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LIFE-CYCLE PERMANENT-INCOME MODEL AND CONSUMER DURABLES

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ABSTRACT

This paper presents an extension of the life-cycle permanent-income model of consumption to the case of a durable good whose purchase involves lumpy transactions costs. Where individual behavior is concerned, the implications of the model are different in some respects from those of standard consumption theory. Specifically, rather than choose an optimal path for the service flow from durables, the optimizing consumer will choose an optimal range and try to keep his service flow inside that range. The dynamics implied by this behavior is different from that of the stock adjustment model. Properties of aggregate durables consumption are derived by explicit aggregation. In particular, it is shown that expenditures on durables display very large short-run elasticity to changes in permanent income. Empirical tests of the sort suggested by Hall (1978) generally produce results that are in line with the predictions of the theory.

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I. THE LIFE CYCLE MODEL AS A THEORY OF CONSUMER SPENDING

In the 1950s, Franco Modigliani and Richard Brumberg (1954) and Milton Friedman (1957) turned the abstract Fisherian model of intertemporal maximization into an operational model of consumption. The model they developed,[1] which drew the crucial distinction between consumption and consumer expenditures and applied only to the former, has three main empirical implications about consumption at the microeconomic level:

(1) Consumption at every age is proportional to permanent income, i.e. to the present value of expected lifetime resources.

(2) Consumption is insensitive to transitory fluctuations in income that do not affect permanent income.

(3) Under the usual assumption that the utility function is CES, consumption grows or shrinks at a rate $(r-\alpha)/b$, where r is the rate of interest, α is the rate of subjective time discounting, and b is a taste parameter.

Item 1 on this list was, of course, the empirical observation that motivated the theory. Item 2 is the distinguishing characteristic of the theory. Both of these apply to aggregate as well as to individual consumption. Item 3 points out an implication of the particular utility function needed to derive item 1; but it does not survive aggregation.[2]

The LCH/PIH is often held up as a model of good economics, and rightly so. A puzzling set of empirical phenomena was explained by a theory based on maximizing behavior. That theory was then translated into econometrically estimatable equations and subjected to a battery of empirical tests--with generally favorable results. Yet, from a business cycle perspective, the model has at least one serious shortcoming: it applies only to consumption, not to consumer expenditures, while spending on durables accounts for most of the cyclical variability. Thus the PIH needs to be supplemented by a model of expenditures on consumer durables (and also, of course, by a way to translate durable stocks into the service flows required by the theory). The usual way to do this is by the stock (partial) adjustment model.

The stock adjustment (SA) model assumes the presence of convex adjustment costs that give rise to a lagged adjustment of actual to "desired" stocks. Each period a constant fraction γ of the gap between the desired and actual stocks is closed,

$$(1)\quad K_{t+1} - K_t = \gamma(K^{\star}_{t+1} - K_t),$$

where the parameter γ, $0 \leq \gamma \leq 1$, is called the "speed of adjustment." K_t denotes the durable stock at the beginning of period t and $K^{\star}_t$ is the desired stock at that time. On the further assumption that depreciation is proportional to the stock, gross expenditures are given by:

$$(2)\quad E_{t+1} = \gamma(K^{\star}_{t+1} - K_t) + \delta K_t$$

where δ is the periodical depreciation rate. According to the LCH/PIH, the desired stock will be proportional to permanent income. But the existence of adjustment costs ($\gamma < 1$) results in a deviation of the actual stock, and thus the service flow of durables, from the desired level.

Despite the SA model's wide acceptance, it has some obvious theoretical and empirical drawbacks.

The first pertains to its microfoundations. It has been known for a long time (Holt, Modigliani, Muth and Simon (1960)) that the SA model can be justified rigorously by quadratic costs of adjusting durable stocks. But this assumption is counterintuitive. There is no apparent reason why it should be less costly to adjust durable holdings in several small steps rather than all at once. In a recent attempt to rationalize the assumption of quadratic adjustment costs, Bernanke (1985) claimed that "it takes time to shop for and acquire a new car." That is no doubt true, but it implies the existence of transactions costs, not adjustment costs. With lumpy transactions costs, we show below, consumers either fully adjust by replacing their old durable good or do not adjust at all. The SA model implies, instead, that they will partially adjust, i.e. purchase successively better durable goods over several consecutive periods. Even if partial adjustment at the aggregate level is conceivable (more on this below), it is hard to accept the conclusion for individuals.

Second, it is well known that an important property of demand for durables is that the purchasing decision can be advanced or postponed. This, many people suspect, is why spending on durables is so volatile. It is difficult to integrate this idea into the SA framework.

Third, the SA model is really a model of expansion demand. Replacement demand is simply grafted on as a fraction of the current stock. But the separation between expansion and replacement demands is an artificial one; consumers make one decision about durable purchases and do not distinguish between the "replacement" and "expansion" parts. In addition, actual data on durables expenditures lump the two components together; unfortunately for the theory, most of the demand is the unmodelled, replacement part.

Fourth, it is generally assumed that the speed of adjustment is constant and independent of any economic variables. In principle, this ought not to be the case; some variables, like credit rationing, interest rates, and supply constraints, should affect γ. However, "in the simple stock-adjustment framework, desired stocks are the only channel through which economic variables can act." (Deaton and Muellbauer (1980), p. 353).

Finally, when the SA model is estimated empirically, the estimated adjustment speeds are very low; in most cases below 30% per quarter.

These problems all suggest that there is room for improvement. In particular, we will derive here a model in which the underlying microeconomics makes better sense; in which postponement/advancement decisions and depreciation are integrated; and in which aggregate behavior is not necessarily a blowup of individual behavior. Instead, the model distinguishes between the individual dynamics, which may be quite discontinuous, and relatively smooth aggregate dynamics.

