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# PRIVATE POLITICS AND PUBLIC REGULATION

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**ABSTRACT**

We develop a dynamic game to explore the interaction between regulation and private policies, such as self-regulation by firms and activism. Without a public regulator, the possibility of self-regulation is bad for the firm, but good for activists who are willing to maintain a costly boycott to raise the likelihood of self-regulation. Results are reversed when the regulator is present: the firm then self-regulates to preempt public regulation, while activists start and continue boycotts to raise the likelihood of such regulation. Our analytical results describe when a boycott is likely, and when it may be expected to be short and/or successful. The model generates a rich set of testable comparative statics.

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# 1 Introduction

For many companies, their business models and business practices result in negative externalities that markets may fail to correct. These externalities may come in different forms, from material ones like water and air pollution, which are relatively easy to measure and quantify, to immaterial, such as direct disutility that some people may experience if a company uses child labor, provides poor workplace conditions, or contributes to melting of the Arctic ice cap because it endangers species like polar bears. A traditional solution to such externalities involves the government in one way or another, but more recently, a phenomenon called *private politics* became widespread. It is now quite common that activist groups that seek to curb or limit certain practices do not necessarily engage in public channels like lobbying or political campaigns; instead, they start activist campaigns and threaten to organize a boycott if their demands are not met. Famous examples of effective and successful boycotts include those of Shell by Greenpeace in 1995 over sinking of the outdated offshore oil storage facility Brent Spar, and of CitiCorp by Rainforest Action Network (RAN) in 2000-05 over lending to companies engaged in non-sustainable mining and logging. The campaign against Shell included organizing a successful boycott in Germany where sales at Shell gas stations fell by as much as 40%, and an occupation of Brent Spar by Greenpeace activists, including Shell's use of water cannons against activists attempting to land, which got a wide coverage on TV. After two months of protests, the company gave in.[1] The campaign by RAN against CitiCorp was much longer, and involved episodes like students cutting their Citibank cards on the campus of Columbia University, as well as picketing the residences of Citi's senior executives.[2] It is not surprising that such campaigns typically target large and visible corporations.

Not every boycott is successful, however. For example, a number of activist groups boycotted Nestlé over its practice of marketing infant feeding formula to mothers in the 1980s-90s. They formed coalitions such as INFACT (Infant Formula Action Coalition) in the U.S.

[1] See Diermeier (1995). The statement released by Shell on June 20, 1995, contained: "Shell's position as a major European enterprise has become untenable. The Spar had gained a symbolic significance out of all proportion to its environmental impact. In consequence, Shell companies were faced with increasingly intense public criticism, mostly in Continental northern Europe. Many politicians and ministers were openly hostile and several called for consumer boycotts."

[2] See Baron and Yurday (2004).

and Canada and IBFAN (International Baby Food Action Network) in many other countries such as Sweden, India and New Zealand. The boycotts started and stopped multiple times; Nestlé would occasionally make minor concessions but it subsequently reneged on them. This sequence of boycotts eventually resulted in government interventions; e.g., in India, the government in 2003 effectively banned Nestlé's promotions of breastmilk substitutes and feeding bottles.[3] On the other hand, there are cases where the government intervened even before any activist campaigns: e.g., in 2010, McDonald's Happy Meals were banned in San Francisco by the city Board of Supervisors on the grounds that including a free toy with an unhealthy meal promotes obesity in children. Such practices by large corporations may be expected to attract activists, but this regulation was apparently the Board of Supervisor's own initiative.[4]

These examples demonstrate a wide spectrum of regulation attempts and their outcomes. The company may opt for self-regulation as a result of an activist campaign (Shell, CitiCorp); it may withstand an activist campaign but may be nevertheless regulated by government body (Nestlé), or the government may regulate it even before activists reveal themselves and start a campaign. This raises several important questions: When and why is (self-) regulation an outcome of private rather than public politics? What are the incentives to initiate or continue a boycott in the precense of a public regulator? What is the regulator's motivation to intervene given the possiblity of private politics?

The purpose of this paper is to address these questions formally. We provide a novel framework in which private politics and public regulation interact. The model is dynamic and we impose no assumptions on the sequence of moves: the length of a corporate campaign or boycott, for example, is endogenously determined. The campaign may fail or succeed; it may end in private or public regulation. The outcome is stochastic and cannot be forecasted

[3] http://www.infactcanada.ca/The%20History%20of%20the%20Campaign.pdf; see also Saunders (1996).

[4] On November 2, 2010, the San Francisco Board of Supervisors supported, with 8-3 vote, a ban on McDonald's Happy Meal. According to the act, no company could give away a free toy with a meal with nutritional value exceeding a certain limit. The Board subsequently overturned the veto of Mayor Gavin Newsom, thereby leaving McDonald's with a list of choices: pull out Happy Meals from the menu, cut the portion, or remove the toy. McDonald's in San Francisco now avoids the effects of the law by charging 10 cents for a toy, a price that parents are reported to be willing to pay. (See http://abcnews.go.com/blogs/health/2011/11/30/mcdonalds-skirts-ban-charges-10-cents-per-happy-meal-toy/)

with certainty by any player. Nevertheless, we can characterize how the probability of success or failure depends on the model's parameters. This way, the analysis provides a large number of testable predictions.

More precisely, we build a dynamic model of interaction between an activist group (henceforth A or *he*), a firm (F or *it*), and also a public regulator (R or *she*). The firm produces and sells a good, but does so in a way that the activist group believes to be wrong or harmful. The firm F is aware of A's concern and may decide to adjust its practice ("self-regulate") at any point in time. Such self-regulation, however, is costly to the firm.

As long as there is no regulation in place, the activist, running a campaign against the firm, can decide to initiate an actual boycott. The boycott is costly for the activist as well as for the firm, and it will end as soon as either the firm decides to self-regulate or the activist decides to give up. This way, we model the boycott as a war of attrition where each player hopes that the other one will give in first.

This war of attrition is anticipated even before the boycott has started. At this stage, the unique Markov Perfect equilibrium (MPE) is in mixed strategies: If the firm were believed to never self-regulate at this stage, then a boycott would be necessary and thus start immediately. If a costly boycott is believed to start immediately, however, the firm would rather self-regulate. In equilibrium, the firm is indifferent and will self-regulate with a positive probability. Taking this probability as given, the activist is indifferent when to initiate a boycott, and a boycott is initiated with some probability. Note that for the activist to be willing to start and continue a boycott, it must be that the likelihood of self-regulation is higher during boycotts than at other times.

As suggested by the description above, we find it helpful to analyze the game without a public regulator first. In the full game with all three players, the regulator may, at any point in time, step in to regulate the firm directly. We assume that such regulation will be more costly to the firm than self-regulation. Intuitively, the regulator might enact legislation that reduces pollution by the same amount but in a clumsy or inefficient way. Thus, it cannot be the case that public regulation occurs with a very high probability, since then the firm would instead self-regulate first. This implies that the equilibrium will again be in mixed strategies.

Although the analyses of boycotts as wars of attrition, both with and without a public regulator, are both interesting and novel, our most important contribution may be in comparing the two settings. The firm self-regulates in order to prevent or end a boycott in the absence of the regulator, but when the regulator is present, self-regulation occurs a rate that makes the regulator just willing to wait and hope for self-regulation. When the regulator is absent, the activist initiates and continues a boycott since this motivates the firm to self-regulate. With the regulator present, however, the activist's motivation to start a boycott is that public regulation is more likely to occur during a boycott than at other times (and this is indeed what would happen in equilibrium). Finally, a boycott is likelier to be shorter and unsuccessful when the regulator is present. This means, in a certain sense, that the activist's campaign and the regulator's action are strategic substitutes: each is less likely in the presence of the other.

This paper, to the best of our knowledge, is among the first to study government regulation and self-regulation, i.e., private and public politics, in a unified framework. In Baron (2013), the government and activists have preferences over the magnitude of self-regulation that the firm imposes. In equilibrium the firm will satisfy the demands of the government, up to the point where the government would reach a gridlock if it attempts further regulation, but it may also move further in order to prevent an activist campaign. In Lyon and Salant (2013), activists target individual firms and force them to self-regulate in order to change their behavior in subsequent lobbying game (e.g., a firm that restricted its pollution would prefer other firms to do so as well, and thus would support rather than oppose regulation). Daubanes and Rochet (2013) assume that activists are less informed but more committed to their cause than regulators, and if regulators are captured by the industry, then activists may improve social welfare. There, the order of moves is also fixed: first, regulator may recommend that the Congress regulate the firm, then activists decide the intensity of a boycott, and then the firm may decide to give in. This allows the authors to introduce the asymmetry of information (the regulator knows more than the activist group). The advantage of our approach is that we remain agnostic about the exact sequence of actions (whether the firm or activists or the government moves first), instead, agents choose themselves when to move. This also allows us to study time variables such as the duration of boycotts without assuming commitment power for any of the agents. More importantly, our approach provides insights that do not depend on a particular bargaining protocol.

The term "private politics" was coined by David Baron (see Baron, 2001 and 2003) to describe non-market interactions between individuals, NGOs, and companies, and has since been in the center of a relatively small but growing literature. Baron (2001) assumes that a company's reputation positively affects demand for its product, and thus is worth investing in. Baron and Diermeier (2007) consider a strategic activist who demands the firm to adopt certain practices, or else he would organize a damaging campaign. Baron (2009) extends the analysis by studying two competing firms, and allowing activist to be an (imperfect) agent of citizens. Feddersen and Gilligan (2001) model activist as a credible source of information about credence good rather than as a campaigner; they show that presence of such an activist may alter the equilibrium, and in particular lead to differentiation of the product. Baron (2012) combines the two approaches by assuming that there are two activist groups, one more moderate and one more aggressive, that never fight each other. In that case, it makes sense of each of the two competing firms to cooperate with the moderate group, as it makes a boycott less likely.[5] In a recent working paper, Besanko, Diermeier and Abito (2011) show that when (flow) investment in Corporate Social Responsibility (CSR) affects the firm's reputation (stock), then activists can increase the firm's investment in CSR by occasionally destroying its reputation if it becomes too good. In general, the idea that socially responsible actions of companies have a positive impact on their reputation and performance has found empirical support. For example, Dean (2004) finds that pre-existing reputation at the time of crisis affects consumers' perception of the brand after the crisis. Minor and Morgan (2011) document that companies with good reputation take a lower hit on their stock price as a result of a crisis.

As one of the most typical, and certainly the most visible, implementation of private politics, boycotts have attracted quite a bit of attention. Delacote (2009) observes that heterogeneity of consumers makes boycotts less efficient, as those consumers who buy a lot and thus could hurt the firm most are also the ones with the highest cost of boycotting.

[5] See also Baron (2010), which looks on cooperative arrangements where various types of activist groups can enforce cooperative behavior.

Diermeier and Van Mieghem (2008) model boycotts as a dynamic process, where each of the (infinitesimal) consumers decides to participate or not depending on the number of other consumers boycotting the product. Innes (2006) builds a theory of boycotts under symmetric information, which suggests that either the activist targets a large firm, and then the boycott is very short (short enough to show that the activist has invested a lot in preparation), or the activist targets a small firm, and in this case the boycott is persistent, as the firm finds it too costly to satisfy the demand of the activist; in the latter case, the reason for boycotting is to redistribute customers to a more responsible, larger firm. For an experimental study of boycotts, see Tyran and Engelman (2005).[6]

As we connect private politics with government regulation, our paper is also related to the large literature in political economy focusing on the incentives and behavior of politicians and legislators (see Persson and Tabellini, 2002). Citizens and consumers are allowed to vote and sometimes run for office (as in Besley and Coate, 1997; Osborne and Slivinski, 1996), but boycotts are rarely considered. Nevertheless, it would be fair to say that activist groups were covered by the literature, at least if they are assumed to lobby governments by providing campaign contributions or information on public opinion (for an overview of this literature, see Grossman and Helpman, 2002). Yu (2005), for example, models activists who can either lobby the government directly, or influence public opinion and thus government's actions.

Our choice of a war-of-attrition game to model boycotts is led by the desire to capture, in the simplest way, both the dynamics of a boycott and its inefficiency for all parties involved (and nevertheless show that they may be part of equilibrium). For similar purposes, wars of attrition are often used in industrial organization and game theory.[7] In political economy, they have also been used to explain gridlock in legislatures (Alesina and Drazen, 1991). As a matter of fact, the first waiting game (before the boycott) between the firm and the activist is *not* a war-of-attrition, since the firm is *not* hoping that the activist makes the move and

[6] Some papers link government regulation and self-regulation by firms without activists. For example, in Maxwell, Lyon, and Hackett (2000) firms can lobby for regulation in order to effectively restrict entry, and self-regulation allows the firm to stay in business. In our paper, the source of (self-)regulation is pressure by activists, which is an integral part of private politics.

[7] War of attrition games were first applied to biological settings (Maynard Smith, 1974). There, as well as in economics, "the object of the fight is to induce the rival to give up. The winning animal keeps the prey; the winning firm obtains monopoly power. The loser is left wishing it had never entered the fight" (Tirole, 1998:311). The definition by Muthoo (1999:241) is similar.

starts a boycott). In this sense, the waiting game before the boycott is more similar to the equilibrium in Harstad (2013), which studies whether and when a private owner of a valuable asset (say, rainforest) is compensated for its conservation.

The rest of the paper is organized as follows. We start with Section 2.1, where we introduce and analyze the model in the absence of a public regulator. This gives us a number of benchmark results. Section 3 introduces the regulator, and we study equilibria and comparative statics of the model with all three players. While Sections 2-3 permit the boycott to be run only once, Section 4 confirms our findings in a model where multiple campaigns may be run sequentially. Section 5 summarizes robust results and discusses, how the presence of a regulator dramatically changes the outcome. Section 6 concludes. The appendices allow for multiple activist groups and multiple regulators and provide a detailed characterization of all Markov Perfect equilibria.

# 2 Private Politics: Boycotts and Self-Regulation

This section shows how boycotts can be analyzed as a dynamic war-of-attrition model with only a firm and an activist group. The results are both of independent interest and they serve as benchmarks when we introduce the regulator in the next section.

## 2.1 A Dynamic Model of Private Politics

The game has two players: the activist A (he) and the firm F (it). Time is continuous and infinite, and the players share a common discount rate $r \in (0, \infty)$. The flow payoffs of F and A are normalized to zero if there is no active boycott and if F has not yet self-regulated. Relative to the status quo, A would benefit if F self-regulates. Such self-regulation could mean, for example, that F installs technology reducing its pollution, switches to organically produced food, or improves working conditions for its employees. If F self-regulates, then A receives a flow benefit $b > 0$ but F faces a flow cost $c > 0$. For simplicity, we assume that self-regulation is an irreversible decision. Thus, the game practically ends after self-regulation, since no further action can be taken, although time continues and $b$ and $c$ measure flow payoffs.[8]

We assume that A cannot pay F to self-regulate (i.e., we rule out Coasian bargaining). The only means that A has is a corporate campaign, which consists with a threat of a boycott (which starts our game) and possibly the boycott itself. The strategic moves that A has are therefore whether and when to start a boycott and when to end it.

The boycott is costly for both parties: F gets a flow payoff of $-h < 0$ due to lower sales, and A gets a flow payoff of $-e < 0$, as it is costly to keep the public interested, organize events interesting to the media, and perhaps forgo one's consumer surplus by participating in the boycott. We assume that $c < h$, so self-regulation is better for the firm than an eternal boycott (otherwise, the outcome would be trivial).

