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AFFIRMATIVE ACTION:
ONE SIZE DOES NOT FIT ALL

Kala Krishna
Alexander Tarasov

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ABSTRACT

This paper identifies a new reason for giving preferences to the disadvantaged using a model of contests. There are two forces at work: the effort effect working against giving preferences and the selection effect working for them. When education is costly and easy to obtain (as in the U.S.), the selection effect dominates. When education is heavily subsidized and limited in supply (as in India), preferences are welfare reducing. The model also shows that unequal treatment of identical agents can be welfare improving, providing insights into when the counterintuitive policy of rationing educational access to some subgroups is welfare improving.

Kala Krishna
Department of Economics
523 Kern Graduate Building
The Pennsylvania State University
University Park, PA 16802
and NBER
kmk4@psu.edu

Alexander Tarasov
Department of Economics
University of Munich
Ludwigstrasse 28, room215
80539 Munich, Germany
alexander.tarasov@lrz.uni-muenchen.de

1 Introduction

Affirmative action, or more generally, preferences are ubiquitous in much of the world. The idea is often put as "levelling the playing field", or even tilting it in favor of certain groups so as to atone for past injustices. In this paper we develop a rationale for affirmative action that is different from that offered so far in the literature. We develop a simple model of contests with large numbers of agents (so that strategic effects are removed) and objects. There are two groups: the disadvantaged and the rest of society and two kinds of ability, native and acquired. The test, which determines placement, values acquired ability more highly than does society. Effort is expended in order to do well in the test and is potentially wasteful.

Our rationale for affirmative action, or giving preferences to the disadvantaged, is based on modelling the trade-off between wasteful effort and selection. On the one hand, conditional on getting in, the disadvantaged tend to put in more effort just to get in, as they need to do so being disadvantaged. Such effort is wasteful, which works against giving preferences to the disadvantaged. On the other hand, conditional on winning, the disadvantaged have higher native ability than the advantaged. This factor works in favor of having preferences. While there is no one size fits all answer, the results obtained suggest *when* affirmative action is likely to be beneficial. The model would apply to the system in place in India as its rules are very clear cut. It would also apply to the US setting in spirit, though the US admission system is very nuanced. Our results suggest that in countries, like India, where education is free and preferences are extreme, such reverse discrimination is very harmful to society. However, in the US, where education is costly and preferences are marginal, such a policy may well be beneficial.

The examples and simulations provide a number of other interesting results. They suggest that even when there are no differences between agents, without imposing any unreasonable conditions, it may be welfare increasing to *create* them by allowing some agents access to education while denying it to others. This could be done by having a lottery that allocates access, or by allowing access to one group and not another. Such artificial differences may be optimal both when there is no difference between what society values and what the test does, and when there is.

We will proceed as follows. In the rest of the introduction, we first provide some background on the prevalence and rationale for preferences in the US and the world. Following this we relate our work to the existing literature. Section 2 lays out the key elements of our model and the *raison d'être* behind them, making it clear how our work differs from the literature. It also works through the properties of basic model. Section 3 defines the "effort effect" and "selection effect" and shows formally that while the former works against giving preferences, the latter works in favor of preferences. Section 4 uses an example and develops some simulations to better understand the case for unequal treatment or preferences. Section 5 discusses extensions and Section 6 concludes.

1.1 Background

As President Lyndon B. Johnson said in a speech at Howard University[1] in 1965:

> "Freedom is not enough. You do not take a person who for years has been hobbled by chains .. bring him to the starting point of a race and then say "you're free to compete" and justly believe that you have been completely fair."

In 1965 President Johnson issued executive order No. 11246 that required all federal contractors to take affirmative action to promote the hiring of blacks and other minorities. Affirmative action was, ironically, seen as way to ensure that "employees are treated ...without regard to their race, creed, color or national origin." In Philadelphia for example, specific quotas were set for each of the building and construction trades for blacks, but by the mid seventies, opposition to quotas was growing. In Bakke vs. the University of California, Davis, the Supreme court ruled that Allan Backke, a white medical school applicant, was denied admission on the basis of his race as a quota was set aside for deprived minorities. However, it upheld the use of race as a legitimate criteria in admissions.

Blacks and Hispanics are still given preferences in higher education. Nevertheless, the scope for giving race based preferences has been considerably reduced over the years. While schools used to actively target a given level of minority presence, i.e., meet quotas, such "quotas" are not acceptable today. In 2003, the U.S. Supreme Court approved the use of "points" to promote a diverse student body. However, a number of states, such as Texas, have moved away from using only race based preferences and added other measures. Texas now gives the top 10% of students from public high schools in Texas automatic admission to the state's flagship public university, UT Austin, in *addition* to using points to help diversify the student body as allowed by the 2003 supreme court ruling. The Supreme Court very recently ruled on Fisher versus the University of Texas which challenged the current policy.[2] The ruling seems to have further reduced the space in which universities could give preferences.

In India, preferences are given in higher education and in public sector jobs to "scheduled castes and tribes" and these preferences are quite extreme. Preferences reserve a fraction of seats, proportional to their population share, for Scheduled Castes and Scheduled Tribes. The former were traditionally relegated to unpleasant tasks as "untouchables", while the latter were outside the traditional caste system in India.[3] India's allocation system is based on performance in open competitive exams. The difference in cutoff scores for admission between those given preferences and the general category is huge. Moreover, India's higher educational system is heavily subsidized. Brazil also approved an affirmative action bill in 2012 that reserves half the spots in federal universities for high school graduates of public schools, and distributes the reserved spots among black, mixed race and indigenous students according to the racial

[1] A leading historically black institution.

[2] Abigail Noel Fisher was denied admission to UT Austin and is white.

[3] See Frisancho and Krishna (2012) for more on this.

makeup of each state.

1.2 Related Literature

Much of the work in economics on the affects of preferences has focused on what is called models of statistical discrimination. If ability is unobserved, but correlated with an observable like race, this line of work argues that race based preferences may be counterproductive or even create inequality where none existed. Another line of work has focused on affirmative action as a way of mitigating differential access to education due to credit constraints. A third line has looked at the issue in the context of contests. We discuss each of these below. While we focus on the theoretical work, we also discuss the relevant empirical work as needed.

1.2.1 Statistical Discrimination

Models of statistical discrimination are the dominant line of research in this area. The key papers are those of Arrow (1973), Phelps (1972) and Coate and Loury (1993). The reason is that preferences create a "culture of dependence". Intuitively, less effort is put in by the group given preferences, precisely because preferences make it easy to get jobs even without effort. As a result, people expect the group given preferences to be worse, and these expectations are validated in equilibrium. Even if there are no differences between groups, quotas giving preferences to one group over the other can end up hurting them through this channel. See Fang and Moro (2011) for a comprehensive survey of this literature.

On the empirical side, Ferman and Assuncao (2005) suggest that the effort expended by those offered preferences falls. They exploit a natural experiment which arose when a racial admissions quota was imposed on two of Rio De Janeiro's top public universities. Using a difference -in-difference approach, they found a 5.5% decrease in standardized test scores among the favored group. This constituted a 25% widening of the achievement gap. Moro (2003) estimates a structural model of statistical discrimination for the US and his work suggests that though wage inequality has declined in the US this is not because of a switch in the equilibrium. A counterfactual exercise suggests that in a color-blind society blacks' wage would have been on average more than 20% higher. Moro and Norman (2004) incorporate general equilibrium effects in the labor market into a model with statistical discrimination. They find that while affirmative action may increase the minority workers' incentive to invest in learning while diminishing the non minority's.

In any case, it is far from clear that affirmative action on the basis of race is optimal. There is concern that admitting students based on preferences creates "mismatch". In extreme cases, this mismatch may result in their being worse off than if they went to a school or program better suited to their preparation level. See Rothstein and Yoon (2008) for work on Law Schools in the US and Frisancho-Robles and Krishna (2012) for work on quotas in India. On the other hand, Alon and Tienda (2007) suggest that students admitted under the 10% rule in Texas had a higher graduation rate. Arcidiacano et. al (2012) shows that this may be due to their choosing easier courses/majors. In addition, giving preferences on the basis of income rather than race seems to be called for. Cestau, Epple and Sieg (2012) empirically investigate the importance of using race as basis for affirmative action. They develop and estimate a structural model of admission to a gifted and talented program for children entering the first grade kindergarten program in a mid sized urban school district in the US. Their work suggests that once admission to the program is allowed to depend on being part of the free lunch program or not[4], further conditioning on race gives little benefit to society.

1.2.2 Credit Constraints

The second line of work, distinct from statistical discrimination, looks at preferences in the presence of credit constraints. The basic idea is that markets work well in allocating agents to seats. Monetary bids are just transfers and as long as those with the most to gain from obtaining the seats are also those that society wants to have the seats, i.e., there is no misalignment in social and private benefit, markets will give the first best allocation at the lowest cost. Contests on the other hand involve wasted resources even if they result in the same allocations. Thus, they are dominated by the market. However, if there are credit constraints, then contests may give a better allocation and so be worth the wasted resources they engender, see Fernandez and Gali (1999). Of course, to the extent that credit constraints are income rather than race based, such arguments also favor income based preferences. Also, basic economics (in the form of the principle of targeting which calls for policies to operate on the same margin as the distortion) would suggest that preferences are not the first best solution if the distortion lies in access to credit.[5]

1.2.3 Contests

There is considerable recent work on the role of preferences in education couched in the contests setup, both with small numbers of agents so that there is a strategic effect, and with large numbers where there is not. Here the main issue is whether contests encourage effort or not and effort is seen as being good per se.

Fu (2007) and Fain (2009) have a model with two agents, a minority one who is handicapped relative to the non minority agent. They compete to get in to a school. Effort raises performance at some cost, though performance has a random component. Affirmative action (giving more weight to the score of minorities) can increase the effort expended by all agents, and if this is the objective, then affirmative action may be desirable.

More recently, Chade, Lewis, and Smith (2011) model a duopoly setting where colleges set admission standards and the ability of students is only partially observable. Students choose the portfolio of schools

[4] This is offered to poor students in the US.

[5] For a nice recent survey on the role of credit constraints in education, see Lochner and Monge-Naranjo (2012).

to apply to and schools choose admission rules. They show a number of interesting asymmetries arise in the comparative statics and argue that their model helps explain the observed fall in minority applicants to top public universities after affirmative action was removed. Such schools were removed from the portfolio of minority students who now had less of a chance of getting in.

These strategic models are less closely related to our work than those based on a competitive setting like that in Fryer and Loury (2013). This paper looks at the least cost way of achieving a given diversity goal. In their model there are a given number of slots that agents want to acquire. In the first period, agents make investment decisions and these decisions result in a distribution of second period abilities. The minority group is seen as having higher costs of such investments and as a result are on average disadvantaged ex-post in terms of getting these slots. As there are no externalities or distortions, the market allocation of slots is efficient. They ask, when should the intervention occur: Ex ante or ex post? Should intervention should be sighted (based on race say) or blind (based on say being in the top 10%). They show that if the policy can be sighted, then ex post intervention (a subsidy for the disadvantaged group) is all that is needed to meet the target as this subsidy also raises the ex ante incentive to invest by the minority group. In contrast to their work, we explicitly model a possible reason for affirmative action: namely that native ability is more valuable to society than acquired ability and look for when a case for unequal treatment can be made.

Chan and Eyster (2003) look at whether sighted or blind intervention is more efficient. They argue that a ban on affirmative action is inefficient in practice. Universities with a wish for diversity will just obtain it using an alternative criterion which is less efficient at identifying student quality. Fryer, Loury and Yuret (2008) estimate these costs to be four to five times as high as color-conscious affirmative action.[6]

Hickman (2011) compares quotas, admission preferences, and a color blind system and shows that they differ in terms of their effects. He uses an all pay auction setting and then takes the limit of the auction as the number of agents rises. In an auction setting, the distribution of costs (or valuations) of those you are competing with plays an essential role in how you bid. With a quota system, minorities compete for slots only with other minorities who have a worse distribution of costs than non minorities. With a points system, where minorities are given some points and then compete with the general population, competitors have better costs. With a color blind system, that does not give preferences there are no points given to minorities. As a result, with quotas, the best minorities put in less effort than under a system of preferences, while the worst minorities put in more effort. For non minorities, the effects are

[6] There is also the question of whether lower admission standards result in minority students being out of their depth and so performing poorly. There is a literature on mismatch and catchup that relates to this question which we will not say much about here. See Frisancho-Robles and Krishna (2012) for a literature review and some evidence on this topic using Indian data. Loury and Garman (1993) show that for a given performance, blacks gain more in subsequent earnings from attending selective colleges than whites. However, for many blacks and some whites at such institutions the gain from attending a selective college may be offset by worse performance.

reversed. Hickman focuses on both the effort elicited and on the gap in attainment and cutoffs. The former is seen as a positive and the latter as a negative. It is worth noting that similar results, though for different reasons, occur in the presence of uncertainty in performance. In this case, the effort profile is hump shaped in ability. Agents that are very good need to put in little effort and those that are very bad know that they have little chance of getting in and so choose to put in little effort. Agents who are in between are highly motivated as their chances of getting in rise steeply with effort. Giving preferences to the disadvantaged reduces their cutoff score. This reduces the effort of the more able in the disadvantaged group but raises that of the less able and as the latter are predominant, tends to raise overall effort.

We focus on anonymous contests[7] and the effect of preferences in such settings. While contests play many roles[8], we abstract from most of these below and zoom in on their effort costs. Contests encourage effort, and to the extent that such effort is under/over supplied, this directly raises/reduces welfare. Examples of wasteful effort are manyfold: managers may undercut their competitors at the cost of the company bottom line and salesmen may steal their rival's customers. Effort itself tends to be privately beneficial but socially wasteful when there are rents being competed for, or agents incentives are misaligned with those of the principal or society. We focus only on the costs of effort and its social versus private benefits in this paper, abstracting from other dimensions such as those above.

Although we use placement in educational institutions as the example, other applications are possible. Our setting is non strategic with large numbers of applicants who understand they cannot affect the equilibrium cutoffs by their actions. This environment makes it much easier to analyze the case for preferences. A number of the insights in Siegel (2009) hold in our framework, albeit in a more transparent manner. There is also recent related work on contests with head starts, see Siegel (2013). Our setup differs from the work on contests with head starts both in terms of being non strategic, and in allowing intrinsic differences between agents in their abilities. With head starts, all agents typically differ only in terms of how far ahead they are, while we allow for interactions between native and acquired ability and effort. We also explicitly focus on social welfare and how policies affect it.

2 The Basic Model

We first lay out the rationale for the basic assumptions made. Then we setup the model and explain how it works.

[7] Anonymous contests provide a non-strategic setting (analogous to that of monopolistic competition) as this makes more sense in the education application we are interested in.

[8] For example, contests identify latent talent that might not otherwise be unearthed. The international Math Olympics or the Putnam Exam in the US (administered by the Mathematical Association of America) for example allows students to discover their math ability at an early age. See Kenderov (2006) for a discussion of the role of competitions on education in mathematics.

2.1 The Setup

Our basic setup makes a few assumptions. First, we posit that performance in the contest is related to total ability which is seen as arising from native ability and acquired ability. This seems reasonable. Both innate ability and training are needed for superior performance, though the extent to which one can substitute for the other is an open question. In this context it is worth mentioning the super 30 program in India which has attracted a lot of attention. Anand Kumar, an India Mathematician from humble beginnings, started a program that tries to level the playing field for scheduled caste/tribe students in the Indian State of Bihar. It trains them to take the JEE (joint entrance exam) for entry into elite engineering schools in India like the IITs (Indian Institutes of Technology).[9] From 2002 onwards, 30 disadvantaged students were chosen on the basis of an aptitude/ability test by the program. What is amazing is that the program has consistently placed over 90%, and often 100% of them these elite institutions.[10]

The results of the super 30 program suggest that natural ability, once combined with some training (acquired ability), yields large improvements in performance, i.e., the cross partial derivatives of performance with respect to native and acquired ability are positive. We assume that groups may differ in terms of the distribution of acquired ability as differences in background create differences in acquired ability so that even if native ability is similarly distributed between groups as posited, total ability may not be.

Second, we assume that (i) total ability and effort give rise to performance in the exam, (ii) that there is no randomness in outcomes, and (iii) that effort to improve exam performance has no innate value. That higher ability and effort raise performance is uncontroversial. That there is no randomness in exams is less so. After all, everyone has a bad day and this affects performance. Or one may get lucky and get exactly what one studied in the exam and do better than someone who focused on topics that were not tested! Though randomness in performance complicates matters, in many ways our basic results remain unchanged as discussed in Section 5. Finally, the assumption that studying for the exam has no innate value needs some discussion. The education literature distinguishes between assessments based on exams which encourage rote learning, where studying may have no innate value,[11] and those based on "formative" and "authentic" assessments. Formative assessments allow students to both develop their abilities and assess their progress. They tend to integrate teaching and learning with assessment. Often, they do not require formal grading, but a demonstration of the ability to complete some task. Authentic assessments involve demonstration of ability in a real-world context. Studying for such exams may indeed be valuable in itself. We show that even if agents obtain private benefits from studying, what is critical

[9] The JEE is fiercely competitive and at a very high level. It is said that it is harder to qualify in this exam than to get into Harvard, MIT or Caltech! Smart upper middle class students with all the advantages of their background and training routinely fail this exam, despite tutoring and maniacal effort. There are preferences for backward castes and tribes that make it much easier for them to get in.

[10] See http://en.wikipedia.org/wiki/Super_30 for more on this.

[11] There is some evidence that high stakes testing does not improves student learning. See Amerein and Berliner (2002).

to our arguments is that they study more than they would otherwise. This excessive effort is what is wasteful.[12]

Third, we assume that society values native ability more than the contest does. The contest allocates seats according to the total ability of the agent. However, society would prefer that seats be allocated to those most likely to contribute to the social good. It is reasonable to expect that agents with high native ability are more likely to make the major breakthroughs of use to society than less able, but highly groomed candidates. In other words, society would want Albert Einstein to be educated even if he was bad at exams. Moreover, agents cannot internalize their social contribution so that there is a mismatch between who gets in and who should get in.

2.2 The Specification

There is a continuum of heterogeneous agents, with measure one, who decide whether to take an exam that will be used as the basis for admission. An agent is admitted if his performance exceeds a cutoff performance level denoted by $\tilde{P}$. This cutoff is determined in equilibrium to fill the available seats. We assume that there is no randomness in outcomes.[13]

As there is a continuum of agents, the environment is non strategic. Agents take a summary statistic (the cutoff score) as given, and maximize their objective function. In equilibrium the cutoff score assumed is validated. This is analogous to models of monopolistic competition where firms take the aggregate price index as given and make their choices (on pricing, entry, etc.) to maximize their profits, and where, in equilibrium, the price index that firms take as given is exactly the price index that emerges from the profit maximizing behavior of firms.[14] While the body of the paper looks at admission to a single school, we explain why the results generalize in Section 5 and show this in an Appendix that is available on request.

Agents differ according to their abilities. Specifically, we distinguish between two types of ability: natural and acquired abilities. Natural ability is the ability an agent was born with, while acquired ability is accumulated prior taking the exam (for instance, acquired in high school, or from tutoring). The performance of an agent is denoted by P where:

$$P = f(a_N + a_A, e), \tag{1}$$

where a_N and a_A are natural and acquired abilities respectively, $a = a_N + a_A$ is total ability. For simplicity, we assume that the two types of ability are perfect substitutes. Finally, e is the effort undertaken by the

[12] We ignore any social benefits of studying for the exam in this paper. Acemoglu and Angrist (2000) suggests that such externalities are limited, at least for secondary school.

[13] The effects of dropping this assumption are discussed in Section 5.

[14] Formally, this is an anonymous game. The game is called anonymous because players' preferences depend only on their own actions and the distribution of all other agents' actions, i.e. on the aggregate behavior of all other agents, not on who plays what.

agent to pass the test. We assume that $f(a, e)$ is increasing in both its arguments and the returns to effort in terms of performance are increasing in ability, i.e., $f_{ae}(\cdot) \geq 0$. Moreover, $f(a, e)$ is concave in e: $f_{ee} \leq 0$. Effort has a private cost denoted by $c(e)$, with $c'(\cdot) > 0$ and $c''(\cdot) \geq 0$ and $c(0) = 0$. As effort is costly, this immediately implies that if an agent decides to take the exam, then the effort expended will be the minimum needed to attain the cutoff denoted by $\tilde{P}$. If he does not take the exam then his effort level is zero. Hence, the effort level for agents who decide to take the exam is implicitly defined by e in equation (2):

$$f(a, e) = \tilde{P}. \tag{2}$$

Let this effort level be denoted by $e^*(a, \tilde{P})$.

