NBER WORKING PAPER SERIES

INFLATIONARY CONSEQUENCES OF ANTICIPATED MACROECONOMIC POLICIES

Allan Drazen

Elhanan Helpman

Working Paper No. 2006

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 1986

This is a combined and shortened version of Drazen & Helpman (1985a, 1985b). We would like to thank Zvi Eckstein, David Pines, and Assaf Razin for comments on a preliminary draft, as well as seminar participants at Chicago, Columbia, The Hebrew University, MIT, Northwestern, Pennsylvania, Stockholm and Tel-Aviv. This research was supported by funds granted to the Foerder Institute for Economic Research by the NUR MOSHE FUND. The research reported here is part of the NBER's research program in Economic Fluctuations. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

We consider a model in which the level of taxes and seignorage are too low to finance government expenditures and debt service. Government debt will therefore grow without bound, implying the eventual need to change policy. Starting with utility maximization, we analyze the effect of the expected switch on equilibrium time paths before the switch takes place. We analyze stabilization via increasing taxes, increasing money growth rates, or cutting expenditures, both under certainty and under uncertainty about the composition or timing of a stabilization.

Under full certainty, inflation may rise, fall, or remain constant before the stabilization, depending on which policy tool is used to stabilize. Uncertainty solely about the composition of the stabilization will yield paths in between the above cases, with a price jump at the time of stabilization. In general there is no simple correlation between changes in the budget deficit and inflation. With uncertainty about the timing of a stabilization, the inflation rate will most likely exhibit fluctuations and may overshoot its steady state value, even when real balances move monotonically. Uncertainty about the timing of a stabilization can therefore itself induce fluctuation in inflation, even if underlying utility and subjective probability functions are smooth.

Allan Drazen
Department of Economics
Tel-Aviv University
Ramat-Aviv, Israel 69978

(03) 420-488

Elhanan Helpman
Department of Economics
Tel-Aviv University
Ramat-Aviv, Israel 69978

(03) 420-712

1. Introduction

What happens when the government follows monetary and fiscal policies which are known to be infeasible, except over the short run? The recognized unsustainability of the policy means that it is known that at some point in the future the policy will be abandoned. What is generally not known is exactly when the regime switch will take place, and the policy mix that will follow. The eventual infeasibility of the given policy yields not a specific abandonment date, but only a presumption that the longer the policy has been in force, the higher the probability that it will be abandoned in the current period. Though most people would accept the above characterization of the increasing probability of regime switch, there has been relatively little work in analyzing the time path of an economy facing a probabilistic abandonment of an infeasible policy (see, however, Flood and Garber (1984)). Given the obvious relevance of such a question in, for example, foreign exchange markets and debt management, such an analysis is highly desirable.

In this paper we analyze this issue by considering a model where for the existing level of government debt, the government's choice of expenditures g, a rate of monetary expansion μ, and level of taxation τ are inconsistent with ever attaining a steady state. Specifically, the level of seignorage and regular tax revenues are too low to finance government expenditures and debt service by themselves. The level of government indebtedness must therefore be continually increased, and, for unchanged macroeconomic policies, government debt will grow without bound.

To analyze the effects of policy mix and timing uncertainty, our strategy is as follows. We begin with the full certainty case. We first set up the individual's maximization problem to derive necessary first-order conditions. Combined with market equilibrium conditions and the government's budget constraint, these will yield dynamic equations for debt and real balances and hence inflation before the regime switch, where the exact path will depend on the necessary characteristics of the time paths after the switch. The case of a known switch date but uncertain post-switch policy mix is a simple extension of the full certainty results.

We then consider the case where the policy mix after the regime switch is known, but the date of the switch is uncertain. We similarly derive time paths for bonds, real balances, and inflation, and illustrate these time paths both via phase diagrams and via computer simulations.

The main results are as follows. Under full certainty, stabilization via an increase in the rate of monetary growth will imply a monotonically increasing inflation rate (and a monotonically declining level of real balances) until the regime switch occurs, while stabilization via increased non-distortionary taxes will yield a constant level of real balances and inflation both before and after the switch. Stabilization via budget cuts implies that inflation must eventually fall, but it could rise or fall prior to stabilization. Hence, the correlation of inflation and the budget deficit depends on the public's expectation about the policies that will effect stabilization.

In the remaining part of the paper we discuss uncertain policies. If the switch date is known, but the post-switch policy mix is not, real balances and inflation before the regime switch will follow a path in between the cases above. The exact nature of the path will depend on the subjective probability assigned to various combinations of policies used to effect the stabilization. There will in general be a one-time price jump (which may be either up or down) at the time of the regime switch.

When there is uncertainty about the timing of the switch, the inflation rate will most likely exhibit fluctuations and may overshoot its steady state value, even when real balances move monotonically. Therefore uncertainty about the timing of stabilization policy can of itself induce fluctuations in the inflation rate even if the underlying utility and subjective probability (of timing or policy mix) functions are smooth. There exist beliefs about the probability of a policy switch which induce very rapid changes in the rate of inflation, and there will almost surely be price jumps following a stabilization effort. Price jumps before a stabilization are also possible if the distribution of the timing of the policy switch has mass points. The results on uncertainty strengthen the result that there will be no simple correlation between changes in the budget deficit on the one hand and in inflation on the other.

Our interest in possible correlations stems from the recent Israeli experience with inflation. Recently there has been no simple correlation between changes in the deficit and inflation. This has been taken by some as proof that the root cause of the Israeli inflation has not been the peresistent deficit. By building a model in which it is clear that the deficit drives inflation, but where no simple correlation emerges, we show that lack of correlation does not mean lack of causation.

2. Regime Switch at a Known Date

We first consider the case where the exact date of a switch away from an infeasible policy is known, although the exact nature of the policy change may not be known. The regime switch at time T can be described as follows. The level of per capita real bond holding achieved at T, namely $b(T)$, will be frozen by choice of a new monetary growth rate μ_s, a new tax rate τ_s and/or a new level of government spending g_s, which imply that government spending plus debt service can be financed by a constant level of taxes (inflation and regular) with no further growth in debt. Uncertain policy mix means that the exact combination of policy changes which will effect the stabilization is unknown before T. As we shall see below, when there is uncertainty about the policy mix, stabilization will require a jump in the price level at T to some post switch level $P_s(T)$, and the relevant uncertainty of the individual is uncertainty about the value of $P_s(T)$. We begin by considering the individual's problem.

a. Individual maximization

The individual is assumed to derive utility from consumption and real money balances, where his instantaneous utility function is assumed separable across commodities and across time. Utility at instant t may then be written as

$$(1) \qquad u(c(t)) + v(\frac{M(t)}{P(t)})$$

where c, M, and P are real consumption, nominal balances, and the price level. The individual can hold either real money balances or real bonds b as assets, the real interest rate on the latter being r. He has a discount rate β and receives after-tax labor income y per period. We assume that total output y_0 is exogeneously given, so that a fixed level of taxes imply that after-tax income is fixed as well.

The individual's objective is to maximize expected discounted utility over an infinite horizon. In the case of full certainty, this is easily represented. Let $V^s(\cdot)$ be the present discounted value of maximized utility from T onwards. V^s will be a function of the real value of an individual's assets at T, namely $b(T) + M(T)/P_s(T)$, and perhaps of T as well. The present discounted welfare from 0 to infinity if a switch occurs at T is then

$$(2) \qquad \int_0^T e^{-\beta t}[u(c(t)) + v(\frac{M(t)}{P(t)})]dt + e^{-\beta T}V^s(b(T) + \frac{M(T)}{P_s(T)}; T)$$

where the characteristics of $V^s(\cdot)$ are to be determined.

If we define by z(t) the addition to nominal cash balances at time t, the individual's choice problem may be thought of as choosing functions c(t), M(t), and z(t) to maximize (3) subject to two constraints -- one on income, the other on the relation of M and z -- and to boundary conditions.

The income constraint is that the present discounted value of income between 0 and t plus the value of wealth at 0 must equal the discounted value of expenditures on c and z/P, plus the value of bonds at t discounted to time zero, plus initial real money balances. This must hold for all t. For t = T we obtain

$$e^{-R(T)}b(T) + \int_0^T e^{-R(t)}\left(c(t) + \frac{z(t)}{P(t)} - y\right)dt + \frac{M(0)}{P(0)} = w(0) \tag{3}$$

where wealth w(0) equals initial bonds plus the initial value of nominal balances evaluated at the initial price level, and where R(t) is the integral of the interest rate from 0 to t.

The second constraint is that the change in money balances between any two points in time is the integral of the z(x), namely, between 0 and t,

$$M(t) = M(0) + \int_0^t z(x)dx. \tag{4}$$

Maximization of (2) subject to constraints (3) and (4) will yield the individual's optimal time paths under full certainty. In the case of uncertainty about the policy mix after T, it will turn out (as shown in section c below) that the individual's relevant uncertainty is about the post switch price level $P_s(T)$. Denoting the distribution of post-switch price level induced by the policy mix by $G(P_s(T))$, the individual maximizes

$$(5) \qquad \int_0^T e^{-\beta t}[u(c(t) + v(\frac{M(t)}{P(t)})]dt$$

$$+ e^{-\beta T} \int_{P_s(T)=0}^{P_s(T)=\infty} V^s(b(T) + \frac{M(T)}{P_s(T)};T)dG(P_s(T))$$

subject to constraints (3) and (4).[1]

The mathematical derivation of the solution to the individual's optimum problem from the first-order conditions is presented in Appendix 1. That derivation makes clear that the basic dynamic equations for debt and real balances turn out to be equivalent for the certainty and uncertainty case, except for the terminal condition. The results that follow in this and the next section will therefore apply to both cases.

Consumption may be written $c = y_0 - g$. The marginal utility of consumption will be denoted by θ before a stabilization and θ_s after. These values will be constant, but unequal if the stabilization includes a change in government spending, and hence a change in consumption.

Optimal consumer behavior implies (see the first-order conditions (A1.2) through (A1.4) in Appendix I)

(6) $$e^{-\beta t}u'(y_0-g) = e^{-\beta T}u'(y_0-g_s)e^{R(T)-R(t)} \qquad 0 \le t < T$$

(7) $$\frac{1}{P(t)} = \int_t^T e^{-\beta(x-t)} \frac{v'(m(x)}{\theta} \frac{1}{P(x)}dx + e^{-[R(T)-R(t)]}\frac{1}{P_s(T)}, \qquad 0 \le t < T$$

where the last equation is derived from combinging (A1.3) and (A1.4) in the Appendix. This is a standard asset pricing equation showing that the value of one unit of money at time t equals the sum of the discounted earnings (here, marginal utility) stream plus the discounted resale value.

Equation (6) implies that the instantaneous interest rate is equal to β before the stabilization, but that there might be a jump in the interest factor at the stabilization date. (It can also be shown that after stabilization the interest is also equal to β). More precisely, from (6) and the initial condition $R(0) = 0$ we obtain (see A1.5):

(8) $$R(T) = \begin{cases} \beta t & \text{for } 0 \le t < T \\ \beta t + \ln \dfrac{u'(y_0-g)}{u'(y_0-g_s)} & \text{for } t = T \end{cases}$$

This may be represented as in Figure 1.

Differentiating (7) with respect to t yields (see the discussion preceding (A1.9) in the appendix)

(9) $$\frac{v'(m)}{\theta} = \beta + \frac{\dot{P}}{P} \qquad 0 \le t < T$$

where the right-hand-side is the real interest rate plus the inflation rate.

A further implication of (7) is that a cut in government expenditure at time T, even if perfectly foreseen, will induce a jump in the price level. This may be seen by noting that (7) implies

$$\frac{P(T)}{P_s(T)} = e^{R(T)-R(T^-)}$$

which, by (8) yields

$$\frac{P(T)}{P_s(T)} = \frac{\theta}{\theta_s}, \tag{10}$$

so that $g_s < g$ implies a (fully anticipated) downward jump in the price level due to the upward jump in consumption necessary to clear the output market.

One may explain this price level jump under perfect foresight, which comes from the jump in the interest factor, as follows. Because bonds are denominated in real terms with a fixed price in terms of the consumption good, the real stock of bonds cannot jump at T (see footnote 3 below). If bonds were nominal or if we allowed their price relative to consumption to vary, then the real stock would jump[2]. Given the fixed real price of bonds, the interest factor R(T) must jump. The infinite return on one asset at the instant T requires an infinite return on the other asset, money, implying a price level jump.

The result of a jump in the price level is robust to a number of reasonable alternative specifications of the output market. First, suppose we opened the economy, so that we could import goods from abroad at fixed terms-of-trade. As long as some part of the cut in government expenditures fell on nontraded goods, there would still be a jump in the price of nontraded goods and a jump in the interest rate in terms of nontradables. The overall price level would therefore also jump (see Drazen and Helpman (1986)).