Initiating a boycott might also require a fixed set-up cost, perhaps A's cost of initially informing and organizing customers. It is convenient to measure this initial cost as $\underline{e}/r$, such that $\underline{e}$ is the flow-cost equivalence. We permit $\underline{e} < 0$, in which case A actually benefits from initiating or announcing a boycott; this allows us to capture activists who are long-term players and care about their reputation, for instance. Similarly, F experiences an instantaneous cost as soon as a boycott starts, in addition to the flow cost $h$. It is convenient to measure also this immediate cost as $\underline{h}/r$, where $\underline{h}$ is the flow-cost equivalence. Intuitively, the firm may suffer some reputation damage as soon as customers are made aware of the issue, independently of how long-losting the boycott is. However, a fraction $\delta \in [0, 1]$ of this cost may be recovered the moment when (and if) the boycott is called off by the activist. If $\delta$ is large, the consumers are quite "forgiving" and the firm's initial loss is soon restored. Note that these assumptions and parameters are roughly in line with Baron (2012).[9]

The following table summarizes the payoffs in this simple stopping game:

| Payoffs | Status quo | Self-regulation | Boycott | At start | At end |
|---|---|---|---|---|---|
| Activist | 0 | $b$ | $-e$ | $-\underline{e}$ | 0 |
| Firm | 0 | $-c$ | $-h$ | $-\underline{h}$ | $\delta\underline{h}$ |

[8]Equivalently, we could assume that when F self-regulates, then A receives the payoff $b/r$ once and for all while F must pay the cost $c/r$ once and for all.

[9]In Baron (2012), $\beta$ denotes the share that may not be recovered if the boycott is called off. With our notation, $\beta = 1 - \delta$.

Each player maximizes the present discounted value of expected payoffs. The strategies of the players are as follows. At any point in time, F can either do nothing or self-regulate. If F self-regulates, the game ends (and this also automatically stops any ongoing boycott).[10] As long as the game has not ended, A chooses, at any point in time, whether to start a boycott or, if the boycott has already started, whether to end it. We also assume, for simplicity, that once a boycott has taken place and ended, it is impossible to start a new boycott (we relax this assumption in Section 4). This implies that the game has three possible subgames, which we refer to as phases: Phase 0 is the initial phase of the campaign where the campaign has not yet started, Phase 1 refers to an ongoing boycott, and if the boycott is recalled by Activist, we enter phase 2.

As in most dynamic games, we have a large set of subgame-perfect equilibria. We thus restrict attention to Markov-perfect equilibria (MPEs), so that the strategies only depend on the payoff-relevant partition of histories, i.e., whether the boycott has started and/or ended. Consequently, each player's probability of acting must be independent of how much time the players have spent in each phase. The MPE can thus be characterized by five Poisson rates: $\{\phi_0, \phi_1, \phi_2, \alpha, \rho\}$. The Poisson rate $\phi_t \in [0, \infty]$ measures the equilibrium rate of self-regulation during phase $t \in \{0, 1, 2\}$. For example, the probability of self-regulating within a small time interval $dt$ during the boycott is $\phi_1 dt$, so $\phi_1 = \infty$ would mean immediate self-regulation. In equilibrium, A starts a boycott at Poisson rate $\alpha \in [0, \infty]$, and during a boycott, A ends it at rate $\rho \in [0, \infty]$.

## 2.2 Boycott as a War of Attrition

We will now solve the game by backward induction. Consider phase 2, the situation *after* the boycott has ended. In this phase, only F is capable of taking an action. Since self-regulation is costly, F prefers to stick to the status-quo and not self-regulate,

$$\phi_2 = 0,$$

implying that both players receive a payoff of zero in phase 2.

[10] Alternatively, we could assume that the decision to call of a boycott must be made by A even after F self-regulated. This would not change any results, because A would call of the boycott immediately.

This is anticipated during phase 1. The boycott is costly for both players, but each of them can unilaterally stop the boycott game. If A ends the boycott, we enter phase 2, where A's flow payoff of 0 is larger than his flow payoff during the boycott, $-e$. But also F can end the boycott by self-regulating and pay the cost $c$, which is assumed to be smaller than the firm's cost of the boycott, $h$.[11] Thus, on the one hand, each player would strictly benefit from acting and stopping the game if the *other* player is *not* expected to end the game anytime soon. On the other, each player would benefit more if the other player acted. Thus, the boycott is a war of attrition where each player hopes that the other one will concede.

The activist's best response depends on $\phi_1$ in the following way. If $\phi_1$ is small, then A realizes that F is highly unlikely to self-regulate during the boycott. In this case, A will stop the boycott immediately. In contrast, if F is very likely to self-regulate, so that $\phi_1$ is large, then A is better off waiting until F indeed self-regulates. If F believes A is unlikely to end the boycott, so that $\rho$ is small, then F is better off by self-regulating immediately. But if $\rho$ is large, F prefers to wait. As in every war of attrition, there are two corner solutions: $(\phi_1, \rho) = (\infty, 0)$ and $(\phi_1, \rho) = (0, \infty)$. In both these equilibria, the boycott ends immediately. The more interesting equilibrium is the one in mixed strategies where the boycott lasts, in expectation, a positive amount of time. Only in this equilibrium can the boycott actually be observed. Since both players are acting with a positive probability in this equilibrium, we call it *interior* and focus on that equilibrium from here on.

**Lemma 1** ***(Boycott game)*** *There is a unique interior equilibrium in the boycott game:*

$$\phi_1 = r\frac{e}{b} \in (0, \infty), \tag{1}$$

$$\rho = r\frac{h-c}{\delta \underline{h} + c} \in (0, \infty), \tag{2}$$

***Proof.*** If A decides to continue and never end the boycott, his expected payoff is driven by F's rate of self-regulation, $\phi_1$:

$$\int_0^\infty \phi_1 \exp(-\phi_1 t) \left( \int_0^t (-e) \exp(-r\tau)\, d\tau + \int_t^\infty b \exp(-r\tau)\, d\tau \right) dt = \frac{1}{r} \left( \frac{\phi_1 b - re}{\phi_1 + r} \right). \tag{3}$$

Here, $t$ denotes the moment at which F self-regulates; this time is distributed exponentially with density $\phi_1 \exp(-\phi_1 t)$. Alternatively, A can receive zero by terminating the boycott. A

[11] If instead $c > h$, F would never give in during a boycott, and thus A would immediately end it.

is thus willing to continue if:

$$\frac{1}{r}\left(\frac{\phi_1 b - re}{\phi_1 + r}\right) \geq 0 \Leftrightarrow \phi_1 \geq r\frac{e}{b}.$$

F's strategy during the boycott depends on how likely it thinks A is to stop. The expected payoff from never self-regulating is:

$$\int_0^{\infty} \rho \exp(-\rho t)\left(\int_0^t (-h)\exp(-r\tau)\,d\tau + \delta\left(\underline{h}/r\right)\exp(-rt)\right)dt = \frac{1}{r}\left(\frac{r}{\rho+r}(-h) + \frac{\rho}{\rho+r}\delta\underline{h}\right),$$

while the payoff from self-regulating immediately is $-c/r$. Thus, F is willing to continue if

$$\frac{1}{r}\left(\frac{-rh}{\rho+r} + \frac{\rho\delta\underline{h}}{\rho+r}\right) \geq -\frac{c}{r} \Leftrightarrow \rho \geq r\frac{h-c}{\delta\underline{h}+c}.$$

If $\phi_1 = re/b$, A is willing to randomize and F is indeed randomizing, so both inequalities must bind, giving (1)-(2). If $\phi_1 > re/b$, A strictly prefers to continue, giving $\rho = 0$, so F strictly prefers to stop, giving $\phi_1 = \infty > re/b$. If $\phi_1 < re/b$, $\rho = \infty$, ensuring $\phi_1 = 0 < re/b$. We thus have three equilibria. ■

While there is no self-regulation after the boycott ($\phi_2 = 0$), F may be willing to self-regulate during the boycott ($\phi_1 > 0$): the boycott is costly and F prefers to end it. As a simple consequence, self-regulation is more likely during than after the boycott. This explains why A is willing to continue the boycott, despite its costs.

Once we have the rates of actions, as described by Lemma 1, it is straightforward to derive the expected length of the boycott as well as its likely outcome. For example, since the boycott ends with rate $\rho+\phi_1$, the expected length is simply $1/(\rho+\phi_1)$. This immediately generates comparative statics results.

**Proposition 1** ***(Duration of boycott)*** *The expected duration of the boycott is short if it is costly (e and h large), but long if self-regulation is costly to F or beneficial to A (c and b large):*

$$\frac{1}{\rho+\phi_1} = \frac{1/r}{e/b + (h-c)/(c+\delta\underline{h})}.$$

*Proof***.** The boycott lasts longer than $t$ if and only if neither A nor F act before $t$, i.e., with probability $e^{-\rho t}e^{-\phi_1 t}$. The duration of the boycott is thus distributed exponentially with density $(\rho+\phi_1)\,e^{-(\rho+\phi_1)t}$ and expected duration $1/(\rho+\phi_1)$. When we substitute from Lemma 1 we arrive at the proposition. ■

In other words, the boycott is more long-lasting if the stakes of either player are high relative to the cost of continuing the boycott. The intuition is the following. Suppose, for example, that self-regulation becomes more costly for F, so $c$ is larger. In this situation, F is willing to self-regulate only if A is less likely to call off the boycott. This implies that $\rho$ must be smaller, and the boycott tends to last longer. In other words, the more costly is self-regulation to F, the more committed must A be to continue for self-regulation to be possible, which in equilibrium leads to longer boycotts. In addition, the smaller $\rho$ implies that F is more likely to self-regulate before the boycott has been called off by A.

If $b$ is large, A has more to gain from continuing the boycott and is willing to stop only if F is very unlikely to self-regulate. This implies that if $b$ is large, $\phi_1$ must be small, the boycott ends later, and it is also more likely to fail, from A's perspective. Now, suppose the boycott is costly for A ($e$ large) or for F ($h$ large). In either case, the boycott is expected to end sooner. The difference is that, if $e$ is large, A is willing to continue the boycott only if F is likely to self-regulate (which implies that $\phi_1$ must increase). Hence, the boycott is *more* likely to succeed if $e$ is large. If $h$ is large, however, F is willing to wait only if A is expected to soon call off the boycott (i.e., $\rho$ must increase), implying that the boycott must be *less* likely to succeed. The next proposition characterizes the probability of a successful boycott (since strategies are Markovian, this probability does not depend on the actual duration).

**Proposition 2** ***(Probability of success)*** *The probability that the boycott succeeds is larger if self-regulation is costly (c large), not very beneficial (b small), and the boycott is costly to A (e large) but not to F (h small):*

$$\frac{\phi_1}{\rho + \phi_1} = \frac{1}{1 + b\left(h - c\right)/e\left(c + \delta \underline{h}\right)}.$$

*Proof.* The probability that F acts (self-regulates) earlier than A is $\int_0^\infty \left(\phi_1 e^{-\phi_1 t}\right) e^{-\rho t} dt = \phi_1 / \left(\phi_1 + \rho\right)$. The result now follows from Lemma 1. ■

Boycotts which happen to be over an issue that A does not care about too much ($b$ small) but which are costly to maintain ($e$ large) tend to be short and effective. In contrast, if A cares deeply about the issue, F will regulate at a lower rate and the boycott will be longer and less effective. This is a consequence of A's inability to commit to a longer boycott from the start. For example, RAN cared deeply about rainforest (saving it was the sole purpose of their existence as an activist group), but was almost ready to give up after a few years of campaigning when Citi got a blow to its reputation due to relations with Enron. At this point, we may assume that the need to improve reputation made $\delta \underline{h}$ higher, which decreased $\rho$, the rate at which Activist (RAN) would give up. This increased the likelihood of success, according to Proposition 2.

The complete the analysis of phase 1, note that the activist's equilibrium payoff is $u_1^A = 0$, since ending the boycott is a best response. The firm's equilibrium payoff is $u_1^F = -c/r$, since self-regulating, generating this payoff, is a best response.

## 2.3 Self-regulation to Preempt a Boycott

Suppose that in the initial phase of the campaign, the players anticipate that starting a boycott will lead to the interior equilibrium analyzed above.[12] The boycott is costly to F, who is thus willing to self-regulate if A is sufficiently likely to initiate a boycott. However, if self-regulation is likely, then A prefers to *wait* rather than to start an expensive boycott. As before, A wishes that F acts, but unlike the previous case, F wants A to wait rather than to act. Thus, this game does not have the war-of-attrition feature, and the equilibrium is in fact unique.

**Lemma 2** ***(Before the boycott)*** *Anticipating the interior equilibrium for phase 1, the unique equilibrium for phase 0 is:*

$$\alpha = r\frac{c}{\underline{h}} \text{ and } \phi_0 = r\frac{(-\underline{e})}{b+\underline{e}}$$

*if* $\underline{e} \in (-b, 0)$. *If* $\underline{e} > 0$, *then* $\alpha = \phi_0 = 0$, *and if* $\underline{e} < -b$, *then* $\alpha = \phi_0 = \infty$.

*Proof.* F is willing to self-regulate in phase 0 if the cost of doing so immediately is smaller than the cost of waiting for a boycott and self-regulating thereafter:

$$\frac{c}{r} \leq \frac{\alpha}{\alpha + r}\left(\frac{\underline{h} + c}{r}\right) \Leftrightarrow \alpha \geq r\frac{c}{\underline{h}}.$$

If $A$ starts the boycott, his payoff is $-\underline{e}$ since in the subgame that follows, it is a best response to end the boycott; he is thus willing to start the boycott if:

$$\frac{b}{r}\frac{\phi_0}{\phi_0 + r} \leq -\frac{\underline{e}}{r}. \tag{4}$$

[12] Appendix B characterizes all the equilibria for phase 0 for each of the three equilibria of phase 1.

If $\underline{e} \in (-b, 0)$, then (4) is equivalent to $\phi_0 \leq r\frac{(-\underline{e})}{b+\underline{e}}$, so best-response functions cross once only, giving a unique interior solution. If $\underline{e} > 0$, (4) cannot hold, so a boycott never starts, andthus $\alpha = 0$ and $\phi_0 = 0$. If $\underline{e} < -b$, (4) holds for any $\phi_0$, thus $\alpha = \infty$ and $\phi_0 = \infty$.[13] ■

Suppose the interior equilibrium is expected during the boycott. The incentive to start a boycott is then quite small: once the costly boycott has started, it is a best response for A to immediately end it, even though this ensures no self-regulation in perpetuity. If $\underline{e} > 0$, a boycott will never start, and F will therefore never self-regulate.[14]. Indeed, If $\underline{e} < 0$, however, A gains from starting a boycott (e.g., reputation-wise), and so a boycott is possible. Moreover, if this gain is very large ($\underline{e} < -b$), it becomes a dominant strategy for A to start a boycott, and fearing this, F would concede immediately. In the more interesting, intermediate case where $\underline{e} \in (-b, 0)$, F must self-regulate at a positive rate to prevent a boycott. The more "aggressive" is A (i.e., if $\underline{e}$ is smaller), the more likely is self-regulation before a boycott starts. If self-regulation is costly ($c$ is high), then it is more likely that A will need a boycott (and $\alpha$ must be higher). However, if the reputational damage from a boycott ($\underline{h}$) is large, then $\alpha$ is lower, and it is more likely that self-regulation will occur before the boycott, not as a result of it. The following proposition verifies these intuitions and adds additional insights (its proof is straightforward and thus omitted).

**Proposition 3** ***(Likelihood of a boycott)*** *Suppose $\underline{e} \in (-b, 0)$. The probability of a boycott is increasing in $b$, $c$, and $\underline{e}$, but decreasing in $\underline{h}$:*

$$\frac{\alpha}{\phi_0 + \alpha} = \frac{1}{1 - \underline{e}\underline{h}/c\,(\underline{e} + b)}.$$

This result suggests that boycotts are likely to occur over "big" issues, which are important for A and expensive for F, while less important issues are likely to be settled in the pre-boycott stage. Furthermore, firms with recognizable brands, which have a lot to lose ($\underline{h}$ large), are more likely to be socially responsible and self-regulate before experiencing a boycott. This goes in line with stylized facts; such companies often choose to self-regulate, invest in CSR, and respond to (reasonable) demands by activists without getting boycotted.

