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EXPERIMENTATION IN FEDERAL SYSTEMS

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ABSTRACT

We present a model where heterogeneous districts choose both whether to experiment and the policies to experiment with. Since districts learn from each other, the first-best requires that policy experiments converge so that innovations are useful also for neighbors. However, the equilibrium implies the reverse – policy divergence – since each district uses its policy choice to discourage free-riding. We then study a clumsy central government that harmonizes final policy choices. This progressive concentration of power induces a policy tournament that can increase the incentive to experiment and encourage policy convergence. We derive the best political regime as well as the optimal levels of heterogeneity, transparency, prizes, and intellectual property rights.

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1 Introduction

Just as people learn from each other, so do governments. Whilst people learn from each other about restaurants and career choices, governments learn from each other about good policies. Governments observe their neighbors, as well as states and countries further afield, and imitate their policy successes while avoiding their policy failures.

The spread of policies in this way – known as policy *diffusion* – has most famously been documented as a strength of federal systems. Yet information need not be constrained by political borders, and policies can diffuse across countries as well, from friends to even sworn enemies.[1] In recent times, researchers have begun to document not just the existence of these flows, but the rate, extent, and channels via which policy information passes. Within the U.S. federal system, Volden (2006) shows that the channel that policy successes spread is across states that are *similar*. This pattern also is prominent across national borders. Buera, Monge-Naranjo, and Primiceri (2011) show how learning from similar countries accounts for a majority of the movement toward market policies and economic growth throughout the late twentieth century.

Despite this progress, much remains unknown about policy experimentation. That policy successes exist at all confirms that free-riding – the classic concern of the literature – doesn't eliminate experimentation altogether. Yet this doesn't speak to whether the efficient amount of experimentation is undertaken. More subtlely, the positive finding that *similar* states can learn from each other hides a deeper pathology of policy experimentation. If policies spread only across similar states, then they do not spread to dissimilar states and, consequently, the informational benefits of policy experimentation are bounded more tightly than previously thought. In fact, that different policies may benefit states unequally raises the novel question of whether the policy experiments undertaken are the *right* type of experiments. That is to say, are the policy experiments that are undertaken those that cast off the most useful information to the broadest array of states? Until now, the theoretical literature on policy experimentation has focused exclusively on the *quantity* of policy experimentation, ignoring the question of which policies are actually experimented with. In a world of similar and dissimilar states – as is typically assumed in models of political economy – this question is of central importance.

The objective of this paper is to shed light on exactly this question. We present a simple model of policy experimentation with political units that we refer to as districts,

[1] The history of warfare, for one, is replete with examples of policies, strategies, and technologies imitated by enemies.

although they can equivalently be interpreted as states, nations, etc. A key novelty of the model of experimentation is that the districts choose both whether to experiment with policy – the quantity – and the policy with which to experiment with, what we refer to as the type. The inclusion of policy choice is important as the districts do not share the same policy preferences – they may be similar or dissimilar – and, therefore, the usefulness of a particular policy experiment varies across the districts. Our primary interest is in how the classic question of free-riding and the quantity of experimentation interacts with the choice of policy with which to experiment. Formally, we model a policy outcome as having two components: a spatial preference component to capture differences across districts, and a quality, public good component, that captures the transferability of successful policy innovation.

We present three sets of results. We begin with decentralized systems in which two districts are free to make their own policy choices, whether due to a weak central government or the non-existence of any formal federation. For this case, we find first that preference heterogeneity delivers some positive news: The incentive to free ride is mitigated by heterogeneity, declining the more different are the districts. This is intuitive. The less similar a neighboring district is to another, the less useful is information revealed by each other's experiments and the more inclined is each to engage in its own experiments.

Within this positive result, however, lurks a deeper inefficiency. The districts, freed to choose the policy with which to experiment, are also freed to take the free-riding problem into their own hands. Whereas efficiency calls for the districts to experiment with policies more favorable to their neighbors, we find that in equilibrium the districts deliberately choose policies that are *less* attractive to each other. We show that they do this even to the degree that they sacrifice their own immediate welfare by experimenting with policies other their own ideal policies. Specifically, the equilibrium policy choices diverge and are Pareto inefficient. The divergence increases as heterogeneity declines as the incentive to free-ride would otherwise increase. This leads to the surprising conclusion that it is not similar districts that benefit each other the most with informational spillovers, but rather that the most efficient decentralized federal systems are those with some heterogeneity across the districts.

The inefficiency of policy experimentation with decentralized authority raises the question of whether a better outcome can be achieved. Our second set of results addresses the question of design. We introduce a central authority into the federal system and follow the literature in supposing that the central authority harmonizes the districts onto one specific choice. We depart from the literature, however, in conceiving of the centralization of authority as a dynamic process. We show that *ex post* policy harmonization is unambiguously inefficient as a district may be forced to implement the less preferred policy of a neighbor. *Ex ante*, on the other hand, policy harmonization can provide a positive benefit by providing the incentives for more efficient policy experiments during the earlier, decentralized phase. This benefit emerges as the looming shadow of future harmonization induces the districts to experiment to ensure that it is their policy – their successful experiment – that is harmonized upon rather than the policy of the other district. The districts compete, therefore, in what might be thought of as a *policy tournament*, so that their policy can win the prize of adoption by the central authority. This implies that the incentive to experiment may increase and policies might converge.

The view of federalism we present is inherently dynamic and, to the best of our knowledge, novel in the academic literature. We sometimes refer to it as *progressive federalism*. Nevertheless, the dynamic concentration of power is evident empirically. The largest – and most successful – federal systems, namely the United States and the European Union, have both followed this dynamic path. Indeed, in a landmark and exhaustive examination of *all* federal systems in place since U.S. independence, Riker (1964) concludes that those that exhibit increased centralization of authority outperform those that moved in the opposite direction. Moreover, our model also predicts that this centralization will proceed in lock-step with experimentation as the policies in effect diffuse from the districts up to the central authority. This comovement is documented by Rabe (2004, 2006) for the case of the United States. Our contribution is to show that this dynamic is more than merely a symptom of changing institutional preferences, but in fact is a key element of the of the federal system as an incentive mechanism.

With a richer conception of federalism in place, our final set of results explores how incentives to experiment interact with several prominent debates in political economy – both within and outside the federalism literature – as well as issues of robustness. In turn, we examine how the inclusion of explicit coordination benefits (a frequent assumption in the literature) affects experimentation, we study the role of transparency and correlation as experimental successes are easier or more difficult to imitate and when the lessons are (partially) transferable to other policies and domains, to the use of prizes and transfers to facilitate efficient experimentation, and finally the impact of larger federal unions, with many districts. The objective of these exercises is to further illuminate how incentives to experiment with policy are driven by political institutions, and also to show how these various features of the political landscape have broader impacts than noted until now.

The idea that a federalist system facilitates policy experimentation has a long history. It has been prominent in academic and policy circles at least since the time of Justice Louis Brandeis, who uttered the famous phrase:

> *"It is one of the happy incidents of the federal system that a single courageous state may, if its citizens choose, serve as a laboratory; and try novel social and economic experiments without risk to the rest of the country."*
> Justice Brandeis, 1932.

The Justice presents an optimistic perspective on federalism. Yet he fails to grasp (or at least to mention) the problem of free-riding, let alone how in a diverse country federalism induces the right experiments to be undertaken. Brandeis may indeed be correct that a policy experiment in a single state is without risk to the rest of the country, but he ignores the possibility that the experiment may also be without benefit.

More subtle – but of no less importance – the Justice fails to explain why a federal system is required at all. If information is unconstrained by physical borders, why must the states formally lash themselves together in a federal union to benefit from the wonders of the policy laboratory? A contribution of our model is to provide an explanation to this question. We show why an *ex ante* commitment to federalism (specifically, to a progressive federal system) may be beneficial. Contra Brandeis, however, this commitment is more akin to entering a tournament – albeit a policy tournament – than it is to joining a laboratory.

The formal study of federalism has produce a large and broad literature that extends well beyond the issue of experimentation. In a prominent, recent contribution, de Figueiredo and Weingast (2005) identify twin dilemmas of federalism as free-riding and the inexorable concentration of power at the center. Both of these forces are central to our model. However, rather than representing threats to the stability and existence of federal systems, with each exacerbating the other, we show how the progressive concentration of power in a federal system actually works to solve the free-riding problem. Thus, progressive federalism is a feature and not a bug of the federalism project.[2]

[2] The relevance of free-riding extends well beyond the issue of experimentation and has given rise to a broader literature that explores how federalist institutions work to align incentives between states and the national interests (for example, Persson and Tabellini 1996; Cremer and Palfrey 1999, 2000). These papers offer very different institutional solutions to ours and do not touch upon policy experimentation, and we do not consider them in detail here.

The formal analysis of policy experimentation in federal systems was pioneered by Rose-Ackerman (1980) who describes the free-riding problem and demonstrates how it dampens the incentives of office-seeking politicians to experiment with policy. Cai and Treisman (2009) explicitly compare this outcome to that possible under a fully centralized government as the candidates seeking national office attempt to construct majority winning coalitions. Their focus is on how the construction of winning majorities can undermine efficiency, a point we do not address here. Closer to our paper is the model of Strumpf (2002). He compares fully decentralized and fully centralized outcomes, modeling the centralized system as one in which the policymaker is compelled to harmonize policies across all districts. Strumpf allows for heterogeneity in district preferences, but in restricting each state to the classic two-armed bandit set-up (with one safe and one risky arm) he abstracts from the question of policy choice that is central to our results. Similarly, Volden, Ting, and Carpenter (2008) allow for preference heterogeneity in the sense we presume here, although they too restrict policy to a binary choice and preclude the inefficiencies in policy choice that are our interest here. Instead, they use the model to make the important point that policy diffusion is often difficult to distinguish from private policy learning.[3]

Our model also connects with the large verbal literature on federalism. Most notable is recent work by Bednar (2011) in which she asks how a federalist system can 'nudge' states toward productive experimentation. She considers a variety of practical inducements, from shifting public attention to offering party-based rewards, that work by aligning states' interests with those of the nation. Bednar explicitly focuses on these 'nudges' as they work outside the formal constitutional structure. Our results show how the constitutional structure can itself align incentives in this way. Interestingly, in a separate survey of the federalism literature, Bednar (2011b, p. 282) argues that "perhaps the most revolutionary research shift moving forward is to develop a theory of the dynamics of federalism's boundaries", a possibility consistent with the dynamic conception of authority that we describe here.

The most prominent research stream on federalism is, without doubt, that originating from Tiebout's (1956) famous model of sorting and policy competition. Our conception of policy competition is very different from that of Tiebout; specifically, we do not allow for inter-district movement of people. Nevertheless, we find that an intriguing connection appears between our work and that of Tiebout. Central to our model is the presence

[3] An interesting computational approach is proposed by Kollman, Miller, and Page (2000). They show that decentralized, parallel, search outperforms centralized search on problems of moderate complexity.

of heterogeneity across districts, the emergence of which is the central prediction of Tiebout's model. We return to this connection – and to the question of whether Tiebout-style sorting can also *improve* policy experimentation – in the discussion section.

The question of efficient experimentation has long been a question of more general interest in economics, captured famously by models of multi-armed bandits. This literature has been extended to environments with two experimenters by Bolton and Harris (1999) who characterize how free riding undermines the efficiency of equilibrium. Keller et al. (2005) permit more than two agents. Our model is a considerable simplification of these general formulations, yet our results suggest that preference heterogeneity may carry more general force in mitigating the free riding problem when multiple agents coexist. Although simple, our model extends the experimentation literature by allowing for heterogeneity of preferences and policy choice, moving beyond the previous focus on binary and predetermined actions. We also take up the question of institutional design and experimentation that has not previously been considered.

The next section describes the basic model and describes the first-best outcome as a benchmark. Thereafter, Section 3 derives the equilibrium under decentralization and the optimal level of heterogeneity. Section 4 analyzes the outcome under centralization, and Section 5 compares the two regimes. Section 6 generalizes the basic model by allowing for multiple districts, multiple periods, coordination benefits, transfers and policies that may be more or less transparent. Section 7 is further discussing the results and Section 8 concludes,. The Appendix contains all proofs as well as more detailed lemmas describing the equilibria.

2 Model and Benchmarks

2.1 The Model

Our basic model consists of two agents and three stages. Each agent could be thought of as a political unit, a state in a federal system or one of two independent countries. We refer to them as districts.

In order to distinguish between the quantity and the type of experimentation, we propose the following timing. First, each district $i \in \{A, B\}$ simultaneously decides the type, or location, of its initial policy or experiment. This location is simply a point on the real line, $x_i \in \mathbb{R}$.

Second, each district decides on the binary quantity of experimentation. That is, a district can play it safe or experiment with the policy. With probability $p \in (0,1)$, an experiment succeeds and raises the value of the policy by 1. The cost of the experiment is $k > 0$. Parameter k can represent the benefit of the safe option relative to the expected benefit of the risky option, but it can also simply measure the investment cost of developing and enhancing the value of a policy. For simplicity, we assume that the probability of success is independent of both the policy location and the other districts' experimental outcome.

Third, after the districts have observed the outcomes of both experiments, each district i decides on its final policy location, y_i, to implement. We assume that a district must implement one of the two policies developed at the first stage, so $y_i \in \{x_A, x_B\}$. This is natural if adopting a completely new policy requires sufficiently high additional costs (these costs can then be abstracted from here).[4]

The districts may have different ideal points regarding the type of policy. For example, districts may disagree regarding how redistributive or pro-market a new health reform ought to be. Mathematically, each ideal point t_i is a point on the real line and $h \geq 0$ measures the heterogeneity or the distance between the ideal points, $h = t_B - t_A$. Without loss of generality, we let $t_A < t_B$ and place the origin half-way between the ideal points. This implies $t_A = -h/2 \leq 0$ and $t_B = h/2 \geq 0$. It is useful to define a_i as the extent to which district i *accommodates* the neighbor by experimenting on a policy that is closer to the center than i's ideal point:

$$a_A \equiv x_A - t_A \text{ and } a_B \equiv t_B - x_B.$$

Putting the pieces together, the payoff to district $i \in \{A, B\}$ is:

$$u_i = I_{y_i} - c(y_i - t_i) - J_i k,$$

where the index-function $I_{y_i} \in \{0,1\}$ equals 1 if the policy, chosen by i at stage three, has proven successful. The index function $J_i \in \{0,1\}$ equals 1 if i decided to experiment at the second stage. To simplify, we let the preference over location be represented by a disutility-function that is symmetric around the ideal point: $c(y_i - t_i) = c(t_i - y_i)$. We assume $c(\cdot)$ to be convex, U-shaped, and to satisfy $c(0) = c'(0) = 0$.

[4] We can endogenize this assumption if selecting a new policy requires a set-up cost larger than $\min\{c(a_i), c(h - a_{-i})\}$, referring to notation (as well as the equilibrium) discussed below.

2.2 The First-Best

To understand the model, consider first the case of autarky with only one district. At the third stage, district i must necessarily stick to the location picked at the first stage, so $y_i = x_i$. Anticipating this, i always prefers to choose the policy at its own ideal point to minimize the distance cost; thus, $y_i = x_i = t_i$. At the second stage, the district finds it optimal to experiment ($J_i = 1$) if the cost k is smaller than the expected benefit from the experiment:

$$p > k,$$

which is henceforth assumed to hold. The equilibrium is thus $(x_i, J_i, y_i) = (t_i, 1, t_i)$ and this implements the first-best outcome when there is only one district.

With two districts the efficient outcome is not so straightforward. Each experiment is a public good that potentially provides a positive externality to the other district. This externality is stochastic and beneficial when one district's experiment succeeds and the other fails (or is not attempted). In this case, the latter district with the failed experiment may abandon its own policy and adopt the successful policy of the other district. This policy imitation will involve a distance cost for the switching district and, consequently, these switching costs must be accounted for in determining the first best. For moderate levels of district heterogeneity, the set of socially optimal experiments actively reduces these costs by requiring positive policy accommodation or *convergence*.

The following proposition presumes that $2p(1-p) > k$, so it is optimal that both experiment even when locations are identical.[5]

Proposition 1 *The first-best outcome is characterized as follows. There exists $\overline{h}_b \in (0, \infty)$ such that:*
(i) For $h \in [0, \overline{h}_b]$, both districts experiment, with the same degree of accommodation, $a \in [0, h/2)$, satisfying:

$$\frac{c'(a)}{c'(h-a) + c'(a)} = p(1-p), \ i \in \{A,B\}. \tag{1}$$

Consequently, $\partial a / \partial h \in (0,1)$ and $a \to 0$ when $h \to 0$. The districts implement the other district's policy only if their own policy fails and the other succeeds.
(ii) For $h \geq \overline{h}_b$, both districts locate and experiment at their ideal points. They imple-

[5] If $2p(1-p) < k$, the first-best requires that only one district experiments when h is sufficiently small.

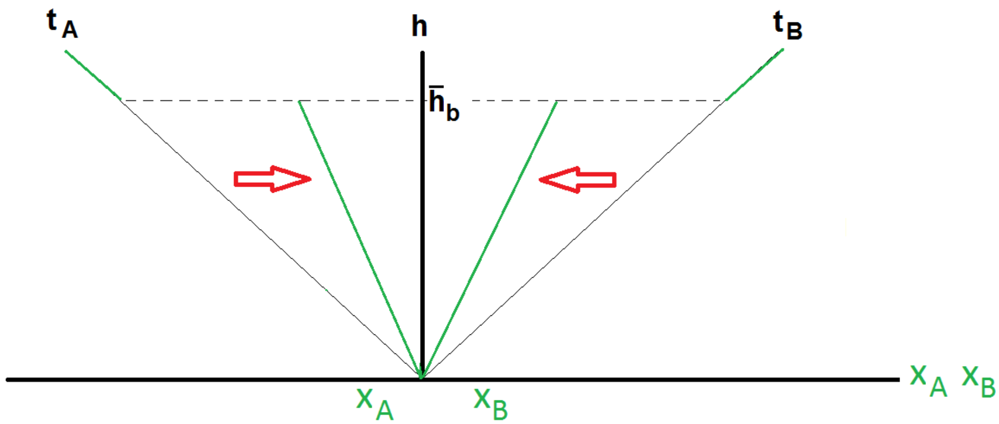


Figure 1: *In the first-best, policy positions converge relative to the ideal points*

ment their own policy regardless of experimental outcomes.

