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ON THE WELFARE AND CYCLICAL IMPLICATIONS OF MODERATE TREND INFLATION

Guido Ascari Louis Phaneuf Eric Sims

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ABSTRACT

We offer a comprehensive evaluation of the welfare and cyclical implications of moderate trend inflation. In an extended version of a medium-scale New Keynesian model, recent proposals to increase trend inflation from 2 to 4 percent would generate a consumption-equivalent welfare loss of 3.7 percent based on the non-stochastic steady state and of 6.9 percent based on the stochastic mean. Welfare costs of this magnitude are driven by four main factors: i) multiperiod nominal wage contracting, ii) trend growth in investment-specific and neutral technology, iii) roundaboutness in the U.S. production structure, and iv) and the interaction between trend inflation and shocks to the marginal efficiency of investment (MEI), insofar that this type of shock is sufficiently persistent. Moreover, moderate trend inflation has important cyclical implications. It interacts much more strongly with MEI shocks than with either productivity or monetary shocks.

Guido Ascari Department of Economics University of Oxford Manor Road Oxford OX1 3UQ United Kingdom guido.ascari@economics.ox.ac.uk

Louis Phaneuf Department of Economics University of Quebec at Montreal Montreal Canada H3C 3P8 phaneuf.louis@uqam.ca Eric Sims Department of Economics University of Notre Dame 723 Flanner Hall South Bend, IN 46556 and NBER esims1@nd.edu

1 Introduction

"Medium-scale" New Keynesian models (e.g., see Christiano, Eichenbaum, and Evans, 2005, and Smets and Wouters, 2007) are now widely used in academic circles and central banks. This class of models has served mostly to identify the sources of business cycle fluctuations and to analyze the effects of aggregate disturbances on macroeconomic variables. In these models, the rate of inflation is often set equal to zero in the steady state. This assumption is, of course, counterfactual considering that the U.S. rate of inflation has averaged nearly 4 percent a year over the entirety of the postwar period.

There exists a large literature which explores the macroeconomic consequences of non-zero trend inflation. The models most often used for this purpose are "small-scale" New Keynesian models with sticky prices only (e.g. see Ascari, 2004; Hornstein and Wolman, 2005; Kiley, 2007; Levin and Yun, 2007; Amano, Ambler, and Rebei, 2007; Ascari and Ropele, 2007; Coibion and Gorodnichenko, 2011).¹ No study has yet fully explored the effects of moderate trend inflation in a framework that meets the current standards of New Keynesian DSGE modeling. Our paper intends to fill this gap.

We want to address two main research questions. The first is: what are the welfare costs of raising trend inflation from 2 to 4 percent? In other words, what is the amount of consumption households would be willing to give away in a low inflation state to have the same welfare as in a high inflation state? The second is: can a moderate rate of trend inflation between 0 and 4 percent alter the business-cycle properties of New Keynesian models in non trivial ways, and if so, in response to what type of shock and through which channels?

The first question is timely given recent proposals to raise the inflation target in light of the Great Recession and the zero lower bound (ZLB) on nominal interest rates. Prominent economists like Blanchard, Dell'Ariccia, and Mauro (2010), Ball (2013) and Krugman (2014) have suggested that the Federal Reserve increase its inflation target from 2 percent to 4 or even 5 percent annually. Presumably, implementing this proposal would help give the Fed the flexibility to lower nominal interest rates, increasing its ability to mitigate the harmful effects of a future recession. Interest for the second question on the cyclical implications of trend inflation is obvious considering the large amount of work devoted in recent years to the identification of the main sources of business cycle fluctuations.

To address these questions, we propose an extended version of a medium-scale DSGE model developed by Christiano, Eichenbaum, and Evans (2005). Our model shares some basic elements of

¹For a survey of this literature, see Ascari and Sbordone (2014) and the references therein.

this model like imperfectly competitive goods and labor markets, nominal wage and price rigidities in the form of Calvo contracts, and real frictions including consumer habit formation, variable capital utilization, and investment adjustment costs.

However, given our main objectives, we develop this framework along four key dimensions. First, steady-state inflation can be positive. Consequently, dispersion variables and markups can vary with trend inflation, and thus play an active role in the model. In standard New Keynesian models with zero trend inflation, price and wage dispersion are constant up to a first approximation. Relaxing the assumption of zero trend inflation allows us to explore the normative and positive implications of positive rates of long-run inflation.

Second, our model embeds real per capita output growth stemming from two distinct sources: trend growth in neutral productivity and investment-specific technology (IST). Previous work by Amano, Moran, Murchison, and Rennison (2009) has examined the interaction between trend inflation and trend growth in neutral productivity and found it to be quantitatively important for understanding the welfare costs associated with positive trend inflation. Our analysis extends their work to also include trend growth arising from IST. Greenwood, Hercowitz, and Krusell (1997) show that investment-specific technological change has been a major source of U.S. economic growth during the postwar era. In our model, trend growth in IST realistically captures the downward secular trend in the relative price of investment observed during the postwar period.

Third, while in existing models with trend inflation aggregate fluctuations are driven primarily by monetary policy and productivity shocks, in our model investment shocks are a major source of business cycle fluctuations. Fisher (2006), Smets and Wouters (2007), Justiniano and Primiceri (2008), and Altig, Christiano, Eichenbaum, and Linde (2011) treat investment shocks as a kind of disturbance identified with trend reductions in the price of investment relative to consumption. Justiniano, Primiceri, and Tambalotti (2011) distinguish instead between two types of investment shocks: an investment-specific technology shock and a shock to the marginal efficiency of investment (MEI). The MEI shock affects the process by which investment goods are transformed into productive capital. This shock is orthogonal to the relative price of investment. A possible interpretation, supported by evidence in Justiniano, Primiceri, and Tambalotti (2011), is that it is a proxy for the effectiveness with which the financial sector channels the flow of household savings into new productive capital. Their evidence suggests MEI shocks account for the bulk of business cycle fluctuations, while investment-specific technology shocks are unimportant for cyclical fluctuations. Our model features deterministic growth in the IST process to account for the downward secular trend in the relative price of investment and stochastic shocks to the marginal efficiency of investment as a major business cycle shock.

Fourth, our model embeds a roundabout production structure (see Basu, 1995, and Huang, Liu, and Phaneuf, 2004), which is consistent with U.S. evidence produced by Hanes (1996) and Hanes (1999), corroborated by Basu and Taylor (1999a, 1999b), showing that more-processed products have become more important in U.S. aggregate output from the interwar period to the postwar era. It is also consistent with postwar evidence in Basu (1995), Huang, Liu, and Phaneuf (2004) and Nakamura and Steinsson (2010), corroborated by a recent dataset gathered through the joints efforts of the NBER and the U.S. Census Bureau's CES covering 473 six-digit 1997 NAICS industries for the years 1959-2009. This evidence shows the degree of roundaboutness in the U.S. production structure has been high during the postwar era.

Our first set of substantive findings pertains to the normative implications of moderate trend inflation. Here, we focus on two welfare cost measures: a consumption-equivalent welfare loss metric based on non-stochastic steady states and an equivalent metric based on stochastic means. In our baseline model, we find that the cost of increasing trend inflation from 2 to 4 percent is 3.7 percent of each period's consumption based on the non-stochastic steady state and 6.9 percent based on stochastic means. These welfare losses are substantially higher than what much of the existing literature has found. Which features of our model are responsible for welfare costs of this magnitude? Our analysis points to four key factors: *i*) staggered wage contracts, *ii*) trend growth in investment-specific and neutral technology, *iii*) roundabout production, and *iv*) the interaction between trend inflation and shocks to the marginal efficiency of investment (MEI), insofar this type of shock is sufficiently persistent.

Staggered wage contracts are a key factor determining the welfare costs of trend inflation in our medium-scale model. In particular, if we assumed that wages were flexible the welfare cost of raising trend inflation from 2 to 4 percent would only be 0.6 percent of consumption based on stochastic means, or about one-tenth of the consumption equivalent welfare loss as in our baseline analysis. Cho, Cooley, and Phaneuf (1997) have shown that multiperiod wage contracting *per se* can be quite costly once included in a macroeconomic framework with explicit microfoundations. Evidence in Amano et al. (2009) suggests this original insight seems to carry over to a framework with positive trend inflation.² With positive trend inflation households would like to reset their wages each period, but only a fraction can. This leads to significant steady state wage dispersion, which drives a wedge between aggregate labor supply and demand. It also results in higher wage

 $^{^{2}}$ Relative to theirs, our model adds capital accumulation, habit formation in consumption, variable capital utilization, investment adjustment costs, trend growth in investment-specific technology, randomness from shocks, a roundabout production structure and a Taylor rule. Furthermore, their analysis is limited to the impact of trend inflation on non-stochastic steady states. Ours is more general and focuses on the normative and positive aspects of non-zero trend inflation.

markups on average, as updating households choose higher wages than they otherwise would to protect their future real wages from inflation. This higher average wage markup moves the economy further from the first best allocation, resulting in significant welfare losses.

Trend growth in IST and neutral technology, in conjunction with wage rigidity, also contributes significantly to the welfare costs of trend inflation. If there is no trend growth, then the welfare cost of moving from 2 to 4 percent inflation is significantly lower relative to our baseline – e.g. the cost of going from 2 to 4 percent trend inflation is only about 2 percent of consumption based on stochastic means when there is no trend growth, compared to nearly 7 percent when trend growth is positive. Positive trend growth means that households would like to adjust their wages each period even if trend inflation is zero.³ This results in steady state wage dispersion and higher than average wage markups than if trend growth were zero. Adding in positive trend inflation exacerbates these distortions, resulting in much larger welfare costs than if there were no trend growth.

Roundabout production also plays an important role in accounting for the high welfare costs of positive trend inflation. The consumption equivalent welfare cost of going from 2 to 4 percent trend inflation is about 1.5 percentage points lower without roundabout production than with it. This effect is much stronger based on stochastic means than on steady states. Roundabout production has two effects in the model: it simultaneously acts as an amplification source for real shocks and also is isomorphic to prices being stickier, because it introduces some strategic complementarity into price-setting. Both of these features make trend inflation relatively more costly.

Finally, there are also potentially interesting interactions between the various shock sources and the consumption equivalent welfare losses based on stochastic means. For our baseline parameterization, trend inflation is significantly more costly conditioned on volatility arising from MEI shocks than from either neutral productivity or monetary shocks. This result depends in an important way on the persistence of these shocks, a point to which we return below.

Our results can be viewed as complementary to those reported by Coibion, Gorodnichenko, and Wieland (2012). While they find that individual ZLB episodes are quite costly, for reasonable frequencies of hitting the ZLB the unconditional cost of the ZLB is nevertheless low. Raising the inflation target would reduce the frequency of hitting the ZLB, but results in welfare costs every period. They therefore argue that the optimal inflation rate considering the effects of the ZLB is low. While we do not explicitly take into account the ZLB, our analysis echoes their arguments in that we find that even a moderate rise in trend inflation can be quite costly. Like those of Coibion,

 $^{^{3}}$ It turns out that the source of trend growth – IST or neutral productivity – is not terribly important for this effect. Either source of growth means that real wages grow along the balanced growth path, which means households would like to increase their nominal wages each period along that balanced growth path.

Gorodnichenko, and Wieland (2012), our findings represent a warning against policy proposals urging central banks to raise their inflation targets.

Our second set of findings pertains to the cyclical implications of positive trend inflation. Whereas trend inflation has relatively minor interactions with the dynamic responses of aggregate variables to productivity and monetary shocks, there are large interactions between trend inflation and MEI shocks. To our knowledge, no previous study has examined the business cycle consequences of the interaction between moderate trend inflation and investment shocks.