[13] The equilibrium need not be unique in the borderline cases, $\underline{e} = 0$ and $\underline{e} = -b$. If $\underline{e} = -b$, any $\alpha \geq rc/\underline{h}$ is an equilibrium if just $\phi_0 = \infty$. If $\underline{e} = 0$, any $\alpha \leq rc/\underline{h}$ is an equilibrium if just $\phi_0 = 0$.

[14] A boycott may be possible even when $\underline{e} > 0$ if the players expect the equilibrium $(\phi_1, \rho) = (\infty, 0)$ for phase 1. Appendix B provides a complete characterization of all (such) equilibria.

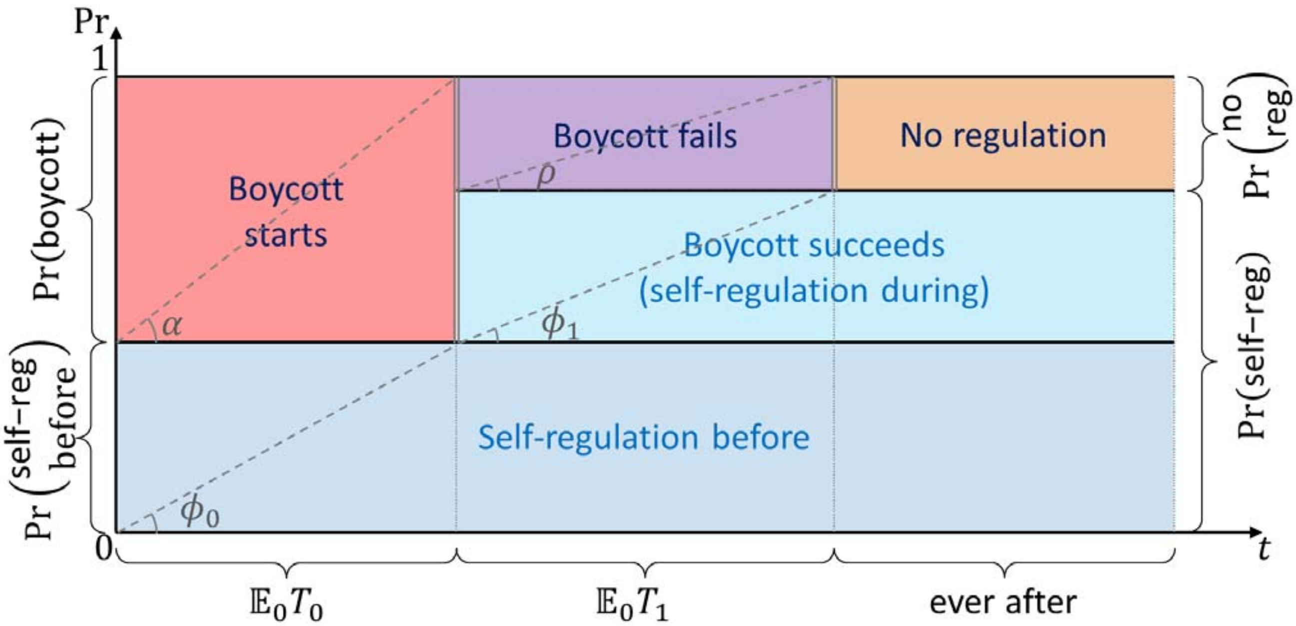


Figure 1: The outcomes of the game without the regulator

The possible histories and corresponding outcomes (failed boycott, successful boycott, self-regulation before the boycott) are illustrated on Figure 1. The horizontal axis corresponds to the time dimension; any line parallel to it illustrates a possible sequence of events, and the vertical axis measures the likelihood of each scenario (for example, the top of Figure 1 shows the case of a failed boycott: it starts, finishes, and then there is no regulation). The expected (unconditional) durations of each phase are shown on the horizontal axis. Below, we will see that in the presence of a regulator, the set of possible scenarios is much richer.

# 3 Private Politics meets Public Regulation

We now repeat the exercise above but with one additional player. The regulator R ("she") is able, at any moment, to step in and impose regulation on F. Like self-regulation, direct regulation is irreversible and ends the game. We also assume that A gets the same flow payoff $b$ regardless of whether F self-regulates or is directly regulated by R.[15] However, direct regulation is assumed to be more costly to F than self-regulation. This is realistic, as

[15] This assumption can be relaxed without affecting the major results, but the analysis would be less transparent.

R might have less information and may be "clumsy" and unable to mandate the cheapest or most efficient technology. In addition, it may be costly to deal with the administrative burden or the red tape involved. F's additional cost is measured by $k$, so regulation imposed by R imposes the total flow cost $c+k$ on the firm. (If $k<0$, then F would actually prefer public regulation and thus would never self-regulate).

We model R's preferences in a simple way. As with F and A, we normalize R's flow payoff to 0 if there is no regulation. If F self-regulates, R gets flow payoff $s>0$, measuring the net benefit of self-regulation. If instead R regulates the firm directly, her flow payoff is $s-q$, where $q\in(0,s)$ by assumption. The assumption $q>0$ may hold for the same reasons as $k>0$: administrating regulation is costly, so self-regulation is preferred. We simplify by assuming that R does not experience any direct benefit or cost from the boycott: her flow payoff during boycotts is just 0.[16] The following table extends the previous one to the case with public regulation:

| Payoffs | Status quo | Self-regulation | Boycott | At start | At end | **Regulation** |
|---|---|---|---|---|---|---|
| Activist | 0 | $b$ | $-e$ | $-\underline{e}$ | 0 | $b$ |
| Firm | 0 | $-c$ | $-h$ | $-\underline{h}$ | $\delta\underline{h}$ | $-(c+k)$ |
| **Regulator** | 0 | $s$ | 0 | 0 | 0 | $s-q$ |

## 3.1 Public Regulation vs. Self-Regulation

We now solve the game by backward induction. Phase 2 is the post-boycott game in which R and F are the only players left in the game. As long as neither regulation nor self-regulation have taken place, the game is practically a stopping game. The firm can stop the game by self-regulating and ensure the payoff $-c/r$ to F and $s/r$ to R, while R can stop the game by directly regulating the firm, giving payoffs $-(c+k)/r$ to F and $(s-q)/r$ to herself. An MPE is characterized by two Poisson rates: $\phi_2$, the rate of self-regulation by F, and $\gamma_2$, the

[16] This assumption is approximately correct if the benefits of regulation and self-regulation are much larger than the cost of a boycott to R. In reality, the firm's lost market share may benefit its competitors, and the activists' forgone consumer surplus may benefit other consumers. So, to the regulator, the costs and benefits of a boycott may approximately cancel. In any case, relaxing this assumption makes the algebra more complicated without generating much additional insight.

rate of direct regulation by R.

**Lemma 3** ***(Firm and regulator after the boycott)*** *There is a unique equilibrium, and it is in mixed strategies:*

$$\begin{aligned}\phi_2 &= r\frac{s-q}{q} \in (0,\infty)\,, \\ \gamma_2 &= r\frac{c}{k} \in (0,\infty)\,.\end{aligned}$$

*Proof.* Expected payoffs of waiting can be calculated with the same method as in the proof of Lemma 1. F is willing to self-regulate if and only if the flow cost of self-regulation is smaller than the expected flow cost when risking direct regulation:

$$c \leq \frac{\gamma_2\,(c+k)}{\gamma_2 + r} \Leftrightarrow \gamma_2 \geq r\frac{c}{k}.$$

R is willing to regulate directly if and only if her expected flow payoff is larger than one from waiting for self-regulation:

$$s - q \geq \frac{\phi_2 s}{r + \phi_2} \Leftrightarrow \phi_2 \leq r\frac{s-q}{q}. \tag{5}$$

The two best-response curves cross only once, which establishes the result. ■

Interestingly, both players would actually prefer self-regulation to direct regulation when $k$ and $q$ are positive. Despite this, there is no equilibrium where F regulates immediately. If F did so, R would simply wait; but if R never regulates, F would not self-regulate.

The comparative statics are interesting. If $q$ decreases, R's cost of regulating F directly becomes smaller. Then, all things equal, R would prefer immediate regulation. The best response for F would then be to self-regulate immediately, which in turn makes R better off waiting. In equilibrium, for R to remain indifferent, F has to self-regulate at a higher rate, which means that $\phi_2$ is a decreasing function of $q$. Thus, as R becomes more efficient, its intervention is less likely to be required, as F will be more likely to self-regulate quickly. Similarly, a larger $s$ makes R more tempted to regulate unless F self-regulates at a higher rate, which is precisely what will happen in equilibrium.

With the same methodology as earlier, it is now straightforward to derive both the probability for regulation as well as the lag.

**Proposition 4** ***(Outcome and delay)*** *(i) Regulation is more likely to be imposed by R rather than F if public regulation is costly to R, relative to her benefit of regulation (q/s high), but inexpensive to F, relative to its cost of regulation (k/c low):*

$$\frac{\gamma_2}{\phi_2+\gamma_2}=\frac{c/k}{c/k+s/q-1}\in(0,\infty)\,.$$

*(ii) The expected delay before regulation or self-regulation is smaller if regulation is beneficial to R (s high), costly to F (c high), and if public regulation has low costs to either party (k or q low):*

$$\frac{1}{\phi_2+\gamma_2}=\frac{1/r}{c/k+s/q-1}\in(0,\infty)\,.$$

In other words, the more costly is the red tape, the longer we should expect to wait before any kind of regulation is introduced. The intuition is as follows. First, R's reluctance to pay the administrative cost is abused by F, which becomes less likely to self-regulate at any point in time. On the other hand, F becomes more eager to preempt direct regulation when $k$ is large, and R can thus regulate at a lower rate while still ensuring that F is willing to act. The likely type of regulation might at first surprise: the more costly administration is to R, the more likely it is that R will have to eventually administer the regulation. The intuition is that F takes advantage of R's cost by self-regulating at a lower rate.

The equilibrium payoffs of phase 2 are the following. The firm gets $-c/r$ since it must be indifferent when considering self-regulation. The regulator must receive $-(s-q)/r$, since direct regulation is a best response for her. The activist's payoff is given by:

$$u_2^A=b\frac{\phi_2+\gamma_2}{r(\phi_2+\gamma_2+r)}=\frac{b}{r}\left(1-\frac{1}{c/k+s/q}\right).\text{[17]} \tag{6}$$

In other words, A is better off if the the stakes are high ($s$ and $c$ are large) while the additional cost of public regulation ($k$ and $q$) are small, i.e., precisely in situation where regulation is not delayed for too long.

## 3.2 The Boycott and its Three Outcomes

During the boycott, each of the three players can decide whether to end the game. While self-regulation or direct regulation will prevent any further action, if the activist "ends" the game by stopping the boycott, then the boycott game ends but the post-boycott phase 2 starts. Thus, the lessons from the previous subsection are anticipated during the boycott.

We will here focus on the interior equilibria where each player acts with some chance.[18]

**Lemma 4** ***(Boycott with regulator)*** *There is a unique interior equilibrium during the boycott. The rate of self-regulation by F is the same as after the boycott; but the rate of public regulation is larger during the boycott than after:*

$$\begin{aligned}
\phi_1 &= r\frac{s-q}{q} = \phi_2, \\
\gamma_1 &= r\left[\left(\frac{c}{k}+\frac{s}{q}\right)\frac{e}{b}+\frac{c}{k}\right] > \gamma_2, \\
\rho &= r\left[\left(\frac{c}{k}+\frac{s}{q}\right)\frac{e}{b}\frac{k}{\delta\underline{h}}+\frac{h}{\delta\underline{h}}\right] \in (0,\infty).
\end{aligned}$$

*Proof***.** Note that R is willing to regulate if and only if (5) holds, just as in phase 2:

$$\phi_1 \le r\frac{s-q}{q} \in (0,\infty). \tag{7}$$

A is willing to end the boycott if his payoff from continuing is smaller than the payoff in the post-boycott game (6):

$$\begin{aligned}
-\frac{e}{r}+\left(\frac{b+e}{r}\right)\frac{\phi_1+\gamma_1}{\phi_1+\gamma_1+r} &\le \frac{b}{r}\left(1-\frac{1}{c/k+s/q}\right) \Leftrightarrow \\
\phi_1+\gamma_1 &\le r\left[\left(\frac{c}{k}+\frac{s}{q}\right)\left(1+\frac{e}{b}\right)-1\right].
\end{aligned} \tag{8}$$

Finally, F is willing to self-regulate if and only if:[19]

$$c \le \frac{rh+\gamma_1(c+k)+\rho(c-\delta\underline{h})}{\rho+\gamma_1+r} \Leftrightarrow \rho \le \frac{r(h-c)+\gamma_1 k}{\delta\underline{h}}. \tag{9}$$

[18] Just as for the case without the regulator, there are two other equilibria in addition to the interior one. Appendix B describes them in detail: in one equilibrium, the firm gives in immediately and the activist (thus) never gives in during a boycott; in the other, the regulator takes no action during the boycott but the firm self-regulates at a higher rate (so high that $\phi_1+\gamma_1$ stays the same as in Lemma 4).

[19] Indeed, Firm's payoff from self-regulation is $-c/r$, while if it never self-regulates, the payoff is equal to

$$\begin{aligned}
&-r\int_0^\infty\left(\int_0^t(-h)\exp(-r\tau)d\tau+\int_t^\infty\left[\frac{\gamma_1}{\rho+\gamma_1}(-c-k)+\frac{\rho}{\rho+\gamma_1}(\delta\underline{h}-c)\right]\exp(-r\tau)\right)(\rho+\gamma_1)\exp(-(\rho+\gamma_1)t)dt \\
&=\int_0^\infty\left[h(1-\exp(-rt))+\frac{\gamma_1}{\rho+\gamma_1}(c+k)\exp(-rt)+\frac{\rho}{\rho+\gamma_1}(c-\delta\underline{h})\exp(-rt)\right](\rho+\gamma_1)\exp(-(\rho+\gamma_1)t)dt \\
&=h-\int_0^\infty[-h(\rho+\gamma_1)+\gamma_1(c+k)+\rho(c-\delta\underline{h})]\exp(-(\rho+\gamma_1+r)t)dt \\
&=h-\frac{-h(\rho+\gamma_1)+\gamma_1(c+k)+\rho(c-\delta\underline{h})}{\rho+\gamma_1+r}=\frac{rh+\gamma_1(c+k)+\rho(c-\delta\underline{h})}{\rho+\gamma_1+r}
\end{aligned}$$

In an interior equilibrium, all these inequalities must hold as equalities; solving the equations completes the proof. ■

Interestingly, the rate of self-regulation during the boycott is the same as in the post-boycott game. In both cases, the rate of self-regulation must be such that R is just indifferent between administering regulation.[20] However, for A to be willing to continue the costly boycott, some kind of regulation must occur at a faster rate during the boycott than after. Consequently, R must step in and regulate at a faster rate during the boycott than after, particularly if the boycott is very costly to A. Thus, the activist is motivated to continue the boycott because public regulation is more likely then than after the boycott has been called off.

While the lemma above describes the Poisson rates for any type of action, it is even more interesting to consider the boycott's expected length and whether it is likely to succeed.

