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THE TERM STRUCTURE OF CURRENCY CARRY TRADE RISK PREMIA

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ABSTRACT

Fixing the investment horizon, the returns to currency carry trades decrease as the maturity of the foreign bonds increases. The local currency term premia, which increase with the maturity, offset the currency risk premia. The time-series predictability of foreign bond returns in dollars similarly declines with the bonds' maturities. Leading no-arbitrage models in international finance cannot match the downward term structure of currency carry trade risk premia. We derive a simple preference-free condition that no-arbitrage models need to satisfy to match the carry trade risk premia on long term bonds.

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Carry trades correspond to simple investment strategies that are funded by borrowing in low interest rate currencies and invest in high interest rate currencies. What are their expected returns? According to the uncovered interest rate parity (U.I.P.) condition, which assumes that investors are risk-neutral, expected carry trade returns should be zero. Yet, empirically, borrowing in low interest rate currencies and investing in Treasury bills of high interest rate currencies delivers large excess returns. This is the U.I.P puzzle, and it gave rise to a large literature that studies the role of systematic risk in exchange rates and expectational errors. Our paper revisits the empirical evidence on carry trades and deepens the puzzle.

Our paper explores the properties of the same carry trade investment strategy implemented with long-term bonds, and compares it to the standard strategy that uses Treasury bills. We focus on the same set of G10 countries and consider the strongest predictors of bond and currency returns: the level and slope of the yield curve. The first strategy we consider goes long the bonds of high interest rate currencies and short the bonds of low interest rate currencies, whereas the second strategy goes long the bonds of flat yield curve currencies and short the bonds of steep yield curve currencies. Most importantly, the investment horizon is one month, as in the classic tests of the U.I.P. condition, not ten years, as in the tests of the U.I.P condition over long horizons. We find that, as the maturity of the bonds increases, the average excess return declines to zero. In other words, whereas the carry trade implemented with Treasury bills is profitable, the carry trade implemented with long-term bonds is not. Similar results emerge in individual country time-series predictability tests: as the maturity of the bonds increases, the predictability of the cross-country differences in dollar bond returns disappears.

The downward-sloping term structure of carry trade risk premia that we uncover represents a challenge for the leading models in international finance. To illustrate this point, we simulate the multi-country model of Lustig, Roussanov, and Verdelhan (2011). This reduced-form model, derived from the term structure literature, offers a flexible and transparent account of the U.I.P puzzle and thus a natural candidate to match the facts. Yet, we show that it implies a counterfactual flat term structure of currency carry trade risk premia.

Our paper explains why other recent no-arbitrage models that replicate the U.I.P. puzzle fail to replicate the carry trade risk premia at the long end of the yield curve. To do so, we derive a preference-free condition that all dynamic no-arbitrage models need to satisfy. We rely on Alvarez and Jermann (2005)'s decomposition of the stand-investor's marginal utility or pricing kernel into a permanent and a transitory component, where

the log of the transitory component is proportional to long term interest rates. To be precise, we show that the difference between domestic and foreign long-term bond risk premia, expressed in domestic currency, is determined by the difference in the volatilities of the permanent components of the stochastic discount factors (SDFs, e.g., the growth rate of the stand-in investor's marginal utility). Therefore, to obtain zero currency carry trade risk premia at the long end of the yield curve, the volatilities of the permanent components need to be equalized across currencies. Our preference-free result is the bond equivalent of the usual expression for the carry trade risk premium in no-arbitrage models: when borrowing and investing in Treasury bills, the carry trade risk premium is equal to the differences in volatilities of the SDFs (see Bekaert, 1996, Bansal, 1997, and Backus, Foresi, and Telmer (2001)). This condition is the basis of most explanations of the U.I.P. puzzles. Our novel condition illustrates the shortcomings of the existing models.

Armed with our preference-free condition, we revisit a large class of dynamic asset pricing models that have been used to study U.I.P. violations, ranging from the reduced-form term structure model of Vasicek (1977) and Cox, Ingersoll, and Ross (1985) to their more recent multi-factor versions, to the Campbell and Cochrane (1999) external habit model, the Bansal and Yaron (2004) long run risks model, the disaster risk model, and the reduced-form model of Lustig, Roussanov, and Verdelhan (2011). We focus on models of the real SDF, given that there is no evidence that inflation risk can account for U.I.P. deviations (if anything, U.I.P. works better in high inflation environments, see Bansal and Dahlquist, 2000) or for cross-country variation in local currency term premia. None of the models we consider can replicate our empirical findings in their standard calibrations. But, when feasible, we show how to modify and calibrate these models to match the downward term structure of carry trade risk premia.

Our results are related to, but different from, the long-run U.I.P condition. The long-run U.I.P condition compares foreign and domestic long-term interest rates to long-term changes in exchange rates. Meredith and Chinn (2005) find that long-run U.I.P is a potentially valid description of the data. However, empirical tests lack power in finite samples: intuitively, there are few non-overlapping observations of 10-year windows available so far. From no-arbitrage conditions, we show that long-run U.I.P. always holds for temporary shocks and thus long-run U.I.P deviations have to come from permanent shocks to exchange rates. For long-run U.I.P to hold at all times, exchange rates must not have any permanent shocks and thus be stationary in levels. Yet, exchange rate stationarity in levels is sufficient but not necessary to satisfy our preference-free condition on the

volatilities of the permanent SDF components. As a result, the carry trade risk premia on long-term bonds could be zero without implying that long-run U.I.P. always holds. Under some additional regularity conditions, in that case, long-run U.I.P. would hold on average in no-arbitrage models.

Recent work has documented a downward sloping term structure of risk premia in equity markets (van Binsbergen, Brandt, and Kojen, 2012), real estate markets (Giglio, Maggiori, and Stroebel, 2015), and volatility markets (Dew-Becker, Giglio, Le, and Rodriguez, 2017). Backus, Boyarchenko, and Chernov (2016) provide a general analysis of the term structure of asset returns. Our work confirms the same pattern in currency markets, and offers a preference-free interpretation. Our theoretical result, although straightforward, has not been derived or used in the literature. On the one hand, at the short end of the maturity curve, currency risk premia are high when there is less overall risk in foreign countries' pricing kernels than at home (Bekaert, 1996, Bansal, 1997, and Backus, Foresi, and Telmer, 2001). On the other hand, at the long end of the maturity curve, local bond term premia compensate investors mostly for the risk associated with transitory innovations to the pricing kernel (Bansal and Lehmann, 1997; Hansen and Scheinkman, 2009; Alvarez and Jermann, 2005; Hansen, 2012; Hansen, Heaton, and Li, 2008; Backus, Chernov, and Zin, 2014; Borovička, Hansen, and Scheinkman, 2016; Backus, Boyarchenko, and Chernov, 2016). In this paper, we combine those two insights to derive general theoretical results under the assumption of complete financial markets. Foreign bond returns allow us to compare the permanent components of the SDFs, which as Alvarez and Jermann (2005) show, are the main drivers of the SDFs. Our preference-free condition does not apply to exchange rate models that allow for market segmentation (see, e.g., Gabaix and Maggiori, 2015, and Bacchetta and van Wincoop, 2005, for leading examples). It remains to be determined whether these models can fit our facts, so we leave this as an open question for future research.

The rest of the paper is organized as follows. Section 1 focuses on the time-series and cross-section of foreign bond risk premia. Section 2 compares recent no-arbitrage models to the empirical term structure of currency carry trade risk premia. In Section 3, we derive the no-arbitrage, preference-free theoretical restriction imposed on bond returns and SDFs. Section 4 links long-term U.I.P. to exchange rate stationarity. Section 5 concludes. An Online Appendix contains supplementary material and all proofs not presented in the main body of the paper.

1 Foreign Bond Returns in the Time-Series and Cross-Section

We first describe the data and the notation, and then turn to our empirical results on the time-series and cross-sectional properties of foreign government bond returns.

1.1 Data

Our benchmark sample, to which we refer as the G-10 sample, consists of a small homogeneous panel of developed countries with liquid bond markets: Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, and the U.K. The domestic country is the United States. We calculate the returns of both coupon and zero-coupon bonds for these countries.

Our data on total return bond indices were obtained from Global Financial Data. The dataset includes a 10-year government bond total return index, as well as a Treasury bill total return index, in U.S. dollars and in local currency. The data are monthly, starting in 1/1951 and ending in 12/2015. We use the 10-year bond returns as a proxy for long maturity bond returns. While Global Financial Data offers, to the best of our knowledge, the longest time-series of government bond returns available, the series have two key limitations. First, they pertain to coupon bonds, while the theory presented in this paper pertains to zero-coupon bonds. Second, they only offer 10-year bond returns, not the entire term structure of bond returns. To address these issues, we also use zero-coupon bond prices. Our zero-coupon bond dataset covers the same benchmark sample of G10 countries, but from at most 1/1975 to 12/2015, with different countries entering the sample at different dates. The details are in the Data Appendix.

Finally, we collect data on inflation rates and sovereign credit ratings. Inflation rates are calculated using monthly Consumer Price Index (CPI) data from Global Financial Data, whereas sovereign credit ratings are from Standard & Poor's, available over the 7/1989 to 12/2015 period. To construct averages of credit ratings, we assign each rating to a number, with a smaller number corresponding to a higher rating. The details are in the Data Appendix.

1.2 Notation

We now introduce our notation for bond prices, exchange rates, and bond and currency returns. In all cases, foreign variables are denoted as the starred version of their U.S. counterpart.

Bonds $P_t^{(k)}$ denotes the price at date t of a zero-coupon bond of maturity k , while $y_t^{(k)}$ denotes its continuously compounded yield: $\log P_t^{(k)} = -ky_t^{(k)}$. The one-period holding return on the zero-coupon bond is $R_{t+1}^{(k)} = P_{t+1}^{(k-1)}/P_t^{(k)}$. The log excess return on the domestic zero-coupon bond, denoted $rx_{t+1}^{(k)}$, is equal to

$$rx_{t+1}^{(k)} = \log \left[R_{t+1}^{(k)} / R_t^f \right], \quad (1)$$

where the risk-free rate is $R_t^f = R_{t+1}^{(1)} = 1/P_t^{(1)}$. Finally, r_t^f denotes the log risk-free rate: $r_t^f = \log R_t^f = y_t^{(1)}$.

Exchange Rates The nominal spot exchange rate in foreign currency per U.S. dollar is denoted S_t . Thus, an increase in S_t implies an appreciation of the U.S. dollar relative to the foreign currency. The log currency excess return, given by

$$rx_{t+1}^{FX} = \log \left[\frac{S_t}{S_{t+1}} \frac{R_t^{f,*}}{R_t^f} \right] = r_t^{f,*} - r_t^f - \Delta s_{t+1}, \quad (2)$$

is the log excess return of a strategy in which the investor borrows at the domestic risk-free rate, R_t^f , invests at the foreign risk-free rate, $R_t^{f,*}$, and converts the proceeds into U.S. dollars at the end of the period.

Bond Risk Premia The log return on a foreign bond position (expressed in U.S. dollars) in excess of the domestic (i.e., U.S.) risk-free rate is denoted $rx_{t+1}^{(k),\$}$. It can be expressed as the sum of the bond log excess return in local currency plus the log excess return on a long position in foreign currency:

$$rx_{t+1}^{(k),\$} = \log \left[\frac{R_{t+1}^{(k),*}}{R_t^f} \frac{S_t}{S_{t+1}} \right] = \log \left[\frac{R_{t+1}^{(k),*}}{R_t^{f,*}} \frac{R_t^{f,*}}{R_t^f} \frac{S_t}{S_{t+1}} \right] = \log \left[\frac{R_{t+1}^{(k),*}}{R_t^{f,*}} \right] + \log \left[\frac{R_t^{f,*}}{R_t^f} \frac{S_t}{S_{t+1}} \right] = rx_{t+1}^{(k),*} + rx_{t+1}^{FX}. \quad (3)$$

Taking conditional expectations, the total term premium in dollars consists of a foreign bond risk premium, $E_t[rx_{t+1}^{(k),*}]$, plus a currency risk premium, $E[rx_{t+1}^{FX}] = r_t^{f,*} - r_t^f - E_t[\Delta s_{t+1}]$.

We are not the first to study the relation between domestic and foreign bond returns. Prior work, from Campbell and Shiller (1991) to Bekaert and Hodrick (2001) and Bekaert, Wei, and Xing (2007), show that investors earn higher returns on foreign bonds from a country in which the slope of the yield curve is currently higher than average for that country. Ang and Chen (2010) and Berge, Jordà, and Taylor (2011) show that yield curve variables can also be used to forecast currency excess returns. These authors, however, do not examine the returns on foreign bond portfolios expressed in domestic currency. The following papers consider foreign

bond returns in U.S. dollars: Dahlquist and Hasseltoft (2013) study international bond risk premia in an affine asset pricing model and find evidence for local and global risk factors, while Jotikasthira, Le, and Lundblad (2015) study the co-movement of foreign bond yields through the lenses of an affine term structure model. Our paper revisits the empirical evidence on bond returns without committing to a specific term structure model.

1.3 Time-Series Predictability of Foreign Bond Returns

To study the properties of the cross-country differences in expected bond excess returns, we first run individual currency predictability regressions on variables that can be used to predict bond and currency returns. We focus on the level and the slope of the term structure, the two predictors that have been shown to forecast both bond and currency returns.

