NBER WORKING PAPER SERIES

THE INTERNATIONAL TRANSMISSION OF CREDIT BUBBLES: THEORY AND POLICY

Jaume Ventura Alberto Martin

Working Paper 20933 http://www.nber.org/papers/w20933

NATIONAL BUREAU OF ECONOMIC RESEARCH 1050 Massachusetts Avenue Cambridge, MA 02138 February 2015

We would like to thank Francisco Queiros for superb research assistance. We acknowledge support from the Spanish Ministry of Science and Innovation (grants ECO2008-01666 and CSD2006-00016), the Generalitat de Catalunya-DIUE (grant 2009SGR1157), and the Barcelona GSE Research Network. In addition, Ventura acknowledges support from the ERC (Advanced Grant FP7-249588), and Martin from the Spanish Ministry of Science and Innovation (grant Ramon y Cajal RYC-2009-04624), the ERC (Consolidator Grant FP7-615651), and the IMF Research Fellowship. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

NBER working papers are circulated for discussion and comment purposes. They have not been peerreviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2015 by Jaume Ventura and Alberto Martin. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

The International Transmission of Credit Bubbles: Theory and Policy Jaume Ventura and Alberto Martin NBER Working Paper No. 20933 February 2015 JEL No. E32,E44,O40

ABSTRACT

We live in a new world economy characterized by financial globalization and historically low interest rates. This environment is conducive to countries experiencing credit bubbles that have large macroeconomic effects at home and are quickly propagated abroad. In previous work, we built on the theory of rational bubbles to develop a framework to think about the origins and domestic effects of these credit bubbles. This paper extends that framework to two-country setting and studies the channels through which credit bubbles are transmitted across countries. We find that there are two main channels that work through the interest rate and the terms of trade. The former constitutes a negative spillover, while the latter constitutes a negative spillover in the short run but a positive one in the long run. We study both cooperative and noncooperative policies in this world. The interest-rate and terms-of-trade spillovers produce policy externalities that make the noncooperative outcome suboptimal.

Jaume Ventura CREI Universitat Pompeu Fabra Ramon Trias Fargas, 25-27 08005-Barcelona SPAIN and Barcelona GSE and also NBER jventura@crei.cat

Alberto Martin CREI Universitat Pompeu Fabra Ramon Trias Fargas, 25-27 08005 Barcelona Spain and Barcelona GSE amartin@crei.cat The last twenty five years can be broadly described as a period of falling interest rates, rising financial integration and frequent credit booms and busts. Figure 1 plots the evolution of the real interest rate and of the share of countries experiencing a credit boom between 1990 and 2012. As the figure shows, the real interest rate has fallen progressively and has become negative towards the end of the sample; the share of countries experiencing a credit boom, in the meantime, has increased over time. In the run-up to the financial crisis of 2008, almost 30% of the world's countries were experiencing a credit boom. Figure 2 plots the international financial integration (IFI) index, defined as the sum of a country's foreign assets and liabilities as a share of GDP: both the top panel, which depicts the evolution of the IFI for advanced economies, and the bottom panel, which depicts the IFI for emerging economies, reflect a substantial increase in financial integration between 1990 and 2012.

It is tempting to view these three stylized facts as part of a general narrative, in which greater financial integration, low and declining interest rates and frequent credit booms (and busts) are different aspects of the same phenomenon. This is exactly the view that many espoused in the aftermath of the 2008 financial crisis, when it was widely argued that low interest rates in advanced economies, which resulted from excessive capital inflows, relaxed lending standards and fueled the credit boom that would eventually give rise to the crisis.¹ Although appealing, this narrative raises a number of questions. What generates these low interest rates? Why should they give rise to credit booms and busts, as opposed to a permanent rise in credit? What are the welfare implications of such low interest rates? Is there a role for policy intervention and, if so, for policy coordination across countries? This paper provides an analytical framework to address these questions.

We build on the closed-economy model of credit bubbles in Martin and Ventura (2014). Our goal now is to study the international transmission of these credit bubbles and the trole of international policy coordination. To do this, we develop a two-country model of the world economy, each of which produces a differentiated input that is used to produce both consumption and investment goods. Each country is populated by entrepreneurs, who can invest productively in capital accumulation, and savers, who cannot. In principle, entrepreneurs could borrow resources from savers in the credit market. We assume that there are weak enforcement institutions, however, which prevent entrepreneurs from pledging their output to savers. This limits the amount of credit in equilibrium, reducing the international interest rate and world savings. As in much of the recent literature, therefore, it is financial frictions that lead to low interest rates in our model: they essentially

¹See, for example, Bernanke (2009a), The Economist (2009), Krugman (2009) and Portes (2009).

constrain the supply of assets by limiting entrepreneurial borrowing (e.g. Caballero et al. 2008). Using this framework as a benchmark, we make two key innovations.

First, we provide a simple characterization of the interaction between low interest rates, financial integration, and credit booms and busts. When a bubble appears in a given country, it enables its entrepreneurs to expand their borrowing beyond the output that can be pledged. In this sense, bubbles have a crowding-in effect, raising entrepreneurial demand for credit and thus the world interest rate, net capital inflows, investment and growth. As usual in this class of models, though, bubbles have a crowding-out effect as well, because at any point in time they make it necessary to divert resources away from investment and use them to pay for the higher credit that has been granted in the past. From a global perspective, the bubble that maximizes long-term welfare is the one that optimally trades-off these two effects.

Second, we show that credit bubbles transmit themselves through two channels. The first one works through the interest rate and constitutes a negative spillover. A credit boom in one country attracts capital from the rest of the world by raising the world interest rate. The rest of the world experiences a capital outflow that lowers investment and growth. The second one works through the terms of trade, and it constitutes a negative spillover in the short run and a positive spillover in the long run. A credit boom in one country raises both the demand and the supply of goods produced by that country. In the short run, before supply has had time to adjust, the demand effect dominates and the credit boom therefore leads to an improvement in the terms of trade and a real appreciation. In this sense, our model reflects a concern that has been repeatedly voiced by policy makers across emerging markets during the last few years, as their countries simultaneously experienced increases in asset prices, capital inflows and large real appreciations. In the long run, though, it is the supply effect that dominates and the credit boom thus leads to a decline in the terms of trade and a real depreciation.

Even though some bubbles are more desirable than others for the world economy, nothing guarantees that they will materialize in equilibrium. In fact, an essential feature of bubbles is that they are driven by investor sentiment or market expectations. Their value today depends on market expectations about their value tomorrow, which in turn depends on tomorrow's market expectations about their value on the day after, and so on. Because of this, the bubble provided by the market may be either too small or too large, or it may be suboptimally distributed across countries. It may, moreover, fluctuate over time as expectations change. Thus, a bubble may appear in a country and lead to capital inflows, and a credit and investment boom. When the bubble bursts, however, the logic is reversed: capital leaves the country, there is a credit bust and investment and output contract.

This leads us to analyze the role of policy in this world. We allow countries to tax and subsidize credit contracted by their own citizens. First, we study the cooperative equilibrium in which countries choose policies in a coordinated fashion so as to maximize the sum of utilities of all individuals. We show that a this equilibrium replicates the optimal bubble allocation by adopting a policy of "leaning against investor sentiment", taxing credit where the bubble is inefficiently large and subsidizing it elsewhere. This policy is what we call *expectationally robust*, in the sense that it stabilizes investment, output and consumption and insulates them from fluctuations in investor sentiment.

Although the cooperative solution provides a useful benchmark, it is not a very realistic description of the real world. Thus, we also analyze a noncooperative equilibrium in which policy solves a Nash problem between the two governments. Since the latter do not take into account policy externalities, the noncooperative equilibrium is in general suboptimal. If the interest-rate spillover is strong and the terms-of-trade spillover is weak, we find that countries tend to subsidize credit too much and the world economy generates excessive credit. If instead the interest-rate spillover is weak and the terms-of-trade spillover is strong, countries tend to tax credit too much and the world economy generates insufficient credit.

Our paper is closely related to three strands of literature. To begin with, it builds on the notion that financial frictions are important determinants of the size and direction of capital flows. This is related to Gertler and Rogoff (1990), Boyd and Smith (1997), Matsuyama (2004) and Aoki et al. (2010), all of which argued that contracting frictions can generate capital outflows even in capital scarce or high-productivity economies. Recently, similar models have been developed to account for global imbalances and low international interest rates. In Caballero et al. (2008), for example, high-growing developing economies may experience capital outflows due to pledgeability constraints that restrict their supply of financial assets. In Mendoza et al. (2007), it is instead the lack of insurance markets in developing economies that fosters precautionary savings and the consequent capital outflows.

The major distinction between our work and this literature is that we show how the low interest rates brought about by financial frictions may give rise to asset bubbles. As such, we are also close to the recent research on bubbles and financial frictions, including Farhi and Tirole (2011), Miao and Wang (2011), and our own previous work (Martin and Ventura (2012, 2014)). Of this literature, we are closest in interest and focus to the branch that has extended the analysis to open economies, including Caballero and Krishnamurthy (2006), Ventura (2011), and Basco (2014). Our work is different from these papers in that it deals with large economies, in its focus on the link between asset bubbles and credit booms and busts, and on the study of cooperative and non-cooperative policy responses.

Finally, our paper is also related to the large body of research that studies fluctuations in credit. On the empirical front, this research has sought to. Some examples of this work are Ruckes (2004), Dell'Ariccia and Marquez (2006), Matsuyama (2007), Gorton and He (2008) and Martin (2008). Like us, these papers model fluctuations in credit. Unlike us, though, these papers emphasize the role of regulation or the incentives in generating and magnifying fluctuations in credit. We take instead a macroeconomic perspective and argue that low interest create the conditions for asset bubbles to arise, which may themselves give rise to credit booms and busts.

The rest of the paper is structured as follows. Sections 1 develops a symmetric two-country model with bubbles. Sections 2 explores bubbly equilibria in low interest rate environments and studies the implications of bubbles for capital flows, investment, output and the terms of trade. Section 3 characterizes the optimal bubble and shows how to implement it with policy. Section 4 analyzes non-cooperative policymaking and shows how it may lead to excessive credit. Section 5 concludes.

1 A two-country model of credit bubbles

This section presents a two-country model of credit bubbles that builds on the closed-economy model developed by Martin and Ventura (2014). The centerpiece of this model is a credit friction that limits the amount of collateral in the economy. As a result, the demand for credit is low and both the interest rate and investment are depressed. This creates the conditions for the economy to experience bubble-driven credit booms and busts. Extending this framework to a two-country world allows us to study how these booms and busts propagate across countries.

1.1 The Model

We consider a world economy with two countries, Home and Foreign. Time is discrete and infinite, $t = 0, ..., \infty$. The world is populated by two-period overlapping generations that are equally sized and evenly distributed across countries. Each country produces one differentiated input using two factors of production, labor and capital. Inputs are traded, but factor services are not. Inputs are combined to produce final goods. Since these goods have a common price in both countries, we use them as the numeraire. The world also contains trees that have no role in production but are used as a store of value. These trees are pure bubbles and we often refer to them as such.

Each generation contains a fraction ε of entrepreneurs and a fraction $1 - \varepsilon$ of savers. Both of them maximize the expected consumption of final goods in their old age. This choice of preferences simplifies the analysis substantially. First, it makes the savings decision trivial as individuals save any income they have when young. Second, it simplifies the portfolio decision since individuals are risk-neutral and choose the portfolio that offers the highest expected return in terms of final goods.

Savers supply one unit of labor when young and earn a wage w_t and w_t^* in Home and Foreign, respectively. As usual, we use asterisks to denote the corresponding Foreign variable. Savers have two options for their savings: to store consumption goods or to purchase credit contracts. Storage of consumption goods yields a gross safe return of $\rho < 1$. Credit contracts offer a, possibly contingent, interest rate. We refer to the expected return to credit contracts, R_{t+1} , as the interest rate. Maximization generates a simple portfolio rule. If $R_{t+1} > \rho$, savers only purchase credit contracts. If $R_{t+1} < \rho$, savers only store goods. If $R_{t+1} = \rho$, savers are indifferent between purchasing credit contracts or storing goods.

