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TAX REFORM AND ADJUSTMENT COSTS: THE IMPACT ON INVESTMENT AND MARKET VALUE

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ABSTRACT

This paper derives analytical measures of the combined effects of tax changes and adjustment costs on investment and market value. Unlike earlier measures, the effective tax rate derived is valid in the presence of adjustment costs and anticipated tax changes. The derived measure of the impact of tax changes on market value permits one to estimate the effects of various tax changes on market value and its components, discounted pure profits and normal returns to capital, and to decompose changes in the value of capital into changes in the marginal value of new capital and changes in the relative value of new and existing capital. These measures are used to evaluate tax changes similar to those introduced by the recent U.S. tax reform.

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1. Introduction

It is recognized that changes in tax policy influence investment behavior and that anticipated tax changes may do so as well. Recent research has shown, however, that the impact of expected future tax changes may push current investment in a different direction than would be suggested by the long-run effects. For example, though a future corporate tax cut would be expected to increase investment in both the long and short runs, an anticipated increase in the investment tax credit would be expected to decrease current investment, as firms delay investment to take advantage of the credit. Likewise, temporary tax changes may be more or less powerful than permanent ones in their impact on investment behavior (e.g., Abel, 1982). In this regard, the structure of taxation is important. For example, a temporary tax cut's power depends crucially on whether depreciation allowances are accelerated relative to economic depreciation.

An important factor influencing the immediate impact of current and future tax changes is the firm's technology. If it is very difficult for the firm to adjust its capital stock, then temporary tax incentives may have little impact on behavior, for example. Thus, the structure of taxation and the structure of production interact in their effects on investment.

A second way in which tax changes and adjustment costs interact is through changes in the value of the firm. Tax policies that encourage investment will also increase the marginal price of capital for the firm facing adjustment costs. The change in firm value that results depends not only on the magnitude of this increase in "marginal q," but also on the tax policy's relative treatment of old and new assets and pure profits. Policies targeted at investment, such as investment tax credits, can have quite different effects from the rate reductions that apply to income from all sources.

This paper derives analytical measures of the combined effects of tax changes and adjustment costs on investment and market value. The measure of the impact on investment is analogous to the "effective tax rate" found in various studies (e.g., Auerbach, 1983; King and Fullerton, 1984). It is based on the same principle of estimating the impact of taxes on new investment, but unlike earlier measures it is valid in the presence of adjustment costs and anticipated tax changes. Using it, one can estimate how a complicated set of current and future tax provisions affects current incentives. The ability to measure current as well as long-run incentives is important. In the recent U.S. tax reform debate, for example, attention was continually focused on the effective tax rates that would eventually prevail under new law even though various phase-in provisions were being considered that would have substantially affected investment behavior in the short run.

The derived measure of the impact of tax changes on market value permits one to estimate the effects of various types of tax changes on market value, as well as the effects of such changes on the different components of market value, pure profits and normal returns to capital. One of the results presented below, for example, is a very simple and intuitive condition under which an increase in the investment tax credit will increase the market value of the firm's existing capital stock.

Since the effects of temporary tax changes on investment depend on the nature of the production technology, the measure to be derived must be based on a specific model of production. The model chosen here is the standard one of a production function with adjustment costs commonly found in the recent "q" theory literature, and used to analyze the effects of tax reform in a number of recent papers (Summers, 1981; Abel, 1982; Auerbach and Hines, 1986). Section 2 presents this model and its solution and derives an analytic expression for the user cost of capital (and the effective tax rate) faced by current investment. Section 3 presents results based on this measure, and Section 4 some simulations of the impact of a tax reform like that recently introduced in the U.S. Section 5 derives the measure of market value, or "average q," based on the same model and considers the impact of particular tax policies on market value and its components. Section 6 offers some concluding comments.

2. The Model

Consider a firm that produces a single output using one factor of production, capital, which depreciates exponentially. The firm is a price-taker in both the capital market and the product market, and incurs adjustment costs with respect to investment. It faces a corporate tax system that includes depreciation allowances and an investment tax credit. Because they have little impact on the problems considered here, personal taxes and the corporate deductibility of interest payments are ignored.

There is no uncertainty in the model, and the firm's planning is done under perfect foresight. Therefore, its objective is to maximize the present value of future cash flows, discounted at the nominal, after-tax cost of capital, r, which is assumed constant:

(1) $$V_t = \int_t^{\infty} e^{-r(s-t)}[p_s F(K_s) - p_s^K C(I_s/K_s)I_s - T_s]ds$$

where

(2) $$T_s = \tau_s[p_s F(K_s) - \int_{-\infty}^{s} p_u^K C(I_u/K_u)I_u D(s,s-u)du] - k_s p_s^K C(I_s/K_s)I_s$$

is the tax bill at date s. The terms p_s and p_s^K are the price levels for output and capital goods at date s, I_s is gross investment at date s, and K_s is the capital stock. The production function $F(\cdot)$ is assumed to be concave, while the investment unit cost function, $C(\cdot)$, is convex in its argument, the rate of investment I/K. The convexity of $C(\cdot)$ means that the unit cost of investment, $p^K C(I/K)$, rises with the rate of investment itself. This introduces the incentive to smooth investment.[1]

The tax variables at date s are τ_s, the corporate tax rate, k_s, the investment tax credit, and D(s,s-u), the depreciation allowance per dollar of date u capital expenditure. Investment and capital are related by:

(3) $$I_s = \delta K_s + \dot{K}_s,$$

where δ is the (assumed to be geometric) rate of economic depreciation of capital.

With (2) and (3), expression (1) may be rewritten as:

(4) $$V_t = \int_t^{\infty} e^{-r(s-t)}[(1-\tau)p_s F(K_s) - p_s^K C(\frac{\delta K_s + \dot{K}_s}{K_s})(\delta K_s + \dot{K}_s)(1-k_s-\Gamma_s]ds + A_t,$$

where

$$(5) \qquad A_t = \int_t^{\infty} e^{-r(s-t)} \tau_s \int_{-\infty}^{t} p_u^K C(I_u/K_u) I_u D(s,s-u)du\, ds$$

is predetermined at date t. A_t is the present value of tax savings due to depreciation of investments made before date t. The term Γ_s in expression (4) represents the present value of tax savings per dollar of date s investment:

$$(6) \qquad \Gamma_s = \int_s^{\infty} e^{-r(u-s)} \tau_u D(u,u-s)du.$$

The Euler condition for the optimal capital stock path based on expression (4) may be written:

$$(7) \qquad F'(K_t) + X_t = c_t = \frac{g_t}{p_t}(r+\delta-(\frac{\dot{g}}{g})_t - (\frac{1-\dot{k}-\Gamma}{1-k-\Gamma})_t)(1-k_t-\Gamma_t)/(1-\tau_t),$$

where:

$$(8) \qquad X_t = \frac{p_t^K}{p_t} C'(I_t/K_t)\cdot(I_t/K_t)^2(1-k_t-\Gamma_t)/(1-\tau_t)$$

and

$$(9) \qquad g_t = p_t^K[C_t'(I_t/K_t) \cdot (I_t/K_t) + C(I_t/K_t)]$$

is the marginal price of capital goods at date t, inclusive of adjustment costs (i.e. the increase in the total cost C(I/K)I with respect to I).

Expression (7) differs from the standard Hall-Jorgenson cost of capital formulation in two respects. It accounts for changes in the effective capital goods price, $g(1-k-\Gamma)$, caused not only by changes in g, but also by changes in k and Γ, and for the fact that the full marginal return to capital includes χ_t, a reduction in current adjustment costs per unit of investment.[2]

Two additional simplifying assumptions facilitate further analysis: that $p^K \equiv p$ and that the cost function $C(\cdot)$ is quadratic, normalized so that the marginal price of capital goods defined in (8) equals p_K when the capital stock is not growing ($\dot{K} = 0$):

$$(10) \qquad C(I/K) = 1 - \phi\delta + \tfrac{1}{2}\phi I/K.$$

These assumptions imply that

$$(11) \qquad \chi_t = \frac{p_t^K}{p_t}(\tfrac{1}{2}\phi(I_t/K_t)^2(1-k_t-\Gamma_t)/(1-\tau_t) = \tfrac{1}{2}\phi(\delta+\dot{K}/K)^2(1-k_t-\Gamma_t)/(1-\tau_t)$$

and

$$(12) \qquad g_t = p_t^K[\tfrac{1}{2}\phi(I_t/K_t) + 1-\phi\delta + \tfrac{1}{2}\phi(I_t/K_t)] = p_t[1-\phi\delta+\phi I_t/K_t] = p_t[1+\phi\dot{K}_t/K_t].$$

