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MODEL

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Raise Rates to Raise Inflation? Neo-Fisherianism in the New Keynesian Model

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ABSTRACT

Increasing the inflation target in a textbook New Keynesian (NK) model may require increasing, rather than decreasing, the nominal interest rate in the short run. We refer to this positive short run co-movement between the nominal interest rate and inflation conditional on a nominal shock as Neo-Fisherianism. We show that the NK model is more likely to be Neo-Fisherian the more persistent is the change in the inflation target and the more flexible are prices. Neo-Fisherianism is driven by the forward-looking nature of the model. Modifications which make the framework less forward-looking make it less likely for the model to exhibit Neo-Fisherianism. As an example, we show that a modest and empirically realistic fraction of "rule of thumb" price-setters may altogether eliminate Neo-Fisherianism in the textbook model.

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1 Introduction

Short term interest rates in the United States and other developed countries have been at or near zero for several years, yet inflation in these countries remains stubbornly low. While conventional wisdom holds that low nominal interest rates are expansionary and lead to an increase in inflation, recent experiences have led several prominent economists, most notably John Cochrane and Steven Williamson, to advance a theory that has been dubbed Neo-Fisherianism.¹ The Neo-Fisherian hypothesis holds that central banks must raise interest rates to raise inflation, and that extended periods of low interest rates are deflationary.

The Neo-Fisherian hypothesis follows from the standard Fisher relationship relating nominal interest rates to real rates and expected inflation, $i_t = r_t + \mathbb{E}_t \pi_{t+1}$, where i_t is the nominal rate, r_t the real rate, and $\mathbb{E}_t \pi_{t+1}$ expected inflation. The Fisher relationship was first derived in Fisher (1896). It is an arbitrage condition between real and nominal assets that holds in virtually every modern macroeconomic model. The majority of economists would likely agree that, in the long run, the real interest rate is independent of nominal factors, so that a long run increase in the nominal interest rate translates into a one-for-one increase in inflation. But in the short run, many economists believe that nominal frictions permit nominal shocks to affect real rates, which in turn affect inflation. Consequently, controversy over the Neo-Fisherian hypothesis is centered mostly on the behavior of nominal interest rates and inflation in the short run.

The objective of this paper is to explore the Neo-Fisherian hypothesis in the textbook New Keynesian (NK) model. This model is used extensively in central banks throughout the world to inform policymaking. In its simplest form, the model is comprised of two principal equations – an IS curve relating spending to the real interest rate, and a Phillips Curve relating inflation to a measure of real activity. We assume that monetary policy is set according to a strict inflation target. We derive an analytic expression showing how the nominal interest rate must adjust to implement changes in the inflation target. For sufficiently transitory changes in the target, the conventional wisdom holds – to implement an increase in inflation, a central bank ought to reduce the nominal interest rate. However, as changes in the inflation target become more persistent, the model may exhibit Neo-Fisherianism. For an otherwise standard parameterization of the model, we show that a persistent but transitory increase in the inflation target with a half-life of 1.5 or more quarters necessitates a short run increase in the nominal interest rate. For a given amount of persistence in the inflation target, the model is more likely to exhibit Neo-Fisherianism the more flexible are prices and

¹For example, see various different posts on their respective blogs: Cochrane (<http://johnhcochrane.blogspot.com/>) and Williamson (<http://newmonetarism.blogspot.com/>).

the higher is the elasticity of intertemporal substitution.

What is the intuition for the Neo-Fisherianism in the model? Consider first a very transitory change in the inflation target, for which it is appropriate to treat expected inflation as approximately fixed. With fixed expected inflation, an increase in current inflation necessitates an increase in output from the Phillips Curve. Higher output requires a lower real interest rate, which necessitates a reduction in the nominal interest rate when expected inflation is fixed. As the change in the inflation target becomes more persistent, expected inflation rises, the more so the more persistent is the change in the target. From the Phillips Curve, this results in a smaller increase in current output. From the IS curve, this requires a smaller decrease in the real interest rate. With expected inflation increasing, the requisite decrease in the nominal interest rate to support a decrease in the real interest rate is smaller the more persistent is the change in the inflation target, and may it be that the nominal interest rate must increase.

