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WAGES, EMPLOYMENT, TRAINING AND  
JOB ATTACHMENT IN LOW WAGE  
LABOR MARKETS FOR WOMEN

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in Low Wage Labor Markets for Women

ABSTRACT

This paper analyzes economic behavior and the effects of training and income support policies in the low wage labor market for women. The opportunity set takes account of nonlinearities and discontinuities associated with career interruption, part-time work, and government programs. There are two sectors, one which rewards training and individual ability, the other which does not and offers only the minimum wage. Effects of policies are found to vary importantly among heterogeneous groups of women according to ability and taste for children and household work. Some preliminary empirical evidence is presented to narrow the choice of specification.

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## I. Introduction

This paper analyzes economic behavior in the low wage labor market for women, and derives implications for training and transfer policies. On the demand side, the opportunity set is based on a two sector model which incorporates the effects of training, career interruption and part-time work on the path of wage offers over the life cycle. On the supply side, women with different abilities and preferences for children and home time sort themselves among available opportunities. The incentive effects of policies such as training, transfer and workfare programs are derived. Implications of the very different effects of policies on women with different abilities and tastes, and the implications of the findings for the design of policy evaluations are discussed. Preliminary empirical evidence is presented to narrow the choice between alternative specifications of the model.

A number of economic relationships have been identified by previous investigators as importantly influencing the life cycle pattern of labor market and household outcomes. Women with different ability levels and different preferences for children and home time (market work) will be making decisions at different margins (Burtless and Hausman, 1978; Heckman, 1974 a and b; Heckman and Willis, 1977; and Moffitt, 1984). One strand in the relevant literature has focused on the linkage between the wage offer and career interruption (Polachek, 1975; Mincer and Polachek, 1974; Weiss and Gronau, 1981; Sandell and Shapiro, 1978 and 1980; Corcoran, 1979; Mincer and Ofek, 1982; Corcoran and Duncan, 1983). Another aspect of the budget equation which has received attention is the the nonlinear and discontinuous relation of the wage offer to hours of work (Rosen, 1976), a relationship that among other things may reflect a

reduction in intensity per hour of market work once women have children and increased household responsibilities (Becker, 1985). Special attention has also been given to the labor supply-fertility relation, especially in reconciling findings from reduced form and structural specifications of life cycle models (Rosensweig and Wolpin, 1980; Lehrer and Nerlove, 1981; Carliner, Robinson and Tomes, 1984).

The model developed in the present paper incorporates those features from previous studies which are relevant to an analysis of policy in the low wage labor market for women. In addition, the opportunity set is expanded to include two separate sectors (as in Dickens and Lang, 1985). The features of the market generate interactions among training opportunities, ability and the minimum wage, and suggest the importance of taking proper account of heterogeneity.

To be more specific about the opportunity set, the model of demand and supply for low wage women specifies two types of full-time jobs. Those in the primary sector offer wages which reflect individual specific differences in productivity. The wage offers in the primary sector also reflect the costs and benefits of general training and any shared costs and benefits of specific training. Jobs in the secondary sector pay all who hold them at or close to the minimum wage and thus do not reward ability or training to any significant degree. Still further complications are assumed to arise for those subject to an effective minimum wage, which for some interferes with on-the-job training, resulting in opportunity sets that differ among women of different abilities not only in degree, but in kind. Jobs in the secondary sector are assumed to be available to all who want them. Thus the model abstracts from the problem of unemployment. However, there is limited access to primary sector jobs, and training subsidies are assumed to be effective in increasing access for workers of

marginal ability. Part-time job opportunities are also considered, and the issue of whether or not wages in such jobs are related to ability is seen to play an important role in the nature of the model which emerges.

A number of insights into the effects of labor market policies emerge from the analysis. Once the relationships of ability and preferences to the choice of the dominant segment of the budget constraint is determined, it becomes possible to analyze how and why a given policy change will affect women in accordance with their abilities and preferences. The model suggests, for example, the possibility that for women with a certain range of abilities and preferences, training programs and policies will work exactly as intended, with training leading these women to return to full-time work earlier than they otherwise would have, and at an increase in earnings. For others, however, training programs which were perceived by the women as working may create an income effect which induces them to prolong the period out of the labor force. Other women who, in the absence of an effective training program, would work when they had children, might instead be induced to drop out of the labor force or reduce hours of work when they had children. Moreover, some of those training programs, if conditioned on parenthood, could even encourage some women to have children. The model also makes clear why it is important to begin policy analysis for the low wage market for women with a behavioral model that is specified in detail. Consider, for example, the persistent finding of evaluation studies of labor market training programs that women receive much higher returns than men, and that much of these additional returns are associated with increased time at work (E.g., see Bloom and McLaughlin, 1982, pp. 20-23; and Bassi et al, 1984, pp. 83-84.) Consistent with the expectations of careful students of training programs, the model readily

indicates that for some but not other groups of women, there is considerable danger of confounding movements along a wage-hours or wage participation locus with shifts in the locus. This analysis explores how these effects will vary among those with different ability and preference combinations, and if fully implemented empirically, would allow separation of true from apparent effects.

In addition to the theoretical discussion, some suggestive empirical results are presented. The frequency and explanations for alternative life cycle patterns, e.g., involving no career interruption, career interruption with no part-time work, or with part-time work are considered and related to measures of ability and ex ante measures of preference for homework and children. The empirical findings help to answer certain questions pertaining to the role of opportunities for part-time work.

The organization of the paper is as follows. The next section discusses the specification of the opportunity set and the utility function for a model of female labor supply and fertility decisions. The following section characterizes the solution to the basic model. Section IV considers how individuals in such a model would react to training subsidies, to changes in the guarantee or benefit reduction rate of a transfer program and to workfare under the assumption of rationing of low wage jobs. Implications for current evaluations of training policies are also noted. The following section discusses various possible extensions to the model. Section VI presents the empirical results. A final section contains further observations about the model and a brief conclusion.

## II. Elements of the Basic Model

The model divides the  $T$  potential working years of a woman into three periods, of durations  $T_1$ ,  $T_2$ , and  $T_3$  years, respectively. The second

period is considered to include the years when any children that the woman might have would be at home. The first period corresponds to the years before any childbearing, and the last period encompasses the years after the children have left. During each of these periods, the woman must choose the level of her labor force participation, and additionally in the second period she must choose whether or not to have children. These decisions are influenced by her earnings possibilities in each of the periods and by her relative preferences for income, vs. children and time in the household.<sup>1</sup>

#### Earnings Opportunities.

Table 1 details the value of net productivity from full-time work. A trained primary sector worker has a productivity denoted by  $\epsilon$ , which reflects the individual's ability and motivation. A primary sector worker with no previous training must undergo training for  $T_t$  years, during which time her productivity is only the fraction  $1 - \tau$  of her post-training productivity. If the worker has been trained previously in a primary sector job, she still must undergo the training for  $T_t$  years, but her productivity is instead the fraction  $1 - \gamma\tau$  of her post-training productivity. In this expression,  $\gamma$  represents the fraction of training that is specific and must be repeated after an interruption of primary sector work. Thus in this model it is not depreciation and restoration of human capital that accounts for reductions in the wage offer after interruption, but only loss of specific human capital.

Previously trained primary sector workers may also work part-time at a wage which depends on their ability and motivation. Denote this part-time wage by  $w_p(\epsilon)$ . Various assumptions may be made about the nature of the relationship between  $w_p$  and  $\epsilon$ . At one extreme, it may be assumed that

$w_p(\epsilon)$  is a constant function whose value is independent of  $\epsilon$ . This would correspond to a situation where part-time work is available only in jobs (perhaps in the secondary sector) where ability is not of real importance. At the other extreme, it may be assumed that  $w_p(\epsilon) = \epsilon$ . In this case, a trained primary sector worker may cut back her hours without incurring any wage penalty. As will be shown shortly, the general nature of the model is somewhat sensitive to the particular assumptions which are made concerning the relationship between part-time wages and ability.

For the secondary sector, all individuals have the same value of productivity,  $w_s$ , a value that is at or slightly above the minimum wage, and any woman who wants work in the secondary sector can get this wage if she works full-time.<sup>2</sup> Wages for part-time work in the secondary sector are given by  $w_p^s$ , which may be taken to be equal to  $w_s$  or may be taken to be somewhat lower.

Not every woman will have enough ability to earn as much in the primary sector as she can in the secondary sector. Furthermore, even among those who could earn more in the primary sector, not all of them will be able to work there because the minimum wage may interfere with the training required for employment in that sector. At first glance, it might appear that firms would not be willing to train any woman whose productivity  $\epsilon(1 - r)$  during her training period falls below the minimum wage  $w_m$ , since if they did so they would have to be paying her a wage above her productivity during the training period. However, firms may be willing to engage in an implicit contract to finance some of the training costs and recoup the costs by paying wages below productivity for a period after the training period. To see this, note that the total productivity of a previously untrained woman over a time period  $T_m$  longer than the training period is given by<sup>3</sup>

$$T_t \varepsilon (1 - \tau) + (T_m - T_t) \varepsilon$$

The first term represents the productivity during training and the second the productivity in the post-training period. The employer knows that in order to retain the individual once she has been trained, he must pay her at least as much in the post training period as she could earn by going to another firm. The amount that an individual could earn at another firm, after having been trained at the first firm, is given by

$$T_t \varepsilon (1 - Y\tau) + (T_m - 2 T_t) \varepsilon$$

Note in this expression that the first term includes only specific and not general training costs, since general training will have already been provided by the first employer. The difference between these two expressions, the value of productivity while in training for the current employer plus the difference between productivity at the current firm once training is completed and net productivity elsewhere, is the maximum amount that an employer would be willing to pay to a woman in training. Dividing the result by  $T_t$  gives the following expression as the wage rate that the employer is willing to pay:<sup>4</sup>

$$\varepsilon [1 - \tau(1 - Y)]$$

This, then, is the quantity which is required to exceed the minimum wage for an employer to be willing to offer a woman training in the primary sector. Let  $\varepsilon_0$  be the value of  $\varepsilon$  which just equates this expression to the minimum wage.  $\varepsilon_0$  thus represents minimum ability level required for training in the primary sector in the absence of any government programs.<sup>5</sup>

As a final consideration regarding earnings opportunities, the model assumes that there are fixed costs  $C$  per time period if the woman engages in either part-time or full-time work. This reflects the costs of getting to and from work and additionally, for women with children, the costs of arranging for child care.  $C$  thus represents the costs that are incurred regardless of the length of the period that is worked. High fixed costs are expected to make part-time work less attractive relative to full-time work, since with part-time work there are fewer hours over which to spread the costs.

### The Utility Function

The utility function summarizing preferences may be expressed as  $U[y, h(t), c; \theta]$ , where  $y$  is total lifetime income,  $h(t)$  is the time path of home time in the second period,  $c$  is a binary variable with a value of unity if the woman has children in the second period, and  $\theta$  is an individual effect indicating relative preferences for children and home time.<sup>6</sup> Individuals with a high value of  $\theta$  place a high value both on having children and on home time spent with them, with the opposite being true for individuals with a low value of  $\theta$ .

To provide a basis for a tractable model, we suppose that this utility function is separable in income:

$$U = u(y) + c \int_{T_1}^{T_1+T_2} \phi(t) v[h(t); \theta] dt$$

The function  $u$ , which describes the utility of income, is taken to be such that the elasticity of marginal utility of income is greater than zero but does not exceed one.<sup>7</sup> The function  $v$  describes how the utility of a woman who has children and home time  $h(t)$  compares to utility when there are no children. For simplicity of exposition, it is assumed that the value of home time is less than the minimum wage, except for women with

children at home. Therefore, all women in the model will work in the first and third periods, with the only question in those periods being the choice of sectors, and women without children will work full time in the second period. The function  $\phi$ , which is assumed to be monotonically declining, allows the value of home time to decline throughout the second period as any children become older.

