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THE INTERRELATIONSHIP BETWEEN ANGEL AND VENTURE CAPITAL MARKETS

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ABSTRACT

This paper develops a theory of how angel and venture capital markets interact. Entrepreneurs first receive angel then venture capital funding. The two investor types are 'friends' in that they rely upon each other's investments. However, they are also 'foes', because at the later stage the venture capitalists no longer need the angels. Using a costly search model we derive the equilibrium deal flows across the two markets, endogenously deriving market sizes, competitive structures, valuation levels, and exit rates. We discuss how the model generates alternative testable hypotheses for the recent rise of angel investing.

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1 Introduction

Investments by wealthy individuals into start-up companies are typically referred to as angel investments. Over the last decade angel investors have become a more important source of early stage funding for entrepreneurs. According to Crunchbase, the US angel market grew at an annual rate of 33% between 2007 and 2013.[1] In a 2011 report of the OECD, the size of the angel market was estimated to be roughly comparable to the venture capital (VC henceforth) market (OECD, 2011).[2] The rise of the angel market coincides with a shift in VC investments towards doing more later-stage deals. As a result the funding path of growth-oriented start-ups typically involves some initial funding from angel investors, with subsequent funding coming from venture capitalists (VCs henceforth). Facebook and Google, two of the most successful start-ups in recent history, both received angel financing prior to obtaining VC.

With this bifurcation in the funding environment of entrepreneurial companies, the question arises how these two types of investors interact, especially whether angels and VCs are friends or foes? Angel investors have limited funds and typically need VCs to provide follow-on funding for their companies. At the same time VCs rely on angel investors for their own deal flow. As they play complementary roles in the process of financing new ventures it might seem that angels and VCs should be friends. However, in practice angels and VCs often see each other as foes. In particular, there is a concern about so-called "*burned angels*". Angel investors frequently complain that VCs abuse their market power by offering unfairly low valuations. Expectations of low valuations at the VC stage then affect the willingness of angel investors to invest in early stage start-ups. Michael Zapata, an angel investor, explains it as follows (Holstein, 2012):

> *In cases where the VCs do see a profit opportunity, they have become increasingly aggressive in low-balling the managements and investors of emerging companies by placing lower valuations on them. [...] Angels call these actions 'cram downs' or 'push downs'. The market has been very rough on the VCs and they are making it tougher on the angels. They are killing their future deal flow by cramming them down, crashing them out.*

The main objective of this paper is to examine the interdependencies between two types of investors, angels and VCs, that focuses on two distinct sequential financing stages. Our goal is to provide a tractable model of the equilibrium dynamics between these two markets

[1] Data based on http://www.crunchbase.com/.

[2] For 2009 the report estimates the US (European) venture capital market at \$18.3B (\$5.3B), and the US (European) angel market at \$17.7B (\$5.6B).

that generates a rich set of empirical predictions. We are particularly interested in identifying the underlying determinants of market size and market competition (which depends on the entry rates of entrepreneurs, angels and VCs), as well as company valuations and success rates. Special attention is given to analyzing the full equilibrium implications of the "*burned angels*" problem.

From a theory perspective, the challenge is to obtain a model of the two connected markets that generates tractable comparative statics for key variables. To this effect we develop a search model with endogenous entry by entrepreneurs, angels and VCs. Companies require angels for seed investments, and may require VC for funding their growth options. The model generates predictions about the level of competition in both the angel and VC market. It predicts the expected length of fundraising cycles (i.e., the time it takes to raise angel and VC funding), as well as the rate at which companies fail, progress from the angel to the VC market, or achieve some kind of exit. The model also derives equilibrium company valuations at both the angel and VC stage.

Our model has three key building blocks that build on previously disparate literatures. First we draw on the staged financing literature (Admati and Pfleiderer, 1994; Berk, Green and Naik, 1999). Specifically we introduce a dynamic investment structure where start-ups first obtain seed funding in the angel market, then follow-up funding in the VC market. This simple dynamic structure allows us to capture the basic interdependencies between angels and VCs: angels invest first but need the VCs to take advantage of a company's growth options. Central to the model are two feedback loops. The first is the forward loop of how the angel market affects the VC market. The key linkage is that outflow of successful deals in the angel market constitute the deal inflow in the VC market. Here we can think of angels and VCs are 'friends'. The second is the backward loop of how the VC market affects the angel market. The key linkage here is that the utilities of the entrepreneurs and angel investors at the VC stage affect the entry rates of entrepreneurs and angel investors at the angel stage. A key insight is that at the VC stage, the VC no longer needs the angel to make the investment. The angel's investment is sunk and he provides no further value to the company. This creates a primal friction between angels and VCs, i.e., this is where angels and VCs become 'foes'.

Second, we draw on the search literature. Inderst and Müller (2004) explain how a search model à la Diamond-Mortensen-Pissaridis allows for a realistic modeling of imperfect competition in the VC market. We expand their model to two interconnected markets. We also augment their specification with a death rate for entrepreneurs to capture the full implications of demand and supply imbalances. Our model highlights the consequences of imperfect competition in the VC market on the angels' bargaining position: while a monopolist VC would have a lot of power over angels, such bargaining power get dissipated in a more competitive VC market.

Third, to examine further determinants of the relative bargaining strengths of entrepreneurs, angels and VCs, we consider the issue of minority shareholder protection (La Porta, Lopez-de-Silanes, Shleifer and Vishny, 2000). In his work on the "burned angels" problem, Leavitt (2005) provides a detailed legal analysis of the vulnerabilities of angels at the time of raising VC. As new investors, VCs can largely dictate terms. They can also use option grants as a way of compensating the entrepreneur for the low valuation offered to angel investors. Leavitt argues that legal minority shareholder protection can mitigate the burned angel problem, but cannot fully resolve it. Based on this, we consider a hold-up problem between the angel and the entrepreneur at the time of the follow-up round.[3] The entrepreneur can try to collude with the VC to pursue the venture alone without the angel. While the threat remains unexercised in equilibrium, the hold-up potential redistributes rents from the angel to the entrepreneur and VC. Our analysis traces out the equilibrium effects that such hold-up has on the returns and investment levels of angels and VCs.

Our model generates a large number of comparative statics results. Throughout the analysis we consider the joint equilibrium across the two markets. We find that our within-market effects are consistent with results in the prior literature (e.g., Inderst and Müller, 2004), so our main contribution is the analysis of cross-market effects. Here we discover several new insights. For example, a standard within-market result is that while higher search costs for the investor lead to less competition, higher search costs for the entrepreneur lead to more competition (because of the fact that investors can capture more of the rents). However, this result does not apply in a cross-market setting. We show that there is less angel competition when there are higher search costs at the VC stage for either the VC or entrepreneur. This is because both of these search costs reduce the utilities of the entrepreneur and angel investor.

One of the most interesting results concern the effects of angel protection. One might conjecture that the ability to take advantage of angel investors increases entrepreneurial entry. However, we show that in equilibrium there is lower entry by entrepreneurs, because the direct benefit for entrepreneurs from holding up angels at a later stage is outweighed by the indirect cost of a thinner angel market. Intriguingly, we find opposite effects for entrepreneurial entry and survival. Weaker angel protection actually leads to better entrepreneurial incentives (because it is allows the entrepreneur to capture additional rents) and therefore to higher success rates.

Our theory implies a series of testable predictions about the size of angel and VC markets, their competitive structure, as well as the valuations obtained. It is useful to present the insights and predictions of the model within the current industry context. Specifically, within the context

[3] We use the term "hold-up" only for the ex-post relationship between angel and entrepreneur, at the time of venture capital financing. The venture capitalist cannot hold up the angel or the entrepreneur, as he has no prior contractual relationship with them.

of our model, we consider a number of alternative explanations for the recent rise of angel markets. This allows us to demonstrate how our theory generates testable predictions for comparing these alternative explanations. Let us consider three leading explanations for the recent rise of angel investments; we informally label them as (i) "angel investing got cheaper", (ii) "angel investing got easier", and (iii) "angel investors got smarter".

The first argument ("angel investing got cheaper") is that the cost of starting a business has dramatically declined in recent years. This phenomenon has been widely discussed in the popular press (The Economist, 2014), and is related to the so-called "lean startup" movement (see e.g. Ries (2011), Blank (2013), and Ewens, Nanda and Rhodes-Kropf (2014)). In our model, lower start-up costs lead to more entry of entrepreneurs, and even more entry of angels. Overall this generates a more competitive angel market, and results in shorter fundraising cycles. Lower start-up costs also imply lower equity stakes for angels, generating better entrepreneurial incentives and thus higher success rates. All this increases the rate at which angel-backed companies enter the VC market. Free entry ensures that the VC market expands in size accordingly.

The second argument ("angel investing got easier") is that angel markets have become more transparent. The main driver is the rise of internet-based platforms that facilitate matching between investors and entrepreneurs. Angellist is currently the market leader, but a number of other "crowdfunding" platforms also compete in this space (Nanda, 2013). So-called "seed accelerators" are likely to further increase the transparency of angel markets.[4] Our search model naturally lends itself to study market transparency. We find that greater transparency in the angel market leads to more entry by entrepreneurs, and even more entry by angel investors. Greater competition leads to higher valuations and better entrepreneurial incentives. Interestingly all of these changes in the angel market imply that the VC market should also expand in size. Moreover, if these electronic platforms also increase the transparency of VC markets, our model further predicts additional entry of VCs, with more competition, shorter fundraising cycles, and higher valuations.

The third argument ("angel investors got smarter") is that angel investors have become more experienced and sophisticated. This could be because of the creation of national angel investor associations, and the rise of organized angel groups and angel networks (OECD, 2011). According to this argument angels learn over time how to better protect themselves against hold-up in a variety of legal and strategic ways (Leavitt, 2005). Our model generates some testable predictions about the effects of better angel protection, namely that it increases the entry of angels as well as entrepreneurs (as discussed above). It also creates a more competitive angel market, and leads to higher valuations. The effects of angel protection on the VC market are nuanced.

[4] A typical accelerator, such as YCombinator, promotes an entire cohort of start-ups in a carefully orchestrated "demo day", so that a large number of angel investors can easily meet the entire cohort at once (Cohen, 2013).

The loss of ex-post rents discourages entry of VCs, which is the ‘foes’ effect. However, there is also a positive ‘friends’ effect from the increased supply of angel-financed companies entering the VC market. While these two effects create an ambiguous prediction on the size of the VC, the model generates the unambiguous prediction that competition in the VC market decreases. Thus we note that better angel protection has opposite effects on the level of competition of angel and VC markets. This also implies shorter fundraising cycles in the angel market, but longer ones in the VC market.

Our theory also generates predictions about the choice of projects, and the timing of exits. Some angel investors have been advocating early exits as an attractive investment approach (Peters, 2009). Accordingly, start-ups focus on projects that can be sold relatively quickly. The advantage of such a strategy is that the entrepreneurs and angels can avoid the various challenges of securing follow-on investments, typically from VCs. The disadvantage is that they may fail to achieve their full potential. Our base model simply assumes that a successful company either has a growth option, in which case it always optimal for the owners to seek VC, or it does not, in which case it is optimal to sell. In a model extension we allow for a choice between two development strategies: a safe strategy, where the venture can simply be sold in an early exit, versus a risky strategy, which either leads to failure, or generates higher returns, namely if the company succeeds in developing a growth option and obtaining VC financing. A key prediction of this model extension is that the safe strategy of early exits becomes more likely when angel protection is low, such as when the value of the start-up resides mainly in the entrepreneur’s human capital. Consistent with this prediction we note that many early exists take the form of so-called “acqui-hires” where the acquirer is mainly interested in the human capital of the start-up, and not in the product itself.

The remainder of the paper is structured as follow. Section 2 discusses the relation of this paper to the literature. Section 3 introduces our main model. We then derive and analyze the angel market equilibrium in Section 4, and the VC market equilibrium in Section 5. Section 6 analyzes how limited legal protection of angel investors affects the angel and VC market equilibrium. Section 7 examines the decision of entrepreneurs and angels to exercise the safe option of selling the venture early, versus the more risky option of seeking VC investments. Section 8 discusses the empirical predictions from our model. Section 9 summarizes our main results and discusses future research directions. All proofs are in the Appendix.

2 Relationship to Literature

The introduction briefly discusses how this paper builds on a variety of literatures. In this section we explain in greater detail the connections to the prior literature. The natural starting point is the seminal paper by Inderst and Müller (2004). They were the first to introduce search into a model of entrepreneurial financing, focussing on how competitive dynamics affect VC valuations. Silveira and Wright (2006) and Nanda and Rhodes-Kropf (2012a) also use similar model specifications for other purposes. One theoretical advance of this paper is that it examines the relationship between two interconnected search markets. A limitation of all these search models (including ours) is that they require homogenous types. Hong, Serfes, and Thiele (2013) consider a single-stage VC financing model with matching among heterogeneous types.

A growing number of papers examine the implications of staged financing arrangements. Neher (1999) and Bergemann, Hege, and Peng (2009) study the design of optimal investment stages. Admati and Pfleiderer (1994) and Fluck, Garrison, and Myers (2005) consider the differential investment incentives of insiders and outsiders at the refinancing stage. Building on recent work about tolerance for failure (Manso, 2011), as well as the literature on soft budget constraints (Dewatripont and Maskin, 1995), Nanda and Rhodes-Kropf (2012a) consider how investors optimally choose their level of failure tolerance in a staged financing model. These models all assume that the original investors can finance the additional round. Our model departs from this assumption by focusing on smaller angel investors who do not have the financial capacity to provide follow-on financing.

The theory closest to ours is the recent work by Nanda and Rhodes-Kropf (2012b) on financing risk. They too assume that the initial investors cannot provide all the follow-on financing. Their analysis focuses on the possibility of multiple equilibria in the late stage market, and shows how different expectations about the risk of refinancing affects initial project choices. Our model does not focus on financing risk, but instead focuses on the hold-up problem at the refinancing stage.

Our model distinguishes between angels and VCs on the basis of the investment stage and the amount of available funding: angels only invest in early stages and have limited funds; VCs only invest in later stages and have sufficient funds to do so. The empirical evidence of Goldfarb, Hoberg, Kirsch and Triantis (2012) and Hellmann, Schure and Vo (2013) is broadly supportive of these assumptions. The latter paper also provides empirical evidence on the financing dynamics, showing how some companies obtain only angel financing (possibly exiting early), whereas others transition from the angel to the VC market.

While we focus on some important distinctions between angels and VCs, our model clearly does not capture all the nuances of reality. First, it is sometimes difficult to draw a precise boundary between what constitutes an angel investor versus a VC. Shane (2008) and the OECD report (OECD, 2011) provide detailed descriptions of angel investing, and the diversity within the angel community. Second, we do not model value-adding activities of angels versus VCs. Chemmanur and Chen (2006) assume that only VCs but not angels can provide value-added services. By contrast, Schwienbacher (2009) argues that both angels and VCs may provide such services, but that angel investors provide more effort because they still need to attract outside investors at a refinancing stage.[5] Third, in our model both angels and VCs are pure profit maximizers. The work of Axelson, Strömberg, and Weisbach (2009) suggests that the behavior of VCs may be influenced by agency considerations. Moreover, angel investors can be motivated by non-financial considerations, such as personal relationships or social causes, as discussed in the work of Shane (2008) and Van Osnabrugge and Robinson (2000). Finally, while we motivate our paper with angels and VCs, our theory applies more broadly to the relationship between early and late investors. Further examples of early investors include friends and family, accelerators, and other specialized early stage investors, such as university-based seed funds. Further examples of late investors include corporate investors, and a variety of other financial institutions such as banks or growth capital funds.

Our paper is also related to the literature on the staged commercialization of new venture ideas. Teece (1986), Anand and Galetovics (2000), Gans and Stern (2000) and Hellmann and Perotti (2011) all consider models where complementary asset holders have a hold-up opportunity at a later stage. They mainly ask how this hold-up problem impacts the optimal organization of the early stage development efforts. This paper focuses on the challenges of financing ventures across the different commercialization stages.

3 The Base Model

Our objective is to build a tractable equilibrium model that endogenously derives the size and competitive structure of the early stage (angel) and late stage (VC) market. Conceptually we want a model with endogenous entry to determine market size, and with a continuum between monopoly and perfect competition to determine the level of competition. This naturally leads us to a search model in the style of Diamond, Mortensen and Pissaridis (see Pissarides (1979, 2000), Mortensen and Pissarides (1994), and Diamond (1982, 1984)). This model has free entry, and it endogenously generates a market density that is a continuous measure of competition. Moreover, the real-life search process of entrepreneurs looking for investors closely resembles the assumptions of pairwise matching used in such search models (Inderst and Müller (2004)).