Entrepreneurs also supply one unit of labor when young and earn a wage. Unlike savers, they have the option of owning capital and trees. Let q_t and q_t^* be the cost of capital in Home and Foreign. Capital earns a rental r_t and r_t^* for one period and then depreciates. Thus, the gross return to investment is $\frac{r_{t+1}}{q_t}$ and $\frac{r_{t+1}^*}{q_t^*}$. Let b_t and b_t^* denote the value all trees in Home and Foreign. Some trees are old since they were planted by previous generations. Some trees are new since they are planted by the current generation. Thus,

$$b_{t+1} = G_{t+1} \cdot b_t + n_{t+1}$$
 and $b_{t+1}^* = G_{t+1}^* \cdot b_t^* + n_{t+1}^*$ (1)

where G_{t+1} and G_{t+1}^* denote the growth in the value of old trees, and n_{t+1} and n_{t+1}^* is the value of new trees. Free disposal implies that $n_t \ge 0$ and $n_t^* \ge 0$. We refer to n_{t+1} and n_{t+1}^* as bubblecreation shocks.

Entrepreneurs sell credit contracts to finance additional holdings of capital and trees. These contracts must be collateralized, that is, they must be backed by credible promises of future payments. This brings us to the only friction that we consider in this paper: weak institutions limit the amount of available collateral. In particular, we assume that entrepreneurs can hide their rental income from enforcement institutions and, as a result, they cannot pledge it to their creditors.² This implies the following credit or collateral constraint:

$$\hat{R}_{t+1} \cdot f_t \le b_{t+1}$$
 and $\hat{R}_{t+1}^* \cdot f_t^* \le b_{t+1}^*$ (2)

where \hat{R}_{t+1} and \hat{R}_{t+1}^* are the contingent interest rates offered by Home and Foreign entrepreneurs; and f_t and f_t^* are the amounts of credit obtained. Equation (2) simply states that promised payments cannot exceed the value of the trees owned by old entrepreneurs.

Throughout, we focus on the case in which the expected return to investment exceeds the interest rate in both countries. Thus, credit is maximized and the collateral constraint is binding in all possible continuations of the economy. Entrepreneurs achieve this by setting $f_t = \frac{E_t b_{t+1}}{R_{t+1}}$ and $\hat{R}_{t+1} = \frac{b_{t+1}}{f_t}$ in Home, and $f_t^* = \frac{E_t b_{t+1}^*}{R_{t+1}}$ and $\hat{R}_{t+1}^* = \frac{b_{t+1}^*}{f_t^*}$ in Foreign. This implies that:

$$q_t \cdot k_{t+1} + b_t = \varepsilon \cdot w_t + \frac{E_t b_{t+1}}{R_{t+1}} \quad \text{and} \quad q_t^* \cdot k_{t+1}^* + b_t^* = \varepsilon \cdot w_t^* + \frac{E_t b_{t+1}^*}{R_{t+1}} \tag{3}$$

since $f_t = q_t \cdot k_{t+1} + b_t - \varepsilon \cdot w_t$ and $f_t^* = q_t^* \cdot k_{t+1}^* + b_t^* - \varepsilon \cdot w_t^*$.

Individuals interact both in markets and within firms and these interactions determine all relevant prices. Consider first the market for trees. Maximization imposes the following restrictions on tree prices:

$$G_{t+1} = R_{t+1} + u_{t+1}$$
 and $G_{t+1}^* = R_{t+1} + u_{t+1}^*$ (4)

where $E_t u_{t+1} = E_t u_{t+1}^* = 0$. We refer to u_{t+1} and u_{t+1}^* as bubble-return shocks. To understand Equation (4), recall that entrepreneurs finance the purchase of trees by selling credit contracts. Note also that the return to holding trees equals the growth in their price. If the expected return to holding trees falls short of the interest rate, young entrepreneurs make a loss when they sell credit contracts to purchase trees and the demand for trees is zero. If the expected return to holding trees exceeds the interest rate, young entrepreneurs make a profit when they borrow to purchase trees and the demand for trees is unbounded. Thus, equilibrium requires that the expected return to holding trees equals the interest rate.

Consider next the credit market. Combining the supply of credit by savers with the demand

²This is obviously an extreme assumption. In Martin and Ventura (2014) we assumed instead that entrepreneurs can pledge a fraction ϕ of their rental income. We focus here on the case $\phi = 0$ to simplify the algebra.

for credit from entrepreneurs we obtain the equilibrium interest rate:

$$R_{t+1} = \max\left\{\rho, \frac{E_t\left\{b_{t+1} + b_{t+1}^*\right\}}{(1-\varepsilon) \cdot (w_t + w_t^*)}\right\}$$
(5)

Equation (5) is quite intuitive and it plays a crucial role in what follows. The economy's collateral is $E_t \{b_{t+1} + b_{t+1}^*\}$. If collateral is low, the demand for credit is low. Only a fraction of the labor income is used to purchase credit contracts and the interest rate equals ρ . When this is the case, we say that there is partial intermediation since there are some savings that are not transferred to entrepreneurs. If collateral is high, the demand for credit is high. All labor income is used to purchase credit contracts and the interest rate exceeds ρ . When this is the case, we say that there is full intermediation since all savings are transferred to entrepreneurs.

Consider next firms, which are assumed to be competitive. To produce inputs, firms combine labor and capital using standard Cobb-Douglas technologies:

$$y_t = l_t^{1-\alpha} \cdot k_t^{\alpha} \quad \text{and} \quad y_t^* = l_t^{*1-\alpha} \cdot k_t^{*\alpha} \tag{6}$$

with $\alpha \in [0, 1]$; where l_t and l_t^* are the labor force in Home and Foreign. Since we have assumed already that the young have one unit of labor in each country, $l_t = l_t^* = 1$. Then, maximization implies that factors are paid the value of their marginal products:

$$w_t = p_t \cdot (1 - \alpha) \cdot k_t^{\alpha} \quad \text{and} \quad w_t^* = p_t^* \cdot (1 - \alpha) \cdot k_t^{*\alpha} \tag{7}$$

$$r_t = p_t \cdot \alpha \cdot k_t^{\alpha - 1}$$
 and $r_t^* = p_t^* \cdot \alpha \cdot k_t^{*\alpha - 1}$ (8)

where p_t and p_t^* are the prices of the Home and Foreign input.

To produce final goods, firms combine Home and Foreign inputs. Competition ensures that price equals cost:

$$1 = \left(0.5 \cdot p_t^{1-\gamma} + 0.5 \cdot p_t^{*1-\gamma}\right)^{\frac{1}{1-\gamma}}$$
(9)

with $\gamma \in (0, \infty)$. Equation (9) says that the price of final goods (which is one since we use these goods as the numeraire) equals the cost of producing them. The cost function is CES, it is symmetric on both inputs, and it has an elasticity of substitution equal to γ . Shepard's lemma implies that the demand for Home and Foreign inputs by final goods producers are given by $0.5 \cdot p_t^{-\gamma} \cdot x_t$ and $0.5 \cdot p_t^{*-\gamma} \cdot x_t$, where x_t is the total production of final goods.

To produce capital, firms combine final goods and the domestic input. The assumption that investment requires the input produced domestically introduces a home bias in spending that will play a role later. Once again, competition implies that price equals cost:

$$q_t = p_t^\beta \quad \text{and} \quad q_t^* = p_t^{*\beta} \tag{10}$$

with $\beta \in [0, 1]$. Equation (10) says that the price of capital equals the cost of producing it. The cost function is Cobb-Douglas with a share of domestic inputs equal to β . Shepard's lemma implies that the demands for Home and Foreign inputs by producers of capital are given by $\beta \cdot p_t^{\beta-1} \cdot k_{t+1}$ and $\beta \cdot p_t^{*\beta-1} \cdot k_{t+1}^*$.

Consider finally the market for inputs. As usual, we obtain prices by imposing that supplies equal demands. In the market for Home inputs this requires that $y_t = 0.5 \cdot p_t^{-\gamma} \cdot x_t + \beta \cdot p_t^{\beta-1} \cdot k_{t+1}$. In the market for Foreign inputs this requires that $y_t^* = 0.5 \cdot p_t^{*-\gamma} \cdot x_t + \beta \cdot p_t^{*\beta-1} \cdot k_{t+1}^*$. Combining these two conditions, we obtain:

$$\frac{k_t^{\alpha} - \beta \cdot p_t^{\beta-1} \cdot k_{t+1}}{k_t^{\ast \alpha} - \beta \cdot p_t^{\ast \beta-1} \cdot k_{t+1}^{\ast}} = \left(\frac{p_t}{p_t^{\ast}}\right)^{-\gamma}$$
(11)

This completes the description of the model.

1.2 Equilibrium dynamics

To construct equilibria for this economy, we propose first a joint stochastic process for bubble return and creation shocks: $\{n_t, n_t^*, u_t, u_t^*\}$ for all t. Naturally, this bubble process must be such that $E_t u_{t+1} = E_t u_{t+1}^* = 0$, $n_t \ge 0$ and $n_t^* \ge 0$ for all t. With this stochastic process at hand, we determine all possible sequences for the state variables $\{k_t, k_t^*, b_t, b_t^*\}$ from a given initial condition using the following set of equations:

$$2 = p_t^{1-\gamma} + p_t^{*1-\gamma}$$
 (12)

$$\frac{k_t^{\alpha} - \beta \cdot p_t^{\beta-1} \cdot k_{t+1}}{k_t^{*\alpha} - \beta \cdot p_t^{*\beta-1} \cdot k_{t+1}^*} = \left(\frac{p_t}{p_t^*}\right)^{-\gamma}$$
(13)

$$R_{t+1} = \max\left\{\rho, \frac{E_t\left\{n_{t+1} + n_{t+1}^*\right\}}{(1-\varepsilon)\cdot(1-\alpha)\cdot(p_t\cdot k_t^{\alpha} + p_t^*\cdot k_t^{*\alpha}) - b_t - b_t^*}\right\}$$
(14)

$$k_{t+1} = p_t^{1-\beta} \cdot \varepsilon \cdot (1-\alpha) \cdot k_t^{\alpha} + p_t^{-\beta} \cdot \frac{E_t n_{t+1}}{R_{t+1}}$$
(15)

$$k_{t+1}^{*} = p_t^{*1-\beta} \cdot \varepsilon \cdot (1-\alpha) \cdot k_t^{*\alpha} + p_t^{*-\beta} \cdot \frac{E_t n_{t+1}^{*}}{R_{t+1}}$$
(16)

$$b_{t+1} = (R_{t+1} + u_{t+1}) \cdot b_t + n_{t+1} \tag{17}$$

$$b_{t+1}^* = \left(R_{t+1} + u_{t+1}^*\right) \cdot b_t^* + n_{t+1}^* \tag{18}$$

If all the sequences generated in this way are such that $k_t \ge 0$, $k_t^* \ge 0$, $b_t \ge 0$ and $b_t^* \ge 0$ for all t, we say that the proposed bubble process is an equilibrium. Otherwise, we say that the proposed bubble process is not an equilibrium.

Bubbly equilibria are possible if the interest rate is below the growth rate. Since this economy exhibits no growth, this requires that the interest rate be less than one.³ Traditional models of rational bubbles generate low interest rates by assuming that investment is dynamically inefficient. That is, these models assume that the interest rate equals the return to investment, and then focus on environments in which the return to investment is below the growth rate. Here we do not follow this path. Instead, we generate low interest rates by assuming financial frictions. That is, we assume that insufficient collateral depresses the interest rate below the growth rate, and then focus on environments in which the return to investment is above the growth rate.