In case (i) each experimenting district accommodates the other, thereby imposing a cost on itself. Whenever it implements its own policy in the final stage, it must pay a distance cost that could have been avoided. This behavior is nevertheless socially optimal as, by the concavity of utility over policy type, the accommodation saves the other district more in distance cost should it wish to imitate the successful district, such that the net effect is positive.

The proposition also exposes the limits of federalism. For sufficiently heterogeneous districts the distance costs overwhelm the externalities from successful experimentation and the benefits of federalism become strained. In this case, the efficient outcome is the same as for an autarky: the districts experiment on their own and learn nothing (useful) from the experiments of others. This result aligns with the conventional wisdom that federalist systems are best composed of homogeneous districts. It is not difficult to deduce from Proposition 1 that, indeed, social welfare is maximized when heterogeneity disappears and the districts are identical ($h = 0$).

The first-best policy choice choices as a function of heterogeneity are depicted in Figure 1: The *horizontal* axis is here measuring ideal points as well as policy positions. Since the *vertical* axis measures heterogeneity, the two ideal points (on the horizontal axis) as a function of h (on the vertical axis) are given by the two straight lines t_A and t_B. For every $h \leq \overline{h}_b$, policy-positions are given by the green/bold lines which are somewhat closer to the center than the lines representing the ideal points.

The logic underlying Proposition 1 is straightforward. Yet as simple as the result is, the convergence we identify is important. It implies that to evaluate federalism in practice, analysis cannot be limited to only the *amount* of policy experimentation observed, but, more subtlely, must also weigh the actual *policies* that are experimented with. Proposition 1 suggests that much of the efficiency gain from federalism may be buried within the convergence of actual policy choices. It is this novel implication of federalism that we expand and explore in the following sections.

Before moving on, it is worth understanding the trade-offs that drive the first-best as it is the same trade-offs that drive equilibrium behavior. Of all the combinations of experimental outcomes, the most important is when the outcomes are mismatched. When both districts succeed – of fail – there is no reason for one to imitate the other. Only in the event where one succeeds and the other fails (or doesn't experiment) – when outcomes are mismatched – does the externality deliver value. It is for this reason that the quantity $p(1-p)$ appears in the first-order-condition of case (i) in the proposition. It is also for this reason that convergence of experimental policies is not complete. The externality is only probabilistic, whereas the distance cost is paid in all events, thus the efficiency of convergence is bounded.

The first-best also permits comparative statics. As the degree of heterogeneity increases, the distance costs grow and, to minimize these effects, the optimal experiments are even more convergent or accommodating (until $h = \overline{h}_b$ is reached). Nevertheless, as heterogeneity increases the degree of accommodation increases at a slower rate. As a result, for increasing heterogeneity, the districts are experimenting at policies further from their own ideal policies but also further from each other.

Example Q: To bring the intuition for Proposition 1 into sharper focus, the following corollary specializes utility to the quadratic functional form:

$$c(a) = qa^2/2.$$

In this case, (1) can be rewritten as:

$$a = p(1-p)h.$$

This special case permits precise comparative statics for relative policy choices. For case (ii), the rate of accommodation can be bounded more tightly at $\frac{1}{4}$, as $\frac{da_i}{dh} = p(1-p) \in \left(0, \frac{1}{4}\right]$. Further, as the distance between the policies is given by: $x_B - x_A = h - 2a$, we have: $\frac{d(x_B - x_A)}{dh} = 1 - 2p(1-p) \in \left[\frac{1}{2}, 1\right)$, so the degree of policy polarization is growing at least half as fast as preference heterogeneity in the first-best outcome.

3 Decentralization

We now turn to equilibrium policy choices when authority is completely decentralized to the districts. The districts will not internalize the informational externality from their own experiment. Nevertheless, this does not imply that the externality is irrelevant to their choices. We proceed by backward induction.

At stage three, the choice facing districts is quite trivial. With sunk investment costs and policies fixed, each districts $i \in \{\text{A,B}\}$ picks the final policy that maximizes its utility:

$$y_i = \arg \max_{x_j \in \{x_A, x_B\}} I_{x_j} - c\left(x_j - t_i\right). \tag{2}$$

Each district will stick to its own policy if it succeeds whenever $|a_i| < |h - a_j|$, which always will hold in equilibrium. The only choice to be made, effectively, is when a district's own experiment fails and the other district succeeds. In this situation, i prefers to switch policy if and only if:

$$c\left(h - a_j\right) - c\left(a_i\right) \leq 1.$$

Note that this set of choices is, conditional on earlier choices, efficient. As district j is indifferent to whether district i imitates it or not (an assumption which is later relaxed), the choice of district i is the same that would be made by a social planner. This equivalence does not extend to earlier stages, as we will now explain.

3.1 The Quantity of Experimentation

At stage two, the decision to experiment depends on whether the other district is experimenting as well as the policy positions. The "incentive compatibility condition" to ensure a district experiments (even when the other does) is given by the following.

Proposition 2 *Taking locations as given, both districts experiment with their policies if and only if:*

$$c\left(h - a_j\right) - c\left(a_i\right) \geq \frac{k - p\left(1 - p\right)}{p^2} \ \forall i, j \in \{A, B\}, i \neq j. \tag{3}$$

Not surprisingly, a district is willing to experiment as long as the distance to the other district's experiment is not too attractive. To understand the expression, it is helpful to rewrite it as:

$$[c(h-a_j)-c(a_i)]\,p^2+p(1-p)\geq k. \tag{4}$$

The right-hand-side is the cost of experimentation and the left-hand-side the marginal benefit. The marginal benefit of experimenting accrues only when the experiment is a success (as a failure leads to a third stage identical to not having experimented) and the size of the benefit depends on whether the other district's experiment is a success or a failure. If the other district's experiment is a success, a success by district i changes the policy it implements (from j's to its own) but doesn't change the quality received. This event saves the distance cost and occurs with probability p^2, corresponding to the first term on the left-hand-side. On the other hand, if the other district's experiment is a failure, then district i does not change the policy it implements at stage 3 (its own) but it does receive a quality boost of one. This event occurs with probability $p(1-p)$ and corresponds to the second term on the left-hand-side.

If the policies fully converged ($x_A = x_B$), the incentive compatibility condition reduces to $p(1-p)-k\geq 0$. To render the free-riding problem meaningful, we hereafter assume that this condition fails and that $k-p(1-p)>0$. This implies that there exists an unique $h_d^*>0$ that satisfies:

$$c(h_d^*)\equiv\frac{k-p(1-p)}{p^2} \tag{5}$$

To interpret this value, h_d^* is the heterogeneity such that if the districts located at their ideal policies, they would both be indifferent between experimenting and not (conditional on the other district experimenting). Note that the larger is the cost of experimenting, k, the larger is the necessary heterogeneity to make sure both districts experiment when located at ideal points. We use the subscript d here and subsequently to denote the decentralized choice.

3.2 The Type of Experimentation

We are now ready to analyze the choice of policy positions. It is immediately clear that, with self-interested districts, the incentive to converge that characterizes the first-best is entirely absent here. Policy convergence delivers an informational externality that not only benefits the other district at the expense of the convergent district, but the very convergence itself may undermine the willingness of the other district to experiment, further harming the convergent district.

In fact, this desire to benefit from the experiment of the other district may induce the districts to deviate from their own ideal policies. However, in contrast to the first-best, the districts deviate to the outside rather than the inside toward each other.

Proposition 3 *There exists a $\underline{h}_d > 0$ such that for each $h \geq \underline{h}_d$, there is a unique equilibrium in which both districts experiment. The equilibrium is symmetric and characterized by $a_i = a_j = a$, such that:*
(i) For $h \in [\underline{h}_d, h_d^*)$, policies diverge:

$$x_A < t_A < t_B < x_B \;\Leftrightarrow\; a < 0, \tag{6}$$

and the level of divergence satisfies

$$c(h-a) - c(a) = \frac{k - p(1-p)}{p^2}, \forall i \in \{A, B\}. \tag{7}$$

(ii) For $h \geq h_d^*$, the districts always experiment at ideal points ($a = 0$).
(iii) For $h < \underline{h}_d$, only one district experiments. The experimental location is at the experimenter's ideal point.

In *Example Q*, when $c(a) = qa^2/2$, we can rewrite (7) as:

$$a = \frac{h}{2} - \frac{k - p(1-p)}{qhp^2}.$$

The proof and the definition of $\underline{h}_d$ are in the Appendix.

In general, the decentralized equilibrium is very different from the first-best. Only for extreme heterogeneity do the requirements coincide. For all other cases, the first-best demands policy convergence whereas in equilibrium they either remain at their ideal policies or they diverge. The equilibrium policy positions as a function of heterogeneity are depicted in Figure 2 (with the same axes as Figure 1). For large h, experiments' locations, given by the green/bold lines, are at ideal points. For smaller heterogeneity, locations diverge, and divergence is larger if h is small. For a sufficiently small h, it becomes too costly to satisfy the incentive constraint. For even lower levels of heterogeneity ($h < \underline{h}_d$), only one district experiments, and it does so at its ideal point.

The amount of divergence is given by the requirements of Proposition 2. Each district diverges enough to ensure that the other district experiments but no more. Divergence in policies arises in the cases where the ideal points of the districts are not different enough

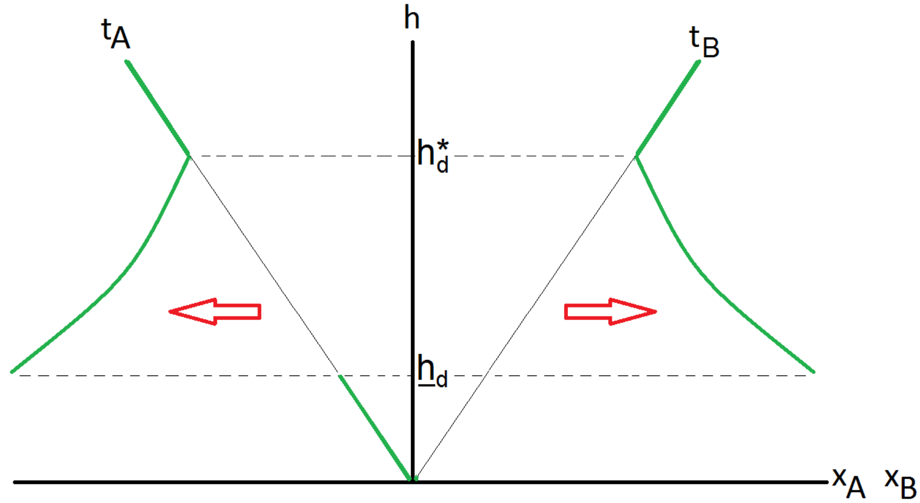


Figure 2: *The green line shows equilibrium experimental locations (on the horizontal axis) as a function of heterogeneity (on the vertical axis): When heterogeneity declines, locations may diverge.*

to ensure this incentive to experiment (and satisfy the requirement of Proposition 2). In this case, each district understands that by making its own policy less attractive to the other district, the other district's incentive to free-ride will be diminished and it will experiment itself. In this way, the informational externality from policy experimentation drives equilibrium behavior, even when the districts are entirely self-interested.

The divergence in policy choice is clearly inefficient. The set of choices is Pareto dominated in the sense that both districts would be better off if they simply agreed to experiment at their own ideal policies (or closer). They do not have the ability to commit to such an agreement, however, and the agreement is not self-enforcing in a decentralized system.

The degree of policy divergence can be significant. Straightforward manipulation of the equilibrium condition in Equation (7) establishes for $h \in (\underline{h}_d, h_d^*)$ that:

$$\frac{\partial a}{\partial h} = \frac{c'(h-a)}{c'(h-a)+c'(a)} > 1,$$

which, in turn, implies that:

$$\frac{\partial (x_B - x_A)}{\partial h} = \frac{\partial (h-2a)}{\partial h} < -1.$$

As heterogeneity declines, not only are the districts more divergent but their policy positions get further apart in absolute terms. This point suggest that efficiency may be lower if the districts are similar, than when they are different. This is confirmed in the next subsection.

The divergence in policies emerges only for moderate levels of heterogeneity. For sufficiently high heterogeneity free-riding is no longer an issue and each district is able to experiment at its ideal policy. Of course, this freedom comes at its own cost as the experiment of the other district provides minimal or no benefit should a district's own experiment fail. At the other end of the spectrum, if the districts are sufficiently homogeneous then the cost of deterring free-riding is so great as to outweighed the benefit that accrues. Thus, one district experiments, letting the other free ride, but the experimenter does not accommodate at all in its policy choice. The critical threshold at which a district prefers to carry the load rather than suffer a policy distortion is given by $\underline{h}_d$.

Finally, a note on equilibria when $h < \underline{h}_d$: In this case, there are multiple equilibria but all require that exactly one district experiments and, in all pure strategy equilibria, that the experimenting district experiments at its ideal point.[6]

3.3 Optimal Heterogeneity

In the first best the effect of heterogeneity is straightforward: The more similar districts are the better off they are. This monotonicity is consistent with the conventional wisdom on federalism. With self-interested and decentralized control, however, the answer may be reversed.

For very heterogeneous districts ($h > h_d^*$), the districts experiment at their own ideal points as the free-riding incentive is sufficiently weak. Districts may still utilize a successful experiment of the other district, but they do not need to distort their policy to ensure the other district experiments. In this case, more heterogeneity leaves the districts worse off, strictly so whilst they still receive an externality from each other's experiments, and weakly so after the point where they are effectively autarkic and the logic of federalism irrelevant.

On the other hand, for more moderate levels of heterogeneity ($h \in (\underline{h}_d, h_d^*)$), decreasing heterogeneity leads to ever greater distortions in policy choice. Proposition 4

[6] A large set of locations for the non-experimenting district can be supported in equilibrium via off-equilibrium-path beliefs. The beliefs necessary are that, following any moderate deviation, it is the deviating district alone that is expected to carry the burden of experimentation. There are certain constraints on locations for the nonexperimenting district in these equilibria, but we have chosen to not report on these here.

shows that this distortion leaves the districts, in aggregate, worse off such that welfare is maximized in this range when $h = h_d^*$. In fact, if the cost of experimentation is low enough, this level of heterogeneity produces the global maximum of welfare, dominating cases in which the districts are arbitrarily similar in preference or even identical. Thus, with self-interested districts and the freedom to choose both policy location and whether to experiment, the conventional wisdom that more similarity among districts produces better federalism is overturned.

Proposition 4 *Equilibrium payoffs are strictly higher at $h = h_d^*$ than at any other $h \geq \underline{h}_d$. In fact, h_d^* is a global optimum if*

$$k \leq 2p\frac{1-p}{2-p}.$$

This result can be readily seen in the region of moderate heterogeneity in which both districts experiment. As heterogeneity increases in this region, each district's experiment is less distorted from its ideal point. From the earlier fact that $\frac{\partial(x_B - x_A)}{\partial h} < -1$, increased heterogeneity also implies that the distance between the policy experiments is also decreasing. In effect, the greater heterogeneity allows free-riding to be avoided more easily, and so much more easily that both districts are better off.

4 Centralization

Modeling centralization in the context of a federal system presents a challenge: If the central government were benevolent and powerful, it could command the districts when to experiment and with which policies, thereby implementing the first-best outcome. We follow the literature in modeling instead a weaker, and somewhat clumsy, version of the central government.

In particular, we assume that policies always emerge locally: stages one and two are exactly as before. Only after locations and experimental outcomes are realized and observed, at stage three, will a central government play any role. This may be reasonable when policies that are not yet developed cannot be contracted on or dictated from the top. Furthermore, if the time lags between the stages in the game are substantial, then a central government that has little power at stage one may have grown to be more powerful at stage three. The history of the European Union, for example, shows that local policies have been developed and once the EU has been sufficiently powerful, some of the local successes have been introduced also elsewhere. This empirical dynamic is at the heart of de Figueiredo and Weingast's (2005) description of federalism.

At stage three, centralization means that the final policy decision, y_i, is not necessarily taken by i. Instead, we consider the case where a median voter, with ideal point t_m, decides on a common federal policy, harmonized across the districts. Note that such harmonization is always suboptimal *ex post*: it is inefficient to require harmonization when either both policies fail or both succeed. In fact, the decentralized system implemented the socially optimal choices at stage 3 (conditional on the choices that had been made at the earlier stages). However, there is a long tradition in the fiscal federalism literature to assume a clumsy central government imposing harmonization (going back to Oates, 1972. See the survey by Oates, 1999). Although the uniformity-assumption has been criticized as ad-hoc (as by Besley and Coate, 2003, or Lockwood, 2002), it has quite a lot of empirical support (Strumpf and Oberholzer-Gee) and it has also been given a theoretical foundation (Harstad, 2007). Note that, conditional on such harmonization, it is socially efficient that locations converge completely ($a = h/2$).

Following the literature on probabilistic voting (for an overview, see Persson and Tabellini (2000)), we assume the median voter's ideal point is a random variable that is distributed symmetrically around zero with density $\sigma > 0$. The realization of t_m is unknown until stage 3, when it is drawn from the uniform distribution:

$$t_m \sim U\left[-\frac{1}{2\sigma}, \frac{1}{2\sigma}\right]. \tag{8}$$

At stage three, the median voter sets policies such as to maximize its payoff:

$$y_A = y_B = \arg \max_{x_j \in \{x_A, x_B\}} I_{x_j} - c\left(t_m - x_j\right).$$

If both (or none) of the experiments $\{x_A, x_B\}$ have proven successful, the chosen policy will be the one closest to the median voter. Given the distribution (8), the probability that A's policy is chosen (when $x_A < x_B$) is given by:

$$\Pr\left(|t_m - x_A| < |t_m - x_B|\right) = \frac{1}{2} + \sigma\left(\frac{x_A + x_B}{2}\right). \tag{9}$$

If one experiment succeeds and the other fails (or is not initiated), then the median voter is assumed to pick the successful policy.[7]

[7]The median voter does indeed prefer to select the successful policy in equilibrium (since locations

4.1 The Quantity of Experimentation

The median voter's strategy in stage 3 sets up a sort-of tournament for the districts in the earlier stages. At the second stage, with policies fixed in place, the districts want to experiment to avoid being compelled to implement the other district's experiment. Nevertheless, this incentive has its limits as the costs of being so compelled can nevertheless still be dominated by the costs of experimenting. This is the basis of the following proposition.