II. MICROFOUNDATIONS OF THE (S,s) MODEL

A. Discrete versus Smooth Adjustment

In deriving a new model of the demand for durable goods, we begin by solving for the decision rule of a single consumer. This decision rule will then be aggregated to yield the consumption behavior of the whole economy, which may be very different from that of any single cosnumer.

The underlying assumption is that the market for durables is characterized by important lumpy transactions costs. The possible origins of these costs are many. Sometimes transactions costs are large and explicit (e.g., in buying a house). Alternatively, since durable goods are characterized by variety of characteristics, potential buyers must spend time and effort finding the right combination for their purposes. So search costs, whether described as time costs, utility costs, or financial costs, may be heavy. A third source of transactions costs is asymmetric information between buyers and sellers of durable goods -- which gives rise to the "lemons' principle (Akerlof, 1970). As a result, the buyer suffers a lumpy cost - in the form of a loss of a fixed percentage of value - as soon as he takes the durable home.

Another property which distinguishes durables from nondurables is noncombinability (Lancaster, 1979), which means, for example, that two used cars cannot be combined to make one new car. Noncombinability implies that if a consumer wants to increase his flow of durable services, he will probably have to replace his old "car" rather than buy a small addition to it. And, because of the "lemons" effect, each replacement will involve a lumpy cost which is a fraction of the purchase.

Despite the apparent similarity between adjustment costs and transactions costs, the two are very different, both in nature and in their empirical implications. The combination of lumpy trnasactions costs and noncombinability implies that durable purchases will be made infrequently. If the deviation between actual and desired stocks is small, people will not find it worthwhile to pay the transactions costs necessary to change their durable good. Instead, they will wait until the deviation is large enough to justify the costs involved in the transaction. Quadratic adjustment costs imply the opposite dynamics: Even if the discrepancy between desired and actual stocks is small, the old "car" should be "replaced" by a slightly better one. Moreover, this should not be done in one shot, but rather spread over several periods.

It is obvious that people purchase durable goods infrequently and, when they do, the additions to their stocks are significant. Unfortunately, lumpy transactions costs are much more difficult to model than the quadratic adjustment costs that underly the SA model. Yet modelling them properly is worth the effort because similar theoretical and empirical problems arise in so many areas of economics.

For example, partial adjustment is often unthinkingly grafted on to models of the demand for money, which then display puzzlingly low speeds of adjustment. But this makes little snse. What possible reason can there be to assume that it costs four times as much to make twice as large a change in one's money holdings? Is it not more likely that marginal adjustment costs are decreasing, or even zero? The same reasoning applies, more generally, to all portfolio adjustments. Contrary to naive theory, investors do not adjust their portfolios continuously, and for good reasons: because there are fixed costs of doing so. Similarly, government officials and business executives may be reluctant to pay the fixed costs of decisionmaking until they are convinced that their current policy is far from optimal. Quadratic adjustment costs have been used to rationalize the Q-theory of investment (see Abel, 1980). But here, just as with consumer durables, lumpy adjustment costs are far more plausible. It is believable that it costs a firm 49 times as much to install seven new drill presses as it costs to install one? We think not.

Indeed, we find it hard to imagine any application in which the (commonly made) assumption of quadratic adjustment costs is more reasonable than the (rarely made) assumption of lumpy transactions costs. Economists' standard theory of gradual adjustment seems to need rethinking. Fortunately, there is a well-known body of analysis in the inventory literature that applies to the case of fixed transactions costs. It leads to the so-called (S,s) or two-bin policy. The basic idea of this approach, which we apply here to consumer durables, is that the optimal plan is defined by a target point S and a trigger point s. If the stock (of money, inventories, or durable goods) falls below level s, an order to restore the stock to level S is made; otherwise, no order is made. We now show how the (S,s) rule applies to the demand for durable goods with lumpy transactions costs.

B. An (S,s) Rule for Durables

Suppose that a consumer consumes two commodities: a perishable good X and a durable good K which depreciates at a constant exponential rate μ.[3] Denote by $q < 1$ the ratio of the selling price of durables to the purchasing price; thus the lumpy transactions cost is a fraction (1-q) of the purchase price, as suggested by the "lemons" principle. We would like to see what effect this parameter has on the consumption plan of a consumer who maximizes lifetime utility subject to a lifetime budget constraint. Assume that the instantaneous utility function is of the standard LCH/PIH form:

(3) $u(K_t, X_t) = aK_t^k + bX_t^k \qquad k < 1,$

where we assume, as is usual, that the flow of services from durables is proportional to the stock. Assuming time separability and an infinite horizon, the consumer wants to maximize:

(4) $U = \int_0^{\infty} u(K_t, X_t)e^{-\alpha t}dt,$

where U is lifetime discounted utility and α is the rate of subjective time discounting.

It is obvious that, because of the lumpy transactions costs, durable purchases will take place only occasionally, for continuous replacement implies infinite transactions cost. Denote by S_n the durable stock immediately after the nth durable purchase which takes place at time t_n. That good will be replaced at time t_{n+1}, when it has deteriorated to a value s_n given by:

(5) $s_n = S_n \exp\,[-\mu(t_{n+1} - t_n)]$

Thus the discounted utility obtained while the nth "car" is held will be,

(6) $\int_{t_n}^{t_{n+1}} u[S_n \exp\,(-\mu(t-t_n)), X_t]e^{-\alpha t}\,dt$

Summation over all lifetime purchases of durables and use of the specific functional form (3) yields the following expression for lifetime utility,

$$(7)\quad U=[a/(\mu k+\alpha)] \sum_{n=1}^{\infty} [\{\exp[-(\mu k+\alpha)t_n]-\exp[-(\mu k+\alpha)t_{n+1}]\} (S_n e^{\mu t_n})^k]+b\int_0^{\infty} e^{-\alpha t}(X_t)^k\, dt$$

which is homogenous of degree k in its arguments S and X.