We regress the 10-year dollar bond log excess return differential ($rx_{t+1}^{(10),\$} - rx_{t+1}^{(10)}$) on the short-term interest rate differential ($r_t^{f,*} - r_t^f$, Panel A of Table 1) and on the yield curve slope differential ($[y_t^{(10,*)} - y_t^{(1,*)}] - [y_t^{(10)} - y_t^{(1)}]$, Panel B of Table 1), focusing on the post-Bretton Woods sample period (1/1975 – 12/2015). Given that the 10-year dollar bond log excess return differential (left columns) can be decomposed into the sum of currency log excess returns (rx_{t+1}^{FX}) and local currency bond log excess return differentials ($rx_{t+1}^{(10),*} - rx_{t+1}^{(10)}$) as noted in Equation (3), we also regress each of those two components on the same predictors (middle and right columns, respectively). By construction, the sum of the slope coefficients in the middle and right columns equals the slope coefficients in the corresponding left column. For each individual country regression, we report Newey-West standard errors with six lags. The panel regressions include country fixed effects and standard errors are calculated using the Driscoll and Kraay (1998) methodology, which corrects for heteroskedasticity, serial correlation, and cross-equation correlation.

When using interest rate differentials as predictors, there is no consistent evidence in support of predictability of 10-year bond return differentials in dollars: indeed, out of nine countries, there is evidence of return predictability only for Japan. The slopes and constants are insignificant for all the other countries in the sample. In a panel regression, the slope coefficient is small and not statistically different from zero. In joint tests of all slope coefficients, they are marginally significant (at the 5% but not the 1% level) because of Japan. In addition, we cannot reject the null that the constants are all zero. This evidence supports the view that there is no difference between expected dollar returns on long bond returns in these countries.

To better understand the lack of predictability, we decompose the dollar excess return differential into currency log excess returns and local currency bond log excess return differentials. As seen in the table, currency log excess returns are strongly forecastable by interest rate differentials (Hansen and Hodrick, 1980; Fama, 1984): as documented in the existing literature, higher than usual interest rate differentials in a given country pair predict higher than usual currency log excess returns. In a joint test of all slope coefficients, we can reject the null that interest rates do not predict currency excess returns. But, while Treasury bill return differentials in U.S. dollars are forecastable, long-term bond return differentials in U.S. dollars are not. The deterioration of return predictability for long-maturity bonds, compared to Treasury bills, appears to be due to the offsetting effect of local currency bond returns: higher than usual interest rate differences in a given country predict lower local currency bond return differences. Again, we can reject the null that interest rate differences do not predict local currency bond return differences at the 1% confidence level. In the panel regression, the local bond return slope coefficient is -1.34 , largely offsetting the 1.98 slope coefficient in the currency excess return regression. The net effect on dollar bond returns is only 65 basis points, the slope coefficient is not statistically significant, and the panel regression adjusted R^2 is -0.05% .¹ Thus, from the perspective of a U.S. investor, the time variation in the currency excess return is largely offset by the variation in the local term premium.

When using yield curve slope differentials as predictors, a similar finding emerges. On the one hand, currency log excess returns are forecastable by yield curve slope differentials: a steeper than usual slope in a given country predicts lower than usual currency log excess returns. On the other hand, a slope steeper than usual in a given country also predicts higher local currency bond excess returns. In the panel regression, the local bond excess return slope coefficient is 3.96 , more than offsetting the -2.02 slope coefficient in the currency excess return regression. The local currency bond return predictability merely confirm the results for U.S. bond excess returns documented by Fama and Bliss (1987), Campbell and Shiller (1991), and Cochrane and Piazzesi (2005). The net effect on dollar bond excess return differences is 194 basis points in a surprising direction: a

¹Note that the local currency log bond excess return differential contains the interest rate differential with a negative sign: $rx_{t+1}^{(10),*} - rx_{t+1}^{(10)} = r_{t+1}^{(10),*} - r_{t+1}^{(10)} - (r_t^{f,*} - r_t^f)$, where $r_{t+1}^{(10),*}$ and $r_{t+1}^{(10)}$ are the foreign and domestic holding-period bond returns. Thus, the local currency log bond excess return differential is highly predictable by the interest rate differential (right column of Panel A in Table 1), simply because the interest rate spread in effect predicts itself, as it is a component (with a negative sign) of the dependent variable. When we regress the local currency log return differential (instead of the excess returns) on the interest rate differential, there is no evidence of predictability (see Online Appendix). Measurement error in the short rate could also give rise to a negative relation, even in the absence of true predictability. The slope of the yield curves predicts the local currency log return differential.

steeper slope seems to weakly forecast *higher* dollar returns for foreign bonds, rather than lower dollar returns as for foreign Treasury Bills. The slope coefficient in the panel regression is statistically significant. From the perspective of a U.S. investor, the time variation in the currency excess return is more than offset by the variation in the local term premium. This reverses the usual carry trade logic: investors want to short the currencies with lower than average slopes to harvest the local bond term premium. We turn now to the economic significance of these results.

To do so, we explore the risk-return characteristics of a simple investment strategy that goes long the foreign bond and shorts the U.S. bond when the foreign short-term interest rate is higher than the U.S. interest rate (or the foreign yield curve slope is lower than the U.S. yield curve slope), and goes long the U.S. bond and shorts the foreign bond when the U.S. interest rate is higher than the foreign country’s interest rate (or the U.S. yield curve slope is lower than the foreign yield curve slope). To assess the risk-return trade-off, investors commonly look at the corresponding Sharpe ratios of an investment strategy, defined as the expected return less the risk-free rate divided by its standard deviation. In the absence of arbitrage opportunities, there is a one-to-one mapping from the R^2 s in predictability regressions to the unconditional Sharpe ratios on investment strategies that exploit the predictability (see Cochrane, 1999, on p.75-76). Table 2 shows this mapping. The top panel reports the results of the interest rate-based strategy, whereas the bottom panel focuses on the slope-based strategy. The very low R^2 s reported in Table 1 lead to low returns and Sharpe ratios in Table 2.

In our sample, none of the individual country dollar returns on the interest rate level strategy are statistically significant. The equally-weighted dollar return on the interest rate strategy is 0.70% per annum, and this return is not statistically significant. The equally-weighted annualized Sharpe ratio is 0.11, not significantly different from zero. Furthermore, with the exception of Sweden, none of the individual country dollar bond returns on the slope strategy are statistically significant either, even though the currency excess returns and the local currency bond returns typically are. The equally-weighted return on the slope strategy is -1.03% per annum, and this dollar return is not statistically significant. The equally-weighted annualized Sharpe ratio is -0.13 , not significantly different from zero. In short, there is no evidence of economically significant dollar return predictability. This is not to say that no significant return predictability exists in our sample. To the contrary, there are large currency excess returns (with a Sharpe ratio of 0.47 for an equally-weighted portfolio in the interest-rate strategy) — these are the well-known deviations from U.I.P., i.e. the carry trade returns — and

Table 1: Dollar Bond Return Differential Predictability

	Bond dollar return difference					Currency excess return					Bond local currency return diff. Slope Diff. Obs.						
	$rx^{(10),\$} - rx^{(10)}$					rx^{FX}					$rx^{(10),*} - rx^{(10)}$						
	α	s.e.	β	s.e.	$R^2(\%)$	α	s.e.	β	s.e.	$R^2(\%)$	α	s.e.	β	s.e.	$R^2(\%)$	p-value	
Panel A: Short-Term Interest Rates																	
Australia	0.01	[0.03]	-0.15	[0.97]	-0.20	-0.02	[0.02]	1.29	[0.62]	0.56	0.03	[0.02]	-1.44	[0.60]	1.51	0.21	492
Canada	0.02	[0.02]	-1.10	[0.69]	0.11	-0.01	[0.01]	1.22	[0.53]	0.46	0.03	[0.01]	-2.32	[0.46]	3.64	0.01	492
Germany	0.01	[0.02]	1.52	[1.21]	0.37	0.02	[0.02]	2.49	[0.99]	1.71	-0.01	[0.01]	-0.97	[0.60]	0.48	0.53	492
Japan	0.06	[0.03]	2.37	[0.84]	1.13	0.07	[0.02]	3.11	[0.67]	3.48	-0.01	[0.02]	-0.74	[0.52]	0.13	0.49	492
New Zealand	-0.03	[0.04]	0.69	[0.87]	-0.03	-0.07	[0.03]	2.23	[0.49]	3.14	0.04	[0.03]	-1.54	[0.66]	1.62	0.12	492
Norway	-0.02	[0.02]	0.72	[0.62]	0.08	-0.02	[0.02]	1.74	[0.57]	2.26	0.01	[0.01]	-1.02	[0.41]	0.97	0.23	492
Sweden	-0.00	[0.02]	-0.64	[0.91]	-0.02	-0.02	[0.02]	0.89	[0.91]	0.25	0.01	[0.01]	-1.53	[0.49]	2.02	0.23	492
Switzerland	0.02	[0.02]	1.16	[0.82]	0.33	0.05	[0.02]	2.45	[0.78]	2.43	-0.03	[0.01]	-1.29	[0.43]	1.69	0.25	492
United Kingdom	-0.02	[0.03]	1.02	[1.18]	0.04	-0.05	[0.02]	2.69	[0.95]	2.44	0.03	[0.02]	-1.67	[0.66]	1.39	0.27	492
Panel	-	-	0.65	[0.49]	-0.05	-	-	1.98	[0.44]	1.82	-	-	-1.34	[0.30]	1.37	0.00	4428
Joint zero (p-value)	0.44		0.04			0.00		0.00			0.00		0.00				0.04
Panel B: Yield Curve Slopes																	
Australia	0.06	[0.02]	3.84	[1.56]	1.54	0.00	[0.02]	-1.00	[1.17]	-0.02	0.05	[0.02]	4.84	[0.92]	7.65	0.01	492
Canada	0.04	[0.02]	4.04	[0.98]	2.25	-0.00	[0.01]	-0.72	[0.66]	-0.07	0.04	[0.01]	4.76	[0.63]	9.09	0.00	492
Germany	0.00	[0.02]	0.50	[1.77]	-0.18	-0.01	[0.02]	-3.05	[1.37]	1.15	0.01	[0.01]	3.55	[0.97]	4.07	0.11	492
Japan	0.00	[0.02]	-0.32	[1.38]	-0.19	-0.01	[0.02]	-4.18	[1.08]	2.91	0.01	[0.01]	3.85	[0.82]	3.96	0.03	492
New Zealand	0.08	[0.04]	2.94	[2.04]	1.26	-0.01	[0.03]	-1.60	[1.18]	0.62	0.09	[0.03]	4.55	[1.19]	7.41	0.05	492
Norway	-0.00	[0.02]	0.59	[1.03]	-0.12	-0.01	[0.02]	-2.03	[0.92]	1.33	0.01	[0.01]	2.62	[0.59]	3.35	0.06	492
Sweden	0.02	[0.02]	3.12	[1.23]	2.12	-0.00	[0.02]	-0.13	[1.14]	-0.20	0.02	[0.01]	3.25	[0.71]	5.29	0.05	492
Switzerland	0.00	[0.02]	0.97	[1.17]	-0.06	-0.02	[0.02]	-3.59	[1.26]	1.97	0.02	[0.01]	4.55	[0.78]	8.82	0.01	492
United Kingdom	0.02	[0.03]	1.59	[1.53]	0.17	-0.02	[0.02]	-3.17	[1.37]	2.11	0.04	[0.01]	4.75	[0.83]	7.95	0.02	492
Panel	-	-	1.94	[0.84]	0.42	-	-	-2.02	[0.73]	0.83	-	-	3.96	[0.50]	6.08	0.00	4428
Joint zero (p-value)	0.07		0.00			0.96		0.00			0.00		0.00				0.00

Notes: The table reports regression results of the bond dollar return difference ($rx_{t+1}^{(10),\$} - rx_{t+1}^{(10)}$, left panel) or the currency excess return (rx_{t+1}^{FX} , middle panel) or the bond local currency return difference ($rx_{t+1}^{(10),*} - rx_{t+1}^{(10)}$, right panel) on the difference between the foreign nominal interest rate and the U.S. nominal interest rate ($r_t^{f,*} - r_t^f$, Panel A) or difference between the foreign nominal yield curve slope and the U.S. nominal yield curve slope ($[y_t^{(10),*} - y_t^{(1,*)}] - [y_t^{(10)} - y_t^{(1)}]$, Panel B). The column “Slope Diff.” presents the p-value of the test for equality between the slope coefficient in the bond dollar return difference regression and the slope coefficient in the currency excess return regression for each country. The last line in each panel presents the p-value of the joint test that all individual-country regression coefficients in the respective column are zero. We use returns on 10-year coupon bonds. The holding period is one month and returns are sampled monthly. The log returns and the yield curve slope differentials are annualized. The sample period is 1/1975–12/2015. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. In individual country regressions, standard errors are obtained with a Newey-West approximation of the spectral density matrix, with the lag truncation parameter (kernel bandwidth) equal to 6. Panel regressions include country fixed effects, and standard errors are obtained using the Driscoll and Kraay (1998) methodology, with the lag truncation parameter (kernel bandwidth) equal to 6.

there are large local currency long-term bond returns (with a Sharpe ratio of 0.51 for an equally-weighted portfolio in the interest-rate strategy) — these are related to the well-known deviations from the expectation hypothesis, i.e. the local term premia. The two corresponding Sharpe ratios are larger than the one on the U.S. aggregate stock market. However, the currency risk premia and the local term premia cancel out, so the Sharpe ratio on foreign bond returns in dollars is not significant. From the vantage point of a U.S. investor, there is no evidence of economically significant return predictability in long foreign bonds.