To evaluate welfare in these equilibria is not straightforward since there are infinitely many generations and each of them contains four different types. The welfare of the savers of generation t is given by:

$$U_{S,t} = R_{t+1} \cdot w_t \text{ and } U_{S,t}^* = R_{t+1} \cdot w_t^*$$
 (19)

in Home and Foreign; while the welfare of the entrepreneurs of generation t is given by

$$U_{E,t} = \varepsilon^{-1} \cdot r_{t+1} \cdot k_{t+1} \text{ and } U_{E,t}^* = \varepsilon^{-1} \cdot r_{t+1}^* \cdot k_{t+1}^*$$
(20)

Some equilibria Pareto dominate others and this allows us to rank them. But some equilibria are better for some individuals and worse for others. In this case, we cannot rank them.

Once the bubble process has been specified, the world economy is a complete dynamical system and Equations (12)-(18) constitute its law of motion. From a given state initial state $\{k_0, k_0^*, b_0, b_0^*\}$, Equations (12)-(18) allow us to obtain the following state $\{k_1, k_1^*, b_1, b_1^*\}$. Before drawing $\{n_1, n_1^*, u_1, u_1^*\}$, Equations (12)-(16) determine the price of the two inputs, the interest

 $^{^{3}}$ Once again, this is an extreme assumption that helps simplify the algebra. In Martin and Ventura (2014) we consider the case in which the economy exibits sustained growth.

rate and the capital stocks next period. After drawing $\{n_1, n_1^*, u_1, u_1^*\}$, Equations (17)-(18) determine the bubbles next period. We can then start the process again using $\{k_1, k_1^*, b_1, b_1^*\}$ as the initial state to obtain $\{k_2, k_2^*, b_2, b_2^*\}$. By repeating this procedure again and again, we find the dynamics of the world economy and determine its properties.

2 Dynamics of credit booms and busts

The world economy developed in the previous section can experience bubble-driven credit booms and busts. In this section, we construct simple equilibria that clarify how these booms and busts work and how their effects are transmitted across borders. These equilibria have been designed to illustrate specific points and they do not to replicate the complex and unpredictable behavior of credit and asset prices observed in real economies. This makes them look somewhat artificial. To remove any temptation to think that all equilibria are so simple, we end the section constructing some equilibria in which credit and tree prices exhibit a high degree of unpredictability and complexity.

2.1 The bubbleless economy

A natural benchmark to keep in mind when going through this section is the bubbleless equilibrium, in which $\{n_t, n_t^*, u_t, u_t^*\} = \{0, 0, 0, 0\}$ for all t and $b_0 = b_0^* = 0$. In this equilibrium, trees have no value so that there is no collateral to sustain credit. Entrepreneurs can only invest their own wages while savers fully store theirs. The world economy converges to a steady state in which:

$$k = k^* = [\varepsilon \cdot (1 - \alpha)]^{\frac{1}{1 - \alpha}} \tag{21}$$

$$R = \rho \tag{22}$$

$$p = p^* = 1 \tag{23}$$

Equations (21)-(23) fully characterize the bubbleless steady state. Because there is no credit, the capital stock and output depend only on the wages of entrepreneurs, and the interest rate equals the return to storage ρ . Moreover, because the steady-state is symmetric and output is equalized across countries, the prices of the Home and Foreign inputs are both equal to one.

As mentioned before, we focus on environments in which the the interest rate is below the

growth rate, while the return to investment is above it:

$$\rho = R < 1 < \frac{r}{q} = \frac{r^*}{q^*} = \frac{\alpha}{\varepsilon \cdot (1 - \alpha)}.$$
(24)

Since the economy is stationary, this requires that $\rho < 1$ (which we have assumed already) and that $\alpha > \frac{\varepsilon}{1+\varepsilon}$ (which we assume from now on). The first inequality creates the conditions for bubbly equilibria to exist. The second inequality ensures that investment is dynamically efficient. To see the latter, note that in each period investment equals the wages of entrepreneurs, which represent a fraction $\varepsilon \cdot (1-\alpha)$ of output. The return to this investment is the economy's capital income, which represents a fraction α of output. Thus, the ratio in Equation (24) represents the rate of return to investment in steady state, which we assume is larger than one.

2.2 Global credit bubbles

We characterize now equilibria with global credit bubbles, which are equally distributed between Home and Foreign. The simplest way to do so is to assume that there are no bubble shocks, so that $u_t = u_t^* = 0$, while bubble creation is a fixed fraction η of income in both countries:

$$n_{t+1} = \eta \cdot p_t \cdot k_t^{\alpha} \quad \text{and} \quad n_{t+1}^* = \eta \cdot p_t^* \cdot k_t^{*\alpha} \tag{25}$$

In this equilibria, collateral and credit represent a fraction $\frac{\eta}{R_{t+1}}$ of world income. We can think of η as the size of the bubble. From any valid initial condition, the world economy converges to a steady state in which:

$$k = k^* = \left[\varepsilon \cdot (1 - \alpha) + \frac{\eta}{R}\right]^{\frac{1}{1 - \alpha}}$$
(26)

$$R = \max\left\{\rho, 0.5 - \sqrt{0.25 - \frac{\eta}{(1-\varepsilon) \cdot (1-\alpha)}}\right\}$$
(27)

$$p = p^* = 1$$
 (28)

Equations (26)-(28) characterize the steady state of the world economy. Figure 3 depicts the steady-state values of the model's main endogenous variables as a function of η . The latter and the associated bubble it generates are shown in Panels A and D.

Panels B and C show the capital stock and storage. At low values of η ,⁴ the steady state is in the partial intermediation region. Collateral is scarce and storage is used. In this range,

⁴In particular, $\eta < (1-\rho) \cdot \rho \cdot (1-\varepsilon) \cdot (1-\alpha)$.

increases in η expand the global stock of collateral and raise investment, output and consumption. At high values of η , the world economy enters the full intermediation region. Collateral is now abundant and storage is no longer used. In this range, increases in η raise the interest rate more than proportionally and reduce the global stock of collateral, thereby contracting investment and output.

Panels E and F show that the rate of return on investment is always higher than the interest rate, which is a direct consequence of the binding credit constraint. Note, however, that this gap is reduced whenever bubble creation raises the global economy's stock of collateral. Also note that, because the equilibrium is symmetric, input prices equal one in Panel G. This is why output in Panel H output follows almost one-to-one the capital stock. Symmetry also explains why the current account is zero in Panel I.⁵

Finally, panels J and K depict the welfare of savers and entrepreneurs. Whereas the welfare of entrepreneurs rises and falls with the capital stock, the welfare of savers is monotonically increasing in η . The reason is that, in the full intermediation region, bubble creation has two distinct effects on the welfare of savers: it lowers the steady-state stock of capital (and thus wages and savings) but it raises the steady-state interest rate (and thus the return on these savings). In our parametrization, this last effect dominates and the welfare of savers is increasing in η .

This example with a constant and global bubble illustrates the effects of bubble creation in our model, but it does not generate credit booms and busts. To create this type of episodes we next introduce global shocks to expectations or investor sentiment, which affect either bubble creation or bubble returns. For theoretical clarity, we analyze each of these two alternatives separately.

We consider first bubble creation shocks. In particular, assume that:

$$n_{t+1} = \eta_{t+1} \cdot p_t \cdot k_t^{\alpha} \quad \text{and} \quad n_{t+1}^* = \eta_{t+1} \cdot p_t^* \cdot k_t^{*\alpha} \tag{29}$$

where $\eta_t \in {\eta_L, \eta_H}$ and $\Pr(\eta_{t+1} = \eta_t) = \lambda > 0.5$ for all t. Figure 4 shows a simulation of this equilibrium.⁶ In this simulation, the economy starts and ends with low tree prices. In between, the economy experiences two episodes of high tree prices. In each of them, the bubble grows very

⁵Strictly speaking, the distribution of storage and lending across the world's savers is indeterminate in our model. We focus throughout on the allocation that minimizes capital flows, i.e., we assume that entrepreneurs borrow first from savers in their own country and only turn to the international market once all domestic savings have been intermediated. Fomally, this is equivalent to adding the additional assumption that there are arbitrarily small but not negligible costs of international transactions.

⁶For all the stochastic examples in the paper, we run the model for 10,000 periods to produce each simulation. We use the data obtained from the simulations to produce the corresponding figures.

quickly and bursts afterwards. So does the capital stock and output. In our parametrization, credit booms are associated with relatively high interest rates (but still below the growth rate) and low returns to investment (but still above the growth rate). Credit busts are instead associated with low interest rates and high returns to investment. Finally, note that both savers and entrepreneurs are better off during a credit boom.

This example with global shocks to bubble creation provides an interesting model of how bubbles in asset prices generate episodes of credit booms and busts. The evolution of the capital stock, the interest rate and the return to investment seem to be broadly consistent with real-world experience. The lack of action in current accounts and the terms of trade (that is, relative input prices) is only due to the symmetric nature of the shocks, as we will see shortly.

We consider next bubble return shocks. We do so by assuming that the rate of bubble creation is constant and equal to η , but there is a positive probability that the bubble bursts in any given period:

$$u_{t+1} = u_{t+1}^* = \begin{cases} \frac{\pi}{1-\pi} \cdot R & \text{with probability } 1-\pi\\ -R & \text{with probability } \pi \end{cases}$$

for all t.⁷ Figure 5 shows a simulation of this example. Initially, savers' wages are fully intermediated by the global bubble and storage is not used. At some point, the global bubble bursts: the immediate effect of this collapse is to reduce the interest rate because entrepreneurs no longer demand funds to purchase old trees. Since the rate of bubble creation is constant, though, this fall in the interest rate boosts entrepreneurial collateral thereby raising investment and the capital stock, reducing the rate of return to investment and increasing entrepreneurial welfare. Once again, due to the symmetry of the exercise, input prices are one and current accounts are zero throughout.

This example of global shocks to bubble returns is interesting from a theoretical perspective because it highlights a key aspect of our model. Namely, it is bubble creation (i.e. the value of neew treees) that provides collateral and fosters investment; while the pre-existing bubble (i.e., the value of old trees) actually reduces investment because it competes with it for the resources of savers. This is why a collapse of the bubble that does not affect bubble creation stimulates growth.

⁷This case might seem unrealistic, as it implies that the value of old trees collapses while the value of new trees remains constant. But it is nonetheless useful to understand the mechanics of the model.

2.3 Country credit bubbles

We characterize now bubbles that are no longer equally distributed across countries. Indeed, we go to the other extreme and now consider equilibria in which bubbles occur only in Home. In particular, we assume that:

$$n_{t+1} = \eta_{t+1} \cdot p_t \cdot k_t^{\alpha}$$
 and $n_{t+1}^* = \eta_{t+1}^* \cdot p_t^* \cdot k_t^{*\alpha}$

where $\eta_t \in {\eta_L, \eta_H}$ and $\Pr(\eta_{t+1} = \eta_t) = \lambda > 0.5$ for all t; while $\eta_{t+1}^* = \eta^*$ for all t. We parametrize the model so that countries are still symmetric in the long run, in the sense that the long-run average rate of bubble creation is the same in Home and Foreign. In the reminder of this section, we choose a given sequence of shocks and illustrate how the economy behaves under different parameter assumptions. Comparing the behavior of endogenous variables in these equilibria allows us to illustrate the key determinants of how credit booms and busts are propagated across countries.

It is useful to start with the case in which inputs of both countries are perfect substitutes, that is $\gamma = \infty$. In this case, their prices are constant and equal to one. This eliminates terms-of-trade effects and allows us to focus on transmission through the interest rate. This special or limiting case is depicted in Figure 6.