A second modification would be to allow investment so that a fixed level of output does not automatically imply a one-to-one relation between changes in private and government consumption. As long as there are convex adjustment costs to investment or disinvestment, utility maximization would yield a jump in consumption at T.

b. <u>Government behavior and the dynamic equations</u>

We may derive the dynamic equations for b and m by embedding the equilibrium conditions in the government behavioral equations. The government budget constraint implies that the basic deficit plus debt service must be financed either by issuing bonds or printing money. We therefore have

$$\begin{aligned} (11) \qquad \dot{b} &= \beta b + d - \dot{M}/P \\ &= \beta b + g - \tau - \mu m \end{aligned}$$

for $0 \le t < T$, where d is the basic real deficit ($d = g - \tau$) and where $\dot{M}/P = \frac{\dot{M}}{M} \cdot M/P = \mu m$. At T the interest rate will jump if g is cut.[3] Combining this with the definitional relation for the change in real balances, namely $\dot{m} = (\mu - \frac{\dot{P}}{P})m$, where we assume population growth is equal to zero, we obtain

$$(12) \qquad \dot{m} = (\beta + \mu)m - \frac{1}{\theta} mv'(m).$$

As indicated above and demonstrated in Appendix 1, the dynamic equations (11) and (12) describe the motion of the system before a known switch date T both for the case of full certainty and for the case of uncertain post-switch policy mix. The difference between the two cases concerns the nature of the terminal condition at T. Before considering the terminal conditions, we can describe the motion of the system until T. The $\dot{b} = 0$ and $\dot{m} = 0$ loci, which follow from (11) and (12), may be represented as in Figure 2, with the implied motion for points off the loci.

The steady state A is a point of unstable equilibrium (a source). For any values of μ, τ, and g there is only one value of debt which is consistent with steady state. If, for given μ, τ, and g, the values of debt and real balances are such that we are to the right of the $\dot{b} = 0$ locus, debt will grow without bound in the absence of a regime switch. We only consider initial values of debt which are to the right of the steady state point described in Figure 2, that is, $b_0 > \underline{b}$.

c. The full time paths under certainty

To derive the full time paths starting from a value of b_0 that will induce a policy switch at T, we consider the transversality condition at T. We first consider the polar cases under certainty, where only one of the policy instruments is changed at T, the others left unchanged. In each case, the relevant policy variables must be chosen so that post-switch real balances and debt are at steady state values.

For each of the policy instruments, the terminal locus (the feasible combinations of m_s and b_s in steady state) may be found by considering the intersection of the $\dot{m} = 0$ and $\dot{b} = 0$ loci as the value of that instrument is varied. We first consider money based stabilization. For increases in μ, both loci shift down, the location of the intersection depending on the elasticity of money demand.[4] When this elasticity is everywhere less than one, this locus will be downward sloping, with the new intersection at point B. More generally, if seignorage revenues μm reach a maximum at some finite value of μ, say μ^*, the locus of intersection points will lie to the southeast of the original point until seignorage revenue reaches its maximum, after which the locus will slope down to the southwest. Associated with maximum seignorage is a maximum value of debt, say $\bar{b}$, which can be financed in steady state by the inflation tax, for a given deficit. Below this maximum a given value of debt is associated with two values of m_s and hence μ_s. We will assume that for values of debt below the maximum the government chooses to stabilize at the lower possible μ_s, so that we have a functional relation between m_s and b_s. We denote this by $m_s^{\mu}(b_s)$ where the superscript indicates which policy variable has been changed to effect the stabilization. This curve is downward sloping along the relevant section, as in Figure 3.[5]

The full time path may now be easily derived (more details may be found in Drazen [1985]). The possibility of stabilization at T for some value of μ means that the values of $b(T)$ and $M(T)/P_s(T)$ must lie on the terminal surface. Since the value of T is known with certainty beforehand, there can be no jump in P at T. This may be seen by inspection of equation (7). Therefore the dynamic path for real balances and debt must just arrive at $m_s^{\mu}(b)$ at T. The path is uniquely specified given: this terminal requirement; the dynamic behavior of debt and real balances from time 0 to T as specified by (11) and (12); and, the initial predetermined value of real per capita bonds b_0. Put another way, there is only one path starting at b_0 at time zero and arriving at $m_s^{\mu}(b)$ at time T which satisfies the dynamic equations. Such a path is illustrated in Figure 3 by the falling arrow path. One notes that along this path debt is rising and real balances are falling monotonically. By condition (9), inflation must therefore be monotonically rising from 0 to T. Specifically, inflation is between μ and μ_s along the path and equals μ_s after a stabilization.

It may be shown that the optimal path must cut the $m_s^{\mu}(b)$ locus from below if T is sufficiently large. Moreover, if the slope of $m_s^{\mu}(b)$ is flat at b_0, the dynamic path will start above $m_s^{\mu}(b)$ and cut it at some $t < T$.[6] The drawing in Figure 3 reflects this possibility.

The case in which non-distortionary taxes τ are used to stabilize with certainty at T may be simply analyzed. Since τ does not appear in the dynamic equation for money balances, changes in τ do not shift the $\dot{m} = 0$ locus when g and μ are held fixed. The locus of intersections of the $\dot{m}=0$ and $\dot{b} = 0$ lines for changes in τ, denoted $m_s^{\tau}(b)$, is then simply the horizontal $\dot{m} = 0$ locus, as indicated in Figure 3. As before, there is a maximum level of debt consistent with tax-based stabilization.

The dynamic path in this case must then always lie on the $m_s^{\tau}(b)$ locus (that is, the $\dot{m} = 0$ locus) if it is to end up there. This is represented by the horizontal arrow path in Figure 3. Along this path real balances and hence inflation will be constant, the latter equal to the unchanged value of μ. A Ricardian equivalence result obtains, since only the timing of non-distortionary taxation is affected.

If taxes are distortionary (for example, labor income taxes) the above result no longer holds. Changes in the tax rate affect the level and marginal utility of consumption, as in the case of stabilization via government expenditure cuts. The results are conceptually similar in most respects to the results for that case and are treated in detail in Drazen and Helpman (1985c).

For a stabilization via expenditure cuts under certainty, the terminal surface, denoted $m_s^g(b)$, is upward sloping, as in Figure 4. A cut in g shifts the $\dot{b} = 0$ locus down and the $\dot{m} = 0$ locus up so that the new intersection lies to the north-east.[7] Hence, real balances unambiguously rise with increases in debt. As with stabilization based on changes in money growth or taxes, there is a maximum feasible level of steady state debt, say $\bar{b}$, which can be supported by a budget cut, as g can drop at most to zero.

The key difference between this and the earlier cases is the behavior of the price level and the real interest rate at T. With stabilization via budget cuts there will be a jump in the real interest factor and in the price level, and hence, in real balances at T.

To derive the dynamic path and see the implications of the jump at T we begin by writing the endpoint condition (10) as

$$\frac{m_s(T)}{m(T)} = \frac{\theta}{\theta_s} \tag{13}$$

If we denote by $\bar{m}$ the value of real balances for which $\dot{m} = 0$, we have that

$$\frac{v'(\bar{m})}{\theta} = \beta + \mu = \frac{v'(m_s)}{\theta_s} .$$

Combining this with (13) and rearranging, we may write

$$\frac{m(T)}{\bar{m}} = \frac{v'(m_s)m_s}{v'(\bar{m})\bar{m}} . \tag{14}$$

It is clear from (14) that the value of its right-hand-side determines whether real balances just before the price jump at T are above, on, or below the horizontal $\dot{m} = 0$ locus. If this value is larger than (equal to, less than) one, real balances have to be above (on, below) the $\dot{m} = 0$ locus. Obviously, since m_s depends on the stabilization date T, so does this value. It is, however, clear from the directions of movement indicated in Figure 2 that if the economy is above $\dot{m} = 0$ at some time it has to be above $\dot{m} = 0$ at all times, and similarly for being on or below $\dot{m} = 0$. Therefore, there exist three possibilities of economic dynamics according to whether the right-hand-side of (14) is larger, equal, or smaller than one. In each case, real balances will jump at T as a result of the downward price jump.

The first possibility is that real balances are rising over time, as indicated by the upward sloping arrow path in Figure 4, until the switch takes place at $b = b(T)$ and real balances jump from A_1 to B. The rate of inflation is falling over time (from (9)) and is lower than the rate of monetary growth (as real balances are rising). After the policy switch, the rate of inflation equals the rate of monetary growth. Panel (i) of Figure 5 displays the resulting time paths of the price level and the rate of inflation.

The second possibility is constant real balances over time (the horizontal path leading to A_2) and a jump up to B. The rate of inflation will be constant, except at T, as in panel (ii) in Figure 5.

Finally, real balances may fall, leading to A_3, followed by an upward jump to B. The rate of inflation is rising before the policy switch, then drops to a lower level, as in panel (iii) of Figure 5.

To summarize these results, one may note that $v'(m)m$ is increasing in real balances when the interest elasticity of demand for real balances is larger than one (in absolute value) and it is declining in real balances when the interest elasticity of demand for real balances is smaller than one. (This is immediately seen by using (8) to write the demand for real balances in implicit form as $v'(m) = \theta i$ or $v'(m_s) = \theta_s i_s$, where i is the nominal interest rate.) Hence, since $m_s > \bar{m}$, the right-hand-side of (14) is larger than one for everywhere interest-elastic demand functions for money and smaller than one for everywhere interest-inelastic demand functions for money. This implies that the time path of inflation and real balances depends on whether the interest elasticity of money demand is larger or smaller than one, but does not depend on the timing of the policy switch. However, the timing of the policy switch does affect the nature of the inflationary process when the interest elasticity of money demand is on both sides of one for real balances above $\bar{m}$.[8]

The certainty framework could be used to analyze multistage stabilization programs, which would imply more complex time paths. For example, suppose the stabilization is known to entail a cut in government expenditures at T followed by a reduction in money growth at $T' > T$, the two changes together leading to no further growth in debt. This program (which resembles those often used to end hyperinflations) can be shown to imply a fall in real balances until T, followed by a rise from T to T', with inflation moving in the opposite direction. (After T', all variables are constant). As noted below, this result mimics a characteristic of hyperinflationary time paths. (A full discussion of this case may be found in Drazen and Helpman (1985b).)

With the three polar cases having been analyzed, known combinations of μ_s, τ_s and g_s to finance a given budget at T may be easily analyzed. Any policy mix rule which determines endogeneous combinations of the policy tools consistent with stabilization will determine a terminal locus relating m_s to b_s. When the policy mix is limited to changes in μ, τ, and g in the direction of a lower deficit (increases in the first two, decreases in the last) this locus will lie between the $m_s(b)$ loci, its precise position depending on the policy mix. (Otherwise, of course, the terminal surface could lie above $m_s^g(b)$ or below $m_s^\mu(b)$). The dynamic path will then be that which starts at b_0 and, following the dynamic equations (11) and (12), just hits the terminal surface at T.

d. <u>The time paths under policy mix uncertainty</u>

The above analysis of the certainty case makes clear that, from the point of view of the individual, the relevant differences between alternative policy mixes at T is in different levels of the post-switch price level $P_s(T)$. Uncertainty about $P_s(T)$ may be represented by a distribution $G(P_s(T))$, which is induced by a subjective probability distribution over possible policy mixes, and knowledge of the structure of the economy linking each policy mix to a value of $P_s(T)$. The optimum problem is then to maximize expected discounted utility over an infinite horizon where T is known and where $G(P_s(T))$ is used to form expected utility. This is exactly the problem set out in equation (5) above and solved in Appendix 1, yielding dynamic equations (11) and (12). Therefore, once behavior at T is specified, the entire path will be known.

After the policy mix is announced at T, we must be on the relevant terminal surface. Since the chosen mix is unknown before T, it is obvious that there will be a jump to the relevant surface at T due to a price level jump.[9] The price level before the jump will be determined by an asset pricing equation, namely (see equation A1.13 in Appendix 1):

$$\frac{1}{P(t)} = \int_0^T e^{-\beta(x-t)} \frac{v'(m(x))}{\theta} \frac{1}{P(x)} dx + e^{-\beta(T-t)} \int \frac{1}{P_s} dG(P_s) \qquad (15)$$

As t approaches T, we therefore have that $\frac{1}{P(t)}$ must approach the expectations of $\frac{1}{P_s}$ taken over $G(P_s(T))$. This ties down the price level along the optimal path the instant before T, which, in turn, ties down the whole path. The equilibrium path may then be represented as in Figure 6, with its exact location at $t=T^-$ dependent on how much weight is assigned to each possible choice of policy mix at T. If, for example, it is considered highly likely that the main adjustment will come in an increase in the rate of monetary growth, with the expectation that there will be a relatively small increase in taxes and cut in expenditures, the path will be relatively closer to the $m_s^{\mu}(b)$ line. At T, when the actual program is announced, we jump to the relevant terminal surface. To continue the example, if the actual program includes a smaller increase in μ than was expected, so that the terminal surface is above the optimal path based on the contrary expectations, there will be an upward jump in m, or, equivalently, a downward price level jump. In this case the stabilization program implies an actual rate of monetary growth lower than was expected, which calls for a downward price adjustment. If the stabilization included a decrease in μ coupled with a sufficiently large value of τ to support it, then the terminal value of real balances may happen to be larger than $\bar{m}$. More generally, policy mixes which include a decrease in taxes or the rate of money growth or increases in government spending imply a post-stabilization value of real balances outside the region between the terminal surfaces.

