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Guido Menzio
Nicholas Trachter

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ABSTRACT

We develop a search-theoretic model of the product market that generates price dispersion across and within stores. Buyers differ with respect to their ability to shop around, both at different stores and at different times. The fact that some buyers can shop from only one seller while others can shop from multiple sellers causes price dispersion across stores. The fact that the buyers who can shop from multiple sellers are more likely to be able to shop at inconvenient times (e.g., on Monday morning) causes price dispersion within stores. Specifically, it causes sellers to post different prices for the same good at different times in order to discriminate between different types of buyers.

Guido Menzio
Department of Economics
University of Pennsylvania
467 McNeil Building
3718 Locust Walk
Philadelphia, PA 19104
and NBER
gmenzio@econ.upenn.edu

Nicholas Trachter
Federal Reserve Bank of Richmond
701 E Byrd Street
Richmond, VA 23219
Nicholas.Trachter@rich.frb.org

1 Introduction

It is a well-known fact that the same product is sold at very different prices, even when one restricts attention to sales taking place in the same geographical area and in the same narrow period of time. For instance, Sorensen (2000) finds that the average standard deviation of the price posted by different pharmacies for the same drug in the same town in upstate New York is 22%. In a more systematic study of price dispersion that covers 1.4 million goods in 54 geographical markets within the US, Kaplan and Menzio (2014b) find that the average standard deviation of the price at which the same product is sold within the same geographical area and the same quarter is 19%. Moreover, it appears that price dispersion is caused by both difference in prices across different stores and difference in prices within each store. For instance, Kaplan and Menzio (2014b) find that approximately half of the overall variance of prices for the same good in the same market and in the same quarter is due to the fact that different stores sell the same good at a different price on average, while the remainder is due to the fact that the same store sells the same good at different prices in different transactions taking place during the same quarter.[1]

In this paper, we develop a model of price dispersion across and within stores by combining the standard theory of price dispersion of, e.g., Butters (1977) and Burdett and Judd (1983) and the standard theory of intertemporal price discrimination of, e.g., Conslik, Gerstner and Sobel (1984) and Sobel (1984).[2] Specifically, we build a model of the market for an indivisible good. On the demand side, there are buyers who differ with respect to their ability to shop at different stores, as well as with respect to their ability to shop at different times: Some buyers can shop from only one seller while others can shop from multiple sellers, and some buyers can shop only during the day while others can

[1] These decomposition follows immediately from Table 3 in Kaplan and Menzio (2014b). For a particular good, the fraction of the price variance coming from across-store variation is given by the sum of the variance of the store component and the store-good component. The fraction of the price variance coming from within store variation is the variance of the transaction component.

[2] Besides intertemporal price discrimination, there are other explanations for why a seller would charge different prices for the same good within the same quarter. First, as in Sheshinski and Weiss (1974), Benabou (1988) or Burdett and Menzio (2013), a seller may change his nominal price during a quarter in order to keep up with the movements in the aggregate price level (more or less frequently depending on inflation and menu costs). Second, as suggested by Aguirregabiria (1999), a seller may change his price during a quarter in response to changes in his inventories of the good. Similarly, Lazear (1986) suggests that a seller will follow a non-stationary pricing policy if the good "expires" at a given date. Finally, as suggested by Menzio and Trachter (2014), a large seller may change his price over time in order to occasionally price out of the market a fringe of small sellers. In this paper, we abstact from these additional sources of high-frequency variations in a seller's price.

shop both during the day and during the night. On the supply side, there are identical sellers and each seller posts a (potentially different) price for the good in the day and at night.

We prove the existence and uniqueness of equilibrium. The equilibrium always features price variation across stores. Moreover, if the buyers who are able to shop both during the day and at night can shop from more stores than the buyers who are only able to shop during the day, the equilibrium also features price variation within stores. In particular, the equilibrium is such that some sellers post a strictly lower price during the night than during the day. On the other hand, if the buyers who are able to shop both during the day and at night can shop from fewer stores, the equilibrium does not features price variation within stores. That is, sellers do not vary their price over time. Intuitively, price dispersion across stores arises because sellers meet some buyers who cannot purchase from any other store and some other buyers who can and—as explained in Butters (1977) and Burdett and Judd (1983)—this heterogeneity induces identical sellers to post different prices for the same good. Price dispersion within stores arises because—if the buyers who are more likely to be able to shop at night are also more likely to be able to shop from multiple stores—a seller can compete more fiercely for these buyers without losing revenues on the other customers by charging a lower price at night than during the day.

The paper's main contribution is to develop a rich theory of price dispersion, where some of the overall dispersion in prices is due to the fact that different sellers set different average prices, and some is due to the fact that the same seller sets a different price at different times of the week. The empirical evidence reported by Kaplan and Menzio (2014b) suggests that both sources of price dispersion are quantitatively important and, hence, any theory of price dispersion should take both of them into account. Moreover, our theory shows that price dispersion within and across stores has the very same origin: heterogeneity across buyers in the ability to shop around (be it at different locations or at different times). The empirical evidence in Aguiar and Hurst (2007) and Kaplan and Menzio (2014a, 2014b) suggests that this type of heterogeneity is important, as, for example, the elderly and the unemployed spend more time shopping than the young and the employed.

The main difference between our model and a standard model of price dispersion is that, in our model, there are multiple times within a period (i.e., day and night) and buyers may differ in both their ability to shop at different stores and at different times.

From the substantive point of view, this novel aspect of our model open the possibility for intertemporal price discrimination and, hence, within store price dispersion.[3] From the technical point of view, this novel aspect of our model implies that sellers post multiple prices (i.e., a day price and a night price) rather than a single price. In turn, this implies that the solution technique for the standard model of price dispersion cannot be applied. Indeed, we develop a new approach to solve for the equilibrium, which is likely to be applicable to several other models in which sellers set multiple prices (e.g., models where sellers trade multiple products or models where seller post screening contracts, etc...).

The main difference between a standard model of intertemporal price discrimination and our model is that we study intertemporal price discrimination in the context of an imperfectly competitive environment in the spirit of Butters (1977) and Burdett and Judd (1983), where identical sellers have an incentive to set different prices. From the substantive point of view, this novel feature of our model can lead to the coexistence of within store and across store price dispersion. From the technical point of view, the novel feature of our model implies that intertemporal price discrimination can emerge even when the only difference across buyers is their ability to shop around (at different locations and different times). In contrast, the existing models of price discrimination (which do not generate price dispersion across sellers) assume that buyers differ both with respect to their valuation of the good and with respect to their ability to shop around (see, e.g., Sobel 1984 and Albrecht, Postel-Vinay and Vroman 2013). Finally, our model admits a unique equilibrium. In contrast, existing equilibrium models of intertemporal price discrimination have typically multiple equilibria, which creates challenges for estimation and counterfactual analysis.