We consider a continuous time model with three different types of risk-neutral agents: entrepreneurs, angel investors, and VCs. The length of one period is $\Delta \to 0$, and the common discount rate is $r > 0$. In each period a number of potential entrepreneurs discover business op-

[5]In terms of related empirics, Hellmann and Puri (2002) provide evidence on the value-adding activities of venture capital versus angel investors. Kerr, Lerner and Schoar (2013) examine the causal effect of angel groups on company performance.

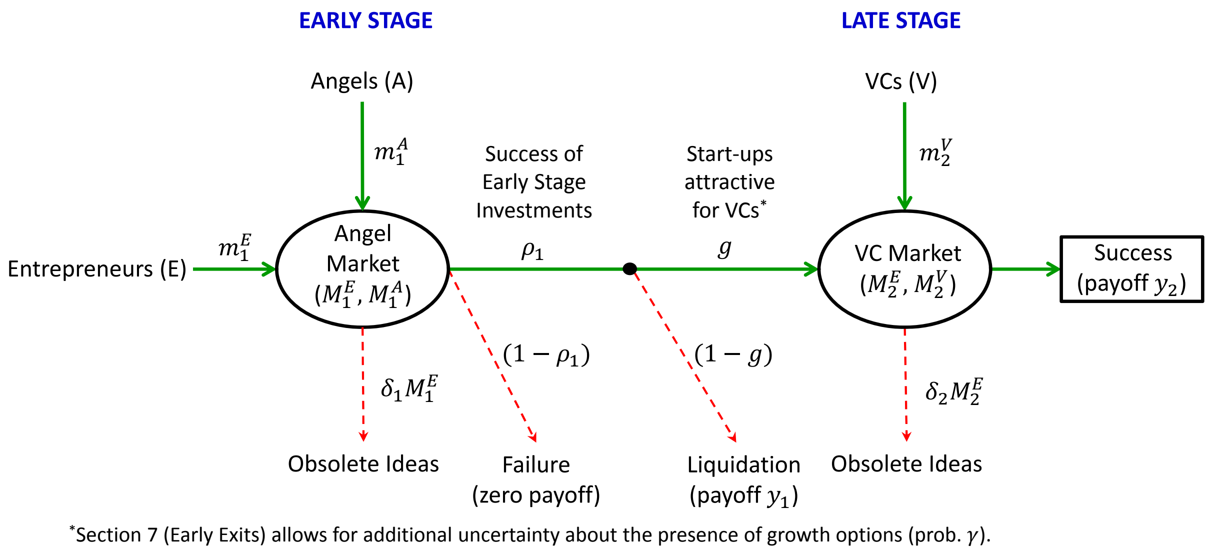

Figure 1: Financing Stages – Angel and VC Markets

portunities. The cost for an entrepreneur to start his business is $l \in [0, \infty)$, which is drawn from the distribution $F(l)$. We interpret l as the personal labor cost associated with establishing the new venture (e.g., the cost of developing a business plan). The endogenous number of start-ups founded in each period is m_1^E; see Figure 1 for a graphical overview.

Entrepreneurs are wealth-constrained, so they require external financing for their start-up companies. Specifically, each entrepreneur needs an early stage investment k_1, and a late stage investment k_2. What we have in mind is that entrepreneurs first need funding to develop prototypes of their products (early stage financing) in order to prove the viability of their business models. They then require follow-on investments to bring their developed products to market (late stage financing). We assume that early and late stage financing is provided by two distinct types of investors. For clarity of exposition we associate angels with early stage investments, and VCs with late stage investments.[6] For simplicity we also assume that early stage investors do no contribute at all to the late stage investment.[7]

In the early stage each entrepreneur needs to find an angel investor, who can make the required investment k_1. We assume a monopolistically competitive search market with free

[6]In reality VCs may sometimes also invest in early stage deals, and angels in late stage deals. This does not affect the basic insights from our model; all that matters is that there is some separation of early and late stage markets, where a company that obtained funding from an investor in the early stage needs to find a new investor at the late stage. This may be either because the early stage investor does not have the funds to fully finance the late stage investments, or because early and late stage investors have different skills and information, making them both essential to the success of the venture.

[7]Allowing them to finance part of the later stage investment would not change anything. All that matters for the model is that angels do not have enough wealth to finance both stages. Conversely we also assume that VCs cannot be the sole investor in both the early and the late stage market. Our assumptions closely matches industry behavior where early stage investors typically seek syndication partners for the later investment stages.

entry.[8] Specifically, in each period m_1^A angel investors enter the early stage market and seek investment opportunities, where m_1^A is endogenous and satisfies the zero-profit condition for angels. We denote M_1^A as the equilibrium stock of angels in the early stage market at a given point in time, and M_1^E as the equilibrium stock of entrepreneurs.

Entrepreneurial opportunities often depend on speedy execution, so that delays in fundraising can be costly for the entrepreneurs. We therefore augment the standard search model with a parameter that measures urgency, i.e., the cost of delay. Specifically we assume that a fraction $\delta_1 M_1^E$ of business ideas becomes obsolete in each period, generating a zero payoff (see Figure 1). We refer to δ_1 as the *death rate* of early stage ventures, which reflects the urgency for start-up companies to receive angel investments. The death rate δ_1 therefore constitutes an indirect cost for entrepreneurs when searching for early stage financing.

Our model includes the standard search model parameters. We denote the individual search cost for entrepreneurs in the early stage market by σ_1^E, and the search cost for angels by σ_1^A. Naturally we focus on the case where the angel market exists, which requires that σ_1^A is not too large. The expected utilities from search in the early stage market is U_1^E for entrepreneurs, and U_1^A for angels. We use a standard Cobb-Douglas matching function $x_1 = \phi_1 \left[M_1^A M_1^E\right]^{0.5}$ with constant returns to scale, where x_1 is the number of entrepreneur-angel matches in each period, and $\phi_1 > 0$ is an efficiency measure of the matching technology in the early stage market.

Once matched, the angel and entrepreneur bargain over the allocation of equity. We use the symmetric Nash bargaining solution to derive the equilibrium outcome of this bilateral bargaining game. The angel then invests k_1 in the new venture, and the entrepreneur exerts private effort e_1. The entrepreneur's disutility of effort $c(e_1)$ is strictly convex, with $c(0) = c'(0) = 0$. Note that our model includes entrepreneurial incentives, which allows us to generate predictions about the rate at which entrepreneurs move from the angel to the VC market.

The early stage investment succeeds with probability $\rho_1(e_1)$, which is increasing and concave in the entrepreneur's private effort e_1, with $\rho_1(0) = 0$, $\rho_1'(0) = \infty$ and $\lim_{e_1 \to \infty} < 1$. With probability $(1 - \rho_1(e_1))$ the early stage investment fails and generates a zero payoff; see Figure 1. With probability $\rho_1(e_1)$ it is successful, in which case there are two possible scenarios: With probability g the venture has a growth option and becomes an attractive candidate for VCs to make the follow-on investment k_2. With probability $1 - g$ the venture has no further growth option, but can be liquidated, generating the payoff $y_1 > 0$. For our base model we assume that the growth option is sufficiently attractive, so that realizing the growth option is always preferred

[8] For tractability we have to assume that all investors in a market are homogenous, and that they face no entry costs. Relaxing this assumption would considerably increase the complexity of the model. Note, however, that our model does account for ongoing investor costs, as captured by their search costs.

to liquidation. In Section 7 we relax this assumption by introducing risky growth options, and analyzing the optimal choice between growth and liquidation (early exit).

The total number of ventures moving into the VC market in each period is given by $m_2^E \equiv g\rho_1(e_1)x_1$. Their owners – namely the respective entrepreneur and angel – then search for a VC investor. Again we assume that the late stage market is monopolistically competitive with free entry. We denote m_2^V as the endogenous number of VCs entering the market in each period. The equilibrium stock of VCs in the late stage market at a given point in time is M_2^V, and the equilibrium stock of ventures seeking VC financing is M_2^E. As for the angel market, we assume that a fraction $\delta_2 M_2^E$ of business ideas becomes obsolete in each period, generating a zero payoff (see Figure 1). The individual search cost in the late stage market is σ_2^i, and the expected utility from search is U_2^i, with $i = E, A, V$. To ensure existence of the VC market, we assume that σ_2^V is not too large. For tractability we assume that entrepreneurs and angels – as joint owners of late stage start-ups – incur the same cost when searching for a VC investor, i.e., $\sigma_2 \equiv \sigma_2^E = \sigma_2^A$. The matching function for the late stage market is $x_2 = \phi_2 \left[M_2^V M_2^E\right]^{0.5}$, where x_2 is the number of start-ups that receive VC financing in each period, and $\phi_2 > 0$ is a measure of the matching efficiency.

Once the owners of a late stage start-up (entrepreneur and angel) found a VC investor, they all bargain over the allocation of equity, as discussed below. After reaching an agreement the VC makes the required follow-on investment k_2.[9] The venture then generates the expected payoff $y_2 > 0$ (with $y_2 > y_1$).

In the late stage market the bargaining game is between three key players. We use the Shapley value to derive the outcome of this trilateral bargaining game.[10] This implicitly assumes that all prior contracts can be renegotiated. While the entrepreneur and angel may want to commit to a specific equity allocation beforehand, the VC never agreed to that, and may therefore ask that all prior arrangements be ignored. In equilibrium the three parties agree on a division of shares that is determined by their outside options.

[9]To keep the analysis of the late stage market with trilateral bargaining games tractable, we abstract from private efforts of entrepreneurs in the late stage. Allowing for private efforts would change the late stage payoff structure; however, this would not qualitatively affect our insights with respect to the interrelationship between both markets – which is the main focus of this paper.

[10]The Shapley value is widely regarded as the most natural extension of the Nash bargaining solution to games with more than two players. It provides an intuitive allocation of equity, which takes into account the marginal contribution of each party to the overall value generation.

Given the simple payoff structure of the model, equity is always an optimal security.[11] The angel investor initially receives an equity stake α in the early stage. This equity stake can be renegotiated into β^A in case of VC financing. The VC receives an equity stake β^V. The so-called post-money valuation is then given by $V_1 = k_1/\alpha$ for angel rounds, and $V_2 = k_2/\beta^V$ for VC rounds.

4 Angel Market

4.1 Bargaining and Deal Values

We start by looking at how entrepreneurs and angels split the expected surplus when making a deal. Our model consists of two investment stages, so the early stage allocation of surplus naturally depends on the partners' expected utilities from search in the late stage market, U_2^E and U_2^A. However, the entrepreneur and angel cannot affect these utilities because of their inability to commit not to renegotiate. The early stage equity allocation therefore only determines how the liquidation value y_1 is split, with the angel receiving αy_1 and the entrepreneur keeping $(1-\alpha)y_1$.

We call the utilities at the time of making a deal the *deal values*, and denote them by D_1^E and D_1^A. In the angel market they are given by

$$\begin{aligned} D_1^E &= \rho_1(e_1)\left[gU_2^E + (1-g)(1-\alpha)y_1\right] - c(e_1) \\ D_1^A &= \rho_1(e_1)\left[gU_2^A + (1-g)\alpha y_1\right] - k_1. \end{aligned}$$

For a given equity allocation α, the entrepreneur then chooses his private effort e_1 to maximize his deal value D_1^E. The entrepreneur's optimal effort choice e_1^* is defined by the following first-order condition:

$$\rho_1'(e_1)\left[gU_2^E + (1-g)(1-\alpha)y_1\right] = c'(e_1). \tag{1}$$

We can immediately see that the entrepreneur's effort e_1^* is increasing in his utility from search in the late stage market, U_2^E, and decreasing in the angel's equity share α.

[11] While our model set-up is suitable for analyzing valuations, it is not meant to generate insights into financial security structures. Angels and VCs sometimes use more elaborate securities, such as preferred equity or convertible notes (see Kaplan and Strömberg (2003) and Hellmann (2006)). In this model none of these more complicated securities can achieve anything better than simple equity. This is because in case of VC financing, the angel's choice of security is irrelevant – the original contract will get renegotiated anyway. Technically we note that none of the coalition values of the Shapley game depend on the securities held by the angel investor. Moreover, in case of liquidation, there is an exogenous liquidation value, rendering security structures unimportant.

The optimal equity share for the angel, α^*, satisfies the symmetric Nash bargaining solution, which accounts for the outside option of each party. The outside option for the entrepreneur is to search for a new angel investor, which would give him the expected utility U_1^E. The angel also searches for a new entrepreneur, but free entry implies $U_1^A = 0$. Thus, α^* maximizes the Nash product $\left\{\left(D_1^E(e_1^*) - U_1\right) D_1^A(e_1^*)\right\}$.

Naturally we focus on the downward sloping portion of the bargaining frontier.[12] In the Appendix we show that the angel's equilibrium equity stake α^* is decreasing in the entrepreneur's outside option U_1^E (which captures the equilibrium of the entire market). This is a well-known result from Nash bargaining, namely that if a party has a better outside option, it receives more utility. In the present setting, this implies that more equity must be left for the entrepreneur, which also makes the venture more likely to succeed (as $de_1^*/d\alpha < 0$).

4.2 Market Equilibrium

We can now characterize the stationary equilibrium of the angel market. Let $q_1^E \equiv x_1/M_1^E$ denote the deal arrival rate for entrepreneurs, which measures the probability that an entrepreneur finds an angel investor at a given point in time. The expected utility of an entrepreneur when searching for an angel investor, U_1^E, is then defined by the following asset equation:

$$rU_1^E = q_1^E(D_1^E - U_1^E) + \delta_1(0 - U_1^E) - \sigma_1^E.$$

This equation says that the discounted expected utility from search equals the expected value of getting a deal ($q_1^E(D_1^E - U_1^E)$), minus the expected costs when searching for an angel investor: the risk of the business idea becoming obsolete ($\delta_1 U_1^E$) and the direct cost of search (σ_1^E). Solving the asset equation we find the entrepreneur's expected utility from search (U_1^E), and likewise the angel's expected utility (U_1^A):

$$U_1^E = \frac{-\sigma_1^E + q_1^E D_1^E}{r + \delta_1 + q_1^E} \qquad\qquad U_1^A = \frac{-\sigma_1^A + q_1^A D_1^A}{r + q_1^A}, \tag{2}$$

where $q_1^A \equiv x_1/M_1^A$ is the deal arrival rate for angels. Because of free entry, the expected utility from search must be zero in equilibrium. Thus, in equilibrium we have

$$q_1^A D_1^A = \sigma_1^A. \tag{3}$$

[12] Formally, let $\widehat{k}_1$ denote the value of k_1 at which the bargaining frontier has a zero slope, i.e., where $dD_1^E(e_1^*)/d\alpha = 0$. We therefore focus on the case where $k_1 < \widehat{k}_1$.

An entrepreneur with entry cost l will only enter the early stage market if $l \leq U_1^E$. Thus, there exists a unique threshold entry cost $\bar{l}$, with $\bar{l} = U_1^E$, so that entrepreneurs enter as long as $l \leq \bar{l}$. The endogenous equilibrium number of entrepreneurs entering the angel market in each period, m_1^{E*}, is then given by

$$m_1^{E*} = F(U_1^E). \tag{4}$$

Moreover, in a stationary equilibrium the stock of entrepreneurs must be constant. This requires that the total outflow of entrepreneurs – because either their business ideas became obsolete ($\delta_1 M_1^E$) or they found an angel investor ($q_1^E M_1^E$) – is equal to the equilibrium inflow of entrepreneurs (m_1^{E*}):

$$\delta_1 M_1^E + q_1^E M_1^E = m_1^{E*}. \tag{5}$$

Likewise, in a stationary equilibrium the outflow of angels ($q_1^A M_1^A$) is equal to the (endogenous) inflow of angels (m_1^A):

$$q_1^A M_1^A = m_1^A. \tag{6}$$

The equilibrium of the angel market is then defined by conditions (2), (3), (4), (5), and (6). Moreover, since the equilibrium is jointly determined in the angel and VC market, it must also satisfy equations (7) - (11), which we will discuss in Section 5.2.

4.3 Results

Entrepreneurs and angel investors are forward-looking decision makers who take into account their expected utilities in the VC market, given by U_2^E and U_2^A. Thus the angel market equilibrium depends on the characteristics of the VC market, which is the backward feedback loop. Our model also features a forward feedback loop, where the outflows from the angel market (excluding liquidations and failures) constitute the inflows into the VC market. All of our comparative statics results take into account these two equilibrium feedback loops, i.e., we always consider the joint equilibrium between the two markets. We now discuss the results for the angel market (we characterize the VC market equilibrium in Section 5.2).