Proposition 5 *Take locations as given and suppose* $|a_j - a_i| \leq 1/\sigma$. *Both districts experiment if:*

$$c(h - a_j) - c(a_i) \geq \frac{k - p(1-p)}{p/2 + \sigma(a_j - a_i)\, p(1/2 - p)},\ i, j \in \{A, B\}, i \neq j. \tag{10}$$

This says that the districts are willing to experiment if the location of the neighbor's experiments is not too attractive, similarly to Proposition 2 for decentralization. If locations are symmetric, the condition in Proposition 5 simplifies to:

$$c(h - a) - c(a) \geq \frac{k - p(1-p)}{p/2}. \tag{11}$$

To understand the condition, it is helpful to rearrange (11):

$$[c(h - a) - c(a)]\frac{1}{2}p^2 + p(1-p)\left[1 + \frac{1}{2}[c(h - a) - c(a)]\right] \geq k. \tag{12}$$

The right-hand side is the cost of experimenting. The left-hand side is the benefit. As before, this benefit has two components and accrues only if the district's experiment is successful, which occurs with probability p. If the other district's experiment succeeds then, with combined probability p^2, the district has its own policy implemented half the time, thereby avoiding the distance cost. If, on the other hand, the other district's experiment fails, which occurs with a combined probability of $p(1-p)$, a successful experiment ensures the successful district's experiment is always implemented, giving it not only a quality boost but also avoiding the distance cost from having to implement the other district's failed experiment half the time.

will be symmetric), and also off the equilibrium path if just the two locations are 'sufficiently similar', meaning that:

$$1 \geq |c(t_m - x_A) - c(x_B - t_m)|\, \forall t_m \in [-1/2\sigma, 1/2\sigma.].$$

The beneficial impact on progressive centralization is evident when the other district's experiment is expected to fail ($p < 1/2$). This strictly increases the incentive of the other district to experiment as it otherwise faces the prospect of implementing the other district's failed experiment half the time. By experimenting it reduces the chance of this fate (it is still possible as the experimenting district cannot guarantee its own success) and, additionally, still receives the benefit of implementing a successful rather than a failed policy.

This benefit, however, must be counterbalanced against the case where the other district's experiment is expected to succeed ($p > 1/2$). In this case, the incentive to experiment is dampened as a district cannot even be guaranteed of implementing its own experiment should it succeed. In this case, an experiment must be discarded by one district, despite proving successful, as the central government imposes harmonization.

The condition in Proposition 5 depends on the heterogeneity and polarization of the districts. Recalling the maintained assumption (from Section 3) that $k > p(1-p)$, there must exist a $h_c^* > 0$ that satisfies:

$$c(h_c^*) \equiv \frac{k - p(1-p)}{p/2}. \tag{13}$$

At heterogeneity level h_c^*, both districts are indifferent between experimenting and not if the policies chosen are the districts' ideal positions. This definition is the analogue of the definition of h_d^* for decentralized federalism.

4.2 The Type of Experimentation

While equation (9) shows that a district's policy is more likely to be chosen at the federal level if the policy position is moderate, the lesson of Proposition 5 is that such accommodation can discourage the other district from experimenting. The threshold at which this occurs is critical to equilibrium behavior. Yet, rather than defining the barrier that must be breached to ensure victory in the policy tournament, the threshold defines the boundary of policy competition.

Proposition 6 *There exist four thresholds $0 < \underline{h}_c < h_c^* < \widetilde{h}$ and $\overline{h}_c$ such that when $h \in (\underline{h}_c, \overline{h}_c)$, there exists a unique symmetric equilibrium in which both districts experiment, given by the following:*

(i) For $h \in \left(\underline{h}_c, \widetilde{h}\right)$, a satisfies:

$$c(h-a) - c(a) = \frac{k - p(1-p)}{p/2}. \tag{14}$$

Consequently, for $h \in (\underline{h}_c, h_c^*)$ policies are divergent ($a < 0$), whereas for $h \in \left(h_c^*, \widetilde{h}\right)$ policies converge ($a > 0$).
(ii) For $h \in \left(\widetilde{h}, \overline{h}_c\right)$, policies converge and $a > 0$ satisfies:

$$c(h-a) - c(a) = \frac{c'(a)}{\sigma\left[p^2 + (1-p)^2\right]}. \tag{15}$$

In *Example Q*, when $c(a) = qa^2/2$, (14) becomes

$$a = \frac{h}{2} - \frac{k - p(1-p)}{qhp/2},$$

while (15) becomes:

$$a = \frac{h/2}{1 + 1/\sigma h\left[p^2 + (1-p)^2\right]}.$$

The policy tournament combines competition with restraint. The districts compete to win the favor of the median voter, but only up to the point at which further competition would drive their opponent from the tournament. The competitive effects of the tournament depend on the incentive to experiment characterized by Proposition 5. For districts of moderate or larger heterogeneity ($h > h_c^*$), this incentive is strong enough to not just overcome the incentive to diverge, but also to induce the districts to converge toward this opponent. In our terminology, this leads to equilibria with positive accommodation, although the incentive for doing so is not exactly hospitable. It is well possible that the level of convergence is larger than what would be required by the first-best outcome (Proposition 1).

These incentives to compete as well as to discourage free-riding explain why convergence is not a universal behavior in centralized federalism. As heterogeneity decreases ($h < h_c^*$), the distance required between experimental policies becomes wider than the distance between the districts' ideal policies. In this case divergence is again necessary to ensure that both districts don't free ride and have the incentive to experiment. The equilibrium policy positions are depicted in Figure 3.

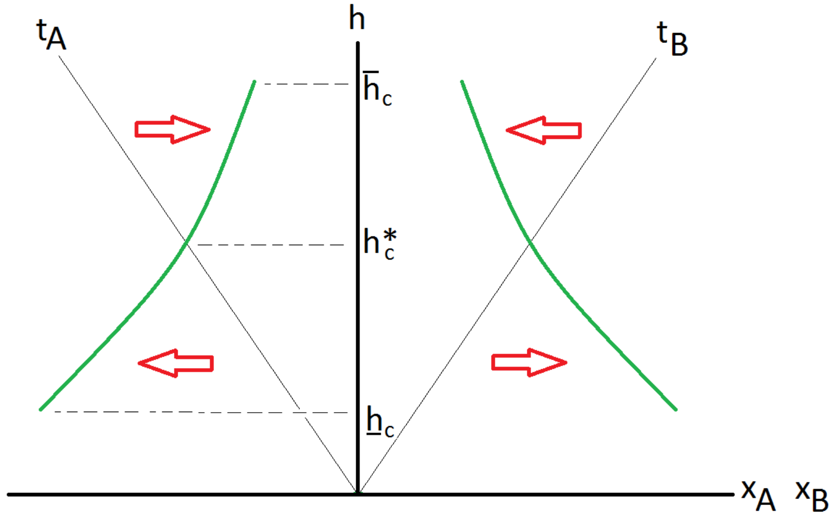


Figure 3: *The green/bold lines show locations of experiments (on the horizontal axis) as functions of h, on the vertical axis. For large h, locations converge.*

In case (ii) of the proposition the districts do not converge so much as to be indifferent between experimenting or not. At this point, further convergence is undesirable even though, if small enough, it would not deter an opponent from experimenting. Driving this result is uncertainty over the location of the median voter. With no uncertainty, a small deviation would not deter the opposing district from experimenting but would guarantee victory in the policy tournament should the experimental outcomes be the same. This would create a discontinuity in district payoffs and ensure convergence, at least to the point where experimentation may be deterred. With uncertainty over the median voter's location, the profitability of small deviations is much smaller and determined by the degree of uncertainty; specifically, the size of σ. For large enough heterogeneity, and high enough σ, the point at which deviations are no longer profitable is reached before the point at which an opponent is deterred from experimentation.

4.3 Optimal Heterogeneity

What is the optimal level of heterogeneity under centralization? Proposition 6 suggests that one gets positive convergence only if h is sufficiently large ($h > h_c^*$), and we know such convergence is necessary in the first-best outcome. If instead h is small, then policies diverge, and more so the smaller is $h \in (\underline{h}_c, h_c^*)$. This reasoning suggest that a large $h > h_c^*$ may increase payoffs.

As a counter-argument, centralization requires policy harmonization and the cost of such uniformity must be larger if the districts are very different. This traditional argument suggests that a small h may be better as the costs of harmonization are then smaller.

It turns out that these arguments are balanced when $h = h_c^*$. At the optimal level of heterogeneity, districts are locating their policies exactly at their ideal points (as was the case, albeit for a different value of h, under decentralization).

Proposition 7 *For all* $h \in \left[\underline{h}_c, \overline{h}_c\right]$, *welfare is maximized when* $h = h_c^*$.

By comparison, note that $h_c^* = h_d^*$ when $p = 1/2$. If p is smaller (larger) than $1/2$, then the optimal heterogeneity is smaller (larger) under centralization than it would be under decentralization. The next section reveals the intuition for this comparison and derives when centralization increases payoffs relative to decentralization.

5 Comparing Centralization and Decentralization

The two institutions differ only at stage three, when centralization imposes harmonization. As this harmonization is often inefficient, whereas decentralization at this stage is equivalent to a benevolent social planner, the comparison between the systems at this stage is straightforward and always favors decentralization. Efficiency, of course, must account for behavior at *all* stages of the game, and when we allow for the incentive effects of centralization's *ex post* inefficiency, the comparison may be reversed.

To understand this possibility, recall that equilibrium policy choices were often determined by the indifference condition between experimenting and not at stage 2. The comparison of institutions, therefore, begins with an analysis of which institution requires more divergence to sustain dual experimentation. A comparison of Propositions 2 and 5, or equations (7) and (11), reveals that decentralization requires more divergence if and only if $p < \frac{1}{2}$ (the intuition is explained in the text following Proposition 5). In this case, centralization moves policy choices closer to the first best and, as long as heterogeneity is not too high such that the costs of harmonization are too great, this effect dominates and progressive centralization is the more efficient federalist institution.

Proposition 8 *Suppose* $h \geq \max\left\{\underline{h}_c, \underline{h}_d\right\}$. *Decentralization dominates centralization if* $p \geq 1/2$ *but centralization might be better if* $p < 1/2$. *When* $c(a) = qa^2$ *and* $p < 1/2$, *progressive centralization dominates decentralization if and only if:*

$$h < \sqrt{\frac{k - p\,(1-p)}{q} \frac{1/4p^2 - 1}{1/2 - p\,(1-p)}},$$

that is, for h small, q small, k large and p small.

The value $p = \frac{1}{2}$ provides the critical threshold as at this value the experimenting districts are equally likely to both succeed as they are to both fail. When $p < \frac{1}{2}$ the districts are more likely to both fail and, therefore, centralization creates a stronger incentive to experiment and, consequently, more policy accommodation. To see this, recall from Equation (12) that when the other district's experiment fails, a successful experiment offers two rewards under centralization: An improved policy quality as well as avoidance of being compelled by the median voter to implement the other district's failed policy. Under decentralization, only the first reward is present (see Equation 4). In contrast, when the other district's experiment is a success, centralization offers only half the reward that decentralization does from a successful experiment. This is because centralization compels each district, despite their own success, to implement the other district's successful policy half of the time. Under decentralization, in contrast, a successful district is always able to enjoy the fruits of this success.

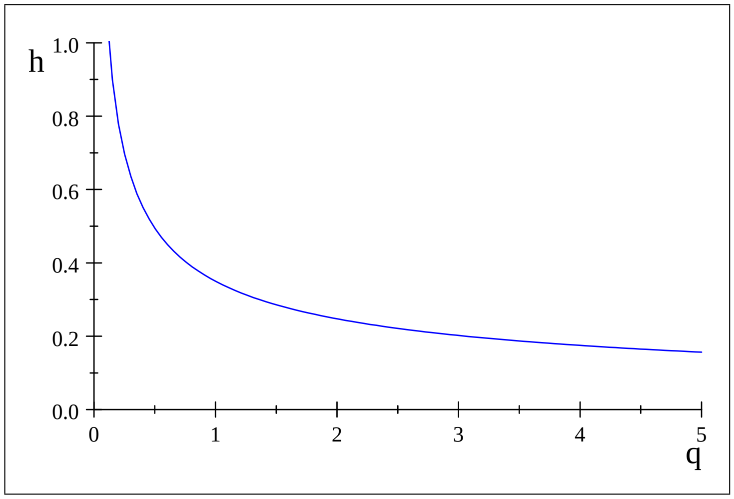
The following numerical example makes this trade-off even clearer. Set $p = 1/3$, $k = 1/4$ and $q = 1$. We then have:

$$\begin{aligned} \underline{h}_d &= 0.24 \text{ and } h_d^* = 0.5, \\ \underline{h}_c &= 0.18 \text{ and } h_c^* = 0.41. \end{aligned}$$

For the domain $h \geq \underline{h}_d = 0,24$, progressive centralization dominates decentralization if and only if:

$$h < 0.35.$$

With $q \neq 1$, the condition is instead $h\sqrt{q} < 0.35$. In the plot, the threshold for h (under which centralization is better) is drawn below as a function of q, measured on the horizontal axis:



Centralization is better than decentralization below the curve

6 Extensions

The model above is intentionally simple so as to illuminate the intuition and demonstrate most clearly the necessary features. A fortuitous by-product of this simplicity is that the model can be easily extended in a number of interesting directions. In this section, we present some of these extensions. Each extension builds on the basic model above (rather than on each other) and can be read isolated from the others. The first three extensions are combined in the Appendix.

6.1 Multiple Periods and Coordination Benefits

So far, a district has been indifferent to whether the other district copies the policy. In reality, there may be "coordination benefits," so that a district receives a benefit, say G, if the two districts end up with the very same policy. A positive G can explain why the median voter under centralization always imposes harmonization at stage three.

Such a coordination benefit would arise naturally in a setting with multiple periods. Suppose we extend the model above by allowing for a fourth stage: After each district has picked its policy at stage three, each may experiment further and the potential rewards can be enjoyed by everyone having implemented that exact policy. This would naturally lead to a coordination benefit to both districts when they adopt the same policy.

To see this, suppose, at stage four, that each district can pay a cost k_2 to experiment and succeed with probability $p_2 > k_2$. If alone, a district's benefit from this fourth stage would be simply $p_2 - k_2$. If two districts have implemented the same policy at stage 3, then we can have (at least) three different cases.

(1) Suppose the success probabilities are uncorrelated and that if both succeeds, then both values (1+1) can be enjoyed by each district. In this case, both districts find it optimal to experiment (when $p_2 > k_2$) and the fourth stage adds the value $2p_2 - k_2$ to each district. Compared to the payoff when policies are different, harmonization gives the additional "coordination benefit" $G = p_2$.

(2) If the value of two innovations at stage four is the same as the value of one, then the fourth stage adds the payoff $p_2 (2 - p_2) - k_2$ if both experiment, since $p_2 (2 - p_2)$ is the probability that at least one district succeeds. In this situation, it is indeed optimal for both to experiment if $(1 - p_2) p_2 > k_2$ and, compared to the payoff when policies are different, harmonization gives the additional coordination benefit $G = p_2 (1 - p_2)$.

(3) If case (2) is modified such that $(1 - p_2) p_2 < k_2$, then only one district will experiment at the fourth stage when policies are harmonized. If each is equally likely to be the one that experiments, then the fourth stage gives the expected payoff $p_2 - k_2/2$. Compared to having different policies, harmonization gives the coordination benefit $G = k_2/2$.

For what follows, it is irrelevant which of these cases (or something else) that determines the coordination benefit G, and we can simply ignore the fourth stage of the game and instead just remember that harmonization gives each district the additional benefit G.

Under decentralization, the incentive to free-ride depends on G. On the one hand, G makes it less costly for one district to fail and adopt the neighbor's policy, particularly if the neighbor is likely to succeed ($p > 1/2$). On the other hand, a large G makes it more important to succeed and attract a failing neighbor (with probability $1-p$). Thus, if $p > 1/2$, a larger G makes free-riding more tempting and, as a response, $a < 0$ must decline. If $p < 1/2$, a larger G makes experimentation more attractive and a can be larger, without discouraging the neighbor from experimenting.

In fact, positive accommodation ($a > 0$) is possible even under decentralization when $G > 0$: When h is so large that even a successful policy is unattractive to a failing neighbor, then a district must accommodate somewhat (by increasing a) to ensure that the coordination benefit may be enjoyed following a success.

Proposition 9 *Suppose $G > 0$.*
(i) Under decentralization, $a < 0$ increases in G if $p < 1/2$ but decreases in G if $p > 1/2$ when $h \in (\underline{h}_d, h_d^*)$. Furthermore, there exist thresholds $\overline{h}_d > \widehat{h}_d > h_d^*$ such that policies

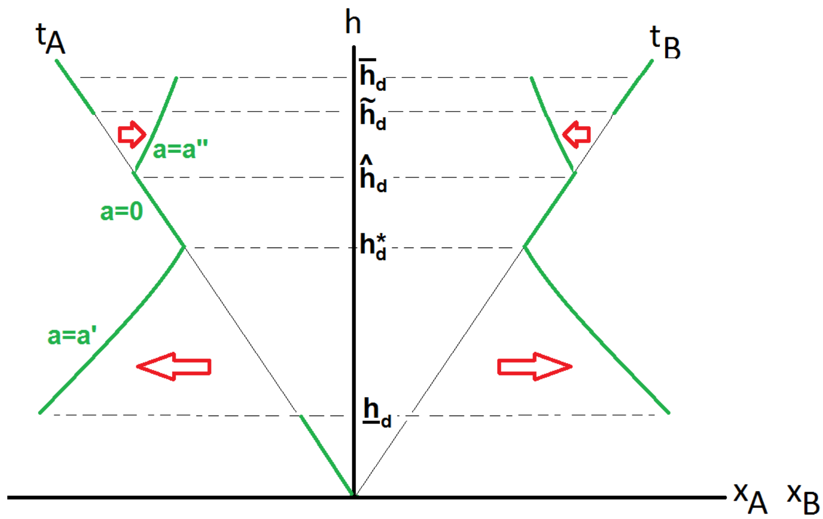


Figure 4: *Convergence is possible even under decentralization when there are benefits of coordinating the policy*

converge $(a > 0)$ *when* $h \in \left(\widehat{h}_d, \overline{h}_d\right)$. *Both thresholds* $\widehat{h}_d$ *and* $\overline{h}_d$, *as well as the size of this interval* $\left(\overline{h}_d - \widehat{h}_d\right)$, *increase in* G. *The optimal level of heterogeneity* h_d^* *decreases in* G *if* $p < 1/2$ *but increases in* G *if* $p > 1/2$.
(ii) Under centralization, the level of G *does not affect the equilibrium choices.*
(iii) By comparison, we always have that centralization is better $(u_c > u_d)$ *when* G *is sufficiently high.*

Figure 4 illustrates that positions converge when h is large. Part (ii) of the proposition is intuitive: under centralization, policies will be harmonized in any case and G can be treated as a lump sum gain which thus does not affect any choices. A larger G only means that the ex post cost of harmonization is reduced and, for a sufficiently large G, harmonization is beneficial. It may not surprise, therefore, that centralization is likely to outperform decentralization when G is sufficiently large.

