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ABSTRACT

A widely held view is that openness to international trade leads to higher GDP volatility, as trade increases specialization and hence exposure to sector-specific shocks. We revisit the common wisdom and argue that when country-wide shocks are important, openness to international trade can lower GDP volatility by reducing exposure to domestic shocks and allowing countries to diversify the sources of demand and supply across countries. Using a quantitative model of trade, we assess the importance of the two mechanisms (sectoral specialization and cross-country diversification) and provide a new answer to the question of whether and how international trade affects economic volatility.

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I Introduction

An important question at the crossroads of macro-development and international economics is whether and how openness to trade affects macroeconomic volatility. A widely held view in academic and policy discussions, which can be traced back at least to Newbery and Stiglitz (1984), is that openness to international trade leads to higher GDP volatility. The origins of this view are rooted in a large class of theories of international trade predicting that openness to trade increases specialization. Because specialization (or lack of diversification) in production tends to increase a country's exposure to shocks specific to the sectors (or range of products) in which the country specializes, it is generally inferred that trade increases volatility. This view seems present in policy circles, where trade openness is often perceived as posing a trade-off between the first and second moments (i.e., trade causes higher productivity at the cost of higher volatility).[1]

This paper revisits the common wisdom on two conceptual grounds. First, the paper points out that the existing wisdom is strongly predicated on the assumption that sector-specific shocks (hitting a particular sector) are the dominant source of GDP volatility. The evidence, however does not support this assumption. Indeed, country-specific shocks (shocks common to all sectors in a given country) are at least as important as sector-specific shocks in shaping countries' volatility patterns (e.g. Koren and Tenreyro, 2007). The first contribution of this paper is to show analytically that when country-specific shocks are an important source of volatility, openness to international trade can lower GDP volatility. In particular, openness reduces a country's exposure to domestic shocks, and allows it to diversify its

[1] See for example the report on "Economic openness and economic prosperity: trade and investment analytical paper" (2011), prepared by the U.K. Department of International Development.

sources of demand and supply, leading to potentially lower overall volatility. This is true as long as the volatility of shocks affecting trading partners are not too big, or the covariance of shocks across countries is not too large. In other words, we show that the sign and size of the effect of openness on volatility depends on the variances and covariances of shocks across countries.

The paper furthermore questions the mechanical assumption that higher sectoral specialization per se leads to higher volatility. Indeed, whether GDP volatility increases or decreases with specialization depends on the intrinsic volatility of the sectors in which the economy specializes in, as well as on the covariance among sectoral shocks and between sectoral and country-wide shocks.

We make these points in the context of a quantitative, multi-sector, stochastic model of trade and GDP determination. The model builds on a variation of Eaton and Kortum (2002), Alvarez and Lucas (2006), and Caliendo and Parro (2012), augmented to allow for country-specific, and sector-specific shocks. In each sector, production combines equipped labour with a variety of tradable inputs. Producers source tradable inputs from the lowest-cost supplier (where supply costs depend on the supplier's productivity as well as trade costs), after productivity shocks have been realized. This generates the potential for trade to "insure" against shocks, as producers can redirect input demand to countries experiencing positive supply shocks. However, (equipped) labor must be allocated to sectors before productivity shocks are realized. This friction allows us to capture the traditional specialization channel, because it reduces a country's ability to respond to sectoral shocks by reallocating resources to other sectors.

We use the model in conjunction with sector-level production and bilateral trade data for a diverse group of countries to quantitatively assess how changes in trading costs since the early 1970s have affected GDP volatility.[2] The quantitative exercise uses as inputs the stochastic properties of country-specific sectoral productivity shocks, which we back out from the model on the basis of sector-level data on gross output, value added, and bilateral trade flows data. To assess the effect of changes in trade barriers since the 1970s we also back out country-and-sector specific paths of trade costs.

We find that the decline in trade costs since the 1970s has caused sizeable reductions in GDP volatility in two-thirds of the countries in our sample, while it led to modest increases in volatility in the other third. The range of changes in volatility varies significantly across countries, with Ireland, the Netherlands, Belgium-Luxembourg, and Norway experiencing the largest reductions in volatility and Greece and Italy experiencing the biggest increases. The general decline in volatility due to trade is the net result of the two different mechanisms discussed above, sectoral specialization, and country-wide diversification. The country-wide diversification mechanism contributed to lower volatility in 90 percent of the countries in our sample, indeed suggesting that there is scope for diversification through trade. Equally interestingly, and against conventional wisdom, higher sectoral specialization does not always lead to higher volatility: Austria, Belgium-Luxembourg, India, the Netherlands, Norway, South Korea, and Sweden, all experienced a decline in volatility due to the trade-induced change in sectoral specialization. For three-quarters of the countries, however, the sectoral-specialization channel contributed to increase volatility. As with the overall net effect of trade on volatility, the relative importance of the two mechanisms we highlight varies across

[2] The data are disaggregated into 24 sectors. We stop the analysis in 2007 as our model abstracts from the factors underlying the financial crisis.

countries, though the effect of the specialization mechanism is on average smaller than the effect of the diversification mechanism.

To summarize, our study challenges the standard view that trade increases volatility. It highlights a new mechanism (country diversification) whereby trade can lower volatility. It also shows that the standard mechanism of sectoral specialization—usually deemed to increase volatility—can in certain circumstances lead to lower volatility. The analysis indicates that diversification of country-specific shocks has generally led to lower volatility during the period we analyze, and has been quantitatively more important than the specialization mechanism.

As the model and quantitative results illustrate, openness to trade does not always causes an unambiguous effect on volatility: the sign and size of the effect varies across countries. This result might partly explain why direct empirical evidence on the effect of openness on volatility has yielded mixed results. Some studies find that trade decreases volatility (e.g., Cavallo, 2008, Strotmann, Döpke, and Buch, 2006, Burgess and Donaldson, 2015, Parinduri, 2011), while others find that trade increases it (e.g., Rodrik, 1998, Easterly, Islam, and Stiglitz, 2000, Kose, Prasad, and Terrones, 2003, and di Giovanni and Levchenko, 2009). The model-based analysis can circumvent the problem of causal identification faced by many empirical studies, allowing for counterfactual exercises that isolate the effect of trade costs on volatility. Moreover, it can cope with highly heterogenous trade effects across countries.

Before proceeding, we should emphasize that we focus the analysis on GDP volatility because for most countries in the world, GDP and consumption fluctuations are almost perfectly correlated. Hence, accounting for GDP volatility goes a long way in accounting for consumption volatility (see Figures 1 and 2).[3] Accordingly, in the modeling section, we abstract from financial trade in assets.[4]

The remainder of the paper is organized as follows. Section II reviews the literature. Section III presents the model and solves analytically for two special cases, autarky and costless free trade. Section IV introduces the data, calibration, and quantitative results. Section V presents concluding remarks. The Appendix contains further derivations and a detailed description of the datasets used in the paper.

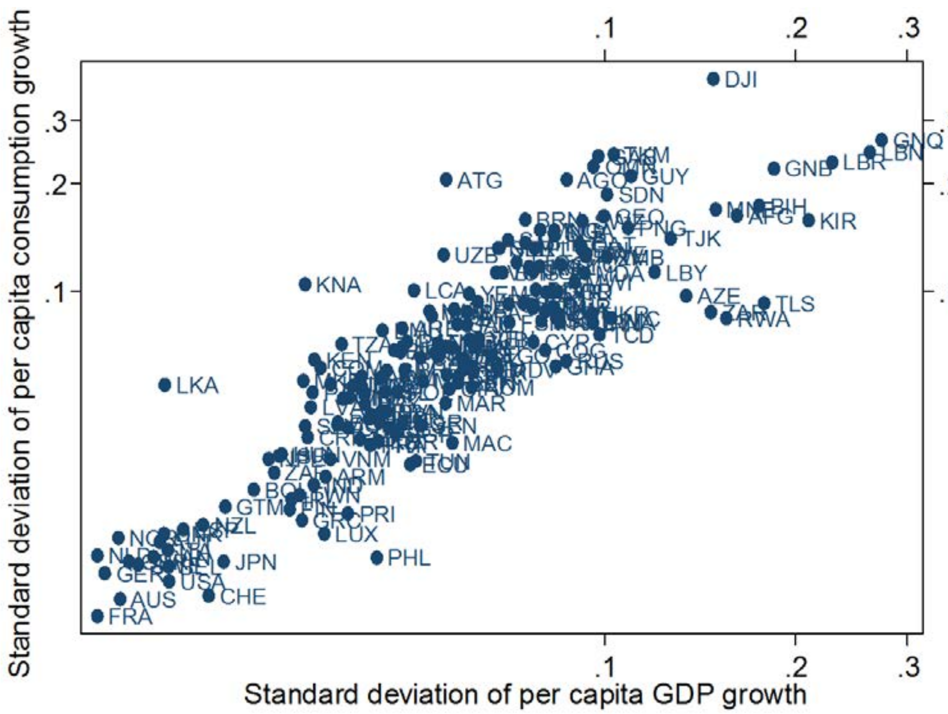

Figure 1: Volatility (standard deviation) of Annual per capita GDP Growth and Annual per capita Consumption Growth. The data come from the World Bank's World Development Indicators 1970–2007.

[3] Figure 1 shows the volatilities of per capita consumption and GDP. Figure 2 shows the volatilities of aggregate consumption and GDP.

[4] There is an obvious analogy between diversification through trade in goods and diversification through trade in financial assets. However, trade in assets stabilizes consumption, not GDP (indeed, trade in assets might increase GDP volatility, as capital would tend to flow to high productivity countries and amplify productivity shocks), whereas in contrast, trade in goods stabilizes GDP—and as a by-product, also consumption. Given the patterns in Figure 1, there appears to be limited asset diversification across countries.

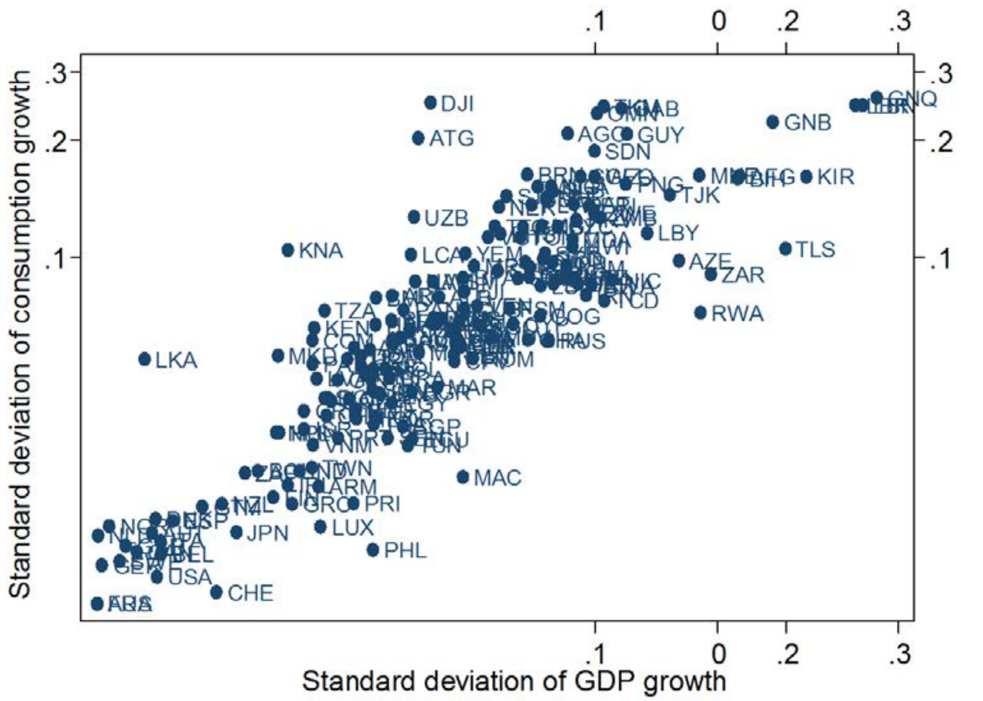

Figure 2: Volatility (standard deviation) of Aggregate Annual GDP Growth and Aggregate Annual Consumption Growth. The data come from the World Bank's World Development Indicators 1970–2007.

II Literature Review

A number of empirical studies have exploited variation across countries to study the effects of trade openness on volatility. Some studies, most notably Rodrik (1998), Easterly, Islam, and Stiglitz (2000), Kose, Prasad, and Terrones (2003) find that trade openness increases volatility, while others, including Haddad, Lim and Saborowski (2010), Cavallo (2008), and Bejan (2006) find that trade openness decreases volatility. Di Giovanni and Levchenko exploit variation across countries and across sectors, concluding that trade openness leads to higher volatility. Strotmann, Döpke, and Buch (2006) exploit variation across firms in

Germany and infer that exposure to international trade increases firm-level and aggregate volatility. While the use of sector- or firm-level data allows researchers to control for a number of country-specific determinants of volatility, omitted-variable biases at lower levels of aggregation, reverse causality, and possibly heterogenous effects of trade openness across countries remain important concerns.[5]

To understand the causal effect of trade openness on volatility, we build on a variation of the theoretical model formulated by Eaton and Kortum (2002), further extended by Alvarez and Lucas (2006) and Caliendo and Parro (2012). The model is amenable to quantitative calibration and has proven useful at replicating trade flows and production patterns across countries. Variations of this model have been used to address a number of questions in international economics—questions related to the effects of trade on the "first moments" of domestic or foreign productivity, but not the trade effects on countries' aggregate volatility. For example, Hsieh and Ossa (2011) study the spillover effects of China's growth on other countries; di Giovanni, Levchenko, and Zhang (2014) study the global welfare impact of China's trade integration and technological change; Levchenko and Zhang (2013) investigate the impact of trade with emerging countries on labour markets; Burstein and Vogel (2012) and Parro (2013) study the effect of international trade on the skill premium; Caliendo, Rossi-Hansberg, Parro, and Sarte (2013) study the impact of regional productivity changes on the U.S. economy, and so on. None of these applications, however, focuses on the impact of openness to trade on volatility. The closest paper to ours, both on question and modelling framework, is Burgess and Donaldson (2012), who use the Eaton-Kortum model in

[5] Trade is by no means the only determinant of volatility. For studies of other determinants of volatility, see Kose, Prasad, and Terrones (2003), Raddatz (2006), Koren and Tenreyro (2007, 2011, 2013), Berrie, Bonomo and Carvalho (2013), and the references therein.

conjunction with data on the expansion of railroads across regions in India to assess whether real income became more or less sensitive to rainfall shocks, as India's regions became more open to trade. The authors find that the decline in transportation costs lowered the impact of productivity shocks on real income, implying a reduction in volatility. Our analysis highlights that, while a reduction in volatility has been experienced by many countries as they became more open to trade, the size and sign of the trade effect on volatility may be— and indeed has been—different across different countries.[6,7]

Our results also relate to Wacziarg and Wallack (2004), who empirically study 25 episodes of trade liberalizations and find a relatively small extent of labour reallocation across sectors. Though the authors do not analyze volatility patterns, their results are consistent with our finding that, on average, the sectoral-specialization channel tends to be of limited quantitative importance; our results, however, point out to significant heterogeneity in the effects, indicating that the sectoral specialization channel played an important role in certain countries, most notably Italy and the Netherlands.