Neo-Fisherianism thus results from the forward-looking nature of the model, and in particular is driven by the jumpiness in inflation expectations. Most empirical research suggests that inflation expectations in the data are not as volatile, nor as influenced by real variables, as the textbook NK model predicts – see, for example, [Kocherlakota \(2016\)](#) and the references therein. We conjecture that the NK model is less likely to exhibit Neo-Fisherianism when ingredients are introduced which make the model less forward-looking. As an illustration, we augment the model to include a fraction of “rule of thumb” price-setters as in [Gali and Gertler \(1999\)](#). For an empirically plausible fraction of rule of thumb price-setters, the model ceases to exhibit Neo-Fisherian properties, regardless of how persistent a change in the inflation target is.

Though much of the discussion surrounding Neo-Fisherianism has taken place in the economics blogosphere, there is a burgeoning academic literature exploring the Neo-Fisherian hypothesis. Ours is not the first paper to explore the Neo-Fisherian hypothesis in the context of the textbook NK model. [García-Schmidt and Woodford \(2015\)](#) and [Cochrane \(2016\)](#) are two recent examples. Both of these papers consider a permanent change in the nominal interest rate and show that the standard NK model predicts that this results in an immediate increase in the inflation rate. [García-Schmidt and Woodford \(2015\)](#) propose a departure from rational expectations which they call “reflective equilibrium,” and show that this different equilibrium concept eliminates the Neo-Fisherian predictions of the basic model. Their proposed “fix” is related to our analysis which suggests that the forward-looking nature of the model drives its Neo-Fisherianism. Unlike them, we propose a simple fix which can eliminate Neo-Fisherianism without departing from rational expectations. [Cochrane \(2016\)](#) provides an extensive overview of the Neo-Fisherian predictions of the textbook NK model, and concludes

that most potential fixes (including adding backward-looking terms to the Phillips Curve) do not break the implication that a permanent increase the nominal interest rate results in an immediate increase in inflation.

The experiments we conduct in our paper differ from both [García-Schmidt and Woodford \(2015\)](#) and [Cochrane \(2016\)](#) in two subtle but important ways. First, whereas both of these papers consider a permanent change in the nominal interest rate, we show that Neo-Fisherian behavior is likely to emerge in the textbook model even after transitory (but sufficiently persistent) nominal shocks. Second, whereas these papers consider the implications of a nominal interest rate peg for inflation, we reverse course and instead examine the implications of a desired change in inflation for the time path of the interest rate. Our approach has the advantage of sidestepping the well-known issues related to equilibrium determinacy of an exogenous interest rate peg.²

2 The Textbook NK Model

We consider the textbook linearized NK model as laid out in [Galí \(2008\)](#). The model is linearized about a zero inflation steady state.³ For ease of exposition, we assume that the flexible price, “natural rate” of output is constant. This means that the output gap and output are the same. The two non-policy equations of the model are:

$$y_t = \mathbb{E}_t y_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \tag{1}$$

$$\pi_t = \lambda y_t + \beta \mathbb{E}_t \pi_{t+1}. \tag{2}$$

In these expressions y_t is the log deviation of output from its steady state, i_t is the deviation of the nominal interest rate from its steady state, and π_t is inflation. The expectation operator conditional on information available at time t is \mathbb{E}_t . Equation (1) is the NK IS equation, derived from a household’s Euler equation. It expresses current spending as a function of expected future spending and the real interest rate, equal to $r_t = i_t - \mathbb{E}_t \pi_{t+1}$. Equation (2) is the NK Phillips Curve. It expresses current inflation as a function of the current output gap (equal to current output with a fixed natural rate) and expected future inflation. The parameter $\sigma \geq 0$ is the elasticity of intertemporal substitution, $0 < \beta < 1$ is a discount factor, and λ is a function of the degree of price rigidity and other deep parameters. In particular,

²For more on potential indeterminacy of interest rate pegs, see the classic reference of [Sargent and Wallace \(1975\)](#). For more recent applications, see [Benhabib et al. \(2001\)](#), [Bullard \(2010\)](#), and [Bullard \(2015\)](#).