The precise conditions can be found in the appendix.

6.2 Transparency and Correlation

So far, we have assumed that a district can perfectly well copy the neighbor's success. In reality, it might be the case that a district can only imperfectly observe and learn the neighbor's policy.

Let $\alpha \in [0, 1]$ measure the fraction (as well as the magnitude) a district can capture of a neighbor's successful experiment. Thus, if A adopts B's successful policy, A's payoff (abstracting from any sunk investment cost) is $\alpha - c(h - a_B)$. Note that parameter α can alternatively be interpreted as the probability at which the policy will succeed for A given that it has already proven successful to B.

Intuitively, more correlation or transparency (a larger α) increases the temptation to free-ride. This makes more divergence necessary in order to satisfy incentive constraints when α is large. Since divergence is costly, a smaller α can be better for both districts.

Proposition 10 *Suppose* $\alpha \in [0,1]$.
(i) Under decentralization, a smaller α *raises* a *and reduces* h_d^*. *The optimal degree of transparency increases in* h *and is given by:*

$$\alpha_d^* = \min\left\{1, c(h) + \frac{p-k}{p^2}\right\}.$$

(ii) Under centralization, a smaller α *raises* a *and reduces* h_c^*. *In Example Q, the optimal degree of transparency increases in* h *and is given by:*

$$\alpha_c^* = 1 - 2\frac{k - p(1-p)}{p^2} + \frac{1}{p}(2-p)\,qh^2 < 1$$

when $h \in \left(\underline{h}_c, \widehat{h}_c\right)$ *and* $h < \widetilde{h}_c \equiv \sqrt{[k - p(1-p)]/q(2-p)p/2}$. *When* $h \geq \min\left\{\widehat{h}_c, \widetilde{h}_c, \overline{h}_c\right\}$, $\alpha = 1$ *is optimal.*
(iii) In Example Q, a decrease in $\alpha \approx 1$ *when* $p \approx 1/2$ *always increases* $u^d - u^c$.

Part (iii) suggests that transparency tends to be more beneficial than under decentralization. This is intuitive since, under centralization, one district is *always* copying the other district's policy. If α is reduced, the cost of imposing harmonization increases and decentralization becomes preferable.

With reduced transparency, the question arises whether district A can pay district B to disclose the remaining information such that A becomes capable to enjoy the full value of the experimental success (and not only the fraction α). Such transfers are analyzed in the following extension.

6.3 Transfers and Patents

The above analysis has assumed away side payments between the districts. This subsection shows how even quite clumsy (or exogenous) side transfers can improve on the equilibrium derived above, and perhaps even implement the first-best outcome.

In short, we simply assume that a district must pay T to the neighbor when copying the neighbor's policy. There may be several motivations for such a transfer. For example, the innovating district may have information that is beneficial for a neighbor trying to copy the successful policy. The innovator may then be able to extract favors (implicit or explicit transfers) before this information is disclosed. To be precise, suppose a fraction α of the innovation can be enjoyed for free (as in the previous subsection), while the remaining fraction $1-\alpha$ can be enjoyed only if the innovator releases all information. One may then refer to $1-\alpha$ as the strength of the innovator's intellectual property right or patent. If the innovator has bargaining power index $\beta \in [0,1]$, then the generalized Nash bargaining solution predicts that the transfer from the neighbor to the successful innovator will be

$$T = \beta (1-\alpha). \tag{16}$$

Another explanation for T is that there may be some exogenous or constitutional prize going to the district that gave birth to the nation-wide policy. Although such a motivation may best fit with the centralized system, we henceforth assume that the transfer T is the same under decentralization as well as under centralization, and we will abstract from the origin of T.

If T increases, it becomes more beneficial to be the innovator and the incentive to experiment increases. This means that there is less of a need to diverge to discourage free-riding and, in equilibrium, a increases. The value of heterogeneity (which also increases a) is then diminished and the optimal level of h decreases. These results hold in both federal systems.

Furthermore, it becomes possible that districts converge ($a > 0$) even under decentralization when T (and h) is large: for a large h, convergence may be necessary to sell a successful policy to a failing neighbor and, for a large T, such a sale is very valuable and worth some convergence.

Proposition 11 *(i) Under decentralization, a larger T raises a and reduces h_d^*. For T sufficiently large, $a > 0$ and the first-best outcome is implemented. In Example Q, the optimal transfer is given by:*

$$T_d^* = \frac{k - p(1-p)}{p} - qh^2 p (1 - 2p(1-p)) \in (0,1).$$

(ii) Under centralization, a larger T raises a and reduces h_c^*. In Example Q, the optimal

transfer is given by:

$$T_c^* = \frac{k - p(1-p)}{p} > T_d^*.$$

(iii) By comparison, if T can be freely chosen, decentralization is always preferable: T_d^* implements the first-best outcome under decentralization; but centralization can never be first-best.

Centralization can never be first-best since it imposes uniform policies. Decentralization, however, can implement the first-best outcome if a sufficiently large T induces the districts to converge rather than diverge. A large T is particularly important when the districts are relatively similar (h low) since the free-riding problem is then large.

Note that if T is the outcome of negotiations and given by (16), then we can implement the first-best $T = T_d^*$ by selecting an appropriate level for intellectual property rights:

$$\alpha^* = 1 - \frac{1}{\beta}\left[1 - qh^2\left(1 - 2p\left(1-p\right)\right)\right].$$

6.4 Multiple Districts

It is simple to permit multiple districts in the model. This can be done in several ways. Perhaps the traditional way would be to place the districts' ideal points on a one-dimensional line. This would destroy symmetry, however, as some districts would be located more centrally than others. Here, we allow for multiple dimensions and we seek symmetry by assuming that the ideal points of n districts are spread out on the rays of a star, as illustrated in Figure 5.

The disutility from distance is assumed to be $c(d)$, where d is now the distance from a district's ideal point to the selected policy given that one must travel along the rays of the star.

When there are many districts, then one district's success can potentially be copied by a larger number of districts. This implies that it is socially optimal that districts converge more (by increasing a) if n is large. That is, in the first-best outcome, a increases in n.

The opposite is the case in equilibrium, however: a will decrease in n in both political regimes. Intuitively, the incentive to free-ride is larger when n is large, since it is then more likely that one of the other $n-1$ districts succeeds. The incentive constraint, which is necessary for a district to still to be willing to experiment, is thus harder to satisfy,

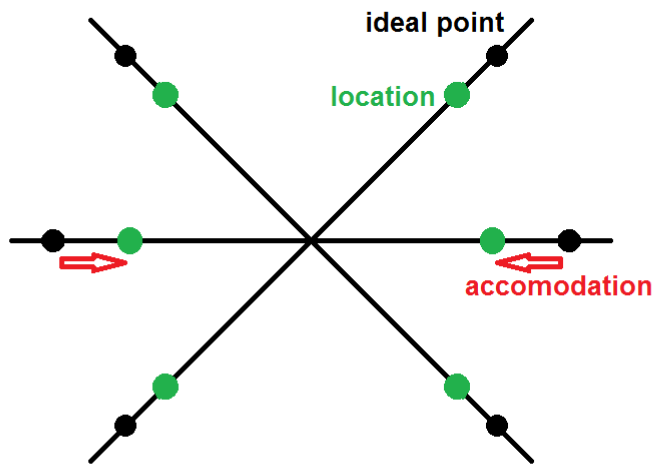


Figure 5: *Ideal points in black; policy locations in green*

and the level of accommodation a which guarantees that all the districts experiment must decline.

Proposition 12 *With a larger number of districts (n), the first-best convergence level ($a > 0$) increases. However, the reverse holds in the symmetric equilibria:*
(i) Under decentralization, a decreases in n.
(ii) Under centralization, a decreases in n.
(iii) By comparison, decentralization is always better if $n \geq 1/p$. Even if $n < 1/p$, decentralization is better if h is sufficiently large in Example Q.

The Appendix contains the details. By comparing decentralization and centralization, there is again a positive and a negative effect of centralization. If it is expected that all the other districts will fail, then the benefit of succeeding is larger than under decentralization, and the incentive to experiment is larger (this is the "positive effect"). If it is instead expected that at least one of the other districts will succeed, then the benefit of succeeding is smaller under centralization than decentralization, and the incentives to experiment is thus also smaller - with the consequence that $a < 0$ must be smaller than it would have been under decentralization (this is the "negative effect"). The positive effect can dominate only if $p < 1/n \Leftrightarrow n < 1/p$, that is, if it is very unlikely that the other experiments will succeed. But for a sufficiently large n, the negative effect will eventually dominate: Almost for sure, at least one other district will succeed and this implies that centralization will reduce the value of succeeding and experimenting. When there are many districts, decentralization is better.

7 Discussion

From a normative perspective, the objective of politics is to identify and implement good policies.[8] Everything else is a means to an end. The focus of political competition, therefore, should be on which policy wins that competition and is implemented rather than which candidate wins office. Besley (2001), in a brief policy paper, emphasizes this distinction and introduces the notion of *policy competition*, in contrast to *political* competition, to shift the focus from candidates to policy. The framework we introduce here pursues this tighter focus and constructs an explicit tournament in which policies compete, placing policy competition at the center of the design of political institutions.

Within this setting, our main contribution is twofold. First, we show that in the absence of federal institutions – in a decentralized system – the incentives to experiment are perverse. Specifically, each district seeks to provide a policy alternative that is *unattractive* to their neighbors, hoping that their fellow districts are inspired to try harder and experiment with a new policy. The end result is that the forces of policy competition drive policy choice in the wrong direction.

Our second contribution is to show how policy competition can be harnessed positively by appropriately-designed political institutions. The key to the design of such institutions is to realign the competitive forces so that competition runs in the right direction. This is done by establishing a sort-of policy tournament. Unlike standard tournaments, however, the winning district earns no accolade or prize other than the freedom to implement its policy of choice. Instead, the tournament carries a negative prize: the loser incurs the punishment of being forced to implement the other district's policy, whether it wishes to or not.

One may wonder, then, why anyone would willingly submit to a tournament with a negative prize. A corollary of our main results is an explanation of just this puzzle. It is only by binding themselves to a negative prize tournament that the states can harness the positive implications of policy competition. From this perspective, the ex post inefficiency of forced policy coordination should not be lamented, rather it should be celebrated as the price of generating more efficient ex ante incentives.

These results offer a reinterpretation of the purpose of federal systems. This reinterpretation can be pushed deeper. One connection is to the practice of coordinating policies across states – frequently referred to as policy *harmonization* in the EU. Popular accounts frequently take harmonization as an end in itself, desirable to avoid the costs

[8] Where a "good" policy is the maximand of some social welfare function.

of coordination or merely to increase continent-wide harmony. From the perspective of a policy tournament, however, policy coordination becomes an incentive mechanism rather than an end in itself.

Our model takes a stark and extreme line on policy coordination and the dual issue of exit from the federal system. Coordination is always imposed in the final stage and states have no option to exit. Once a design perspective is adopted, though, focus quickly moves beyond what is possible with institutional design to what is optimal. At the heart of the design question is how to most efficiently provide the incentives for states to experiment and converge. It may be that these incentives are induced most cheaply and effectively by forcing coordination only when both policies fail or when they both succeed. Such a design may appear unrealistic, yet the same ends may be achieved by more recognizable mechanisms; for instance, by allowing states the option to exit the federal system in some circumstances (generally or on specific issues).[9] Of course, other complications may arise from these features – such as verification of the outcomes of experiments. The dimensions of choice available to the designer are rich, extending well beyond coordination and exit. It is possible that legislative institutions (such as bicameralism) shape experimentation, as well as supermajority rules and the strategic setting of the scope of the federal union itself (*a la* Mundell (1961) for currency areas and Alesina and Spolaore (1997) for federalism).

Although the overall pattern in the history of the U.S. and E.U. is toward greater centralization of authority, it is notable that some issues have moved in the opposite direction, with authority devolved from the center back to the states. A prominent example of this decentralization of authority in the U.S. is welfare reform policy in the 1990's.[10] Although this example seemingly runs counter to our model, we would like to suggest that it, in fact, reaffirms our theory. It is not difficult to see that once a political issue is centralized, it becomes relatively stagnant with minimal experimentation. This stability is fine in a stable world, yet it is restrictive in a changing world when shocks require that further experimentation be undertaken. Our conjecture is that when uncertainty is present – be it due to a shock or insufficiently attractive experimentation earlier in the process – the central authority will devolve an issue back to the states to restart the policy tournament and reignite policy experimentation. For welfare reform in the 1990's, it appears this is an accurate description of the state of the environment. As Bednar (2011, p. 511) argues, on welfare policy "By the early 1990's, there was a

[9] See Bednar (2007) for an interesting discussion of the incentive effects of exit options in federal systems.

[10] We thank Craig Volden for this example.

sense that the federal government had run out of ideas, and that the country needed much more diverse experimentation in order to discover policy improvement." If our conjecture is true, the logic suggests not only that power will accumulate in the center in federal systems, but that a natural limit to the size or authority of the central government will obtain, where the natural limit is determined by the degree of stability in the environment over which policy rules.

An intriguing feature of our results is that the collective welfare of the states is maximized when the states are similar but not identical. If states are too similar, they are plagued by free riding and inefficiently low levels of experimentation. This result creates a surprising connection to the famous model of Tiebout. The implication is that in setting up a federal system, a degree of heterogeneity across the districts is preferable, and this is exactly what is generated by Tiebout style sorting. An interesting open question, therefore, is whether Tiebout sorting generates the efficient degree of inter-district heterogeneity that our model predicts.

The connection to Tiebout also suggests how a decentralized federal system may meaningfully differ from a collection of districts with no formal attachments. The difference may be in labor mobility. That is, if people within a decentralized federal system are able to relocate, Tiebout sorting may generate greater heterogeneity and more efficiency than would the same states without a formal federalist pact.

Finally, although we apply our ideas exclusively to policy competition, the issue of free riding and experimentation is of broad importance. Our framework and results can be adapted to these other settings. Theorists within organizational economics are interested in joint control problem within firms where they players – such as the boss and workers or managers and owners – have preference differences. Experimentation and learning is also key to joint ventures and research partnerships between firms, such as oil exploration ventures and pharmaceutical research into new drug compounds. Our key insight into the use of tournaments to reconcile incentives in collective choice environments is as applicable in these settings as it is in politics.

8 Conclusion

The prominence of policy experimentation in policy and popular discourse has not been matched by development of a formal understanding of the underlying phenomenon. The objective of this paper has been to close this gap, if only a small degree.

We have allowed districts to choose both the type and quantity of policy experimentation. To discourage free-riding, each district may prefer a policy that is less useful to the other districts, particularly when the districts are relatively similar. This implies that policies may be more different than the districts' ideal points would suggest, and that heterogeneity is beneficial since it discourages free-riding.

In this setting, we have found a new role for even a very clumsy central government. If policies are harmonized at the end of the game, each district may face a stronger incentive to experiment and policy positions may converge (as in the first-best) rather than diverge (as in the decentralized equilibrium).

Our workhorse model is both simple and tractable and thus possible to extend in several directions. So far, we have explored the effects of multiple districts, multiple periods, coordination benefits, transparency or imperfect correlation, and transfers, prizes and intellectual property rights. Needless to say, many possible important extensions are left unpursued and these should be explored in future in subsequent work.

9 Appendix: Proofs and Generalizations

The following proofs (except for Lemma 1) are based on the most general two-district model (i.e., with the extensions from sections 6.1-6.3 incorporated). To refer to an arbitrary district, we use $i \in \{A, B\}$ and $j \equiv \{A, B\} \setminus i$. Hence, i succeeds with probability p, and then i enjoys the quality-increase

$$V \equiv 1 + G(1-p),$$

where G is the coordination benefit i receives if the other district copies the policy (which can happen if the other district fails, i.e., with probability $1-p$). If i instead adopts j's successful policy, then i enjoys the quality-benefit

$$v \equiv \alpha + g,$$

where $\alpha \in [0,1]$ is the transparency-index while g is the coordination benefit for the looser. By employing the parameters v and V, we incorporate all our first three extensions as well as many others. For centralization, we also use $W \equiv 1 + G$.

The following lemmas describe the results for this "generalized model". Each proposition above follows directly from the corresponding lemma (with the same number as the associated proposition).

Lemma 1: *Suppose $p(1-p) > k/2$ and define $\overline{h}_b > 0$ by:*

$$p(1-p)\, c\left(h - f_b^{-1}\left(\overline{h}_b\right)\right) + [1 - p(1-p)]\, c\left(f_b^{-1}\left(\overline{h}_b\right)\right) = p(1-p),$$

where $a = f_b^{-1}(\cdot)$ is the inverse of:

$$f_b(a) \equiv a + c'^{-1}\left(\left[\frac{1}{p(1-p)} - 1\right] c'(a)\right).$$

(i) If $h \leq \overline{h}_b$, the first-best is implemented by $a_i = a_j = a > 0$ satisfying:

$$\frac{c'(a)}{c'(h-a) + c'(a)} = p(1-p).$$

(ii) If $h \geq \overline{h}_b$, the first-best is implemented by $a_i = a_j = 0$.
(iii) If $h = \overline{h}_b$, both outcomes [(i) and (ii)] give the same payoffs.