Our paper is also related to the seminal contribution of Backus, Kehoe, and Kydland (1992). The authors show that in a real-business-cycle setting, GDP volatility is higher in the open economy than in the closed economy, as capital inputs are allocated to production in the country with the most favorable technology shock. In other words, GDP fluctuations are amplified in an open economy. (In contrast, consumption volatility decreases in the

[6]Though similar in question and modelling framework, the quantiative approach carried out in our paper is very different from that adopted for India by Burgess and Donaldson (2012).

[7]See also Donaldson (2015), where the question also is addressed in the context of India's railroad expansion. There is also a growing literature on the effect of globalization on income risk and inequality. We do not focus on distributional effects within countries in this paper, though it is obviously a very important issue, and a natural next step in our research. For theoretical developments in that area, see for example, Anderson (2011) and the references therein.

open economy, as financial markets allow countries to smooth the impact of GDP shocks on consumption; this result generates a large cross-country correlation of consumption relative to the cross-country correlation of output, which, as the authors point out, is not borne out by the data.) In our multi-country, multi-sector setting, instead, GDP volatility can—and often does—decrease with openness, as intra-temporal trade in inputs allows countries with less favorable productivity shocks to source inputs from abroad, thus reducing GDP (as well as consumption) volatility.[8] Also related is the empirical literature initiated by Frankel and Rose (1998), who documented a strong correlation between bilateral trade flows and GDP comovements between pairs of countries. Our main focus in this paper is on the *causal* effect of trade on *volatility*—and the channels mediating this effect—but the quantitative approach we follow in our counterfactual exercise can potentially be extended to also identify the *causal* effect of trade on bilateral comovement—and indeed, other higher-order moments. We keep the focus on volatility, which is our main motivation, and speak to the perennial question of how trade might affect it.[9]

Readers of Kehoe and Ruhl (2008) may wonder whether changes in terms of trade caused by (broadly construed) foreign productivity shocks affect *measured* real GDP—and hence real GDP volatility. In the Appendix we explain that this is indeed the case, given the way in which statistical offices construct real GDP measures in practice.

[8] A number of papers have tried to address the comovement "anomaly" pointed out by Backus, Kehoe, and Kydland (1992), that is, the result that cross-country consumption correlations increase vis-à-vis cross output correlations in the open economy; see, for example, Stockman and Tesar (1995). In this paper, we focus on the effect of trade on output volatility and refer readers interested in the comovement puzzle to the complementary literature.

[9] For studies on the effect of bilateral trade on bilateral comovement, see Kose and Yi (2001), Arkolakis and Ramanarayanan (2008), and the references therein.

III A Model of Trade with Stochastic Shocks

The baseline model builds on a multi-sector variation of Eaton and Kortum (2002), Alvarez and Lucas (2006), and Caliendo and Parro (2012), augmented to allow for stochastic shocks, as well as frictions to the allocation of non-produced (and non-traded) inputs across sectors.

A Model Assumptions

The world economy is composed of N countries. At a given point in time t, each country n is endowed with L_{nt} units of a primary (non produced) input, which we interpret as equipped labour. There are J sectors (or broad classes of goods) in the economy, whose output is combined into a final good through a Cobb-Douglas aggregate. In formulas, aggregate gross output in the economy is given by:

$$Q_{nt} = \prod_{j=1}^{J} \left(Q_{nt}^{j}\right)^{\alpha^j} \tag{1}$$

where Q_t^j is the gross output in sector j and $\sum_{j=1}^{J} \alpha^j = 1$. Competitive firms in each sector j produce a composite good according to the following constant-elasticity-of-substitution (CES) technology:

$$Q_{nt}^{j} = \left[\int_0^1 q_{nt}(\omega^j)^{\frac{\eta-1}{\eta}} d\omega^j\right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $q_{nt}(\omega^j)$ is the quantity of good ω^j used by country n in sector j at time t, and $\eta > 0$ is the elasticity of substitution across goods within a given sector. The intermediate goods ω^j can be produced locally or imported from other countries. Delivering a good from country n to country m in sector j and time period t results in $0 < \kappa_{mnt}^{j} \leq 1$ goods arriving at

m; we assume that $\kappa^j_{mnt} \geq \kappa^j_{mkt}\kappa^j_{knt} \quad \forall m, n, k, j, t$ and $\kappa^j_{nnt} = 1$. All costs incurred are net losses.[10] Under the assumption of perfect competition, goods are sourced from the lowest-cost producer, after adjusting for transport costs. The technology for producing $q_{nt}(\omega^j)$ is given accordingly by the country of origin (m) with the lowest cost (with $m = n$ when the good is produced locally):

$$x_{mt}(\omega^j) = A^j_{mt} z_m(\omega^j) l_{mt}(\omega^j)^{\beta^j} M_{mt}(\omega^j)^{1-\beta^j} \tag{3}$$

where $x_{mt}(\omega^j)$ is the production of good ω^j by country m at time t, $M_{mt}(\omega^j)$ is the amount of the aggregate composite good used by country m to produce $x_{mt}(\omega^j)$ units of good ω^j and $l_{mt}(\omega^j)$ is the corresponding amount of equipped labour. Total factor productivity (TFP) levels vary across countries, sectors, and goods. Specifically, each intermediate good ω^j in sector j of country n has a time-invariant idiosyncratic productivity factor $z_n(\omega^j)$ and a time-varying factor A^j_{nt} common to all the goods ω^j in sector j. Building on the literature, we assume the productivities $z_n(\omega^j)$ follow a sector-specific, time-invariant Fréchet distribution $F^j_n(z) = \exp(-T^j_n z^{-\theta})$. A higher T^j_n shifts the distribution of productivities to the right, that is leading to probabilistically higher productivities. A higher θ decreases the dispersion of the productivity distribution, and hence reduces the scope for comparative advantage. Shocks to A^j_{nt} over time are interpreted as standard sectoral total factor productivity (TFP) shocks.

The single final good can be used both as input in the production of intermediaries ω^j

[10] In the calibration, the κs will reflect all trading costs, including tariffs; so implicitly we adopt the extreme assumption that tariff revenues are wasted—or at least not rebated back to agents in a way that would interact with the allocation of resources in the economy.

or for final consumption, C_{nt}. Hence, market clearing in the good markets implies:

$$Q_{nt} = C_{nt} + \sum_{j=1}^{J} \int_0^1 M_{nt}(\omega^j) d\omega^j,$$

where the integral aggregates over the unit-size continuum of goods ω^j entering in the production of each sector's j aggregate good.

Clearing in the input market within a sector implies:

$$L_{nt}^j = \int_0^1 l_{nt}(\omega^j) d\omega^j,$$

where $l_{nt}(\omega^j)$ denotes the amount of equipped labour used in the production of good ω^j by country n. The (equipped) labour allocated to each sector, L_{nt}^j, with $\sum_{k=1}^{J} L_{nt}^k = L_{nt}$, are determined ex ante (before the realization of the shocks). Specifically, we assume there is perfect risk-sharing within a country, but no risk-sharing across countries.[11] At the beginning of each period, a representative consumer decides on the optimal allocation of the primary input L_{nt} into different sectors in order to maximize the expected value of utility; then (stochastic) shocks to productivity A_{nt}^j are realized, equipped labour is reallocated within a sector (but not across sectors), and production and consumption take place. The lack of ex-post reallocation across sectors in a given period aims at capturing the idea that in the short run, it is costly to reallocate productive factors across sectors. Hence, ex post, L_t^j is fixed until $t+1$.[12]

[11] To motivate the lack of risk-sharing across countries, see our discussion of Figures 1 and 2.

[12] In the quantification, a period will be one year. This amounts to assuming that it takes at least one year for resources to be reallocated across sectors.

The representative consumer's budget constraint in each period is:

$$P_{nt}C_{nt} = \sum_{j=1}^{J} w_{nt}^{j} L_{nt}^{j},$$

where P_{nt} is the price of the aggregate good (1), $w_{nt}^{j} L_{nt}^{j}$ is the nominal value-added generated in sector j. Lifetime utility is given by

$$U_n = \sum_{t=0}^{\infty} \delta^{t} u(C_{nt}),$$

where $u' > 0$, $u'' \leq 0$ and δ is the discount factor. Because there is no intertemporal trade and no capital in the economy, each period consumers maximize the expected static utility flow $E\left[u\left(C_{nt}\right)\right]$ and the equilibrium is simply a sequence of static equilibria (in the quantitative section, we allow for trade imbalances). In making his labor allocation decisions the representative consumer takes into account the joint probability distribution function of sectoral productivities, A_{nt}^{j}s.

In the analysis, we assume log utility and therefore the consumer solves:

$$L_{nt}^{j} = \arg\max E_{t-1}\left[\ln\left(\frac{\sum_{j=1}^{J} w_{nt}^{j} L_{nt}^{j}}{P_{nt}}\right)\right], s.t.: \sum\nolimits_{j=1}^{J} L_{nt}^{j} = L_{nt}, \tag{4}$$

where E_{t-1} indicates that the expectation is taken before the realization of period t shocks. This maximization problem leads to the following first-order conditions for the allocation of inputs to sectors:

$$\frac{L_{nt}^{j}}{L_{nt}} = E_{t-1}\left[\frac{w_{nt}^{j} L_{nt}^{j}}{\sum_{k} w_{nt}^{k} L_{nt}^{k}}\right], \qquad \forall j, t. \tag{5}$$

In words, the share of resources allocated to a given sector equals its expected share in value added. To gain intuition on this expression note that $1/\sum_k w_{nt}^k L_{nt}^k$ is the marginal utility of consumption in period t; thus, more resources are allocated to higher value-added sectors, after appropriately weighting by marginal utility. Consider, for further intuition, a (small) sector whose productivity is negatively correlated with the rest of the economy (that is, it has high value added when the rest of the economy has low value added); in states of the world in which overall income is low, the marginal utility of consumption $1/\sum_k w_{nt}^k L_{nt}^k$ will be high and hence the optimal allocation entails allocating more resources to this sector. (Log-linearizing this expression makes the role of second moments on the allocation of resources clearer.) In the closed economy, the value-added share is pinned down by the Cobb–Douglas coefficients $\alpha^j \beta^j$, as with Cobb-Douglas technology there is no variation on expenditures (and sales) shares—and log-utility implies the shares determine the sectoral allocation of resources. (In the open economy this result no longer holds as a country's sectoral shares depend on its absolute and comparative advantage as well as trading costs vis-à-vis other countries.

B Model Solution

We first discuss the solution under autarky, and then turn to the solution under free trade.

Solution under Autarky We solve the model backwards in two stages. First, we solve the model taking the sectoral allocation of nonproduced inputs L_t^j as fixed. We then solve for the ex-ante optimal $L_t^{j\prime} s$ before the shocks are realized. In the analysis of the autarky case, we omit the country-specific subscripts n for convenience.

The demand for each sector's composite good is given by

$$Q_t^j = \alpha_t^j \left(\frac{P_t^j}{P_t} \right)^{-1} Q_t, \tag{6}$$

and the demand for each intermediate good ω^j is

$$q_t(\omega^j) = \left[\frac{p_t(\omega^j)}{P_t^j} \right]^{-\eta} Q_t^j,$$

where

$$P_t^j = \left[\int_0^1 p_t(\omega^j)^{1-\eta} d\omega^j \right]^{\frac{1}{1-\eta}} \tag{7}$$

is the aggregate price index in sector j, and the economy-wide price index is given by:

$$P_t = \prod_{j=1}^{J} \alpha_t^{j^{-\alpha_t^j}} \left(P_t^j \right)^{\alpha_t^j}. \tag{8}$$

The demand for non-produced inputs $l_t(\omega^j)$ and produced inputs $M_t(\omega^j)$ are given, respectively, by $l_t^j(\omega^j) = \beta^j \frac{p_t(\omega^j) q_t(\omega^j)}{w_t^j}$ and $M_t(\omega^j) = (1 - \beta^j) \frac{p_t(\omega^j) q_t(\omega^j)}{P_t}$. Aggregating over all goods ω^j in a given sector, we obtain

$$w_t^j L_t^j = \beta^j P_t^j Q_t^j \tag{9}$$

and, correspondingly, $P_t M_t^j = (1 - \beta^j) P_t^j Q_t^j$. Using the input demand functions and the zero profit condition the autarky prices of intermediate goods are given by:

$$p_t(\omega^j) = B^j \left[A_t^j \cdot z(\omega^j) \right]^{-1} \left(w_t^j \right)^{\beta^j} P_t^{1-\beta^j}, \tag{10}$$

where $B^j = \left(\beta^j\right)^{-\beta^j}(1-\beta^j)^{-(1-\beta^j)}$. Using (10) and the properties of the Fréchet distribution, we can express the sectoral price index as:

$$P_t^j = \xi B^j \left[A_t^j \cdot \left(T^j\right)^{\frac{1}{\theta}}\right]^{-1} \left(w_t^j\right)^{\beta^j} P_t^{1-\beta^j} \tag{11}$$

where $\xi = \left[\Gamma\left(\frac{\theta+1-\eta}{\theta}\right)\right]$, and Γ is the gamma function. Using (6), (9), and (8) we obtain real GDP:

$$Y_t = \sum_{j=1}^{J} \frac{w_t^j L_t^j}{P_t} = \prod_{j=1}^{J} R_j \left[A_t^j\right]^{\frac{\alpha^j}{\bar{\beta}}} \left(L_t^j\right)^{\frac{\alpha^j \beta^j}{\bar{\beta}}}, \tag{12}$$

where $\bar{\beta} = \sum_{j=1}^{J} \alpha^j \beta^j$ and $R_j \propto \prod_{j=1}^{J} \left(\beta^j \alpha^j\right)^{-\frac{\alpha^j \beta^j}{\bar{\beta}}} \left(B^j\right)^{-\frac{\alpha^j}{\bar{\beta}}} \left(T^j\right)^{\frac{\alpha^j}{\bar{\beta}\cdot\theta}}$ is a time-invariant product.