2 Environment

We consider the market for an indivisible good that operates in two subperiods: *day* and *night*. On one side of the market, there is a measure 1 of identical sellers who can produce the good on demand at a constant marginal cost, which we normalize to zero. Each seller

[3] In Butters (1977), Varian (1980) and Burdett and Judd (1983), each seller is indifferent between posting any price on the support of the equilibrium price distribution. Therefore, if sellers choose different prices on different days, these models would generate price dispersion both across and within stores. However, this result is not robust to the introduction of menu costs (which would discourage sellers from resetting their prices if there is not a strictly positive benefit from doing so) or to the introduction of heterogeneity in the sellers' cost of production (which would break the seller's indifference between any price on the support of the equilibrium distribution).

simultaneously and independently posts a pair of prices (p_d, p_n), where $p_d \in [0, u]$ is the price of the good during the day, $p_n \in [0, u]$ is the price of the good during the night, and $u > 0$ is the buyers' valuation of the good. We denote as $G(p_d, p_n)$ the distribution of prices across sellers. Similarly, we denote as F_d the marginal distribution of day prices and as F_n the marginal distribution of night prices. Finally, we denote as F_m the marginal distribution of the lowest price of each seller.

On the other side of the market, there is a measure $\theta_x > 0$ of buyers of type x, and a measure $\theta_y > 0$ of buyers of type y. The two types of buyers differ with respect of their ability to shop from different sellers, as well as with respect of their ability to shop at different points in time. In particular, a buyer of type x is in contact with only one seller with probability $\alpha_x \in (0, 1)$ and with multiple (for the sake of simplicity, two) sellers with probability $1 - \alpha_x$. The buyer observes both the day and night price of the sellers with whom he is contact. The buyer is able to shop from these sellers during both day and night with probability $1-\beta_x$. With probability $\beta_x \in (0, 1)$, the buyer is able to shop only during the day. Similarly, a buyer of type y is in contact with one seller with probability α_y, and with multiple (two) sellers with probability $1 - \alpha_y$. A buyer of type y is able to shop from the contacted sellers during both day and night with probability $1 - \beta_y$, and only during the day with probability β_y. Both types of buyers enjoy a utility of $u - p$ if they purchase the good at the price p, and a utility of zero if they do not purchase the good. Without loss in generality, we assume that buyers of type x are in contact with fewer sellers than buyers of type y, i.e. $\alpha_x \geq \alpha_y$.

The definition of equilibrium for this model is standard (see, e.g., Burdett and Judd 1983 or Head et alii 2012).

Definition 1: *An equilibrium is a price distribution G such that the seller's profit is maximized everywhere on the support of G.*

A couple of comments about the environment are in order. First, we assume that some buyers are in contact with one seller and others are in contact with multiple sellers. We think of the contacts as the network of sellers that the buyer can easily access (e.g., the sellers on the way between the buyer's home and his workplace, the sellers that are close to his children's preschool, etc...), rather than the set of sellers whose price is known to the buyer. In this sense, we think of our model as a version of Hotelling (1929) where the buyer's transportation cost to different sellers is either zero or infinity, rather than as a model in which buyers discover sellers through a search process. Yet, just as in the search models of Butters (1977) and Burdett and Judd (1983), the fraction of buyers in contact with multiple sellers is the parameter that controls the degree of competitiveness of the product market.

Second, we assume that some buyers can shop only during the day, while others can shop both during the day and during the night. We do not think of day and night literally. Instead, we interpret the day as the time when all buyers are able to shop and the night as the time when only a subset of buyers is able to shop. For instance, the day might be the time when both employed and non-employed people can shop (i.e., after 5 PM), and the night might be the time when only the non-employed people can shop (i.e., before 5 PM). Alternatively, the night might be one particular day of the week, when only people with a flexible schedule can shop, and the day might be any other day of the week, when people with prior commitments can shop. As in Conslik, Gerstner and Sobel (1984) or Albrecht, Postel-Vinay and Vroman (2013), the fact that some buyers are flexible with respect to their shopping time and others are not gives sellers the opportunity to price discriminate.[4]

3 Characterization of equilibrium

In this section, we show that the equilibrium of our model exists and is unique, and we find necessary and sufficient conditions under which the equilibrium features both price dispersion across sellers and within sellers. Solving for the equilibrium of our model is not as simple as in Burdett and Judd (1983). In Burdett and Judd, every seller posts one price and must attain the same profit everywhere on the support of the price distribution. It is easy to prove that the support of the price distribution must be some connected interval $[p_\ell, p_h]$ with $p_h = u$. In light of this property, solving for the equilibrium amounts to solving one equation (i.e., the seller's equal profit condition on the support of the price distribution) for one unknown (i.e., the price distribution). In our model, every seller posts two prices and must attain the same profit everywhere on the support of the price distribution. Again, it is easy to show that both the support of the marginal distribution of day and night prices must be a connected interval. However, this property of the marginals is not enough to identify the support of the joint distribution. Hence, we

[4]It is easy to generalize the model to allow for some buyers to be able to shop only during the day, some only during the night, and others at both times. The analysis would be nearly identical, but the algebra would become more involved.

cannot solve for the equilibrium by simply solving the equal profit condition with respect to the distribution.

We develop an alternative solution technique. In Subsection 3.1, we show that we can restrict attention to equilibria in which every seller chooses a price for the good in the night that is non-greater than the price for the good in the day. In Subsection 3.2, we look for an equilibrium in which the seller's profit from day sales is constant everywhere on the support of the marginal distribution of day prices, and the seller's profit from night sales is constant everywhere on the support of the marginal distribution of night prices. We show that—if and only if the type of buyers who is in contact with fewer sellers is also less likely to be able to shop at night—this equilibrium exists and it features price dispersion both across and within sellers. In Subsection 3.3, we consider an equilibrium in which every seller posts the same price in the day and at night. We show that—if and only if the type of buyers who is in contact with fewer sellers is also more likely to be able to shop at night—this equilibrium exists and it features price dispersion across sellers but not within sellers. Finally, in Subsection 3.4, we rule out the existence of any other type of equilibrium by showing that the seller's profit from a nighttime trade must be either constant with respect to the night price (in which case the first type of equilibrium emerges), or strictly increasing with respect to the night price (in which case the second type of equilibrium emerges). The solution techniques developed here are likely to be applicable to other versions of Burdett and Judd where sellers post multiple prices.[5]

3.1 A general property of equilibrium

As a preliminary step, we show that we can restrict attention to equilibria in which sellers post prices (p_d, p_n) such that $p_n \leq p_d$. This is the case because—by assumption—all of the buyers who can shop at night can also shop during the day and, hence, a seller posting a higher price during the night as during the day enjoys the same profit and exerts the same competition on other seller as if he were to post the same price at both times of day.