3. Uncertainty about the Timing of the Regime Switch

a. The basic structure

The basic structure is the same as above, but we now assume that the switch may occur at any time between 0 and some T_{max}, where the cumulative distribution of a switch occurring until T is F(T). Clearly $F(0) = 0$ and $F(T_{max}) = 1$. We consider the case where only one switch takes place. For simplicity, we assume the policy mix is known.

The expectation of discounted utility over the horizon is taken over dF(T). The individual's present discounted utility if a switch occurs at T is given by (2). Expected welfare over all possible realizations of T may then be written

$$(16)\qquad \int_0^{T_{max}} \left\{ \int_0^T e^{-\beta t}[u(c(t)) + v(\frac{M(t)}{P(t)})]dt + e^{-\beta T} V^s(b(T) + \frac{M(T)}{P_s(T)};T) \right\} dF(T)$$

The individual's problem is therefore to maximize (16) subject to constraints (3) and (4). The derivation of the first-order conditions for this case (which are derived in Appendix 2) are

$$(17)\qquad [1-F(t)]e^{-\beta t}\theta = \int_t^{T_{max}} e^{-\beta T} e^{R(T)-R(t)} \theta_s(T) dF(T)$$

$$(18)\qquad \frac{1}{P(t)} = \frac{1}{1-F(t)} \int_t^{T_{max}} [\int_t^T e^{-\beta(x-t)} \frac{v'(m(x)}{\theta} \frac{1}{P(x)} dx + $$

$$+ e^{-\beta(T-t)} \frac{\theta_s(T)}{\theta} \frac{1}{P_s(T)}] dF(T)$$

Differentiating (17) with respect to t yields

$$(19)\qquad dR = \beta dt + (1 - \frac{\theta_s}{\theta}) \frac{dF}{1-F}$$

In the case of a (non-distortionary) tax-based or money-based stabilization, the marginal utility of consumption is equal before and after a stabilization, so that the real interest rate is constant and equal to β with uncertainty about timing. In the case of a budget cut, the <u>real</u> interest rate will include a risk premium reflecting the probability of a jump in the marginal utility of consumption. The marginal utility of consumption after a stabilization, θ_s, is falling over time, since the longer that no stabilization has taken place, the higher is the level of debt at the time of stabilization, the lower must be government post-stabilization expenditures, and hence the higher is private consumption. The risk premium is the product of the instantaneous probability of a regime switch at t conditional on one not having previously taken place $(\frac{dF}{1-F})$ and the percentage fall in marginal utility of consumption which the regime switch induces $(\frac{\theta-\theta_s}{\theta})$. Since typically both of these terms are rising over time until a switch occurs, the real interest rate dR will be rising as well. (The hazard rate may in some circumstances be falling over time).

Differentiating (18) with respect to t and rearranging, we obtain (see equation (A2.8) in Appendix 2)

$$\text{(20a)} \qquad \frac{v'(m)}{\theta}dt = \beta dt + \frac{dP}{P} + \frac{dF}{1-F}\left(1 - \frac{\theta_s/P_s}{\theta/P}\right)$$

$$\text{(20b)} \qquad = \beta dt + \frac{dF}{1-F}\left(1 - \frac{\theta_s}{\theta}\right) + \frac{dP}{P} + \frac{dF}{1-F}\left(1 - \frac{P}{P_s}\right)\frac{\theta_s}{\theta}$$

The right-hand-side is the nominal interest rate, which includes both a nominal and real risk premium. The second right-hand-side term in (20b) is the real risk premium associated with jumps in the real interest rate. The last term is a nominal risk premium reflecting changes in the real value of money due to a regime switch. It includes three effects: the hazard rate, the percentage change in the real value of nominal balances from a price jump $(\frac{1/P - 1/P_s}{1/P})$, and the change in the utility value of real balances (θ_s/θ). The form of (20) allows for the possibility of a jump in F. It makes clear that a jump in F induces a jump in P. For example, if P_s is less than P, a jump in F induces an upward jump in P.

b. The dynamic equations for m and b

Equation (20) allows us to derive dynamic equations for debt and real balances, precisely as in the previous section. When F is differentiable, we obtain from (11) and (12)

$$\dot{b} = (\beta + \frac{F'}{1-F}[1 - \frac{\theta_s}{\theta}])b + g - \tau - \mu m \tag{21}$$

$$\frac{\dot{m}}{m} = \beta + \mu - \frac{v'(m)}{\theta} + \frac{F'}{1-F}[1 - \frac{m_s(b)}{m}\frac{\theta_s}{\theta}] \tag{22}$$

where $m_s(b)$ refers to the relevant terminal surface for the policy chosen.

These equations may be expressed in a time-autonomous form. Note first that θ_s may be written as a function of b since $\theta_s = v'(m_s(b))/(\beta + \mu)$ (see footnote 7). The conditional probability of a switch should logically also be a function of debt. Since there is a maximum level of debt consistent with stabilization and since b grows without bound in the absence of stabilization, one must expect that if no regime switch has occurred before b hits some b_{max} (less than or equal to maximum feasible steady state b attained, for example, at $g_s = 0$), then a regime switch must occur at that time. More generally, one may argue that the probability of a regime switch grows as b(t) approaches b_{max}, with a regime switch occurring with certainty sometime between 0 and the time b(t) hits b_{max}.[10] We represent this by writing the conditional density of a switch as a function of b

$$\text{(23)} \qquad \frac{F'(t)}{1-F(t)} = \phi(b(t)).$$

where $\phi'(b) \geq 0$. The restriction that $F(T_{max}) = 1$ will imply that ϕ becomes infinite as b approaches b_{max}, unless the distribution has a mass point at T_{max}.[11]

Given these specifications of θ_s and $\frac{F'}{1-F}$, (21) and (22) may be written

$$\text{(24)} \qquad \dot{b} = (\beta + \phi(b)[1 - \frac{\theta_s(b)}{\theta}])b + g - \tau - \mu m$$

$$\text{(25)} \qquad \frac{\dot{m}}{m} = \beta + \mu - \frac{v'(m)}{\theta} + \phi(b)[1 - \frac{m_s(b)}{m} \frac{\theta_s(b)}{\theta}]$$

Equations (24) and (25) form an autonomous system of two equations in m and b which yield equilibrium time paths for any specification of $\phi(\cdot)$ satisfying the above-noted restrictions. It is the existence of both real and nominal risk premia which complicate these equations.

c. <u>The time paths for debt and real balances under monetary stabilization</u>

To derive the characteristics of the full-time paths when a regime switch is effected solely by changing the rate of money growth, we begin by deriving phase diagrams in m-b space. Since $\theta = \theta_s$, the dynamic equation for debt is equivalent to the certainty case, as is the $\dot{b} = 0$ locus. The dynamic equation for real balances becomes

$$(26) \qquad \dot{m} = (\beta+\mu)m - \frac{1}{\theta}mv'(m) + \phi(b)(m - m_s^{\mu}(b))$$

Since b enters the dynamic equation for real balances, the $\dot{m} = 0$ locus is no longer horizontal and will depend on the sign of $m - m_s$ along $\dot{m} = 0$. One can show that the $\dot{m} = 0$ locus must lie below the $\dot{m} = 0$ locus under certainty.[12]

The phase diagram for uncertain T when only μ changes with a regime switch can be represented as in Figure 7. As will be demonstrated below, equilibrium will require that if we are not already in a steady state, we must be in the unshaded region to the right of $\underline{b}$ where b is rising and m is falling.[13]

We may now specify the equilibrium dynamic path. After a stabilization at some T and associated b(T), we must be on $m_s^{\mu}(b)$. We therefore jump at T to the value of m_s consistent with b(T) according to the terminal surface. What is the path before T? As in the previous section, we consider the terminal condition and work backwards.

Suppose the regime switch occurs only at the last possible moment (rather than considering this final instant in the time domain, let us consider it in terms of b and b_{max}). There can then be no jump in real balances before b_{max}. But if a switch occurs with positive probability at b_{max}, there can also be no jump at b_{max}. Intuitively this is easy to see.

If we know that with non-zero probability there will be a regime switch at the last instant (conditional on the switch not having taken place before), a path which implies a jump at this last instant means that immediately before one may expect with certainty an infinite rate of capital gain (or loss) on money holdings. In such a case, money holdings would adjust before the last instant to eliminate this possibility. Hence, on a path where the regime switch occurs at b_{max} with non-zero probability, we must approach $m_s^{\mu}(b_{max})$ smoothly.[14]

Since the path before a regime switch must arrive smoothly at $(b_{max}, m_s^{\mu}(b_{max}))$ if a switch occurs only at T_{max}, it is clear from the slope of $m_s^{\mu}(b)$ that starting at any $b_0 < b_{max}$, debt must be rising and real balances must be falling. This is why the initial value of real balances consistent with initial debt and equilibrium at every point in time must be in the unshaded region in Figure 7. The exact value of initial balances m_0 may be found by starting at b_{max} and "running the dynamic equations backwards."

At $T \leq T_{max}$, when the regime switch actually occurs, we jump to the $m_s^{\mu}(b)$ line, meaning the price level jumps to $P_s(T)$. The direction of the jump depends on whether the dynamic path is above or below the $m_s^{\mu}(b)$ line (where we assume as before that b_{max} is strictly less than $\bar{b}$). If b_{max} is sufficiently close to $\bar{b}$, one can show that for $b(t)$ close to b_{max}, the optimal dynamic path must be below $m_s^{\mu}(b)$, as in Figure 7.[15] A regime switch "late in the game" will therefore induce an upward jump in real balances meaning a downward price jump. On the other hand, if the horizon is sufficiently long, the dynamic path may start above $m_s^{\mu}(b)$ and a regime switch fairly early on will imply an upward jump in P.

d. Fluctuation in inflation and the risk premium under monetary stabilization

Monotonically declining real balances until the switch takes place implies a rising nominal interest rate, which in the case where the date of the switch is known with certainty, implies a monotonically rising inflation rate. This result does not carry over. Uncertainty about the switch date may induce fluctuations in the inflation rate even when $\phi(\cdot)$ is well-behaved and real balances move monotonically.

Analytically, such a possibility is easy to see. The first-order condition equates the ratio of marginal utilities of real balances and consumption to the nominal interest rate, where under uncertainty the latter is the sum of the real interest rate, the inflation rate, and a "risk premium" $\phi(1 - P/P_s)$. The risk premium reflects the possibility that at any instant a regime change may take place which would induce a price jump and a capital loss (or gain) on nominal balances. Solving for the inflation rate $\pi(t)$ we obtain (in the case where F(t) is differentiable, except perhaps at T_{max})

$$\pi(t) = -\beta + \frac{v'(m(t))}{\theta} - \phi(b(t))[1 - \frac{m_s^{\mu}b(t))}{m(t)}] \tag{27}$$

One notes that a large negative risk premium implies a large positive rate of inflation. Therefore, during the pre-switch period the rate of inflation may happen to be higher than in the post-switch equilibrium. Differentiating (27) with respect to time we obtain

$$(28)\quad \dot{\pi} = \frac{\dot{m}}{m}\left[\frac{mv''}{\theta} - \phi\frac{m_s^{\mu}}{m}\right] + \dot{b}\left[-\phi'\left(1 - \frac{m_s^{\mu}}{m}\right) + \frac{\phi dm_s^{\mu}/db}{m}\right]$$

The term multiplying $\dot{m}/m$ is unambiguously negative, so that the first term is unambiguously positive along the dynamic path, pushing inflation upwards. The term in the second parentheses and hence the whole second term, is, however, negative if $\phi'(b) > 0$ and $m(t)$ is larger than $m_s^{\mu}(b(t))$, and ambiguous if it is smaller. Therefore, $d\pi/dt$ may be of either sign and can change sign as the relative strength of the various influences changes. π therefore need not be monotonic and may fluctuate. (One may note that the expected inflation rate <u>inclusive</u> of the expected jump in P is monotonically rising). Comparison with the case where T is known shows that it is the uncertainty about the switch date (and the implied risk premium) which is the source of these fluctuations.

The importance of the risk premium will be especially great as we approach T_{max} when the instantaneous probability of a regime switch approaches infinity. Here, instantaneous movements in the risk premium will be the dominant factor in determining movements in the rate of inflation and may well induce "overshooting." Consider the first term in the second brackets of (28). As b approaches b_{max}, m will approach $m_s^{\mu}(b_{max})$ (from below if b_{max} is close to $\bar{b}$, given our arguments in section 2c above), so that $(1 - m_s/m)$ will approach zero from below. Since $\phi(b)$ is infinitely large at b_{max} when there is no mass point of F at T_{max}, $\phi'(b)$ will approach infinity as b approaches b_{max}. If ϕ' approaches infinity faster than $(1 - m_s/m)$ approaches zero, the negative contribution of this term will dominate the other terms and the inflation rate will fall rapidly as debt approaches its maximum feasible value. Earlier in the path, when the risk premium is small, the behavior of π will be closer to the case of certainty, so that inflation will be rising. Therefore, one may anticipate overshooting if no regime switch has occurred as t approaches T_{max}. This argument does not, of course, rule out earlier fluctuations in the rate of inflation.