**Proposition 5** ***(Expected duration).*** *The expected duration of the boycott is large if regulation is beneficial (b large), inexpensive (c small), and if the boycott itself is inexpensive to both A and F (e and h small):*

$$\frac{1}{\phi_1+\gamma_1+\rho}=\frac{1/r}{(c/k+s/q)\left[1+(1+k/\delta\underline{h})\,e/b\right]-1+h/\delta\underline{h}}.$$

The costlier the boycott, the sooner it ends. Intuitively, if F finds the boycott costly, then it is willing to delay self-regulation only if it expects the boycott to end relatively soon. If regulation is very beneficial to A, then he is willing to end the boycott only if the regulation is less likely to be introduced. Since the rate of self-regulation does not depend on $b$, it must be the case that R ends the game later if $b$ is large, which implies a longer-lasting boycott. Comparing this to the case without R, we can see that in expecteation, the boycott is shorter when R is present. The reason is that A is less reluctant to end the boycott when the R is present than when she is not, because he can still hope for eventual regulation after he calls off the boycott.

Say that the boycott *succeeds* if some kind of regulation is introduced before A gives in by stopping his boycott.

[20]This result is driven by assumption that R is indifferent whether the boycott continues or ends. If R disliked the boycott, F would self-regulate at an even lower rate during the boycott.

**Proposition 6** ***(Probability of success).*** *The probability that the boycott succeeds is larger if public regulation is efficient (k and q small), regulation is expensive to F (c large), and beneficial to R (s large):*

$$\frac{\phi_1+\gamma_1}{\phi_1+\gamma_1+\rho}=\frac{(c/k+s/q)\,[1+e/b]-1}{(c/k+s/q)\,[1+(1+k/\delta\underline{h})\,e/b]-1+h/\delta\underline{h}}.$$

Intuitively, if direct regulation is costly, then A expects regulation to be delayed if he ends the boycott (according to Proposition 4-ii). He is then less willing to call it off, unless the boycott is anyway unlikely to succeed. Likewise, if $s$ and $c$ are small, A's continuation value from ending the boycott is small, and A may want to end the boycott only if the boycott is unlikely to succeed.

The equilibrium payoffs in this subgame are the following. The firm receives $-c/r$ since it must be indifferent when considering self-regulation. The regulator receives $(s-q)/r$, since direct regulation is a best response for her. The payoff to the activist is again given by $u_2^A$ and equation (6), since a best response is to end the boycott and enter the post-boycott game.

## 3.3 Self-regulation to Preempt Regulation

Before the boycott has started, the firm can self-regulate and thus prevent future boycotts. Similarly, R can decide to regulate and, in this way, end the game. If A decides to start the boycott, we enter phase 1 described above. As before, we here focus on the interior equilibrium where all players act with some chance. This requires $\underline{e}\geq 0$, as explained below.

**Lemma 5** *Suppose $\underline{e}\in(0,bq/s\,[1+ks/cq])$. There is a unique interior equilibrium in the pre-boycott game. The rate of self-regulation is the same as during and after the boycott but the rate of public regulation is lower before the boycott than during or after the boycott:*

$$\begin{aligned}
\phi_0 &= \phi_1=\phi_2=r\frac{s-q}{q}\in(0,\infty)\,,\\
\gamma_0 &= r\frac{bc/k\underline{e}-(c/k+s/q)\,s/q}{b/\underline{e}+c/k+s/q}\in(0,\gamma_2)\,,\\
\alpha &= r\frac{k}{\underline{h}}\frac{(c/k+s/q)^2}{b/\underline{e}+c/k+s/q}\in(0,\infty)\,.
\end{aligned}$$

*Proof.* Just as in phase 1, R is willing to regulate if and only if (5) holds. A is willing to initiate the boycott if and only if his phase 1 payoff, less the cost of initiating the boycott, is larger than the payoff of waiting:

$$\frac{b}{r}\left(1-\frac{kq}{ks+cq}\right)-\frac{\underline{e}}{r}\geq\frac{b\left(\phi_0+\gamma_0\right)}{r\left(r+\phi_0+\gamma_0\right)}\Leftrightarrow\phi_0+\gamma_0\leq\frac{r}{1+\frac{\underline{e}}{b}\left(\frac{c}{k}+\frac{s}{q}\right)}\left(\frac{c}{k}+\frac{s}{q}\right)-r.$$

Substituting for $\phi_0=r\frac{s-q}{q}$, we find that F is willing to self-reguate if the flow cost of doing so is smaller than the flow cost of risking regulation or boycott:

$$c\leq\frac{\alpha\left(c+\underline{h}\right)+\gamma_0\left(c+k\right)}{\alpha+\gamma_0+r}.$$

All inequalities bind in the interior solution; solving the equations completes the proof. ■

Just as before, the rate of self-regulation is such that R is indifferent between regulating and postponing regulation. A is willing to start a costly boycott only if this increases the chance for either kind of regulation. Consequently, R must impose regulation at a lower rate before the boycott has started if $\underline{e}>0$. Then, the motivation for A to start a boycott is because direct regulation becomes more likely, not because F will be more likely to self-regulate (it won't).

The lemma, describing the equilibrium rates of actions, generates rich comparative statics. For example, if $\underline{e}$ is small, then A is quite tempted to initiate the boycott, unless R regulates at a higher rate. Thus, an "aggressive" activist with a small cost of initiating a boycott will in fact be less likely to have to start one. If $b$ is large, A has more to gain from initiating a boycott, because he wants regulation more, and boycott is a way to get it faster. To keep A willing to wait, regulation must be more likely. Thus, if $b$ is large, it is also more likely that regulation will take place before the boycott.

**Proposition 7** ***(Likelihood of a boycott).*** *The probability for a boycott is larger if costly for A ($\underline{e}$ large), inexpensive for F ($\underline{h}$ small), and if regulation is not very beneficial for A ($b$ small):*

$$\frac{\alpha}{\phi_0+\gamma_0+\alpha}=\frac{\left(c/k+s/q\right)^2\underline{e}/\underline{h}}{\left(c/k+s/q\right)^2\underline{e}/\underline{h}+\frac{1}{k}\left(c/k+s/q\right)\left(b-\underline{e}\right)-b}.$$

Parameter $\underline{e}$ deserves some further discussion. If $\underline{e}$ decreases, it becomes less expensive to initiate a boycott and A is willing to wait only if $\gamma_0$ increases, which must thus be the case in equilibrium. Then, for a larger $\gamma_0$, F becomes more eager to self-regulate unless the boycott is less likely to start. Thus, $\alpha$ must increase in $\underline{e}$ to keep F indifferent. This intuition

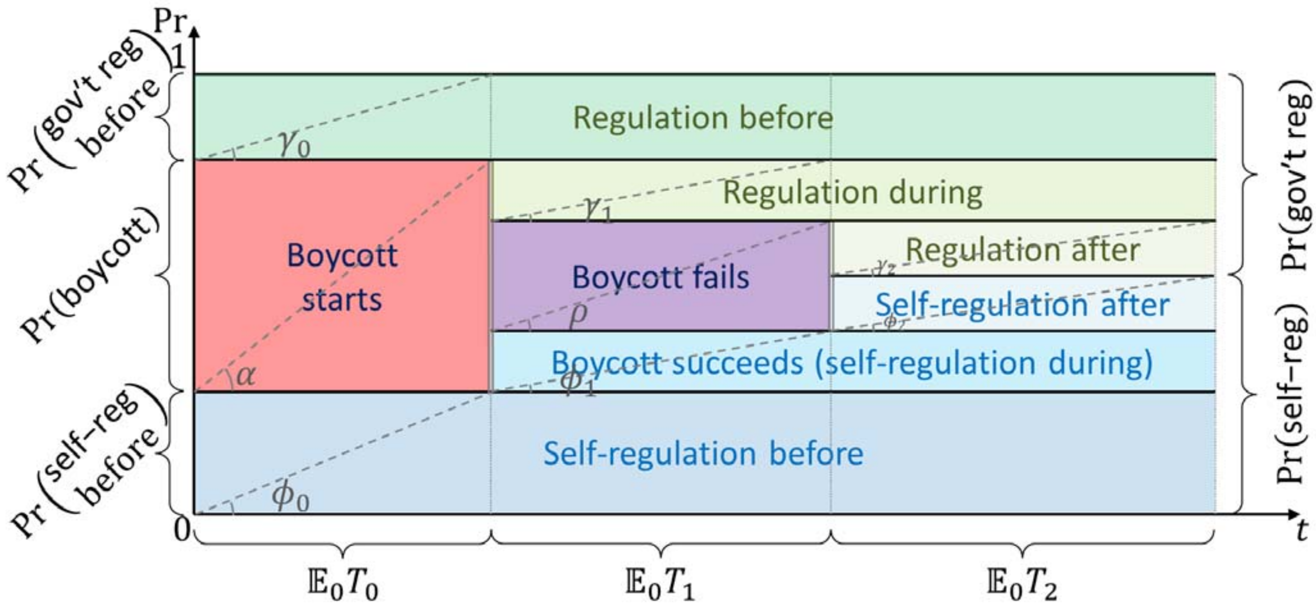


Figure 2: The outcomes of the game with the regulator

also suggests that if $\underline{e}$ goes down to 0, $\gamma_0$ converges to $\gamma_2$ from above, and $\alpha$ converges to 0. If $\underline{e} < 0$, A would be willing to postpone the boycott only if $\phi_0 + \gamma_0 > \phi_2 + \gamma_2$, which can only hold if $\gamma_0 = 0$, implying that the equilibrium is not strictly "interior".[21] In this case, the pre-boycott game between F and A is similar to the one in the previous section, without government regulator.

The equilibrium payoffs at the very start of the game are the following. The firm gets $-c/r$ since self-regulation is a best response. The regulator receives $(s - q)/r$, since public regulation is a best response. The payoff to the activist is given by (6), minus the cost $\underline{e}/r$ of starting the boycott.

As in the case without the regulator, we can illustrate the set of possible outcomes graphically. Figure 2 suggests that there are six scenarios, and all of them end in either self-regulation or regulation in finite time.

[21] Indeed, $\phi_0 + \gamma_0 > \phi_2 + \gamma_2$ implies that either $\gamma_0 > \gamma_2$ or $\phi_0 > \phi_2$. In the first case, F would stricty prefer to self-regulate (it is indifferent in phase 2, without a threat of boycott!), which means that R would not regulate, a contradiction. So, $\phi_0 > \phi_2$, but then we know (from the definition of $\phi_2$) that R strictly prefers to wait, so $\gamma_0 = 0$.

# 4 Multiple campaigns and equilibria

We now show that our main results continue to hold if multiple and sequential corporate campaigns are permitted. The analysis above could employ backward induction to solve the game because we assumed that activists can organize only one boycott. This assumption is quite weak: given the rates of public regulation characterized earlier, A would never *want* to start another boycott, because the probability of public regulation is larger after the boycott than before.

If a corporate campaign can be re-started, however, then Markov-perfection must imply that the equilibrium rates of actions must be the same before and after the boycott, since the two subgames are equivalent (the rates can be different *during* the boycott).

Let phase 0 refer to the situation without a boycott and phase 1 refer to a situation with a boycott currently in place. We do not need to assume any fixed cost of starting the boycott, so we set $\underline{e} = 0$.[22] There are multiple Markov-perfect equilibria since what matters for A is the rate of actions during boycotts *relative* to other times: For example, if $\phi_0$ is large, A may be willing to start a boycott if just $\phi_1$ is even larger.

Consider first the situation where R is not a player in the game.

**Lemma 6** *Suppose multiple boycotts are possible and there is no regulator. For every $\phi_0 > 0$ there is an interior equilibrium where*

$$\begin{aligned}
\phi_1 &= \phi_0 \left(1 + \frac{e}{b}\right) + r\frac{e}{b} > \phi_0, \\
\alpha &= r\frac{c}{\underline{h}}, \\
\rho &= r\frac{h - c}{\delta \underline{h}}.
\end{aligned}$$

*Proof.* For either $\alpha \in (0, \infty)$ or $\rho \in (0, \infty)$, A must be indifferent between stopping or not starting a boycott, giving the expected payoff $(b/r)\,\phi_0/\,(\phi_0 + r)$, and continuing a boycott, giving the payoff (3). The latter payoff is larger if and only if

$$\phi_1 \geq \phi_0 + \frac{e\,(\phi_0 + r)}{b}. \tag{10}$$

[22] When rates must be identical before and after a boycott, A would never start a boycott if $\underline{e} > 0$, given that a best response is to immediately end it: this is costly, and nothing would be achieved. If $\underline{e} < 0$, A would prefer to start and end boycotts as often as possible. Thus, the only interesting case is $\underline{e} = 0$, which is henceforth assumed.

If (10) holds with equality, A is willing to randomize in both phases. Since (10) implies that, $\phi_1 \in (0,\infty)$, F must be indifferent during the boycott, requiring:

$$\frac{c}{r} = \frac{h}{r} + \frac{\rho}{\rho + r}\left(-\frac{h + \delta \underline{h}}{r} + \frac{\alpha}{\alpha + r}\left(\frac{c + \underline{h}}{r}\right)\right). \tag{11}$$

When there is no boycott, F is willing to wait whenever

$$\frac{c}{r} \geq \frac{\alpha}{\alpha + r}\left(\frac{c + \underline{h}}{r}\right) \Leftrightarrow \alpha \leq r\frac{c}{\underline{h}}. \tag{12}$$

In an interior equilibrium, all inequalities bind, giving the results. Note that we also have non-interior equilibria, e.g., $\phi_0 = 0$, $\phi_1 = re/b$, $\alpha \leq rc/\underline{h}$ if $\rho = r(h-c)/[c + \delta\underline{h} - (c + \underline{h})\alpha/(\alpha + r)]$, so that (11) holds. ■

The lemma confirms the main findings from Section 2. The firm is self-regulating at a higher rate during boycotts than at other times, particularly if the boycott is costly to A relative to his benefit from regulation. If self-regulation is costly to the firm, the boycott must start at a higher rate and end at a slower rate (i.e., more time will be spent in the boycott phase).

Consider now the situation with the government regulator. It is easy to show that there is no equilibrium where $\gamma_0 > rc/k$.

**Lemma 7** *Suppose multiple boycotts are possible and there is a regulator. For every $\gamma_0 \in (0, rc/k)$ there is an interior equilibrium where:*

$$\begin{aligned}
\phi_1 &= \phi_0 = r\frac{s-q}{q}, \\
\gamma_1 &= \gamma_0\left(1 + \frac{e}{b}\right) + r\frac{es}{bq} > \gamma_0, \\
\alpha &= r\frac{c - k\gamma_0/r}{\underline{h}}, \\
\rho &= r\frac{h-c}{\underline{h}\delta} + r\frac{sek/bq + (1 + e/b)\,k\gamma_0/r}{\underline{h}\delta}.
\end{aligned}$$

*Proof.* In an interior equilbrium, A is indifferent between boycotting and not and (10) must hold as equality after replacing $\phi$ with the total Poisson rate of any kind of regulation, so

$$\gamma_1 + \phi_1 = \gamma_0 + \phi_0 + (\gamma_0 + \phi_0 + r)\,e/b.$$

When R is indifferent (and thus willing to regulate) in phase 0 as well as phase 1, the start/end of the boycott is not affecting R's payoff, and to make R indifferent, we must have $\phi_0 = \phi_1 = r(s-q)/q$, just as in Section 3. When we substitute this into the first equality, we get $\gamma_1 = \gamma_0 + (\gamma_0 + rs/q)\, e/b$.

Finally, when there is no boycott, F is indifferent to self-regulation if:

$$\frac{c}{r} = \frac{\alpha}{\alpha+\gamma_0+r}\left(\frac{c+\underline{h}}{r}\right) + \frac{\gamma_0}{\alpha+\gamma_0+r}\frac{c+k}{r},$$

which gives $\alpha$ as a function of $\gamma_0$. When there is a boycott, F is indifferent to self-regulation if (9) holds as equality, as before, so $\rho = [r(h-c)+\gamma_1 k]/\delta\underline{h}$, which gives $\rho$ as a function of $\gamma_0$ (since $\gamma_1$ is a function of $\gamma_0$). Solving the equations concludes the proof. ■

Just as in Section 3, the rate of self-regulation is constant over time and independent of presence of a boycott at the moment. Furthermore, the rate of public regulation must be larger during a boycott than at other times, particularly if the boycott is costly to A relative to his potential gain (i.e., if $e/b$ is large). As before, boycotts and public regulation are strategic substitutes: If $\gamma_0$ increases (then $\gamma_1$ increases, as well), then the boycott starts at a lower rate and ends at a higher rate. Boycotts are thus both rarer and shorter when the R's activity (measured by $\gamma_0$) is large. Similarly, note that if the probability of a boycott ($\alpha$) is large, then the probabilities of public regulation (both $\gamma_0$ and $\gamma_1$) must decrease.