1.4 Cross-Sectional Properties of Foreign Bond Returns

After focusing on time-series predictability, we turn now to cross-sectional evidence, as in Lustig and Verdelhan (2007). There is no mechanical link between the time-series and cross-sectional evidence. The time-series regressions test whether a predictor that is higher than its average implies higher returns, while the cross-sectional tests show whether a predictor that is higher in one country than in others implies higher returns in that country (see Hassan and Mano, 2014). Our cross-sectional evidence echoes some papers that study the cross-section of bond returns: Koijen, Moskowitz, Pedersen, and Vrugt (2016) and Wu (2012) examine the currency-hedged returns on ‘carry’ portfolios of international bonds, sorted by a proxy for the carry on long-term bonds. But these papers do not examine the interaction between currency and term risk premia, the topic of our paper.

We sort countries into three portfolios on the level of the short-term interest rates or the slope of their yield curves. Portfolios are rebalanced every month and those formed at date t only use information available at that date. Portfolio-level log excess returns are obtained by averaging country-level log excess returns across all countries in the portfolio. We first describe results obtained with the 10-year bond indices, reported in Table 3, and then turn to the zero-coupon bonds to study the whole term structure, presented in Figure 2.

1.4.1 Sorting by Interest Rates

We start with the currency portfolios sorted by short-term interest rates. In order to focus on the conditional carry trade, we use interest rates in deviation from their past 10-year rolling mean as the sorting variable.² Thus, the first (third) portfolio includes the conditionally low (high) interest rate currencies. Clearly, the

²The unconditional carry trade goes long in currencies with an average high interest rates (Hassan and Mano, 2014). That is not the focus of our paper.

Table 2: Dynamic Long-Short Foreign and U.S. Bond Portfolios

	Bond dollar return difference					Currency excess return					Bond local currency return diff.				
	$rx^{(10),\$} - rx^{(10)}$					rx^{FX}					$rx^{(10),*} - rx^{(10)}$				
	Mean	s.e.	Std.	SR	s.e.	Mean	s.e.	Std.	SR	s.e.	Mean	s.e.	Std.	SR	s.e.
Panel A: Short-Term Interest Rates															
Australia	1.28	[2.23]	14.27	0.09	[0.16]	3.55	[1.75]	11.40	0.31	[0.16]	-2.27	[1.34]	8.46	-0.27	[0.15]
Canada	-0.46	[1.42]	9.10	-0.05	[0.16]	0.86	[1.13]	6.97	0.12	[0.16]	-1.31	[0.85]	5.50	-0.24	[0.15]
Germany	2.19	[1.93]	12.45	0.18	[0.16]	3.86	[1.72]	11.13	0.35	[0.16]	-1.67	[1.16]	7.30	-0.23	[0.16]
Japan	0.93	[2.19]	14.31	0.07	[0.16]	1.54	[1.73]	11.31	0.14	[0.16]	-0.61	[1.43]	9.03	-0.07	[0.16]
New Zealand	0.65	[2.56]	16.91	0.04	[0.16]	3.84	[1.87]	12.25	0.31	[0.17]	-3.19	[1.76]	11.47	-0.28	[0.16]
Norway	0.69	[1.98]	13.00	0.05	[0.16]	3.17	[1.60]	10.61	0.30	[0.16]	-2.48	[1.42]	8.99	-0.28	[0.16]
Sweden	-0.40	[2.03]	12.85	-0.03	[0.15]	2.30	[1.73]	11.14	0.21	[0.16]	-2.71	[1.38]	8.68	-0.31	[0.16]
Switzerland	0.57	[2.05]	12.84	0.04	[0.16]	1.14	[1.97]	12.22	0.09	[0.15]	-0.57	[1.13]	7.62	-0.07	[0.16]
United Kingdom	0.89	[2.00]	12.76	0.07	[0.15]	3.09	[1.59]	10.26	0.30	[0.16]	-2.20	[1.25]	8.21	-0.27	[0.15]
Equally-weighted	0.70	[1.00]	6.36	0.11	[0.16]	2.59	[0.87]	5.55	0.47	[0.17]	-1.89	[0.60]	3.73	-0.51	[0.16]
Panel B: Yield Curve Slopes															
Australia	-1.88	[2.22]	14.26	-0.13	[0.16]	2.89	[1.80]	11.41	0.25	[0.17]	-4.76	[1.34]	8.37	-0.57	[0.14]
Canada	-2.07	[1.41]	9.08	-0.23	[0.15]	1.23	[1.10]	6.97	0.18	[0.16]	-3.30	[0.83]	5.43	-0.61	[0.16]
Germany	1.98	[1.92]	12.46	0.16	[0.16]	5.17	[1.71]	11.09	0.47	[0.16]	-3.19	[1.14]	7.25	-0.44	[0.15]
Japan	-0.71	[2.21]	14.31	-0.05	[0.16]	4.60	[1.73]	11.24	0.41	[0.16]	-5.31	[1.42]	8.90	-0.60	[0.16]
New Zealand	-0.18	[2.57]	16.91	-0.01	[0.16]	3.49	[1.90]	12.26	0.28	[0.17]	-3.67	[1.77]	11.45	-0.32	[0.15]
Norway	-0.56	[2.05]	13.00	-0.04	[0.15]	2.84	[1.73]	10.61	0.27	[0.16]	-3.40	[1.39]	8.97	-0.38	[0.15]
Sweden	-3.62	[1.99]	12.81	-0.28	[0.16]	1.32	[1.75]	11.15	0.12	[0.17]	-4.94	[1.38]	8.60	-0.57	[0.16]
Switzerland	0.47	[2.00]	12.84	0.04	[0.15]	4.80	[1.91]	12.15	0.40	[0.15]	-4.33	[1.17]	7.51	-0.58	[0.16]
United Kingdom	-2.73	[1.95]	12.73	-0.21	[0.16]	2.06	[1.61]	10.29	0.20	[0.16]	-4.79	[1.30]	8.12	-0.59	[0.16]
Equally-weighted	-1.03	[1.24]	7.82	-0.13	[0.16]	3.16	[1.08]	6.68	0.47	[0.16]	-4.19	[0.80]	5.04	-0.83	[0.16]

Notes: For each country, the table presents summary return statistics of investment strategies that go long the foreign country bond and short the U.S. bond when the foreign short-term interest rate is higher than the U.S. interest rate (or the foreign yield curve slope is lower than the U.S. yield curve slope), and go long the U.S. bond and short the foreign country bond when the U.S. interest rate is higher than the country's interest rate (or the U.S. yield curve slope is lower than the foreign yield curve slope). Results based on interest rate levels are reported in Panel A and results based on interest rate slopes are reported in Panel B. The table reports the mean, standard deviation and Sharpe ratio (denoted SR) for the currency excess return (rx^{FX} , middle panel), for the foreign bond excess return on 10-year government bond indices in foreign currency ($rx^{(10),*} - rx^{(10)}$, right panel) and for the foreign bond excess return on 10-year government bond indices in U.S. dollars ($rx^{(10),\$} - rx^{(10)}$, left panel). The holding period is one month. The table also presents summary return statistics for the equally-weighted average of the individual country strategies. The slope of the yield curve is measured by the difference between the 10-year yield and the one-month interest rate. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns. The log returns are annualized. The data are monthly and the sample is 1/1975–12/2015.

classic uncovered interest rate parity condition fails in the cross-section: the currencies in the third portfolio only depreciate by 60 basis points per year on average, not enough to offset the 2.65% interest rate difference and thus delivering a 2.05% return. As a result, average currency excess returns increase from low- to high-interest-rate portfolios, ranging from -0.61% to 2.05% per year over the last 40 years. Thus, the long-short currency carry trade (invest in Portfolio 3, short Portfolio 1) implemented with Treasury bills delivers an average annual log excess return of $2.05\% - (-0.61\%) = 2.66\%$ and a Sharpe ratio of 0.36 (Panel B), higher than the Sharpe ratio on the U.S. S&P500 equity index over the same period.

However, implementing the carry trade with long-term bonds does not yield a similar performance, as local currency bond premia decrease from low- to high-interest-rate portfolios, from 3.53% to -0.25% , implying that the 10-year bond carry trade entails a local currency bond premium of $-0.25\% - 3.53\% = -3.78\%$ (Panel C). Therefore, the long-short currency carry trade implemented with long-term government bonds, which is the sum of the two long-short returns above, delivers a negative average return of $-3.78\% + 2.66\% = -1.12\%$ (Panel D) that is not statistically significant. This contrasts with the equivalent trade using Treasury bills. The dollar bond risk premia (-1.12%) are statistically different from the carry trade risk premia (2.66%) because the local term premia (-3.78%) are statistically significant. Investors have no reason to favor the long-term bonds of a particular set of countries on the basis of average returns after converting the returns into the same currency (here, U.S. dollars).

Inflation risk is not a good candidate explanation for local currency bond excess returns, because inflation is higher and more volatile in countries that are in the last portfolio than in the first portfolio (Panel A of Table 3, left section). Similarly, sovereign default risk is not a plausible explanation, given that the countries in the first portfolio tend to have slightly better credit ratings than the countries in the last portfolio. The relatively high term premium of 3.53% in the first portfolio thus corresponds to relatively low inflation and default risk.

1.4.2 Sorting by Slopes

Similar results emerge when we sort countries into portfolios by the slope of their yield curves. There is substantial turnover in these portfolios, more so than in the usual interest rate-sorted portfolios. On average, the flat slope currencies (first portfolio) tend to be high interest rate currencies, while the steep slope currencies (third portfolio) tend to be low interest rate currencies. As expected, average currency log excess returns decline

Table 3: Cross-sectional Predictability: Bond Portfolios

Portfolio	Sorted by Short-Term Interest Rates				Sorted by Yield Curve Slopes				
	1	2	3	3 – 1	1	2	3	1 – 3	
Panel A: Portfolio Characteristics									
Inflation rate	Mean	2.90	3.45	4.81	1.91	4.89	3.41	2.87	2.02
	s.e.	[0.16]	[0.19]	[0.23]	[0.20]	[0.23]	[0.19]	[0.18]	[0.19]
	Std	1.03	1.23	1.48	1.30	1.39	1.16	1.20	1.26
Rating	Mean	1.45	1.25	1.49	0.04	1.54	1.38	1.28	0.25
	s.e.	[0.02]	[0.02]	[0.02]	[0.04]	[0.02]	[0.02]	[0.02]	[0.03]
Rating (adj. for outlook)	Mean	1.50	1.37	1.84	0.33	1.84	1.50	1.37	0.47
	s.e.	[0.03]	[0.02]	[0.02]	[0.04]	[0.02]	[0.02]	[0.02]	[0.03]
$y_t^{(10),*} - r_t^{*,f}$	Mean	1.52	0.92	-0.44	-1.96	-0.81	0.85	1.96	-2.76
Panel B: Currency Excess Returns									
$-\Delta s_{t+1}$	Mean	-0.44	0.11	-0.60	-0.16	-0.95	0.38	-0.36	-0.58
$r_t^{f,*} - r_t^f$	Mean	-0.17	0.54	2.65	2.81	3.35	0.55	-0.88	4.23
rx_{t+1}^{FX}	Mean	-0.61	0.66	2.05	2.66	2.41	0.92	-1.24	3.65
	s.e.	[1.35]	[1.44]	[1.36]	[1.14]	[1.48]	[1.38]	[1.40]	[1.18]
	SR	-0.07	0.07	0.23	0.36	0.26	0.11	-0.14	0.49
Panel C: Local Currency Bond Excess Returns									
$rx_{t+1}^{(10),*}$	Mean	3.53	2.60	-0.25	-3.78	-1.01	2.29	4.61	-5.61
	s.e.	[0.69]	[0.69]	[0.73]	[0.77]	[0.76]	[0.69]	[0.70]	[0.74]
	SR	0.80	0.58	-0.05	-0.77	-0.21	0.53	1.00	-1.18
Panel D: Dollar Bond Excess Returns									
$rx_{t+1}^{(10),\$}$	Mean	2.92	3.26	1.80	-1.12	1.40	3.21	3.36	-1.96
	s.e.	[1.56]	[1.58]	[1.57]	[1.33]	[1.64]	[1.57]	[1.62]	[1.38]
	SR	0.29	0.32	0.18	-0.13	0.14	0.33	0.32	-0.22
$rx_{t+1}^{(10),\$} - rx_{t+1}^{(10)}$	Mean	0.14	0.48	-0.98	-1.12	-1.38	0.43	0.59	-1.96
	s.e.	[1.64]	[1.64]	[1.73]	[1.33]	[1.81]	[1.63]	[1.75]	[1.38]

Notes: The countries are sorted by the level of their short term interest rates in deviation from the 10-year mean into three portfolios (left section) or the slope of their yield curves (right section). The slope of the yield curve is measured by the difference between the 10-year yield and the one-month interest rate. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns. The table reports the average inflation rate, the standard deviation of the inflation rate, the average credit rating, the average credit rating adjusted for outlook, the average slope of the yield curve ($y^{(10),*} - r^{*,f}$), the average change in exchange rates (Δs), the average interest rate difference ($r^{f,*} - r^f$), the average currency excess return (rx^{FX}), the average foreign bond excess return on 10-year government bond indices in foreign currency ($rx^{(10),*}$) and in U.S. dollars ($rx^{(10),\$}$), as well as the difference between the average foreign bond excess return in U.S. dollars and the average U.S. bond excess return ($rx^{(10),\$} - rx^{(10)}$). For the excess returns, the table also reports their Sharpe ratios (denoted SR). The holding period is one month. The log returns are annualized. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The data are monthly and the sample is 1/1975–12/2015.

from 2.41% per annum on the first portfolio (low slope, high short-term interest rates) to -1.24% per annum on the third portfolio (high slope, low short-term interest rates) over the last 40 years (Panel B). Therefore, a long-short position of investing in flat-yield-curve currencies (Portfolio 1) and shorting steep-yield-curve currencies (Portfolio 3) delivers a currency excess return of 3.65% per annum and a Sharpe ratio of 0.49. Our findings confirm those of Ang and Chen (2010): the slope of the yield curve predicts currency excess returns at the short end of the maturity spectrum.