The interaction between trend inflation and the cyclical responses to MEI shocks depends heavily on the persistence of the shock. When the shocks are sufficiently persistent, we find that higher levels of trend inflation significantly dampen the responses of output and other aggregate variables to MEI shocks. When the autocorrelation parameter in the MEI process is 0.95, at a ten quarter forecast horizon the impulse response of output to a MEI shock is about one-quarter as large with 4 percent trend inflation than with 2 percent trend inflation. The interaction between trend inflation and the cyclical response to MEI shocks flips signs at lower levels of persistence. For example, when the autoregressive parameter in the MEI process is 0.8 instead of 0.95, the impulse response of output at a ten quarter horizon is about 15 percent larger with 4 percent trend inflation relative to 2 percent. The effects of trend inflation on the response of output to these shocks – whether the interaction is positive or negative – are substantially larger than the interaction between trend inflation and responses to productivity and monetary shocks.

What is the intuition for the interaction between trend inflation and MEI shocks, and why does it depend on the persistence of the shock? The MEI shock is an intertemporal demand shock, pushing output, inflation, and wages in the same direction. The key insight to understanding the relationship between the persistence of the shock and trend inflation is that positive trend inflation makes price- and wage-setting more forward-looking. Higher trend inflation causes households and firms to adjust prices and wages even more to the shock than they would if trend inflation were zero. This interaction is particularly strong when the shock is very persistent, with the increase in wages in particular sufficiently strong that monopoly distortions are exacerbated and the response of output to the shock is dampened with higher trend inflation. When the shock is not very persistent, the sensitivity to trend inflation of updated prices and wages is not very strong, and output and other aggregate variables respond more to an MEI shock at higher levels of trend inflation.

As noted above, an increasing body of research suggests that MEI shocks are a major driver of the business cycle – Justiniano, Primiceri, and Tambalotti (2010, 2011) find that these shocks account for 50 percent or more of the business cycle volatility in output.⁴ Conventional wisdom in the literature has been that trend inflation might matter in a normative sense, but that it is innocuous to ignore it for the purposes of understanding positive aspects of the business cycle. Our results suggest that this is not the case – trend inflation interacts strongly with MEI shocks, whether that effects is to dampen or amplify the effects of these shocks.

The remainder of the paper is organized as follows. Section 2 lays out our medium-scale DSGE model. Section 3 discusses some issues related to calibration. Section 4 examines the steady-state and mean welfare implications of moderate trend inflation. Section 5 studies the cyclical implications of trend inflation. Section 6 contains concluding remarks.

2 A Medium-Scale Macro Model with Trend Inflation

This section lays out a medium-scale DSGE model in the spirit of Christiano, Eichenbaum, and Evans (2005). It includes nominal rigidities in the form of Calvo (1983) wage and price contracts, habit formation in consumption, investment adjustment costs, variable capital utilization, and monetary policy is governed by a Taylor rule. We add to this model non-zero steady-state inflation, trend growth in IST and neutral technology, and a roundabout production structure. Aggregate fluctuations are driven by shocks to neutral technology, the marginal efficiency of investment, and monetary policy. The subsections below lay out the decision problems, while the optimality conditions of the relevant model agents are in the Appendix.

2.1 Good and Labor Composites

There is a continuum of firms, indexed by $j \in [0, 1]$, and producing differentiated goods with the use of a composite labor input. The composite labor input is aggregated from differentiated labor supplied by a continuum of households, indexed by $i \in [0, 1]$. The differentiated goods are bundled into a gross output good, X_t . As we discuss below, some of this gross output good is used as a factor of production by firms. Net output is therefore measured as gross output less intermediates. The households can either consume or invest the final net output good. The composite gross output and labor input respectively are:

$$X_t = \left(\int_0^1 X_t(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}},\tag{1}$$

⁴Justiniano, Primiceri, and Tambalotti (2010) do not distinguish between MEI and IST shocks, referring instead only to an "investment shock." They do not associate this shock with the relative price of investment goods to consumption, and so in the parlance of their 2011 paper, the investment shock in their 2010 paper is best interpreted as a MEI shock.

$$L_t = \left(\int_0^1 L_t(i)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}.$$
(2)

The parameters $\theta > 1$ and $\sigma > 1$ are the elasticities of substitution between goods and labor. The demand curves for goods and labor are:

$$X_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} X_t, \quad \forall j,$$
(3)

$$L_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\sigma} L_t, \quad \forall i.$$
(4)

The aggregate price and wage indexes are:

$$P_t^{1-\theta} = \int_0^1 P_t(j)^{1-\theta} dj,$$
(5)

$$W_t^{1-\sigma} = \int_0^1 W_t(i)^{1-\sigma} di.$$
 (6)

2.2 Households

There is a continuum of households, indexed by $i \in [0, 1]$, who are monopoly suppliers of labor. They face a downward-sloping demand curve for their particular type of labor given in (4). Following Calvo (1983), each period, there is a fixed probability, $(1 - \xi_w)$, that households can adjust their nominal wage. As in Erceg, Henderson, and Levin (2000), we assume that utility is separable in consumption and labor. State-contingent securities insure households against idiosyncratic wage risk arising from staggered wage-setting. With this setup, households will be identical along all dimensions other than labor supply and wages.

The problem of a typical household, omitting dependence on i except for these two dimensions, is:

$$\max_{C_t, L_t(i), K_{t+1}, B_{t+1}, I_t, Z_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln \left(C_t - b C_{t-1} \right) - \eta \frac{L_t(i)^{1+\chi}}{1+\chi} \right),\tag{7}$$

subject to the following budget constraint,

$$P_t\left(C_t + I_t + \frac{a(Z_t)K_t}{\varepsilon_t^{I,\tau}}\right) + \frac{B_{t+1}}{1+i_t} \le W_t(i)L_t(i) + R_t^k Z_t K_t + \Pi_t + B_t + T_t,\tag{8}$$

and the physical capital accumulation process,

$$K_{t+1} = \varepsilon_t^{I,\tau} \vartheta_t \left(1 - S\left(\frac{I_t}{I_{t-1}}\right) \right) I_t + (1-\delta)K_t.$$
(9)

Here, P_t is the nominal price of goods, C_t is consumption, I_t is investment measured in units of consumption, K_t is the physical capital stock, and Z_t is the level of capital utilization. $W_t(i)$ is the nominal wage paid to labor of type i, and R_t^k is the common rental price on capital services (the product of utilization and physical capital). Π_t and T_t are, respectively, distributed dividends from firms and lump sum taxes from the government, both of which households take as given. B_t is a stock of nominal bonds that the household enters the period with. $a(Z_t)$ is a resource cost of utilization, satisfying a(1) = 0, a'(1) = 0, and a''(1) > 0. This resource cost is measured in units of physical capital. $S\left(\frac{I_t}{I_{t-1}}\right)$ is an investment adjustment cost, satisfying $S(g_I) = 0$, $S'(g_I) = 0$, and $S''(g_I) > 0$, where $g_I \ge 1$ is the steady state (gross) growth rate of investment. i_t is the nominal interest rate. $0 < \beta < 1$ is a discount factor, $0 < \delta < 1$ is a depreciation rate, and $0 \le b < 1$ is a parameter for internal habit formation. χ is the inverse Frisch labor supply elasticity.

 $\varepsilon_t^{I,\tau}$, which enters the capital accumulation equation by multiplying investment and the budget constraint in terms of the resource cost of capital utilization, measures the level of IST. We assume that it follows a deterministic trend with no stochastic component. The deterministic trend is necessary to match the observed downward trend in the relative price of investment goods in the data. The exogenous variable ϑ_t , which enters the capital accumulation equation in the same way as the IST term, is a stochastic MEI shock.

Justiniano, Primiceri, and Tambalotti (2011) distinguish between these two types of investment shocks, showing that IST shocks map one-to-one into the relative price of investment goods, while MEI shocks do not impact the relative price of investment.⁵ They find that MEI shocks are critical for business cycles, while stochastic shocks to IST virtually have no effect on output at business cycle frequencies. These findings form the basis for our modeling choice of having the MEI component stochastic while the IST term only affects trend growth.

A household given the opportunity to adjust its wage in period t will choose a "reset wage" to maximize the expected value of discounted flow utility, where discounting in period t + s is $(\beta \xi_w)^s$, ξ_w^s being the probability that a wage chosen in period t will still be in effect in period t + s. Given our assumption on preferences and wage-setting, all updating households will choose the same reset wage, denoted in real terms by w_t^* .

2.3 Firms

The production function for a typical producer j is:

⁵In the model, the relative price of investment goods is easily seen to be $\frac{1}{\varepsilon_t^{I,\tau}}$. The division by $\varepsilon_t^{I,\tau}$ in the resource cost of utilization is therefore necessary so that capital is priced in terms of consumption goods.

$$X_t(j) = \max\left\{A_t\Gamma_t(j)^{\phi} \left(\widehat{K}_t(j)^{\alpha} L_t(j)^{1-\alpha}\right)^{1-\phi} - \Upsilon_t F, 0\right\},\tag{10}$$

where F is a fixed cost, and production is required to be non-negative. Υ_t is a growth factor, to be discussed later. Given Υ_t , F is chosen to keep profits zero along a balanced growth path, so the entry and exit of firms can be ignored. $\Gamma_t(j)$ is the amount of intermediate input, and $\phi \in (0, 1)$ is the intermediate input share. Intermediate inputs come from aggregate gross output, X_t . $\widehat{K}_t(j)$ is capital services (the product of utilization and physical capital), while $L_t(j)$ is labor input.

The firm gets to choose its price, $P_t(j)$, as well as quantities of intermediates, capital services, and labor input. It is subject to Calvo (1983) pricing, where each period there is a $(1 - \xi_p)$ probability that a firm can re-optimize its price.

2.4 Monetary Policy

Monetary policy follows a Taylor rule:

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\rho_i} \left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi} \left(\frac{Y_t}{Y_{t-1}}g_Y^{-1}\right)^{\alpha_y} \right]^{1-\rho_i} \varepsilon_t^r.$$
(11)

The nominal interest rate responds to deviations of inflation from an exogenous steady-state target, π , and to deviations of output growth from its trend level, g_Y . ε_t^r is an exogenous shock to the policy rule. The parameter ρ_i governs the smoothing-effect on nominal interest rates while α_{π} and α_y are control parameters.

2.5 Shock Processes

Neutral productivity obeys a process with both a trending and stationary component. A_t^{τ} is the deterministic trend component, where g_A is the gross growth rate:

$$A_t = A_t^{\tau} \widetilde{A}_t, \tag{12}$$

$$A_t^\tau = g_A A_{t-1}^\tau. \tag{13}$$

The initial level in period 0 is normalized to 1: $A_0^{\tau} = 1$. The stationary component of neutral productivity follows an AR(1) process in the log, with the non-stochastic mean level normalized to unity, and innovation, u_t^A , drawn from a mean zero normal distribution with known standard deviation equal to s_A :

$$\widetilde{A}_t = \left(\widetilde{A}_{t-1}\right)^{\rho_A} \exp\left(s_A u_t^A\right), \quad 0 \le \rho_A < 1, \tag{14}$$

The IST term obeys the following deterministic trend, where $g_{\varepsilon^{I}}$ is the gross growth rate and the initial level in period 0 is normalized to unity:

$$\varepsilon_t^{I,\tau} = g_{\varepsilon^I} \varepsilon_{t-1}^{I,\tau} \tag{15}$$

The MEI shock follows a stationary AR(1) process, with innovation drawn from a mean zero normal distribution with standard deviation s_I :

$$\vartheta_t = (\vartheta_{t-1})^{\rho_I} \exp(s_I u_t^I), \quad 0 \le \rho_I < 1 \tag{16}$$

The only remaining shock in the model is the monetary policy shock, ε_t^r . We assume that is drawn from a mean zero normal distribution with known standard deviation s_r .