We may summarize and compare the two lemmas as follows:

**Proposition 8** ***(Multiple campaigns)*** *Consider the equilibria of Lemmas 6-7.*
*(i) Without a regulator, $\phi_1 > \phi_0$.*
*(ii) With the regulator, $\phi_1 = \phi_0$ but $\gamma_1 > \gamma_0$.*
*(iii) The rate of starting a boycott, $\alpha$ is smaller, and the rate of ending it $\rho$ is larger with than without the regulator.*

# 5 Results and Comparisons

This section provides the main contribution of the paper. In our view, the above mathematical description of the equilibria are interesting, but still more important is the comparison *between* them. After summarizing important and robust comparative statics, we discuss the effects of including or removing each of the three players in the game. This leads to a deeper understanding of how public and private politics interact.

## 5.1 Comparative Statics

Most of the comparative statics are robust and hold whether there is a regulator or not. If regulation is important to A ($b$ is large), then the boycott tends to last longer. Intuitively, A is then willing to continue the boycott even when F is less likely to self-regulate (or R is less likely to regulate). If regulation is costly to F ($c$ is large), then the boycott is more likely to be successful. Intuitively, when regulation is costly, then F is willing to self-regulate only if the boycott is unlikely to be called off by A. If the boycott is costly for A to run ($e$ is large), then it is likely to be shorter. If the boycott is costly to F ($h$ large), then it is both shorter, and more likely to fail. Intuitively, a costly boycott means that F may continue to hold on if it believes that A will soon give up anyway. If F expects a large gain when the boycott ends (if $\delta \underline{h}$ is large), e.g., because it expects customers to "forgive" it, then A must end the boycott and give up at a lower rate. Thus, forgiving consumers (large $\delta \underline{h}$) imply that the boycott is likely to be long-lasting as well as successful. All these comparative statics results hold both with and without a regulator.

Other comparative statics results may depend on the presence of a public regulator. This suggests that the presence of an active public regulator plays an important role in the private politics game. The next subsection explains how fundamental qualitative results are reversed if the regulator is introduced in the game.

## 5.2 The impact of public regulation

In the absence of public regulator (Section 2), F self-regulates at a higher rate during boycotts than after ($\phi_1 > \phi_2$). This is precisely the reason for why A is willing to maintain and continue a costly boycott.

When the regulator is present, the rate of self-regulation is constant throughout the game: before, during and after a boycott. This rate is pinned down by R's indifference condition, which is the same in all phases. However, regulation takes place at a higher rate during a boycott than after the boycott or before it. Thus, in the presence of a regulator, A's motivation to start and continue a costly boycott is that R is more likely to act during a boycott than at other times. The rationale for a boycott is therefore quite different in these two cases.

For A, actions by F and by R are perfect substitutes. If the probability of one type of regulation increases, the other can decrease without altering A's decisions. Similarly, for F, actions by R and by A (in phase 0) are both costly, and if one of these probabilities increases, the other may decrease without changing F's preferences. This explains why the boycott ends at a faster rate if R is present than if she is not.

Finally, note that introducing public regulation to the model is beneficial for the activist but not for the firm. The activist is better off because even if a boycott starts and ends without success, there is still a chance that R would step in or that F will self-regulate in phase 2.

| | **Without regulator** | **With regulator** |
|---|---|---|
| **Self-regulation:** | *more during boycotts;* | *independent of boycotts.* |
| **Public regulation:** | | *more during boycotts.* |
| **Why boycott?** | *raise self-regulation;* | *raise public regulation.* |
| **Private politics:** | *benefits activist but harms firm;* | *harms activist but benefits firm.* |

## 5.3 The impact of self-regulation

Just as we can study the game with and without public regulation, we can look at the consequences of removing the firm's ability to self-regulate. For example, the firm would never self-regulate if such actions could not be verified by third parties, or if it cannot commit to stay self-regulated once the threat of a boycott is lifted. Either of these considerations may prevent F from getting credit for doing good.

If there is no regulator, then a firm that is unable to self-regulate is committed to the status quo. In this case, of course, a boycott makes no sense and the activist will remain passive as well. Introducing the possibility to self-regulate is thus good for the activist and bad for the firm, since now self-regulation is a possibility, and this motivates A to start a campaign that is costly to the firm. In other words, in the absence of a regulator, F would be better off if it could commit to never self-regulate.

These conclusions are very different in the presence of a regulator. In this case, if F were unable to self-regulate, then R would impose regulation immediately. This would give A the largest possible utility. Allowing F to self-regulate is reducing R's inclination to regulate fast, since she hopes that F will instead regulate itself. The firm is thus better off both because the regulator is less inclined to act, and because the final outcome might be self-regulation rather than more costly public regulation. In sum, when public regulation is possible, the ability to self-regulate is good for the firm and bad for activists, in stark contrast with the case without public regulation.

### 5.4 The impact of activism

Finally, let us consider the impact of A's existence. If there is no regulator in place either, then F will never self-regulate. This means that activism is necessary for self-regulation. On the other hand, we saw above that the possibility to self-regulate is necessary to make A willing to act. In this sense, the two types of private politics (boycotts by activists and self-regulation by firms) are "*strategic complements*": one type of private policy arises if and only if also the other type of private policy exists. None of the two will be observed in isolation.

If there is a regulator, then the game between the firm and the regulator is exactly as described in Section 3.1. When the activist is added, the firm does not change the rate of self-regulation at any point in time; instead the regulator becomes less active before a boycott but more active during a boycott. At the same time, we have learned that the presence of a regulator makes the activist less likely to start a boycott, and more likely to call it off. In this sense, therefore, public policy and private politics (in the form of activism) are *strategic substitutes*.

## 6 Conclusion

This paper develops a unified framework for studying regulation, self-regulation, activism, and interactions between them. The model is dynamic and does not impose restrictions on the sequence of moves. Our model of the boycott is therefore a war of attrition: the firm hopes that the activists give in by ending the boycott, while the activists hope the firm gives in by self-regulating. The firm self-regulates to preempt or end a boycott, while the boycott is started or continues because the firm is more likely to self-regulate during a boycott than at other times. With such strategic complementarity, the firm's possibility to self-regulate is good for the activist but bad for the firm: if the firm could not self-regulate, activists would never boycott the firm.

In the presence of a regulatory agency, these results are reversed. Being able to self-regulate is then good for the firm, as the regulator will be willing to wait in hope that the firm self-regulates. This is, in turn, bad for the activists, who prefer regulation without delay. Furthermore, with a regulator, self-regulation is *not* more likely during boycotts than at other times. Instead, public regulation is more likely during boycotts, and this motivates the activists to start and continue the campaign in our model.

Our analytical results allow us to characterize the length and likelihood of boycotts, the probabilities of success, and the probabilities for self-regulation versus public regulation. These results generate a rich set of testable comparative statics. For example, we predict that boycotts tend to last longer if regulation is beneficial to the activists, but shorter if the boycott is costly to the activist or the firm. If consumers are "forgiving," the boycotts last longer but succeed with a higher probability. These and other comparative statics are robust in that they hold whether or not the regulator is present.

Our main results are also robust in that they continue to hold if we allow for multiple sequential boycotts. The Appendix is also permitting multiple activist groups and multiple firms, and it characterizes all equilibria (not only the interior ones which we focused on above). Our workhorse model has thus proven to be sufficiently flexible to allow extensions in several directions. Future research should permit multiple competing firms, collaboration between activists and firms, and also more complicated interaction (such as lobbying) between the firms, the activists, and the regulator. This is necessary to get a more complete understanding of private politics and public regulation.

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# Appendix A: Multiple Activists

The basic model is flexible enough to allow for multiple activists (and even for multiple regulators). In fact, there is an interesting parallel between activists and regulators. The game between R and F is similar to the game between A and F, provided that in the latter, both players anticipate the equilibrium where F gives in immediately after the boycott has started (the games are then identical if we set $s = b$, $q = \underline{e}$, and $k = \underline{h}$). This suggests that R can be interpreted as a "tough" (or credible) activist which is always successful in achieving regulation if he decides to do so.

With this interpretation of R in mind, Section 3 may be thought of as a game between a firm F, a "soft" activist A, and a tough activist R; the latter is committed to never give up on a campaign, which prompts F to give in immediately to R, but not to A. Our results of Section 5 may thus be interpreted as follows: A becomes even "softer" (in that his boycotts are rarer and shorter) in presence of R, and R is less active before A has started a boycott than after it has ended, i.e., when A is no longer a player in the game. This suggests an effect of crowding out of activists by other activists.

Below, we study a setting where A and R are both tough or soft (before we permit $n > 2$ activist groups or regulators). We start with the first case: Consider the same game as in Section 3.3, but assume that once A starts the campaign, F is expected to give in immediately (this is indeed an equilibrium, albeit different than the interior one studied above). If, however, R acts first (at phase 0), then R pays the extra cost $q/r$ (as before). Thus, A hopes that R stops the game while R hopes that A stops instead, just like in a war of attrition. The firm, however, prefers the status quo. We will again focus on the interior equilibrium for phase 0.

**Proposition A1.** *When both A and F are "tough", there is a unique interior equilibrium in phase 0:*

$$\begin{aligned}
\phi_0 &= r\frac{bk/\underline{e} - c + \underline{h}\frac{s-q}{q} - k}{\underline{h} + k}, \\
\alpha &= r\frac{sk/q + c - bk/\underline{e}}{\underline{h} + k}, \\
\gamma_0 &= r\frac{c - \underline{h}\left[\frac{s}{q} - \frac{b}{\underline{e}}\right]}{\underline{h} + k},
\end{aligned}$$

A full characterization of the equilibria, and the corresponding proofs, can be found in the next appendix. By comparing to the case without A or without F, we can see that the presence of both A and R means that F is less likely to self-regulate. Players A and R both hope that the other player will act, and each is willint to take the burden and end the game only if F self-regulates at a low rate. Intuitively, the free-riding problem between A and R means that F can relatively safely postpone self-regulation. Of course, rather than interpreting A and R as both being "tough" activists, we could interpret both as (different) regulators, each trying to free-ride on the other. Alternatively, one can obviously think of R as a regulator and A as a tough activist. For any of these situations, the outcome is described by the proposition above.

Since the regulator, as mentioned, may be interpreted as a tough activist, the proposition implies that we have the following equilibrium for two symmetric activist groups (where $s = b$, $q = \underline{e}$ and $k = \underline{h}$):

$$\begin{aligned}
\phi_0 &= r\frac{b}{\underline{e}} - r - r\frac{c}{2\underline{h}}, \\
\alpha_1 &= r\frac{c}{2\underline{h}}, \\
\alpha_2 &= r\frac{c}{2\underline{h}},
\end{aligned}$$

which implies $\alpha \equiv \alpha_1 + \alpha_2 = r\frac{c}{\underline{h}}$, just as before. The result in the case of $n$ symmetric activist groups would be similar:

**Proposition A2.** *With $n$ symmetric "tough" activists and no regulator, there is a unique interior equilibrium where each activist starts a boycott at rate $\alpha_i$ where:*

$$\begin{aligned}
\alpha_i &= r\frac{c}{n\underline{h}}, \\
\phi_0 &= r\frac{b}{\underline{e}} - r - r\frac{c}{\underline{h}}\left(1 - \frac{1}{n}\right).
\end{aligned}$$

*Proof.* To see this, note that with $n$ symmetric activist groups, the total $\alpha = n\alpha_i$ is the same (to make F indifferent), and so for one activist to stay indifferent,

$$\begin{aligned}
b - \underline{e} &= \frac{\phi_0 + \alpha(1 - 1/n)}{\phi_0 + \alpha(1 - 1/n) + r} b \Rightarrow \phi_0 + \alpha(1 - 1/n) = r\frac{b - \underline{e}}{\underline{e}} = r\frac{b}{\underline{e}} - r \Rightarrow \\
\phi_0 &= r\frac{b}{\underline{e}} - r - \alpha(1 - 1/n) = r\frac{b}{\underline{e}} - r - r\frac{c}{\underline{h}}(1 - 1/n).
\end{aligned}$$

■

Now, suppose instead that there are several "soft" activist groups (or regulators), meaning that when one of them targets the firm by ending phase 0, then this player and the firm starts a war or attrition as studied in Section 2.2. Furthermore, suppose that the *interior* equilibrium is expected once a boycott starts: each of the two players hopes the other one will give in. If one group has ended its boycott, another group can take over and start a new boycott. This, of course, increases the incentive to stop a boycott unless, as will be the case in equilibrium, the rate of self-regulation is much smaller when there is no boycott compared to the times when there is a boycott. Likewise, the incentives to start a boycott are diminished, unless the firm is very unlikely to self-regulate without an active boycott.

To show this result in a simple way, we make the following assumptions: there is no regulator (or that she is "soft" as well); the activist groups are identical; one cannot start a boycott if another activist group is already running one, but everyone can run multiple sequential boycotts (i.e., we are within the world of Section 4); moreover, there is no cost of starting a boycott ($\underline{e} = 0$). Although we have multiple equilibria, we can still do some comparisons.

**Proposition A3.** *Suppose there are* $n$ *activist groups. Then we must have* $\phi_0 < \phi_1$. *Furthermore, as* $n$ increases, $\phi_0$ is decreasing for any fixed $\phi_1$ (and likewise, $\phi_1$ is increasing for any fixed $\phi_0$). *Moreover, for every* $\phi_1 > 0$ we have an interior equilibrium where:

$$\begin{aligned}
\alpha_i &= r\frac{c}{n\underline{h}}, \\
\rho &= r\frac{h-c}{\delta\underline{h}}, \\
\phi_0 &= \frac{b(\phi_1+r)}{b+e}\left(1-\frac{(1-1/n)\,ce/b\underline{h}}{\phi_1/r+1+(h-c)/\delta\underline{h}}\right)-r.
\end{aligned}$$

*Proof.* The firm's indifference conditions self-regulate pin down $\alpha$ and $\rho$, just as in Lemma *6*. An activist must be indifferent between starting and continuing a boycott and doing nothing until another activist group starts it (at total rate $\alpha(1-1/n)$), ends it (at rate $\rho$), and only then start and continue a boycott. This gives rise to the following equation:

$$\begin{aligned}
\frac{\phi_1}{\phi_1+r}\frac{b}{r}-\frac{r}{\phi_1+r}\frac{e}{r} &= \frac{\phi_0}{\phi_0+\alpha(1-1/n)+r}\frac{b}{r} \\
&\quad+\frac{\alpha(1-1/n)}{\phi_0+\alpha(1-1/n)+r}\left[\frac{\phi_1}{\phi_1+\rho+r}+\frac{\rho}{\phi_1+\rho+r}\left[\frac{\phi_1}{\phi_1+r}-\frac{r}{\phi_1+r}\frac{e}{b}\right]\right]\frac{b}{r}.
\end{aligned}$$

Substituting for $\alpha$ and $\rho$ and rearranging, we get:

$$\begin{aligned}
\frac{\phi_1}{\phi_1+r}\left(1-\frac{r}{\phi_1}\frac{e}{b}\right) &= \frac{\phi_0}{\phi_0+r\frac{c}{\underline{h}}(1-1/n)+r} \\
&\quad+\frac{r\frac{c}{\underline{h}}(1-1/n)}{\phi_0+r\frac{c}{\underline{h}}(1-1/n)+r}\left[\frac{\phi_1}{\phi_1+r}\left(1-\frac{r}{\phi_1}\frac{e}{b}\left(1-\frac{\phi_1+r}{\phi_1+r\frac{h-c}{\delta\underline{h}}+r}\right)\right)\right],
\end{aligned}$$

and thus

$$\phi_0=\frac{\phi_1}{\phi_1+r}\left(1-\frac{r}{\phi_1}\frac{e}{b}\right)[\phi_0+r]-r\frac{c}{\underline{h}}(1-1/n)\left[\frac{e}{b}\left(\frac{r}{\phi_1+r\frac{h-c}{\delta\underline{h}}+r}\right)\right].$$

Rearranging again, we get

$$\begin{aligned}
\frac{\phi_1}{\phi_1+r}\left(1-\frac{r}{\phi_1}\frac{e}{b}\right)[\phi_0+r]-\phi_0 &= r\frac{c}{\underline{h}}(1-1/n)\left[\frac{e}{b}\left(\frac{r}{\phi_1+r\frac{h-c}{\delta\underline{h}}+r}\right)\right] \Rightarrow \\
\frac{\phi_0+r}{\phi_1+r} &= \frac{b}{b+e}\left(1-\frac{(1-1/n)\,ce/b\underline{h}}{\phi_1/r+1+(h-c)/\delta\underline{h}}\right)<1,
\end{aligned}$$

which yields $\phi_0$ as a function of $\phi_0$. ■

# 7 Appendix B: Characterization of all equilibria

The main text focused on "interior" equilibria, where all players have a positive chance of acting in every phase of the game. These equilibria are the empirically relevant (and thus the most interesting) ones, if we are to study boycotts. Moreover, this restriction left us with a unique MPE for each case considered in Sections 2 and 3, which helped us study comparative statics. In this appendix, we characterize all equilibria for the models of Sections 2 and 3, for completeness.