It is useful to distinguish between market-level and firm-level effects. On the market level we are interested in the extent of entrepreneurial activities within the economy, which in our model is measured by the equilibrium entry of entrepreneurs m_1^{E*}. We are also interested in the equilibrium number of angel investors m_1^{A*} that enter the market in each period to search for investment opportunities. Related to these two variables of interest is the number of angel-backed deals x_1^* in each period. It turns out that in equilibrium the inflow of angels equals the number of early stage deals, i.e., $m_1^{A*} = x_1^*$.

We also want to understand the determinants of the degree of competition among angel investors. In a search model this can be readily measured by the number of angels relative to the number of entrepreneurs in the market, also known as market thickness. Formally we denote the degree of angel market competition by θ_1^*, where $\theta_1^* = M_1^{A*}/M_1^{E*}$. It is interesting to note that this measure of competition is closely related to the expected time for entrepreneurs to find an angel, which is given by $1/[\phi_1\sqrt{\theta_1^*}]$.[13] We call this the fundraising cycle, which is negatively related to the market transparency ϕ_1 and to the level of competition θ_1^*.

On the firm level we want to understand the determinants of the valuation of angel-backed start-ups. We focus on the post-money valuation as given by $V_1 = k_1/\alpha$. We also want to examine the implications for the success rate of angel investments, as reflected by $\rho_1(e_1)$.

For parsimony we derive the equilibrium properties of all these variables in the Appendix. The next proposition summarizes our comparative statics results, focusing on the effects of the early stage parameters.

Proposition 1 (Angel Market – Early Stage Parameters) *Consider the angel market.*

Market-level Effects:

(i) *The equilibrium inflow of entrepreneurs m_1^{E*} is increasing in ϕ_1, and decreasing in δ_1, σ_1^E, σ_1^A and k_1.*

(ii) *The equilibrium inflow of angels m_1^{A*}, and therefore the equilibrium number of early stage deals x_1^*, is increasing in ϕ_1, and decreasing in σ_1^A and k_1. The effects of δ_1 and σ_1^E are ambiguous.*

(iii) *The equilibrium degree of competition θ_1^* is increasing in ϕ_1, δ_1 and σ_1^E, and decreasing in σ_1^A and k_1.*

Firm-level Effects:

(i) *The equilibrium valuation of early stage start-up companies V_1^* is increasing in ϕ_1, and decreasing in δ_1, σ_1^E and σ_1^A. The effect of k_1 is ambiguous.*

(ii) *The equilibrium success rate of angel investments $\rho_1(e_1^*)$ is increasing in ϕ_1, and decreasing in δ_1, σ_1^E, σ_1^A, and k_1.*

Our model generates intuitive comparative statics. Higher costs for angels (σ_1^A and k_1) lead to less angel entry (lower m_1^{A*}), and therefore to fewer early stage deals (lower x_1^*), so that

[13] Formally, the probability that an entrepreneur finds an angel in a given period is x_1^*/M_1^{E*}. Using the definitions of x_1 and θ_1 we find that the equilibrium probability of finding an angel is $\phi_1\sqrt{\theta_1^*}$. Thus, the expected time for entrepreneurs to find an investor is $1/[\phi_1\sqrt{\theta_1^*}]$.

entrepreneurs on average need longer to secure early stage financing. And because search is costly, fewer entrepreneurs then find is worthwhile to enter the market (lower m_1^{E*}). This makes the angel market less competitive, suggesting that the effect on angel entry is more pronounced (as σ_1^A and k_1 have a first-order effect on m_1^{A*}, and only a second-order effect on m_1^{E*}).

Higher direct and indirect search costs for entrepreneurs (σ_1^E and δ_1) result in fewer entrepreneurs entering the early stage market (lower m_1^{E*}). This implies on the one hand fewer investment opportunities for angels (which has a negative effect on m_1^{A*}), and, on the other hand, a better bargaining position and higher deal values for angels (which has positive effect on m_1^{A*}). While the net effect on angel entry (m_1^{A*}) is ambiguous, we find that the angel market overall becomes more competitive (i.e., the angel/entrepreneur ratio increases).

The firm-level effects are best understood by looking at the equilibrium allocation of equity between entrepreneurs and angels. We know that higher direct and indirect search costs for entrepreneurs, σ_1^E and δ_1, weaken their outside option when negotiating a deal with an angel investor, explaining the lower valuations. Higher costs for angels, σ_1^A and k_1, also decrease valuations. This is because fewer angels then enter the market in equilibrium, so that entrepreneurs have a weaker outside option, as they need on average longer to find an alternative investor. A lower valuation means that the entrepreneur needs to give up more equity, which in turn curbs his effort incentives (lower e_1^*), and therefore results in a lower probability of success (lower $\rho_1(e_1^*)$).[14]

The next proposition provides a comprehensive summary of how the equilibrium of the angel market depends on the determinants of the VC market.

Proposition 2 (Angel Market – Late Stage Parameters) *Consider the angel market.*

Market-level Effects: *The equilibrium inflow of entrepreneurs m_1^{E*}, the inflow of angels m_1^{A*} (and therefore the number of early stage deals x_1^*), and the early stage degree of competition θ_1^*, are all increasing in ϕ_2, and decreasing in δ_2, σ_2, σ_2^V, and k_2.*

Firm-level Effects: *The equilibrium valuation of early stage start-up companies V_1^*, and the success rate of angel investments $\rho_1(e_1^*)$, are increasing in ϕ_2, and decreasing in δ_2, σ_2, σ_2^V, and k_2.*

All cross-market effects are driven by the backward feedback loop. Higher utilities for the entrepreneur and angel investor at the VC stage also benefit the angel market. To fully

[14] Intuitively we would expect k_1 to have a positive effect on the equilibrium valuation V_1^*. Because k_1 also affects α^*, which in turn is only implicitly defined by (12), we do not get a clear comparative statics result. However, one can show that $dV_1^*/dk_1 > 0$ when (i) k_1 is sufficiently small, or (ii) the entrepreneur's effort e_1 is exogenous (so that surplus can be perfectly transferred between entrepreneurs and angels through α). The proof is available from the authors upon request.

understand these results we need to draw on some insights about the degree of VC market competition, which we discuss in more detail in Section 5.3. Intuitively a more competitive VC market has the following two effects: First, it reduces the expected time for entrepreneurs and angels to secure follow-on investments from VCs. Second, it improves their bargaining position when striking a deal with a VC, allowing them to capture more of the expected surplus from the investment. Hence a more efficient VC market, as captured by an increase in the market transparency ϕ_2, improves entrepreneurs' and angels' expected utilities from search (U_2^E and U_2^A). This in turn explains why the angel market variables, such as m_1^{E*}, m_1^{A*}, θ_1^*, V_1^*, and $\rho_1(e_1^*)$, are all increasing in ϕ_2. A similar argument also applies for lower values of σ_2^V and k_2. However, even though δ_2 and σ_2 both have a positive effect on the level of VC competition (as we will formally show in Section 5.3), we find that the net effect on U_2^E and U_2^A is negative. This is because the direct negative effects of these parameters on U_2^E and U_2^A always dominate. The angel market variables, such as m_1^{E*}, m_1^{A*}, θ_1^*, V_1^*, and $\rho_1(e_1^*)$, are therefore decreasing in δ_2 and σ_2.

It is worth pointing out that cross-market effects fundamentally have a different logic than within-market effects. This becomes most obvious when looking at the entrepreneurs' search costs. Proposition 1 shows that higher search costs at the angel stage increase angel market competition. This is because of a market power effect, where the angels' stronger bargaining position encourages more angels and fewer entrepreneurs to enter. In contrast Proposition 2 shows that higher search costs at the VC stage decrease angel market competition. This is because of the backward feedback loop, where lower utilities at the VC stage discourage entry and competition at the angel stage.[15]

5 VC Market

5.1 Bargaining and Deal Values

In the late stage market the owners of each start-up company – the entrepreneur and angel – seek an investment k_2 from a VC. Naturally the angel investment k_1 is now sunk. The total surplus, which we denote by π, is then defined by $\pi = y_2 - k_2$. We use the Shapley value to derive the outcome of this tripartite bargaining game. The outside option for the entrepreneur and angel is to go back to the market and search for a new VC investor; the joint value of their outside option is therefore given by $U_2^E + U_2^A$. The VC can also search for a new investment

[15]Note also that the results about the entrepreneurs' search costs are different from the comparative statics for investors' search costs. Higher VC search costs at the VC stage dampen competition in the angel market; and this is also true for higher angel search costs at the angel stage.

opportunity; however, free entry implies $U_2^V = 0$. For now we assume that the success of a late stage start-up requires the involvement of all three parties, i.e., it is impossible to exclude any of the three partners. In Section 6 we consider a model extension where the entrepreneur and VC can establish a new venture based on the same business idea, thereby excluding the angel investor.

In the Appendix we derive the following late stage deal values for the entrepreneur (D_2^E), angel (D_2^A), and VC (D_2^V), using the Shapley value:

$$D_2^E = \tfrac{1}{3}\pi + \tfrac{1}{6}\left[U_2^E + U_2^A\right], \quad D_2^A = \tfrac{1}{3}\pi + \tfrac{1}{6}\left[U_2^E + U_2^A\right], \quad D_2^V = \tfrac{1}{3}\pi - \tfrac{1}{3}\left[U_2^E + U_2^A\right]$$

Each entrepreneur and angel gets one-third of the total surplus π, plus a premium which reflects the joint value of their outside option ($U_2^E + U_2^A$). The VC receives the remaining surplus. The equilibrium deal values define the late stage equity shares that all parties agree on, which we denote by β^i, $i = E, A, V$. For parsimony we state the equilibrium equity shares in the Appendix. A better outside option for the entrepreneur and angel – as reflected by $U_2^E + U_2^A$ – gives them more equity in equilibrium, and therefore higher deal values.

5.2 Market Equilibrium

We can now characterize the equilibrium of the VC market. The inflow of start-up companies, which we denote by m_2^{E*}, is endogenously determined by the angel market equilibrium. This forward loop is characterized by the following equilibrium condition:

$$m_2^{E*} = g\rho_1(e_1^*)x_1^* \tag{7}$$

Let $q_2^E \equiv x_2/M_2^E$ denote the deal arrival rate for entrepreneur-angel pairs, and $q_2^V \equiv x_2/M_2^V$ the deal arrival rate for VCs. In addition to (7), the stationary VC market equilibrium is then defined by the following conditions:

$$U_2^A = U_2^E = \frac{-\sigma_2 + q_2^E D_2^E}{r + \delta_2 + q_2^E} \tag{8}$$

$$q_2^V D_2^V = \sigma_2^V \tag{9}$$

$$\delta_2 M_2^E + q_2^E M_2^E = g\rho_1(e_1)x_1 \tag{10}$$

$$q_2^V M_2^V = m_2^V. \tag{11}$$

The expected utility for entrepreneurs and angels from search in the late stage market is given by (8). The expected utilities are the same because entrepreneurs and angels – as co-owners of a late stage start-up company – have identical search costs ($\sigma = \sigma_2^E = \sigma_2^A$), in addition to identical deal values ($D_2^E = D_2^A$).[16] Condition (9) is the free-entry condition for VCs, which implies that $U_2^V = 0$ in equilibrium. Condition (10) ensures that the total outflow of new ventures equals the total inflow. Likewise, condition (11) guarantees that the outflow of VCs equals the inflow. The VC market equilibrium is therefore determined by equations (7) - (11), but for our analysis we actually use the joint equilibrium between both markets, which in fact is characterized by equations (2) - (11).

5.3 Results

For the market-level effects we focus on the equilibrium entry of VCs (m_2^{V*}), the number of VC-backed deals (x_2^*), and the degree of competition ($\theta_2^* \equiv M_2^{V*}/M_2^{E*}$). Again we find that $m_2^{V*} = x_2^*$ in the stationary equilibrium. Moreover, the measure of competition is inversely related to the VC fundraising cycle, as given by $1/[\phi_2\sqrt{\theta_2^*}]$. On the firm level we are interested in how the determinants of the VC market affect the equilibrium valuation of late stage start-up companies. We focus again on the post-money valuation as given by $V_2 = k_2/\beta^V$.

The next proposition summarizes the comparative statics results, focusing on the parameters that are associated with the VC market.

Proposition 3 (VC Market – Late Stage Parameters) *Consider the VC market.*

Market-level Effects:

(i) *The equilibrium inflow of VCs m_2^{V*}, and therefore the equilibrium number of late stage deals x_2^*, is increasing in ϕ_2, and decreasing in σ_2^V and k_2. The effects of δ_2 and σ_2 are ambiguous.*

(ii) *The equilibrium degree of competition θ_2^* is increasing in ϕ_2, δ_2 and σ_2, and decreasing in σ_2^V and k_2.*

Firm-level Effect: *The equilibrium valuation of late stage start-up companies V_2^* is increasing in ϕ_2 and k_2, and decreasing in δ_2, σ_2 and σ_2^V.*

The comparative statics results resemble those for the angel market equilibrium (effects of early stage parameters). We therefore do not provide a detailed discussion of the results here, and refer the reader to the explanations right after Proposition 1 in Section 4.3.

[16] This greatly simplifies many of our basic comparative statics. However, in Section 6 we consider a model extension where the late stage expected utilities of angels and entrepreneurs differ.

The next proposition provides a comprehensive summary of how the equilibrium of the VC market depends on the parameters associated with the angel market.

Proposition 4 (VC Market – Early Stage Parameters) *Consider the VC market.*

Market-level Effects:

(i) *The equilibrium inflow of start-up companies m_2^{E*} and VCs m_2^{V*} are both increasing in ϕ_1, and decreasing in σ_1^A and k_1. The effects of δ_1 and σ_1^E are ambiguous.*

(ii) *The characteristics of the angel market do not affect the equilibrium degree of VC market competition θ_2^*.*

Firm-level Effect: *The characteristics of the angel market do not affect the equilibrium valuation of late stage start-ups V_2^*.*

The main driver of these results is the forward feedback loop, i.e., the fact that the angel market generates the deal flow to the VC market as show by equation (7). We know from Proposition 1 that the early stage matching efficiency ϕ_1 has a positive effect on these two equilibrium variables, while the direct and indirect costs for angels, σ_1^A and k_1, have a negative effect. In equilibrium, a higher inflow of start-ups also encourages more VCs to enter the market. This explains why we obtain identical comparative statics results for m_2^{V*} and m_2^{E*}. We also discussed in Section 4.3 why the direct and indirect search costs for entrepreneurs, σ_1^E and δ_1, have an ambiguous effect on the number of angel investments, which naturally extends to the inflow of start-ups into the VC market.

Another important insight from Proposition 4 is that while the characteristics of the angel market determine the absolute inflow of start-ups and VCs into the late stage market, they do not affect the equilibrium ratio of investors to companies (θ_2^*). In other words, when the inflow of start-ups increases by x percent, then the number of VCs entering the market also increases by x percent, so that the investor/company ratio remains constant in equilibrium. As a consequence, angel market parameters do not affect the valuation of late stage companies (V_2^*), which depends on the level of competition, and not on the size, of the VC market.[17]

[17]Naturally one may ask what additional assumptions are required to obtain a model where the angel market parameters directly affect the level of VC competition. It turns out that such models are no longer tractable. The main problem is that search models have tractable solutions as long as types are homogenous. Suppose we let choices in the angel market (e.g., entrepreneurial effort) affect VC market variables other than market size (e.g., the profitability at the VC stage). Any first-order condition at the angel stage then has to allow for the possibility that the optimizing entrepreneur will differ from all other entrepreneurs at the VC stage. This means that one would have to solve a search model with heterogeneous types, which is well beyond the scope of this paper.

6 Angel Protection

6.1 Bargaining and Deal Values

We already noted that one of the fundamental problems for angel investors is their limited bargaining power at the VC stage. Their investments are sunk and they cannot finance the deals by themselves. At the bargaining table they solely rely on their contractual rights, especially on their right to refuse the VCs' deal. So far the model assumes that angels enjoy full legal protection, in the sense that they can always prevent the entrepreneur from pursuing the venture without them. In this section we allow for the possibility that the entrepreneur colludes with the VC to exclude the angel investor by implementing the growth option in a new venture that the angel is not part of. From the VCs' perspective, such an exclusion is an opportunistic exercise of their market power. From the entrepreneurs' perspective, this essentially constitutes a hold-up of their original angel investors.