2.6 Functional Forms

We assume that the resource cost of utilization and the investment adjustment cost function have the following functional forms:

$$a(Z_t) = \gamma_1(Z_t - 1) + \frac{\gamma_2}{2}(Z_t - 1)^2,$$
(17)

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - g_I\right)^2,\tag{18}$$

where $\gamma_2 > 0$ is a free parameter; as $\gamma_2 \to \infty$ utilization becomes fixed at unity. γ_1 must be restricted so that the optimality conditions are consistent with the normalization of steady state utilization of 1. $\kappa \ge 0$ is a free parameter. The functional form for the investment adjustment cost is standard in the literature (e.g. see Christiano, Eichenbaum, and Evans, 2005).

2.7 Growth

Most variables in the model will inherit trend growth from the deterministic trends in neutral and investment-specific productivity. Let this trend factor be Υ_t . Output, consumption, investment, intermediate inputs, and the real wage will all grow at the rate of this trend factor on a balanced growth path: $g_Y = g_I = g_{\Gamma} = g_w = g_{\Upsilon}$. The capital stock will grow faster due to growth in investment-specific productivity, with $\widetilde{K}_t \equiv \frac{K_t}{\Upsilon_t \varepsilon_t^{I,\tau}}$ being stationary. Given our specification of preferences, labor hours will be stationary. The full set of equilibrium conditions re-written in stationary terms can be found in the Appendix. One can show that the trend factor that induces stationarity among transformed variables is:

$$\Upsilon_t = (A_t^{\tau})^{\frac{1}{(1-\phi)(1-\alpha)}} \left(\varepsilon_t^{I,\tau}\right)^{\frac{\alpha}{1-\alpha}}.$$
(19)

This reverts to the conventional trend growth factor in a model with growth in neutral and investment-specific productivity when $\phi = 0$. Under this restriction, intermediates are irrelevant for production, and the model reduces to the standard New Keynesian model. Interestingly, from (19), it is evident that a higher value of ϕ amplifies the effects of trend growth in neutral productivity on output and its components. For a given level of trend growth in neutral productivity, the economy will grow faster the larger is the share of intermediates in production.

3 Calibration

We split the baseline calibration of the model's parameters into two groups: non-shock and shock parameters.

3.1 Non-Shock Parameters

The values of non-shock parameters are summarized in Table 1. $\beta = 0.99$ is the discount factor, b = 0.8 is the habit formation parameter, $\chi = 1$ is the inverse Frisch elasticity, and $\eta = 6$ is the weight on disutility of labor set so that steady-state labor hours are around 1/3. The parameters in the production function are the share of capital services $\alpha = 1/3$ and the share of intermediate inputs $\phi = 0.61$.

The parameter ϕ is obtained as follows. As in Nakamura and Steinsson (2010), the weighted average revenue share of intermediate inputs in the U.S. private sector using Consumer Price Index (CPI) expenditure weights is roughly 51% in 2002. The cost share of intermediate inputs is equal to the revenue share times the markup. Our calibration of θ implies a markup of 1.2. Therefore, our estimate of the weighted average cost share of intermediate inputs is roughly 61%.⁶

The parameter $\delta = 0.025$ is the depreciation rate on physical capital, $\kappa = 3$ is the investment adjustment cost parameter consistent with the estimates reported in Christiano, Eichenbaum, and Evans (2005) and Justiniano, Primiceri, and Tambalotti (2010), $\gamma_2 = 0.05$ relates to the squared term in the cost of utilization and is consistent with Basu and Kimball (1997) and Dotsey and King (2006), and γ_1 is set so that steady-state utilization is 1.

⁶The steady-state price markup is for a trend inflation of zero. We find that this markup is almost insensitive to trend inflation between 0 and 4 percent leaving ϕ unaffected as trend inflation rises.

The parameters θ and σ are the elasticities for goods and labor which are both set at 6 (Woodford, 1997, and Liu and Phaneuf, 2007). The Calvo price and wage probabilities, ξ_p and ξ_w , are set at 2/3. Using a dataset covering the frequency of price changes for 350 categories of consumer goods and services for the years 1995-1997, Bils and Klenow (2004) find that the median duration of U.S. prices ranges between 4.3 and 5.5 months. Cogley and Sbordone (2008) link the median duration of prices to the Calvo probability of price non-reoptimization by $-\ln(2)/\ln(\xi_p)$. Setting $\xi_p = 2/3$ therefore implies a median duration of prices of 5 months, which is broadly consistent with the evidence presented by Bils and Klenow.

By setting $\xi_w = 2/3$, we adopt a conservative stand. While this value is broadly consistent with the macro estimate reported by Christiano, Eichenbaum, and Evans (2005), it is somewhat lower than the micro evidence offered by Barattieri, Basu, and Gottschalk (2014), and also somewhat lower than the estimates in Justiniano, Primiceri, and Tambalotti (2010,2011).

The last three parameters are for the Taylor rule, and include the smoothing parameter set at 0.8, the coefficient on inflation at 1.5, and the coefficient on output growth at 0.2. These values are fairly standard in the literature.

3.2 Trend Inflation, Trend Growth, and Shock Parameters

We now explain the calibration of parameters governing trend inflation, trend growth, and the parameters governing the shock processes. Table 2 summarizes the values assigned to these parameters. The baseline model includes three types of shocks: neutral technology, MEI and monetary policy shocks. In Christiano, Eichenbaum, and Evans (2005), aggregate fluctuations are driven only by shocks to monetary policy.⁷

Mapping the model to the data, the trend growth rate of the IST term, g_{ε^I} , equals the negative of the growth rate of the relative price of investment goods. To measure this in the data, we define investment as expenditures on new durables plus private fixed investment, and consumption as consumer expenditures of nondurables and services. These series are from the BEA and cover the period 1960:I-2007:III.

Let $C_{nd,t}^n, C_{s,t}^n, D_t^n$, and $I_{f,t}^n$ denote nominal non-durable consumption, services consumption, expenditure on durables, and fixed investment. Let $P_{nd,t}, P_{s,t}, P_{d,t}$, and $P_{f,t}$ denote the corresponding

⁷Smets and Wouters (2007) assume seven different types of shocks. The increasing number of disturbances in recent New Keynesian models has been criticized. For example, referring to the seven shocks included in the model of Smets and Wouters (2007), Chari, Kehoe, and McGrattan (2009) argue that only three, the TFP shock, the investment shock and the monetary policy shock, are structural. The other four are dubiously structural and do not have a clear economic interpretation.

price indexes. Nominal consumption and nominal investment are then:

$$C_t^n = C_{nd,t}^n + C_{s,t}^n, (20)$$

$$I_t^n = D_t^n + I_{f,t}^n. aga{21}$$

Let $g_{nd,t}$, $g_{s,t}$, $g_{d,t}$, and $g_{f,t}$ denote the real growth rates of the series:

$$g_{nd,t} = \ln C_{nd,t}^n - \ln C_{nd,t-1}^n - \left(\ln P_{nd,t} - \ln P_{nd,t-1}\right), \qquad (22)$$

$$g_{s,t} = \ln C_{s,t}^n - \ln C_{s,t-1}^n - (\ln P_{s,t} - \ln P_{s,t-1}), \qquad (23)$$

$$g_{d,t} = \ln C_{t,t}^n - \ln D_{t,t-1}^n - \left(\ln P_{d,t} - \ln P_{d,t-1}\right), \qquad (24)$$

$$g_{f,t} = \ln I_{f,t}^n - \ln I_{f,t-1}^n - (\ln P_{f,t} - \ln P_{f,t-1}).$$
(25)

The real growth rate of non-durable and services consumption is the share-weighted growth rates of the real component series:

$$g_{c,t} = \left(\frac{C_{nd,t-1}^n}{C_{t-1}^n}\right)g_{nd,t} + \left(\frac{C_{s,t-1}^n}{C_{t-1}^n}\right)g_{s,t}.$$
(26)

The real growth rate of investment is the share-weighted growth rates of the real components:

$$g_{i,t} = \left(\frac{D_{t-1}^n}{I_{t-1}^n}\right) g_{d,t} + \left(\frac{I_{f,t-1}^n}{I_{t-1}^n}\right) g_{f,t}.$$
(27)

The log-level real series is computed by cumulating the growth rates starting from a base of 1. To put them in levels, we exponentiate the log-levels. Then they are re-scaled so that the real and nominal series are equal in the third quarter of 2009. The price indexes for consumption and investment are computed as the ratios of the nominal to the real series. The relative price of investment is the ratio of the implied price index for investment goods to the price index for consumption goods. The average growth rate of the relative price from the period 1960:I-2007:III is -0.00472. This implies a calibration of $g_{\varepsilon^I} = 1.00472$.

We compute aggregate output in a similar way. Define nominal output as the sum of the nominal components:

$$Y_t^n = C_{nd,t}^n + C_{s,t}^n + D_t^n + I_{f,t}^n.$$
 (28)

The growth rate of real GDP is calculated by using the share-weighted real growth rates of the constituent series:

$$g_{y,t} = \left(\frac{C_{nd,t-1}^n}{Y_{t-1}^n}\right)g_{nd,t} + \left(\frac{C_{s,t-1}^n}{Y_{t-1}^n}\right)g_{s,t} + \left(\frac{D_{t-1}^n}{Y_{t-1}^n}\right)g_{d,t} + \left(\frac{I_{f,t-1}^n}{Y_{t-1}^n}\right)g_{f,t}.$$
(29)

Then, we cumulate to get in log-levels, and exponentiate to get in levels. The price deflator is obtained as the ratio between the nominal and real series. The average growth rate of the price index over the period 1960:I-2007:III is 0.008675. This implies $\pi^* = 1.0088$ or 3.52 percent annualized.

Real per capita GDP is computed by subtracting from the log-level the log civilian noninstitutionalized population. The average growth rate of the resulting output per capita series over the period is 0.005712. The standard deviation of output growth over the period is 0.0078. The calculations above imply that $g_Y = 1.005712$ or 2.28 percent a year. Given the calibrated growth of IST from the relative price of investment data ($g_{\varepsilon^I} = 1.00472$), we then pick $g_A^{1-\phi}$ to generate the appropriate average growth rate of output. This implies $g_A^{1-\phi} = 1.0022$ or a measured TFP growing at about 1 percent per year.

To get the parameters governing the shock processes, we proceed as follows. We first fix the autoregressive parameters in both the neutral and MEI processes at 0.95, e.g. $\rho_A = \rho_I = 0.95$. Given trend inflation of $\pi^* = 1.0088$, we then pick s_I , s_A , and s_r so that our baseline model matches the actual volatility of output growth of 0.0078 during our sample period.

For this, we need to assign a percentage contribution of each type of shock to output fluctuations. Justiniano, Primiceri, and Tambalotti (2011) find that MEI shocks account for nearly 60 percent of output fluctuations, while TFP shocks contribute to about 25 percent. Several other studies find that investment shocks explain a larger fraction of output fluctuations than TFP shocks (Fisher, 2006; Justiniano and Primiceri, 2008; Justiniano, Primiceri, and Tambalotti, 2010; and Altig, et al, 2011).

We take a conservative stand and set the percentage contribution of each type of shock to output fluctuations as follows: 50 percent to MEI shocks, 35 percent to TFP shocks and 15 percent to monetary policy shocks.⁸ With the AR(1) coefficients ρ_I and ρ_A set at 0.95, the resulting shock variances satisfying these conditions are $s_I = 0.0272$, $s_A = 0.0029$ and $s_r = 0.0020$.

⁸Our results regarding the mean welfare costs are not significantly affected by varying the percentage contribution of shocks in some reasonable range. Steady-state inflation costs are not affected at all. Since there is a consense that monetary policy shocks explain a relatively modest percentage of output fluctuations–25 percent or less–this leaves 75 percent or more to be split between MEI and neutral technology shocks.