We can now move one step backward and solve for the allocation of the primary input across sectors, L_t^j, $j = 1, ..., J$. From (4) and (5), $L_t^j = \frac{\alpha^j \beta^j}{\bar{\beta}} L_t$.[13] Substituting into (12), GDP is given by:

$$Y_t = \prod_{j=1}^{J} R_j \left(\alpha^j \frac{\beta^j}{\bar{\beta}}\right)^{\frac{\alpha^j \beta^j}{\bar{\beta}}} \left(A_t^j\right)^{\frac{\alpha^j}{\bar{\beta}}} L_t \tag{13}$$

In words, GDP varies with fluctuations in sectoral productivity, A_t^j, and aggregate resources, L_t.

Solution with International Trade The key difference in the internationally open economy is that inputs can potentially be sourced from different countries. Delivering a unit of

[13] The FOC are $\frac{\alpha^j \beta^j}{\bar{\beta}} E[u'(C_t)C_t] = \upsilon L_t^j$ where υ is the multiplier for the resource constraint.

good ω^j produced in country m to country n costs:

$$p^j_{nmt}(\omega^j) = \frac{B^j \left(w^j_{mt}\right)^{\beta^j} P^{1-\beta^j}_{mt}}{A^j_{mt}\kappa^j_{nmt} z_m(\omega^j)}$$

where $B^j \left(w^j_{mt}\right)^{\beta^j} P^{1-\beta^j}_{mt}$ is the cost of the input bundle in country of origin m, sector j, at time t. Because of perfect competition, the price paid in country n, denoted $p_{nt}(\omega^j)$, will be the minimum price across all N potential trading partners: $p^j_{nt}(\omega^j) = \min\left\{p^j_{nmt}(\omega^j);\ m = 1, ..., N\right\}$. Producers of the aggregate good in (1) minimize production costs taking prices as given. We assume the distribution of efficiencies for any good ω^j in sector j and country n are independent across countries and sectors and follow a time-invariant Fréchet distribution: $F^j_n(z) = \exp(-T^j_n z^{-\theta})$. Under this assumption, the distribution of prices in sector j of country n, conditional on $\left\{A^j_{mt}\right\}_{m=1,...N}$ is given by $G^j_{nt}(p)|_{\left\{A^j_t\right\}} = \Pr(P^j_{nt} < p) = 1 - \exp\left[-\Phi^j_{nt}p^\theta\right]$ where $\Phi^j_{nt} = \sum_{m=1}^{N} T^j_m \left(\frac{B^j\left(w^j_{mt}\right)^{\beta^j} P^{1-\beta^j}_{mt}}{A^j_{mt}\kappa^j_{nmt}}\right)^{-\theta}$. Given that there is a continuum of ω^j in each sector, by the law of large numbers the probability that country m provides a good in sector j at the lowest price in country n equals the fraction of goods that country n buys from country m in sector j:

$$d^j_{nmt} = \frac{T^j_m \left(\frac{B^j\left(w^j_{mt}\right)^{\beta^j} P^{1-\beta^j}_{mt}}{A^j_{mt}\kappa^j_{nmt}}\right)^{-\theta}}{\Phi^j_{nt}} \tag{14}$$

that is, d^j_{nmt} is the fraction of country n's total spending on sector-j goods from country m at time t. The equilibrium in the open economy can be defined as following.

Equilibrium Definition. An equilibrium in the open economy is defined as a set of resource allocations $\left\{L^j_{nt}\right\}$, import shares $\left\{d^j_{nit}\right\}$, prices $\{P_{nt}\}$, $\left\{P^j_{nt}\right\}$, and $\{w^j_n\}$ such that, given technology $\left\{A^j_{it}\right\}, \left\{T^j_{it}\right\}$, aggregate endowments $\{L_{nt}\}$ and trading costs $\left\{\kappa^j_{int}\right\}$

i) consumers maximize expected utility, ii) firms minimize costs and, iii) markets for goods and inputs clear, and iv) trade is balanced. In equilibrium, prices and quantities satisfy (15)-(21):

$$P_{nt} = \prod_{j}^{J} \left(\frac{1}{\alpha_n^j}\right)^{\alpha^j} \left(P_{nt}^j\right)^{\alpha^j} \tag{15}$$

$$P_{nt}^j = \xi \Phi_{nt}^{j^{-1/\theta}} \tag{16}$$

$$\Phi_{nt}^j = \left(B^j\right)^{-\theta} \sum_{i=1}^{N} T_i^j \left(A_{it}^j\right)^{\theta} \left[\frac{P_{it}^{1-\beta^j} \left(w_{it}^j\right)^{\beta^j}}{\kappa_{nit}^j}\right]^{-\theta} \tag{17}$$

$$d_{nmt}^j = \frac{\left(B^j\right)^{-\theta} T_m^j \left(A_{mt}^j\right)^{\theta} \left(\frac{P_{mt}^{1-\beta^j} w_{mt}^{j\beta^j}}{\kappa_{nmt}^j}\right)^{-\theta}}{\Phi_{nt}^j}; \sum_{m=1}^{N} d_{nmt}^j = 1 \tag{18}$$

$$w_{nt}^j L_{nt}^j = \beta^j \sum_{m=1}^{N} d_{mnt}^j \left[\alpha^j + \frac{1-\beta^j}{\beta^j} \cdot \frac{w_{mt}^j L_{mt}^j}{w_{mt} L_{mt}}\right] w_{mt} L_{mt} \tag{19}$$

$$w_{nt} L_{nt} = \sum_{j=1}^{J} w_{nt}^j L_{nt}^j \tag{20}$$

$$\frac{L_{nt}^j}{L_{nt}} = E_{t-1}\left[\frac{w_{nt}^j L_{nt}^j}{\sum_{k=1}^{J} w_{nt}^k L_{nt}^k}\right] \tag{21}$$

Equations (15)–(17) show the equilibrium prices as a function of technology and input costs resulting from firms' cost minimization and consumers' maximization problems. The first equation in (18) shows the value of goods from sector j bought by country n from country m as a share of total spending on goods j by country n. The second equation says that the sum of spending shares on goods j from all countries m by country n (including n itself) add to 1, that is, imports plus domestic expenditures on goods j by country n, add up to the overall spending value on goods j by country n. Equation (19) gives the value of total sales accruing to the primitive factor in sector j of country n; it already incorporates the balanced trade condition, i.e., total payments for goods flowing out of country m to the rest of the world equal payments flowing in country m from the rest of the world.[14] Equation (18) expresses total value added in the economy as the sum of sectoral value added. (Real value added is given by $Y_{nt} = \frac{w_{nt}L_{nt}}{P_{nt}}$.) Finally, (21) expresses the resource shares as a function of expected shares, following the first order conditions in (5).

The model can conceptually be solved backwards in two steps. First, for any given set of values for L_{nt}^j, the first five sets of equations can be solved for P_{nt}, w_{nt}^j, P_{nt}^j, d_{nmt}^j as a function of the $\kappa_{mnt}^j s$ and the augmented productivity factors defined as:

$$Z_{nt}^j \equiv T_n^j \left[L_{nt} \left(A_{nt}^j \right)^{1/\beta^j} \right]^{\beta^j \theta}. \tag{22}$$

Then in a first stage, we can solve for the shares $\frac{L_{nt}^j}{L_{nt}}$. As seen, with log utility the solution for $\frac{L_n^j}{L_n}$ simplifies significantly as it is the expected value of sectoral value-added shares; in the implementation, we will use the data to help pin down these expectations.

C Two Illustrative Cases: Autarky and Costless Trade

To illustrate the mechanism of diversification through trade, we analyze a one-sector version of the model (that is, the Eaton-Kortum model) under two extreme cases for which we have closed-form analytical solutions for GDP: autarky and costless trade. We accordingly drop

[14] In formulas, $\sum_{j=1}^J X_{mt}^j = \sum_{n=1}^N \sum_{j=1}^J d_{nmt}^j X_{nt}^j$, where X_{nt}^j is total expenditure by country n on sector-j goods. The right-hand side is the total demand by all N countries for goods produced in country m. The left-hand side is the total expenditures by country m, which, under trade balance also equals its total sales. Recall that $P_m^j Q_m^j$ is the total purchases of goods from sector j by country m. Note $P_{mt}^j Q_{mt}^j$, the total purchases of goods from sector j by country m. Hence: $P_{mt}^j Q_{mt}^j = \alpha^j w_{mt} L_{mt} + \frac{1-\beta^j}{\beta^j} w_{mt}^j L_{mt}^j$.

the sector subscripts.

C.1 Volatility under Autarky

Under complete autarky, value added in the one-sector economy is given by (13), which can be rewritten as:

$$Y_{nt} \propto (Z_{nt})^{\frac{1}{\beta\theta}}$$

where $Z_{nt} \equiv T_n \left(L_{nt} A_{nt}^{1/\beta}\right)^{\beta\theta}$. Taking log-differences around the mean (or trend value in the empirics), we obtain,

$$\hat{Y}_{nt} = \frac{1}{\beta\theta}\hat{Z}_{nt}.$$

Thus, in the one-sector economy under autarky, shocks to value added are driven exclusively by domestic shocks to the productive capacity of the economy, $\hat{Z}_{nt}$. The variance of GDP, $V(\hat{Y}_{nt})$ thus depends on the variance of the shocks $V(\hat{Z}_{nt})$:

$$V(\hat{Y}_{nt}) = \frac{1}{(\beta\theta)^2} V(\hat{Z}_{nt}).$$

C.2 Volatility under Costless Trade

Under costless trade in the one-sector economy ($\kappa_{nmt} = 1$), GDP per capita simplifies to:[15]

$$Y_{nt} = (\xi B)^{1/\beta} Z_{nt}^{\frac{1}{1+\beta\theta}} \left(\sum_{m=1}^{N} Z_{mt}^{\frac{1}{1+\beta\theta}}\right)^{\frac{1}{\beta\theta}}$$

[15]See derivations in the Appendix.

and hence GDP fluctuations are given by:

$$\hat{Y}_{nt} = \frac{1}{1+\beta\theta}\left[\hat{Z}_n + \frac{1}{\beta\theta}\sum_{m=1}^{N}\gamma_m \hat{Z}_m\right]$$

where $\gamma_m = \frac{\bar{Z}_m^{\frac{1}{1+\beta\theta}}}{\sum_{i=1}^{N}\bar{Z}_i^{\frac{1}{1+\beta\theta}}}$ is the relative size of country j evaluated at the mean of $Z_j s$. Rearranging, we obtain:

$$\hat{Y}_{nt} = \frac{1}{\beta\theta}\left[\frac{\gamma_n + \beta\theta}{1+\beta\theta}\hat{Z}_n + \frac{1}{1+\beta\theta}\sum_{m\neq n}^{N}\gamma_m \hat{Z}_m\right] \tag{23}$$

Volatility under free trade is hence given by:

$$Var(\hat{Y}_{nt}) = \left(\frac{1}{\beta\theta}\right)^2 \left\{ \begin{array}{c} \left(\frac{\gamma_n+\beta\theta}{1+\beta\theta}\right)^2 Var(\hat{Z}_{nt}) + \left[\frac{1}{1+\beta\theta}\right]^2 \sum_{m\neq i}\gamma_m^2 Var(\hat{Z}_{mt}) \\ 2\frac{\gamma_n+\beta\theta}{1+\beta\theta}\frac{1}{1+\beta\theta}\sum_{m\neq n}\gamma_m Cov(\hat{Z}_{m,}\hat{Z}_n) \end{array} \right\} \tag{24}$$

Compared to the variance in autarky, $V(\hat{Y}_{nt}) = \frac{1}{(\beta\theta)^2}V(\hat{Z}_{nt})$, it is clear that the volatility due to domestic productivity fluctuations, $Var(\hat{Z}_{nt})$, now receives a smaller loading, as $\left(\frac{\gamma_n+\beta\theta}{1+\beta\theta}\right)^2 < 1$ since $\gamma_n < 1$. The smaller the country (as gauged by its share γ_n), the smaller the impact of domestic volatility of shocks, $\hat{Z}_n$, on its GDP, when compared to autarky. Openness to trade, however, exposes the economy to other countries' productivity shocks, which will also contribute to the country's overall volatility. Whether or not the gain in diversification (given by lower exposure to domestic productivity) is bigger than the increased exposure to new shocks depends on the variance-covariance matrix of shocks across countries. If all countries have the same constant variance $Var(\hat{Z}_{nt}) = \sigma$, and the $\hat{Z}_{nt}$ are uncorrelated, volatility under free trade becomes:

$$Var(\hat{Y}_{nt}) = \left(\frac{1}{\beta\theta}\right)^2 \left\{ \left(\frac{\gamma_n + \beta\theta}{1+\beta\theta}\right)^2 + \left[\frac{1}{1+\beta\theta}\right]^2 \sum_{m\neq i} \gamma_m^2 \right\} \sigma \tag{25}$$

which is unambiguously lower than the volatility in autarky given that[16]

$$\left(\frac{\gamma_n + \beta\theta}{1+\beta\theta}\right)^2 + \left[\frac{1}{1+\beta\theta}\right]^2 \sum_{m\neq i} \gamma_m^2 < 1 \tag{26}$$

(recall $\gamma_m < 1$ and $\sum_{m=1}^{N} \gamma_m^2 \leq 1$). Of course, if other countries have higher variances or the covariance terms are important, then the weights countries receive matter and the resulting change in volatility cannot be unambiguously signed.