We model the hold-up opportunity in the following way. Suppose the initial investment was successful, and the venture has a growth opportunity which makes it attractive for VC financing (this happens with probability $\rho_1 g$). The angel has a legal stake in the company, but is not needed for the remaining value creation process. The entrepreneur may now consider closing down the existing company and incorporating a new venture to implement the growth option, thereby excluding the angel investor. This new venture still needs the late stage investment k_2 from a VC. Closing down the existing venture and starting a new venture is obviously not without challenges. Most notably, the angel could mount a legal claim that the growth option belongs to the existing venture. If starting such a new venture is inefficient, the entrepreneur and VC do not actually start a new venture in equilibrium. However, the threat of doing so improves their bargaining position. This changes the payoffs at the VC stage, which has repercussions for the entire equilibrium across both the angel and the VC market.

In the model we define $\lambda\pi$ as the total surplus that an entrepreneur and VC can obtain when excluding the angel investor by implementing the growth option in a newly incorporated venture. The parameter $\lambda \in [0, 1]$ measures how dispensable the angel is for the late stage value creation process, and therefore measures the hold-up power of the entrepreneur. For example, if the business model is based on a patented idea, and the patent belongs to the original firm, then the entrepreneur cannot incorporate a new venture based on the same idea without the angel. The angel is then completely indispensable, so that $\lambda = 0$; this was the assumption in the base model. However, if the business idea cannot be fully protected by patents or contracts, the entrepreneur can hold up the angel by incorporating the new venture, generating a surplus $\lambda\pi$, with $\lambda \in (0, 1)$. The angel is completely dispensable for $\lambda = 1$.

Consider now the case of $\lambda \in (0,1)$ where angel protection is imperfect. The threat of a new incorporation weakens the angel's bargaining position, and forces him to agree to a lower equity stake (β^A) compared to our base model with $\lambda = 0$. Formally, for the Shapley value the joint surplus for the entrepreneur-VC subcoalition (which excludes the angel investor) is now given by $\lambda\pi$. In the Appendix we derive the following new deal values (and related new equity shares) for the late stage market:

$$\begin{aligned} D_2^E &= \frac{1}{6}\left[2+\lambda_A\right]\pi + \frac{1}{6}\left[U_2^E + U_2^A\right], \quad D_2^A = \frac{1}{3}\left[1-\lambda_A\right]\pi + \frac{1}{6}\left[U_2^E + U_2^A\right], \\ D_2^V &= \frac{1}{6}\left[2+\lambda_A\right]\pi - \frac{1}{3}\left[U_2^E + U_2^A\right]. \end{aligned}$$

For now fix the expected joint utility from search for the entrepreneur and angel ($U_2^E + U_2^A$). We can then see that weaker angel protection (higher λ) leads to a lower deal value for the angel (D_2^A), and to higher deal values for the entrepreneur (D_2^E).[18] For the VC there is no prior contractual relationship with the angel, so we cannot talk of hold-up power. Still, lower angel protection improves the VC's bargaining position and leads to a higher deal value (D_2^V). Of course, the equilibrium effect of λ is more complex, as λ also affects the outside options of entrepreneurs and angels (U_2^E, U_2^A).

6.2 Results

The next proposition sheds light on how the level of angel protection affects the angel and VC market equilibrium.

Proposition 5 *The effect of late stage hold-up of angels, as measured by λ, is as follows:*

(i) ***Early stage:*** *The equilibrium inflow of entrepreneurs m_1^{E*}, the inflow of angels m_1^{A*} (and therefore the number of early stage deals x_1^*), the early stage degree of competition θ_1^*, and the equilibrium valuation of early stage start-ups V_1^*, are all decreasing in λ. In contrast, the equilibrium success rate of angel investments $\rho_1(e_1^*)$ is increasing in λ.*

(ii) ***Late stage:*** *The late stage degree of competition θ_2^* is increasing in λ, while the valuation of late stage start-ups V_2^* is decreasing. The effect of λ on the equilibrium inflow of start-up companies m_2^{E*} and inflow of VCs m_2^{V*} is ambiguous.*

[18] Interestingly we note that D_2^A remains positive even as $\lambda_A \to 1$. This is true in our model because the entrepreneur still needs the angel to find an alternative VC. This search role preserves some of the angel's bargaining power.

The analysis of angel protection highlights the core tension in the relationship between angels and VCs, as summarized by Holstein's quote in the introduction. At the individual deal level there is a relationship of 'foes', where VCs are happy to collude with entrepreneurs to weaken the angels' bargaining position. Yet for the market level there is also a need for 'friendly' relations, as VCs collectively rely on the angel market for their deal flow. A similar observation pertains to the relationship between angels and entrepreneurs, where at the ex-post deal level, entrepreneurs may like the idea of taking advantage of their angel investors. However, from an ex-ante perspective, this behavior makes it less attractive for angels to invest in the first place, undermining the collective self-interest of entrepreneurs.

Proposition 5 generates several important insights into the net effects of these competing forces. Consider first the relationship between angels and entrepreneurs. We already saw that weaker angel protection (higher λ) provides entrepreneurs with higher deal values at the VC stage. By itself this should encourage more of them to enter the early stage market. Then, why do fewer entrepreneurs choose to enter the market in equilibrium? The reason for this is the diminished supply of angel capital. Weaker protection at the VC stage makes angel investing less attractive, so fewer angels enter the market. This means that each entrepreneur needs to search on average longer to secure start-up financing, with more ventures dying before ever raising any angel capital. This makes entry less attractive for entrepreneurs. The key insight from Proposition 5 is that the ex-post rent-capture effect is dominated by the ex-ante effect of a thinner angel market. In equilibrium we observe not only less entry by angels, but also less entry by entrepreneurs. Moreover, the angel market is less competitive with longer fundraising cycles.

One might have expected that a less competitive angel market would have resulted in lower valuations and weaker entrepreneurial incentives. Proposition 5 indeed finds that higher λ leads to lower angel valuations. This is because angels are compensated for the expected hold-up by receiving larger equity stakes upfront (i.e., higher α^*). However, we also find that a higher λ actually strengthens entrepreneurial incentives (i.e., higher e_1^*), and therefore also improves the success rates at the angel stage (i.e., higher $\rho_1(e_1^*)$). The reason is that entrepreneurs are only partly motivated by their payoffs in case of liquidation (given by $(1-\alpha^*)y_1$). The other relevant payoff is in case of entering the VC market, where a higher λ gives them a higher ex-post deal value (D_2^E). Our analysis shows that the latter effect dominates the former. We therefore obtain the surprising conclusion that while weaker angel protection reduces entrepreneurial entry, it actually increases entrepreneurial incentives and success rates at the angel stage.

One interesting consequence of this tension between lower entry rates and higher success rates is that the effect of λ on the outflow into the VC market (as measured by m_2^{E*}) is ambiguous. This is precisely because λ has a negative on the size of the angel market x_1^*, but a positive effect on the success rate $\rho_1(e_1^*)$. We can identify cases where one effect dominates the other. If, for example, entrepreneurial incentives are unimportant so that ρ_1' is close to zero, then the market size effect always dominates, and m_2^{E*} is decreasing in λ. However, if entrepreneurial incentives are important, the supply of entrepreneurs is sufficiently inelastic, and there is little urgency (δ_1 close to zero), then the market size effect is small; the success effect dominates, and m_2^{E*} becomes increasing in λ.

We are now in a position to understand the effect of angel protection on the VC market. With weaker angel protection (i.e., higher λ), VCs can strike better deals, so investing in late stage start-ups becomes more attractive. This is a 'foes' effect and encourages entry into VC. However, there is also a 'friends' effect through the supply of new deals into the VC market (m_2^{E*}). As noted in the previous paragraph, the effect of λ on m_2^{E*} is ambiguous, so we cannot sign the effect of λ on x_2^* in general. For the particular case with little urgency and an inelastic supply of entrepreneurs, both the 'foes' and the 'friends' effect point in the same direction, so that the VC market size x_2^* is increasing in λ. For all other cases, the 'foes' and 'friends' effects point in different directions, and therefore the effect of λ on x_2^* depend largely on the elasticity of entrepreneurial entry.

While the effect of λ on the size of the VC market remains ambiguous, Proposition 5 shows that our model generates the unambiguous result that weaker angel protection makes the VC market more competitive. The key intuition is that the degree of competition is a relative measure, and does not depend on the level of inflows into the VC market (m_2^{E*}). Instead, the level of competition is driven by the modified late stage deal values in Section 6.1, with higher rents for VCs attracting more VC entry per unit of deal inflow.

Proposition 5 finally shows that weaker angel protection also leads to lower VC valuations. The main intuition is simply that weaker angel protection gives the VCs more bargaining power. Note, however, that this is a different rationale from the lower valuations in the angel market, which was because angels are being compensated for future hold-up problems.

Overall we note that the level of angel protection has profound and sometimes surprising effects on all aspects of the entire equilibrium, both in the angel and the VC market.

7 Early Exits

So far our analysis takes it for granted that angels always want to bring their companies to the VC market. However, some angels argue that it is better to avoid the VC market altogether, and instead take an early exit (see Peters, 2009). Egan (2014) also shows that start-up companies may redirect their technology strategies towards getting acquired when opportunities for follow-on funding diminish. In this section we consider the endogenous choice of angels and entrepreneurs to take the more risky option of seeking VC financing, versus the safer option of selling the company at an early stage.

To model the endogenous choice between entering the VC market versus an early exit, we consider the simplest possible model extension. In our base model we assumed that a successful project faces one of two scenarios: either the company has a growth option, in which case it is always optimal to raise VC funding; or the company does not have a growth option, in which case it is liquidated (generating the payoff y_1). In this section we augment our base model by allowing entrepreneurs and angels to choose between two different commercialization strategies. After achieving success with the initial investment (which happens with $g\rho_1$), entrepreneurs and angels can either choose the safe strategy of selling what they have and taking the liquidation value y_1 (early exit). Or they can pursue the risky option of developing their growth option and seeking out VC financing – see Figure 1. We assume that the entrepreneur and angel receive a signal about the success probability of developing a growth option. Let γ be the probability that the venture develops the growth option and becomes ready for VC financing. However, with probability $1 - \gamma$ the growth option does not work out, so that the venture fails and is worth nothing. That is, by exploiting growth opportunities, the entrepreneur and angel forgo the chance to sell a more market-ready project at price y_1. In our base model we essentially assumed that $\gamma = 1$. Now we assume that γ is a random variable with some distribution $\Omega(\gamma)$ and support $[0, 1]$. For simplicity we assume that the realization of γ is verifiable.[19]

The entrepreneur and angel investor agree ex-ante on an optimal γ^*, so that they choose an early exit if and only if $\gamma \leq \gamma^*$. It is easy to see that γ^* satisfies $\gamma^* = y_1/(U_2^E + U_2^A)$. A higher value of γ^* means that the entrepreneur and angel are more likely to choose the safe over the risky option. The next proposition examines how γ^* depends on the characteristics of the VC market.

Proposition 6 *The project choice threshold γ^* is increasing in δ_2, σ_2, σ_2^V and k_2, and decreasing in ϕ_2.*

Proposition 6 establishes how the commercialization strategies of entrepreneurs are affected by the structure of the VC market. Entrepreneurs and angels avoid the VC market more often when securing follow-on investments is more costly (higher σ_2 and δ_2). The same applies when fewer VCs search for investment opportunities (due to higher σ_2^V and k_2), so that on average entrepreneurs and angels search longer before receiving VC. In equilibrium we observe more

[19] Assuming that the signal is observable but not verifiable adds some technical complications, but does not affect the main insight. With observability, the entrepreneur and angel can always renegotiate an inefficient decision. The main issue is that the entrepreneur is wealth constrained, which constrains renegotiation choices. This renegotiation closely resembles the analysis in Hellmann (2006).

small entrepreneurial projects, and fewer risky projects that rely on large scale (VC) investments.

A related and interesting question is how the level of angel protection affects the early stage project choice. Consider again our model extension from Section 6, where the parameter λ measures the entrepreneur's late stage hold-up power. We obtain the following result:

Proposition 7 *More severe hold-up of angels in the late stage market leads to more safe projects being implemented in equilibrium (i.e., $d\gamma^*/d\lambda > 0$).*

Proposition 7 shows how the level of angel protection affects the commercialization strategies of early stage ventures. We know from the modified late stage deal values in Section 6.1 that, ceteris paribus, less protection means that angels get a smaller and entrepreneurs a larger share of the total surplus π. We formally show in the Appendix (see Proof of Proposition 5) that this translates into a lower expected utility for angels (U_2^A), and a higher expected utility for entrepreneurs (U_2^E). However, VCs also capture a part of the surplus from angels, so that the net effect on the joint utility of angels and entrepreneurs ($U_2^A + U_2^E$) is negative. This implies that the anticipation of more severe hold-up of angels in the late stage market discourages owners of early stage ventures to exploit growth opportunities through raising VC.

Overall we find that the commercialization strategies of entrepreneurs, and therefore the timing of exit, is influenced by VC market parameters, such as the level of competition in the VC market, or the strength of angel protection.

8 Empirical Predictions

Our theory has a large number of comparative statics results. In principle all of these generate empirical predictions. In this section we explore in depth those predictions that are empirically most relevant.

8.1 Dependent Variables

Our model makes predictions about several endogenous outcomes that all lend themselves to be used as dependent variables. At the market level, our model makes predictions about the size of the angel and VC market, i.e. the rate at which angels and VCs make investments. Such market activities are regularly tracked by governments and commercial data providers. While there are important measurement challenges with tracking private deals, electronic data collection has vastly improved the quality of such data in recent years. Our model also endogenizes the number of entrepreneurs, distinguishing between those that are seeking funds and those that actually succeed in raising funds. On electronic matchmaking platforms, such as Angellist, it is now possible to empirically distinguish between companies merely seeking versus actually finding investments.

Another important endogenous variable is the level of competition in the angel and the VC market. The most immediate measure is the ratio of investors willing to invest, relative to the number of entrepreneurs seeking funds. The challenge is that this requires estimates for the number of potential investors, as opposed to actual investors. As mentioned above, it is sometimes possible to measure the number of entrepreneurs seeking funds. However, estimating the number of investors that are truly willing to invest remains challenging. Intriguingly, our search model also suggests an alternative measurement approach. As shown in Sections 4.2 and 5.2, the level of competition in a market is inversely related to the length of fundraising cycles, i.e., the expected time to complete a fundraising campaign. This is simply because for a given level of market transparency, greater competition implies shorter expected search time. The interesting observation is that online search markets naturally record the time it takes to raise funds on the platform. This might therefore be used as a proxy for the level of competition.

Beyond this market-level analysis, our model also generates predictions at the company level. In particular, the model allows for endogenous pricing of deals, expressed as post-money valuations. These valuations in turn imply equity stakes for the entrepreneurs, which can be used as a measure for entrepreneurial incentives. In addition, our model generates predictions about entrepreneurial outcomes, namely the rate at which start-ups survive, the rate at which they move from the angel to the VC market, and the rate at which they experience early or late exits. All of these are standard measurable outcomes.

8.2 Key Independent Variables and Their Predictions

Our theory contains a large number of exogenous model parameters. We focus here on those that are particularly relevant for empirical evaluations. Returning to the arguments from the introduction, we consider (i) variations in the strength of angel protection, as measured by λ ('angel investors got smarter') (ii) variations in the cost of starting a business, as measured by k_1 ('angel investing got cheaper') and (iii) variations in the transparency of angel markets, as measured by ϕ_1 ('angel investing got easier'). We also consider two additional sets of predictions, one concerning the cost of venture capital (σ_2^V), and one concerning the urgency of entrepreneurial opportunities (δ_1 and δ_2).

8.2.1 Angel Protection

Consider first the role of angel protection. Our model makes the following key predictions: Better angel protection leads to a larger and more competitive angel market, more entrepreneurial entry, higher angel valuations, and a higher probability of success. The effect on the size of the VC market is ambiguous, but valuations are higher and the VC market becomes less competitive. Finally, better angel protection encourages late over early exits.

