NBER WORKING PAPER SERIES

PRICES, CONSUMPTION, AND DIVIDENDS OVER THE BUSINESS CYCLE: A TALE OF TWO REGIMES

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Working Paper 20678
http://www.nber.org/papers/w20678

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2014

We thank Frank Schorfheide, Amir Yaron, other colleagues, and, in particular, Stefano Giglio for helpful comments. Constantinides acknowledges financial support from the Center for Research in Security Prices of the University of Chicago Booth School of Business. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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JEL No. D03,E44,G12,G14,G3

ABSTRACT

An economy in which investors know the true model and its parameters and filter the regime probability from aggregate consumption history has been empirically rejected. Hypothesizing that prices partly reflect investors belief about the regime, we infer beliefs from prices. The model fits well the moments of the market return, risk free rate, and price-dividend ratio. Consistent with the data, it implies higher mean and lower volatility of consumption and dividend growth rates, lower mean and volatility of the market return and equity premium, and higher mean of the price-dividend ratio in the first regime compared with the second one. The probability of recession in a year is 62:5% (23:7%) if the probability of being in the first regime at the beginning of the year is lower (higher) than 50%. The results support the hypothesis that investors employ a broader information set than just aggregate consumption history in forming their beliefs.

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1 Introduction

An overload of worldwide economic, business, and political news inundates investors. Little is known as to how investors cope with this vast amount of information, which subset of information they pay attention to, which heuristics they apply, and to what extent they process information rationally. It is unlikely that investors, or even an enlightened subset of them, formally quantify this vast array of information as a compact vector, specify its dynamics, and formally filter their beliefs about the economic regime. Researchers typically model investors as focusing on the histories of a limited number of economic variables, for example, aggregate consumption, and applying a filter to rationally extract relevant information about the economy.

We fix ideas in a parsimonious Lucas (1978) economy with two regimes. The regime is a Markov process with fixed transition probabilities. The aggregate consumption and dividend growth processes have conditional means and volatilities that depend on the regime. Investors have Epstein-Zin recursive preferences. The investors know the true model and its parameters but do not observe the regime. Each period investors receive new information and revise their probability that the economy is in the first regime.

Johannes, Lochstoer, and Mou (2014, Table 4) considered such a model in the case where the investors' information set consists of the history of past consumption and strongly rejected the model. The model does a poor job in generating the observed unconditional moments of the risk free rate, equity premium, Sharpe ratio, and the price-dividend ratio. They then proceeded to study the important problem of model and parameter uncertainty, in addition to uncertainty about the regime. They showed that considerations of model and parameter uncertainty substantially improve the ability of the model to match these moments, with the exception of the mean risk free rate and the volatility of the price-dividend ratio.

In our paper, we maintain the assumption that investors know the true model and its parameters but relax the assumption that the investors' information set consists of the history of past consumption. We refrain from ascribing to investors super-rationality or the ability and willingness to process vast amounts of information. We, therefore, do not take a stand on the content of the information set that investors use or the filter that they apply to form their beliefs. Instead, we recognize that, to some extent, prices reflect investors' beliefs about the economic regime and use asset prices to infer these beliefs. In particular, we demonstrate that investors employ a broader information set than just the aggregate consumption history in forming their beliefs.

We estimate the model, test it, and do not reject it. The model provides a good fit to the historically observed average levels of the market return and the risk free rate, thereby offering an explanation of the equity premium and risk free rate puzzles. It also matches well the volatility of the market return and market-wide price-dividend ratio, thereby explaining the excess volatility puzzle.

Using the point estimates of the model parameters and the time series of the price-dividend ratio and interest rate, we extract the time series of the investors' probability that the economy is in the first regime. The model implies higher mean and lower volatility of the consumption and dividend growth rates in the first regime compared to the second one, higher mean and volatility of the market return and the equity premium in the second regime, and higher mean of the market-wide price-dividend ratio in the first regime. These features are consistent with the data. Finally, the regimes are related to the business cycle: the probability of a recession in a year is 62.5% if the probability of being in the first regime at the beginning of the year is lower than 50%; and 23.7% if the probability of being in the first regime exceeds 50%.

The paper draws on several strands of the literature. It draws on the literature of regime-switching models, for example, Bekaert and Engstrom (2010), Bonomo and Garcia (1994), Cecchetti, Lam, and Mark (1990), Johannes, Lochstoer, and Mou (2014), Kandel and Stambaugh (1990), Mehra and Prescott (1985), Rietz (1988), and Whitelaw (2000), to mention only a few. It also draws on the literature of models where the agent learns about the regime (or state) from observables. This literature is reviewed in Pastor and Veronesi (2009).

Our model incorporates recursive preferences introduced by Epstein and Zin (1989), Kreps and Porteus (1978), and Weil (1989) which address investors' attitudes toward the timing of resolution of uncertainty of future consumption and cash flows and which break the link between risk aversion and the intertemporal elasticity of substitution. Bansal, Dittmar, and Lundblad (2005), Bansal and Yaron (2004), Hansen, Heaton, and Li (2008), amongst others, incorporated long-run risks – low frequency properties of the time series of dividends and aggregate consumption – in asset pricing models, in conjunction with recursive preferences. Specifically, Bansal and Yaron (2004) calibrated the preference parameters such that the model focuses on long-run risks of very low frequency, whereas our estimates of the preference parameters are such that the model focuses on intermediate-run risks at the business cycle frequency.

The paper is organized as follows. In Section 2, we present the regime shifts model. We express the price-dividend ratio, risk free rate, expected equity premium, and expected consumption and dividend growth rates as functions of the state variable, the probability that the economy is in the first regime. In Section 3, we discuss the data. In Section 4, we estimate the model parameters with GMM. Using the point estimates of the model parameters, we invert the expressions for the price-dividend ratio and risk free rate as functions of the state variable and express the state variable as a function of the price-dividend ratio and risk free rate. Armed with the time series of the state variable, we address the questions raised in this paper. In Section 5, we discuss the economic interpretation of the two regimes. Section 6 provides robustness tests and Section 7 concludes. The Appendix contains the derivation of the main results.

2 Model

We model the aggregate consumption and dividend growth rates as having different mean and volatility across two latent regimes:

$$\Delta c_{t+1} = \mu_{c,s_{t+1}} + \sigma_{s_{t+1}} \varepsilon_{c,t+1}, \tag{1}$$
$$\Delta d_{t+1} = \mu_{d,s_{t+1}} + \phi \sigma_{s_{t+1}} \varepsilon_{d,t+1}, \tag{2}$$

where c_{t+1} is the logarithm of aggregate consumption; d_{t+1} is the logarithm of the aggregate stock market dividends; and $s_t = 1, 2$ is a variable that denotes the economic regime. The means of consumption and dividend growth, μ_{c,s_t} and μ_{d,s_t}, and the level of their volatilities, σ_{s_t} and $\phi\sigma_{s_t}$, are generally different in the two regimes. The shocks, $\varepsilon_{c,t+1}$ and $\varepsilon_{d,t+1}$, are assumed to be normally distributed with mean 0 and variance 1 and independent of each other as well as of the past.

We assume that s_t follows a Markov process with the following transition probability matrix:

$$\Pi = \begin{pmatrix} \pi_1 & 1-\pi_2 \\ 1-\pi_1 & \pi_2 \end{pmatrix}, \tag{3}$$

where $0 < \pi_i < 1$ for $i = 1, 2$. Therefore, the unconditional probability of $s_t = 1$ is $\frac{1-\pi_2}{2-\pi_1-\pi_2}$.

The investor does not observe the regime s_t. Given his information set, $\digamma(t)$, he calculates the probability, p_t, at time t of being in regime $s_t = 1$:

$$p_t \equiv Prob\left(s_t = 1 | \digamma(t)\right) \tag{4}$$

We do not take a stand on the content of the information set, $\digamma(t)$. In one extreme case, it may be limited to the history of consumption and dividends. In the other extreme case, it may include all publicly available information. In general, investors filter noisy signals of the state s_t to infer p_t. These signals include a wide variety of publicly available information which we do not model in this paper. Any specification of this publicly available information is controversial and ultimately ad hoc. Therefore, we do not take a stand on the nature of these signals. Furthermore, we do not take a stand on the optimality of the filter that the investor applies to form his belief, p_t.

We model the innovation in p_{t+1}, $p_{t+1} - f\left(p_t\right)$, as

$$p_{t+1} - f\left(p_t\right) = \begin{cases} \frac{1-f(p_t)}{1+e^{\varepsilon_{p,t+1}}}, & \text{if } s_{t+1} = 1 \\ \frac{-f(p_t)}{1+e^{\varepsilon_{p,t+1}}}, & \text{if } s_{t+1} = 2 \end{cases} \tag{5}$$

where $\varepsilon_{p,t} \sim N\left(0, \sigma_p\right)$ and *i.i.d.* and is independent of $\varepsilon_{c,t}$ and $\varepsilon_{d,t}$; and $f(p_t) \equiv (1-\pi_2) + (\pi_1 + \pi_2 - 1)\, p_t$. In the special case $\varepsilon_{p,t+1} \to -\infty$, $p_{t+1} = 1$, if $s_{t+1} = 1$, and $p_{t+1} = 0$, if $s_{t+1} = 2$; that is, investors observe the true state without noise. Even though $\varepsilon_{p,t}$ is modeled to be independent of $\varepsilon_{c,t}$ and $\varepsilon_{d,t}$, the innovation in beliefs is correlated with the innovations in consumption and dividend growth.[1]

Note that this specification respects the constraint $0 \leq p_{t+1} \leq 1$. Also, the innovation $p_{t+1} - f(p_t)$ is consistent with the definition of $f(p_t)$, since

$$\begin{aligned} E\left[p_{t+1} - f(p_t)\,|p_t\right] &= f(p_t)\left(1 - f(p_t)\right) E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right] \\ &\quad - \left(1 - f(p_t)\right) f(p_t) E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right] \\ &= 0. \end{aligned}$$

Finally, since $E\left(E\left[p_{t+1} - f(p_t)\,|p_t\right]\right) = \overline{p} - (1-\pi_2) - (\pi_1 + \pi_2 - 1)\,\overline{p} = 0$, the unconditional expectation of p_t equals the unconditional probability of $s_t = 1$, $\overline{p} = \frac{1-\pi_2}{2-\pi_1-\pi_2}$.

We assume that the consumer has the version of Kreps and Porteus (1978) preferences adopted by Epstein and Zin (1989) and Weil (1989). These preferences allow for a separation between the coefficient of relative risk aversion and the elasticity of intertemporal substitution. Furthermore, they allow for the expression of the preference for early versus late resolution of uncertainty. The utility function is defined recursively as

$$V_t = \left[(1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta\left(E\left[V_{t+1}^{1-\gamma} | \mathcal{F}(t)\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{6}$$

where δ denotes the subjective discount factor, $\gamma > 0$ is the coefficient of relative risk aversion, $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$, and $\psi > 0$ is the elasticity of intertemporal substitution. Note that the sign of θ depends on the relative magnitudes of γ and ψ. The standard time-separable power utility is obtained as a special case when $\theta = 1$, i.e. $\gamma = \frac{1}{\psi}$.

For this specification of preferences, Epstein and Zin (1989) and Weil (1989) showed that, for any asset j, the first-order conditions of the consumer's utility maximization yield the following Euler equations,

$$E\left[\exp(m_{t+1} + r_{j,t+1}) | \mathcal{F}(t)\right] = 1, \tag{7}$$

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{8}$$

where m_{t+1} is the natural logarithm of the intertemporal marginal rate of substitution, $r_{j,t+1}$ is the continuously compounded return on asset j, and $r_{c,t+1}$ is the unobservable continuously compounded return on an asset that delivers aggregate consumption as its dividend each period.

[1] The covariances are

$$\mathrm{Cov}\left[p_{t+1}, \Delta c_{t+1} | p_t\right] = f(p_t)\left(1 - f(p_t)\right) E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1} - \mu_{c,2}\right),$$

and

$$\mathrm{Cov}\left[p_{t+1}, \Delta d_{t+1} | p_t\right] = f(p_t)\left(1 - f(p_t)\right) E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{d,1} - \mu_{d,2}\right).$$

We rely on log-linear approximations for the return on the consumption claim, $r_{c,t+1}$, and that on the market portfolio (the observable return on the aggregate dividend claim), $r_{m,t+1}$, as in Campbell and Shiller (1988),

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1}, \quad (9)$$

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + \Delta d_{t+1}, \quad (10)$$

where z_t is the log price-consumption ratio and $z_{m,t}$ is the log price-dividend ratio. In equation (9), $\kappa_1 = \frac{e^{\overline{z}}}{1+e^{\overline{z}}}$ and $\kappa_0 = \log(1 + e^{\overline{z}}) - \kappa_1 \overline{z}$ are log-linearization constants, where $\overline{z}$ denotes the long-run mean of the log price-consumption ratio. Similarly, in equation (10), $\kappa_{1,m} = \frac{e^{\overline{z}_m}}{1+e^{\overline{z}_m}}$ and $\kappa_{0,m} = \log(1 + e^{\overline{z}_m}) - \kappa_{1,m}\overline{z}_m$, where $\overline{z}_m$ denotes the long-run mean of the log price-dividend ratio.