The function  $v$  is illustrated in Figure 1. In this figure,  $l_f$ ,  $l_p$ , and  $l_n$  refer to the amounts of home time associated with full-time market work, part-time market work, and no market work at all, respectively. For convenience, the actual argument of  $v$  is the amount of working time, defined as  $h_i = l_n - l_i$ . The reference utility level for each woman is point A, representing utility with no children and working full-time. A woman with a high desire for children will obtain a greater utility with children than without even if she has to work full-time, as indicated by the fact that point B lies above point A. This same individual would obtain more utility if she could be home part-time with her children, as at point C, and even more utility if she could be home full-time, as at point D. A woman with a moderate desire for children, in contrast, might find it preferable not to have children if she were to work full-time, as indicated by the fact that point E lies below point A, but would prefer to have children if she were to work only part-time or not at all, and thus enjoy utility from children and home time as indicated either by point F or point G. Finally, a woman who has little desire at all for children might be characterized by HIJ, wherein utility actually rises when she is working and is away from children (but note that the utility of this individual never is as high with children as can be obtained without children).

The function  $v$  is characterized by the relations:

$$(d/d\theta) [v(h_p; \theta) - v(h_f; \theta)] > 0$$

$$(d/d\theta) [v(h_n; \theta) - v(h_p; \theta)] > 0$$

These relations suggest that the greater the desire for children, the more valuable additional home time will be.

### III. The Base Solution to the Model

The base solution to the model relates the work and fertility decisions of an individual to her ability, as reflected in the parameter  $\epsilon$ , and her preferences, as reflected in the parameter  $\theta$ . More specifically, the woman must decide in the second period whether to have children and if so, what parts of the period she wishes to work full-time, part-time, or not at all. It will be assumed that  $\phi$ , which is monotonic, is large enough relative to the difference between the real wage and real interest rate to insure bunching of work at the beginning of the second period.<sup>8</sup>

In this circumstance, the work decisions during the second period can be characterized by two numbers:  $t_p$ , the amount of time that passes in the second period before the return to the labor force, and  $t_f$ , the amount of time before the return to full-time work. If  $t_p = t_f$ , then there is no part-time work; otherwise  $t_f - t_p$  represents the amount of time spent in part-time work. The decisions regarding both  $t_p$  and  $t_f$ , and also the decision regarding children, are functions of  $\epsilon$  and  $\theta$ . Perhaps the easiest way to characterize the solution is to look at the choices made by women with different combinations of  $\epsilon$  and  $\theta$  at some particular moment in time, as illustrated in Figure 2. The two panels in this figure correspond to the two extreme assumptions regarding part-time

wages which were mentioned before. Panel (a) represents the situation where  $w_p$  is a constant independent of  $\epsilon$ , and panel (b) represents the situation where  $w_p(\epsilon) = \epsilon$  and  $w_p^S = w_s$ , so that part-time wages are equal to full-time wages. There are corresponding figures for every moment in time during the second period, and it is of interest to investigate how these figures change as the women move through the second period. First, though, let us discuss how the different areas in Figure 2 can be derived from the model.

Suppose that Figure 2 corresponds to an instant of time  $t_0$  after the second period has begun. In the left-hand panel, the curve JL represents combinations of  $\epsilon$  and  $\theta$  for which, at the specified moment in time, the women will have chosen to have children and will be just on the borderline between being out of the labor force and working full-time. Note that JL is to the right of  $\epsilon_0$ , so that all full-time work by these women will be in the primary sector. For women along this particular borderline, the possibility of part-time work is irrelevant, and they are solving the problem of maximizing

$$u(y) + c \left[ \int_{T_1}^{T_1+t_f} \phi(t) v(h_n; \theta) dt + \int_{T_1+t_f}^{T_1+T_2} \phi(t) v(h_f; \theta) dt \right]$$

subject to

$$y = \epsilon h_f (T - t_f) - \epsilon h_f [(1 + Y) \tau T_t] - (T - t_f) C$$

where  $t_f$  (which is equal to  $t_p$  in this case) is the time within the second period that the woman shifts from being out of the labor force to working full-time and  $h_f$  is the number of hours in a full-time work period. The middle term in the definition of  $y$  reflects the fact that the fraction  $Y$  of the training must be done again when the woman reenters

the labor force, and the latter term reflects the fixed costs of working.

The marginal condition which emerges from this problem is given by

$$(1) \quad -u'(y) (\varepsilon h_f - C) + \phi(T_1 + t_f) [v(h_n; \theta) - v(h_f; \theta)] = 0$$

Differentiating this condition with respect to  $\varepsilon$  and  $\theta$ , respectively, yields

$$\frac{\partial t_f}{\partial \varepsilon} = \frac{h_f u'(y) + (\varepsilon h_f - C) u''(y) s}{\phi'(T_1 + t_f) [v(h_n; \theta) - v(h_f; \theta)] + (h_f \varepsilon - C)^2 u''(y)} < 0$$

$$\frac{\partial t_f}{\partial \theta} = \frac{-\phi'(T_1 + t_f) (\partial/\partial \theta) [v(h_n; \theta)/\partial \theta - \partial v(h_f; \theta)]}{\phi'(T_1 + t_f) [v(h_n; \theta) - v(h_f; \theta)] + (h_f \varepsilon - C)^2 u''(y)} > 0$$

where  $s = \partial y/\partial \varepsilon$ . The sign of  $\partial t_f/\partial \varepsilon$  is strictly required only when  $C$  is small, so that  $(\varepsilon h_f - C)s \approx y$ .<sup>9</sup> These signs imply that along this margin, returns to full-time work begin earlier the greater the level of ability (and hence wage) of the individual and begin later the greater the desire of the woman for children. Both are results which would be expected. In order to derive the slope of JL in the diagram, however, what we want is  $d\theta/d\varepsilon$ , holding  $t_f$  constant at  $t_0$ . Since  $t_f$  is a function of both  $\varepsilon$  and  $\theta$ , the derivative of interest can be established by the implicit function theorem as

$$(d\theta/d\varepsilon)|_{t_f=t_0} = -\frac{\partial t_f/\partial \varepsilon}{\partial t_f/\partial \theta} > 0$$

so that the slope of JL is upward sloping.

The curves IJ and JK in the left panel and IJ and KL in the right panel represent combinations of  $\varepsilon$  and  $\theta$  for which women with children

will be on the borderlines between working full-time or part-time and between working part-time or not at all, respectively, at some time  $t_0$ . Let  $t_p$  and  $t_f$  be defined as above. The algebra in this case is considerably more complicated than in the previous example and is left to the appendix, but there it is shown that  $\partial t_p / \partial \varepsilon$  is positive for panel (a) and negative for the panel (b), and  $\partial t_f / \partial \varepsilon$  is negative for both cases.<sup>10</sup> This implies that a woman with higher ability will reenter the labor force earlier or later, depending on whether or not the part-time wage depends on ability, than will a woman with lower ability, holding constant the desire for children. In either case the higher ability woman will start full-time work again sooner. With regard to the derivatives  $\partial t_p / \partial \theta$  and  $\partial t_f / \partial \theta$ , it can be shown only that at least one of them is positive (The conditions for both of them to be positive are derived in the appendix). However, these are the derivatives of the dates of reentering the labor force and of resuming full-time work with respect to the desires for children, and we would expect that in the normal case both of these derivatives would be positive. Hence, in the remainder of the analysis (except where noted to the contrary) we will assume that the conditions are in fact met for these derivatives to be positive and refer to such preferences as "normal," remembering that cases are theoretically possible for one (but not both) of them to be negative.

Given the signs of these derivatives, the slopes of IJ, JK, and KL are established in much the same manner as that of JL. Again, both  $t_p$  and  $t_f$  are functions of  $\varepsilon$  and  $\theta$ , so that the implicit function theorem yields

$$\left. \frac{d\theta}{d\varepsilon} \right|_{t_n=t_0} = - \frac{\partial t_n / \partial \varepsilon}{\partial t_n / \partial \theta} \begin{matrix} < 0 & \text{for Panel (a)} \\ > 0 & \text{for Panel (b)} \end{matrix}$$

$$(d\theta/d\varepsilon)|_{t_p=t_0} = - \frac{\partial t_p / \partial \varepsilon}{\partial t_p / \partial \theta} > 0$$

Thus for panel (a), and in the case of normal preferences toward children, IJ is upward sloping and JK is downward sloping. Where they meet, at point J, defines a point where the woman is indifferent at time  $t_0$  between working full-time, part-time, or not at all. For points beyond J, the choice is between working full-time or not at all, as defined along the curve JL. For panel (b), both IJ and KL are upward sloping. Which has the steeper slope is theoretically indeterminate, and the panel is drawn for the situation where KL is steeper.

For the area to the left of  $\varepsilon_0$ , the analysis is much the same, except that neither the full-time wage nor the part-time wage depends on the ability level  $\varepsilon$ . For that reason the lines EF and GH, which indicate the boundaries between full-time work and part-time work and between part-time work and being out of the labor force, respectively, are horizontal. The line CD, which represents the boundary between women who have children and those who do not, is also horizontal, primarily as a result of the fact that the level of  $\theta$  for which the woman is indifferent between full-time work with and without children in Figure 1 does not depend on  $\varepsilon$ .<sup>11</sup>

The exact positions of these boundaries are sensitive to, among other things, the fixed costs of employment. In the appendix, it is shown that  $\partial t_p / \partial C$  is positive and  $\partial t_f / \partial C$  is negative. Thus, women with higher fixed costs of employment will begin part-time work later and full-time work sooner than will otherwise identical women with lower fixed costs. This is to be expected, since the higher fixed costs have a higher proportional impact on the returns to part-time work than on the returns to full-time work. The resulting effect of higher fixed costs is shown in

Figure 3. In this figure, higher fixed costs shift the boundaries from the dashed lines to the solid lines. The figure indicates that the higher fixed costs shrinks the areas of part-time employment in both panels. More formally, the horizontal direction of the shift of a segment such as IJ in the panel (a) due to a higher value of  $C$ , holding  $\theta$  and  $t_f$  constant, is given by

$$(d\epsilon/dC)|_{\theta, t_f} = - \frac{\partial t_f / \partial C}{\partial t_f / \partial \epsilon} < 0$$

Hence, this segment shifts leftward with an increase in  $C$ . The formal derivations for the shifts of the other segments are similar.

In combination, these boundaries serve to separate the women at time  $t_0$  into four groups: those with children who are working full-time, part-time, and not at all, and those without children who are working full-time. Over time during the second period, the boundaries demarcating the area of full-time work from the areas of either part-time work or nonparticipation, and the boundary separating the areas of part-time work and nonparticipation, must be moving uniformly downward. The only exception occurs at the beginning of the second period, where the boundaries involving full-time work in the primary sector [along IJL in Panel (a) of Figure 2 or along IJ in Panel (b) of the figure] will remain stationary for a while. This occurs because the retraining costs for dropping out of and then reentering the primary sector will imply a minimum length for any periods of nonparticipation and/or part-time work at the beginning of the second period, with the result that the corresponding boundaries do not move until this minimum length of time has passed.

#### IV. Analysis of the Effects of Policy Changes

This section examines the effects upon fertility and labor force participation decisions of: a subsidy for the training of mothers with children, an increase in the income guarantee available to low-income mothers with children, and an increase in the marginal tax rate on earnings of individuals who are receiving benefit payments. In addition, reductions in the guarantee and tax rate, or more directly a return to the market situation analyzed earlier where there is no transfer program, may be taken as an indication of the effects of a simple workfare program which replaces the transfer for the full term of the life cycle. Note, however, that because minimum wage jobs are available to all who want them, the "workfare" is provided by low wage firms in the private sector. For each of these policy changes, the effects are to cause some individuals near particular margins to change their behavior, which in Figure 2 amounts to shifting some of the boundaries separating the regions at a particular point in time. If the policy changes are restricted to some subset of the general population, then the analysis of this section will apply only to the potentially eligible subpopulation. In particular, most of the programs of the type under consideration apply only to women who are heads of households. Although the model does consider the decision to have children as endogenous, marital status is not considered and hence is effectively taken to be exogenous in determining who is eligible for a program and who is not.