The main empirical challenge for testing these predictions is to find credible proxies for angel protection. We propose three different approaches. First, one can examine cross-country variations in the quality of legal protection. A large prior literature focuses on the legal origin in terms of common versus civil law, as well as indices of legal enforcement and minority shareholder protection (see Glaeser and Shleifer (2002) and La Porta et al. (2000)).[20] Second, within countries, entrepreneurs' hold-up power is likely to differ across industry sectors. For example, ideas in the software sector are likely to be more portable across ventures, whereas ideas in the life sciences tend to be associated with patents and physical devices that can be better protected. Third, even within sectors there is likely a variation in the relative importance of contractible assets (e.g., physical assets, patents) versus "non-contractible"assets (e.g., non-patented ideas, customer knowledge, entrepreneurial skills). For each of these three approaches it is possible to exploit cross-sectional (and possibly time-varying) variations in the level of angel protection.

8.2.2 Start-up Costs

Consider now the role of start-up costs. Our model predicts that lower seed investment costs lead to a larger and more competitive angel market, more entrepreneurial entry, and a higher probability of success. The VC market also becomes larger.

For the empirical measurement we can imagine several approaches. One simple approach is to directly exploit the variation in the size of seed investment rounds to look at the role of start-up costs. Another approach is to exploit time variation. Ewens, Nanda and Rhodes-Kropf (2014), for example, argue that the introduction of Amazon's Elastic Compute Cloud (EC2) services lowered the cost of starting a company. Yet another approach is to combine the time variation with some cross-sectional variations. For example, Amazon's EC2 is likely to have a bigger effect on web-based start-ups rather than 'bricks and mortar' start-ups.

[20] A related but different approach is to focus on cross-country or inter-state differences in the strength of intellectual property protection. The main hypothesis is that stronger IP protection better protects angels from hold-up by entrepreneurs. Hyde (1998) discusses differences in the enforcement of trade secrets; the work of Gilson (1999) and Marx, Strumsky and Fleming (2009) emphasizes the role of non-compete agreements.

8.2.3 Market Transparency

Next we consider the role of market transparency. Our theory predicts that entry by entrepreneurs and angels both increase as angel markets become more transparent. There is a net increase in competition, angel valuations rise, and the probability of success goes up. The size of the VC market also increases. If transparency also increases in the VC market, our theory also predicts an increase in VC competition and valuations.

The advent of online investment platforms provides a unique natural experiment for testing these predictions. We can think of two alternative approaches. One approach is to consider a market before and after the adoption of online platforms. This approach might also leverage regional variations in the speed of adoption of online platforms. A second approach is to consider variations in transparency within an online platform. Online platforms themselves are evolving in the way they work, changing their rules of what information can or has to disclosed.

8.2.4 Entrepreneurial Urgency

We augment the standard search model with the urgency parameter δ_1 (δ_2) that measures the probability that a new venture fails before raising angel (VC) funding. The most interesting prediction is that greater urgency leads to a more competitive angel market with shorter fundraising cycles, yet it also leads to lower angel valuations. This stands in contrast to the results about greater transparency, where the model predicted *higher* valuations alongside a more competitive angel market. The same observation applies to the VC market. Cross-market effects are also interesting. Greater urgency at the VC stage unambiguously hurts the angel market, resulting in lower entry by entrepreneurs and angels, less competition, lower valuations and lower success rates. By contrast, the effect of greater urgency at the angel market has an ambiguous effect on the size of the VC market. On the one hand fewer entrepreneurs enter, on the other hand fewer ventures fail before getting funded, precisely because of shorter fundraising cycles. The overall effect on the size of the angel market, and thus on the size of the VC market, is ambiguous.

The most direct approach to empirically measure urgency is to look at the failure rate of *unfunded* entrepreneurs. Again, this is much easier to observe in online platforms than in traditional data sources. However, because higher failure rates could also be due to lower competition, it is important to separately control for the level of competition (as discussed in Section 8.1 above). Another more indirect approach would be to look at measures of entry and product market competition in fairly narrowly defined industries. Or one could try to measure the length of product development, or the speed at which products or patents become obsolete. For example, it is widely believed that markets moves faster in software than in the life sciences.

8.2.5 VC Supply Shocks

Finally we can also use the model to examine the impact of changes in the VC market. Historically this market has been experiencing significant supply shocks (see Gompers and Lerner (2001)). We capture this somewhat simplistically as changes in the VCs' costs, as measured by σ_2^V. Higher VC costs naturally lead to a smaller supply of VC funding. The model then shows that this also implies a less competitive VC market. Most importantly, a supply shock to VC leads to a smaller and less competitive angel market. Valuations are also lower in both markets. The model also predicts a relative increase in early over late exits.

We immediately note that the main empirical predictions do not seem to fit the recent market patterns, where a retrenchment from the VC market was accompanied by a rise in angel investing. This suggests that a supply shock to VC is unlikely to be the sole or main driver behind the recent rise of angel markets.

Our model also provides a warning that it is not appropriate to measure VC supply shocks directly from the number of realized deals, because all market sizes are endogenous. Several approaches have been suggested in the prior literature to identify supply shocks to VC, focusing mainly on investment conditions at the limited partner level. Samila and Sorenson (2011), for example, use the returns to local university endowments as an instrument for local VC availability. Mollica and Zingales (2007) adopt a similar approach using the returns of state pension funds in the US.

Overall we note that our theory generates a rich set of empirical predictions. We also provide ample suggestions for how to empirically measure many of the key variables. Any empirical analysis would naturally also have to tackle issues of sample selection and econometric identification. However, that is clearly beyond the scope of this theory paper.

9 Conclusion

In this paper we develop a theory of the interactions between angel and venture capital markets. Entrepreneurs receive their initial funding from angel investors, but may need follow-on funding from venture capitalists. On the one hand the two investor types are 'friends', in the sense that they rely upon each other's investments. On the other hand they are 'foes', because venture capitalists no longer need the angel investors when they make their follow-up investments. The venture capitalists' bargaining power depends on how competitive venture markets are, and how well angels are legally protected. Using a costly search model we establish a joint equilibrium across the two markets, and endogenously derive the size and competitive structure of both markets. We also identify determinants of angel and venture capital valuations, and generate predictions about the rate at which entrepreneurs proceed across the two markets, and whether they choose strategies to exit early or late. We relate the theory to the recent rise of angel investments, exploring alternative explanations concerning the cost of starting a business, the transparency of angel markets, and the ability of angels to better protect their investments.

Our analysis opens doors for further research into the relationship between early and late investors. One limiting assumption of standard search models is that all types are homogenous. Allowing for heterogenous types is technically much more complicated, but it would allow for a richer set of feedback loops, in particular introducing the possibility that entrepreneurs make endogenous decisions at the angel stage that affect the quality of their deals at the VC stage. A different but equally interesting issue is to what extent investors can build reputations and networks that limit the extent of counter-productive hold-up. Finally, our framework raises some interesting policy questions. For instance, if a government wanted to subsidize VC (presumably because of other market failures), would a subsidy to angels be more or less efficient than a subsidy to VCs? We hope that future research, by ourselves and others, will help to illuminate these important set of next questions.

Appendix

Angel Market: Equilibrium Equity Shares and Entrepreneur's Outside Option.

According to the Nash product, α^* is implicitly defined by

$$\frac{dD_1^E(e_1^*)}{d\alpha}D_1^A(e_1^*) + (D_1^E(e_1^*) - U_1^E)\frac{dD_1^A(e_1^*)}{d\alpha} = 0. \tag{12}$$

Applying the Envelope Theorem we find that $dD_1^E(e_1^*)/d\alpha < 0$. We can then infer from (12) that $dD_1^A(e_1^*)/d\alpha > 0$ must hold for $\alpha = \alpha^*$.

Using (12) we can implicitly differentiate α^* w.r.t. U_1^E:

$$\frac{d\alpha^*}{dU_1^E} = \frac{\frac{dD_1^A}{d\alpha}}{\frac{d}{d\alpha}\left[\frac{dD_1^E}{d\alpha}D_1^A + (D_1^E - U_1^E)\frac{dD_1^A}{d\alpha}\right]}.$$

Note that the denominator is strictly negative due to the second-order condition for α^*. Moreover, recall that $dD_1^A/d\alpha > 0$. Thus, $d\alpha^*/dU_1^E < 0$.

Derivation of Angel Market Equilibrium.

Using $q_1^A = x_1/M_1^A$ and $x_1 = \phi_1\left[M_1^E M_1^A\right]^{0.5}$, we can write (3) as

$$\phi_1 D_1^A\left[\frac{M_1^E}{M_1^A}\right]^{0.5} = \sigma_1^A. \tag{13}$$

Using $\theta_1 = M_1^A/M_1^E$ we then get the equilibrium degree of competition for the angel market: $\theta_1^* = \left[\phi_1 D_1^A/\sigma_1^A\right]^2$. Next, note that we can write (13) as

$$M_1^A = M_1^E\left[\frac{\phi_1\widetilde{D}_1^A}{\sigma_1^A}\right]^2 = M_1^E\theta_1.$$

Solving (5) for M_1^E and using $q_1^E = \phi_1\left[M_1^E M_1^A\right]^{0.5}/M_1^E = \phi_1\left[M_1^A/M_1^E\right]^{0.5}$, we get the equilibrium stock of entrepreneurs in the early stage market:

$$M_1^{E*} = \frac{F(U_1^E)}{\delta_1 + q_1^E} = \frac{F(U_1^E)}{\delta_1 + \phi_1\left[M_1^A/M_1^E\right]^{0.5}} = \frac{F(U_1^E)}{\delta_1 + \phi_1\sqrt{\theta_1^*}}.$$

Thus, the equilibrium stock of angels is given by

$$M_1^{A*} = M_1^{E*}\theta_1^* = \frac{F(U_1^E)\theta_1^*}{\delta_1 + \phi_1\sqrt{\theta_1^*}}.$$

Using $M_1^{E*} = M_1^{A*}/\theta_1^*$ we can then write x_1^* as

$$x_1^* = \phi_1\left[M_1^{A*}M_1^{E*}\right]^{0.5} = \frac{\phi_1 M_1^{A*}}{\sqrt{\theta_1^*}} = F(U_1^E)\frac{\phi_1\sqrt{\theta_1^*}}{\delta_1 + \phi_1\sqrt{\theta_1^*}}.$$

Moreover, using (6) and $q_1^A = x_1/M_1^A$ we get $m_1^{A*} = q_1^A M_1^{A*} = x_1^*$.

Proof of Proposition 1.

Recall that the equilibrium of the angel market is determined by the deal values D_1^E and D_1^A, and therefore by the late stage utilities U_2^E and U_2^A, as well as by the entrepreneur's outside option U_1^E (through α^*). We will show in Proof of Proposition 4 that U_2^E and U_2^A do not depend on ϕ_1, δ_1, σ_1^E, σ_1^A, and k_1. Next we need to derive a condition which defines U_1^E. The equilibrium condition (2) can be written as

$$U_1^E\left[r+\delta_1\right] = -\sigma_1^E + q_1^E\left[D_1^E - U_1^E\right].$$

Using $q_1^E = \phi_1\left[M_1^{A*}/M_1^{E*}\right]^{0.5} = \phi_1\sqrt{\theta_1^*} = \phi_1^2 D_1^A/\sigma_1^A$ we get the following condition which defines U_1^E:

$$U_1^E\left[r+\delta_1\right] - \frac{\phi_1^2}{\sigma_1^A}D_1^A\left[D_1^E - U_1^E\right] + \sigma_1^E = 0. \tag{14}$$

Now consider the equilibrium degree of competition θ_1^*. Differentiating θ_1^* w.r.t. δ_1 yields

$$\frac{d\theta_1^*}{d\delta_1} = 2\frac{\phi_1^2 D_1^A}{\left[\sigma_1^A\right]^2}\frac{dD_1^A}{d\delta_1} = 2\frac{\phi_1^2 D_1^A}{\left[\sigma_1^A\right]^2}\frac{dD_1^A}{d\alpha}\frac{d\alpha^*}{dU_1^E}\frac{dU_1^E}{d\delta_1}.$$

Next we define $\Gamma \equiv D_1^A\left[D_1^E - U_1^E\right]$. We then get

$$\frac{dU_1^E}{d\delta_1} = -\frac{U_1^E}{r+\delta_1 - \frac{\phi_1^2}{\sigma_1^A}\left[\frac{d\Gamma}{d\alpha}\frac{d\alpha^*}{dU_1^E} + \frac{\partial\Gamma}{\partial U_1^E}\right]}.$$

Note that $d\Gamma/d\alpha = 0$ due to the first-order condition for α^*. Moreover, $\partial\Gamma/\partial U_1^E = -D_1^A$. Consequently,

$$\frac{dU_1^E}{d\delta_1} = -\frac{U_1^E}{r + \delta_1 + \frac{\phi_1^2}{\sigma_1^A} D_1^A} < 0.$$

This in turn implies that $d\theta_1^*/d\delta_1 > 0$. Likewise,

$$\frac{d\theta_1^*}{d\sigma_1^E} = 2\frac{\phi_1^2 D_1^A}{\left[\sigma_1^A\right]^2} \frac{dD_1^A}{d\alpha} \frac{d\alpha^*}{dU_1^E} \frac{dU_1^E}{d\sigma_1^E},$$

with $dD_1^A/d\alpha > 0$, $d\alpha^*/dU_1^E < 0$, and

$$\frac{dU_1^E}{d\sigma_1^E} = -\frac{1}{r + \delta_1 + \frac{\phi_1^2}{\sigma_1^A} D_1^A} < 0.$$

Thus, $d\theta_1^*/d\sigma_1^E > 0$. Moreover, note that $dD_1^A/dk_1 < 0$. Consequently, $d\theta_1^*/dk_1 < 0$. For the remaining comparative statics it is useful to express the condition for U_1^E in terms of θ_1^*:

$$U_1^E\left[r + \delta_1\right] - \phi_1\sqrt{\theta_1^*}\left[D_1^E - U_1^E\right] + \sigma_1^E = 0,$$

so that

$$\frac{dU_1^E}{d\theta_1^*} = \frac{\phi_1 \frac{1}{2\sqrt{\theta_1^*}}\left[D_1^E - U_1^E\right]}{r + \delta_1 + \phi_1\sqrt{\theta_1^*}} > 0.$$

Moreover, using the definition of θ_1^* we define

$$G \equiv \theta_1^* - \left[\frac{\phi_1}{\sigma_1^A} D_1^A\right]^2 = 0 \tag{15}$$

where $D_1^A = D_1^A(\alpha^*(U_1^E(\theta_1^*)))$. We get

$$\frac{d\theta_1^*}{d\phi_1} = \frac{2\frac{\phi_1}{\left[\sigma_1^A\right]^2}\left[D_1^A\right]^2}{1 - 2\left[\frac{\phi_1}{\sigma_1^A}\right]^2 D_1^A \frac{dD_1^A}{d\alpha} \frac{d\alpha^*}{dU_1^E} \frac{dU_1^E}{d\theta_1^*}}.$$

Recall that $dD_1^A/d\alpha > 0$, $d\alpha^*/dU_1^E < 0$, and $dU_1^E/d\theta_1^* > 0$. Thus, the denominator is positive, which implies that $d\theta_1^*/d\phi_1 > 0$. Likewise, using (15), we get

$$\frac{d\theta_1^*}{d\sigma_1^A} = -\frac{2\frac{\phi_1^2}{\left[\sigma_1^A\right]^3}\left[D_1^A\right]^2}{1 - 2\left[\frac{\phi_1}{\sigma_1^A}\right]^2 D_1^A \frac{dD_1^A}{d\alpha}\frac{d\alpha^*}{dU_1^E}\frac{dU_1^E}{d\theta_1^*}}.$$

Again, the denominator is positive, which implies that $d\theta_1^*/d\sigma_1^A < 0$.

Next, note that $dm_1^{E*}/dU_1^E = F'(U_1^E) > 0$, and recall that $dU_1^E/d\delta_1$, $dU_1^E/d\sigma_1^E < 0$. Moreover, using (14) we find

$$\begin{aligned}\frac{dU_1^E}{d\phi_1} &= \frac{2\frac{\phi_1}{\sigma_1^A}D_1^A\left[D_1^E - U_1^E\right]}{r + \delta_1 + \frac{\phi_1^2}{\sigma_1^A}D_1^A} > 0\\ \frac{dU_1^E}{d\sigma_1^A} &= -\frac{\frac{\phi_1^2}{\left[\sigma_1^A\right]^2}D_1^A\left[D_1^E - U_1^E\right]}{r + \delta_1 + \frac{\phi_1^2}{\sigma_1^A}D_1^A} < 0.\end{aligned}$$

Likewise, using $\Gamma = D_1^A\left[D_1^E - U_1^E\right]$,

$$\frac{dU_1^E}{dk_1} = \frac{\frac{\phi_1^2}{\sigma_1^A}\frac{d\Gamma}{dk_1}}{r + \delta_1 + \frac{\phi_1^2}{\sigma_1^A}D_1^A},$$

with

$$\frac{d\Gamma}{dk_1} = \underbrace{\frac{d\Gamma}{d\alpha}}_{=0}\frac{d\alpha^*}{dk_1} + \frac{\partial\Gamma}{\partial k_1} = -\left[D_1^E - U_1^E\right] < 0.$$

Thus, $dU_1^E/dk_1 < 0$. All this implies that m_1^{E*} is increasing in ϕ_1, and decreasing in δ_1, σ_1^E, σ_1^A, and k_1.