However, those currency premia are offset by term premia, as local currency bond excess returns and currency excess returns move in opposite directions across portfolios. In particular, the first portfolio produces negative bond average excess returns of -1.01% per year, compared to 4.61% on the third portfolio (Panel C), so the slope carry trade generates an average local currency bond excess return of -5.61% per year. This result is not mechanical: the spread in the slopes is about half of the spread in local currency excess returns. The corresponding average dollar bond excess returns range from 1.40% to 3.36% , so the slope carry trade implemented with 10-year bonds delivers an average annual dollar excess return of -1.96% (Panel D), which is not statistically significant.

Importantly, this strategy involves long positions in bonds issued by countries with slightly lower average inflation, less inflation risk and slightly better average sovereign credit rating, despite the higher term premium. Therefore, the offsetting effect of local currency term premia is unlikely to be due to inflation or credit risk.

1.4.3 Looking Across Subsamples

To summarize the portfolio results and test their robustness across subsamples, Figure 1 plots the cumulative returns on interest rate and slope carry strategies over the entire sample. The full line is the cumulative local currency bond excess return (in logs), while the dashed line is the cumulative currency excess return (in logs). The cumulative dollar excess return is the sum of these two (not shown). The top panels sort by interest rates on the left and interest rate deviations on the right. The bottom panel sort by slope and slope deviations. Overall, the same patterns reappear in each these plots, but there are three noticeable differences.

First, when we sort on interest rates, the off-setting effect of local currency bond excess returns is slightly weaker, and the currency excess returns are larger, than when we sort on interest rate deviations. The latter sort only captures the conditional carry premium, while the former captures the entire carry premium. The carry

trade risk premium is somewhat larger when sorting on interest rates, as documented by Lustig, Roussanov, and Verdelhan (2011) and Hassan and Mano (2014), as these sorts capture both the conditional carry trade premium (long in currencies with currently high interest rates) and the unconditional carry trade premium (long in currencies with high average interest rates). Since our paper is about the conditional currency carry trade, we focus mainly on the demeaned sorts. Second, when we use interest rate (deviations) sorts, the negative contribution of local currency bond premia weakens starting in the mid 80s (90s), compared to the earlier part of the sample. This is not surprising given that interest rates started to converge across G10 countries in the second part of the sample (Wright, 2011). In other words, when G10 countries have similar low interest rates, these rates do not predict large differences in term premia across countries. Third, the offsetting effect of local currency bond premia is larger when we sort on the slope of the yield curve, and it does not weaken in the second part of the sample.

1.4.4 Looking Across Maturities

The results we discussed previously focus on the 10-year maturity. We now turn to the full maturity spectrum, using the zero-coupon bond dataset. The panel is unbalanced, and because of data limitations, we can only examine three-month holding period returns over the 4/1985–12/2015 sample, which mitigates predictability (4/1985 is the first month for which we have data for at least three foreign bonds). As a result, the standard errors are larger than in the benchmark sample. Building on the previous results, countries as sorted by the slope of their term structures because the slope appears as the strongest predictor of both currency and term premia over this sample. We sort only on the slope of the yield curve, not on the slope deviation from its 10-year mean, because of the shorter sample.

Our findings are presented graphically in Figure 2, which shows the dollar log excess returns as a function of the bond maturity, using the same set of funding and investment currencies. Investing in the short-maturity bills of countries with flat yield curves (mostly high short-term interest rate countries), while borrowing at the same horizon in countries with steep yield curves (mostly low short-term interest rate countries) leads to an average dollar excess return of 2.67% per year (with a standard error of 1.50%). This is the slope version of the standard currency carry trade. However, when we implement the same strategy using longer maturity bonds instead of short-term bills, the dollar excess return decreases monotonically as the maturity of the bonds

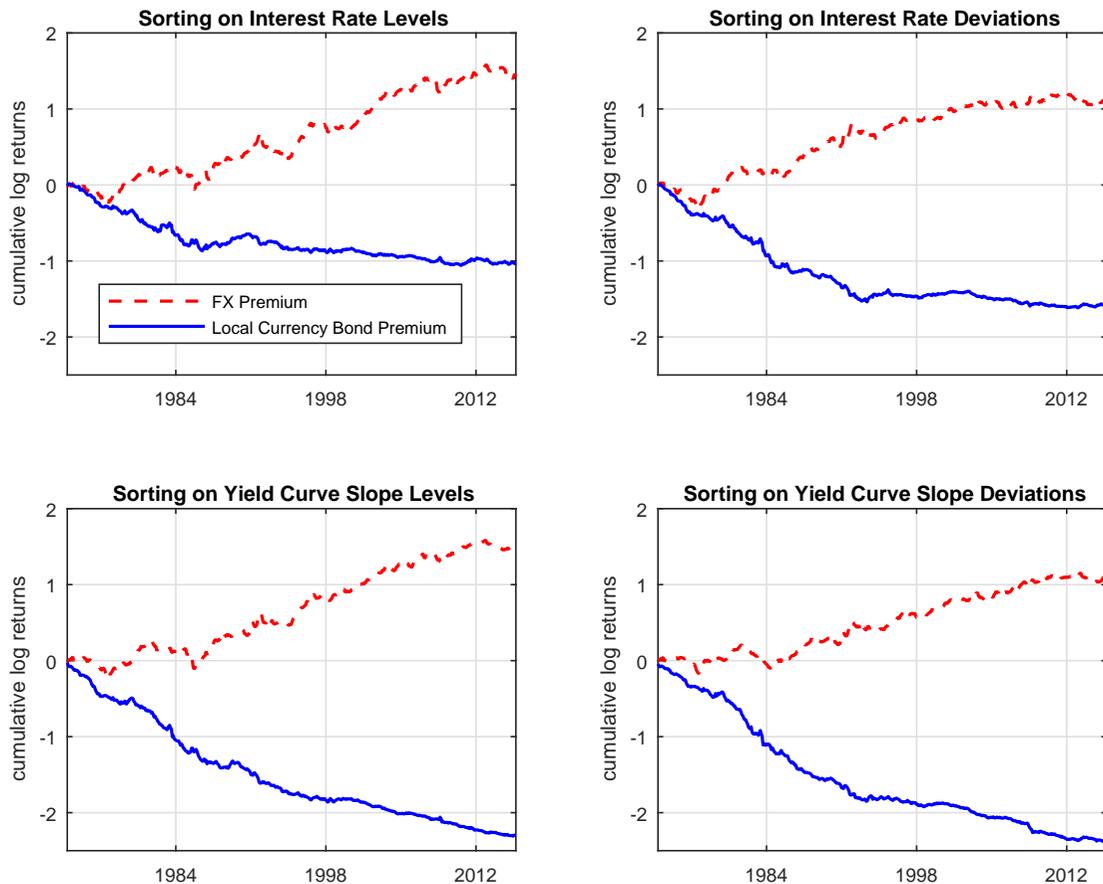


Figure 1: FX Premia and Local Currency Bond Premia— The figure shows the cumulative log excess returns on interest rate and slope carry investment strategies that go long in high interest rate (flat slope) currencies and short low interest rate (steep slope) currencies. The full line is the cumulative log currency excess return. The dashed line is the cumulative log local currency bond excess return. The cumulative log dollar excess return is represented by the sum of these 2 lines. At each date t , the countries are sorted by the interest rate (slope of the yield curves) into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t . The holding period is one month. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The data are monthly and the sample is 1/1975–12/2015.

increases. The zero-coupon findings confirm our previous results and seem to rule out measurement error in the 10-year coupon bond indices as an explanation. At the long end (15-year maturity), the bond term premium more than offsets the currency premium, so the slope carry trade yields a (non-significant) average annual dollar return of -2.18% (with a standard error of 2.28%). The average excess returns at the long-end of the yield curve are statistically different from those at the short-end: the difference between those returns correspond to the local term premia, which are equal to -4.85% with a standard error of 1.82% . Therefore, carry trade strategies that yield positive average excess returns when implemented with short-maturity bonds yield lower

(or even negative) excess returns when implemented using long-maturity bonds.³

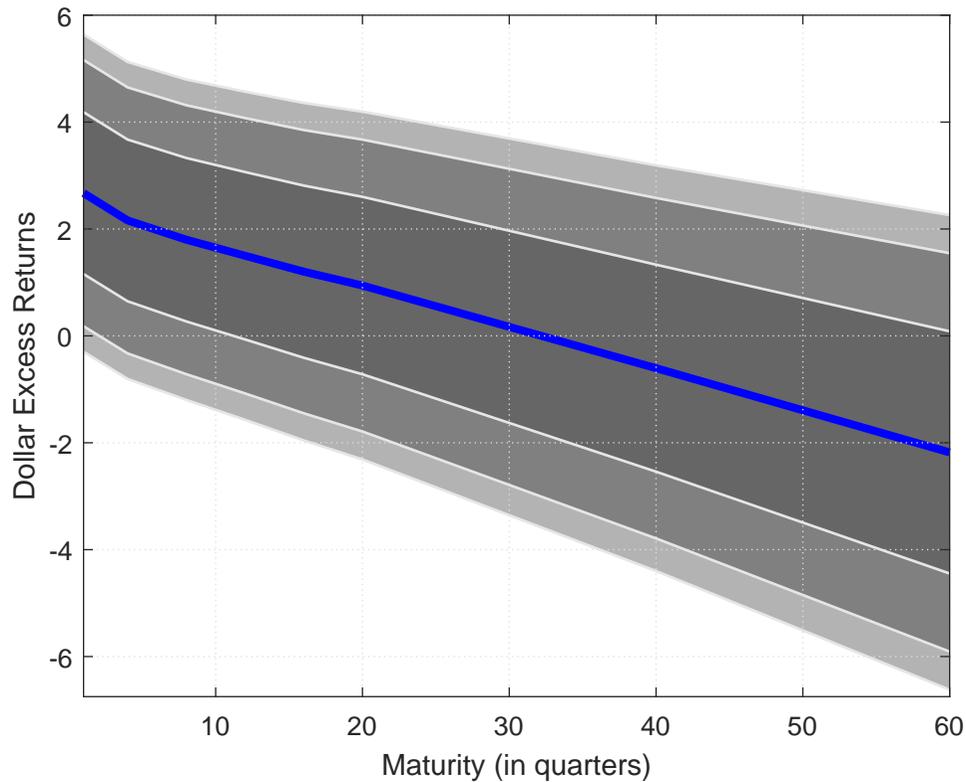


Figure 2: Long-Minus-Short Foreign Bond Risk Premia in U.S. Dollars— The figure shows the dollar log excess returns as a function of the bond maturities. Dollar excess returns correspond to the holding period returns expressed in U.S. dollars of investment strategies that go long and short foreign bonds of different countries. The unbalanced panel of countries consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. At each date t , the countries are sorted by the slope of their yield curves into three portfolios. The first portfolio contains countries with flat yield curves while the last portfolio contains countries with steep yield curves. The slope of the yield curve is measured by the difference between the 10-year yield and the three-month interest rate at date t . The holding period is one quarter. The returns are annualized. The dark shaded area corresponds to one-standard-error bands around the point estimates. The gray and light gray shaded areas correspond to the 90% and 95% confidence intervals. Standard deviations are obtained by bootstrapping 10,000 samples of non-overlapping returns. Zero-coupon data are monthly, and the sample window is 4/1985–12/2015.

1.5 Robustness Checks

We consider many robustness checks, both regarding our time-series results and our cross-sectional results. The time-series predictability robustness checks are reported in Section A of the Online Appendix, whereas the cross-sectional portfolio robustness checks can be found in Section B of the Online Appendix.

³When we use interest rate sorts, the term structure is flat: The carry premium is 3.71% per annum (with a standard error of 1.80%), while the local 10-year bond premium is only -0.21% per annum (with a standard error of 1.76%), so the dollar bond premium at the 15-year maturity is 3.50% (with a standard error of 2.32%). As noted in the previous subsection and apparent in the top panels of Figure 1, interest rates in levels do not predict bond excess returns in the cross-section over the 4/1985–12/2015 sample.

As regards time-series predictability, we consider predictability regressions using inflation and sovereign credit as additional controls, we consider an alternative decomposition of dollar bond returns into an exchange rate component and a local currency bond return difference, we include predictability results with GBP as the base currency and we report predictability results using different time-windows (10/1983–12/2007, 1/1975–12/2007, 10/1983–12/2015) and investment horizons (three months). We find that our main results are robust to those alternative specifications.