Higher ability agents need to put in less effort to attain a given performance cutoff so that effort levels are decreasing in ability. This follows from totally differentiating equation (2) to get

$$\frac{\partial e^*(a, \tilde{P})}{\partial a} = -\frac{f_a(\cdot)}{f_e(\cdot)} < 0. \tag{3}$$

Moreover, effort rises as the performance cutoff rises,

$$\frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} = \frac{1}{f_e(.)} > 0. \tag{4}$$

2.3 Equilibrium

The private returns from education of an agent with ability a are given by $s(a) - T$, where T is the tuition level. Without loss of generality, the outside option is normalized to zero. In the text, we assume that effort expended in studying for the exam has no intrinsic value as it does not affect the payoffs, $s(a)$, which depend only on a. In the Appendix, we incorporate an intrinsic value of effort by letting payoffs when educated depend on effort and show that the essence of our results remains. Though effort has two roles in this augmented model; it raises payoffs when educated and it helps you get in, only the former has social value. When there are rents to be appropriated from getting in, agents exert more effort than socially optimal in order to be admitted which is wasteful.

An individual decides to take the exam if doing so is better than not doing so. That is, an agent with total ability a takes the exam if and only if

$$s(a) - T \geq c(e^*(a, \tilde{P})).$$

We assume that $s(a)$ is increasing in a: more able individuals gain more from education.

Define a^* by:

$$s(a^*) - T = c(e^*(a^*, \tilde{P})).$$

The marginal agent (the agent with ability a^*) is indifferent between getting into university or not. Agents with ability below the cutoff a^* choose not to put in the effort required to pass the exam while students above this cutoff choose to put in the required effort.[15]

We assume that the natural and acquired abilities are independently distributed across agents. The distribution functions are given by $H_N(a_N)$ and $H_A(a_A)$ on $[0, a^N_{\max}]$ and $[0, a^A_{\max}]$, respectively. Then, the distribution function for the total ability a is $H(a)$ on $[0, a_{\max}]$, where

$$H(a) = \int_0^{a^A_{\max}} H_N(a - y) dH_A(y) \tag{5}$$

and $a_{\max} = a^N_{\max} + a^A_{\max}$.[16]

The equilibrium in the model is determined by two conditions. First, the total mass of agents accepted to the university equals the number of available seats. Second, the agent with ability a^* is indifferent between going to university and not. The former condition means

$$1 - H(a^*) = \alpha, \tag{6}$$

where α is the number of seats (which is exogenously given and strictly less then one). The latter condition means that

$$s(a^*) - T = c(e^*(a^*, \tilde{P})). \tag{7}$$

Hence, we have two equations with two unknowns: a^* and $\tilde{P}$. In particular, we can solve (6) for a^* and then, given $a^*(\alpha)$, solve (7) for $\tilde{P}$.[17] Since $s(a)$ is increasing in a and $e^*(a, \tilde{P})$ is decreasing, $s(a) - T - c(e^*(a, \tilde{P}))$ is increasing in a. Note that the equilibrium value of a^* depends solely on α, while $\tilde{P}$ depends not only on α but also on the tuition level T.

As depicted in Figure 1, in equilibrium, the effort level is zero till ability a^*. At this point it jumps to $e^*(a^*, \tilde{P})$, the effort required to get in. Then, effort falls with ability and at some point may go to zero as depicted if able enough agents can attain the performance cutoff with no effort.[18]

The next lemma shows what happens if the number of seats is increased or the tuition level rises.

Lemma 1 *A greater number of available seats or a higher tuition level decreases the performance cutoff $\tilde{P}$. That is, $\frac{d\tilde{P}}{d\alpha} < 0$ and $\frac{d\tilde{P}}{dT} < 0$. Moreover, a greater number of available seats reduces the ability cutoff though a higher tuition has no effect on it. That is, $\frac{da^*}{d\alpha} < 0$ and $\frac{da^*}{dT} = 0$.*

[15] This corresponds to Siegel (2009) where the reach and power of an agent are critical in defining the equilibrium. In our setup, the "reach" of an agent is the score at which his gain from admission is zero, the cutoff score or "threshold" is the reach of the marginal agent, and the "power" of an agent is his surplus from choosing a score equal to the threshold score. This is also his payoff in equilibrium.

[16] Note that $\Pr(a_N + a_A \leq y) = \sum_{a_A=0}^{a_A = a^{\max}_A} \Pr(a_N < y - a_A)\Pr(a_A)$. Replacing the sum with the integral gives the equation. In other words, the area under the line $a^N + a^A = a$, $H(a)$, is the same as calculating the density of the population below the line $a^N + a^A = a$ at a given a^A, $(H_N(a - a^A)h_A(a^A))$ and then integrating over all a^A.

[17] We assume that the number of seats is small enough that an interior equilibrium occurs.

[18] As the cost of this effort at a^* is just compensated for by the increase in earnings from going to university, the payoffs depicted in Figure 2 are continuous, though there may be a kink at a^{**}, the ability at which no effort is needed to be admitted. They are also increasing in ability.

Figure 1: Effort in Equilibrium

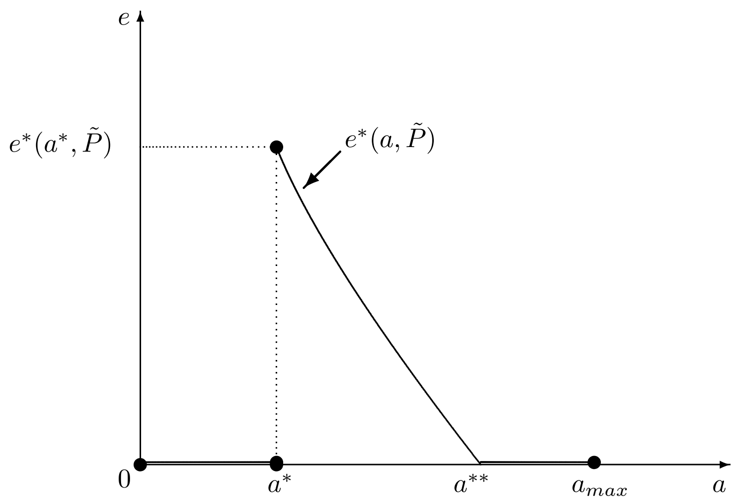
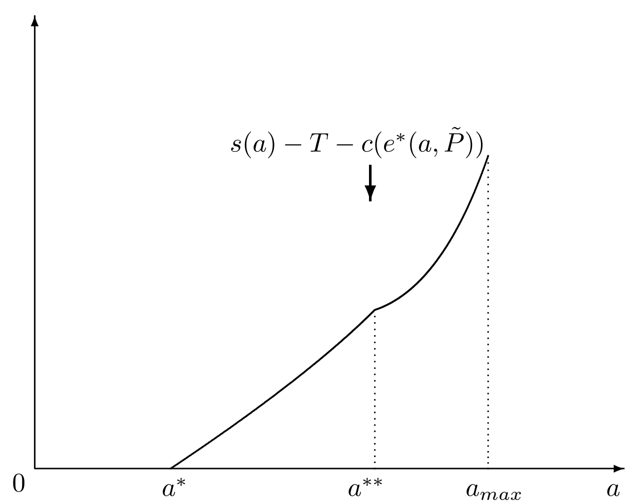


Figure 2: Payoffs from Education



Proof. In the Appendix. ■

A greater number of seats lets agents with lower ability in ($\frac{da^*}{d\alpha} < 0$) and as there are more seats, the cutoff performance falls $\frac{d\tilde{P}}{d\alpha} < 0$. In Figure 1, the effort curve shifts down and to the left with the marginal agent being of lower ability and putting in less effort.

Similarly, a rise in the tuition level decreases the return from being educated of all agents at a given effort level. In order to make the marginal agent indifferent between getting in and not, the effort need to get in, and hence the performance cutoff, must fall as the tuition rises ($\frac{d\tilde{P}}{dT} < 0$). As the performance cutoff falls, all agents need to put in less effort to be admitted. In Figure 1, the effort curve would shift down, with the identity of the marginal agent being unaffected ($\frac{da^*}{dT} = 0$), as the tuition rose.

Due to the lack of randomness in outcomes, agents are able to put in just the effort needed to ensure they attain the cutoff performance, $\tilde{P}$. However, since effort does not improve ability, these efforts are a social waste, even though they are privately valuable and are necessary to allocate seats.[19] Effort expended is then the maximum of the effort needed to get in and the effort chosen for other reasons. Effort is excessive when the effort needed to get in exceeds that which would be chosen for other reasons. As the effort needed to get in falls with ability, unless the effort chosen for other reasons falls even faster (which is hard to motivate) lower ability agents are the ones who expend excessive effort and the essence of our results goes through. See the Appendix for details.

2.4 Social Welfare

We postulate that natural ability is more important than acquired ability for society. To capture this we assume that the social gains from education of an agent with abilities a_N and a_A are given by $s(a_N+\beta a_A)$, where $\beta \in [0, 1]$. Here, β represents the relative importance of the acquired ability for the society. If β is equal to zero, then the society cares only about natural ability. If β is equal to one, then natural and acquired abilities are of the same importance for the society. Social welfare is obtained by integrating social surplus over those with total ability over the cutoff level a^*.

$$W = \int_{a_N+a_A \geq a^*} \Big(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P})) - F \Big) \, dH_N(a_N) dH_A(a_A), \tag{8}$$

where F is the social cost of education per student. As tuition is a lump-sum transfer, T does not directly affect welfare. It only affects it via the effort put in by agents. In addition, as there is no uncertainty in our setup, agents below the cutoff ability do not expend any effort.

Given the number of seats, there are two distortions in the economy. In the model, the effort expended to pass the exam is determined by the tuition level: if the tuition is low, then getting in is very valuable and the performance cutoff is high. To make the cutoff, much effort is needed. This effort is wasteful

[19] While assuming efforts have no effect on ability or payoffs might seem a bit extreme, it is not an unreasonable characterization of many entrance exams which involve studying for the test and emphasize memorization rather than deep understanding. In addition, relaxing this assumption does not change the flavor of the results as shown in the Appendix.

which results in welfare losses. In (8), this loss is equal to $c(e^*(a_N + a_A, \tilde{P}))$ for an agent with ability $a = a_N + a_A$.

The other distortion, given the number of seats, arises due to social benefit deviating from private benefit. In particular, it is optimal for society that the agents with the highest social benefit get in. However, in practice, those with the highest private gain enter. For example, if admission is on the basis of exam performance, and the latter depends on the sum of natural and acquired ability, but social benefit comes from a function that puts more weight on natural ability than acquired ability, then the wrong people will be admitted. Note that, in the first best case, the number of seats should be set so that the social cost of an additional seat equals the social benefit generated. If the number of seats differs from this level there are welfare costs.

Next, we explore the effects of the tuition level and the number of seats on social welfare. In many countries the best public education is much cheaper and often far better than private education. However, it is rationed by strict performance cutoffs. In India, for example, until recently, all higher education institutions were public, close to free, and seats were allocated by performance in a school leaving exam. Even now, the best colleges remain public. The alternative to a bad domestic placement is to go abroad, where admission to comparable institutions is much easier, and to pay non-resident tuition. As a result, those going abroad to study from India seem to fall into two categories: those admitted with funding who tend go to the best places abroad, and those without funding who pay their own way, often at less prestigious places as the best places fund whoever cannot pay. Turkey has a similar system. In fact, in most continental European countries, higher education is public and free. In some countries, students even get a government stipend to go to school. It is easy to see that this system encourages agents to put in more effort than is socially optimal when the number of seats is small.

Next we build our understanding of the model by first considering what happens when $\beta = 1$ and find the optimal tuition level given the number of seats and the optimal number of seats. Then we look at the effect of $\beta \neq 1$.

2.4.1 Optimal Tuition and Seats with $\beta = 1$

We show that the following proposition holds.

Proposition 1 *For any given number of seats, the welfare maximizing tuition elicits zero effort from the marginal agent. When $\beta = 1$, that is the society values native and acquired ability equally, and conditional on no preferences, the first best can be achieved by setting tuition at the full cost of education and setting the number of seats so that all seats are demanded and the marginal agent puts in no effort.*

Proof. In the Appendix. ■

The intuition behind the result is straightforward. An increase in T has no affect on the identity of the marginal agent as long as the equilibrium effort level of the marginal agent is positive. In addition, as it is a transfer, it has no direct effect on welfare. Its welfare consequences arise through its effects on agent's actions. As an increase in T reduces the payoffs from admission, it reduces the effort expended by the marginal agent (whose identity is unchanged by the tuition increase) and so reduces the performance cutoff. This reduction in the performance cutoff in turn reduces the effort each agent choosing to become educated needs to incur, which raises welfare. This is the case until T is such that zero effort is expended by the marginal agent. In this way, the optimal tuition removes the distortion caused by wasted efforts.

The optimal tuition level, T^{opt}, is given by

$$T^{opt} = s(a^*) \tag{9}$$

and the equilibrium performance cutoff is determined by

$$e^*(a^*, \tilde{P}) = 0 \iff \tilde{P} = f(a^*, 0).$$

It is worth noting that higher tuition does have a redistributive effect. The least able agents are unaffected as the alternative to going to university is the same and they are indifferent between these two options. The more able lose from an increase in tuition. When tuition rises, there are two effects on the payoffs from education (given by $s(a) - T - c(e^*(a, \tilde{P}))$). First, keeping effort fixed, the increase in tuition shifts the surplus curve down in a parallel fashion. Second, less effort is expended by all agents in order to get in (as $\tilde{P}$ falls) and this shifts the surplus curve up till the surplus of agent a^* is again zero. As the less able put in more effort, this fall in effort is more valuable to them, so that the shift up is greater for less able agents and this flattens the surplus curve. As a result, individual welfare falls with tuition increases, and more so for the more able. Figure 3 illustrates this reasoning. Of course, as wasted effort is reduced, social welfare rises.

2.5 Bringing in Selection

As discussed above, the distortion caused by wasted effort can be eliminated by setting a sufficiently high tuition level. However, the distortion caused by selection into education can not be completely removed. Only the agents with the highest social gains from education should fill the available seats. Specifically, given the number of seats to be filled, agents with abilities a_N and a_A, such that $s(a_N + \beta a_A) \geq b^*$, should be accepted where b^* is determined by

$$\int_{s(a_N+\beta a_A)\geq b^*} dH_N(a_N)dH_A(a_A) = \alpha.$$

However, competition results in the acceptance condition $a_N + a_A \geq a^*$. As a result, some agents, who should not be accepted to the university, are accepted and vice versa, which in turn leads to welfare losses.

Figure 4 illustrates the distortion for the case when $s^{-1}(b^*) < a^* < s^{-1}(b^*)/\beta$. In Figure 4, agents

Figure 3: Payoffs from Education: a Rise in T

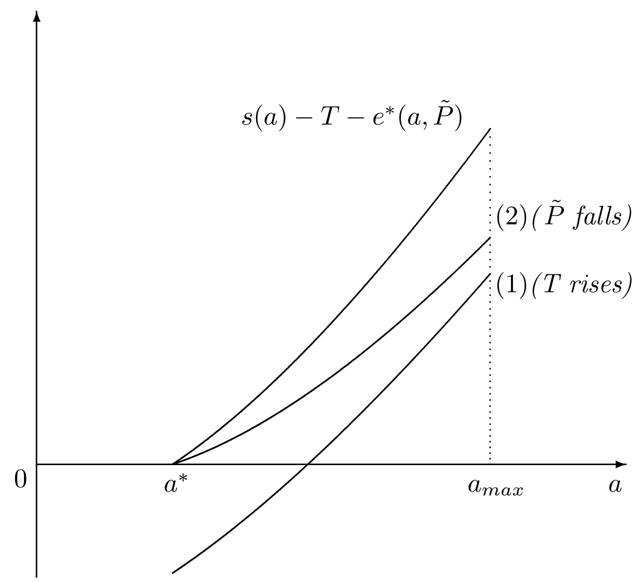
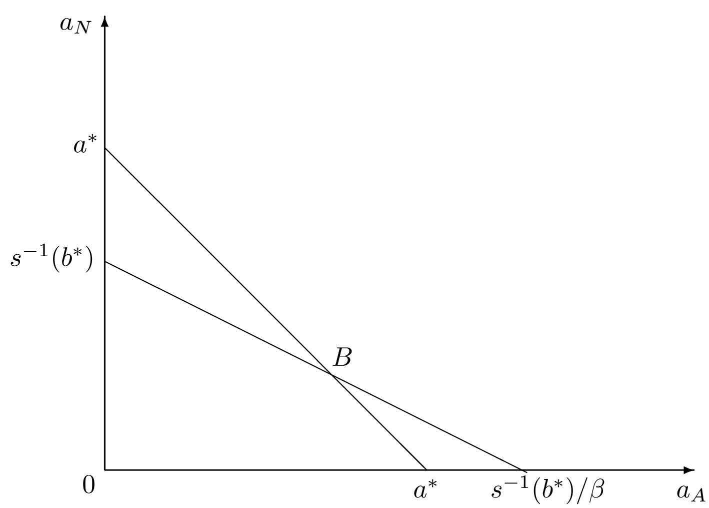


Figure 4: The Selection Distortion



with abilities in the triangle $a^*Bs^{-1}(b^*)$ should be accepted to the university on the basis of maximizing social welfare, but they do not apply, as their individual gains from education are less than their outside option. Instead, agents with abilities in the triangle $a^*B\ s^{-1}(b^*)/\beta$ take the exam and get in, while the social gains from their education are lower than those of the agents in the triangle $a^*Bs^{-1}(b^*)$. As the cutoff ability a^* is determined by the number of seats α, the optimal choice of α can limit the welfare losses caused by the selection distortion, but can not completely eliminate them.

Notice that if β is equal to one, private gains are equal to social gains, so that the first best outcome can be achieved. Specifically, the number of seats should be such that only agents with $s(a_N + a_A) \geq F$ take the exam, which implies that the equilibrium value of a^* must be equal to $s^{-1}(F)$. In this case, the optimal number of seats, α^{opt}, is such that the solution $a^*(\alpha^{opt})$ of

$$1 - H(a^*) = \alpha^{opt}$$

is equal to $s^{-1}(F)$. Finally, the optimal tuition T^{opt} in this case is equal to the social cost of education, F.

3 Reservations and Welfare

In this section, we ask if there is a case for treating groups of agents differently. We assume there are two groups of agents indexed by $i \in \{1, 2\}$, which have identical distributions of natural ability and potentially different distributions of acquired ability. The latter is motivated by the fact that agents with different social backgrounds have had different educational inputs prior to taking the exam, which in turn results in different acquired abilities on their part. In particular, we assume that $H_N^1(a_N) = H_N^2(a_N) \equiv H_N(a_N)$, while $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$ where $\succeq_{LR}$ stands for the likelihood stochastic order. Hence,

$$\frac{h_A^1(a_A)}{h_A^1(x)} > \frac{h_A^2(a_A)}{h_A^2(x)} \text{ for any } a_A, x : a_A > x.$$

This means that group 1 is more favored in terms of acquired ability than group 2. In addition, we assume that the distribution of natural ability has a log-concave density. This assumption is needed to ensure the likelihood stochastic order of the distributions of total ability: i.e., $H^1(a) \succeq_{LR} H^2(a)$.[20]

The share of each group in the total mass of agents (which is normalized to unity) is denoted by γ_i, where $\gamma_1 + \gamma_2 = 1$. A share of available seats is reserved for each group of agents. We denote this reservation quota by θ_i, where $\theta_1 + \theta_2 = 1$. If these quotas are binding, then the cutoffs for the two groups will differ.

Rewriting the equilibrium conditions (6) and (7) to reflect this, let

$$H^i(a) = \int_0^{a^A_{\max,i}} H_N(a - y) dH_A^i(y) \ i = 1, 2. \tag{10}$$

[20] See Theorem 1.C.9 in Shaked and Shanthikumar (2007) for the proof. This assumption is not very restrictive, as a number of commonly used distributions such as the normal, uniform, Gamma, and Beta distributions satisfy it.

where $a^A_{\max,i}$ is the upper bound of $H^i_A(a_A)$. Then

$$\begin{aligned}\gamma_i\left(1 - H^i(a_i^*)\right) &= \theta_i \alpha, &(11)\\ s(a_i^*) - T &= c(e^*(a_i^*, \tilde{P}_i)), \text{ where } i = 1, 2. &(12)\end{aligned}$$

Here, a_i^* is the total ability of the marginal agent from group i and $\tilde{P}_i$ is the performance cutoff for agents from group i.