3.3 Selected Moments

We use our benchmark model to generate a number of moments capturing key features of the business cycle. The model is solved via second order perturbation about the non-stochastic steady state. The moments are summarized in Table 3. The reported volatility statistics are for variables measured in growth rates or as deviations from stochastic trends obtained using the HP filter.

The mean value of real per capita output growth implied by the model, $E(\Delta Y)$, matches the data at 0.0057 or 2.28 percent annualized. The volatility of output growth equals the actual volatility by assumption. The volatility of HP-filtered log output is only slightly higher than the actual volatility. The volatility of consumption, whether measured in growth rate or the filtered log-level, matches the data. The volatility of investment is somewhat overestimated in the model, but remains plausible.

The model exactly matches the variability of inflation in the data, although this moment was not explicitly targeted. The first-order autocorrelation of inflation predicted by the model is high at 0.892 and close to the actual one.⁹ Note that the model predicts inflation is highly persistent in spite of the fact that we abstract from wage and price indexation to past inflation. The model also generates a positive autocorrelation of output growth, meeting the test of endogenous business-cycle propagation proposed by Cogley and Nason (1995). Overall, the baseline model performs very well along usual business-cycle dimensions.

4 The Welfare Costs of Trend Inflation

This section examines the normative implications of moderate trend inflation. We assess the consumption-equivalent welfare losses of changes in trend inflation. Our analysis emphasizes the welfare costs of raising trend inflation from 2 to 4 percent (annualized), a scenario which is consistent with recent proposals to raise the inflation target.

In our analysis, we focus on the following two statistics: i) a consumption-equivalent welfare loss metric denoted λ_{ss} conditioned on steady states and measuring how much consumption needs to be taken away in a low inflation state for households to have the same welfare as in a high inflation state, and ii) an equivalent metric denoted λ_m and conveying information conditioned on stochastic means.¹⁰ Table 4 reports the welfare costs implied by the benchmark model, with panel

⁹The autocorrelations of inflation implied by the model are positive at lags of 1 to 6 quarters (not reported).

 $^{^{10}}$ To do this we include a recursive measure of aggregate welfare as an equilibrium condition and compute its mean value under different levels of trend inflation after solving the model via a second order approximation, as in Schmitt-Grohé and Uribe (2004).

(a) showing welfare losses computed from non-stochastic steady states while panel (b) presents welfare losses computed from stochastic means.

Our benchmark model predicts that increasing trend inflation from 2 to 4 percent would generate a consumption-equivalent welfare loss of 3.7 percent conditioned on non-stochastic steady states and 6.9 percent conditioned on stochastic means. That the welfare loss based on stochastic means is higher than the steady state loss is consistent with the analysis in Amano, Ambler, and Rebei (2007). It is worth pointing out that the welfare cost of increasing trend inflation is non-linear. Based on stochastic means, the cost of doing from 0 to 2 percent trend inflation is only 3.3 percent, whereas the cost of going from 2 to 4 percent is nearly twice as large. This non-linearity is important when thinking about policies to raise the inflation target in light of the zero lower bound, as in Coibion, Gorodnichenko, and Wieland (2012). While very small amounts of trend inflation might be desirable to reduce the frequency of ZLB episodes, going from 2 to 4 percent trend inflation would result in substantially larger welfare costs.

The welfare costs of increasing trend inflation that we report are higher than conventionally thought. We therefore consider different model ingredients and how they impact the costs of trend inflation. These exercises are reported in Table 5, which is similar to Table 4 but shuts off different model features to isolate their roles.

Panel (i) considers the case where wages are flexible, $\xi_w = 0$. The welfare costs of increasing trend inflation are substantially smaller with flexible wages. Based on the non-stochastic steady state, the consumption equivalent welfare loss of going from 2 to 4 percent trend inflation is only 0.4 percent. Focusing on stochastic means, this cost is higher but still moderate at 0.6 percent. That wage rigidity is more important than price stickiness in delivering large welfare costs of trend inflation is consistent with Amano et al. (2009). Whereas the steady state and mean price markups vary little with the level of trend inflation, the wage markup increases markedly with trend inflation. This extra distortion has large effects on welfare.

We next consider the role of trend growth. Panel (ii) of Table 5 sets the trend growth rates of both IST and neutral productivity to zero. This results in much smaller welfare costs of trend inflation – the cost of going from 2 to 4 percent trend inflation is 1.8 percent based on the nonstochastic steady state and 2.1 percent based on stochastic means. Trend growth interacts with wage stickiness to produce volatile wage dispersion and higher than average wage markups. When trend growth is positive, households would like to adjust their wage every period. If wages are sticky, positive trend growth forces updating households to adjust their wages more. This results in higher average wage markups and large welfare losses. The exact source of trend growth is not all that important. Panel (i) of Table 6 assumes that all trend growth in output comes from IST, with no trend growth in neutral productivity. This results in welfare costs that are slightly higher (based on both steady states and means) than our baseline analysis, as well as when the only source of growth is neutral productivity, as shown in panel (ii). Nevertheless, these differences are relatively minor.

Panel (iii) of Table 5 considers the role of roundabout production. The absence of roundabout production has a fairly strong impact on the welfare costs based on stochastic means – without roundabout production, the cost of going from 2 to 4 percent trend inflation is 5.5 percent of consumption, as opposed to 6.9 percent in our baseline analysis. The differences in welfare cost based on the non-stochastic steady state with and without roundabout production are much smaller. Roundabout production has two related roles in the model. First, it serves as an amplification source for real shocks, and so shutting roundabout production off results in less overall volatility, which tends to make an increase in trend inflation less costly. Second, roundabout production serves as a source of strategic complementarity, with effects isomorphic to assuming stickier prices. The welfare costs of rasing the inflation target are naturally larger the stickier are prices and/or wages.

We also consider the roles of the different shock processes in generating welfare costs from trend inflation. For these exercises we naturally focus on the welfare metric based on stochastic means. Table 7 considers two different cases. In panel (i), we re-calibrate the model such that there are no neutral productivity shocks. In doing so we target the same volatility of output growth but pick the standard deviation of the MEI shock to account for 75 percent of the unconditional variance of output. Here we observe that the welfare costs are substantially higher than our baseline analysis, with the welfare cost of going from 2 to 4 percent trend inflation 8.7 percent (as opposed to 6.9 percent under our baseline calibration). In panel (ii) we shut off the MEI shock, choosing the standard deviation of the productivity shock to generate 75 percent of the targeted variance of output. Here the welfare cost of increasing trend inflation is substantially smaller than our baseline analysis at 4 percent of consumption. These exercises suggest that there may be a significant cyclical interaction between trend inflation and MEI shocks, a point to which we return below.

Our analysis suggests that wage rigidity is the key nominal friction giving rise to large welfare costs of trend inflation. Two key parameters influencing how wage rigidity interacts with trend inflation are the Frisch labor supply elasticity and any indexation of wages to inflation. In our model the inverse Frisch labor supply elasticity is given by the parameter χ . Panel (i) of Table 8 computes welfare costs of trend inflation when this parameter is set to zero, implying an infinite Frisch elasticity and making the model isomorphic to the indivisible labor model of Rogerson (1988) and Hansen (1985). Here we see that the welfare costs of higher trend inflation are substantially smaller than in our baseline. The reasoning for the effect of χ on the results is straightforward. As $\chi \to 0$, preferences become linear in labor. Trend inflation distorts the relative allocation of labor across households through an effect on wage dispersion. With curvature in preferences over labor, this misallocation can be quite costly. But if this curvature is absent, then misallocated labor arising from wage dispersion has very modest effects on welfare.

Panel (ii) of Table 8 considers the role of wage indexation. Christiano, Eichenbaum, and Evans (2005) assume that wages and prices that cannot be reoptimized in a quarter are fully indexed to the lagged inflation rate. In terms of driving the welfare costs of trend inflation, price indexation is relatively unimportant, while wage indexation is critical. This should not be surprising since wage rigidity is the key nominal friction driving our results. When wages are fully indexed to lagged inflation, the welfare costs of trend inflation are significantly smaller than in our baseline analysis. The use of indexation mechanisms in New Keynesian models has attracted its share of criticism (Woodford, 2007; Cogley and Sbordone, 2008; and Chari, Kehoe, and McGrattan, 2009). One line of criticism is that backward indexation lacks microfoundations. Another is that it is at odds with the data since it implies that *all* wages and prices change every quarter (Bils and Klenow, 2004; Barattieri, Basu, and Gottschalk, 2014). DSGE models which estimate the degree of wage indexation typically find the indexing parameter is generally low, between 0 and 0.15 (Justiniano and Primiceri, 2008; Justiniano, Primiceri, and Tambalotti, 2010; Justiniano, Primiceri, and Tambalotti, 2011). Our baseline results would be virtually the same with degrees of indexation in this range.

5 The Cyclical Effects of Trend Inflation

This section analyzes the positive implications of moderate trend inflation. We begin by showing impulse responses of key macroeconomic variables to the three shocks in the model for different levels of trend inflation and then focus in on how different levels of trend inflation impact some unconditional moments from the model.

Figure 1 reports the impulse responses of several variables to a one standard deviation positive shock to productivity for trend inflation varying between 0 and 4 percent. These impulse responses are broadly consistent with those reported, for example, by Galí (1999) and Basu, Fernald, and Kimball (2006). Trend inflation going from 0 to 4 percent has a relatively modest impact on the responses of most variables. The main effect is that the expansionary effects of the shock on consumption, output, and hours become somewhat stronger with trend inflation. Ascari and Sbordone (2014) report a similar finding, showing that the expansionary effect of a productivity improvement on output gets stronger with higher trend inflation. In contrast to their results, we find that the fall in inflation is slightly smaller with higher levels of trend inflation. This arises due to the different real and nominal frictions in our medium-scale model compared to the small-scale model with only price rigidity on which they focus.

The impulse responses to a monetary policy shock with different levels of trend inflation are shown in Figure 2. As in Ascari and Sbordone (2014), a monetary policy shock has a larger depressing effect on output at higher levels of trend inflation. The differences in the output response for different levels of trend inflation are not particularly large in an absolute sense but the effects of trend inflation seem somewhat stronger here compared to a productivity shock. The response of inflation is very similar for the three different levels of trend inflation depicted in the figure.

Figure 3 plots impulse responses of key variables to a shock to the marginal efficiency of investment for different levels of trend inflation. It is clear from the figure that trend inflation has much larger effects on the reactions of macroeconomic variables to a MEI shock than to either the productivity or monetary policy shocks. In particular, the response of output is significantly lower at all forecast horizons, as are the responses of consumption and investment, for higher levels of trend inflation. The depressing effect of trend inflation on these responses stands in contrast to the amplifying effect of trend inflation conditional on productivity and monetary shocks. Quantitatively, the effects here are also very large – with four percent trend inflation, at most forecast horizons the output response is roughly one-half its value with zero trend inflation. Inflation also increases by more to a MEI shock for higher levels of trend inflation.

Table 9 shows a few select second moments for the model under different levels of trend inflation. The first panel, labeled (i), does so for our baseline parameterization. For 3.52 percent annualized trend inflation, these moments are identical to those shown in Table 3. There are quantitatively large effects of trend inflation on the volatilities of output and inflation. In particular, the volatility of output is close to 20 percent lower with four percent trend inflation than with zero percent trend inflation. This is true whether one measures volatility by focusing on the growth rate of output (volatility goes from 0.0090 at zero trend inflation to 0.0077 at 4 percent) or on HP filtered output (volatility goes from 0.0201 at zero trend inflation to 0.0164 at 4 percent). The volatility of inflation rises by roughly the same percentage magnitude as trend inflation goes from 0 to 4 percent (from 0.0056 at zero trend inflation to 0.0066 at 4 percent).