## 7.1 The case without a public regulator

For the case without R, there was a unique equilibrium in phase 2, and in that equilibrium, F would never self-regulate and A could no longer act.

In phase 1, however, we noted in Section 2.2 that there were exactly three equilibria (as in a typical war of attrition game between two players). It thus remains to characterize all equilibria of phase 0, before the boycott has started. It turns out that for each of the equilibria played in phase 1, the corresponding equilibrium of phase 0 is determined uniquely. If the equilibrium $(\phi_1, \rho) = (0, \infty)$ is expected, then A never starts a boycott (unless A receives a large benefit from simply starting one). If the equilibrium $(\phi_1, \rho) = (\infty, 0)$ is expected, so that F will give in immediately once the boycott has started, then A is tempted to start a boycott even if this is costly. In that case, F is willing to self-regulate in phase 0, and we end up having a unique equilibrium in mixed strategies if $\underline{e} > 0$. More precisely, we have the following:

**Lemma 8** ***(Before the boycott)***

*(i) If the interior equilibrium of phase 1 is anticipated (Lemma 1), the phase 0 equilibrium is as in Lemma 2:*

$$\alpha = r\frac{c}{\underline{h}} \text{ and } \phi_0 = r\frac{-\underline{e}}{b+\underline{e}},$$

*if $\underline{e} \in (-b, 0)$. If $\underline{e} > 0$, then $\alpha = \phi_0 = 0$, and if $\underline{e} < -b$, then $\alpha = \phi_0 = \infty$.*[23]

*(ii) If equilibrium $(\phi_1, \rho) = (0, \infty)$ is played in phase 1, then in phase 0, the equilibrium is*

$$\alpha = r\frac{c}{\underline{h}(1-\delta) - c} \text{ and } \phi_0 = r\frac{-\underline{e}}{b+\underline{e}},$$

*provided that $\underline{h}(1-\delta) > c$ and $\underline{e} \in (-b, 0)$. If $\underline{h}(1-\delta) > c$ and $\underline{e} < -b$, then $\phi_0 = \alpha = \infty$; if $\underline{h}(1-\delta) > c$ and $\underline{e} > 0$, then $\phi_0 = \alpha = 0$. If $\underline{h}(1-\delta) \leq c$ and $\underline{e} > 0$, then $\phi_0 = \alpha = 0$; if $\underline{h}(1-\delta) \leq c$ and $\underline{e} < 0$, then $\phi_0 = 0$, $\alpha = \infty$.*

*(iii) If equilibrium $(\phi_1, \rho) = (\infty, 0)$ is played in phase 1, then in phase 0, the equilibrium is*

$$\alpha = r\frac{c}{\underline{h}} \text{ and } \phi_0 = r\frac{b-\underline{e}}{\underline{e}},$$

*provided that $\underline{e} \in (0, b)$. If $\underline{e} > b$, then $\alpha = \phi_0 = 0$, and if $\underline{e} < 0$, then $\alpha = \phi_0 = \infty$.*

*Proof of Lemma 8**:** (i)* This is proved in Lemma 2.

[23]If $\underline{e} = -b$, then any $\alpha \geq rc/\underline{h}$, $\phi_0 = \infty$ is an equilibrium. If $\underline{e} = 0$, then any $\alpha \leq rc/\underline{h}$, $\phi_0 = 0$ is an equilibrium.

(ii) Anticipating $(\phi_1, \rho) = (0, \infty)$, A prefers to start a boycott if:

$$(-\underline{e}) \geq \frac{\phi_0 b}{\phi_0 + r} \Leftrightarrow \phi_0 (b + \underline{e}) \leq -r\underline{e},$$

while F prefers self-regulation if

$$c \leq \frac{\alpha}{\alpha + r} \underline{h} (1 - \delta) \Leftrightarrow \alpha [\underline{h} (1 - \delta) - c] \geq rc$$

So, if $\underline{e} < -b$, A starts a boycott immediately; if $\underline{e} > 0$, A never starts a boycott. If $\underline{h} (1 - \delta) < c$, F never self regulates, but if $\underline{h} (1 - \delta) > c$, F will if $\alpha$ is sufficiently high. This produces the results.

(iii) The proof is analoguous and thus omitted. ■

## 7.2 The case with a public regulator: During the boycott

Consider the case with a public regulator. In Section 3.1, we showed that the equilibrium in phase 2 is unique, with F and R acting with positive rates. We use this to study equilibria in phase 1.

It turns out that with R, we have three equilibria in phase 1, just like without R (see above). One is the interior one, studied in Section 3. In the second one is where F gives in immediately during a boycott ($\phi_1 = \infty$), just as in the case without R. In this equilibrium, A will never end the boycott ($\rho = 0$), since he anticipates to win. Similarly, there is no reason for R to step in, so $\gamma_1 = 0$. Thus, F is then self-regulating not because of the threat of public regulation, but because of A's commitment to continue, just as in the analoguous equilibrium without R.

There is a third equilibrium, which is similar to the interior one without R: there, $\gamma_1 = 0$, $\phi_1 \in (0, \infty)$, which is large enough so that A is just indifferent between continuing or stopping the boycott, which implies $\phi_1 > \phi_0 + \gamma_0$ (where $\phi_0$ are $\gamma_0$ are for an equilibrium where phase 1 can be reached), and this, in turn, ensures that $\phi_1 > \phi_0$ and that R is not willing to step in during a boycott. Similarly, A must end the boycott at a rate that makes F indifferent between self-regulating and not.[24]

More precisely, we have the following characterization.

[24] In principle, there is another equilibrium (or even family of equilibria), similar to the other corner equilibrium in the case without R. In these equilibria, F never gives in during the boycott ($\phi_1 = 0$), A ends the boycott as soon as it can ($\rho = \infty$), and R either does not regulate, or regulates at a rate low enough so that A does not want to continue the boycott; more precisely, $\gamma_1 \leq r \left[ \left( \frac{c}{k} + \frac{s}{q} \right) \left( 1 + \frac{e}{b} \right) - 1 \right]$. However, these equilibria are non-robust, in the sense that R is indifferent between regulating immediately and waiting for A to end the boycott and regulating after that only because this happens instantly. If there was some upper limit on how fast A can end the boycott, then R would strictly prefer to regulate ($\gamma_1 = \infty$), which would, in turn, prompt F to self-regulate immediately, contradicting $\phi_1 = 0$. Because of this non-robustness, we exclude these equilibria from further consideration.

**Lemma 9** ***(During the boycott)*** *There are three equilibria in phase 1:*

$$\begin{aligned} \text{(I) } (\phi_1, \gamma_1, \rho) &= \left( r\frac{s-q}{q}, r\frac{cq(b+e)+kse}{bkq}, r\frac{hb+e(c+sk/q)}{\delta \underline{h} b} \right); \\ \text{(II) } (\phi_1, \gamma_1, \rho) &= (\infty, 0, 0); \\ \text{(III) } (\phi_1, \gamma_1, \rho) &= \left( r\left[ \left( \frac{c}{k} + \frac{s}{q} \right) \left( 1 + \frac{e}{b} \right) - 1 \right], 0, r\frac{h-c}{\delta \underline{h}} \right). \end{aligned}$$

*Proof of Lemma 9.* Consider the following possibilities:

(1) Suppose $\gamma_1 \in (0, \infty)$. Then R randomizes so (7) must bind, implying that F randomizes so (9) binds. This means that $\rho \in (0, \infty)$ and that A randomizes, requiring (8) to bind. Thus, when $\gamma_1 \in (0, \infty)$, the equilibrium must be interior, and thus it coincides with the one in Lemma 4.

(2) Suppose $\gamma_1 = \infty$. Then (8) is violated, thus A prefers to wait, so $\rho = 0$; but then F prefers to self-regulate immediately. If so, R prefers to wait, implying $\gamma_1 = 0$; a contradiction.

(3) Suppose $\gamma_1 = 0$. If we pick $\phi_0$ and $\rho$ that make (8)-(9) hold as equalities, then R would indeed prefer to wait; this yields equilibrium III. If F acts at a faster rate, A prefers $\rho = 0$, making F prefer $\phi_1 = \infty$, which is thus equilibrium II (again, R prefers to wait). If F acts at a slower rate, A prefers $\rho = \infty$, making F prefer $\phi_1 = 0$, but then R will prefer to act, so it is impossible to have $\gamma_1 = 0$. Thus, only equilibria I-III exist. ■

## 7.3 Before the boycott: Anticipating equilibrium (I)

Section 3.3 assumed that the players anticipated the interior equilibrium (I) when they played the game in phase 0. In that case, there is a unique interior equilibrium in phase 0. However, depending on the parameters, there may be up to two other equilibria. If $\underline{e}$ is large enough, there is an equilibrium in phase 0 where A never takes an action (this equilibrium is identical to the one in phase 2, since this becomes, in fact, a game without a boycott). In this equilibrium, R regulates at a rate higher than the rate in the interior equilibrium, and this gives A an incentive to abstain from a costly boycott.

If $\underline{e}$ is small, there is an equilibrium which resembles the phase-0 equilibrium in the case without R (as in Section 2.3), where $\gamma_0 = 0$, and the game is between A and F only. Since R does not take any action, F must self-regulate at a higher rate to make A willing to postpone a boycott. This ensures, in turn, that R is willing to remain passive. But for F to be willing to self-regulate when R is passive, A must initiate the boycott with a probability that is larger than when R were an active player. This confirms our previous finding that A is more active when R is absent than when R is present.

**Lemma 10** ***(Before the boycott I)*** *If the interior equilibrium (I) is anticipated during the boycott, the equilibria in phase 0 are:*

*(i)* $\underline{e} < -\frac{b}{c/k+s/q}$, *the unique equilibrium is* $\gamma_0 = 0$ *and* $\phi_0 = \alpha = \infty$.

*(ii) If* $\underline{e} \in \left(-\frac{b}{c/k+s/q}, 0\right)$, *the unique equilibrium is "without R":*

$$(\phi_0, \gamma_0, \alpha) = \left(r\frac{ks+cq}{kq+(ks+cq)\,\underline{e}/b} - r, 0, r\frac{c}{\underline{h}.}\right).$$

*(iii) If* $\underline{e} > \frac{bcq^2}{(ks+cq)s}$, *the unique equilibrium is "without A":* $(\phi_0, \gamma_0, \alpha) = (\phi_2, \gamma_2, 0)$.

*(iv) If* $\underline{e} \in \left[0, \frac{bcq^2}{(ks+cq)s}\right]$, *we have three equilibria: one as in (ii); another as in (iii); and a third one, characterized by Lemma 5.*

*Proof.* There are three possibilities.

(1) If $\phi_0 < r\frac{s-q}{q}$, R regulates immediately, making F willing to self-regulating immediately, which violates the assumption $\phi_0 < r\frac{s-q}{q}$.

(2) If $\phi_0 = r\frac{s-q}{q}$, R is willing to randomize. We have three subcases. Let $\hat{R}_0^A$ be the sum of regulation, $\hat{R}_0^A \equiv \phi_0 + \gamma_0$, such that A is indifferent:

$$\begin{aligned}
\frac{\hat{R}_0^A}{\hat{R}_0^A + r}\frac{b}{r} &= \frac{R_2}{R_2+r}\frac{b}{r} - \frac{\underline{e}}{r} \Rightarrow \\
\hat{R}_0^A &= \frac{r\left(\frac{R_2}{R_2+r}b - \underline{e}\right)}{b - \left(\frac{R_2}{R_2+r}b - \underline{e}\right)} = r\frac{1}{\frac{1}{c/k+s/q} + \frac{\underline{e}}{b}} - r. \qquad (13)
\end{aligned}$$

(a) If $\phi_0 + \gamma_0 < \hat{R}_0^A$, then A prefers to start the boycott immediately, making F willing to self-regulate immediately, which violates $\phi_0 = r\frac{s-q}{q}$.