Having binding reservations is equivalent to setting different performance cutoffs across groups for acceptance to the university. Social welfare in this case is given by

$$\begin{aligned}W(.) &= \gamma_1 \int_{a_N + a_A \geq a_1^*} \Big(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}_1)) - F\Big)\, dH_N(a_N) dH_A^1(a_A)\\ &+\gamma_2 \int_{a_N + a_A \geq a_2^*} \Big(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}_2)) - F\Big)\, dH_N(a_N) dH_A^2(a_A).\end{aligned}$$

Next, we explore how imposing reservations affects social welfare. To reduce notation, let $\theta_2 = \theta$, so that $\theta_1 = 1 - \theta$. The reservation quota θ has two effects on the social welfare. First, changes in θ have an impact on the performance cutoffs $\tilde{P}_1$ and $\tilde{P}_2$ and, therefore, on the effort put in by the agents. For example, a rise in θ (i.e. more seats for the "underprivileged" second group) decreases $\tilde{P}_2$ and increases $\tilde{P}_1$. This in turn implies that agents from group 2 need to put in less effort, while agents from group 1 need to put in more effort to get in. We call this the *effort* effect on the social welfare. Second, there is an impact on selection into education, as a_1^* and a_2^* are affected by θ as well. Specifically, a rise in θ decreases a_2^* and increases a_1^*. That is, more agents from group 2 and fewer agents from group 1 are accepted. We call this the *selection* effect. Hence, we have the following:

$$\begin{aligned}\frac{dW(a_1^*, a_2^*, \tilde{P}_1, \tilde{P}_2)}{d\theta} &= \left[\frac{\partial W(a_1^*, a_2^*, \tilde{P}_1, \tilde{P}_2)}{\partial \tilde{P}_1}\frac{\partial \tilde{P}_1}{\partial \theta} + \frac{\partial W(a_1^*, a_2^*, \tilde{P}_1, \tilde{P}_2)}{\partial \tilde{P}_2}\frac{\partial \tilde{P}_2}{\partial \theta}\right]\\ &+ \left[\frac{\partial W(a_1^*, a_2^*, \tilde{P}_1, \tilde{P}_2)}{\partial a_1^*}\frac{\partial a_1^*}{\partial \theta} + \frac{\partial W(a_1^*, a_2^*, \tilde{P}_1, \tilde{P}_2)}{\partial a_2^*}\frac{\partial a_2^*}{\partial \theta}\right]\\ &= EE + SE,\end{aligned}$$

where EE and SE stand for the effort (changes in θ affect the performance cutoff and via it effort) and selection effects (changes in θ affect the total ability cutoff and affect selection) on the welfare, respectively. In the Appendix, we derive exact expressions for EE and SE.

3.1 Identical Groups

Before we proceed to the analysis of the general case, we first ask whether there is a case for slight preferences if the groups of agents are the same: i.e., $H_A^1(a_A) \equiv H_A^2(a_A)$, even if social and private benefits from education differ. Not surprisingly, the answer is no. The intuition is that on the margin, the gains of one group are exactly made up for by the losses of the other in this case.

Let us define the non-discrimination quota as the quota which results from market clearing so that $\tilde{P}_1 = \tilde{P}_2$. Note that this immediately implies that $a_1^* = a_2^*$. The following proposition holds.

Proposition 2 *If $H_A^1(a_A) \equiv H_A^2(a_A)$, then whatever be β, welfare is locally unchanged in response to the quota when it is set at the non-discrimination level, i.e., such that $\theta_i = \gamma_i$.*

Proof. In the Appendix. ■

The intuition behind this is straightforward. Given a change in θ, agents in one group have their performance cutoff fall (the cutoff falls by more if the group is small so that asymmetry in the size of the groups does not affect this result), which reduces the effort for all abilities and raises the welfare of this group. The opposite happens for the other group. If the initial equilibrium is non discriminatory and the groups have the same distributions, then the gains to one group exactly cancel the losses to the other so that welfare is unchanged. In other words, the effort effect evaluated at the non-discrimination quota is equal to zero. Similarly, there is no effect on the total welfare through selection. The welfare losses due to the selection effect of one group are completely offset by the gains of the other group.

Even though welfare does not change when a "marginal quota" is imposed when groups are identical in their acquired as well as native abilities, this *does not* mean that quotas are always welfare reducing with identical groups. Strictly binding quotas may well raise welfare. The reason is that under certain conditions, the non-discrimination quota gives a minimum, not a maximum of welfare! We explore this result using simulations in the next section.

3.2 Non Identical Groups

When groups are not identical, even marginal quotas can affect welfare. That is, starting from a point where the two groups have the same cutoff, but differ in terms of their ability distributions, discriminating in favor of one group can raise the social welfare. To better understand when this can occur we evaluate the effort and selection effects (EE and SE) when the initial equilibrium is non discriminatory. We show that the effort effect is negative, which calls for discrimination against the weaker group, but that the selection effect is positive which calls for discrimination in favor of the weaker group.

The following proposition holds.

Proposition 3 *If $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$, and the distribution of natural ability has a log-concave density, then the effort effect of discriminating in favor of the weaker group (group 2), evaluated at the non-discrimination quota, is negative. That is,*

$$EE_{\theta=\theta^*} < 0.$$

Proof. In the Appendix. ■

Moving seats from group 1 to group 2 increases the performance cutoff for group 1 and decreases it for group 2, implying that agents from group 1 put in more effort to get in, while agents from group 2 put in less effort. As the distribution of total ability in group 1 stochastically dominates that in group 2, in the non-discrimination equilibrium, agents from group 1 put in less effort on average than agents from group 2**.** As a result, discriminating in favor of group 2 (a rise in θ) raises total wasted effort and decreases the social welfare compared to the non-discrimination equilibrium.

Next we examine the selection effect evaluated at the non-discrimination quota. Specifically, the following proposition holds.

Proposition 4 *If $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$, then the selection effect of discriminating in favor of the weaker group (group 2) evaluated at the non-discrimination quota is positive. That is,*

$$SE_{\theta=\theta^*} > 0.$$

Proof. In the Appendix. ■

The intuition is simple. When group 2 is more disadvantaged in terms of acquired ability, those agents from group 2 who do get must on average have a higher average natural ability.[21] In fact, we can show that if $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$, then in the non-discrimination equilibrium, the distribution of natural ability among agents from group 2 accepted to the university stochastically dominates (in the likelihood ratio order) that among accepted agents from group 1.[22] Since likelihood ratio dominance ensures first order stochastic dominance, accepted agents from group 2 have a higher natural ability on average than do accepted agents from group 1. As society cares more about the natural ability, than does the exam, a quota in favor of group 2 increases welfare by raising the average natural ability of admitted students.

>From the above considerations, we can see that a quota in favor of the disadvantaged group can decrease or increase welfare depending on the magnitudes of the effort and selection effects. Group 2 is the disadvantaged group (because of a worse distribution of acquired ability), and reservations in favor of this group make sense as they allow talented but poorly educated agents to get higher education (the selection effect in the model). However, this comes at a cost, as agents from the disadvantaged group put in on average more effort (the effort effect), which is socially useless. If the selection effect dominates, as is likely when β is low so that acquired ability is worth little to society, then discriminating in favor of the disadvantaged is socially optimal. If β is close to 1, so that acquired ability can easily compensate for native ability in social welfare, then it is actually optimal to discriminate *against* the less advantaged!

In the next section we explore when discriminating in favor of the less advantages is welfare improving using a parametric model as an example as well as simulations.

[21] This is the equivalent of saying in casual conversation that if someone from a bad High School got into Harvard, he/she must be really good!

[22] The proof is available on request.

4 Example and Simulations

In this section we look at an example and some simulations to better understand the model and its implications for policy.

4.1 A Special Case

We parametrize the model to derive a closed-form solution and thereby compare the magnitudes of the effort and selection effects. To simplify the analysis, we assume linearity so $s(a) = Sa$, $f(a,e) = a + e$, and $c(e) = Ce$, where S and C are parameters. In this case, the effort put in is given by

$$e^*(a, \tilde{P}) = \max\left(\tilde{P} - a, 0\right),$$

which implies that agents with total ability greater than $\tilde{P}$ do not put in any effort. The equilibrium conditions under the non-discrimination quota are given by

$$\gamma_1\left(1 - H^1(a^*)\right) + \gamma_2\left(1 - H^2(a^*)\right) = \alpha, \tag{13}$$

$$Sa^* - T - C\left(\tilde{P} - a^*\right) = 0, \tag{14}$$

where the first equation determines the cutoff of total ability, a^*, while the second one determines the performance cutoff:

$$\tilde{P} = \frac{(S+C)a^* - T}{C}.$$

We also assume that both types of ability are uniformly distributed across the agents. That is, $H_N(a_N) = a_N/a^N_{\max}$ on $\left[0, a^N_{\max}\right]$ and $H^i_A(a_A) = a_A/a^A_{\max,i}$ on $\left[0, a^A_{\max,i}\right]$ where $a^A_{\max,1} \geq a^A_{\max,2}$. Note that under this assumption, $H^1_A(a_A) \not\succeq_{LR} H^2_A(a_A)$ (but $H^1_A(a_A) \succeq_1 H^2_A(a_A)$) so that our assumption about the likelihood stochastic order does not hold anymore. However, as shown below, with uniform distributions of abilities, first-order stochastic dominance is sufficient for all the results formulated in the previous section to hold.

In equilibrium, the marginal ability a^* is pinned down by the number of seats α (see (13)). In our analysis, we consider the case when $a^* \geq a^N_{\max}$ and $a^* \geq a^A_{\max,i}$ for $i = 1, 2$. That is, the number of seats is so low that an agent needs both types of ability to get in. Next, we derive explicit expressions for the effort and selection effects evaluated at the non-discrimination quota.

Proposition 5 *The effort and selection effects evaluated at the non-discrimination quota are given by*

$$EE_{\theta=\theta^*} = -\frac{\alpha\,(S+C)}{2}\left(a^A_{\max,1} - a^A_{\max,2} + \frac{\left(a_{\max,2} - \min(\tilde{P}, a_{\max,2})\right)^2}{a_{\max,2} - a^*} - \frac{\left(a_{\max,1} - \min(\tilde{P}, a_{\max,1})\right)^2}{a_{\max,1} - a^*}\right),$$

$$SE_{\theta=\theta^*} = \frac{\alpha(1-\beta)S\left(a^A_{\max,1} - a^A_{\max,2}\right)}{2},$$

where

$$a_{\max,i} = a^A_{\max,i} + a^N_{\max}.$$

Proof. In the Appendix. ■

As can be seen, the magnitude of the effort effect positively depends on the parameters describing the returns from education and the cost of effort, S and C. Moreover, it is straightforward to see that the magnitude of the effort effect is increasing in the performance cutoff $\tilde{P}$. Indeed, if $\tilde{P} < a_{\max,i}$ for $i = 1, 2$, so that some of both abilities get in, then

$$EE_{\theta=\theta^*} = -\frac{\alpha(S+C)}{2}\left(\tilde{P} - a^*\right)^2 \frac{a^A_{\max,1} - a^A_{\max,2}}{(a_{\max,2} - a^*)(a_{\max,1} - a^*)},$$

which is negative and decreasing in $\tilde{P}$. If $a_{\max,2} \leq \tilde{P} \leq a_{\max,1}$, then

$$EE_{\theta=\theta^*} = -\frac{\alpha(S+C)}{2}\left(a^A_{\max,1} - a^A_{\max,2} - \frac{\left(a_{\max,1} - \tilde{P}\right)^2}{a_{\max,1} - a^*}\right),$$

which is also decreasing in $\tilde{P}$. Finally, if $\tilde{P} > a_{\max,1}$, then the effort effect is given by

$$EE_{\theta=\theta^*} = -\frac{\alpha(S+C)}{2}\left(a^A_{\max,1} - a^A_{\max,2}\right),$$

and, therefore, does not depend on $\tilde{P}$. Thus, all else equal, the effort effect is strictly decreasing in $\tilde{P}$ on $[a^*, a_{\max,1})$ and then is flat with respect to $\tilde{P}$.

The selection effect depends only on S and the parameter β that describes the difference between the social and private gains from education. Note that if $\tilde{P} > a_{\max,1}$, the overall effect on social welfare is negative:

$$\frac{\partial W}{\partial \theta}_{\theta=\theta^*} = EE_{\theta=\theta^*} + SE_{\theta=\theta^*} = -\frac{\alpha\left(a^A_{\max,1} - a^A_{\max,2}\right)}{2}(\beta S + C).$$

In this case, the effort effect dominates over the selection effect. Hence, we can conclude that, for sufficiently high values of the performance cutoff (which represents the level of competition for seats, which in turn depends on tuition and the availability of seats), the effort effect is stronger than the selection effect and, as a result, a quota in favor of disadvantaged results in welfare losses. Whereas, for sufficiently low values of $\tilde{P}$, the selection effect prevails over the effort effect and a reservation quota in favor of disadvantaged can increase the social welfare. Note that the tuition fee T affects the effort and selection effects only through $\tilde{P}$. Moreover, a rise in T reduces the performance cutoff $\tilde{P}$. Thus, we know:

Proposition 6 *There exists a value of the tuition fee, T^{tr}, such that an increase in the quota for the disadvantaged group evaluated at the non-discrimination level, θ^*, raises welfare if and only if $T > T^{tr}$.*

Intuitively, a higher tuition level reduces the magnitude of the effort effect and, as a result, a quota in favor of the disadvantaged is more likely to be welfare improving. For this reason, the model suggests that affirmative action is likely to reduce welfare in a setting where education is subsidized. In India, for example, backward castes and tribes have a share of seats (given by their population share) reserved for them in publicly funded higher education. These reservations result in cutoff entrance exam scores that are much lower for these groups than for the general category.[23] As public higher education is not only much cheaper than private, and as the very best institutions are public and seats are scarce, competition to get in is extreme. In such a setting, reservations are likely to be welfare reducing. Higher education is also subsidized in many European countries. However, supply is abundant, and as a result, effort expended to get in is far less than in the Indian context. In the U.S., State Universities tend to be cheaper than private ones of a similar quality. However, the emphasis on need blind admissions and the availability of financial aid significantly reduces the difference in price.

In the next section we turn to some simulations that help us better understand the model. We continue to use the linear setup used in the example in our simulations.

4.2 Simulations

In this subsection, we simulate the model for a number of different values of the parameters. In particular, we assume that the production function $f(a,e)$ and the payoff function $s(a)$ are again linear: $f(a,e) = a+e$ and $s(a) = Sa$. In this case,

$$e^*(a, \tilde{P}_i) = \max(0, \tilde{P}_i - a).$$

In addition, we assume that the cost of effort is quadratic: $c(e) = Ce^2$. As a result, the equilibrium conditions are given by

$$\begin{aligned} \gamma_i \left(1 - H^i(a_i^*)\right) &= \theta_i \alpha, \\ Sa_i^* - T &= C\left(\max(0, \tilde{P}_i - a_i^*)\right)^2 \text{ for } i = 1, 2. \end{aligned}$$

The first condition gives the ability cutoff for each group, while the second equates the benefit from getting in with its full cost for the marginal agent and defines the cutoff performance level for each group. Note that the above implies that a proportional change in S, T, and C does not change either the cutoff ability or the performance cutoff. Therefore, we set C and T to one and leave S to vary in the simulations. We also assume that $F = 1$. In fact, the choice of F does not affect the qualitative implications of the model, as changes in F only shift the welfare function downward or upward and do not affect the equilibrium.

In our simulations, we assume that the distributions of natural and acquired ability take a Gamma form. A Gamma distribution has a density function:

$$f(x, \xi, \upsilon) = \frac{1}{\upsilon^{\xi}} \frac{1}{\Gamma(\xi)} x^{\xi - 1} e^{-x/\upsilon}.$$

[23] In the celebrated Indian Institutes of Technology, the entrance exam marks for the general category are in the high nineties and while they are in the low fifties for the reserved category.

It is characterized by the shape parameter ξ and the scale parameter υ. It has a mean of $\xi\upsilon$ and a variance of $\xi\upsilon^2$. There are two advantages of using a Gamma distribution. First, a Gamma distribution has the following property: if X_i has a Gamma distribution with ξ_i and υ, then $\sum_i X_i$ has a Gamma distribution with $\sum_i \xi_i$ and υ. That is, if the distributions of natural and acquired ability are Gamma with the same scale parameter, then the distribution of total ability is Gamma as well. The second advantage is that if the shape parameter ξ is less than or equal to one, then the density function is decreasing, i.e., the cumulative distribution function (c.d.f.) is concave. For $\xi = 1$, we have the exponential distribution as a special case. Its c.d.f. is also concave. However, for $\xi > 1$, the density at zero is zero, and the density function is first increasing and then decreasing. Thus, the cumulative density is locally convex and then concave. This property is important, as the curvature of the c.d.f. of total ability seems to determine the curvature of the welfare function (see the experiments below). We will assume that $\upsilon = 1$ for all distributions. Then, the density functions are given by

$$\begin{aligned} h_N(a_N) &= \frac{1}{\Gamma(\xi_N)} x^{\xi_N - 1} e^{-x}, \\ h_A^i(a_A) &= \frac{1}{\Gamma(\xi_A^i)} x^{\xi_A^i - 1} e^{-x}, \text{ and} \\ h^i(a) &= \frac{1}{\Gamma(\xi_A^i + \xi_N)} x^{\xi_A^i + \xi_N - 1} e^{-x}, \end{aligned}$$

where ξ_N and ξ_A^i take on different values in the simulations. Table 1 summarizes the parametrization. Parameters that are varied in the simulations are in italics.

It is useful to write welfare slightly differently as done below. Given the form we use, welfare is easily broken down into its value when $\beta = 1$ plus an adjustment factor to account for the different weights placed on acquired ability by society and by individual agents. Specifically,

$$\begin{aligned} W &= \sum_i \gamma_i \int_{a_N + a_A \geq a_i^*} \Big(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}_i)) - F \Big) \, dH_N(a_N) dH_A^i(a_A) \\ &= \sum_i \gamma_i \int_{a_i^*}^{a_{\max,i}} \Big(Sa - c(e^*(a, \tilde{P}_i)) - T \Big) \, dH^i(a) \\ &\quad -(1-\beta) S \sum_i \gamma_i \int_0^{a_{\max,i}^A} a_A \, (1 - H_N(a_i^* - a_A)) dH_A^i(a_A) + \alpha \, (T - F) \, . \end{aligned}$$

Note that we use the fact that $s(.)$ is linear in the above. The first part in the expression above stands for private welfare, which is the sum of the private payoffs from being educated. Let us call this PW for private welfare. The second term in the welfare function, which is given by

$$SC = -(1-\beta) S \sum_i \gamma_i \int_0^{a_{\max,i}^A} a_A \, (1 - H_N(a_i^* - a_A)) dH_A^i(a_A) + \alpha \, (T - F) \, ,$$

is the difference between social welfare and private welfare. For instance, if the private gains from education are equal to the social gains (i.e., β is equal to one), then the welfare is equal to private welfare

Table 1: Parametrization

Variable	Parametrization
Score Production Function:	$f(a, e) = a + e$
Cost of Effort:	$c(e) = Ce^2$, $C = 1$
Tuition:	$T = 1$
Social Cost of Education:	$F = 1$
Scale Parameter of Gamma:	$\nu = 1$
Share of Population in Group i :	$\gamma_1 = \gamma_2 = 0.5$
Shape Parameters of Gamma:	ξ_A^i, $i = 1, 2$. $\xi_N = .5$
Payoff Function:	$s(a) = Sa$
Weight on Acquired Ability in Welfare:	β
Share of Seats in Group i :	θ_i, $i = 1, 2$
Seats Available:	α

if $T = F$. We call this term the "selection" component of welfare and denote it by SC. Thus,

$$W = PW + SC.$$

Next, we construct the welfare as a function of quota θ for different values of $\left\{\beta, S, \alpha, \xi_A^i\right\}_{i=1,2}$.

