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TESTING RICARDIAN NEUTRALITY WITH AN INTERTEMPORAL STOCHASTIC MODEL

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ABSTRACT

The purpose of this paper is to develop and estimate a stochastic-intertemporal model of consumption behavior and to use it for testing a version of the Ricardian-equivalence proposition with time series data. Two channels that may give rise to deviations from this proposition are specified: Finite horizons and liquidity constraints. In addition, the model incorporates explicitly the roles of taxes, substitution between public and private consumption, and different degrees of consumer goods' durability. The evidence, based on data for Israel in the first half of the 1980s, supports the Ricardian neutrality specification, yielding plausible estimates for the behavioral parameters of the aggregate consumption function.

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1. Introduction

The impact of government budget variables on private-sector consumption is a key issue in assessing the implications of fiscal and monetary policy on the real side of the economy. In fact there are sharp controversies on this topic, most of which center around the Ricardian-equivalence proposition.[1]

The purpose of this paper is to develop and estimate a stochastic-intertemporal model of consumption behavior and to use it for testing a version of the Ricardian-equivalence proposition with time series data. Our framework allows for two channels that may give rise to deviations from Ricardian neutrality: Finite horizons and liquidity constraints. In addition, it incorporates explicitly the roles of taxes, substitution between public and private consumption, and different degrees of consumption durability.

The standard approach in empirical studies of the neutrality hypothesis is based on directly specifying regression equations linking consumption to disposable income, measures of non-human wealth, government spending, taxes, government transfers, etc. (see for example Kochin (1974), Tanner (1979), Feldstein (1982), Seater (1982), Kormendi (1983), Reid (1985)). While the results from applying this approach are informative, a limitation, which makes the interpretation of the results ambiguous, is that the connection between the estimated equations and the underlying theoretical model is not specified explicitly. Although the theoretical model typically specifies that current consumption is influenced by current **and** expected future changes in labor income, taxes, etc., most of the empirical applications focus mainly on current explanatory variables and ignore expected future ones. Therefore, the estimated coefficients of a given explanatory variable (such as current government spending or taxes) in a consumption equation may reflect not only direct effects of this variable, but also its effects as a predictor of future relevant variables. Moreover, these results cannot be used to assess the effects of policy changes, as for example a change in taxation, on consumption (Lucas's (1976) critique).[2] In contrast, the present study adopts an intertemporal optimizing framework whose implications, derived explicitly in the analysis, are the subject of empirical tests.

Since the seminal contribution of Hall (1978), numerous studies have applied the intertemporal optimizing approach to examine consumption behavior. However, almost none of these studies focus on the comovements of consumption and government-budget variables.[3] Moreover, these studies typically assume an infinite-horizon representative consumer. This assumption restricts the economic channels through which government-budget finance exerts its effects on consumption, resulting in an extreme case in which the model exhibits Ricardian properties. To move away from this case, Blanchard (1985) extended the intertemporal framework by relaxing the infinite-horizon assumption. His formulation allows for a richer set of interactions between government-budget-deficit variables and consumption, with Ricardian implications emerging only as a special case.[4] Another factor that may give rise to deviations from Ricardian neutrality is the existence of liquidity constraints that prevent some consumers from free access to capital markets (see the early work by Tobin and Dolde (1971), and the more recent contributions by Hayashi (1985) and Hubbard and Judd (1986)). In the present study, we develop a testable model which allows for deviations from neutrality through both these channels.

By virtue of the assumption of rational expectations, our framework results in a set of cross-equation restrictions. These restrictions are taken into account in the joint estimation of the consumption-behavior parameters and those of the stochastic processes governing the evolution of the forcing variables. We implement the model on monthly time series data for Israel covering the 1980-1985 period.[5] This case is of particular interest in testing the Ricardian-equivalence hypothesis because of the high volatility of movements in the budget deficit, taxes, and private consumption in an economy with an unusually high government budget deficits, amounting to 15% of aggregate output, on average, during this period. The sizeable deficits have resulted in a relatively large government debt, which was twice the size of GNP at the end of the period. These characteristics differ from those of the more stable environments studied in previous empirical works. They, therefore, enable a potentially more powerful test of hypotheses related to the comovements of private-sector consumption and taxes and public-sector spending.

The paper is organized as follows. Section 2 outlines the model. Empirical specifications and implementation of the model are presented in section 3. Section 4 extends the basic model to account for direct effects of public consumption on private consumption. Last, section 5 concludes the paper.

2. Theoretical Framework

We assume that there are overlapping generations of rational agents that have finite horizons. Specifically, there is a probability γ, smaller than unity, that individuals living in the present period will survive to the next period. A small open economy is considered, one that takes as given the world interest rate. We begin by considering the choice problem of an individual consumer.

2.1. Individual Consumer

The consumer is assumed to face a given safe interest factor R (where $R = (1+r)$ and r denotes the safe rate of interest), but due to lifetime uncertainty the effective (risk adjusted) interest factor is R/γ.[6] Disposable income is assumed to be stochastic and is denoted by y. Viewed from the standpoint of period t, consumer's utility from his stock of consumption goods during period $t+\tau$, $c_{t+\tau}$, is given by $\delta^{\tau}U(c_{t+\tau})$, where δ is the subjective discount factor. The probability of survival from period t through period $t+\tau$ is γ^{τ}, and therefore expected lifetime utility as of period t is

$$(1) \qquad E_t\sum_{\tau=0}^{\infty}(\gamma\delta)^{\tau}U(c_{t+\tau}),$$

where E_t is the conditional expectations operator. Individuals are assumed to maximize (1) subject to