(b) If $\phi_0 + \gamma_0 = \hat{R}_0^A$, then A is willing to randomize. This equation pins down $\gamma_0$:

$$\gamma_0 = r\frac{ks+cq}{kq+(ks+cq)\,\underline{e}/b} - r - r\frac{s-q}{q} = r\frac{cq-(ks+cq)\,\underline{e}s/bq}{kq+(ks+cq)\,\underline{e}/b}; \qquad (14)$$

this is positive only if

$$\underline{e} \leq \frac{bcq^2}{(ks+cq)\,s}.$$

Furthermore, note that, in this situation, $\gamma_0 < rc/k = \gamma_2$ if and only if $\underline{e} > 0$. So, F is then willing to randomize only if A starts the boycott with a positive probability. F is indifferent if:

$$\begin{aligned}
\frac{c}{r} &= \frac{\alpha}{\alpha+\gamma_0+r}\left(\frac{c+\underline{h}}{r}\right) + \frac{\gamma_0}{\alpha+\gamma_0+r}\left(\frac{c+k}{r}\right) \Rightarrow \\
\alpha\underline{h} &= cr - \gamma_0 k = r\frac{(c+ks/q)^2\,\underline{e}/b}{k+(c+ks/q)\,\underline{e}/b}. \qquad (15)
\end{aligned}$$

Thus, we have an equilibrium here if

$$0 \leq \underline{e} \leq \frac{bcq^2}{(ks+cq)\,s}.$$

Otherwise, this case does not yield an equilibrium. Indeed, if $\underline{e} < 0$, then from (14), $\gamma_0 > rc/k = \gamma_2$, so F prefers immediate self-regulation, which violates $\phi_0 = r\frac{s-q}{q}$. In the case $\underline{e} > \frac{bcq^2}{(ks+cq)s}$, we have $\phi_0 + \gamma_0 > \hat{R}_0^A$ for all $\gamma_0 \geq 0$, again a contradiction

(c) If $\phi_0 + \gamma_0 > \hat{R}_0^A$, then A never starts a boycott, and then F is willing to randomize if $\gamma_0 = \gamma_2$. This is an equilibrium if the resulting $\phi_0 + \gamma_0$ is indeed larger than $\hat{R}_0^A$, which requires:

$$r\frac{s-q}{q} + r\frac{c}{k} > r\frac{ks+cq}{kq+(ks+cq)\,\underline{e}/b} - r \Rightarrow \underline{e} > 0.$$

Thus, if $\underline{e} > 0$, there is an equilibrium with

$$\alpha = 0,\, \phi_0 = \phi_2,\, \gamma_0 = \gamma_2.$$

(3) Suppose $\phi_0 > r\frac{s-q}{q}$. In this case, R prefers not to intervene, so $\gamma_0 = 0$. Immediate self-regulation ($\phi_0 = \infty$) is possible only if A prefers to boycott even in that case, which requires $\underline{e} < -\frac{b}{c/k+s/q}$; in this case, we have a (unique) equilibrium $\gamma_0 = 0$, $\alpha = \infty$, $\phi_0 = \infty$. If $\underline{e} > -\frac{b}{c/k+s/q}$, then $\phi_0$ cannot be too high, for then A would prefer to wait and then, in the absence of any threat, F would prefer to wait as well. On the other hand, $\phi_0$ cannot be too low either, since then A would prefer to start a boycott, thus making F willing to self-regulate immediately. We thus must have an equilibrium where both A and F randomize. A is indifferent if

$$\phi_0 = r\frac{ks+cq}{kq+(ks+cq)\,\underline{e}/b} - r,$$

which, as noted, is larger than $r\frac{s-q}{q}$ iff

$$r\frac{ks+cq}{kq+(ks+cq)\,\underline{e}/b} - r > r\frac{s-q}{q} \Rightarrow \frac{ks+cq}{kq+(ks+cq)\,\underline{e}/b} > \frac{s}{q},$$

which is satisfied if $\underline{e} \leq 0$, or if $\underline{e} > 0$ is so small that

$$cq^2 > s\,(ks+cq)\,\underline{e}/b \Rightarrow \underline{e} < \frac{bcq^2}{s\,(ks+cq)}.$$

In its turn, F is willing to randomize, whenever $\alpha$ satisfies

$$\frac{c}{r} = \frac{\alpha}{\alpha+r}\left(\frac{c+\underline{h}}{r}\right) \Rightarrow \alpha = r\frac{c}{\underline{h}}.$$

Thus, if $\underline{e} < \frac{bcq^2}{(ks+cq)s}$, we have an equilibrium $\gamma_0 = 0$, $\phi_0 = r\frac{ks+cq}{kq+(ks+cq)\underline{e}/b} - r$ and $\alpha = r\frac{c}{\underline{h}}$. The lemma summarizes all three cases. ■

## 7.4 Before the boycott: Anticipating equilibrium (II)

Suppose the players anticipate that, if a boycott starts, then F gives in immediately (equilibrium II in Lemma 9). If A benefits from starting a boycott, A will immediately start one and thus F will immediately self-regulate; in turn, R will remain passive). But if starting the boycott is costly to A, then A hopes that R regulates so that A does not need to pay the set-up cost, while R hopes that A initiates a boycott, so that F self-regulates. The game between A and R is similar to a war of attrition where, in addition to an interior equilibrium in mixed strategies, we can have equilibria where one of A and R stays passive.

If A and R are both active players, but when both these players gain so little from the boycott ($s/q$ and $b/\underline{e}$ are small) that they are willing to act only if the other player is acting with a low probability, then A and R are taking an action with such a small probability that F remains passive ($\phi_0 = 0$). This is thus a fourth type of equilibrium, although it cannot exist for the same parameters which permits the interior equilibrium. Whether F is active when both A and R are active depends on the sign of

$$\kappa \equiv \frac{s-q}{q} - \frac{c}{\underline{h}}.$$

Note that $\kappa$ is positive if $s/q$ is sufficiently large but negative if $c$ is sufficiently large.

**Lemma 11** ***(Before the boycott II)*** *If the equilibrium* $(\phi_1 = \infty, \rho = 0, \gamma_1 = 0)$ *is anticipated during the boycott, the equilibria in phase 0 are:*[25]

*(i) If* $\underline{e} < 0$*, the unique equilibrium is* $\phi_0 = \infty, \alpha = \infty, \gamma_0 = 0$.

*(ii) If* $b/\underline{e} \in (0, 1 + \max\{0, \kappa\})$*, the unique equilibrium is "without A":* $\phi_0 = \phi_2, \alpha = 0, \gamma_0 = \gamma_2$.

*(iii) If* $b/\underline{e} > \frac{s}{q} + \frac{c}{k}$*, the unique equilibrium is "without R":* $\phi_0 = r\frac{b-\underline{e}}{\underline{e}}, \alpha = r\frac{c}{\underline{h}}, \gamma_0 = 0$.

*(iv) If* $b/\underline{e} \in [1, 1 + \max\{0, -\kappa\}\, \underline{h}/k]$*, there are three equilibria: one is as in (ii); another is as in case (iii), and the third one is "without F":*

$$\begin{aligned} \phi_0 &= 0 \\ \alpha &= r\frac{s-q}{q} \\ \gamma_0 &= r\frac{b-\underline{e}}{\underline{e}} \end{aligned}$$

*(v) If* $b/\underline{e} \in \left[1 + \max\{\kappa, -\kappa \underline{h}/k\}, \frac{s}{q} + \frac{c}{k}\right]$*, there are three equilibria: One is as in case (ii); another is as in case (iii), and the third one is interior:*

$$\begin{aligned} \phi_0 &= r\frac{bk/\underline{e} - c + \underline{h}\frac{s-q}{q} - k}{\underline{h}+k} \\ \alpha &= rk\frac{s/q + c/k - b/\underline{e}}{\underline{h}+k} \\ \gamma_0 &= r\frac{c - \underline{h}\left[\frac{s}{q} - \frac{b}{\underline{e}}\right]}{\underline{h}+k} \end{aligned}$$

[25]In the borderline case $\underline{e} = 0$, there are a continuum of equilibria of the form $\alpha \geq r\frac{c}{\underline{h}}$, $\phi_0 = \infty$, and $\gamma_0 = 0$.

*Proof.* Obviously, if $\underline{e} < 0$, A prefers to boycott immediately, so the unique equilibrium is $\phi_0 = \alpha = \infty$, $\gamma_0 = 0$. Hereinafter, assume $\underline{e} > 0$. There are five possibilities regarding $\phi_0$.

(1) Suppose $\phi_0 = r\frac{s-q}{q}$. A cannot strictly prefer a boycott, since then F would self-regulate immediately. Furthermore, if we have $\alpha > 0$, R strictly prefers to wait, but then A is indifferent only $\phi_0 = r\frac{b-\underline{e}}{\underline{e}}$, which equals $r\frac{s-q}{q}$ only when $\underline{e} = bq/s$ (in that exact case, there is an equilibrium $\phi_0 = r\frac{s-q}{q}$, $\gamma_0 = 0, \alpha = r\frac{c}{\underline{h}}$). If $\underline{e} \neq bq/s$, we must have $\alpha = 0$, meaning that $\gamma_0 = \gamma_2 = rc/k$. A is then actually willing to wait if and only if he would get weakly less from initiating the boycott:

$$b - \underline{e} \leq \frac{R_0}{R_0 + r} b = b\left(\frac{s/q - 1 + c/k}{s/q + c/k}\right) \Leftrightarrow \underline{e} \geq \frac{b}{s/q + c/k}.$$

So, there is an equilibrium "without A" (similar to phase 2) if $\underline{e} \geq \frac{b}{s/q+c/k} \Leftrightarrow \frac{b}{\underline{e}} \leq \frac{s}{q} + \frac{c}{k}$ and $\underline{e} > 0$. If exactly $\underline{e} = bq/s$, we also have an equilibrium $\phi_0 = r\frac{s-q}{q}$, $\gamma_0 = 0, \alpha = r\frac{c}{\underline{h}}$.

(2) If $\phi_0 \in \left(r\frac{s-q}{q}, \infty\right)$, then $\gamma_0 = 0$. For F to be indifferent, it must be that

$$\alpha = r\frac{c}{\underline{h}}.$$

A must therefore be indifferent, so,

$$\phi_0 = r\frac{b - \underline{e}}{\underline{e}},$$

which is indeed larger than $r\frac{s-q}{q}$ if

$$\frac{b}{\underline{e}} > \frac{s}{q},$$

So, if $\frac{b}{\underline{e}} > \frac{s}{q}$, there is an equilibrium $\gamma_0 = 0$, $\alpha = r\frac{c}{\underline{h}}$, $\phi_0 = r\frac{b-\underline{e}}{\underline{e}}$.

(3) If $\phi_0 = 0$, we must have

$$cr \geq \gamma_0 k + \alpha \underline{h}. \tag{16}$$

A and R must both be indifferent, which implies

$$\alpha = r\frac{s-q}{q}, \tag{17}$$

$$\gamma_0 = r\frac{b - \underline{e}}{\underline{e}}, \tag{18}$$

and for F to be willing to choose $\phi_0 = 0$, we must have

$$\begin{aligned}
\frac{c}{r} &\geq \frac{\alpha}{\alpha + \gamma_0 + r}\left(\frac{c + \underline{h}}{r}\right) + \frac{\gamma_0}{\alpha + \gamma_0 + r}\left(\frac{c + k}{r}\right) \Leftrightarrow \\
c &\geq \alpha\underline{h}/r + \gamma_0 k/r = \frac{s-q}{q}\underline{h} + \frac{b - \underline{e}}{\underline{e}} k \Leftrightarrow \\
b/\underline{e} &\leq \frac{c}{k} - \frac{s-q}{qk}\underline{h} + 1.
\end{aligned}$$

If this holds and $\gamma_0 = r\frac{b-\underline{e}}{\underline{e}} \geq 0$, i.e., if $b/\underline{e} \in \left[1, \frac{c}{k} - \frac{s-q}{qk}\underline{h} + 1\right]$, then we have an equilibrium with $\phi_0 = 0$ and (17)-(18).

(4) If $\phi_0 \in \left(0, r\frac{s-q}{q}\right)$, F must be indifferent, so:

$$cr = \gamma_0 k + \alpha \underline{h}. \tag{19}$$

R regulates if:

$$\frac{s-q}{r} \geq \frac{\alpha+\phi_0}{\alpha+\phi_0+r}\frac{s}{r} \Rightarrow \alpha+\phi_0 \leq r\frac{s-q}{q}.$$

A starts the boycott if

$$b-\underline{e} \geq \frac{R_0}{R_0+r}b \Rightarrow b \geq (R_0+r)\,\underline{e}/r \Rightarrow R_0 \leq r\frac{b-\underline{e}}{\underline{e}}.$$

Consider the following subcases:

(a) If $\alpha+\phi_0 < r\frac{s-q}{q}$, R regulates immediately, violating (19).

(b) If $\alpha+\phi_0 > r\frac{s-q}{q}$, R prefers to wait, so $\gamma_0 = 0$. From (19), we get

$$\alpha = r\frac{c}{\underline{h}}.$$

For A to be willing to randomize, we must have

$$\phi_0 = r\frac{b-\underline{e}}{\underline{e}},$$

which is indeed in $\left(0, r\frac{s-q}{q}\right)$ if

$$0 < r\frac{b-\underline{e}}{\underline{e}} < r\frac{s-q}{q} \Rightarrow b/\underline{e} \in (1, s/q)\,.$$

Furthermore, the condition for this case (b), $\alpha+\phi_0 > r\frac{s-q}{q}$, is satisfied if

$$r\frac{b-\underline{e}}{\underline{e}} + r\frac{c}{\underline{h}} > r\frac{s-q}{q} \Rightarrow \frac{b}{\underline{e}} > \frac{s}{q} - \frac{c}{\underline{h}}.$$

Thus, if $\max\left\{\frac{s}{q} - \frac{c}{\underline{h}}, 1\right\} < b/\underline{e} < s/q$, then we have an equilibrium $\gamma_0 = 0$, $\phi_0 = r\frac{b-\underline{e}}{\underline{e}}$, $\alpha = r\frac{c}{\underline{h}}$.

(c) If $\alpha+\phi_0 = r\frac{s-q}{q} \Rightarrow \alpha \in (0,\infty)$, A must be willing to randomize, so we must have $R_0 = r\frac{b-\underline{e}}{\underline{e}}$. We then have three indifference conditions:

$$\begin{aligned}\alpha+\phi_0 &= r\frac{s-q}{q},\\ \phi_0+\gamma_0 &= r\frac{b-\underline{e}}{\underline{e}},\\ cr &= \gamma_0 k + \alpha \underline{h}.\end{aligned}$$

Solving for the rates explicitly:

$$\begin{aligned}
\alpha - \gamma_0 &= r\frac{s-q}{q} - r\frac{b-\underline{e}}{\underline{e}} = r\frac{s}{q} - r\frac{b}{\underline{e}}, \\
cr &= \gamma_0 k + \underline{h}\left[r\frac{s}{q} - r\frac{b}{\underline{e}} + \gamma_0\right] \\
\gamma_0 &= r\frac{c - \underline{h}\left[\frac{s}{q} - \frac{b}{\underline{e}}\right]}{\underline{h}+k} > 0 \text{ if} \\
c &> \underline{h}\left[\frac{s}{q} - \frac{b}{\underline{e}}\right] \text{ or } \frac{b}{\underline{e}} > \frac{s}{q} - \frac{c}{\underline{h}}.
\end{aligned}$$

Furthermore,

$$\begin{aligned}
\phi_0 &= r\frac{b-\underline{e}}{\underline{e}} - \frac{cr - r\underline{h}\left[\frac{s}{q} - \frac{b}{\underline{e}}\right]}{\underline{h}+k} = r\frac{bk/\underline{e} - c + \underline{h}\frac{s-q}{q} - k}{\underline{h}+k} \\
&> 0 \text{ if } b/\underline{e} > 1 + c/k - \underline{h}\frac{s-q}{qk}.
\end{aligned}$$

And,

$$\begin{aligned}
\alpha &= r\frac{s-q}{q} - r\frac{b-\underline{e}}{\underline{e}} + \frac{cr - r\underline{h}\left[\frac{s}{q} - \frac{b}{\underline{e}}\right]}{\underline{h}+k} \\
&= rk\frac{s/q + c/k - b/\underline{e}}{\underline{h}+k} > 0 \text{ if } \frac{b}{\underline{e}} < s/q + c/k.
\end{aligned}$$

So, this interior equilibrium exists if

$$\begin{aligned}
\max\left\{\frac{s}{q} - \frac{c}{\underline{h}}, 1 + c/k - \underline{h}\frac{s-q}{qk}\right\} &< \frac{b}{\underline{e}} < s/q + c/k \Rightarrow \\
1 + \max\left\{\frac{s-q}{q} - \frac{c}{\underline{h}}, -\frac{\underline{h}}{k}\left[\frac{s-q}{q} - \frac{c}{\underline{h}}\right]\right\} &< \frac{b}{\underline{e}} < s/q + c/k.
\end{aligned}$$

(5) If $\phi_0 = \infty$, then A does not want to start the boycott ($\underline{e} > 0$), and R does not want to regulate either. If so, F would prefer to wait, which is a contradiction.

The lemma follows from combining the above cases. ■

## 7.5 Before the campaign: Anticipating equilibrium (III)

Suppose the players anticipate that, if a boycott starts, then R stays passive while F and A play the mixed strategy equilibrium III in Lemma 9. In that case, R may be strictly better off during a boycott compared to his utility from regulation. In phase 0, the the start of the boycott may therefore be good news to R, even if she does not care about the boycott per se. This can happen only if F is very likely to self-regulate during a boycott.

In this situation, there may be multiple equilibria in phase 0, before the boycott has started. In addition to an interior solution, there may be equilibria where only two players are active.