Note that the current model specification involves a single state variable p_t. We conjecture the following approximate expressions for the log price-consumption ratio, log price-dividend ratio and log risk free rate and derive expressions for their parameters in Appendices A.3, A.5, and A.4, respectively:

$$z_t = A_0 + A_1 p_t, \quad (11)$$

$$z_{m,t} = A_{0,m} + A_{1,m} p_t, \quad (12)$$

$$r_{f,t} = A_{0,f} + A_{2,f} p_t. \quad (13)$$

Note that we do not directly observe p_t and, therefore, it is latent. However, under the model assumptions, the probability of being in the first regime in each period can be extracted from observable macroeconomic and financial variables, like the market-wide price-dividend ratio and risk free rate using equations (12) and (13).

3 Data

Throughout our investigation, we use annual and quarterly data over the entire available sample periods $1929 - 2013$ and $1947{:}Q1 - 2013{:}Q4$, respectively. The asset menu consists of the market return and the risk free rate. Our market proxy is the Center for Research in Security Prices (CRSP) value-weighted index of all stocks on the NYSE, AMEX, and NASDAQ. The proxy for the real risk free rate is obtained as follows: the quarterly nominal yield on three-month Treasury Bills is deflated using the realized growth in the Consumer Price Index to obtain the ex post real three-month Treasury-Bill rate. The ex ante quarterly risk free rate is then obtained as the fitted value from the regression of the ex post three-month Treasury-Bill rate on the three-month nominal yield and the realized growth in the Consumer Price Index over the previous year. The ex ante quarterly risk free rate at the beginning of the year is annualized to obtain the ex ante annual risk free rate. The equity premium is the difference in average log returns on the market and the risk free rate.

Also used in the empirical analysis are the price-dividend ratio and dividend growth rate of the market portfolio. These two time series are computed using the monthly returns with and without dividends on the market portfolio obtained from the CRSP files. The monthly dividend payments within a year are added to obtain the annual aggregate dividend, i.e. we do not reinvest dividends either in T-Bills or in the aggregate stock market. The annual price-dividend ratio is computed as the ratio of the price at the end of each calendar year to the annual aggregate dividends paid out during that year. Finally, the consumption data consists of the per capita personal consumption expenditure on nondurable goods and services obtained from the Bureau of Economic Analysis. All nominal quantities are converted to real, using the personal consumption deflator.

4 Parameter Estimation and Interpretation

The model has a total of 13 parameters, including 3 preference parameters (γ, ψ, δ) and 10 parameters of the time-series processes of aggregate consumption and dividend growth $\left(\mu_{c,1}, \mu_{c,2}, \mu_{d,1}, \mu_{d,2}, \phi, \sigma_1, \sigma_2, \pi_1, \pi_2, \sigma_p\right)$. We estimate the parameters using the GMM approach to match the following sample moments: the unconditional mean, variance, and first-order autocorrelation of consumption growth, dividend growth, market-wide price-dividend ratio and risk free rate, the unconditional mean and variance of the market return, and the correlation between consumption and dividend growth rates. Therefore, we have an over-identified system of 15 restrictions and 13 parameters. The weighting matrix used in the estimation is a diagonal matrix with unit entries corresponding to all the moments except for the weights on the mean risk free rate and market return that each equal 100. Similar results are obtained using the efficient weighting matrix and are available from the authors upon request.

The estimation results are reported in Table 1 for annual data over the entire available sample period 1929 – 2013. The point estimates of the parameters, along with the associated standard errors (Newey-West (1987) corrected using two lags) in parentheses, are displayed in the first two rows. Row 2 reveals that the preference parameters are precisely estimated. The point estimates, 1.86 and 1.99, respectively, of the coefficient of relative risk aversion and the elasticity of intertemporal substitution suggest preference for early resolution of uncertainty.

Row 1 shows that the consumption growth rate has mean $\mu_{c,1} = 2.7\%$ and volatility $\sigma_1 = 1.0\%$ in regime 1. The mean and volatility in the second regime are $\mu_{c,2} = -3.2\%$ and $\sigma_2 = 3.5\%$, respectively. The mean and volatility of the dividend growth rate are $\mu_{d,1} = 5.4\%$ and $\phi\sigma_1 = 5.62\%$, respectively, in regime 1 and $\mu_{d,2} = -9.9\%$ and $\phi\sigma_2 = 19.7\%$, respectively, in regime 2. The point estimates of the transition probabilities imply that the first regime has a duration longer than that of the second regime. Also, note that the point estimates of the transition probabilities strongly suggest the existence of at least two regimes, since π_1 is very different from $1 - \pi_2$. Therefore, the first regime is characterized by higher mean and lower volatility of the consumption and dividend growth rates and lasts longer than the second regime.

In Table 1, we also report the historical and model-generated moments of the consumption and dividend growth rates, market return, risk free rate, and market-wide price-dividend ratio. The "Data" column reports the moments computed from historical data along with standard errors in parentheses. The "Model" column presents the model-generated moments along with the 95% confidence intervals in square brackets. We calculate the model-generated moments from the analytical expressions for these moments at the point estimates of the parameters. We calculate their 95% confidence intervals from 10,000 simulations of eighty-four years each, the same size as the historical sample.

The model does a good job matching the unconditional moments of the aggregate consumption and dividend growth rates. The unconditional mean, volatility, and autocorrelation of consumption growth are 0.019, 0.021, and 0.501, respectively, in the historical data, while their model-implied values are 0.020, 0.025, and 0.512, respectively. The corresponding numbers for the dividend growth rate are 0.013, 0.114, and 0.183, respectively in the data and 0.035, 0.100, and 0.299, respectively, in the model. The model-implied correlation between the consumption and dividend growth rates is 0.395, smaller than its sample value of 0.584. This shows that the model does not rely on high correlation between consumption and dividends. This is a desirable feature of the model as this correlation is difficult to measure precisely and is quite sensitive to small changes in timing or time aggregation (see e.g., Campbell and Cochrane (1999)).

The model provides a good fit to the historically observed low average level and volatility of the risk free rate. It also rationalizes the high mean market return and the volatility of the market return observed in the data. Therefore, it offers an explanation of the equity premium and risk free rate puzzles. It also matches well the mean and, more importantly, the volatility of the market-wide price-dividend ratio, thereby accounting for the excess volatility puzzle. Overall, the model fits well the moments of consumption and dividend growth and returns, and does so without requiring implausible dynamics in the consumption and dividend growth processes. The J-stat is 7.61 and the model is not rejected at the 10% level of significance.

Finally, note that the point estimate of the elasticity of intertemporal substitution parameter, ψ, is similar to that used in the literature while the value of the risk aversion parameter, γ, is smaller than is typically required in models with recursive preferences. For example, in the Bansal and Yaron (2004) long-run risks model, the calibrated values of these preference parameters are $\gamma = 10$ and $\psi = 1.5$; in the Nakamura, Steinsson, Barro, and Ursua (2013) rare disasters model, they are $\gamma = 6.5$ and $\psi = 2.0$. The estimated value of γ in Table 1, on the other hand, is only 1.86. The specific feature of our model that allows risk aversion to be smaller is that it puts less weight on extremely low frequencies, and more weight on business cycle frequencies (2-10 years). For instance, the frequency domain calculations in Dew-Becker and Giglio (2014) reveal that, at the Bansal and Yaron (2004) calibrated values of the preference parameters γ, ψ, and δ, cycles longer than 235 years account for half of the total risk premium of a permanent consumption shock. In other words, removing shocks above these frequencies would reduce the risk premium on a permanent shock by half. Instead, at our estimates of the preference parameters, the total mass of the pricing function of such long frequencies is reduced to 35%. Another way to see this is that the Bansal and Yaron (2004) calibration puts only 7.5% of the weight on business cycle fluctuations while our model puts a higher weight of 24% on those cycles. Thus, while the Bansal and Yaron (2004) model focuses on long-run risks of very low frequency, our model focuses on intermediate-run risks at the business cycle frequency.[2]

5 Economic Interpretation of the Two Regimes

Note that the model implies that the market-wide price-dividend ratio is an affine function of the probability p_t of being in the first regime, with coefficients that are known functions of the underlying model parameters. Therefore, using the point estimates of the model parameters in Table 1 and the historical time series of the price-dividend ratio, we extract the time series of the probability p_t. Figure 1 plots the extracted time series of p_t, along with the NBER-designated recession periods (grey shaded areas) and the major stock market crashes (vertical dashed lines) identified in Mishkin and White (2001).

The figure reveals that the investors' regime probabilities are correlated with the business cycle. The correlation between the probability series and a dummy variable that takes the value one in a recession year and zero otherwise is -31.4%. A year is classified as a recession year if at least two of its quarters are in NBER-designated recessions. Conditional on lower than 50% probability that the economy is in the first regime at the beginning of the year ($p_t < 0.5$), the probability of a recession in that year is 62.5%; conditional on higher than 50% probability that the economy is in the first regime ($p_t > 0.5$), the probability of a recession in that year is 23.7%. The association of the second regime with recessions is consistent with our earlier finding that the second regime is associated with lower mean and higher volatility of consumption growth and has a shorter duration compared to the first regime.

— Figure 1 about here —

The investors' regime probabilities are also correlated with major stock market downturns. The correlation between the probability series and a dummy variable that takes the value one in years with a stock market downturn and zero otherwise is -23.7%. The years corresponding to stock market downturns are taken from Mishkin and White (2001) who identify them as periods in which either the Dow Jones Industrial, the S&P500, or the NASDAQ index drops by at least 20 percent in a time window of either one day, five days, one month, three months, or one-year. Conditional on lower than 50% probability that the economy is in the first regime, the probability of a recession or a stock market downturn in that year is 75.0%; conditional on higher than 50% probability that the economy is in the first regime, the probability of a recession or stock market downturn in that year is 36.8%. Note that the second regime does double duty by capturing both economic recessions and periods of stock market downturns. This rendition is necessarily imperfect because economic recessions and stock market downturns are related but distinct economic phenomena. In fact, the correlation between a dummy variable that takes the value one in a recession year and zero otherwise and a dummy variable that takes the value one in years with a stock market downturn and zero otherwise over the period $1929 - 2013$ is only 0.22.

[2]We thank Stefano Giglio for pointing this out.

In Figure 2, we plot the historical and model-implied time series of the market-wide price-dividend ratio.[3] Note that the two series are indistinguishable, except starting with the mid nineties when asset prices rose to unprecedented levels.[4]

— Figure 2 about here —

For comparison purposes, we also filter the probability of being in the first regime from the consumption history as

$$p_{t+1}|\left(\Delta c_{t+1};p_t\right)=\frac{N\left(\Delta c_{t+1};\mu_{c,1},\sigma_1\right)f\left(p_t\right)}{N\left(\Delta c_{t+1};\mu_{c,1},\sigma_1\right)f\left(p_t\right)+N\left(\Delta c_{t+1};\mu_{c,2},\sigma_2\right)\left(1-f\left(p_t\right)\right)},$$

where $N\left(x;\mu,\sigma\right)$ denotes the density of a normal random variable with mean μ and standard deviation σ, evaluated at x. We set p_t in year 1929 equal to its unconditional mean. We then filter p_t for the years $1930-2013$ and present the time series in Figure 3.[5] The correlation of p_t and a dummy variable that takes the value of one in a recession year and zero otherwise is -28.8%. Conditional on $p_t<0.5$, the probability of a recession in that year is 46.7%; conditional on $p_t>0.5$, the probability of a recession in that year is 23.2%. Given that NBER-designated recessions are *defined* primarily in terms of negative realized consumption growth, it is remarkable that the results are, in fact, a little worse than, the results obtained earlier in the case where p_t is filtered from the market-wide price-dividend ratio.

— Figure 3 about here —

In Table 2, we report the sample mean and volatility, along with the associated asymptotic standard errors in parentheses, of the consumption and dividend growth rates, the price-dividend ratio, risk free rate, and market return, in the two regimes. In Panel A, we present these summary statistics for the 76 years over the period $1930-2013$ in which the probability that the economy is in the first regime exceeds 50%. In Panel B, we present these summary statistics for the 8 years in which the probability that the economy is in the first regime is below 50%. Given the small size of these subsamples, particularly the second one, the standard errors are large and differences in the point estimates across the two regimes are often statistically insignificant. However, consistent with the interpretation of the second regime as the

[3]Figure 1 suggests that the model fails to account for the movements in the price-dividend ratio starting with the mid nineties. However, the corresponding figure 4 for the post war subperiod shows that the historical and model-implied time series of the price-dividend ratio are indistinguishable over the entire period, except in the mid nineties.

[4]Researchers have suggested several possible reasons for the high stock market valuations during this period, including a decline in the equity premium (e.g., Blanchard (1993), Jagannathan, McGrattan, and Scherbina (2000), Fama and French (2002)); reductions in the costs of stock market participation and diversification (e.g., Heaton and Lucas (1999), Siegel (1999), Calvet, Gonzalez-Eiras, and Sodini (2004)); decline in macroeconomic risk (e.g., Lettau, Ludvigson, and Wachter (2008)); and irrational exuberance (e.g., Shiller (2000)).