(2a) $$c_t = (1-\phi)c_{t-1} + x_t,$$

(2b) $$x_t = b_t + y_t - (\frac{R}{\gamma})b_{t-1},$$

and the solvency condition $\lim_{t\to\infty} (\gamma/R)^t b_t = 0$. The variable x_t denotes the flow of consumption purchases, c_t denotes the stock of consumer goods, and ϕ denotes the rate of depreciation of this stock. The variable b_t is the one period debt issued in period t. Consolidating eqs.(2a) and (2b), the expected value of the lifetime budget constraint is given by

$$\left[1-(\frac{\gamma}{R})(1-\phi)\right]E_t\sum_{\tau=0}^{\infty}(\frac{\gamma}{R})^{\tau}c_{t+\tau} = E_t\sum_{\tau=0}^{\infty}(\frac{\gamma}{R})^{\tau}y_{t+\tau} - (\frac{R}{\gamma})b_{t-1} + (1-\phi)c_{t-1} = E_t w_t,$$

where $E_t w_t$ is (a specific definition of) expected wealth. This consolidated budget constraint is implied from the equality of the expected value of the discounted sum of the flow of consumption purchases and the corresponding discounted sum of the flow of disposable income, minus initial debt commitment.

With a view towards empirical implementation, we specify the utility function to be quadratic. That is,

(3) $$U(c_t) = \alpha c_t - \frac{1}{2}c_t^2,$$

where $\alpha > 0$ and $c_t < \alpha$.

It is shown in the Appendix that the solution to the optimization problem is

$$(4) \qquad c_t = \beta_0 + \beta_1 E_t w_t,$$

where

$$\beta_0 = \gamma\alpha \frac{1-\delta R}{\delta R(R-\gamma)} ,$$

and

$$\beta_1 = \left[1 - \frac{\gamma}{\delta R^2}\right]\left[1 - (\frac{\gamma}{R})(1-\phi)\right]^{-1}.$$

Equation (4) is a linear consumption function, relating the stock of consumer goods c_t to the expected value of wealth, where β_1 is the marginal propensity to consume out of wealth.

2.2. **Aggregate Consumption**

The economy consists of overlapping generations. The size of each cohort is normalized to 1, there are γ^a individuals of age a, and the size of population is constant at the level $1/(1-\gamma)$.

From equation (4), the consumption of an individual of age a at time t is

$$(5) \qquad c_{t,a} = \beta_0 + \beta_1\left[E_t\sum_{\tau=0}^{\infty}(\frac{\gamma}{R})^{\tau} y_{t+\tau} - \frac{R}{\gamma}b_{t-1,a-1} + (1-\phi)c_{t-1,a-1}\right].$$

Aggregating consumption over all cohorts and dividing by the size of population, yields per-capita aggregate consumption, C_t, as

$$(6)\quad C_t = (1-\gamma)\sum_{a=0}^{\infty}\gamma^a c_{t,a} = \beta_0 + \beta_1\left[E_t\sum_{\tau=0}^{\infty}(\tfrac{\gamma}{R})^\tau y_{t+\tau} - RB_{t-1} + \gamma(1-\phi)C_{t-1}\right],$$

where B_{t-1} is aggregate per-capita debt issued in period t-1.

It is shown in the Appendix that Eq.(6) can be rearranged as follows:

$$(7a)\quad C_t = \gamma\alpha(R-1)\,\frac{\delta R-1}{\delta R(R-\gamma)}$$
$$+ (1-\gamma)(1 - \frac{\gamma}{\delta R^2})\left[1-(\tfrac{\gamma}{R})(1-\phi)\right]^{-1}E_{t-1}\sum_{\tau=0}^{\infty}(\tfrac{\gamma}{R})^\tau(Y_{t+\tau} - T_{t+\tau}) + \Gamma C_{t-1} + \epsilon_t,$$

where $\Gamma = \left[\frac{\gamma}{\delta R} + \gamma(1-\phi)\left[1-\gamma(1 + \frac{1}{\delta R^2})\right]\right]\left[1 - (\frac{\gamma}{R})(1-\phi)\right]^{-1}$, and where Y is gross income and T is the level of taxes (both in per-capita terms), and ϵ_t is a zero mean, finite variance, error term. In order to express the consumption equation in terms of observed consumer purchases, we use recursively the per capita aggregate version of eq.(2a) applied to aggregate per-capita consumption, and substitute it into eq.(7a). This yields:

$$(7b)\quad X_t = \gamma\alpha(R-1)\,\frac{\delta R-1}{\delta R(R-\gamma)} + (1-\gamma)(1 - \frac{\gamma}{\delta R^2})\left[1-(\tfrac{\gamma}{R})(1-\phi)\right]^{-1}E_{t-1}\sum_{\tau=0}^{\infty}(\tfrac{\gamma}{R})^\tau(Y_{t+\tau} -$$
$$T_{t+\tau}) + (\Gamma - \gamma(1-\phi))\sum_{\tau=0}^{\infty}\gamma^\tau(1-\phi)^\tau X_{t-\tau-1} + \epsilon_t,$$

where X_t is the aggregate per-capita value of consumer purchases.

Equation (7b) is the focal relation for our empirical work. It expresses aggregate consumption purchases (per-capita) as a function of a constant term, expected human wealth, lagged purchases, and an error term. The present formulation is general enough to encompass both Ricardian and non-Ricardian systems as special cases. The key parameter, in this context, is γ. When $\gamma = 1$ the system possesses Ricardian neutrality, and eq.(7b) indicates that only lagged consumer purchases can be used to predict current purchases (similar to Hall (1978)). However, when $\gamma < 1$, expected human wealth affects current consumption purchases over and beyond the impact of lagged consumption purchases. For example, a current-period cut in taxes raises expected human wealth and thus results in an increase in consumption. The reason is that the future tax hike, needed in order to balance the intertemporal budget constraint of the government, is given a smaller weight, by finite-horizon consumers, than the weight attached by them to the current cut in taxes.