To examine these presumptions about fluctuations in π, we ran numerous simulations of the model. We considered the utility function specified in footnote 5 and the function $\phi(b) = 1 + 100(\bar{b}-b)^{-2}$ which satisfies the restrictions on ϕ in footnote 11. For a range of specified values for the exogenous variable, we indeed found there to be overshooting in π as b approached b_{max}. The results of one simulation may be seen in Table 1 (this is a sample from several thousand points) which presents the relationship between debt, real balance, the rate of inflation and the risk premium. The relationship between debt and the rate of inflation is plotted in Figure 8, showing a rise in the rate of inflation followed by a rapid fall as debt approaches its upper limit.

It is also of interest to note from equation (20a) (or (A2.5) in Appendix 2) that if the distribution function F(t) is discontinuous, so is the price function P(t). This implies that if at some point in time people believe that there is a probability mass of a switch taking place, this will result in a price jump independently of whether the switch does in effect take place. If real balances are above $m_s^{\mu}(b)$, that is, if the risk premium is positive, there is a downward price jump just before it becomes known whether a policy switch takes place. If it does not take place the price level changes continuously, and if it does take place, the price level jumps upwards. Jumps in opposite directions result at times at which the risk premium is negative.

A situation such as this may arise if there is a threshold debt level (or a point in time) at which a switch is most likely to occur. This describes, therefore, a realistic possibility. Since a discontinuity in the distribution function can be approximated by a very fast increase in F(t), this shows also that in the presence of uncertainty about marcoeconomic policies there can be rapid changes in the rate of inflation. For example, suppose the distribution function rises very quickly over a very short time interval and flattens out subsequently. If the risk premium is positive during this time interval, inflation will rapidly decline initially and rapidly increase subsequently, but if it is negative, inflation will rapidly increase initially and rapidly decline subsequently, provided that no policy change has taken place.

Our analysis has a bearing on the issue of tight money and inflation that was raised by Sargent and Wallace (1981). They consider an economy initially in steady state with a constant rate of inflation and money growth in which the government unexpectedly reduces the rate of money growth without changing taxes or expenditure. This induces a growth in debt which is stopped only when the rate of monetary growth is raised to the necessary level. They ask what are the inflationary consequences of a current tightening of monetary growth, given the anticipation of an eventual inevitable increase in the money growth rate. As Drazen (1985) has shown in an environment of the type considered in this paper, the temporary tightening of money growth can bring about an initial increase or reduction in the rate of inflation, but thereafter the rate of inflation will rise over time until it reaches its new higher steady state level. There may, but need not be, short-run decreases in the rate of inflation. It is in fact possible for the tight money policy to bring about higher information at all times. In that analysis the date at which debt is stopped from growing is known with certainty.

Our analysis implies that in the presence of uncertainty about the date in which debt growth will be stopped, a tight money policy can generate temporarily very high inflation rates, with the rate of inflation declining over time as long as debt is not prevented from growing (see the last part of the graph in Figure 8). This is more likely the larger the initial debt level and the larger the instantaneous conditional probability of a policy change, that is, the less confidence there is in the tight money policy.[16]

e. Dynamic behavior with expenditure cuts

When stabilization is effected via a cut in expenditures, the dynamic equations are of the general form of (24) and (25) with $m_s(b)$ set equal to

$m_s^g(b)$. To derive phase diagrams, the equation for the $\dot{b} = 0$ locus may be written

$$(29) \qquad \beta b + \phi(b)(1 - \frac{\theta_s(b)}{\theta})b + g - \tau = \mu m$$

Assuming that $\phi(\cdot)$ is rising in b the second left-hand-side term is unambiguously increasing in b. Hence, the $\dot{b} = 0$ locus is upward sloping with a slope greater than in the certainty case. $\dot{b}$ is rising to the right of this locus, falling to the left of it.

The $\dot{m} = 0$ locus is defined by

$$(30) \qquad \frac{v'(m)}{\theta} - (\beta + \mu) = \phi(b)[1 - \frac{\theta_s(b)m_s^g(b)}{\theta m}]$$

As steady state b rises, θ_s falls and m_s rises. The derivative of the right-hand-side of (30) is therefore ambiguous with respect to b. Therefore the $\dot{m} = 0$ locus may either rise or fall in m-b space and in fact need not be single-signed. Real balances will be falling below the locus, rising above it. The locus will lie below $m_s^g(b)$, since along $m_s^g(b)$ the probability ϕ of a regime switch is zero, so that $\dot{m} > 0$ from (25). The phase diagram may be represented as in Figure 9, where we show two possible loci for $\dot{m} = 0$.

We may now derive the dynamic path. The ambiguity about the possible slope of the $\dot{m} = 0$ locus (discussed in the previous paragraph) combined with the jump in real balances at T_{max} (due to a jump in the marginal utility of consumption and hence the price level at this point) will lead to a variety of possible paths. Specifically, at T_{max} we have (see equation (A2.7) in Appendix 2)

$$(31) \qquad \frac{\theta}{\theta_s(b_{max})} = \frac{P(T_{max})}{P_s(T_{max})} = \frac{m_s^g(b_{max})}{m_{max}}$$

where m_{max} denotes the pre-switch level of real balances at T_{max} (analogous to $m(T)$ in the certainty case). Since $\theta > \theta_s(b_{max})$ (as government spending is lower and therefore consumption higher after a stabilization), m_{max} must lie below $m_s^g(b_{max})$.

The key to the behavior of the dynamic path is the location of m_{max} relative to $\dot{m} = 0$ locus. If m_{max} lies above this locus, the dynamic path will be rising as we approach T_{max}; if it lies below, it will be falling. As demonstrated in Appendix 3, where possible paths are discussed in detail, the location of m_{max} relative to the $\dot{m} = 0$ locus depends on whether it is greater or lesser than $\bar{m}$, the value of real balances for which $\dot{m} = 0$ in the certainty case (see the discussion preceding equation (14)). m_{max} will lie below the $\dot{m} = 0$ locus under uncertainty when m_{max} is less than $\bar{m}$ and will lie above when it is greater.

The other crucial characteristic is whether the dynamic path crosses the $\dot{m} = 0$ locus. If it does, real balances will not move monotonically along the path. If it does not cross, they will. (Debt is monotonically rising along every path). Of course, whether or not the dynamic path crosses the $\dot{m} = 0$ locus depends on the characteristics of that locus. Leaving a more detailed discussion of the paths to be presented in Appendix 3, the two possibilities about the location of m_{max} and the two possibilities about crossing $\dot{m} = 0$ yield four general types of paths for real balances: they may rise monotonically; they may rise near T_{max}, falling or oscillating beforehand; they may fall monotonically; and, they may fall near T_{max}, rising or oscillating beforehand. The conditions for each case are summarized in Table 2, and the cases are illustrated in the four panels of Figure 10. One should, of course, remember that these are paths only until a stabilization takes place. When a stabilization takes place at some t before T_{max}, the value of real balances jumps to $m_s^g(b(t))$, due to a price level jump.

The behavior of inflation along any path depends not only on the behavior of real balances, but also on the existence of the risk premia, which reflect the possibility that at any instant a regime change may take place which would induce a jump in both the price level and the real interest rate. The first-order condition (20) equates the ratio of marginal utilities of real balances and consumption to the nominal interest rate, which reflects risk premia which may fluctuate. Solving for the inflation rate, we obtain (where $F(\cdot)$ is differentiable):

$$(32) \qquad \pi(t) = -\beta + \frac{v'(m(t))}{\theta} - \phi(b(t))(1 - \frac{\theta_s(b(t))}{\theta})$$

$$- \phi(b(t))(1 - \frac{P(t)}{P_s(t)})\frac{\theta_s(b(t))}{\theta}$$

$$= -\beta + \frac{v'((t))}{\theta} - \phi(b(t))[1 - \frac{\theta_s(b(t))}{\theta} \frac{m_s^g(b(t))}{m(t)}]$$

Differentiating with respect to time we obtain

$$(33) \qquad \dot{\pi} = \frac{\dot{m}}{m}[\frac{mv''}{\theta} - \phi \frac{\theta_s}{\theta} \frac{m_s^g}{m}]$$

$$+ \dot{b}[-\phi'(1 - \frac{\theta_s}{\theta} \frac{m_s^g}{m}) + \phi(\frac{m_s^g}{m} \cdot \frac{d\theta_s/db}{\theta} + \frac{\theta_s}{\theta} \frac{dm_s^g/db}{m})]$$

The term multiplying $\dot{m}/m$ is unambiguously negative, so that the first term is unambiguously positive along the dynamic path, when real balances are falling, negative if they are rising. The term in the second brackets is of ambiguous sign, so that $d\pi/dt$ may be of either sign and can change sign. Therefore, inflation and real balances need not move in opposite directions.

In Drazen and Helpman (1985b) we present an example that replicates the time path shown in panel b of Figure 10. This may be of especial interest in connection with the European hyperinflations of the 1920's, which were ended by fiscal reforms including sharp cuts in government expenditures whose timing was uncertain ex-ante. In a number of cases the sharp drop in real balances which characterize hyperinflations was reversed before the fiscal reforms which led to a lower rate of growth of money were enacted. (Austria and Hungary provide two examples). We think it useful to show that such behavior can arise in this sort of model, in particular in view of the fact that in that example it was assumed that the instantaneous probability of a switch to constant, so that this variable is not the main driving force behind the result. This lengthy example is not presented here in order to save space.

g. Dynamic Behavior under tax stabilization

When stabilization is effected via a change in non-distortionary taxes, the dynamic equations are identical to the case of money-based stabilization. Since the dynamic path must intersect $m_s^{\tau}(b)$ at T_{max}, which is identical to the $\dot{m} = 0$ locus under certainty, the path must always be on this locus. The dynamic equations and results therefore turn out to be identical to the certainty case.

5. SUMMARY AND CONCLUSIONS

The purpose of this paper was to consider situations where current macroeconomic policies are known to be infeasible, implying an eventual regime switch, but where the exact timing or nature of this switch is unknown. Our goal was to consider the effects of this uncertainty on macroeconomic variables.

We found that under full certainty, stabilization via an increase in the money growth rate implies a monotonically increasing inflation rate before a regime switch occurs, while tax-based stabilization induces a constant inflation rate, everywhere lower than in the previous case. Stabilization via budget cuts induces a jump in the price level and a "blip" in the real interest rate at the date of stabilization. This occurs because the upward jump in private consumption means that the price level and the real interest rate must jump to ensure market clearing at every point in time. Real balances may rise, stay constant, or fall before a stabilization, with inflation moving in the opposite direction. Under uncertainty only about the policy mix, there will be a one-time price jump when the switch occurs whose sign and magnitude depend on the actual policy mix adopted relative to what was expected.

In contrast to these results, when it is known with certainty that the regime switch will entail an increase in the rate of monetary growth, but the timing of the switch is uncertain, not only will there be a jump in the price level at the actual switch date, but fluctuations in the rate of inflation may also result. Therefore, uncertainty about the timing of an inevitable policy change may of itself induce fluctuations in the inflation rate. Furthermore, when budget cuts whose timing is uncertain are used to stabilize, the real balances may rise monotonically, fall monotonically, or may oscillate. The rate of inflation may similarly exhibit a wide range of paths.

A further characteristic of the time paths which emerges is the lack of any necessary contemporaneous correlation between budget deficits and the rate of monetary growth on the one hand and the rate of inflation on the other. Depending on the expected policy mix which effects the regime switch, a rising deficit (inclusive of debt service) may induce a rising, constant, or falling inflation rate. With uncertainty about the timing of a policy change, a constant rate of money growth and a constant deficit (exclusive of debt service) may be associated with a fluctuating inflation rate. This lack of correlation arises even though it is clear that the budget deficit is the ultimate cause of inflation.

