (0.001)	-0.0672 (0.0771)	-0.0685 (0.0761)
Chair (1=Yes)	0.3667*** (0.097)	0.3959*** (0.0020)	0.482*** (0.1336)	0.4759*** (0.1321)
Seniority	-0.0143*** (0.0034)	-0.0153*** (0.00001)	-0.0178*** (0.0048)	-0.0169*** (0.0047)
Margin of Victory	-0.9233*** (0.0893)	-0.8959*** (0.0019)	-0.7757*** (0.1217)	-0.7793*** (0.1202)
Per capita Income	0.0059** (0.0026)	0.0062*** (0.00004)	0.0066* (0.0036)	0.0067* (0.0035)
DW_ideology	-1.0948*** (0.1251)	-1.0817*** (0.0031)	-1.1011*** (0.169)	-1.1171*** (0.1670)
Relevant Committee (1=Yes)	0.1143*** (0.0415)	0.0998*** (0.0007)	0.1085* (0.0582)	0.1135** (0.0575)
Joint Committee (1=Yes)	0.1704* (0.087)	0.1669*** (0.0016)	0.07 (0.1142)	0.0810 (0.1128)
Top 10 university (1=Yes)	0.0559 (0.0818)	0.0579*** (0.0011)	0.1037 (0.0907)	0.0790 (0.0900)
Unobservables ( $\psi$ )		-0.1132*** (0.0016)		
Intercept	1.4642*** (0.0948)	1.2949*** (0.0629)	1.3531*** (0.1329)	1.2895*** (0.1330)
Time dummies	Yes	Yes	Yes	Yes
<i>AIC</i>	5686.323	5679.196	3201.537	3193.922
<i>Lik-ratio test (<math>\phi=0</math>)</i>		11.127 ***		9.615***
<i>p-value</i>		[0.003]		[0.001]
N. obs.	2,128	2,128	1,166	1,166

Notes: OLS estimated coefficients and standard errors (in parentheses) are reported in column (1) and (3). ML estimated coefficients and standard errors (in parentheses) are reported in columns (2) and (4). A precise definition of control variables can be found in Table A.1. \*, \*\*, \*\*\* indicate statistical significance at the 10, 5 and 1 percent levels. *Lik-ratio test*  $\sim \chi_1$

**TABLE 4. Explicative power of traditional network measures**

	Dep. Var.: PAC contributions (\$mil)							
	Committee network				Alumni network			
	OLS (1)	OLS (2)	OLS (3)	OLS (4)	OLS (5)	OLS (6)	OLS (7)	OLS (8)
Centrality Measure:								
Degree		0.0004 (0.0003)				-0.0018 (0.0053)		
Between			-4.9e-05* (2.9e-05)				-3.2e-05 (5e-05)	
Closeness				0.5902 (0.4711)				0.5903 (0.4771)
Party (1=Republican)	0.1568*** (0.0576)	0.1497*** (0.0578)	0.1582*** (0.0576)	0.1561*** (0.0576)	0.2157*** (0.081)	0.2145*** (0.0811)	0.2148*** (0.0811)	0.1561*** (0.0576)
Gender (1=Female)	-0.0944* (0.054)	-0.0974* (0.0541)	-0.0925* (0.054)	-0.0967* (0.0541)	-0.0672 (0.0771)	-0.065 (0.0773)	-0.065 (0.0772)	-0.0967* (0.0541)
Chair (1=Yes)	0.3667*** (0.097)	0.3904*** (0.0986)	0.3555*** (0.0972)	0.3789*** (0.0975)	0.482*** (0.1336)	0.4799*** (0.1338)	0.4785*** (0.1338)	0.3789*** (0.0975)
Seniority	-0.0143*** (0.0034)	-0.0153*** (0.0035)	-0.0136*** (0.0035)	-0.0149*** (0.0035)	-0.0178*** (0.0048)	-0.0177*** (0.0048)	-0.0177*** (0.0048)	-0.0149*** (0.0035)
Margin of Victory	-0.9233*** (0.0893)	-0.9146*** (0.0896)	-0.9231*** (0.0893)	-0.919*** (0.0894)	-0.7757*** (0.1217)	-0.7783*** (0.1219)	-0.7804*** (0.1219)	-0.919*** (0.0894)
Per capita Income	0.0059** (0.0026)	0.0059** (0.0026)	0.0059** (0.0026)	0.0059** (0.0026)	0.0066* (0.0036)	0.0066* (0.0036)	0.0067* (0.0036)	0.0059** (0.0026)
DW_ideology	-1.0948*** (0.1251)	-1.0835*** (0.1253)	-1.0982*** (0.125)	-1.0943*** (0.125)	-1.1011*** (0.169)	-1.0969*** (0.1695)	-1.1012*** (0.1691)	-1.0943*** (0.125)
Relevant Committee (1=Yes)	0.1143*** (0.0415)	0.1364*** (0.0448)	0.1241*** (0.0419)	0.1402*** (0.0465)	0.1085* (0.0582)	0.1095* (0.0583)	0.1091* (0.0582)	0.1402*** (0.0465)
Joint Committee (1=Yes)	0.1704* (0.087)	0.179** (0.0872)	0.1677* (0.0869)	0.1767** (0.0871)	0.07 (0.1142)	0.0707 (0.1142)	0.0706 (0.1142)	0.1767** (0.0871)
Top 10 University (1=Yes)	0.0559 (0.0818)	0.0567 (0.0818)	0.0538 (0.0818)	0.0591 (0.0818)	0.1037 (0.0907)	0.1113 (0.0934)	0.1154 (0.0926)	0.0591 (0.0818)
Intercept	1.4642*** (0.0948)	1.4737*** (0.0949)	1.1448*** (0.275)	1.4642*** (0.0948)	1.3531*** (0.1329)	1.3641*** (0.1367)	1.3587*** (0.1333)	1.1448*** (0.275)
Time dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R2	0.148	0.149	0.148	0.148	0.148	0.148	0.148	0.148
N. obs.	2,128	2,128	2,128	2,128	1,166	1,166	1,166	1,166