2.3. Liquidity-Constrained Consumers

The foregoing specifications hold under the assumption that all consumers have free access to the capital market and thus can borrow against future incomes. In that case, Ricardian neutrality breaks down due to finite horizons (as captured by $\gamma < 1$). In this subsection, we extend the model to allow for an additional channel through which nonequivalence results may arise: The existence of liquidity constraints. Accordingly, we allow here for the possibility that while a fraction Π of aggregate consumption is due to consumers that have access to capital markets, a fraction $(1-\Pi)$ is due to consumers that are liquidity constrained in their consumption purchases. Formally,

$$(8) \qquad X_t = \Pi X_{ut} + (1-\Pi)X_{ct},$$

where X_{ut} denotes consumption purchases of liquidity-unconstrained individuals, and X_{ct} denotes purchases of those that are subject to liquidity constraints. For X_{ut}, we use the specification in equation (7b), and for X_{ct} we use the following simple specification,

$$(9) \qquad X_{ct} = Y_{t-1} + v_t.$$

That is, consumption purchases under liquidity constraints are modelled as the sum of two components: Last period's net income and an error term.[7]

It can be easily verified that in this augmented version of the model, Ricardian equivalence holds only under the restriction that $\gamma = 1$ and $\Pi = 1$. This restriction is tested in the next section.

3. Empirical Implementation

3.1. Specifications

To implement equation (7b) it is necessary to specify, under rational expectations, the stochastic processes which govern the evolution of gross income and taxes. Accordingly, we stipulate simple first-order auto regressive processes for these variables,[8]

$$(10) \qquad Y_t - Y_{t-1} = \rho_Y(Y_{t-1} - Y_{t-2}) + \eta_{Yt}$$

$$(11) \qquad T_t - T_{t-1} = \rho_T(T_{t-1} - T_{t-2}) + \eta_{Tt},$$

where the ρ's are time-independent, and the η's are serially uncorrelated zero-mean stochastic terms that are orthogonal to variables dated t-1 and previously.[9]

Using equations (10) and (11) to calculate expected human wealth yields, as shown in the Appendix, the following expression for consumption purchases:

$$(12) \qquad X_t = d_0 + \sum_{i=1}^{n} d_{1i}X_{t-i} + d_2Y_{t-1} + d_3Y_{t-2} + d_4T_{t-1} + d_5T_{t-2} + \eta_{xt},$$

where n is the number of lagged-purchases terms, and the d-coefficients satisfy the following restrictions

$$d_0 = \frac{\gamma\alpha(R-1)(\delta R-1)}{\delta R(R-\gamma)}$$

$$d_{1i} = \left[\Gamma - \gamma(1-\phi)\right]\gamma^{i-1}(1-\phi)^{i-1} \quad , \qquad \text{for } i = 1,\dots,n$$

$$d_{1,n+1} = \Gamma\gamma^n(1-\phi)^n$$

$$d_2 = (1-\gamma)(1 - \frac{\gamma}{\delta R^2})\left[1 - \frac{\gamma}{R}(1-\phi)\right]^{-1}\left[(\frac{R}{R-\gamma})(1+\rho_y) + \frac{\rho^2_Y\gamma}{R} + \frac{\gamma^2\rho_Y^2}{R(1-\rho_Y)(R-\gamma)} - \right.$$
$$\left. - \frac{\rho_Y^4\gamma^2}{(1-\rho_Y)R(R-\rho_Y\gamma)}\right],$$

$$d_3 = (1-\gamma)(1 - \frac{\gamma}{\delta R^2})\left[1 - \frac{\gamma}{R}(1-\phi)\right]^{-1}(\frac{R}{R-\gamma}) - d_2,$$

$$d_4 = -(1-\gamma)(1 - \frac{\gamma}{\delta R^2})\left[1 - \frac{\gamma}{R}(1-\phi)\right]^{-1}\left[(\frac{R}{R-\gamma})(1+\rho_T) + \frac{\rho_T^2\gamma}{R} + \frac{\gamma^2\rho_T^2}{R(1-\rho_T)(R-\gamma)} - \frac{\rho_T^4\gamma^2}{(1-\rho_T)R(R-\rho_T\gamma)}\right],$$

$$d_5 = (1-\gamma)(1 - \frac{\gamma}{\delta R^2})\left[1 - \frac{\gamma}{R}(1-\phi)\right]^{-1}(\frac{R}{R-\gamma}) - d_4.$$

Equations (8)-(12), form the system to be empirically analyzed.

3.2. **Findings**

Several versions of the system consisting of eqs.(8)-(12) are estimated using Israeli monthly data covering the period 1980-1985. The use of monthly data clearly limits our choice of the actual time series that serve as counterparts for the variables in the model. For consumption purchases, X, we use an index of purchases within the organized retail trade.[10] Total wage bill is used for income, Y, and government tax receipts are used for T. The data source is Bank of Israel's Publication **Main Economic Indicators** (various issues).