The next three panels of the table, labeled (ii)-(iv), present these moments conditional on one shock at a time. For these exercises, we set the variances of two of the three shocks to zero, and solve for the variance of the listed shock to generate the observed volatility of output growth of 0.0078 at 3.52 percent trend inflation. It is immediately clear that the effects of trend inflation on volatilities in the full model are almost entirely driven by an interaction between trend inflation

and the MEI shock. In particular, the volatility of output conditional on productivity shocks is roughly invariant to the level of trend inflation while it is increasing in trend inflation for monetary shocks.¹¹ In contrast, conditional on the MEI shock, the volatility of output sharply decreases with trend inflation, while the volatility of inflation increases.

Given the important role ascribed to MEI shocks in our calibration, and given their very strong interaction with trend inflation, it is important to investigate the sensitivity of these results to other parameters. In our baseline analysis we assume that the persistence parameter in the MEI process is 0.95. Justiniano, Primiceri, and Tambalotti (2011) estimate this parameter to be much smaller at 0.81. Figure 4 shows impulse responses to the MEI shock for different levels of trend inflation when the persistence parameter is 0.81. On the one hand there is an important similarity to our baseline analysis in that there is a much stronger effect of trend inflation on output and other aggregate variables conditional on a MEI shock, but relative to Figure 3 there is a critical difference – the responses of output and other aggregate variables to a MEI shock are larger for a higher levels of trend inflation, not smaller.

The second to last panel in Table 9, marked (v), computes model moments (when all three shocks are included in the model) for the different parameterization of the persistence parameter in the stochastic process for the marginal efficiency of investment. In contrast to our baseline analysis, the volatility of output is increasing in the level of trend inflation when MEI shocks are less persistent. While the effects on volatility are not as large in an absolute sense as in our baseline analysis, they are nevertheless quantitatively non-negligible. The effects on second moments with all shocks turned on are again almost entirely driven by the interaction between trend inflation and MEI shocks, as can be gleaned from the last panel of the table, labeled (vi). There we see that volatility is increasing in trend inflation conditioning only on MEI shocks, more so than the increase in volatility when all three shocks are included in the model. Like the case when the persistence parameter in the MEI process is 0.95, the volatility of inflation is increasing in the level of trend inflation conditioning on MEI shocks, but the effect is not as strong as with a higher persistence parameter.

Given the large interaction between the persistence of the MEI process and trend inflation, it is worthwhile to investigate how the welfare analysis from the previous section might be impacted by assuming a lower level of persistence in the MEI process. We do this in Table 10. The persistence

¹¹Although not apparent to four decimal places, the volatility of output growth is increasing in trend inflation conditional on productivity shocks. The volatility of HP filtered output is actually decreasing in trend inflation conditional on productivity shocks, which is somewhat difficult to square with the impulse response functions. This appears to be an artifact of filtering, as the volatility of the (deterministically detrended) level of output is increasing in the level of trend inflation conditional on productivity shocks.

parameter in the MEI process is set to 0.81, and the standard deviations of the three shocks are chosen to generate a standard deviation of output growth of 0.0078, with the investment shock accounting for 50 percent of the unconditional variance of output, the productivity shock 35 percent, and the monetary shock 15 percent, just as in our baseline analysis. We see that the consumption equivalent welfare cost of moving from 2 to 4 percent trend inflation is 4.1 percent. This number is quite high, but is nevertheless significantly lower than in our baseline analysis.

What is the intuition for the "sign flip" in the effect of trend inflation on the responses to MEI shocks for different levels of trend inflation? The MEI shock is an intertemporal demand shock, pushing output, prices, and wages in the same direction. These features can clearly be seen in either Figures 3 or 4. Trend inflation has the effect of making price- and wage-setting relatively more forward-looking, for the reason that with positive trend inflation there is a heightened cost of being stuck with a price or wage chosen today far into the future. Regardless of its persistence, priceand wage-setters respond relatively more to the MEI shock with higher levels of trend inflation, as evidenced by the responses of the relative reset price and reset wage for updating firms and households in the graphs (this is also true for both the productivity and monetary shocks).¹² However, when comparing Figures 3 and 4, one notices that the responses of the reset wage and inflation rate are substantially more sensitive to the trend inflation rate when the investment shock is more persistent. This is particular true for the reset wage response. The heightened sensitivity of the reset wage to the MEI shock when the shock is very persistent is sufficiently strong such that at high levels of trend inflation the average wage markup goes up, not down. This heightened monopoly distortion dampens the responses of output and other aggregate variables to the MEI shock. When the shock is much less persistent, while the updating firms and households do respond more to the shock, this effect is not very strong, and on net trend inflation is expansionary.

6 Conclusion

Economists have recently debated over the desirability for the Federal Reserve and other major central banks of the world to raise inflation targets. This debate is the result of economic pain experienced through the Great Recession and after. It would strain credulity to deny that a greater flexibility in lowering the nominal interest rate would have alleviated the burden of the last recession. Ireland (2011), for instance, argues that because of the ZLB on nominal interest rates, the Federal Reserve was prevented from stabilizing the U.S. economy as it previously did. With

¹²The relative reset price is defined as the optimal reset price for updating firms relative to the aggregate price level, while the reset wage is the optimal real wage for updating households.

the same flexibility the Federal Reserve had in previous in the two previous recessions, the last one might have been shorter and less severe.

But proposals to raise the inflation target are built on the premise that it would not be costly to increase trend inflation by a moderate amount of 2 to 4 percent. Paradoxically, despite the practical importance of this question, few efforts have been devoted prior to our study to address this question in the context of the empirically realistic medium-scale DSGE models that central bankers and academics use to study the macroeconomy. The evidence we have provided here offers a comprehensive benchmark against which these costs can be gauged in future research, and serves as a cautionary warning that the welfare costs of increasing an inflation target may be substantially higher than many think.

Another important side to our findings is their implications for the business cycle. For more than three decades now since the work of Kydland and Prescott (1982) and Nelson and Plosser (1982), macroeconomists have tried to identify the sources of business cycles. Moderate trend inflation can have strong distorting effects conditioned on shocks to the marginal efficiency of investment. These findings are thus complementary to those of Justiniano, Primiceri, and Tambalotti (2011), who show that MEI shocks are the most important driving force behind business cycles. The strong interaction between trend inflation and MEI shocks has been heretofore overlooked in the literature.

Given that much recent research ascribes an important role to MEI shocks, our analysis suggests that it is not innocuous to ignore trend inflation and that trend inflation may have larger effects on business cycle dynamics than previously thought. We are aware of no other paper which has explored the interaction between these shocks and trend inflation. Because there is an interesting dependence between the persistence of these shocks and the sign of the interaction between trend inflation and the responses of key aggregate variables to MEI shocks, we do not wish to take a stand on which direction trend inflation affects business cycle volatility. We simply note that there is a large interaction between trend inflation and these shocks, much larger than for either productivity or monetary shocks. Researchers ought to take this dependence into account when evaluating the quantitative effects and importance of these shocks.

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β	δ	α	η	χ	b	κ	γ_2
0.99	0.025	1/3	6	1	0.8	3	0.05
θ	σ	ξ_p	ξ_w	ϕ	$ ho_i$	α_{π}	α_y
6	6	0.66	0.66	0.61	0.8	1.5	0.2

Table 1: Non-Shock Parameters

Note: this table gives the baseline values of the parameters unrelated to the stochastic processes used in our quantitative simulations.

Table 2: Shock Parameters

$\begin{array}{cccccccccccccccccccccccccccccccccccc$	g_A	<i>a</i> . <i>i</i>	0	S.,	01	s_I	0.4	S A
-1.0022 -1.0041 0 0.0020 0.90 0.0212 0.90 0.0							0.95	0.0029

Note: this table gives the baseline values of the parameters of the stochastic processes used in our quantitative simulations. The trend growth rate of the IST process is chosen to match the average growth rate of the relative price of investment goods in the data. The trend growth growth of the neutral productivity processes is chosen to match the average growth rate of output observed in the sample conditional on the growth rate of the IST process. Given the assumed values of autoregressive parameters governing the stochastic processes, the shock standard deviations are chosen to match the observed volatility of output growth in the data, with the MEI shock accounting for 50 percent of the variance of output growth, the neutral shock 35 percent, and the monetary shock 15 percent.

Table 3: Moments

	$E(\Delta Y)$	$\sigma(\Delta Y)$	$\sigma(\Delta I)$	$\sigma(\Delta C)$	$\sigma(Y^{hp})$	$\sigma(I^{hp})$	$\sigma(C^{hp})$	$\sigma(\pi)$	$\rho_1(\pi)$	$\rho_1(\Delta Y)$
Model	0.0057	0.0078	0.0247	0.0045	0.0169	0.0555	0.0089	0.0064	0.892	0.715
Data	(0.0057)	(0.0078)	(0.0202)	(0.0047)	(0.0162)	(0.0386)	(0.0086)	(0.0064)	(0.907)	(0.363)

Note: this table shows selected moments generated from the baseline model. These moments are generated using the parameter values shown in the tables above with annualized trend inflation of 3.52 percent. " σ " denotes standard deviation, " Δ " refers to the first difference operator, ρ_1 is a first order autocorrelation coefficient, and a superscript "hp" stands for the HP detrended component of a series using smoothing parameter of 1600. The variables Y, I, and C are the natural logs of these series; π is quarter-over-quarter inflation. Moments in the data are computed on the sample 1960q1-2007q3 and are shown in parentheses.

	π^*	$1.00 \rightarrow$	$1.02 \rightarrow$	$1.0352 \rightarrow$
(a) Steady States				
	1.0000	0	n/a	n/a
	1.0200	0.0171	0	n/a
	1.0352	0.0424	0.0258	0
	1.0400	0.0534	0.0370	0.0115
(b) Means				
	1.0000	0	n/a	n/a
	1.0200	0.0336	0	n/a
	1.0352	0.0802	0.0482	0
	1.0400	0.1003	0.0690	0.0219

Table 4: Welfare Effects of Trend Inflation

Note: this table shows consumption equivalent welfare losses from increasing the trend inflation rate using the benchmark parameterization of our model. Panels marked (a) show losses based on the non-stochastic steady state, while panels marked (b) present losses based on stochastic means. A number in the table has the interpretation as the fraction of consumption the representative household would be willing to give up to avoid changing the trend inflation rate from the level in the columns to the level shown in the rows.

		π^*	$1.00 \rightarrow$	$1.02 \rightarrow$	$1.0352 {\rightarrow}$
(i) Flexible wages					
	(a) Steady States	1 0000	0	/-	/-
		1.0000	$0 \\ 0.0006$	n/a	n/a
		1.0200 1.0252		0	n/a
		$1.0352 \\ 1.0400$	$0.0034 \\ 0.0049$	$0.0029 \\ 0.0043$	$\begin{array}{c} 0 \\ 0.0014 \end{array}$
	(b) Means	1.0400	0.0049	0.0045	0.0014
	(b) Means	1.0000	0	n/a	n/a
		1.0000 1.0200	0.0020	$\frac{11}{a}$	n/a
		1.0200 1.0352	0.0020	0.0044	0
		1.0400	0.0083	0.0063	0.0020
(ii) No trend growth		1.0100	0.0000	0.0000	0.0020
(ii) ito ticila growth	(a) Steady States				
	(a) Steady States	1.0000	0	n/a	n/a
		1.0200	0.0052	0	n/a
		1.0352	0.0176	0.0125	0
		1.0400	0.0235	0.0183	0.0060
	(b) Means				
		1.0000	0	n/a	n/a
		1.0200	0.0068	0	n/a
		1.0352	0.0211	0.0144	0
		1.0400	0.0278	0.0211	0.0068
(iii) No RP					
	(a) Steady States				
		1.0000	0	n/a	n/a
		1.0200	0.0167	0	n/a
		1.0352	0.0404	0.0241	0
	(-)	1.0400	0.0506	0.0345	0.0106
	(b) Means				
		1.0000	0	n/a	n/a
		1.0200	0.0276	0	n/a
		1.0352	0.0652	0.0386	0
		1.0400	0.0814	0.0553	0.0173
(iv) Only Sticky Wages					
	(a) Steady States	1 0000	0	/-	/-
		1.0000 1.0200	0	n/a	n/a
		$1.0200 \\ 1.0352$	$0.0047 \\ 0.0142$	$\begin{array}{c} 0\\ 0.0096 \end{array}$	n/a 0
		1.0352 1.0400	0.0142 0.0187	0.0090 0.0141	0.0045
	(b) Means	1.0400	0.0107	0.0141	0.0040
	(b) means	1.0000	0	n/a	n/a
				11 / G	11 / Cl
		1.0200 1.0352	$0.0093 \\ 0.0244$	$0 \\ 0.0152$	n/a 0

Table 5: Welfare Effects of Trend Inflation, Alternative Model Specifications

Note: this table presents consumption equivalent welfare losses from higher trend inflation under four different deviations from the benchmark model: flexible wages, $\xi_w = 0$, in panel (i); no trend growth in panel (ii); no roundabout production, $\phi = 0$, in panel (ii); and no trend growth, flexible prices, and no roundabout production in panel (iv). The rows and columns are organized in the same manner as Table 4.