Proof. Consider the basic model. Suppose that $2p(1-p) > k$, so it is optimal that both experiment even when locations are identical.

(i) Suppose both districts experiment and switching constraints are satisfied: $c(h-a_i) - c(a_j) \leq 1$. The first-best then maximizes $u_{i,b} + u_{j,b}$, where

$$\begin{aligned} u_{i,b} &= p^2(1-c(a_i)) - (1-p)^2 c(a_i) + p(1-p)(2-c(a_i)-c(h-a_j)) - k \\ &= p(2-p) - c(a_i) - p(1-p)[c(h-a_j) - c(a_i)] - k. \end{aligned}$$

Clearly, first-order conditions imply $a_i = a_j = a$ satisfying:

$$c'(a) = p(1-p)[c'(h-a) + c'(a)] \Rightarrow (1).$$

The second-order condition is $-c''(a) - p(1-p)[c''(h-a) - c''(a)] < 0$, which always hold.

(ii) If j's switching constraints are not satisfied, then $a_i = 0$ is the trivial local optimum, giving $u_i = p - k$. By comparison, payoffs are higher under (1) when

$$p - k < p(2-p) - c(a) - p(1-p)[c(h-a) - c(a)] - k \Leftrightarrow$$

$$p(1-p)c(h-a) + [1-p(1-p)]c(a) < p(1-p). \tag{17}$$

Note that we can rewrite (1) to:

$$h = f_b(a) \equiv a + c'^{-1}\left(\left[\frac{1}{p(1-p)} - 1\right] c'(a)\right),$$

where $f_b(a)$, as well as $h - a$, are increasing in a. Inserted into (17), we get

$$p(1-p)c\left(h - f_b^{-1}(h)\right) + [1-p(1-p)]c\left(f_b^{-1}(h)\right) < p(1-p), \tag{18}$$

where the left-hand side is increasing in h. Define $\overline{h}_b$ such that (18) binds when $h = \overline{h}_b$, and note that $\overline{h}_b > 0$ since when $h = 0$, the left-hand side of (18) is zero. It follows that (18) holds if and only if $h < \overline{h}_b$.

It is easy to check that it cannot be optimal with an asymmetric solution such that one switching constraint is satisfied but not the other. *QED*

9.1 Decentralization

Lemma 2:

(i) Suppose that j's switching constraint at stage three is satisfied:

$$c\left(h-a_i\right)-c\left(a_j\right)\leq v\ (\mathrm{SC}_j).$$

If j experiments, i is also willing to experiment if and only if IC_i holds:

$$c\left(h-a_j\right)-c\left(a_i\right)\geq \frac{k-p\left(V-pv\right)}{p^2}\ (\mathrm{IC}_i). \tag{19}$$

(ii) If SC_j is violated and j experiments, i is also willing to experiment if and only if:

$$c\left(h-a_j\right)-c\left(a_i\right)\geq \frac{k-p\left(1-pv\right)}{p^2}.$$

Proof. At stage 3, $i\in\{A,B\}$ prefers j's success to i's own failed experiment if and only if the following switching or selection constraint (SC) for i (SC_i) is satisfied:

$$c\left(h-a_j\right)-c\left(a_i\right)\leq v\quad (\mathrm{SC}_i) \tag{20}$$

If (20) fails, i will certainly experiment (since we have assumed $p>k$). Thus, presume, first, that (20) is satisfied.

It is easy to check that there is no equilibrium where both experiments if i actually prefer j's location, i.e., if $|h-a_j|\leq|a_i|$ for some $i\in\{A,B\}$. From now we thus consider the natural case where $|h-a_j|>|a_i|$ for both districts.

If the other district j experiments and SC_j is satisfied, i experiments as well if this gives i a higher payoff than by not experimenting:

$$\begin{aligned} & p\left(V-c\left(a_i\right)\right)+\left(1-p\right)\left[p\left(v-c\left(h-a_j\right)\right)-\left(1-p\right)c\left(a_i\right)\right]-k \qquad (21)\\ \geq\ & p\left(v-c\left(h-a_j\right)\right)-\left(1-p\right)c\left(a_i\right)\Leftrightarrow (19). \end{aligned}$$

This gives the incentive constraint (IC) for i (IC_i). If SC_j is violated, V in (19) should be replaced by 1, since i's success will then never create any coordination benefit (as j will never switch).

Assume from now that SC_j is satisfied as well (as it must be in the *symmetric* equilibrium - and it can be shown that all equilibria where both experiment must be symmetric; see below).

Note that when IC_i holds (binds), SC_i can still be (is) satisfied if

$$v - \frac{pV - k}{p^2} \leq v \Leftrightarrow pV > k,$$

which is already assumed to hold.

Finally, when SC_i is violated, then i always prefers to experiment and (19) holds also in this case. *QED*

Lemma 3:

Define the thresholds $h_d^* < \widehat{h}_d \leq \widetilde{h}_d \leq \overline{h}_d$ *and* $\underline{h}_d$ *by:*

$$\begin{aligned}
h_d^* &= c^{-1}\left(v - \frac{pV-k}{p^2}\right), \\
\widehat{h}_d &= c^{-1}(v), \\
\widetilde{h}_d &\equiv c^{-1}(p[1-p]G) + c^{-1}(v), \\
\overline{h}_d &\equiv c^{-1}(p(1-p)G) + c^{-1}(v + p(1-p)G), \\
\underline{h}_d &\equiv c^{-1}\left[v - \frac{pV-k}{p^2} + (1-p)\left(V - \frac{k}{p}\right) - p^2 G\right] \\
&\quad - c^{-1}\left[(1-p)\left(V - \frac{k}{p}\right) - p^2 G\right],
\end{aligned}$$

where the function $c^{-1}(x)$ *is the inverse of* $c(|x|)$.

There exist pure-strategy equilibria where both districts experiment if and only if $h \geq \underline{h}_d$. *All equilibria are symmetric and they are characterized as follows:*

(i) If $h \in [\underline{h}_d, h_d^*)$, *the equilibrium is unique: policies diverge and* $a < 0$ *increases in* h *and satisfies:*

$$c(h-a) - c(a) = \frac{k - pV + p^2 v}{p^2}. \tag{22}$$

(ii) If $h \in \left[h_d^*, \widehat{h}_d\right]$, *the equilibrium is unique: districts experiment at their ideal points, so* $a = 0$.

(iii) If $h \in \left(\widehat{h}_d, \widetilde{h}_d\right)$, *the equilibrium is unique: policies converge and* $a > 0$ *increases in* h *and ensures that (SC) binds:*

$$c(h-a) - c(a) = v. \tag{23}$$

(iv) If $h \in \left[\widetilde{h}_d, \overline{h}_d\right]$, *there are two equilibria in pure strategies: one is as described by (iii), and in the other,* $a = 0$.

(v) If $h > \overline{h}_d$, the equilibrium is unique: districts experiment at their ideal points, so $a = 0$.

Some comments: Note that we have $\overline{h}_d > \widehat{h}_d$ if and only if the coordination benefit is positive, $G > 0$, and the larger is G, the larger is the range at which a district would like to accommodate ($\overline{h}_d$ increases in G). For a given $G > 0$, it is also necessary to accommodate more if α (and thus $v = \alpha + g$) is small. At the end of the proof, there are explicit formulations for the thresholds for the case where $c(.)$ is quadratic.

Proof. Objectives and constraints: Anticipating the behavior of district i, described in the previous proof, we now derive district j's optimal location, given by a_j. Just as i's utility is given by (21), we can write

$$u_j = p\left(V - c\left(a_j\right)\right) + (1-p)\left[p\left(v - c\left(h - a_i\right)\right) - (1-p)\, c\left(a_i\right)\right] - k$$

as long as both districts experiment (IC holds) and both selection constraints hold. For a given a_i, IC_i is satisfied when $a_j \leq a_j'(a_i)$, where $a_j'(a_i)$ is defined such that (19) binds:

$$c\left(h - a_j'\left(a_i\right)\right) - c\left(a_i\right) = v - \frac{pV - k}{p^2}, \tag{24}$$

while SC_i requires that $a_j \geq a_j''(a_i)$, where $a_j''(a_i)$ is defined such that (20) binds:

$$c\left(h - a_j''\left(a_i\right)\right) - c\left(a_i\right) = v. \tag{25}$$

The left-hand sides of (24) and (25) are (identical and) both decreasing in a_j. Since the right-hand side of (24) is larger than that of (25), we have $a_j''(a_i) < a_j'(a_i)$. When both IC_i and SC_i hold, $a_j \in \left[a_j''(a_i), a_j'(a_i)\right]$ and u_j is increasing whether $a_j \uparrow 0$ or $a_j \downarrow 0$. Hence, $a_j = 0$ is preferred by j if both IC_i and SC_i are satisfied. If $a_j''(a_i) < 0 < a_j'(a_i)$, j can indeed set $a_j = 0$ without violating IC_i or SC_i. If $a_j''(a_i) < a_j'(a_i) < 0$, j needs to diverge, by selecting $a_j = a_j'(a_i) < 0$, to satisfy IC_i. If $0 < a_j''(a_i) < a_j'(a_i)$, j needs to accommodate, by selecting $a_j = a_j''(a_i) > 0$, if j wants i to copy j's success if i fails (i.e., to satisfy SC_i). It follows that for the set of equilibria where both IC_i and SC_i hold, we have:

$$a_j = \min\left\{a_j'\left(a_i\right), \max\left\{0, a_j''\left(a_i\right)\right\}\right\}. \tag{26}$$

Fixed points: To further characterize j's best response, as a function of a_i, note that a_j, as given by (26), is a continuous function. Then, differentiate the left-hand side of (24) and (25). In both cases (i.e., regardless of whether $a_j = a_j'(a_i)$ or $a_j = a_j''(a_i)$), we get:

$$-c'(h-a_j)\,da_j - c'(a_i)\,da_i = 0 \Leftrightarrow \frac{da_j}{da_i} = \frac{-c'(a_i)}{c'(h-a_j)},$$

which is an element in the interval $(0,1)$ when $a_i < 0 \Rightarrow c'(a_i) < 0$, but in $(-1,0)$ when $a_i > 0 \Rightarrow c'(a_i) > 0$ since $|c'(h-a_j)| > |c'(a_i)|$ when $|h-a_j| > |a_i|$, which we have already assumed. When neither IC$_i$ nor SC$_i$ binds, $a_j = 0$ so $da_j/da_i = 0$. Thus, the best-response functions are flat (flatter than the 45-degree line) and they cross exactly once. The equilibrium is thus unique. Since the best-response curves are identical, the unique equilibrium is also symmetric, so simply write $a_i = a_j = a$ and define the fixed points $a' = a'_j(a)$ and $a'' = a''_j(a) < a'$, using (24)-(25). Note that both a' and a'' increase in h. By differentiating (24) or (25), we can see that a smaller h leads to more divergence regardless of whether $a = a'$ or $a = a''$:

$$c'(h-a)(dh-da) - c'(a)\,da = 0 \Leftrightarrow \frac{da}{dh} = \frac{c'(h-a)}{c'(h-a)+c'(a)} > 0. \tag{27}$$

By again referring to (24)-(25), note that $a' = 0$ when $h = h_d^*$ while $a'' = 0$ when $h = \widehat{h}_d$, when these thresholds are implicitly defined by:

$$\begin{aligned} c(h_d^*) &= v - \frac{pV-k}{p^2}, \\ c\left(\widehat{h}_d\right) &= v. \end{aligned} \tag{28}$$

Consequently, when $h \in \left[h_d^*, \widehat{h}_d\right]$, $a'' \leq 0 \leq a'$ and, from (26), the equilibrium is $a = 0$. If $h < h_d^*$, $a'' < a' < 0$ (the IC-constraint is violated when $a = 0$) and $a = a' < 0$ is necessary to satisfy IC (19). If $h > \widehat{h}_d$, $0 < a'' < a'$ (the SC-constraint is violated when $a = 0$) and $a = a'' > 0$ is necessary to satisfy SC.

Lower threshold: So far, we have simply assumed that j wants to satisfy IC$_i$ and SC$_i$. But as $h < h_d^*$ decreases and also $a_j < 0$ decreases to satisfy IC$_i$ (remember that SC$_i$ does not bind in this case), u_j decreases and, for a sufficiently small h (and a_j) it might be that j finds it too costly to satisfy IC$_i$. By instead increasing a_j, j is, at worst, risking that i will stop experimenting because (19) is violated. If so, j's payoff would be $p(1+G) - c(a_j) - k$, since the coordination benefit G is now guaranteed if j succeeds. Given this payoff, $a_j = 0$ would be the best choice for j. Thus, it can be optimal to pick

$a_j = a'$ satisfying IC only if j' payoff in this case (21) is larger than $p(1+G) - k$:

$$\begin{aligned}
p(V - c(a)) + (1-p)[p(v - c(h-a)) - (1-p)c(a)] - k &\geq p(1+G) - k \Leftrightarrow \\
p[(1 + G(1-p)) - c(a)] + (1-p)[pv - p[c(h-a) - c(a)] - c(a)] &\geq p(1+G) \Leftrightarrow \\
(1-p)\left[pv - p\left(v - \frac{pV-k}{p^2}\right)\right] - c(a) &\geq p^2 G \Leftrightarrow \\
(1-p)\left[V - \frac{k}{p}\right] - p^2 G &\geq c(a), \qquad (29)
\end{aligned}$$

where $a = a' < 0$, ensuring that IC binds, is an increasing function of h. We can make this function explicit by rewriting (24) to get:

$$\begin{aligned}
h &= f(a') \equiv a + c^{-1}\left(v - \frac{pV-k}{p^2} + c(a')\right) \Rightarrow \\
a' &= f^{-1}(h).
\end{aligned}$$

Note that $h_d^* = f(0)$. Further, (27) implies that f^{-1}, and thus f, are increasing functions. Inserting $a = f^{-1}(h)$ into (29), we get:

$$\begin{aligned}
c\left(-f^{-1}(h)\right) &\leq (1-p)\left(V - \frac{k}{p}\right) - p^2 G \Leftrightarrow \\
f^{-1}(h) &\geq -c^{-1}\left[(1-p)\left[V - \frac{k}{p}\right] - p^2 G\right] \Leftrightarrow \\
h &\geq \underline{h}_d \equiv f\left[-c^{-1}\left((1-p)\left(V - \frac{k}{p}\right) - p^2 G\right)\right] \\
&= -c^{-1}\left[(1-p)\left(V - \frac{k}{p}\right) - p^2 G\right] \\
&\quad + c^{-1}\left[v - \frac{pV-k}{p^2} + c\left(-c^{-1}\left[(1-p)\left(V - \frac{k}{p}\right) - p^2 G\right]\right)\right] \\
&= c^{-1}\left[v - \frac{pV-k}{p^2} + (1-p)\left(V - \frac{k}{p}\right) - p^2 G\right] - c^{-1}\left[(1-p)\left(V - \frac{k}{p}\right) - p^2 G\right].
\end{aligned}$$

So, there is an equilibrium where both districts experiment and locations strictly diverge ($a = a' < 0$) if and only if $h \in [\underline{h}_d, h_d^*)$. Note that we always have $\underline{h}_d > 0$ (since when $h \downarrow 0$, $a' \to -\infty$, violating (29)). Also, note that $\underline{h}_d < h_d^*$ if and only if $(1-p)\left[V - \frac{k}{p}\right] - p^2 G > 0 \Leftrightarrow G\left(p^2 - (1-p)^2\right) < (p-k)(1/p - 1)$.

Upper threshold: If $h > \widehat{h}_d$, j must accommodate by choosing $a_j = a'' > 0$ in order to satisfy SC_i (remember that IC_i does not bind in this situation). As h increases, and $a_j = a'' > 0$ increases to satisfy SC_i, j may reach the point where j instead wants to reduce a_j, even though SC_i (and SC_j) will then be violated. If so, j's payoff is $p - c(a_j) - k$, and $a_j = 0$ would be the optimal choice (IC_i will still be satisfied for such a reduced a_j, but that is irrelevant to j because j does not strictly prefer to switch to x_i is only i succeeds).

Assume first that i selects $a_i = a'' > 0$. Then, it can be optimal for j to pick $a = a'' > 0$ satisfying SC_i if and only if j's payoff (21) is larger than the best alternative payoff, $p - k$:

$$\begin{aligned} p(V - c(a)) + (1-p)\left[p(v - c(h-a)) - (1-p)c(a)\right] - k &\geq p - k \Leftrightarrow \\ p\left[(1 + G(1-p)) - c(a)\right] + (1-p)\left[pv - p\left[c(h-a) - c(a)\right] - c(a)\right] &\geq p \Leftrightarrow \\ p(1-p)G + (1-p)\left[pv - pv\right] &\geq c(a) \Leftrightarrow \\ p(1-p)G &\geq c(a), \quad (30) \end{aligned}$$

where $a = a'' > 0$, ensuring that SC binds, is an increasing function of h. We can make this function explicit by rewriting (25) to get:

$$\begin{aligned} h &= g(a'') \equiv a'' + c^{-1}(v + c(a'')) \Rightarrow \\ a'' &= g^{-1}(h). \end{aligned}$$

Note that when $a > 0$, c^{-1} is an increasing function, and g, and thus g^{-1}, are both increasing functions (this fact also follows from (27)). Inserting $a = g^{-1}(h)$ into (30), we get:

$$\begin{aligned} c\left(g^{-1}(h)\right) &\leq p(1-p)G \Leftrightarrow \\ g^{-1}(h) &\leq c^{-1}(p(1-p)G) \Leftrightarrow \\ h &\leq \overline{h}_d \equiv g\left(c^{-1}(p(1-p)G)\right) \\ &= c^{-1}(p(1-p)G) + c^{-1}\left(v + c\left(c^{-1}(p(1-p)G)\right)\right) \\ &= c^{-1}(p(1-p)G) + c^{-1}(v + p(1-p)G). \end{aligned}$$

So, there is an equilibrium where both districts experiment and locations converge according to SC if and only if $h \in \left[\widehat{h}_d, \overline{h}_d\right]$. Note that such an equilibrium with strictly positive accommodation requires $G > 0$, since if $G = 0$, then $\overline{h}_d \equiv g(c^{-1}(0)) = g(0) = c^{-1}(v) = \widehat{h}_d$.