Estimation was performed by non-linear least squares (from the TSP program) jointly applied to the system. The estimator is based on computing maximum likelihood, and the estimates are obtained by concentrating variance parameters out of the multivariate likelihood and then maximizing the negative of the logdeterminant of the residual covariance matrix. As is well known, the estimates are efficient if the disturbances are multivariate normal and identically distributed. Table 1 reports different versions of the estimated model, allowing for seven lags in the estimation of the durability parameter and setting the monthly risk-free real interest factor to 1.002.[11] Column (1) gives the parameter estimates of the model. The likelihood ratio test of the model against its unrestricted counterpart yields a χ^2 statistic of 12.3 (with 8 degrees of freedom), which is not significant at the one-percent-significance level. The statistic for the test equals twice the difference between the unrestricted and restricted values of the log-likelihood function. Estimates of the unrestricted version are given in the Appendix. While this indicates that the data do not reject the model, some of the parameters obtain somewhat implausible estimated values. In particular, δ and Π seem to be too high relative to what is commonly expected. The parameter γ is smaller but close to unity. Under Blanchard's formulation, this parameter stands for the survival probability. A monthly $\gamma = 0.989$ implies under this interpretation an expected life of $\gamma^{12}/\left[(1-\gamma^{12})\right]^{-2} = 58$ years. Although viewed from the time of birth this is a low life expectancy, it seems more plausible when viewed from the point-of-view of the average horizon for consumption-decision-making of the mature population.

Columns (2) and (3) impose further restrictions on the estimated model. In column (2), we set consumer's time horizon to infinity ($\gamma = 1.0$) and the estimated model is not rejected when compared to the unrestricted model. (The likelihood ratio is 18.96, with 9 degrees of freedom). Interestingly, more plausible parameter estimates obtain in this column than in the previous one, including an estimated value for the fraction of liquidity-unconstrained consumption close to (and below) unity. Column (3) allows for estimation of γ, but sets the parameter Π equal to unity. Again, this version of the model is not rejected using a likelihood ratio test (whose value is about the same as the one for column (2)). The parameter γ obtains a value of 0.999 which is larger than the one reported in column (1). Notice that in moving from column (1) to the next columns the estimated values of δ decline and become closer to unity.

The Ricardian-equivalence proposition implies the $\gamma = \Pi = 1.0$ restriction, which is tested in column (4). The likelihood ratio for testing this restriction against the unrestricted counterpart of the model is 19.32 (with 10 degrees of freedom). This is lower than the one-percent chi-square critical value of 23.2. Thus, Ricardian neutrality is not rejected by the data.[12]

Having established this result, we can now discuss the parameter estimates for the specification of the model that embodies the neutrality properties. The parameters generally obtain the hypothesized signs and are significantly different from zero. The estimated first-order autoregressive parameters of the processes for $(Y_t - Y_{t-1})$ and $(T_t - T_{t-1})$ are negative indicating that shocks to these variables tend to be reversed in subsequent months. Shocks to the gross income variable show a larger degree of persistence than shocks to the tax variable. The estimated monthly subjective discount factor is slightly above unity; however, we have tested for $\delta = 1.0$ and the test does not reject this hypothesis.[13] The utility function parameter α is positive and equal to 301.8. An important feature of this value is that it satisfies the assumption that marginal utility of consumption is positive, i.e., $\alpha > c$. Specifically, the maximal value of consumption purchases in the sample implies, using a durability parameter of 0.79, for seven lags, a maximal stock of consumption goods of about 85 (index units) which is smaller than the estimated α. Further, this estimated parameter can be used to calculate the implied degree of relative risk aversion $(C/(\alpha-C))$, which turns out to be equal to 0.3 (at the mean sample value of consumption purchases).[14] The parameter estimate for ϕ implies that twenty one percent of the stock of consumer goods depreciates from month to month. Since, due to lack of more refined monthly data, our measure of consumption purchases includes goods with different degrees of durability, this parameter ϕ should be interpreted as an average depreciation rate.

TABLE 1 - ESTIMATED VERSIONS OF THE MODEL
(ISRAEL: 1980:9 - 1985:12)

	Model's Restrictions	As in Column (1) and γ=1.0	As in Column (1) and Π=1.0	As in Columns (2) and (3)
Parameters	(1)	(2)	(3)	(4)
ρ_Y	-0.17	-0.23	-0.23	-0.24
	(0.05)	(0.10)	(0.10)	(0.10)
ρ_T	-0.57	-0.58	-0.58	-0.58
	(0.09)	(0.09)	(0.09)	(0.09)
δ	1.20	1.03	1.04	1.03
	(0.06)	(0.02)	(0.02)	(0.02)
α	104.81	233.19	220.54	301.80
	(105.89)	(112.11)	(110.03)	(62.79)
ϕ	0.24	0.20	0.20	0.21
	(0.04)	(0.06)	(0.06)	(0.06)
γ	0.986	1.00[b]	0.999	1.00[b]
	(0.01)		(0.0002)	
Π	2.09	0.99	1.00[b]	1.00[b]
	(0.45)	(0.02)		
L	-618.94	-622.27	-622.19	-622.45

Notes:[a] The basic model consists of eqs.(8)-(12). Its parameter estimates are reported in Column (1). L denotes the value of the log-likelihood function. Figures in parentheses are estimated standard errors. The value of L for the unrestricted system is 612.79 (free parameters).

[b] Imposed value.

4. Substitution Between Public and Private Consumption

We now extend the model by allowing direct effects of government spending on private consumption. The model's specification in section 2 can be interpreted as one that incorporates public goods in the utility function in a separable way, implying that public goods have neutral effects on the consumption of private goods. The present extension differs from the foregoing specifications since it allows for substitutability between public and private consumption. When the degree of substitution approaches zero we are back to the original model.