APPENDIX I - KNOWN SWITCH DATES

In this appendix we derive the first-order conditions when the switch date is known. These are necessary and sufficient conditions. The problem of maximizing present discounted utility (2) subject to constraints (3) and (4) may be written

$$\max_{\{c(t),z(t),M(t)\}} \int_0^T e^{-\beta t}[u(c(t)) + v(\frac{M(t)}{P(t)})]dt$$

$$\text{(A.1.1)} \qquad + e^{-\beta T}V^s[e^{R(T)}w(0) - e^{R(T)}\int_0^T e^{-R(t)}(c(t) + \frac{z(t)}{P(t)} - y)dt$$

$$+ \frac{M(0) + \int_0^T Z(x)dx}{P_s(T)}] + \int_0^T \gamma(t)[M(0) + \int_0^t z(x)dx - M(t)]dt$$

where $\gamma(t)$ is the multiplier on constraint (4)) in the text. Maximization of (A1.1) with respect to each of the $c(t)$, $z(t)$ and $M(t)$ yields, respectively, (where V_w^s is the derivative of $V^s(\cdot)$ with respect to wealth which equals $u'(y_0-g_s)$):

$$\text{(A1.2)} \qquad e^{-\beta t}u'(y_0-g) = e^{-\beta T}u'(y_0-g_s)e^{R(T)-R(t)}$$

$$\text{(A1.3)} \qquad \int_t^T \gamma(x)dx = e^{-\beta T}u'(y_0-g_s)[e^{R(T)-R(t)}\ \frac{1}{P(t)} - \frac{1}{P_s(T)}]$$

$$\text{(A1.4)} \qquad e^{-\beta t} v'\left(\frac{M(t)}{P(t)}\right) \cdot \frac{1}{P(t)} = \gamma(t)$$

Condition (A1.2) implies that (using $R(0) = 0$)

$$\text{(A1.5)} \quad R(t) = \begin{matrix} \beta t & \text{for } 0 \le t < T \\ \beta T + \ln\frac{u'(y_0 - g)}{u'(y_0 - g_s)} & \text{for } t = T \end{matrix} \qquad dR(t) = \begin{matrix} \beta dt & \text{for } 0 \le t < T \\ \beta dt + \ln\frac{u'(y_0 - g)}{u'(y_0 - g_s)} & \text{for } t = T \end{matrix}$$

The conditions (A1.3) through (A1.5) also imply

$$\text{(A1.6)} \quad \frac{1}{P(t)} = \int_t^T e^{-\beta(x-t)} \frac{v'(m(x))}{\theta} \frac{1}{P(x)} dx + e^{-(R(T)-R(t))} \frac{1}{P_s(T)}, \quad 0 \le t < T$$

where $\theta = u'(y_0 - g)$. Thisis a standard asset pricing equation. (A1.6) implies that at T there is a price jump since we have that

$$\frac{P(T)}{P_s(T)} = e^{R(T) - R(T^-)}$$

which from (A1.5) yields

$$\text{(A1.7)} \qquad \frac{P(T)}{P_s(T)} = \frac{u'(y_0 - g)}{u'(y_0 - g_s)} .$$

Differentiating (A1.6) we obtain

$$\text{(A1.8)} \qquad \frac{v'(m)}{\theta}dt = dR + \frac{dP}{P} \quad , \qquad 0 \leq t < T$$

APPENDIX 2 - UNCERTAIN DATE OF A REGIME SWITCH

In this appendix we derive the first-order conditions when the date T of a switch is unknown. When the cumulative distribution of a switch occurring until T is F(T), the individual maximizes (16) in the text, subject to (3) and (4). The choice problem may be written

$$\max_{\{c(t),z(t),M(t)\}} \int_0^T \max\{\int_0^T e^{-\beta T}[u(c(t)) + v(\frac{M(t)}{P(t)})]dt$$

$$\text{(A2.1)} \qquad + e^{-\beta T}V^s[e^{R(T)}w(0) - \int_0^T e^{R(T)-R(t)}(c(t) + \frac{z(t)}{P(t)} - y)dt$$

$$+ \frac{M(0) + \int_0^T z(x)dx}{P_s(T)}] + \int_0^T \gamma(t)[M(0) + \int_0^t z(x)dx - M(t)]dt\}dF(T)$$

where $\gamma(t)$ is the multiplier on the constraint (4)in the text. Maximization of (A2.1) with respect to each of the c(t), z(t), and M(t) yields, respectively, where $\theta_s = u'(y_0-g_s)$ is the derivative of $V^s(\cdot)$ with respect to wealth and where $\theta = u'(y_0-g)$,

$$\text{(A2.2)} \qquad [1 - F(t)]e^{-\beta t}\theta = \int_t^{T_{max}} e^{-\beta T} e^{R(T)-R(t)} \theta_s(T)\, dF(T)$$

$$\text{(A2.3)} \qquad \int_t^{T_{max}} \{-\theta_s(T) e^{-\beta T} e^{R(T)-R(t)} \frac{1}{P(t)} + \frac{\theta_s(T)}{P_s(T)} e^{-\beta T} + \int_t^T \gamma(x)\,dx\}\, dF(T) = 0$$

$$\text{(A2.4)} \qquad e^{-\beta T} \frac{1}{P(t)} v'(m(t)) = \gamma(t)$$

Differentiating (A2.2) with respect to t we obtain

$$\text{(A2.5)} \qquad dR = \beta dt + (1 - \frac{\theta_s}{\theta})\frac{dF}{1-F}$$

We may rewrite (A2.3) as

$$\text{(A2.6)} \qquad e^{-R(t)} \frac{1}{P(T)} \int_t^{T_{max}} \theta_s(T) e^{R(T)-\beta T} dF(T) = \int_t^{T_{max}} \int_t^T e^{-\beta x} \frac{v'(m(x))}{P(x)} dx\, dF(T) +$$

$$+ \int_t^{T_{max}} e^{-\beta t} \frac{\theta_s(T)}{P_s(T)} dF(T)$$

where by (A2.2) the left-hand-side of (A2.6) is simply $[1-F(t)]\theta\frac{e^{-\beta t}}{P(t)}$. Upon substitution of the expression on the left-hand-side of (A2.6) we obtain the asset pricing equation for money balances. As t approaches T_{max} we therefore have

$$\text{(A2.7)} \qquad \frac{\theta}{P(T_{max})} = \frac{\theta_s(T_{max})}{P_s(T_{max})}$$

Differentiating (A2.6) with respect to t (after the substitution of (A2.2)) we obtain

$$-dF\theta\frac{e^{-\beta t}}{P} - \beta(1-F)\theta\frac{e^{-\beta t}}{P}dt - (1-F)\theta\frac{e^{-\beta t}}{P}\frac{dP}{P}$$

$$= -(1-F)e^{-\beta t}v'(m)\frac{1}{P}dt - e^{-\beta t}\frac{\theta_s}{P_s}dF$$

which, upon rearranging becomes

$$\text{(A2.8)} \qquad \frac{v'(m)}{\theta}dt = \beta + \frac{dP}{P} + \frac{dF}{1-F}\left(1 - \frac{\theta_s/P_s}{\theta/P}\right)$$

APPENDIX 3: POSSIBLE DYNAMIC PATHS UNDER UNCERTAINTY

In this appendix we consider in detail possible dynamic paths under uncertainty about the timing of the regime switch. We will derive four general types of paths which may obtain if no switch occurs before T_{max}. Two sets of characteristics are key: first, whether the value of real balances at T_{max} prior to a switch, denoted m_{max}, lies above or below the $\dot{m} = 0$ locus; second, whether the dynamic path crosses the $\dot{m} = 0$ locus.

On the first, if we denote by $\bar{m}$ the value of real balances when $\dot{m} = 0$ in the certainty case, m_{max} will lie below the $\dot{m} = 0$ locus under uncertainty when m_{max} is less than $\bar{m}$, and will lie above it when it is greater. This follows from noting that since $\frac{v'(\bar{m})}{\theta} = \beta + \mu$ we have (using (31) in the text) that

(A3.1)
$$\frac{\dot{m}}{m}(m = m_{max}) = \beta + \mu - \frac{v'(m_{max})}{\theta} \gtrless 0 \text{ as } m_{max} \gtrless \bar{m}$$

Whether or not the dynamic path crosses the $\dot{m} = 0$ locus will clearly depend on the slope of the locus and the relative position of m_{max}. If the locus is monotonically upward sloping, the dynamic path cannot cross if m_{max} lies above the locus. This follows from noting that the dynamic path must end up above the $\dot{m} = 0$ locus, which will be true when the locus is always rising only if it is everywhere above the locus. In this case the dynamic path itself is monotonically rising (Figure 10a). One may note that even if the locus is downward sloping, the dynamic path may still be everywhere rising, though a negatively sloped $\dot{m} = 0$ combined with $m_{max} > \bar{m}$ is not sufficient for a rising dynamic path as was true in the first case.

Analogously, if m_{max} lies below the $\dot{m} = 0$ locus ($m_{max} < \bar{m}$), the dynamic path must be monotonically falling if $\dot{m} = 0$ is monotonically falling (Figure 10c). When $m_{max} < \bar{m}$, a positively sloped $\dot{m} = 0$ locus can, but need not yield a monotonically falling dynamic path.

In other cases, especially those where the $\dot{m} = 0$ locus itself changes sign, the dynamic path may change sign, or equivalently, the dynamic path will cross the $\dot{m} = 0$ locus. For example, when m_{max} lies above the $\dot{m} = 0$ locus, a path along which real balances first fall and then rise is consistent either with $\dot{m} = 0$ first rising and then falling, or with the locus everywbere falling (Figure 10b). When m_{max} lies below the $\dot{m} = 0$ locus, real balances first rising and then falling is consistent with $\dot{m} = 0$ first falling and then rising, or with it rising throughout (Figure 10d).

If the dynamic path crosses the $\dot{m} = 0$ locus more than once, real balances will oscillate, the number of sign changes obviously being equal to the number of crossings. Multiple crossings may occur even when the slope of the $\dot{m} = 0$ locus change sign only once.

All of these possibilities are summarized in Table 2.

FOOTNOTES

1. We assume that the distribution of expected policy mixes does not change with the passage of time, so that $G(P_s(T))$ is time autonomous. We consider below the effects of changing this assumption.

2. The stock of real debt could jump as the result of an open-market operation, a policy tool we exclude from consideration here. Open-market operations are considered in Drazen and Helpman (1985c).

3. The government budget constraint shows that although $\dot{b}$ is not continuous, b will be for the case of a stabilization via an expenditure cut. This may be seen by writing the budget constraint in integral form

$$b(t) = b_0 + \int_0^t e^{R(x)}(g(t) - \tau - \mu m(x))dx \quad \text{for all } t$$

where

$$g(t) = \begin{cases} g & \text{for } 0 \le t < T \\ g_s & \text{for } t \ge T \end{cases}$$

4. The mathematical expression for the terminal surface may be derived by combining the $\dot{m} = 0$ locus with the $\dot{b} = 0$ locus (the government budget constraint in steady state) to obtain

$$\frac{v'(m_s)}{\theta} = \beta + \frac{\beta b_s + g - \tau}{m_s}$$

Diagramatically, when the elasticity of money demand is less than one, an increase in μ shifts the $\dot{m} = 0$ locus downward less than the $\dot{b} = 0$ locus so that the new intersection is to the southeast of the original intersection.

5. For example, if the instantaneous utility of money balances is

$$v(m) = \begin{cases} \int_m^1 \ln x dx & 0 \leq m \leq 1 \\ 0 & m \geq 1 \end{cases}$$

the money demand function is $m = e^{-\theta(\beta + \dot{P}/P)}$, so that inflation tax revenue reaches a maximum at $\dot{P}/P = 1/\theta$. Maximum sustainable bond holdings are $\bar{b} = \frac{1}{\beta\theta} e^{-(1+\beta\theta)} - \frac{d}{\beta}$.

6. Adding (11) and (12) together, one notes that the locus $\dot{m} + \dot{b} = 0$ is identical to the $m_s^{\mu}(b)$ locus in footnote 4, so that total assets are rising above the $m_s^{\mu}(b)$ locus, falling below it, and locally constant

on the curve. Since the absolute value of the slope of the path relative to unity depends on whether $m + b$ is rising or falling, the slope of the path will be less than, equal to, or greater than one in absolute value depending on whether we are above, on, or below $m_s^{\mu}(b)$, whose slope approaches infinity as b approaches $\bar{b}$. The optimal path must therefore cut this curve from below if T is sufficiently large, and will be above $m_s^{\mu}(b)$ if it is flat.

7. Algebraically, we may derive the equation for the terminal surface as follows. After a budget cut, we have

$$c_s = y_0 - g_s$$

$$= y_0 - \beta b - \tau - \mu m_s$$

from the steady state government budget constraint. Along the $\dot{m} = 0$ locus this implies

$$\beta + \mu = \frac{v'(m_s)}{u'(y_0 + \beta b - \tau - \mu m_s)}$$

which yields an upward sloping $m_s^g(b)$ curve.

8. This point is treated more fully in Drazen and Helpman (1985b).

9. The jump in the price level will reflect not only the "arrival of new information" about the actual stabilization program, but also any change in c and θ if the stabilization includes a budget cut.

10. The previous section indicated that the maximum level of debt under each policy mix is fixed by the underlying behavioral relations. It is therefore independent of F(t). The time it takes for debt to hit this maximum will, however, depend on F(t) under uncertainty about timing. T_{max} therefore depends on F(t) via the behavioral relations, though the individual takes T_{max} as given in his optimization problem.

11. This may be shown by solving the differential equation (26) to yield

$$F(t) = 1 - \exp(-\int_0^{T_{max}} \phi(b(x))dx),$$

where we have used the boundary condition $F(0) = 0$. The other boundary condition, $F(T_{max}) = 1$, obviously implies that $\int_0^{T_{max}} \phi(b(x))dx = +\infty$, unless the distribution has a mass point at $t = T_{max}$. Using equation (9) for $\dot{b}$ we find

$$\int_0^{T_{max}} \phi(b(t))dt = \int_{b_0}^{b_{max}} \phi(b)\ (1/\dot{b})db$$

$$= \int_{b_0}^{b_{max}} \frac{\phi(b)}{\beta b + g - \tau - \mu m(b)} db = +\infty$$

Since $\int_{b_0}^{b_{max}} \frac{\theta(b)}{\beta b + g - \tau - \mu m(b)} db >$

$$(\beta b_{max} + g - \tau - \mu m(b_{max}))^{-1}\int_{b_0}^{b_{max}} \phi(b)db,$$

a sufficient condition for $\int_0^t \phi(b(t))dt$ to approach $+\infty$ as t approaches T_{max} is that $\int_{b_0}^{b} \phi(b)db$ approaches $+\infty$ as b approaches b_{max}. This can also be shown to be a necessary condition.