³We have also experimented with higher order approximations or by approximating about a non-zero inflation steady state. The results which follow are very similar under both modifications, with the exception that output rises by less after a shock to the target inflation rate, because higher inflation raises price dispersion, which is isomorphic to a reduction in productivity.

$\lambda = (1 - \theta)(1 - \theta\beta)\theta^{-1}(\sigma^{-1} + \eta^{-1})$, where $\theta \in [0, 1)$ is the fraction of firms unable to adjust their price in a given period and η is the Frisch labor supply elasticity.

Monetary policy is characterized by an exogenous inflation target.⁴ The exogenous inflation target follows a stationary AR(1) process. The monetary authority adjusts the nominal interest rate in such a way as to be consistent with inflation equaling its target. In particular:

$$\pi_t = \pi_t^* \tag{3}$$

$$\pi_t^* = \rho_\pi \pi_{t-1}^* + \epsilon_t \tag{4}$$

with $0 < \rho_\pi < 1$ and $\epsilon_t \sim N(0, s^2)$.

Given the specification of monetary policy characterized by (3)–(4), one can solve for an analytic expression for the nominal interest rate as a function of the inflation target:

$$i_t = \left[\frac{(1 - \rho_\pi\beta)}{\sigma\lambda} (\rho_\pi - 1) + \rho_\pi \right] \pi_t^*. \tag{5}$$

The sign of the coefficient multiplying π_t^* in this expression is ambiguous. Given our assumptions, the first term, $\frac{(1 - \rho_\pi\beta)}{\sigma\lambda} (\rho_\pi - 1)$, is negative while the second term, ρ_π , is positive. This coefficient is more likely to be positive (i) the bigger is ρ_π (i.e. the more persistent is the change in the inflation target), (ii) the bigger is λ (i.e. the more flexible are prices, with θ closer to zero), and (iii) the bigger is σ .⁵

Figure 1 plots the coefficient on π_t^* as a function of ρ_π for different values of θ , the parameter governing price stickiness. We assume that $\sigma = \eta = 1$ and $\beta = 0.99$. The coefficient on π_t^* is everywhere increasing in ρ_π – that is, it is more likely that i_t must increase to generate an increase in inflation the more persistent is the desired change in inflation. When prices are close to flexible, as with $\theta = 0.01$, the model exhibits Neo-Fisherianism for any value of ρ_π . As prices get stickier, it is less likely that i_t must increase in order to generate an increase in π_t . For $\theta = 0.7$, a common value used in the literature, the model exhibits Neo-Fisherianism if ρ_π is greater than about 0.6. An autoregressive coefficient of 0.6 implies a half-life of inflation of only about one and a half quarters.⁶ When prices are very rigid, as with $\theta = 0.9$, the model only exhibits Neo-Fisherian behavior for nearly permanent changes in the inflation target.

⁴Our results are not dependent upon assuming a strict inflation targeting regime. As we show in Appendix A, qualitatively the same results emerge when monetary policy is instead characterized by a Taylor rule with persistent policy shocks.

⁵Some care needs to be taken with regards to the effect of σ , as λ is a function of σ . However, $\sigma\lambda = (1 - \theta)(1 - \theta\beta)\theta^{-1}(1 + \sigma\eta^{-1})$, so $\sigma\lambda$ is in fact increasing in σ .

⁶The half-life of an AR(1) process is equal to $\frac{\ln(0.5)}{\ln \rho_\pi}$. Thus, for $\rho_\pi = 0.6$, the half-life is 1.36.