IV Mapping the Model into Observables

In this section, we connect the model to the data and use it to quantitatively assess the effect of historical changes in trade barriers on GDP volatility for a diverse sample of 24 core countries and an aggregate of the remaining countries to which we refer as "rest of the world" (ROW).

The equilibrium of the model is characterized by equations (15)-(21). We solve the model numerically, for which we need to calibrate the values of the exogenous trading costs κ_{nmt}^j, the productivity process Z_{nt}^j, and the parameters α^j, β^j, θ, and η. We consider 24 sectors

[16]since $(\beta\theta)^2 + 2\beta\theta\gamma_n + \sum_{j=1} \gamma_j^2 < (1+\beta\theta)^2$ as

$$2\beta\theta\gamma_n + \sum_{j=1} \gamma_j^2 < 2\beta\theta + 1$$

in the analysis (agriculture, 22 manufacturing sectors, and services). Throughout the study, services are treated as a nontradable sector (that is, $\kappa^j_{nmt} = 0$ for all $n \neq m$ and $\kappa^j_{nmt} = 1$ for $n = m$), whereas agriculture and all manufacturing sectors are treated as tradables, with potentially different trading costs.

We set α^j so as to match the average share of each sector on total final uses in the OECD Input-Output tables across all countries. The betas for each sector are calculated as the ratio of value added to total output. A detailed description of the data and the calculations are available in the Appendix.

We allow for a relatively broad parametric range for θ, from $\theta = 2$ to $\theta = 8$, consistent with the estimates in the literature (see Eaton and Kortum, 2003, Donaldson 2015, and Simonovska and Waugh, 2011). We use $\theta = 4$ as the baseline case, and report the results for other values when discussing the sensitivity of our results. We calibrate the elasticity of substitution across varieties $\eta = 2$, consistent with Broda and Weinstein (2006). The results are not sensitive to this parametric choice.

We explain next how we obtain the processes for κ^j_{nmt} and Z^j_{it} using data on sectoral bilateral trade flows, value added, output, and prices. Before we specify the details, a quick intuition on how these series are backed-out from the model is as follows. We recover trade costs κ^j_{nmt} using information on bilateral trade shares and gross output at the sectoral level. Intuitively, if two countries trade little with one another in a given sector (relative to the sectoral gross output of these countries), this will signal high trade costs between the countries in that sector. Second, we recover productivities relative to a benchmark country using the market share of each exporter. If a country has a high export share in a sector, that is a sign of revealed comparative advantage, meaning a high relative productivity in the sector relative to the benchmark country. To calibrate the absolute level of productivities, we use price data for a benchmark country. We explain the procedure in detail and with formulas in the next section.

A Implementation

Kappas In order to perform counterfactual experiments we need to back out the historical realizations of the exogenous processes. Following the idea in Head and Ries (2011), we assume that sectoral bilateral trading costs are symmetric, that is: $\kappa^j_{nmt} = \kappa^j_{mnt}$, and hence bilateral trade costs at the sectoral level can be backed out from the data. Indeed, inverting the structural model, we obtain:

$$\frac{d^j_{nmt} d^j_{mnt}}{d^j_{mmt} d^j_{nnt}} = \left(\kappa^j_{nmt}\right)^{2\theta}. \tag{27}$$

The left hand side objects can be measured using data on bilateral imports and gross output at the sectoral level. Specifically, d^j_{nmt} is the value of exports from m to n in sector j at t relative to total spending by n on sector j at time t, where total spending is measured as gross output plus imports minus exports by that sector and country at time t. The share d^j_{mmt} is obtained as a residual from the accounting restriction:

$$d^j_{mmt} = 1 - \sum_{n \neq m}^{N} d^j_{mnt}$$

Hence, for a given value of θ, we can obtain the time series of trading costs by sector and country-pairs $\left\{\kappa^j_{nmt}\right\}$.

Productivity in Tradable Sectors To back out the productivities, we proceed as follows. First, using the formula for d^j_{nm} in equation (18), after some algebra, we obtain:

$$d^j_{nm} = \frac{\left(B^j\right)^{-\theta} \left(\psi^j_m\right)^{\beta^j \theta} Z^j_m \left(\kappa^j_{nm}\right)^{\theta} \left(y^j_m\right)^{-\theta\beta^j}}{P^{\theta}_m \Phi^j_n}, \tag{28}$$

where $\psi^j_m \equiv \frac{L^j_m}{L_m}$ and $y^j_m \equiv \frac{L^j_m w^j_m}{P_m}$. We can exploit this to recover Z^j_m. In particular, inverting (28) we have:

$$Z^j_{mt} = {B^j}^{\theta} \xi^{\theta} d^j_{nmt} \left(y^j_m\right)^{\theta\beta^j} \left(\kappa^j_{nmt}\right)^{-\theta} \left(\frac{P^j_{nt}}{P_{mt}}\right)^{-\theta} \left(\psi^j_{mt}\right)^{-\theta\beta^j} \tag{29}$$

To approximate terms on the right hand side we use data on sectoral import shares d^j_{nmt}, sectoral value added y^j_m, sectoral shares ψ^j_{mt}, and aggregate prices P_{nt} along with the calibrated parameters. (See the Appendix for more details.) The only terms we cannot back out directly from data are sectoral prices. We thus use the model in conjunction with the data to infer them. Note first that equation (29) holds for all (n, k) pairs of countries and all sectors j (except for services). The procedure becomes clear when we collect known and unknown terms as follows:

$$\begin{aligned} Z^j_{nt} &= \underbrace{\xi^{\theta} {B^j}^{\theta} d^j_{k,n,t} \left(\kappa^j_{k,n,t}\right)^{-\theta} \left(w^j_{n,t} L^j_{n,t}\right)^{\theta\beta^j} \left(\psi^j_{n,t}\right)^{-\theta\beta^j} {P_{n,t}}^{\theta(1-\beta^j)}}_{\equiv \exp(\zeta^j_{k,n,t})} {P^j_{k,t}}^{-\theta} \\ &= \exp(\zeta^j_{k,n,t}) {P^j_{k,t}}^{-\theta} \end{aligned} \tag{30}$$

Note in particular that this relationship holds for any choice of country k. Note also that the factor $\exp(\zeta^j_{k,n,t})$ can be constructed from observable data. We decompose $\exp(\zeta^j_{k,n,t}) = Z^j_{nt} \left(P^j_{k,t}\right)^{\theta}$ according to the following procedure:

1. Take logs and rename terms for brevity.

$$\zeta^j_{k,n,t} = \ln Z^j_{nt} + \theta \ln P^j_{k,t} \tag{31}$$

$$\equiv \chi^j_{nt} + \tau^j_{k,t} \tag{32}$$

where $\chi^j_{nt} \equiv \ln Z^j_{nt}$ and $\tau^j_{k,t} \equiv \theta \ln P^j_{k,t}$.

2. To proceed we need a benchmark country, so we use sectoral prices in the US.

$$\tau^j_{US,t} \equiv \theta \ln P^j_{US,t}$$

We choose units of accounts for each sector so that U.S. nominal sectoral prices are equal to 1 in 1972.

3. Obtain $\tau^j_{k,t}$ for all other countries as:

$$\tau^j_{k,t} = \frac{1}{N} \sum_{n=1}^{N} \left(\zeta^j_{k,n,t} - \zeta^j_{US,n,t} \right) + \tau^j_{US,t} \tag{33}$$

(Note that this equation holds with and without the averaging operator, $\frac{1}{N}\sum_{n=1}^{N}$, as $\tau^j_{k,t}$ and $\tau^j_{US,t}$ do not depend on the exporter n.[17])

4. Back-out χ^j_{nt} for all other countries:

$$\chi^j_{nt} = \frac{1}{N} \sum_{k=1}^{N} \left(\zeta^j_{k,n,t} - \tau^j_{k,t} \right) \tag{34}$$

[17] We use the average in the quantitative analysis to minimize measurement error.

5. Recover shocks and prices:

$$Z_{n,t}^{j} = \exp\left(\chi_{nt}^{j}\right) \tag{35}$$

$$P_{k,t}^{j} = \exp\left(\frac{\tau_{kt}^{j}}{\theta}\right) \tag{36}$$

At the end of the procedure we end up with augmented productivity factors $Z_{n,t}^{j}$ and sectoral prices for agriculture and all manufacturing sectors $P_{k,t}^{j}$.

Productivity in Nontradables To compute the productivities in the services sector for each country, we use equilibrium equations (15), (16) and (28).

1. As we already have sectoral prices of tradables we can use (15) to recover the price of services as follows:

$$P_{n,t}^{s} = \left(\frac{P_{n,t}}{P_{US,t}} P_{US,t}\right)^{\frac{1}{\alpha^{s}}} \left(\prod_{j=1}^{J} \alpha^{j-\alpha^{j}}\right)^{-\frac{1}{\alpha^{s}}} \left[\prod_{j\neq s} \left(P_{n,t}^{j}\right)^{\alpha^{j}}\right]^{-\frac{1}{\alpha^{s}}} \tag{37}$$

   Note that we observe data on the price of country n relative to the price in the United States, $\frac{P_{n,t}}{P_{US,t}}$, from the Penn World Tables.

2. Now we recover $Z_{n,t}^{s}$ using (16), (28), and $n = m$.

$$Z_{n,t}^{s} = \xi^{\theta} B^{s\theta} \left(\frac{w_{n,t}^{s} L_{n,t}^{s}}{\psi_{n,t}^{s}}\right)^{\theta\beta^{s}} \left(\frac{P_{n,t}}{P_{US,t}} P_{US,t}\right)^{\theta(1-\beta^{s})} P_{n,t}^{s\ -\theta} \tag{38}$$

Sectoral versus Aggregate Shocks Note that the changes in productivity retrieved above,

$$\frac{1}{\beta^j \theta}\hat{Z}^j_m \equiv \frac{1}{\beta^j}\hat{A}^j_{mt} + \hat{L}_{mt}, \tag{39}$$

can be decompose into two factors: a sectoral factor, $\frac{1}{\beta^j}\hat{A}^j_{mt}$, and an aggregate factor $\hat{L}_{mt}$. The interpretation of L_{mt} as "equipped labour" means that it embeds a productivity component too. Given the functional form, the split between pure productivity and resources in L_{mt} is not relevant from the point of view of aggregate volatility. (A shock to L_{mt} will be equivalent to an aggregate shock to A^j_{mt}s that leaves the relative productivities $A^j_{mt}/A^{j'}_{mt}$ unchanged $\forall j, j'$.) For identification, we impose the restriction that

$$\sum \frac{\alpha^j}{\beta^j}\hat{A}^j_{mt} = 0. \tag{40}$$

Thus, changes in the sectoral productivity will correspond to changes in the relative value of A^j_{mt}, while changes in aggregate productivity (affecting all sectors equally), as well as changes in overall resources, will be subsumed in L_{mt}. We hence call sectoral shocks, those affecting $\hat{A}^j_{mt}$ and aggregate shocks those affecting the aggregate factor $\hat{L}_{mt}$. The identification restriction implies that any primitive aggregate shock affecting all sectors will be collected in $\hat{L}_{mt}$.

Summary of the Procedure We can summarize the procedure as follows.

1. Obtain the inverse of trade costs, κs, from (27).

2. Compute ψ^j_{mt} as the sectoral value-added share at time t.

3. Retrieve the panel of sectoral and country productivities$\left\{Z^j_{mt}\right\}$ from the procedure described above.[18]

4. Retrieve L_{mt} from (39) using (40) and compute $L^j_{mt} = \psi^j_{mt} L_{mt}$.

5. Solve the equilibrium values of $\left\{d^j_{nit}\right\}$, $\{P_{nt}\}$, $\left\{P^j_{nt}\right\}$, and $\{w^j_n\}$ using equations (15) through (20).

A.1 Counterfactual Equilibria

We discuss next how we compute the equilibrium in the counterfactual exercise and how we identify the two theoretical mechanisms.

Numerical Counterfactual Equilibria For each new value of (inverse) trading cost κ, and the estimated sequence of sectoral productivities $\left\{Z^j_{mt}\right\}$, we need to solve for the sequence of equipped labour allocated to each sector $\left\{L^j_{mt}\right\}$. The rational-expectations equilibrium is a fixed point of a below mapping on the space of all possible sequences $\left\{L^j_{mt}\right\}$. We proceed as follows.

1. We start from the initial value $(L^j_{nt})^0 = \alpha^j L_{nt}$.

2. In iteration i for the actual $(L^j_{nt})^i$ we get sectoral and aggregate (equipped labour) wages, $(w^j_{nt})^i$ and $(w_{nt})^i$, from the equilibrium equations.

3. We calculate the implied total value added and the sectoral value added shares as

$$\left(\frac{w^j_{nt} L^j_{nt}}{w_{nt} L_{nt}}\right)^i = \frac{(w^j_{nt})^i (L^j_{nt})^i}{(w_{nt})^i L_{nt}}.$$

[18] Units of accounts are chosen so that nominal sectoral prices in the US in 1972 equal 1.