First consider the introduction of a training subsidy to be made available to mothers with children. It is assumed that such a subsidy is not available in the first period, before the fertility decision is made, but is available in both the second and third periods to individuals who elect to have children in the second period. The effects of the training

subsidy are illustrated in Figure 4. In this diagram, the dashed curves represent the situation before the subsidy is introduced and are copied from Figure 2. The solid lines represent the situation after the subsidy and hence illustrate how behavior reacts in response to the subsidy.

Along J'L' in panel (a), the only change from the maximization problem analyzed at the beginning of the last section is to include the subsidy amount  $S_t$  in the equation defining  $y$ :

$$y = \varepsilon h_f [T - t_f] - \varepsilon h_f [(1 + \gamma) \tau T_t] - (T - t_f) C + S_t$$

Differentiating the marginal condition given in equation (1) of the last section with respect to  $S_t$  then gives

$$\frac{\partial t_f}{\partial S_t} = \frac{u''(y) (h_f \varepsilon - C)}{\phi'(T_1 + t_f) [v(h_3; \theta) - v(h_1; \theta)] + (h_f \varepsilon - C)^2 u''(y)} > 0$$

The horizontal movement of J'L' can be calculated as the change in  $\varepsilon$  necessary to maintain the equilibrium relation in response to a change in  $S_t$ , holding  $\theta$  and  $t_f$  constant:

$$\left(\frac{d\varepsilon}{dS_t}\right)_{\theta, t_f} = - \frac{\partial t_f / \partial S_t}{\partial t_f / \partial \varepsilon} > 0$$

Thus, in response to an increase in  $S_t$ , J'L' will shift to the right.

The same kind of exercise based on the analysis of the appendix also establishes that in panel (a) I'J' will shift to the right and J'K' will shift to the left, while in panel (b) I'J' and K'L' will both shift to the right. Thus, among individuals with ability levels above  $\varepsilon_0$ , a somewhat perverse result emerges. Individuals will tend to stay out of the labor force longer than without a subsidy, and they will return to full-time work

later. This result stems primarily from the fact that among this group the training subsidy induces only an income effect, since these individuals would have been retrained anyway when they returned to full-time work. It may be noted that this result cannot be avoided by restricting the training subsidies to individuals who have been earning no more than the minimum wage immediately prior to the training, since even these higher ability individuals will have been out of the labor force or engaged in part-time work in the time span before they wish to be retrained for their return to full-time work in the primary sector.

For some individuals with an ability level just below  $\varepsilon_0$ , the subsidy may enable them to overcome the minimum wage obstacle and be trained for work in the primary sector when they return to full-time work after having their children. Whether or not this happens depends upon whether an individual would be eligible for another subsidy if she were to be trained by a second employer. If the subsidy were available for both employers, then the equations in the first part of Section II which describe the total value of the individual to each employer would have to be augmented by  $S_t$ . Since the amount the first employer is willing to pay during the training period is related to the difference between these two amounts, in this case the first employer would not be willing to pay the individual any more in the training period, and the minimum skill level for training in the primary sector would still be  $\varepsilon_0$ .<sup>12</sup> If, on the other hand, the subsidy were available only to the first employer, then the minimum skill level for training in the primary sector would have to satisfy:

$$\varepsilon [1 - \tau(1 - \gamma)] + (S_t / T_t) = W_m$$

Figure 4 is drawn for this second case, with the minimum skill level

for a woman with children to be trained denoted by  $\epsilon_s$ . However, although employers are willing to train all women with skill levels between  $\epsilon_s$  and  $\epsilon_0$ , only women with a sufficiently strong taste for children, that is above M'N' in the diagram will in fact obtain training. Below that line, the disutility of working full-time while raising children implies that the additional income available as a result of being trained is insufficient to compensate for the fact that the receipt of the training subsidy is conditional on having children. The analysis thus implies that there is a group of women with abilities between  $\epsilon_s$  and  $\epsilon_0$ , above M'N' and below CD, who will find it advantageous to have children they would not have otherwise had in order to qualify for the subsidy.

In summary, then, the effects of a training subsidy depend critically on the individual's ability level. For ability levels between  $\epsilon_s$  and  $\epsilon_0$ , the effects are as intended, with the trained women returning to full-time work earlier than they otherwise would and earning substantially more than they otherwise would have.<sup>13</sup> For ability levels above  $\epsilon_0$ , the effects of the training subsidy may well be perverse, while for ability levels below  $\epsilon_s$ , there is no effect because the minimum wage will still prevent these women from obtaining training.

There are implications of this discussion for econometric studies designed to evaluate training programs. In the analysis of training subsidies, the sharply different effects among different groups of women imply that attempts to evaluate the effects of training programs which do not carefully consider the differences and discontinuities in the wage offer with ability interacted with sector of employment, stage of the life cycle and full-time or part-time work, will produce numbers which are not measuring what is intended. If such studies are to isolate true program

impacts from the effects of voluntary choices with regard to sector, hours of work and career interruption, the training program effects will have to be modeled in the context of a structural model specified along the line of the model outlined above so that those in each group can be distinguished. Program effects can only be measured by comparing outcome differences between those in the same ability-preference group, and even then, the estimates should standardize for the effects of voluntary changes in labor supply. (For related discussions, see Heckman and Robb, 1985.)<sup>14</sup>

The other related policy changes to be considered are the effects of changes in transfer policy, and symmetrically, reductions of transfers associated with the introduction of workfare. An increase in the guarantee of an income transfer program and a change in the benefit reduction rate of such a program, are more straightforward to analyze, and the results of the analysis do not appear to contain any real surprises. These two changes are presumed to be made to a program which pays benefits to mothers who are not working or are working part-time at relatively low wages but which does not pay benefits to women who are working full-time or are working part-time at relatively high wages. In such a program, an increase in the guarantee amount, holding the benefit reduction rate constant, will increase the effective returns to part-time work and to nonparticipation by the same amount but will not affect the returns to full-time work. The effect of this increase in the guarantee amount is illustrated in Figure 5. In this figure and in the next, the dashed lines represent the boundaries before the change and the solid lines represent the boundaries after the change. In Panel (b), the kinks in the right part of the figure occur at the point where the part-time wage is high enough that a woman working part-time at that wage is no longer eligible to receive benefits from the program. As might be expected, the net result is to push the boundary

between nonparticipation and part-time work downward, since the increased guarantee does not generate any substitution effects between nonparticipation and part-time work, and the income effect favors remaining out of the labor force longer. The boundaries involving full-time work are also shifted down, both because of the income effect just mentioned and because the fact that full-time workers are not eligible for benefits means that an increase in the guarantee amount will generate substitution effects away from full-time work.

The result of an increase in the benefit reduction rate are illustrated in Figure 6. The net effect of such a change is to reduce the effective returns to part-time work at relatively lower wages. The returns to full-time work or part-time work at relatively higher wages are not affected, since there are no benefits to be reduced, and the returns to nonparticipation are likewise not affected, since there is no income upon which the benefit reduction rate will operate. The result is to generate substitution effects along all the boundaries involving part-time work at relatively lower wages, so that individuals will begin part-time work later and end part-time work in favor of full-time work sooner. The analyses of both an increase in the benefit reduction rate and of an increase in the guarantee illustrate the importance of the distinction emphasized by Levy (1981) and others between those whose earnings are beyond the breakeven and those whose earnings are low enough to leave them eligible for the program.

#### V. Extensions of the Model.

The model that has been analyzed in the previous sections appears to be amenable to several refinements which would increase its complexity without changing its basic nature. First, note that the first period could

be eliminated without major changes in the analysis. This would occur if the woman made the choice between having and not having children at the beginning of her potential work career. Technically, this would do two things to the model. One is to eliminate the  $1 + Y$  term from the budget constraint, since under these circumstances any work in the primary sector would not be broken up and as a result retraining costs would be irrelevant. The second would be to eliminate the short period discussed at the end of the last paragraph during which the boundaries would not move, since this period of nonmovement was motivated by the retraining costs. Overall, though, omitting the first period would not change the general nature of the model discussed in this section nor of the results to be discussed in following sections.

A second refinement would be to allow for productivity to be a function of tenure, at least in the primary sector. Such an allowance would have the effect of changing the exact expression of the marginal conditions defining the boundaries between the various areas but should not change the character of the previous analysis very much. One change that would be expected is that in such a setting the penalty for dropping out of the primary sector and then reentering would clearly be much larger than simply incurring the retraining costs already explicitly included in the model. As a result, the minimum length of a period of nonparticipation and/or part-time work at the beginning of the second period should be considerably longer than without a tenure effect, and the length of time during which  $IJL$  in panel (a) or  $IJ$  in panel (b) of Figure 2 would remain stationary would be correspondingly longer.

A third refinement would be to allow for more than one birth in the first part of the second period. For a fixed sequence of births, it seems

fairly clear that the analysis before the first birth and after the last birth would be largely unchanged, and that during the childbearing period a model may well predict alternating labor force states. For instance, a woman might drop out of the labor force for a year immediately succeeding the birth of each child and then work part-time until the birth of the next child. Considerably more complex would be attempts to model additional behavior within the actual childbearing period, including possibly attempts to model the joint decisions regarding how many children to have, the spacing between children, and the total length of the childbearing period.

Another refinement would consider the value of home time during periods when children are not at home. For a woman who chooses not to have children, for whom the value of home time is likely to be changing relatively smoothly over time, a traditional life-cycle analysis of labor force decisions should be applicable. For a woman who does choose to have children, it is clear that the utility function used in the above analysis could be extended to cover the first and third periods, probably without severely affecting the analysis. In particular, this refinement would only affect those women with children who are still at home or working part-time at the end of the second period, since any woman working full-time toward the end of the second period will presumably find it advantageous to work full-time during the first and third periods, when the value of home time is presumably less. A complete decision would then entail calculating the optimal labor force behavior conditional both on having and not having children, and making the choice regarding children so as to pick whichever of the two paths yields a higher overall utility.

Finally, the model could introduce education as an alternative means to acquire the necessary general training for primary sector work. A relatively simple but informative case would allow both the intensity and

duration of required general training to be inversely related to the amount of education an individual has acquired. The monetary cost of the education would be reflected in the budget constraint, and the time costs would be reflected in a reduced amount of time available in the first period. Such a modification would not affect the signs of the slopes calculated for the various segments in Figures 2 and 3 or the directions of movement in Figures 4 through 6. However, the modification does have implications for the effectiveness of a training subsidy. In the original model, there is a discrete utility difference between individuals just below and just above the critical ability level required for primary sector employment, resulting in the implication that a marginal training subsidy may result in non-marginal utility improvements for some individuals. With endogenous education, the lower ability individuals can use education to get around minimum wage constraints which are restraining on-the-job general training in the primary sector. This process will be carried to the point where the individual with marginal ability is just indifferent between getting the education and working in the primary sector and remaining in the secondary sector. In this setting, marginal training subsidies will result only in marginal utility improvements for the individuals it induces into primary sector employment.

## VI. Empirical Analysis

In this section we will begin the job of exploring the empirical implications of the model. Because the model speaks to labor force patterns, training, wage offers, the role of ability and tastes for children, and the interrelations among these variables, there are a large number of empirical implications which are testable, and a number of parameters which may be estimated by using increasingly complex econometric

techniques. Our hope is to provide sufficient information to determine whether the outcomes highlighted by this approach are important, whether the general structure of the model seems reasonable, and to provide guidance for specification to be used in estimating a full structural version of the model.