To formalize the above argument, consider an equilibrium in which the marginal price

[5]Indeed, the results in Subsection 3.4 are used by Kaplan et al. (2015) in the context of a model of multiproduct retailing. Lester et al. (2015) develop similar arguments to solve for the equilibrium distribution of multiple screening contracts in the context of a model of the asset market. In contrast, these techniques are very different from those used to solve for the equilibrium in the models of multiproduct retailing of Zhou (2013) and Rhodes (2015). In fact, in these models, the seller's optimal pricing strategy is generically unique and there is no need to solve for the equilibrium price distribution.

distributions are continuous functions[6] F_d, F_n and F_m. A seller who posts prices $(p_d, p_n) \in [0, u]^2$ enjoys a profit

$$\begin{aligned} V(p_d, p_n) \quad &= [\mu_{1d} + \mu_{2d}(1 - F_d(p_d))]\, p_d \\ &+ [\mu_{1n} + \mu_{2n}(1 - F_m(\min\{p_d, p_n\}))] \min\{p_d, p_n\}, \end{aligned} \tag{1}$$

where the constants μ_{1d} and μ_{1n} are defined as

$$\begin{aligned} \mu_{1d} &= \theta_x \alpha_x \beta_x + \theta_y \alpha_y \beta_y, \\ \mu_{1n} &= \theta_x \alpha_x (1 - \beta_x) + \theta_y \alpha_y (1 - \beta_y), \end{aligned} \tag{2}$$

and the constants μ_{2d} and μ_{2n} are defined as

$$\begin{aligned} \mu_{2d} &= 2\theta_x (1 - \alpha_x)\beta_x + 2\theta_y (1 - \alpha_y)\beta_y, \\ \mu_{2n} &= 2\theta_x (1 - \alpha_x)(1 - \beta_x) + 2\theta_y (1 - \alpha_y)(1 - \beta_y). \end{aligned} \tag{3}$$

Let us briefly explain (1). The seller meets μ_{1d} buyers who are not in contact with any other seller and who can only shop during the day. Each one of these buyers will purchase the good from the seller at the price p_d. The seller meets μ_{1n} buyers who are not in contact with any other seller and who can shop both in the day and at night. Each one of these buyers will purchase the good from the seller at the price $\min\{p_d, p_n\}$. The seller meets μ_{2d} buyers who are in contact with a second seller and who can only shop during the day. Each one of these buyers will purchase the good from the seller if p_d is lower than the day price posted by the second seller they contacted, an event that occurs with probability $1 - F_d(p_d)$. Finally, the seller meets μ_{2n} buyers who are in contact with a second seller and can shop both during the day and at night. Each one of these buyers will purchase the good from the seller if $\min\{p_d, p_n\}$ is lower than the lowest price posted by the second seller they meet, an event that occurs with probability $1 - F_m(\min\{p_d, p_m\})$.

A seller posting the prices $(p_d, p_n) \in [0, u]^2$ with $p_n > p_d$ enjoys a profit

$$\begin{aligned} V(p_d, p_n) \quad &= [\mu_{1d} + \mu_{2d}(1 - F_d(p_d))]\, p_d \\ &+ [\mu_{1n} + \mu_{2n}(1 - F_m(p_d))]\, p_d. \end{aligned} \tag{4}$$

Notice that the seller's profit does not depend on the night price. Indeed, if $p_n > p_d$, the seller never makes a sale at night. The customers who can only shop during the day will purchase at the price p_d. The customers who can shop at both times will choose to

[6] The assumption that the distribution functions F_d, F_n and F_m are continuous is for the sake of exposition only. It is straightforward to generalize Lemma 1 to the case in which these distribution functions have mass points.

purchase during the day at the price p_d. Therefore, the seller enjoys the same profit if he were to post the prices (p_d, p_d) rather than (p_d, p_n).

Now, suppose that there is an equilibrium G in which some sellers post (p_d, p_n) with $p_n > p_d$. Consider an alternative price distribution $\hat{G}$ in which the sellers posting (p_d, p_n) with $p_n > p_d$ change their prices to (p_d, p_d), while the sellers posting (p_d, p_n) with $p_n \leq p_d$ keep their prices unchanged. Clearly, the marginal price distributions $\hat{F}_d$ and $\hat{F}_m$ associated with $\hat{G}$ are the same as the marginal price distributions F_d and F_m associated with G. Since the prices (p_d, p_n) with $p_n \leq p_d$ maximize the profit of the seller given G, they also maximize the profit of the seller given $\hat{G}$, as the profit function (1) only depends on the marginals F_d and F_m. Moreover, since the prices (p_d, p_n) with $p_n > p_d$ maximize the profit of the seller given G, the prices (p_d, p_d) maximize the profit of the seller given $\hat{G}$, as the seller's profit function (1) only depends on the marginals F_d and F_m and, as shown in (4), the seller is indifferent between posting (p_d, p_n) and (p_d, p_d). Thus, the joint price distribution $\hat{G}$ is an equilibrium and it is—along all relevant dimensions—equivalent to the equilibrium joint price distribution G.

We have therefore established the following Lemma.

Lemma 1: *Without loss in generality, one can restrict attention to equilibria G in which every seller posts a price$(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$, and the marginal distribution of lowest prices, F_m, is equal to the marginal distribution of night prices, F_n.*

3.2 Equilibrium with price dispersion across and within stores

In this section, we look for an equilibrium G in which every seller posts prices $(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$ and such that the marginal distribution of day prices, F_d, and the marginal distribution of night prices, F_n, are respectively given by

$$F_d(p) = 1 - \frac{\mu_{1d}}{\mu_{2d}} \frac{u - p}{p}, \forall p \in [p_{d\ell}, p_{dh}], \tag{5}$$

and

$$F_n(p) = 1 - \frac{\mu_{1n}}{\mu_{2n}} \frac{u - p}{p}, \forall p \in [p_{n\ell}, p_{nh}], \tag{6}$$

where the boundaries of the support of the distributions are

$$\begin{aligned} p_{t\ell} &= \frac{\mu_{1t}}{\mu_{1t} + \mu_{2t}} u, \quad && \text{for } t = \{d, n\} \\ p_{th} &= u, && \text{for } t = \{d, n\}. \end{aligned} \tag{7}$$

As we shall see momentarily, F_d is the distribution that makes the seller's profit from day trades constant for all day prices $p_d \in [p_{d\ell}, p_{dh}]$, and F_n is the distribution that makes the seller's profit from night trades constant for all night prices $p_n \in [p_{n\ell}, p_{nh}]$.

Given the marginal price distributions F_d and F_n in (5) and (6), we can identify the region where the profit of the seller attains its maximum. In general, a seller posting prices $(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$ attains a profit of

$$\begin{aligned} V(p_d, p_n) \quad &= [\mu_{1d} + \mu_{2d}(1 - F_d(p_d))]\, p_d \\ &+ [\mu_{1n} + \mu_{2n}(1 - F_n(p_n))]\, p_n. \end{aligned} \tag{8}$$

The first line in (8) denotes the seller's profit from day trades and the second line in (8) denotes the seller's profit from night trades. If the seller post prices $(p_{d,}p_n)$ such that $p_d \in [p_{d\ell}, p_{dh}]$, $p_n \in [p_{n\ell}, p_{nh}]$ and $p_n \leq p_d$, his profit is given by

$$V(p_d, p_n) = [\mu_{1d} + \mu_{1n}]\, u, \tag{9}$$

where (9) follows from (8) and from the expressions for the marginal price distributions F_d and F_n in (5) and (6). Notice that (9) is a constant. Indeed, the seller's profit from day trades attains the same value for all prices p_d on the support of the marginal distribution F_d, and the profit from night trades attains the same value for all prices p_n on the support of the marginal price distribution F_n. Moreover, the overall profit is equal to the profit that the seller would attain if he were to charge the buyer's reservation price u at both times and sell only to those buyers who are not in contact with any other seller.