In order to derive the budget constraint, assume that the nondurable good X is the numeraire and that the relative price of durables to nondurables is constant. Denote the purchase price of one "unit" of the durable good by p; therefore the resale price is qp. The discounted cost of the nth durable good is,

$$(8)\quad pS_n \exp(-rt_n)$$

where $r > 0$ is the interest rate. The discounted income from selling this "car" at time t_{n+1} will be,

$$(9)\quad qp\, S_n \exp\,[-rt_{n+1} - \mu\, (t_{n+1} - t_n)]$$

The difference between (8) and (9) is the net expenditure on the nth car. Summation over all durable transactions and inclusion of spending on nondurables yield the following lifetime budget constraint:

$$(10)\quad W = p \sum_{n=1}^{\infty} \left\{[e^{-rt_n} - q \exp[-rt_{n+1} - \mu\, (t_{n+1} - t_n)]]S_n\right\} + \int_0^{\infty} e^{-rt}\, X_t\, dt$$

where W denotes total (human and nonhuman) lifetime wealth. Notice that the budget constraint is homogenous of degree 1 in its arguments X and S_1, S_2, ...

The intertemporal optimization problem of the consumer is maximization of total discounted utility subject to the lifetime budget constraint. The solution consists of a plan for nondurable consumption, X_t, and two infinite series of trigger pionts (S_1, S_2, ...) and (s_1, s_2, ...) which denote the stocks immediately after the purchase and just before resale, respectively. However, the homogeneity of lifetime utility and the linearity of the budget constraint simplify the solution significantly and reduce the infinite number of parameters in the s and S series to only three: S_1, S_{n+1}/S_n and s_n/S_n. Similarly, the nondurable consumption plan X_t is characterized by only two parameters: the initial consumption and a constant exponential growth rate. Moreover, the growth rates of the consumption plans of both goods are the same, which reduces the total number of parameters to four. All this is summarized in the following theorem:

Theorem 1: The optimal consumption plan (S, s, X) exhibits the following properties:[4]

(i) X_o, all the S_n, and all the s_n are proportional to total lifetime wealth.

(ii) The ratio s_n/S_n defined by (5) is constant, so the interval between purchases, τ, is constant

(iii) The ratio S_{n+1}/S_n is constant and equal to $e^{\tau g}$ where $g = \frac{r-\alpha}{1-k}$.

(iv) The growth rate of nondurable consumption is constant and equals the growth rate g in (iii).

The inclusion of lumpy transactions costs in the durable goods market changes the durable transactions plan substantially from a continuous to a discrete one. Except for isolated points in time at which a purchase is made, the consumer will not be active in the market for durables. Do the key features of the PIH/LCh still hold with transactions costs? The answer is that these properties do hold in the "long run," but not in the "short run." Specifically, the pattern of durables stock, and therefore the service flow from durables, follows a ratchet path, as shown in Figure 1. This path, of course, differs in details from the predictions of the strict PIH. However, the envelope curve which connects the S_i levels in Figure 1 does follow the PIH/LCH predictions: It is proportional to permanent income; the rate of growth of consumption (g) is identical to that implied by the PIH; and transitory income affects

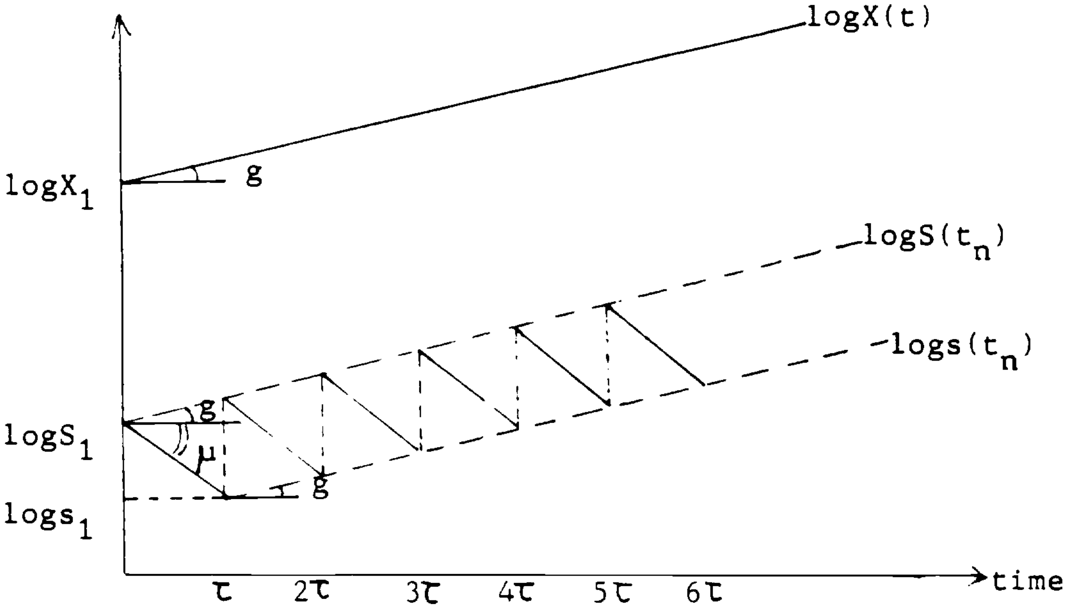
logX(t)
logX1
g
logS(tn)
logs(tn)
logS1
g
μ
g
logs1
τ
2τ
3τ
4τ
5τ
6τ
time

Figure 1

consumption only insofar as it changes permanent income.[5] Thus in the short run, between purchases, there are deviations from the strict PIH/LCH predictions, deviations which are larger the larger are transactions costs. However, in the long run these deviations are rectified and the consumption plan returns to the PIH/LCH path. Notice also that since each purchase of a durable good involves a lumpy transactions cost, each change lowers the value of lifetime wealth. Thus total lifetime wealth, W, will follow a discontinuous ratchet pattern. At a time of a durable purchase, when the durable stock jumps up, wealth jumps down.