		π^*	$1.00 \rightarrow$	$1.02 \rightarrow$	$1.0352 \rightarrow$
(i) Only IST					
	(a) Steady States				
	() 0	1.0000	0	n/a	n/a
		1.0200	0.0171	0	n/a
		1.0352	0.0426	0.0259	0
		1.0400	0.0536	0.0371	0.0115
	(b) Means				
		1.0000	0	n/a	n/a
		1.0200	0.0347	0	n/a
		1.0352	0.0830	0.0501	0
		1.0400	0.1040	0.0718	0.0229
(ii) Only Neutral					
	(a) Steady States				
		1.0000	0	n/a	n/a
		1.0200	0.0170	0	n/a
		1.0352	0.0423	0.0258	0
		1.0400	0.0533	0.0369	0.0115
	(b) Means				
		1.0000	0	n/a	n/a
		1.0200	0.0327	0	n/a
		1.0352	0.0779	0.0467	0
		1.0400	0.0974	0.0669	0.0212

Table 6: Welfare Effects of Trend Inflation, Sources of Trend Growth

Note: this table presents consumption equivalent welfare losses from higher trend inflation under two different sources of trend growth: in panel (i) trend growth only comes from investment-specific technological change (IST), whereas in panel (ii) trend output growth results solely from neutral productivity. The rows and columns are organized in the same manner as Table 4.

		π^*	$1.00 \rightarrow$	$1.02 \rightarrow$	$1.0352 \rightarrow$
(i) No Neutral Shocks					
	Means				
		1.0000	0	n/a	n/a
		1.0200	0.0429	0	n/a
		1.0352	0.1010	0.0607	0
		1.0400	0.1259	0.0868	0.0278
(ii) No MEI Shocks					
	Means				
		1.0000	0	n/a	n/a
		1.0200	0.0187	0	n/a
		1.0352	0.0461	0.0279	0
		1.0400	0.0580	0.0400	0.0124

Table 7: Welfare Effects of Trend Inflation, Shock Sources

Note: this table presents consumption equivalent welfare losses from higher trend inflation when different shocks are absent. In panel (i), we report losses when there are no neutral productivity shocks, re-calibrating the magnitudes of the MEI and monetary shocks to jointly match the observed volatility of output growth, with the MEI shock accounting for 75 percent of the variance of output growth and the monetary shock 25 percent. In panel (ii), we shut the MEI shock off and re-calibrate the standard deviations of the neutral productivity and monetary shock to match the observed volatility of output growth, with the neutral shock accounting for 75 percent of the variance of output growth and the monetary shock to match the observed volatility of output growth, with the neutral shock accounting for 75 percent of the variance of output growth and the monetary shock the remainder. Since the steady state welfare losses are independent of the magnitudes of the shocks, these are not reported in this table. The rows and columns are otherwise organized in the same manner as Table 4.

		π^*	$1.00 \rightarrow$	$1.02 \rightarrow$	$1.0352 \rightarrow$
(i) Infinite Frisch elasticity					
	(a) Steady States				
		1.0000	0	0	0
		1.0200	0.0028	0	0
		1.0352	0.0084	0.0056	0
		1.0400	0.0109	0.0081	0.0025
	(b) Means				
		1.0000	0	0	0
		1.0200	0.0045	0	0
		1.0352	0.0118	0.0074	0
		1.0400	0.0150	0.0106	0.0032
(ii) Full Wage Indexation					
	(a) Steady States				
		1.0000	0	0	0
		1.0200	0.0006	0	0
		1.0352	0.0034	0.0029	0
		1.0400	0.0049	0.0043	0.0014
	(b) Means				
		1.0000	0	0	0
		1.0200	0.0019	0	0
		1.0352	0.0062	0.0043	0
		1.0400	0.0082	0.0063	0.0020

Table 8: Welfare Effects of Trend Inflation, Labor Supply Elasticity and Wage Indexation

Note: this table presents consumption equivalent welfare losses (both in terms of the steady state state as well as stochastic means) under two deviations from our benchmark parameterization: an infinite Frisch labor supply elasticity, $\chi = 0$, in panel (i), and full wage indexation to lagged inflation in panel (ii). The rows and columns are organized in the same manner as Table 4.

Specification	π^*	$\sigma(\Delta Y)$	$\sigma(Y^{hp})$	$\sigma(\pi)$
(i) Baseline	1.0000	0.0090	0.0201	0.0056
	1.0200	0.0084	0.0184	0.0060
	1.0352	0.0078	0.0169	0.0064
	1.0400	0.0077	0.0164	0.0066
(ii) Only Productivity Shocks	1.0000	0.0078	0.0191	0.0039
	1.0200	0.0078	0.0189	0.0037
	1.0352	0.0078	0.0188	0.0036
	1.0400	0.0078	0.0187	0.0035
(iii) Only MEI Shocks	1.0000	0.0101	0.0218	0.0070
	1.0200	0.0089	0.0185	0.0077
	1.0352	0.0078	0.0155	0.0084
	1.0400	0.0074	0.0144	0.0087
(iv) Only Monetary Shocks	1.0000	0.0075	0.0156	0.0030
	1.0200	0.0076	0.0159	0.0030
	1.0352	0.0078	0.0163	0.0030
	1.0400	0.0079	0.0164	0.0031
(v) All Shocks, Less Persistent MEI	1.0000	0.0075	0.0171	0.0030
	1.0200	0.0076	0.0173	0.0030
	1.0352	0.0078	0.0175	0.0029
	1.0400	0.0079	0.0176	0.0029
	•			
(vi) Only MEI Shocks, Less Persistent	1.0000	0.0073	0.0160	0.0021
() = 5 = 5 = 5 = 5 = 5 = 5 = 5 = 5 = 5 =	1.0200	0.0076	0.0164	0.0022
	1.0352	0.0078	0.0169	0.0024
	1.0400	0.0079	0.0171	0.0024
	1.0 200			

Table 9: Trend Inflation and Model Moments

Note: this table shows selected moments generated from the model for different levels of trend inflation. These moments are generated using the parameter values shown in the tables above with annualized trend inflation of 3.52 percent. " σ " denotes standard deviation, " Δ " refers to the first difference operator, and a superscript "hp" stands for the HP detrended component of a series using smoothing parameter of 1600. The panel labeled "Baseline" is parameterized in our baseline specification. In the remaining panels the parameters of the shock processes are re-calibrated as indicated. In all of these specifications the relevant shock standard deviations are chosen to generate volatility of output growth of 0.0078 when annualized gross trend inflation is $\pi^* = 1.0352$. For example, in the "Only Productivity Shocks" panel the standard deviations of the MEI and monetary shocks are set to zero, and the standard deviation of the productivity shock is chosen to generate volatility of output growth of 0.0078 when $\pi^* = 1.0352$. In the panels labeled "Less Persistent MEI" we set $\rho_I = 0.81$ instead of 0.95.

	π^*	$1.00 \rightarrow$	$1.02 \rightarrow$	$1.0352 \rightarrow$
Means				
	1.0000	0	n/a	n/a
	1.0200	0.0194	0	n/a
	1.0352	0.0477	0.0289	0
	1.0400	0.0600	0.0414	0.0129

Table 10: Welfare Effects of Trend Inflation, Less Persistence in MEI Process

Note: this table shows consumption equivalent welfare losses from increasing the trend inflation rate using the the alternative parameterization of the model in which the persistence parameter of the MEI shock process is 0.81 instead of 0.95. The table is otherwise structured similar to Table 4, although we only show welfare losses based on stochastic means because the parameters of the shock processes are irrelevant for the welfare losses based on the non-stochastic steady state.

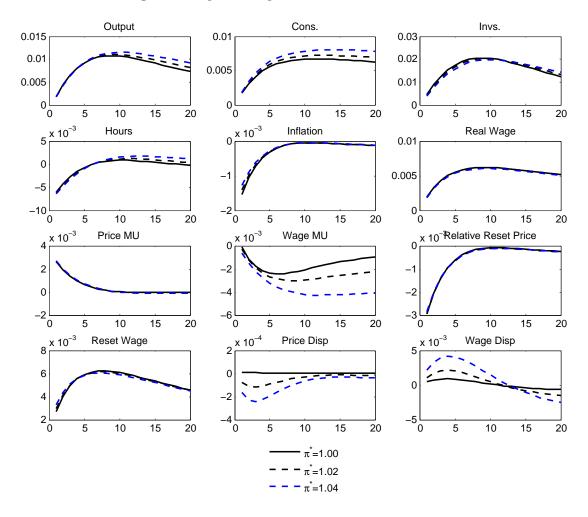


Figure 1: Impulse Responses to Neutral Prod. Shock

Note: this figures plots the average impulse responses to a neutral productivity shock using our baseline parameterization for the three different levels of trend inflation indicated in the legend. "Price MU" and "Wage MU" stand for the average price and wage markups, respectively, while "Price Disp" and "Wage Disp" stand for price and wage dispersion, respectively. "Relative Reset Price" and "Reset Wage" are the optimal reset price relative to the aggregate price level and the optimal real wage for updating price- and wage-setters, respectively.

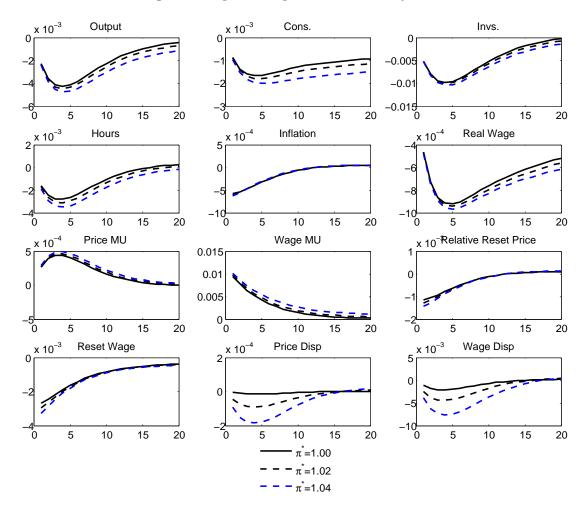


Figure 2: Impulse Responses to Monetary Shock

Note: this figures plots the average impulse responses to a monetary shock using our baseline parameterization for the three different levels of trend inflation indicated in the legend. "Price MU" and "Wage MU" stand for the average price and wage markups, respectively, while "Price Disp" and "Wage Disp" stand for price and wage dispersion, respectively. "Relative Reset Price" and "Reset Wage" are the optimal reset price relative to the aggregate price level and the optimal real wage for updating price- and wage-setters, respectively.