Expressions (7), (11) and (12) yield a system of first-order, nonlinear differential equations in the capital stock, K, and the relative capital goods price, g/p, which may be rewritten (suppressing subscripts) as:

$$(13a) \qquad \dot{K} = \frac{(\frac{g}{p})-1}{\phi} K$$

$$(13b) \qquad (\dot{\frac{g}{p}}) = -F'(K)(\frac{1-\tau}{1-k-\Gamma}) - \tfrac{1}{2}\phi(\delta+\frac{g/p-1}{\phi})^2 + (\frac{g}{p})(r+\delta-\frac{\dot{p}}{p})) - \frac{g}{p}(\frac{1-\dot{k}-\Gamma}{1-k-\Gamma}).$$

For notational simplicity, let the real interest rate $r - \dot{p}/p$ equal ρ, and the relative capital goods price g/p equal q. Then (13a) and (13b) become:

$$(14a)\qquad \dot{K} = \left(\frac{q-1}{\phi}\right)K$$

$$(14b)\qquad \dot{q} = -F'(K)\left(\frac{1-\tau}{1-k-\Gamma}\right) - \tfrac{1}{2}\phi\left(\delta+\frac{q-1}{\phi}\right)^2 + q(\rho+\delta) + q\left(\frac{\dot{k}+\dot{\Gamma}}{1-k-\Gamma}\right),$$

This system does not in general have an analytical solution. It may be examined graphically using phase diagrams, as in Abel (1982). Such an approach is very helpful in understanding how the model works and how K and q will respond to various tax changes. For the present purposes, however, a sense of magnitudes is also important. To obtain an analytic solution, one may consider the behavior of the system near a steady state equilibrium, where the local behavior of K and q can be approximated by the version of (14a) and (14b) linearized around the steady state. This approach is common in the literature on dynamic models. It has been used in a related analysis of the impact of tax changes by Judd (1985), for example.

Linearizing (14a) and (14b) around the steady state, one obtains (using the facts that $q = 1$ and $\dot{k} = \dot{\Gamma} = \dot{K} = 0$ in the steady state)

$$(15a)\qquad \dot{K} \doteq \frac{K^*}{\phi}(q-1)$$

$$(15b)\qquad \dot{q} \doteq -F''(K^*)\left(\frac{1-\tau^*}{1-k^*-\Gamma^*}\right)(K-K^*) - \delta(q-1) + (\rho+\delta)(q-1)$$
$$+ \frac{F'(K^*)}{1-k^*-\Gamma^*}(\tau-\tau^*) - \frac{F'(K^*)}{(1-k^*-\Gamma^*)^2}[(k+\Gamma) - (k^*+\Gamma^*)]$$
$$+ \frac{\dot{k}+\dot{\Gamma}}{1-k^*-\Gamma^*}.$$

where the "*" superscript denotes the steady state value of a variable.

Using the fact that, in the steady state, (7) becomes:

$$(7') \qquad F'(K^*) = (\rho+\delta-\tfrac{1}{2}\phi\delta^2)(1-k^*-\Gamma^*)/(1-\tau^*) = (\rho+\hat{\delta})(1-k^*-\Gamma^*)/(1-\tau^*)$$

where

$$(16) \qquad \hat{\delta} = \delta(1-\tfrac{1}{2}\phi\delta),$$

expression (15b) may be rewritten:

$$(15b') \qquad \dot{q} = \frac{-F''(K^*)}{F'(K^*)}(\rho+\hat{\delta})(K-K^*) + \rho(q-1) + (\rho+\hat{\delta})\left(\frac{\tau-\tau^*}{1-\tau^*}\right)$$

$$- (\rho+\hat{\delta})\left[\frac{(k+\Gamma)-(k^*+\Gamma)^*}{1-k^*-\Gamma^*}\right] + \left(\frac{\dot{k}+\dot{\Gamma}}{1-k^*-\Gamma^*}\right)$$

The term $\hat{\delta}$ defined in (16) is the rate of economic depreciation of the capital stock, in the presence of adjustment costs.[3] Expressions (15a) and (15b') form a first-order linear system in K and q. It can be represented as a second-order linear equation in K by substituting q from (15a) and $\dot{q}$ from the derivative of (15a) into (15b'). Doing so yields (with subscripts):

$$(17) \qquad \ddot{K}_t - \rho\dot{K}_t - \frac{\alpha(\rho+\hat{\delta})}{\phi}K_t = -\frac{\alpha(\rho+\hat{\delta})}{\phi}K^*[1-\frac{1}{\alpha}a_t]$$

where:

$$(18) \qquad \alpha = \frac{-F''(K^*)K^*}{F'(K^*)}$$

and

$$(19) \qquad a_t = \frac{(k^*+\Gamma^*)-(k_t+\Gamma_t)}{1-k^*-\Gamma^*} - \frac{\tau^*-\tau_t}{1-\tau^*} + \frac{1}{\rho+\hat{\delta}} \cdot \frac{\dot{k}_t+\dot{\Gamma}_t}{1-k^*-\Gamma^*}.$$

The term α equals the elasticity of F' with respect to K, $-d\ln F'/d\ln K$, evaluated at K^*. This term is important in the translation of capital cost changes into capital stock changes and vice versa. The term a_t represents the proportional deviation in the cost of capital at time t from its long-run value due directly to taxation (see (7)). If $a_t > 0$, the cost of capital will be higher in the short run, given the levels of investment and capital (since the variables χ_t and g_t depend on I_t and K_t).[4] Thus, a_t is the exogenous component of the cost of capital variation.

Factorization of (17) yields

$$(20) \qquad (D-\lambda_1)(D-\lambda_2)K_t = \frac{-\alpha(\rho+\hat{\delta})}{\phi}K^*(1-\frac{1}{\alpha}a_t),$$

where $DX_t = \dot{X}_t$ and λ_1 and λ_2 are the equation's characteristic roots, satisfying:

$$(21) \qquad \lambda_1 = \frac{\rho - \sqrt{\rho^2+\frac{4\alpha(\rho+\hat{\delta})}{\phi}}}{2}; \quad \lambda_2 = \frac{\rho + \sqrt{\rho^2+\frac{4\alpha(\rho+\hat{\delta})}{\phi}}}{2}.$$

As long as the marginal product of capital is positive, then, since α and ϕ are also positive, $\lambda_1 < 0 < \lambda_2$, and the model has one stable root (λ_1) and one unstable root (λ_2). This is the standard result in such models. Given the initial value of K and the transversality condition ruling out the explosion of q, there will be a unique saddlepath equilibrium for the system.

To incorporate these two boundary conditions, we solve the unstable root "forward" and the stable root "backward." Let $M_t = (D-\lambda_1)K_t$. Then (20) may be written as a first-order equation in M:

$$(22) \qquad (D-\lambda_2)M_t = \frac{-\alpha(\rho+\hat{\delta})}{\phi}K^*(1-\frac{1}{\alpha}a_t).$$

Solving (22) forward, and then substituting for M_t using its definition, yields the first-order equation in K:

(23) $$\dot{K}_t = \lambda_1 K_t + \int_t^{\infty} e^{-\lambda_2(s-t)} \frac{\alpha(\rho+\hat{\delta})}{\phi} K^*(1-\frac{1}{\alpha}a_s)ds.$$

Expression (23) could, in turn, be solved for K_t using the initial condition with respect to the capital stock. However, it is more easily interpreted in its present form.

Since $\lambda_1\lambda_2 = \frac{-\alpha(\rho+\hat{\delta})}{\phi}$, (23) may be rewritten:

(24) $$\dot{K}_t = (-\lambda_1)(\hat{K}_t - K_t),$$

where

(25) $$\hat{K}_t = K^*(1 - \frac{\Omega_t}{\alpha})$$

and

(26) $$\Omega_t = \lambda_2 \int_t^{\infty} e^{-\lambda_2(s-t)} a_s ds.$$

Thus, the firm's investment behavior at time t may be described by a partial adjustment process, at rate $-\lambda_1$, which closes the gap between the actual capital stock, K_t, and the "desired" capital stock $\hat{K}_t$. This desired capital stock differs from the long-run capital stock, K*, due to the existence of temporary tax provisions between date t and the steady state. The presence of adjustment costs means that λ_1 is finite and that future as well as current tax-induced cost of capital effects influence current investment. The intuition is clear. If, absent adjustment costs, the firm wished to invest a substantial amount in the near future, the desire to smooth capital accumulation may lead to increased investment today. The term Ω_t is a weighted average of the current and future tax effects, a_s, with weights summing to one and declining at rate λ_2. As ϕ gets smaller, λ_2 increases (see (21)), making future tax effects less important because of a reduced incentive to smooth investment.