Next, recall that $m_1^{A*} = x_1^*$ is given by

$$m_1^{A*} = x_1^* = F(U_1^E)\underbrace{\frac{\phi_1\sqrt{\theta_1^*}}{\delta_1 + \phi_1\sqrt{\theta_1^*}}}_{\equiv T}.$$

It is straightforward to show that $dT/d(\phi_1\sqrt{\theta_1^*}) > 0$. Because $dU_1^E/d\phi_1 > 0$ and $d\theta_1^*/d\phi_1 > 0$, we then have $dm_1^{A*}/d\phi_1 = dx_1^*/d\phi_1 > 0$. Likewise, we know that $dU_1^E/d\sigma_1^A$, $dU_1^E/dk_1 < 0$ and $d\theta_1^*/d\sigma_1^A$, $d\theta_1^*/dk_1 < 0$. Thus, $dm_1^{A*}/d\sigma_1^A = dx_1^*/d\sigma_1^A < 0$ and $dm_1^{A*}/dk_1 = dx_1^*/dk_1 < 0$.

Moreover, we have shown that $dU_1^E/d\delta_1$, $dU_1^E/d\sigma_1^E < 0$, while $d\theta_1^*/d\delta_1$, $d\theta_1^*/d\sigma_1^E > 0$. Thus, the effects of δ_1 and σ_1^E on $m_1^{A*} = x_1^*$ are ambiguous.

Now consider the equilibrium valuation V_1^*. Note that V_1^* is decreasing in the angel's equilibrium equity share α^*, which is defined by (12). Recall that $d\alpha^*/dU_1^E < 0$, $dU_1^E/d\phi_1 > 0$ and $dU_1^E/d\delta_1$, $dU_1^E/d\sigma_1^E$, $dU_1^E/d\sigma_1^A < 0$. Consequently, $d\alpha^*/d\phi_1 < 0$ and $d\alpha^*/d\delta_1$, $d\alpha^*/d\sigma_1^E$, $d\alpha^*/d\sigma_1^A > 0$. All this implies that V_1^* is increasing in ϕ_1, and decreasing in δ_1, σ_1^E and σ_1^A. Furthermore, note that k_1 affects D_1^A and U_1^A. Using (12) we get

$$\frac{d\alpha^*}{dk_1} = -\frac{\frac{dD_1^E}{d\alpha}\frac{\partial D_1^A}{\partial k_1} - \frac{dU_1^E}{dk_1}\frac{dD_1^A}{d\alpha} + (D_1^E - U_1^E)\frac{d^2 D_1^A}{d\alpha dk_1}}{\frac{d}{d\alpha}\left[\frac{dD_1^E}{d\alpha}D_1^A + (D_1^E - U_1^E)\frac{dD_1^A}{d\alpha}\right]},$$

where the denominator is strictly negative due to the second-order condition for α^*. Thus, to prove that $d\alpha^*/dk_1 > 0$, we need to show that the numerator is positive. We know that $dD_1^E/d\alpha < 0$, $dD_1^A/d\alpha > 0$, and $dU_1^E/dk_1 < 0$. Moreover, $\partial D_1^A/\partial k_1 = -1$ and $d^2 D_1^A/(d\alpha dk_1) = 0$. Thus, the numerator is strictly positive, so that $d\alpha^*/dk_1 > 0$. This in turn implies that the effect of k_1 on $V_1^* = k_1/\alpha^*$ is ambiguous.

Finally consider the equilibrium success probability $\rho_1(e_1^*)$, with $\rho_1'(e_1^*) > 0$. Using (1) we get

$$\frac{de_1^*}{d\alpha} = \frac{\rho_1'(e_1)(1-q)y_1}{\frac{d}{de_1}\left[\rho_1'(e_1)\left[qU_2^A + (1-q)(1-\alpha)y_1\right] - c'(e_1)\right]},$$

where the denominator is strictly negative due to the second-order condition for e_1^*. Thus, $de_1^*/d\alpha < 0$. Our comparative statics results for α^* then imply that $d\rho_1(e_1^*)/d\phi_1 > 0$ and $d\rho_1(e_1^*)/d\delta_1$, $d\rho_1(e_1^*)/d\sigma_1^E$, $d\rho_1(e_1^*)/d\sigma_1^A$, $d\rho_1(e_1^*)/dk_1 < 0$. □

Proof of Proposition 2.

In equilibrium, $U_2^E = U_2^A$. Moreover, we will show in Proof of Proposition 3 that $dU_2^E/d\phi_2 > 0$ and $dU_2^E/d\delta_2$, $dU_2^E/d\sigma_2$, $dU_2^E/d\sigma_2^V$, $dU_2^E/dk_2 < 0$. Consider the equilibrium degree of competition θ_1^*. With $U_2^E = U_2^A$ note that

$$\frac{d\theta_1^*}{dU_2^E} = 2\frac{\phi_1^2}{\left[\sigma_1^A\right]^2}D_1^A\frac{dD_1^A}{dU_2^A}.$$

For a given α we find that

$$\frac{dD_1^A}{dU_2^E} = \rho_1'(e_1^*)\frac{de_1^*}{dU_2^E}\left[gU_2^E + (1-g)\alpha y_1\right] + \rho_1(e_1^*)g > 0.$$

Moreover, applying the Envelope Theorem we get $dD_1^E/dU_2^E = g\rho_1(e_1^*) > 0$. Thus, the bargaining frontier shifts outwards, so that $dD_1^E/dU_2^E > 0$ and $dD_1^A/dU_2^E > 0$ at the equilibrium equity share α^*. This implies that $d\theta_1^*/dU_2^E > 0$, and consequently, $d\theta_1^*/d\phi_2 > 0$ and $d\theta_1^*/d\delta_2$, $d\theta_1^*/d\sigma_2$, $d\theta_1^*/d\sigma_2^V$, $d\theta_1^*/dk_2 < 0$.

Now consider the equilibrium inflow of entrepreneurs $m_1^{E*} = F(U_1^E)$, with $F'(U_1^E) > 0$. Using (14) we get

$$\frac{dU_1^E}{dU_2^E} = \frac{\frac{\phi_1^2}{\sigma_1^A}\frac{d\Gamma}{dU_2^E}}{r + \delta_1 + \frac{\phi_1^2}{\sigma_1^A}D_1^A},$$

where $\Gamma = D_1^A\left[D_1^E - U_1^E\right]$. Note that

$$\frac{d\Gamma}{dU_2^E} = \frac{d\Gamma}{de_1}\frac{de_1^*}{U_2^E} + \frac{d\Gamma}{d\alpha}\frac{d\alpha^*}{U_2^E} + \frac{\partial\Gamma}{\partial U_2^E},$$

where $de_1^*/dU_2^E > 0$ and $d\Gamma/d\alpha = 0$ (see (12)). Moreover,

$$\frac{d\Gamma}{de_1} = \frac{dD_1^A}{de_1}\left[D_1^E - U_1^E\right] + D_1^A\underbrace{\frac{D_1^E}{de_1}}_{=0} = \rho_1'(e_1^*)\underbrace{\left[gU_2^A + (1-g)\alpha y_1\right]}_{>0}\underbrace{\left[D_1^E - U_1^E\right]}_{>0} > 0$$

$$\frac{\partial\Gamma}{\partial U_2^E} = \underbrace{\frac{\partial D_1^A}{\partial U_2^E}}_{>0}\underbrace{\left[D_1^E - U_1^E\right]}_{>0} + D_1^A\underbrace{\frac{\partial D_1^E}{\partial U_2^E}}_{>0} > 0$$

This implies that $dU_1^E/dU_2^E > 0$, and therefore, $dF(U_1^E)/dU_2^E > 0$. Our comparative statics results for U_2^E (see Proof of Proposition 3) then imply that $dm_1^{E*}/d\phi_2 > 0$ and $dm_1^{E*}/d\delta_2$, $dm_1^{E*}/d\sigma_2$, $dm_1^{E*}/d\sigma_2^V$, $dm_1^{E*}/dk_2 < 0$.

Next consider the equilibrium inflow of angels, m_1^{A*}, which is defined by

$$m_1^{A*} = x_1^* = \underbrace{F(U_1^E)}_{=m_1^{E*}}\underbrace{\frac{\phi_1\sqrt{\theta_1^*}}{\delta_1 + \phi_1\sqrt{\theta_1^*}}}_{\equiv T}.$$

One can show that $dT/d\sqrt{\theta_1^*}) > 0$. Our comparative statics results for m_1^{E*} and θ_1^* then imply that $dm_1^{A*}/d\phi_2 > 0$ and $dm_1^{A*}/d\delta_2$, $dm_1^{A*}/d\sigma_2$, $dm_1^{A*}/d\sigma_2^V$, $dm_1^{A*}/dk_2 < 0$.

Now consider the equilibrium equity share α^* for angels. Recall that $dD_1^E/dU_2^E > 0$ and $dD_1^A/dU_2^E > 0$ at the equilibrium equity share α^*. Moreover, using the Envelope Theorem it is straightforward to show that $dD_1^A/dU_2^E > dD_1^E/dU_2^E$. The Nash bargaining solution then implies that $d\alpha^*/dU_2^E < 0$. Thus, $d\alpha^*/d\phi_2 < 0$ and $d\alpha^*/d\delta_2$, $d\alpha^*/d\sigma_2$, $d\alpha^*/d\sigma_2^V$, $d\alpha^*/dk_2 >$

0. For the equilibrium valuation $V_1^* = k_1/\alpha^*$ we can then infer that $dV_1^*/d\phi_2 > 0$ and $dV_1^*/d\delta_2$, $dV_1^*/d\sigma_2$, $dV_1^*/d\sigma_2^V$, $dV_1^*/dk_2 < 0$.

Finally consider the equilibrium success rate $\rho_1(e_1^*)$, with $\rho_1'(e_1^*) > 0$. Using (1) it is straightforward to show that $\partial e_1^*/\partial U_2^E > 0$ and $\partial e_1^*/\partial \alpha < 0$. Using our comparative statics results for U_2^E and α^* we can then infer that $de_1^*/d\phi_2 > 0$ and $de_1^*/d\delta_2$, $de_1^*/d\sigma_2$, $de_1^*/d\sigma_2^V$, $de_1^*/dk_2 < 0$. Consequently, $d\rho_1(e_1^*)/d\phi_2 > 0$ and $d\rho_1(e_1^*)/d\delta_2$, $d\rho_1(e_1^*)/d\sigma_2$, $d\rho_1(e_1^*)/d\sigma_2^V$, $d\rho_1(e_1^*)/dk_2 < 0$.
□

VC Market: Derivation of Deal Values and Equity Shares.

Let CV_i denote the value generated by the coalition $i = EAV, EV, EA, AV, E, A, V$. Using the Shapley value we get the following general deal values from the tripartite bargaining game:

$$D_2^E = \frac{1}{3}\left[CV_{EAV} - CV_{AV}\right] + \frac{1}{6}\left[CV_{EA} - CV_A\right] + \frac{1}{6}\left[CV_{EV} - CV_V\right] + \frac{1}{3}CV_E \quad (16)$$

$$D_2^A = \frac{1}{3}\left[CV_{EAV} - CV_{EV}\right] + \frac{1}{6}\left[CV_{EA} - CV_E\right] + \frac{1}{6}\left[CV_{AV} - CV_V\right] + \frac{1}{3}CV_A \quad (17)$$

$$D_2^V = \frac{1}{3}\left[CV_{EAV} - CV_{EA}\right] + \frac{1}{6}\left[CV_{EV} - CV_E\right] + \frac{1}{6}\left[CV_{AV} - CV_A\right] + \frac{1}{3}CV_V \quad (18)$$

We note that $CV_{EAV} = \pi$ and $CV_{AV} = CV_{EV} = CV_E = CV_A = CV_V = 0$. Moreover, by assumption we have $U_2^E + U_2^A > y_1$, so that $CV_{EA} = U_2^E + U_2^A$. Thus,

$$D_2^E = \frac{1}{3}\pi + \frac{1}{6}\left[U_2^E + U_2^A\right]$$

$$D_2^A = \frac{1}{3}\pi + \frac{1}{6}\left[U_2^E + U_2^A\right]$$

$$D_2^V = \frac{1}{3}\pi - \frac{1}{3}\left[U_2^E + U_2^A\right]$$

The deal values then allow us to derive the equilibrium equity shares β^{E*}, β^{A*}, and β^{V*}. The equilibrium equity share for entrepreneurs, β^{E*}, ensures that their actual net payoff equals their deal value from the bargaining game: $\beta^{E*}y_2 = D_2^E$. Solving this for β^{E*} yields

$$\beta^{E*} = \frac{D_2^E}{y_2} = \frac{1}{6y_2}\left[2\pi + U_2^E + U_2^A\right].$$

Likewise we get

$$\beta^{A*} = \frac{D_2^A}{y_2} = \frac{1}{6y_2}\left[2\pi + U_2^E + U_2^A\right]$$

$$\beta^{V*} = \frac{k_2 + D_2^V}{y_2} = \frac{1}{3y_2}\left[3k_2 + \pi - \left(U_2^E + U_2^A\right)\right].$$

Derivation of VC Market Equilibrium.

The first part of the derivation follows along the lines of the derivation of the angel market equilibrium: Using (9) we get $\theta_2^* = \left[\phi_2 D_2^V/\sigma_2^V\right]^2$. Moreover, using (10) and the relationship $M_2^{V*} = M_2^{E*}\theta_2^*$ we find

$$M_2^{V*} = g\rho_1(e_1^*)x_1^* \frac{\theta_2^*}{\delta_2 + \phi_2\sqrt{\theta_2^*}}.$$

Using $M_2^{E*} = M_2^{V*}/\theta_2^*$ and the definition of M_2^{V*}, we can write x_2^* as

$$x_2^* = \phi_2\left[M_2^{V*}M_2^{E*}\right]^{0.5} = \frac{\phi_2 M_2^{V*}}{\sqrt{\theta_2^*}} = m_2^{E*}\frac{\phi_2\sqrt{\theta_2^*}}{\delta_2 + \phi_2\sqrt{\theta_2^*}},$$

where $m_2^{E*} = g\rho_1(e_1^*)x_1^*$. Furthermore, using (11) and $q_2^V = x_2/M_2^V$ we find that $m_2^{V*} = q_2^V M_2^{V*} = x_2^*$.

Finally, using the equilibrium equity share β^{V*} for VCs we can write V_2^* as follows:

$$V_2^* = \frac{k_2}{\beta^{V*}} = \frac{k_2 y_2}{k_2 + D_2^V} = \left(\frac{3k_2}{3k_2 + \pi - (U_2^E + U_2^A)}\right)y_2.$$

Proof of Proposition 3.