[5]Note that the first few values of p_t are probably influenced by the starting value and should, therefore, be interpreted with caution.

regime associated with recessions, we find that the sample mean of consumption and dividend growth are higher in the first regime and their volatilities are higher in the second regime. The same pattern across regimes is reflected in the simulated model-implied moments. The sample mean and volatility of the market return are higher in the second regime than in the first one, consistent with the interpretation of the second regime as being the regime associated with recessions. The same pattern across regimes is reflected in the simulated moments. The sample mean of the price-dividend ratio is higher in the first regime than in the second one; the simulated mean of the price-dividend ratio is also higher in the first regime than the second one.

Forecasting regressions of the consumption and dividend growth rates and the market return on the market-wide price-dividend ratio lend further support to the risk channels highlighted in the model. These results are presented in Table 3. Panels A, B, and C present the forecasting results at the one-year, 5 year, and 10 year horizons, respectively. Consider first Panel A. Row 1 shows that, in the historical sample, a forecasting regression of the one-year ahead market return on the lagged market-wide price-dividend ratio produces a slope coefficient of -0.065. The model-implied slope coefficient, -0.107, is within one-standard error of the point estimate in the data. Moreover, the model-implied R^2 of 2.9% is very close to its sample counterpart of 2.2%. Similar results are obtained for the regression of the dividend growth on the price-dividend ratio in Row 2. Row 3 shows, however, that the model implies higher forecastability of the consumption growth rate by the price-dividend ratio than that observed in the data – the R^2 from a forecasting regression of the consumption growth on the price-dividend ratio is 4.5% in the historical sample while the model-implied R^2 is 32.0%. Since p_t is a (truncated) affine function of the price-dividend ratio, we obtain similar results for the R^2 if we use p_t instead of the price-dividend ratio as the forecasting variable.

Consistent with the data, Panels B and C show that model-implied slope coefficients in the forecasting regressions of the 5-year and 10-year market returns on the lagged price-dividend ratio are -0.368 and -0.527, respectively – similar to the corresponding historical values of -0.380 and -0.632, respectively. The R^2 of the regressions also increase with the horizon, as in the data. The table shows that, while the R^2 of the forecasting regression of the market return on the price-dividend ratio increases monotonically with the return horizon in the historical data, the same is not true with the aggregate consumption and dividend growth rates. With the consumption growth rate, the R^2 falls from 4.5% to 1.3% as we go from the one-year to the five-year horizon, and then rises to 5.7% at the 10-year horizon. Similarly with the dividend growth rate, the R^2 falls from 10.2% to 3.1% as we go from the one-year to the five-year horizon, and then rises to 3.9% at the 10-year horizon. Consistent with the data, the model-implied R^2 is not monotone in the forecast horizon for the consumption and dividend growth rates, but is generally higher than that in the data.

Note that the results in this section are based on extracting the time series of the regime probability p_t from the market-wide price-dividend ratio. However, other observables, like the risk free rate, are also functions of the state variable p_t under the model assumptions. Therefore, to ensure that our results are not entirely driven by the choice of the price-dividend ratio as the sole observable from which to filter

p_t, we repeated the analysis of this section when the probability is extracted from the price-dividend ratio and the risk free rate.[6] The results remain virtually unchanged and are omitted for the sake of brevity.

6 Robustness Tests

The period prior to 1947 was one of great economic uncertainty, including the Great Depression and World War II. It has also been argued that pre-war macroeconomic data has substantial measurement error. Therefore, we address the robustness of our results to the post-war subperiod. Tables 4-6 present results analogous to those in Tables 1-3 using post-war annual data over $1947-2013$. Table 4 shows that the model matches well the unconditional mean, volatility, and autocorrelation of the consumption growth, dividend growth, risk free rate, market return, and the market-wide price-dividend ratio. The point estimates of the coefficient of risk aversion and the elasticity of intertemporal substitution are reasonable at 1.503 and 1.442, respectively. The point estimates of the time series parameters suggest the presence of two regimes. Aggregate consumption and dividend growth rates have higher mean and lower volatility in the first regime than in the second one. The average duration of the first regime is longer than that of the second regime. Finally, the overidentifying restrictions test statistic fails to reject the model at the 5% level of significance. As with the full sample, we find that the regimes are related to the business cycle - the correlation between the time series of the probability and a recession dummy variable is -28.4%.

Table 5 shows that, consistent with the implications of the model, the consumption growth, market-wide price-dividend ratio, and the risk free rate have higher mean in the first regime compared to the second, while the market return has a higher mean in the second regime compared to the first one. Table 6 shows that the slope coefficient and R^2 from a forecasting regression of the market return on the lagged price-dividend ratio are very similar in the data and in the model. However, the model implies greater predictive power of the price-dividend ratio for the ahead aggregate consumption and dividend growth rates than that observed in the data. Finally, Figure 4 shows that the historical and model-implied time series of the price-dividend ratio are indistinguishable, except in the mid nineties. Overall, the results obtained using post-war data are very similar to those obtained using the entire available sample, suggesting that our results are not driven solely by the characteristics of the pre-war period.

— Figure 4 about here —

Tables 7-9 present results using quarterly data over 1947:$Q1-2013$:$Q4$. Note that, because of the strong seasonality in dividend payouts, aggregate dividend growth has

[6] Since this approach consists of two equations in the single unknown p_t, we extract p_t period-by-period using a least squares criterion. This gives

$$p_t = \frac{(z_{m,t} - A_{0,m})\, A_{1,m} + (r_{f,t} - A_{0,f})\, A_{1,f}}{A_{1,m}^2 + A_{1,f}^2}.$$

a large negative first order autocorrelation of -0.581 in the data at the quarterly frequency. Since we do not incorporate the seasonality of dividends in the model, we eliminate this moment restriction from our GMM estimation of the model parameters. The results are, once again, very similar to those obtained in Tables 1-3. The parameter estimates in Table 7 suggest that the first regime is characterized by higher mean and lower volatility of the consumption and dividend growth rates and lasts longer than the second regime. The estimates of the risk aversion and elasticity of intertemporal substitution parameters are economically plausible and suggest preference for early resolution of uncertainty. Also, note that, consistent with their economic interpretation, the point estimates of these preference parameters are very similar irrespective of the assumed decision frequency of the investors or the sample period used in the estimation. The overidentifying restrictions test statistic has an asymptotic p-value of 3.7%. Once again, we find that the regimes are correlated with the business cycle. The correlation between the time series of the probability and a recession dummy variable is -30.3%.

Table 8 shows that, consistent with the implications of the model, the consumption growth, market-wide price-dividend ratio, and the risk free rate have higher mean in the first regime compared to the second, while the market return has a higher mean in the second regime compared to the first one. Finally, Table 9 shows that the model implies that the price-dividend ratio forecasts the market return with slope coefficient and R^2 similar to that observed in the data, but implies higher forecasting ability of the price-dividend ratio for the consumption and dividend growth rates than that observed in the historical data. Overall, the results obtained using quarterly data are similar to those obtained at the annual data frequency, suggesting that the results are not sensitive to the assumed decision frequency of investors.

7 Concluding Remarks

We presented a parsimonious exchange economy with one Markovian latent state variable that signifies the economic regime. We recognized that investors are inundated with an overload of news and explored the hypothesis that they cope with this vast amount of information by focusing on a subset of information they pay attention to, apply heuristics, and process information in a way that may or may not be fully rational. We refrained from taking a stand on the content of the information set that investors use or the filter that they apply to form their beliefs. Instead, we recognized that, to some extent, prices reflect investors' beliefs about the economic regime and we relied on asset prices to infer these beliefs. In particular, we demonstrated that investors employ a broader information set than just the aggregate consumption history in forming their beliefs.

We estimated the model, tested it, and did not reject it. The model provides a good fit to the unconditional and conditional moments of the market return, the risk free rate, market-wide price-dividend ratio, and consumption and dividend growth. We identified the second regime as the one associated with recessions and market downturns along several criteria. The second regime has a shorter duration than the first one. In the second regime, the realized consumption and dividend growth rates have lower mean and higher volatility; the market return has higher mean and volatility; and the mean market-wide price-dividend ratio and risk free rate are lower. High on our agenda is the investigation of the information subset that investors actually use to infer the economic regime and the extent to which their filtering of this information is rational.

A Appendix

A.1 Moments of the State Variable p_t

The conditional mean and variance of p_t are given by

$$E\left[p_{t+1}|\mathcal{F}(t)\right] = f(p_t), \tag{14}$$

and

$$\begin{aligned}
\operatorname{Var}\left[p_{t+1}|p_t\right] &= E\left[\left(p_{t+1} - f(p_t)\right)^2|p_t\right] \\
&= f(p_t)\left(1 - f(p_t)\right)^2 E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right] \\
&\quad + \left(1 - f(p_t)\right)\left(f(p_t)\right)^2 E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right] \\
&= f(p_t)\left(1 - f(p_t)\right) E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right].
\end{aligned} \tag{15}$$

Equation (14) implies that the unconditional mean of p_t is $\overline{p} \equiv E(p_t) = E\left(E\left[p_{t+1}|p_t\right]\right) = E\left[f(p_t)\right] = (1-\pi_2) + (\pi_1 + \pi_2 - 1)\,E(p_t) \Rightarrow E(p_t) = \frac{1-\pi_2}{2-\pi_1-\pi_2}$. The unconditional variance is calculated as follows:

$$\begin{aligned}
E(p_{t+1}^2) &= E\left(E\left[p_{t+1}^2|p_t\right]\right) \\
&= E\left\{\left(E\left[p_{t+1}|p_t\right]\right)^2 + \operatorname{Var}\left[p_{t+1}|p_t\right]\right\} \\
&= E\left\{\left[f(p_t)\right]^2 + f(p_t)\left(1 - f(p_t)\right) E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right]\right\} \\
&= (1-\pi_2)^2 + (\pi_1+\pi_2-1)^2 E(p_t^2) + 2\,(1-\pi_2)\,(\pi_1+\pi_2-1)\,E(p_t) \\
&\quad + E\left[f(p_t)\left(1-f(p_t)\right)\right] E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right] \\
\Rightarrow \quad E(p_t^2) &= \frac{\begin{pmatrix} (1-\pi_2)^2 + 2\,(1-\pi_2)\,(\pi_1+\pi_2-1)\,E(p_t) \\ +E\left[f(p_t)\left(1-f(p_t)\right)\right] E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right] \end{pmatrix}}{1-(\pi_1+\pi_2-1)^2}
\end{aligned} \tag{16}$$

Note that $E(p_t^2)$ depends on $E\left[f(p_t)\left(1-f(p_t)\right)\right]$. Now,

$$\begin{aligned}
E\left[f(p_t)\left(1-f(p_t)\right)\right] &= E\left[f(p_t)\right] E\left[1-f(p_t)\right] + \operatorname{Cov}\left(f(p_t), 1-f(p_t)\right) \\
&= f(\overline{p})\left[1-f(\overline{p})\right] - \operatorname{Var}\left(f(p_t)\right) \\
&= f(\overline{p})\left[1-f(\overline{p})\right] - (\pi_1+\pi_2-1)^2\left[E(p_t^2) - \overline{p}^2\right]
\end{aligned} \tag{17}$$

Equations (16) and (17) can be solved for $E(p_t^2)$ and $E\left[f(p_t)\left(1-f(p_t)\right)\right]$. The variance can then be obtained as $\mathrm{Var}(p_t)=E(p_t^2)-\overline{p}^2$.