4.2.1 Simulations with only the Effort Effect

In this subsection, we consider the case when β is equal to one. In this case, social welfare is given by

$$W = PW + \alpha\left(T - F\right) = \sum_i \gamma_i \int_{a_i^*}^{a_{\max,i}} \left(Sa - c(e^*(a, \tilde{P}_i))\right) dH^i(a) - \alpha F.$$

We examine two subcases: when the groups have identical distributions of acquired ability (the symmetric case) and when the groups are different in terms of the distributions of acquired ability (the asymmetric case).[24]

The Symmetric Case When the groups are identical ($\xi_A^1 = \xi_A^2$), our analytical results show that the derivative of welfare with respect to the quota, evaluated at the non-discrimination quota, is equal to zero (see Proposition 2). This implies that if the welfare function is concave (or single-peaked), then the non-discrimination quota is optimal. Recall that the non-discrimination quota when the groups are symmetric is equal to the size of each group (given by γ_i). Simulations verify this result when $\xi_N + \xi_A^i \leq 1$, which ensures that the c.d.f. of total ability is concave which seems to make the welfare function concave as well.

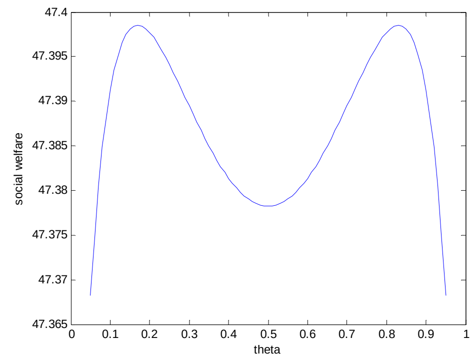
When $\xi_N + \xi_A^i > 1$, interesting things happen. In the figures in this section, the horizontal axis is the quota in favor of the disadvantaged group. In Figures 5 and 6, we consider the case when $\xi_A^1 = \xi_A^2 = 1$. In this case, the shape parameter of the distribution of total ability is 1.5 and, therefore, the c.d.f. is not concave, nor is welfare. The values of S are 110 and 130 in Figure 5 and 6, respectively. The welfare function still has a zero derivative at the non-discrimination quota (as predicted), but this could deliver a local minimum rather than a local maximum.

As can be seen from Figures 5 and 6, there are two global maxima so that it is optimal to discriminate in favor of one group or the other as the two groups are of the same size. In Figure 5 it is optimal not to give all seats to one group. As S rises, we move to Figure 6 and it becomes optimal to exclude one group completely.

Why does this happen? Let us consider what happens, for example, when we move a seat from group 1 to group 2 when the density function is decreasing at the cutoff ability under the non-discrimination quota. As a result of this, the cutoff score for group 2 falls and that for group 1 rises, and the ability of the marginal agent rises in group 1 and falls in group 2. As the density function is decreasing in ability at the cutoff ability, the cutoff for group 2 falls by a smaller amount than the cutoff for group 1 increases

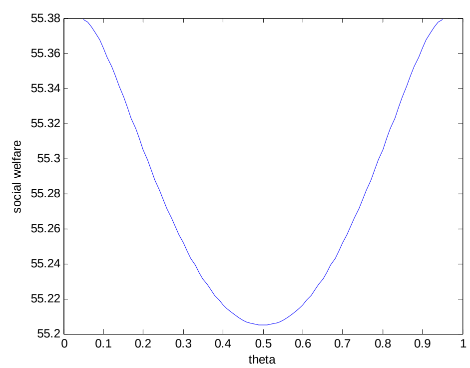
[24] Note that only the distribution of total ability matters for welfare in this case.

Figure 5: Social Welfare (no selection effect, the symmetric case)



The parametrization: $\beta = 1$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 1$, and $S = 110$.

Figure 6: Social Welfare (no selection effect, the symmetric case)



The parametrization: $\beta = 1$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 1$, and $S = 130$.

even when the two groups are of the same size (as illustrated in Figure 7). This happens because there are fewer agents to the right of the cutoff (as the density is decreasing) than to the left. The more the cutoff falls (rises) the more the effort expended falls (rises). As a result, the increase in effort by the agents in group 1 (which reduces surplus and so welfare) is more than the decrease in effort by agents in group 2 (which raises surplus and welfare). Consequently, welfare falls as this reallocation is made so that non discrimination is optimal.

This is depicted in Figure 8. The surplus

$$s(a) - c(e^*(a, \tilde{P}_i)) - T$$

Figure 7: Decreasing Density: Cutoff Effects

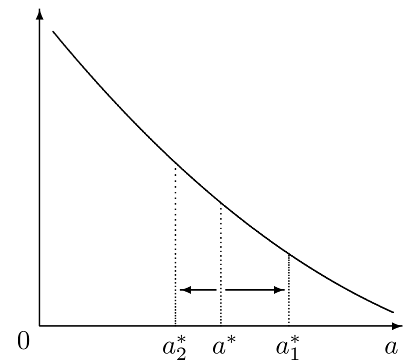


Figure 8: Decreasing Density: Surplus Changes

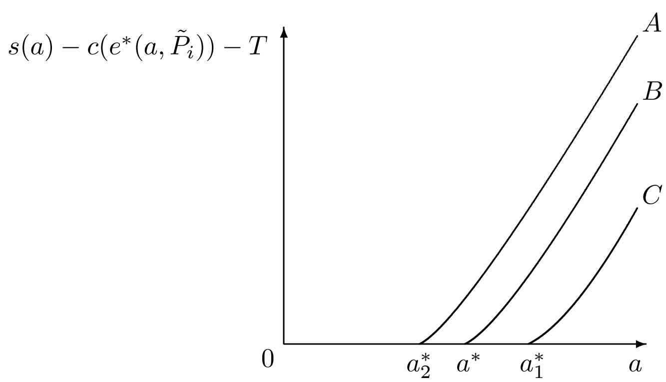
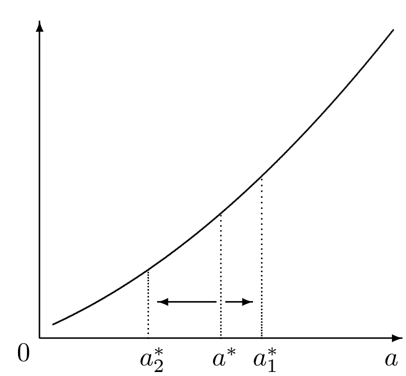


Figure 9: Increasing Density: Cutoff Effects



is zero for the marginal agent and is increasing in ability. With the above reallocation the effort function for group 2 shifts down while that for group 1 shifts up so that their surplus moves in the opposite direction from their effort. As can be seen, the total losses of group 1 (the area $Ca_1^*a^*B$) are greater than the total gains of group 2 (the area $Aa_2^*a^*B$).

However, if the density function is increasing in ability at the cutoff ability under the non-discrimination quota, then the opposite happens (see Figure 9). The increase in effort by the agents in group 1 (which reduces welfare) is less than the decrease in effort by agents in group 2 (which raises welfare), so that welfare rises as this reallocation is made. Thus, non discrimination is a minimum of welfare not a maximum as depicted in Figures 5 and 6. To summarize, if the density function has a single peak (like the normal distribution), then whether welfare locally rises or falls depends on what side of the peak the non discrimination cutoff ability lies. Of course, this cutoff falls as the number of available seats rises.

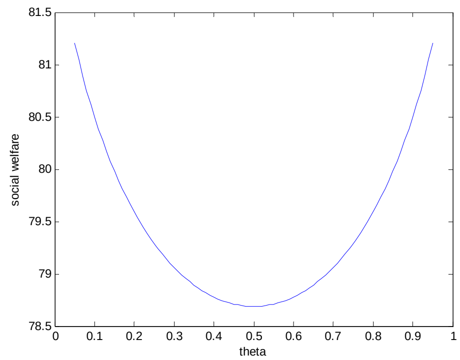
Our argument also explains why greater discrimination is optimal when S rises (see Figures 5 and 6). As S rises, the effort expended to get in rises and this is increasingly costly due to the convex cost of effort posited. As a result, the saving from reduced effort by discriminating in favor of one group is higher and more discrimination is called for as S rises.

Armed with this understanding let us see what happens when α rises from being 0.3. At $\alpha = 0.3$, we have two peaks as in Figure 5. It may actually become optimal to exclude one group or the other as α rises. The reason is that giving all the seats to one group reduces the effort of that group a lot. It also makes the effort of the other group zero as that group has no seats. This is what is depicted in Figure 10 for $\alpha = 0.5$. If α is even higher, then if all seats are given to one group, there may not be enough agents in this group who wish to avail themselves of the seats (given the tuition level) so that some seats are wasted. In this case, giving fewer seats to this group will raise welfare as depicted in Figure 11 where $\alpha = 0.8$.

More generally, the simulations suggest that even when there are no differences between agents, it may be welfare increasing to create them by allowing some agents access to education while denying it to others. By granting a smaller share of agents access, surplus per capita in the resulting competition is higher, though of course, the number granted access is lower. What happens to total welfare depends on how fast surplus per capita rises as access is restricted, i.e., its elasticity. This observation is interesting in terms of policy. In practice, differential access to education is the norm and the welfare implications of such differential access is poorly understood. Advantaged groups tend to have better access to education than others. For example, the better off and better connected may have superior information about how and where to apply. These simulations suggest a novel reason why (even when groups are identical ex ante) differential access (perhaps via a lottery) may be welfare improving.

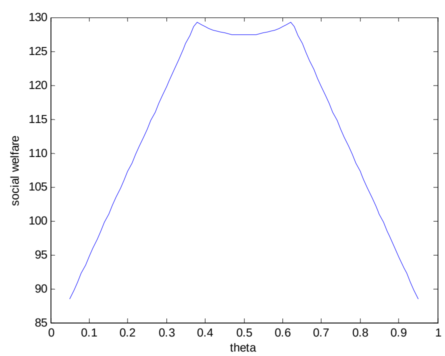
The Asymmetric Case If groups have different distributions of acquired ability and β is equal to one, then, by Propositions 3 and 4, the derivative of the social welfare function at the non-discrimination

Figure 10: Social Welfare (no selection effect, the symmetric case)



The parametrization: $\beta = 1$, $\alpha = 0.5$, $\xi_A^1 = \xi_A^2 = 1$, and $S = 110$.

Figure 11: Social Welfare (no selection effect, the symmetric case)



The parametrization: $\beta = 1$, $\alpha = 0.8$, $\xi_A^1 = \xi_A^2 = 1$, and $S = 110$.

quota is negative. If the welfare function is concave, this means that the optimal quota should be smaller than the non discrimination quota[25], i.e., it should discriminate in favor of the advantaged group. Thus, in this case, θ should be less than non discrimination level.

Under our parametrization, $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$ if $\xi_A^1 > \xi_A^2$. In our experiments, we assume that $\xi_A^1 = 0.5$, $\xi_A^2 = 0.2$, and $\xi_N = 0.5$. Thus, $\xi_A^i + \xi_N < 1$ implying that the density functions of total ability for both groups are decreasing. We also assume that the groups are of the same size: $\gamma_1 = \gamma_2 = 0.5$.

[25] Recall that θ is the quota in favor of the disadvantaged group.

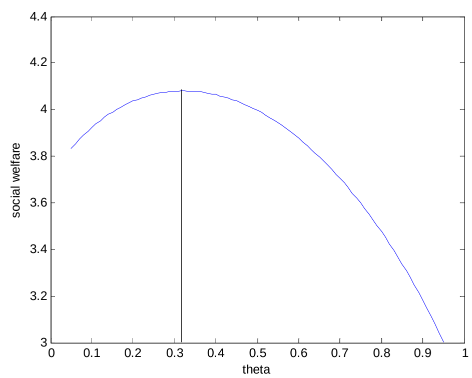
Then, the non-discrimination quota solves the following system of equations:

$$\begin{cases} \alpha = 1 - \gamma_1 H^1(a^*) - \gamma_2 H^2(a^*) \\ \theta^* = \gamma_2(1 - H^2(a^*))/\alpha. \end{cases}$$

The first equation sets demand for seats under non discrimination equal to the supply to obtain the common cutoff. The second equation sets the fraction of seats that go to the less privileged group at this common cutoff equal, by definition, to the non-discrimination quota. Given the values of the parameters, the non-discrimination quota is equal to 0.3932.

Figures 12 and 13 depict the welfare function for different values of S. In Figure 12, $S = 10$ and in Figure 13, $S = 100$. As expected, the optimal quota is always less than the non-discrimination quota (as suggested by the theory). Moreover, as can be seen in 12 and 13, the higher is S, the lower the optimal quota. This is explained by the fact that a higher S implies higher performance cutoffs and, thereby, more effort put in by the agents. This in turn increases the welfare losses due to wasted effort. As a result, a larger quota for the advantaged group (which on average put in less effort) is optimal as S rises.

Figure 12: Social Welfare (no selection effect, the asymmetric case)



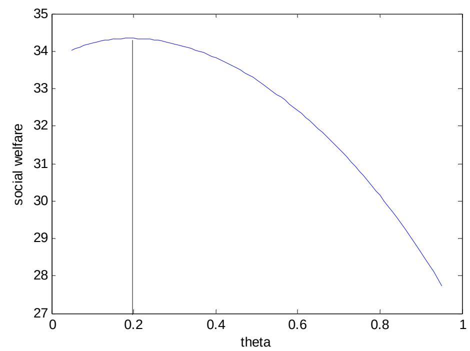
The parametrization: $\beta = 1$, $\alpha = 0.3$, $\xi_A^1 = 0.5$, $\xi_A^2 = 0.2$, and $S = 10$.

Note that if the distributions of total ability were not concave, then, similar to the symmetric case, the welfare function could be not concave as well. However, in this case, imposing a quota in favor of the advantaged group is still optimal, though the optimal quota is very likely to be equal to zero, i.e., it is optimal to ban the disadvantaged group.

4.2.2 Simulations with the Selection Effect

In this subsection, we assume that β is strictly less than one. In this case, the selection component of the welfare is not zero and, as a result, the optimal quota comes from the interplay of the effort and

Figure 13: Social Welfare (no selection effect, the asymmetric case)

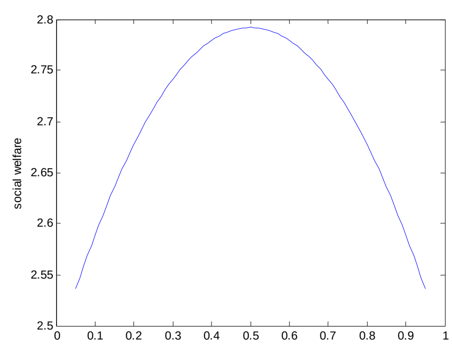


The parametrization: $\beta = 1$, $\alpha = 0.3$, $\xi_A^1 = 0.5$, $\xi_A^2 = 0.2$, and $S = 100$.

selection effects. In particular, the selection effect provides a rationale to discriminate in favor of the less advantaged group. We will first consider the symmetric case.

The Symmetric Case When the distribution of acquired ability is the same in both groups, the selection effect evaluated at the non-discrimination quota is equal to zero. This implies that if the total welfare function is concave, then the non-discrimination quota is optimal. Figure 14 illustrates this idea. It depicts the welfare function for the following set of parameters: $\xi_A^1 = \xi_A^2 = 0.4$, $\xi_N = 0.5$, $\gamma_1 = \gamma_2 = 0.5$, $S = 10$, $\alpha = 0.3$, and $\beta = 0.5$. As can be seen, the non-discrimination quota maximizes the welfare.

Figure 14: Social Welfare (with the selection effect, the symmetric case)



The parametrization: $\beta = 0.5$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 0.4$, and $S = 10$.

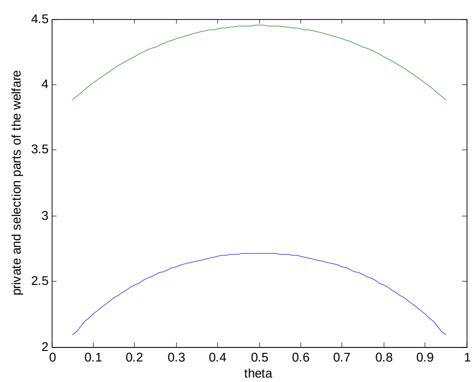
However, in this case it is not enough for the density function to be decreasing for total welfare to be concave. The selection part of the welfare is an additional source of non-concavity of the welfare function (besides the non-concavity of the distributions of total ability). Even though, the distributions of total ability are concave (as in this example), the presence of the selection part of the welfare can make the welfare function non-concave. The idea behind this is as follows. The selection part of welfare is given by

$$SW = -(1-\beta)S\sum_i \gamma_i \int_0^{a^A_{\max,i}} a_A \left(1 - H_N(a_i^* - a_A)\right) dH_A^i(a_A) + \alpha\left(T - F\right),$$

which according to our experiments is usually convex in θ. Thus, the welfare function is the sum of a concave (the private component of welfare) and convex (the selection component of welfare) function. As a result, the curvature of the social welfare function is not pinned down when $\beta < 1$ even if private welfare is concave. Moreover, the lower is the value of β, the more likely it is that the convex component dominates, so that the social welfare function is not concave.

Figure 15 depicts the private part of the welfare (the upper curve) and the negative of the selection components of welfare taken $(-SW)$ for $\beta = 0$. As can be inferred, both of these curves are concave with a maximum at the non-discrimination quota. However, the welfare function is the difference between these two curves and, therefore, could be convex or concave. Figure 16 depicts the resulting total welfare function which, as can be seen, is maximized at the corners.

Figure 15: Private and Selection Components of Welfare (the symmetric case)

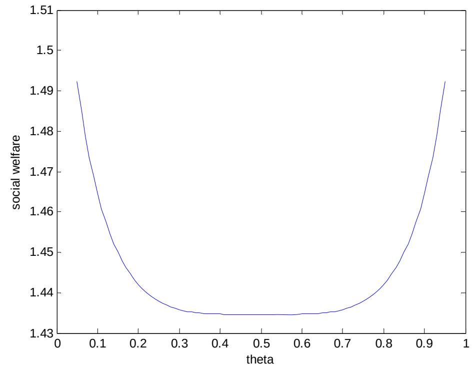


The parametrization: $\beta = 0$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 0.4$, and $S = 10$.

Thus, if agents are symmetric, the density of ability is decreasing, and β is close to unity, then (by continuity arguments) no discrimination will be optimal as depicted in Figure 14. If β is very small, so that acquired ability does not matter, then giving all the seats to one group or the other could be optimal.

When is such a policy optimal? Intuitively, if there is a lot of wasted effort (wasted effort is large

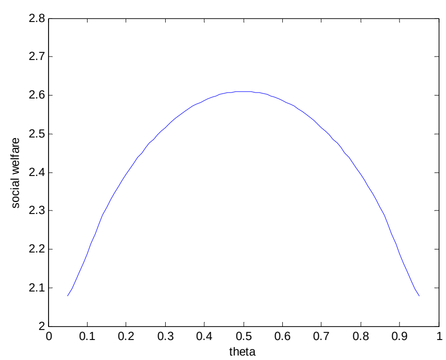
Figure 16: Social Welfare (with the selection effect, the symmetric case)



The parametrization: $\beta = 0$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 0.4$, and $S = 10$.

when S is high and/or the number of seats is low), then the private component of welfare is relatively flat. Once it is flat enough, and β is small enough, then the selection component dominates so that giving all seats to one group or the other increases welfare. Figures 17 and 18 illustrate this intuition. In Figure 17 we only increase α from 0.3 to 0.5, in Figure 18 we only decrease S from 10 to 5. As can be seen, in both cases the welfare function is concave and, as a result, the non-discrimination quota is optimal.

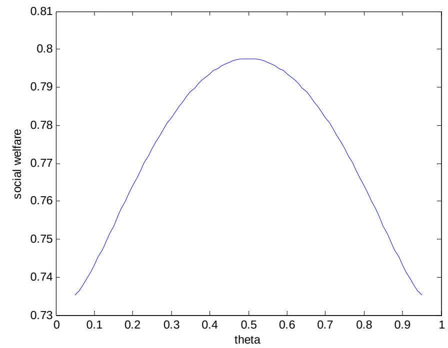
Figure 17: Social Welfare (with the selection effect, the symmetric case)



The parametrization: $\beta = 0$, $\alpha = 0.5$, $\xi_A^1 = \xi_A^2 = 0.4$, and $S = 10$.