From expression (25), it follows that $-(\frac{\Omega_t}{\alpha})$ is the proportional deviation of the desired capital stock from K* due to short-run tax factors. Given the definition of α, it follows that Ω represents the proportional increase in the short-run cost of capital per dollar due to tax changes. That is:

$$(27) \qquad \frac{\Delta F'}{F'}\Big|_{K^*} = \frac{1}{F'^*}\frac{dF'}{dK}\Big|_{K^*}(\hat{K}-K^*) = -\alpha(\frac{\hat{K}-K^*}{K^*}) = \Omega.$$

The current cost of capital effect, Ω_t, combines future tax changes and adjustment costs in a particularly simple way. One first estimates the date s impact of tax changes on the user cost of capital ignoring adjustment costs, a_s, for $s > t$, then weights these with the factors $\lambda_2 e^{-\lambda_2(s-t)}$ to account for the presence of adjustment costs. The term Ω_t differs from a_t in that the former includes cost of capital effects due to changes in the rate of investment that make $\dot{q} \neq 0$. This difference would vanish if $\phi = 0$, for then there would be no change in q due to investment.

One may also relate the effect on investment to changes in effective tax rates. Define the effective tax rate, θ, to be that tax rate on true economic income which, if applied without change over time, would yield the level of investment that actually occurs at a given date. Then, by definition,

$$(28) \qquad \theta = \frac{F'-(\rho+\hat{\delta})}{F'-\hat{\delta}}.$$

In the long run, the value of θ, θ^*, is the standard measure of the effective tax rate found in the literature (e.g., Auerbach, 1983; King and Fullerton, 1984). The short run value, $\hat{\theta}_t$, will differ from θ because of the term Ω_t. In the neighborhood of the steady state, we have (using (28)):

$$(29) \qquad \hat{\theta}_t - \theta^* \doteq \frac{d\theta}{dF'}\Big|_* \Delta F' = \Omega_t(1-\theta^*) \cdot \frac{F'}{F'-\hat{\delta}}\Big|_*.$$

Note that, because it is based on Ω_t and not a_t, this effective tax rate measure incorporates the impact of adjustment costs on the efficacy of future tax changes.

These results apply in general for small changes in the tax system, and Ω and θ are quite easily calculated. In addition, one may simplify the expressions for Ω in particular important cases, making possible the further analysis of the impact of anticipated tax changes in the next section.

3. The Impact of Tax Reform

This section considers the impact, Ω, on the short-run cost of capital of anticipated temporary and permanent tax changes. It focuses on changes in the corporate tax rate τ and in the investment tax credit k, although other experiments, such as changes in the schedule of depreciation allowances, could also be examined. An important issue when considering the impact of a change in τ is the pattern of depreciation allowances that prevails during the tax reform. The extent to which such allowances are accelerated relative to economic depreciation is important in determining short-run investment incentives and changes in market value. To allow for different degrees of acceleration, assume that the depreciation allowance function $D(\cdot)$ is invariant with respect to time, and described by:

$$D(a) = \delta' e^{-\delta' a} \tag{30}$$

where δ' is the rate of declining balance depreciation permitted for tax purposes. Thus, the present value of tax savings per dollar of date s investment is (from (6)):

$$\Gamma_s = \int_s^{\infty} e^{-r(u-s)} \tau_u \delta' e^{-\delta'(u-s)} du. \tag{31}$$

Note that these tax savings are discounted at the nominal interest rate, $r = \rho + \pi$ (where $\pi = \dot{p}/p$), since depreciation allowances are expressed in nominal terms and not indexed for inflation. Under a constant tax system (and hence in the steady state), $\Gamma_s = \tau z$, where $z = \frac{\delta'}{r+\delta'} = \frac{\delta'}{\rho+\pi+\delta'}$ is the present value of the depreciation allowances themselves.

It is now possible to consider the effects of changes in the corporate tax rate, τ, and the investment tax credit, k, on Ω and current investment.

A. Anticipated Tax Rate Change

Suppose the tax rate is currently (at date t) equal to $\bar{\tau}$, and will remain at $\bar{\tau}$ until switching permanently to τ^* at date T > t. This will affect all three terms on the right-hand side of (19), the expression for a_t. Solving for Γ_t from (31) yields:

$$\begin{aligned} \Gamma_t &= (\bar{\tau}[1-e^{-(\rho+\pi+\delta')(T-t)}] + \tau^* e^{-(\rho+\pi+\delta')(T-t)})z \\ &= \Gamma^* - \Delta\tau z[1-e^{-(\rho+\pi+\delta')(T-t)}] \end{aligned} \tag{32}$$

where $\Delta\tau = \tau^* - \bar{\tau}$ and $z = \frac{\delta'}{\rho+\pi+\delta'}$. Differentiating (32) with respect to t yields:

$$\dot{\Gamma} = \Delta\tau z(\rho+\pi+\delta') e^{-(\rho+\pi+\delta')(T-t)}, \tag{33}$$

which has the same sign as $\Delta\tau$, since depreciation allowances increase in value as $t \to T$, for $\tau^* > \bar{\tau}$.

The last impact on a_t of the impending change in τ is the direct effect on after-tax cash flows. Combining these three effects yields:

$$(34) \qquad a_t = \Delta\tau\{-\frac{1}{1-\tau^*} + \frac{z}{1-k^*-\Gamma^*}[1 - e^{-(\rho+\pi+\delta')(T-t)} + (\frac{\rho+\pi+\delta'}{\rho+\hat{\delta}})e^{-(\rho+\pi+\delta')(T-t)}]\}.$$

To interpret this expression, it is useful to rewrite it in the following manner:

$$(35) \qquad a_t = \Delta\tau[-\frac{1}{1-\tau^*} + \frac{z}{1-k^*-\Gamma^*}] + \frac{1}{\rho+\hat{\delta}} \frac{\Delta\tau z}{1-k^*-\Gamma^*} e^{-(\rho+\hat{\delta})(T-t)} \frac{d}{dt} e^{-(\delta'+\pi-\hat{\delta})(T-t)}$$

The first term on the right-hand side of (35) is the percent change in the long-run cost of capital $q(\rho+\hat{\delta})(1-k-\Gamma)/(1-\tau)$, holding q fixed, that results from the change in the tax regime at date T. It is a long-run change in that it compares the costs of capital under the two systems in the absence of anticipated tax changes.

The second term on the right-hand side of (35) accounts for the additional impact on a_t due to the anticipated change to the new tax system at date T. It is nonzero if and only if $\delta' + \pi \neq \hat{\delta}$. That is, an anticipated change in τ affects the no-adjustment-cost user cost (a_t) in earlier years if and only if depreciation allowances do not have the same time pattern as actual economic appreciation. If depreciation allowances are accelerated ($\delta' + \pi > \hat{\delta}$), then existing capital will be worth less than the equivalent amount of new capital due to its already having received a disproportionate share of its depreciation allowances. This gap in value will depend not only on the degree of acceleration, but also on the tax rate at which depreciation allowances are deducted. An increase in τ will widen the gap between new and old capital values in the presence of accelerated depreciation. One would expect the prospect of this to discourage current investment further. The second term in (35) accounts for capital gains or losses that capital goods purchased at date t will experience when the tax rate changes at date T as a result in the change in relative value of existing to new capital goods. This is now demonstrated.