Initially, Home is in a credit slump with low bubble creation, high storage, and low capital accumulation. The value of trees in foreign, in turn, is just enough to intermediate the wages of its savers: as a consequence, the current account is initially balanced. At some point, Home enters a credit boom as bubble creation rises and expands entrepreneurial collateral. As growth picks up in Home, storage is reduced as savings are reallocated towards entrepreneurial investment: once storage is eliminated, though, Home entrepreneurs begin borrowing from Foreign savers, raising the world interest rate, crowding out capital accumulation in Foreign and leading to a current account deficit in Home. This benefits savers in Foreign despite the fall in the country's capital and wages, but it hurts Foreign entrepreneurs. Naturally, when the credit booms ends, the process is reversed. Home growth collapses, the world interest rate falls, and Home once again runs a current account surplus as there is capital flight to Foreign.⁸

This example illustrates the first transmission channel of credit bubbles which works through

⁸Note, for instance, that Foreign savers fare worst right at the end of a credit boom in Home. At this point, they face both low wages (because, in the past, the credit boom in Home has crowded-out investment in Foreign) and a low interest rate (because, looking forward, the credit bust in Home has reduced the global demand for investment).

the interest rate and the current account. This interest-rate effect constitutes a negative spillover for Foreign. A credit boom in Home attracts Foreign savings by raising the world interest rate. Foreign experiences a capital outflow that lowers investment and growth. A credit bust at Home releases Home savings lowering the world interest rate. Foreign experiences a capital inflow that raises investment and growth.

Once we make the Home and Foreign inputs imperfect substitutes, we have a second transmission channel which works through the terms of trade or the real exchange rate. Note that these two concepts are linked in this model. Home's terms of trade is the ratio of the export and import prices, i.e. $\frac{p_t}{p_t^*}$. Home's real exchange rate is the ratio of the prices of nontraded and traded goods. Since consumption goods have the same prices in both countries, Home's real exchange rate is positively linked to the relative price of investment goods, i.e. $\left(\frac{p_t}{p_t^*}\right)^{\beta}$. Thus, the real exchange rate fluctuates with the terms of trade, except in the limit when the home bias in spending, i.e. $\beta \to 0$.

Figure 7A depicts the case in which $\gamma = 2$ and $\beta = 0$. That is, we assume that there is no home bias in spending. The main difference with Figure 6 is that credit booms in Home, which expand the supply of the Home input, are now accompanied by a deterioration in the country's terms of trade. The reason, of course, is that the expansion in the capital stock in Home raises the relative supply of Home inputs. This terms-of-trade effect constitutes a positive spillover effect for Foreign. In the simulation, this spillover partly compensates for the negative interest-rate spillover. Thus, relative to the case in which $\gamma = \infty$, the increase in the relative output of Home is smaller than before and the benefits of Home's credit boom are more equally distributed throughout the world.

In the presence of a home bias in spending, the terms-of-trade effect becomes more complex and interesting. This is shown in Figure 7B which depicts the case in which $\gamma = 2$ and $\beta = 0.8$. Now, we see that the credit boom initially improves Home's terms-of-trade and only to worsen them gradually. The reason is that now the expansion in the capital stock in Home also raises the demand for Home inputs. This effect happens immediately and leads to a an initial terms-of-trade improvement. Over time, as the capital stock expands, the supply of Home inputs grows and the terms-of-trade deteriorate as in the previous example. We see then an interesting pattern emerging: credit booms lead to a real exchange rate appreciation in the short run that is slowly reversed into a real exchange rate depreciation in the long run.

This example illustrates the second transmission channel of credit bubbles which works through

the terms of trade. This terms-of-trade effect constitutes a negative spillover to Foreign in the short run, but a positive spillover in the long run. A credit boom in Home raises both the demand and the supply for Home inputs. The demand effect dominates in the short run and worsens Foreign's terms of trade. The supply effect dominates in the long run and improves Foreign's terms of trade. Naturally, these effects are reversed if Home experiences a credit bust.

The importance of the terms-of-trade effect depends on the elasticity of substitution between Home and Foreign inputs. So far, we have assumed that this elasticity is high. Figures 8A and 8B assume that $\gamma = 1$, while Figures 9A and 9B assume that $\gamma = 0.6$. Note that, the lower are both γ and β , the larger is the positive spillover to Foreign. A low β reduces the demand effect, while a low γ translates a given supply effect into a larger change in the terms-of-trade effect. An interesting case is that in which $\gamma = 1$ and $\beta = 0$. This is the famous Cole-Obstfeld case in which countries share expansions and contractions equally and their outputs are perfectly correlated.

This concludes our exploration of simple examples, which exhibit either global shocks to investor sentiment or shocks in only one country. These examples are meant to illustrate how the theory works and, in particular, the channels through which credit booms and busts are propagated around the world. To show the type of complex dynamics that the model can generate, we turn next to an example in which credit and asset prices exhibit a rich and unpredictable behavior.

2.4 Complex equilibria

We now simulate an economy that starts in a normal state in which $\eta_t = \eta_t^* = \eta$. With probability $\lambda < 0.5$, the world economy transitions from the normal state to a credit episode in which $\eta_t = \tilde{\eta}$ and $\eta_t^* = \tilde{\eta}^*$ where $\tilde{\eta}$ and $\tilde{\eta}^*$ are drawn from a uniform distribution centered in η . With probability λ , the episode ends and both countries return to the normal state. We also assume that, during each credit episode, there are bubble return shocks that are i.i.d. both across countries and time periods.

Figure 10 illustrates a sample of this simulation. The sample contains three credit episodes We can refer to the first episode as a global depression. Indeed, there is a credit bust in both countries even though it is more pronounced at Home. As investment collapses in both countries, the use of storage surges all over the world. Welfare falls for Foreign and Home savers and entrepreneurs, even though residents of Foreign are relatively better off since they suffer a smaller contraction in output. Once the depression is over, the world economy returns to a normal state for a few periods before entering a second credit episode.

The second episode combines a credit boom in Home with a credit slump in Foreign. As asset prices and investment surge in Home, its entrepreneurs start borrowing also from Foreign savers. As a consequence, Home runs a current account deficit as the world interest rate begins to rise. Although the relative fall of Foreign's output is somewhat mitigated by the improvement in its terms of trade, welfare falls for all residents of Foreign while it rises for residents of Home.

Finally, the third episode in the sample is a global credit boom, driven by a simultaneous rise in the value of trees in Home and in Foreign. The boom is particularly strong in Foreign, though, so that its entrepreneurs borrow also from Home's savers and Foreign runs a current account deficit. The most interesting aspect of this episode is that this simultaneous boom turns out to be excessive from a global perspective. The global bubble, and thus the demand for credit, is so large that it generates a sharp increase in the world interest rate and a collapse in global investment and output! Simply put, the large bubble crowds out investment. As a consequence, the world's entrepreneurs lose from the boom despite the higher value of their trees. The world's savers, in the meantime, benefit from the high interest rates.

This example shows that a world economy characterized by low interest rates can experience rich and volatile patterns of credit, capital flows, and economic activity. Moreover, even though all different types and combinations of credit booms and busts have different implications for welfare, it is not possible to determine whether the market will select any specific one over the rest. Thus, sometimes the global credit boom is too small, leading to weak investment and growth. At other times, the global credit boom may be too large, sustaining high interest rates that also undermine growth. Finally, the global credit boom may be inefficiently distributed across countries, sustaining too much credit in some economies and too little in others. These observations raise a natural and important question: can policies be used to improve upon the market equilibrium? If so, is their a role for policy coordination across countries? We turn to these questions next.

3 Managing credit booms and busts

This section considers the problem of Home and Foreign governments that can influence credit markets by taxing and/or subsidizing entrepreneurs. We first describe the set of policies that we consider and show how they affect the evolution of credit, investment and output. We then characterize a type of cooperative policies, in which both governments jointly maximize a measure of global welfare, and non-cooperative policies, in which governments design policy independently of one another.

Before turning to the analysis, though, it is important to ponder briefly on what it means to design policy in our environment. In our world, the competitive equilibrium is essentially driven by investor sentiment or expectations, which determine the processes $\{n_t, n_t^*, u_t, u_t^*\}$ and thus investment and prices. This poses a problem for the evaluation of policy, since the implementation of a policy may influence expectations themselves and change the underlying process $\{n_t, n_t^*, u_t, u_t^*\}$. We avoid this problem by focusing on policies that are *expectationally robust*, in the sense that they implement the same allocations regardless of agent expectations or investor sentiment.

3.1 What can governments do?

We introduce governments in Home and Foreign and assume that they can influence the equilibrium allocation by subsidizing and/or taxing entrepreneurs. In particular, the Home and Foreign governments respectively promise to give entrepreneurs of generation t a transfer equal to s_{t+1} and s_{t+1}^* units of the consumption good during old age. These transfers can be negative and/or contingent on the state of the economy. Whenever a transfer is positive in a given country, its government finances it by imposing a tax of z_t to entrepreneurs of generation t in that same country. If a transfer is negative, its benefit is distributed to young savers in the same country.

The key aspect of this policy is that it affects entrepreneurial wealth through taxes and subsidies, and thus their ability to borrow. To see this, define n_{t+1}^s and n_{t+1}^{s*} as the net resources that the policies provide to entrepreneurs of generation t in Home and Foreign:

$$n_{t+1}^{s} \equiv s_{t+1} - R_{t+1} \cdot z_t \quad \text{and} \quad n_{t+1}^{s*} \equiv s_{t+1}^* - R_{t+1} \cdot z_t^*.$$
(30)

That is, n_{t+1}^s and n_{t+1}^{s*} reflect the difference between subsidies that will be obtained in old age and the taxes paid in young age by Home and Foreign entrepreneurs, respectively. This transfer of wealth naturally affects entrepreneurs' ability to borrow and thus – as we show below – the demand for credit, expanding it when it is positive and contracting it otherwise.

To close the model, we just need to specify how governments balance their budget constraints. To this effect, as we already mentioned, we require each government to run a balanced budget. That is, we assume throughout that,

$$s_t = z_t \quad \text{and} \quad s_t^* = z_t^*. \tag{31}$$

This enables us to express the dynamics of the policies as follows:

$$s_{t+1} = R_{t+1} \cdot s_t + n_{t+1}^s \text{ and } s_{t+1}^* = R_{t+1} \cdot s_t^* + n_{t+1}^{s*}$$
 (32)

The formal similarity between Equation (32) and Equations (17) and (18) describing the dynamics of the Home and Foreign bubbles hints at some of the results that follow.

We are now ready to analyze the effects of policies $\{n_t^s, n_t^{s*}\}$. These effects are circumscribed to the credit market, and the analysis of the input and final good markets in section 1 remains unchanged. Remember that we focus throughout on the case in which the expected return to investment exceeds the interest rate in both countries. Thus, credit is maximized and the collateral constraint of Equation (2) is binding in all possible continuations of the economy. Entrepreneurs now achieve this by setting $f_t = \frac{E_t b_{t+1} + E_t s_{t+1}}{R_{t+1}}$ and $\hat{R}_{t+1} = \frac{b_{t+1} + s_{t+1}}{f_t}$ in Home, and $f_t^* = \frac{E_t b_{t+1}^* + E_t s_{t+1}^*}{R_{t+1}}$ and $\hat{R}_{t+1} = \frac{b_{t+1}^* + s_{t+1}}{f_t}$ in Home, and $f_t^* = \frac{E_t b_{t+1}^* + E_t s_{t+1}^*}{R_{t+1}}$ and $\hat{R}_{t+1}^* = \frac{b_{t+1}^* + s_{t+1}^*}{f_t^*}$ in Foreign, i.e., by borrowing both against the value of trees and against government subsidies. Given policies $\{n_t^s, n_t^{s*}\}$, this implies that:

$$q_t \cdot k_{t+1} + b_t + s_t = \varepsilon \cdot w_t + \frac{E_t b_{t+1} + E_t s_{t+1}}{R_{t+1}} \quad \text{and} \quad q_t^* \cdot k_{t+1}^* + b_t^* + s_t^* = \varepsilon \cdot w_t^* + \frac{E_t b_{t+1}^* + E_t s_{t+1}^*}{R_{t+1}},$$
(33)

since $f_t = q_t \cdot k_{t+1} + b_t + s_t - \varepsilon \cdot w_t$ and $f_t^* = q_t^* \cdot k_{t+1}^* + b_t^* + s_t^* - \varepsilon \cdot w_t^*$ and we have replaced governments' budget constraints from Equation (31).