4. (a) Decompose all $N \cdot J$ value-added-share series into trend and cycle components using an annual band-pass filter.

$$\log \left(\frac{w_{nt}^j L_{nt}^j}{w_{nt} L_{nt}} \right)^i = trend_{nt}^j + cycle_{nt}^j.$$

   (b) Normalize the trend values so that in each period and each country the trend values add up to 1:

$$\widehat{\left(\frac{w_{nt}^j L_{nt}^j}{w_{nt} L_{nt}} \right)}^i = \frac{\exp(trend_{nt}^j)}{\sum_k \exp(trend_{nt}^k)}$$

   (c) Replace the expectation with the adjusted trend value.

$$E_{t-1} \left(\frac{w_{nt}^j L_{nt}^j}{w_{nt} L_{nt}} \right) = \widehat{\left(\frac{w_{nt}^j L_{nt}^j}{w_{nt} L_{nt}} \right)}^i$$

5. Update the resource allocations

$$(L_{nt}^j)^{i+1} = L_{nt} \widehat{\left(\frac{w_{nt}^j L_{nt}^j}{w_{nt} L_{nt}} \right)}^i$$

6. Repeat the procedure until convergence.

Productivities in Counterfactual Scenario We are interested in decomposing the trade effect on volatility on the contributions of the two mechanisms, specialization and diversification. To achieve that, we need to identify the sources of shocks to productivity. We resort to a factor model that decomposes productivity shocks into sector- and country-specific components in a way described in Koren and Tenreyro (2007). To separate per period shocks from trends we use a band pass filter to detrend each $\left\{\log Z_{n,t}^{j}\right\}_{t=1}^{T}$ series. Then we calculate the time average of the shocks for each (n, j) pair and subtract it from the growth rate to get the object to be decomposed, $\tilde{Z}_{nt}^{j}$.

$$\tilde{Z}_{nt}^{j} = \hat{Z}_{n,t}^{j} - (T-1)^{-1} \sum_{t=2}^{T} \hat{Z}_{n,t}^{j}$$

Without loss of generality, we decompose $\tilde{Z}_{nt}^{j}$ as:

$$\tilde{Z}_{nt}^{j} = \lambda_{t}^{j} + \mu_{nt} + \epsilon_{nt}^{j},$$

where $\mu_{n,t}$ is the country-specific factor, affecting all sectors within the country; λ_{t}^{j} is the global sectoral factor, affecting sector j in all countries; and the residual $\epsilon_{n,t}^{j}$ is the idiosyncratic component, specific to the country and sector. The three factors, λ, μ, and ϵ are estimated as:

$$\begin{aligned}
\hat{\lambda}_{t}^{j} &= N^{-1} \sum_{n=1}^{N} \tilde{Z}_{nt}^{j} \\
\hat{\mu}_{nt} &= J^{-1} \sum_{j=1}^{J} \left(\tilde{Z}_{nt}^{j} - \hat{\lambda}_{t}^{j}\right) \\
\hat{\epsilon}_{nt}^{j} &= \tilde{Z}_{nt}^{j} - \hat{\lambda}_{t}^{j} - \hat{\mu}_{nt},
\end{aligned}$$

with the restriction $\sum_{n} \mu_{n} = 0$ implying that the country-specific effect is expressed relative to the world's aggregate. In the counterfactual exercises, we can mute the sector- or country-specific factors by setting the corresponding components equal to 0, in order to identify the separate effects of the two trade channels affecting volatility.

V Quantifying the Effect of Trade on Volatility

This section uses the framework developed above to quantitatively assess how historical changes in trade costs from the early 1970s have affected volatility patterns in a sample of countries at different levels of development.

Trade barriers have declined significantly since the early 1970s. This decline in barriers (or increase in κ) is illustrated in Figures 3 and 4, which show, correspondingly, the histograms of bilateral κs in manufacturing and agriculture in the first and last year of our sample. As the figures show, the distribution of κ has moved to the right, indicating a decline in trading costs in both sectors.

Figure 3: Histogram of bilateral κ in Manufacturing sectors. Years 1972 and 2007

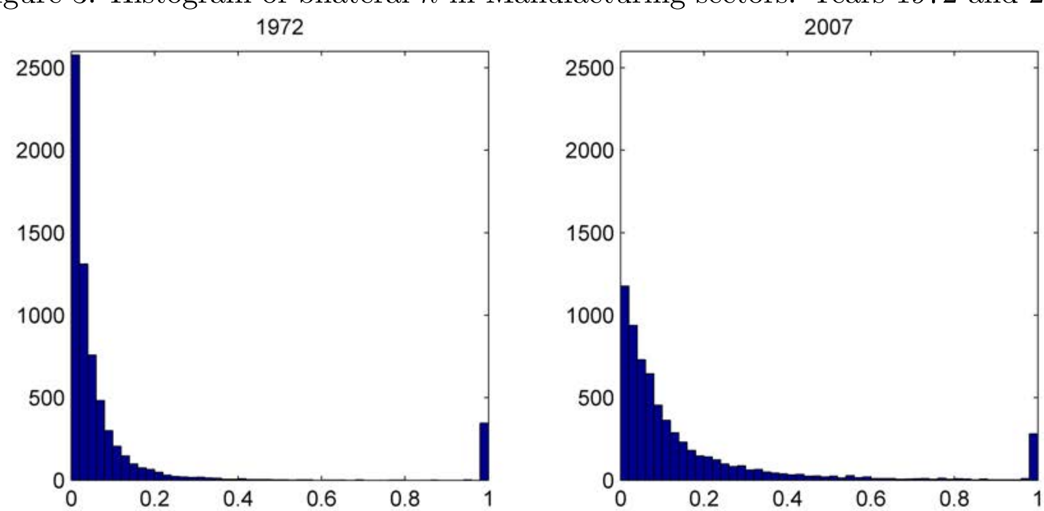

Figure 4: Histogram of bilateral κ in Agriculture. Years 1972 and 2007

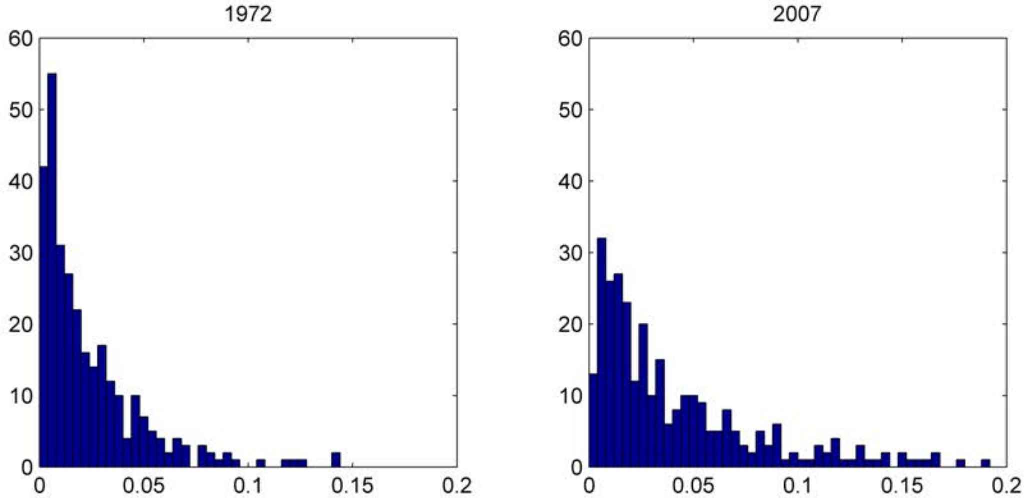

Table 1 investigates how the changes in trading costs have affected volatility in the 25 countries in our sample. (The list of countries can be seen in Table 1.) The results in the table correspond to our benchmark calibration, based on $\theta = 4$. Column (1) in the table shows the volatility generated by the model. Volatility is computed as the variance of annual growth rates over 35 years (we focus on the variance rather than the standard deviation, as the variance allows for an additive decomposition into the two mechanisms we are interested in). The value reported in Column (1) is very close to the actual volatility experienced by these economies from 1972 through 2007, since both the trading costs and productivity processes fed into the model are backed out from the data. Column (2) shows the volatility that would be observed if there were no global sectoral shocks. (The latter is generally smaller than the benchmark volatility, though there are some exceptions, as global sectoral shocks can covary negatively with country-specific shocks in some countries.) To compute this counterfactual measure, we mute the global sector-specific shocks in the decomposition of $\tilde{Z}^j_{nt}$. (This measure of volatility is useful to identify and quantify the two trade channels, as it will become clear next.) Column (3) shows the country's volatility in the counterfactual scenario that trading costs (κ) stayed at their 1972 levels. Column (4) shows this latter measure in the absence of global sector-specific shocks.

Column (5) shows the percent change in average volatility due to actual changes in trading costs since 1972, that is, the percent difference between columns (1) and (3). Column (6) shows the contribution of the specialization channel to the change in volatility in (5) and Column (7) shows the corresponding contribution of the diversification channel. The contribution of diversification to the change in volatility is computed as the difference between the volatility in the absence of sectoral shocks (Column 2) and the volatility under 1970's trading costs, in the absence of sectoral shocks (Column 4); the difference is expressed relative to the volatility under the 1972's trading cost levels (Column 3). This measures captures the trade in volatility due exclusively to the country-diversification effect, as the sectoral shocks are muted. The volatility due to specialization is computed as the difference between Columns (5) and (7).

As Table 1 shows, two thirds of the countries in our sample experienced a decline in volatility due to the decline in trade barriers since 1972, while the other third of the countries experienced an increase in volatility. The biggest decreases in volatility caused by trade occurred in Belgium-Luxemburg, Ireland, the Netherlands, and Norway, all of which saw volatility reductions of over 90 percent, meaning their current volatility is 90 percent lower than it would have been if trading costs stayed at their 1972 levels. The biggest increases in volatility due to trade were witnessed by Greece (14 percent increase) and Italy (12 percent increase). These results are the net effect stemming from the contribution of the two separate channels we study. The diversification channel contributed to lower volatility in nearly 90 percent of the countries. The specialization channel contributed to increase volatility in two-thirds of the countries, with the biggest increases experienced by Italy, Spain, the Netherlands and Greece. In some countries, sectoral specialization actually contributed to lower volatility; this is the case of Austria, Belgium-Luxembourg, India, Norway, South Korea, and Sweden. This is possible, as the model illustrates, when the sector (or sectors) in which the economy specializes in comoves negatively (or less positively) with the country's aggregate shocks—or other sectoral shocks. Interestingly, the United Kingdom did not experience a sizeable change in its volatility due to changes in trade costs. However, this result masks the contribution of a sizeable reduction in volatility due to the diversification channel and a comparably sizeable increase in volatility due to the sectoral specialization channel.

In absolute terms, the diversification effect was in general larger than the specialization effect, and hence, on net, two-thirds of the countries saw a reduction in volatility, while the remaining third saw a more modest increase in volatility. The heterogeneity in the trade effects across countries is remarkable.

Table 2 shows the change in volatility due to free trade and its decomposition for two other (more extreme) values of θ, $\theta = 2$ and $\theta = 8$. The general message is qualitatively robust: i) the effect of trade on volatility varies across countries; ii) the diversification channel tends to reduce volatility; and iii) sectoral specialization tends to increase volatility. Interestingly, for $\theta = 2$, the case of high scope for comparative advantage, volatility always declines with trading costs, with the declines being significant even for countries like the United States. The decline in volatility is driven almost exclusively by the large effects stemming from the country-wide diversification channel. The effects of sectoral specialization are also sizeable, but smaller than the diversification effect. These results imply that, on average, trade leads to a reduction in volatility. On the other extreme of $\theta = 8$, the results are qualitatively similar to the benchmark case, although in general the results are quantitatively smaller. Taken together, the findings appear robust to increases in θ and suggest that the effect of country diversification on volatility would be stronger for lower values of θ, meaning that trade will reduce volatility even further as the scope for comparative advantage increases. (This result holds despite the fact that sectoral specialization would also increase in this case.)[19]

VI Conclusions

How does openness to trade affect GDP volatility? This paper revisits the common wisdom that trade increases volatility by causing higher sectoral specialization. It argues that when country-specific shocks are an important source of volatility, openness to international trade can lower GDP volatility, as it reduces exposure to domestic shocks and allows countries to diversify the sources of demand and supply across countries. Building on Eaton and Kortum (2002)'s quantifiable model for trade, the paper assesses the effect of trade on volatility and the role played by these two mechanisms, sectoral specialization and country diversification.

A key finding of the paper is that the historical decline in trade barriers in agriculture and manufacturing has led to a reduction in volatility in two-thirds of the countries analyzed, and to modest increases in volatility in the remaining third. The quantitative change in volatility varies significantly across countries. The overall volatility change due to trade openness is

[19] Our exercise underscores the importance of the parameter θ, and adds to the message of Arkolakis, Costinot, and Rodriguez-Clare (2012): in order to assess the effects of trade on key aggregate variables, the elasticity of trade to trade costs plays a key role.

Table 1: Baseline and counterfactual change in volatility (measured as variance) under free trade. Baseline calibration with $\theta = 4$.