Let the utility function be specified by

$$U(c_t, G_t) = \alpha(c_t + \theta G_t) - \frac{1}{2}(c_t + \theta G_t)^2, \tag{13a}$$

$$G_t = (1-\phi)G_{t-1} + g_t, \tag{13b}$$

where G denotes the stock of public consumption, g denotes the flow of government purchases, and θ is a parameter that measures the weight of public consumption in total private **effective** consumption, $c_t + \theta G_t$ (see Aschauer (1985)). For tractability, the rates of depreciation of the stocks of private and public consumption goods are assumed to be identical and are denoted by ϕ. As shown in the Appendix, in this case, the analogue of equation (6), expressing aggregate per-capita consumption, is:

$$C_t = \beta_0 + \beta_1\Big[E_t \sum_{\tau=0}^{\infty} (\tfrac{\gamma}{R})^{\tau}(y_{t+\tau} + \theta g_{t+\tau}) - RB_{t-1} + \gamma(1-\phi)(C_{t-1} + \theta g_{t-1})\Big] - \theta G_t. \tag{14}$$

Similarly, the analogue of eq.(7a) of above is,

(15) $$C_t = \gamma\alpha(R-1)\frac{\delta R-1}{\delta R(R-\gamma)} + (1-\gamma)(1 - \frac{\gamma}{\delta R^2})\Big[1 - \frac{\gamma}{R}(1-\phi)\Big]^{-1} E_{t-1}\sum_{\tau=0}^{\infty}(\frac{\gamma}{R})^{\tau}\Big[Y_{t+\tau} - T_{t+\tau} + \theta g_{t+\tau}\Big] + \Gamma(C_{t-1} + \theta G_{t-1}) - \theta G_t + \epsilon_t'.$$

We assume that the expected flow of future public consumption evolves according to the simple proceess

(16) $$g_t - g_{t-1} = \rho_g(g_{t-1} - g_{t-2}) + \eta_{gt}.$$

Equation (15) can then be rewritten as

(17) $$X_t = d_0' + \sum_{i=1}^{n} d_{1i}(X_{t-i} + \theta g_{t-i}) + d_2 Y_{t-1} + d_3 Y_{t-2} + d_4 T_{t-1} + d_5 T_{t-2} + d_6 g_{t-1} + d_7 g_{t-2} + \epsilon_t',$$

where d_1 through d_5 are as in equation (12) above, and

$$d_6 = \theta(1-\gamma)(1 - \frac{\gamma}{\delta R^2})\Big[1 - \frac{\gamma}{R}(1-\phi)\Big]^{-1}\Big[\frac{R}{R-\gamma}(1+\rho_g) + \frac{\rho_g^2\gamma}{R} + \frac{\gamma^2\rho_g^2}{R(1-\rho_g)(R-\gamma)} - \frac{\rho_g^4\gamma^2}{(1-\rho_g)R(R-\rho_g\gamma)}\Big] - \theta(1+\rho_g),$$

$$d_7 = \theta(1-\gamma)(1 - \frac{\gamma}{\delta R^2})\Big[1 - \frac{\gamma}{R}(1-\phi)\Big]^{-1}(\frac{R}{R-\gamma}) + \theta\rho_g - d_6.$$

Note that eq.(17) holds for liquidity-unconstrained consumers. As in section (2.3) we embed this equation in a more general framework in which aggregate consumption includes also another component which is due to liquidity-constrained individuals. Accordingly, eqs.(8)-(11), and (16)-(17) constitute the more general system to be implemented in this section.

Table 2 reports the results of estimating two versions of the system. To save degrees of freedom under this augmented version of the model, the number of lags used in estimating the durability parameter is set equal to 3. Column (1) gives the parameter estimates under the model's restrictions. These restrictions are not rejected against the unrestricted version of the model; the pertinent likelihood ratio is 14.52 (with seven degrees of freedom), a value that is below the critical one-percent value of 18.5. Column (2) can be used to test Ricardian neutrality which implies the $\gamma = \Pi = 1.0$ restriction. As before, this hypothesis is not rejected by the data. In extending the model and going from Table 1 to Table 2 it can be observed that most of the parameter estimates do not change noticeably. However, in contrast to the notion of government consumption yielding positive marginal utility, the estimated value of θ is negative.[15] Thus, although statistically the specification underlying column (2) is not rejected by the data, the public consumption variable has effects that do not conform with the theoretical model.

TABLE 2 - THE MODEL WITH PUBLIC GOODS
(ISRAEL: 1980:9-1985:12)

Parameters	Model's Restrictions (1)	As (1) and $\gamma = \Pi = 1.0$ (2)
ρ_Y	-0.23 (0.08)	-0.22 (0.10)
ρ_T	-0.59 (0.07)	-0.59 (0.07)
ρ_g	-0.56 (0.08)	-0.55 (0.07)
δ	1.17 (0.12)	1.04 (0.04)
α	152.66 (218.47)	128.78 (36.64)
ϕ	0.41 (0.08)	0.39 (0.09)
γ	0.989 (0.02)	1.00[b]
Π	1.37 (0.29)	1.00[b]
θ	-0.52 (0.20)	-0.47 (0.26)
L	-781.87	-782.52

Notes: [a] The model consists of eqs.(8)-(11), (16)-(17). Its parameter estimates are reported in column (1). L denotes the value of the log-likelihood function. Figures in parentheses are estimated standard errors. The value of L for the unrestricted system is - 774.61 (16 free parameters).

[b] Imposed value

5. **Conclusions**

In this paper, we have developed a stochastic framework in which the intertemporal implications of the Ricardian-equivalence proposition can be tested with aggregate time series data. The framework allows for two types of deviations from Ricardian neutrality. The first is due to finite consumers' planning horizons, and is modelled as an extension of Blanchard (1985) to a stochastic environment. The second is due to the existence of liquidity constraints on consumption behavior. In addition, our framework allows for direct substitutability between private and public consumption, and treats explicitly the degree of durability of aggregate consumption.

The model was implemented on monthly data for Israel during the first half of the 1980's, a period of high and volatile government budget deficits. Our main findings are that the restrictions implied by the Ricardian-neutrality hypothesis are not rejected by the sample information, and that the resulting parameter estimates generally conform with the theoretical model. These features held up when the model was extended to allow for public goods consumption, with the exception that the parameter capturing the direct effects of public consumption on private utility turned out to be implausible.