A.2 Moments of Consumption and Dividend Growth

The conditional mean of consumption growth is

$$E\left[\Delta c_{t+1}|p_t\right]=f(p_t)\mu_{c,1}+\left(1-f(p_t)\right)\mu_{c,2}. \tag{18}$$

Therefore, the unconditional mean is given by

$$E\left(\Delta c_{t+1}\right)=\frac{1-\pi_2}{2-\pi_1-\pi_2}\mu_{c,1}+\frac{1-\pi_1}{2-\pi_1-\pi_2}\mu_{c,2}. \tag{19}$$

The conditional variance is given by

$$\begin{aligned}
\mathrm{Var}\left[\Delta c_{t+1}|p_t\right] &= E\left[\left(\Delta c_{t+1}-E\left[\Delta c_{t+1}|p_t\right]\right)^2|p_t\right]\\
&= E\left[\left(\mu_{c,s_{t+1}}-E\left[\Delta c_{t+1}|p_t\right]+\sigma_{s_{t+1}}\varepsilon_{c,t+1}\right)^2\right]\\
&= E\left[\left(\mu_{c,s_{t+1}}-E\left[\Delta c_{t+1}|p_t\right]\right)^2\right]+E\left[\sigma^2_{s_{t+1}}\varepsilon^2_{c,t+1}\right]\\
&= f(p_t)\mu^2_{c,1}+\left(1-f(p_t)\right)\mu^2_{c,2}-\left[f(p_t)\mu_{c,1}+\left(1-f(p_t)\right)\mu_{c,2}\right]^2\\
&\quad +f(p_t)\sigma_1^2+\left(1-f(p_t)\right)\sigma_1^2\\
&= f(p_t)\left(1-f(p_t)\right)\left(\mu_{c,1}-\mu_{c,2}\right)^2+f(p_t)\sigma_1^2+\left(1-f(p_t)\right)\sigma_1^2.
\end{aligned} \tag{20}$$

The unconditional variance is given by

$$\begin{aligned}
Var\left(\Delta c_{t+1}\right) &= E\left[\Delta c^2_{t+1}\right]-\left(E\left[\Delta c_{t+1}\right]\right)^2\\
&= \frac{1-\pi_2}{2-\pi_1-\pi_2}\left(\mu^2_{c,1}+\sigma_1^2\right)+\frac{1-\pi_1}{2-\pi_1-\pi_2}\left(\mu^2_{c,2}+\sigma_2^2\right)\\
&\quad -\left\{\frac{1-\pi_2}{2-\pi_1-\pi_2}\mu_{c,1}+\frac{1-\pi_1}{2-\pi_1-\pi_2}\mu_{c,2}\right\}^2
\end{aligned} \tag{21}$$

The first order autocovariance is obtained as follows:

$$\begin{aligned}
E(\Delta c_{t+1},\Delta c_{t+2}) &= \frac{1-\pi_2}{2-\pi_1-\pi_2}E\left[\left(\mu_{c,1}+\sigma_1\varepsilon_{c,t+1}\right)\Delta c_{t+2}|s_{t+1}=1\right]\\
&\quad +\frac{1-\pi_1}{2-\pi_1-\pi_2}E\left[\left(\mu_{c,2}+\sigma_2\varepsilon_{c,t+1}\right)\Delta c_{t+2}|s_{t+1}=2\right]\\
&= \frac{1-\pi_2}{2-\pi_1-\pi_2}\mu_{c,1}E\left[\Delta c_{t+2}|s_{t+1}=1\right]+\frac{1-\pi_1}{2-\pi_1-\pi_2}\mu_{c,2}E\left[\Delta c_{t+2}|s_{t+1}=2\right]\\
&= \frac{1-\pi_2}{2-\pi_1-\pi_2}\mu_{c,1}\left[\pi_1\mu_{c,1}+\left(1-\pi_1\right)\mu_{c,2}\right]\\
&\quad +\frac{1-\pi_1}{2-\pi_1-\pi_2}\mu_{c,2}\left[\left(1-\pi_2\right)\mu_{c,1}+\pi_2\mu_{c,2}\right]
\end{aligned}$$

Therefore,

$$
\begin{aligned}
\mathrm{Cov}(\Delta c_{t+1}, \Delta c_{t+2}) &= \frac{1-\pi_2}{2-\pi_1-\pi_2}\mu_{c,1}\left[\pi_1\mu_{c,1} + (1-\pi_1)\,\mu_{c,2}\right] \\
&\quad + \frac{1-\pi_1}{2-\pi_1-\pi_2}\mu_{c,2}\left[(1-\pi_2)\,\mu_{c,1} + \pi_2\mu_{c,2}\right] \\
&\quad - \left\{\frac{1-\pi_2}{2-\pi_1-\pi_2}\mu_{c,1} + \frac{1-\pi_1}{2-\pi_1-\pi_2}\mu_{c,2}\right\}^2 \qquad (22)
\end{aligned}
$$

Finally, the conditional covariance between p_{t+1} and Δc_{t+1} is

$$
\begin{aligned}
\mathrm{Cov}\left[p_{t+1}, \Delta c_{t+1}|p_t\right] &= E\left[\left(p_{t+1} - f(p_t)\right)\Delta c_{t+1}|p_t\right] \\
&= f(p_t)E\left[\left(\frac{1-f(p_t)}{1+e^{\varepsilon_{p,t+1}}}\right)\left(\mu_{c,1} + \sigma_1\varepsilon_{c,t+1}\right)|p_t\right] \\
&\quad + (1-f(p_t))\,E\left[\left(-\frac{f(p_t)}{1+e^{\varepsilon_{p,t+1}}}\right)\left(\mu_{c,2} + \sigma_2\varepsilon_{c,t+1}\right)|p_t\right] \\
&= f(p_t)\left(\mu_{c,1}\left(1-f(p_t)\right)E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\right) \\
&\quad + (1-f(p_t))\left(-\mu_{c,2}f(p_t)\,E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\right) \\
&= f(p_t)\left(1-f(p_t)\right)E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1} - \mu_{c,2}\right). \qquad (23)
\end{aligned}
$$

Similar expressions are obtained for the moments of the dividend growth rate. Finally, the conditional covariance between the consumption and dividend growth rates is

$$
\begin{aligned}
\mathrm{Cov}(\Delta c_{t+1}, \Delta d_{t+1}|p_t) &= \left\{f(p_t)\mu_{c,1}\mu_{d,1} + (1-f(p_t))\,\mu_{c,2}\mu_{d,2}\right\} \\
&\quad - \left\{f(p_t)\mu_{c,1} + (1-f(p_t))\,\mu_{c,2}\right\}\left\{f(p_t)\mu_{d,1} + (1-f(p_t))\,\mu_{d,2}\right\}, \qquad (24)
\end{aligned}
$$

and the unconditional covariance is

$$
\begin{aligned}
\mathrm{Cov}(\Delta c_{t+1}, \Delta d_{t+1}) &= \left\{\frac{1-\pi_2}{2-\pi_1-\pi_2}\mu_{c,1}\mu_{d,1} + \frac{1-\pi_1}{2-\pi_1-\pi_2}\mu_{c,2}\mu_{d,2}\right\} \\
&\quad - \left\{\frac{1-\pi_2}{2-\pi_1-\pi_2}\mu_{c,1} + \frac{1-\pi_1}{2-\pi_1-\pi_2}\mu_{c,2}\right\}\left\{\frac{1-\pi_2}{2-\pi_1-\pi_2}\mu_{d,1} + \frac{1-\pi_1}{2-\pi_1-\pi_2}\mu_{d,2}\right\}. \qquad (25)
\end{aligned}
$$

A.3 Consumption Claim

We rely on the log-linear approximation for the continuous return on the consumption claim, $r_{c,t+1}$,

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1},$$

where z_t is the log price-consumption ratio. Note that the current model specification involves the single latent state variable p_t. We conjecture that the log price-consumption ratio at date t takes the form,

$$z_t = A_0 + A_1 p_t.$$

The Euler equation for the consumption claim is,

$$E\left[\exp\left(m_{t+1} + r_{c,t+1}\right)|F(t)\right] = 1, \tag{26}$$

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}.$$

Substituting the above expression for m_{t+1} into (26), we have,

$$E\left[\exp\left(\theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta r_{c,t+1}\right)|F(t)\right] = 1,$$

which implies

$$E\left[\exp\left(\theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta\left(\kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1}\right)\right)|F(t)\right] = 1$$

By Taylor series expansion up to quadratic terms, we obtain the following:

$$\begin{aligned} 0 &= \theta \log \delta + \theta\left(\kappa_0 - z_t\right) + \left(\theta - \frac{\theta}{\psi}\right) E\left[\Delta c_{t+1}|F(t)\right] + \theta\kappa_1 E\left[z_{t+1}|F(t)\right] \\ &\quad + \frac{1}{2} var\left(\left(\theta - \frac{\theta}{\psi}\right)\Delta c_{t+1} + \theta\kappa_1 z_{t+1}|F(t)\right) \end{aligned}$$

We use equations (18), (20), (14), (15), and (23) to simplify the above expression as follows:

$$\begin{aligned} &\theta \log \delta + \left(\theta - \frac{\theta}{\psi}\right)\left(f(p_t)\mu_{c,1} + \left(1 - f(p_t)\right)\mu_{c,2}\right) + \theta\kappa_0 + \theta\kappa_1 A_0 + \theta\kappa_1 A_1 f\left(p_t\right) \\ &-\theta A_0 - \theta A_1 p_t + \frac{1}{2}\left(\theta - \frac{\theta}{\psi}\right)^2 f(p_t)\left(1 - f(p_t)\right)\left(\mu_{c,1} - \mu_{c,2}\right)^2 \\ &+\frac{1}{2}\left(\theta - \frac{\theta}{\psi}\right)^2\left[f(p_t)\sigma_1^2 + \left(1 - f(p_t)\right)\sigma_1^2\right] + \frac{1}{2}\left(\theta\kappa_1 A_1\right)^2 f(p_t)\left(1 - f(p_t)\right) E\left[\left(\frac{1}{1 + e^{\varepsilon_{p,t+1}}}\right)^2\right] \\ &+\left(\theta - \frac{\theta}{\psi}\right)\theta\kappa_1 A_1 f(p_t)\left(1 - f(p_t)\right) E\left[\frac{1}{1 + e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1} - \mu_{c,2}\right) \\ = \;& 0 \end{aligned}$$

Collecting terms, we obtain

$$
\begin{aligned}
&\left(\begin{array}{c}
\theta\log\delta+\left(\theta-\frac{\theta}{\psi}\right)\left((1-\pi_2)\mu_{c,1}+\pi_2\mu_{c,2}\right)+\theta\kappa_0+\theta\kappa_1A_0+\theta\kappa_1A_1(1-\pi_2)\\
-\theta A_0+\frac{1}{2}\left(\theta-\frac{\theta}{\psi}\right)^2\pi_2(1-\pi_2)\left(\mu_{c,1}-\mu_{c,2}\right)^2+\frac{1}{2}\left(\theta-\frac{\theta}{\psi}\right)^2\left((1-\pi_2)\sigma_1^2+\pi_2\sigma_2^2\right)\\
+\frac{1}{2}\left(\theta\kappa_1A_1\right)^2\pi_2(1-\pi_2)E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right]\\
+\left(\theta-\frac{\theta}{\psi}\right)\theta\kappa_1A_1\pi_2(1-\pi_2)E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1}-\mu_{c,2}\right)
\end{array}\right)\\
&+\left(\begin{array}{c}
\left(\theta-\frac{\theta}{\psi}\right)\left(\pi_1+\pi_2-1\right)\left(\mu_{c,1}-\mu_{c,2}\right)+\theta\kappa_1A_1\left(\pi_1+\pi_2-1\right)-\theta A_1\\
+\frac{1}{2}\left(\theta-\frac{\theta}{\psi}\right)^2\left(\pi_1+\pi_2-1\right)\left(2\pi_2-1\right)\left(\mu_{c,1}-\mu_{c,2}\right)^2\\
+\frac{1}{2}\left(\theta-\frac{\theta}{\psi}\right)^2\left(\pi_1+\pi_2-1\right)\left(\sigma_1^2-\sigma_2^2\right)\\
+\frac{1}{2}\left(\theta\kappa_1A_1\right)^2\left(\pi_1+\pi_2-1\right)\left(2\pi_2-1\right)E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right]\\
+\left(\theta-\frac{\theta}{\psi}\right)\theta\kappa_1A_1\left(\pi_1+\pi_2-1\right)\left(2\pi_2-1\right)E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1}-\mu_{c,2}\right)
\end{array}\right)p_t\\
&+\left(\begin{array}{c}
-\frac{1}{2}\left(\theta-\frac{\theta}{\psi}\right)^2\left(\pi_1+\pi_2-1\right)^2\left(\mu_{c,1}-\mu_{c,2}\right)^2-\frac{1}{2}\left(\theta\kappa_1A_1\right)^2\left(\pi_1+\pi_2-1\right)^2E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right]\\
-\left(\theta-\frac{\theta}{\psi}\right)\theta\kappa_1A_1\left(\pi_1+\pi_2-1\right)^2E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1}-\mu_{c,2}\right)
\end{array}\right)p_t^2\\
&= 0
\end{aligned}
$$

We approximate the above expression to order p_t. Therefore, we expand the term p_t^2 as a Taylor series to first order around the unconditional mean, $\overline{p}$, to obtain the following:

$$
\begin{aligned}
p_t^2 &\approx \overline{p}^2+2\overline{p}\left(p_t-\overline{p}\right)\\
&= -\overline{p}^2+2\overline{p}p_t
\end{aligned}
$$

Since the Euler equation holds for all observable states p_t, we obtain the following 2 parameter restrictions:

Constant:

$$
\left(\begin{array}{c}
\theta\log\delta+\left(\theta-\frac{\theta}{\psi}\right)\left((1-\pi_2)\mu_{c,1}+\pi_2\mu_{c,2}\right)+\theta\kappa_0+\theta\kappa_1A_0+\theta\kappa_1A_1(1-\pi_2)\\
-\theta A_0+\frac{1}{2}\left(\theta-\frac{\theta}{\psi}\right)^2\pi_2(1-\pi_2)\left(\mu_{c,1}-\mu_{c,2}\right)^2+\frac{1}{2}\left(\theta-\frac{\theta}{\psi}\right)^2\left((1-\pi_2)\sigma_1^2+\pi_2\sigma_2^2\right)\\
+\frac{1}{2}\left(\theta\kappa_1A_1\right)^2\pi_2(1-\pi_2)E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right]+\left(\theta-\frac{\theta}{\psi}\right)\theta\kappa_1A_1\pi_2(1-\pi_2)E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1}-\mu_{c,2}\right)\\
+\frac{1}{2}\left(\theta-\frac{\theta}{\psi}\right)^2\left(\pi_1+\pi_2-1\right)^2\left(\mu_{c,1}-\mu_{c,2}\right)^2\overline{p}^2+\frac{1}{2}\left(\theta\kappa_1A_1\right)^2\left(\pi_1+\pi_2-1\right)^2E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right]\overline{p}^2\\
+\left(\theta-\frac{\theta}{\psi}\right)\theta\kappa_1A_1\left(\pi_1+\pi_2-1\right)^2E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1}-\mu_{c,2}\right)\overline{p}^2
\end{array}\right)=0 \tag{27}
$$

Coefficient of p_t:

$$\left(\begin{array}{c}\left(\theta-\frac{\theta}{\psi}\right)(\pi_1+\pi_2-1)\left(\mu_{c,1}-\mu_{c,2}\right)+\theta\kappa_1 A_1(\pi_1+\pi_2-1)-\theta A_1\\ +\frac{1}{2}\left(\theta-\frac{\theta}{\psi}\right)^2(\pi_1+\pi_2-1)(2\pi_2-1)\left(\mu_{c,1}-\mu_{c,2}\right)^2+\frac{1}{2}\left(\theta-\frac{\theta}{\psi}\right)^2(\pi_1+\pi_2-1)(\sigma_1^2-\sigma_2^2)\\ +\frac{1}{2}(\theta\kappa_1 A_1)^2(\pi_1+\pi_2-1)(2\pi_2-1)E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right]\\ +\left(\theta-\frac{\theta}{\psi}\right)\theta\kappa_1 A_1(\pi_1+\pi_2-1)(2\pi_2-1)E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1}-\mu_{c,2}\right)\\ -\left(\theta-\frac{\theta}{\psi}\right)^2(\pi_1+\pi_2-1)^2\left(\mu_{c,1}-\mu_{c,2}\right)^2\overline{p}-(\theta\kappa_1 A_1)^2(\pi_1+\pi_2-1)^2E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right]\overline{p}\\ -\left(\theta-\frac{\theta}{\psi}\right)\theta\kappa_1 A_1(\pi_1+\pi_2-1)^2E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1}-\mu_{c,2}\right)2\overline{p}\end{array}\right)=0 \tag{28}$$

Equations (27) and (28) can be solved to obtain the parameters A_0 and A_1. Equation (28) is quadratic in A_1. We set A_1 equal to the smaller root of the quadratic equation as this minimizes the GMM objective function in the estimation.

A.4 Risk free Rate

The risk free rate, $r_{f,t}$, is priced using the Euler equation,

$$E\left[\exp(m_{t+1}+r_{f,t})|\mathcal{F}(t)\right]=1.$$

Hence,

$$\begin{aligned}\exp\left(-r_{f,t}\right) &= E\left[\exp(m_{t+1})|\mathcal{F}(t)\right]\\ &= E\left[\exp(\theta\log\delta-\frac{\theta}{\psi}\Delta c_{t+1}+(\theta-1)r_{c,t+1})|\mathcal{F}(t)\right]\end{aligned}$$

By Taylor series expansion up to quadratic terms, we obtain the following:

$$\begin{aligned}-r_{f,t} &= \theta\log\delta+(\theta-1)(\kappa_0-z_t)+\left(-\frac{\theta}{\psi}+\theta-1\right)E\left[\Delta c_{t+1}|\mathcal{F}(t)\right]+(\theta-1)\kappa_1 E\left[z_{t+1}|\mathcal{F}(t)\right]\\ &\quad \frac{1}{2}var\left(\left(-\frac{\theta}{\psi}+\theta+1\right)\Delta c_{t+1}+(\theta-1)\kappa_1 z_{t+1}|\mathcal{F}(t)\right).\end{aligned}$$

Therefore, we obtain

$$
\begin{aligned}
r_{f,t} &= \left(\begin{array}{c}
\theta\log\delta + \left(-\frac{\theta}{\psi}+\theta-1\right)\left((1-\pi_2)\mu_{c,1}+\pi_2\mu_{c,2}\right) \\
+(\theta-1)\kappa_0+(\theta-1)\kappa_1 A_0+(\theta-1)\kappa_1 A_1(1-\pi_2) \\
-(\theta-1)A_0+\frac{1}{2}\left(-\frac{\theta}{\psi}+\theta-1\right)^2\pi_2(1-\pi_2)\left(\mu_{c,1}-\mu_{c,2}\right)^2 \\
+\frac{1}{2}\left(-\frac{\theta}{\psi}+\theta-1\right)^2\left((1-\pi_2)\sigma_1^2+\pi_2\sigma_2^2\right) \\
+\frac{1}{2}\left((\theta-1)\kappa_1 A_1\right)^2\pi_2(1-\pi_2)E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right] \\
+\left(-\frac{\theta}{\psi}+\theta-1\right)(\theta-1)\kappa_1 A_1\pi_2(1-\pi_2)E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1}-\mu_{c,2}\right)
\end{array}\right) \\
&+\left(\begin{array}{c}
\left(-\frac{\theta}{\psi}+\theta-1\right)\left(\pi_1+\pi_2-1\right)\left(\mu_{c,1}-\mu_{c,2}\right)+(\theta-1)\kappa_1 A_1\left(\pi_1+\pi_2-1\right)-(\theta-1)A_1 \\
+\frac{1}{2}\left(-\frac{\theta}{\psi}+\theta-1\right)^2\left(\pi_1+\pi_2-1\right)\left(2\pi_2-1\right)\left(\mu_{c,1}-\mu_{c,2}\right)^2 \\
+\frac{1}{2}\left(-\frac{\theta}{\psi}+\theta-1\right)^2\left(\pi_1+\pi_2-1\right)\left(\sigma_1^2-\sigma_2^2\right) \\
+\frac{1}{2}\left((\theta-1)\kappa_1 A_1\right)^2\left(\pi_1+\pi_2-1\right)\left(2\pi_2-1\right)E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right] \\
+\left(-\frac{\theta}{\psi}+\theta-1\right)(\theta-1)\kappa_1 A_1\left(\pi_1+\pi_2-1\right)\left(2\pi_2-1\right)E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1}-\mu_{c,2}\right)
\end{array}\right)p_t \\
&+\left(\begin{array}{c}
-\frac{1}{2}\left(-\frac{\theta}{\psi}+\theta-1\right)^2\left(\pi_1+\pi_2-1\right)^2\left(\mu_{c,1}-\mu_{c,2}\right)^2 \\
-\frac{1}{2}\left((\theta-1)\kappa_1 A_1\right)^2\left(\pi_1+\pi_2-1\right)^2E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right] \\
-\left(-\frac{\theta}{\psi}+\theta-1\right)(\theta-1)\kappa_1 A_1\left(\pi_1+\pi_2-1\right)^2E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{c,1}-\mu_{c,2}\right)
\end{array}\right)p_t^2
\end{aligned}
$$

As before, we approximate $p_t^2 \approx -\overline{p}^2 + 2\overline{p}p_t$ to express the equilibrium risk free rate as an affine function of the state variable p_t.

A.5 Dividend Claim

The market portfolio is defined as the claim to the aggregate dividend stream. We rely on the log-linear approximation for the continuous return on the aggregate dividend claim, $r_{m,t+1}$,

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + \Delta d_{t+1},$$

where $z_{m,t}$ is the market-wide log price-dividend ratio. We conjecture that the log price-dividend ratio at date t takes the form,

$$z_{m,t} = A_{0,m} + A_{1,m} p_t$$

Similar calculations as in Appendix A.3 give:

$$
\begin{aligned}
0 \;=\; & \left(\begin{array}{c}
\theta \log \delta + \left(-\frac{\theta}{\psi} + \theta - 1\right) \left((1-\pi_2)\mu_{c,1} + \pi_2 \mu_{c,2}\right) + (\theta-1)\kappa_0 \\
+(\theta-1)\kappa_1 A_0 + (\theta-1)\kappa_1 A_1 (1-\pi_2) - (\theta-1) A_0 + \\
\kappa_{0,m} + \kappa_{1,m} A_{0,m} + \kappa_{1,m} A_{1,m} (1-\pi_2) - A_{0,m} + \left((1-\pi_2)\mu_{d,1} + \pi_2 \mu_{d,2}\right) \\
+\frac{1}{2}\left(-\frac{\theta}{\psi} + \theta - 1\right)^2 \pi_2 (1-\pi_2) \left(\mu_{c,1} - \mu_{c,2}\right)^2 \\
+\frac{1}{2}\left(-\frac{\theta}{\psi} + \theta - 1\right)^2 \left((1-\pi_2)\sigma_1^2 + \pi_2 \sigma_2^2\right) \\
+\frac{1}{2}\pi_2 (1-\pi_2) \left(\mu_{d,1} - \mu_{d,2}\right)^2 + \frac{1}{2}\phi^2 \left((1-\pi_2)\sigma_1^2 + \pi_2 \sigma_2^2\right) \\
+\frac{1}{2}\left((\theta-1)\kappa_1 A_1 + \kappa_{1,m} A_{1,m}\right)^2 \pi_2 (1-\pi_2) E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right] \\
+\left(-\frac{\theta}{\psi} + \theta - 1\right) \left((\theta-1)\kappa_1 A_1 + \kappa_{1,m} A_{1,m}\right) \pi_2 (1-\pi_2) E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right] \left(\mu_{c,1} - \mu_{c,2}\right) \\
+\left((\theta-1)\kappa_1 A_1 + \kappa_{1,m} A_{1,m}\right) \pi_2 (1-\pi_2) E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right] \left(\mu_{d,1} - \mu_{d,2}\right) \\
+\left(-\frac{\theta}{\psi} + \theta - 1\right) \left[(1-\pi_2) \mu_{c,1}\mu_{d,1} + \pi_2 \mu_{c,2}\mu_{d,2}\right] \\
+\left(-\frac{\theta}{\psi} + \theta - 1\right) \left[-(1-\pi_2)^2 \mu_{c,1}\mu_{d,1} - \pi_2 (1-\pi_2) \left(\mu_{c,1}\mu_{d,2} + \mu_{c,2}\mu_{d,1}\right) - \pi_2^2 \mu_{c,2}\mu_{d,2}\right]
\end{array} \right) + \\
& \left(\begin{array}{c}
\left(-\frac{\theta}{\psi} + \theta - 1\right) (\pi_1 + \pi_2 - 1) \left(\mu_{c,1} - \mu_{c,2}\right) + (\theta-1)\kappa_1 A_1 (\pi_1 + \pi_2 - 1) - (\theta-1) A_1 \\
+\kappa_{1,m} A_{1,m} (\pi_1 + \pi_2 - 1) - A_{1,m} + (\pi_1 + \pi_2 - 1) \left(\mu_{d,1} - \mu_{d,2}\right) \\
+\frac{1}{2}\left(-\frac{\theta}{\psi} + \theta - 1\right)^2 (\pi_1 + \pi_2 - 1)(2\pi_2 - 1) \left(\mu_{c,1} - \mu_{c,2}\right)^2 \\
+\frac{1}{2}\left(-\frac{\theta}{\psi} + \theta - 1\right)^2 (\pi_1 + \pi_2 - 1)(\sigma_1^2 - \sigma_2^2) \\
+\frac{1}{2}(\pi_1 + \pi_2 - 1)(2\pi_2 - 1) \left(\mu_{d,1} - \mu_{d,2}\right)^2 + \frac{1}{2}\phi^2 (\pi_1 + \pi_2 - 1)(\sigma_1^2 - \sigma_2^2) \\
+\frac{1}{2}\left((\theta-1)\kappa_1 A_1 + \kappa_{1,m} A_{1,m}\right)^2 (\pi_1 + \pi_2 - 1)(2\pi_2 - 1) E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right] + \\
\left(-\frac{\theta}{\psi} + \theta - 1\right) \left((\theta-1)\kappa_1 A_1 + \kappa_{1,m} A_{1,m}\right) (\pi_1 + \pi_2 - 1)(2\pi_2 - 1) E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right] \left(\mu_{c,1} - \mu_{c,2}\right) \\
+\left((\theta-1)\kappa_1 A_1 + \kappa_{1,m} A_{1,m}\right) (\pi_1 + \pi_2 - 1)(2\pi_2 - 1) E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right] \left(\mu_{c,1} - \mu_{c,2}\right) \\
+\left(-\frac{\theta}{\psi} + \theta - 1\right) (\pi_1 + \pi_2 - 1) \left(\mu_{c,1}\mu_{d,1} - \mu_{c,2}\mu_{d,2}\right) \\
-2(1-\pi_2) \mu_{c,1}\mu_{d,1} \left(-\frac{\theta}{\psi} + \theta - 1\right) (\pi_1 + \pi_2 - 1) \\
+\left(-\frac{\theta}{\psi} + \theta - 1\right) (\pi_1 + \pi_2 - 1) \left[-(2\pi_2 - 1) \left(\mu_{c,1}\mu_{d,2} + \mu_{c,2}\mu_{d,1}\right) + 2\pi_2 \mu_{c,2}\mu_{d,2}\right]
\end{array} \right) p \\
& + \left(\begin{array}{c}
-\frac{1}{2}\left(-\frac{\theta}{\psi} + \theta - 1\right)^2 (\pi_1 + \pi_2 - 1)^2 \left(\mu_{c,1} - \mu_{c,2}\right)^2 - \frac{1}{2}(\pi_1 + \pi_2 - 1)^2 \left(\mu_{d,1} - \mu_{d,2}\right)^2 \\
-\frac{1}{2}\left((\theta-1)\kappa_1 A_1 + \kappa_{1,m} A_{1,m}\right)^2 (\pi_1 + \pi_2 - 1)^2 E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right] \\
-\left(-\frac{\theta}{\psi} + \theta - 1\right) \left((\theta-1)\kappa_1 A_1 + \kappa_{1,m} A_{1,m}\right) (\pi_1 + \pi_2 - 1)^2 E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right] \left(\mu_{c,1} - \mu_{c,2}\right) \\
-\left((\theta-1)\kappa_1 A_1 + \kappa_{1,m} A_{1,m}\right) (\pi_1 + \pi_2 - 1)^2 E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right] \left(\mu_{d,1} - \mu_{d,2}\right) \\
+\left(-\frac{\theta}{\psi} + \theta - 1\right) (\pi_1 + \pi_2 - 1)^2 \left[-\mu_{c,1}\mu_{d,1} + \left(\mu_{c,1}\mu_{d,2} + \mu_{c,2}\mu_{d,1}\right) - \mu_{c,2}\mu_{d,2}\right]
\end{array} \right) p_t^2
\end{aligned}
$$