At any date, arbitrage must fix the relative values of otherwise identical old and new capital goods so that they differ to the extent that they have different tax attributes. Consider a unit of capital purchased at date t, holding all future capital expenditures constant. This yields an increase in the level of productive capital at date T of $e^{-\hat{\delta}(T-t)}$ units, taking account of additional subsequent capital expenditures made possible by reduced adjustment costs (see footnote 3). This increased capital purchase at date t also has depreciation allowances of $q_t\Gamma^{(T-t)}$ remaining, where $\Gamma^{(T-t)}$ is the present value of the allowances per unit of capital expenditure made (T-t) years earlier. Since a new unit of capital with the same future quasirents costs $q_Te^{-\hat{\delta}(T-t)}$ and receives investment tax credits and depreciation allowances equal to $q_Te^{-\hat{\delta}(T-t)}(k+\Gamma)$, the unit value of the capital bought in year t, say $q_t^{(T-t)}$, must satisfy:

$$(36) \qquad q_t^{(T-t)}e^{-\hat{\delta}(T-t)} + q_t\Gamma^{(T-t)} = q_T(1-k-\Gamma)e^{-\hat{\delta}(T-t)}.$$

Since $\Gamma = \tau z$ and $\Gamma^{(T-t)} = \tau z e^{-(\delta'+\pi)(T-t)}$, (36) may be rewritten (holding the price of new capital goods constant at $q_T = q_t = q$):

$$(37) \qquad q^{(T-t)}/q = 1 - k - \tau z(1-e^{-(\delta'+\pi-\hat{\delta})(T-t)})$$

the derivative of which with respect to time at date t is:

$$(38) \qquad \frac{d(q^{(T-t)}/q)}{dt} = \tau z \frac{d}{dt}e^{-(\delta'+\pi-\hat{\delta})(T-t)}$$

The change in this value due to the tax change, discounted back to date t at a rate $(\rho+\hat{\delta})$ to account not only for the interest rate but also the fact that only $e^{-\hat{\delta}(T-t)}$ of the capital purchased at date t will be present at date T, yields the impact on the date t real capital gain of the change in relative tax treatment of new and old assets at date T. This value,

$$(39) \qquad e^{-(\rho+\hat{\delta})(T-t)}\Delta\tau z \frac{d}{dt} e^{-(\delta'+\pi-\hat{\delta})(T-t)}$$

differs from the second term in (35) by the factor $\frac{1}{(\rho+\hat{\delta})(1-k^*-\Gamma^*)}$, which puts the price change into capital user cost units. The term is positive if $\delta' + \pi > \hat{\delta}$, this discouraging investment if a tax increase is anticipated because of the expected capital losses that will occur.

Integrating a_t to obtain Ω_t, according to (26), yields:

$$(40) \qquad \Omega_t = -\Delta\tau(\frac{1}{1-\tau^*} - \frac{z}{1-k^*-\Gamma^*})(1-e^{-\lambda_2(T-t)})$$

$$+ \frac{\Delta\tau z}{1-k^*-\Gamma^*}(\frac{\lambda_2}{\lambda_2-(\rho+\pi+\delta')})(\frac{\delta'+\pi-\hat{\delta}}{\rho+\hat{\delta}})(e^{-(\rho+\pi+\delta')(T-t)}-e^{-\lambda_2(T-t)}).$$

If $\hat{\delta} = \delta' + \pi$, Ω_t becomes

$$(41) \qquad \Omega_t = -\Delta\tau\left(\frac{1}{1-\tau^*} - \frac{z}{1-k^*-\Gamma^*}\right)(1-e^{-\lambda_2(T-t)}) = -\frac{\Delta\left(\frac{1-k-\Gamma}{1-\tau}\right)}{\left(\frac{1-k^*-\Gamma^*}{1-\tau}\right)}(1-e^{-\lambda_2(T-t)}).$$

As long as the tax increase increases the long-run cost of capital (i.e., $\Delta(\frac{1-k-\Gamma}{1-\tau})$ has the same sign as $\Delta\tau$), this expression calls for a higher value of Ω_t, and a lower value of current investment, the sooner a tax increase occurs (and a higher value the sooner a tax cut occurs). The intuition is that, after T, the desired capital stock will be lower. Given the incentive to smooth investment, the firm will reduce investment immediately. The sooner the tax change, the stronger the incentive to smooth investment. The strength of the smoothing incentive depends on λ_2. If there were no adjustment costs, λ_2 would be infinite (see (21)); the firm would wait until date T to reduce its capital stock. Similarly, if the production function were very concave, α and hence λ_2 would be quite large (see (19) and (21)). Again, little adjustment would be optimal before date T, in this case because little capital investment would be required to move the marginal product of capital to its new optimal level.

It must be stressed that these particular results depend on the assumption that $\hat{\delta} = \delta' + \pi$. If $\hat{\delta} < \delta' + \pi$, as is more common in actual tax systems, depreciation allowances decline more rapidly. This acceleration may be a legislated one (i.e., $\delta' > \hat{\delta}$), but may also be attributable in part to the fact that depreciation allowances on existing assets decline over time due to inflation. In this case, this second term in (40) reinforces the reduction in investment associated with a future tax increase. Indeed, the reduction in investment today may be so great that it exceeds that which would occur under an immediate increase in τ to τ^* (i.e., T = t). Alternatively, a delayed cut in τ may increase investment more than an immediate one. This possibility was demonstrated by Abel (1982) for the case of instantaneous tax depreciation ($\delta' = \infty$),[5] but there is a much weaker and more intuitive necessary condition for the result to hold.

Consider the effect on Ω_t of an increase in T:

$$(42) \qquad \frac{d\Omega_t}{dT} = \Delta\tau\lambda_2 e^{-\lambda_2(T-t)}\{[-\frac{1}{1-\tau^*} + \frac{z}{1-k^*-\Gamma^*}(1 + (\frac{\delta'+\pi+\hat{\delta}}{\rho+\hat{\delta}}))]$$

$$- \frac{z}{1-k^*-\Gamma^*}(\frac{\rho+\pi+\delta'}{\lambda_2-(\rho+\pi+\delta')})(\frac{\delta'+\pi+\hat{\delta}}{\rho+\hat{\delta}})(e^{(\lambda_2-(\rho+\pi+\delta')(T-t)}-1)\}.$$

For a delay in a tax increase to reduce current investment, this derivative must have the same sign as $\Delta\tau$. At T = t, the second part of (42) equals zero, so the condition that $\mathrm{sgn}(\frac{d\Omega_t}{dT}) = \mathrm{sgn}(\Delta\tau)$ is (given the definitions of Γ and z)

$$(43) \qquad \frac{(\rho+\hat{\delta})(1-k^*-\Gamma^*)}{1-\tau^*} < \delta'.$$

The intuition for this result is quite simple. At time t, the new investment's tax base is negative if its gross marginal product of capital is less than its depreciation allowance. The left-hand side of (43) is the long-run cost of capital per dollar under the new tax system, to which the marginal product will eventually converge, while the right-hand side of (43) is the instantaneous depreciation allowance per dollar. Although the matter is complicated by the fact that the actual value of F' won't equal the left-hand side of (43) immediately (because of adjustment costs), the intuition is that if the underlying tax base is negative just after investment, then a delay in a tax cut raises taxes and investment.

For T > t, the second term in (42) is nonzero. For accelerated depreciation ($\delta' + \pi > \hat{\delta}$), it has the opposite sign of $\Delta\tau$, making it less likely that $\frac{d\Gamma_t}{dT}$ will have the same sign as $\Delta\tau$. The intuition is that with accelerated depreciation the tax base will increase, eventually approaching the entire marginal product of capital. Thus, if $\frac{d\Omega_t}{dT}$ and $\Delta\tau$ have different signs at T = 0, their signs remain opposite for T > 0.

The following result may also be demonstrated.

Proposition: Suppose $\Delta\tau > 0$ (< 0) and $\frac{d\Omega_t}{dT} > 0$ (< 0) at T = t. If $(\rho+\hat{\delta})$, π, and θ^* are all nonnegative, and $\delta' + \pi > \hat{\delta}$, then there exists a finite value $\bar{T} > t$ at which Ω_t is maximized (minimized).

Proof: There are two cases to consider. If $\lambda_2 > \rho + \pi + \delta'$, it is clear that the second term in brackets in (42) is negative and becomes arbitrarily large in absolute value as T increases. This must make the entire term in brackets negative for all T above some critical value. If $\lambda_2 < \rho + \pi + \delta'$, the second term again is negative and increases in absolute value with T, but is bounded by the value it approaches asymptotically at $T = \infty$. At $T = \infty$, the term in brackets in (42) equals:

$$(44) \qquad -\frac{1}{1-\tau^*} + \frac{z}{1-k^*-\Gamma^*}[1 + (\frac{\delta'+\pi-\hat{\delta}}{\rho+\hat{\delta}})(1 + \frac{\rho+\pi+\delta'}{\lambda_2-(\rho+\pi+\delta')})],$$

which has the same sign as

$$(45) \qquad -\frac{(\rho+\hat{\delta})(1-k^*-\Gamma^*)}{(1-\tau^*)} + \delta'(\frac{\lambda_2-(\rho+\hat{\delta})}{\lambda_2-(\rho+\pi+\delta')}).$$

By assumption, $\lambda_2 < \rho + \pi + \delta'$. From the definition of λ_2 (in (21)), it is easily shown that $\lambda_2 > \rho$, since α and ϕ are ≥ 0 and $(\rho+\hat{\delta})$ was assumed > 0. Since by assumption $\delta' + \pi > \hat{\delta}$, expression (45) is bounded above by:

$$(46) \qquad -\frac{(\rho+\hat{\delta})(1-k^*-\Gamma^*)}{(1-\tau^*)} + \delta'\frac{\hat{\delta}}{\pi+\delta'}.$$

A sufficient condition for this to be negative is for the assumed conditions to hold, that $(\rho+\hat{\delta}) \geq 0$, $\pi \geq 0$, and the effective tax rate θ^* is nonnegative (and hence $(1-k^*-\Gamma^*) > (1-\tau^*)$). Therefore, expression (46) is negative and so is the term in brackets in (42). Hence, this term is zero for some finite value of T, and negative for all higher values.