If the seller post prices $(p_{d,}p_n)$ such that $p_d \in [p_{d\ell}, p_{dh}]$, $p_n \in [0, p_{n\ell})$ and $p_n \leq p_d$, his profit is given by

$$\begin{aligned} V(p_d, p_n) \quad &= \mu_{1d}u + (\mu_{1n} + \mu_{2n})p_n \\ &< [\mu_{1d} + \mu_{1n}]\, u, \end{aligned} \tag{10}$$

where the first line on the right-hand side of (10) follows from (8) and the fact that $F_n(p_n) = 0$ for all $p_n \leq p_{n\ell}$, and the second line on the right-hand side of (10) follows from $p_n < p_{n\ell}$ and $p_{n\ell} = u\mu_{1n}/(\mu_{1n} + \mu_{2n})$. Therefore, for any (p_d, p_n) such that $p_d \in [p_{d\ell}, p_{dh}]$, $p_n \in [0, p_{n\ell})$ and $p_n \leq p_d$, the profit of the seller is lower than in (9). This result is intuitive as lowering the price p_n below $p_{n\ell}$ reduces the profit per sale without increasing the probability of making a sale to a night shopper. Similarly, for any (p_d, p_n) such that $p_d \in [0, p_{d\ell})$, $p_n \in [0, p_{nh}]$ and $p_n \leq p_d$, the profit of the seller is lower than in (9), as lowering the price p_d below $p_{d\ell}$ reduces the profit per sale without increasing the probability of making a sale to a day shopper. Finally, as established in section 3.1, the seller is indifferent between posting the prices (p_d, p_n) with $p_n > p_d$ and the prices (p_d, p_d).

Taken together, the above observations imply that the seller's profit attains its maximum for all prices (p_d, p_n) such that $p_d \in [p_{d\ell}, p_{dh}]$, $p_n \in [p_{n\ell}, p_{nh}]$ and $p_n \leq p_d$, and it attains strictly less than the maximum for all other prices such that $p_n \leq p_d$. The profit maximization region is the shaded area in Figure 1. In light of this finding, it follows that an equilibrium such that all sellers post a night price lower than the day price and where the marginal price distributions F_d and F_n are given as in (5) and (6) exists if and only if we can find a joint price distribution G such that: (a) the support of G lies in the region of prices (p_d, p_n) with $p_d \in [p_{d\ell}, p_{dh}]$, $p_n \in [p_{n\ell}, p_{nh}]$ and $p_n \leq p_d$ (i.e., the support of G lies in the profit maximizing region), (b) the joint price distribution G generates the marginals F_d and F_n.

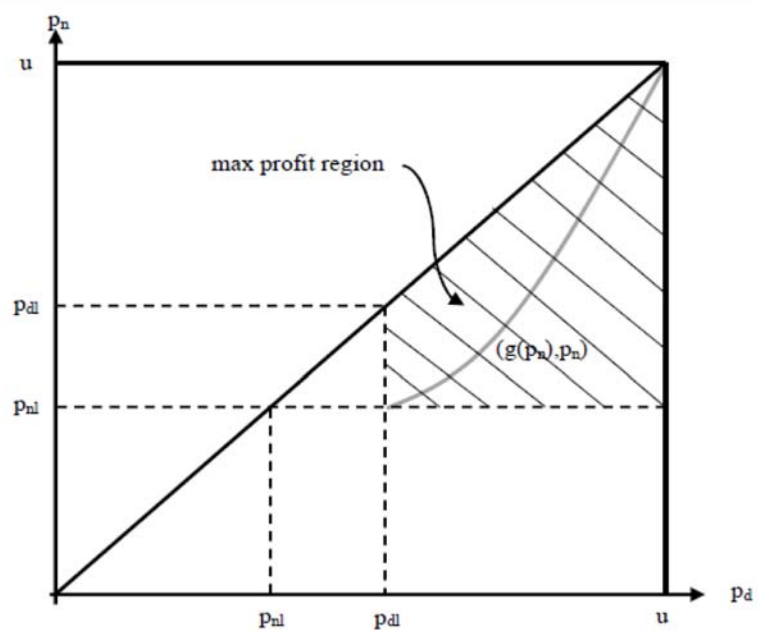


Figure 1: Equilibrium Profits

Clearly, a necessary condition for the existence of the desired equilibrium is that the marginal distribution of day prices first-order stochastically dominates the marginal distribution of night prices, i.e. $F_d(p) \leq F_n(p)$ for all $p \in [0, u]$. From (5) and (6), it follows that $F_d(p) \leq F_n(p)$ is equivalent to $\mu_{1d}/\mu_{2d} \geq \mu_{1n}/\mu_{2n}$. Moreover, the condition

$F_d(p) \leq F_n(p)$ or, equivalently, $\mu_{1d}/\mu_{2d} \geq \mu_{1n}/\mu_{2n}$ is also sufficient for the existence of the desired equilibrium. To see why this is the case, suppose that the distribution of sellers over night prices is the F_n in (6) and a seller with a night price of p_n posts the day price $g(p_n)$, where

$$g(p_n) = \left[\frac{\mu_{1n}}{\mu_{2n}}u + \left(\frac{\mu_{1d}}{\mu_{2d}} - \frac{\mu_{1n}}{\mu_{2n}}\right)p_n\right]^{-1}\frac{\mu_{1d}}{\mu_{2d}}p_n. \tag{11}$$

Given that sellers post $(g(p_n), p_n)$, it is immediate to verify that the marginal distribution of day prices is the F_d in (5). Moreover, if $\mu_{1d}/\mu_{2d} \geq \mu_{1n}/\mu_{2n}$, it is easy to verify that the support of the joint price distribution G lies in the region $p_d \in [p_{d\ell}, u]$, $p_n \in [p_{n\ell}, u]$, and $p_n \leq p_d$. The support of the joint price distribution G is represented by the gray curve in Figure 1.

Overall, the necessary and sufficient condition for the existence of the desired equilibrium is

$$\frac{\mu_{1d}}{\mu_{2d}} \geq \frac{\mu_{1n}}{\mu_{2n}}. \tag{12}$$

In words, the necessary and sufficient condition (12) states that the ratio of captive buyers—i.e. buyers who are in contact with a particular seller and nobody else—to non-captive buyers—i.e. buyers who are in contact with a particular seller and a second one—must be greater in the day than at night.

In what follows, we vary the parameters of the model $(\alpha_x, \beta_x, \alpha_y, \beta_y)$ and verify whether condition (12) is satisfied.

Case 1: *Buyers of type x are in contact with fewer sellers than buyers of type y and are less likely to shop at night, i.e. $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$.* Using (2)-(3) and (5)-(6), it is straightforward to verify that $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$ imply $\mu_{1d}/\mu_{2d} > \mu_{1n}/\mu_{2n}$ and $F_d < F_n$. Since condition (12) is satisfied, there exists a joint price distribution G whose support lies on the required region and that generates the marginals F_d and F_n in (5) and (6). Moreover, since $F_d < F_n$, any such joint price distribution G must be such that a positive measure of sellers posts a strictly lower price at night than during the day. Hence, the equilibrium features price dispersion both across stores and within stores, in the sense that a positive measure of sellers posts different prices at different times. Price dispersion across stores emerges because the equilibrium price distribution makes sellers indifferent between posting a high price, enjoying a high profit margin and selling a small quantity of the good and posting a low price, enjoying a low profit margin and selling a large quantity of the good. Price dispersion within stores emerges because, when $\alpha_x > \alpha_y$

and $\beta_x > \beta_y$, sellers have the incentive and the opportunity to price discriminate between different types of buyers. Indeed, since the two types of buyers differ in their likelihood to shop at night, sellers face a different composition of buyers in the two times of the day. Moreover, since the type of buyer who is more likely to shop at night is also the type of buyer who is in contact with more sellers, sellers face more competition at night. As a result, sellers find it optimal to post lower prices—in the sense of first-order stochastic dominance—at night than during the day.