Equation (33) captures the fact that entrepreneurial borrowing must be used to invest, to purchase existing trees and to pay the taxes prescribed by the policy. It also confirms that the collateral constraints of Home and Foreign entrepreneurs are relaxed by the policies of their respective countries if and only if $E_t n_{t+1}^s > 0$ and $E_t n_{t+1}^{s*} > 0$, i.e., if and only if the policies raise the expected wealth of entrepreneurs. Taking into account that the supply of credit, which depends on the wages of savers, is unaffected by the policies, the equilibrium interest rate is given by:

$$R_{t+1} = \max\left\{\rho, \frac{E_t\left\{b_{t+1} + b_{t+1}^* + s_{t+1} + s_{t+1}^*\right\}}{(1-\varepsilon) \cdot (w_t + w_t^*)}\right\}$$
(34)

To construct equilibria for this economy, we propose first a joint stochastic process for bubble return and creation shocks, $\{n_t, n_t^*, u_t, u_t^*\}$ for all t; and a policy process $\{n_t^s, n_t^{s*}\}$. With these stochastic processes at hand, we determine all possible sequences for the variables $\{k_t, k_t^*, b_t, b_t^*, s_t, s_t^*\}$ from a given initial condition using Equations (12)-(13) and (17)-(18) together with:

$$R_{t+1} = \max\left\{\rho, \frac{E_t\left\{n_{t+1} + n_{t+1}^* + n_{t+1}^s + n_{t+1}^{s*}\right\}}{(1-\varepsilon)\cdot(1-\alpha)\cdot(p_t\cdot k_t^{\alpha} + p_t^*\cdot k_t^{*\alpha}) - b_t - b_t^* - s_t - s_t^*}\right\}$$
(35)

$$k_{t+1} = p_t^{1-\beta} \cdot \varepsilon \cdot (1-\alpha) \cdot k_t^{\alpha} + p_t^{-\beta} \cdot \frac{E_t \left\{ n_{t+1} + n_{t+1}^s \right\}}{R_{t+1}}$$
(36)

$$k_{t+1}^* = p_t^{*1-\beta} \cdot \varepsilon \cdot (1-\alpha) \cdot k_t^{*\alpha} + p_t^{*-\beta} \cdot \frac{E_t \left\{ n_{t+1}^* + n_{t+1}^{**} \right\}}{R_{t+1}}$$
(37)

Relative to the model without policy interventions, Equations (36) and (37) highlight that, for a given interest rate, investment is now determined by $E_t n_{t+1} + E_t n_{t+1}^s$ in Home and by $E_t n_{t+1}^* + E_t n_{t+1}^{s*}$ in Foreign. When the policies prescribe $E_t n_{t+1}^s > 0$ and/or $E_t n_{t+1}^{s*} > 0$, they raise the expected wealth of Home and/or Foreign entrepreneurs, respectively, and allow them to expand their investment. Naturally, the opposite is true when $E_t n_{t+1}^s < 0$ and/or $E_t n_{t+1}^{s*} < 0$. Equation (35) in turn highlights that, like bubbles, these policies affect the world interest rate by raising or reducing the maximum amount of resources that entrepreneurs can pledge to their creditors, which now becomes $E_t \{b_{t+1} + b_{t+1}^* + s_{t+1} + s_{t+1}^*\}$.

Equations (35)-(37) are key to understand the role of policy in our model. A comparison with the corresponding Equations (14)-(16) of the baseline model reveals that the only effect of policy is to replace b_t with $b_t + s_t$ and b_t^* with $b_t^* + s_t^*$ (and hence, n_t and n_t^* are replaced with $n_t + n_t^S$ and $n_t^* + n_t^{s*}$, respectively). This immediately implies that any competitive equilibrium of the global economy can be implemented through the appropriate design of policies in Home and Foreign. Suppose, for instance, that both governments want to implement the allocation corresponding to an equilibrium in which the value of Home and Foreign trees equals \bar{b}_t and \bar{b}_t^* , respectively. Then, they can do so simply by following a policy that is contingent on the realized value of trees in both countries, setting $s_t = \bar{b}_t - b_t$ and $s_t^* = \bar{b}_t^* - b_t^*$ for all t. This leads immediately to the following proposition.

Proposition 1 Suppose that the governments of Home and Foreign can subsidize their respective entrepreneurs by giving them transfers s_t and s_t^* during old age. Suppose moreover that each government finances these transfers by taxing its young entrepreneurs. Then, the allocations of any equilibrium with bubble processes $\left\{\overline{b}_t, \overline{b}_t^*\right\}$ can be replicated by policies that set,

$$s_t = \overline{b}_t - b_t,$$

$$s_t^* = \overline{b}_t^* - b_t^*,$$

in all periods.

Proof. Follows from previous discussion.

Proposition 1 tells us that governments seeking to replicate the equilibrium allocations associated with bubble processes $\{\bar{b}_t, \bar{b}_t^*\}$ should follow a policy of "leaning against investor sentiment", promising to set s_t and s_t^* in a state-contingent fashion so that $s_t + b_t = \bar{b}_t$ and $s_t^* + b_t^* = \bar{b}_t^*$ in all periods. A salient feature of such policies is that they are expectationally robust, in the sense that they implement the desired allocations regardless of investor sentiment, i.e., no matter what expectations are regarding the price of trees in Home and Foreign. In the countries and times where investor sentiment and the price of trees is low relative to the desired benchmark, the policy requires the corresponding government to subsidize entrepreneurs. Instead, in the countries and times where investor sentiment and the price of trees are too high relative to the desired benchmark, the policy requires the corresponding government to tax entrepreneurial income from the sale of trees in order to reduce borrowing.

3.2 What should governments do?

Proposition 1 tells us that, given the class of policies that we consider, governments can in principle replicate any equilibrium allocation that they desire. Determining which allocation they would actually want to implement, though, is a different matter. First and foremost, it depends naturally on the objective function that we assign to governments. In what follows, we will assume that they maximize steady-state welfare. This seems like a sensible goal and it suits our purpose, which is to illustrate the effects of policy. Moreover, given that the only shocks in our economy are expectation or investor sentiment shocks, we characterize the deterministic allocation that achieves this goal, i.e. we characterize the pair $\{n, n^*\}$ that maximizes steady-state welfare.

To gain a better understanding of the problem, note that there are four types of agents in the world, entrepreneurs and savers both in Home and in Foreign. In a deterministic steady state, the welfare of Home entrepreneurs and savers is respectively given by:

$$c_E = \varepsilon^{-1} \cdot \alpha \cdot p \cdot k^{\alpha}$$
 and $c_S = R \cdot (1 - \alpha) \cdot p \cdot k^{\alpha}$.

This expression simply states that the consumption of entrepreneurs equals the economy's capital income, whereas the consumption of savers equals their labor income times the world interest rate. The corresponding expression for Foreign is given by,

$$c_E^* = \varepsilon^{-1} \cdot \alpha \cdot p^* \cdot k^{*\alpha}$$
 and $c_S^* = R \cdot (1 - \alpha) \cdot p^* \cdot k^{*\alpha}$

The problem that we consider is one in which the governments of Home and Foreign respectively choose the values of n and/or n^* that maximize welfare. They can then implement the desired allocation through the appropriate use of subsidies and taxes as stated in Proposition 1. Given the previous expressions for consumption, we can define the welfare of Home and Foreign by adding the utilities of their residents:

$$W = [R \cdot (1 - \varepsilon) \cdot (1 - \alpha) + \alpha] \cdot p \cdot k^{\alpha}$$
(38)

$$W^* = [R \cdot (1 - \varepsilon) \cdot (1 - \alpha) + \alpha] \cdot p^* \cdot k^{*\alpha}$$
(39)

This completes the characterization of governments' objective functions and of the policy tools at their disposal. The solution to this optimization problem, however, depends on the equilibrium concept that we use. In particular, we characterize a cooperative equilibrium, in which governments choose the policies that maximize their joint welfare; and a non-cooperative equilibrium, in which each government chooses the policy that maximizes its own welfare. Characterizing both types of equilibria clarifies the spillovers and potential externalities that are associated to the design of policy. We turn to each of them next.

3.3 Cooperative policy

This section solves for the cooperative equilibrium, which is defined as follows:

Definition 1 The cooperative equilibrium of our economy is characterized by the pair $\{n, n^*\}$ that solves

$$\max\left[W + W^*\right] = \left[R \cdot (1 - \varepsilon) \cdot (1 - \alpha) + \alpha\right] \cdot \left[p \cdot k^{\alpha} + p^* k^{*\alpha}\right].$$

The pair $\{n, n^*\}$ is obtained by solving

$$\frac{\partial W}{\partial n} + \frac{\partial W^*}{\partial n} = 0,$$
$$\frac{\partial W}{\partial n^*} + \frac{\partial W^*}{\partial n^*} = 0,$$

subject to Equations (12)-(16).

Thus, the cooperative equilibrium is defined as the competitive equilibrium that corresponds to the bubble processes for Home and Foreign that maximize global welfare in steady state. The following proposition, which is formally proved in the appendix, characterizes the solution to this problem.

Proposition 2 The cooperative equilibrium of our economy is characterized by a constant and symmetric rate of bubble creation

$$n = n^* = (1 - \rho) \cdot \rho \cdot (1 - \varepsilon) \cdot (1 - \alpha)^{\frac{1}{1 - \alpha}} \cdot (1 - \rho(1 - \varepsilon))^{\frac{\alpha}{1 - \alpha}}.$$

This level of bubble creation simultaneously guarantees that $R = \rho$ and that wages are fully intermediated, and it sustains a steady state with a capital stock equal to

$$k = k^* = \{ [1 - \rho(1 - \varepsilon)] \cdot (1 - \alpha) \}^{\frac{1}{1 - \alpha}}$$

Proof. See appendix. \blacksquare

Proposition 2 tells us two important yet intuitive features of the "optimal bubble". The first feature is that it is distributed equally across both countries. This arises naturally in our model given the symmetry of the setup and the existence of decreasing returns, which imply that any redistribution of bubble creation away from the symmetric allocation reduces the return to global investment. It is true that, by affecting the relative output of both countries, any such redistribution of the global bubble also has effects on their terms of trade. But this adds an additional source of decreasing returns, since the terms of trade shift against the country with the highest output. The second important feature of the optimal bubble is that it is just large enough to intermediate all wages. In effect, once all wages have been intermediated, investment cannot be raised any further and any additional increases in the bubble can only lead to a higher world interest rate and thus to a lower capital stock, output and, ultimately, overall consumption. By combining propositions 1 and 2, we can characterize the optimal policy when both governments seek to maximize global welfare in steady state:

Corollary 1 To maximize steady-state welfare, the governments of Home and Foreign should set $\{s_t, s_t^*\}$ so that

$$s_t = \rho \cdot (1-\varepsilon) \cdot (1-\alpha)^{\frac{1}{1-\alpha}} \cdot [1-\rho(1-\varepsilon)]^{\frac{\alpha}{1-\alpha}} - b_t,$$

$$s_t^* = \rho \cdot (1-\varepsilon) \cdot (1-\alpha)^{\frac{1}{1-\alpha}} \cdot [1-\rho(1-\varepsilon)]^{\frac{\alpha}{1-\alpha}} - b_t^*,$$

in all periods.

Proof. Follows directly from Propositions 1 and 2.

Corollary 1 is a specific application of Proposition 1, and it tells us the exact manner in which governments must lean against investor sentiment to replicate the optimal bubble.

To see how the effects of this policy in practice, we can return to the example of Section 2.4. In that example, the world economy was subject to bubble creation and bubble return shocks that generated rich dynamics. We now analyze the dynamics in the presence of policy. Figure 11 reproduces in black the evolution of the main variables in the absence of policy, whereas it depicts in red the evolution of these same variables under the policy specified in the corollary. Figure 12 shows the specific policy interventions that underlie these results, assuming that investor expectations are unaffected by the presence of a policy.