The Asymmetric Case When the groups are different in terms of their acquired ability distribution and $\beta < 1$, then both forces are at play. The presence of the selection effect shifts the optimal quota

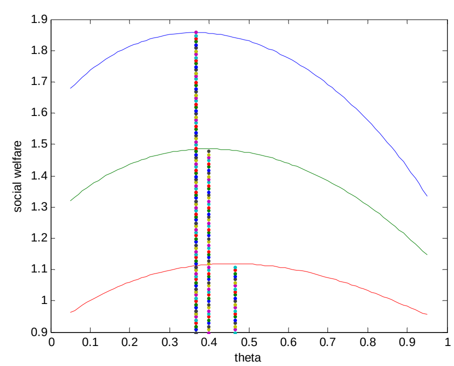
Figure 18: Social Welfare (with the selection effect, the symmetric case)



The parametrization: $\beta = 0$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 0.4$, and $S = 5$.

towards the disadvantaged group. However, the effort effect shifts the quota in favor of the advantaged group. Figure 19 depicts the social welfare function for different values of β. The upper curve is the welfare function when $\beta = 0.8$. As can be seen, in this case the optimal quota is in favor of the advantaged group (optimal θ is lower than the non-discrimination quota given by 0.3932). That is, the effort effect is stronger than the selection effect. When β is equal to 0.5 (the middle curve), the two effects are of the same magnitude and, therefore, the optimal quota is close to the non-discrimination quota. Finally, when $\beta = 0.2$, one can see that the optimal quota is in favor of the disadvantaged group as the selection effect prevails over the effort effect.

Figure 19: Social Welfare (with the selection effect, the asymmetric case)



The parametrization: $\beta = \{0.8, 0.5, 0.2\}$, $\alpha = 0.3$, $\xi_A^1 = 0.5$, $\xi_A^2 = 0.2$, and $S = 5$.

5 Extensions

In our model, we make several assumptions and it is important to understand the extent to which our results depend on them. First, we assume no uncertainty in terms of the performance in the exam. As a result, agents expend just enough effort to get in, so that more able agents exert less effort, and all agents above an ability threshold get in. Those agents with abilities below this threshold do not get in, nor do they exert any effort. The effort effect follows basically from this: reallocating seats to the advantaged group reduces total effort as this group has higher average total ability and, therefore, tends to expend less (wasteful) effort. This intuition goes through for small enough levels of randomness in performance.

In general, with randomness the effort function becomes hump shaped over ability. The least able put in little or no effort and rely on luck to get in, while the most able need to put in little effort to get in. Those in between need to work to get in. As a result, the disadvantaged could put in more or less effort than the advantaged on average so that the results depend on the position of the hump and the distribution of abilities. If the less advantaged put in more effort, then discriminating against them helps and vice versa.

The selection effect is more general. The exam selects on the basis of total ability which is on average higher in the advantaged group. Society values native ability more than the exam does. Conditional on getting in, the native ability of those from the disadvantaged group is higher and this is true with or without uncertainty. Consequently, in general, the selection effect makes it welfare increasing to discriminate in favor of the less advantaged.

Second, we assume that there is only one school. It is relatively straightforward to show that the intuition remains valid with more schools. When one discriminates in favor of the less advantaged, their cutoffs fall in each school. As a result, they put in less effort. The opposite occurs in the more advantaged group. As the more advantaged on average put in less effort, welfare falls due to the effort effect. The selection effect of course generalizes (these results are available in the Appendix).

6 Conclusion

Most of the work on preferences and affirmative action has focused on statistical discrimination. Given preferences, blacks may work less hard and so on average be worse than whites precisely because of preferences. Employers then prefer to hire whites as race is observable. However, if the basis on which preferences are given is not observable, then this model is less valid. In India for example, it is not always easy to tell caste, especially in the cities, as caste based last names are not always used. If preferences are based on background, or parental income, as has been proposed, then again those given preferences may not be easily identifiable.[26] In such settings we show that while there is no one size fits all policy, we can provide some guidance on when affirmative action might be welfare improving.

We identify two distortions: an effort distortion that arises from wasteful effort, and a selection distortion that arises from society placing a greater weight on native ability than does the placement system. How these two play off against each other and interact results in a one size does not fit all answer. Preferences may be good for one society and bad for another. The disadvantaged group puts in more effort to get in on average than the advantaged one. Given the number of seats, preferences in favor of the disadvantaged group raise the average effort level and this reduces welfare. This effect is large when tuition is low and potential surplus is dissipated via effort. However, if society puts more weight on natural ability than does the placement algorithm, then there is an additional selection effect that operates. The disadvantaged group, *that gets in,* on average has greater natural ability. This makes preferences in their favor desirable. The former effect dominates when tuition is low and the latter when it is high. As a result, our work suggests that while preferences may be a good idea in the US where tuition tends to be high, it may be a very bad idea in India where tuition is very low.

[26] After all, after going through a university education a native accent is often shed in favor of a one that blends in more easily.

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Appendix A

This Appendix contains the proofs of the results in the body of the paper.

Proof of Lemma 1. Follows from performing comparative statics on equations

$$\begin{aligned} 1 - H(a^*) &= \alpha, \\ s(a^*) - T &= c(e^*(a^*, \tilde{P})). \end{aligned}$$

Totally differentiating them gives

$$\begin{aligned} -h(a^*)da^* &= d\alpha \\ \{s'(a^*) - c'(e^*(a^*, \tilde{P}))e^*_a(a^*, \tilde{P})\}da^* - c'(e^*(a^*, \tilde{P}))e^*_{\tilde{P}}(a^*, \tilde{P})d\tilde{P} &= dT, \end{aligned}$$

where $s'(a^*) - c'(e^*(a^*, \tilde{P}))e^*_a(a^*, \tilde{P}) > 0$ as $e^*_a(a^*, \tilde{P}) < 0$ and $e^*_{\tilde{P}}(a^*, \tilde{P}) > 0$. Thus,

$$\begin{bmatrix} -h(a^*) & 0 \\ s'(a^*) - c'(e^*(a^*, \tilde{P}))e^*_a(a^*, \tilde{P}) & -c'(e^*(a^*, \tilde{P}))e^*_{\tilde{P}}(a^*, \tilde{P}) \end{bmatrix} \begin{bmatrix} da^* \\ d\tilde{P} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d\alpha \\ dT \end{bmatrix}$$

so that

$$\begin{bmatrix} \frac{da^*}{d\alpha} & \frac{da^*}{dT} \\ \frac{d\tilde{P}}{d\alpha} & \frac{d\tilde{P}}{dT} \end{bmatrix} = \begin{bmatrix} -\frac{1}{h(a^*)} & 0 \\ -\frac{s'(.) - c'(e^*(.))e^*_a(.)}{h(a^*)c'(e^*(.))e^*_{\tilde{P}}(.)} & -\frac{1}{c'(e^*(.))e^*_{\tilde{P}}(.)} \end{bmatrix}.$$

■

Proof of Proposition 1. 1. Basics

Recall that social welfare is given by

$$W = \int_{a_N + a_A \geq a^*} \Big(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P})) - F \Big) \, dH_N(a_N) dH_A(a_A).$$

As the cutoff ability a^* does not depend on T, the derivative of the welfare function with respect to T is

$$\frac{\partial W}{\partial T} = -\frac{\partial \tilde{P}}{\partial T} \int_{a_N + a_A \geq a^*} c'(e^*(a_N + a_A, \tilde{P})) \frac{\partial e^*(a_N + a_A, \tilde{P})}{\partial \tilde{P}} dH_N(a_N) dH_A(a_A).$$

>From Lemma 1 in the paper, $\partial \tilde{P}/\partial T < 0$. In addition, $\partial e^*(a, \tilde{P})/\partial \tilde{P} > 0$. This immediately implies that $\partial W/\partial T > 0$. Thus, increasing T, as long as this leaves the ability cutoff unaffected, raises welfare. Once tuition is such that

$$s(a^*) - T = 0,$$

that is, if tuition is so high that with zero effort needed to get in and the number of seats is just filled, further increases in tuition will result in an excess supply of seats. The optimal tuition, given the share of seats going to each group, is therefore equal to $s(a^*)$. If the tuition is set at this level, then the equilibrium is determined by the following equations:

$$\begin{aligned} 1 - H(a^*) &= \alpha, \\ e^*(a^*, \tilde{P}) &= 0 \iff \tilde{P} = f(a^*, 0). \end{aligned}$$

The first line gives the identity of the marginal agent, a^*. The second line defines the performance cutoff $\tilde{P}$.

2. The Optimal Number of Seats

Given the optimal choice of T, social welfare is given by

$$W = \int_{a_N + a_A \geq a^*} \left(s(a_N + \beta a_A) - F\right) dH_N(a_N) dH_A(a_A).$$

The latter can be rewritten in the following way:

$$W = \int_0^{a_{\max}^A} \left(\int_{a^* - a_A}^{a_{\max}^N} \left(s(a_N + \beta a_A) - F\right) dH_N(a_N) \right) dH_A(a_A).$$

This implies that the derivative of the social welfare with respect to α is given by

$$\frac{\partial W}{\partial \alpha} = -\frac{\partial a^*}{\partial \alpha} \int_0^{a_{\max}^A} h_N(a^* - a_A) \left(s(a^* - (1-\beta)a_A) - F\right) dH_A(a_A).$$

As $-\frac{\partial a^*}{\partial \alpha}$ is positive, the optimal (if it is interior) α solves

$$\int_0^{a_{\max}^A} h_N(a^* - a_A) \left(s(a^* - (1-\beta)a_A) - F\right) dH_A(a_A) = 0,$$

where a^* is $H^{-1}(1-\alpha)$. We assume that the functions are such that the second order condition is satisfied.

Note that if $\beta = 1$, then

$$\begin{aligned} \int_0^{a_{\max}^A} h_N(a^* - a_A) \left(s(a^* - (1-\beta)a_A) - F\right) dH_A(a_A) &= 0 \iff \\ s(a^*) &= F. \end{aligned}$$

That is, the optimal α is such that

$$s(H^{-1}(1-\alpha)) = F.$$

In other words, the optimal number of seats is such that, when tuition is set at the full cost of education, all seats are demanded and no one puts in any effort. It is straightforward to check that the second order condition is satisfied in this case. ■

Proof of Proposition 2

Recall that the social welfare is given by

$$\begin{aligned} W &= \gamma_1 \int_{a_N + a_A \geq a_1^*} \left(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}_1)) - F \right) dH_N(a_N) dH_A^1(a_A) \\ &\quad + \gamma_2 \int_{a_N + a_A \geq a_2^*} \left(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}_2)) - F \right) dH_N(a_N) dH_A^2(a_A). \end{aligned}$$

This can be rewritten in the following way (by adding and subtracting T for each agent):

$$\begin{aligned} W &= \gamma_1 \int_0^{a^A_{\max,1}} \left(\int_{a_1^*-a_A}^{a^N_{\max}} \left(s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}_1)) - T \right) dH_N(a_N) \right) dH^1_A(a_A) \\ &\quad + \gamma_2 \int_0^{a^A_{\max,2}} \left(\int_{a_2^*-a_A}^{a^N_{\max}} \left(s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}_2)) - T \right) dH_N(a_N) \right) dH^2_A(a_A) \\ &\quad + \alpha(T-F). \end{aligned}$$

Then, adding and subtracting $s(a_N+a_A)$ to each of the above terms under the integral gives

$$\begin{aligned} W &= \gamma_1 \int_0^{a^A_{\max,1}} \left(\int_{a_1^*-a_A}^{a^N_{\max}} \left(\begin{array}{c} s(a_N+\beta a_A) - s(a_N+a_A) \\ +s(a_N+a_A) - c(e^*(a_N+a_A,\tilde{P}_1)) - T \end{array} \right) dH_N(a_N) \right) dH^1_A(a_A) \\ &\quad + \gamma_2 \int_0^{a^A_{\max,2}} \left(\int_{a_2^*-a_A}^{a^N_{\max}} \left(\begin{array}{c} s(a_N+\beta a_A) - s(a_N+a_A) \\ +s(a_N+a_A) - c(e^*(a_N+a_A,\tilde{P}_2)) - T \end{array} \right) dH_N(a_N) \right) dH^2_A(a_A) \\ &\quad + \alpha(T-F). \end{aligned}$$

Noting that

$$\begin{aligned} & \frac{\partial}{\partial a_i^*} \int_{a_i^*-a_A}^{a^N_{\max}} \left(s(a_N+\beta a_A) - s(a_N+a_A) + s(a_N+a_A) - c(e^*(a_N+a_A,\tilde{P}_i)) - T \right) dH_N(a_N) \\ = & - \left(s(a_i^* - (1-\beta)a_A) - s(a_i^*) + s(a_i^*) - c(e^*(a_i^*,\tilde{P}_i)) - T \right) h_N(a_i^*-a_A) \end{aligned}$$

and that at the cutoff for total ability, the marginal agent is indifferent between paying tuition and getting in and not, so that $c(e^*(a_i^*,\tilde{P}_i)) = s(a_i^*) - T$, it follows that

$$\begin{aligned} \frac{\partial W}{\partial \theta} &= \Bigg\{ -\gamma_1 \frac{\partial \tilde{P}_1}{\partial \theta} \int_0^{a^A_{\max,1}} \left(\int_{a_1^*-a_A}^{a^N_{\max}} c'(e^*(a_N+a_A,\tilde{P}_1)) \frac{\partial e^*(a_N+a_A,\tilde{P}_1)}{\partial \tilde{P}} dH_N(a_N) \right) dH^1_A(a_A) \\ &\quad - \gamma_2 \frac{\partial \tilde{P}_2}{\partial \theta} \int_0^{a^A_{\max,2}} \left(\int_{a_2^*-a_A}^{a^N_{\max}} c'(e^*(a_N+a_A,\tilde{P}_2)) \frac{\partial e^*(a_N+a_A,\tilde{P}_2)}{\partial \tilde{P}} dH_N(a_N) \right) dH^2_A(a_A) \Bigg\} \\ &\quad \Bigg\{ +\gamma_1 \frac{\partial a_1^*}{\partial \theta} \int_0^{a^A_{\max,1}} \left(s(a_1^*) - s(a_1^* - (1-\beta)\, a_A) \right) h_N(a_1^*-a_A) dH^1_A(a_A) \\ &\quad + \gamma_2 \frac{\partial a_2^*}{\partial \theta} \int_0^{a^A_{\max,2}} \left(s(a_2^*) - s(a_2^* - (1-\beta)\, a_A) \right) h_N(a_2^*-a_A) dH^2_A(a_A) \Bigg\}. \end{aligned}$$

Hence,

$$\frac{\partial W}{\partial \theta} = EE + SE,$$

where EE, the effort effect, is the first term in curly brackets and SE, the selection effect is the second.

Let us define θ^* as the non-discrimination quota. Under the non-discrimination quota, $\tilde{P}_1 = \tilde{P}_2 \equiv \tilde{P}$ and $a_1^* = a_2^* \equiv a^*$. Moreover, if $H^1_A(a_A) \equiv H^2_A(a_A)$, then $H^1(a) \equiv H^2(a)$. Then, from the equilibrium conditions given by

$$\gamma_i \left(1 - H^i(a_i^*)\right) = \theta_i \alpha, \tag{15}$$

$$s(a_i^*) - T = c(e^*(a_i^*,\tilde{P}_i)), \text{ where } i = 1,2, \tag{16}$$

we obtain that

$$\frac{\gamma_1}{\gamma_2} = \frac{\theta_1}{\theta_2} \Leftrightarrow \theta_i = \gamma_i.$$

That is, the non-discrimination quota is equal to the share of the group in the total mass of agents. This makes sense as if the two groups are identical then their non discriminating quota share will be the same as their population weight.

Next, we assume that $H_A^1(a_A) \equiv H_A^2(a_A)$ and evaluate $\frac{\partial W}{\partial \theta}$ at the non-discrimination quota. >From (15), it is straightforward to show that

$$\frac{\partial a_1^*}{\partial \theta}_{\theta=\theta^*} = \frac{\alpha}{\gamma_1 h^1(a^*)} > 0 \text{ and} \tag{17}$$

$$\frac{\partial a_2^*}{\partial \theta}_{\theta=\theta^*} = -\frac{\alpha}{\gamma_2 h^2(a^*)} < 0. \tag{18}$$

Hence, if $H^1(a) \equiv H^2(a)$,

$$\gamma_1 \frac{\partial a_1^*}{\partial \theta}_{\theta=\theta^*} = -\gamma_2 \frac{\partial a_2^*}{\partial \theta}_{\theta=\theta^*}.$$

This implies that SE evaluated at $\theta = \theta^*$ is equal to zero. From (16), it is also possible to show that

$$\frac{\partial \tilde{P}_1}{\partial \theta}_{\theta=\theta^*} = \frac{s'(a^*) - c'(e^*(a^*, \tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial a}}{c'(e^*(a^*, \tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial \tilde{P}}} \frac{\partial a_1^*}{\partial \theta}_{\theta=\theta^*} > 0, \text{ and} \tag{19}$$

$$\frac{\partial \tilde{P}_2}{\partial \theta}_{\theta=\theta^*} = \frac{s'(a^*) - c'(e^*(a^*, \tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial a}}{c'(e^*(a^*, \tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial \tilde{P}}} \frac{\partial a_2^*}{\partial \theta}_{\theta=\theta^*} < 0. \tag{20}$$

This then implies that

$$\gamma_1 \frac{\partial \tilde{P}_1}{\partial \theta}_{\theta=\theta^*} = -\gamma_2 \frac{\partial \tilde{P}_2}{\partial \theta}_{\theta=\theta^*}.$$

As a result, EE is also equal to zero. To summarize, if $H_A^1(a_A) \equiv H_A^2(a_A)$, then for all β,

$$\frac{\partial W(\theta)}{\partial \theta}_{\theta=\theta^*} = 0.$$

Thus, if W is well-behaved, i.e., it has a single peak as a function of θ, social welfare is maximized at $\theta = \theta^*$. However, this could also be a minimum, not a maximum as explored in the simulations.

The Proof of Proposition 3

Using the results derived in Proposition 2, the effort effect evaluated at the non-discrimination quota θ^* is given by

$$\begin{aligned} EE_{\theta=\theta^*} = & -\gamma_1 \frac{\partial \tilde{P}_1}{\partial \theta} \int_0^{a^A_{\max,1}} \left(\int_{a^*-a_A}^{a^N_{\max}} c'(e^*(a_N + a_A, \tilde{P})) \frac{\partial e^*(a_N + a_A, \tilde{P})}{\partial \tilde{P}} dH_N(a_N) \right) dH_A^1(a_A) \\ & -\gamma_2 \frac{\partial \tilde{P}_2}{\partial \theta} \int_0^{a^A_{\max,2}} \left(\int_{a^*-a_A}^{a^N_{\max}} c'(e^*(a_N + a_A, \tilde{P})) \frac{\partial e^*(a_N + a_A, \tilde{P})}{\partial \tilde{P}} dH_N(a_N) \right) dH_A^2(a_A), \end{aligned}$$

as when θ is non discriminatory, $a_1^* = a_2^* = a^*$ and $\tilde{P}_1 = \tilde{P}_2 = \tilde{P}$. Note that[27]

$$\begin{aligned}
&\int_0^{a^A_{\max,i}} \left(\int_{a^*-a_A}^{a^N_{\max}} c'(e^*(a_N + a_A, \tilde{P})) \frac{\partial e^*(a_N + a_A, \tilde{P})}{\partial \tilde{P}} dH_N(a_N) \right) dH^i_A(a_A) \\
= &\int_{a_N+a_A \geq a^*} c'(e^*(a_N + a_A, \tilde{P})) \frac{\partial e^*(a_N + a_A, \tilde{P})}{\partial \tilde{P}} dH_N(a_N) dH^i_A(a_A) \\
= &\int_{a^*}^{a_{\max,i}} c'(e^*(a, \tilde{P})) \frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} dH^i(a),
\end{aligned}$$

where $H^i(a)$ is the distribution of the total ability (on $[0, a_{\max,i}]$) in group i.