As regards cross-sectional currency portfolios, we consider different lengths of the bond holding period (three and twelve months), different time windows, different samples of countries, sorts by (non-demeaned) interest rate levels, and other potential explanations of excess returns. Our results appear robust to the choice of the bond holding period and across time windows. Furthermore, our results appear robust across several samples of countries. Introducing more countries adds power to the experiment, but forces us to consider less liquid and more default-prone bond markets. In what may be of particular interest, we show that inflation risk or credit risk are unlikely explanations for differences in term premia even in larger sets of countries and different time windows. For both our benchmark sample (1/1975 – 12/2015) and a longer sample (1/1951 – 12/2015), term premia are higher in low inflation countries. Thus, assuming that there is a positive association between average inflation rates and exposure to inflation risk, inflation risk does not account for our findings. This is true not only for our benchmark set of countries, but also for the extended sets of countries. Similarly, the cross-sectional patterns in term premia we observe empirically are not likely to be due to sovereign default risk. As seen in Table 3, countries with high average local currency bond premia have average credit ratings (both unadjusted and adjusted for outlook) that are lower than or similar to the ratings of countries with low average local currency bond premia. That finding is robust to considering different sample periods: it holds both in the long sample period (1/1951 – 12/2015) and in the 7/1989–12/2015 period, during which full ratings are available. Therefore, we find no empirical evidence in favor of an inflation- or credit-based explanation of our findings in this sample of G-10 currencies, and thus pursue a simple interest rate risk interpretation that seems the most relevant, especially over the last thirty years.

2 The Term Structure of Currency Carry Trade Risk Premia: A Challenge

In this section, we show that the downward sloping term structure of currency carry trade risk premia is a challenge even for a reduced-form model.

2.1 The Necessary Condition for Replicating the U.I.P Puzzle

We start with a review of a key necessary condition for replicating the U.I.P puzzle, established by Bekaert (1996) and Bansal (1997) and generalized by Backus, Foresi, and Telmer (2001). To do so, we first introduce some additional notation.

Pricing Kernels and Stochastic Discount Factors The nominal pricing kernel is denoted by $\Lambda_t(\varpi)$; it corresponds to the marginal value of a currency unit delivered at time t in the state of the world ϖ . The nominal SDF M is the growth rate of the pricing kernel: $M_{t+1} = \Lambda_{t+1}/\Lambda_t$. Therefore, the price of a zero-coupon bond that promises one currency unit k periods into the future is given by

$$P_t^{(k)} = E_t \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right). \quad (4)$$

SDF Entropy SDFs are volatile, but not necessarily normally distributed. In order to measure the time-variation in their volatility, it is convenient to use entropy, rather than variance (Backus, Chernov, and Zin, 2014). The conditional entropy L_t of any random variable X_{t+1} is defined as

$$L_t(X_{t+1}) = \log E_t(X_{t+1}) - E_t(\log X_{t+1}). \quad (5)$$

If X_{t+1} is conditionally lognormally distributed, then the conditional entropy is equal to one half of the conditional variance of the log of X_{t+1} : $L_t(X_{t+1}) = (1/2)var_t(\log X_{t+1})$. If X_{t+1} is not conditionally lognormal, the entropy also depends on the higher moments: $L_t(X_{t+1}) = \kappa_{2t}/2! + \kappa_{3t}/3! + \kappa_{4t}/4! + \dots$, where $\{\kappa_{it}\}_{i=2}^{\infty}$ are the cumulants of $\log X_{t+1}$.⁴

⁴The literature on disaster risk in currency markets shows that higher order moments are critical for understanding currency returns (see Brunnermeier, Nagel, and Pedersen, 2009; Gourio, Siemer, and Verdelhan, 2013; Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan, 2013; Chernov, Graveline, and Zviadadze, 2011).

Exchange Rates When markets are complete, the change in the nominal exchange rate corresponds to the ratio of the domestic to foreign nominal SDFs:

$$\frac{S_{t+1}}{S_t} = \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\Lambda_t^*}{\Lambda_{t+1}^*}. \quad (6)$$

The no-arbitrage definition of the exchange rate comes directly from the Euler equations of the domestic and foreign investors, for any asset return R^* expressed in foreign currency terms: $E_t[M_{t+1}R_{t+1}^*S_t/S_{t+1}] = 1$ and $E_t[M_{t+1}^*R_{t+1}^*] = 1$.

Currency Risk Premia As Bekaert (1996) and Bansal (1997) show, in models with lognormally distributed SDFs the conditional log currency risk premium $E_t(rx^{FX})$ equals the half difference between the conditional variance of the log domestic and foreign SDFs. This result can be generalized to non-Gaussian economies. When higher moments matter and markets are complete, the currency risk premium is equal to the difference between the conditional entropy of two SDFs (Backus, Foresi, and Telmer, 2001):

$$E_t(rx_{t+1}^{FX}) = r_t^{f,*} - r_t^f - E_t(\Delta s_{t+1}) = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right). \quad (7)$$

According to the U.I.P. condition, expected changes in exchange rates should be equal to the difference between the home and foreign interest rates and, thus, the currency risk premium should be zero. In the data, the currency risk premium is as large as the equity risk premium. Any complete market model that addresses the U.I.P. puzzle must thus satisfy a simple necessary condition: high interest rate countries must exhibit relatively less volatile SDFs. In the absence of differences in conditional volatility, complete market models are unable to generate a currency risk premium and the U.I.P. counterfactually holds in the model economy.

Why is the downward term structure of currency carry trade risk premia a challenge for arbitrage-free models? Intuitively, the models need to depart from risk-neutrality in order to account for the U.I.P. deviations at the short end of the yield curve and large carry trade risk premia. Yet, for the exact same investment horizon, the models need to deliver zero risk premia at the long end of the yield curve, thus behaving as if investors are risk-neutral. In the rest of the paper, we highlight this tension and describe necessary conditions for arbitrage-free models to replicate our empirical evidence.

2.2 An Example: A Reduced-Form Factor Model

We start by showing that even a flexible, N -country reduced-form model calibrated to match the currency carry trade risk premia does not replicate the evidence on long-term bonds. Several two-country models satisfy the condition described in Equation (7) and thus replicate the failure of the U.I.P. condition, but they cannot replicate the portfolio evidence on carry trade risk premia. The reason is simple: when those models are extended to multiple countries, investors in the models can diversify away the country-specific exchange rate risk and there are no cross-sectional differences in carry trade returns across portfolios. To the best of our knowledge, only two models can so far replicate the portfolio evidence on carry trades: the multi-country long-run risk model of Colacito, Croce, Gavazzoni, and Ready (2017) and the multi-country reduced-form factor model of Lustig, Roussanov, and Verdelhan (2011). We focus on the latter because of its flexibility and close forms, and revisit the long-run risk model in Section E of the Appendix, along with other explanations of the U.I.P. puzzle. Moreover, in the Online Appendix, we cover a wide range of term structure models, from the seminal Vasicek (1977) model to the classic Cox, Ingersoll, and Ross (1985) model and to the most recent, multi-factor dynamic term structure models. To save space, we focus here on their most recent international finance version, illustrated in Lustig, Roussanov, and Verdelhan (2014). To replicate the portfolio evidence on carry trades, Lustig, Roussanov, and Verdelhan (2011, 2014) show that no-arbitrage models need to incorporate global shocks to the SDFs along with country heterogeneity in the exposure to those shocks. Following Lustig, Roussanov, and Verdelhan (2014), we consider a world with N countries and currencies in a setup inspired by classic term structure models.

Using their benchmark calibration, we calculate the model-implied term structure of currency risk premia when implementing the slope carry trade strategy (invest in low yield slope currencies, short the high yield slope interest rate currencies). This is very similar to investing in high interest rate countries while borrowing in low interest rate countries. The simulation details are provided in the Appendix. Figure 3, obtained with simulated data, is the model counterpart to Figure 2, obtained with actual data. A clear message emerges: while this model produces U.I.P. deviations (and thus currency risk premia) at the short end of the yield curve, the model produces a flat term structure of currency carry trade risk premia. We turn now to a novel necessary condition that dynamic asset pricing models need to satisfy in order to generate a downward-sloping term structure.

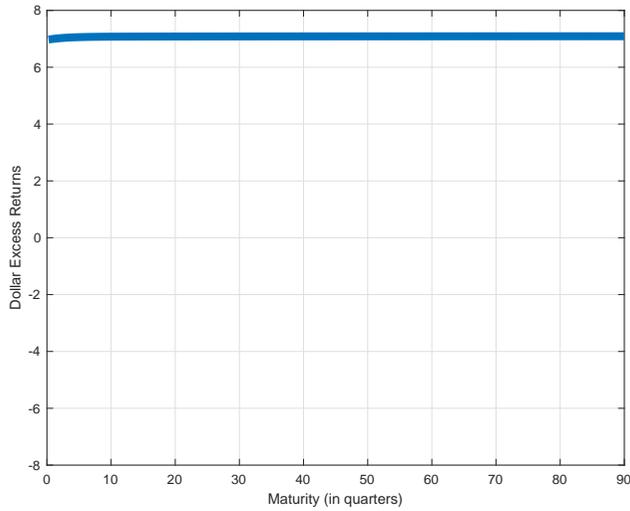


Figure 3: Simulated Long-Minus-Short Foreign Bond Risk Premia in U.S. Dollars— The figure shows the simulated average dollar log excess return of the slope carry trade strategy as a function of the bond maturities in the reduced-form model of Lustig, Roussanov, and Verdelhan (2014). At each date t , currencies are sorted into three portfolios by the slope of their yield curve (measured as the difference between the 10-year and the three-month yields). The first portfolio contains the currencies of countries with low yield slopes, while the third portfolio contains the currencies of countries with high yield slope. The slope carry trade strategy invests in the first portfolio and shorts the third portfolio. The model is simulated at the monthly frequency. The holding period is one month and returns are annualized.

3 Foreign Long-Term Bond Returns and the Properties of SDFs

In this section, we derive a novel, preference-free necessary condition that complete market models need to satisfy in order to reproduce the downward sloping term structure of currency carry trade risk premia. To do so, we first review a useful decomposition of the pricing kernel.

3.1 Pricing Kernel Decomposition

Our results build on the Alvarez and Jermann (2005) decomposition of the pricing kernel Λ_t into a permanent component $\Lambda_t^{\mathbb{P}}$ and a transitory component $\Lambda_t^{\mathbb{T}}$ using the price of the long-term bond:

$$\Lambda_t = \Lambda_t^{\mathbb{P}} \Lambda_t^{\mathbb{T}}, \text{ where } \Lambda_t^{\mathbb{T}} = \lim_{k \rightarrow \infty} \frac{\delta^{t+k}}{P_t^{(k)}}, \quad (8)$$

where the constant δ is chosen to satisfy the following regularity condition: $0 < \lim_{k \rightarrow \infty} \frac{P_t^{(k)}}{\delta^k} < \infty$ for all t . Note that $\Lambda_t^{\mathbb{P}}$ is equal to:

$$\Lambda_t^{\mathbb{P}} = \lim_{k \rightarrow \infty} \frac{P_t^{(k)}}{\delta^{t+k}} \Lambda_t = \lim_{k \rightarrow \infty} \frac{E_t(\Lambda_{t+k})}{\delta^{t+k}}.$$

The second regularity condition ensures that the expression above is a martingale. Alvarez and Jermann (2005) assume that, for each $t + 1$, there exists a random variable x_{t+1} with finite expected value $E_t(x_{t+1})$ such that almost surely $\frac{\Lambda_{t+1}}{\delta^{t+1}} \frac{P_{t+1}^{(k)}}{\delta^k} \leq x_{t+1}$ for all k . Under those regularity conditions, the infinite-maturity bond return is:

$$R_{t+1}^{(\infty)} = \lim_{k \rightarrow \infty} R_{t+1}^{(k)} = \lim_{k \rightarrow \infty} P_{t+1}^{(k-1)} / P_t^{(k)} = \frac{\Lambda_t^{\mathbb{T}}}{\Lambda_{t+1}^{\mathbb{T}}}. \quad (9)$$

The permanent component, $\Lambda_t^{\mathbb{P}}$, is a martingale and is an important part of the pricing kernel: Alvarez and Jermann (2005) derive a lower bound of its volatility and, given the size of the equity premium relative to the term premium, conclude that it accounts for most of the SDF volatility.⁵ In other words, a lot of persistence in the pricing kernel is needed to jointly deliver a low term premium and a high equity premium. Throughout this paper we assume that stochastic discount factors $\frac{\Lambda_{t+1}}{\Lambda_t}$ and returns R_{t+1} are jointly stationary.

3.2 Main Preference-Free Result on Long-Term Bond Returns

We now use this pricing kernel decomposition to understand the properties of the dollar returns of long-term bonds. Recall that the dollar term premium on a foreign bond position, denoted by $E_t[rx_{t+1}^{(k),\$}]$, can be expressed as the sum of foreign term premium in local currency terms, $E_t[rx_{t+1}^{(k),*}]$, plus a currency risk premium, $E_t[rx_{t+1}^{FX}] = r_t^{f,*} - r_t^f - E_t[\Delta s_{t+1}]$. Here, we consider the dollar term premium of an infinite-maturity foreign bond, so we let $k \rightarrow \infty$.