The policy fully stabilizes the economy throughout the sample. In the first episode, which was a global credit bust, it does so by subsidizing entrepreneurs both in Home and in Foreign. Because entrepreneurs borrow against these promised subsidies, the policy raises global investment and output while eliminating storage. It stabilizes the global interest rate and sets it equal to ρ , and it raises the welfare of savers and entrepreneurs all over the world.

In the second episode, which combines a credit boom in Home with a bust in Foreign, the policy also stabilizes the world economy but it has important redistributional consequences. Namely, due to diminishing returns, the market bubble is suboptimally distributed between Home and Foreign in this episode. By taxing Home entrepreneurs and subsidizing Foreign ones, the policy reestablishes symmetry. This raises global investment and output, while hurting residents of Home and benefiting those of Foreign. Finally, the last episode in the example entailed a global credit boom, which raised global interest rates and led to a fall in global investment and output. Here, the policy prescribes taxing entrepreneurs both in Home and in Foreign. This reduces the interest rate and boosts investment and output. Interestingly, this tax raises the welfare of entrepreneurs and reduces that of savers.

This example shows how the policy outlined in corollary 1 is successful in stabilizing the world economy and raising global welfare. This casts a positive light on the possibilities of policy intervention, but these appear to rely on two crucial features of the analysis. The first is the assumption that governments can impose unlimited lump-sum taxes on young entrepreneurs. This assumption may seem central to our results, since it is not entirely clear whether governments with more limited powers of taxation (i.e., access only to distortionary taxes or subject to a maximum tax rate) would be able to implement the optimal allocation. Although this presumption is intuitive, we have shown elsewhere that it is not true, since governments subject to such limitations could finance the policy equally well by issuing debt.⁹ The second crucial feature of our analysis is, of course, that we have shown the effects of a policy that is optimally designed and implemented by governments seeking to maximize global welfare. This may raise the question of whether such a policy would also be implemented if governments behaved in a non-cooperative fashion.

3.4 Non-cooperative policy

This section solves for the cooperative equilibrium, which is defined as follows:

Problem 2 In the non-cooperative solution n maximizes W and n^* maximizes W^* . In this case, n and n^* are obtained by solving,

$$\frac{\partial W}{\partial n} = 0,$$
$$\frac{\partial W^*}{\partial n^*} = 0,$$

subject to Equations (12)-(16).

Thus, the non-cooperative equilibrium is defined as the Nash equilibrium of a game between governments in which each of them tries to maximize its own welfare. In this Nash equilibrium, countries take the policies of the other country as given.

In general, the non-cooperative equilibrium does not replicate the cooperative equilibrium. The reason, of course, is that countries do not take into account the spillovers that their policies create.

⁹For a formal demonstration of this, see Martin and Ventura (2014).

We saw in Section 2.3 that these spillovers work through the world interest rate and the terms-of trade. The interest-rate spillover is negative and this implies that individual countries have an incentive to generate too much credit. The terms-of-trade spillover is positive in the long run and this implies that countries have an incentive to generate too little credit. These two forces distort country incentives and create a global inefficiency. The sign of this inefficiency depends on the relative importance of the two distortions.

We illustrate these insights by solving the noncooperative problem under three different parameter assumptions. Figure 13 shows the case in which $\gamma = 2$ and $\beta = 0$. In this case, the terms-of-trade spillover is weak and does not compensate the interest-rate inefficiency. As a result, the noncooperative equilibrium leads to too much credit, too high interest and too low investement and output. In this world economy, credit is too high. Figure 14 shows the Cole-Obstfeld case in which $\gamma = 1$ and $\beta = 0$. In this case, the two distortions offset each other and the noncooperative equilibrium coincides with the cooperative one. Figure 15 shows the case in which $\gamma = 0.01$ and $\beta = 0$. In this case, the terms-of-trade is so powerful that governments eliminate all credit and the noncooperative equilibrium delivers the bubbleless equilibrium.

4 Concluding remarks

We live in a world of low financial integration, low interest rates, and frequent credit booms. In this paper, we have developed a framework to think about these three stylized facts as part of a general narrative. In our model, low interest rates are the result of weak enforcement institutions, which limit collateral and thus credit and investment in the global economy. For simplicity, we have assumed throughout that enforcement institutions are equally weak in all countries: nothing substantial would change, however, if we assumed that they are particularly weak in developing countries. In this case, the decline in the world interest rate could be attributed to the latter's increasing financial integration with the global economy.

Our model shows that low interest rates create the conditions for asset bubbles to arise, which in turn give rise to credit booms and busts. When a bubble raises the market value of assets in an economy, it relaxes borrowing constraints and fuels a credit boom that attracts foreign capital and expands investment and economic growth. The effects of this expansion propagate across the world economy through two channels: the interest rate, which increases for the rest of the world due to the higher demand for credit, and the terms of trade, which initially worsen but then improve for the rest of the world as the country going through the credit boom first experiences an increase in the relative demand for its products and eventually an increase in their relative supply. Of course, credit booms sustained by asset bubbles ultimately rest on expectations or investor sentiment, which are prone to sudden shifts. When this happens, the boom turns into a credit bust and asset prices, intermediation, investment and output all fall as capital flows abroad.

This general description of credit booms and busts seems quite conventional. But it turns the standard perception of the link between asset prices, investment and capital flows on its head. The conventional way of thinking about these episodes is that, in any given country, investment responds to increases in productivity: it is this increase in investment that raises credit, capital inflows and asset prices. Our model provides a different way of interpreting these phenomena. In a global economy characterized by low interest rates, it may very well be it is changes in expectations or investor sentiment that drive changes in asset prices, which in turn translate into fluctuations in credit, investment and capital inflows. Insofar as investor sentiment can change abruptly and independently of economic fundamentals, our model provides a rationale for the incidence of credit booms and busts in a low-interest world.

By providing an alternative interpretation of the world that we live in, our model also provides a different role for policy. In particular, it shows that a global planner should adopt a policy of "leaning-against-investor-sentiment", taxing credit in those times and countries where credit is excessive and subsidizing it elsewhere. An important characteristic of this policy is that it is *expectationally robust*, in the sense that it isolates the world economy from fluctuations in investor sentiment. This policy may be hard to implement in a decentralized fashion, though, as individual countries are unlikely to internalize the effects of their policies on the world interest rate and on other countries' terms-of-trade.

Of course, our model provides a first approach to many of these questions. One important issue that we have left aside is the study of policy, both at the global and at the country level, in an asymmetric world. What would change in our analysis if countries differed in size or productivity? What if they differed in the credibility of their institutions, for instance, and thus in their ability to implement policy? Is it reasonable to assume that larger and more credible countries will be better able to adopt policies like the one studied here, thereby capturing the rents that arise from hosting bubbles in a low-interest global economy? These are fascinating questions that provide an exciting avenue for future research.

References

- Aoki, K., Benigno, G. and N. Kiyotaki (2010), "Adjusting to Capital Account Liberalization", mimeo, LSE.
- [2] Basco, Sergi (2014), "Globalization and financial development: A model of the Dot-Com and the Housing Bubbles", Journal of International Economics, 92(1), 78-94.
- [3] Bernanke, B. (2009), "Financial reform to address systemic risk," Speech at the Council on Foreign Relations, Washington, D.C, 10 March.
- Boyd, John, and B. Smith (1997), "Capital Market Imperfections, International Credit Markets, and Nonconvergence", *Journal of Economic Theory*, 335-64.
- [5] Caballero, R., E. Farhi and P.O. Gourinchas (2008), "An Equilibrium Model of 'Global Imbalances' and Low Interest Rates", *American Economic Review*, 98 (1), 358–393.
- [6] Caballero, R. and A. Krishnamurthy (2006), "Bubbles and Capital Flow Volatility: Causes and Risk Management", *Journal of Monetary Economics* 53(1), 33-53.
- [7] Claessens, S., A. Kose and M. Terrones (2011), "Financial Cycles: What? How? When?", *NBER International Seminar on Macroeconomics*, University of Chicago Press, 7(1), 303 -344.
- [8] Dell'Ariccia, G., D. Igan, L. Laeven and H. Tong, with B. Bakker and J. Vandenbussche (2012),
 "Policies for Macrofinancial Stability: How to Deal with Credit Booms", IMF Staff Discussion Note, SDN 12/06.
- [9] Dell'Ariccia, G., D. Igan and L. Laeven (2012), "Credit Booms and Lending Standards: Evidence from the Subprime Mortgage Market", *Journal of Money, Credit, and Banking* 44(2-3), 367-384..
- [10] Dell'Ariccia, G. and R. Marquez (2006), "Lendong Booms and Lending Standards", The Journal of Finance 51(5), 2511–2546.
- [11] Economist, The (2009), "When a flow becomes a flood", January 22.
- [12] Farhi, E. and J. Tirole (2011), "Bubbly Liquidity", Review of Economic Studies 79, 678-706.

- [13] Gertler, M., and K. Rogoff (1990), "North-south lending and endogenous domestic capital market inefficiencies", *Journal of Monetary Economics* 26, 245–266.
- [14] Gorton, G. and P. He (2008), "Bank Credit Cycles", Review of Economic Studies 75(4), 1181-1214.
- [15] Gourinchas, P.O., R. Valdes and O. Landerretche (2001), "Lending Booms: Latin America and the World", *Economia* 1(2), 47-99.
- [16] Krugman, P (2009): "Revenge of the glut," The New York Times, 1 March.
- [17] Martin, A. (2008), "Endogenous Credit Cycles", mimeo, CREI.
- [18] Martin, A. and J. Ventura (2012), "Economic Growth with Bubbles", American Economic Review 102, 3033-3058.
- [19] Martin, A. and J. Ventura (2014), "Managing Credit Bubbles", mimeo, CREI.
- [20] Matsuyama, K. (2004), "Financial market globalization, symmetry-breaking and endogenous inequality of nations", *Econometrica* 72 (3), 853–884.
- [21] Matsuyama, K. (2007), "Credit Traps and Credit Cycles", American Economic Review 97(1), 503-516.
- [22] Mendoza, E., J. Rios-Rull and V. Quadrini (2009), "Financial Integration, Financial Deepness and Global Imbalances", Journal of Political Economy 117 (3), 371-410
- [23] Mendoza, E. and M. Terrones (2012), "An Anatomy of Credit Booms and their Demise", WP 18379, NBER.
- [24] Miao, J. and P. Wang (2011), "Bubbles and Credit Constraints", mimeo, Boston University.
- [25] Portes, R (2009): "Global imbalances", in M. Dewatripont, X. Freixas, and R. Portes (eds), Macroeconomic stability and financial regulation: Key issues for the G20, London, CEPR.
- [26] Ruckes, M. (2004), "Bank Competition and Credit Standards", The Review of Financial Studies 17(4), 1073-1102.
- [27] Samuelson, P. (1958), "An Exact Consumption-loan Model of Interest with or without the Social Contrivance of Money", *Journal of Political Economy* 66, 467-482.

- [28] Schularick, M. and A. Taylor (2012), "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles and Financial Crises, 1870-2008", American Economic Review 102(2), 1029–1061.
- [29] Ventura, J. (2011), "Bubbles and Capital Flows", Journal of Economic Theory 147 (2), 738-758.