Case 2: *Buyers of type x are in contact with fewer sellers than buyers of type y and they are more likely to shop at night, i.e. $\alpha_x > \alpha_y$ and $\beta_x < \beta_y$.* Using (2)-(3) and (5)-(6), one can verify that $\alpha_x > \alpha_y$ and $\beta_x < \beta_y$ imply $\mu_{1d}/\mu_{2d} < \mu_{1n}/\mu_{2n}$. Since condition (12) is violated, there exists no joint price distribution G whose support is on the required region and that generates the marginals F_d and F_n in (5) and (6). Let us explain this result. Since the two types of buyers differ with respect to their ability to shop at night, sellers face a different composition of buyers in the two times of the day. Moreover, since the type of buyer who is more likely to shop at night is in contact with fewer sellers, sellers face less competition at night. Hence, sellers would like to post higher prices at night than during the day but this is not compatible with equilibrium, as it would induce buyers who can shop at night to purchase during the day.

In between cases 1 and 2, there are two knife-edge cases.

Case 3: *Buyers of type x are in contact with fewer sellers than buyers of type y, but are equally likely to shop at night, i.e. $\alpha_x \geq \alpha_y$ and $\beta_x = \beta_y$.* Using (2) and (3), it is straightforward to verify that $\alpha_x \geq \alpha_y$ and $\beta_x = \beta_y$ imply that $\mu_{1d}/\mu_{2d} = \mu_{1n}/\mu_{2n}$. In turn, using (5) and (6), it is immediate to verify that $\mu_{1d}/\mu_{2d} = \mu_{1n}/\mu_{2n}$ implies $F_d = F_n$. Since $\mu_{1d}/\mu_{2d} = \mu_{1n}/\mu_{2n}$, we know that there exists a joint price distribution G whose support lies in the required region and that generates the marginals F_d and F_n. Moreover, since $F_d = F_n$ and all sellers must post a nighttime price non-smaller than their daytime price, the only equilibrium G is the one where every seller posts the same price at both times of day. Hence, while the equilibrium features price dispersion across sellers, it does not feature price dispersion within sellers. This result is intuitive. Since the two types of buyers are equally likely to shop at night, sellers faces the same composition of buyers and, hence, the same amount of competition in the two subperiods. For this reason, the equilibrium marginal price distribution is the same in the two times of day. And, since sellers post a lower price at night than during the day, this implies that every individual seller must always post the same price.

Case 4: *Buyers of type x are in contact with the same number of sellers as buyers of type y, but they are less likely to shop at night, i.e. $\alpha_x = \alpha_y$ and $\beta_x > \beta_y$.* Again, it is straightforward to verify that $\alpha_x = \alpha_y$ and $\beta_x > \beta_y$ imply $\mu_{1d}/\mu_{2d} = \mu_{1n}/\mu_{2n}$ and $F_d = F_n$ which, in turn, implies that every seller posts the same price during the daytime and the nighttime. This result is also intuitive. Since the two types of buyers differ with respect to their ability to shop at night, a seller faces a different composition of buyers in the two subperiods. However, since the two types of buyers are in contact with the same number of sellers, this difference in composition does not translate into a difference in competition. As a result, the equilibrium marginal price distribution during the day is the same as during the night, and every individual seller must always post the same price.

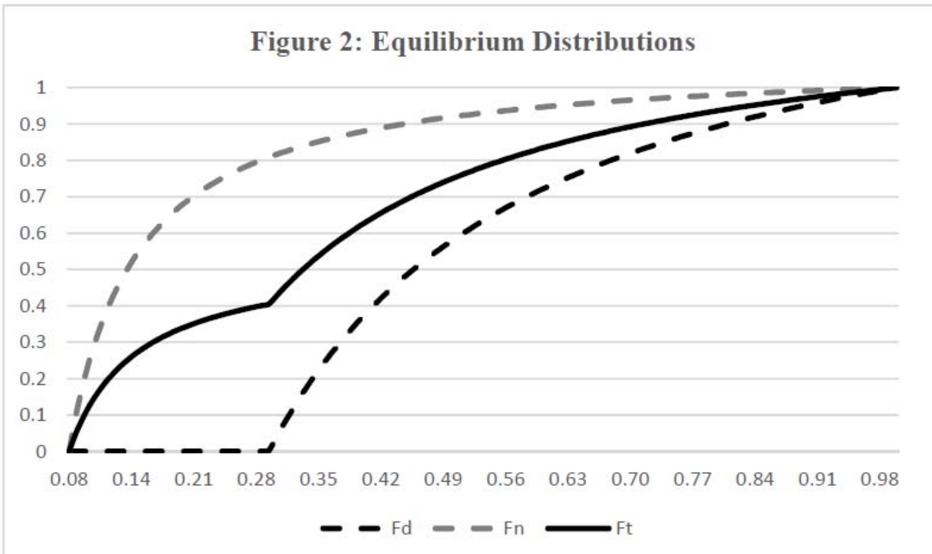
The above analysis is summarized in Proposition 1.

Proposition 1: *An equilibrium G in which sellers post prices $(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$ and such that the marginal price distributions F_d and F_n are as in (5) and (6) exists if and only if $\alpha_x = \alpha_y$ or $\beta_x \geq \beta_y$. If $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$, the equilibrium features price dispersion across and within sellers. If either $\alpha_x = \alpha_y$ or $\beta_x = \beta_y$, the equilibrium features price dispersion across sellers, but not within sellers.*

Figure 2 illustrates the type of equilibrium described by Proposition 1 for $\theta_x = \theta_y = 1/2$, $\alpha_x = 0.5$, $\alpha_y = 0.1$, $\beta_x = 0.9$, $\beta_y = 0.1$ and $u = 1$. The black dashed line is the distribution of day prices F_d. The gray dashed line is the distribution of night prices F_n. As explained above, the distribution of day prices first-order stochastically dominates the distribution of night prices. The black solid line is the overall price distribution, $F_t \equiv (F_d + F_n)/2$. All of these marginal distributions are uniquely pinned down. In contrast, the joint price distribution G is not pinned down uniquely. However, for any choice of G that is consistent with equilibrium, the model must generate both price dispersion across sellers—in the sense that there are differences in the average price of different sellers—and within sellers—in the sense that some sellers must post different prices during the day and during the night.[7] For a particular choice of G that is consistent with equilibrium, we can decompose the overall variance of prices into the variance in the average price of different stores (i.e., across store variance) and the variance of the price

[7] Here we illustrate the distribution of posted prices and decompose the associated variance into across and within store components. We could do the same for transaction prices.

of each store (i.e., within store variance). For instance, if sellers post prices $(g(p_n), p_n)$, we find that the overall standard deviation of prices is 23%, the across store variance is 56% of the overall variance, and the within store variance is 44%. These numbers are very close to those found by Kaplan and Menzio (2015), which shows that our simple model has the potential to match some of the key empirical facts about price dispersion.