	Average volatility				Changes in average volatility due to measured changes in trade barriers		
	Benchmark volatility (1)	Volatility absent sectoral shocks (2)	Volatility at 1972s trade barriers (3)	Volatility absent sectoral shocks, at 1972s trade barriers (4)	Volatility change due to change in trade barriers (5)	Volatility change due to specialization (6)	Volatility change due to diversification (7)
Australia	0.00085	0.00081	0.00090	0.00090	-5.6%	4.8%	-10.4%
Austria	0.00023	0.00020	0.00037	0.00033	-37.5%	-3.5%	-34.0%
Belgium and Luxembourg	0.00035	0.00019	0.00465	0.00426	-92.4%	-4.8%	-87.5%
Canada	0.00019	0.00014	0.00040	0.00037	-53.0%	4.2%	-57.2%
China	0.00631	0.00581	0.00630	0.00582	0.2%	0.3%	-0.1%
Colombia	0.00113	0.00089	0.00106	0.00084	6.2%	1.3%	4.9%
Denmark	0.00031	0.00013	0.00049	0.00032	-35.5%	5.5%	-41.0%
Finland	0.00038	0.00034	0.00046	0.00045	-16.3%	7.2%	-23.5%
France	0.00022	0.00012	0.00023	0.00014	-7.5%	4.1%	-11.6%
Germany	0.00028	0.00014	0.00029	0.00018	-5.3%	6.0%	-11.3%
Greece	0.00032	0.00023	0.00028	0.00022	13.9%	10.4%	3.5%
India	0.00087	0.00082	0.00159	0.00150	-45.7%	-2.9%	-42.7%
Ireland	0.00078	0.00055	0.06890	0.06919	-98.9%	0.8%	-99.6%
Italy	0.00017	0.00009	0.00015	0.00010	12.4%	19.5%	-7.1%
Japan	0.00027	0.00011	0.00025	0.00011	8.2%	7.4%	0.8%
Mexico	0.00066	0.00076	0.00186	0.00202	-64.3%	3.3%	-67.6%
Netherlands	0.00021	0.00012	0.00239	0.00260	-91.4%	12.1%	-103.5%
Norway	0.00055	0.00046	0.01116	0.01078	-95.1%	-2.7%	-92.4%
Portugal	0.00115	0.00082	0.00193	0.00170	-40.3%	5.4%	-45.6%
ROW	0.00164	0.00173	0.00163	0.00173	0.6%	0.8%	-0.2%
South Korea	0.00094	0.00069	0.00097	0.00072	-3.3%	-0.9%	-2.4%
Spain	0.00018	0.00015	0.00017	0.00016	9.3%	14.7%	-5.4%
Sweden	0.00020	0.00020	0.00030	0.00029	-32.7%	-2.1%	-30.6%
United Kingdom	0.00020	0.00016	0.00020	0.00018	0.4%	9.2%	-8.8%
United States	0.00028	0.00017	0.00027	0.00018	2.1%	3.2%	-1.1%
Average	0.00075	0.00063	0.00429	0.00420	-26.8%	4.1%	-31.0%

Note: Column (1) shows the average volatility in the baseline model using the calibrated kappas and shocks from 1972-2007. Column (2) is the volatility in (1) after removing common sectoral shocks. Column (3) shows the average volatility using the calibrated shocks from 1972-2007 under the assumption that trading costs in manufacturing and agriculture remain at their 1970 levels. Column (4) is similar to (3), after removing common sectoral shocks. Column (5) shows the percent change in average volatility as economies lowered their trading costs (move from (3) to (1)). Column (6) shows the contribution of specialization to the change in volatility in (5). Column (7) shows the contribution of diversification to the change in volatility in (5).

Table 2: Counterfactual change in volatility (measured as variance) under free trade. Alternative calibrations with $\theta = 2$ and $\theta = 8$.

Changes in average volatility due to measured changes in trade barriers

	$\theta = 2$			$\theta = 8$		
	Volatility change due to change in trade barriers	Volatility change due to specialization	Volatility change due to diversification	Volatility change due to change in trade barriers	Volatility change due to specialization	Volatility change due to diversification
Australia	-30.1%	7.9%	-38.0%	-1.2%	2.2%	-3.4%
Austria	-60.2%	-9.8%	-50.4%	-22.6%	2.7%	-25.3%
Belgium and Luxembourg	-94.4%	-4.3%	-90.1%	-86.4%	-6.1%	-80.4%
Canada	-82.1%	5.4%	-87.5%	-13.4%	-0.3%	-13.1%
China	-1.5%	0.7%	-2.3%	0.4%	0.1%	0.3%
Colombia	-2.2%	1.4%	-3.6%	3.3%	0.8%	2.5%
Denmark	-58.7%	-2.8%	-55.9%	-24.8%	8.5%	-33.3%
Finland	-55.1%	4.7%	-59.9%	-4.4%	4.1%	-8.6%
France	-32.8%	11.5%	-44.2%	-4.4%	0.5%	-4.9%
Germany	-21.6%	12.8%	-34.4%	-3.8%	2.3%	-6.0%
Greece	-22.4%	19.2%	-41.5%	5.0%	2.2%	2.7%
India	-66.1%	-3.1%	-63.0%	-17.6%	-1.3%	-16.3%
Ireland	-98.8%	0.2%	-99.0%	-97.7%	1.7%	-99.4%
Italy	-10.4%	44.9%	-55.3%	4.2%	6.5%	-2.3%
Japan	-5.5%	16.3%	-21.8%	4.4%	3.4%	1.0%
Mexico	-83.0%	0.8%	-83.8%	-37.6%	3.8%	-41.4%
Netherlands	-92.3%	11.6%	-103.9%	-87.4%	13.5%	-101.0%
Norway	-96.8%	-2.8%	-94.0%	-91.2%	-3.0%	-88.2%
Portugal	-72.4%	3.7%	-76.1%	-1.5%	3.0%	-4.5%
ROW	-6.1%	2.6%	-8.7%	0.6%	0.2%	0.5%
South Korea	-14.5%	1.7%	-16.2%	0.7%	-0.2%	0.9%
Spain	-27.8%	30.2%	-58.0%	3.0%	4.0%	-1.0%
Sweden	-75.2%	-3.6%	-71.6%	-9.3%	-0.9%	-8.5%
United Kingdom	-37.5%	12.7%	-50.2%	-2.5%	1.6%	-4.2%
United States	-20.8%	6.2%	-27.0%	1.3%	1.2%	0.2%
Average	-46.7%	6.7%	-53.4%	-19.3%	2.0%	-21.3%

the net result of the two different mechanisms, sectoral specialization, and country-wise diversification. The first mechanism tends to decrease volatility, while the second tends to increase it (though, as we point out, this general tendency finds a number of exceptions). The diversification effect is, on average, quantitatively stronger than the specialization effect; this result explains why, on average, volatility tends to decline with trade. The model sheds light on why the magnitude of the trade effects may differ across countries. The sizeable heterogeneity in the trade effects on volatility can contribute to understand the diversity of results documented by the existing empirical literature.

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VII Appendix:

The following Appendix provides details on the derivations of the model, the data, and the quantitative approach. Next, it addresses the problem raised by Kehoe and Ruhl (2008) and shows that given the way price indexes are computed in practice by statistical offices, changes in terms of trade affect measured real GDP.

A Derivation of GDP under free trade

In the one-sector economy, under free trade, prices are equalized across countries.

$$P_t = P_{nt} = (\xi B)^{1/\beta} \left\{ \sum_{m=1}^{N} T_m (A_{mt})^{\theta} (w_{mt})^{-\beta\theta} \right\}^{\frac{-1}{\beta\theta}} \tag{41}$$

Thus, from $d_{nmt} = (\xi B)^{-\theta} T_m (A_{mt})^{\theta} (w_{mt})^{-\beta\theta} (P_{mt})^{\beta\theta}$ we obtain:

$$d_{mnt} = T_n (A_{nt})^{\theta} (w_{nt})^{-\beta\theta} \left\{ \sum_{m=1}^{N} T_m (A_{mt})^{\theta} (w_{mt})^{-\beta\theta} \right\}^{-1} \tag{42}$$

and from $w_{nt} L_{nt} = \sum_{m=1}^{N} d_{mnt} w_{mt} L_{mt},$, we have:

$$w_{nt} = \left(\frac{T_n (A_{nt})^{\theta}}{L_{nt}} \right)^{\frac{1}{1+\beta\theta}} V_t \tag{43}$$

where $V_t \equiv \left[\sum_{m=1}^{N} \frac{w_{mt} L_{mt}}{\sum_{i=1}^{N} T_i (A_{it})^{\theta} (w_{it})^{-\beta\theta}} \right]^{\frac{1}{1+\beta\theta}}$ is common to all countries. Therefore, using the definition of Z_{nt},

$$\begin{aligned}
\frac{w_{nt} L_{nt}}{P_{nt}} &= L_{nt} \left(\frac{T_n (A_{nt})^{\theta}}{L_{nt}} \right)^{\frac{1}{1+\beta\theta}} V_t (\xi B)^{1/\beta} \left\{ \sum_{i=1}^{N} T_i (A_{it})^{\theta} \left(\left(\frac{T_i (A_{it})^{\theta}}{L_{it}} \right)^{\frac{1}{1+\beta\theta}} V_t \right)^{-\beta\theta} \right\}^{\frac{1}{\beta\theta}} \\
&= (\xi B)^{1/\beta} \left(T_n A_{nt}^{\theta} L_{nt}^{\beta\theta} \right)^{\frac{1}{1+\beta\theta}} \left[\sum_{i=1}^{N} \left(T_i (A_{it})^{\theta} L_{it}^{\beta\theta} \right)^{\frac{1}{1+\beta\theta}} \right]^{\frac{1}{\beta\theta}} \\
Y_{nt} &= (\xi B)^{1/\beta} Z_{nt}^{\frac{1}{1+\beta\theta}} \left(\sum_{m=1}^{N} Z_{mt}^{\frac{1}{1+\beta\theta}} \right)^{\frac{1}{\beta\theta}}
\end{aligned}$$

B Derivation of diversification result

We now prove the result in inequality (26). Start with the original condition that shows that GDP under costless trade less is volatile than under autarky.

$$\frac{(\beta+\theta\gamma_i)^2+\theta^2\sum_{j\neq i}^{N}\gamma_j^2}{(\beta+\theta)^2}<1$$

The first line below expands the numerator and adds terms while the second line collect terms. The last line adds the $(\theta\gamma_i)^2$ term to the expression in square brackets (note the change of the index under the sum). The inequality holds since $\gamma_i < 1$ for all i.

$$\begin{aligned}
\frac{\beta^2+(\theta\gamma_i)^2+2\beta\theta\gamma_i+\theta^2-\theta^2+2\beta\theta-2\beta\theta+\theta^2\sum_{j\neq i}^{N}\gamma_j^2}{(\beta+\theta)^2} &< 1\\
\frac{(\beta+\theta)^2+(\theta\gamma_i)^2+2\beta\theta(\gamma_i-1)+\theta^2\left[\sum_{j\neq i}^{N}\gamma_j^2-1\right]}{(\beta+\theta)^2} &< 1\\
2\beta\theta(\gamma_i-1)+\theta^2\left[\sum_{j=i}^{N}\gamma_j^2-1\right] &< 0
\end{aligned}$$

C Mapping Lw_i/p_i into constant-price GDP in PPP

It is instructive to start with variable P_i that in the Penn World Tables denotes the price level of GDP, or more precisely the USD value of local expenditures over expenditures evaluated in international prices. While the PWT variables are originally defined (and computed) in terms of expenditures and relative prices, it is possible to cast them in terms of prices and quantities as follows:

$$P_i=\frac{\sum_g p_{g,i}\,q_{g,i}}{\sum_g p_g\,q_{g,i}}$$

with $p_{g,i}$ and $q_{q,i}$ represent the USD price and quantity of good g respectively and p_g is the price of the same good in an international currency. Index g represents spending groups (basic headings in PWT slang), which are constructed in a way that the sum of these expenditure groups adds to total GDP. One of these groups are net exports, valuation of which follows the assumption that

$$p_{nx,i}\, q_{nx,i} = p_{nx}\, q_{nx,i} = S_i$$

where S_i is in USD. In our model, consumers buy all individual goods $q(x)$ and bundle them using the CES aggregator in a final good q_f. Hence, a PWT statistician would be able to sample only from this one final good in each country and the quantity P_i measured becomes

$$P_{i,t} = \frac{p_{i,t}\, q_{f,i,t}\, L_{i,t} + S_{i,t}}{p_t\, q_{f,i,t}\, L_{i,t} + S_{i,t}}$$

Setting $P_{US,t} = 100$ as is the case in the PWT implies $p_t = p_{US,t}/100$ for all t. The denominator of $P_{i,t}$ is the current-price GDP in international prices

$$CGDP_{i,t} = p_{US,t}\, q_{f,i,t}\, L_{i,t} + S_{i,t}$$

and the real-price (Laspeyres) GDP in international prices is defined as

$$RGDP_{i,t} = p_{US,T}\, q_{f,i,t}\, L_{i,t} + S_{i,t}^{T}$$

where the last term captures real net exports in year t valued at prices from base year T.

Using the income-expenditure identity $L_{i,t}\, w_{i,t} = p_{i,t}\, q_{f,i,t}\, L_{i,t} + S_{i,t}$ and simple algebra we get

$$
\begin{aligned}
RGDP_{i,t} &= p_{US,T}\,\frac{(L_{i,t}\, w_{i,t} - S_{i,t})}{p_{i,t}} + S^T_{i,t} \\
&= p_{US,T}\,\frac{L_{i,t}\, w_{i,t}}{p_{i,t}} - \frac{p_{US,T}}{p_{i,t}} S_{i,t} + S^T_{i,t} \\
&= p_{US,T}\,\frac{L_{i,t}\, w_{i,t}}{p_{i,t}} - \frac{p_{US,T}}{p_{i,T}}\frac{p_{i,T}}{p_{i,t}} S_{i,t} + S^T_{i,t} \\
&= p_{US,T}\,\frac{L_{i,t}\, w_{i,t}}{p_{i,t}} + S^T_{i,t}\left(1 - \frac{p_{US,T}}{p_{i,T}}\right) \\
&\approx \mu\frac{L_{i,t}\, w_{i,t}}{p_{i,t}}
\end{aligned}
$$

The last equality follows the PWT convention of valuing net exports by the price index of domestic absorption for years other than the base year. By dropping the last term in the approximation we assume that changes in *real* net exports are small for most countries compared to the role of domestic absorption. Given the weight attached to $S^T_{i,t}$ this assumption will be of importance only for countries with price level far off the US one in the base year.

D Data Sources

We first describe the sample of countries and then the various sources of data.

D.1 Sample of Countries

Our sample consists of 24 core countries, for which we were able to collect all the information needed to carry out the quantitative analysis with no need—or very limited need—of estimation. Other countries, for which data are nearly complete and estimation of some sectors' output or value added was needed, are grouped as "Rest of the World" (ROW); the sectoral trade data are available for virtually all countries. Some countries were aggregated (for example Belgium and Luxembourg, and, before making to ROW, Former USSR, Former Yugoslavia.). In particular, the minimum condition to keep a country (or an aggregation of countries) in the sample is the availability of complete series for sectoral value added and the presence of trade data.