Figure 2 plots impulse responses of the nominal interest rate, output, and inflation to an inflation target shock. The different panels correspond to different values of ρ_π . In generating this figure we assume that $\theta = 0.70$. For low values of ρ_π , the model responses accord with conventional wisdom about the effects of nominal shocks – the nominal interest rate decreases, while output and inflation increase. The output response is smaller the more persistent is the shock. Output and inflation still increase after the inflation target shock for higher values of ρ_π , but the behavior of the nominal interest rate is different. In particular, for $\rho_\pi = 0.7$ or $\rho_\pi = 0.9$, the nominal interest rate increases. In other words, implementing a sufficiently persistent increase in inflation requires increasing, rather than decreasing, the nominal interest rate.

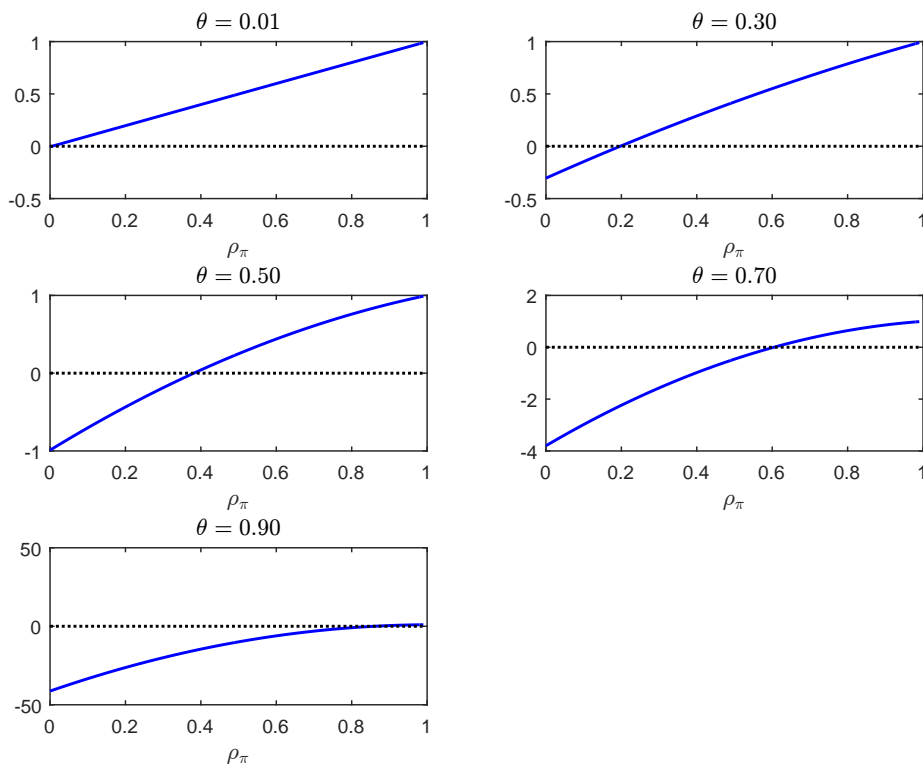


Figure 1: Coefficient of i_t on π_t^* as a Function of ρ_π and θ

Notes: This figure plots the coefficient of i_t on π_t^* (i.e. the term in brackets in (5)) as a function of ρ_π for different values of θ , the parameter governing price stickiness. In generating this figure, we assume that $\sigma = 1$, $\eta = 1$, and $\beta = 0.99$.