Approximating $p_t^2 \approx -\overline{p}^2 + 2\overline{p} p_t$ gives 2 equations that can be solved to obtain the 2 parameters $A_{0,m}$ and $A_{1,m}$. As with the price-consumption ratio, we set $A_{1,m}$ equal to the smaller root of the quadratic equation.

A.6 Model-Implied Predictive Regressions

In this Appendix, we derive the slope coefficient and the R^2 from model implied forecasting regressions of the one-year ahead market return, consumption growth, and dividend growth on the market-wide price dividend ratio. Consider first the market return. The model implies

$$\begin{aligned}
E\left(r_{m,t+1}|p_t\right) &= \kappa_{0,m}+\kappa_{1,m}E\left(z_{m,t+1}|p_t\right)-z_{m,t}+E\left(\Delta d_{t+1}|p_t\right) \\
&= \kappa_{0,m}+\kappa_{1,m}\left[A_{0,m}+A_{1,m}f\left(p_t\right)\right]-A_{0,m}-A_{1,m}p_t \\
&\quad +f(p_t)\mu_{d,1}+\left(1-f(p_t)\right)\mu_{d,2} \\
&= \kappa_{0,m}+\kappa_{1,m}A_{0,m}+\kappa_{1,m}A_{1,m}\left(1-\pi_2\right)-A_{0,m}+\left[\left(1-\pi_2\right)\mu_{d,1}+\pi_2\mu_{d,2}\right] \\
&\quad +\left[\kappa_{1,m}A_{1,m}\left(\pi_1+\pi_2-1\right)-A_{1,m}+\left(\pi_1+\pi_2-1\right)\left(\mu_{d,1}-\mu_{d,2}\right)\right]p_t \quad (29)
\end{aligned}$$

Since, $p_t=\frac{z_{m,t}-A_{o,m}}{A_{1,m}}$, the model-implied slope coefficient from a forecasting regressions of the market return on the market-wide price dividend ratio is

$$\left[\kappa_{1,m}\left(\pi_1+\pi_2-1\right)-1+\frac{\left(\pi_1+\pi_2-1\right)\left(\mu_{d,1}-\mu_{d,2}\right)}{A_{1,m}}\right]$$

Therefore, the model-implied R^2 is

$$\left[\kappa_{1,m}1\left(\pi_1+\pi_2-1\right)-1+\frac{\left(\pi_1+\pi_2-1\right)\left(\mu_{d,1}-\mu_{d,2}\right)}{A_{1,m}}\right]^2\frac{\mathrm{Var}\left(z_{m,t}\right)}{\mathrm{Var}\left(r_{m,t+1}\right)}$$

Now, $\mathrm{Var}(z_{m,t})=A_{1,m}^2\mathrm{Var}(p_t)$, where $\mathrm{Var}(p_t)$ was derived in Section A.1. We calculate $\mathrm{Var}(r_{m,t+1})$ as follows.

$$\mathrm{Var}\left(r_{m,t+1}\right)=\mathrm{Var}\left[E\left(r_{m,t+1}|p_t\right)\right]+E\left[\mathrm{Var}\left(r_{m,t+1}|p_t\right)\right]$$

Note that Equation (29) implies that

$$Var\left[E\left(r_{m,t+1}|p_t\right)\right]=\left[\kappa_{1,m}A_{1,m}\left(\pi_1+\pi_2-1\right)-A_{1,m}+\left(\pi_1+\pi_2-1\right)\left(\mu_{d,1}-\mu_{d,2}\right)\right]^2Var\left(p_t\right).$$

And,

$$\begin{aligned}
\mathrm{Var}\left(r_{m,t+1}|p_t\right) &= \kappa_{1,m}^2A_{1,m}^2\mathrm{Var}\left(p_{t+1}|p_t\right)+\mathrm{Var}\left(\Delta d_{t+1}|p_t\right)+2\kappa_{1,m}A_{1,m}\mathrm{Cov}\left(p_{t+1},\Delta d_{t+1}|p_t\right) \\
&= \kappa_{1,m}^2A_{1,m}^2f(p_t)\left(1-f(p_t)\right)E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right] \\
&\quad +f(p_t)\left(1-f(p_t)\right)\left(\mu_{d,1}-\mu_{d,2}\right)^2+\phi^2\left[f(p_t)\sigma_1^2+\left(1-f(p_t)\right)\sigma_1^2\right] \\
&\quad +2\kappa_{1,m}A_{1,m}f(p_t)\left(1-f(p_t)\right)E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{d,1}-\mu_{d,2}\right)
\end{aligned}$$

which implies

$$\begin{aligned} E\left[\operatorname{Var}\left(r_{m,t+1}|p_t\right)\right] &= \kappa_{1,m}^2 A_{1,m}^2 E\left[f\left(p_t\right)\left(1-f\left(p_t\right)\right)\right] E\left[\left(\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right)^2\right] \\ &\quad + E\left[f\left(p_t\right)\left(1-f\left(p_t\right)\right)\right]\left(\mu_{d,1}-\mu_{d,2}\right)^2 + \phi^2\left[f(p_t)\sigma_1^2 + \left(1-f\left(p_t\right)\right)\sigma_1^2\right] \\ &\quad + 2\kappa_{1,m} A_{1,m} E\left[f\left(p_t\right)\left(1-f\left(p_t\right)\right)\right] E\left[\frac{1}{1+e^{\varepsilon_{p,t+1}}}\right]\left(\mu_{d,1}-\mu_{d,2}\right). \end{aligned}$$

The slope coefficient and R^2 from the forecasting regression of the one-year ahead consumption growth rate on the market-wide price-dividend ratio are given by

$$\frac{\left(\pi_1+\pi_2-1\right)\left(\mu_{c,1}-\mu_{c,2}\right)}{A_{1,m}}$$

and

$$\left[\frac{\left(\pi_1+\pi_2-1\right)\left(\mu_{c,1}-\mu_{c,2}\right)}{A_{1,m}}\right]^2 \frac{\operatorname{Var}\left(z_{m,t}\right)}{\operatorname{Var}\left(\Delta c_{t+1}\right)},$$

respectively. Note that $\operatorname{Var}(\Delta c_{t+1})$ was derived in Equation (21).

Similarly, the slope coefficient and R^2 from the forecasting regression of the one-year ahead dividend growth rate on the market-wide price-dividend ratio are given by

$$\frac{\left(\pi_1+\pi_2-1\right)\left(\mu_{d,1}-\mu_{d,2}\right)}{A_{1,m}}$$

and

$$\left[\frac{\left(\pi_1+\pi_2-1\right)\left(\mu_{d,1}-\mu_{d,2}\right)}{A_{1,m}}\right]^2 \frac{\operatorname{Var}\left(z_{m,t}\right)}{\operatorname{Var}\left(\Delta d_{t+1}\right)},$$

respectively.

The slope coefficients and R^2s from the five-year ahead and ten-year ahead forecasting regressions are obtained similarly and are not included here for the sake of brevity.

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Table 1: Parameter Estimates and Model Fit, Annual Data, 1929-2013

$\mu_{c,1}$	$\mu_{c,2}$	$\mu_{d,1}$	$\mu_{d,2}$	ϕ	σ_1	σ_2	σ_p	π_1	π_2
0.027	-0.032	0.054	-0.099	5.615	0.010	0.035	12.89	0.980	0.857
(0.020)	(0.114)	(0.043)	(0.312)	(0.159)	(0.075)	(0.173)	(0.341)	(0.082)	(0.125)
γ	ψ	δ							
1.856	1.991	0.972							
(0.178)	(0.117)	(0.002)							

	Data	*Model*		*Data*	*Model*
$E\left(r_f\right)$	0.005	0.014	$E(\Delta c)$	0.019	0.020
	(0.005)	[−0.027,0.032]		(0.003)	[0.002,0.028]
$\sigma\left(r_f\right)$	0.030	0.046	$sd(\Delta c)$	0.021	0.025
	(0.005)	[0.003,0.070]		(0.004)	[0.009,0.039]
$AC_1\left(r_f\right)$	0.675	0.837	$AC1(\Delta c)$	0.501	0.512
	(0.201)	[0.083,0.930]		(0.284)	[−0.154,0.704]
$E\left(r_m\right)$	0.070	0.069	$E(\Delta d)$	0.013	0.035
	(0.019)	[0.029,0.094]		(0.013)	[−0.016,0.062]
$\sigma\left(r_m\right)$	0.188	0.275	$sd(\Delta d)$	0.114	0.100
	(0.018)	[0.060,0.436]		(0.019)	[0.049,0.162]
$E\left(p/d\right)$	3.400	3.354	$AC_1\left(\Delta d\right)$	0.183	0.299
	(0.080)	[2.968,3.520]		(0.145)	[−0.215,0.496]
$\sigma\left(p/d\right)$	0.452	0.436	$AC\left(\Delta c, \Delta d\right)$	0.584	0.395
	(0.048)	[0.032,0.663]		(0.316)	[−0.164,0.649]
$AC_1\left(p/d\right)$	0.881	0.837			
	(0.210)	[0.083,0.930]			
$Jstat$	7.607				
	(11.07)				

The table reports GMM estimates (asymptotic standard errors in parentheses) of the model parameters, obtained using annual data over the entire available sample period 1929-2013. It also reports the model-implied moments, obtained from a single simulation of 1 million observations, and the 95% confidence interval (in square brackets), obtained through 10000 simulations of the same length as the historical data, and the corresponding sample values (asymptotic standard errors in parentheses) of the mean, volatility, and first-order autocorrelation of the risk free rate, price-dividend ratio, market return, and the consumption and dividend growth rates. Finally, the last row reports the J-stat for overidentifying restrictions, along with its asymptotic p-value in parentheses.

Table 2: Summary Statistics in the Two Regimes, Annual Data 1929-2013

	Panel A: Regime 1				*Panel B: Regime2*			
	Data		*Model*		*Data*		*Model*	
	$E(.)$	$sd(.)$	$E(.)$	$sd(.)$	$E(.)$	$sd(.)$	$E(.)$	$sd(.)$
Δc	0.021	0.017	0.025	0.016	-0.009	0.036	-0.018	0.040
	(0.002)	(0.002)	[0.017,0.028]	[0.010,0.029]	(0.016)	(0.006)	[-0.041,0.011]	[0.023,0.054]
Δd	0.030	0.080	0.049	0.071	-0.152	0.220	-0.064	0.188
	(0.011)	(0.013)	[0.020,0.063]	[0.050,0.122]	(0.087)	(0.022)	[-0.171,0.052]	[0.090,0.268]
$\log(P/D)$	3.448	0.432	3.487	0.199	2.867	0.323	2.367	0.455
	(0.080)	(0.048)	[3.387,3.503]	[0.152,0.398]	(0.107)	(0.062)	[2.176,2.661]	[0.226,0.668]
r_f	0.005	0.027	0.031	0.007	0.007	0.052	-0.108	0.022
	(0.005)	(0.005)	[0.026,0.032]	[0.003,0.017]	(0.020)	(0.004)	[-0.122,-0.092]	[0.010,0.030]
r_m	0.055	0.190	0.055	0.215	0.113	0.288	0.179	0.490
	(0.019)	(0.017)	[-0.037,0.076]	[0.149,0.418]	(0.089)	(0.075)	[-0.009,0.494]	[0.240,0.743]

Panel A reports the sample mean and volatility (asymptotic standard errors in parentheses) of consumption and dividend growth rates, the log price-dividend ratio, risk free rate, and market return in the first regime. It also reports the model-implied values, obtained from a single simulation of 1 million observations, and the 95% confidence interval (in square brackets), obtained through 10000 simulations of the same length as the historical data, of these moments. Panel B reports the corresponding moments in the second regime. Regime 1 is defined as corresponding to those data points at which $p_t > 0.5$ while Regime 2 corresponds to those data points at which $p_t < 0.5$.