This result says that in the presence of accelerated depreciation there is a finite date of introduction (perhaps the current one) at which the impact of a tax change on current investment is maximized. For a tax cut, this date is the date which maximizes current investment. For a tax increase, it is the date which minimizes current investment.

To summarize the results of this section, an anticipated tax change influences current investment in two ways. The first relates to the desire to smooth the investment path to a new desired long-run capital stock. As the enactment date, T, becomes more distant, the current impact on investment of this incentive declines. The effect disappears entirely when there are no adjustment costs. The second effect, which is present even when there are no adjustment costs, comes from anticipated capital gains or losses associated with changes in the relative treatment of new and old capital goods. When there is an impending tax cut ($\Delta\tau < 0$) and accelerated depreciation ($\delta' + \pi > \hat{\delta}$), a delay of the tax cut may increase current investment if initial depreciation allowances are sufficiently large. Eventually, however, the first effect must dominate. (Even if $\phi = 0$, this is true, since the second effect changes sign once assets have been substantially depreciated.)

Considered next are the effects of changes in the investment tax credit, both temporary and permanent.

B. Anticipated Permanent Change in the Investment Tax Credit

Suppose the investment tax credit changes from $\bar{k}$ to k^* at date $T > t$. Then, for $s > T$, $a_s = 0$ (see (19)). For $s < T$,

$$a_s = \frac{k^*-\bar{k}}{1-k^*-\Gamma^*} = \frac{\Delta k}{1-k^*-\Gamma^*}. \tag{47}$$

The contribution of this term to Ω_t is:

$$\lambda_2\int_t^T e^{-\lambda_2(T-t)}\frac{\Delta k}{1-k^*-\Gamma^*}ds = \frac{\Delta k}{1-k^*-\Gamma^*}(1-e^{-\lambda_2(T-t)}) \tag{48}$$

$$= \frac{\Delta(\frac{1-k-\Gamma}{1-\tau})}{(\frac{1-k^*-\Gamma^*}{1-\tau^*})}(1-e^{-\lambda_2(T-t)}).$$

Comparing (47) and (48) to (35) and (41), one observes that this effect of the credit is equivalent to that of an anticipated tax cut in the presence of economic depreciation allowances. A future increase in k leads to more investment today to smooth the accumulation of the larger capital stock desired after date T. Just as in the case of accelerated depreciation, however, there is an additional effect associated with a change in the relative values of new and old capital. When k increases it decreases the value of existing capital, which does not qualify for the credit, relative to new capital, which does. This effect, which discourages current investment if k is expected to increase, is accounted for by the $\dot{k}_t$ term appearing in (19).

Because of the jump in k at T, $\dot{k}_T$ is undefined. Its impact on Ω_t is massed at T in a_T. However, its effect can be calculated as the limit of the effects of the $\dot{k}$ terms associated with a change from $\bar{k}$ to k* over an arbitrarily short interval around T. This yields an effect on Ω_t of:

$$(49) \qquad \frac{\lambda_2}{\rho+\hat{\delta}} e^{-\lambda_2(T-t)} \frac{\Delta k}{1-k^*-\Gamma^*},$$

which has the same sign as Δk, discouraging investment if $\Delta k > 0$. Combining (48) and (49) yields the total effect of the change in k:

$$(50) \qquad \Omega_t = \frac{\Delta k}{1-k^*-\Gamma^*}\left(1 + \frac{\lambda_2-(\rho+\hat{\delta})}{\rho+\hat{\delta}} e^{-\lambda_2(T-t)}\right).$$

If $\lambda_2 > (\rho+\hat{\delta})$, Ω_t exceeds the value it would have for no change in k (i.e., for $T \to \infty$). In this case, the expectation of an increase in the investment tax credit reduces current investment. The capital loss effect in (49) outweighs the smoothing effect in (48). However, λ_2 may be less than $(\rho+\hat{\delta})$. From the definition of λ_2 in (21), it follows that $\lambda_2 < (\rho+\hat{\delta})$ if and only if $\alpha < \phi\hat{\delta}$.[6] When the elasticity of quasirents with respect to changes in the capital stock, α, is low, and adjustment costs, φ, are high, smoothing can outweigh the capital loss term. As shown below, this is the same condition for the value of existing capital goods to <u>increase</u> with an increase in the investment tax credit. The increase indicates that marginal q increases by more than the gap between marginal and average q does. Since a capital gain occurs in such an event, the anticipation of such a gain encourages current investment.

Because $\hat{\delta}$ depends on φ, the condition $\alpha < \phi\hat{\delta}$ will not be satisfied for all φ above some critical level. In fact, it will be satisfied for a possibly empty interval of ϕ, given δ and α. Since $\hat{\delta} = \delta(1-\frac{1}{2}\phi\delta)$, α is less than $\phi\hat{\delta} = \phi\delta - \frac{1}{2}(\phi\delta)^2$ if and only if $\alpha < .5$ and ϕ is in the interval $(\frac{1 - \sqrt{1-2\alpha}}{\delta}, \frac{1 + \sqrt{1-2\alpha}}{\delta})$.

C. A Temporary Tax Credit

In this case, we imagine a shift from k^* to $\bar{k}$ at some date $T' < T$ before the shift back to k^* at T. The shift at T has the effect on Ω_t just estimated, while the effect of the earlier shift is:

$$(51) \qquad -\frac{\Delta k}{1-k^*-\Gamma^*}\left(1 + \frac{\lambda_2-(\rho+\hat{\delta})}{\rho+\hat{\delta}}\, e^{-\lambda_2(T'-t)}\right).$$

Combining (50) and (51) yields the full impact of the temporary change from T' to T:

$$(52) \qquad \Omega_t = \frac{\Delta k}{1-k^*-\Gamma^*}\left(\frac{\lambda_2-(\rho+\hat{\delta})}{(\rho+\hat{\delta})}\right)\left(e^{-\lambda_2(T-t)} - e^{-\lambda_2(T'-t)}\right).$$

Since $T > T'$, this has the same sign as $-\Delta k = \bar{k} - k^*$ if and only if $\lambda_2 > (\rho+\hat{\delta})$. In this case, an anticipated temporary increase in the investment tax credit from $\bar{k}^*$ to $\bar{k}$ raises the current cost of capital. Once again, the desire to smooth higher investment during the interval (T', T) is outweighed by the anticipated capital losses at T' (net of the gains at T) caused by changes in the relative value of existing capital. Similarly, if $\lambda_2 < (\rho+\hat{\delta})$, an anticipated temporary tax credit increases current investment.

Using (52), one can estimate the impact of adjustment costs on the current impact of an anticipated temporary change in k. Since $\frac{d\lambda_2}{d\phi} < 0$, the effect on Ω_t of an increase in λ_2 is opposite that of an increase in ϕ.

Differentiating (52) with respect to λ_2, on obtains:

$$(53) \qquad \frac{d\Omega_t}{d\lambda_2} = - \frac{\Delta k}{1-k^*-\Gamma^*}$$

$$\cdot \frac{e^{\lambda_2(T-t)}}{\rho+\hat{\delta}} (\Delta T[\lambda_2 - (\rho+\hat{\delta})] + (e^{\lambda_2 \Delta T} -1)[1 - (T'-t)[\lambda_2 - (\rho+\hat{\delta})]])$$

where $\Delta T = T - T' > 0$. Consider the "normal" case where $(\rho+\hat{\delta}) < \lambda_2$, where an anticipated credit discourages current investment. For a temporary increase in k ($\Delta k = k^* - k < 0$), the expression in (53) is positive if $(T'-t) < \frac{1}{\lambda_2-(\rho+\hat{\delta})}$. (Otherwise, the two terms in brackets on the right-hand on the right-hand side of (53) are of opposite sign and the overall sign is ambiguous.) Thus, for a temporary policy that starts sufficiently soon ($T' \rightarrow t$), an increase in ϕ (which decreases λ_2) will reduce Ω_t, thereby lessening the negative impact on current investment. For a more distant policy, the inability to adjust quickly to changes in incentives when ϕ is large is opposed by the greater relevance of future tax changes.