There are several interesting possible extensions of the present research. First, it would be important to allow for additional sources of deviations from Ricardian neutrality, such as the existence of distortionary taxes, (e.g., income, value added and inflation taxes). In this context, it is desirable to decompose taxes into at least two categories, consumption and income taxes.[16] Second, another channel through which government policies can affect private consumption is related to monetary and exchange rate policies.[17] Third, the model's specifications can be modified to allow for different effects on private consumption of various components of government spending, potentially capturing substitutability as well as complementarity with private consumption. Fourth, the model could be extended to allow for a bequest motive. Since negative bequests are not feasible, individuals may become bequest constrained. In such a case Ricardian neutrality breaks down. In this specification the key factor to be tested in the context of Ricardian neutrality is the strength of the bequest motive relative to the path of income growth in the economy. These extensions to the intertemporal framework of consumption determination are necessary before policy recommendations based on the Ricardian neutrality theme are advanced.

APPENDIX

1. Derivation of the Consumption Function (Equation (4)).

The maximization problem described in section 2.1 can be expressed in dynamic programming terms by the value function V as

$$V(y_t - \frac{R}{\gamma}b_{t-1}) = \underset{x_t}{\text{Max}}\{U(x_t+(1-\phi)c_{t-1})+\gamma\delta E_t V(y_{t+1}+\frac{R}{\gamma}(y_t - x_t - \frac{R}{\gamma}b_{t-1}))\}. \tag{A.1}$$

Differentiating the right-hand-side of (A.1) and equating to zero yields

$$U'(c_t) - \delta R E_t V'(\cdot) = 0, \tag{A.2}$$

where primes denote derivatives.

Totally differentiating (A.1) yields

$$V'(y_t - \frac{R}{\gamma}b_{t-1}) = [U'(c_t) - \delta R E_t V'(\cdot)]\frac{dx_t}{dy_t} + \delta R E_t V'(\cdot) = \delta R E_t V'(\cdot), \tag{A.3}$$

where use has been made of (A.2). Equations (A.2) and (A.3) imply

$$U'(c_t) = \delta R E_t U'(c_{t+1}). \tag{A.4}$$

Using the quadratic utility function specified in equation (3), eq.(A.4) can be expressed as

(A.5) $$\alpha - c_t = \delta R E_t(\alpha - c_{t+1}).$$

Define expected human wealth by

(A.6) $$E_t h_t = E_t \sum_{T=0}^{\infty} (\frac{\gamma}{R})^{\tau} y_{t+\tau}.$$

From eq.(A.6) we obtain

(A.7) $$y_t = E_t h_t - \frac{\gamma}{R} E_t h_{t+1}.$$

Define expected wealth by

(A.8) $$E_t w_t = E_t h_t - \frac{R}{\gamma} b_{t-1} + (1-\phi) c_{t-1}$$

Then, from the constraints (eqs.(2a) and (2b)) and from eq.(A.7) we get

(A.9) $$a c_t = E_t w_t - (\frac{\gamma}{R}) E_t w_{t+1}.$$

where $a = 1 - (\frac{\gamma}{R})(1-\phi)$.

Postulating that the solution to the maximization problem is of the form

(A.10) $$c_t = \beta_0 + \beta_1 E_t w_t,$$

equations (A.9) and (A.10) imply

$$E_t w_{t+1} = \frac{R}{\gamma}\left[-\beta_0 a + (1 - \beta_1 a)E_t w_t\right]. \tag{A.11}$$

Substituting (A.10) into (A.5) yields

$$\alpha - (\beta_0 + \beta_1 \; E_t w_t) = \delta R\left[\alpha - (\beta_0 + \beta_1 \; E_t w_{t+1})\right]. \tag{A.12}$$

Substituting (A.11) into (A.12) yields

$$\alpha - (\beta_0 + \beta_1 \; E_t w_t) = \delta R\left[\alpha - (\beta_0 + \beta_1 \; \frac{R}{\gamma}(- \; \beta_0 a + (1-\beta_1 a)E_t w_t))\right]. \tag{A.13}$$

Rearranging terms in equation (A.13) yields

$$\left[(1-\delta R)\alpha - (1 - \delta R(1 - \frac{R}{\gamma}\beta_1 a))\beta_0\right] + \tag{A.14}$$

$$+ \left[-1 + \frac{\delta R^2}{\gamma}(1-\beta_1 a)\right]\beta_1 E_t w_t = 0.$$

The solution specified in equation (A.10) is confirmed when (A.14) holds for all $E_t w_t$. This requirement is fulfilled when the bracketed terms in (A.14) equal zero. Thus, (A.15) $$1 - \beta_1 a = \frac{\gamma}{\delta R^2}, \qquad (\beta_1 = \frac{1}{a}\,(1 - \frac{\gamma}{\delta R^2}))$$

(A.16) $$\beta_0 = \alpha\,\frac{\gamma(1 - \delta R)}{\delta R(R - \gamma)}$$

2. Derivation of Equations (7a) and (7b)

Aggregating eq.(2b) over all cohorts, the per-capita flow budget constraint lagged one period is

(A.17) $$B_{t-1} = X_{t-1} - Y_{t-1} + RB_{t-2},$$

where X_t denotes aggregate purchases per-capita. Substituting eqs.(2a), (A.7) and (6) into (A.17) yields

(A.18) $$B_{t-1} = \beta_0 + (\beta_1 - 1)E_{t-1}h_{t-1} + \frac{\gamma}{R}E_{t-1}h_t + R(1-\beta_1)B_{t-2} + \\ + \gamma(1-\phi)(\beta_1-1)C_{t-2}.$$