Table 3: Forecasting with log$(P/D)_t$, Annual Data 1929-2013

	Data		*Model*	
	Panel A: 1-year			
	Coefficient	R^2 (%)	Coefficient	R^2 (%)
$r_{m,t,t+1}$	-0.065 (0.048)	2.17	-0.107	2.89
$\Delta d_{t,t+1}$	0.080 (0.026)	10.19	0.084	13.35
$\Delta c_{t,t+1}$	0.010 (0.005)	4.54	0.032	32.00
	Panel B: 5-year			
	Coefficient	R^2 (%)	Coefficient	R^2 (%)
$r_{m,t,t+5}$	-0.380 (0.079)	22.69	-0.368	8.04
$\Delta d_{t,t+5}$	0.080 (0.050)	3.14	0.313	21.61
$\Delta c_{t,t+5}$	-0.013 (0.013)	1.28	0.119	32.40
	Panel C: 10-year			
	Coefficient	R^2 (%)	Coefficient	R^2 (%)
$r_{m,t,t+10}$	-0.632 (0.109)	31.62	-0.527	10.03
$\Delta d_{t,t+10}$	0.094 (0.055)	3.89	0.445	16.53
$\Delta c_{t,t+10}$	-0.033 (0.016)	5.67	0.169	21.74

The table reports the slope coefficient and the R^2 from forecasting regressions of the market return, the aggregate consumption and dividend growth rates on the price dividend ratio at the 1-year (Panel A), 5-year (Panel B), and 10-year (Panel C) frequencies, in the model and in the historical data.

Table 4: Parameter Estimates and Model Fit, Annual Data, 1947-2013

$\mu_{c,1}$	$\mu_{c,2}$	$\mu_{d,1}$	$\mu_{d,2}$	ϕ	σ_1	σ_2	σ_p	π_1	π_2
0.043 (0.007)	0.009 (0.030)	0.052 (0.018)	-0.021 (0.060)	3.184 (0.192)	0.005 (0.044)	0.044 (0.043)	4.616 (0.075)	0.986 (0.013)	0.848 (0.199)
γ	ψ	δ							
1.503 (0.531)	1.442 (0.420)	0.988 (0.016)							

	Data	*Model*		*Data*	*Model*
$E(r_f)$	0.008 (0.004)	0.018 [-0.034,0.034]	$E(\Delta c)$	0.019 (0.002)	0.040 [0.029,0.044]
$\sigma(r_f)$	0.027 (0.006)	0.049 [0.003,0.088]	$sd(\Delta c)$	0.013 (0.001)	0.017 [0.004,0.033]
$AC_1(r_f)$	0.607 (0.268)	0.834 [0.037,0.933]	$AC1(\Delta c)$	0.341 (0.165)	0.274 [-0.323,0.612]
$E(r_m)$	0.069 (0.019)	0.070 [0.043,0.081]	$E(\Delta d)$	0.022 (0.010)	0.046 [0.020,0.056]
$\sigma(r_m)$	0.175 (0.018)	0.238 [0.027,0.404]	$sd(\Delta d)$	0.070 (0.008)	0.048 [0.014,0.097]
$E(p/d)$	3.497 (0.086)	3.695 [3.273,3.819]	$AC_1(\Delta d)$	0.229 (0.077)	0.368 [-0.370,0.531]
$\sigma(p/d)$	0.427 (0.050)	0.398 [0.022,0.709]	$AC(\Delta c, \Delta d)$	0.181 (0.157)	0.244 [-0.464,0.741]
$AC_1(p/d)$	0.914 (0.231)	0.834 [0.037,0.933]			
$Jstat$	9.42 (0.096)				

The table reports GMM estimates (asymptotic standard errors in parentheses) of the model parameters, obtained using annual data over the post-war period 1947-2013. It also reports the model-implied moments, obtained from a single simulation of 1 million observations, and the 95% confidence interval (in square brackets), obtained through 10000 simulations of the same length as the historical data, and the corresponding sample values (asymptotic standard errors in parentheses) of the mean, volatility, and first-order autocorrelation of the risk free rate, price-dividend ratio, market return, and the consumption and dividend growth rates. Finally, the last row reports the J-stat for overidentifying restrictions, along with its asymptotic p-value in parentheses.

Table 5: Summary Statistics in the Two Regimes, Annual Data 1947-2013

	Panel A: Regime 1				*Panel B: Regime2*			
	Data		*Model*		*Data*		*Model*	
	$E(.)$	$sd(.)$	$E(.)$	$sd(.)$	$E(.)$	$sd(.)$	$E(.)$	$sd(.)$
Δc	0.019 (0.002)	0.014 (0.001)	0.042 [0.036,0.044]	0.010 [0.005,0.025]	0.016 (0.004)	0.009 (0.001)	0.016 [−0.012,0.042]	0.041 [0.020,0.061]
Δd	0.020 (0.010)	0.068 (0.009)	0.050 [0.034,0.057]	0.029 [0.015,0.073]	0.029 (0.026)	0.092 (0.018)	−0.006 [−0.085,0.073]	0.128 [0.060,0.189]
$\log(P/D)$	3.574 (0.082)	0.391 (0.046)	3.793 [3.671,3.797]	0.175 [0.142,0.413]	2.932 (0.083)	0.188 (0.031)	2.540 [2.302,2.835]	0.481 [0.183,0.690]
r_f	0.012 (0.004)	0.025 (0.006)	0.033 [0.024,0.034]	0.008 [0.003,0.025]	−0.016 (0.006)	0.026 (0.004)	−0.146 [−0.165,−0.120]	0.028 [0.013,0.040]
r_m	0.057 (0.021)	0.176 (0.019)	0.057 [−0.026,0.064]	0.169 [0.132,0.402]	0.155 (0.039)	0.147 (0.023)	0.224 [0.034,0.493]	0.481 [0.208,0.753]

Panel A reports the sample mean and volatility (asymptotic standard errors in parentheses) of consumption and dividend growth rates, the log price-dividend ratio, risk free rate, and market return in the first regime. It also reports the model-implied values, obtained from a single simulation of 1 million observations, and the 95% confidence interval (in square brackets), obtained through 10000 simulations of the same length as the historical data, of these moments. Panel B reports the corresponding moments in the second regime. Regime 1 is defined as corresponding to those data points at which $p_t > 0.5$ while Regime 2 corresponds to those data points at which $p_t < 0.5$.

Table 6: Forecasting with $\log(P/D)_t$, Annual Data 1947-2013

	Data		Model	
	Panel A: 1-year			
	Coefficient	R^2 (%)	Coefficient	R^2 (%)
$r_{m,t,t+1}$	-0.103 (0.049)	6.24	-0.151	6.38
$\Delta d_{t,t+1}$	0.011 (0.020)	0.42	0.036	8.85
$\Delta c_{t,t+1}$	0.002 (0.004)	0.39	0.017	16.1
	Panel B: 5-year			
	Coefficient	R^2 (%)	Coefficient	R^2 (%)
$r_{m,t,t+5}$	-0.463 (0.090)	30.2	-0.506	19.85
$\Delta d_{t,t+5}$	0.023 (0.046)	0.40	0.137	17.52
$\Delta c_{t,t+5}$	-0.016 (0.012)	3.16	0.064	25.23
	Panel C: 10-year			
	Coefficient	R^2 (%)	Coefficient	R^2 (%)
$r_{m,t,t+10}$	-0.862 (0.127)	45.2	-0.739	28.09
$\Delta d_{t,t+10}$	0.068 (0.062)	2.11	0.193	13.64
$\Delta c_{t,t+10}$	-0.044 (0.015)	13.3	0.091	18.30

The table reports the slope coefficient and the R^2 from forecasting regressions of the market return, the aggregate consumption and dividend growth rates on the price dividend ratio at the 1-year (Panel A), 5-year (Panel B), and 10-year (Panel C) frequencies, in the model and in the historical data.

Table 7: Parameter Estimates and Model Fit, Quarterly Data, 1947:Q1-2013:Q4

$\mu_{c,1}$	$\mu_{c,2}$	$\mu_{d,1}$	$\mu_{d,2}$	ϕ	σ_1	σ_2	σ_p	π_1	π_2
0.003 (0.001)	-0.008 (0.007)	0.021 (0.009)	-0.024 (0.044)	9.562 (0.037)	0.001 (0.127)	0.020 (0.035)	1.172 (0.199)	0.984 (0.010)	0.914 (0.124)
γ	ψ	δ							
1.995 (0.090)	1.344 (0.040)	0.990 (0.007)							

	Data	*Model*		*Data*	*Model*
$E(r_f)$	0.003 (0.0006)	0.001 [-0.009,0.008]	$E(\Delta c)$	0.005 (0.0003)	0.001 [-0.001,0.003]
$\sigma(r_f)$	0.006 (0.0005)	0.016 [0.004,0.022]	$sd(\Delta c)$	0.005 (0.0003)	0.009 [0.002,0.014]
$AC_1(r_f)$	0.865 (0.203)	0.898 [0.738,0.977]	$AC1(\Delta c)$	0.304 (0.100)	0.176 [-0.180,0.473]
$E(r_m)$	0.018 (0.005)	0.043 [0.032,0.053]	$E(\Delta d)$	0.006 (0.005)	0.014 [-0.001,0.023]
$\sigma(r_m)$	0.084 (0.005)	0.209 [0.072,0.231]	$sd(\Delta d)$	0.140 (0.012)	0.079 [0.020,0.123]
$E(p/d)$	3.494 (0.044)	3.515 [3.244,3.701]	$AC_1(\Delta d)$	-0.581 (0.106)	0.070 [-0.298,0.355]
$\sigma(p/d)$	0.421 (0.027)	0.420 [0.114,0.585]	$AC(\Delta c, \Delta d)$	-0.036 (0.091)	0.093 [-0.283,0.466]
$AC_1(p/d)$	0.980 (0.127)	0.898 [0.738,0.977]			
$Jstat$	28.26 (0.037)				

The table reports GMM estimates (asymptotic standard errors in parentheses) of the model parameters, obtained using quarterly data over the entire available sample period 1947-2013. It also reports the model-implied moments, obtained from a single simulation of 1 million observations, and the 95% confidence interval (in square brackets), obtained through 10000 simulations of the same length as the historical data, and the corresponding sample values (asymptotic standard errors in parentheses) of the mean, volatility, and first-order autocorrelation of the risk free rate, price-dividend ratio, market return, and the consumption and dividend growth rates. Finally, the last row reports the J-stat for overidentifying restrictions, along with its asymptotic p-value in parentheses.

Table 8: Summary Statistics in the Two Regimes, Quarterly Data 1947:Q1-2013:Q4

	Panel A: Regime 1				*Panel B: Regime2*			
	Data		*Model*		*Data*		*Model*	
	$E(.)$	$sd(.)$	$E(.)$	$sd(.)$	$E(.)$	$sd(.)$	$E(.)$	$sd(.)$
Δc	0.005	0.004	0.003	0.004	0.003	0.008	-0.007	0.019
	(0.0004)	(0.0003)	[0.002,0.003]	[0.001,0.007]	(0.001)	(0.001)	[-0.013,0.001]	[0.013,0.024]
Δd	0.006	0.117	0.020	0.032	0.005	0.232	-0.020	0.182
	(0.005)	(0.011)	[0.014,0.024]	[0.012,0.057]	(0.021)	(0.032)	[-0.082,0.050]	[0.118,0.230]
$\log(P/D)$	3.598	0.365	3.694	0.130	2.904	0.128	2.569	0.260
	(0.041)	(0.025)	[3.660,3.711]	[0.066,0.201]	(0.031)	(0.014)	[2.499,2.784]	[0.161,0.409]
r_f	0.003	0.005	0.008	0.003	-0.001	0.009	-0.037	0.005
	(0.0005)	(0.0006)	[0.007,0.009]	[0.001,0.004]	(0.002)	(0.001)	[-0.039,-0.032]	[0.003,0.008]
r_m	0.013	0.086	0.035	0.119	0.041	0.065	0.085	0.304
	(0.006)	(0.005)	[0.019,0.044]	[0.057,0.189]	(0.011)	(0.006)	[0.018,0.221]	[0.228,0.462]

Panel A reports the sample mean and volatility (asymptotic standard errors in parentheses) of consumption and dividend growth rates, the log price-dividend ratio, risk free rate, and market return in the first regime. It also reports the model-implied values, obtained from a single simulation of 1 million observations, and the 95% confidence interval (in square brackets), obtained through 10000 simulations of the same length as the historical data, of these moments. Panel B reports the corresponding moments in the second regime. Regime 1 is defined as corresponding to those data points at which $p_t > 0.5$ while Regime 2 corresponds to those data points at which $p_t < 0.5$.