What is the intuition for the Neo-Fisherian behavior in the model? This intuition can be best understood by focusing on the IS equation and Phillips curve, (1) and (2). Consider a completely transitory change in the inflation target, with $\rho_\pi = 0$. Given that there are no endogenous state variables in the model, this means that $\mathbb{E}_t \pi_{t+1}$ and $\mathbb{E}_t y_{t+1}$ will be unaffected. From the Phillips Curve, higher inflation necessitates an increase in output. The increase in output is larger the smaller is λ . From the IS equation, an increase in y_t , holding $\mathbb{E}_t \pi_{t+1}$ and

$\mathbb{E}_t y_{t+1}$ fixed, requires a reduction in i_t . As the inflation target shock becomes more persistent, $\mathbb{E}_t \pi_{t+1}$ rises by more and more. From the Phillips Curve, this results in a smaller increase in y_t . For a given $\mathbb{E}_t y_{t+1}$, a smaller increase in y_t , coupled with an increase in $\mathbb{E}_t \pi_{t+1}$, results in a smaller required decrease in i_t for the IS equation to hold. For a sufficiently large increase in $\mathbb{E}_t \pi_{t+1}$, and consequently a smaller increase in y_t , i_t may need to rise for the IS equation to hold. There is also an effect operating through $\mathbb{E}_t y_{t+1}$, though this effect is non-monotonic. One can show that $\mathbb{E}_t y_{t+1} = \rho_\pi(1 - \rho_\pi\beta)\lambda^{-1}\pi_t^*$. For low values of ρ_π , the effect of π_t^* on $\mathbb{E}_t y_{t+1}$ is increasing in ρ_π , which means that an inflation target shock requires a smaller decrease in i_t to be consistent with a given increase in y_t .

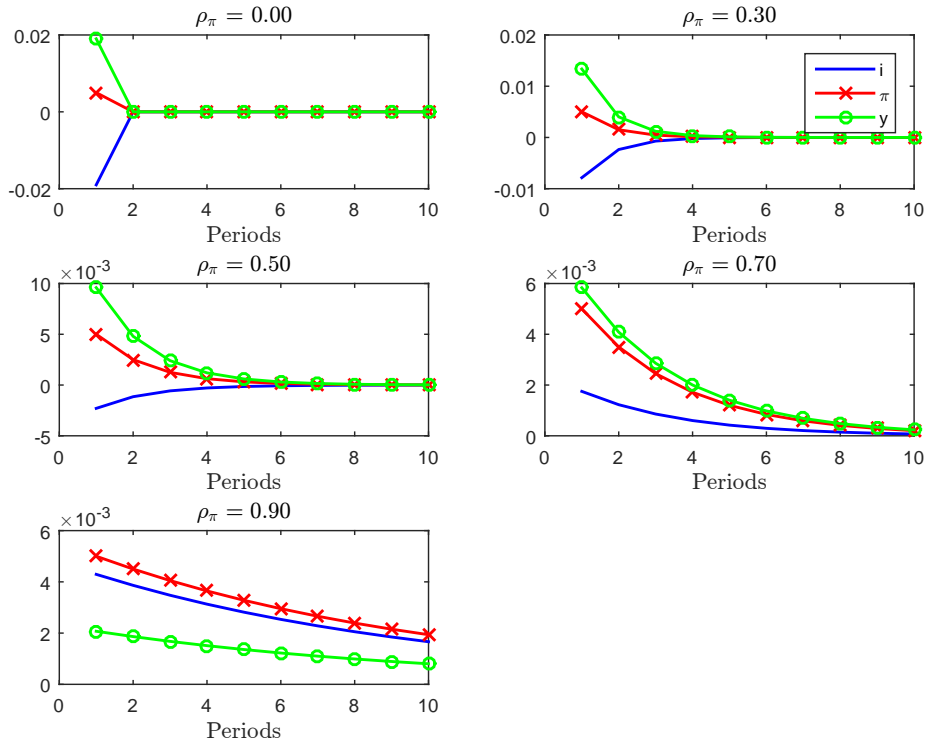


Figure 2: Impulse Responses to Inflation Target Shock

Notes: This figure plots impulse responses of i_t , π_t , and y_t to a shock to the inflation target. Different panels correspond to different values of ρ_π . In generating this figure, we assume that $\sigma = 1$, $\eta = 1$, $\beta = 0.99$, and $\theta = 0.70$.