Proposition 1. *The foreign term premium on the long-term bond in dollars is equal to the domestic term premium plus the difference between the domestic and foreign entropies of the permanent components of the pricing kernels:*

$$E_t[rx_{t+1}^{(\infty),\$}] = E_t[rx_{t+1}^{(\infty)}] + L_t \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} \right) - L_t \left(\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}} \right). \quad (10)$$

⁵Proposition 2 in Alvarez and Jermann (2005) establishes that $L_t \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} \right) \geq E_t \log R_{t+1} - \log R_{t+1}^{\infty}$ for any return R_{t+1} and that $\frac{L \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} \right)}{L \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right)} \geq \text{Min} \left\{ 1, \frac{E \log R_{t+1} / R_t^f - E \log R_{t+1}^{\infty} / R_t^f}{E \log R_{t+1} / R_t^f + L(1/R_t^f)} \right\}$ for any positive return R_{t+1} such that $E \log R_{t+1} / R_t^f + L(1/R_t^f) > 0$. Alvarez and Jermann (2005) take the latter expression to the data and report several lower bounds for the relative variance of the permanent component in their Table 2, page 1989. These lower bounds, obtained with either yields or holding-period returns on long-term bonds, range from 0.76 to 1.11. Thus, the variance of the permanent component is at least 76% of the total variance of the SDF.

To intuitively link the long-run properties of pricing kernels to foreign bond returns and exchange rates, let us consider the simple benchmark of countries represented by stand-in agents with power utility and *i.i.d.* consumption growth rates. In that case, all SDF shocks are permanent ($\Lambda_t = \Lambda_t^{\mathbb{P}}$ for all t). As we shall see, such model is counterfactual. In this model, the risk-free rate is constant, so bonds of different maturities offer the same returns. Foreign bond investments differ from domestic bond investments only because of the presence of exchange rate risk and, since consumption growth rates are *i.i.d.*, exchange rates are stationary in changes but not in levels. Finally, carry trade excess returns are the same at the short end, (see Equation (7)) and at the long end (see Equation (10)) of the yield curve, so the term structure of currency carry trade risk premia is flat. A power utility model with only permanent shocks cannot match the facts.

Let us now turn to the opposite case: a model without permanent shocks in the SDF ($\Lambda_t = \Lambda_t^{\mathbb{T}}$ for all t). In case of an adverse temporary innovation to the foreign pricing kernel, the foreign currency appreciates, so a domestic position in the foreign bond experiences a capital gain. However, this capital gain is exactly offset by the capital loss suffered on the long-term bond as a result of the increase in foreign interest rates. Hence, interest rate exposure completely hedges the temporary component of the currency risk exposure. In this case, as Equation (10) shows, the long-term bond risk premium in dollars should be equal to the domestic term premium.

Beyond these two polar cases, Proposition 1 shows that in order to have differences across countries in bond risk premia, once converted in the same currency, no-arbitrage models need conditional entropy differences in the permanent component of their pricing kernels. If the domestic and foreign pricing kernels have identical conditional entropy, then high local currency term premia are always associated with low currency risk premia and vice-versa, so dollar term premia are identical across currencies.

Proposition 1 is thus the bond equivalent to the usual currency carry trade condition. We gather them below to emphasize their similarities:

$$\begin{aligned} E_t(r x_{t+1}^{FX}) &= L_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) - L_t \left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*} \right), \\ E_t[r x_{t+1}^{(\infty), \$}] - E_t[r x_{t+1}^{(\infty)}] &= L_t \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} \right) - L_t \left(\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}} \right). \end{aligned}$$

To reproduce large currency carry trade risk premia, no-arbitrage models need large differences in the volatilities

of their SDFs. To reproduce the absence of dollar bond risk premia, no-arbitrage models need to feature the same volatilities of the martingale components of their SDFs. As will shall see, this condition is a key tool to assess existing international finance models.

3.3 Additional Assumptions and Interpretation

Dynamic asset pricing models that generate small amounts of dollar return differential predictability may produce moments in small samples that fall within the confidence intervals, but it is useful to have a clear benchmark: Ours is no predictability in dollar bond return differentials for long bonds. This has been the null hypothesis in this literature. Our paper shows that this null cannot be rejected at longer maturities.

In order to use Proposition 1 to interpret our empirical findings, two additional assumptions are required.

Assumption 1: First, since very long-term bonds are rarely available or liquid, we assume that infinite-maturity bond returns can be approximated in practice by 10 and 15-year bond returns. The same assumption is also present in Alvarez and Jermann (2005), Hansen, Heaton, and Li (2008), Hansen and Scheinkman (2009), and Hansen (2012). It is supported by the simulation of the state-of-the-art Joslin, Singleton, and Zhu (2011) term structure model (see section H of the Online Appendix).

Assumption 2: Second, we assume that the level and slope of the yield curve summarize all the relevant information that investors use to forecast dollar bond excess returns. Proposition 1 pertains to *conditional* risk premia and is, thus, relevant for interpreting our empirical time series predictability results and the average excess returns of currency portfolios sorted by conditioning information (the level of the short-term interest rate or the slope of the yield curve). Building portfolios sorted by conditioning variables is a flexible, non-parametric approach to bringing in conditioning information. We cannot definitively rule out the possibility that there other predictors, but in all of the models that we consider, the *spanning hypothesis* holds, and all relevant information for forecasting returns is contained in the yield curve, except in pathological, knife-edge cases. Hence, there are no ‘missing’ predictors in these models that are orthogonal to the level and the slope. There is a lively empirical debate on whether there are unspanned macro variables that have incremental out-of-sample forecasting power for bond returns (see Bauer and Hamilton, 2017, for a thorough evaluation of the empirical evidence). In our models, there are no unspanned macro variables, except in knife-edge cases.

Under those two assumptions, a simple condition illustrates our empirical findings:

Condition 1. *In order for the conditional dollar term premia on infinite-maturity bonds to be identical across countries, the conditional entropy of the permanent SDF component has to be identical across countries: $L_t \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} \right) = L_t \left(\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}} \right)$, for all t .*

If this condition fails, under Assumptions 1 and 2, portfolios sorted on conditioning variables produce non-zero currency carry trade risk premia at the long end of the term structure, as the conditional dollar term premia of long-maturity bonds differ across countries.⁶ Condition 1 is satisfied when permanent shocks are common across countries ($\Lambda_{t+1}^{\mathbb{P}} = \Lambda_{t+1}^{\mathbb{P},*}$ for all t) and thus, in the absence of permanent shocks, when exchange rates are stationary in levels. But note that the stationarity is sufficient but not necessary to satisfy Condition 1.

To develop some intuition for this condition, we rely on an example from Alvarez and Jermann (2005), who consider a model with conditionally log-normally distributed pricing kernels driven by both permanent and transitory shocks.

Example 1. *Consider the following pricing kernel (Alvarez and Jermann, 2005):*

$$\begin{aligned} \log \Lambda_{t+1}^{\mathbb{P}} &= -\frac{1}{2}\sigma_P^2 + \log \Lambda_t^{\mathbb{P}} + \varepsilon_{t+1}^P, \\ \log \Lambda_{t+1}^{\mathbb{T}} &= \log \beta^{t+1} + \sum_{i=0}^{\infty} \alpha_i \varepsilon_{t+1-i}^T, \end{aligned}$$

where α is a square summable sequence, and ε^P and ε^T are serially independent and normally distributed random variables with mean zero, variance σ_P^2 and σ_T^2 , respectively, and covariance σ_{TP} . A similar decomposition applies to the foreign pricing kernel.

In this economy, Alvarez and Jermann (2005) show that the domestic term premium is given by the following expression: $E_t \left[rx_{t+1}^{(\infty)} \right] = \frac{1}{2}\sigma_T^2 + \sigma_{TP}$. Only transitory risk is priced in the market for long-maturity bonds: when marginal utility is transitorily high, interest rates increase because the representative agent wants to borrow, so long-term bonds suffer a capital loss. Permanent shocks to marginal utility do not affect the prices of long-term

⁶For some countries (Australia, Canada, and Sweden), time-series regressions show that the yield slopes predict dollar bond returns with the “wrong” sign: while an increase in the yield slope decreases short-bond carry trade excess returns, it increases the dollar long-bond excess returns (cf Table 1). To match this particular evidence, one may replace Condition 1 with the following condition: if $E_t \left(rx_{t+1}^{FX} \right) = L_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) - L_t \left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*} \right) > 0$, then $E_t [rx_{t+1}^{(\infty),S}] - E_t \left[rx_{t+1}^{(\infty)} \right] = L_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) - L_t \left(\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}} \right) < 0$. We do not study this stricter condition as the amount of predictability on long-bond dollar excess returns is not economically significant, as can be seen in Table 2.

bonds at all. Similarly, the foreign term premium, in local currency terms, is $E_t \left[rx_{t+1}^{(\infty),*} \right] = \frac{1}{2} (\sigma_T^*)^2 + \sigma_{TP}^*$. The currency risk premium is the difference in the two countries' conditional SDF entropy:

$$E_t \left[rx_t^{FX} \right] = r_t^{f,*} - r_t^f - E_t[\Delta s_{t+1}] = \frac{1}{2} (\sigma_T^2 + 2\sigma_{TP} + \sigma_P^2) - \frac{1}{2} \left((\sigma_T^*)^2 + 2\sigma_{TP}^* + (\sigma_P^*)^2 \right).$$

As a result, the foreign term premium in dollars, given by Equation (7), is:

$$E_t \left[rx_{t+1}^{(\infty),\$} \right] = E_t \left[rx_{t+1}^{(\infty),*} \right] + E_t \left[rx_t^{FX} \right] = \frac{1}{2} \sigma_T^2 + \sigma_{TP} + \frac{1}{2} (\sigma_P^2 - (\sigma_P^*)^2).$$

In the Alvarez and Jermann (2005) example, Condition 1 is satisfied, provided that $\sigma_P^2 = (\sigma_P^*)^2$. Then the foreign term premium in dollars equals the domestic term premium:

$$E_t \left[rx_{t+1}^{(\infty),\$} \right] = \frac{1}{2} \sigma_T^2 + \sigma_{TP} = E_t \left[rx_{t+1}^{(\infty)} \right].$$

This example illustrates our theoretical and empirical findings. It shows how SDFs can deliver carry trade risk premia with Treasury bills, when $\sigma_T \neq \sigma_T^*$ or $\sigma_{TP} \neq \sigma_{TP}^*$, while producing no carry trade risk premia with long-term bonds when $\sigma_P = \sigma_P^*$. It also shows that exchange rate stationarity ($\varepsilon_{t+1}^P = \varepsilon_{t+1}^{*,P}$) is a sufficient but not a necessary condition to produce no carry trade risk premia with long-term bonds. This example, however, lacks the time variation in risk premia that has been extensively documented in equity, bond, and currency markets. We turn now to a second example, with time-varying risk premia, to better understand the achievements and drawback of the reduced-form model presented in Section 2.2.

The Lustig, Roussanov, and Verdelhan (2014) model simulated in Section 2.2 builds on the seminal Cox, Ingersoll, and Ross (1985) model and introduces multiple common and local shocks. In order to illustrate the SDF decomposition and Condition 1 in a very transparent setting, we focus here on a simple two-country version of the Cox, Ingersoll, and Ross (1985) model with only one kind of shocks per country.

Example 2. *The two-country Cox, Ingersoll, and Ross (1985) model is defined by the following law of motions*

for the SDFs:

$$\begin{aligned}
-\log \frac{\Lambda_{t+1}}{\Lambda_t} &= \alpha + \chi z_t + \sqrt{\gamma z_t} u_{t+1}, \\
z_{t+1} &= (1 - \phi)\theta + \phi z_t - \sigma \sqrt{z_t} u_{t+1}, \\
-\log \frac{\Lambda_{t+1}^*}{\Lambda_t^*} &= \alpha^* + \chi^* z_t^* + \sqrt{\gamma^* z_t^*} u_{t+1}^*, \\
z_{t+1}^* &= (1 - \phi^*)\theta^* + \phi^* z_t^* - \sigma^* \sqrt{z_t^*} u_{t+1}^*,
\end{aligned}$$

where z_t and z_t^* are the two state variables that govern the volatilities of the normal shocks u_{t+1} and u_{t+1}^* .