The empirical evidence comes from the National Longitudinal Survey of Young Women, which has surveyed for fifteen years women who were 14 to 24 years old in 1968, the initial year of the survey. About two-thirds of the individuals remained in the survey in 1983, the last year for which the data have been made available. Thus for individuals who remained with the survey for the full fifteen years, the survey covers the age spans from 14-29 to 24-39. Particularly for women who were in their late teens when the survey began, this fifteen year span covers just the age range which is of particular interest in terms of evaluating this model. In evaluating the statistics presented in this section, however, a word of caution is in order. One might expect the general levels of participation to be even higher today than in the period covered by the survey. Ultimately the model should be able to account for these changes, but significant aspects of behavior remain exogenous to our analysis. Accordingly, simple extrapolation of the relations fitted here to future periods may be inappropriate.

In the results which follow, the sample is restricted in several ways. First, individuals are eliminated if they lack information on critical variables. This most frequently arises in the cases of the ability (IQ) variable and the taste variable. Regarding the ability variable, the NLS lacks information on this variable for about 35 percent of the sample. As for the taste variable, since this variable is constructed on the basis of

questions asked in the 1972 survey, it is missing for individuals who did not remain with the survey until at least this time.<sup>15</sup> The sample further omits individuals who had not reached the age of 30 by the time of the last survey which the particular individual completed. This is done to eliminate cases in which the observed pattern of work behavior is too short to impart meaningful information.

For each individual, the labor force behavior is examined in every survey year following the survey year in which the individual last reported full-time enrollment in school. In each of these surveys, the individual is classified as working full-time (ft), working part-time (pt), or not in the labor force (nlf). The resulting sequences of labor force behavior were then separated into four groups: (i) full-time work in all applicable survey years, (ii) either full-time work or part-time work in all applicable years, with at least some part-time work, (iii) sequences which include at least some part-time work and some years not in the labor force, and (iv) either full-time work or a not in the labor force status in all applicable years, with at least some years not in the labor force. These four groups are denoted in the tables by ft, ft/pt, ft/pt/nlf, and ft/nlf, respectively.

Table 2 presents regressions of each of these four groups on a set of IQ and taste variables, with standardization for birth year and race. In each regression, the dependent variable is a dummy variable taking on a value of unity if the individual had a labor force participation pattern of the indicated type. The IQ and taste variables are each separated into three categories so that roughly equal numbers of those with a high school education or less fall into each category. For the IQ variable, the low category is 1 and the high category is 3, whereas for the taste variable a value of 1 indicates a strong taste for children and home time and a value

of 3 indicates a weak taste. It should be kept in mind that IQ is admittedly an imperfect measure of the underlying theoretical construct of ability-related earnings power.

Table 3 is derived from the estimates of Table 2 and attempts to present the information in a more useful form. For each of the regressions in Table 2, the coefficients of each of the IQ-taste combinations is adjusted up or down so that the weighted sum of the coefficients is zero. Then the resulting coefficients are arranged in a grid corresponding to the various IQ-taste combinations, and corresponding also in a rough manner to the axes of Figure 2 of Section III. The actual entries in Table 3 indicate the amounts by which the average fraction of individuals in a particular sequence must be adjusted up or down for the particular cell. For example, the first entry indicated that for the low IQ and high taste for children and home life combination, there would be 11.6 percent fewer individuals with continuous full-time participation than there would be for the complete sample (again, correcting for birth year and race).

In comparing the entries of Table 3 with the two panels of Figure 2, it is evident that they correspond much better to panel (b) of the figure than to panel (a). To see this, consider the figure to be pictured for the time immediately after the second period begins. In this case, any individuals below the lines EFIJL in panel (a) and EFIJ in panel (b) should always be observed in the full-time status. Sequences involving only full-time and part-time work should originate from areas of the figure in which the individual elects part-time work at the beginning of the second period. Sequences involving some part-time work and some time not in the labor force should arise if the individual is in the area above GHKJ in panel (a) or above GHKL in panel (b). Finally, sequences involving some time not in

the labor force but no part-time work can arise from an individual initially above JL in panel (a). These sequences can also arise in panel (b) if the fixed costs of employment are sufficiently high so that the left-hand portions of EF and GH and of IJ and KL become coincident. In this regard, it may be more appropriate in an empirical model to regard the fixed costs of employment as a stochastic variable which varies from individual to individual. In a situation such as panel (b), lower ability individuals with high fixed costs who begin the second period not working would proceed directly to full-time work, while individuals with similar ability but lower fixed costs would go through a period of part-time work. For higher ability individuals, high fixed costs are less likely to induce a transition directly from not working to full-time work. The empirical implication is that in this panel, lower ability workers as a group should be less likely to have work patterns with at least some part-time work.

In looking at Table 3, it is evident that continuous full-time work is depressed in the upper left part of the table and enhanced in the lower right part of the table. This is exactly what one would expect from either panel of the figure, however, and although it is consistent with the model it does little to distinguish which panel in the figure is closer to the truth. The situation is much different when the last kind of sequence is examined, however. This is the sequence which contains some years not in the labor force but no part-time work. In the table, it is seen that this kind of behavior is considerably more common among individuals with lower IQ scores, contrary to what would be expected if panel (a) of the figure were correct. Given relatively high fixed costs of employment, however, this behavior is consistent with panel (b) of the figure. In this case, what is happening is that the high fixed costs of employment make part-time work unattractive for those with very low part-time wages but less

unattractive for those with higher ability and correspondingly higher part-time wages.

The results for the two kinds of sequences involving some part-time work are less consistent than the other results, but they too are broadly consistent with the model. There is no clear relationship between the part-time sequences and the IQ variable, but particularly in the last two columns of Table 3 the fraction of part-time sequences also involving some years not in the labor force increases as taste for home and family time increases and the fraction of part-time sequences in which the individual is always in the labor force declines. Again, this corresponds well to what might be expected on the basis of the theoretical model, since sequences involving some time not working should begin in the upper part of the diagram where tastes for home and family time are high.

There is another piece of evidence which also suggests that Panel (b) of Figure 2 may be the closer to the true model. For any woman who experienced both full-time and part-time work, and for whom valid wages could be calculated for both, the full-time wages are averaged into a single number, and the same is done for all part-time wage observations. Given a single full-time average wage and a single part-time average wage for each woman, the correlation between the two was calculated for the women in the sample. The simple correlation is 0.41 over 1586 individuals, and the implied regression coefficient of part-time on full-time wages is 0.70. These imply a fairly strong positive relationship between the full-time and part-time wages, which is somewhat more consistent with Panel (b) than Panel (a) in the figure.

## VII. Summary and Conclusions

This paper has presented a model which incorporates a number of

features from the low wage labor market for women. The model has been used to analyze alternative patterns of labor market participation and home time over the life cycle. The critical labor market choices are whether or not to interrupt the career, whether or not to work part-time, and when to return to full-time work, with all these choices conditioned on the individual's own ability, on the penalties for career interruption and for part-time work, and on the woman's preference for home time and children. Reasons for differences in wage offers associated with each regime were analyzed in the context of a two sector model of the labor market. In one sector there was training, which was a mix of specific and general, while the other offered jobs with wages at or just above the minimum wage. Ability differences were assumed to be associated with wage offer differences for full-time work in the primary sector, but not for full-time work in the secondary sector.

Among the policy measures which can be analyzed in the context of this model, the most interesting results are obtained for a training subsidy for mothers with children. Such a subsidy does permit some women with marginal ability levels to qualify for training programs whereas they would not be able to obtain training without the subsidies. For higher ability women, however, the result is perverse in the sense that it delays both the return to the labor force and the resumption of full-time work after the childbearing period. On the other hand, changes in the guarantee and/or marginal tax rates on benefits for low income mothers have the effects which might be more or less expected from casual observation. An increase in the guarantee will tend to reduce full-time work and increase both part-time work and nonparticipation, at least on the assumption that full-time workers earn sufficient income to be ineligible for the benefit. On the

same assumption, a reduction in the marginal tax rate will tend to increase part-time work at the expense of both full-time work and nonparticipation.

Perhaps the strongest implications of the analysis pertain to evaluations of training programs. These evaluations frequently attempt to isolate program impacts by comparing outcomes for program completers with those who are thought to be comparable due to their demographic characteristics and earnings histories. There are a couple of particularly treacherous problems for evaluation studies suggested by the model. First, it is important not to confuse changes in wage rates which are brought about by training programs with changes which are due to changes in labor supply. This is particularly likely to be a problem for an individual who is working part-time before entering into the program. Secondly, the analysis shows that a training program affects two quite different groups. There is a fairly small group whose members would not obtain training at all without the program, and for these individuals the effect of training is a good measure of the effect of the program. However, there is a larger group for whom the program affects only the timing of training and work decisions, and for this group the relation between having participated in a training program and the subsequent wage is likely to overstate substantially the effect of the program.

The empirical analysis is more suggestive than conclusive, but it does appear to indicate that the general implications of the model are broadly consistent with data drawn from the National Longitudinal Survey of Young Women. It indicates fairly strongly that a more complete empirical model of the sequences of work and home decisions of young women should embed a part-time wage that is correlated with full-time wages, and that such a model should probably also consider fixed employment costs which are possibly different from one woman to the next. In view of these results,

it would appear appropriate to pursue further this analysis with the eventual aim of developing a structural empirical model suitable for policy analysis.

## Footnotes

1. In addition to the choices about labor supply and whether or not to have children, choices analyzed in the model, a young woman makes related choices about schooling, marriage, and concerning number, spacing and quality of children, perhaps making preliminary plans simultaneously for each, some more tentatively than others, and then modifying these plans in light of realizations of stochastic variables. Considerable work has been done on these issues. (For two examples of many, see Weiss and Gronau, 1981; and Heckman, Hotz and Walker, 1985.) In order to focus on the basic labor market decisions and their interactions with the decision as to whether or not to have children, the model abstracts from the marriage and schooling decisions, treats fertility as certain, ignores questions about number and spacing of children, and adopts certain assumptions that will have the effect of fixing the woman's age at first birth. Thus while the model brings together many important determinants of behavior, there is a long way to go.

Some of the problems created by conditioning parts of the empirical analysis on what are endogenous outcomes are discussed by Heckman and Willis (1977). Although the model conditions on formal schooling, note that its structure is compatible with the model developed by Lehrer and Nerlove (1981), where the endogeneity of the schooling decision is emphasized. It would be straight forward to extend our model to include the decision, but would, of course, add to the complexity of the solution.

2.  $w_s$  may be above the minimum wage if women are more productive than some other group (e.g., teenagers) in the secondary sector. In this case, unemployment would be concentrated among the other group, whose members would be paid just the minimum wage. The growing importance of women in minimum wage jobs is documented by Gramlich (1976).

3. This expression supposes that the discounted value of productivity remains constant over time, which in turn implies that real productivity is growing at the same rate as the real interest rate. This assumption is made for expositional convenience.

4. In this expression, the employer is willing to finance all of the specific training costs but none of the costs of general training. The exact percentage of specific training costs that the employer is willing to finance would be different if the growth rate of productivity were not equal to the real interest rate, but the general nature of the analysis would not be changed.

5. A second possible constraint might arise if the period  $T_m$  were short enough relative to the training period that the employer would not be able to recover his share of the training costs in the post-training period while still paying a wage at least as great as the wage in the secondary sector. This constraint would seem to be more of a potential problem in the first period than after the woman returns to full-time work in the second and third periods. The major results discussed below do not appear to be substantially different if this second constraint is binding in the first period, and so the discussion will proceed on the presumption that it is the constraint discussed in the text is the binding one. For a further discussion of this type of implicit contract in a somewhat different context, see Gustman and Steinmeier (1985). Further complications, which are not considered, would address the possibility that the rate of deterioration of skills is reduced by part-time work compared to no market work, and that the mix between specific and general skills varies among jobs and thus is subject to choice when the initial occupation is chosen. (For related discussions, see Polachek 1979.)