First we need to derive a condition which defines U_2^E. We can write (8) as

$$U_2^E\left[r + \delta_2\right] = -\sigma_2 + q_2^E\left[D_2^E - U_2^E\right].$$

Note that $D_2^E - U_2^E = \pi/3 - 2U_2^E/3 = D_2^V$. Using $q_2^E = \phi_2\left[M_2^{V*}/M_2^{E*}\right]^{0.5} = \phi_2\sqrt{\theta_2^*} = \phi_2^2 D_2^V/\sigma_2^V$, we get the following condition which defines U_2^E:

$$U_2^E\left[r + \delta_2\right] - \frac{\phi_2^2}{\sigma_2^V}\left[D_2^V\right]^2 + \sigma_2 = 0. \tag{19}$$

Consider the equilibrium degree of competition θ_2^*. Recall that $U_2^A = U_2^E$ in equilibrium; thus,

$$\frac{d\theta_2^*}{dU_2^A} = \frac{d\theta_2^*}{dU_2^E} = 2\frac{\phi_2^2 D_2^V}{\left[\sigma_2^V\right]^2}\frac{dD_2^V}{dU_2^E} = -\frac{4}{3}\frac{\phi_2^2 D_2^V}{\left[\sigma_2^V\right]^2} < 0.$$

Note that δ_2 only affects U_2^E in the definition of θ_2^*. Implicitly differentiating U_2^E w.r.t. δ_2 yields

$$\frac{dU_2^E}{d\delta_2} = -\frac{U_2^E}{r + \delta_2 + \frac{4}{3}\frac{\phi_2^2}{\sigma_2^V}D_2^V} < 0,$$

which implies that $d\theta_2^*/d\delta_2 > 0$. Likewise, σ_2 only affects U_2^E in the definition of θ_2^*. We get

$$\frac{dU_2^E}{d\sigma_2} = -\frac{1}{r + \delta_2 + \frac{4}{3}\frac{\phi_2^2}{\sigma_2^V}D_2^V} < 0.$$

Thus, $d\theta_2^*/d\sigma_2 > 0$. Next, differentiating U_2^E w.r.t. ϕ_2 yields

$$\frac{d\theta_2^*}{d\phi_2} = 2\frac{\phi_2 D_2^V}{\left[\sigma_2^V\right]^2}\left[D_2^V + \phi_2\frac{dD_1^V}{dU_2^E}\frac{dU_2^E}{d\phi_2}\right] = 2\frac{\phi_2 D_2^V}{\left[\sigma_2^V\right]^2}\left[D_2^V - \frac{2}{3}\phi_2\frac{dU_2^E}{d\phi_2}\right],$$

with

$$\frac{dU_2^E}{d\phi_2} = \frac{2\frac{\phi_2}{\sigma_2^V}\left[D_2^V\right]^2}{r + \delta_2 + \frac{4}{3}\frac{\phi_2^2}{\sigma_2^V}D_2^V} > 0.$$

Therefore,

$$\frac{d\theta_2^*}{d\phi_2} = 2\frac{\phi_2 D_2^V}{\left[\sigma_2^V\right]^2}\frac{(r+\delta_2)D_2^V}{r + \delta_2 + \frac{4}{3}\frac{\phi_2^2}{\sigma_2^V}D_2^V} > 0.$$

Likewise,

$$\frac{d\theta_2^*}{d\sigma_2^V} = 2\frac{\phi_2^2 D_2^V}{\sigma_2^V}\frac{1}{\left[\sigma_2^V\right]^2}\left[-\frac{2}{3}\frac{dU_2^E}{d\sigma_2^V}\sigma_2^V - D_2^V\right], \quad \text{with} \quad \frac{dU_2^E}{d\sigma_2^V} = -\frac{\frac{\phi_2^2 D_2^V D_2^V}{\left[\sigma_2^V\right]^2}}{r + \delta_2 + \frac{4}{3}\frac{\phi_2^2}{\sigma_2^V}D_2^V} < 0.$$

Consequently,

$$\frac{d\theta_2^*}{d\sigma_2^V} = -2\frac{\phi_2^2 D_2^V}{\sigma_2^V}\frac{1}{\left[\sigma_2^V\right]^2}\frac{\frac{2}{3}\frac{\phi_2^2}{\sigma_2^V}\left[D_2^V\right]^2 + (r+\delta_2)D_2^V}{r + \delta_2 + \frac{4}{3}\frac{\phi_2^2}{\sigma_2^V}D_2^V} < 0.$$

Moreover, we get

$$\frac{d\theta_2^*}{dk_2} = 2\frac{\phi_2^2 D_2^V}{\left[\sigma_2^V\right]^2}\frac{dD_2^V}{dk_2} = 2\frac{\phi_2^2 D_2^V}{\left[\sigma_2^V\right]^2}\left[-\frac{1}{3} - \frac{2}{3}\frac{dU_2^E}{dk_2}\right], \quad \text{with} \quad \frac{dU_2^E}{dk_2} = -\frac{\frac{2}{3}\frac{\phi_2^2}{\sigma_2^V}D_2^V}{r+\delta_2+\frac{4}{3}\frac{\phi_2^2}{\sigma_2^V}D_2^V} < 0.$$

Thus,

$$\frac{d\theta_2^*}{dk_2} = -\frac{2}{3}\frac{\phi_2^2 D_2^V}{\left[\sigma_2^V\right]^2}\frac{r+\delta_2}{r+\delta_2+\frac{4}{3}\frac{\phi_2^2}{\sigma_2^V}D_2^V} < 0.$$

Next, recall that $m_2^{V*} = x_2^*$ is given by

$$m_2^{V*} = x_2^* = \underbrace{g\rho_1(e_1^*)x_1^*}_{=m_2^{E*}}\underbrace{\frac{\phi_2\sqrt{\theta_2^*}}{\delta_2+\phi_2\sqrt{\theta_2^*}}}_{\equiv T}.$$

We have shown in Proof of Proposition 2 that $dx_1^*/d\phi_2 > 0$ and $dx_1^*/d\delta_2$, $dx_1^*/d\sigma_2$, $dx_1^*/d\sigma_2^V$, $dx_1^*/dk_2 < 0$. Likewise, we have shown that $d\rho_1(e_1^*)/d\phi_2 > 0$ and $d\rho_1(e_1^*)/d\delta_2$, $d\rho_1(e_1^*)/d\sigma_2$, $d\rho_1(e_1^*)/d\sigma_2^V$, $d\rho_1(e_1^*)/dk_2 < 0$. Moreover, it is straightforward to verify that $dT/d(\phi_2\sqrt{\theta_2^*}) > 0$. Using our comparative statics results for θ_2^*, we can infer that $dT/d\phi_2$, $dT/d\delta_2$, $dT/d\sigma_2 > 0$, and $dT/d\sigma_2^V$, $dT/dk_2 < 0$. All this implies that $dm_2^{V*}/d\phi_2 > 0$ and $dm_2^{V*}/d\sigma_2^V$, $dm_2^{V*}/dk_2 < 0$, while the effects of δ_2 and σ_2 on m_2^{V*} are ambiguous.

Now consider the equilibrium late stage valuation V_2^*:

$$V_2^* = \left(\frac{3k_2}{3k_2+\pi-2U_2^E}\right)y_2.$$

Recall that $dU_2^E/d\phi_2 > 0$, and $dU_2^E/d\sigma_2$, $dU_2^E/d\sigma_2^V$, $dU_2^E/d\delta_2 < 0$. Thus, $dV_2^*/d\phi_2 > 0$ and $dV_2^*/d\sigma_2$, $dV_2^*/d\sigma_2^V$, $dV_2^*/d\delta_2 < 0$. Furthermore, recall that V_2^* can also be written as $V_2^* = k_2 y_2/(k_2+D_2^V)$. Taking the first derivative of V_2^* w.r.t. k_2 yields

$$\frac{dV_2^*}{dk_2} = \frac{k_2+D_2^V-k_2\left[1-\frac{1}{3}-\frac{2}{3}\frac{dU_2^E}{dk_2}\right]}{\left[k_2+D_2^V\right]^2}y_2 = \frac{\overbrace{\frac{1}{3}k_2+D_2^V+\frac{2}{3}k_2\frac{dU_2^E}{dk_2}}^{\equiv N}}{\left[k_2+D_2^V\right]^2}y_2.$$

Note that the denominator is always positive. Moreover, we have $N > 0$ for $k_2 \to 0$. Thus, $dV_2^*/dk_2 > 0$ for $k_2 \to 0$. To verify that $dV_2^*/dk_2 > 0$ for all $k_2 > 0$, it is sufficient to show that $dN/dk_2 > 0$:

$$\frac{dN}{dk_2} = \frac{1}{3} - \frac{1}{3} - \frac{2}{3}\frac{dU_2^E}{dk_2} + \frac{2}{3}\left[\frac{dU_2^E}{dk_2} + k_2\frac{d^2U_2^E}{dk_2^2}\right] = \frac{2}{3}k_2\frac{d^2U_2^E}{dk_2^2}.$$

It remains to identify the sign of $d^2U_2^E/dk_2^2$. Using $a_2 \equiv \phi_2^2/\sigma_2^V$ we can write dU_2^E/dk_2 as

$$\frac{dU_2^E}{dk_2} = -\frac{\frac{2}{3}a_2D_2^V}{r + \delta_2 + \frac{4}{3}a_2D_2^V} = -\frac{\frac{2}{3}}{(r+\delta_2)\left[a_2D_2^V\right]^{-1} + \frac{4}{3}}.$$

Thus,

$$\frac{d^2U_2^E}{dk_2^2} = \frac{\frac{2}{9}a_2\left(r+\delta_2\right)\left[a_2D_2^V\right]^{-2}\left[1 + 2\frac{dU_2^E}{dk_2}\right]}{\left[(r+\delta_2)\left[a_2D_2^V\right]^{-1} + \frac{4}{3}\right]^2}.$$

Note that

$$1 + 2\frac{dU_2^E}{dk_2} = 1 - \frac{\frac{4}{3}a_2D_2^V}{r + \delta_2 + \frac{4}{3}a_2D_2^V} = \frac{r+\delta_2}{r + \delta_2 + \frac{4}{3}a_2D_2^V} > 0.$$

Hence, $d^2U_2^E/dk_2^2 > 0$, so that $dN/dk_2 > 0$. Consequently, $dV_2^*/dk_2 > 0$. □

Proof of Proposition 4.

We can see from (19) that U_2^E (and therefore U_2^A) does not depend on the early stage parameters ϕ_1, δ_1, σ_1^E, σ_1^A, and k_1. This also implies that D_2^V, and therefore θ_2^* and V_2^*, do not depend on these parameters.

Now consider the equilibrium inflow of start-ups $m_2^{E*} = g\rho_1(e_1^*)x_1^*$. Recall from Proposition 1 that $dx_1^*/d\phi_1 > 0$ and $dx_1^*/d\sigma_1^A$, $dx_1^*/dk_1 < 0$, while the effects of δ_1 and σ_1^E are ambiguous. Moreover, we know from Proposition 1 that $d\rho_1(e_1^*)/d\phi_1 > 0$ and $d\rho_1(e_1^*)/d\delta_1$, $d\rho_1(e_1^*)/d\sigma_1^E$, $d\rho_1(e_1^*)/d\sigma_1^A$, $d\rho_1(e_1^*)/dk_1 < 0$. This implies that $dm_2^{E*}/d\phi_1 > 0$ and $dm_2^{E*}/d\sigma_1^A$, $dm_2^{E*}/dk_1 < 0$, while the effects of δ_1 and σ_1^E are ambiguous.

Finally consider the equilibrium inflow of VCs m_2^{V*}, as defined by

$$m_2^{V*} = x_2^* = m_2^{E*}\frac{\phi_2\sqrt{\theta_2^*}}{\delta_2 + \phi_2\sqrt{\theta_2^*}}.$$

Recall that θ_2^* does not depend on the early stage parameters. Our comparative statics results for m_2^{E*} then imply that $dm_2^{V*}/d\phi_1 > 0$ and $dm_2^{V*}/d\sigma_1^A$, $dm_2^{V*}/dk_1 < 0$, while the effects of δ_1

and σ_1^E are ambiguous. □

Angel Protection: Derivation of Deal Values and Equity Shares.

The new coalition values are given by $CV_{EAV} = \pi$, $CV_{EA} = U_2^E + U_2^A$, $CV_{EV} = \lambda\pi$, and $CV_{AV} = CV_E = CV_A = CV_V = 0$. Using the general deal values (16), (17), and (18), we get

$$\begin{aligned} D_2^E &= \frac{1}{6}[2+\lambda]\pi + \frac{1}{6}\left[U_2^E + U_2^A\right] \\ D_2^A &= \frac{1}{3}[1-\lambda]\pi + \frac{1}{6}\left[U_2^E + U_2^A\right] \\ D_2^V &= \frac{1}{6}[2+\lambda]\pi - \frac{1}{3}\left[U_2^E + U_2^A\right] \end{aligned}$$

The new equilibrium equity share for entrepreneurs, β^{E*}, ensures that their actual net payoff equals their deal value from the bargaining game: $\beta^{E*} y_2 = D_2^E$. Solving this for β^{E*} yields

$$\beta^{E*} = \frac{D_2^E}{y_2} = \frac{1}{6y_2}\left[(2+\lambda)\pi + U_2^E + U_2^A\right].$$

Likewise we get

$$\begin{aligned} \beta^{A*} &= \frac{D_2^A}{y_2} = \frac{1}{6y_2}\left[2(1-\lambda)\pi + U_2^E + U_2^A\right] \\ \beta^{V*} &= \frac{k_2 + D_2^V}{y_2} = \frac{1}{6y_2}\left[6k_2 + (2+\lambda)\pi - 2\left(U_2^E + U_2^A\right)\right]. \end{aligned}$$

Proof of Proposition 5.

We first show that $dU_2^A/d\lambda < 0$. Note that $D_2^A \neq D_2^E$ for $\lambda > 0$, and recall that $q_2^E = \phi_2\left[M_2^{V*}/M_2^{E*}\right]^{0.5} = \phi_2^2 D_2^V/\sigma_2^V$. Thus, using (8) we define

$$\begin{aligned} F &\equiv U_2^E(r+\delta_2) + \sigma - a_2 D_2^V\left[D_2^E - U_2^E\right] = 0 \\ G &\equiv U_2^A(r+\delta_2) + \sigma - a_2 D_2^V\left[D_2^A - U_2^A\right] = 0, \end{aligned}$$

where $a_2 = \phi_2^2/\sigma_2^V$. Using Cramer's rule we get

$$\frac{dU_2^A}{d\lambda} = \frac{\begin{vmatrix} -\frac{\partial F}{\partial \lambda} & \frac{\partial F}{\partial U_2^E} \\ -\frac{\partial G}{\partial \lambda} & \frac{\partial G}{\partial U_2^E} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F}{\partial U_2^A} & \frac{\partial F}{\partial U_2^E} \\ \frac{\partial G}{\partial U_2^A} & \frac{\partial G}{\partial U_2^E} \end{vmatrix}} = \frac{-\frac{\partial F}{\partial \lambda}\frac{\partial G}{\partial U_2^E} + \frac{\partial G}{\partial \lambda}\frac{\partial F}{\partial U_2^E}}{\frac{\partial F}{\partial U_2^A}\frac{\partial G}{\partial U_2^E} - \frac{\partial G}{\partial U_2^A}\frac{\partial F}{\partial U_2^E}}.$$

The denominator is negative if

$$\frac{\partial F}{\partial U_2^A}\frac{\partial G}{\partial U_2^E} < \frac{\partial G}{\partial U_2^A}\frac{\partial F}{\partial U_2^E},$$

which is equivalent to

$$\begin{aligned} & \frac{1}{6}a_2\left[2\left[D_2^E - U_2^E\right] - D_2^V\right]\frac{1}{6}a_2\left[2\left[D_2^A - U_2^A\right] - D_2^V\right] \\ < \; & \left[r + \delta_2 + \frac{1}{6}a_2\left[2\left[D_2^A - U_2^A\right] + 5D_2^V\right]\right]\left[r + \delta_2 + \frac{1}{6}a_2\left[2\left[D_2^E - U_2^E\right] + 5D_2^V\right]\right]. \end{aligned}$$

If this condition holds for $r + \delta_2 = 0$, then it also holds for all $r + \delta_2 > 0$. Setting $r + \delta_2 = 0$ we get

$$-2\left[D_2^E - U_2^E\right]D_2^V - 2\left[D_2^A - U_2^A\right]D_2^V < 10\left[D_2^A - U_2^A\right]D_2^V + 10D_2^V\left[D_2^E - U_2^E\right] + 24\left[D_2^V\right]^2.$$

This condition is satisfied as $D_2^E > U_2^E$ and $D_2^A > U_2^A$. Thus, the denominator of $dU_2^A/d\lambda$ is strictly negative. Likewise, the numerator is positive if

$$\frac{\partial G}{\partial \lambda}\frac{\partial F}{\partial U_2^E} > \frac{\partial F}{\partial \lambda}\frac{\partial G}{\partial U_2^E},$$

which is equivalent to

$$\begin{aligned} & \frac{1}{6}\pi a_2\left[\left[D_2^A - U_2^A\right] - 2D_2^V\right]\left[r + \delta_2 + \frac{1}{6}a_2\left[2\left[D_2^E - U_2^E\right] + 5D_2^V\right]\right] \\ < \; & \frac{1}{6}\pi a_2\left[\left[D_2^E - U_2^E\right] + D_2^V\right]\frac{1}{6}a_2\left[2\left[D_2^A - U_2^A\right] - D_2^V\right]. \end{aligned}$$

This condition can be written as

$$\frac{2}{a_2}(r + \delta_2)\left[\left[D_2^A - U_2^A\right] - 2D_2^V\right] + \left[D_2^A - U_2^A\right]D_2^V - D_2^V\left[D_2^E - U_2^E\right] < 3\left[D_2^V\right]^2. \quad (20)$$

From F and G we know that

$$D_2^V \left[D_2^E - U_2^E\right] = \frac{U_2^E (r+\delta_2) + \sigma}{a_2} \quad \text{and} \quad D_2^V \left[D_2^A - U_2^A\right] = \frac{U_2^A (r+\delta_2) + \sigma}{a_2},$$

so that we can write condition (20) as follows:

$$\begin{aligned} \frac{2}{a_2}(r+\delta_2)\left[\left[D_2^A - U_2^A\right] - 2D_2^V\right] + \frac{U_2^A (r+\delta_2)+\sigma}{a_2} - \frac{U_2^E (r+\delta_2)+\sigma}{a_2} \quad &< \quad 3\left[D_2^V\right]^2 \\ \Leftrightarrow \qquad (r+\delta_2)\underbrace{\left[2D_2^A - U_2^A - 4D_2^V - U_2^E\right]}_{\equiv T} \quad &< \quad 3\left[D_2^V\right]^2 a_2. \end{aligned}$$

We now show that $T < 0$. Using the definitions of D_2^A and D_2^V we can write $T < 0$ as

$$\begin{aligned} \frac{2}{3}[1-\lambda]\pi + \frac{1}{3}\left[U_2^E + U_2^A\right] - U_2^A - \frac{2}{3}[2+\lambda]\pi + \frac{4}{3}\left[U_2^E + U_2^A\right] - U_2^E \quad &< \quad 0 \\ \Leftrightarrow \qquad U_2^E + U_2^A \quad &< \quad [1+2\lambda]\pi. \end{aligned}$$

This condition is satisfied for all $\lambda \geq 0$ because $\pi > U_2^E + U_2^A$. Thus, the numerator of $dU_2^A/d\lambda$ is strictly positive. Consequently, $dU_2^A/d\lambda < 0$. Finally note that $\partial D_2^E/\partial\lambda = \pi/6 < \left|\partial D_2^A/\partial\lambda\right| = \pi/3$. Thus, $d\left[U_2^E + U_2^A\right]/d\lambda < 0$, which implies that $dD_2^V/d\lambda > 0$.