The domestic risk-free rate is given by $r_t^f = \alpha + (\chi - \frac{1}{2}\gamma)z_t$. The log bond prices are affine in the state variable z : $p_t^{(n)} = -B_0^n - B_1^n z_t$, where the bond price coefficients evolve according to the second-order difference equations:

$$\begin{aligned}
B_0^n &= \alpha + B_0^{n-1} + B_1^{n-1}(1 - \phi)\theta, \\
B_1^n &= \chi - \frac{1}{2}\gamma + B_1^{n-1}\phi - \frac{1}{2}(B_1^{n-1})^2 \sigma^2 + \sigma \sqrt{\gamma} B_1^{n-1}
\end{aligned}$$

The temporary and martingale components of the pricing kernel are:

$$\begin{aligned}
\Lambda_t^{\mathbb{T}} &= \lim_{n \rightarrow \infty} \frac{\delta^{t+n}}{P_t^{(n)}} = \lim_{n \rightarrow \infty} \delta^{t+n} e^{B_0^\infty + B_1^\infty z_t}, \\
\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} &= \frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} \left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} \right)^{-1} = \beta^{-1} e^{-\alpha - \chi z_t - \sqrt{\gamma z_t} u_{t+1}} e^{-B_1^\infty [(\phi-1)(z_t - \theta) - \sigma \sqrt{z_t} u_{t+1}]}.
\end{aligned}$$

where the constant β is chosen in order to satisfy : $0 < \lim_{n \rightarrow \infty} \frac{P_t^{(n)}}{\delta^n} < \infty$. The limit of $B_0^n - B_0^{n-1}$ is finite: $\lim_{n \rightarrow \infty} B_0^n - B_0^{n-1} = \alpha + B_1^\infty(1 - \phi)\theta$, where B_1^∞ is defined implicitly in the second order equation $B_1^\infty = \chi - \frac{1}{2}\gamma + B_1^\infty\phi - \frac{1}{2}(B_1^\infty)^2 \sigma^2 + \sigma \sqrt{\gamma} B_1^\infty$. As a result, B_0^n grows at a linear rate in the limit. We choose the constant β to offset the growth in B_0^n as n becomes very large. Setting $e^\beta = e^{-\alpha - B_1^\infty(1-\phi)\theta}$ guarantees that Assumption 1 in Alvarez and Jermann (2005) is satisfied. The temporary pricing component of the SDF is thus equal to: $\Lambda_t^{\mathbb{T}} = e^{\beta t - B_1^\infty z_t}$. In this two-country model, Condition 1 requires that

$$(\sqrt{\gamma} - B_1^\infty \sigma) z_t = \left(\sqrt{\gamma^*} - B_1^{\infty*} \sigma^* \right) z_t^*.$$

There are two ways to ensure that this condition is satisfied, depending on whether the shocks are either country-specific or common.

First, let us consider a model with only country-specific shocks and factors. Let us assume that these countries share all of the parameters. Since z_t and z_t^* will differ, a necessary and sufficient condition is that $B_1^\infty = \sqrt{\gamma}/\sigma$, and $B_1^{\infty,*} = \sqrt{\gamma^*}/\sigma^*$. In this case, there are no permanent shocks to the pricing kernel. Long bond prices absorb the full, cumulative effect of the shock the pricing kernel. To see why, note that in this case $B_1^\infty = \chi/(1 - \phi)$. The log currency risk premium is given by $E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma(z_t - z_t^*)$ and the expected term premium is simply $E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}\gamma z_t$. The expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to the U.S. term premium: $E_t[rx_{t+1}^{(\infty),*}] + E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma z_t$. In a two-country Cox, Ingersoll, and Ross (1985) model with country-specific shocks, Condition 1 implies some restrictions on the model parameters, and more crucially, the absence of permanent shocks in the SDFs and thus in exchange rates: exchange rates are stationary in levels. The case of country-specific shocks, however, is not the most interesting as such shocks can be diversified away.

Second, let us consider a model with common shocks and common factors: $z_t = z_t^*$ is a global state variable. In this case, the two countries share the parameters $\sigma = \sigma^*$, $\phi = \phi^*$, $\theta = \theta^*$ which govern the dynamics of z_t and z_t^* . Condition 1 then requires that $\sqrt{\gamma} + B_1^\infty \sigma = \sqrt{\gamma^*} + B_1^{\infty*} \sigma$. Note that B_1^∞ depends on χ and γ , as well as on the global parameters ϕ and σ . Hence, we also need $\gamma = \gamma^*$ and $\chi = \chi^*$. In this case, Condition 1 requires that both countries have the same pricing kernel. This case illustrates the tension between the carry trade at the short and the long end of the yield curve: in order to replicate the carry trade on Treasury bills, the two-country Cox, Ingersoll, and Ross (1985) model needs to feature heterogeneous exposure to common shocks; yet, in order to replicate the absence of carry trade returns on long term bonds, this model needs to satisfy Condition 1 that prohibits such heterogeneous exposure to common shocks.

This reduced-form model shows the difference between unconditional and conditional risk premia. In the case of a symmetric model ($\sigma = \sigma^*$, $\phi = \phi^*$, $\theta = \theta^*$), the unconditional risk currency risk premium is zero: $E[rx_{t+1}^{FX}] = \frac{1}{2}\gamma(E[z_t] - E[z_t^*]) = 0$, while the conditional currency risk premium is not: $E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma(z_t - z_t^*) \neq 0$. The conditional risk premium moves with the two state variables z_t and z_t^* . These variables take different values across countries because countries experience different shocks. Our work is about the conditional risk premium: the portfolios are built by sorting on the level and slope of interest rates, which in this model are

driven by the state variables z_t and z_t^* . When taking the average of all returns in the high interest rate portfolio for example, we are averaging over low values of the state variables z_t , not over all possible values of z_t . In this symmetric model, the average currency risk premium obtained by simply averaging all returns would be zero. Yet, as its simulation show in the previous section, the average currency risk premium obtained on the high interest rate portfolio (for example) is as large as in the data. Our portfolio and predictability tests thus are about conditional risk premia, as is our preference-free condition.

Condition 1 is a diagnostic tool that can be applied to richer models. In the Online Appendix, we derive parametric restrictions to implement Condition 1 in four classes of dynamic term structure models, from the simple one-factor Vasicek (1977) and Cox, Ingersoll, and Ross (1985) models to their multi-factor versions. In order to save space, we summarize the implications of Condition 1 in Table A18.

In the Online Appendix, we also consider leading structural macro-finance models. We focus on models of the real SDF, because (i) there is no evidence to suggest that inflation risk can account for UIP deviations, and (ii) inflation risk does not seem to account for the cross-country variation in local currency bond excess returns, as shown in Section 1. We consider two-country versions of the habit, long-run risk, and disaster risk models. In a nutshell, among the reduced-form term structure models we consider, Condition 1 implies novel parameter restrictions for all models (and in some cases, it rules out all permanent shocks or the time-variation in the price of risk). In order to save space, we summarize the implications of Condition 1 in Table 4. All the intermediary steps to determine the two components of the pricing kernels are reported in the Appendix. In the habit model with common shocks, the carry trade risk premia and Condition 1 requires that countries exhibit the same risk-aversion and the same volatility of consumption growth shocks but they can differ in the persistence of their habit levels. The long-run risk models satisfy Condition 1 only with common shocks and for knife-edge parameter values. This model generically produces non-stationary real exchange rates. For the disaster risk models, common shocks are also necessary for Condition 1 to hold and the downward term structure of carry trade risk premia is consistent with heterogeneity in the rate of time preference, the rate of depreciation, or the country-specific growth rate, but no heterogeneity in the coefficient of risk aversion, the common and country-specific consumption drops in case of a disaster, and the probability of a disaster. Overall, the term structure of carry trade raises the bar for international finance models.

Table 4: Long-Run Risk-Neutrality: Dynamic Asset Pricing Model Scorecard

	<i>Symmetric Models with Country-specific Shocks</i>	<i>Asymmetric Models with Common Shocks</i>
External Habit Model	✓	✓ Only heterogeneity in ϕ No heterogeneity in (γ, σ^2)
Long Run Risks Model	No	Only knife-edge cases
Disaster Model	No	✓ Only heterogeneity in (R, λ, g_w) No heterogeneity in (γ, B, F, p_t)

This table summarizes whether each class of models can satisfy Condition 1. The left section of the table focuses on models with only country-specific shocks in which all countries have the same parameters. The right section focuses on models with only common shocks and heterogeneity in the parameters. In the external habit model (Campbell and Cochrane, 1999; Wachter, 2006; Verdelhan, 2010; Stathopoulos, 2017), the parameters ϕ and B govern the dynamics of the surplus consumption ratio process. In the long run risk model (Bansal and Yaron, 2004; Colacito and Croce, 2011; Bansal and Shaliastovich, 2013; Engel, 2016), Condition 1 is always violated except in knife-edge cases. In the disaster model (Farhi and Gabaix, 2016; Gabaix, 2012; Wachter, 2013), the parameters R , λ , and g_w govern the rate of time preference, the rate of depreciation and the country-specific growth rate, while the parameters γ , B , F , p_t are the coefficient of risk aversion, the common consumption drop in case of a disaster, the country-specific consumption drop in case of a disaster, and the probability of a disaster. The details are in section E of the Online Appendix.

4 U.I.P. in the Long Run

Examining the conditional moments of one-period returns on long-maturity bonds, the focus of our paper, is not equivalent to studying the moments of long-maturity bond yields in tests of the long-horizon U.I.P. condition. In this section, we show the links and differences between these moments. To do so, we use again the decomposition of the pricing kernel proposed by Alvarez and Jermann (2005). Exchange rate changes can be represented as the product of two components, defined below:

$$\frac{S_{t+1}}{S_t} = \left(\frac{\Lambda_{t+1}^{\mathbb{P}} \Lambda_t^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P}} \Lambda_{t+1}^{\mathbb{P},*}} \right) \left(\frac{\Lambda_{t+1}^{\mathbb{T}} \Lambda_t^{\mathbb{T},*}}{\Lambda_t^{\mathbb{T}} \Lambda_{t+1}^{\mathbb{T},*}} \right) = \frac{S_{t+1}^{\mathbb{P}} S_{t+1}^{\mathbb{T}}}{S_t^{\mathbb{P}} S_t^{\mathbb{T}}}. \quad (11)$$

Exchange rate changes reflect differences in both the transitory and the permanent component of the two countries' pricing kernels.⁷ If two countries share the same martingale component of the pricing kernel, then

⁷Note that $S_{t+1}^{\mathbb{P}}$, the ratio of two martingales, is itself not a martingale in general. However, in the class of affine term structure models, this exchange rate component is indeed a martingale.

the resulting exchange rate is stationary and Condition 1 is trivially satisfied. However, as already mentioned, exchange rate stationarity is obviously not necessary for long-run risk neutrality. This exchange rate decomposition implies a lower bound on the cross-country correlation of the permanent components of the SDFs. In the interest of space, we present it in the Online Appendix and focus in the main text on its implications for long-run U.I.P.

The definition of the transitory component of exchange rate changes, given by $\Delta s_{t+1}^{\mathbb{T}} = (\lambda_{t+1}^{\mathbb{T}} - \lambda_t^{\mathbb{T}}) - (\lambda_{t+1}^{\mathbb{T},*} - \lambda_t^{\mathbb{T},*})$, where $\lambda_t^{\mathbb{T}} \equiv \log \Lambda_t^{\mathbb{T}} = \lim_{k \rightarrow \infty} (t+k) \log \delta + \lim_{k \rightarrow \infty} k y_t^{(k)}$, implies that a currency experiences a temporary appreciation when its long-term interest rates increase more than the foreign ones:

$$\Delta s_{t+1}^{\mathbb{T}} = \log \delta - \log \delta^* + \lim_{k \rightarrow \infty} k \left(\Delta y_{t+1}^{(k)} - \Delta y_{t+1}^{(k),*} \right). \quad (12)$$

By backward substitution, it follows that the transitory component of the exchange rate in levels is given by the spread in long-term yields:

$$s_t^{\mathbb{T}} = s_0 + t(\log \delta - \log \delta^*) + \lim_{k \rightarrow \infty} k \left(y_t^{(k)} - y_t^{(k),*} \right) - \lim_{k \rightarrow \infty} k \left(y_0^{(k)} - y_0^{(k),*} \right). \quad (13)$$

This decomposition of exchange rates implies that deviations from the long-run UIP are due to the permanent component of exchange rates. We use $rx_{t \rightarrow t+k}^{FX} = k \left(y_t^{(k),*} - y_t^{(k)} \right) - \Delta s_{t \rightarrow t+k}$ to denote the currency excess return over a holding period of k years.

Proposition 2. *The expected rate of transitory depreciation is equal to the spread in the long-term interest rates:*

$$\lim_{k \rightarrow \infty} E_t \Delta s_{t \rightarrow t+k}^{\mathbb{T}} = - \lim_{k \rightarrow \infty} k \left(y_t^{(k)} - y_t^{(k),*} \right).$$

Thus, the deviation from long-run U.I.P is the permanent component of the exchange rates:

$$\lim_{k \rightarrow \infty} E_t rx_{t \rightarrow t+k}^{FX} = \lim_{k \rightarrow \infty} k \left(y_t^{(k),*} - y_t^{(k)} \right) - \lim_{k \rightarrow \infty} E_t \Delta s_{t \rightarrow t+k} = - \lim_{k \rightarrow \infty} E_t \Delta s_{t \rightarrow t+k}^{\mathbb{P}}.$$

If exchange rates are stationary in levels, in which case the permanent component of exchange rate changes is zero, then long-run U.I.P. holds. In this case, the slope coefficient in the regression of long-run exchange

rate changes on yield differences converges to one and the intercept converges to zero. This result is previewed in Backus, Boyarchenko, and Chernov (2016), who show that claims to stationary cash flows earn a zero log risk premium over long holding periods. It follows that long-run deviations from U.I.P. are consistent with no arbitrage only if the exchange rate is not stationary in levels.

Let us go back to the symmetric two-country CIR model with country-specific factors presented in Example 2. In that model, the transitory component of the exchange rate is given by: $s_t^{\mathbb{T}} = s_0 + B_1^{\infty} ((z_t - z_0) - (z_t^* - z_0^*))$. As already noted, the pricing kernel is not subject to permanent shocks when $B_1^{\infty} = \frac{\sqrt{\gamma}}{\sigma} = \frac{\chi}{1-\phi}$.⁸ In that case, the exchange rate is stationary and hence $s_t = s_t^{\mathbb{T}}$. The expected rate of depreciation is then equal to

$$\lim_{k \rightarrow \infty} E_t[\Delta s_{t \rightarrow t+k}] = \frac{\chi}{1-\phi} (z_t - z_t^*) = - \lim_{k \rightarrow \infty} k \left(y_t^{(k)} - y_t^{(k),*} \right).$$

Even if exchange rates are not stationary in levels, as long as Condition 1 holds, then (under some additional regularity conditions), long-run U.I.P. holds on average, as the following proposition shows.