4. Numerical Simulations of Effective Tax Rates

This section uses the expression derived in the last section to illustrate the effects, Ω, on the cost of capital associated with tax reforms such as those recently enacted in the U.S., and translate these estimates into the effective tax rates on current investment with expression (29).

Performing these experiments requires values for several economic and technological parameters. The real discount rate is set at 4 percent, as is the inflation rate. For the constant elasticity specification $F(K) = AK^{\gamma}$, the parameter $\alpha = 1 - \gamma$ (see (18)). Since this production function corresponds to the Cobb-Douglas function with other factors (such as labor) held constant, one may view α as the complement of the capital share of gross output. Since depreciation in the U.S. is typically about 10 percent of GNP, and capital's share of net income is about one quarter, it is reasonable to set $\alpha = .65$. This value of α guarantees that $\lambda_2 > (\hat{\rho+\delta})$, so that an anticipated cut in the investment tax credit stimulates investment. The remaining technological parameters, δ and ϕ, are varied to estimate the impact of tax changes for different types of asset under different adjustment cost conditions. Values of $\delta = .03$ and $.10$ are used to represent structures and equipment, respectively, and values of $\phi = .5$ and 20 are considered. The latter value of ϕ is much more consistent with findings in the empirical literature, though there are arguments one can make suggesting that such estimates may be biased upward.[7]

Consider a change in τ and k such as that recently adopted in the U.S. under the Tax Reform Act of 1986. The reform lowered the statutory corporate tax rate from .46 to .34 and repealed the investment tax credit, which had been .10 for equipment only. Thus, $\bar{\tau} = .46$, $\tau^* = .34$, $\bar{k} = .10$ and $k^* = 0$ in the base simulations for equipment and $\bar{k} = k^* = 0$ for structures. Tax depreciation parameters of $\delta' = .20$ for equipment and $\delta' = .05$ for structures are used to represent the fact that each asset has accelerated depreciation under both old and new tax systems, relative to the direct measures of economic depreciation, δ.[8]

The tax reform was enacted after several years of discussion, during which it became progressively more likely that the reform would occur. In recent years before 1986, important changes in investment incentives were introduced in 1981, 1982 and 1984. In 1981, as well as in 1986, the reform eventually introduced was discussed and debated for at least two years. Thus, it is quite important to consider the potential impact of anticipated tax changes on current investment.

To assess the impact of expectations, we consider permanent transitions to the new tax system enacted immediately and with prior announcements of one and five years. The simulation results are given in Table 1, which gives effective tax rates $\hat{\theta}$ based on expression (29). For purposes of comparison, simulations that consider the impact on equipment of the changes in τ and k alone are also presented. Results for each simulation include the values of $\hat{\delta}$, the true rate of economic depreciation, $-\lambda_1$, the speed of adjustment, and λ_2, the discount factor applied to the terms a_s (s>t) in computing Ω_t.

The impact of the tax changes in the long run is given by the tax rates under immediate adoption, since there are no anticipated tax changes in these Oimulations. As has been pointed out by many who analyzed the recent tax changes, the long-run effective tax rates rise for equipment (when the investment tax credit is removed) and fall for structures, with both new rates very close to the statutory tax rate (at least when $\delta \approx \hat{\delta}$). The fact that $\theta^* \approx \tau^*$ does not, however, deny the presence of accelerated depreciation. It simply indicates that the acceleration via δ' is roughly offset in present value by the lack of inflation indexing. Indeed, the measure of acceleration that matters in the current context is $\delta' + \pi$, since this is the rate at which real depreciation allowances decline. Thus, a delay in the tax cut provision need not by itself increase the current effective tax rate.

When the tax change is delayed, the short-run results are different. For structures, the present value of depreciation allowances is small, so the positive effect of investment due to increased value of depreciation deductions does not outweigh the negative effect of a reduction in the long-run cost of capital. When $\phi = .5$, the value of $\hat{\theta}$ under a five-year delay nearly equals its value under the old tax system. The change is heavily discounted due to the rapid speed of adjustment ($\lambda_2 = .32$). For equipment, the acceleration of depreciation itself, described in the second set of simulations, is enough to make a delay in the tax cut increase current investment and lower the current effective tax rate. This is because the present value of depreciation allowance, z, is much higher for this more rapidly depreciating asset. The anticipated removal of the investment tax credit described in the third set of simulations increases current investment as firms wish to invest more to take advantage the investment tax credit. These two effects combined give the announced policy shifts a powerful effect on current investment.

As has been pointed out elsewhere (Auerbach and Hines, 1986), it is not necessary that reductions in the effective tax rate associated with a delay in tax changes also reduce tax revenues. Indeed, the delayed reduction in τ encourages equipment investment while raising revenue. This is because, while taxes collected on new investment may be reduced (if δ' exceeds the marginal product of capital), taxes collected on existing assets will be increased by keeping the higher tax rate.

This ability to increase current investment and revenue at the same time is another way of presenting the fact that windfalls are being given existing assets when the tax rate is cut. Such windfalls have been seen as an important shortcoming of the tax reform, since if revenue is being held constant the effective tax rate on new investment rises. However, one must stop short of characterizing as superior or more efficient policies that increase current investment without decreasing current tax revenue, since taxing existing capital may well have an impact on expectations about the shape of future "reforms." Nevertheless, the short-run impact of tax reforms, as well as their long-run consequences, should be considered in light of the frequency with which new provisions have been introduced.

5. Tax Reform and Market Value

Tax changes affect the value of the firm as well as the incentive to invest. The close relation of these two effects has already been brought out in showing how anticipated capital gains and losses are incorporated into current incentives. It is also possible to calculate how tax reforms affect the value of the firm as a whole, not just investments undertaken at a specific date.

At any given time, the value of the firm will depend not only on the capital stock but also on the previous path of capital accumulation, since depreciation allowances do not follow economic depreciation. This section's analysis is limited to cases in which previous accumulation has been in a steady state. Thus, the tax reform experiment is one not only near a steady-state, but where the steady state has been disturbed.

With a single factor of production, capital, and decreasing returns to scale, there are two sources of firm value in the absence of taxes: normal returns to capital and pure economic profits. As always, it is possible to reinterpret a decreasing returns technology with one factor as a constant returns technology with two factors, the second being a fixed factor owned by the firm that "earns" the economic profits as a factor reward. This is especially helpful in the current context, for then it is possible to apply the result of Hayashi (1982), adjusted for taxes by Summers (1981), that the value of the firm's capital stock per unit equals the marginal cost of new capital, adjusted for differences in tax attributes. Thus, the firm's value has two components: this tax-adjusted value of marginal q, multiplied by the capital stock, plus the discounted value of pure profits. This can be expressed per unit of capital, yielding a value of average q that includes not only the tax adjusted value of capital but also the discounted profits per unit of capital. One can then consider the effects of tax reform on the total as well as the components, a particularly useful exercise if one wishes to consider the effects of tax reform on the value of the firm under different assumptions about whether the firm has any pure profits.

To begin, consider the value of the firm's capital stock. The marginal price of new capital goods is, from (12) and the definition of q, $q = 1 + \phi\dot{K}/K$. This capital receives investment credits per unit of kq and depreciation allowances worth Γq, and yield a stream of after-tax quasirents in the future. Hence, the existing capital stock, K, which has the same future productivity per unit, must be worth $q(1-k-\Gamma)K + \Delta$, where Δ is the present value (in terms of taxes saved) of this capital stock's depreciation allowance deductions.