12. It turns out that all we need to know about the $\dot{m} = 0$ locus is its position relative to the $m_s^{\mu}(b)$ locus and the $\dot{b} = 0$ locus. On the first, note that along $m_s^{\mu}(b)$, the final term in (26) vanishes, so that the movement of real balances is identical to the certainty case. Since $m_s^{\mu}(b)$ lies below the $\dot{m} = 0$ locus under certainty, we therefore know that on $m_s^{\mu}(b)$, $\dot{m} < 0$ (and $\dot{b} > 0$ to the right of $\underline{b}$ in Figure 7), for any $\phi(\cdot)$ function. Therefore, the $m_s^{\mu}(b)$ curve must lie below the $\dot{m} = 0$ curve when T is uncertain. Since m is therefore greater than $m_s^{\mu}(b)$ along $\dot{m} = 0$, $\phi(b)(m-m_s)$ is unambiguously positive and the $\dot{m} = 0$ curve is downward sloping when ϕ is increasing in b. In this case it must also lie below the $\dot{b} = 0$ curve.

13. We have drawn the $\dot{m} = 0$ locus as horizontal to the left of the $\dot{b} = 0$ locus, since when b is falling, there is no inevitability of stabilization, implying that $\phi(b) = 0$ and $\dot{m} = 0$ is horizontal.

14. Technically, the impossibility of a jump at T_{max} may be seen by examining the individual's first-order conditions (A2.3) and (A2.4) in Appendix 2 as t approaches T_{max}. From the definition of $\gamma(t)$ in (A2.4) we see that the second integral in (A2.3) must be zero at $t = T_{max}$. This means that as long as $dF(T_{max})$ is non-zero, the term in brackets in the first integral must be zero at T_{max}, so that $P_s(T_{max})$ is equal to $P(T_{max})$. That is, there is no price jump.

15. This may be seen by noting first that for b_{max} strictly less than $\bar{b}$, $\phi(b)$ will be finite at $\bar{b}$ from our earlier assumptions. Since the dynamic path must intersect $m_s^{\mu}(b)$ at b_{max}, meaning $m - m_s$ approaches zero, the equations for the dynamic path and $m_s^{\mu}(b)$ approach those of the certainty case, so the slope of the dynamic path approaches one at b_{max}. For b_{max} close to $\bar{b}$, the slope of $m_s^{\mu}(b)$ at b_{max} approaches infinity, so the dynamic path must cut from below.

16. One may note that if the utility function is log-linear in real balances, the results are identical to the certainty case. This arises because if the dynamic path ever crosses the $m_s^{\mu}(b)$ locus, it must remain on this locus. The dynamic path must therefore always be on this locus. This case is discussed in Drazen and Helpman (1985a). See also Liviatan (1984) and Drazen (1985).

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TABLE 1

$$v'(m) = \begin{cases} -\ln(m/100), & 0 \le m \le 1 \\ 0, & m \ge 1 \end{cases}, \qquad \phi(b) = 1 + 100(\bar{b}-b)^{-2}$$

$\theta = 1, \quad \beta = .5, \quad \gamma = .5, \quad g - \tau = 10$

$\bar{b} = 24.626, \quad b_{max} = 23.788$

b public debt	m real balances	$\dot{P}/P$ inflation	$\phi(1 - \frac{m_s}{m})$ risk premium
21.885	30.415	.8018	-.1116
22.002	30.227	.8170	-.1206
22.244	29.983	.8516	-.1418
22.370	29.612	.8719	-.1549
22.500	29.385	.8946	-.1699
22.633	29.147	.9202	-.1874
22.771	28.895	.9490	-.2074
22.912	28.627	.9823	-.2315
23.207	28.034	1.0677	-.2960
23.351	27.702	1.1232	-.3395
23.362	27.738	1.1794	-.3840
23.701	26.928	1.1751	-.3629
23.753	26.823	1.0393	-.2233
23.788	26.776	.8177	0

TABLE 2

POSSIBLE DYNAMIC PATHS

	Path does not cross $\dot{m} = 0$ locus[1]	Path crosses $\dot{m} = 0$ locus
$m_{max} > \bar{m}$ (m_{max} above $\dot{m}=0$ locus)	Real balances rise monotonically (Figure 10a)	Real balances fall, then rise[2], or, oscillate and then rise (Figure 10b)
$m_{max} < \bar{m}$ (m_{max} below $\dot{m}=0$ locus)	Real balances fall monotonically (Figure 10c)	Real balances rise, and then fall[3], or, oscillate and then fall (Figure 10d)

1. This is consistent with the $\dot{m} = 0$ being either positively or negatively sloped.

2. This is consistent with the $\dot{m} = 0$ locus rising and then falling, or falling throughout.

3. This is consistent with the $\dot{m} = 0$ locus falling and then rising, or rising throughout.

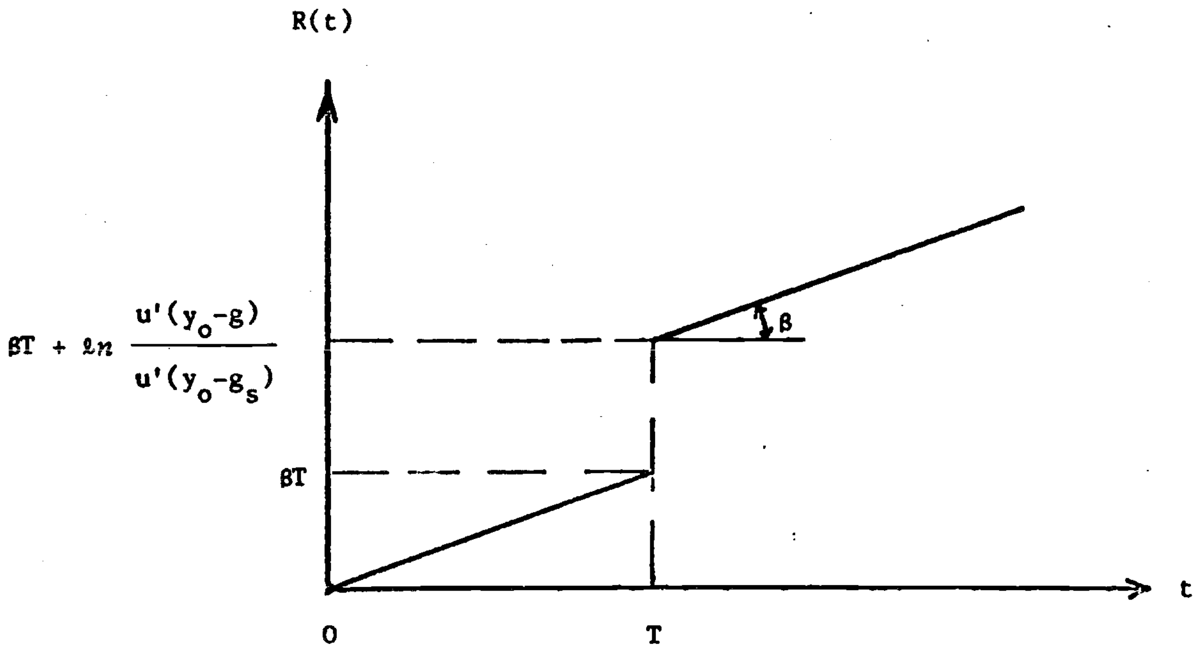
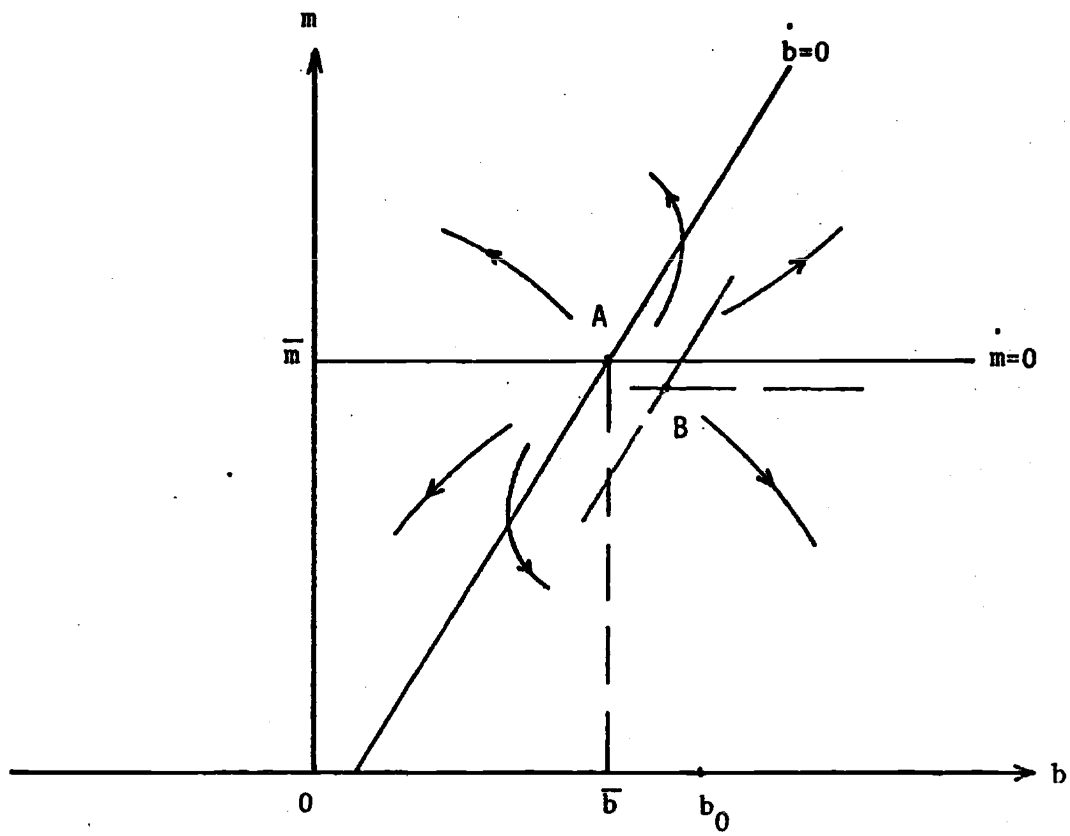