Define

$$\kappa' \equiv \frac{s-q}{q} - \frac{c}{\underline{h}}\left(\frac{s/q+(b/e+1)\,c/k}{(s/q+c/k)\,(b/e+1)+\frac{h-c}{\delta\underline{h}}\frac{b}{e}}\right) > \kappa.$$

**Lemma 12** ***(Before the boycott III)*** *If the equilibrium III ("without R") is anticipated during the boycott, then equilibria at phase 0 are the following:*

*(i) If* $\underline{e} \leq -\frac{b}{c/k+s/q}$*, the unique equilibrium is* $\gamma_0 = 0$*,* $\alpha = \infty$*,* $\phi_0 = \infty$*.*

*(ii) If* $\underline{e} \in \left(-\frac{b}{c/k+s/q}, 0\right)$*, the unique equilibrium is "without R":*

$$\gamma_0 = 0,\ \alpha = r\frac{c}{\underline{h}} \text{ and } \phi_0 = r\frac{c/k+s/q}{1+(c/k+s/q)\,\underline{e}/b} - r.$$

*(iii) If* $\kappa' > 0$*, then there exists*

$$\underline{e}_R \equiv \frac{b}{1+\kappa'} - \frac{b}{c/k+s/q} < \underline{e}_A \equiv b\left(1-\frac{1}{c/k+s/q}\right)$$

*such that:*

*(iii-a) If* $\underline{e} > \underline{e}_R$*, the unique equilibrium is "without A" as in phase 2.*

*(iii-b) If* $\underline{e} \in [0, \underline{e}_R]$*, there are three equilibria: One as in (ii), another as in (iii-a), and the third is interior:*

$$\begin{aligned}
\gamma_0 &= r\frac{c}{k} - r\frac{(c/k+s/q)^2}{1+(s/q-1-\kappa')\,k/c}\left(\frac{\underline{e}}{b+\underline{e}\,(c/k+s/q)}\right) \in (0,\infty),\\
\alpha &= \frac{k}{\underline{h}}r\left[\frac{(c/k+s/q)^2}{1+(s/q-1-\kappa')\,k/c}\left(\frac{\underline{e}}{b+\underline{e}\,(c/k+s/q)}\right)\right] \in (0,\infty),\\
\phi_0 &= r\left(\frac{c/k+s/q}{1+(c/k+s/q)\,\underline{e}/b}\right)\left(1+\frac{(c/k+s/q)\,\underline{e}/b}{1+(s/q-1-\kappa')\,k/c}\right) - r\frac{c+k}{k} \in (0,\infty).
\end{aligned}$$

*(iv) If* $\kappa' < 0$*, then there exists*

$$\underline{e}_F \equiv \frac{b\,(s/q-1-\kappa')}{s/q-1-\kappa'\,(1+c/k)} - \frac{b}{c/k+s/q} < \underline{e}_A$$

*such that:*

*(iv-a) If* $\underline{e} > \underline{e}_A$*, the unique equilibrium is "without A" as in phase 2.*

*(iv-b) If* $\underline{e} \in [0, \underline{e}_F)$*, there are three equilibria: one is as in (ii), one as in (iii-a), and the third is interior as in (iii-b).*

*(iv-c) If $\underline{e} \in [\underline{e}_F, \underline{e}_A]$, there are three equilibria: one is as in (ii), one is as in (iii-a), and the third is "without F":*

$$\begin{aligned}\phi_0 &= 0,\\ \alpha &= r\frac{h}{c}\left(\frac{s-q}{q}\right)\left(\frac{s-q}{q}-\kappa'\right),\\ \gamma_0 &= r\frac{c/k+s/q}{1+(c/k+s/q)\,\underline{e}/b}-r.\end{aligned}$$

*Proof of Lemma 12:*

Suppose that equilibrium III, $(\phi_1, \gamma_1, \rho) = \left(\frac{(ks+cq)(b+e)}{bkq}r - r, 0, \frac{r(h-c)}{\delta\underline{h}}\right)$, is anticipated. Then in phase 1, R receives the following expected continuation payoff:

$$\begin{aligned}u^R_{1,\underline{h}} &= \frac{\phi_1}{\phi_1+\rho+r}\frac{s}{r}+\frac{\rho}{\phi_1+\rho+r}\frac{s-q}{r}=\frac{s}{r}-\frac{s+q\rho/r}{\phi_1+\rho+r}\\ &= \frac{s}{r}-\frac{s}{r}\frac{1+(h-c)\,q/s\delta\underline{h}}{(s/q+c/k)\,(1+e/b)+(h-c)\,/\delta\underline{h}}.\end{aligned}$$

Given this, R is willing to stay passive by not regulating in phase 0 only if

$$\begin{aligned}\frac{s-q}{r} &\leq \frac{\phi_0}{\phi_0+\alpha+r}\frac{s}{r}+\frac{\alpha}{\phi_0+\alpha+r}u^R_{1,\underline{h}}\Rightarrow\\ \frac{s-q}{r} &\leq \frac{\phi_0}{\phi_0+\alpha+r}\frac{s}{r}+\frac{\alpha}{\phi_0+\alpha+r}\left(\frac{s}{r}-\frac{s}{r}\frac{1+\frac{h-c}{\delta\underline{h}}\frac{q}{s}}{(s/q+c/k)\,(1+e/b)+\frac{h-c}{\delta\underline{h}}}\right)\Rightarrow\\ \phi_0 &\geq \left(\frac{s-q}{q}\right)r-\alpha\left(\frac{se/bq-(1+e/b)\,c/k}{(s/q+c/k)\,(1+e/b)+\frac{h-c}{\delta\underline{h}}}\right)\Rightarrow\\ \left(\frac{s-q}{q}\right)r &\leq \phi_0+\alpha\left(\frac{s/q+(b/e+1)\,c/k}{(s/q+c/k)\,(b/e+1)+\frac{h-c}{\delta\underline{h}}\frac{b}{e}}\right).\end{aligned} \tag{20}$$

From (15), it follows that F is willing to remain passive by not self-regulating if

$$c \geq \alpha\frac{h}{r}+\gamma_0\frac{k}{r}, \tag{21}$$

while from (13), A is willing to remain passive if

$$\phi_0+\gamma_0 \geq \hat{R}^A_0 = r\frac{1}{\frac{1}{c/k+s/q}+\frac{e}{b}}-r, \tag{22}$$

whenever $\frac{e}{b} \in \left(-\frac{1}{c/k+s/q}, 1-\frac{1}{c/k+s/q}\right)$. If $\underline{e} > b\left(1-\frac{1}{c/k+s/q}\right) \equiv \underline{e}_A$, the dominant strategy of A is to never start a boycott (in that case, the unique equilibrium is as in phase 2 with $\alpha = 0$). If $\frac{e}{b} < -\frac{1}{c/k+s/q}$, his dominant strategy is to start a boycott immediately, regardless of $\phi_0+\gamma_0 \leq \infty$ (in that case, the unique equilibrium is $\alpha = \phi_0 = \infty$, $\gamma_0 = 0$). From now on, consider the case $\frac{e}{b} \in \left(-\frac{1}{c/k+s/q}, 1-\frac{1}{c/k+s/q}\right)$. Note that none of the rates may be infinite in this interval (for example, if $\alpha = \infty$, F prefers immediate self-regulation but then $\alpha = 0$ would be optimal for A). Thus, if $\gamma_0 > 0$, (20) binds; if $\phi_0 > 0$, (21) binds; and if $\alpha > 0$, (22) binds. This also implies that at most one rate may equal 0.

Consider the following possibilities:

(i) Equilibria "without A", where $\alpha = 0$. Then, $\phi_0$ and $\gamma_0$ must be as in phase two. From (22), A is willing to remain passive if and only if $\underline{e} \geq 0$.

(ii) Equilibria "without R", where $\gamma_0 = 0$. Then the best response functions of A and R cross only once, yielding:

$$\alpha = \frac{c}{\underline{h}} \text{ and } \phi_0 = r\frac{1}{\frac{1}{c/k+s/q}+\frac{e}{b}} - r.$$

From (20), we know that R is willing to remain passive only if (substituting for $\alpha$ and $\phi_0$)

$$\begin{aligned}
\left(\frac{s-q}{q}\right)r &\leq r\frac{1}{\frac{1}{c/k+s/q}+\frac{e}{b}} - r + r\frac{c}{\underline{h}}\left(\frac{s/q+(b/e+1)\,c/k}{(s/q+c/k)\,(b/e+1)+\frac{h-c}{\delta\underline{h}}\frac{b}{e}}\right) \Rightarrow \\
\frac{1}{\frac{1}{c/k+s/q}+\frac{e}{b}} - 1 &\geq \frac{s-q}{q} - \frac{c}{\underline{h}}\left(\frac{s/q+(b/e+1)\,c/k}{(s/q+c/k)\,(b/e+1)+\frac{h-c}{\delta\underline{h}}\frac{b}{e}}\right) \equiv \kappa' \Rightarrow \qquad (23) \\
\underline{e} &\leq \underline{e}_R \equiv \frac{b}{1+\kappa'} - \frac{b}{c/k+s/q}.
\end{aligned}$$

So, this is an equilibrium if $\underline{e} \in \left(-\frac{b}{c/k+s/q}, \min\{\underline{e}_A, \underline{e}_R\}\right)$. Note that $\underline{e}_R < \underline{e}_A$ if and only if $\kappa' > 0$.

(iii) Equilibria "without F", where $\phi_0 = 0$. In this case, equilibrium must have interior rates for A and R (otherwise, one of them would act immediately and $\phi_0 = \infty$ would be the best response). This implies, given (20) and (22):

$$\begin{aligned}
\alpha &= r\left(\frac{s-q}{q}\right)\frac{(s/q+c/k)\,(b/e+1)+\frac{h-c}{\delta\underline{h}}\frac{b}{e}}{s/q+(b/e+1)\,c/k} \\
&= r\frac{\underline{h}}{c}\left(\frac{s-q}{q}\right)\left(\frac{s-q}{q}-\kappa'\right), \\
\gamma_0 &= r\frac{1}{\frac{1}{c/k+s/q}+\frac{e}{b}} - r.
\end{aligned}$$

From (21), we know that for F to be willing to remain passive, we must have

$$\begin{aligned}
c &\geq \alpha\frac{\underline{h}}{r}+\gamma_0\frac{k}{r} \\
&= \underline{h}\left(\frac{s-q}{q}\right)\frac{(s/q+c/k)(b/e+1)+\frac{h-c}{\delta\underline{h}}\frac{b}{e}}{s/q+(b/e+1)c/k}+\frac{k}{r}\left(r\frac{1}{\frac{1}{c/k+s/q}+\frac{\underline{e}}{b}}-r\right)\Rightarrow \\
&\frac{1}{\frac{1}{c/k+s/q}+\frac{\underline{e}}{b}}-1\leq\frac{c}{k}-\frac{\underline{h}}{k}\left(\frac{s-q}{q}\right)\frac{(s/q+c/k)(b/e+1)+\frac{h-c}{\delta\underline{h}}\frac{b}{e}}{s/q+(b/e+1)c/k} \qquad (24)\\
&= \frac{c}{k}\left(\frac{s-q}{q}\right)\left[(-\kappa')/\left[\frac{c}{\underline{h}}\left(\frac{s/q+(b/e+1)c/k}{(s/q+c/k)(b/e+1)+\frac{h-c}{\delta\underline{h}}\frac{b}{e}}\right)\left(\frac{s-q}{q}\right)\right]\right]\Rightarrow \qquad (25)\\
\underline{e} &\geq \underline{e}_F\equiv\frac{b}{\frac{-\kappa' c}{k(s/q-1-\kappa')}+1}-\frac{b}{c/k+s/q}=\frac{b\,(s/q-1-\kappa')}{s/q-1-\kappa'(1+c/k)}-\frac{b}{c/k+s/q}.
\end{aligned}$$

We know that $\underline{e}_F>0$ (if $\underline{e}\downarrow 0$, the left-hand side of (24) approaches $c/k+(s-q)/q$, while the right-hand side is less than $c/k$, violating the equation). Furthermore, $\underline{e}_F<\underline{e}_A$ if and only if $\kappa'<0$ (as can be seen from (25)). So, the equilibrium "without F" exists if and only if $\underline{e}\in(\underline{e}_F,\underline{e}_A)$, requiring $\kappa'<0$.

(iv) Interior equilibria: If $\underline{e}\in(\underline{e}_F,\underline{e}_A)$, we have an equilibrium "without F". If, in this situation, $\phi_0$ were to be positive, then $\alpha$ would have to be lower in order to keep R willing to regulate, and $\gamma_0$ would have to decrease to make A willing to postpone the boycott. Then, however, F would stricly prefer to remain passive; this contradicts the assumption that $\phi_0$ is positive. Consequently, there is no interior equilibrium if $\underline{e}\geq\min\{\underline{e}_F,\underline{e}_A\}$. If $\underline{e}=\underline{e}_F<\underline{e}_A$, we know that all the equations hold with equality for $\phi_0=0$:

$$\begin{aligned}
\left(\frac{s-q}{q}\right)r &= \phi_0+\alpha\left(\frac{s/q+(b/e+1)c/k}{(s/q+c/k)(b/e+1)+\frac{h-c}{\delta\underline{h}}\frac{b}{e}}\right), \\
c &= \alpha\frac{\underline{h}}{r}+\gamma_0\frac{k}{r}, \\
\phi_0+\gamma_0 &= r\frac{1}{\frac{1}{c/k+s/q}+\frac{\underline{e}}{b}}-r.
\end{aligned}$$

For $\phi_0$ to become positive, $\alpha$ must decrease, and, in its turn, $\gamma_0$ must increase, but this is possible only if $\underline{e}$ becomes lower than $\underline{e}_F<\underline{e}_A$, which is thus an upper boundary for the interior equilibrium. Solving the equations gives:

$$\begin{aligned}
\left(\frac{s-q}{q}\right)r &= \phi_0+\alpha\left(\frac{s/q+(b/e+1)c/k}{(s/q+c/k)(b/e+1)+\frac{h-c}{\delta\underline{h}}\frac{b}{e}}\right), \\
c &= \alpha\frac{\underline{h}}{r}+\gamma_0\frac{k}{r}, \\
\gamma_0 &= r\frac{c}{k}-r\frac{(c/k+s/q)^2}{1+(s/q-1-\kappa')k/c}\left(\frac{\underline{e}}{b+\underline{e}(c/k+s/q)}\right).
\end{aligned}$$

Similarly,

$$
\begin{aligned}
\alpha &= r\frac{c-\gamma_0\frac{k}{r}}{\underline{h}} = r\frac{c}{\underline{h}} - \frac{k}{\underline{h}}r\left[\frac{c}{k} - \frac{(c/k+s/q)^2}{1+(s/q-1-\kappa')\,k/c}\left(\frac{\underline{e}}{b+\underline{e}\,(c/k+s/q)}\right)\right] \\
&= \frac{k}{\underline{h}}r\left[\frac{(c/k+s/q)^2}{1+(s/q-1-\kappa')\,k/c}\left(\frac{\underline{e}}{b+\underline{e}\,(c/k+s/q)}\right)\right].
\end{aligned}
$$

Finally,

$$
\begin{aligned}
\phi_0 &= r\frac{1}{\frac{1}{c/k+s/q}+\frac{\underline{e}}{b}} - r - \gamma_0 \\
&= r\left(\frac{c/k+s/q}{1+(c/k+s/q)\,\underline{e}/b}\right)\left(1+\frac{(c/k+s/q)\,\underline{e}/b}{1+(s/q-1-\kappa')\,k/c}\right) - r\frac{c+k}{k}.
\end{aligned}
$$

Thus, if $\underline{e} \in (0, \min\{\underline{e}_F, \underline{e}_A\})$, all these rates are positive, confirming an interior equilibrium. The lemma summarizes the findings. ■