Since it has been assumed that this capital was accumulated in a steady state, a constant amount of capital, δK, was purchased at each prior date, at an <u>average</u> price of $(1-\frac{1}{2}\phi\delta)$. This average cost is relevant for calculating the total value of depreciation allowances, since allowances are based on total capital expenditures. Thus, the value of depreciation allowances on existing capital at the current date, zero, is:

$$(54)\qquad \Delta = \int_{-\infty}^{0} \delta K(1-\tfrac{1}{2}\phi\delta)\Gamma^{(-t)}dt = \hat{\delta}K\int_{-\infty}^{0}\Gamma^{(-t)}dt$$

where $\Gamma^{(-t)}$ is, as before, the present value of depreciation allowances remaining for an asset of age -t. If one assumes, as above, that depreciation allowances are at a constant proportional rate δ' but not indexed, then (54) may be rewritten:

$$(55)\qquad \Delta = \hat{\delta}K\int_{-\infty}^{0} e^{(\delta'+\pi)t}\Gamma dt = \frac{\hat{\delta}}{\delta'+\pi}\Gamma K.$$

It follows that the average value of the capital stock is:

$$(56)\qquad q^K = q(1-k-\Gamma) + \Delta/K = q(1-k-\Gamma) + \frac{\hat{\delta}}{\delta'+\pi}\Gamma$$

To simplify expression (56), note that, in the steady state, $K = K^*$, the optimal capital stock under the steady state's tax system. Thus, (24) and (25) may becombined to yield, at t = 0,

$$(57)\qquad \dot{K} = (-\lambda_1)(\hat{K}-K^*) = (-\lambda_1)(\frac{\Omega}{\alpha})K^*.$$

Using (57) and the definition of q, one may rewrite (56) as:

$$(58)\qquad q^K = (1-k-\Gamma+\frac{\hat{\delta}}{\delta'+\pi}) + \lambda_1\phi(1-k-\Gamma)\Omega/\alpha.$$

In the steady state, $k = k^*$, $\Gamma = \tau^* Z$, and $\Omega = 0$. Thus, the deviation of q^K from its steady state value at time zero due to an unannounced tax policy change is:

$$(59) \qquad \Delta q^K = q^K - q^{K*} = -\Delta k_0 - \Delta\tau_0 z(1-\frac{\hat{\delta}}{\delta'+\pi}) + \lambda_1\phi(1-k^*-\Gamma^*)\frac{\Omega_0}{\alpha}$$

Where $\Delta k_0 = k_0 - k^*$ and $\Delta\tau_0 = \tau_0 - \tau^*$. This value will reflect both changes in marginal q (through Ω_0) and changes in the relative valuation of new and old capital.

Next, consider the impact of tax reform on the discounted value of pure profits. By construction, these profits equal the after-tax quasirents in each year in excess of the capital stock's marginal product, or, normalized by the current capital stock,

$$(60) \qquad q^P = \frac{1}{K^*}\int_0^\infty e^{-\rho t}(1-\tau_t)[F(K_t) - K_t F'(K_t)]dt.$$

For a small change in tax policy around the steady state, the change in q^P at time zero is:

$$(61) \qquad \Delta q^P = \frac{1}{K^*}\int_0^\infty e^{-\rho t}(-\Delta\tau_t[F(K^*)-K^*F'(K^*)] - (1-\tau^*)K^*F''(K^*)(K_t-K^*))dt$$

where $\Delta\tau_t = \tau_t - \tau^*$. This has two components, due to the change in the taxation of existing profits, and the change in profits. Given (7') and the definition of α in (18) this may also be written:

$$(62) \qquad \Delta q^P = \frac{(\rho+\hat{\delta})(1-k^*-\Gamma^*)}{(1-\tau^*)}[-\frac{\alpha}{1-\alpha}\int_0^\infty e^{-\rho t}\Delta\tau_t dt + \alpha\int_0^\infty e^{-\rho t}(1-\tau_t)(\frac{K_t-K^*}{K^*})dt]$$

An expression for K_t is obtained by solving the first-order differential equation (24), using the initial condition that $K_0 = K^*$:

$$(63) \qquad K_t = e^{\lambda_1 t}[K^* - \lambda_1 \int_0^t e^{-\lambda_1 s}\hat{K}_s ds]$$

which, given the definition of $\hat{K}_s$ in (25), yields:

$$(64) \qquad \frac{K_t - K^*}{K^*} = \frac{\lambda_1}{\alpha}\int_0^t e^{\lambda_1 (t-s)}\Omega_s ds.$$

Substitution of (64) into (62) yields a solution for Δq^p in terms of exogenous parameters alone:

$$(65) \qquad \Delta q^p = \frac{(\rho+\hat{\delta})(1-k^*-\Gamma^*)}{(1-\tau^*)}[-\frac{\alpha}{1-\alpha}\int_0^\infty e^{-\rho t}\Delta\tau_t dt + \lambda_1 \int_0^\infty e^{-\rho t}(1-\tau_t)\int_0^t e^{\lambda_1 (t-s)}\Omega_s ds dt].$$

Expressions (59) and (65) provide the component changes in market value resulting from any change in tax policy initiated at date zero. For immediate, permanent tax changes q, they simplify considerably.

For a permanent change in k and τ, it follows from (40) and (50) that:

$$(66) \qquad \Omega \equiv -\frac{1}{1-k^*-\Gamma^*}[\Delta k - \Delta\tau(\frac{1-k^*-\Gamma^*}{1-\tau^*} - z)]$$

which is simply the proportional change in the long run cost of capital, $(\rho+\hat{\delta})(1-k-\Gamma)/(1-\tau)$. Substituting (66) into (59) and (65) (and using the facts that $\lambda_1 + \lambda_2 = \rho$ and $\lambda_1\lambda_2 = -\frac{\alpha(\rho+\hat{\delta})}{\phi}$) yields:

$$(67a) \qquad \Delta q^K = [\Delta k - \Delta\tau(\frac{1-k^*-\Gamma^*}{1-\tau^*} - z)](\frac{\rho+\hat{\delta}}{\lambda_2}) - [\Delta k + \Delta\tau z(1-\frac{\hat{\delta}}{\delta'+\pi})]$$

(67b) $$\Delta q^{P} = [\Delta k - \Delta\tau(\frac{1-k^{*}-\Gamma^{*}}{1-\tau^{*}} - z)](\frac{\rho+\hat{\delta}}{\rho} - \frac{\rho+\hat{\delta}}{\lambda_2}) - \Delta\tau\frac{\alpha}{1-\alpha}(\frac{1-k^{*}-\Gamma}{1-\tau^{*}})(\frac{\rho+\hat{\delta}}{\rho})]$$

(67c) $$\Delta q^{T} = \Delta q^{k} + \Delta q^{P} = \Delta k\frac{\hat{\delta}}{\rho} - \Delta\tau[(\frac{1-k^{*}-\Gamma^{*}}{1-\tau^{*}})(\frac{1}{1-\alpha})(\frac{\rho+\hat{\delta}}{\rho}) - z(\frac{\hat{\delta}}{\delta'+\pi} + \frac{\hat{\delta}}{\rho})]$$

From these expressions, a number of points about the effects of changes in τ and k may be made. Each tax change affects the value of the capital stock, q^K, in two ways, represented by the two bracketed terms in (67a). The first is the change due to the change in marginal q, the second the change in the relative value of new and existing assets. Any policy that increases marginal q (a cut in τ or an increase in k) increases the first term, while with accelerated depreciation, the credit increase and the tax cut affect the second term in opposite directions ways. The credit increase causes a capital loss by increasing the distinction between old and new capital, while the tax cut narrows the difference associated with differences in prospective depreciation allowances.

A second difference between the two policies appears in expression (67b), the impact on the present value of pure profits. This is due to the extra windfall given to the firm by a tax cut as the result of the reduced taxation of existing profits. Since $\lambda_2 > \rho$ (for F' and hence $\hat{\rho} + \delta > 0$), policies of either type that encourage investment also increase profits through an expansion of output. This profit increase depends on the assumption that the firm faces fixed output prices. In a more general model, with other factors of production or profits bid down by declining output prices, one might expect all or part of this increase in profits to be absent.

The division of changes in the total value of the firm between changes in the value of capital and changes in the value of profits depends on the technology of adjustment. For an investment tax credit, the total change in the value of the firm is simply $\Delta k\frac{\hat{\delta}}{\rho}$, the discounted value of additional investment credits. With high adjustment costs, $\lambda_2 \to \rho$, so this appears entirely as an increase in q^K; there is little change in output or profits and the firm simply receives the additional credits as a windfall to capital. At the other extreme, with no adjustment costs, $\Delta q^K = -\Delta k$, as the value of marginal q doesn't change at all. At the critical intermediate value of $\lambda_2 = \rho + \hat{\delta}$ (where, as shown above, $\alpha = \phi\hat{\delta}$), the effect on q^K is zero, as the two effects in (67a) cancel.

For a tax cut, the situation is more complicated, depending on the extent to which depreciation allowances are accelerated. Total value increases by more per unit increase in marginal q than in the case of the investment tax credit for three reasons: reduced taxation of normal returns to existing capital, reduced taxation of the component of the tax base associated with recapture of previous accelerated depreciation, and reduced taxation of preexisting profits. The first two effects are present in (67a), the last in (67b). Even with no adjustment costs ($\lambda_2 = \infty$), the value of the capital stock increases because of the reduced tax on recapture of accelerated depreciation, by $\Delta\tau z(1-\frac{\hat{\delta}}{\delta'+\pi})$.

Since much of the criticism of the recent tax reform has focused on shifts in the tax burden between new and old capital, it is interesting to consider the effects on the value of the existing capital stock, q^K, of such a change.