Appendix: Proof of proposition 2

The cooperative equilibrium of our economy is characterized by the pair $\{n, n^*\}$ that solves

$$\max\left[W+W^*\right] = \left[R \cdot (1-\varepsilon) \cdot (1-\alpha) + \alpha\right] \cdot \left[p \cdot k^{\alpha} + p^* k^{*\alpha}\right].$$
(40)

Let

$$\eta = \frac{n_t}{(1-\varepsilon)\cdot(1-\alpha)\cdot p_t\cdot k_t^{\alpha}} = \frac{n_t^*}{(1-\varepsilon)\cdot(1-\alpha)\cdot p_t^*\cdot k_t^{*\alpha}}$$

denote the rate of bubble creation in Home and Foreign as a function of saver wages. Given this notation, maximization of Equation (40) amounts to maximizing

$$A(R) \cdot \hat{y}(\eta, R) \tag{41}$$

where

$$A(R) = (1 - \alpha) \cdot (1 - \varepsilon) \cdot R + \alpha$$

and

$$\widehat{y} = \left(p^{\frac{1-\beta\alpha}{1-\alpha}} + p^{*\frac{1-\beta\alpha}{1-\alpha}}\right) \cdot \left[\varepsilon \cdot (1-\alpha) + (1-\varepsilon) \cdot (1-\alpha) \cdot \frac{\eta}{R}\right]^{\frac{\alpha}{1-\alpha}}.$$

Taking the first-order condition of (41) with respect to η tells us that a larger global bubble is desirable as long as

$$\xi_{A,R} \cdot \xi_{R,\eta} + \xi_{\widehat{y},\eta} + \xi_{\widehat{y},p} \xi_{p,\eta} + \xi_{\widehat{y},p^*} \xi_{p^*,\eta} + \xi_{\widehat{y},R} \xi_{R,\eta} \ge 0,$$

where $\xi_{x,m} = \frac{dx}{dm} \cdot \frac{m}{x}$ denotes an elasticity between variables x and m. From Equation (14) for the interest rate, it can be shown that

$$\xi_{R,\eta} = \begin{cases} 0 & \text{if } \eta < (1-\rho) \cdot \rho \\ \frac{1-R}{1-2 \cdot R} & \text{otherwise} \end{cases},$$

in any symmetric allocation. It can also be shown that $\xi_{\hat{y},p}\xi_{p,\eta} = -\xi_{\hat{y},p^*}\xi_{p^*,\eta}$ in any symmetric allocation.

It follows from these observations that it is always optimal to raise η when the economy is in the partial intermediation region, i.e., as long as $\xi_{R,\eta} = 0$. Once $\eta \ge (1-\rho) \cdot \rho$, and taking into account that $\xi_{\hat{y},\eta} = -\xi_{\hat{y},R} = \frac{\alpha}{1-\alpha} \cdot \frac{(1-\varepsilon) \cdot \frac{\eta}{R}}{\varepsilon + (1-\varepsilon) \cdot \frac{\eta}{R}}$, the first order condition becomes

$$\frac{(1-\varepsilon)\cdot(1-\alpha)\cdot R}{(1-\varepsilon)\cdot(1-\alpha)\cdot R+\alpha}\cdot\frac{1-R}{R}-\frac{\alpha}{1-\alpha}\cdot\frac{(1-\varepsilon)\cdot\frac{\eta}{R}}{\varepsilon+(1-\varepsilon)\cdot\frac{\eta}{R}}\geq 0,$$

which, evaluated at $\eta = (1 - R) \cdot R$, can be rewritten as

$$\frac{(1-\alpha)}{(1-\varepsilon)\cdot(1-\alpha)\cdot R+\alpha} \ge \frac{\alpha}{1-\alpha}\cdot\frac{1}{\varepsilon+(1-\varepsilon)\cdot(1-R)}.$$

This condition can never be satisfied under our assumption that $\alpha > \frac{1 + \varepsilon - \rho}{1 + \varepsilon}$. To see this, simply note that the left hand side of the expression is decreasing in both α and R, and it is therefore lower than $\frac{\rho}{1 + \varepsilon \cdot (1 - \rho)} < 1$. The right hand side of the expression is instead increasing in both α and R, and it is therefore higher than $\frac{1 + \varepsilon - \rho}{\rho \cdot (1 + \rho \cdot \varepsilon - \rho)} > 1$.

Taking this into account, it is immediate to compute the bubble $n = n^*$ that maximizes Equation (40). It must simulatenously satisfy $R = \rho$ and $R = \frac{1}{1-R} \cdot \eta$, which implies $\eta = \rho(1-\rho)$ and thus

$$n = n^* = (1 - \rho) \cdot \rho \cdot (1 - \varepsilon) \cdot (1 - \alpha)^{\frac{1}{1 - \alpha}} \cdot (\varepsilon + (1 - \varepsilon) \cdot (1 - \rho))^{\frac{\alpha}{1 - \alpha}}.$$

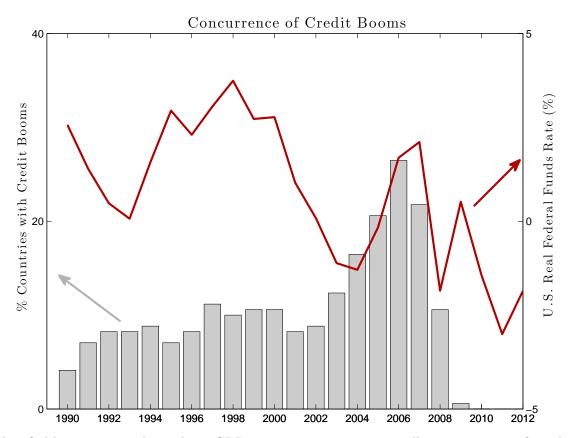


Figure 1 : Credit booms are identified by comparing the credit-to-GDP ratio in a given year to a rolling, country-specific, cubic trend estimated over a 10-year period (see Dell'Ariccia et al. [2012]). In particular, a credit booms occurs if the deviation from trend is greater than 1.5 times its standard deviation and the annual growth rate of the credit-to-GDP ratio exceeds 10 percent; or if the growth rate of the credit-to-GDP ratio exceeds 20 percent. There is a total of 170 countries in the sample.

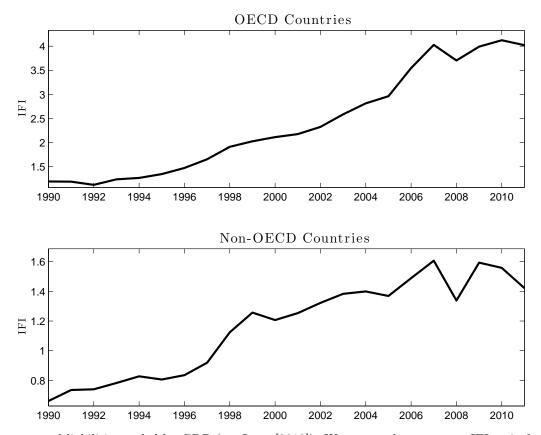


Figure 2 : IFI is the sum foreign assets and liabilities scaled by GDP (see Lane [2012]). We report the aggregate IFI ratio for OECD and Non-OECD countries. Data are taken from the Lane and Milesi-Feretti database and low-income countries, oil producers and countries with population less than one million were excluded.

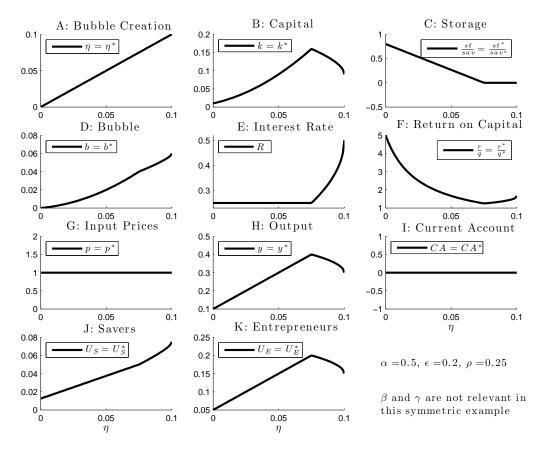


Figure 3 : This panel plots the steady-state values of key variables as a function of η .

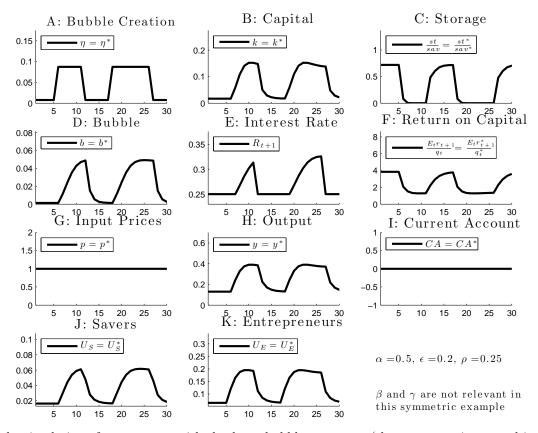


Figure 4 : This panel shows the simulation of an economy with shocks to bubble returns η_t (the two countries are subject to the same shocks, so that $\eta_t = \eta_t^*$).

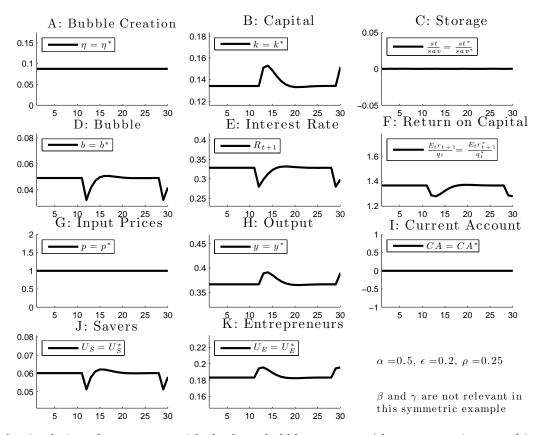


Figure 5: This panel shows the simulation of an economy with shocks to bubble returns u_t (the two countries are subject to the same shocks, so that $u_t = u_t^*$).

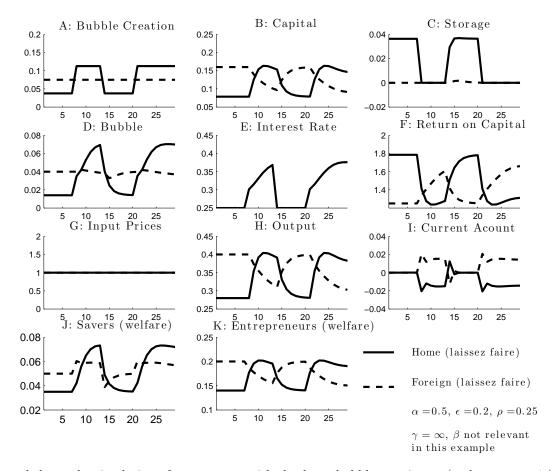


Figure 6 : This panel shows the simulation of an economy with shocks to bubble creation η_t (and constant η_t^*). In this example, $\gamma = \infty$.

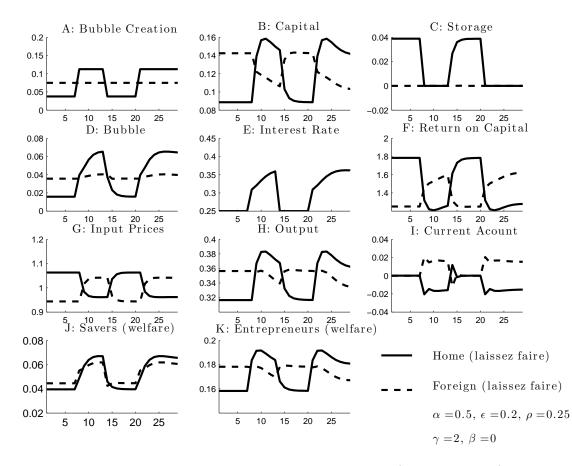


Figure 7.A : This panel shows the simulation of an economy with shocks to bubble creation η (and constant η^*). In this example, $\gamma = 2$ and $\beta = 0$.