6. There are two reasons for specifying the consumption argument in the utility function in terms of lifetime income. First, preferences are specified in terms of lifetime income rather than in terms of income spent in each period separately because the model is not particularly concerned with the pattern of consumption over time. (Problems created by borrowing constraints and the implications of these problems for this analysis are discussed in footnote 13 below.) Second, an assumption that the minimum wage is fixed in terms of primary sector output allows us to specify this argument as a function of income rather than of consumption of each of the two goods produced by the economy. The reason is that the least capable group of secondary sector workers must be paid the minimum wage, and fixing its value in terms of primary sector output has the effect of dictating the price ratio between the outputs of the two sectors. Then the composite commodity theorem implies that preferences may be expressed in terms of income rather than in terms of the consumption amounts of the two goods separately.

7. This elasticity is  $-u''y/u'$ . If the elasticity exceeds unity everywhere, then utility has an absolute upper bound. To see this, suppose  $-u''y/u' = k > 1$ . This differential equation has the solution  $u = a + by^{1-k}$ , with  $b < 0$  required because marginal utility must be positive. With  $k > 1$  and  $b < 0$ ,  $u$  has an absolute maximum of  $a$  as  $y$  tends toward  $\infty$ . A further characteristic of interest is that if the elasticity of marginal utility is between zero and unity for  $u(y)$ , the same must be true for  $s(y) = u(y+m)$ , where  $m$  is any positive amount. This is because

$$-s''y/s' = [y/(y+m)][-u''(y+m)(y+m)/u'(y+m)]$$

and since both factors on the right hand side are between zero and unity in absolute value, the left hand side must be also. This means that some

amount  $m$ , say representing the husband's income, can be added to  $y$  without invalidating the assumption that  $0 < -u''y/u' \leq 1$ .

8. In section II, on the job training costs were assumed to involve a fraction of the real wage. In this circumstance, as long as the rate of real wage growth and the real interest rate are similar, OJT itself creates no incentive to work earlier in the second period. Otherwise, if the woman were to leave the labor force for the same amount of time later in the period, she would not earn any more income, but would lose more utility than she would were she to leave the labor force for an equal length of time at the beginning of the period. Similarly, if the woman works part-time at any time during the second period, it should be after she has taken her time (if any) out of the labor force, but before any full-time work during the period.

9. The derivation of the sign of the numerator of  $\partial t_f / \partial \epsilon$  also uses the assumption that the elasticity of the marginal utility of income does not exceed unity.

10. As is shown in the appendix, the condition  $\partial t_p / \partial \epsilon < 0$  in panel (b) is strictly required only if the fixed costs of employment are small. Otherwise, the sign of this derivative is indeterminate.

11. Another general solution exists for the model, as illustrated in Appendix Figure A1. This solution, however, corresponds to a situation in which no low-wage women (i.e., those who can only work in the secondary sector) with children are working full-time at the beginning of the second period. For further discussion of this solution, see the last part of the appendix.

12. This would not be true if the discounted value of the subsidy to the second employer were less than the discounted value to the first employer.

In that case, the minimum level of ability necessary for training in the primary sector would be reduced, though not by as much as it would be if the subsidy were completely unavailable to the second employer.

13. In specifying the consumption term in the utility function in terms of lifetime income, the model has assumed that the individual is not subject to a borrowing constraint. Such a constraint is most likely to be binding when an individual with ability between  $\epsilon_s$  and  $\epsilon_0$  is offered a training subsidy, since this individual is restricted to the lower part-time wage until after she returns to full-time work in the second period, with the higher wages available only then. (In all other cases, individuals who could expect higher earnings after the childbearing period would find them available in the first period as well.) However, there are a couple of considerations which might mitigate the effects of any borrowing constraints. First, there is the characteristic of children that younger children tend to be expensive in terms of time while older children tend to be expensive in terms of resources. This means that by the time the financial demands of children are reaching their peaks, the woman may well have already returned to full-time work. And secondly, the period of children is not the only stage of the life cycle in which expenditures may exceed income; there is also the phase of retirement, which may be regarded as occurring after the third period in the model. Hence, some of the increased earning power after the individual returns to full-time work may simply enable individuals to service their expected retirement needs. Despite these considerations, it may nevertheless be the case that individuals in the second period are constrained by borrowing constraints. If so, the utility function must be written in terms of income available in each period rather than in terms simply of total income. In this circumstance, one might expect the major points of the paper, including the

anomalous effects of training subsidies and the cautions regarding evaluation studies of training subsidies, to persist even in a borrowing constrained model if the utility function entails a reasonable degree of substitutability of income in different periods. The analysis would become more complicated, though, and consideration would have to be given to the possibility that there may be particular sets of circumstances in which these findings might no longer hold.

14. Our analysis assumes that training programs are permanent and well understood. Evaluation studies, based either on econometric techniques or on a classic experimental design, must also deal with the difficulties created by temporary programs. The introduction of new programs and subsequent reoptimization, and the inability to count on current programs being around in future years will lead to behavior different from what would be observed with a permanent program available over the long term. If a training program is perpetually available, there is an optimal time for the individual to enroll that may not be available when the program is temporary.

15. The taste variable is formed on the basis of twelve attitudinal questions which were asked in the 1972 survey. (Specifically, these variables are reference numbers 3867-3878 in the survey.) For each question, the five possible responses are arranged so that the lowest response corresponds to an attitude of wanting to be home during periods when children are present and the highest response reflects an attitude that work is all right and even desirable during periods when young children are present in the household. The responses so ordered are assigned values of 1 to 5 for each question and are summed across the twelve questions. The resulting sums are broken into three categorical

variables.

Table 1

## Value of Productivity For Full-Time Work

Sector	Circumstances	Value of Productivity
Primary	Training completed on current job	$\epsilon$
Primary	In training on current job, with no previous training	$\epsilon(1-\tau)$
Primary	In training on current job, with previous training on a different primary sector job	$\epsilon(1-\gamma\tau)$
Secondary	All cases	$w_s$

Table 2

## Work Pattern Regressions

Variable	Pattern			
	ft	ft/pt	ft/pt/nlf	ft/nlf
constant	-0.601 (0.169)	0.111 (0.105)	1.226 (0.193)	0.265 (0.193)
birth year	0.012 (0.003)	-0.001 (0.002)	-0.014 (0.004)	0.003 (0.004)
race	0.125 (0.026)	-0.009 (0.016)	-0.147 (0.029)	0.031 (0.029)
IQ1 - T1	-0.025 (0.046)	-0.003 (0.028)	-0.030 (0.052)	0.058 (0.052)
IQ2 - T2	-0.066 (0.045)	0.014 (0.028)	0.052 (0.051)	0.000 (0.051)
IQ3 - T3	0.052 (0.433)	0.003 (0.027)	-0.011 (0.050)	-0.044 (0.050)
IQ2 - T1	0.029 (0.042)	-0.002 (0.026)	0.034 (0.048)	-0.061 (0.048)
IQ2 - T3	0.112 (0.040)	0.087 (0.025)	-0.062 (0.046)	-0.136 (0.046)
IQ3 - T1	0.073 (0.040)	0.026 (0.025)	0.034 (0.045)	-0.133 (0.045)
IQ3 - T2	0.129 (0.039)	0.072 (0.024)	-0.078 (0.045)	-0.123 (0.045)
IQ3 - T3	0.235 (0.036)	0.087 (0.022)	-0.127 (0.041)	-0.196 (0.041)
R <sup>2</sup>	0.058	0.024	0.034	0.029
number of observations	2292			

Table 3

Deviations from Average Work Patterns  
By IQ and Taste

		IQ			
		low	medium	high	
	high	-0.116	-0.062	-0.018	ft
		-0.045	-0.045	-0.017	ft/pt
		0.006	0.070	0.071	ft/pt/nlf
		0.155	0.036	-0.036	ft/nlf
taste for home and family life	medium	-0.156	-0.091	0.038	ft
		-0.029	-0.043	0.030	ft/pt
		0.089	0.037	-0.041	ft/pt/nlf
		0.097	0.097	-0.026	ft/nlf
	low	-0.039	0.021	0.144	ft
		-0.040	0.044	0.045	ft/pt
		0.026	-0.026	-0.090	ft/pt/nlf
		0.053	-0.039	-0.099	ft/nlf

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## Appendix

In the first part of this appendix we will derive the impact of changes in  $\epsilon$  and  $\theta$  on the dates of entering and leaving part-time work in the second period. The utility problem in this case involves maximizing

$$u(y) + c \left[ \int_{T_1}^{T_1+t_p} \phi(t) v(h_n; \theta) dt + \int_{T_1+t_p}^{T_1+t_f} \phi(t) v(h_p; \theta) dt \right] \\ + \int_{T_1+t_f}^{T_1+T_2} \phi(t) v(h_f; \theta) dt$$

subject to the budget constraint

$$y = \epsilon h_f [T - t_f] + w_p(\epsilon) h_p (t_f - t_p) - \epsilon h_f [(1 + \gamma) \tau T_t] - (T - t_p) C$$

for primary sector workers. Secondary sector workers solve the same problem with  $w_s$  and  $w_p^s$  (both independent of  $\epsilon$ ) replacing  $\epsilon$  and  $w_p(\epsilon)$  in the budget constraint.

The following notation will help to reduce the notational burden in the derivations:

$$\begin{aligned} u, u', u'' &: u(y), u'(y), u''(y) \\ \phi_p, \phi'_p &: \phi(T_1+t_p), \phi'(T_1+t_p) \\ \phi_f, \phi'_f &: \phi(T_1+t_f), \phi'(T_1+t_f) \\ v_n, v'_n &: v(h_n; \theta), \partial v(h_n; \theta) / \partial \theta \\ v_p, v'_p &: v(h_p; \theta), \partial v(h_p; \theta) / \partial \theta \\ v_f, v'_f &: v(h_f; \theta), \partial v(h_f; \theta) / \partial \theta \\ w_p, w'_p &: w_p(\epsilon), w'_p(\epsilon) \end{aligned}$$

Using this notation, the conditions for utility maximization can be found by substituting the definition of  $y$  into the utility function, differentiating with respect to  $t_p$  and  $t_f$ , and setting the results equal to zero. These conditions are:

$$-u'(w_p h_p - C) + \phi_p (v_n - v_p) = 0$$

$$-u'(\epsilon h_f - w_p h_p) + \phi_f (v_p - v_f) = 0$$

To find how  $t_p$  and  $t_f$  are affected by changes in  $\epsilon$  and  $\theta$ , totally differentiate the above equations to obtain the matrix equation

$$\begin{bmatrix} u''(w_p h_p - C)^2 + \phi_p'(v_n - v_p) & u''(\epsilon h_f - w_p h_p)(w_p h_p - C) \\ u''(\epsilon h_f - w_p h_p)(w_p h_p - C) & u''(\epsilon h_f - w_p h_p)^2 + \phi_f'(v_p - v_f) \end{bmatrix} \begin{bmatrix} dt_p \\ dt_f \end{bmatrix} = \begin{bmatrix} u''(w_p h_p - C)s + u'w_p' h_p \\ u''(\epsilon h_f - w_p h_p)s + u'(h_f - w_p' h_p) \end{bmatrix} d\epsilon + \begin{bmatrix} -\phi_p'(v_n - v_p) \\ -\phi_f'(v_p - v_f) \end{bmatrix} d\theta$$

where

$$s = \partial y / \partial \epsilon = h_f [T - t_f - (1 + \gamma) \tau T_t] + h_p w_p' (t_f - t_p)$$

By Cramer's rule,  $\partial t_p / \partial \epsilon = |A_{p\epsilon}| / |A|$ , where  $A_{p\epsilon}$  is the matrix

$$\begin{bmatrix} u''(w_p h_p - C)s + u'w_p' h_p & u''(\epsilon h_f - w_p h_p)(w_p h_p - C) \\ u''(\epsilon h_f - w_p h_p)s + u'(h_f - w_p' h_p) & u''(\epsilon h_f - w_p h_p)^2 + \phi_f'(v_p - v_f) \end{bmatrix}$$

and  $A$  is the matrix on the left hand side of the matrix equation above.