It is of interest to compare the predictions of the (S,s) model with those of the SA model. The inclusion of transactions costs increases the number of choice variables from one (the "desired" stock) to two, S and s -- a change which has considerable implications for the microdynamics. To see this, assume that the rate of time discounting (α) equals the discount rate (r) so that the trigger points, S and s, do not change unless new information is received. The SA model implies that the individual will hold a constant durable stock by replacing the depreciated amount each period. The (S,s) dynamics, by contrast, follow a ratchet path. Even with no new information the stock in different periods may be different, and durable purchases will not be made each period. If an unanticipated income shock takes place, the SA model predicts a smooth exponential convergence toward the new desired stock. No such smooth partial adjustment is predicted by the (S,s) model. Everyone adjusts either fully or not at all. Thus unanticipated changes may induce large contemporaneous changes for those consumers who adjust right away. But, for other consumers, no adjustment whatsoever will be observed for some time.

The microdynamics are therefore not the smooth paths described by the SA model. The "speed of adjustment" may vary considerably among individuals, depending on their initial durables stock. Thus explicit aggregation over the entire population is necessary in order to find the aggregate adjustment speed. Note also that, although the dynamics implied by the (S,s) model are more complicated than those of the SA model, the underlying theoretical basis is more solid. The postponement/ advancement decision is naturally integrated into the model. By optimizing over the choice variables S and s, the consumer decides not only how much to spend on durables (as in the SA model), but also when to spend; when the durables stock hits the level s, he spends p(S-qs). In principle every piece of relevant information is taken into account in determining the levels S and s. This means that such important timing factors as intertemporal price substitution and variable interest rates are in principle captured by the model.[6] Another advantage relative to the SA model is that the (S,s) model envisions one unified decision about how much to spend on durables; this avoids the artificial distinction between expansion and replacement expenditures.

III. FROM MICROFOUNDATIONS TO MACRO IMPLICATIONS: AGGREGATION.

Clearly, individuals do not "partially adjust" to "desired" stocks. Does the aggregate economy? In order to aggregate over the population, assume that all consumers have the same lifetime income but differ in their durable stocks, i.e. their position between the (common) levels S and s. An unexpected change in income will set in motion the following dynamics. Consumers who find themselves outside their (new) desired (S,s) region will react by moving into that region.[7] This is the short-run effect. These rescheduled purchases will also change the distribution of durable stocks in the population, which will lead to long-run dynamics.

Assume that consumers monitor their durable stocks continuously while the econometrician observes the data only periodically, at intervals of length θ. Denote the depreciation rate for one observation period by δ; thus $\delta = 1 - e^{-\mu\theta}$. Durable goods that depreciate to the new trigger point, s_t, within the period will be replaced. The following is the decision rule:

$$(11)\quad \begin{array}{ll} \text{If } K_t\,(1-\delta) < s_t, & \text{buy } S_t - qs_t \\ \text{If } K_t\,(1-\delta) \geq s_t, & \text{buy nothing,} \end{array}$$

where K_t is the stock at the start of a period.

Hence the number of consumers N_t who purchase a durable good during period t is,[8]

$$(12)\quad N_t = \int_{s_{t-1}}^{s_t/(1-\delta)} f_t(K_t)\, dK_t = F_t(s_t/(1-\delta)) - F_t(s_{t-1})$$

where $f_t\,(K_t)$ is the density function of durables at the beginning of period t and $F_t(K_t)$ is the corresponding cumulative density function. Those who buy a new "car" during period t spend:

$$(13)\quad C_t = S_t - qs_t,$$

where we have normalized $p = 1$.

Denote by E_t the average economy-wide expenditure on durables during period t. E_t is the product of C_t times N_t or:

$$(14)\quad E_t \equiv C_t N_t = (S_t - qs_t)[F_t\,(\frac{s_t}{1-\delta}) - F_t(s_{t-1})]$$

At this point a simple example may be helpful. In the steady state, the number of durable goods purchased is the same each period (except for a possible trend). Hence the age distribution of cars is uniform. If cars are held for at most T periods we have,

$$(15)\quad f(h) = \begin{cases} \frac{1}{T} & \text{for } 0 \leq h \leq T \\ 0 & \text{otherwise} \end{cases}$$

where h is the age of the car. To derive the density of K_t from that of age, recall that:

$$(16)\quad f(K) = f(h)\ \left|\frac{dh}{dK}\right|,$$

and that K and h are related by,

$$(17)\quad K = Se^{-\mu h} \quad \text{for } 0 \leq h \leq T$$

Equations (15), (16), (17) yield the following expression for the density function of the durables stock:

$$(18)\quad f(K) = \begin{cases} 1/(\mu TK) = 1/[K(\ln S - \ln s)] & \text{for } s \leq K \leq S \\ 0 & \text{otherwise} \end{cases}$$

since $s = Se^{-\mu T}$.

The function f(K) is depicted in Figure 2. Although the age distribution is uniform, the distribution of stocks is monotonically decreasing due to exponential depreciation. The distribution in (18) implies that the average stock is:

$$(19)\quad \bar{K} = \frac{S-s}{\ln S - \ln s}$$

It is clear from the fact that the distribution is skewed towards the lower end that $\bar{K}$ is below (S+s)/2. for example, if S = 10 (thousand dollars) $\mu = .2$ and T = 5 then s = 3.68 and $\bar{K} = 6.32$ even though the midpoint of the (s,S) range is 6.84. The cumulative distribution function corresponding to the density function (18) is,

$$(20)\quad F(K) = \begin{cases} 0 & \text{for } K \leq s \\ \dfrac{\ln K - \ln s}{\ln S - \ln s} & \text{for } s \leq K \leq S \\ 1 & \text{for } K \geq S \end{cases}$$

Since $\ln S - \ln s = \mu T$ and $\ln(1-\delta) = -\mu\theta$, equations (14) and (20) yield the following simple expression for average expenditures on durables at periot t,