Proposition 3. *If the stochastic discount factors $\frac{\Lambda_{t+1}}{\Lambda_t}$ and $\frac{\Lambda_{t+1}^*}{\Lambda_t^*}$ are strictly stationary, and $\lim_{k \rightarrow \infty} \frac{1}{k} L \left(E_t \left[\frac{\Lambda_{t+k}}{\Lambda_t} \right] \right) = 0$ and $\lim_{k \rightarrow \infty} \frac{1}{k} L \left(E_t \left[\frac{\Lambda_{t+k}^*}{\Lambda_t^*} \right] \right) = 0$, then the per period long-run currency risk premium is given by:*

$$\lim_{k \rightarrow \infty} \frac{1}{k} E[rx_{t \rightarrow t+k}^{FX}] = \lim_{k \rightarrow \infty} E \left(y_t^{(k),*} - y_t^{(k)} \right) - \lim_{k \rightarrow \infty} \frac{1}{k} E[\Delta s_{t \rightarrow t+k}] = \left[L \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} \right) - L \left(\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}} \right) \right].$$

This immediately implies that the per period currency risk premium converges to zero on average, and therefore long-run U.I.P. holds on average, if Condition 1 is satisfied. Importantly, unconditional long-run U.I.P. does not necessarily require stationary exchange rates, as Condition 1 is a weaker condition than exchange rate stationarity. Moreover, Condition 1 may hold while deviations from long-run U.I.P. still exist. Proposition 3 only implies that long-run U.I.P. holds on average, not at any date. As a result, the study of holding period returns of long-term bonds, the topic of this paper, is not the same as the study of U.I.P. in the long-run.

⁸The restrictions $B_1^{\infty} = \frac{\sqrt{\gamma}}{\sigma} = \frac{\chi}{1-\phi}$ have a natural interpretation as restrictions on the long-run loadings of the exchange rate on the risk factors: $\sum_{i=1}^{\infty} E_t[\Delta s_{t+i}] = \sum_{i=1}^{\infty} E_t[m_{t+i} - m_{t+i}^*] = \sum_{i=1}^{\infty} \phi^{i-1} \chi (z_t^* - z_t)$.

5 Conclusion

While holding period bond returns, expressed in a common currency, differ across G10 countries at the short end of the yield curve (the U.I.P. puzzle), they are rather similar at the long end. In other words, the term structure of currency carry trade risk premia is downward-sloping. Replicating such a term structure is a non-trivial for most models: recent no-arbitrage models of international finance that are able to address the U.I.P. puzzle fail to replicate the downward-sloping term structure of carry trade risk premia.

We derive a preference-free result that helps assess existing models and guides future theoretical and empirical work. In order to exhibit similar long-term bond returns when expressed in the same units, complete market models need to exhibit the same volatility of the permanent components of their pricing kernels. This condition implies novel parameter restrictions in the workhorse no-arbitrage models of international finance.

Our results show that exchange rate risk is different from equity and bond risk. In order to account for the high equity premium and the low term premium, most of the variation in the marginal utility of wealth in no-arbitrage models must come from *permanent* shocks. Yet, differences across countries in how *temporary* shocks affect the marginal utility of wealth — and thus exchange rates — appear as natural way to align no-arbitrage models with the downward-sloping term structure of carry trade risk premia.

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APPENDIX

A Data

Our zero-coupon bond dataset covers the same benchmark sample of G10 countries, but from at most 1/1975 to 12/2015. We use the entirety of the dataset in Wright (2011) and complement the sample, as needed, with sovereign zero-coupon curve data sourced from Bloomberg, estimated from government notes and bonds as well as interest rate swaps of different maturities. The panel is unbalanced: for each currency, the sample starts with the beginning of the Wright (2011) dataset. The starting dates for each country are as follows: 2/1987 for Australia, 1/1986 for Canada, 1/1973 for Germany, 1/1985 for Japan, 1/1990 for New Zealand, 1/1998 for Norway, 12/1992 for Sweden, 1/1988 for Switzerland, 1/1979 for the U.K., and 12/1971 for the U.S. For New Zealand, the data for maturities above 10 years start in 12/1994. Yields are available for bond maturities ranging from three months to 15 years, in three-month increments.

To construct averages of credit ratings, we assign each rating to a number, with a smaller number corresponding to a higher rating. In particular, a credit rating of AAA corresponds to a numerical value of 1, with each immediately lower rating getting assigned the immediately higher numerical value: AA+ corresponds to a numerical value of 2 and AA to 3, all the way down to CC-(22) and SD (23). We also construct rating series adjusted for outlook, as follows: a ‘Negative’ outlook corresponds to an upward adjustment of 0.5 in the numerical value of the rating, a ‘Watch Negative’ outlook to an upward adjustment of 0.25, a ‘Stable’ or ‘Satisfactory’ outlook to no adjustment, and a ‘Positive’, ‘Strong’ or ‘Very Strong’ outlook to a downward adjustment of 0.5. For example, a credit rating of BB (coded as 12) receives a numerical value of 12.5 with a ‘Negative’ outlook and a value of 11.5 with a ‘Positive’ outlook. In order to construct credit rating averages for portfolios formed before 7/1989, we backfill each country’s credit rating by assuming that the country’s rating before 7/1989 is equal to its rating at the first available observation.

B Proofs

- Proof of Proposition 1:

Proof. The proof builds on some results in Backus, Foresi, and Telmer (2001) and Alvarez and Jermann (2005). Specifically, Backus, Foresi, and Telmer (2001) show that the foreign currency risk premium is equal to the difference between domestic and foreign total SDF entropy:

$$(f_t - s_t) - E_t[\Delta s_{t+1}] = L_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) - L_t \left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*} \right).$$

Furthermore, Alvarez and Jermann (2005) establish that total SDF entropy equals the sum of the entropy of the permanent pricing kernel component and the expected log term premium:

$$L_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) = L_t \left(\frac{\Lambda_{t+1}^p}{\Lambda_t^p} \right) + E_t \left(\log \frac{R_{t+1}^{(\infty)}}{R_t^f} \right).$$

Applying the Alvarez and Jermann (2005) decomposition to the Backus, Foresi, and Telmer (2001) expression yields the desired result.

To derive the Backus, Foresi, and Telmer (2001) expression, consider a foreign investor who enters a forward position in the currency market with payoff $S_{t+1} - F_t$. The investor’s Euler equation is:

$$E_t \left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*} (S_{t+1} - F_t) \right) = 0.$$

In the presence of complete, arbitrage-free international financial markets, exchange rate changes equal the ratio of the domestic and foreign pricing kernels:

$$\frac{S_{t+1}}{S_t} = \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\Lambda_t^*}{\Lambda_{t+1}^*},$$

Dividing the investor’s Euler equation by S_t and applying the no arbitrage condition, the forward discount is:

$$f_t - s_t = \log E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) - \log E_t \left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*} \right).$$

The second component of the currency risk premium is expected foreign appreciation; applying logs and conditional expectations to the no arbitrage condition above leads to:

$$E_t[\Delta s_{t+1}] = E_t \left(\log \frac{\Lambda_{t+1}}{\Lambda_t} \right) - E_t \left(\log \frac{\Lambda_{t+1}^*}{\Lambda_t^*} \right).$$

Combining the two terms of the currency risk premium leads to:

$$(f_t - s_t) - E_t[\Delta s_{t+1}] = \log E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) - E_t \left(\log \frac{\Lambda_{t+1}}{\Lambda_t} \right) - \log E_t \left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*} \right) + E_t \left(\log \frac{\Lambda_{t+1}^*}{\Lambda_t^*} \right)$$

Applying the definition of conditional entropy in the equation above yields the Backus, Foresi, and Telmer (2001) expression.

To derive the Alvarez and Jermann (2005) result, first note that since the permanent component of the pricing kernel is a martingale, its conditional entropy can be expressed as follows:

$$L_t \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} \right) = -E_t \left(\log \frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} \right).$$

The definition of conditional entropy implies the following decomposition of total pricing kernel entropy:

$$L_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) = \log E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) - E_t \left(\log \frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} \frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} \right)$$

or, using the above expression for the conditional entropy of the permanent pricing kernel component:

$$L_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) = -\log R_t^f - E_t \left(\log \frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} \right) + L_t \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} \right).$$

The Alvarez and Jermann (2005) result hinges on:

$$\lim_{k \rightarrow \infty} R_{t+1}^{(k)} = \Lambda_t^{\mathbb{T}} / \Lambda_{t+1}^{\mathbb{T}}.$$

Under the assumption that $0 < \lim_{k \rightarrow \infty} \frac{P_t^{(k)}}{\delta^k} < \infty$ for all t , one can write:

$$\lim_{k \rightarrow \infty} R_{t+1}^{(k)} = \lim_{k \rightarrow \infty} \frac{E_{t+1} \left(\frac{\Lambda_{t+k}}{\Lambda_{t+1}} \right)}{E_t \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right)} = \frac{\lim_{k \rightarrow \infty} \frac{E_{t+1}(\Lambda_{t+k}/\delta^{t+k})}{\Lambda_{t+1}}}{\lim_{k \rightarrow \infty} \frac{E_t(\Lambda_{t+k}/\delta^{t+k})}{\Lambda_t}} = \frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = \Lambda_t^{\mathbb{T}} / \Lambda_{t+1}^{\mathbb{T}}.$$

Thus, the infinite-maturity bond is exposed only to transitory pricing kernel risk. \square

- Proof of Proposition 2:

Proof. The transitory component of the exchange rate in levels is given by the spread in long-term yields:

$$s_t^{\mathbb{T}} = s_0 + t(\log \delta - \log \delta^*) + \lim_{k \rightarrow \infty} k \left(y_t^{(k)} - y_t^{(k),*} \right) - \lim_{k \rightarrow \infty} k \left(y_0^{(k)} - y_0^{(k),*} \right).$$

This follows directly from the definition of the transitory component of the pricing kernel. Note that $\lim_{k \rightarrow \infty} k \left(y_t^{(k)} - y_t^{(k),*} \right)$ is not a constant. If it was, the transitory component of the exchange rate would always be a constant.

This, in turn, implies that the rate of appreciation over longer holding periods is given by:

$$\lim_{k \rightarrow \infty} \Delta s_{t \rightarrow t+k}^{\mathbb{T}} = \lim_{k \rightarrow \infty} k(\log \delta - \log \delta^*) + \lim_{k \rightarrow \infty} k \left(y_{t+k}^{(k)} - y_{t+k}^{(k),*} \right) - \lim_{k \rightarrow \infty} k \left(y_t^{(k)} - y_t^{(k),*} \right).$$

After taking conditional expectations, this expression simplifies to:

$$\lim_{k \rightarrow \infty} E_t \Delta s_{t \rightarrow t+k}^{\mathbb{T}} = - \lim_{k \rightarrow \infty} k \left(y_t^{(k)} - y_t^{(k),*} \right),$$

where we have used $\log \delta = -\lim_{k \rightarrow \infty} E_t y_{t+k}^{(k)} = -\lim_{k \rightarrow \infty} E(y_t^k)$; the average long yield in logs is equal to $\log \delta$ under regularity conditions given in Borovička, Hansen, and Scheinkman (2016) on page 2515 and 2516. \square

- Proof of Proposition 3:

Proof. We start from the following equation:

$$\lim_{k \rightarrow \infty} \frac{1}{k} E[rx_{t \rightarrow t+k}^{FX}] = \lim_{k \rightarrow \infty} E\left(y_t^{(k),*} - y_t^{(k)}\right) - \lim_{k \rightarrow \infty} \frac{1}{k} E[\Delta s_{t \rightarrow t+k}] = \lim_{k \rightarrow \infty} (1/k) E \left[L_t \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right) - L_t \left(\frac{\Lambda_{t+k}^*}{\Lambda_t^*} \right) \right].$$

This follows from the definition of the currency risk premium $E_t[rx_{t \rightarrow t+k}] = L_t \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right) - L_t \left(\frac{\Lambda_{t+k}^*}{\Lambda_t^*} \right)$ in equation (7), extended to longer horizons. Next, we note that $L(x_{t+1}) = EL_t(x_{t+1}) + L(E_t(x_{t+1}))$. Given the stationarity of the stochastic discount factor, we know that $\lim_{k \rightarrow \infty} (1/k) L \left(E_t \frac{\Lambda_{t+k}}{\Lambda_t} \right) = 0$. Hence $\lim_{k \rightarrow \infty} (1/k) L \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right) = \lim_{k \rightarrow \infty} (1/k) EL_t \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right)$. It then follows that

$$\lim_{k \rightarrow \infty} (1/k) E \left[L_t \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right) - L_t \left(\frac{\Lambda_{t+k}^*}{\Lambda_t^*} \right) \right] = \lim_{k \rightarrow \infty} (1/k) \left[L \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right) - L \left(\frac{\Lambda_{t+k}^*}{\Lambda_t^*} \right) \right] = \left[L \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} \right) - L \left(\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}} \right) \right].$$

The last equality follows directly from the Alvarez-Jermann decomposition of the pricing kernel (see Alvarez and Jermann (2005)'s proposition 6). \square