Define

(A.19) $$E_tW_t = E_th_t - RB_{t-1} + \gamma(1-\phi)C_{t-1} = E_{t-1}h_t - RB_{t-1} + \\ + \gamma(1-\phi)C_{t-1} + \epsilon'_t,$$

where $\epsilon^*_t = (E_th_t - E_{t-1}h_t)$. Substituting (A.18) into (A.19) yields

(A.20) $$E_tW_t = (1-\gamma)E_{t-1}h_t - R\beta_0 - R(\beta_1-1)E_{t-1}W_{t-1} + \gamma(1-\phi)C_{t-1} + \epsilon^*_t.$$

Eq.(6) in the text is rewritten as

$$C_t = \beta_0 + \beta_1 E_t W_t . \tag{A.21}$$

Lagging (A.21) and rearranging yields

$$E_{t-1} W_{t-1} = \frac{1}{\beta_1}(C_{t-1} - \beta_0). \tag{A.22}$$

Substituting (A.22) into (A.20) yields

$$E_t W_t = (1-\gamma)E_{t-1}h_t + \gamma(1-\phi)C_{t-1} - R\beta_0 - \frac{R(\beta_1 - 1)}{\beta_1}(C_{t-1} - \beta_0) + \epsilon_t, \tag{A.23}$$

which can be substituted into (A.21) to yield

$$C_t = \beta_0(1-R) + \beta_1(1-\gamma)E_{t-1}h_t + \left[\gamma(1-\phi)\beta_1 - R(\beta_1 - 1)\right]C_{t-1} + \epsilon_t, \tag{A.24}$$

where $\epsilon_t = \beta_1 \epsilon_t^*$.

Equation (A.24) corresponds to eq.(7a) in the text.

The solution to the individual maximization problem is, therefore, given by equation (4) in the text.

3. **Derivation of the Estimated Consumption Equation (Eq.(12)).**

Here we incorporate the stochastic processes governing the evolution of disposable income into eq.(7b) in the text. For brevity, we illustrate the calculations for the case in which there is a single auto regressive process applicable to Y and T, given by

$$(A.25) \qquad Y_t - Y_{t-1} = \rho(Y_{t-1} - Y_{t-2}) + \eta_t \quad , \quad E_t\eta_t = 0.$$

Notice that here we allow for a constant term λ, which is dropped later on in the empirical analysis.

Let $z_t = Y_t - Y_{t-1}$. Eq.(A.25) yields

$$(A.26) \qquad z_{t+i} = \rho^i z_t + \sum_{\tau=0}^{i} \rho^{i-\tau}\eta_{t+\tau} \quad , \quad i \geq 1.$$

Substituting eq.(A.26) into $E_t h_t$ yields

$$(A.27) \quad E_{t-1}h_t = E_{t-1}\left[Y_t + \frac{\gamma}{R}(Y_t + \rho(Y_t - Y_{t-1})) + (\frac{\gamma}{R})^2(Y_{t+1}+\rho(Y_{t+1}-Y_t))+\dots\right]$$

$$= E_{t-1}\left[Y_t + \frac{\gamma}{R}(Y_t + \rho(Y_t - Y_{t-1})) + (\frac{\gamma}{R})^2(Y_t + \rho(Y_t - Y_{t-1}) + \right.$$

$$\left. + \rho^2(Y_t - Y_{t-1}) + \dots \right].$$

Using (A.26)

$$\text{(A.28)} \quad E_{t-1}h_t = E_{t-1}\left[Y_t + \frac{\gamma}{R}(Y_t + \rho z_t) + (\frac{\gamma}{R})^2(Y_t + (\rho + \rho^2)z_t) + \ldots\right]$$

$$= (\frac{R}{R-\gamma})z_{t-1}+\rho(z_{t-1}-z_{t-2})) + \rho\,\frac{\gamma}{R}\left[1 + (\frac{\gamma}{R}\,\frac{1}{1-\rho})(\frac{R}{R-\gamma}) - \frac{\gamma}{R}\,\frac{\rho^2}{1-\rho}(\frac{R}{R-\rho\gamma})\right]\rho(z_{t-1}-z_{t-2})$$

Finally, noting that $E_{t-1}h_t = E_{t-1}\sum_{\tau=0}^{\infty}(\frac{\gamma}{R})^{\tau}(Y_{t+\tau} - T_{t+\tau})$, allowing for separate stochastic processes for Y_t and T_t as in eqs.(8) and (9), substituting formulas such as (A.28) for both expected gross income and taxes into equation (7) yields equation (12) in the text.

4. **Derivation of Equations (14) and (15).**

The solution method applied in section 1 of this appendix is now applied to the extended utility function, equation (13a). The analogue of (A.5) is

$$\text{(A.29)} \quad \alpha - (c_t + \theta G_t) = \delta R E_t(\alpha - (c_{t+1} + \theta G_{t+1})).$$

The solution to the maximization problem is of the form

$$\text{(A.30)} \quad (c_t + \theta G_t) = \beta_0 + \beta_1\, E_t\tilde{W}_t,$$

where

$$E_t\tilde{w}_t = E_t \left[\sum_{\tau=0}^{\infty} (\frac{\gamma}{R})^{\tau} (Y_{t+\tau}+\theta g)_{t+\tau} - RB_{t-1} + \gamma(1-\phi)(C_{t-1}+\theta G_{t-1})\right]$$

This gives equation (14) of the text.

To see this, one can use equations (A.9) and (A.21) to obtain

$$\text{(A.31)} \qquad \left[(1 - \delta R)\alpha - (1 - \delta R(1 - \frac{R}{\gamma}\beta_1))\beta_0\right]$$

$$+ \left[-1 + \delta \frac{R^2}{\gamma}(1-\beta_1)\right]\beta_1 E_t\tilde{w}_t = 0,$$

which holds for any values of $\tilde{w}_t$ and G_t when β_0 and β_1 are chosen so that the bracketed terms are zero.