The core sample of countries include the United States, Mexico, Canada, Australia, China, Japan, South Korea, India, Colombia, the United Kingdom, a composite of France and its overseas departments, Germany, Italy, Spain, Portugal, a composite of Belgium and Luxembourg, the Netherlands, Finland, Sweden, Norway, Denmark, Greece, Austria and Ireland. While some important countries appear only in our ROW group (most notably Brazil, Russia, Turkey, Indonesia, Malaysia and oil exporters), the selection of core countries is meaningful both in terms of geographic location (covering all continents) and in terms of their share in global trade and GDP. The time period we study covers years from 1972 to 2007. (1970–1971 are slightly problematic for trade data, as there are many missing observations; hence the decision to start in 1972. The end period is chosen in order to avoid confounding the trade effects we are after with the financial crisis, which had other underlying causes.)We focus on annual data.

The rest of the section describes our data sources and estimation methods.

D.2 Sectoral Gross Output

The data are disaggregated into 24 sectors: agriculture (including mining and quarrying), 22 manufacturing sectors, and services, all expressed in millions of US dollars for the core countries and the Rest of the world (ROW). The 22 manufacturing sectors correspond to the industries numbered 15 to 37 in the ISIC Rev. 3 classification (36 and 37 are bundled together).

The final dataset is obtained by combining different sources and some estimation. Data on agriculture, aggregate manufacturing and services for core countries come mostly from the EU KLEMS database. There is no available series for services output in China and India, so they are obtained as residuals. Additional data come from the UN National Accounts.

Data on manufacturing subsectors come from UNIDO and EU KLEMS. For some subsectors, EU KLEMS data are available only at a higher level of aggregation (i.e. sector 15&16 instead of the two separately); in those cases, we use the country specific average shares from UNIDO for the years in which they are available to impute values for each subsectors.

For the countries in the ROW, the output dataset is completed through estimation, using sectoral value added, aggregate output, GDP and population (the latter two from the Penn World Table 7.1) using Poisson regressions. For every country for which sectoral value added and PWT data are available, we estimate gross output using Possion regressions. Finally, for the few countries for which we have value added data but no PWT data, we estimate sectoral output by calculating for each year and sector the average value added/output ratio,

$$\bar{\beta}_t^j = \frac{1}{N}\sum_{i=1}^{N}\frac{VA_{i,t}^j}{Output_{i,t}^j}$$

and then use it in

$$\widehat{Output}_{i,t}^j = \frac{VA_{i,t}^j}{\bar{\beta}_t^j}$$

Data collection notes on the core countries are as follows:

- USA: missing years 1970-76 generated using a growth rate of each sector from EU KLEMS (March 2008 edition).

- Canada: 1970-04 EU KLEMS (March 2008 edition), for 2005-06 sectoral growth rates from the Canadian Statistical Office's National Economic Accounts (table Provincial gross output at basic prices by industries).

- China: data are from the Statistical yearbooks of China. Output in agriculture is defined as gross output value of farming, forestry, animal husbandry and fishery and is available for all years. Mining and manufacturing is reported as a single unit labelled output in industry, which apart from the extraction of natural resources and manufacture of industrial products includes sectors not covered by other countries: water and gas production, electricity generation and supply and repair of industrial products (no adjustment was made). The primary concern was the methodological change initiated around 1998, when China stopped reporting *total* industrial output and limited the coverage to industrial output of firms with annual sales above 5m yuan (USD 625 000). The sectoral coverage remained the same in both series. There were 5 years of overlapping data of both series over which the share of the 5m+ firms on total output decreased from 66 to 57 percent. The chosen approach to align both series was to take the levels of output from the pre-1999 series (output of all firms) and apply the growth rate of output of 5m+ firms in the post-1999 period. This procedure probably exaggerates the level of output in the last seven years and leads to an enormous increase in the output/GDP in industry ratio (from 3.5 in 1999 to 6.0 in 2006). Our conjecture is that the ratio would be less steep if the denominator was value added in industry (unavailable on a comparable basis) because the GDP figure includes net taxes, which might take large negative values. Output in industry of all firms reflects the 1995 adjustment with the latest economic census.

There is no available estimate for output in services, so we use the predicted values from a Poisson regression on the other core countries, with sectoral value added (see below for details on the source), output in agriculture, output in manufacturing, GDP and population (the latter two from the Penn World Table 7.1) and year dummies as regressors.

- India: data are from the Statistical Office of India, National Accounts Statistics. Years 1999-06 are reported on the SNA93 basis. Earlier years were obtained using the growth rates of sectoral output as defined in their 'Back Series' database. The main issue with India was the large share of 'unregistered' manufacturing that is reported in the SNA93 series but missing in the pre-1999 data. The 'unregistered' manufacturing covers firms employing less than 10 workers and is also referred to as the informal or unorganized sector. We reconstructed the total manufacturing output using the assumption that the share of registered manufacturing output in total manufacturing output mirrors the share of value added of the registered manufacturing sector in total value added in manufacturing (available from the 'Back Series' database).

  As for China, output in services was estimated through a Poisson regression method.

- Mexico: data are from the System of National Accounts published by the INEGI and from the UN National Accounts Database. 2003-06 Sistema de cuentas nacionales, INEGI (NAICS), 1980-03 growth rate from the UN National Accounts Data, 1978-79 growth rate from Sistema de cuentas nacionales, INEGI, 1970-1978 growth rate from System of National Accounts (1981), Volumen I issued by the SPP.

- Japan: data for 1973-06 are from EU KLEMS (November 2009 Edition), for 1970-72 the source is the OECD STAN database (growth rate).

- Colombia and Norway: data are from the UN National Accounts Database.

- Germany: the series is EU KLEMS' estimate for both parts of Germany.

D.3 Sectoral Value Added

The data on sectoral value added is obtained by combining data from the World Bank, UN National Accounts, EU KLEMS and UNIDO. For the World Bank and UN cases, the format of the data does not allow to have exactly the same sectoral classification as the output data: namely, mining here is not included in agriculture.

The World Bank and UN data are cleaned (we note a contradiction in the UN data for Ethiopia and Former Ethiopia, which we correct to include in ROW final sample; see the file for more details).

Data on manufacturing subsectors come from UNIDO and EU KLEMS. UNIDO. For some subsectors, EU KLEMS data are available only at a higher level of aggregation (i.e. sector 15&16 instead of the two separately); in those cases, we use the country specific average shares from UNIDO for the years in which they are available to impute values for each subsectors; if no such data are available in UNIDO, we use the average shares for the whole sample. We use the UNIDO data as baseline and complete it with EU KLEMS when necessary (in these cases the growth rates of the EU KLEMS series are used to impute values;

this is done because sometimes the magnitudes are quite different in the two datasets). If an observation is missing in both datasets, we impute it using the country specific average sectoral shares for the years in which data are available.

D.4 Trade and dij's

These bilateral import shares in gross output are obtained through several steps. We use the SITC1 classification for all the sample. This is made in order to ensure a consistent definition of the sectors throughout the whole time period. In order to construct the agricultural sector we aggregate the subsectors in the SITC1 classification corresponding to the BEC11 group. For the manufacturing sectors, we use the correspondence tables available on the UN website to identify the SITC1 groups corresponding to the ISIC 3 groups used for output and value added. Re-exports and re-imports are not included in the exports and imports figures. We use bilateral imports and exports at the sectoral level from 1972 to 2007 from the UN COMTRADE database. This dataset contains the value of all the transactions with international partners reported by each country. Since every transaction is potentially recorded twice (once reported by the exporter and once by the importer) we use the values reported by the importer when possible and integrate with the corresponding values reported by the exporter if only those are available.

As discussed in the paper, the d^j_{nmt}'s are computed as the ratio between the value of exports from m to n in sector j and total spending by n on sector j at time t, where total spending is measured as gross output plus imports minus exports of that sector. The share

d^j_{mmt} is obtained as a residual from the accounting restriction:

$$d^j_{mmt} = 1 - \sum_{n \neq m}^{N} d^j_{mnt}$$

We compute the surpluses as Total Exports - Total Imports, merge with the output dataset and calculate the dij's and dii using the formulas in the text.

D.5 Prices

The sectoral price indices come from the EU KLEMS database and data are available for most of the core countries. We construct the sectoral deflators using a chain weighted index. In particular, we compute for every year a weighted average of the growth rate of the subsectoral price indexes, where the weights are the output shares in that year; then, we apply this growth rate to the previous year's sectoral price index (where the first year's price index is a weighted average of the subsectors). We then rescale so that the index is 100 in 1995 for all countries and sectors.

The aggregate price of GDP relative to that of the United States are obtained from the PWT 5.6 for Former USSR, Former Czechoslovakia and Former Yugoslavia and PWT 7.1 for the other countries. For the ROW, we compute a weighted average of the relative prices of GDP for all the countries for which the PWT data are available (most of the ROW countries), where the weights are each country's share of total output. Similarly, for Belgium-Luxemobourg, we compute the weighed average of the two.

D.6 Exchange Rates

The exchange rates used for the conversion of output data come from the IMF.

D.7 Alphas

To calibrate the α^js we use sectoral value added data according to the following procedure:

1. $s_t^j \doteq \frac{\sum_n w_{nt}^j L_{nt}^j}{\sum_k \sum_n w_{nt}^k L_{nt}^k}$

2. $\alpha_t^j \doteq \frac{s_t^j / \beta^j}{\sum_k (s_t^k / \beta^k)}$

To allow for more flexibility and accomodate world-wide structural changes over time we do the calibration every year and then use a smoothed trend from the resulting time series. Then renormalize so that the sum of alphas is 1 in all periods.

D.8 Prices

We use the sectoral price indexes from the EU KLEMS database to compute price deflators for our three sectors (using a chain weighted index).

The relative price of GDP comes from the PWT 7.1 for all countries except Former Soviet Union, Former Czechoslovakia and Former Yugoslavia, for which we use the PWT 5.6. For the ROW, we compute a weighted average of the relative prices of GDP for all the countries for which the PWT data are available, where the weights are each country's share of total output.

E Numerical Procedure for Model Equilibrium

We use nested loops to compute the model equilibrium.

E.1 Inner loop

For a given pair of sectoral resource allocation (L^j_{nt}) and sectoral wages (w^j_{nt}) solve the system below for the aggregate price indexes P_{nt}.

$$
\begin{aligned}
P_{nt} &= \prod_{j=1}^{J} \alpha^{j\,-\alpha^j} {P^j_{nt}}^{\alpha^j} && (44)\\
P^j_{nt} &= \xi {\Phi^j_{nt}}^{-\frac{1}{\theta}} && (45)\\
\Phi^j_{nt} &= B^{j\,-\theta} \sum_{i=1}^{N} T^j_i {A^j_{it}}^{\theta} \left(\frac{{P_{it}}^{1-\beta^j} {w^j_{it}}^{\beta^j}}{\kappa^j_{nit}} \right)^{-\theta} && (46)
\end{aligned}
$$

Where

$$\xi = \Gamma\left(\frac{\theta + 1 - \eta}{\theta}\right)$$

and

$$B^j = \beta^{j\,-\beta^j} (1-\beta^j)^{-(1-\beta^j)}.$$

Algebraic manipulations to arrive at a system of N equations and N unknowns. Simplify Φ^j_{nt} :

$$
\begin{aligned}
\Phi^j_{nt} &= B^{j\,-\theta} \sum_{i=1}^{N} \underbrace{T^j_i {A^j_{it}}^{\theta}}_{\frac{Z^j_{it}}{{L_{it}}^{\beta^j\theta}}} \left(\frac{{P_{it}}^{1-\beta^j} {w^j_{it}}^{\beta^j}}{\kappa^j_{nit}} \right)^{-\theta} \\
&= B^{j\,-\theta} \sum_{i=1}^{N} Z^j_{it} {L_{it}}^{-\beta^j\theta} {w^j_{it}}^{-\beta^j\theta} {\kappa^j_{nit}}^{\theta} {P_{it}}^{\theta(\beta^j-1)} \\
&= B^{j\,-\theta} \sum_{i=1}^{N} \underbrace{Z^j_{it} \left((L_{it} w^j_{it})^{-\beta^j} \kappa^j_{nit} \right)^{\theta}}_{D^j_{nit}} {P_{it}}^{\theta(\beta^j-1)} \\
&= B^{j\,-\theta} \sum_{i=1}^{N} D^j_{nit} {P_{it}}^{\theta(\beta^j-1)}
\end{aligned}
$$

Notice that we can compute the coefficients of the equation (the D values) before starting the search for the price vector.

Use equation (45) and then the expression for Φ^j_{nt}:

$$
\begin{aligned}
P_{nt} &= \prod_{j=1}^{J} \alpha^{j-\alpha^j} P^{j\ \alpha^j}_{nt} \\
&= \prod_{j=1}^{J} \alpha^{j-\alpha^j} \left(\xi \Phi^{j\ -\frac{1}{\theta}}_{nt}\right)^{\alpha^j} \\
&= \prod_{j=1}^{J} \alpha^{j-\alpha^j} \xi^{\alpha^j} \Phi^{j\ -\frac{\alpha^j}{\theta}}_{nt} \\
&= \prod_{j=1}^{J} \alpha^{j-\alpha^j} \xi^{\alpha^j} \left(B^{j-\theta} \sum_{i=1}^{N} D^j_{nit} P_{it}^{\theta(\beta^j-1)}\right)^{-\frac{\alpha^j}{\theta}} \\
&= \prod_{j=1}^{J} \underbrace{\alpha^{j-\alpha^j} \xi^{\alpha^j} B^{j\alpha^j}}_{K^j} \left(\sum_{i=1}^{N} D^j_{nit} P_{it}^{\theta(\beta^j-1)}\right)^{-\frac{\alpha^j}{\theta}} \\
&= \left(\prod_{j=1}^{J} K^j\right) \cdot \prod_{j=1}^{J} \left(\sum_{i=1}^{N} D^j_{nit} P_{it}^{\theta(\beta^j-1)}\right)^{-\frac{\alpha^j}{\theta}}
\end{aligned}
$$

Notice that $\prod_{j=1}^{J} K^j$ can be computed before the whole procedure.