Notes: OLS Estimated coefficients and standard errors (in parentheses) are reported. A precise definition of control variables can be found in Table A.1. \*, \*\*, \*\*\* indicate statistical significance at the 10, 5 and 1 percent levels.

TABLE 5. Robustness checks

	<u>Committee network weighted by</u>				<u>Alumni network with graduation within</u>	
	<u>Political affiliation</u>		<u>Chairmanship</u>		<u>4 years</u>	<u>2 years</u>
	(1)	(2)	(3)	(4)	(5)	(6)
$\Phi$	0.2486*** (0.0656)	0.2588*** (0.0755)	0.3575*** (0.0866)	0.2152*** (0.0745)	0.07700*** (0.0290)	0.06694** (0.0306)
Party (1=Republican)	0.1387** (0.0571)	0.1415*** (0.0015)	0.1513*** (0.0568)	0.1472*** (0.0011)	0.2894*** (0.103)	0.23436** (0.1188)
Gender (1=Female)	-0.0928* (0.0534)	-0.0930*** (0.0013)	-0.0919* (0.0532)	-0.0948*** (0.0011)	-0.1291 (0.0918)	-0.18105* (0.1023)
Chair (1=Yes)	0.3988*** (0.0963)	0.3933*** (0.002)	0.3736*** (0.0954)	0.3876*** (0.002)	0.4843*** (0.1684)	0.48828*** (0.1813)
Seniority	-0.0153*** (0.0034)	-0.0152*** (0.00004)	-0.0147*** (0.0034)	-0.0153*** (0.00004)	-0.0133** (0.0057)	-0.01158* (0.0064)
Margin of Victory	-0.8988*** (0.0883)	-0.8985*** (0.0021)	-0.8846*** (0.088)	-0.8959*** (0.0021)	-0.7064*** (0.148)	-0.63747*** (0.1687)
Per capita Income	0.0062** (0.0025)	0.0062*** (1e-04)	0.0063** (0.0025)	0.0062*** (0.0535)	0.0059 (0.0043)	0.00516 (0.0046)
DW_ideology	-1.0619*** (0.1239)	-1.0664*** (0.0037)	-1.0818*** (0.1233)	-1.0817*** (0.0029)	-1.1594*** (0.2159)	-1.02709*** (0.2484)
Relevant Committee (1=Yes)	0.1017** (0.0412)	0.0970*** (0.00008)	-0.0992** (0.0411)	0.0995*** (0.00009)	0.2207*** (0.0715)	0.24384*** (0.0801)
Joint Committee (1=Yes)	0.1664* (0.0859)	0.1628*** (0.0019)	0.1609* (0.0856)	0.1672*** (0.0022)	0.1665 (0.1412)	0.05271 (0.1627)
Top 10 university (1=Yes)	0.0592 (0.0808)	0.0585*** (0.0019)	0.0596 (0.0805)	0.0581*** (0.0014)	0.1427 (0.0993)	0.17826* (0.1068)
Unobservables ( $\psi$ )		-0.1242*** (0.0020)		-0.1158*** (0.0019)		
Intercept	1.26805*** (0.1062)	1.2608*** (0.0679)	0.2055*** (0.0624)	1.2966*** (0.0672)	1.1913*** (0.1674)	1.11578*** (0.185)
Time dummies	Yes	Yes	Yes	Yes	Yes	Yes
N. obs.	2,128	2,128	2,128	2,128	767	597

Notes: ML estimated coefficients and standard errors (in parentheses) are reported. In columns (2) and (4) standard errors are bootstrapped with 1000 replications. A precise definition of control variables can be found in Table A.1. \*, \*\*, \*\*\* indicate statistical significance at the 10, 5 and 1 percent levels.