3.3 Equilibrium without within-store price dispersion

In this section, we look for an equilibrium in which the joint price distribution, G, is such that every seller posts the same price for the good during the day and during the night and such that the marginal distribution of day and night prices is given by

$$F_d(p) = F_n(p) = 1 - \frac{\mu_{1n} + \mu_{1d}}{\mu_{2n} + \mu_{2d}} \frac{u - p}{p}, \forall p \in [p_\ell, p_h], \tag{13}$$

where the boundaries of the support of the distributions are

$$p_\ell = \frac{\mu_{1n} + \mu_{1d}}{\mu_{1n} + \mu_{1d} + \mu_{2n} + \mu_{2d}} u, \quad p_h = u. \tag{14}$$

As we shall see momentarily, $F_d = F_n$ is the marginal distribution that makes the profit of a seller posting the same price p in the day and at night constant for all $p \in [p_\ell, p_h]$.

First, consider a seller posting the prices (p, p) with $p \in [p_\ell, p_h]$. This seller obtains a profit of

$$\begin{aligned} V(p,p) &= \left[\mu_{1d} + \mu_{1n} + (\mu_{2d} + \mu_{2n}) \frac{\mu_{1d} + \mu_{1n}}{\mu_{2d} + \mu_{2n}} \frac{u-p}{p}\right] p \\ &= [\mu_{1d} + \mu_{1n}]\, u. \end{aligned} \tag{15}$$

The first line on the right-hand side of (15) follows from (8) and the expression for the marginal price distributions F_d and F_n in (13). The second line on the right-hand side of (15) follows from algebraic manipulation of the first. Notice that the second line on the right-hand side of (15) is a constant. That is, the seller attains the same profit by posting any prices (p, p) on the support of the joint distribution G. Moreover, this profit is equal to the profit that the seller would attain if he were to charge the buyer's reservation price u both in the day and in the night and sell only to those buyers who are not in contact with any other seller.

Second, consider a seller posting prices (p_d, p_n), with $p_d \in [p_\ell, p_h]$, $p_n \in [p_\ell, p_h]$ and $p_n \leq p_d$. This seller obtains a profit of

$$\begin{aligned} V(p_d, p_n) &= \left[\mu_{1d} + \mu_{2d} \frac{\mu_{1d} + \mu_{1n}}{\mu_{2d} + \mu_{2n}} \frac{u - p_d}{p_d}\right] p_d \\ &+ \left[\mu_{1n} + \mu_{2n} \frac{\mu_{1d} + \mu_{1n}}{\mu_{2d} + \mu_{2n}} \frac{u - p_n}{p_n}\right] p_n. \end{aligned} \tag{16}$$

Notice that the derivative of the seller's profit with respect to the night price, p_n, is strictly positive if $\mu_{1n}/\mu_{2n} < \mu_{1d}/\mu_{2d}$; it is zero if $\mu_{1n}/\mu_{2n} = \mu_{1d}/\mu_{2d}$; and it is negative if $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$. Hence, if the seller posts prices (p_d, p_n) with $p_d \in [p_\ell, p_h]$, $p_n \in [p_\ell, p_h)$ and $p_n \leq p_d$, he attains a profit non-greater than (15) if and only if $\mu_{1n}/\mu_{2n} \leq \mu_{1d}/\mu_{2d}$.

Third, consider a seller posting prices (p_d, p_n), with $p_d \in [p_\ell, p_h]$, $p_n \in [0, p_\ell]$ and $p_n \leq p_d$. This seller's profit is lower than what he could attain by posting the prices (p_d, p_ℓ), as lowering the price p_n below p_ℓ reduces the profit per sale without increasing the probability of making a sale to a night shopper. Similarly, for any (p_d, p_n) such that $p_d \in [0, p_\ell)$, $p_n \in [0, p_h]$ and $p_n \leq p_d$, the seller's profit is lower than what he could attain by posting the prices (p_ℓ, p_n), as lowering the price p_d below p_ℓ reduces the profit per sale without the probability of making a sale to a day shopper. Finally, as established in section 3.1, the seller is indifferent between posting the prices (p_d, p_n) with $p_n > p_d$ and the prices (p_d, p_d).

From the above observations, it follows that the seller's profit is maximized everywhere on the support of the joint price distribution G if and only if

$$\frac{\mu_{1n}}{\mu_{2n}} \leq \frac{\mu_{1d}}{\mu_{2d}}. \tag{17}$$

In words, the necessary and sufficient condition (17) states that the ratio of captive buyers to non-captive buyers must be greater at night than during the day. Notice that condition (17) is the opposite as condition (12) and, hence, for any values of the parameters, there exists either the type of equilibrium studied in Subsection 3.2 or the type of equilibrium studied in this subsection. Moreover, the two types of equilibria coexist only when $\mu_{1n}/\mu_{2n} = \mu_{1d}/\mu_{2d}$, which is a knife-edge configuration of parameters.

In particular, we have the following cases.

Case 1: *Buyers of type x are in contact with fewer sellers than buyers of type y and are less likely to shop at night, i.e. $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$.* When $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$, condition (17) is violated and, hence, there is no equilibrium in which all sellers post the same price at both times of day, and the marginal price distributions F_d and F_n are given by (13). Intuitively, when $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$, sellers face more competition at night than during the day. For this reason, sellers have an incentive to post lower prices—in the sense of first-order stochastic dominance—at night than during the day.

Case 2: *Buyers of type x are in contact with fewer sellers than buyers of type y and they are more likely to shop at night, i.e. $\alpha_x > \alpha_y$ and $\beta_x < \beta_y$.* When $\alpha_x > \alpha_y$ and $\beta_x < \beta_y$, condition (17) is satisfied and, hence, there exists an equilibrium in which all sellers post the same price at both times of day, and the marginal price distributions F_d and F_n are given by (13). In this equilibrium, there is price dispersion across stores—in the sense that different sellers post different prices—but no price dispersion within stores—in the sense that every seller posts the same price in the two subperiods. Intuitively, when $\alpha_x > \alpha_y$ and $\beta_x < \beta_y$, sellers face more competition during the day than at night. For this reason, sellers want to post a night price as high as possible. However, sellers cannot post a night price higher than the day price or, else, buyers who can shop at night will purchase the good during the day. As a result, sellers post a night price equal to the day price.

In between cases 1 and 2, there are two knife-edge cases. In these cases, the type of equilibrium that we considered in Subsection 3.2 and the type of equilibrium that we are considering here coexist and coincide.

Case 3: *Buyers of type x are in contact with fewer sellers than buyers of type y, but are* *equally likely to shop at night, i.e.* $\alpha_x \geq \alpha_y$ *and* $\beta_x = \beta_y$. In this case, condition (17) holds with equality. Therefore, there exists an equilibrium in which sellers post the same price during the day and during the night, and the marginal price distributions F_d and F_n are given as in (13). Intuitively, when $\alpha_x \geq \alpha_y$ and $\beta_x = \beta_y$, sellers face the same composition of buyers during the day and during the night and, hence, they have no incentive to vary their price over time. Notice that, when $\alpha_x \geq \alpha_y$ and $\beta_x = \beta_y$, condition (12) holds as well and, hence, there exists also an equilibrium in which the marginal price distributions F_d and F_n are given as in (5) and (6). However, as discussed in the previous subsection, this equilibrium is also such that sellers post the same price in the two subperiods. Moreover, it is immediate to see that the marginal price distributions F_d and F_n in (5) and (6) are the same as in (13). Hence, the two types of equilibria coexist and are identical.