Evaluating the determinant of  $A_{p\epsilon}$  yields

$$\begin{aligned} |A_{p\epsilon}| &= [u''(w_p h_p - C)s + u'w_p' h_p] \phi_f'(v_p - v_f) \\ &\quad + u'u''(\epsilon h_f - w_p h_p) [h_p h_f (w_p' \epsilon - w_p) + C(h_f - w_p' h_p)] \end{aligned}$$

In one extreme case, this determinant is positive if  $w_p' = 0$ , since  $u'' < 0$ ,  $\phi_f' < 0$ ,  $v_p > v_f$  (if nonparticipation is a viable alternative to part-time work), and  $w_p h_p > C$  (if part-time work is a viable alternative to

nonparticipation). In the other extreme case, the determinant is negative if  $w_p = \epsilon$ , since  $-u''y/u' < 1$  by assumption, and  $(w_p h_p - C)s < h_p y$ . The latter inequality follows from  $(h_f - h_p)/h_f > (1+Y)\tau T_t / (T - t_f)$ , which will be satisfied as long as part-time hours are nontrivially shorter than full-time hours and the training period is relatively short as compared to the lifetime amount of full-time work.

Similarly, the determinant of  $A$  can be evaluated as

$$|A| = \phi'_p \phi'_f (v_p - v_f)(v_n - v_p) + u''[(w_p h_p - C)^2 \phi'_f (v_p - v_f) + (\epsilon h_f - w_p h_p)^2 \phi'_p (v_n - v_p)] > 0$$

Together, these two determinants imply that  $\partial t_p / \partial \epsilon$  must be positive in the case of  $w'_p = 0$  and negative in the case of  $w_p = \epsilon$ .

For  $t_f$ ,  $\partial t_f / \partial \epsilon = |A_{f\epsilon}| / |A|$ , where  $A_{f\epsilon}$  is the matrix

$$\begin{bmatrix} u''(w_p h_p - C)^2 + \phi'_p (v_n - v_p) & u''(w_p h_p - C)s + u'w'_p h_p \\ u''(\epsilon h_f - w_p h_p)(w_p h_p - C) & u''(\epsilon h_f - w_p h_p)s + u'(h_f - w'_p h_p) \end{bmatrix}$$

with the determinant given by

$$|A_{f\epsilon}| = u''u'(w_p h_p - C)[h_p h_f (w_p - w'_p \epsilon) - C(h_f - w'_p h_p)] + \phi'_p (v_n - v_p)[u''(\epsilon h_f - w_p h_p)s + u'(h_f - w'_p h_p)]$$

If  $C$  is sufficiently small, this determinant is negative for both of the extreme cases,  $w'_p = 0$  and  $w_p = \epsilon$ . For  $w'_p = 0$ , the last term is less than zero because  $\phi'_p < 0$  and

$$u''(\epsilon h_f - w_p h_p)s + u'h_f > u''h_f y + u'h_f = u'h_f [(u''y/u') + 1] > 0$$

with the last inequality arising because  $-u''y/u' < 1$ . Similar reasoning applies if  $w_p = \epsilon$ . Thus, in both extreme cases  $\partial t_f / \partial \epsilon$  is negative for

sufficiently small  $C$ , and otherwise it is of indeterminate sign.

For the extreme case  $w_p(\varepsilon) = \varepsilon$ , both  $|A_{p\varepsilon}|$  and  $|A_{f\varepsilon}|$  are negative (with  $C = 0$ ), and it will be of interest to ask which is larger in magnitude. Under these circumstances, the two determinants will reduce to

$$|A_{p\varepsilon}| = (u''y + u')h_p\phi'_f(v_p - v_f)$$

$$|A_{f\varepsilon}| = (u''y + u')(h_f - h_p)\phi'_p(v_n - v_p)$$

For small differences in income, the first terms will be approximately equal, and the first derivative will be larger in absolute magnitude if the following condition is fulfilled:

$$|\phi'_f| \frac{v_p - v_f}{h_f - h_p} > |\phi'_p| \frac{v_n - v_p}{h_p}$$

The first term on each side of this expression is the rate at which the weight on  $v$  in the utility function is declining throughout the second period, and one would expect this decline to be larger earlier in the period, so that  $|\phi'_p|$  would be larger than  $|\phi'_f|$ . However, the fractions in the second term on each side represent the marginal utility per hour of hours worked full-time and hours worked part-time, and under the assumption of diminishing marginal utility one would expect the marginal utility of full-time hours to be greater. Hence, whether the above relation holds or does not hold depends upon the specific parameters in the utility function.

Evaluation of the derivatives with respect to  $\theta$  proceeds in much the same manner.  $\partial t_p / \partial \theta$  is given by  $|A_{p\theta}| / |A|$ , where  $A_{p\theta}$  is

$$\begin{bmatrix} -\phi'_p(v_n - v_p) & u''(\varepsilon h_f - w_p h_p)(w_p h_p - C) \end{bmatrix}$$

$$\left[ \begin{array}{cc} -\phi_f'(v_p' - v_f') & u''(\epsilon h_f - w_p h_p)^2 + \phi_f'(v_p - v_f) \\ \phi_f'(v_p' - v_f') & u''(\epsilon h_f - w_p h_p) \phi_f(v_p - v_f) / u' \end{array} \right]$$

Taking the determinant and substituting in from the two marginal conditions yields

$$|A_{p\theta}| = -\phi_p'(v_n' - v_p') \{ [u''(\epsilon h_f - w_p h_p) \phi_f(v_p - v_f) / u'] + \phi_f'(v_p - v_f) \} \\ + \phi_f'(v_p' - v_f') u''(\epsilon h_f - w_p h_p) \phi_p(v_n - v_p) / u'$$

which means that  $\partial t_p / \partial \theta$  will be positive as long as

$$\frac{v_n' - v_p'}{v_n - v_p} \left[ 1 + \frac{\phi_f'}{\phi_f} \frac{u'}{u''(\epsilon h_f - w_p h_p)} \right] > \frac{v_p' - v_f'}{v_p - v_f}$$

Similarly,  $\partial t_f / \partial \theta = |A_{f\theta}| / |A|$ , where  $A_{f\theta}$  is given by

$$\left[ \begin{array}{cc} u''(w_p h_p - C)^2 + \phi_p'(v_n - v_p) & -\phi_p'(v_n' - v_p') \\ u''(\epsilon h_f - w_p h_p)(w_p h_p - C) & -\phi_f'(v_p' - v_f') \end{array} \right]$$

Substituting from the two marginal conditions gives a determinant of

$$|A_{f\theta}| = -\{ [u''(w_p h_p - C) \phi_p(v_n - v_p) / u'] + \phi_p'(v_n - v_p) \} \phi_f'(v_p' - v_f') \\ + \phi_p'(v_n' - v_p') u''(w_p h_p - C) \phi_f(v_p - v_f) / u'$$

This yields the condition that  $\partial t_f / \partial \theta$  will be positive if

$$\frac{v_p' - v_f'}{v_p - v_f} \left[ 1 + \frac{\phi_p'}{\phi_p} \frac{u'}{u''(w_p h_p - C)} \right] > \frac{v_n' - v_p'}{v_n - v_p}$$

Note that either this condition or the previous one must be fulfilled, so that at least one (and possibly both) of the two derivatives  $\partial t_p / \partial \theta$  and  $\partial t_f / \partial \theta$  must be positive.

To analyze the effects of higher fixed costs of employment, it is necessary first to calculate how these costs affect  $t_p$  and  $t_f$ , holding other things constant. Using the same methodology as above, the relevant derivatives may be calculated as  $\partial t_p / \partial C = |A_{pC}| / |A|$  and  $\partial t_f / \partial C = |A_{fC}| / |A|$ , where the denominators are the same matrices as before. For the numerator of  $\partial t_p / \partial C$ , we have the matrix

$$\begin{bmatrix} -u' - u''(w_p h_p - C)(T - t_p) & u''(\epsilon h_f - w_p h_p)(w_p h_p - C) \\ -u''(\epsilon h_f - w_p h_p)(T - t_p) & u''(\epsilon h_f - w_p h_p)^2 + \phi'_f(v_p - v_f) \end{bmatrix}$$

whose determinant is given by

$$|A_{pC}| = -u'u''(\epsilon h_p - w_p h_p)^2 - \phi'_f(v_p - v_f)[u' + u''(w_p h_p - C)(T - t_p)] > 0$$

with the inequality following because  $u' + u''(w_p h_p - C)(T - t_p) > u' + u''y > 0$ .

For the numerator of  $\partial t_f / \partial C$ , the matrix is

$$\begin{bmatrix} u''(w_p h_p - C)^2 + \phi'_p(v_n - v_p) & -u' - u''(w_p h_p - C)(T - t_p) \\ u''(\epsilon h_f - w_p h_p)(w_p h_p - C) & -u''(\epsilon h_f - w_p h_p)(T - t_p) \end{bmatrix}$$

whose determinant is given by

$$|A_{fC}| = u'u''(\epsilon h_p - w_p h_p)(w_p h_p - C) - \phi'_p(v_n - v_p)u''(\epsilon h_f - w_p h_p)(T - t_p) < 0$$

Hence,  $\partial t_p / \partial C$  is positive and  $\partial t_f / \partial C$  is negative.

As a final topic in the appendix, we consider the situation where no low-wage women with children elect to work full-time in the early part of the second period. This is the case illustrated in Figure A1, and it corresponds to an instance in which the slope of the segments in Figure 1 between full-time and part-time work are relatively steeper, so that at

sufficiently low wage levels women would not find it advantageous to work full-time if they have children. As time passes in the second period, the areas corresponding to nonparticipation and to part-time work will move upward in the diagram, for exactly the same reasons as discussed in the text and earlier in this appendix. However, the boundary separating women with children and women without children must remain fixed throughout the period, so that although later on in the period the boundaries between full-time work, part-time work, and nonparticipation will look much like Figure 2, the boundary between women with and without children will not be the horizontal segment pictured in Figure 2 but will continue to be the segment pictured in Figure A1. Note that in this case there is a positive association between the  $\epsilon$  and the minimum level of  $\theta$  for which the woman will choose to have children.