Table 9: **Forecasting with log$(P/D)_t$, Quarterly Data 1947:Q1-2013:Q4**

	Data		*Model*	
	Panel A: 1-year			
	Coefficient	R^2 (%)	Coefficient	R^2 (%)
$r_{m,t,t+1}$	−0.026 (0.012)	1.79	−0.046	1.54
$\Delta d_{t,t+1}$	0.003 (0.020)	0.00	0.030	2.46
$\Delta c_{t,t+1}$	0.0004 (0.0007)	0.10	0.007	10.9
	Panel B: 5-year			
	Coefficient	R^2 (%)	Coefficient	R^2 (%)
$r_{m,t,t+5}$	−0.138 (0.027)	9.24	−0.335	12.34
$\Delta d_{t,t+5}$	0.005 (0.023)	0.02	0.134	9.71
$\Delta c_{t,t+5}$	−0.003 (0.002)	0.58	0.032	29.64
	Panel C: 10-year			
	Coefficient	R^2 (%)	Coefficient	R^2 (%)
$r_{m,t,t+10}$	−0.254 (0.034)	17.6	−0.595	21.51
$\Delta d_{t,t+10}$	0.0002 (0.026)	0.00	0.212	10.89
$\Delta c_{t,t+10}$	−0.010 (0.004)	2.72	0.051	27.28

The table reports the slope coefficient and the R^2 from forecasting regressions of the market return, the aggregate consumption and dividend growth rates on the price dividend ratio at the 1-year (Panel A), 5-year (Panel B), and 10-year (Panel C) frequencies, in the model and in the historical data.

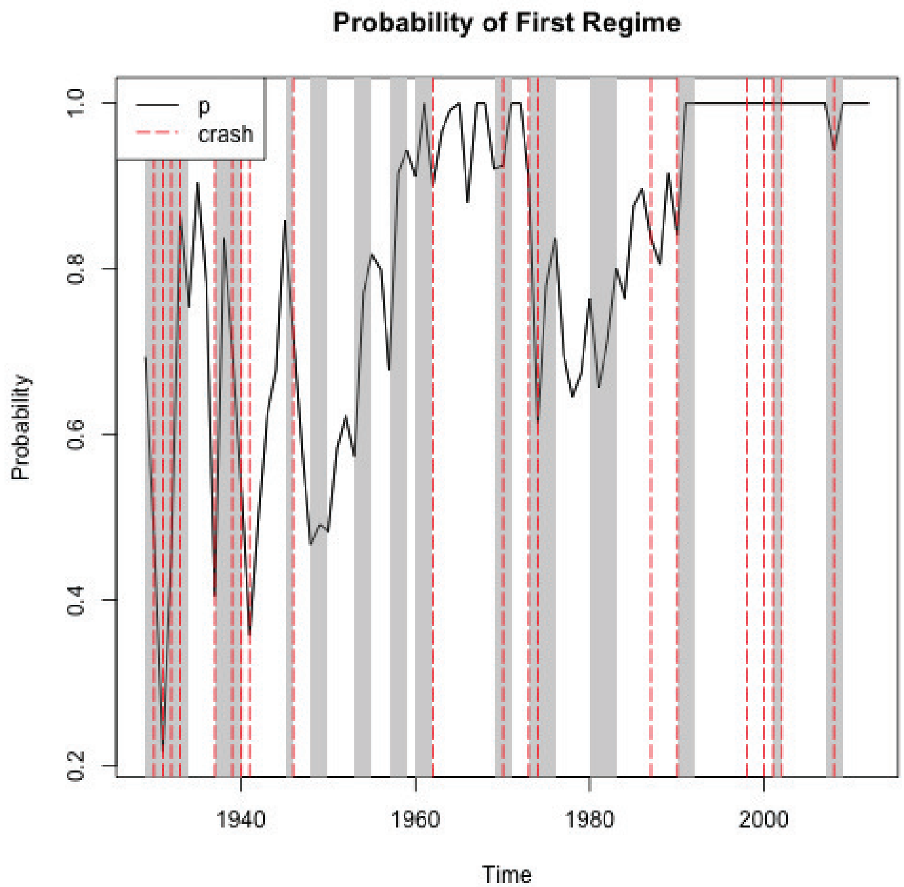

Figure 1: The figure plots the time series of the probability of being in the first regime over 1930-2013. The shaded areas denote NBER-designated recessions and the dashed vertical lines denote the major stock market crashes identified in Mishkin and White (2001).

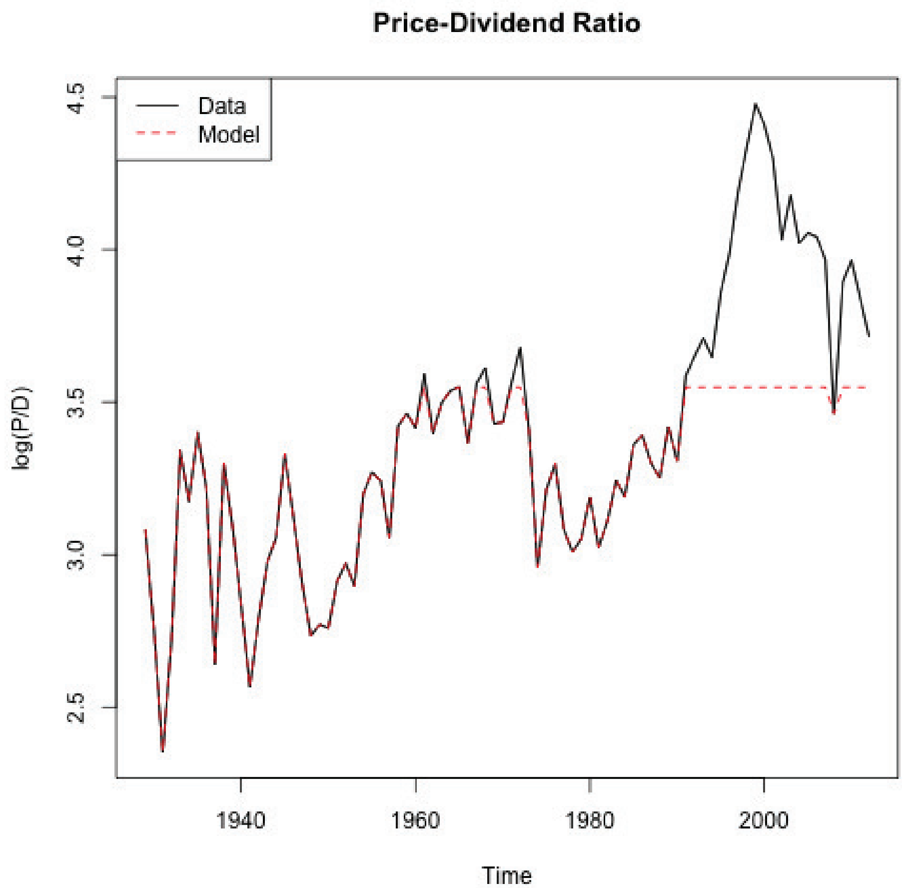

Figure 2: The figure plots the time series of the market-wide log price-dividend ratio in the historical data as well as that implied by the model over 1930-2013.

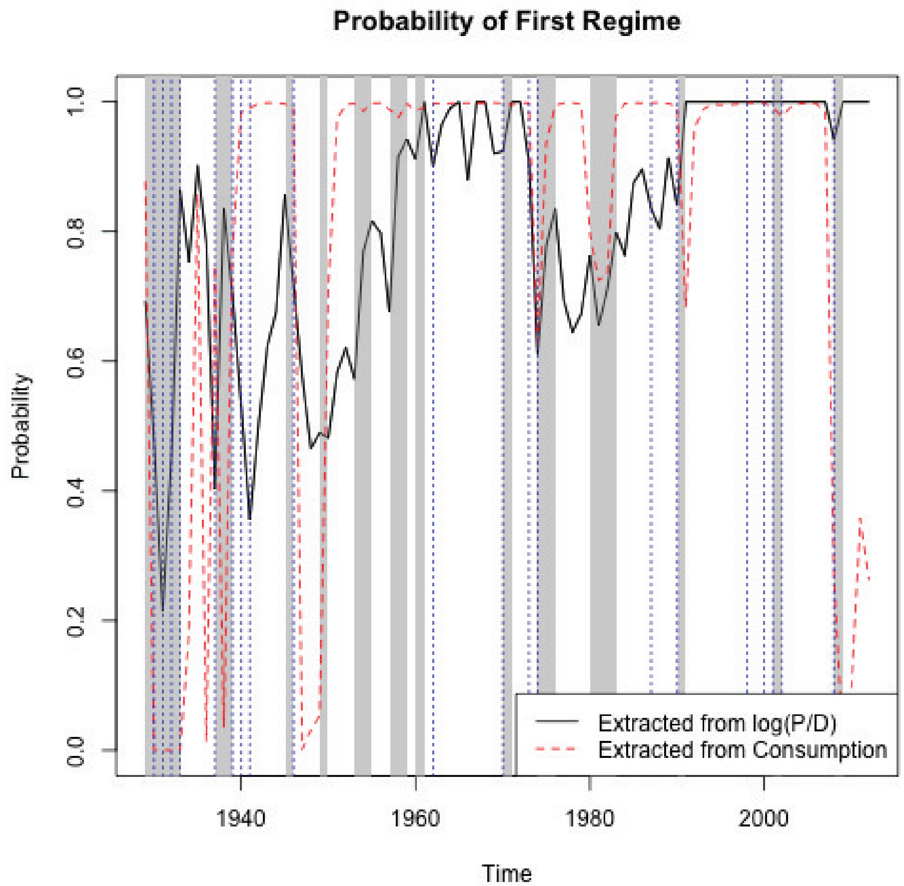

Figure 3: The figure plots the time series of the probability of being in the first regime over 1930-2013 extracted from the market-wide price-dividend ratio (black solid line) and the history of consumption (red dashed line). The shaded areas denote NBER-designated recessions and the dashed vertical lines denote the major stock market crashes identified in Mishkin and White (2001).

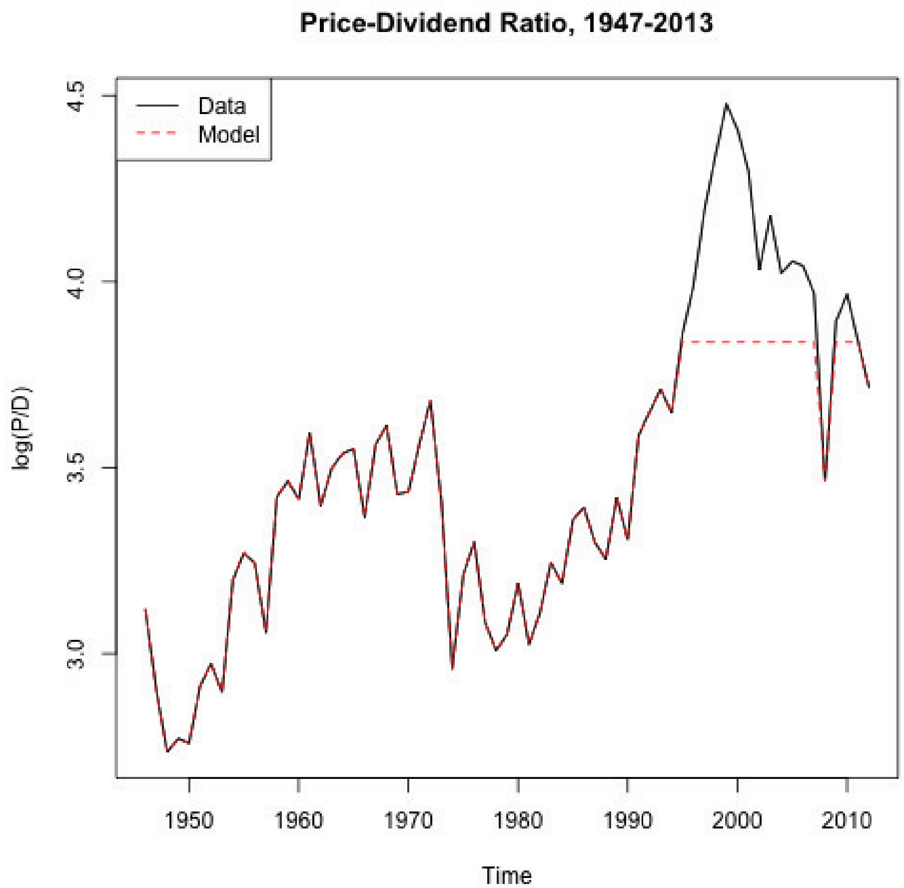

Figure 4: The figure plots the time series of the market-wide log price-dividend ratio in the historical data as well as that implied by the model over the post war subperiod 1947-2013.