FIGURE 1



<u>FIGURE 2</u>

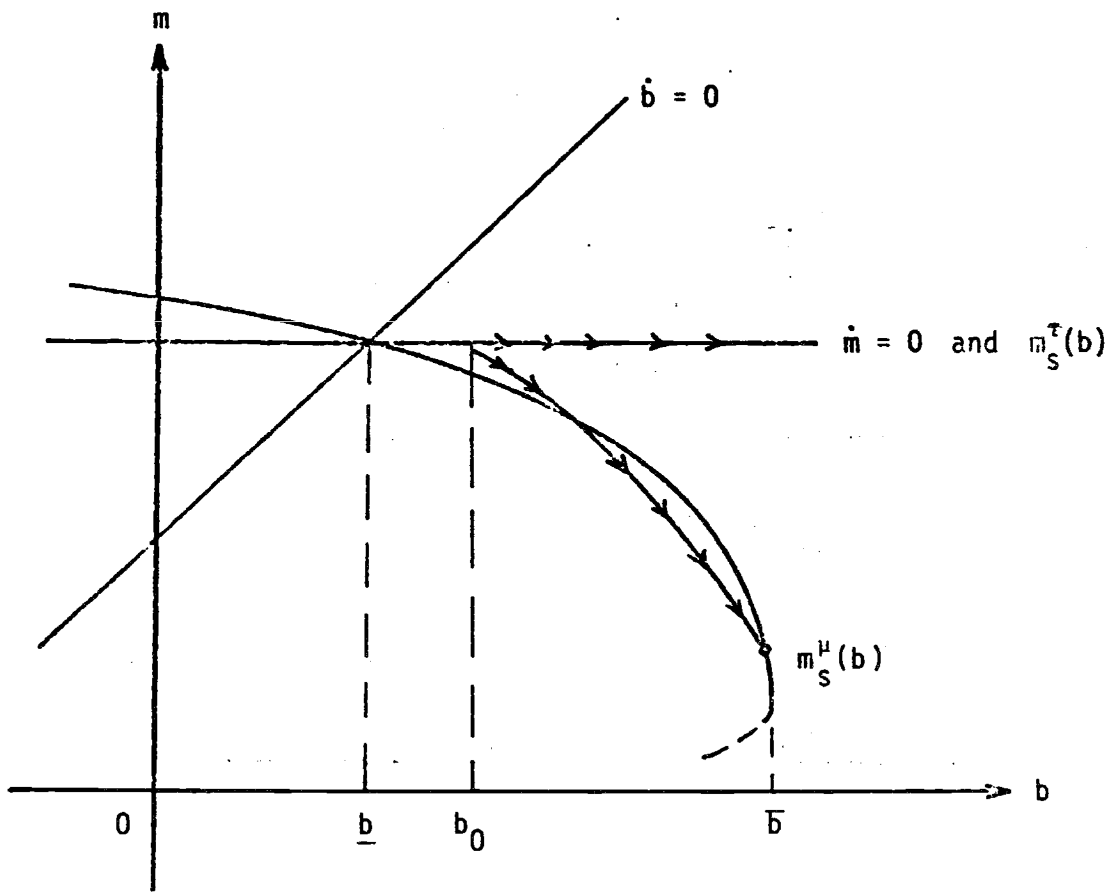


Figure 3

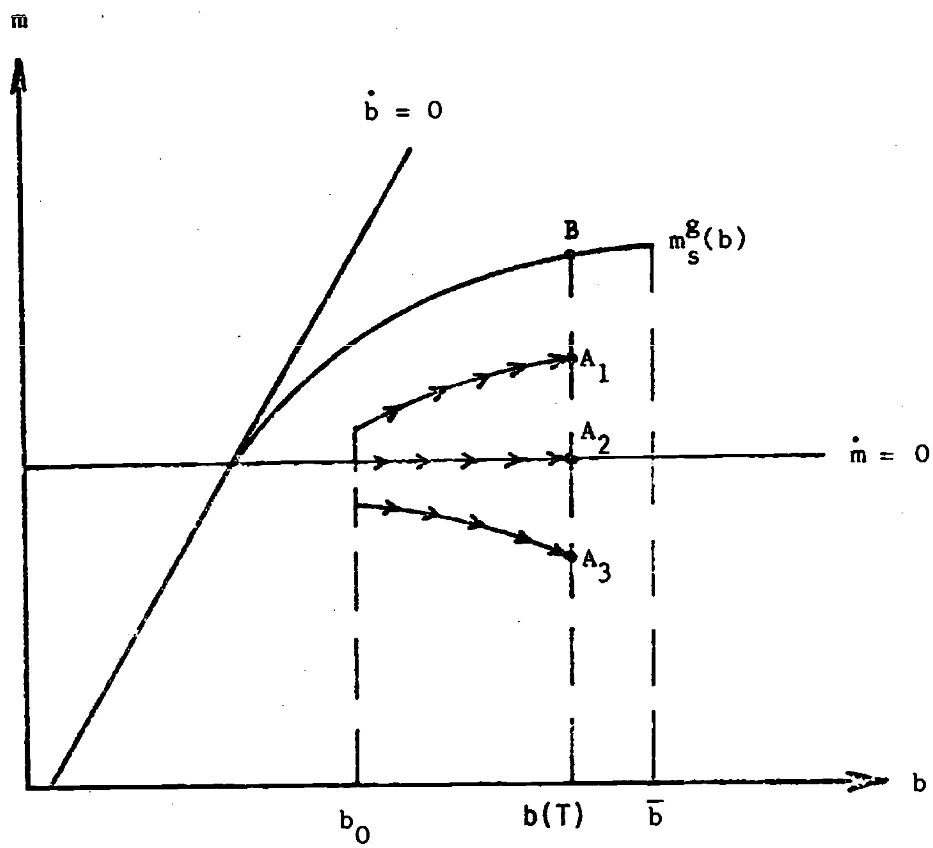
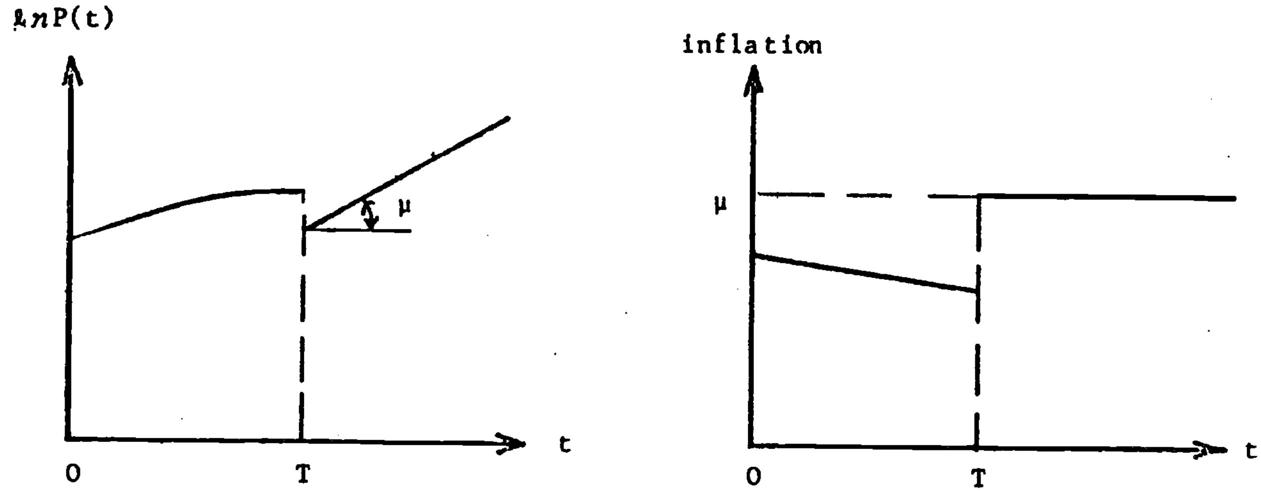
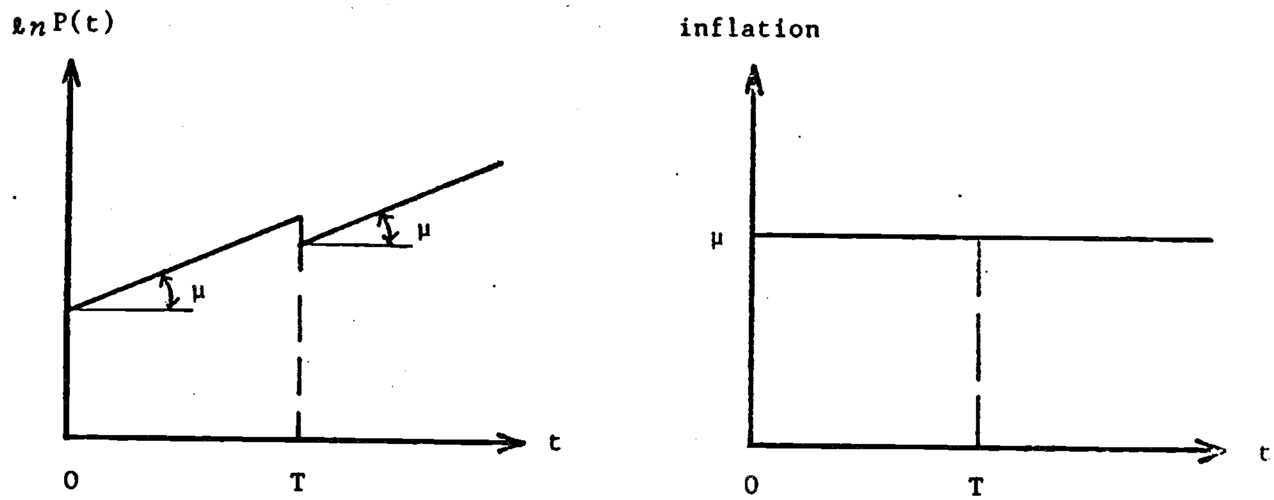