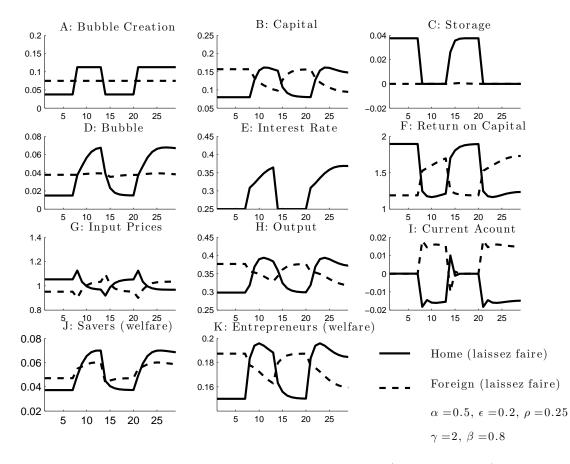


Figure 7.B : This panel shows the simulation of an economy with shocks to bubble creation η (and constant η^*). In this example, $\gamma = 2$ and $\beta = 0.8$.

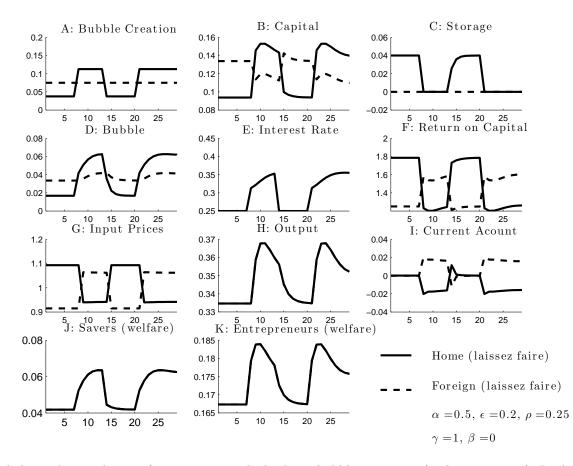


Figure 8.A : This panel shows the simulation of an economy with shocks to bubble creation η (and constant η^*). In this example, $\gamma = 1$ and $\beta = 0$.

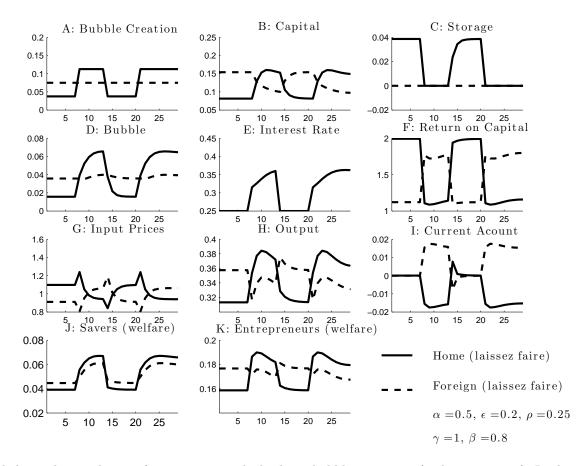


Figure 8.B : This panel shows the simulation of an economy with shocks to bubble creation η (and constant η^*). In this example, $\gamma = 1$ and $\beta = 0.8$.

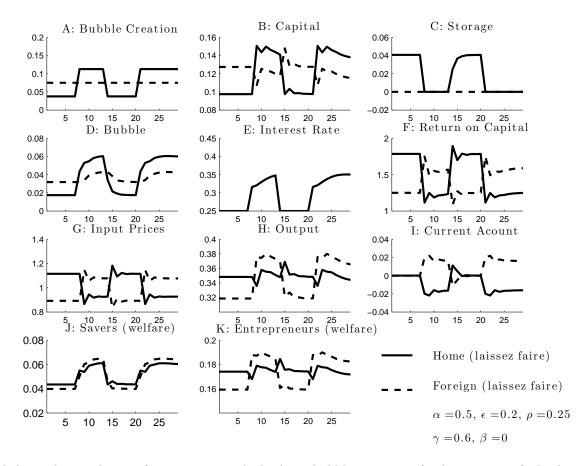


Figure 9.A : This panel shows the simulation of an economy with shocks to bubble creation η (and constant η^*). In this example, $\gamma = 0.6$ and $\beta = 0$.

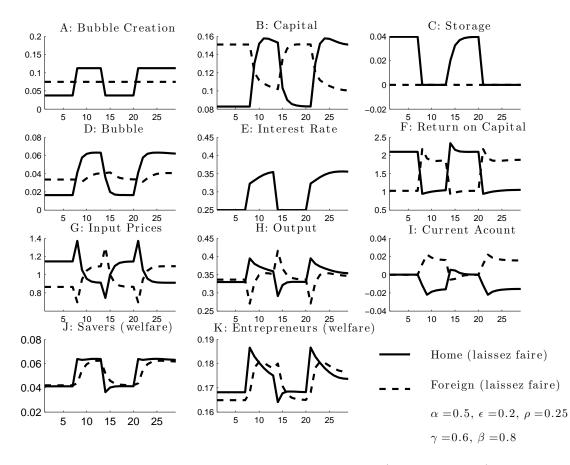


Figure 9.B : This panel shows the simulation of an economy with shocks to bubble creation η (and constant η^*). In this example, $\gamma = 0.6$ and $\beta = 0.8$.

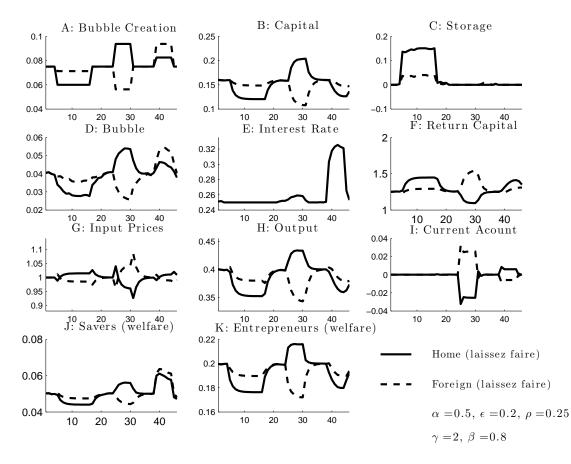


Figure 10 : This panel shows the simulation of an economy with shocks to bubble creation (η_t and η_t^*) and bubble returns (u_t and u_t^*). There are two states of the world: a normal state and a credit episode. In the normal state, $\eta_t = \eta_t^* = \eta$. In an episode, $\eta_t = \tilde{\eta}$ and $\eta_t^* = \tilde{\eta}^*$, where $\tilde{\eta}$ and $\tilde{\eta}^*$ are independently drawn from a uniform distribution with support $[(1 - \sigma)\eta, (1 + \sigma)\eta]$ at the beginning of the episode. There is a constant transition probability λ across states. In this simulation, $\sigma = 0.25$, $\lambda = 0.1$ and $\eta = \eta^{opt} = (1 - \epsilon)(1 - \alpha)\rho(1 - \rho)$. The bubble return shocks u_t and u_t^* are i.i.d. across time and countries and are drawn from a uniform distribution with support [-0.02, 0.02].

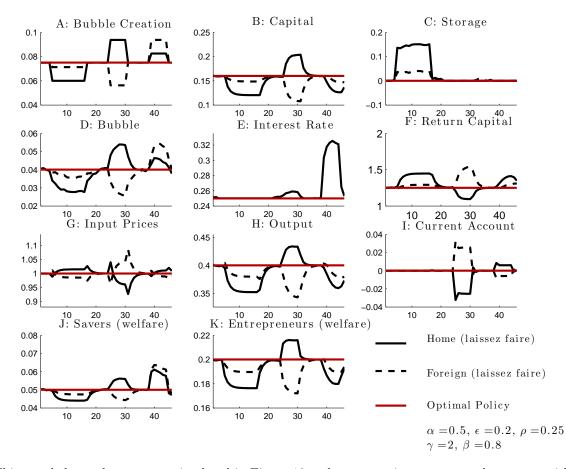


Figure 11 : This panel shows the economy simulated in Figure 10 and compares it to a managed economy with the optimal policy.

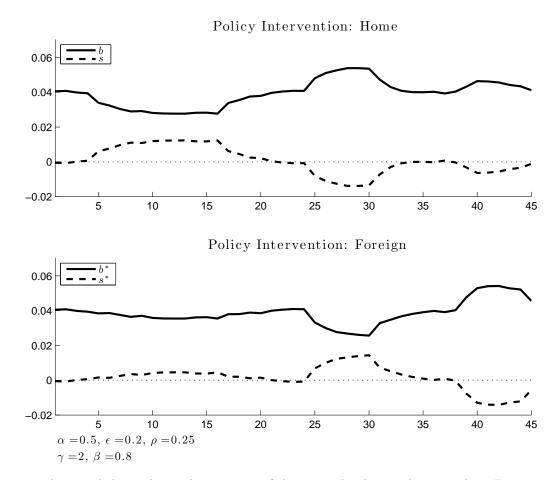


Figure 12 : This panel shows the implementation of the optimal policy in the example in Figure 11.

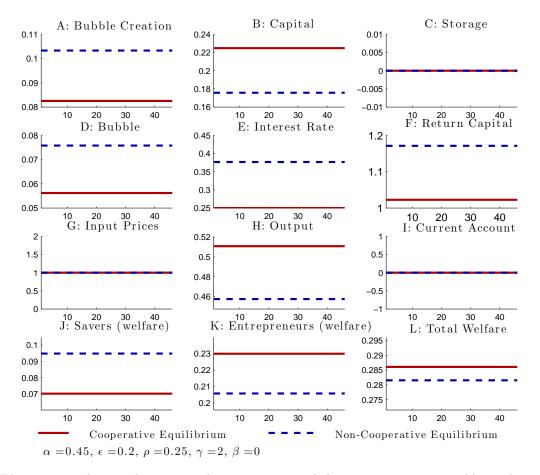


Figure 13 : This panel compares the cooperative and the non-cooperative equilibria when $\gamma > 1$.

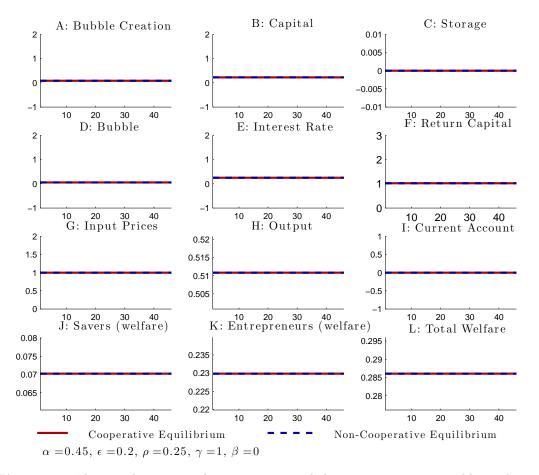


Figure 14 : This panel compares the cooperative and the non-cooperative equilibria when $\gamma = 1$.

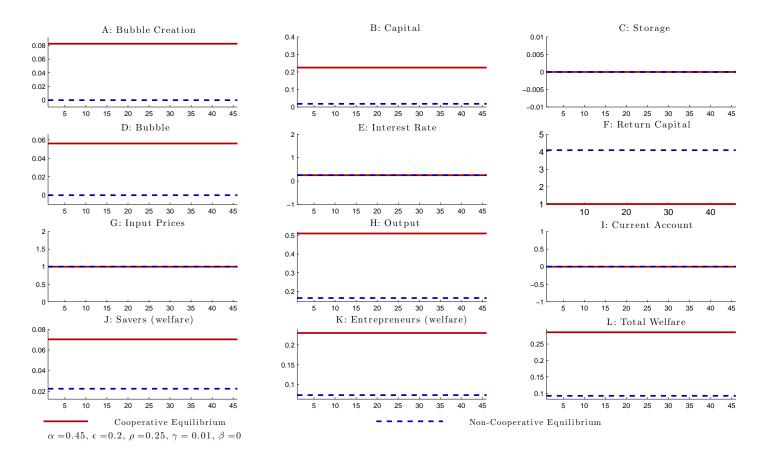


Figure 15 : This panel compares the cooperative and the non-cooperative equilibria when $\gamma < 1$.