3 Introducing Backward-Looking Elements into the NK Model

The analysis from the previous section suggests that Neo-Fisherianism is driven by the forward-looking nature of the NK model. If there were no forward-looking terms in the IS equation and Phillips Curves, the intuition from a completely transitory change in the

inflation target would hold regardless of the persistence of the inflation target shock – an increase in inflation would generate an increase in y_t , which would require a reduction in i_t . This suggests that breaking the Neo-Fisherianism of the model requires modifications to the model which make it less forward-looking.

One could envision many such modifications, such as adaptive expectations, habit formation, or sticky information in place of sticky prices, as in [Mankiw and Reis \(2003\)](#). In this section we consider one such modification shown to fit the data well – “rule of thumb” price-setters. We follow [Gali and Gertler \(1999\)](#) in assuming that a fraction of firms, $0 \leq \omega < 1$, use a simple rule of thumb when setting prices, whereas the remaining fraction $1 - \omega$ firms behave according to the standard Calvo model. This modification to the model does not affect the demand side, governed by the IS equation (1). The modified Phillips curve is:

$$\pi_t = \lambda_y y_t + \gamma_f \mathbb{E}_t \pi_{t+1} + \gamma_b \pi_{t-1}. \quad (6)$$

The coefficients λ_y , γ_f , and γ_b relate to the underlying deep parameters of the model via:

$$\phi = \theta + \omega [1 - \theta(1 - \beta)] \quad (7)$$

$$\lambda_y = (1 - \omega)(1 - \theta)(1 - \theta\beta)(\sigma^{-1} + \eta^{-1})\phi^{-1} \quad (8)$$

$$\gamma_f = \beta\theta\phi^{-1} \quad (9)$$

$$\gamma_b = \omega\phi^{-1}. \quad (10)$$

It is straightforward to show that (6) reduces to the standard, purely forward-looking Phillips curve, (2), when $\omega = 0$. As ω gets bigger, coefficient on output gets smaller (in a way isomorphic to having stickier prices), the coefficient on expected future inflation gets smaller, and the coefficient on lagged inflation gets bigger.

We again assume that monetary policy is set according to the exogenous inflation target described in the previous section. One can express the nominal interest rate as a function of the inflation target as:

$$i_t = \left[\frac{(\rho_\pi - 1)(1 - \rho_\pi \gamma_f) - \gamma_b}{\sigma \lambda_y} + \rho_\pi \right] \pi_t^* + \frac{\gamma_b}{\sigma \lambda_y} \pi_{t-1}^*. \quad (11)$$

This expression shares some similarities with (5) and reduces to it in the special case of $\omega = 0$. The nominal interest rate now depends both on the current inflation target as well as the lagged inflation target. Focus first on the coefficient on π_t^* , which shows how i_t must react on impact to a surprise change in the inflation target. The term $\frac{(\rho_\pi - 1)(1 - \rho_\pi \gamma_f) - \gamma_b}{\sigma \lambda_y}$ is negative while ρ_π is positive, meaning that it is ambiguous how the nominal interest rate must react on impact in order to implement an increase in inflation. As in the purely forward-looking

version of the model, it is more likely that the coefficient on π_t^* is positive, and that the model displays Neo-Fisherianism, the bigger is ρ_π . The model is again less likely to be Neo-Fisherian the stickier are prices (so the lower is λ_y) and the bigger is the elasticity of intertemporal substitution, σ .