Thus, the effort effect can be written as:

$$EE_{\theta=\theta^*} = -\gamma_1 \frac{\partial \tilde{P}_1}{\partial \theta} \int_{a^*}^{a_{\max,1}} c'(e^*(a, \tilde{P})) \frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} dH^1(a) - \gamma_2 \frac{\partial \tilde{P}_2}{\partial \theta} \int_{a^*}^{a_{\max,2}} c'(e^*(a, \tilde{P})) \frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} dH^2(a).$$

Substituting the expressions for $\frac{\partial \tilde{P}_i}{\partial \theta}$ (see (19), (20) and (17), (18)), we obtain that

$$\begin{aligned}
EE_{\theta=\theta^*} = &-\left[\frac{s'(a^*) - c'(e^*(a^*, \tilde{P})) \frac{\partial e^*(a^*, \tilde{P})}{\partial a}}{c'(e^*(a^*, \tilde{P})) \frac{\partial e^*(a^*, \tilde{P})}{\partial \tilde{P}}} \right] \{ \gamma_1 \left[\frac{\alpha}{\gamma_1 h^1(a^*)} \right] \int_{a^*}^{a_{\max,1}} c'(e^*(a, \tilde{P})) \frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} dH^1(a) \\
&-\gamma_2 \left[\frac{\alpha}{\gamma_2 h^2(a^*)} \right] \int_{a^*}^{a_{\max,2}} c'(e^*(a, \tilde{P})) \frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} dH^2(a) \}
\end{aligned}$$

Thus,

$$EE_{\theta=\theta^*} = D \left(\frac{\int_{a^*}^{a_{\max,2}} c'(e^*(a, \tilde{P})) \frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} dH^2(a)}{h^2(a^*)} - \frac{\int_{a^*}^{a_{\max,1}} c'(e^*(a, \tilde{P})) \frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} dH^1(a)}{h^1(a^*)} \right) \tag{21}$$

$$= D \left(\frac{\int_{a^*}^{a_{\max,2}} g(a, \tilde{P}) dH^2(a)}{h^2(a^*)} - \frac{\int_{a^*}^{a_{\max,1}} g(a, \tilde{P}) dH^1(a)}{h^1(a^*)} \right), \tag{22}$$

[27] By definition,

$$\begin{aligned}
1 - H^i(a) &= \int_a^{a_{\max,i}} h^i(a) da \\
&= \int_a^{a_{\max,i}} \left[\int_0^{a^A_{\max,i}} h_N(a - a_A) h^i_A(a_A) da_A \right] da \\
&= \int_0^{a^A_{\max,i}} h^i_A(a_A) \left[\int_a^{a_{\max,i}} h_N(a - a_A) da \right] da_A \\
&= \int_0^{a^A_{\max,i}} h^i_A(a_A) \left[H_N(a_{\max,i} - a_A) - H_N(a - a_A) \right] da_A.
\end{aligned}$$

As $a_{\max,i} - a_A \geq a^N_{\max}$ for any a_A, $H_N(a_{\max,i} - a_A) = 1$. Hence,

$$\begin{aligned}
\int_a^{a_{\max,i}} h^i(a) da &= \int_0^{a^A_{\max,i}} h^i_A(a_A) \left[1 - H_N(a - a_A) \right] da_A \\
&= \int_0^{a^A_{\max,i}} h^i_A(a_A) \left[\int_{a-a_A}^{a^N_{\max}} h_N(a_N) da_N \right] da_A \\
&= \int_0^{a^A_{\max,i}} \int_{a-a_A}^{a^N_{\max}} h^i_A(a_A) h_N(a_N) da_N da_A.
\end{aligned}$$

Thus, when we integrate the change in effort over all agents whose effort changes, there is more than one way to do so: as above or alternatively, we could integrate $h^i(a)$ over all a above a^*.

where recall $g(a,\tilde{P}) = c'(e^*(a,\tilde{P}))\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}}$ is positive (as $\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}} = \frac{1}{f_e(.)} > 0$) and

$$D = \alpha \frac{s'(a^*) - c'(e^*(a^*,\tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial a}}{c'(e^*(a^*,\tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial \tilde{P}}} > 0.$$

Recall that, as $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$ and the distribution of the natural ability is log-concave, we know that $H^1(a) \succeq_{LR} H^2(a)$ so that

$$\frac{h^1(a)}{h^1(a^*)} > \frac{h^2(a)}{h^2(a^*)} \text{ for any } a, a^* : a > a^*.$$

Moreover, as $a_{\max,i} = a_{\max}^N + a_{\max,i}^A$ and $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$, $a_{\max,1} \geq a_{\max,2}$. Hence, we have that

$$\begin{aligned}
\frac{\int_{a^*}^{a_{\max,1}} g(a,\tilde{P})dH^1(a)}{h^1(a^*)} &\geq \frac{\int_{a^*}^{a_{\max,2}} g(a,\tilde{P})dH^1(a)}{h^1(a^*)} \\
&= \int_{a^*}^{a_{\max,2}} g(a,\tilde{P})\frac{h^1(a)}{h^1(a^*)}da \\
&> \int_{a^*}^{a_{\max,2}} g(a,\tilde{P})\frac{h^2(a)}{h^2(a^*)}da \\
&= \frac{\int_{a^*}^{a_{\max,2}} g(a,\tilde{P})dH^2(a)}{h^2(a^*)}.
\end{aligned}$$

This implies that $EE_{\theta=\theta^*} < 0$.

The Proof of Proposition 4

>From previous sections, the selection effect evaluated at the non-discrimination quota (which ensures $a_1^* = a_2^* = a^*$) is given by

$$\begin{aligned}
SE_{\theta=\theta^*} &= \gamma_1 \frac{\partial a_1^*}{\partial \theta} \int_0^{a_{\max,1}^A} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) h_N(a^* - a_A) dH_A^1(a_A) \\
&+ \gamma_2 \frac{\partial a_2^*}{\partial \theta} \int_0^{a_{\max,,2}^A} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) h_N(a^* - a_A) dH_A^2(a_A).
\end{aligned}$$

Substituting the expressions for $\frac{\partial a_i^*}{\partial \theta}$ (see (17) and (18)), we obtain that

$$\begin{aligned}
SE_{\theta=\theta^*} &= \frac{\alpha}{h^1(a^*)} \int_0^{a_{\max,1}^A} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) h_N(a^* - a_A) dH_A^1(a_A) \qquad (23) \\
&- \frac{\alpha}{h^2(a^*)} \int_0^{a_{\max,2}^A} \left(s(a^*) - s(a_2^* - (1-\beta)\, a_A)\right) h_N(a^* - a_A) dH_A^2(a_A).
\end{aligned}$$

Taking into account that

$$h^i(a^*) = \int_0^{a_{\max,i}^A} h_N(a^* - y) h_A^i(y) dy, \qquad (24)$$

the selection effect can be written as follows:

$$\begin{aligned}
SE_{\theta=\theta^*} &= \alpha \int_0^{a_{\max,1}^A} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) \frac{h_A^1(a_A) h_N(a^* - a_A)}{\int_0^{a_{\max,1}^A} h_N(a^* - y) h_A^1(y) dy} da_A \\
&- \alpha \int_0^{a_{\max,2}^A} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) \frac{h_A^2(a_A) h_N(a^* - a_A)}{\int_0^{a_{\max,2}^A} h_N(a^* - y) h_A^2(y) dy} da_A.
\end{aligned}$$

Next, we define

$$\tilde{h}_A^i(a_A, a^*) \equiv \frac{h_A^i(a_A) h_N(a^* - a_A)}{\int_0^{a^A_{\max,i}} h_N(a^* - y) h_A^i(y) dy}.$$

Suppressing a^* in the notation, we replace $\tilde{h}_A^i(a_A, a^*)$ with $\tilde{h}_A^i(a_A)$. Notice that $\tilde{h}_A^i(a_A)$ is a density function. Let $\tilde{H}_A^i(a_A)$ be its associated distribution function. As $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$,

$$\frac{\tilde{h}_A^1(a_A)}{\tilde{h}_A^1(x)} = \frac{h_A^1(a_A) h_N(a^* - a_A)}{h_A^1(x) h_N(a^* - x)} \geq \frac{h_A^2(a_A) h_N(a^* - a_A)}{h_A^2(x) h_N(a^* - x)} = \frac{\tilde{h}_A^2(a_A)}{\tilde{h}_A^2(x)} \text{ for any } a_A,\, x:\; a_A \geq x.$$

That is, $\tilde{H}_A^1(a_A) \succeq_{LR} \tilde{H}_A^2(a_A)$ implying $\tilde{H}_A^1(a_A) \succeq_1 \tilde{H}_A^2(a_A)$.

Then, the selection effect can be rewritten in the following way:

$$\begin{aligned} SE_{\theta=\theta^*} &= \alpha \int_0^{a^A_{\max,1}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) \\ &\quad -\alpha \int_0^{a^A_{\max,2}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^2(a_A). \end{aligned}$$

Equivalently, as $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$ implies $a^A_{\max,1} \geq a^A_{\max,2}$

$$\begin{aligned} SE_{\theta=\theta^*} &= \alpha \int_{a^A_{\max,2}}^{a^A_{\max,1}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) \\ &\quad +\alpha \int_0^{a^A_{\max,2}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) d\left(\tilde{H}_A^1(a_A) - \tilde{H}_A^2(a_A)\right). \end{aligned}$$

Integrating the second term above by parts implies that

$$\begin{aligned} SE_{\theta=\theta^*} &= \alpha \int_{a^A_{\max,2}}^{a^A_{\max,1}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) \\ &\quad -\alpha \left(s(a^*) - s(a^* - (1-\beta)\, a^A_{\max,2})\right) \left(1 - \tilde{H}_A^1(a^A_{\max,2})\right) \\ &\quad +\alpha (1-\beta) \int_0^{a^A_{\max,2}} \left(\tilde{H}_A^2(a_A) - \tilde{H}_A^1(a_A)\right) s'(a^* - (1-\beta)\, a_A) da_A \\ &> 0 \end{aligned}$$

Why? The third term is positive as $s'(\cdot)$ is positive and $\tilde{H}_A^2(a_A) \geq \tilde{H}_A^1(a_A)$ for any a_A (recall that $\tilde{H}_A^1(a_A) \succeq_1 \tilde{H}_A^2(a_A)$).

The sum of the first two terms is also positive as shown next. As $s(\cdot)$ is increasing, we know that

$$k(a_A) \equiv s(a^*) - s(a^* - (1-\beta)\, a_A) > 0$$

and $k(a_A)$ is increasing in a_A. Thus, the average area under the curve $k(a_A)$,

$$\int_{a^A_{\max,2}}^{a^A_{\max,1}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) \frac{\tilde{h}_A^1(a_A)}{\left(1 - \tilde{H}_A^1(a^A_{\max,2})\right)} da_A > s(a^*) - s(a^* - (1-\beta)\, a^A_{\max,2})$$

which is its value at the lowest point. Thus, we have that

$$\alpha \int_{a^A_{\max,2}}^{a^A_{\max,1}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) \geq \alpha \left(s(a^*) - s(a^* - (1-\beta)\, a^A_{\max,2})\right) (1 - \tilde{H}_A^1(a^A_{\max,2})).$$

Thus, it is straightforward to see that $SE_{\theta=\theta^*} > 0$. Notice that if $\beta = 1$, then $SE_{\theta=\theta^*} = 0$.

The Proof of Proposition 5 (Example)

>From (21), the effort effect evaluated at the non-discrimination quota is given by

$$EE_{\theta=\theta^*} = D\left(\frac{\int_{a^*}^{a_{\max,2}} c'(e^*(a,\tilde{P}))\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}}dH^2(a)}{h^2(a^*)} - \frac{\int_{a^*}^{a_{\max,1}} c'(e^*(a,\tilde{P}))\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}}dH^1(a)}{h^1(a^*)}\right),$$

where

$$D = \alpha\frac{s'(a^*) - c'(e^*(a^*,\tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial a}}{c'(e^*(a^*,\tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial \tilde{P}}}.$$

Given the assumptions, the effort effect can be written as follows (recall that agents with total ability higher than $\tilde{P}$ put in zero effort and, therefore, the upper bound of the integrals is $\min(\tilde{P}, a_{\max,i})$) :

$$EE_{\theta=\theta^*} = \alpha\,(S+C)\left(\frac{\int_{a^*}^{\min(\tilde{P},a_{\max,2})} h^2(a)da}{h^2(a^*)} - \frac{\int_{a^*}^{\min(\tilde{P},a_{\max,1})} h^1(a)da}{h^1(a^*)}\right).$$

Recall that

$$H^i(a) = \int_0^{a^A_{\max,i}} H_N(a-y)dH^i_A(y),$$

implying that

$$h^i(a) = \int_0^{a^A_{\max,i}} h_N(a-y)h^i_A(y)dy.$$

Since $H_N(\cdot)$ and $H^i_A(\cdot)$ are uniform,

$$h^i(a) = \frac{1}{a^A_{\max,i}a^N_{\max}}\int_{\max(0,a-a^N_{\max})}^{\min(a^A_{\max,i},a)} dy = \frac{\min(a^A_{\max,i},a) - \max(0,a-a^N_{\max})}{a^A_{\max,i}a^N_{\max}}.$$

This implies that if $a \geq a^* \geq \max\left[a^A_{\max,1}, a^A_{\max,2}, a^N_{\max}\right]$ (as assumed), then

$$\begin{aligned} h^i(a) &= \frac{a^A_{\max,i} + a^N_{\max} - a}{a^A_{\max,i}a^N_{\max}} \\ &= \frac{a_{\max,i} - a}{a^A_{\max,i}a^N_{\max}}, \end{aligned}$$

where $a_{\max,i} = a^A_{\max,i} + a^N_{\max}$. Hence,

$$\int_{a^*}^{\min(\tilde{P},a_{\max,i})} h^i(a)da = \frac{(a_{\max,i} - a^*)^2 - \left(a_{\max,i} - \min(\tilde{P},a_{\max,i})\right)^2}{2a^A_{\max,i}a^N_{\max}}.$$

Substituting the latter in the expression for the effort effect, we derive that

$$\begin{aligned} EE_{\theta=\theta^*} &= -\frac{\alpha\,(S+C)}{2}\frac{(a_{\max,1} - a^*)^2 - \left(a_{\max,1} - \min(\tilde{P},a_{\max,1})\right)^2}{a_{\max,1} - a^*} \\ &\quad + \frac{\alpha\,(S+C)}{2}\frac{(a_{\max,2} - a^*)^2 - \left(a_{\max,2} - \min(\tilde{P},a_{\max,2})\right)^2}{a_{\max,2} - a^*} \\ &= -\frac{\alpha\,(S+C)}{2}\left(a^A_{\max,1} - a^A_{\max,2} + \frac{\left(a_{\max,2} - \min(\tilde{P},a_{\max,2})\right)^2}{a_{\max,2} - a^*} - \frac{\left(a_{\max,1} - \min(\tilde{P},a_{\max,1})\right)^2}{a_{\max,1} - a^*}\right). \end{aligned}$$

>From (23), the selection effect is given by

$$\begin{aligned} SE_{\theta=\theta^*} &= \frac{\alpha}{h^1(a^*)}\int_0^{a^A_{\max,1}} \left(s(a^*) - s(a^* - (1-\beta)\,a_A)\right) h_N(a^* - a_A) dH^1_A(a_A) \\ &\quad -\frac{\alpha}{h^2(a^*)}\int_0^{a^A_{\max,2}} \left(s(a^*) - s(a^* - (1-\beta)\,a_A)\right) h_N(a^* - a_A) dH^2_A(a_A). \end{aligned}$$

Taking into account the assumptions about the functional forms, the latter can be written as follows:

$$SE_{\theta=\theta^*} = \alpha(1-\beta)S\left(\frac{\int_0^{a^A_{\max,1}} a_A h_N(a^*-a_A)dH^1_A(a_A)}{h^1(a^*)} - \frac{\int_0^{a^A_{\max,2}} a_A h_N(a^*-a_A)dH^2_A(a_A)}{h^2(a^*)}\right).$$

We have that (recall $a^* > a^A_{\max,i}$)

$$\begin{aligned} \frac{\int_0^{a^A_{\max,i}} a_A h_N(a^*-a_A)dH^i_A(a_A)}{h^i(a^*)} &= \frac{\int_{a^*-a^N_{\max}}^{a^A_{\max,i}} a_A da_A}{a^A_{\max,i} + a^N_{\max} - a^*} \\ &= \frac{a^A_{\max,i} + a^* - a^N_{\max}}{2}. \end{aligned}$$

Therefore,

$$\begin{aligned} SE_{\theta=\theta^*} &= \alpha(1-\beta)S\left(\frac{a^A_{\max,1} + a^* - a^N_{\max}}{2} - \frac{a^A_{\max,2} + a^* - a^N_{\max}}{2}\right) \\ &= \frac{\alpha(1-\beta)S\left(a^A_{\max,1} - a^A_{\max,2}\right)}{2}. \end{aligned}$$

When Effort Affects the Payoffs from Education

In this section, we modify the model so that the effort put in is not fully wasted. We assume that the private gains from education are given by $s(a) - T + q(a,e)$, where $q(a,e)$ represents additional payoffs from effort.[28] We assume that $q(a,e)$ is increasing in both a and e, concave in e ($q_{ee}(a,e) < 0$), and the cross derivative $q_{ea}(a,e)$ is positive (which means that more able agents gain more from putting in more effort).

The sequence of actions is the same as in the benchmark model. An agent decides whether to take the exam or not and how much effort to put in (if she takes the exam). Let $e^*(a,\tilde{P})$ be the effort required to get in. It is defined as the solution of

$$\tilde{P} = f(a,e).$$

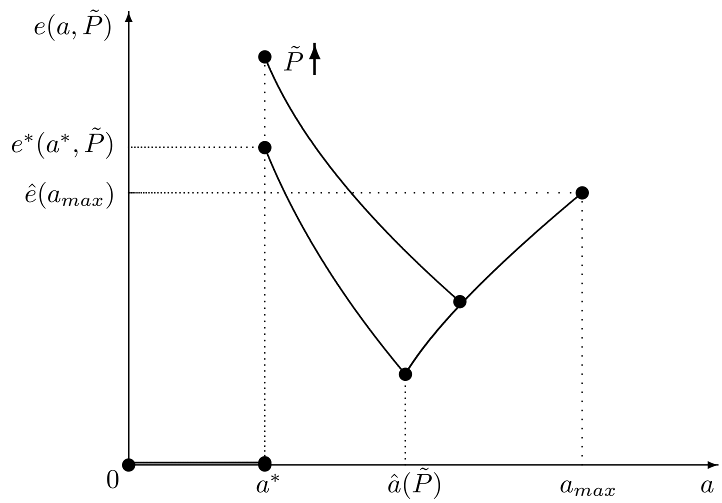
As can be seen, it is decreasing in a.

Let $\hat{e}(a)$ be the effort chosen if admission was ensured, i.e., effort independent of any considerations of admission. It is the solution of

$$\max_e \left\{s(a) - T + q(a,e) - c(e)\right\},$$

[28] This setup is equivalent to that where the function $s(\cdot)$ depends not only on a, but also on e.

Figure 20: Effort in the Model with Additional Payoffs from Effort



which is defined by

$$q_e(a, \hat{e}) - c'(\hat{e}) = 0.$$

Also, $q_{ee}(a, e) - c''(e) < 0$ so that $q_e(a, e) - c'(e) < 0$ for $e > \hat{e}(a)$. Also, as $q_{ea}(a, e)$ is positive, $\hat{e}(a)$ is increasing in a. Since $e^*(a, \tilde{P})$ is decreasing in a, there exists a unique ability where the two are equal. This cutoff ability is implicitly defined by

$$e^*(a, \tilde{P}) = \hat{e}(a)$$

and denoted by $\hat{a}(\tilde{P})$.

The equilibrium effort function is a composite one made up of $e^*(a, \tilde{P})$ and $\hat{e}(a)$. Agents with ability above $\hat{a}(\tilde{P})$ want to put in more effort than they need to to get in and so choose to put in what they want to independent of admission considerations. Agents with ability below $\hat{a}(\tilde{P})$ want to put in less effort than they need to to get in and are forced to put in what is needed to be admitted. Hence, the agent with total ability a expends effort

$$e(a, \tilde{P}) = \max\{e^*(a, \tilde{P}), \hat{e}(a)\} \quad (25)$$

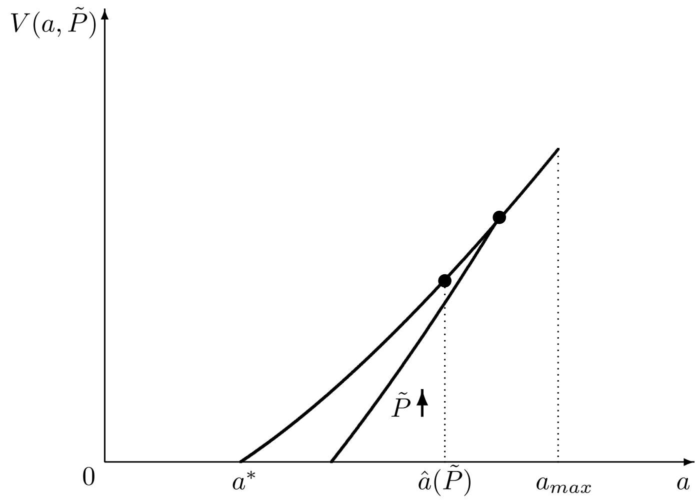
$$= \begin{cases} e^*(a, \tilde{P}) & \text{if } a \leq \hat{a}(\tilde{P}) \\ \hat{e}(a) & \text{if } a > \hat{a}(\tilde{P}). \end{cases} \quad (26)$$

As depicted in Figure 20, $e(a, \tilde{P})$ is decreasing in a till $\hat{a}(\tilde{P})$ and then increasing. A higher cutoff performance shifts the decreasing part of the curve upwards and to the right and does not effect the increasing part.