Next we analyze the effects of λ on the early stage equilibrium variables. Consider the equilibrium degree of competition θ_1^*. We get

$$\frac{d\theta_1^*}{d\lambda} = 2\frac{\phi_1^2}{\left[\sigma_1^A\right]^2} D_1^A \frac{dD_1^A}{d\lambda}.$$

Recall that

$$\frac{d}{d\lambda}\left(U_2^A + U_2^E\right) = \underbrace{\frac{dU_2^A}{d\lambda}}_{<0} + \underbrace{\frac{dU_2^E}{d\lambda}}_{>0} < 0.$$

This implies

$$\frac{dD_1^A}{d\lambda} + \frac{dD_1^E}{d\lambda} < 0 \quad \Rightarrow \quad \frac{dD_1^A}{d\lambda} < 0.$$

Thus, $d\theta_1^*/d\lambda < 0$.

Now consider the equilibrium entry of entrepreneurs m_1^{E*}. Using (14), we get

$$\frac{dU_1^E}{d\lambda} = \frac{\frac{\phi_1^2}{\sigma_1^A}\left[\frac{dD_1^A}{d\lambda}\left[D_1^E - U_1^E\right] + D_1^A \frac{dD_1^E}{d\lambda}\right]}{r + \delta_1 - \frac{\phi_1^2}{\sigma_1^A}\left[\frac{d\Gamma}{d\alpha}\frac{d\alpha^*}{dU_1^E} + \frac{\partial\Gamma}{\partial U_1^E}\right]},$$

where $\Gamma = D_1^A \left[D_1^E - U_1^E \right]$. Note that $d\Gamma/d\alpha = 0$; see (12). Thus,

$$\frac{dU_1^E}{d\lambda} = \frac{\frac{\phi_1^2}{\sigma_1^A} \left[\frac{dD_1^A}{d\lambda} \left[D_1^E - U_1^E \right] + D_1^A \frac{dD_1^E}{d\lambda} \right]}{r + \delta_1 + \frac{\phi_1^2}{\sigma_1^A} D_1^A},$$

where the denominator is positive. Consequently, $dU_1^E/d\lambda < 0$ if

$$\frac{dD_1^A}{d\lambda} \left[D_1^E - U_1^E \right] + D_1^A \frac{dD_1^E}{d\lambda} < 0. \tag{21}$$

Using (12) we can derive the following expression for $D_1^E - U_1^E$:

$$D_1^E - U_1^E = -\frac{\frac{dD_1^E}{d\alpha}}{\frac{dD_1^A}{d\alpha}} D_1^A,$$

so that (21) can be written as

$$\frac{dD_1^A}{d\lambda} \underbrace{\frac{\left(-\frac{dD_1^E}{d\alpha} \right)}{\frac{dD_1^A}{d\alpha}}}_{\equiv X} + \frac{dD_1^E}{d\lambda} < 0.$$

Recall that $d(D_1^A + D_1^E)/d\lambda < 0$, with $dD_1^A/d\lambda < 0$; thus, this condition is satisfied when $X \geq 1$. Note that $dD_1^E/d\alpha < 0$ and $dD_1^A/d\alpha > 0$. Hence, $X \geq 1$ if

$$0 \geq \frac{dD_1^A}{d\alpha} + \frac{dD_1^E}{d\alpha} = \frac{d}{d\alpha} \left[D_1^A + D_1^E \right].$$

It is easy to show that the joint surplus is maximized when $\alpha = 0$ (which maximizes effort incentives for the entrepreneur); thus

$$\left. \frac{d \left[D_1^A + D_1^E \right]}{d\alpha} \right|_{\alpha = \alpha^* > 0} < 0,$$

so that $X \geq 1$. Consequently, $dU_1^E/d\lambda < 0$, and therefore $dm_1^{E*}/d\lambda = dF(U_1^E)/d\lambda < 0$.

Next consider the equilibrium inflow of angels, m_1^{A*}, which is defined by

$$m_1^{A*} = x_1^* = \underbrace{F(U_1^E)}_{= m_1^{E*}} \underbrace{\frac{\phi_1 \sqrt{\theta_1^*}}{\delta_1 + \phi_1 \sqrt{\theta_1^*}}}_{\equiv T}.$$

Note that $dT/d\sqrt{\theta_1^*}) > 0$. Our comparative statics results for m_1^{E*} and θ_1^* then imply that $dm_1^{A*}/d\lambda = dx_1^*/d\lambda < 0$.

Now consider the angel's equilibrium equity share α^*, which is defined by (12). We get

$$\frac{d\alpha^*}{d\lambda} = \frac{d\alpha^*}{dU_2^E}\frac{dU_2^E}{d\lambda} + \frac{d\alpha^*}{dU_2^A}\frac{dU_2^A}{d\lambda},$$

where $dU_2^E/d\lambda > 0$ and $dU_2^A/d\lambda < 0$. Moreover, the Nash bargaining solution implies that $d\alpha^*/dU_2^E > 0$ and $d\alpha^*/dU_2^A < 0$. Thus, $d\alpha^*/d\lambda > 0$. For the equilibrium valuation $V_1^* = k_1/\alpha^*$ this concurrently implies that $dV_1^*/d\lambda < 0$. Finally we know that $dD_1^E/d\lambda > 0$ in equilibrium. Using the Envelope Theorem we get

$$\frac{dD_1^E}{d\lambda} = \rho_1(e_1)\underbrace{\frac{d}{d\lambda}\left[gU_2^E + (1-g)(1-\alpha^*)y_1\right]}_{\equiv T} > 0,$$

which implies that $T > 0$. Using (1) we find

$$\frac{de_1^*}{d\lambda} = -\frac{\rho_1'(e_1)\overbrace{\frac{d}{d\lambda}\left[gU_2^E + (1-g)(1-\alpha)y_1\right]}^{=T}}{\frac{d}{de_1}\left[\rho_1'(e_1)\left[gU_2^E + (1-g)(1-\alpha)y_1\right] - c'(e_1)\right]},$$

where $T > 0$, and the denominator is negative due to the second-order condition for e_1^*. Thus, $de_1^*/d\lambda > 0$. This in turn implies that $d\rho_1(e_1^*)/d\lambda > 0$.

Finally we analyze the effects of λ on the late stage equilibrium variables. Note that $d\left(U_2^E + U_2^A\right)/d\lambda < 0$ also implies that $dD_2^V/d\lambda > 0$. Using the definitions of θ_2^*, β^{V*} and V_2^*, we can then infer that $d\theta_2^*/d\lambda > 0$, $d\beta^{V*}/d\lambda > 0$ and $dV_2^*/d\lambda < 0$. Moreover,

$$\frac{dm_2^{E*}}{d\lambda} = \frac{d}{d\lambda}\left[g\rho_1(e_1^*)x_1^*\right] = g\left[\rho_1'(e_1^*)\frac{de_1^*}{d\lambda}x_1^* + \rho_1(e_1^*)\frac{dx_1^*}{d\lambda}\right].$$

In general, the effect on m_2^{E*} is ambiguous as $de_1^*/d\lambda > 0$ and $dx_1^*/d\lambda < 0$. However, we can see that $dm_2^{E*}/d\lambda < 0$ when $\rho_1'(e_1^*) \to 0$. Moreover, for $\delta_1 \to 0$ we have $m_1^{A*} = m_1^{E*}$; with m_1^{E*} being sufficiently inelastic, we have $dx_1^*/d\lambda \to 0$, so that $dm_2^{E*}/d\lambda > 0$. Next, recall that m_2^{V*} is defined by

$$m_2^{V*} = x_2^* = m_2^{E*}\underbrace{\frac{\phi_2\sqrt{\theta_2^*}}{\delta_2 + \phi_2\sqrt{\theta_2^*}}}_{\equiv T}.$$

One can show that $dT/d\sqrt{\theta_2^*} > 0$, so that $dT/d\lambda > 0$. Recall, however, that the sign of $dm_2^{E*}/d\lambda$ is ambiguous. Thus, the effect of λ on $m_2^{V*} = x_2^*$ is also ambiguous. $\square$

Proof of Proposition 6.

Recall that $U_2^E = U_2^A$ in equilibrium. Moreover, as shown in Proof of Proposition 3, $dU_2^E/d\phi_2 > 0$, and $dU_2^E/d\sigma_2$, $dU_2^E/d\delta_2$, $dU_2^E/d\sigma_2^V$, $dU_2^E/dk_2 < 0$. Consequently, $d\gamma^*/d\phi_2 < 0$, and $d\gamma^*/d\sigma_2$, $d\gamma^*/d\delta_2$, $d\gamma^*/d\sigma_2^V$, $d\gamma^*/dk_2 > 0$. $\square$

Proof of Proposition 7.

Recall from Proof of Proposition 5 that $d\left[U_2^E + U_2^A\right]/d\lambda < 0$. Thus, $d\gamma^*/d\lambda > 0$. $\square$

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Appendix for Referees (Not for Publication)

Early Stage Investment and Valuation.

Consider first our base model with endogenous effort. Differentiating V_1^* w.r.t. k_1 yields

$$\frac{dV_1^*}{dk_1} = \frac{d}{dk_1}\left(\frac{k_1}{\alpha^*}\right) = \frac{\alpha^* - k_1 \frac{d\alpha^*}{dk_1}}{[\alpha^*]^2}.$$

Note that $dV_1^*/dk_1 > 0$ when $k_1 \to 0$. Thus, the equilibrium valuation V_1^* is decreasing in k_1 when k_1 is sufficiently small.

Next, suppose the entrepreneur's effort e_1 is exogenous, and define $\rho_1 \equiv \rho_1(e_1)$. The early stage deal values are then given by

$$\begin{aligned} D_1^E &= \rho_1 \left[gU_2^E + (1-g)(1-\alpha)y_1\right] - c \\ D_1^A &= \rho_1 \left[gU_2^A + (1-g)\alpha y_1\right] - k_1, \end{aligned}$$

where c is the entrepreneurs disutility of providing effort e_1. The optimal equity share for the angel, α^*, then satisfies the symmetric Nash bargaining solution, which accounts for the outside option of each party (U_1^E for the entrepreneur, and 0 for the angel because of free entry). Let $\widetilde{D}_1^E$ and $\widetilde{D}_1^A$ denote the deal values reflecting the Nash bargaining solution, which are given by

$$\begin{aligned} \widetilde{D}_1^E &= \frac{1}{2}\left[\rho_1\left[g\left(U_2^E + U_2^A\right) + (1-g)y_1\right] - k_1 - c + U_1^E\right] \\ \widetilde{D}_1^A &= \frac{1}{2}\left[\rho_1\left[g\left(U_2^E + U_2^A\right) + (1-g)y_1\right] - k_1 - c - U_1^E\right]. \end{aligned}$$

The equilibrium equity share for the angel, α^*, then satisfies $D_1^E(\alpha^*) = \widetilde{D}_1^E$ and $D_1^A(\alpha^*) = \widetilde{D}_1^A$. Recall that $U_2^A = U_2^E$ in equilibrium. Thus,

$$\alpha^* = \frac{1}{2} + \frac{k_1 - c - U_1^E}{2\rho_1(1-g)y_1}.$$

The equilibrium early stage valuation is $V_1^* = k_1/\alpha^*$. Taking the first derivative w.r.t. k_1 we get

$$\frac{dV_1^*}{dk_1} = \frac{\overbrace{\alpha^* - k_1\frac{d\alpha^*}{dk_1}}^{\equiv N}}{[\alpha^*]^2}.$$

The denominator is always non-negative. Moreover, note that $N \geq 0$ for $k_1 \to 0$, which implies that $dV_1^*/dk_1 \geq 0$ for $k_1 \to 0$. To show that $dV_1^*/dk_1 > 0$ for all $k_1 > 0$, it is thus sufficient to verify that $dN/dk_1 > 0$:

$$\frac{dN}{dk_1} = \frac{d\alpha^*}{dk_1} - \left(\frac{d\alpha^*}{dk_1} + k_1 \frac{d^2\alpha^*}{dk_1^2}\right) = -k_1 \frac{d^2\alpha^*}{dk_1^2}.$$

We need to find the sign of $d^2\alpha^*/dk_1^*$. We start by taking the first derivative of α^* w.r.t. k_1:

$$\frac{d\alpha^*}{dk_1} = \frac{1}{2\rho_1(1-g)y_1}\left[1 - \frac{dU_1^E}{dk_1}\right].$$

It is easy to see that $\widetilde{D}_1^E - U_1^E = \widetilde{D}_1^A$. Thus, the condition defining U_1^E simplifies to

$$U_1^E\left[r + \delta_1\right] - \frac{\phi_1^2}{\sigma_1^A}\left[\widetilde{D}_1^A\right]^2 + \sigma_1^E = 0.$$

Thus,

$$\frac{dU_1^E}{dk_1} = -\frac{a_1 \widetilde{D}_1^A}{r + \delta_1 + a_1 \widetilde{D}_1^A},$$

where $a_1 = \phi_1^2/\sigma_1^A$. Consequently,

$$\frac{d\alpha^*}{dk_1} = \frac{1}{2\rho_1(1-g)y_1}\left[1 + \frac{1}{(r+\delta_1)\left[a_1\widetilde{D}_1^A\right]^{-1} + 1}\right].$$

We then get

$$\frac{d^2\alpha^*}{dk_1^2} = \frac{1}{2\rho_1(1-g)y_1} \frac{-\frac{1}{2}a_1\left(r+\delta_1\right)\left[a_1\widetilde{D}_1^A\right]^{-2}\left[1 + \frac{dU_1^E}{dk_1}\right]}{\left[(r+\delta_1)\left[a_1\widetilde{D}_1^A\right]^{-1} + 1\right]^2}.$$

Note that

$$1 + \frac{dU_1^E}{dk_1} = 1 - \frac{a_1\widetilde{D}_1^A}{r + \delta_1 + a_1\widetilde{D}_1^A} = \frac{r + \delta_1}{r + \delta_1 + a_1\widetilde{D}_1^A} > 0.$$

Thus, $d^2\alpha^*/dk_1^2 < 0$. This implies that $dN/dk_1 > 0$, and therefore $dV_1^*/dk_1 > 0$.