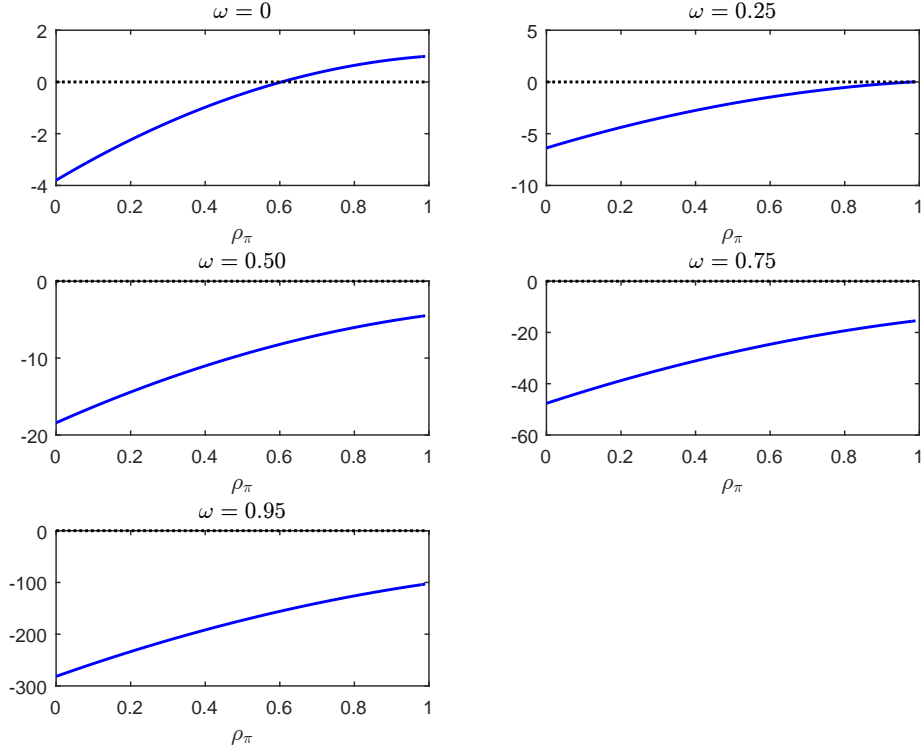


Figure 3: Coefficient of i_t on π_t^* as a Function of ρ_π and ω

Notes: This figure plots the coefficient of i_t on π_t^* (i.e. the term in brackets in (11)) as a function of ρ_π . Different panels correspond to different values of ω . In generating this figure, we assume that $\sigma = 1$, $\eta = 1$, $\beta = 0.99$, and $\theta = 0.70$.

In the hybrid specification of the model with some backward-looking elements, it is less likely for the inflation target to have a positive effect on the nominal interest rate on impact for larger values of ω . The fraction of firms who follow the rule of thumb affects this expression in four different ways. First, a higher value of ω makes γ_f smaller, which makes the first term in brackets more negative. Second, a higher value of ω makes γ_b larger, which also makes it more likely that the coefficient on π_t^* in (11) is negative. Third, a higher value of ω flattens the Phillips curve. A smaller value of λ_y also makes it more likely that i_t must rise on impact so as to implement an increase in the inflation target. Finally, in a dynamic sense, future nominal interest rates will depend on the lagged inflation target via the coefficient $\frac{\gamma_b}{\sigma\lambda_y}$. Larger values of ω make this coefficient bigger. This means that while large backward-looking elements in the Phillips Curve make it less likely that the nominal interest rate must increase

on impact to implement an increase in the inflation target, in future periods the nominal rate may have to increase.

Figure 3 plots the coefficient on π_t^* as a function of ρ_π for different values of ω . In other words, this figure plots the impact response of inflation to an increase in the inflation target. In generating this figure we continue to assume that $\sigma = \eta = 1$, $\beta = 0.99$, and we assume that $\theta = 0.7$. As in Figure 1, the coefficient on π_t^* is everywhere increasing in ρ_π regardless of ω . When $\omega = 0$, the model exhibits Neo-Fisherianism for values of ρ_π greater than roughly 0.6. Interestingly the model does not exhibit Neo-Fisherianism, regardless of how persistent the inflation target shock is, for values of ω greater than about 0.15. Gali and Gertler (1999) estimate values of ω in the range of 0.25 to 0.50. This suggests that the model ceases to exhibit Neo-Fisherian properties, regardless of the persistence of the inflation target, for an empirically plausible fraction of rule of thumb price-setters.