Case 4: *Buyers of type* x *are in contact with the same number of sellers as buyers of type* y*, but they are less likely to shop at night, i.e.* $\alpha_x = \alpha_y$ *and* $\beta_x \geq \beta_y$. In this case, condition (17) holds with equality. Therefore, there exists an equilibrium in which sellers post the same price during the day and during the night, and the marginal price distributions F_d and F_n are given as in (13). Intuitively, when $\alpha_x = \alpha_y$ and $\beta_x \geq \beta_y$, sellers face a different composition of buyers during the day and during the night but this difference in composition does not translate into a difference in competition because both types of buyers are in contact with the same number of sellers. For this reason, sellers have no incentive to vary their price over time. Notice that, also when $\alpha_x = \alpha_y$ and $\beta_x \geq \beta_y$, this equilibrium coexists and coincides with the one studied in Subsection 3.2.

The above analysis is summarized in Proposition 2.

Proposition 2: *An equilibrium* G *in which all sellers post the same price in the day as in the night and the marginal price distributions* F_d *and* F_n *are given as in (13) exists if and only if* $\alpha_x = \alpha_y$ *or* $\beta_x \leq \beta_y$.

3.4 Other equilibria

The final step of the analysis is to rule out the existence of any type of equilibrium different from those studied in Subsections 3.2 and 3.3. To this aim, consider an equilibrium distribution of sellers over prices, $G(p_d, p_n)$. Let $F_d(p_d)$ denote the marginal distribution of sellers over day prices and as $m_d(p_d)$ the measure of sellers who post a day price of p_d, i.e. the mass point associated with the price p_d. Similarly, let $F_n(p_n)$ denote the marginal distribution of sellers over night prices and as $m_n(p_n)$ the measure of sellers who post a night price of p_n. In light of Lemma 1, we can restrict attention to equilibria in which all sellers post a price $p_n \leq p_d$ and, consequently, such that the marginal distribution of sellers over their lowest price, F_m, is equal to F_n.

In equilibrium, a seller posting prices (p_d, p_n) with $p_n \leq p_d$ attains a profit of

$$V(p_d, p_n) = V_d(p_d) + V_n(p_n), \tag{18}$$

where V_d and V_n are respectively defined as

$$V_d(p_d) = \left[\mu_{1d} + \mu_{2d}\left(1 - F_d(p_d) + \frac{1}{2}m_d(p_d)\right)\right] p_d, \tag{19}$$

and

$$V_n(p_n) = \left[\mu_{1n} + \mu_{2n}\left(1 - F_n(p_n) + \frac{1}{2}m_n(p_n)\right)\right] p_n. \tag{20}$$

In words, $V_d(p_d)$ denotes the seller's profit from trades that take place during the day. In fact, during the day, the seller meets μ_{1d} captive buyers and μ_{2d} non-captive buyers. A captive buyer purchases the good from the seller with probability one. A non-captive buyer purchases the good from the seller with probability one if he is in contact with a second seller whose price is strictly greater than p_d, an event that occurs with probability $1 - F_d(p_d)$, or with probability $1/2$ if he is contact with a second seller whose price is equal to p_d, an event that occurs with probability $m_d(p_d)$. Similarly, $V_n(p_n)$ denotes the seller's profit from trades that take place during the night.

Every price pair (p_d, p_n) on the support of the distribution G must maximize the profit $V(p_d, p_n)$ of the seller. We use this property to establish several features of the equilibrium.

Claim 1: *The marginal price distributions F_d and F_n have no mass points.*

Proof: We begin by proving that F_d has no mass points. On the way to a contradiction, suppose that there exists an equilibrium G in which F_d has a mass point at p_d^*. Consider a seller posting the prices (p_d^*, p_n) with $p_n < p_d^*$. From (18), it follows that this seller can attain a strictly higher profit by posting the prices $(p_d^* - \epsilon, p_n)$ for some $\epsilon > 0$ sufficiently small. Hence, no prices (p_d^*, p_n) with $p_n < p_d^*$ can be on the support of G. Next, consider a seller posting prices (p_d^*, p_d^*). From (18), it follows that this seller can attain a strictly higher profit by choosing the prices $(p_d^* - \epsilon, p_d^* - \epsilon)$ for some $\epsilon > 0$ sufficiently small. Hence, the prices (p_d^*, p_d^*) cannot be on the support of G. Finally, since G is such that every seller posts a price p_n smaller than p_d, no prices (p_d^*, p_n) with $p_n > p_d^*$ can be on the support of G. We have thus reached a contradiction. The proof that F_n has no mass points is analogous $\blacksquare$

Claim 2: *The marginal price distribution F_d has no gaps and $p_{dh} = u$.*

Proof: We first establish that F_d has no gaps. On the way to a contradiction, suppose that F_d has a gap between p_0 and p_1 with $p_1 > p_0$. Since $F_d(p_1) = F_d(p_0)$, a seller posting prices (p_0, p_n) with $p_n \leq p_0$ can attain a strictly higher profit by choosing the prices (p_1, p_n) instead. Hence, the prices (p_0, p_n) with $p_n \leq p_0$ cannot be on the support of G. Similarly, since G is such that every seller posts a price p_n smaller than p_d, no prices (p_0, p_n) with $p_n > p_0$ can be on the support of G. We have thus reached a contradiction. The proof that $p_{dh} = u$ is analogous. $\blacksquare$

Claim 3: *Let $p_{n\ell}$ be the lower bound of the support of the marginal price distribution F_n. The profit function $V_n(p_n)$ is weakly increasing in p_n over the interval $[p_{n\ell}, u]$.*

Proof: On the way to a contradiction, suppose $V_n(p_n)$ is strictly decreasing over the interval (p_0, p_1), with $p_{n\ell} \leq p_0 < p_1 \leq u$. If this is the case, $V_n(p_n) < V_n(p_0)$ for all $p_n \in (p_0, p_2)$ where $p_2 > p_1$. Any seller with a day price $p_d \geq p_2$, will choose a night price p_n such that $p_n \leq p_d$ and $p_n \notin (p_0, p_2)$. Any seller with a day price $p_d \in (p_0, p_2)$, will choose a night price $p_n \leq p_0$. And any seller with a day price $p_d \leq p_0$, will choose a night price smaller than p_d. Therefore, the marginal price distribution F_n has a gap between p_0 and p_2, i.e. $F_n(p_n) = F_n(p_0)$ for all $p \in (p_0, p_2)$. From (20), it follows that, if F_n is constant over the interval (p_0, p_2), then $V_n(p)$ is strictly increasing over the interval (p_0, p_2) which contradicts the assumption that $V_n(p)$ is strictly decreasing over the interval (p_0, p_1). $\blacksquare$

Claim 4: *The function $V_n(p_n)$ is either strictly increasing for all $p_n \in [p_{n\ell}, u]$, or it is constant for all $p_n \in [p_{n\ell}, u]$.*