Multiple equilibria: When deriving $\overline{h}_d$, we assumed that i selected $a_i = a'' > 0$ to satisfy SC_j. But suppose instead that i prefers to abandon SC_j and instead go for i's best alternative, $a_i = 0$. To determine j's best response in this case, note that if j would like to satisfy SC_i, j certainly prefers the smallest $a_j > 0$ satisfying (25), giving $a_j = a_j''(0)$, defined (as above) by $c\left(h - a_j''(0)\right) = v$. It is easy to check that SC_j is not satisfied at this point, so j receives the payoff

$$pV - c\left(a_j''(0)\right) - k.$$

By comparison, if j gives up on satisfying SC_i, j gets $p - k$, which larger if

$$\begin{aligned} p(V-1) &\leq c\left(a_j''(0)\right) \Leftrightarrow \\ c^{-1}\left(p[1-p]G\right) &\leq a_j''(0) = h - c^{-1}(v) \Leftrightarrow \\ h &\geq \widetilde{h}_d \equiv c^{-1}\left[p(1-p)G\right] + c^{-1}(v). \end{aligned}$$

So, if $h \geq \widetilde{h}_d$, there is an equilibrium where $a_i = a_j = 0$ and both experiment (IC are satisfied but not SC). Note that $\widetilde{h}_d \leq \overline{h}_d$, but this inequality is strict if and only if $G > 0$. Thus, when $G > 0$ and $h \in \left[\widetilde{h}_d, \overline{h}_d\right]$, j prefers to set $a_j = a''$ if i does the same, but to set $a_j = 0$ if $a_i = 0$. Consequently, we have two equilibria in pure strategies where both district experiment when $h \in \left[\widetilde{h}_d, \overline{h}_d\right]$.

Example Q: If $c(a) = qa^2$, then $c^{-1}(\varphi) = \sqrt{\varphi/q}$, so

$$\begin{aligned} h_d^* &= \sqrt{\frac{k - pV + p^2 v}{qp^2}}, \\ \widehat{h}_d &= \sqrt{v/q}, \\ \widetilde{h}_d &\equiv \sqrt{\frac{p(1-p)G}{q}} + \sqrt{\frac{v}{q}}, \\ \overline{h}_d &= \sqrt{\frac{p(1-p)G}{q}} + \sqrt{\frac{p(1-p)G}{q} + \frac{v}{q}}, \\ \underline{h}_d &= \sqrt{\frac{(1-p)\left(V - \frac{k}{p}\right) - p^2 G}{q} + \frac{k - pV + p^2 v}{qp^2}} - \sqrt{\frac{(1-p)\left(V - \frac{k}{p}\right) - p^2 G}{q}}. \end{aligned}$$

Furthermore, $c(h-a) - c(a) = qh(h-2a)$, giving:

$$\begin{aligned} h(h - 2a') &= \frac{k - pV + p^2 v}{qp^2} \Leftrightarrow -a' = \frac{k - pV + p^2 v}{2qhp^2} - \frac{h}{2}, \\ h(h - 2a'') &= \frac{v}{q} \Leftrightarrow a'' = \frac{h}{2} - \frac{v}{2qh}. \end{aligned}$$

QED

Lemma 4: *(i) Of all $h \geq \underline{h}_d$, payoffs are maximized at $h = h_d^*$. (ii) In fact, h_d^* is a global maximum if:*

$$V\left(1 - \frac{p\left(1 + v/V\right)}{2}\right) \geq k\left(\frac{1}{p} - \frac{1}{2}\right). \tag{31}$$

Proof. (i) When $h \in [\underline{h}_d, h_d^*]$ and IC binds, each district receives the equilibrium payoff

$$\begin{aligned} u^d &= p\left(V - c\left(a^d\right)\right) + (1-p)\left[p\left(v - c\left(h - a^d\right)\right) - (1-p)\,c\left(a^d\right)\right] - k \\ &= pV + (1-p)\,p\left[v - c\left(h - a^d\right) + c\left(a^d\right)\right] - c\left(a^d\right) - k \\ &= pV + (1-p)\,p\left[\frac{pV - k}{p^2}\right] - c\left(a^d\right) - k \\ &= V - k/p - c\left(a'\right), \end{aligned} \tag{32}$$

which we can see is maximized (and equal to $V - k/p$) at $h_d^* \Rightarrow a' = 0$.

For $h > h_d^*$, IC does not bind but both districts are worse off since $c(h)$ increases: when $h \in \left(h_d^*, \widehat{h}_d\right)$, we have $a = 0$ so

$$u^d = pV + (1-p)\,p\,(v - c(h)) - k, \tag{33}$$

decreasing in h and approaching $V - k/p$ when $h \downarrow h_d^*$.

When $h \in \left[\widehat{h}_d, \overline{h}_d\right]$ and SC binds (so that one weakly best response is to not copy a successful neighbor), we have

$$u^d = pV - c\left(a''\right) - k, \tag{34}$$

which is also decreasing in h. When $h > \overline{h}_d$, we have $u^d = p - k$, which is independent of h and smaller than $V - k/p$.

Example Q:

$$u^d = \left\{ \begin{array}{c} V - k/p - q\left(\frac{k - pV + p^2 v}{2qhp^2}\right)^2 \text{ if } h \in \left[\underline{h}_d, h_d^*\right], \\ pV - k + (1-p)\,p\left(v - qh^2\right) \text{ if } h \in \left(h_d^*, \widehat{h}_d\right), \\ pV - k - q\left(\frac{h}{2} - \frac{v}{2qh}\right)^2 \text{ if } h \in \left[\widehat{h}_d, \overline{h}_d\right], \\ p - k \text{ if } h > \overline{h}_d. \end{array} \right\}$$

(ii) If $h < \underline{h}_d$, only one district experiments and it does so at its own ideal point.

The utilitarian sum of payoffs is therefore maximized at $h = 0$, giving an average payoff equal to $p(V + v)/2 - k/2$. By comparing to the payoff when $h = h_d^*$, $V - k/p$, we can conclude that h_d^* gives the highest payoff when (31) holds. *QED*

9.2 Centralization

Assume that if only j succeeds, then the median voter selects j's policy with probability one.

Lemma 5l
(i) Suppose $a_j - a_i \leq 1/\sigma$. If j experiments, i does as well if and only if

$$c(h - a_j) - c(a_i) \geq \frac{k - p[1 - p(1 + \alpha)/2] - \sigma(a_i - a_j)p^2(1 - \alpha)/2}{p/2 + \sigma(a_i - a_j)p(p - 1/2)} - (G - g) \quad (\mathrm{IC}_i). \tag{35}$$

(ii) *Consequently, if $a_A = a_B = a$ and j experiments, i does as well if and only if*

$$c(h - a) - c(a) \geq \frac{k - p[1 - p(1 + \alpha)/2]}{p/2} - (G - g). \tag{36}$$

Proof. (i) Consider a district's decision whether to experiment. When both districts succeed or fail, we let $z_A \in [0, 1]$ represent the probability that A's policy is chosen by the median voter when both experiments have the same outcomes. If the ideal point of the decision-maker (median voter) is uniformly distributed with mean 0 and density σ, i.e., $t_m \sim U[-1/2\sigma, 1/2\sigma]$, then the probability that A's policy is chosen is

$$z_A = \frac{1}{2} + \sigma\left(\frac{x_A + x_B}{2}\right) = \frac{1}{2} + \sigma\left(\frac{a_A - a_B}{2}\right),$$

when this is in $[0, 1]$ (requiring $|a_A - a_B| \leq 1/\sigma$). Then $z_B = 1 - z_A \in [0, 1]$.

If i's policy is chosen, i enjoys the value $W = 1 + G$ if i's experiment succeeded but the benefit G if i failed. Note that $W > V = 1 + G(1 - p)$, since i receives the coordination benefit with certainty under centralization.

If the other district experiments, i does, as well, if this gives i a higher payoff than by not experimenting:

$$
\begin{aligned}
& \left[z_i p^2 + p\left(1-p\right)\right]\left(W - c\left(a_i\right)\right) + \left[\left(1-z_i\right)p^2 + p\left(1-p\right)\right]\left(v - c\left(h-a_j\right)\right) \quad (37)\\
& +\left(1-p\right)^2 z_i\left[G - c\left(a_i\right)\right] + \left(1-p\right)^2\left(1-z_i\right)\left[g - c\left(h-a_j\right)\right] - k\\
\geq \; & p\left(v - c\left(h-a_j\right)\right) + \left(1-p\right)\left[\left(1-z_i\right)\left[g - c\left(h-a_j\right)\right] + z_i\left[G - c\left(a_i\right)\right]\right],
\end{aligned}
$$

which can be rewritten as:

$$
\begin{aligned}
& \left[c\left(h-a_j\right) - c\left(a_i\right)\right]\left[\frac{p}{2} + \left(z_i - \frac{1}{2}\right)p\left(2p-1\right)\right] \quad (38)\\
\geq \; & k - \frac{1}{2}\left[p\left(2+G-g\right) - p^2\left(1+\alpha\right)\right]\\
& -\left(z_i - \frac{1}{2}\right)\left[p^2\left(1-\alpha+2\left(G-g\right)\right) - p\left(G-g\right)\right].
\end{aligned}
$$

Note that (38) becomes (35) when we assume $z_i \in [0,1] \Leftrightarrow |a_i - a_j| \leq 1/\sigma$ (then, the inequality-sign when transitioning from (38) to (35) is preserved since the second bracket in (38), which we divide both sides on, is always positive).

(ii) follows straightforwardly. *QED*

Remark: If $a_j - a_i \leq 1/\sigma$ is violated, j accommodates so much more than i that j's policy is always selected when the experiments have identical outcomes. (It may still be that both districts experiment in this case.)

Lemma 6: *Under centralization, there exists four thresholds $\underline{h}_c$, h_c^*, $\widehat{h}_c > h_c^*$ and $\overline{h}_c > h_c^*$ such that, for every $h \geq \underline{h}_c$ there is a symmetric equilibrium where both districts experiment and locations are characterized as follows:*

$$
\begin{aligned}
a &= a_{IC}^c < 0 \text{ if } h \in \left[\underline{h}_c, h_c^*\right)\\
a &= 0 \text{ if } h = h_c^*\\
a &= a_{IC}^c > 0 \text{ if } h \in \left(h_c^*, \widehat{h}_c\right]\\
a &= a_{foc}^c > 0 \text{ if } h \in \left[\widehat{h}_c, \overline{h}_c\right]\\
a &= 0 \text{ if } h \geq \overline{h}_c,
\end{aligned}
$$

where a^c_{IC}, a^c_{foc}, and h^*_c are defined implicitly by:

$$
\begin{aligned}
c\left(h-a^c_{IC}\right)-c\left(a^c_{IC}\right) &= \frac{k-p\left[1-p+\left(G-g\right)/2+p\left(1-\alpha\right)/2\right]}{p/2} \\
c\left(h-a^c_{foc}\right)-c\left(a^c_{foc}\right) &= \frac{c'\left(a^c_{foc}\right)/\sigma-p^2\left(1-\alpha+G-g\right)-\left(1-p\right)^2\left(G-g\right)}{p^2+\left(1-p\right)^2} \\
c\left(h^*_c\right) &= \frac{k-p\left[1-p+\left(G-g\right)/2+p\left(1-\alpha\right)/2\right]}{p/2}.
\end{aligned}
$$

The thresholds $\widehat{h}_c$, $\underline{h}_c$ and $\overline{h}_c$ are defined in the proof. Note that both a^c_{IC} and a^c_{foc} increase in h, and so does therefore $a=\min\left\{a^c_{IC},a^c_{foc}\right\}$.

Proof. We now investigate j's choice of a_j given a_i and that i's subsequent action is as described by Lemma 5 and its proof. Just as the left-hand-side of (37) describes i's payoff, j's payoff can be written similarly:

$$
\begin{aligned}
u^c_j &= \left[z_jp^2+p\left(1-p\right)\right]\left(W-c\left(a_j\right)\right)+\left[\left(1-z_j\right)p^2+p\left(1-p\right)\right]\left(v-c\left(h-a_i\right)\right) \quad (39)\\
&\quad +\left(1-p\right)^2z_j\left[G-c\left(a_j\right)\right]+\left(1-p\right)^2\left(1-z_j\right)\left[g-c\left(h-a_i\right)\right]-k,\ \text{where} \\
z_j &= \frac{1}{2}+\sigma\left(\frac{a_j-a_i}{2}\right). \quad (40)
\end{aligned}
$$

A first-order-condition approach: By taking the derivative of (39) we find that $\partial u^c_j/\partial a_j\geq 0$ if and only if

$$
\begin{aligned}
&\frac{dz_j}{da_j}\left[p^2\left(W-c\left(a_j\right)-v+c\left(h-a_i\right)\right)+\left(1-p\right)^2\left[G-c\left(a_j\right)-g+c\left(h-a_i\right)\right]\right] \quad (41)\\
&-c'\left(a_j\right)\left[z_jp^2+p\left(1-p\right)+\left(1-p\right)^2z_j\right]\geq 0,
\end{aligned}
$$

while the second-order condition $\partial^2u^c_j/\left(\partial a_j\right)^2<0$ always holds. By inspection, and because $dz_j/da_j=\sigma/2>0$, u^c_j is strictly increasing in a_j for all $a_j\leq 0$. When the analogous f.o.c. holds for a_i, we have two equations in two unknown, permitting the unique (and symmetric) solution $a_i=a_j=a^c_{foc}>0$ satisfying:

$$
\begin{aligned}
&c'\left(a^c_{foc}\right)-\sigma\left[c\left(h-a^c_{foc}\right)-c\left(a^c_{foc}\right)\right]\left[p^2+\left(1-p\right)^2\right] \\
=\ &\sigma p^2\left(1-\alpha\right)+\sigma\left(G-g\right)\left[p^2+\left(1-p\right)^2\right]\Leftrightarrow
\end{aligned}
$$

$$
c\left(h-a^c_{foc}\right)-c\left(a^c_{foc}\right)=\frac{c'\left(a^c_{foc}\right)/\sigma-p^2\left(1-\alpha\right)}{p^2+\left(1-p\right)^2}-\left(G-g\right). \quad (42)
$$

Note that the l.h.s. of (42) decreases as $a^c_{foc} < h/2$ increases, so a^c_{foc} increases in σ and $(G-g)$ but decreases in α. When $\alpha = 1$, a^c_{foc} is minimized (over p) when $p = 1/2$ (then, the denominator in (42) is smallest), so, the larger is the probability that the two experiments have the same outcome, the more important it is to pander to the median voter and so the larger is a^c_{foc}.

Adding IC-constraints: The first-order-condition approach is ignoring the IC-constraint for i, (35) or, equivalently, (38). As a_j increases towards a^c_{foc}, (38) might be violated. When both the IC-constraints bind in the symmetric model, the outcome is symmetric $a_i = a_j = a^c_{IC}$ and satisfies:

$$c\left(h - a^c_{IC}\right) - c\left(a^c_{IC}\right) = \frac{k - p\left[1 - p + (G-g)/2 + p(1-\alpha)/2\right]}{p/2}. \tag{43}$$

Remark: When (38) binds, a marginally larger a_j does violate (38) if and only if:

$$\begin{aligned}-c'(h-a_j)\left[\frac{p}{2} + \left(z_i - \frac{1}{2}\right)p(2p-1)\right]& \\ -\left[c(h-a_j) - c(a_i)\right]\left[\frac{\sigma}{2}p(2p-1)\right]& \\ -\frac{\sigma}{2}\left[p^2(1-\alpha+2(G-g)) - p(G-g)\right] &< 0,\end{aligned}$$

which, in the symmetric equilibrium, holds if and only if

$$\begin{aligned}\frac{c'\left(h-a^c_{IC}\right)}{\sigma} &> \left[c\left(h-a^c_{IC}\right) - c\left(a^c_{IC}\right) + G - g\right](1-2p) - p(1-\alpha), \\ &= \left[\frac{k - p\left[1 - p(1+\alpha)/2\right]}{p/2}\right](1-2p) - p(1-\alpha),\end{aligned}$$

which is always satisfied when the r.h.s. is negative (for example when $p > 1/2$), but if it is positive, we must require:

$$\sigma < \overline{\sigma}(h) \equiv \frac{c'\left(h - a^c_{IC}\right)}{\left[k - p\left[1 - p(1+\alpha)/2\right]\right](2/p - 4) - p(1-\alpha)},$$

where we may note that $c'\left(h - a^c_{IC}\right) \geq c'(h/2)$ for all $a^c_{IC} \leq h/2$. We henceforth assume $\sigma < \overline{\sigma}(h)$. If $\sigma > \overline{\sigma}(h)$, there would be no pure strategy equilibrium where both invested.

Returning to (43), note that $a^c_{IC} \leq 0$ if $h \leq h^*_c$ and $a^c_{IC} \geq 0$ if $h \geq h^*_c$, where h^*_c is defined by:

$$c\left(h^*_c\right) = \frac{k - p\left[1 - p + (G-g)/2 + p(1-\alpha)/2\right]}{p/2}.$$

Combined with the lessons from the first-order approach, we can conclude that as long as j indeed prefers that also i experiments, we have in a symmetric equilibrium that:

$$a^c = \min\left\{a^c_{IC}, a^c_{foc}\right\}.$$

So, for $h < h^*_c$, $a^c = a^c_{IC} < 0 < a^c_{foc}$. Both a^c_{IC} and a^c_{foc} are increasing functions of h and so is thus $a^c = \min\left\{a^c_{IC}, a^c_{foc}\right\}$, which we may write as $a^c = \varphi(h)$. Note that a^c_{IC} increases in h faster than does a^c_{foc} when $a > 0$ (this follows from the term $c'\left(a^c_{foc}\right)$ in (42)): we have $a^c_{foc} > a^c_{IC}$ if and only if the l.h.s. of (42) is smaller than the l.h.s. of (43), and this requires that $c'\left(a^c_{foc}\right)$, and therefore a^c_{foc} and h, are sufficiently small.