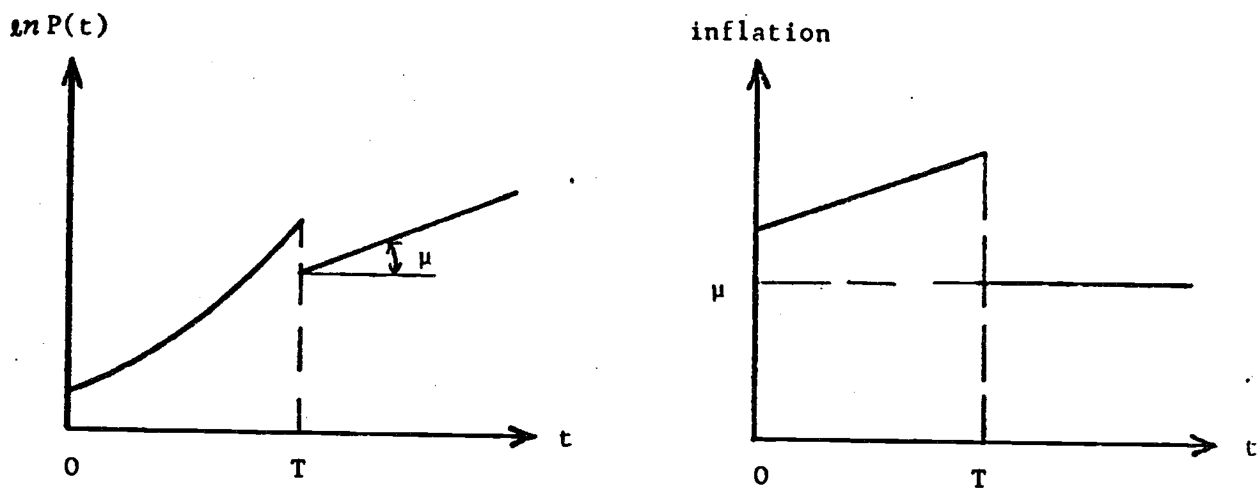
Figure 4



(i) $v'(m_s)m_s > v'(\bar{m})\bar{m}$



(ii) $v'(m_s)m_s = v'(\bar{m})\bar{m}$



(iii) $v'(m_s)m_s < v'(\bar{m})\bar{m}$

FIGURE 5

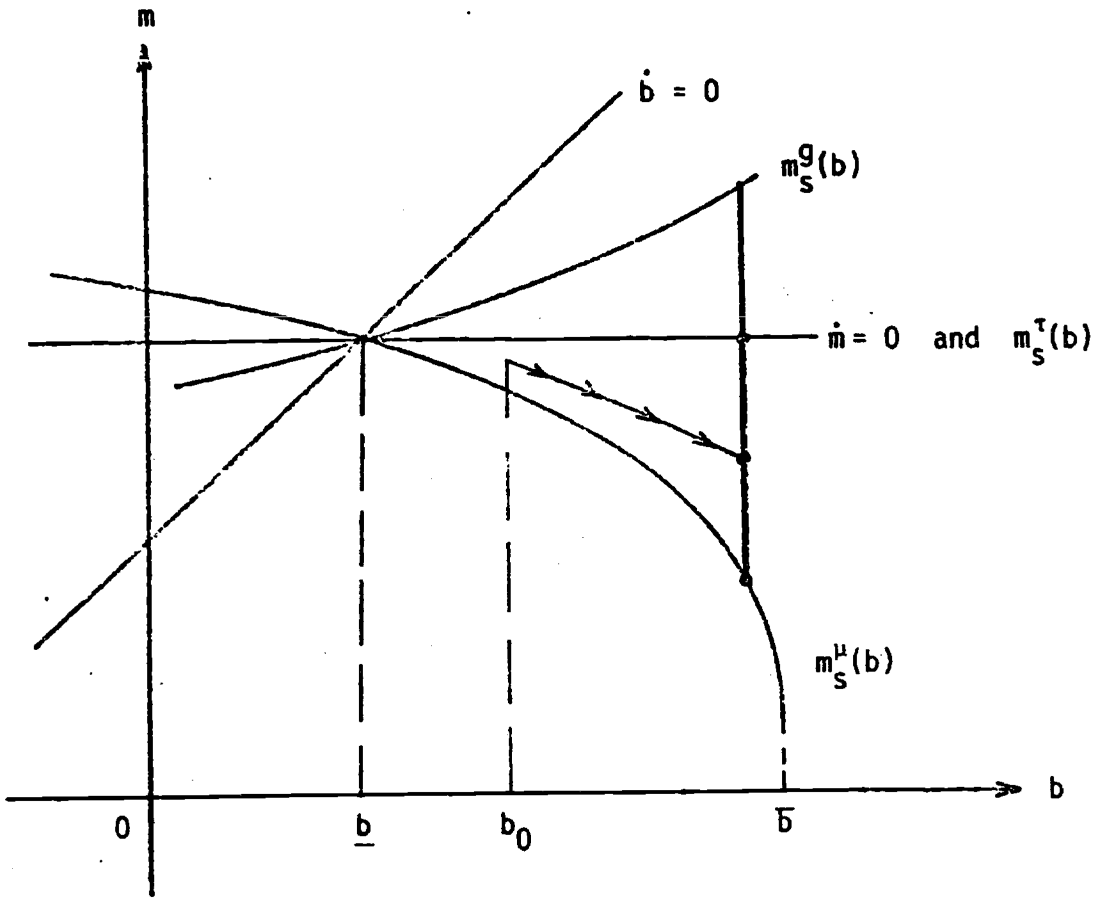


Figure 6

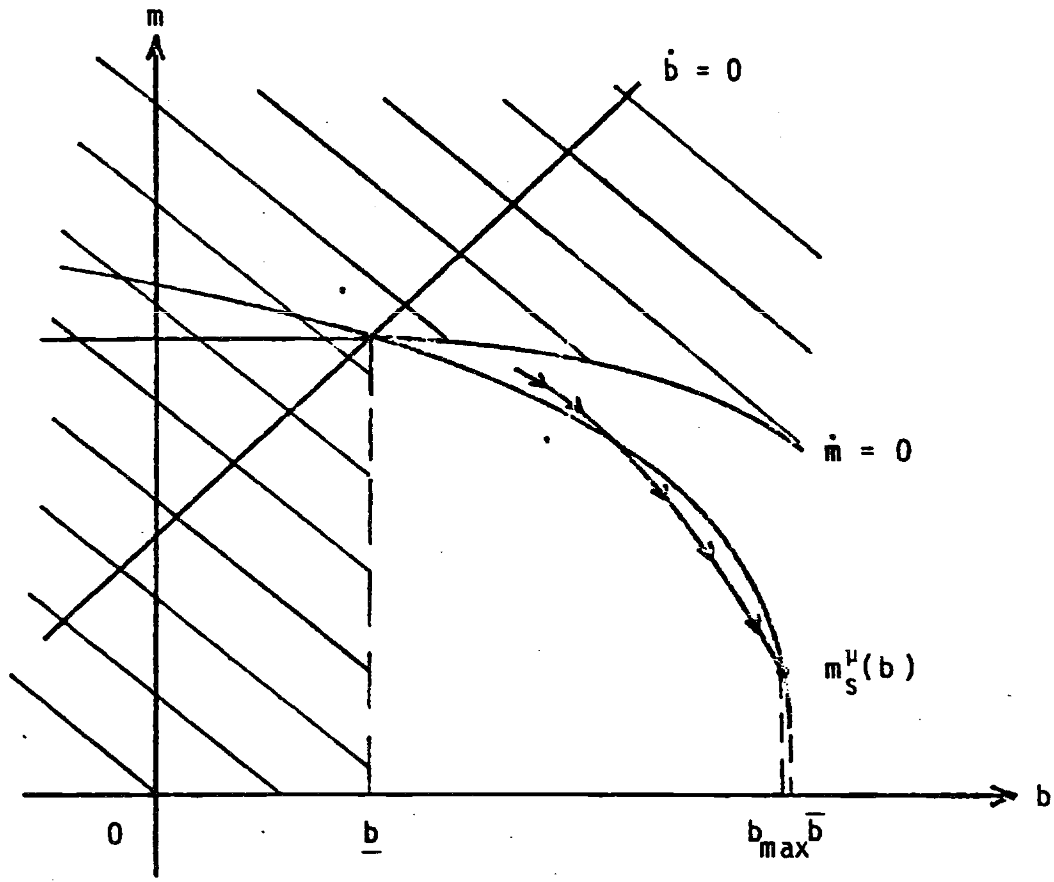


Figure 7

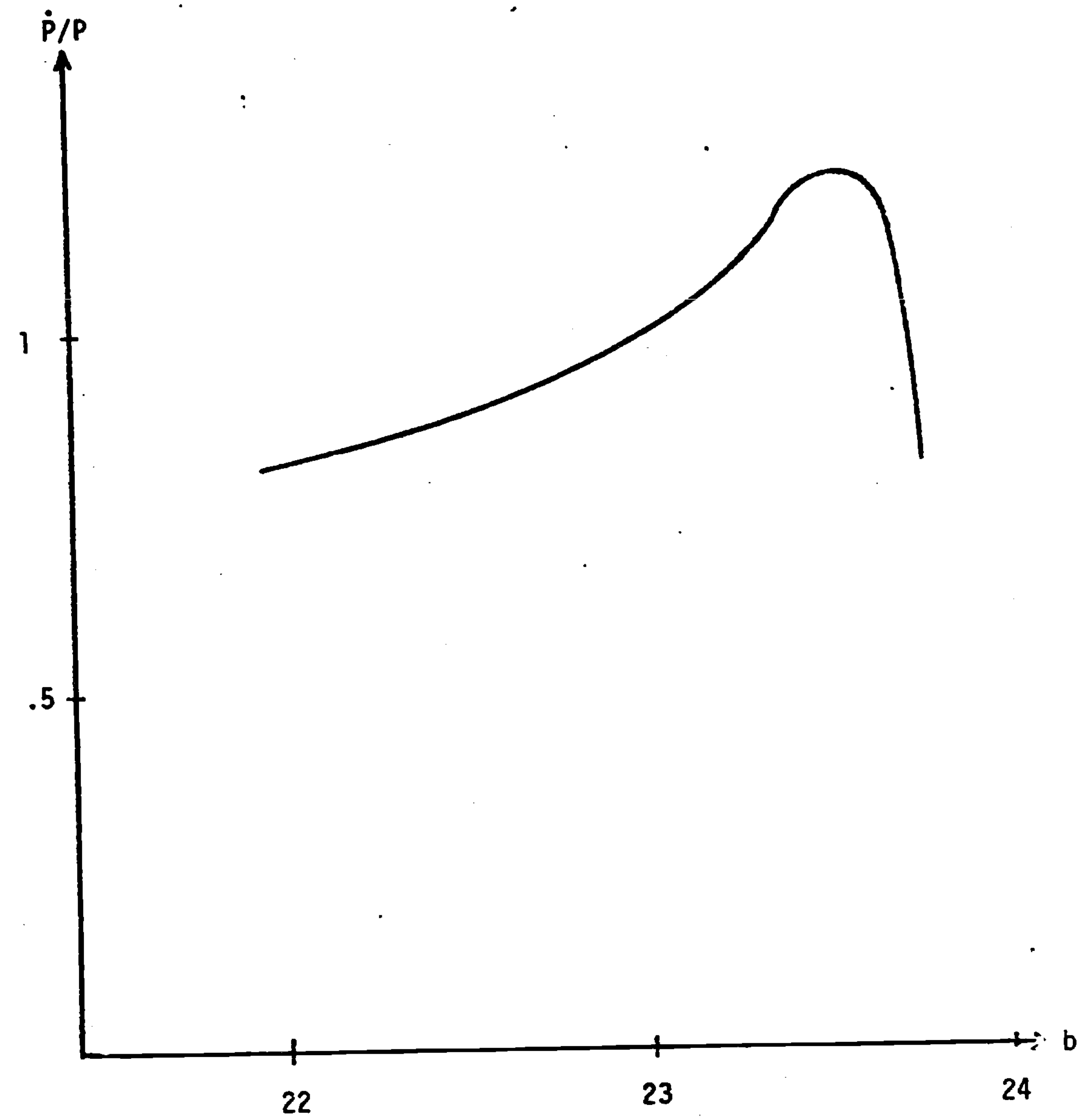


Figure 8

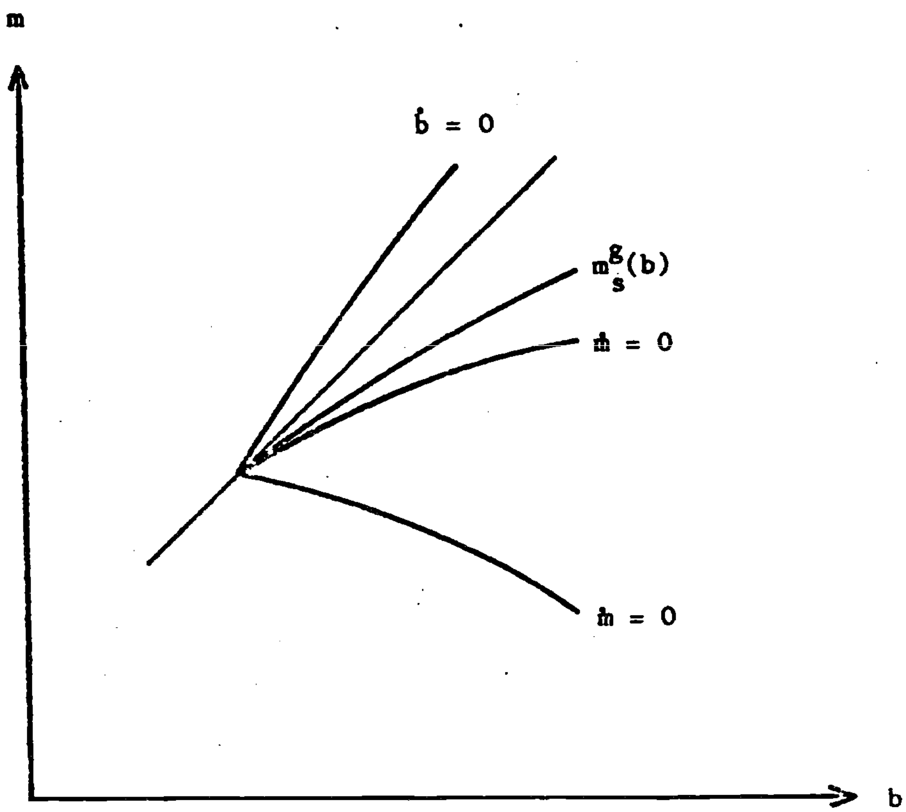
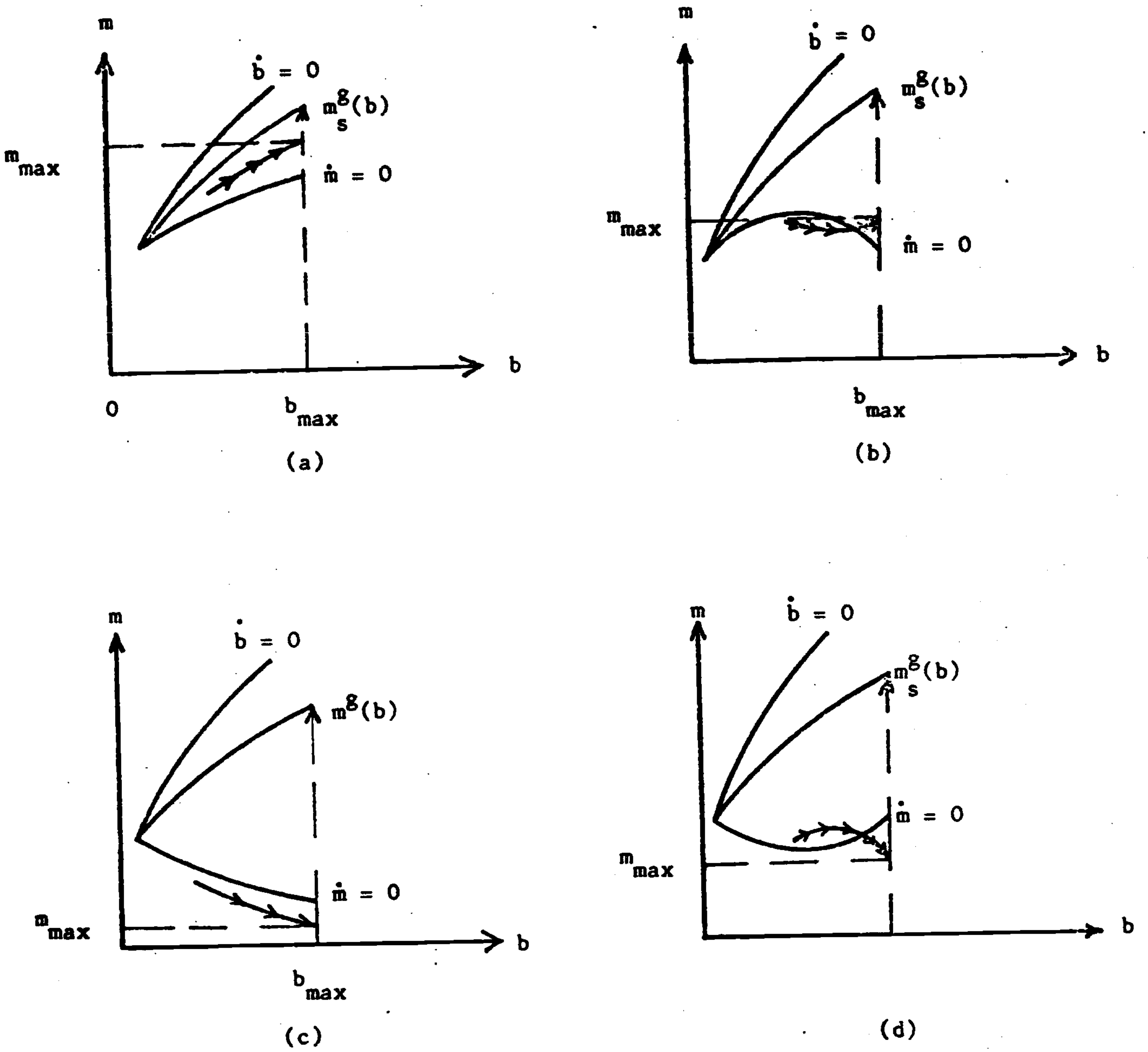


FIGURE 9



m
$\dot{b} = 0$
$m_s^g(b)$
m_{max}
$\dot{m} = 0$
b
0
b_{max}
(a)
m
$\dot{b} = 0$
$m_s^g(b)$
m_{max}
$\dot{m} = 0$
b
b_{max}
(b)
m
$\dot{b} = 0$
$m^g(b)$
$\dot{m} = 0$
m_{max}
b
b_{max}
(c)
m
$\dot{b} = 0$
$m_s^g(b)$
$\dot{m} = 0$
m_{max}
b
(d)

FIGURE 10