What about surplus? The surplus of an agent with ability a who decides to take the exam is given by

$$V(a, \tilde{P}) = s(a) - T + q(a, e(a, \tilde{P})) - c(e(a, \tilde{P})).$$

Figure 21: The Surplus Function in the Model with Additional Payoffs from Effort



Taking into account the expression for $e(a,\tilde{P})$,

$$V(a,\tilde{P}) = \begin{cases} s(a) - T + q(a, e^*(a,\tilde{P})) - c(e^*(a,\tilde{P})) & \text{if } a \leq \hat{a}(\tilde{P}) \\ s(a) - T + q(a, \hat{e}(a)) - c(\hat{e}(a)) & \text{if } a > \hat{a}(\tilde{P}). \end{cases}.$$

It is straightforward to show that if $a > \hat{a}(\tilde{P})$, $V(a,\tilde{P})$ is increasing in a. Indeed, by the envelope theorem, for $a > \hat{a}(\tilde{P})$

$$V_a(a,\tilde{P}) = s'(a) + q_a(a,\hat{e}(a)) > 0.$$

Next we show that $V(a,\tilde{P})$ is increasing in a for $a \leq \hat{a}(\tilde{P})$ as well. For $a \leq \hat{a}(\tilde{P})$,

$$V_a(a,\tilde{P}) = s'(a) + q_a(a, e^*(a,\tilde{P})) + (q_e(.) - c'(.))\, e^*_a(a,\tilde{P}).$$

Note that for $a \leq \hat{a}(\tilde{P})$, $e(a,\tilde{P})) = e^*(a,\tilde{P}) > \hat{e}(a)$. That is, in this region effort is excessive so that $(q_e(.) - c'(.)) < 0$. Since $e^*_a(a,\tilde{P}) < 0$, $(q_e(.) - c'(.))\, e^*_a(a,\tilde{P}) > 0$. As a result, it follows that for $a \leq \hat{a}(\tilde{P})$, $\hat{V}_a(a,\tilde{P}) > 0$. Thus, we have shown that $V(a,\tilde{P})$ is increasing in a. Notice that a rise in $\tilde{P}$ raises the effort needed to get in and shifts $V(a,\tilde{P})$ downwards (for $a \leq \hat{a}(\tilde{P})$) and $\hat{a}(\tilde{P})$ up. This is depicted in Figure 21.

Finally, an agent with total ability a takes the exam if and only if her surplus from doing so is positive. Since this surplus is increasing in a, all agents with ability more that a some level take the exam. The cutoff ability, a^*, satisfies

$$V(a^*,\tilde{P}) = s(a^*) - T + q(a^*, e(a^*,\tilde{P})) - c(e(a^*,\tilde{P})) = 0, \tag{27}$$

as the outside option has been set at 0.

Hence, in the model when effort can be useful, there is still some wasted effort (when $a^* < \hat{a}(\tilde{P})$ in equilibrium). This occurs among the lower ability agents taking the exam. As a result, the effort distortion will again suggest that one discriminates in favor of the advantaged as they put in less wasteful effort. Hence, our results regarding the effort effect on welfare derived in the benchmark model can be derived in this modification of the model as well.

Appendix B

In this Appendix, we consider the extension of the benchmark model with two universities of different quality.

The Model

We assume that the universities are different in that they offer education of different qualities, which affects the payoffs from being educated. As a result, in equilibrium, the performance cutoff for a better university is higher so that it takes more effort to be accepted to the higher quality university. The payoffs from being educated at university i are given by $q^i s(a)$, where q^i is the measure of quality of university i (as before, a is the total ability). Here, $i \in \{H, L\}$. The net payoffs are given by

$$q^i s(a) - T_i - c(e^*(a, \tilde{P}^i)),$$

where T_i is the tuition fee and $\tilde{P}^i$ is the performance cutoff at university i ($\tilde{P}^H$ is assumed to be higher than $\tilde{P}^L$ (see the discussion below)), $e^*(a, \tilde{P}^i)$ is the effort level put in to be accepted. $c(.)$ is weakly convex.

Lemma 2 below shows that the difference between the net payoffs from studying in the better university is increasing in ability. As a result, more able agents are matched with better universities.

Lemma 2 *For any given performance cutoffs,*

$$\begin{aligned} D(a; \tilde{P}^H, \tilde{P}^L) &= q^H s(a) - T_H - c(e^*(a, \tilde{P}^H)) - \left(q^L s(a) - T_L - c(e^*(a, \tilde{P}^L))\right) \\ &= \triangle q s(a) - \triangle T - c(e^*(a, \tilde{P}^H)) + c(e^*(a, \tilde{P}^L)) \end{aligned}$$

where $\triangle q = q^H - q^L > 0$ *and* $\triangle T = T_H - T_L$. *Then,*

$$\begin{aligned} \frac{\partial D(a; \tilde{P}^H, \tilde{P}^L)}{\partial a} &= \triangle q s'(a) + \left[-c'(e^*(a, \tilde{P}^H)) \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} - \left(-c'(e^*(a, \tilde{P}^L)) \frac{\partial e^*(a, \tilde{P}^L)}{\partial a} \right) \right] && (28) \\ &= \triangle q s'(a) - \left[c'(e^*(a, \tilde{P}^H)) - c'(e^*(a, \tilde{P}^L)) \right] \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} && (29) \\ &\quad + c'(e^*(a, \tilde{P}^L)) \left(\frac{\partial e^*(a, \tilde{P}^L)}{\partial a} - \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} \right) && (30) \\ &> 0. && (31) \end{aligned}$$

Proof. Using the fact that

$$f(a, e^*(a, \tilde{P})) = \tilde{P},$$

it is easy to see that

$$e^*_{\tilde{P}}(a, \tilde{P}) = \frac{1}{f_e(a, e^*(a, \tilde{P}))} > 0.$$

That is, meeting a higher cutoff requires greater effort from any agent. Thus, as $c(\cdot)$ is convex, $c'(e^*(a, \tilde{P}^H)) > c'(e^*(a, \tilde{P}^L))$. In addition,

$$e^*_a(a, \tilde{P}) = -\frac{f_a(a, e^*(a, \tilde{P}))}{f_e(a, e^*(a, \tilde{P}))} < 0.$$

This implies that

$$-\left[c'(e^*(a, \tilde{P}^H)) - c'(e^*(a, \tilde{P}^L))\right] \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} > 0.$$

Finally,

$$e^*_{a\tilde{P}}(a, \tilde{P}) = -\frac{f_{ea}(a, e^*(a, \tilde{P})) + f_{ee}(a, e^*(a, \tilde{P}))e^*_a(a, \tilde{P})}{\left(f_e(a, e^*(a, \tilde{P}))\right)^2} < 0,$$

as $f_{ee} < 0$, $e^*_a(a, \tilde{P}) < 0$, and $f_{ea} > 0$. This in turn means that $e^*_a(a, \tilde{P}^L) > e^*_a(a, \tilde{P}^H)$, implying that

$$c'(e^*(a, \tilde{P}^L)) \left(\frac{\partial e^*(a, \tilde{P}^L)}{\partial a} - \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} \right) > 0.$$

Summarizing the above findings, it follows that

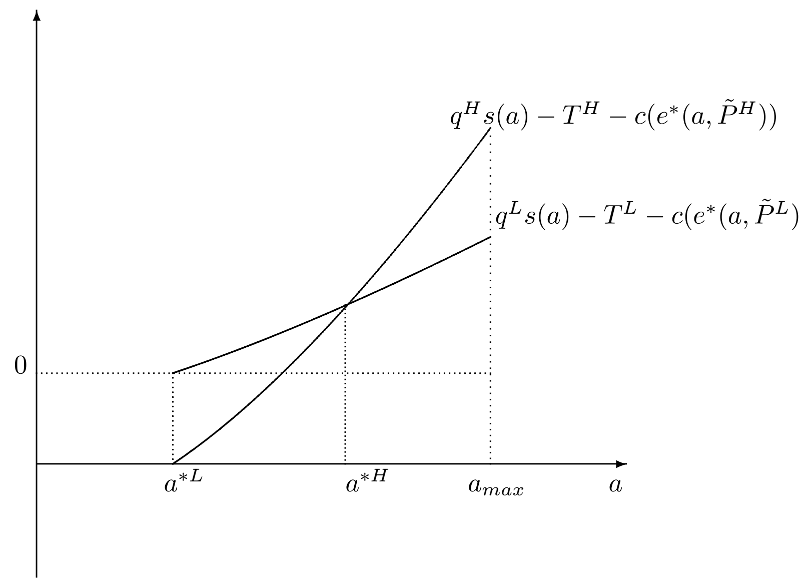
$$\frac{\partial D(a; \tilde{P}^H, \tilde{P}^L)}{\partial a} > 0.$$

■

It is probably easiest to see what we have in a picture like Figure 22. We have that $q^H s(a)$ is steeper than $q^L s(a)$, as $q^H > q^L$ and $s(a)$ is increasing in ability so that more able individuals earn more at any given education level, and this is more so at better institutions. In order to get in to school H (or L), each agent must put in $e^*(a, \tilde{P}^H)$ (or $e^*(a, \tilde{P}^L)$) and this means costs of $c(e^*(a, \tilde{P}^H))$ (and $c(e^*(a, \tilde{P}^L))$) be incurred. These costs are decreasing in ability as the more able need to put in less effort to meet any given performance cutoff. Moreover, they decrease faster in ability when the cutoff is higher (as shown in Lemma 2). This happens because the higher performance cutoff requires more effort from all individuals, but due to the complementarity between ability and effort in creating performance, more able agents need to put in less effort to attain the higher cutoff. As they are putting in less effort to get the lower performance cutoff anyway, this increased effort to meet a higher cutoff is also less costly for them.

Thus, the net surplus (the benefit less the cost) from going to school is increasing in ability, and more so for the better school as depicted in Figure 22. This means that when we add tuition cost which are independent of ability, the net benefit of going to the better school rises faster than that of the worse school so that these curves can cross at most once and better students must select into the better school.

Figure 22: Payoffs from Education: Two Schools



Note this is independent of tuition, though too high a tuition could make the payoff from that school lie entirely below that of the other so no one goes there.

Next we consider the equilibrium and comparative statics of the model. In the equilibrium, there are two total ability cutoffs: a_H^* and a_L^*. Agents with ability lower than a_L^* choose the outside option. The cutoffs are determined by taking the number of seats in the better school and finding a_H^* such that these seats are filled. a_L^* is then defined by its seats being filled by lower ability agents. This gives the equilibrium conditions:

$$1 - H(a_H^*) = \alpha_H, \tag{32}$$

$$H(a_H^*) - H(a_L^*) = \alpha_L, \tag{33}$$

where α_i is the number of seats in university i and $H(.)$ is the distribution of total ability. As before, we assume that the natural and acquired abilities are independently distributed across the agents. The distribution functions are given by $H_N(a_N)$ and $H_A(a_A)$ on $[0, a_{\max}^N]$ and $[0, a_{\max}^A]$, respectively. Then, the distribution function for the total ability a is $H(a)$ on $[0, a_{\max}]$, where

$$H(a) = \int_0^{a_{\max}^A} H_N(a - y) dH_A(y)$$

and $a_{\max} = a_{\max}^N + a_{\max}^A$.

The agent at a_L^* must be indifferent between the worse school and the outside option of zero which pins down $c(e^*(a_L^*, \tilde{P}^L))$ and defines $\tilde{P}^L$. The agent at a_H^* must be indifferent between the two schools which pins down $c(e^*(a_H^*, \tilde{P}^H))$ and defines $\tilde{P}^H$. Thus

$$\begin{aligned} q^L s(a_L^*) - T_L - c(e^*(a_L^*, \tilde{P}^L)) &= 0, & (34) \\ \triangle q s(a_H^*) - \triangle T - c(e^*(a_H^*, \tilde{P}^H)) + c(e^*(a_H^*, \tilde{P}^L)) &= 0. & (35) \end{aligned}$$

Thus, we have four unknowns: a_H^*, a_L^*, $\tilde{P}^H$, $\tilde{P}^L$; and four equilibrium equations. Note that the condition, $\tilde{P}^H > \tilde{P}^L$, is equivalent to $c(e^*(a_H^*, \tilde{P}^H)) - c(e^*(a_H^*, \tilde{P}^L)) > 0$. Therefore, from the equilibrium conditions, we can infer that $\tilde{P}^H > \tilde{P}^L$ if and only if $\triangle q s(a_H^*) - \triangle T > 0$. In other words, that the difference in the tuition levels is not set too high relative to the difference in quality.

Next, we explore how changes in T_i affect the equilibrium outcome. The following lemma holds.

Lemma 3 *1) The cutoffs, a_H^* and a_L^*, do not depend on the tuition fees, T_H and T_L.*

2) A rise in T_H does not affect $\tilde{P}^L$ and decreases $\tilde{P}^H$.

3) A rise in T_L decreases $\tilde{P}^L$ and increases $\tilde{P}^H$.

4) A rise in T_H and T_L (such that $\triangle T$ does not change) decreases $\tilde{P}^L$ and $\tilde{P}^H$.

Proof. 1)-2) and 4) directly follow from the equilibrium equations. Let us prove the third statement in the lemma. From the equilibrium, we have that

$$\frac{\partial \tilde{P}^L}{\partial T_L} = -\frac{1}{c'\left(e^*(a_L^*, \tilde{P}^L)\right) \frac{\partial e^*(a_L^*, \tilde{P}^L)}{\partial \tilde{P}}} < 0.$$

In addition,

$$\frac{\partial \tilde{P}^H}{\partial T_L} = -\frac{-c'\left(e^*(a_H^*, \tilde{P}^L)\right) \frac{\partial e^*(a_H^*, \tilde{P}^L)}{\partial \tilde{P}} \frac{\partial \tilde{P}^L}{\partial T_L} - 1}{c'\left(e^*(a_H^*, \tilde{P}^H)\right) \frac{\partial e^*(a_H^*, \tilde{P}^H)}{\partial \tilde{P}}} = \frac{c'\left(e^*(a_H^*, \tilde{P}^L)\right) \frac{\partial e^*(a_H^*, \tilde{P}^L)}{\partial \tilde{P}} \frac{\partial \tilde{P}^L}{\partial T_L} + 1}{c'\left(e^*(a_H^*, \tilde{P}^H)\right) \frac{\partial e^*(a_H^*, \tilde{P}^H)}{\partial \tilde{P}}}.$$

Hence, the sign of the derivative is the same as the sign of the numerator (as the denominator is positive). The numerator is in turn equal to

$$c'\left(e^*(a_H^*, \tilde{P}^L)\right) \frac{\partial e^*(a_H^*, \tilde{P}^L)}{\partial \tilde{P}} \frac{\partial \tilde{P}^L}{\partial T_L} + 1 = 1 - \frac{c'\left(e^*(a_H^*, \tilde{P}^L)\right) \frac{\partial e^*(a_H^*, \tilde{P}^L)}{\partial \tilde{P}}}{c'\left(e^*(a_L^*, \tilde{P}^L)\right) \frac{\partial e^*(a_L^*, \tilde{P}^L)}{\partial \tilde{P}}}.$$

Note that as $a_L^* < a_H^*$, $e^*(a_H^*, \tilde{P}^L) < e^*(a_L^*, \tilde{P}^L)$ implying that $c'\left(e^*(a_H^*, \tilde{P}^L)\right) < c'\left(e^*(a_L^*, \tilde{P}^L)\right)$. Moreover, since $\frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} = 1/f_e\left(a, e^*(a, \tilde{P})\right)$,

$$\frac{\frac{\partial e^*(a_H^*, \tilde{P}^L)}{\partial \tilde{P}}}{\frac{\partial e^*(a_L^*, \tilde{P}^L)}{\partial \tilde{P}}} = \frac{f_e\left(a_L^*, e^*(a_L^*, \tilde{P}^L)\right)}{f_e\left(a_H^*, e^*(a_H^*, \tilde{P}^L)\right)} < 1,$$

as $f_{ea} > 0$ and $f_{ee} < 0$. That is, the sign of the numerator is positive. This proves the statement. ■

The intuition behind 1) and 2) in the lemma is straightforward. The idea behind 3) is as follows. Keep the performance cutoffs fixed. An increase in T_L shifts the payoff curve for L down and reduces a_H^*

while raising a_L^*. As a result, more agents apply for the seats in the high-quality university and fewer for the low quality one. As the number of seats remains the same, $\tilde{P}^H$ must rise and $\tilde{P}^L$ fall.

The intuition behind 4 is simple. Suppose we increase tuition fees by the same amount. At given performance cutoffs, this change does not affect the intersection of the two curves as both sift down by the same amount, but raises a_L^*. This reduces the demand for school L below its capacity which reduces the performance cutoff of L. This fall in L's performance cutoff must shift the payoff for L back up so that it goes through the original level of a_L^*. However, the fall in L's performance cutoff has a smaller impact on the payoff for higher ability agents and so makes it flatter. This reduces a_H^* from its original level, requiring a fall in H's performance cutoff as well.[29]

Next, we explore the effects of changes in the number of available seats on the equilibrium outcomes. Using the equilibrium equations we see that a rise in α_L does not change a_H^* (as it is pinned down by α_H) and decreases a_L^*. This in turn means that $\tilde{P}^L$ and $\tilde{P}^H$ fall. Intuitively, more available seats in the low-quality university reduces the performance cutoff in that university, making it more attractable compared to the high-quality university. As the number of seats in the high-quality university does not change, the performance cutoff, $\tilde{P}^H$, must fall to compensate for the decrease in $\tilde{P}^L$.

A rise in α_H in turn decreases both the ability cutoffs, a_L^* and a_H^*. The decrease in a_L^* in turn results in lower $\tilde{P}^L$. The low-quality university has to reduce its performance cutoff in order to fill in the all available seats. The impact on $\tilde{P}^H$ is also straightforward. The direct effect of a rise in α_H decreases a_H^*, reducing $\tilde{P}^H$. In addition, the rise in α_H decreases $\tilde{P}^L$, which further reduces $\tilde{P}^H$ (see (34)). As can be seen, both effects work in the same direction. As a result, $\tilde{P}^H$ falls. The following lemma summarizes the above reasoning.

Lemma 4 *1) A rise in α_L does not change a_H^* and decreases a_L^*, $\tilde{P}^L$, and $\tilde{P}^H$.*

2) A rise in α_H decreases a_H^* and a_L^* and $\tilde{P}^L$ and $\tilde{P}^H$.

Next, we examine the welfare implications of changes in the parameters in the model.

Social Welfare

As before, we allow the private gains from education to differ from the social gains. Specifically, for an individual, natural and acquired abilities are of the same importance but for the society natural ability is more important than acquired ability.

Social welfare is given by (the outside option is normalized to zero)

$$\begin{aligned} W \;=\; & \int_{a_N+a_A\geq a_H^*}\Big(q^H s(a_N+\beta a_A)-c(e^*(a_N+a_A,\tilde{P}^H))-F\Big)\,dH_N(a_N)dH_A(a_A) \\ & +\int_{a_L^*\leq a_N+a_A<a_H^*}\Big(q^L s(a_N+\beta a_A)-c(e^*(a_N+a_A,\tilde{P}^L))-F\Big)\,dH_N(a_N)dH_A(a_A). \end{aligned} \tag{36}$$

[29] Formally, this result comes from the impact of T_H on $\tilde{P}^H$ being stronger than that of T_L.

where F is the social cost of education per student. Note that as the tuition is a lump-sum transfer, T does not directly affect the welfare. It only affects it via the effort put in by agents.

Next, we explore the effects of the tuition fees on the social welfare. First, we examine how changes in T_L and T_H affect the welfare. Then, we find the values of T_L and T_H that maximize the social welfare function. It is straightforward to see that

$$\begin{aligned}\frac{\partial W}{\partial T_i} &= \int_{a_N+a_A\geq a_H^*} \frac{\partial\left(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^H)) - F\right)}{\partial T_i} dH_N(a_N)dH_A(a_A) \\ &+ \int_{a_L^*\leq a_N+a_A<a_H^*} \frac{\partial\left(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^L)) - F\right)}{\partial T_i} dH_N(a_N)dH_A(a_A).\end{aligned}$$

Here, we take into account that the ability cutoffs do not depend on the tuition fees (see Lemma 2)).