To be precise, we have $a^c_{IC} > a^c_{foc}$ if and only if $h > \widehat{h}_c$, where $\widehat{h}_c$ ensures that $a^c_{IC} = a^c_{foc}$. Note that we can rewrite (43) to

$$\begin{aligned} h &= f_c\left(a^c_{IC}\right) \equiv a^c_{IC} + c^{-1}\left[\frac{k - p\left[1 - p + (G-g)/2 + p(1-\alpha)/2\right]}{p/2} + c\left(a^c_{IC}\right)\right] \Rightarrow \\ a^c_{IC} &= f_c^{-1}(h). \end{aligned}$$

So, $a^c_{IC} < a^c_{foc}$ when $h < \widehat{h}_c$, implicitly defined such that $a^c_{IC} = a^c_{foc}$ by the following:

$$\frac{c'\left(f_c^{-1}\left(\widehat{h}_c\right)\right)/\sigma - p^2(1-\alpha)}{p^2 + (1-p)^2} - (G-g) = \frac{k - p(1-p)}{p/2} - (G-g) - p(1-\alpha) \Rightarrow \quad (44)$$

$$c'\left(f_c^{-1}\left(\widehat{h}_c\right)\right)/\sigma - p^2(1-\alpha) = \left[\frac{k - p(1-p)}{p/2} - p(1-\alpha)\right]\left[p^2 + (1-p)^2\right] \Rightarrow$$

$$c'\left(f_c^{-1}\left(\widehat{h}_c\right)\right)/\sigma = \frac{k - p(1-p)}{p/2}\left[p^2 + (1-p)^2\right] - p(1-\alpha)\left[p^2 + (1-p)^2 - p\right] \Rightarrow$$

$$f_c^{-1}\left(\widehat{h}_c\right) = c'^{-1}\left[\sigma\frac{k - p(1-p)}{p/2}\left[p^2 + (1-p)^2\right] - \sigma p(1-\alpha)\left[(1-p)(1-2p)\right]\right] \Rightarrow$$

$$\begin{aligned} \widehat{h}_c &= f_c\left[c'^{-1}\left[\sigma\frac{k - p(1-p)}{p/2}\left[p^2 + (1-p)^2\right] - \sigma p(1-\alpha)\left[(1-p)(1-2p)\right]\right]\right] \Rightarrow \\ &= c'^{-1}\left[\sigma\frac{k - p(1-p)}{p/2}\left[p^2 + (1-p)^2\right] - \sigma p(1-\alpha)\left[(1-p)(1-2p)\right]\right] \\ &\quad + c^{-1}\left[\begin{array}{c} \frac{k - p[1 - p + (G-g)/2 + p(1-\alpha)/2]}{p/2} + \\ c\left(c'^{-1}\left[\sigma\frac{k - p(1-p)}{p/2}\left[p^2 + (1-p)^2\right] - \sigma p(1-\alpha)\left[(1-p)(1-2p)\right]\right]\right) \end{array}\right]. \end{aligned}$$

This equation is explicitly defining $\widehat{h}_c \in (h^*_c, \infty)$.

Lower threshold: As $h < h_c^*$ decreases, $a = a_{IC}^c < 0$ decreases in order to satisfy IC. This is costly for district j, which may be tempted to increase a_j, even if then IC_i should be violated and i would stop experiment. In this case,

$$z_i = \min\left\{0, \frac{1}{2} + \frac{a_{IC}^c \sigma}{2}\right\}.$$

if the two outcomes are the same. The payoff to j would be:

$$p + G - (1-p)\, z_i \left(c\left(h - a_{IC}^c\right) + G - g\right) - k,$$

In contrast, the payoff from selecting $a_j = a_i = a_{IC}^c$ is:

$$\left[\frac{p^2}{2} + p\,(1-p)\right](1+\alpha) + \frac{G+g}{2} - \frac{c\left(a_{IC}^c\right) + c\left(h - a_{IC}^c\right)}{2} - k. \tag{45}$$

By comparison, j prefers to stick to a_{IC}^c if and only if:

$$\begin{aligned}
p\,(1-p/2)\,(1+\alpha) - \frac{G-g}{2} - \frac{c\left(a_{IC}^c\right) + c\left(h - a_{IC}^c\right)}{2} &> p - (1-p)\, z_i \left[c\left(h - a_{IC}^c\right) + G - g\right] \Rightarrow \\
\alpha p\left(1 - \frac{p}{2}\right) - \frac{p^2}{2} - \frac{G-g}{2} &> c\left(h - a_{IC}^c\right) - \frac{c\left(h - a_{IC}^c\right) - c\left(a_{IC}^c\right)}{2} \\
- (1-p)\, z_i \left[c\left(h - a_{IC}^c\right) + G - g\right] &\Rightarrow
\end{aligned}$$

$$\begin{aligned}
\alpha p\left(1 - \frac{p}{2}\right) - \frac{p^2}{2} - \frac{G-g}{2} &> c\left(h - a_{IC}^c\right) - \frac{k - p\left[1 - p + (G-g)/2 + p\,(1-\alpha)/2\right]}{p} \\
&\quad - (1-p)\, z_i \left[c\left(h - a_{IC}^c\right) + G - g\right] \Rightarrow
\end{aligned}$$

$$\frac{k}{p} - (1-p)^2 - \frac{p\,(3-p)}{2}(1-\alpha) > c\left(h - a_{IC}^c\right) - (1-p)\, z_i \left[c\left(h - a_{IC}^c\right) + G - g\right], \tag{46}$$

where the r.h.s. becomes arbitrarily high (and the condition will fail) when $h \downarrow 0$ since then $a_{IC}^c \downarrow -\infty$. Thus, there exists a $\underline{h}_c > 0$ (implicitly defined such that (46) binds) such that both districts prefer to stick to $a_{IC}^c < 0$ only if $h \geq \underline{h}_c$.

In the basic model, the condition simplifies to

$$\frac{k - p\,(1-p)^2}{p} > c\left(h - a_{IC}^c\right)\left[1 - (1-p)\, z_i\right].$$

Upper threshold: As $h > h_c^*$ increases, $a > 0$ increases since both districts try to please the median voter. Suppose j considers to give up on pleasing the median voter for the case where both experiments have the same outcomes. If, in this case, the uncertainty regarding the median voter's ideal point is so large (i.e., σ is so small) that $z_j > 0$ even when $a_j = 0$, then we know that this cannot be an optimal choice for j: from (41) we have that $\partial u_j^c / \partial a_j > 0$ when $a_j = 0$. However, if $z_j = 0$ when $a_j \downarrow 0$, then it is no longer the case that $\partial u_j^c / \partial a_j > 0$ at $a_j = 0$, since j's policy would then never be chosen when $a_i = a^c = \min \left\{ a_{IC}^c, a_{foc}^c \right\}$ and both experimental outcomes are the same. Instead, j's policy will only (and always) be chosen if i fails but j succeeds, and, for this situation, $a_j = 0$ is indeed the best choice, giving j the payoff:

$$\widetilde{z}_j p (1-p) W + pv - [1 - \widetilde{z}_j p (1-p)] c (h - a^c) - k.$$

By choosing a^c, j could instead receive the payoff:

$$\left[\frac{p^2}{2} + p (1-p) \right] (1 + \alpha) + \frac{G + g}{2} - \frac{c (a^c) + c (h - a^c)}{2} - k.$$

By comparison, j prefers a^c if

$$\begin{aligned} & p \left(1 - \frac{p}{2} \right) (1 + \alpha) + \frac{G+g}{2} - \frac{c(a^c) + c(h - a^c)}{2} > \\ & p(1-p) W + pv - [1 - p(1-p)] c (h - a^c), \end{aligned} \tag{47}$$

which fails if $h > \overline{h}_c \in (h_c^*, \infty)$, where $\overline{h}_c$ is implicitly defined such that (47) holds with equality. So, when $h \in \left(h_c^*, \overline{h}_c \right)$ and i selects $a^c > 0$, then j prefers $a_j = a^c$ to $a_j = 0$.

Example Q: Rewriting (42) and (43), we get

$$\begin{aligned} qh^2 - 2qha_{foc}^c &= \frac{2qa_{foc}^c/\sigma - p^2 (1-\alpha)}{p^2 + (1-p)^2} - (G - g) \Rightarrow \\ a_{foc}^c &= \frac{qh^2 + G - g + \frac{p^2(1-\alpha)}{p^2 + (1-p)^2}}{2hq + \frac{2q/\sigma}{p^2+(1-p)^2}} \\ &= \frac{(qh^2 + G - g)\left(p^2 + (1-p)^2\right) + p^2 (1-\alpha)}{2hq \left(p^2 + (1-p)^2\right) + 2q/\sigma}, \end{aligned} \tag{48}$$

$$\begin{aligned} qh^2 - q2ha_{IC}^c &= \frac{k - p\left[1 - p + (G-g)/2 + p(1-\alpha)/2\right]}{p/2} \Rightarrow \\ a_{IC}^c &= \frac{h}{2} - \frac{k - p\left[1 - p + (G-g)/2 + p(1-\alpha)/2\right]}{qhp}. \end{aligned} \tag{49}$$

QED

LemmaŁWZ *Of every* $h \geq \underline{h}_c$, *payoffs are highest when* $h = h_c^*$.

Proof. If $h \in \left[\underline{h}_c, \widehat{h}_c\right]$,

$$
\begin{aligned}
u^c &= \left[\frac{p^2}{2} + p(1-p)\right](1+\alpha) + \frac{G+g}{2} - \frac{c\left(a_{IC}^c\right) + c\left(h - a_{IC}^c\right)}{2} - k \qquad (50) \\
&= p\left(1 - \frac{p}{2}\right)(1+\alpha) + \frac{G+g}{2} - \frac{2k - p\left[2 - 2p + G - g + p(1-\alpha)\right]}{2p} - c\left(a_{IC}^c\right) - k \qquad (51) \\
&= 1 + p(1-p) - p(1-p)(1-\alpha)/2 + G - k\frac{1+p}{p} - c\left(a_{IC}^c\right),
\end{aligned}
$$

which is increasing as $h \to h_c^*$ and $a^c \to 0$. If instead $h > \widehat{h}_c$, $a < a_{IC}^c$, smaller than what is presumed in (50), and thus u^c is smaller as well.

Example Q: So, when $h \in \left[\underline{h}_c, \widehat{h}_c\right]$, we get

$$
\begin{aligned}
u^c &= 1 + p(1-p) - p(1-p)(1-\alpha)/2 + G - k\frac{1+p}{p} \\
&\quad - q\left(\frac{h}{2} - \frac{k - p\left[1 - p + (G-g)/2 + p(1-\alpha)/2\right]}{qhp}\right)^2.
\end{aligned}
$$

QED

Lemma 8: *(i) Consider Example Q and suppose* $h \in \left[\underline{h}_c, \widehat{h}_c\right] \cap \left[\underline{h}_d, h_d^*\right]$. *We have* $u^c > u^d$ *if and only if:*

$$
\begin{aligned}
& k - p(1-p) + p(1-p)(1-\alpha)/2 - pG + q\left(\frac{k - p\left[1 - p + p(1-\alpha)/2\right]}{qhp} - \frac{G-g}{2qh} - \frac{h}{2}\right)^2 \\
<\ & q\left(\frac{k - p\left(1 - p\alpha + G(1-2p)\right)}{2qhp^2} - \frac{G-g}{2qh} - \frac{h}{2}\right)^2.
\end{aligned}
$$

(ii) *Consider the basic model and suppose* $h \geq \max\left\{\underline{h}_c, \underline{h}_d\right\}$. *We have* $u^c > u^d$ *if and only if:*

$$
h < \sqrt{\frac{k - p(1-p)}{q}\frac{1/4p^2 - 1}{1/2 - p(1-p)}}.
$$

Proof. (i) Consider first the case where IC binds for centralization as well as decentralization: $h \in \left[\underline{h}_c, \widehat{h}_c\right] \cap \left[\underline{h}_d, h_d^*\right]$ .Under centralization, we had from above that when

$h \in \left[\underline{h}_c, \widehat{h}_c\right]$:

$$\begin{aligned} u^c &= 1 + p(1-p) - p(1-p)(1-\alpha)/2 + G - k\frac{1+p}{p} \\ &\quad -q\left(\frac{h}{2} - \frac{k - p\left[1 - p + (G-g)/2 + p(1-\alpha)/2\right]}{qhp}\right)^2. \end{aligned} \tag{52}$$

Under decentralization when $h \in [\underline{h}_d, h_d^*]$,

$$u^d = V - \frac{k}{p} - q\left(\frac{k - pV + p^2 v}{2qhp^2} - \frac{h}{2}\right)^2. \tag{53}$$

By comparison, $u^c > u^d$ if and only if:

$$\begin{aligned} & p(2-p) - p(1-p)(1-\alpha)/2 + \frac{G+g}{2} - \left[\frac{k - p\left[1 - p + (G-g)/2\right]}{p}\right] \\ & -q\left(\frac{k - p\left[1 - p + (G-g)/2 + p(1-\alpha)/2\right]}{qhp} - \frac{h}{2}\right)^2 - k \\ > \; & V - \frac{k}{p} - q\left(\frac{k - pV + p^2 v}{2qhp^2} - \frac{h}{2}\right)^2 \Leftrightarrow \end{aligned}$$

$$\begin{aligned} & p(2-p) - p(1-p)(1-\alpha)/2 + G + 1 - p \\ & -q\left(\frac{k - p\left[1 - p + (G-g)/2 + p(1-\alpha)/2\right]}{qhp} - \frac{h}{2}\right)^2 - k \\ > \; & 1 + G(1-p) - q\left(\frac{k - pV + p^2 v}{2qhp^2} - \frac{h}{2}\right)^2 \Leftrightarrow \end{aligned}$$

$$\begin{aligned} & p(1-p) - p(1-p)(1-\alpha)/2 + pG - k \\ & > q\left(\frac{k - p\left[1 - p + (G-g)/2 + p(1-\alpha)/2\right]}{qhp} - \frac{h}{2}\right)^2 \\ & -q\left(\frac{k - p(1-p) - p^2(1-\alpha-g) - p(1-p)G}{2qhp^2} - \frac{h}{2}\right)^2. \end{aligned} \tag{54}$$

The condition is more likely to hold if α is large (less transparency favors decentralization).

Regarding G and/or g, we should take the derivative... But it seems like if T increases, then condition is less likely to hold (more transfers favours decentralization) and if p roughly $1/2$, then a larger $G = g$ makes condition more likely to hold (more coordination benefits favor centralization). All this can be intuitively explained.

Set $G = g$ and $\alpha = 1$, then the condition becomes:

$$k - p(1-p) + q\left(\frac{k - p[1-p]}{qhp} - \frac{h}{2}\right)^2 < pG + q\left(\frac{k - p(1-P) + Gp(2p-1)}{2qhp^2} - \frac{h}{2}\right)^2,$$

where the derivative of the r.h.s. w.r.t. G is positive if

$$\begin{aligned}
p + 2q\left(\frac{k - p(1-p) + Gp(2p-1)}{2qhp^2} - \frac{h}{2}\right)\frac{p(2p-1)}{2qhp^2} &> 0 \Leftrightarrow \\
\left(\frac{k - p(1-p) + Gp(2p-1)}{2qhp^2} - \frac{h}{2}\right)\frac{(1-2p)}{hp} &< p \Leftrightarrow \\
\left(\frac{k - p(1-p) + Gp(2p-1)}{qh^2p^2}\right)(1-2p) &< 2p^2 + (1-2p) \Leftrightarrow \\
\left(\frac{k - p(1-p) + Gp(2p-1)}{qh^2p^2}\right)(1-2p) &< p^2 + (1-p)^2,
\end{aligned}$$

which holds if $p > 1/2$, for example (the parenthesis must be positive when $h < h_d^* \Leftrightarrow a^d < 0$). If $p < 1/2$

(ii) In the simplest model, the condition above becomes

$$\begin{aligned}
k - p(1-p) + q\left(\frac{k - p[1-p]}{qhp} - \frac{h}{2}\right)^2 &< q\left(\frac{k - p(1-p)}{2qhp^2} - \frac{h}{2}\right)^2 \Rightarrow \\
h &< \sqrt{\frac{k - p(1-p)}{q}\frac{1/4p^2 - 1}{1/2 - p(1-p)}}.
\end{aligned}$$

For this case, consider also $h \geq h_d^*$. Suppose $h \geq \underline{h}_c$. If $a = a_{IC}^c$, we have from (51):

$$u^c = p(2-p) - \left[\frac{k - p[1-p]}{p}\right] - c(a_{IC}^c) - k$$

If $h \geq h_d^*$,

$$u^d = p + p(1-p)\left(1 - qh^2\right) - k.$$

In this case, $u^d > u^c$, when (using (49)):

$$\begin{aligned}
p\left(2-p\right)-\left[\frac{k-p\left[1-p\right]}{p}\right]-c\left(a_{IC}^{c}\right)-k &< p+p\left(1-p\right)\left(1-qh^2\right)-k \Rightarrow \\
\left[\frac{k-p\left[1-p\right]}{p}\right]+q\left(\frac{h}{2}-\frac{k-p\left[1-p\right]}{qhp}\right)^2 &> p\left(1-p\right)qh^2 \Rightarrow \\
qh^2\left(\frac{1}{4}-p\left(1-p\right)\right)+q\left(\frac{k-p\left[1-p\right]}{qhp}\right)^2 &> 0,
\end{aligned}$$

which always hold. Furthermore, if $h > \widehat{h}_c$, we know $a^c < a_{IC}^c$, which reduces u^c still further and, again, we must have $u^d > u^c$. Thus, it is possible that $u^c > u^d$ only when $h < h_d^*$. *QED*

9.3 Extensions

Proof of Proposition 9. Departing from the generalized model, set $V = 1 + (1-p)\,G$, $v = W = 1 + G$, $g = G$ and $\alpha = 1$.