Figure 1

Utility of Home Time for Women with children

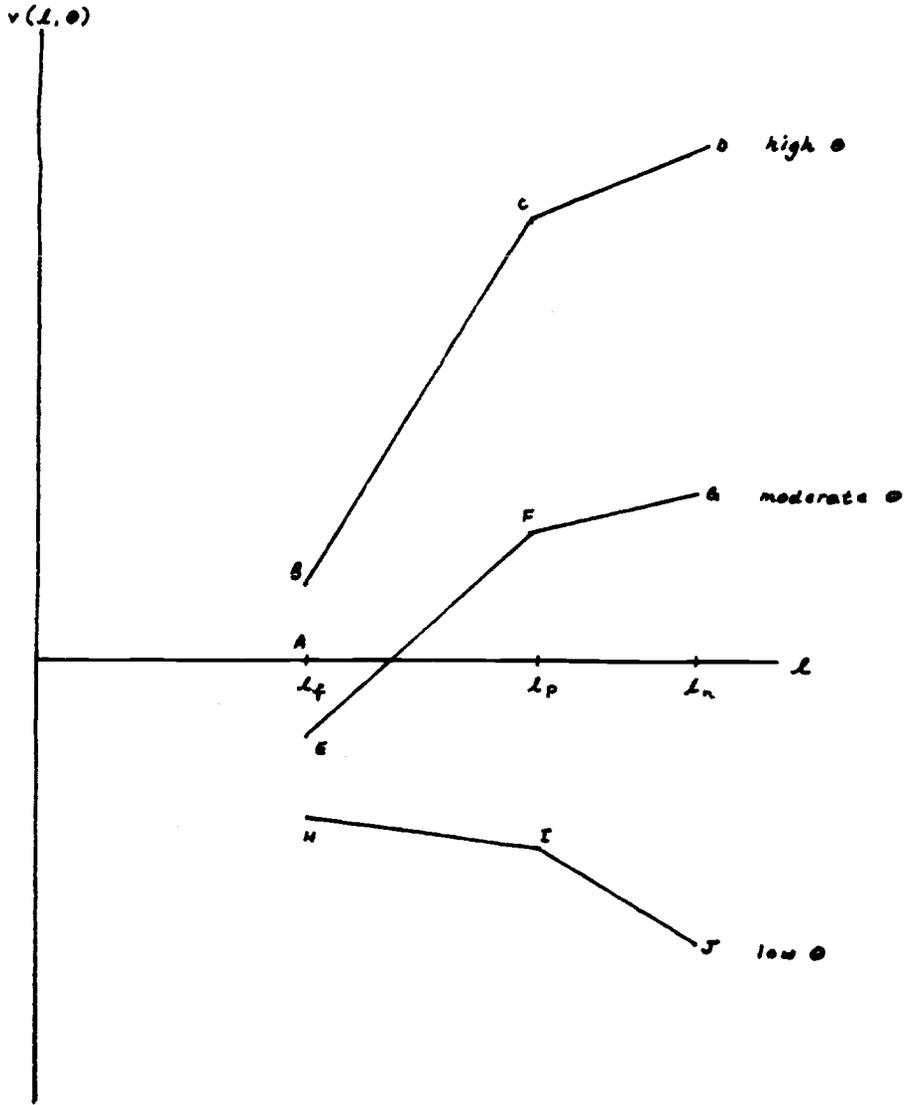
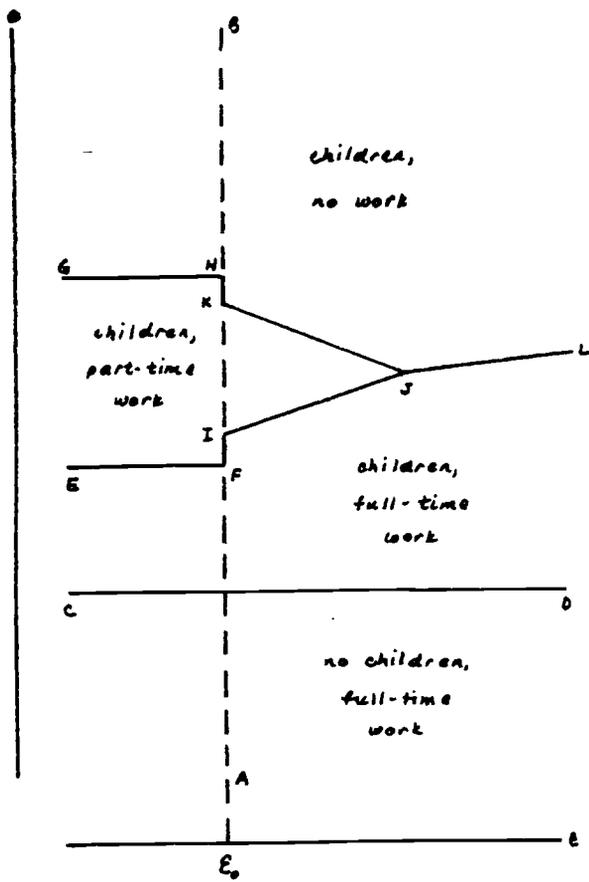
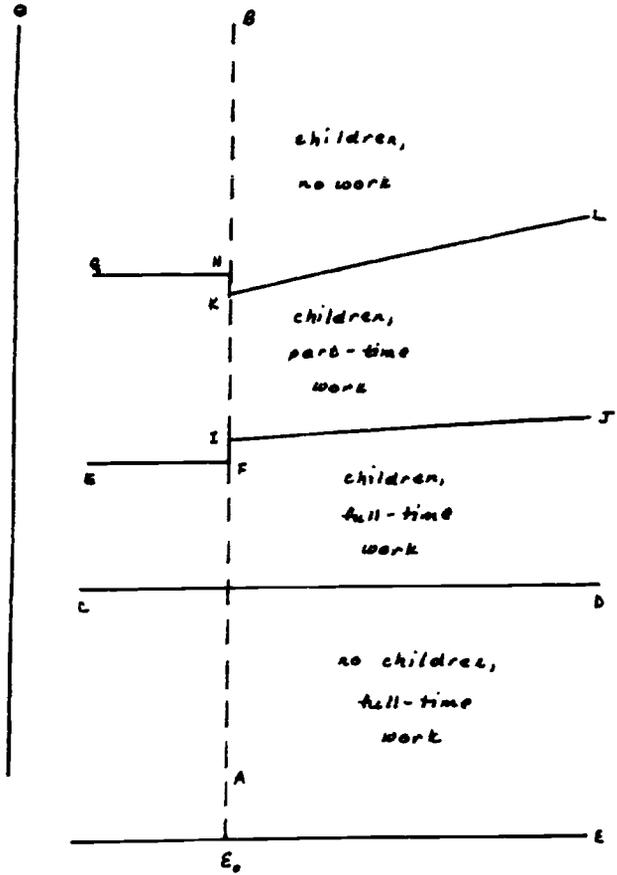


Figure 2

Patterns of Fertility and Work  
Decisions at Time  $t_0$



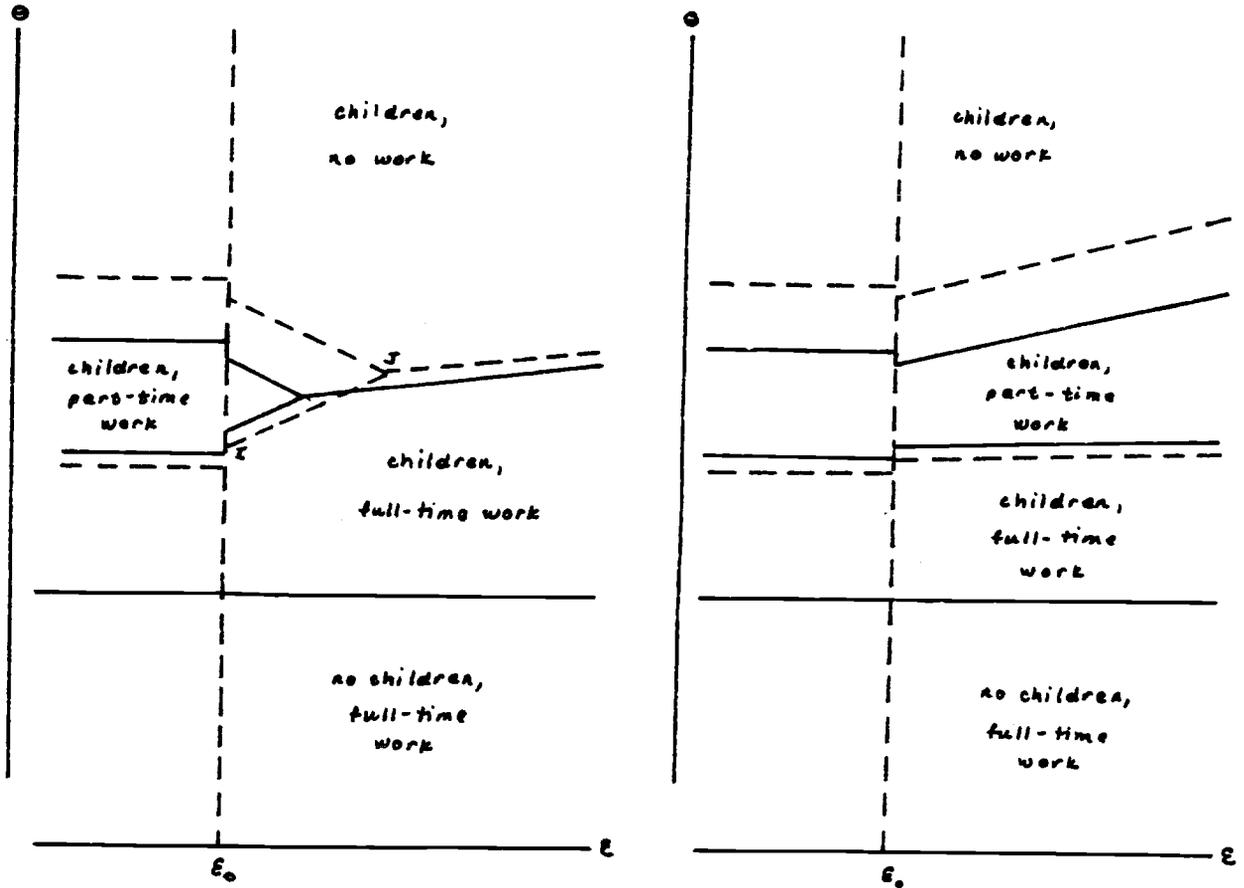
(a) Constant Part-Time Wage



(b) Part-time Wage Equal to Full-Time Wage

Figure 3

Effect of Higher Fixed Costs of Employment on Patterns of Fertility and Work Decisions at Time  $t$ .

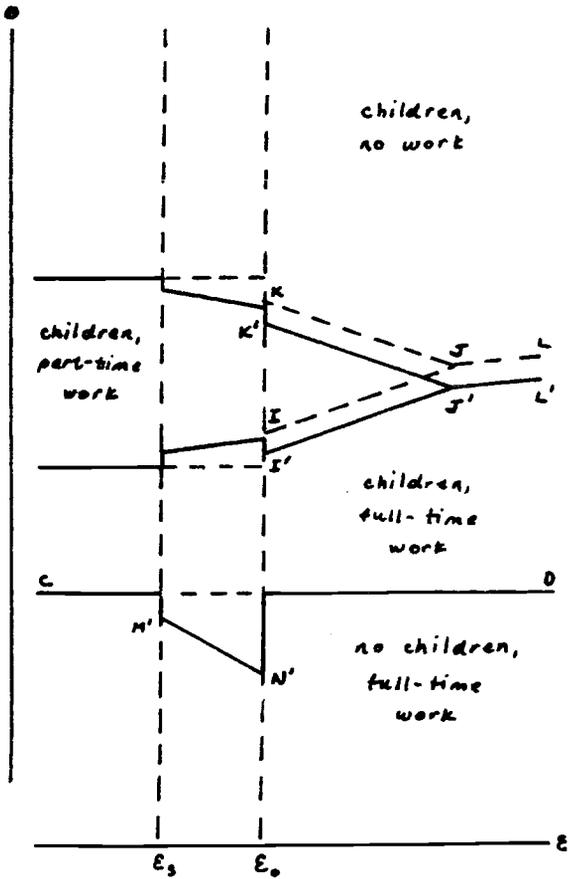


(a) Constant Part-time Wage

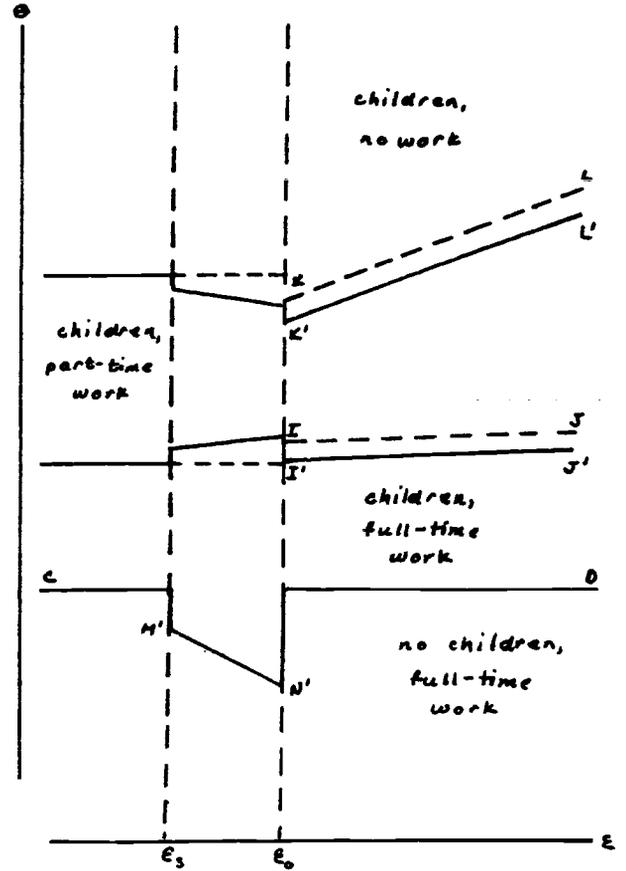
(b) Part-Time Wage Equal to Full-Time Wage

Figure 4

Effect of Training Subsidies on Patterns of Fertility and Work Decisions at Time  $t_0$ .