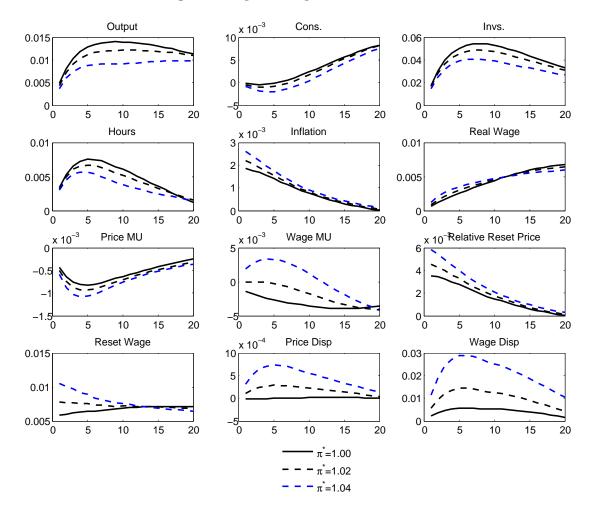


Figure 3: Impulse Responses to MEI Shock

Note: this figures plots the average impulse responses to a MEI shock using our baseline parameterization for the three different levels of trend inflation indicated in the legend. "Price MU" and "Wage MU" stand for the average price and wage markups, respectively, while "Price Disp" and "Wage Disp" stand for price and wage dispersion, respectively. "Relative Reset Price" and "Reset Wage" are the optimal reset price relative to the aggregate price level and the optimal real wage for updating price- and wage-setters, respectively.

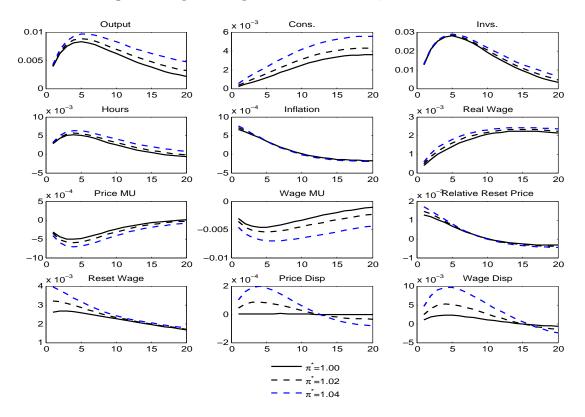


Figure 4: Impulse Responses to MEI Shock, Less Persistence

Note: this figures plots the average impulse responses to a nMEI shock assuming an AR parameter of $\rho_A = 0.81$. The other parameters are set at their baseline values and we report responses for three different levels of trend inflation as indicated in the legend. The standard deviations of the three shocks have been re-chosen to hit observed output growth volatility and match the 50-35-15 variance share split as described in the text. "Price MU" and "Wage MU" stand for the average price and wage markups, respectively, while "Price Disp" and "Wage Disp" stand for price and wage dispersion, respectively. "Relative Reset Price" and "Reset Wage" are the optimal reset price relative to the aggregate price level and the optimal real wage for updating price- and wage-setters, respectively.

A The Model

A.1 Households

For the household problem, the optimality conditions over non-labor choices, which will be the same for all households, are:

$$\lambda_t^r = \frac{1}{C_t - bC_{t-1}} - E_t \frac{\beta b}{C_{t+1} - bC_t},$$
(A1)

$$r_t^k = \frac{a'(Z_t)}{\varepsilon_t^{I,\tau}},\tag{A2}$$

$$\lambda_t^r = \mu_t \varepsilon_t^{I,\tau} \vartheta_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \right] + \beta E_t \mu_{t+1} \varepsilon_{t+1}^{I,\tau} \vartheta_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right) \left[\frac{I_{t+1}}{I_t}\right]^2, \tag{A3}$$

$$\mu_t = \beta E_t \lambda_{t+1}^r \left(r_{t+1}^k Z_{t+1} - \frac{a(Z_{t+1})}{\varepsilon_{t+1}^{I,\tau}} \right) + \beta (1-\delta) E_t \mu_{t+1}, \tag{A4}$$

$$\lambda_t^r = \beta E_t \lambda_{t+1}^r (1+i_t) \pi_{t+1}^{-1}.$$
 (A5)

For the above equations, λ_t is the multiplier on the flow budget constraint and μ_t is the multiplier on the accumulation equation. We write the optimality conditions in terms of $\lambda_t^r \equiv \lambda_t P_t$ so that this multiplier can be interpreted as the marginal value of an extra good, as opposed to an extra dollar. Similarly, we define $r_t^k \equiv \frac{R_t^k}{P_t}$ and $w_t(i) \equiv \frac{W_t(i)}{P_t}$ as real factor prices. $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is gross inflation.

The reset wage is determined by the following first-order condition:

$$w_t^* = \frac{\sigma}{\sigma - 1} \frac{f_{1,t}}{f_{2,t}}.$$
 (A6)

The terms $f_{1,t}$ and $f_{2,t}$ can be written recursively as:

$$f_{1,t} = \eta \left(\frac{w_t}{w_t^*}\right)^{\sigma(1+\chi)} L_t^{1+\chi} + \beta \xi_w E_t(\pi_{t+1})^{\sigma(1+\chi)} \left(\frac{w_{t+1}^*}{w_t^*}\right)^{\sigma(1+\chi)} f_{1,t+1},\tag{A7}$$

and

$$f_{2,t} = \lambda_t^r \left(\frac{w_t}{w_t^*}\right)^{\sigma} L_t + \beta \xi_w E_t (\pi_{t+1})^{\sigma-1} \left(\frac{w_{t+1}^*}{w_t^*}\right)^{\sigma} f_{2,t+1}.$$
 (A8)

A.2 Firms

Regardless of whether a firm can re-optimize its price, it will always choose inputs so as to minimize cost, subject to the constraint of meeting demand at its price. The cost-minimization problem of a typical firm is:

min
$$P_t \Gamma_t(j) + R_t^k \widehat{K}_t + W_t L_t(j),$$
 (A9)

subject to the constraint,

$$A_t \Gamma_t(j)^{\phi} \left(\widehat{K}_t(j)^{\alpha} L_t(j)^{1-\alpha}\right)^{1-\phi} - \Upsilon_t F \ge \left(\frac{P_t(j)}{P_t}\right)^{-\theta} X_t.$$
(A10)

It would be straightforward to show that all firms will hire capital services and labor in the same ratio, which will also equal the aggregate ratio. Similarly, all firms will hire capital services and intermediate inputs in the same ratio. Together, this means that all firms will have the same real marginal cost, mc_t . The factor demands can be written:

$$L_t(j) = (1 - \alpha)(1 - \phi)\frac{mc_t}{w_t} (X_t(j) + \Upsilon_t F),$$
(A11)

$$\widehat{K}_t(j) = \alpha (1 - \phi) \frac{mc_t}{r_t^k} \left(X_t(j) + \Upsilon_t F \right), \tag{A12}$$

$$\Gamma_t(j) = \phi mc_t \left(X_t(j) + \Upsilon_t F \right). \tag{A13}$$

Firms given the opportunity to adjust their price in period t will do so to maximize the expected present discounted value of profits, where discounting is the usual stochastic discount factor of the household as well as ξ_p^s , since this is the probability that a price chosen in period t will still be in effect in period t + s. All updating firms will choose the same reset price. Let $p_t^* \equiv \frac{P_t^*}{P_t}$ be the optimal reset price relative to the aggregate price index. The optimal pricing condition can be written:

$$p_t^* = \frac{\theta}{\theta - 1} \frac{x_{1,t}}{x_{2,t}}.$$
(A14)

The auxiliary variables $x_{1,t}$ and $x_{2,t}$ can be written recursively:

$$x_{1,t} = \lambda_t^r m c_t X_t + \beta \xi_p E_t(\pi_{t+1})^\theta x_{1,t+1},$$
(A15)

$$x_{2,t} = \lambda_t^r X_t + \beta \xi_p E_t(\pi_{t+1})^{\theta - 1} x_{1,t+1}.$$
(A16)

A.3 Aggregation

Given properties of Calvo (1983) price- and wage-setting, aggregate inflation and the real wage evolve according to:

$$1 = \xi_p(\pi_t)^{\theta - 1} + (1 - \xi_p) (p_t^*)^{1 - \theta}, \qquad (A17)$$

$$w_t^{1-\sigma} = \xi_w \left(\frac{w_{t-1}}{\pi_t}\right)^{1-\sigma} + (1-\xi_w) \left(w_t^*\right)^{1-\sigma}.$$
 (A18)

Market-clearing for capital services, labor, and intermediate inputs requires that $\int_0^1 \widehat{K}_t(j) dj = \widehat{K}_t$, $\int_0^1 L_t(j) dj = L_t$, and $\int_0^1 \Gamma_t(j) dj = \Gamma_t$. This means that aggregate gross output can be written:

$$s_t X_t = A_t \Gamma_t^{\phi} \left(\widehat{K}_t^{\alpha} L_t^{1-\alpha} \right)^{1-\phi} - \Upsilon_t F, \tag{A19}$$

where s_t is a price dispersion variable that can be written recursively:

$$s_t = (1 - \xi_p) (p_t^*)^{-\theta} + \xi_p (\pi_t)^{\theta} s_{t-1}.$$
 (A20)

Using the market-clearing conditions, the aggregate factor demands can be written:

$$L_t = (1 - \alpha)(1 - \phi)\frac{mc_t}{w_t} \left(s_t X_t + \Upsilon_t F\right), \qquad (A21)$$

$$\widehat{K}_t = \alpha (1 - \phi) \frac{mc_t}{r_t^k} \left(s_t X_t + \Upsilon_t F \right), \tag{A22}$$

$$\Gamma_t = \phi m c_t \left(s_t X_t + \Upsilon_t F \right). \tag{A23}$$

Aggregate net output, Y_t , is gross output minus intermediate input:

$$Y_t = X_t - \Gamma_t. \tag{A24}$$

Integrating over household budget constraints yields the aggregate resource constraint:

$$Y_t = C_t + I_t + \frac{a(Z_t)K_t}{\varepsilon_t^{I,\tau}}.$$
(A25)

We can define aggregate welfare using a utilitarian social welfare function as the integral over household welfare:

$$\mathcal{W}_t = \int_0^1 V_t(i)di,\tag{A26}$$

where:

$$V_t(i) = \ln \left(C_t - bC_{t-1} \right) - \eta \frac{L_t(i)^{1+\chi}}{1+\chi} + \beta E_t V_{t+1}(i).$$
(A27)

Integrating across households, and making use of the demand for labor of type i, (4), we can write aggregate welfare in terms of aggregate variables only:

$$\mathcal{W}_{t} = \ln \left(C_{t} - bC_{t-1} \right) - \eta v_{t}^{w} \frac{L_{t}^{1+\chi}}{1+\chi} + \beta E_{t} W_{t+1}, \tag{A28}$$

where v_t^w is a wage dispersion variable that can be written recursively in terms of aggregate variables only:

$$v_t^w = (1 - \xi_w) \left(\frac{w_t}{w_t^*}\right)^{\sigma(1+\chi)} + \xi_w \left(\frac{w_t \pi_t}{w_{t-1}}\right)^{\sigma(1+\chi)} v_{t-1}^w.$$
 (A29)

The last expression summarizes the different factors that may affect wage dispersion, and thus the welfare costs of long-run inflation.