(i) By differentiating the simplified version of (22), we get:

$$\begin{aligned}
-c'\left(h-a\right)da-c'\left(a\right)da &= \frac{-p\left(1-p\right)dG+p^2dG}{p^2} \Rightarrow \\
\frac{da}{dG} &= \frac{1/p-2}{c'\left(h-a\right)+c'\left(a\right)},
\end{aligned}$$

where the numerator is always positive. The rest of the proof follows from Lemma 3 - except for the claim on h_d^*, which follows from its definition (28) and Lemma 4.

(ii) follows from Lemma 5 and Lemma 6.

(iii) Since we know $\partial u_c/\partial G = 1$, we only need to consider $\partial u_d/\partial G$. From the proof of Lemma 4 and (33)-(34), we can check that $\partial u_d/\partial G < 1$ if $h > h_d^*$. If $h \in [\underline{h}_d, h_d^*]$, we have

$$\frac{\partial u_d}{\partial G} = 1-p+c'\left(a'\right)\frac{\partial a'}{\partial G} \geq 1 \text{ if}$$

$$\begin{aligned}
\left(1-p\right)-2q\left(\frac{k-p\left[1+\left(1-p\right)G\right]+p^2\left(1+G\right)}{2qhp^2}-\frac{h}{2}\right)\left(\frac{2p^2-p}{2qhp^2}\right) &\geq 1 \Leftrightarrow \\
\left(\frac{k-p\left(1-p\right)-Gp\left(1-2p\right)}{2qhp^2}-\frac{h}{2}\right)\left(\frac{1-2p}{h}\right) &\geq 1,
\end{aligned}$$

which is never satisfied if $p > 1/2$ (since then the l.h.s. is negative). If $p < 1/2$, as well, the inequality fails if G is sufficiently large (which again makes the l.h.s. negative). *QED*

Proof of Proposition 10. For the transparency-extension, we can simplify the generalized model by setting $v = \alpha$, $V = W = 1$ and $g = G = 0$.

(i) We can differentiate the simplified version of (22) to get

$$\frac{da}{d\alpha} = \frac{-1}{c'(a) + c'(h-a)} < 0.$$

From Lemma 4 and (28), we have that the optimal h is given by

$$c(h_d^*) = \alpha - \frac{p-k}{p^2},$$

which we can differentiate to get $dh_d^*/d\alpha = 1/c'(h_d^*) > 0$.

If $h \in [\underline{h}_d, h_d^*]$, u^d follows from (32). Taking the derivative w.r.t. α, we find that u^d is maximized when a'

is minimized (at zero), which from (28) requires $\alpha = c(h) + (p-k)/p^2$. If this expression is larger than one (which it is if $h > h_d^*$), then $\alpha = 1$ is optimal. Equation (33) is also uncovering that $\alpha = 1$ is optimal when $h > h_d^*$.

(ii) The claim on a follows from the definitions of a_{IC}^c and a_{foc}^c in Lemma 6 (or by differentiating these definitions). The claim on h_c^* follows from Lemma 7 and the definition of h_d^* in Lemma 6.

Regarding the optimal α in Example Q, when $h \in \left[\underline{h}_c, \widehat{h}_c\right]$ the utility under centralization becomes:

$$\begin{aligned} u^c &= 1 + p(1-p) - p(1-p)(1-\alpha)/2 - k\frac{1+p}{p} \\ &\quad - q\left(\frac{h}{2} - \frac{k - p[1-p+p(1-\alpha)/2]}{qhp}\right)^2, \end{aligned}$$

which is maximized over α when the following f.o.c. holds (the second-order condition holds trivially):

$$\frac{p(1-p)}{2} - 2q\left(\frac{k - p[1-p+p(1-\alpha)/2]}{qhp} - \frac{h}{2}\right)\left(\frac{p}{2qh}\right) = 0 \Rightarrow$$

$$\begin{aligned} k-p\left[1-p+p\left(1-\alpha\right)/2\right] &= p\left(1-\frac{p}{2}\right)qh^2 \Rightarrow \\ -p\left(1-\alpha\right)/2 &= \left(1-\frac{p}{2}\right)qh^2-\frac{k-p\left(1-p\right)}{p} \Rightarrow \\ \alpha &= 1-\frac{k-p\left(1-p\right)}{p^2/2}+\frac{1}{p/2}\left(1-\frac{p}{2}\right)qh^2. \end{aligned}$$

But this $\alpha < 1$ only if

$$\frac{k-p\left(1-p\right)}{p} > \left(1-\frac{p}{2}\right)qh^2 \Rightarrow h < \widetilde{h}_c \equiv \sqrt{\frac{k-p\left(1-p\right)}{q\left(2-p\right)p/2}}.$$

Thus, optimally we have

$$\begin{aligned} \alpha &= 1-\frac{k-p\left(1-p\right)}{p^2/2}+2\frac{p-k}{p^2}+\left(\frac{2}{p}-1\right)qh^2-2\frac{k-p\left(1-p\right)}{p^2} < 1 \text{ if } h \in \left[\underline{h}_c, \widetilde{h}_c\right). \\ \alpha &= 1 \text{ if } h \in \left[\widetilde{h}_c, \widehat{h}_c\right]. \end{aligned}$$

Consider next $h \in \left[\widehat{h}_c, \overline{h}_c\right]$ and note that (48) simplifies to:

$$\begin{aligned} a^c_{foc} &= \frac{qh^2\left(p^2+\left(1-p\right)^2\right)+p^2\left(1-\alpha\right)}{2hq\left(p^2+\left(1-p\right)^2\right)+2q/\sigma} \Rightarrow \\ \frac{da^c_{foc}}{d\alpha} &= \frac{-p^2}{2hq\left(p^2+\left(1-p\right)^2\right)+2q/\sigma} < 0. \end{aligned}$$

So the payoff, from (50), is

$$u^c = \frac{1+\alpha}{2}\left[1-\left(1-p\right)^2\right]-\frac{c\left(a^c_{foc}\right)+c\left(h-a^c_{foc}\right)}{2}-k.$$

The derivative of u^c w.r.t. α is then:

$$\begin{aligned} & \frac{1-\left(1-p\right)^2}{2}+q\left(h-2a^c_{foc}\right)\frac{da^c_{foc}}{d\alpha} \\ = \ & \frac{1-\left(1-p\right)^2}{2}-\left(h-2a^c_{foc}\right)\frac{p^2}{2h\left(p^2+\left(1-p\right)^2\right)+2/\sigma} \\ = \ & p^2+2p\left(1-p\right)-\frac{p^2}{p^2+\left(1-p\right)^2+1/\sigma}\left(1-\frac{2a^c_{foc}}{h}\right), \end{aligned}$$

which is always positive since $a^c_{foc} > 0$. If $h \geq \overline{h}_c$, $a^c = 0$ and u^c is maximized when

$\alpha = 1$.

(iii) From (52) we have

$$\frac{du_c}{d\alpha} = \frac{p(1-p)}{2} - \left(\frac{k - p[1 - p + p(1-\alpha)/2]}{qhp} - \frac{h}{2}\right)\frac{p}{h},$$

and from (53) we have

$$\frac{du_d}{d\alpha} = -\left(\frac{k - p + p^2\alpha}{2qhp^2} - \frac{h}{2}\right)\frac{1}{h}.$$

Thus, when $\alpha = 1$,

$$\frac{du_c}{d\alpha} - \frac{du_d}{d\alpha} = \left[\frac{k - p + p^2}{qh^2p^2}\right]\left[\frac{1}{4} + \frac{1}{4} - p^2\right] - \frac{(1-p)^2}{2},$$

which is always positive when $p \approx 1/2$ since the first bracket is larger than one when $h < h_d^*$. *QED*

Proof of Proposition 11. For this extension we can simplify by setting $G = -g = T$ and $\alpha = 1$, so $V = 1 + (1-p)T$, $v = 1 - T$, and $W = 1 + T$.

(i) The first sentence follows from the definition of a_d and h_d^* when simplifying the model. From Lemma 1, we know that the first-best requires $a = a_b \in [0, h/4]$. In equilibrium, Lemma 3 reveals that, since the thresholds for h depend on T, we can always find T such that $h \in \left(\widehat{h}_d, \overline{h}_d\right)$ and $a > 0$, given by (23). In fact any $a \in (0, h/2)$ is achievable for some T, implying that the first-best can be implemented. (Since i captures the entire surplus if j switches when (23) binds, i will not jump to $a = 0$ before this is socially optimal).

Under (Q) and if $h \in \left(\widehat{h}_d, \overline{h}_d\right)$,(23) becomes:

$$\begin{aligned} h^2 - 2ah &= \frac{k - p(1-p) - pT}{qp^2} \Rightarrow \\ T &= \frac{k - p(1-p)}{p} - qp\left(h^2 - 2ah\right). \end{aligned}$$

Substituting in the first-best a from Corollary 1 completes the proof.

(ii) The first sentence follows from the definitions of a_{IC}^c, a_{foc}^c and h_c^*. Furthermore, we have:

$$u^c = 1 + p(1-p) + T - k\frac{1+p}{p} - 2q\left(\frac{h}{2} - \frac{k - p[1 - p + T]}{qhp}\right)^2.$$

So the f.o.c. when maximizing u^c w.r.t. T is

$$1-2q\left(\frac{h}{2}-\frac{k-p\left[1-p+T\right]}{qhp}\right)\left(\frac{1}{qh}\right)=0$$

giving

$$T=\frac{k-p\left(1-p\right)}{p}$$

(iii) follows from that above. *QED*

Proof of Proposition 12. Let p' be the probability that one of the other $n-1$ district, different from i, succeeds:

$$p'=1-\left(1-p\right)^{n-1}.$$

We will consider equilibria where all districts experiment. If every other district accommodates by a_{-i}, then district i's payoff is:

$$u_{d,n}=p-c\left(a_i\right)+\left(1-p\right)p'\left[1-c\left(h-a_{-i}\right)+c\left(a_i\right)\right]-k.$$

Maximizing this payoff w.r.t. $a=a_i=a_{-i}$ gives the *first-best* level of a:

$$-c'\left(a\right)+\left(1-p\right)\left[1-\left(1-p\right)^{n-1}\right]\left[c'\left(h-a\right)+c'\left(a\right)\right]=0\Leftrightarrow$$

$$\frac{c'\left(a\right)}{c'\left(h-a\right)+c'\left(a\right)}=\left(1-p\right)p'=\left(1-p\right)\left[1-\left(1-p\right)^{n-1}\right].$$

So, a larger n (increasing the r.h.s.) implies more accommodation (increasing the l.h.s.) in the first-best.

(i) If the other districts experiments, i does as well if

$$\begin{aligned}u_n &\geq p'\left[1-c\left(h-a_{-i}\right)+c\left(a_i\right)\right]-c\left(a_i\right)\Leftrightarrow\\ p\left(1-p'\left[1-c\left(h-a_{-i}\right)+c\left(a_i\right)\right]\right) &\geq k\Leftrightarrow\\ c\left(h-a_{-i}\right)-c\left(a_i\right) &\geq \frac{k-p\left(1-p'\right)}{pp'}=1-\frac{p-k}{pp'}\text{ (IC).}\end{aligned}\tag{55}$$

Naturally, the IC-constraint becomes harder to satisfy when n is large since it is quite likely that a nonexperimenter can copy another district's success (p' increases in n and so does the r.h.s. of (55)).

Note that IC holds even when $a = 0$ if $h \geq h^*_{d,n}$, given by:

$$c\left(h^*_{d,n}\right) = \frac{k - p\left(1 - p'\right)}{pp'} = 1 - \frac{p-k}{pp'},$$

increasing in n. But when $h < h^*_{d,n}$, it is necessary with divergence ($a < 0$) for all districts to be motivated to experiment. Just as before, in an equilibrium where all districts will experiment, the IC will bind for every district. This requires $a_i = a_d$, given by:

$$c\left(h - a_d\right) - c\left(a_d\right) = \frac{k - p\left(1 - p'\right)}{pp'} = 1 - \frac{p-k}{p\left[1 - (1-p)^{n-1}\right]}.$$

Since the right-hand side increases in n, so must the left-hand side and therefore $-a_d$. It is easy to check (just as before) that among all these equilibria, payoffs are highest at $h = h^*_{d,n}$.

Example Q gives

$$\begin{aligned} qh\left(h - 2a_d\right) &= \frac{k - p\left(1 - p'\right)}{pp'} \Leftrightarrow \\ a_d &= \frac{h}{2} - \frac{k - p\left(1 - p'\right)}{2qhpp'} \Leftrightarrow \\ h - a_d &= \frac{h}{2} + \frac{k - p\left(1 - p'\right)}{2qhpp'} \end{aligned}$$

and $u_{d,n}$ becomes

$$u_{d,n} = p - qa_d^2 + (1-p)\,p'\left[1 - q\left(h - a_d\right)^2 + qa_d^2\right] - k.$$

(ii) In a symmetric equilibrium where everyone experiments, each district's payoff is

$$\begin{aligned} u_{c,n} &= 1 - (1-p)^n - \frac{c(a) + c(h-a)(n-1)}{n} - k \\ &= p + (1-p)\,p' - \frac{c(a) + c(h-a)(n-1)}{n} - k. \end{aligned}$$

If instead i decides to not experiment, then i's payoff is:

$$p'\left(1 - c(h-a)\right) - \left(1 - p'\right)\frac{c(a) + c(h-a)(n-1)}{n}.$$

By comparison, i experiments only if

$$\begin{aligned} p\left(1-p'\right)+p'\left[c\left(h-a\right)-\frac{c\left(a\right)+c\left(h-a\right)\left(n-1\right)}{n}\right] &\geq k \Leftrightarrow \\ c\left(h-a\right)-c\left(a\right) &\geq n\frac{k-p\left(1-p'\right)}{p'}, \end{aligned}$$

where the r.h.s. is again increasing in n (even more than under decentralization because of the n-effect). Thus, a must decrease in n also under centralization. When the IC-constraints bind, the inequality binds.

Example Q gives:

$$\begin{aligned} qh\left(h-2a_c\right) &= \frac{k-p\left(1-p'\right)}{p'/n} \Leftrightarrow \\ a_c &= \frac{h}{2}-\frac{k-p\left(1-p'\right)}{2qhp'/n} \Leftrightarrow \\ h-a_c &= \frac{h}{2}+\frac{k-p\left(1-p'\right)}{2qhp'/n}, \end{aligned}$$

and payoff $u_{c,n}$ becomes:

$$u_{c,n}=p+\left(1-p\right)p'-\frac{qa_c^2+q\left(h-a_c\right)^2\left(n-1\right)}{n}-k.$$

(iii) By comparison, centralization is better if

$$\begin{aligned} u_{c,n} &> u_{d,n} \Leftrightarrow \\ -\frac{qa_c^2+q\left(h-a_c\right)^2\left(n-1\right)}{n} &> -qa_d^2-\left(1-p\right)p'q\left[\left(h-a_d\right)^2-a_d^2\right] \Leftrightarrow \\ -\frac{n-1}{n}\left[q\left(h-a_c\right)^2-qa_c^2\right]-qa_c^2 &> -qa_d^2-\left(1-p\right)p'q\left[\left(h-a_d\right)^2-a_d^2\right] \Leftrightarrow \\ -\frac{n-1}{n}\left[\frac{k-p\left(1-p'\right)}{p'/n}\right]-qa_c^2 &> -qa_d^2-\left(1-p\right)p'\left[\frac{k-p\left(1-p'\right)}{pp'}\right] \Leftrightarrow \end{aligned}$$

$$
\begin{aligned}
qa_d^2 - qa_c^2 &> [k - p(1-p')]\left(\frac{n-1}{p'} - \frac{1-p}{p}\right) \Leftrightarrow \\
q\left(\frac{h}{2} - \frac{k-p(1-p')}{2qhpp'}\right)^2 - q\left(\frac{h}{2} - \frac{k-p(1-p')}{2qhp'/n}\right)^2 &> [k - p(1-p')]\left(\frac{n-1}{p'} - \frac{1-p}{p}\right) \Leftrightarrow \\
\frac{k-p(1-p')}{q(2hpp')^2} - \frac{k-p(1-p')}{q(2hp'/n)^2} - h\left(\frac{1}{2hpp'} - \frac{1}{2hp'/n}\right) &> \frac{n-1}{p'} - \frac{1-p}{p} \Leftrightarrow \\
\frac{k-p(1-p')}{4qh^2p'^2}\left(\frac{1}{p^2} - n^2\right) - \frac{1}{2p'}\left(\frac{1}{p} - n\right) &> \frac{n-1}{p'} - \frac{1-p}{p} \Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
\frac{k-p(1-p')}{4qh^2p'^2}\left(\frac{1}{p^2} - n^2\right) &> \frac{1}{2p'}\left(\frac{1}{p} - n\right) + \frac{n-1}{p'} - \frac{1-p}{p} \Leftrightarrow \\
\frac{k-p(1-p')}{2qh^2p'}\left(\frac{1}{p^2} - n^2\right) &> \frac{1}{p} + n - 2 - \frac{2p'}{p}(1-p) \Leftrightarrow \\
\frac{k-p(1-p')}{qh^2p'p}(1-np)\left(\frac{1+np}{2}\right) &> 2(1-p)^n - (1-np). \qquad (56)
\end{aligned}
$$

It is easy to see that the inequality can never hold if $np > 1$. If $np < 1$, such that the l.h.s. of (56) is positive, then the inequality still fails if h is sufficiently large, i.e., if:

$$
h < \sqrt{\frac{k-p(1-p')}{qp'p}\frac{1-n^2p^2}{4(1-p)^n - 2(1-np)}}. \qquad (57)
$$

This claim follows since the r.h.s. of (56), and thus the denominator in (57), is always positive.

Remark: To see that the r.h.s. of (56) is always positive, note that its derivative w.r.t. n is

$$
2(\ln((1-p)))(1-p)^n + p,
$$

which is increasing in n. When $n = 2$, the r.h.s. of (56) becomes

$$
\begin{aligned}
2(1-p)^2 - (1-2p) &= p + (1-p)(2-2p-1) \\
&= p + (1-p)(1-2p) > 0
\end{aligned}
$$

when $p < 1/n$. Thus, the r.h.s. of (56) must be positive when $n \geq 2$. *QED*

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