Proof: Suppose $V_n(p_n)$ is strictly increasing over some region (p_0, p_1), where $p_{n\ell} \leq p_0 < p_1 \leq u$. This implies that a seller posting a day price $p_d \geq p_1$ chooses a night price $p_n \geq p_1$. A seller posting a day price $p_d \in (p_0, p_1)$ chooses a night price $p_n = p_d$. And a seller posting a day price $p_d \leq p_0$ must post a night price $p_n \leq p_0$. Therefore, for all $p \in (p_0, p_1)$, the fraction of sellers with a night price smaller than p is equal to the fraction of sellers with a day price smaller than p, i.e. $F_n(p) = F_d(p)$ for all $p \in (p_0, p_1)$. Using this fact and $V(p,p) = V(p_1, p_1)$ for all $p \in (p_0, p_1)$, we obtain

$$F_n(p) = F_n(p_1) - \frac{\mu_{1n} + \mu_{1d} + (\mu_{2n} + \mu_{2d})(1 - F_n(p_1))}{\mu_{2n} + \mu_{2d}} \frac{p_1 - p}{p}, \forall p \in (p_0, p_1). \tag{21}$$

Given the expression for F_n in (21), we can compute the derivative of the function $V_n(p_n)$, which is given by

$$V_n'(p_n) = \mu_{1n} - \mu_{2n} \frac{\mu_{1n} + \mu_{1d}}{\mu_{2n} + \mu_{2d}}, \forall p \in (p_0, p_1). \tag{22}$$

The derivative is strictly positive if and only if $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$. Thus, if $\mu_{1n}/\mu_{2n} \leq \mu_{1d}/\mu_{2d}$, the function $V_n(p_n)$ cannot be strictly increasing over the region (p_0, p_1) and, in light of Claim 3, it must be constant for all $p \in [p_{n\ell}, u]$.

Conversely, suppose $V_n(p_n)$ is constant over some region (p_0, p_1) with $p_0 \geq p_{n\ell}$. In this case, we can prove that $\mu_{1n}/\mu_{2n} \leq \mu_{1d}/\mu_{2d}$. Thus, if $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$, the function $V_n(p_n)$ cannot be constant over some region (p_0, p_1) and, in light of Claim 3, it must be strictly increasing for all $p \in [p_{n\ell}, u]$. ■

Now, suppose that the equilibrium is such that $V_n(p_n)$ is constant over the interval $[p_{n\ell}, u]$. In this case, it is straightforward to verify that the marginal distribution of night prices, F_n, is given as in (5). Moreover, since $V_n(p_n)$ is constant, the function $V_d(p_d)$ must also be constant over the interval $[p_{d\ell}, u]$. It is also straightforward to verify that this implies that the marginal distribution of day prices, F_d, is given as in (6). Thus, the only equilibrium with a constant $V_n(p_n)$ is the one characterized in Subsection 3.2.

Next, suppose that the equilibrium is such that the function $V_n(p_n)$ is strictly increasing over the interval $[p_{n\ell}, u]$. In this case, every seller posts the same price in the day as in the night and the marginal price distributions F_d and F_n are identical. In turn, this implies that the marginal price distributions F_d and F_n are given as in (13). Thus, the only equilibrium with a strictly increasing $V_n(p_n)$ is the one characterized in Subsection 3.3.

Thus, we have established the following result.

Proposition 3: *Any equilibrium G is such that either: (i) the marginal price distributions F_d and F_n are given as in (6) and (7); or (ii) the marginal price distributions F_d and F_n are given as in (13).*

4 Conclusions

We developed a search-theoretic framework that generates equilibrium price dispersion across sellers and within sellers. Price dispersion across sellers obtains because of the buyers are heterogeneous in their ability to shop at different stores. Price dispersion within sellers obtains when the buyers who are better at shopping at different stores are also better at shopping at less different times and, hence, sellers can discriminate between different types of buyers by varying their price over time. Our model is richer than standard models of intertemporal price discrimination and standard models of price dispersion. Naturally, it would be interesting to estimate our model using the econometric techniques developed by Hong and Shum (2006) and Moraga-Gonzales and Wildenbeest (2009). Also, it would be useful to intergrate our model with the multiproduct model of Kaplan et al. (2015) to build a unified framework for studying pricing in the retail market.

References

[1] Aguiar, M., and E. Hurst. 2007. "Life-Cycle Prices and Production." *American Economic Review*, 97: 1533-1559.

[2] Aguirregabiria, V. 1999. "The Dynamics of Markups and Inventories in Retailing Firms." *Review of Economic Studies*, 66: 275-308.

[3] Albrecht, J., Postel-Vinay, F., and S. Vroman. 2013. "An Equilibrium Search Model of Synchronized Sales." *International Economic Review*, 54: 472-493.

[4] Benabou R. 1988. "Search, Price Setting and Inflation." *Review of Economic Studies*, 55: 353-376.

[5] Burdett, K., and K. Judd. 1983. "Equilibrium Price Dispersion." *Econometrica* 955-969.

[6] Burdett, K., and G. Menzio. 2014. "The (Q, S, s) Pricing Rule." NBER Working Paper 19094.

[7] Butters, G. 1977. "Equilibrium Distributions of Sales and Advertising Prices." *Review of Economic Studies*, 44: 465-491.

[8] Conlisk, J, Gerstner, E., and J. Sobel. 1984. "Cyclical Pricing by a Durable Goods Monopolist." *Quarterly Journal of Economics*, 99: 489-505.

[9] Head, A., Liu, L., Menzio, G, and R. Wright. 2012. "Sticky Prices: A New Monetarist Approach." *Journal of the European Economic Association*, 10: 939-973.

[10] Hong, J., and M. Shum. 2006. "Using Price Distributions to Estimate Search Costs." *Rand Journal of Economics*, 37: 257-275.

[11] Hotelling, H. 1929. "Stability in Competition". *Economic Journal*, 39: 41-57.

[12] Kaplan, G., and G. Menzio. 2014a. "Shopping Externalities and Self-Fulfilling Unemployment Fluctuations." *Journal of Political Economy*. Forthcoming.

[13] Kaplan, G., and G. Menzio. 2014b. "The Morphology of Price Dispersion." *International Economic Review*. Forthcoming.

[14] Kaplan, G., Menzio, G., Rudanko, L., and N. Trachter. 2015. "Relative Price Dispersion: Evidence and Theory." Manuscript, Princeton University.

[15] Lazear, E. 1986. "Retail Pricing and Clearance Sales." *American Economic Review*, 76: 14-32.

[16] Lester, B., Shourideh, A., Venkateswaran, V., and A. Zeitlin-Jones. 2015. "Screening and Adverse Selection in a Frictional Market." Manuscript, University of Pennsylvania.

[17] Menzio, G., and N. Trachter. 2014. "Equilibrium Price Dispersion with Sequential Search." NBER Working Paper 19990.

[18] Moraga-Gonzales, J., and M. Wildenbeest. 2008. "Maximum Likelihood Estimation of Search Costs." *European Economic Review*, 52: 820-848.

[19] Rhodes, A. 2015. "Multiproduct Retailing." *Review of Economic Studies*, 82: 360-390.

[20] Sheshinski, E., and Y. Weiss. 1982. "Inflation and Costs of Price Adjustment." *Review of Economic Studies*, 44: 287-303.

[21] Sobel, J. 1984. "The Timing of Sales." *Review of Economic Studies*, 51: 353-368.

[22] Sorensen, A. 2000. "Equilibrium Price Dispersion in Retail Markets for Prescription Drugs." *Journal of Political Economy*, 108: 833-850.

[23] Varian, H. 1980. "A Model of Sales." *American Economic Review*, 70: 651-659.

[24] Zhou, J. 2014. "Multiproduct Search and the Joint Search Effort." *American Economic Review*, 104: 2918-2939.