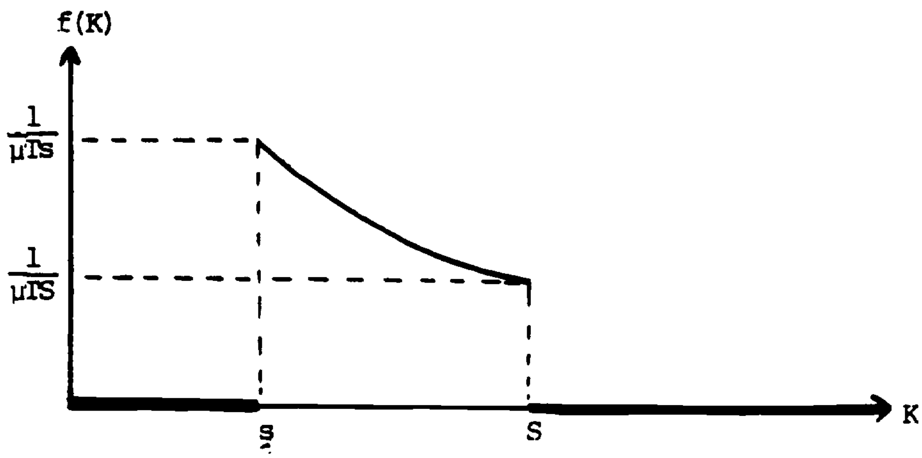

Figure 2

(21) $E_t = \frac{\theta}{T}(S_t - qs_t)$

If, for example, the average durable good is held for T = 5 years, and the observation period is a quarter ($\theta = 1/4$), then about 5% of the cars are replaced each period in the steady state ($\delta = .049$). If S = 10 (thousand dollars), s = 3.68, and q = 0.7, average expenditure per capita is \$371.

Returning to the general case, we would like to understand the dynamics implied by equation (14), and in particular whether the standard implications of the PIH/LCH hold for aggregate expenditures on durables. The contemporaneous effect of permanent income change on E_t is,

(22) $$\frac{dE_t}{dy_t^p} = F\left(\frac{s_t}{1-\delta}\right)\frac{d(S_t-qs_t)}{dy_t^p} + (S_t-qs_t)\frac{1}{1-\delta}f\left(\frac{s_t}{1-\delta}\right)\frac{ds_t}{dy_t^p}$$

The first term represents the increase in expenditures per consumer (C_t) times the number of buyers (N_t). The second term is the average expenditure per buyer times the increase in the number of buyers. Recall from Theorem 1 that both S_t and s_t are proportional to permanent income y_t^p. Therefore, the difference (S_t-qs_t) is also proportional to permanent income. That means that S_t-qs_t is unit elastic with respect to y_t^p. So the first term in (22) is:

(23) $$\frac{d(S_t-qs_t)}{dy_t^p}F\left(\frac{s_t}{1-\delta}\right) = \frac{S_t-qs_t}{y_t^p}F\left(\frac{s_t}{1-\delta}\right).$$

Similarly, the second term in equation (22) simplifies to

(24) $$(S_t-qs_t)\frac{1}{(1-\delta)}f\left(\frac{s_t}{1-\delta}\right)\frac{s_t}{y_t^p}$$

Adding (23) and (24), and converting to an elasticity of aggregate expenditures on durables with respect to permanent income yields:

$$(25)\quad \frac{dE_t}{dy_t^p}\,\frac{y_t^p}{E_t} = 1 + x_t\,\frac{f(x_t)}{F(x_t)} \equiv 1 + \eta_t > 1$$

where $x_t \equiv \frac{S_t}{1-\delta} \geq s_t > 0$, and η_t is the elasticity of $F(x_t)$ with respect to its argument.

Thus the elasticity of durable expenditures with respect to permanent income is larger than one, rather than the unit elasticity normally associated with the PIH/LCH. The actual number depends on the density function. However, if the distribution of durables is close to its steady state, the elasticity must be much larger than 1. For example, consider again the case of a uniform age distribution. Using the previous expressions, the second term in (25) becomes:

$$x_t\,\frac{f(x_t)}{F(x_t)} = \frac{-1}{\ln(1-\delta)} \simeq \frac{1}{\delta} \text{ (for small } \delta),$$

which gives the following (large) elasticity:

$$(26)\quad \frac{dE_t}{dy_t^p}\,\frac{y_t^p}{E_t} \simeq 1 + \frac{1}{\delta}$$

For a quarterly depreciation rate $\delta = 5\%$, the elasticity will be 21, meaning that a 1% increase in permanent income yields a short-run increase of 21% in durable expenditures!

What is the reason for this very large elasticity? Look at the two terms in equation (22). The first term, the increase in expenditures per transaction (C_t), is unit elastic to permanent income because both S_t and s_t are. The second term describes the increase in the number of purchasers, which might be very large because of advancement of the purchasing decision. In the case of a periodical depreciation rate of $\delta = .05$, 5% of the population are replacing their durable good each period in the steady state. If permanent income now rises by 1%, the trigger point s will increase by 1%. This temporarily raises the percentage of the population buying cars from the normal 5% up to 6%. Thus the increase in the flow rate of transactions is 20% (6% instead of 5% of the population). The other 1% comes from the larger purchase size.