4 Conclusion

A textbook NK model may exhibit Neo-Fisherian behavior, by which we mean that a central bank must increase, rather than decrease, nominal interest rates in the short run in order to implement higher inflation. The standard NK model is more likely to have Neo-Fisherian implications the more persistent is a change in the inflation target and the more flexible are prices.

Neo-Fisherianism in the New Keynesian model is a consequence of the forward-looking nature of the model, wherein inflation expectations are volatile and spending depends not just on current real interest rates but also on expectations of future real rates. The extreme forward-looking behavior in the model is at the heart of other potential “puzzles” in the NK model, including its tendency to produce large and explosive fiscal multipliers under an interest rate peg (see, e.g. Carlstrom et al. (2014)) as well as the so-called “forward guidance puzzle” (Del-Negro et al. (2015)), wherein extended periods of low interest rates can be wildly expansionary. Features which dampen the forward-looking nature of the model have been shown to help resolve some of these puzzles. In the context of our paper, we show that an empirically realistic fraction of “rule of thumb” price-setters may eliminate Neo-Fisherianism from the model altogether. We suspect, but have not verified, that other features – such as habit formation, rule of thumb consumers, adaptive expectations, or sticky information – would have qualitatively similar effects.

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A Analysis Under a Taylor Rule

This appendix shows that our conclusions about whether a central bank must raise the nominal interest rate to raise inflation do not depend upon our assumption that policy is characterized by a strict inflation target. The IS equation and Phillips Curve are the same as in Section 2, given by (1) and (2), respectively. Instead of an exogenous inflation target, we assume that monetary policy is governed by a Taylor rule of the sort:

$$i_t = \phi_\pi \pi_t + \phi_y y_t + u_t. \quad (\text{A.1})$$

Variables here denote deviations from steady state. We assume that $\phi_\pi > 1$ and $\phi_y \geq 0$, which is sufficient to assure equilibrium determinacy. u_t is a persistent but stationary, monetary policy shock, obeying:

$$u_t = \rho u_{t-1} + \epsilon_t, \quad 0 \leq \rho < 1. \quad (\text{A.2})$$

Using the method of undetermined coefficients, we solve for analytic expressions for π_t, y_t , and i_t :

$$\pi_t = \frac{-\lambda\sigma}{\lambda\sigma(\phi_\pi - \rho) + (1 - \beta\rho)(1 + \phi_y - \rho)} u_t. \quad (\text{A.3})$$

$$y_t = \frac{-\sigma(1 - \beta\rho)}{\lambda\sigma(\phi_\pi - \rho) + (1 - \beta\rho)(1 + \sigma\phi_y - \rho)} u_t. \quad (\text{A.4})$$

$$i_t = \frac{-\lambda\sigma\rho + (1 - \beta\rho)(1 - \rho)}{\lambda\sigma(\phi_\pi - \rho) + (1 - \beta\rho)(1 + \sigma\phi_y - \rho)} u_t. \quad (\text{A.5})$$

A positive policy shock decreases both inflation and output on impact. It could result in either an increase or a decrease in the nominal interest rate. This depends on the sign of the numerator in (A.5). If ρ is small, it is likely that the coefficient in (A.5) is positive, so that the conventional wisdom holds – to raise inflation, the central bank must lower the nominal interest rate. For a given value of ρ , it is more likely that the nominal interest rate must decrease to increase the inflation rate the lower is λ (i.e. the stickier are prices) and the lower is σ (i.e. the lower is the elasticity of intertemporal substitution). These conclusions are qualitatively the same as what is presented in Section 2 with policy characterized by a strict inflation target. As ρ increases, it is increasingly likely that i_t must move in the same direction as the desired change in inflation.