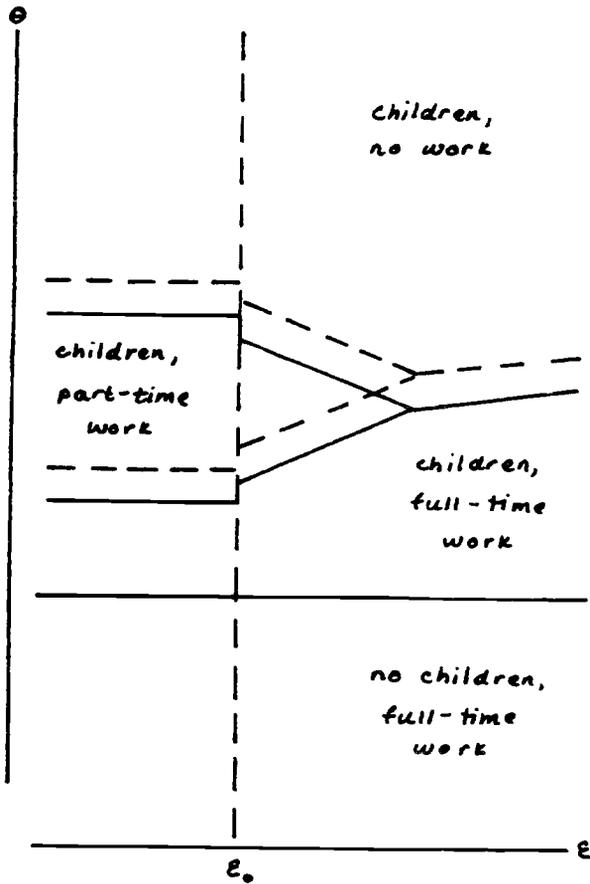
(a) Constant Part-Time Wage



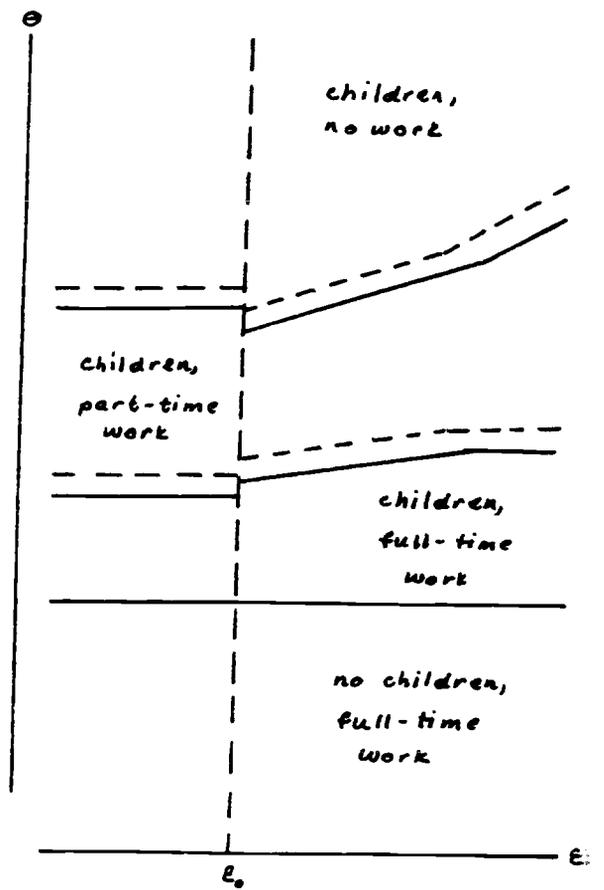
(b) Part-Time Wage Equal to Full-Time Wage

Figure 5

Effect of Increase in Guarantee on Patterns of Fertility and Work Decisions at Time  $t_0$ .



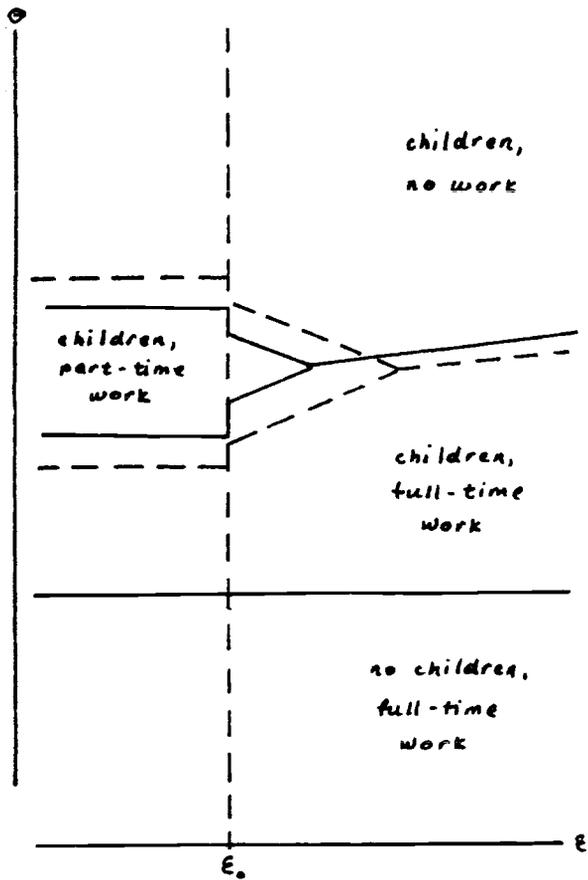
(a) Constant Part-Time Wage



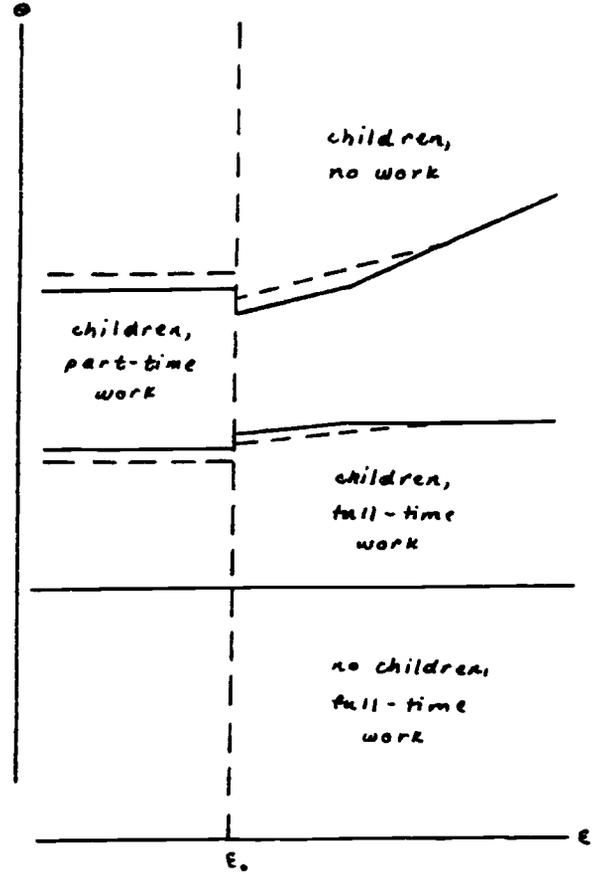
(b) Part-Time Wage Equal to Full-Time Wage

Figure 6

Effect of Increase in Implicit Tax Rate on Patterns of Fertility and Work Decisions at Time  $t_0$ .



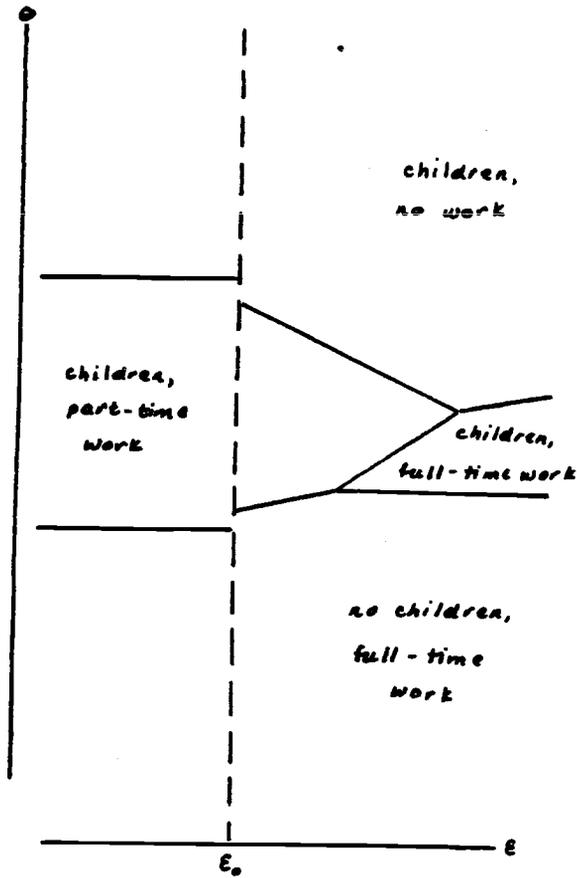
(a) Constant Part-Time Wage



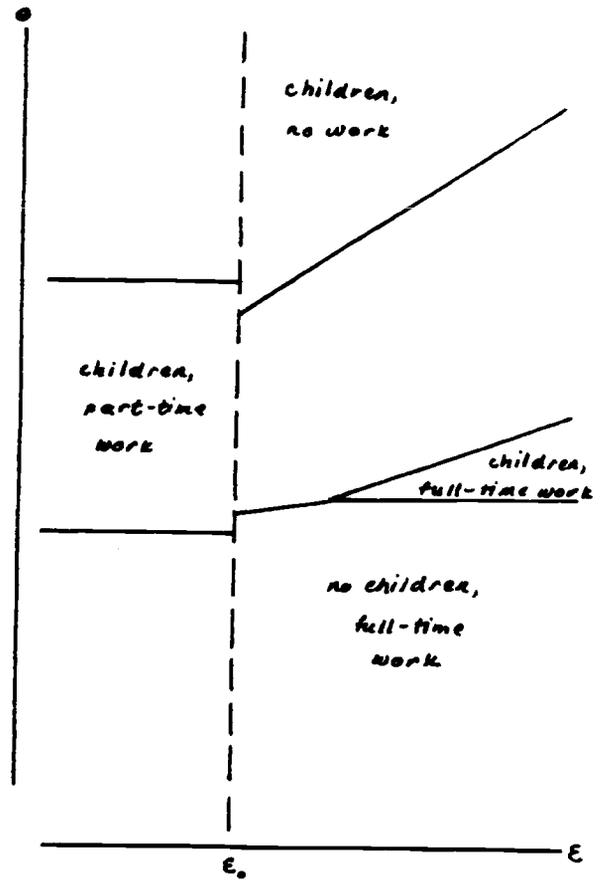
(b) Part-Time Wage Equal to Full-Time Wage

Figure A1

Alternative Patterns of Fertility and Work Decisions at Beginning of Childbearing Period



(a) Constant Part-Time Wage



(b) Part-Time Wage Equal to Full-Time Wage