This discussion shows why breaking the data on durable expenditures into C_t and N_t is important. The average transaction size C_t describes the behavior of a single consumer. As such it should satisfy the predictions of the PIH/LCH; and, in fact, C_t is proportional to permanent income, is not sensitive to transitory income, and the growth rate of C_t is identical to that implied by the PIH.[9] The number of transactions N_t may vary widely in the short run with changes in permanent income. Since changes in aggregate durable expenditures E_t are dominated by variations in N_t, we cannot expect E_t to be proportional to permanent income. In the next section we shall make an empirical test of the different theoretical predictions for C_t, N_t and E_t. Although aggregate expenditures are not described very well by permanent income on a period by period basis, E is proportional to y^P in the long run. To see this, recall that:

$$(14)\quad E_t = (S_t - qs_t)\ [F_t\ (\frac{s_t}{1-\delta}) - F_t(s_{t-1})].$$

In the long run $[F_t\ (\frac{s_t}{1-\delta}) - F_t(s_{t-1})]$, the proportion of the population purchasing durables in a certain period, equals the steady state level -- which is independent of permanent income. Thus in the long run E_t changes only with $C_t = (S_t - qs_t)$, which is ~~proportional to permanent income.~~

The conclusion is that durable expenditures will exhibit a very high short-run income elasticity and a long run elasticity of unity. What can be said about the dynamics between these two extreme cases? The aggregate demand for durables depends crucially on the initial distribution of stocks and, in particular, on the lower tail of that distribution. An increase in permanent income leads to advancement of purchases, and hence to a change in the distribution. The echoes may reverberate for a long period and are not easy to characterize. We can, however, get a rough idea about the nature of these subsequent changes in the distribution of stocks. After a rise in y^P, there are more "new cars" and fewer "old cars". So the density in the lower tail will be smaller and fewer people will purchase new cars. Thus an unexpected income increase will lead to the following dynamics. Initially, there is a large short-run increase in durable expenditures. If the initial distribution is uniform, then the distribution after the income shock will not be uniform anymore. Then there is a long period in which expenditures may change as the distribution of stocks adjust. Spending will tend to be low until convergence to the long-run steady state is achieved, which might take a long time.

These dynamics are quite different from those implied by the stock-adjustment ~~model, which predicts smooth, exponential convergence of actual to desired stocks.~~ The closest analogue to the "desired" stock in the SA model is the mean of the steady-state distribution in the (S,s) model. But according to the (S,s) model, the mean of the actual distribution does not converge smoothly to this "desired" level. For example, we have just noted that if the initial age distribution is uniform, then the average stock will "overshoot" the "desired" level. Subsequently, it will fluctuate in long damped oscillations around the steady state until convergence is achieved.[10] Thus the SA and (S,s) models imply very different micro and macrodynamics.

At this point, it may be useful to summarize the aggregate implications of the (S,s) model:

1. The variable which is most closely related to the PIH/LCH is average

expenditure per transaction, C_t. This variable should satisfy the key properties of the PIH. However, total expenditures on durables E_t should not be predicted very well by standard results of the PIH. The reason is that the typical application of the PIH is based on a representative consumer. This abstraction can capture only one dimension of the consumption decision: how much to spend. But because of the existence of transaction costs, there is another dimension: when to spend. This advancement/postponement decision cannot be captured by a model of a representative agent.

2. The average durable stock, K_t, and total expenditures, E_t, will not necessarily be proportional to permanent income; neither will they follow the growth rate implied by the PIH. Instead, changes in permanent income might lead to very large changes in durable expendtirues with echo effects which might last for a long time. Only the long-run, steady-state levels of expenditures and stock will follow the predictions of the PIH. This means that the market for durable goods is inherently more volatile than the markets for nondurable goods and services. Even with no new information, E_t and K_t might vary across periods. Only when there are no surprises and the distribution of stocks is in a steady state will durable expenditures and stocks not fluctuate.

3. The high short-run income elasticity of expenditures implied by the (S,s) model opens up an avenue through which small impulses in perceived permanent income may lead to large business cycles. Suppose a small, negative innovation to income leads people to write down their estimated permanent incomes by small amounts. By the logic of the (S,s) model, spending on durables may fall by a much larger percentage than permanent income, thereby kicking off a recession. (Supposing, of course, that prices and wages are not perfectly flexible.)[11]

where u_t and v_t are stochastic terms which represent factors that are not included in the analysis. The amount spent on durable purchases C_t is, in real terms:

$$(28) \quad C_t = S_t - qs_t = \Omega\, y_t^p + \epsilon_t$$

where the coefficient Ω is defined by $\Omega \equiv a_1 - qa_2$, q is the ratio of selling to purchasing price, and the transitory component ϵ_t satisfies $\epsilon_t \equiv u_t - qv_t$. Equation (28) is the conventional way of modelling the PIH. (See, for example, Flavin (1981), p. 978). It states that, apart from transitory consumption, consumption is proportional to permanent income.

In order to estimate the number of purchases, N_t, begin with equation (12). The integral can be approximated by using The Theorem of the Mean for Integral which states that if f(x) is a continuous function on a closed interval [a,b], then there is a number c, $a \leq c \leq b$, such that

$$\int_a^b f(x)\, dx = f(c)\ (b-a).$$

Applying this theorem to (12) yields,

$$(29) \quad N_t = f_t(\hat{K}_t)\ [s_t/(1-\delta) - s_{t-1}]$$

where $s_{t-1} \leq \hat{K}_t \leq s_t/(1-\delta)$

Equation (29) is exact because no approximation was involved in its derivation. However, it is not operational since the theorem does not specify the exact location of the point $\hat{K}_t$. In general $f_t(\hat{K}_t)$ depends on the distribution of stocks in the lower tail. The simplest assumption to permit empirical work is that $f_t(\hat{K}_t)$ is constant through time. Call that constant B. Using the approximation $1/(1-\delta) \simeq 1 + \delta$ for $\delta << 1$, we get,

$$(30) \quad N_t = \delta A y_t^p + A\ (y_t^p - y_{t-1}^p) + (1+\delta)\ Bv_t - Bv_{t-1}$$

where $A \equiv Ba_2$. Notice that N_t depends not only on y^p_t, but also on y^p_{t-1}.