B Full Set of Stationarized Equilibrium Conditions

Output, consumption, investment, intermediate inputs, and the real wage will all grow at the rate of this trend factor on a balanced growth path: $g_Y = g_I = g_{\Gamma} = g_w = g_{\Upsilon}$. Given the source of

trend growth in the model is deterministic, $g_{\Upsilon} = \frac{\Upsilon_t}{\Upsilon_{t-1}}$, it will be the same for any two adjacent periods. For these variables, the transformation $\tilde{m}_t \equiv \frac{m_t}{\Upsilon_t}$ will be stationary. The capital stock will grow faster due to growth in investment-specific productivity, with $\tilde{K}_t \equiv \frac{K_t}{\Upsilon_t \varepsilon_t^{I,\tau}}$ being stationary. The stationary rental rate on capital will be $\tilde{r}_t^k \equiv r_t^k \varepsilon_t^{I,\tau}$, and the stationary transformation of the marginal utility of income is $\tilde{\lambda}_t^r \equiv \lambda_t^r \Upsilon_t$. Given our specification of preferences, labor hours will be stationary, as will real marginal cost and capital utilization. The inflation rate and the relative reset price will also be stationary. The full set of equilibrium conditions re-written in stationary terms are as follows.

$$\widetilde{\lambda}_t^r = \frac{1}{\widetilde{C}_t - bg_{\Upsilon}^{-1}\widetilde{C}_{t-1}} - E_t \frac{\beta b}{g_{\Upsilon}\widetilde{C}_{t+1} - b\widetilde{C}_t}$$
(A30)

$$\widetilde{r}_t^k = \gamma_1 + \gamma_2(Z_t - 1) \tag{A31}$$

$$\widetilde{\lambda}_{t}^{r} = \widetilde{\mu}_{t}\vartheta_{t} \left(1 - \frac{k}{2} \left(\frac{\widetilde{I}_{t}}{\widetilde{I}_{t-1}}g_{\Upsilon} - g_{\Upsilon}\right)^{2} - \kappa \left(\frac{\widetilde{I}_{t}}{\widetilde{I}_{t-1}}g_{\Upsilon} - g_{\Upsilon}\right) \frac{\widetilde{I}_{t}}{\widetilde{I}_{t-1}}g_{\Upsilon}\right) + \beta E_{t}g_{\Upsilon}^{-1}\widetilde{\mu}_{t+1}\vartheta_{t+1}\kappa \left(\frac{\widetilde{I}_{t+1}}{\widetilde{I}_{t}}g_{\Upsilon} - g_{\Upsilon}\right) \left(\frac{\widetilde{I}_{t+1}}{\widetilde{I}_{t}}g_{\Upsilon}\right)^{2}$$
(A32)

$$g_I g_{\Upsilon} \tilde{\mu}_t = \beta E_t \tilde{\lambda}_{t+1}^r \left(\tilde{r}_{t+1}^k Z_{t+1} - \left(\gamma_1 (Z_{t+1} - 1) + \frac{\gamma_2}{2} (Z_{t+1} - 1)^2 \right) \right) + \beta (1 - \delta) E_t \tilde{\mu}_{t+1}$$
(A33)

$$\widetilde{\lambda}_t^r = \beta g_{\Upsilon}^{-1} E_t (1+i_t) \pi_{t+1}^{-1} \widetilde{\lambda}_{t+1}^r$$
(A34)

$$\widetilde{w}_t^* = \frac{\sigma}{\sigma - 1} \frac{f_{1,t}}{\widetilde{f}_{2,t}} \tag{A35}$$

$$\widetilde{f}_{1,t} = \eta \left(\frac{\widetilde{w}_t}{\widetilde{w}_t^*}\right)^{\sigma(1+\chi)} L_t^{1+\chi} + \beta \xi_w E_t(\pi_{t+1})^{\sigma(1+\chi)} \left(\frac{\widetilde{w}_{t+1}^*}{\widetilde{w}_t^*}\right)^{\sigma(1+\chi)} g_{\Upsilon}^{\sigma(1+\chi)} \widetilde{f}_{1,t+1}$$
(A36)

$$\widetilde{f}_{2,t} = \widetilde{\lambda}_t^r \left(\frac{\widetilde{w}_t}{\widetilde{w}_t^*}\right)^{\sigma} L_t + \beta \xi_w E_t (\pi_{t+1})^{\sigma-1} \left(\frac{\widetilde{w}_{t+1}^*}{\widetilde{w}_t^*}\right)^{\sigma} g_{\Upsilon}^{\sigma-1} \widetilde{f}_{2,t+1}$$
(A37)

$$\widetilde{\widetilde{K}}_{t} = g_{I}g_{\Upsilon}\alpha(1-\phi)\frac{mc_{t}}{\widetilde{r}_{t}^{k}}\left(s_{t}\widetilde{X}_{t}+F\right)$$
(A38)

$$L_t = (1 - \alpha)(1 - \phi)\frac{mc_t}{\widetilde{w}_t} \left(s_t \widetilde{X}_t + F\right)$$
(A39)

$$\widetilde{\Gamma}_t = \phi m c_t \left(s_t \widetilde{X}_t + F \right) \tag{A40}$$

$$p_t^* = \frac{\theta}{\theta - 1} \frac{x_t^1}{x_t^2} \tag{A41}$$

$$x_t^1 = \tilde{\lambda}_t^r m c_t \tilde{X}_t + \xi_p \beta \left(\frac{1}{\pi_{t+1}}\right)^{-\theta} x_{t+1}^1 \tag{A42}$$

$$x_t^2 = \tilde{\lambda}_t^r \tilde{X}_t + \xi_p \beta \left(\frac{1}{\pi_{t+1}}\right)^{1-\theta} x_{t+1}^2$$
(A43)

$$1 = \xi_p \left(\frac{1}{\pi_t}\right)^{1-\theta} + (1-\xi_p) p_t^{*1-\theta}$$
 (A44)

$$\widetilde{w}_t^{1-\sigma} = \xi_w g_{\Upsilon}^{\sigma-1} \left(\frac{\widetilde{w}_{t-1}}{\pi_t}\right)^{1-\sigma} + (1-\xi_w) \widetilde{w}_t^{*1-\sigma} \tag{A45}$$

$$\widetilde{Y}_t = \widetilde{X}_t - \widetilde{\Gamma}_t \tag{A46}$$

$$s_t \widetilde{X}_t = \widetilde{A}_t \widetilde{\Gamma}_t^{\phi} \widetilde{\widehat{K}}_t^{\alpha(1-\phi)} L_t^{(1-\alpha)(1-\phi)} g_{\Upsilon}^{\alpha(\phi-1)} g_I^{\alpha(\phi-1)} - F$$
(A47)

$$\widetilde{Y}_{t} = \widetilde{C}_{t} + \widetilde{I}_{t} + g_{\Upsilon}^{-1} g_{I}^{-1} \left(\gamma_{1} (Z_{t} - 1) + \frac{\gamma_{2}}{2} (Z_{t} - 1)^{2} \right) \widetilde{K}_{t}$$
(A48)

$$\widetilde{K}_{t+1} = \vartheta_t \left(1 - \frac{\kappa}{2} \left(\frac{\widetilde{I}_t}{\widetilde{I}_{t-1}} g_{\Upsilon} - g_{\Upsilon} \right)^2 \right) \widetilde{I}_t + (1 - \delta) g_{\Upsilon}^{-1} g_I^{-1} \widetilde{K}_t$$
(A49)

$$\frac{1+i_t}{1+i} = \left(\left(\frac{\pi_t}{\pi}\right)^{\alpha_{\pi}} \left(\frac{\widetilde{Y}_t}{\widetilde{Y}_{t-1}}\right)^{\alpha_y} \right)^{1-\rho_i} \left(\frac{1+i_{t-1}}{1+i}\right)^{\rho_i} \varepsilon_t^r \tag{A50}$$

$$\widetilde{\widetilde{K}}_t = Z_t \widetilde{K}_t \tag{A51}$$

$$s_t = (1 - \xi_p) p_t^{*-\theta} + \xi_p \left(\frac{1}{\pi_t}\right)^{-\theta} s_{t-1}$$
(A52)

$$v_t^w = (1 - \xi_w) \left(\frac{\widetilde{w}_t^*}{\widetilde{w}_t}\right)^{-\sigma(1+\chi)} + \xi_w \left(\frac{\widetilde{w}_{t-1}}{\widetilde{w}_t} g_{\Upsilon}^{-1} \frac{1}{\pi_t}\right)^{-\sigma(1+\chi)} v_{t-1}^w$$
(A53)

$$\widetilde{V}_{t}^{c} = \ln\left(\widetilde{C}_{t} - bg_{\Upsilon}^{-1}\widetilde{C}_{t-1}\right) + \beta E_{t}\widetilde{V}_{t+1}^{c}$$
(A54)

$$V_t^n = -\eta \frac{L_t^{1+\chi}}{1+\chi} v_t^w + \beta E_t V_{t+1}^n$$
 (A55)

$$\mathcal{W}_t = \tilde{V}_t^c + \tilde{V}_t^n + \Psi_t \tag{A56}$$

$$\Psi_t = \frac{\beta \ln g_{\Upsilon}}{(1-\beta)^2} \tag{A57}$$

$$\widetilde{A}_{t} = \left(\widetilde{A}_{t-1}\right)^{\rho_{A}} \exp\left(s_{A} u_{t}^{A}\right) \tag{A58}$$

$$\vartheta_t = (\vartheta_{t-1})^{\rho_I} \exp\left(s_I u_t^I\right) \tag{A59}$$

In these equations $g_{\Upsilon} = \frac{\Upsilon_t}{\Upsilon_{t-1}}$, or the growth rate of the deterministic trend. We require that $\beta g_{\Upsilon} < 1$. The recursive representation of social welfare above is written as the sum of three components: utility from consumption, labor, and a third term. The third term, defined as Ψ_t in (A57), is essentially a shift term that arises because of trend growth and appears when rewriting flow utility from consumption in terms of stationarized consumption. Recursive utility from consumption and labor in the levels, are, respectively:

$$V_t^c = \ln \left(C_t - bC_{t-1} \right) + \beta E_t V_{t+1}^c \tag{A60}$$

$$V_t^L = -\eta v_t^w \frac{L_t^{1+\chi}}{1+\chi} + \beta E_t V_{t+1}^L$$
 (A61)

Writing the recursive representation for utility from consumption in terms of stationary variables, one has:

$$V_t^c = \ln\left(\tilde{C}_t - bg_{\Upsilon}^{-1}\tilde{C}_{t-1}\right) + \ln\Upsilon_t + \beta E_t V_{t+1}^c \tag{A62}$$

Define:

$$\widetilde{V}_t^c = \ln\left(\widetilde{C}_t - bg_{\Upsilon}^{-1}\widetilde{C}_{t-1}\right) + \beta E_t \widetilde{V}_{t+1}^c \tag{A63}$$

Similarly, define Ψ_t as:

$$\Psi_t = E_t \sum_{s=0}^{\infty} \beta^s E_t \ln \Upsilon_{t+s}$$
(A64)

If we normalize $\Upsilon_t = 1$, then $E_t \ln \Upsilon_{t+s} = sg_{\Upsilon}$. Then we can write:

$$\Psi_t = \frac{\beta \ln g_{\Upsilon}}{(1-\beta)^2} \tag{A65}$$

Then aggregate welfare can be written as:

$$V_t = \widetilde{V}_t^c + \Psi_t + V_t^L \tag{A66}$$

We define the consumption equivalent, λ , as the fraction of consumption that would have to be sacrificed in each period in a benchmark case (e.g. two percent inflation) to have the same welfare as in an alternative case (e.g. four percent inflation). As discussed in the text, we consider two different consumption equivalents – one based on steady states, λ_{ss} , and one based on stochastic means. Given the definition of welfare and the fact that utility over consumption is logarithmic, it is straightforward to derive either measure, which are both shown below.

$$\lambda_{ss} = 1 - \exp\left[(1 - \beta)(V^{SS} - V_B^{SS})\right] \tag{A67}$$

$$\lambda_m = 1 - \exp\left[(1 - \beta)(E(V) - E(V_B))\right] \tag{A68}$$

In the above expressions a subscript "B" denotes the "base" scenario (e.g. two percent inflation) while the absence of a subscript denotes the alternative scenario (e.g. four percent inflation). A superscript "SS" stands for the non-stochastic steady state, while $E(\cdot)$ is the unconditional expectations operator.