It can be verified that the expressions for β_0 and β_1 which solve (A.31) are given in equations (A.15) and (A.16), respectively. Equation (15) of the text is obtained from equation (A.30) using similar calculations as done in section 1 of the appendix.

5. Estimates of the unrestricted versions of the model

A. The Model in Table 1

$$Y_t - Y_{t-1} = \underset{(0.13)}{-0.23}\,(Y_{t-1} - Y_{t-2})$$

$$T_t - T_{t-1} = \underset{(0.08)}{-0.61}\,(T_{t-1} - T_{t-2})$$

$$X_t = \underset{(7.70)}{4.27} + \underset{(0.14)}{0.25}\,X_{t-1} + \underset{(0.15)}{0.04}\,X_{t-2} + \underset{(0.12)}{0.14}\,X_{t-3} + \underset{(0.10)}{0.07}\,X_{t-4}$$

$$- \underset{(0.11)}{0.02}\,X_{t-5} - \underset{(0.12)}{0.14}\,X_{t-6} + \underset{(0.12)}{0.11}\,X_{t-7} + \underset{(0.17)}{0.31}\,X_{t-8}$$

$$+ \underset{(0.03)}{0.03}\,Y_{t-1} - \underset{(0.03)}{0.04}\,Y_{t-2} - \underset{(0.27)}{0.06}\,T_{t-1} + \underset{(0.23)}{0.65}\,T_{t-2}$$

B. The Model in Table 2

$$Y_t - Y_{t-1} = -0.22\,(Y_{t-1} - Y_{t-2})$$
$$T_t - T_{t-1} = -0.61\,(T_{t-1} - T_{t-2})$$
$$g_t - g_{t-1} = -0.56\,(g_{t-1} - g_{t-2})$$
$$X_t = \underset{(7.85)}{11.26} + \underset{(0.15)}{0.37}\,X_{t-1} - \underset{(0.13)}{0.04}\,X_{t-2} + \underset{(0.12)}{0.26}\,X_{t-3} + \underset{(0.13)}{0.18}\,X_{t-4}$$

$$- \underset{(0.20)}{0.04}\,g_{t-1} - \underset{(0.26)}{0.40}\,g_{t-2} - \underset{(0.20)}{0.25}\,g_{t-3} - \underset{(0.14)}{0.13}\,g_{t-4}$$

$$+ \underset{(0.03)}{0.01}\,Y_{t-1} + \underset{(0.05)}{0.02}\,Y_{t-2} - \underset{(0.32)}{0.12}\,T_{t-1} + \underset{(0.31)}{0.81}\,T_{t-2}$$

(Numbers in parentheses are estimated standard errors. Log-Likelihood values for these unrestricted systems are provided in tables 1 and 2.)

FOOTNOTES

1. See Barro (1974).

2. For a recent survey of empirical tests of Ricardian equivalence, see Leiderman and Blejer (1986).

3. For an exception, see Aschauer (1985).

4. For analysis of effects of fiscal policy in open economies using this type of model, see Frenkel and Razin (1986). For an empirical implementation motivated by a model of this type, see van Wijnbergen (1985).

5. The present paper addresses the issue of consumption and government finance that was analyzed also in Leiderman and Razin (1986). However, the present framework is different from the previous one in five major respects: (1) It explicitly models the stochastic environment (by using a quadratic utility function); (2) It allows for a joint estimation of the consumption and the driving-force equations; (3) It allows for consumption durability; (4) It allows for substitution between public and private consumption; and (5) It permits the existence of liquidity constraints.

6. See Blanchard (1985). Throughout we use the assumption of a constant real rate. While this is a restrictive assumption, it need not be very unrealistic in an economy with widespread indexation in financial markets.

7. We use Y_{t-1} in this formulation because earned income (wages) during period t-1 is typically paid at the beginning of period t. We also allow for a stochastic component of payment, v_t, during period t.

8. On the sensitivity of the empirical results with respect to alternative specifications see fn.7, below.

9. Experimentation with univariate and multivariate autoregressive processes with longer lag structures for the forcing variables and with constant terms yielded results that do not reject the present first-difference univariate system (with no constants).

10. These monthly measures of consumption are closely correlated with the national-accounts series for consumption. Using quarterly moving averages of the monthly purchases data we obtain a correlation coefficient of 0.9 between our time series and the national-accounts quarterly consumption series. (See also Fisher (1986)) In conformity with the theoretical model one should have used per-capita data. However, in view of the small changes in population during the short sample period and the unavailability of these data on a monthly basis we use aggregate data in the study..

11. Experimenting with different lags as well as different realistic values of R did not yield noticeably different results from those reported in Table 1. We a-priori set the value of R in order to identify the other parameters.

12. This result is different from that in Leiderman and Razin (1986). It turns out that once we allow for some degree of durability in consumption (as in the present paper) the results become more favourable to Ricardian neutrality.

13. Interestingly, Hansen and Singleton (1983) also found that the point estimate of δ (with monthly U.S. data) is close to (and sometimes above) unity.

14. This estimate for the degree of relative risk aversion falls within the range of those reported in studies for the U.S.

15. This may reflect improper measurement of public consumption in our data set. This measure is derived from cash-flow accounts of the Treasury, which partly include transfer payments such as consumption subsidies. In a study based on U.S. data, Aschauer (1985) reports estimated value for θ of 0.23.

16. As shown by Frenkel and Razin (1986), private spending responds differently to cuts in alternative types of taxes.

17. For a theoretical analysis, see Helpman and Razin (1987, forthcoming).

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