In his influential (1978) paper, Hall pointed out that if the PIH holds then lagged information other than lagged consumption will be useless for predicting consumption. In order to see if this result is robust to the inclusion of transactions costs difference equation (30) to obtain:

$$(31)\quad N_t = N_{t-1} + A(1+\delta)\ (y^p_t - y^p_{t-1}) - A(y^p_{t-1} - y^p_{t-2}) + (1+\delta)B(v_t - v_{t-1}) - B(v_{t-1} - v_{t-2})$$

And similarly equation (28) implies,

$$(32)\quad C_t = C_{t-1} + \Omega(y^p_t - y^p_{t-1}) + \epsilon_t - \epsilon_{t-1}$$

Equation (32) is Hall's well known result. Expenditures per transaction follow a random walk process if we assume away the transitory element, or ARMA (1,1) ~~without this assumption. However, equation (31) implies that the number of durables~~ sold, N_t, is far from a random walk. Even ignoring the moving average error term, ΔN_t depends on <u>both</u> Δy^p_t <u>and</u> Δy^p_{t-1}. So past income does have a value in predicting N_t, and thus future consumption expenditures.

We test these implications with quarterly data on automobile purchases because good data are available on both the average price of a new car, C_t, and the number of new cars purchased by consumers, N_t.[13]

First, following Hall, we ask if E_t can be predicted by its own past values, other than E_{t-1}. The result is (with t-ratios in parentheses):

$$(33)\quad E_t = \underset{(3.44)}{28.52} + \underset{(5.66)}{.571}E_{t-1} + \underset{(3.35)}{.387}E_{t-2} - \underset{(-.98)}{.113}E_{t-3} - \underset{(-.68)}{.067}E_{t-4}$$

$$R^2 = .635;\ DW = 1.987;\ F(3,98) = 3.799$$

The F test rejects the omission of longer lags at the 2% level, and the equation makes it clear that it is E_{t-2} that matters. According to our theory, the rejection should come from N_t, not from C_t. That turns out to be the case, as the following two regressions show.

$$(34)\quad C_t = \underset{(.11)}{11.01} + \underset{(10.22)}{1.017}C_{t-1} - \underset{(-.18)}{.025}C_{t-2} + \underset{(.71)}{.094}C_{t-3} - \underset{(-.87)}{.086}C_{t-4}$$

$$R^2 = .956;\ DW = 2.006;\ F(3,98) = .265$$

$$(35)\quad N_t = \underset{(2.76)}{533.7} + \underset{(5.76)}{.583}N_{t-1} + \underset{(3.55)}{.414}N_{t-2} - \underset{(-1.07)}{.124}N_{t-3} - \underset{(-.41)}{.041}N_{t-4}$$

$$R^2 = .687;\ DW = 1.984,\ F(3,98) = 4.511$$

Longer lags are inconsequential in the C_t equation, but N_{t-2} matters in the N_t equation. (The F-statistic for omitting the longer lags rejects the null hypothesis at well beyond the 1% level.)

Next, again following Hall, we ask if lagged values of disposable income can predict expenditures on autos. The result is:

$$(36)\quad E_t = \underset{(4.02)}{38.14} + \underset{(7.87)}{.622}E_{t-1} + \underset{(1.90)}{.642}Y_{t-1} - \underset{(-.54)}{.244}Y_{t-2} + \underset{(.52)}{.234}Y_{t-3} - \underset{(-1.83)}{.615}Y_{t-4}$$

$$R^2 = .621;\ DW = 2.350;\ F(4,97) = 1.856$$

or, if only y_{t-1} is allowed to enter the equation:

$$(37)\quad E_t = \underset{(3.23)}{28.80} + \underset{(11.12)}{.737}E_{t-1} + \underset{(.724)}{.013}Y_{t-1}$$

$$R^2 = .594;\ DW = 2.340;\ F(1,100) = .534$$

In both (36) and (37), the null hypothesis that all lagged y's can be excluded cannot be rejected. In this case, the failure to reject characterizes both the equation for the number of cars and the equation for average expenditure per car:

$$(38)\quad C_t = \underset{(.24)}{22.02} + \underset{(28.44)}{.978}C_{t-1} - \underset{(-.18)}{.043}Y_{t-1} - \underset{(-.93)}{.313}Y_{t-2} + \underset{(1.78)}{.604}Y_{t-3} - \underset{(-.94)}{.223}Y_{t-4}$$

$$R^2 = .958;\ DW = 1.970;\ F(4,97) = 1.295$$

$$(39)\quad N_t = \underset{(3.65)}{952.4} + \underset{(11.82)}{.751}N_{t-1} + \underset{(1.06)}{.796}Y_{t-1} - \underset{(-.32)}{.338}Y_{t-2} + \underset{(.26)}{.280}Y_{t-3} - \underset{(-1.02)}{.795}Y_{t-4}$$

$$R^2 = .659;\ DW = 2.487;\ F(4,97) = 1.088$$

We conclude that lagged disposable income is of no use in predicting expenditures on cars. In the case of (39) this is contrary to our model. A similar conclusion was obtained by Hall, and also by Mankiw (1982), who tested Hall's hypothesis using data on total expenditures on durable goods.

Finally, we ask whether lagged wealth has any predictive power. In this case, the answer is significantly yes (at a 1% level) for E_t and N_t, but barely so (significant at 10%, but not 5% level) for C_t,

$$(40)\quad E_t = \underset{(3.73)}{34.64} + \underset{(10.80)}{.692}E_{t-1} + \underset{(3.53)}{.234}W_{t-1} - \underset{(-2.78)}{.381}W_{t-2} + \underset{(2.41)}{.330}W_{t-3} - \underset{(-2.69)}{.182}W_{t-4}$$

$$R^2 = .657;\ DW = 2.462;\ F(4,97) = 4.621$$

$$(41)\quad C_t = \underset{(.70)}{64.22} + \underset{(22.10)}{.930}C_{t-1} + \underset{(2.41)}{1.300}W_{t-1} - \underset{(-1.98)}{2.196}W_{t-2} + \underset{(1.33)}{1.483}W_{t-3} - \underset{(-.83)}{0.451}W_{t-4}$$

$$R^2 = .960;\ DW = 1.924;\ F(4,97) = 2.283$$

$$(42)\quad N_t = \underset{(4.30)}{1213.6} + \underset{(12.52)}{.738}N_{t-1} + \underset{(2.97)}{4.828}W_{t-1} - \underset{(-2.44)}{8.20}W_{t-2} + \underset{(2.16)}{7.243}W_{t-3} - \underset{(-2.49)}{4.128}W_{t-4}$$

$$R^2 = .699;\ DW = 2.573;\ F(4,97) = 4.480$$

which echoes Hall's finding. It is pretty clear that the strongest rejection of the PIH comes from N_t, not from C_t.