Table 2 presents calculations of Δq^K based on (67a) for the same permanent tax reform considered in Table 1, the removal of the 10 percent investment tax credit for equipment and a cut in the corporate tax rate from .46 to .34. The calculations use the same economic parameters as before (δ = .1, δ' = .2 for equipment, δ = .03, δ' = .05 for structures, and $\rho = \pi$ = .04). For each case, the change in q^K is broken down into four components: the changes in marginal q and the gap between marginal and average q caused by the changes in k and τ.

As expected, the combined effects coming through marginal q are negative for equipment and positive for structures, with the effects larger when adjustment costs are large. This impact of adjustment costs is especially strong for structures because of the lower rate at which structures depreciate; existing capital gets the benefit of increased after-tax returns over a longer period. The "windfall" effects represented by increases in the value of old relative to new capital are positive for both parts of the policy, the removal of the tax credit and the reduction in the tax rate. For equipment, each part of the policy raises the market value of capital, with the total impact relatively insensitive to the size of adjustment costs. For structures, the total increase value is substantially higher with higher adjustment costs.

Given the differences in modelling approaches (analytical linear approximation versus exact calculations derived from a numerical simulation model) and economic assumptions these results are quite consistent with those found by Auerbach and Hines (1986).

6. Conclusion

This paper has presented an analytical discussion of the impact of tax reforms on current investment and market value, taking account not only of the nature of the tax law but also the production and adjustment cost technology.

Its main contribution has been the derivation of analytical expressions for the impact of future tax provisions on the value of the firm and the user cost of capital. These expressions are helpful in understanding the impact of particular tax changes and the importance of investment smoothing and announcement effects.

Many important considerations have been omitted from the analysis. For example, in recent years, tax losses and other constraints have been an important phenomenon. The impact of such constraints varies across assets and can either encourage or discourage investment (Auerbach, 1983, 1986; Auerbach and Poterba, 1986; Altshuler and Auerbach, 1986). In an environment without perfect loss offset, the effects of immediate or delayed tax reforms may be quite different than those portrayed here. In considering assets individually, one ignores the spillover effects that changes in one type of investment may have on another through complementarity in the production function and shared adjustment costs. A multiple capital stock model is too complicated for the derivation of interpretable analytical expressions, although Auerbach and Hines (1986) have considered the effects of large anticipated tax changes in a numerical simulation model with two capital stocks.

The analysis has emphasized the important relation between investment incentives and changes in the firm's market value, both present and anticipated. For these to be useful in evaluating tax incentives, a better positive model of the dynamic process of tax reform is needed.

Footnotes

1. The expression for taxes in (2) treats all capital costs $p_K C(I/K)I$ as part of capital expenditures for tax purposes. This is consistent with the U.S. tax treatment calling for the addition of indirect costs (such as installation) to basis. In reality, some of the indirect costs associated with adding capital, such as retraining of labor, would normally not be capitalized but simply deducted as an expense.

2. This latter effect would be absent if adjustment costs depended only on the level of investment, rather than the ratio of investment to the capital stock. The ratio specification is typically used in the empirical literature estimating adjustment costs. An additional reason for using it here is that it makes analysis of the effects on market value easier. This choice of specification has some impact on the results concerning investment behavior. An earlier version of the paper used the level rather than ratio specification. The differences in results are discussed below.

3. The total cost to the firm of new capital goods is $(1-\phi\delta+\frac{1}{2}\phi I/K)I = (1-\frac{1}{2}\phi\delta)\delta K$ in the steady state. The steady state value of the firm's capital stock is constant. Thus, depreciation, which is the reduction in capital value plus expenditure on new capital goods, is $(1-\frac{1}{2}\phi\delta)\delta K = \hat{\delta}K$. Another way of viewing the same result is that an increase in capital expenditure today, holding future expenditure constant, yields an asset that depreciates at rate δ plus additional capital at each date in the future because of the reduced unit price of capital induced by the current expenditure. The increase (in the steady state) is $\frac{1}{2}\phi\delta^2$ per unit of capital, compounded at each date,

yielding a net rate of depreciation of $\delta - \frac{1}{2}\phi\delta^2$.

As shown by Abel (1982), neutrality of the tax system in such a case would require netting these gains against primary depreciation in computing depreciation allowances. That such a correction to the measurement of economic depreciation is appropriate does not appear to be widely recognized in discussions about measuring depreciation properly. Given typical estimated magnitudes of the proportional adjustment cost parameter ϕ, the correction may be quite large.

4. Note that (7), holding I and K constant, may be rewritten:

$$c_t = q(\rho + \hat{\delta} + \frac{\dot{k}+\dot{\Gamma}}{1-k-\Gamma})(1-k-\Gamma)/(1-\tau)$$

Thus, for small changes one obtains:

$$\frac{\Delta c}{c} = -(\frac{\Delta k+\Delta\Gamma}{1-k-\Gamma} - \frac{\Delta\tau}{1-\tau} + \frac{1}{\rho+\hat{\delta}}\frac{\Delta(\dot{k}+\dot{\Gamma})}{1-k-\Gamma}).$$

5. Abel actually considered the case where only a fraction of new investment could be written off immediately, so that $z = \frac{\delta}{\rho+\delta}$ despite the accelerated write-off. The crucial issue, however, is the timing of the allowances.

6. A similar ambiguity was found in a general equilibrium model without adjustment costs, but with the interest rate influenced by individual savings decisions by Judd (1985). Implicit in the fixed interest rate assumption made here is the notion that that the assets being considered are small relative to the (perhaps international) capital market. The choice of adjustment cost specification is also crucial here. Under the level adjustment cost specification used in an earlier paper (C(I) instead of C(I/K)), λ_2 must exceed $\rho+\hat{\delta}$ and the ambiguity disappears. The condition $\alpha < \phi\hat{\delta}$ may be shown to imply that the reduction in the marginal product of existing capital caused by new investment is less than the increase in adjustment cost rents earned by the same capital. Such rents are zero in the C(I) specification.

7. For further discussion, see Auerbach and Hines (1986).

8. The law also included changes in depreciation provisions that were less important than the changes in τ and k.

9. Since perterbations around the steady state are being assumed, these "permanent" changes are, strictly speaking, temporary changes of a very long duration.

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Table 1
The Effects of Tax Reform on
Investment Incentives

Case	$\hat{\delta}$	$-\lambda_1$	λ_2	Effective Tax Rates: Old Law	New Law With: No Delay	1-Year Delay	5-Year Delay
Equipment (δ =.1, δ'=.2, $\bar{k}$=.1)							
k^*=0, τ^*=.34							
ϕ=20	0	.02	.06	.055	.128	-.101	-.130
ϕ=.5	.0975	.40	.44	.167	.336	-.534	-.048
k^*=.1, τ^*=.34							
ϕ=20	0	.02	.06	.055	-.004	-.042	-.098
ϕ=.5	.0975	.40	.44	.167	-.015	-.096	-.031
k^*=0, τ^*-.46							
ϕ=20	0	.02	.06	.055	.196	-.028	-.011
ϕ=.5	.0975	.40	.44	.167	.456	-.385	.025
Structures (δ=.03, δ'=.05, $\bar{k}$=k^*=0, τ^*=.34)							
ϕ=20	.021	.03	.07	.444	.326	.332	.360
ϕ=.5	.0296	.28	.32	.478	.356	.391	.472

All simulations assume τ = .46, ρ = .04, π = .04, and α = .65.

Table 2
The Effects of Tax Reform on The Value of Existing Capital

	Proportional Change in Capital Value Resulting From:						
	Removal of Investment Credit			Cut in Corporate Tax Rate			
Asset	Marginal Effect	New-Old Capital Effect	Overall Effect	Marginal Effect	New-Old Capital Effect	Overall Effect	Total
Equipment							
ϕ=20	-.067	.100	.033	.028	.086	.114	.147
ϕ=.5	-.032	.100	.068	.013	.051	.064	.132
Structures							
ϕ=20	--	--	--	.119	.035	.154	.154
ϕ=.5	--	--	--	.030	.031	.061	.061

Calculations are based on parameters used for simulations presented in Table 1.

Analytically, the effects are defined (based on (67a)) by:

investment credit cut:

marginal: $\Delta k(\rho+\hat{\delta})/\lambda_2$
new-old capital: $-\Delta k$

corporate tax cut:

marginal: $-\Delta\tau\left(\frac{1-k^*-\Gamma^*}{1-\tau^*} - z\right)(\rho+\hat{\delta})/\lambda_2$

new-old capital: $-\Delta\tau z\left(1 - \frac{\delta}{\delta'+\pi}\right)$