We could simplify this further by solving for $\mathcal{P}_{nt} \equiv P^{\theta}_{nt}$ instead of P_{nt}:

$$
\mathcal{P}_{nt} = K \prod_{j=1}^{J} \left(\sum_{i=1}^{N} D^j_{nit} \mathcal{P}_{it}^{(\beta^j-1)}\right)^{-\alpha^j}, \tag{47}
$$

where

$$
K \equiv \left(\prod_{j=1}^{J} K^j\right)^{\theta} \equiv \left(\prod_{j=1}^{J} \alpha^{j-\alpha^j} \xi^{\alpha^j} B^{j\alpha^j}\right)^{\theta}
$$

and

$$
D^j_{nit} \equiv Z^j_{it}\left((L_{it} w^j_{it})^{-\beta^j} \kappa^j_{nit}\right)^{\theta}.
$$

Then we solve the system for the vector $\mathcal{P}_{\cdot t}$ by iterating on the right hand side of (47) starting from $\mathcal{P}_{\cdot,t-1}$.

E.2 Middle loop

For a given resource allocation, L_{nt}^j, this loop searches for sectoral wages w_{nt}^j that solve the nonlinear system of equations below.

$$
\begin{aligned}
w_{nt}^j L_{nt}^j &= \beta^j \sum_{m=1}^{N} d_{mnt}^j \left(\alpha^j w_{mt} L_{mt} + \frac{1-\beta^j}{\beta^j} w_{mt}^j L_{mt}^j \right) && (48) \\
w_{nt} L_{nt} &= \sum_{j=1}^{J} w_{nt}^j L_{nt}^j && (49)
\end{aligned}
$$

There are three important remarks:

- The system is separable in t, so we can solve the corresponding subsystem for each t separately.
- The system is nonlinear because d depends on sectoral wages by definition through

$$
d_{mnt}^j \equiv \frac{B^{j\,-\theta} T_n^j {A_{nt}^j}^{\theta} \left(\frac{{P_{nt}}^{1-\beta^j} {w_{nt}^j}^{\beta^j}}{\kappa_{mnt}^j} \right)^{-\theta}}{B^{j\,-\theta} \sum_{i=1}^{N} T_i^j {A_{it}^j}^{\theta} \left(\frac{{P_{it}}^{1-\beta^j} {w_{it}^j}^{\beta^j}}{\kappa_{mit}^j} \right)^{-\theta}}.
$$

- This is a system of the form $x = A(x)x$, where the matrix $A(x)$ depends on x nonlinearly. To solve for x we can use the following iterative procedure.

    1. Start from an initial x^0.

2. Iterate $x^{i+1} = \lambda A(x^i)x^i + (1-\lambda)x^i \ \forall i = 0, 1, \ldots$ until x^i converges to some x^*, where $\lambda \in (0, 1]$ is a dampening parameter.

Nonlinear part To facilitate computation we introduce D, the coefficients from the inner loop. We can rewrite the definition of d as

$$
\begin{aligned}
d^j_{mnt} &= \frac{\overbrace{T^j_n {A^j_{nt}}^\theta}^{\frac{Z^j_{nt}}{{L_{nt}}^{\beta^j\theta}}} \left(\frac{{P_{nt}}^{1-\beta^j} {w^j_{nt}}^{\beta^j}}{\kappa^j_{mnt}} \right)^{-\theta}}{\sum_{i=1}^N \underbrace{T^j_i {A^j_{it}}^\theta}_{\frac{Z^j_{it}}{{L_{it}}^{\beta^j\theta}}} \left(\frac{{P_{it}}^{1-\beta^j} {w^j_{it}}^{\beta^j}}{\kappa^j_{mit}} \right)^{-\theta}} \\
&= \frac{Z^j_{nt} {L_{nt}}^{-\beta^j\theta} {w^j_{nt}}^{-\beta^j\theta} {\kappa^j_{mnt}}^\theta {P_{nt}}^{\theta(\beta^j-1)}}{\sum_{i=1}^N Z^j_{it} {L_{it}}^{-\beta^j\theta} {w^j_{it}}^{-\beta^j\theta} {\kappa^j_{mit}}^\theta {P_{it}}^{\theta(\beta^j-1)}} \\
&= \frac{\overbrace{Z^j_{nt} {L_{nt}}^{-\beta^j\theta} {w^j_{nt}}^{-\beta^j\theta} {\kappa^j_{mnt}}^\theta}^{D^j_{mnt}} {P_{nt}}^{\theta(\beta^j-1)}}{\sum_{i=1}^N \underbrace{Z^j_{it} {L_{it}}^{-\beta^j\theta} {w^j_{it}}^{-\beta^j\theta} {\kappa^j_{mit}}^\theta}_{D^j_{mit}} {P_{it}}^{\theta(\beta^j-1)}} \\
&= \frac{D^j_{mnt} {P_{nt}}^{\theta(\beta^j-1)}}{\sum_{i=1}^N D^j_{mit} {P_{it}}^{\theta(\beta^j-1)}}
\end{aligned}
$$

Note that d does not depend on the resource allocation.

Linear part Substitute the sum from the second equation into the first one to get the expression below:

$$w_{nt}^{j}L_{nt}^{j} = \beta^{j}\sum_{m=1}^{N} d_{mnt}^{j}\left(\alpha^{j}\left(\sum_{k=1}^{J} w_{mt}^{k}L_{mt}^{k}\right) + \frac{1-\beta^{j}}{\beta^{j}}w_{mt}^{j}L_{mt}^{j}\right)$$
$$w_{nt}^{j}L_{nt}^{j} = \alpha^{j}\beta^{j}\sum_{m=1}^{N} d_{mnt}^{j}\sum_{k=1}^{J} w_{mt}^{k}L_{mt}^{k} \quad + \quad (1-\beta^{j})\sum_{m=1}^{N} d_{mnt}^{j}w_{mt}^{j}L_{mt}^{j}$$

We gather the coefficients on the right hand side to matrix A and then iterate on the sectoral value added term according to the procedure described above. That is in iteration $i+1$ the new value of the value added term is calculated as

$$\left(w_{nt}^{j}L_{nt}^{j}\right)^{i+1} = \lambda A\Big(\left(w_{nt}^{j}\right)^{i}\Big)\left(w_{nt}^{j}L_{nt}^{j}\right)^{i} + (1-\lambda)\left(w_{nt}^{j}L_{nt}^{j}\right)^{i}.$$

E.3 Wage Normalization

Once the procedure converged, we get sectoral wages from dividing $w_{nt}^{j}L_{nt}^{j}$ by L_{nt}^{j}. Then we scale sectoral wages so that we match the corresponding aggregate price with the observed aggregate price index in the benchmark country.

E.4 Outer Loop

The goal of this loop is to find the sectoral resource allocations L_{nt}^{j} that satisfy

$$\frac{L_{nt}^{j}}{L_{nt}} = E_{t-1}\left(\frac{w_{nt}^{j}L_{nt}^{j}}{w_{nt}L_{nt}}\right).$$

This loop runs over iterations of L_{nt}^{j} until it converges up to a predefined threshold. We use a band pass filtered trend that allows for breaks in growth rates to approximate the expectation. The rest of this loop can be found in the main text.

F The Kehoe–Ruhl Critique

Kehoe and Ruhl (2008) argue that if real GDP is measured as a chain weighted quantity index of value added, then terms-of-trade changes (and as a consequence, import price changes) do not affect real GDP up to a first order. The basic argument in Kehoe and Ruhl (2008) is as follows:

Real GDP is output minus the cost of inputs, both evaluated at past-period prices

$$y_t = f(l, m_t) - p_{t-1} m_t.$$

We have chosen output as the numeraire. Labour is the only "final" input, and is fixed over time (or, at least, not responding to p_t), as in our case. Chain-weighed GDP growth is simply

$$\frac{f(l, m_t) - p_{t-1} m_t}{f(l, m_{t-1}) - p_{t-1} m_{t-1}}.$$

Clearly, m_t is chosen in response to p_t, so that

$$f_m(l, m_t) = p_t.$$

Taking a first-order Taylor approximation of $f(l, m_t)$ around m_{t-1},

$$f(l, m_t) \approx f(l, m_{t-1}) + f_m(l, m_{t-1})(m_t - m_{t-1}) = f(l, m_{t-1}) + p_{t-1}(m_t - m_{t-1}),$$

current GDP is

$$y_t \approx f(l, m_{t-1}) - p_{t-1}m_{t-1},$$

which is *last period's* quantities at last period's prices, so there is no first-order change in the chain-weighted quantity index!

F.1 Multiple goods

Suppose now that the input bundle consists of a continuum of goods. Each good can be sourced from the home country as well as the foreign country.

$$m_t = \left\{ \int_0^1 [m_{Ft}(i) + m_{Ht}(i)]^\alpha di \right\}^{1/\alpha}.$$

The relevant input price index of the firm is

$$p_t = \left[\int_0^1 \min\{p_{Ft}(i), p(i)_{Ht}(i)\}^{1-\theta} di \right]^{1/(1-\theta)}.$$

The nominal GDP is still

$$f(l, m_t) - p_t m_t,$$

where

$$p_t m_t = \int_0^1 [p_{Ft}(i)m_{Ft}(i) + p_{Ht}(i)m_{Ht}(i)]di$$

is the total spending on inputs. Let $\mathcal{M}_t$ denote the set of products in which the foreign supplier is cheaper, so that the goods are imported. Total input spending can be split among the two source countries,

$$p_t m_t = \int_{i \in \mathcal{M}_t} p_{Ft}(i) m_{Ft}(i) di + \int_{i \notin \mathcal{M}_t} (i) p_{Ht} m_{Ht}(i) di.$$

F.2 Real GDP in the model

What does real GDP look like in the model? It would be tempting to replace current-period prices in nominal GDP with base-period prices, but this is not what is reported in NIPA. In NIPA, nominal quantities are deflated by *price indices* to obtain the real quantities. It is therefore important to see how these price indices would be calculated in our model.

More specifically, real GDP is

$$f(l, m_t) - \frac{1}{P_{Ft}} \int_{i \in \mathcal{M}_t} p_{Ft}(i) m_{Ft}(i) di - \frac{1}{P_{Ht}} \int_{i \notin \mathcal{M}_t} p_{Ht}(i) m_{Ht}(i) di,$$

where P_{Ft} is the import price index, and P_{Ht} is the producer price index of the domestic inputs.

Both price indices only measure a subset of goods, and, more importantly, a *non-random* subset of goods. To be included in the price index, the good had to be transacted both in the base period, as well as in the current period. Intuitively, this biases the price index towards *no change*. If there is too big a price increase, firms stop buying the product, and it will be dropped from the index. In what follows, we try to quantify this bias. We show that in the case the EK model, the bias is so severe that the price index is constant.

F.3 What does the Bureau of Labor Statistics (BLS) measure?

The price index is a weighted average of price changes for goods that are in the sample in both periods,

$$P_{Ft} \equiv \frac{\int_{i \in \mathcal{M}_0 \cup \mathcal{M}_t} p_{Ft}(i) m_{F0}(i) di}{\int_{i \in \mathcal{M}_0 \cup \mathcal{M}_t} p_{F0}(i) m_{F0}(i) di}$$

for imports, and a similar expression for domestic prices.

Suppose all products i are symmetric ex ante. Each draw a price at random from a common distribution. Let

$$\phi_t = \Pr(p_{Ft} \leq p_{Ht})$$

denote the probability that a good is imported at time t. By the law of large numbers, this is also the measure of $\mathcal{M}_t$.

Assume that the prices are independent across periods. This will be true if the productivity shocks in EK are independent. (For our argument to go through, we only need that they are imperfectly correlated across periods.) Then the probability of being exported in both periods is simply $\phi_0 \phi_t$.

F.4 Why terms-of-trade changes affect measured real GDP? An answer

A crucial assumption in Kehoe and Ruhl is that the BLS perfectly measures the prices that are relevant for input demand. This is not the case, however. In particular, the BLS does not (and cannot) measure the cost savings arising from *input substitution*, which are the key driving force of the Eaton-Kortum model.

What does the BLS measure? The BLS compares current import prices to the past prices of the *same good* from the *same supplier*. If a firm switches from a Mexican to a Chinese supplier, because it is cheaper, the BLS will miss the input price savings associated with it. Moreover, the BLS certainly does not mix up import prices with domestic prices, they create two separate price indices. So if the substitution is from an American supplier to a Chinese supplier, the BLS will certainly miss it. This means that for models like EK in which the *extensive margin* of suppliers plays a big role, the BLS would capture these terms of trade changes as affecting real GDP. The specific assumptions of the EK model actually ensure that *all* cost saving will occur on the extensive margin, and the BLS will not record any of the import price savings. That is, the Kehoe–Ruhl critique does not apply in this setting.

The intuition is as follows. Suppose Chinese goods have become 20 percent cheaper. Due to the winner-take-all nature of Ricardian competition, they will be competitive for a wider range of goods. These goods are off the radar from the BLS—they have been switched from an American or higher-cost foreign supplier. In fact, the goods that the BLS *does* measure are not a random sample from all goods—these are the ones in which China is still relatively expensive and/or other suppliers are still relatively cheap (otherwise, a switch would have occurred). The BLS will measure Chinese prices with an upward bias, all other prices with a downward bias. Hence its estimate of the terms of trade will biased toward no change.

In fact, in the EK model, the *measured* terms of trade remains constant. In any other model, with only a partial role for the extensive margin, some of the price change would show up in the terms of trade, some would go directly in measured productivity. This could be the explanation for the observed correlation of terms of trade and productivity in the data.