These results hardly can be said to support the (S,s) model. More exacting tests of some of the implications mentioned in the previous section are necessary for that.

But they are encouraging in that rejections of the simple PIH/LCH using data on durables do seem to stem more from the behavior of N_t than from the behavior of C_t which is what our model predits.

V. SUMMARY

We have presented here an extension of the life-cycle permanent-income model of consumption to the case of a durable good whose purchase involves lumpy transactions costs. The micro-theoretic foundation of the model is a particular application of what might be called "the general optimality of doing nothing" in that fixed costs of decisionmaking generally make it optimal to make large changes in behavior at sporadic intervals, but to do nothing most of the time.

Where individual behavior is concerned, the implications of the model match those of the PIH/LCH in some respects, but not in others. Specifically, rather than choose an optimal path for the service flow from durables, the optimizing consumer will choose an optimal range and try to keep his service flow inside that range. When the durable good deteriorates to the bottom of the range, s, he will buy enough to restore the stock to the top of the range, S; he will not "partially adjust" toward some "desired level" of the durable stock. The (S,s) range itself, however, evolves precisely as prescribed by the PIH/LCH, as does the consumption of nondurable goods and services. The model naturally integrates replacement and expansion investment in a unified framework, and also automatically takes account of the opportunities to postpone or advance purchases that may make expenditures on durables so volatile.

Because there is no "representative consumer" in the (S,s) model, aggregation is more difficult than in the standard PIH/LCH. Building from microfoundations to macro aggregates suggests separate treatment of the number of durable goods purchased and the purchase size. According to the theory, the latter follows the implications of the PIH. The former displays higher-order dynamics and a potentially huge short-run elasticity to changes in permanent income (despite a long-run elasticity of unity).

Empirical tests of the sort suggested by Hall (1978), carried out on quarterly data on new purchases of automobiles by U.S. consumers, generally produce results that are in line with the predictions of the theory. In particular, the time series behavior of the number of cars purchased differs substantially (and in the predicted way) from that of the average purchase size. However, these are not very powerful tests for discriminating between the (S,s) model and the stock adjustment model. Much more detailed empirical work is necessary before anyone can really say that the data support or reject the (S,s) model.

Footnotes

[1]For purposes of this paper, the differences between the life cycle hypothesis (LCH) and the permanent income hypothesis (PIH) are inconsequential.

[2]There is much confusion on this point. If population is constant and the age distribution is uniform, then the LCH/PIH implies that aggregate consumption is constant regardless of the time profile of individual consumption, that is, regardless of r and α.

[3]This section is an extention of earlier work by Flemming (1969).

[4]For the proof, see Bar-Ilan (1985). The consumption plan depends also on the initial durable stock. The solution stated in Theorem 1 and described in Figure 1 holds when the consumer adjusts his initial stock immediately to level S. This will be the case, for example, when the initial stock is zero. However, if the initial stock is different from zero, there is a possibility that the optimal policy is not to purchase anything for some time. In this case the first holding period may be different from the other periods. Hence property (ii), Theorem 1, holds only after the first purchase of durables had been made.

[5]In an uncertainty model an innovation to permanent income will induce a consumer to advance his purchase and spend more on durables. Both of these effects will be larger the more permanent the income shock.

[6]However, in practice analytical solutions for S and s are hard to obtain; so implementing these features analytically is difficult.

[7]Either immediately, by making a durable transaction, or by letting their "excessive" durable stock depreciate to the new region.

[8] $N_t = 0$ when $\dot{s}_t/(1-\delta) < s_{t-1}$

[9] C_t is depicted in Figure 1 by the difference between the upper and lower envelope curves. Since both curves grow at the same rate, this is also the growth rate of C_t. Notice that unlike the discrete purchasing behavior of a specific consumer, C_t is observed every period and its growth path is continuous, i.e. the predictions of the PIH hold each period and not only right after a purchasing by a specific individual (as was the case in the former section).

[10] The convergence is gauranteed when the depreciation is stochastic. For a more rigorous analysis of the dynamics in the case of stochastic depreciation see Bar-Ilan (1985).

[11] The stock-adjustment model can also produce a large short-run income elasticity, though for very different reasons. In the SA model, the short-run elasticity arises from the stock/flow distinction; each consumer's <u>flow</u> rate of expenditure depends on his desired <u>stock</u>, and the desired stock may be very large relative to the flow of expenditures. In the (S,s) model, the high short-run elasticity arises naturally from the postponement/advancement decision.

[12] This is very similar to the explanation given by Bils (1985) to the countercyclical aggregation bias in computing average real wage. Since the income of low income people is most volatile, they should be weighted highly in studying the cyclical behavior of wages, not given the low (or even zero, when they are unemployed) weight assigned to them automatically by their income.

[13] The data are unpublished and were kindly furnished by the bureau of Economic analysis. The period of observation is 1958:1 through 1984:3, and all data are

seasonally adjusted. C_t is average expenditure per new car purchased by consumers. N_t is retail sales of new passenger cars to consumers (business and government expenditures are excluded). E_t is the product C_tN_t.

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