>From the results stated in Lemma 2, we can conclude that (recall that $\frac{\partial \tilde{P}^L}{\partial T_H} = 0$)

$$\frac{\partial W}{\partial T_H} = -\frac{\partial \tilde{P}^H}{\partial T_H} \int_{a_N+a_A\geq a_H^*} c'(e^*(a_N+a_A,\tilde{P}^H)) \frac{\partial e^*(a_N+a_A,\tilde{P}^H)}{\partial \tilde{P}} dH_N(a_N)dH_A(a_A) > 0,$$

while

$$\begin{aligned}\frac{\partial W}{\partial T_L} &= -\frac{\partial \tilde{P}^H}{\partial T_L} \int_{a_N+a_A\geq a_H^*} c'(e^*(a_N+a_A,\tilde{P}^H)) \frac{\partial e^*(a_N+a_A,\tilde{P}^H)}{\partial \tilde{P}} dH_N(a_N)dH_A(a_A) \\ &\quad -\frac{\partial \tilde{P}^L}{\partial T_L} \int_{a_L^*\leq a_N+a_A<a_H^*} c'(e^*(a_N+a_A,\tilde{P}^H)) \frac{\partial e^*(a_N+a_A,\tilde{P}^L)}{\partial \tilde{P}} dH_N(a_N)dH_A(a_A).\end{aligned}$$

The sign of the latter expression is ambiguous, as $\frac{\partial \tilde{P}^H}{\partial T_L} > 0$ and $\frac{\partial \tilde{P}^L}{\partial T_L} < 0$.

As can be seen, the impact of T_H on welfare is similar to that in the model with one university: $\frac{\partial W}{\partial T_H} > 0$. The intuition is similar as well. A rise in T_H reduces the effort put in the agents who decide to apply for the high-quality university and does not change the effort of the agents who apply for the low-quality university. As a result, welfare rises. The impact of T_L is ambiguous in general. A rise in T_L reduces the effort put in by the agents applying for the low-quality university and increases the effort put in by the agents applying for the high-quality university. As a result, given T_H, there exists a certain optimal level of T_L such that $\frac{\partial W}{\partial T_L} = 0$ (unless the condition $\frac{\partial W}{\partial T_L} = 0$ delivers the minimum).

However, if the goal is to describe the value of the pair (T_H, T_L) that delivers the maximum, the outcome will be exactly the same as in the case with one university. In other words, the social welfare as a function of T_H and T_L is maximized when the effort put in by the marginal agents is equal to zero. That is, $e^*(a_H^*, \tilde{P}^H) = 0$ and $e^*(a_L^*, \tilde{P}^L) = 0$. This can be obviously seen from the expression for the social welfare (36), which is maximized when there is no wasted effort. The conditions of having zero effort put in by the marginal agents can be written as follows:

$$s(a_H^*) = \frac{T_H - T_L}{\triangle q}, \tag{37}$$

$$s(a_L^*) = T_L/q^L. \tag{38}$$

Since the ability cutoffs are determined by the number of seats in the universities, from the above equations we can find the optimal values of the tuition levels.

Note that if we assume that $T_H = T_L = T$, then the welfare function will be increasing in T. However, the optimal value of T does not elicit the zero effort put in by all agents. Indeed, a rise in T reduces $\tilde{P}^L$ and, thereby, $\tilde{P}^H$ (recall that $\triangle T = 0$). In this case, it is straightforward to show that the social welfare is increasing in T. Therefore, we keep increasing T till the effort put in by the marginal agent a_L^* (a further increase in T does not affect welfare, as $\tilde{P}^L$ is not affected anymore). The equilibrium conditions in this case are

$$\begin{aligned} \triangle q s(a_H^*) - c(e^*(a_H^*, \tilde{P}^H)) + c(e^*(a_H^*, \tilde{P}^L)) &= 0, \\ q^L s(a_L^*) - T &= 0. \end{aligned}$$

As $c(e^*(a_L^*, \tilde{P}^L)$ is equal to zero, $c(e^*(a_H^*, \tilde{P}^L))$ is equal to zero as well. Therefore, the equilibrium conditions can be written as follows:

$$\begin{aligned} \triangle q s(a_H^*) - c(e^*(a_H^*, \tilde{P}^H)) &= 0, \\ q^L s(a_L^*) - T &= 0. \end{aligned}$$

As can be seen, $e^*(a_H^*, \tilde{P}^H)$ is strictly positive in the equilibrium. That is, the agents applying for the high-quality university put in some positive effort. The corresponding value of $\tilde{P}^H$ can be found from the first equation in the latter system of equations.

Finally, similar to the benchmark case with one university, the distortion caused by selection into education can not be completely removed, as the social gains from education are different from the private gains.

The Case with Quotas

In this section, we introduce educational quotas in the above framework. We assume there are two groups of agents indexed by $i \in \{1, 2\}$, which have identical distributions of natural ability and potentially different distributions of acquired ability. The latter is motivated by the fact that agents with different social backgrounds have had different educational inputs prior to taking the exam, which in turn results in different acquired abilities on their part. In particular, we assume that $H_N^1(a_N) = H_N^2(a_N) \equiv H_N(a_N)$, while $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$ where $\succeq_{LR}$ stands for the likelihood stochastic order. Hence,

$$\frac{h_A^1(a_A)}{h_A^1(x)} > \frac{h_A^2(a_A)}{h_A^2(x)} \text{ for any } a_A, x : a_A > x.$$

This means that group 1 is more favored in terms of acquired ability than group 2. In addition, we assume that the distribution of natural ability has a log-concave density. This assumption is needed to ensure the likelihood stochastic order of the distributions of total ability: i.e., $H^1(a) \succeq_{LR} H^2(a)$.

The share of each group in the total mass of agents (which is normalized to unity) is denoted by γ_i, where $\gamma_1 + \gamma_2 = 1$. We then define θ_{ik} as a share of available seats reserved for group i in university k: $\theta_{1k} + \theta_{2k} = 1$ for $k \in \{H, L\}$. If these quotas are binding, then the cutoffs for the two groups will differ. Note that the quota given to a certain group can be in general different in different universities. The equilibrium conditions can be then written as follows:

$$\begin{aligned}
\gamma_i \left(1 - H^i(a^*_{iH})\right) &= \theta_{iH}\alpha_H, \\
\gamma_i \left(H^i(a^*_{iH}) - H^i(a^*_{iL})\right) &= \theta_{iL}\alpha_L, \\
q^L s(a^*_{iL}) - T_L - c(e^*(a^*_{iL}, \tilde{P}^{iL})) &= 0, \\
\triangle q s(a^*_{iH}) - \triangle T - c(e^*(a^*_{iH}, \tilde{P}^{iH})) + c(e^*(a^*_{iH}, \tilde{P}^{iL})) &= 0,
\end{aligned}$$

where $i \in \{1, 2\}$.

We define by a non-discrimination quota in university k, θ^*_k (the quota in favor of group 1, the corresponding quota in favor of group 2 is $1 - \theta^*_k$), such that the quota leads to $\tilde{P}^{1k} = \tilde{P}^{2k}$: i.e., the performance cutoffs are the same for both groups. If both universities set the non-discrimination quotas, then it is straightforward to see that

$$\begin{aligned}
a^*_{1H} &= a^*_{2H}, \\
a^*_{1L} &= a^*_{2L}.
\end{aligned}$$

If in addition $H^1(a) \equiv H^2(a) = H(a)$, then

$$\theta^*_H = \theta^*_L = \gamma_1.$$

This is similar to the case with one university.

Next, we write down the social welfare under the presence of two groups of agents. In particular, we have the following expression:

$$\begin{aligned}
W &= \sum_i \gamma_i \int_{a_N + a_A \geq a^*_{iH}} \left(q^H s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}^{iH})) - F\right) dH_N(a_N) dH^i_A(a_A) \\
&\quad + \sum_i \gamma_i \int_{a^*_{iL} \leq a_N + a_A < a^*_{iH}} \left(q^L s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}^{iL})) - F\right) dH_N(a_N) dH^i_A(a_A) \\
&= \sum_i \gamma_i \int_{a_N + a_A \geq a^*_{iH}} \left(q^H s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}^{iH})) - T_H\right) dH_N(a_N) dH^i_A(a_A) \\
&\quad + \sum_i \gamma_i \int_{a^*_{iL} \leq a_N + a_A < a^*_{iH}} \left(q^L s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}^{iL})) - T_L\right) dH_N(a_N) dH^i_A(a_A) \\
&\quad + \alpha_H (T_H - F) + \alpha_L (T_L - F).
\end{aligned} \tag{39}$$

In the next sections, we explore the behavior of social welfare (as a function of the quotas) around the non-discrimination quotas.

Symmetric Groups with no Selection Effect

In this subsection, we assume that the groups are symmetric: $H^1(a) \equiv H^2(a) = H(a)$; and examine how *uniform* changes in the quotas set by the universities locally affect the social welfare in the case of no selection effect. In particular, we assume that

$$\begin{aligned} \theta_{1k} &= \mu\theta_k^*, \text{ implying that} \\ \theta_{2k} &= 1 - \mu\theta_k^*. \end{aligned}$$

This specification allows us consider uniform changes in the quotas set by the universities. Moreover, if $\mu = 1$, then both universities set the non-discrimination quotas θ_k^*. If $\mu = 0$, then in both universities all seats are given to the second group. Finally, if $\mu = 1/\gamma_1 > 1$, then all seats in both universities are given to the first group (recall that if $H^1(a) \equiv H^2(a)$, $\theta_H^* = \theta_L^* = \gamma_1$). Next, we consider the social welfare as a function of μ in the case of no selection effect: $\beta = 1$.

Taking into account (39), the derivative of the social welfare function with respect to μ can be written as follows. Note that $W = W(\tilde{P}^{iH}, \tilde{P}^{iL}, a_{iH}^*, a_{iL}^*)$. Thus:

$$\begin{aligned} \frac{\partial W(.)}{\partial \mu} &= \frac{\partial W(.)}{\partial \tilde{P}^{iH}} \frac{\partial \tilde{P}^{iH}}{\partial \mu} + \frac{\partial W(.)}{\partial \tilde{P}^{iL}} \frac{\partial \tilde{P}^{iL}}{\partial \mu} \\ &\quad + \frac{\partial W(.)}{\partial a_{iH}^*} \frac{\partial a_{iH}^*}{\partial \mu} + \frac{\partial W(.)}{\partial a_{iL}^*} \frac{\partial \tilde{P}^{iL}}{\partial \mu}. \end{aligned}$$

$$\begin{aligned} \frac{\partial W(.)}{\partial \mu} &= -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a_{iH}^*} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\ &\quad - \sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a_{iL}^* \leq a < a_{iH}^*} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) \\ &\quad - \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \left(q^H s(a_{iH}^*) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) - T_H \right) h^i(a_{iH}^*) \\ &\quad + \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \left(q^L s(a_{iH}^*) - c(e^*(a_{iH}^*, \tilde{P}^{iL})) - T_L \right) h^i(a_{iH}^*) \\ &\quad - \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} \left(q^L s(a_{iL}^*) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L \right) h^i(a_{iL}^*). \end{aligned}$$

Taking into account the equilibrium conditions for the marginal agents, these can be written as follows:

$$\begin{aligned}\frac{\partial W(.)}{\partial \mu} &= -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a^*_{iH}} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\ &\quad -\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a^*_{iL} \leq a < a^*_{iH}} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) \\ &\quad -\sum_i \gamma_i \frac{\partial a^*_{iH}}{\partial \mu} \left(q^L s(a^*_{iH}) - c(e^*(a^*_{iH}, \tilde{P}^{iL})) - T_L \right) h^i(a^*_{iH}) \\ &\quad +\sum_i \gamma_i \frac{\partial a^*_{iH}}{\partial \mu} \left(q^L s(a^*_{iH}) - c(e^*(a^*_{iH}, \tilde{P}^{iL})) - T_L \right) h^i(a^*_{iH}) \\ &\quad -\sum_i \gamma_i \frac{\partial a^*_{iL}}{\partial \mu} (0)\, h^i(a^*_{iL}).\end{aligned}$$

(as agent a^*_{iH} is indifferent between schools) so that

$$\begin{aligned}\frac{\partial W}{\partial \mu} &= -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a^*_{iH}} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\ &\quad -\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a^*_{iL} \leq a < a^*_{iH}} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a).\end{aligned}$$

Let us then find the derivative of $\tilde{P}^{ik}$ with respect to μ. >From the equilibrium conditions, we have

$$\frac{\partial \tilde{P}^{iL}}{\partial \mu} = \frac{q_L s'(a^*_{iL}) - c'(e^*(a^*_{iL}, \tilde{P}^{iL})) \frac{\partial e^*(a^*_{iL}, \tilde{P}^{iL})}{\partial a}}{c'(e^*(a^*_{iL}, \tilde{P}^{iL})) \frac{\partial e^*(a^*_{iL}, \tilde{P}^{iL})}{\partial \tilde{P}}} \frac{\partial a^*_{iL}}{\partial \mu}.$$

Taking into account that

$$\theta_{iH}\alpha_H + \theta_{iL}\alpha_L = \gamma_i \left(1 - H^i(a^*_{iL})\right),$$

we derive that

$$\begin{aligned}\frac{\partial a^*_{1L}}{\partial \mu} &= -\frac{\theta^*_H \alpha_H + \theta^*_L \alpha_L}{\gamma_1 h(a^*_{1L})} < 0, \\ \frac{\partial a^*_{2L}}{\partial \mu} &= \frac{\theta^*_H \alpha_H + \theta^*_L \alpha_L}{\gamma_2 h(a^*_{2L})} > 0.\end{aligned}$$

This implies that

$$\begin{aligned}\frac{\partial \tilde{P}^{1L}}{\partial \mu} &= -\frac{q_L s'(a^*_{1L}) - c'(e^*(a^*_{1L}, \tilde{P}^{1L})) \frac{\partial e^*(a^*_{1L}, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1L}, \tilde{P}^{1L})) \frac{\partial e^*(a^*_{1L}, \tilde{P}^{1L})}{\partial \tilde{P}}} \frac{\theta^*_H \alpha_H + \theta^*_L \alpha_L}{\gamma_1 h(a^*_{1L})} < 0, \\ \frac{\partial \tilde{P}^{2L}}{\partial \mu} &= \frac{q_L s'(a^*_{2L}) - c'(e^*(a^*_{2L}, \tilde{P}^{2L})) \frac{\partial e^*(a^*_{2L}, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2L}, \tilde{P}^{2L})) \frac{\partial e^*(a^*_{2L}, \tilde{P}^{2L})}{\partial \tilde{P}}} \frac{\theta^*_H \alpha_H + \theta^*_L \alpha_L}{\gamma_2 h(a^*_{2L})} > 0.\end{aligned}$$

As can be seen, at the non-discrimination quotas (when $\mu = 1$),

$$\gamma_1 \frac{\partial \tilde{P}^{1L}}{\partial \mu} = -\gamma_2 \frac{\partial \tilde{P}^{2L}}{\partial \mu},$$

implying that (recall that $H^1(a) \equiv H^2(a)$)

$$\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a_{iL}^* \leq a < a_{iH}^*} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) = 0.$$

Next, we consider the derivative of $\tilde{P}^{iH}$ with respect to μ. >From the equilibrium conditions, we have that

$$\begin{aligned} \frac{\partial \tilde{P}^{iH}}{\partial \mu} &= \frac{\left[\triangle q s'(a_{iH}^*) - c'(e^*(a_{iH}^*, \tilde{P}^{iH})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iH})}{\partial a} + c'(e^*(a_{iH}^*, \tilde{P}^{iL})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iL})}{\partial a}\right] \frac{\partial a_{iH}^*}{\partial \mu}}{c'(e^*(a_{iH}^*, \tilde{P}^{iH})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iH})}{\partial \tilde{P}}} \\ &\quad + \frac{c'(e^*(a_{iH}^*, \tilde{P}^{iL})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iL})}{\partial \tilde{P}} \frac{\partial \tilde{P}^{iL}}{\partial \mu}}{c'(e^*(a_{iH}^*, \tilde{P}^{iH})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iH})}{\partial \tilde{P}}}. \end{aligned}$$

In addition, we have that

$$\frac{\partial a_{1H}^*}{\partial \mu} = -\frac{\theta_H^* \alpha_H}{\gamma_1 h(a_{1H}^*)} < 0, \ \frac{\partial a_{2H}^*}{\partial \mu} = \frac{\theta_H^* \alpha_H}{\gamma_2 h(a_{2H}^*)} > 0.$$

Summarizing all the previous results, we can see that

$$\frac{\partial \tilde{P}^{1H}}{\partial \mu} < 0 \text{ and } \frac{\partial \tilde{P}^{2H}}{\partial \mu} > 0.$$

The latter follows from the fact that $\frac{\partial \tilde{P}^{1L}}{\partial \mu} < 0$, $\frac{\partial \tilde{P}^{2L}}{\partial \mu} > 0$, and

$$\triangle q s'(a_{iH}^*) - c'(e^*(a_{iH}^*, \tilde{P}^{iH})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iH})}{\partial a} + c'(e^*(a_{iH}^*, \tilde{P}^{iL})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iL})}{\partial a} > 0.$$

Moreover, if $\mu = 1$, it is straightforward to see that

$$\gamma_1 \frac{\partial \tilde{P}^{1H}}{\partial \mu} = -\gamma_2 \frac{\partial \tilde{P}^{2H}}{\partial \mu},$$

implying that

$$\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a_{iH}^*} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) = 0.$$

Thus, we have that

$$\frac{\partial W}{\partial \mu}|_{\mu=1} = 0.$$

That is, non-discrimination delivers a local extremum. In the case of concave welfare social welfare, non-discrimination is globally optimal. Next, we explore the case when the groups are asymmetric in terms of the distribution of total ability.

Intuitively the logic is exactly the same. When the two groups are the same, the losses of one group exactly make up for the gains of the other for slight changes. Thus, if welfare is concave, this is a local maximum.

Asymmetric Groups with no Selection Effect

Assume now that $H^1(a) \succeq_{LR} H^2(a)$. Using the results derived in the above section, the derivative of welfare with respect to μ is given by

$$\begin{aligned}\frac{\partial W}{\partial \mu} &= -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a^*_{iH}} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\ &\quad - \sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a^*_{iL} \leq a < a^*_{iH}} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a).\end{aligned}$$

Consider the second component of the derivative:

$$\begin{aligned}&-\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a^*_{iL} \leq a < a^*_{iH}} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) \\ &= (\theta^*_H \alpha_H + \theta^*_L \alpha_L) \left[\begin{array}{c} \frac{q_L s'(a^*_{1L}) - c'(e^*(a^*_{1L}, \tilde{P}^{1L})) \frac{\partial e^*(a^*_{1L}, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1L}, \tilde{P}^{1L})) \frac{\partial e^*(a^*_{1L}, \tilde{P}^{1L})}{\partial \tilde{P}}} \int_{a^*_{1L} \leq a < a^*_{1H}} c'(e^*(a, \tilde{P}^{1L})) \frac{\partial e^*(a, \tilde{P}^{1L})}{\partial \tilde{P}} \frac{h^1(a)}{h^1(a^*_{1L})} da \\ - \frac{q_L s'(a^*_{2L}) - c'(e^*(a^*_{2L}, \tilde{P}^{2L})) \frac{\partial e^*(a^*_{2L}, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2L}, \tilde{P}^{2L})) \frac{\partial e^*(a^*_{2L}, \tilde{P}^{2L})}{\partial \tilde{P}}} \int_{a^*_{2L} \leq a < a^*_{2H}} c'(e^*(a, \tilde{P}^{2L})) \frac{\partial e^*(a, \tilde{P}^{2L})}{\partial \tilde{P}} \frac{h^2(a)}{h^2(a^*_{2L})} da \end{array} \right].\end{aligned}$$

At the non-discrimination quota, $a^*_{2L} = a^*_{1L}$ and $\tilde{P}^{2L} = \tilde{P}^{1L}$. This implies that

$$\frac{q_L s'(a^*_{2L}) - c'(e^*(a^*_{2L}, \tilde{P}^{2L})) \frac{\partial e^*(a^*_{2L}, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2L}, \tilde{P}^{2L})) \frac{\partial e^*(a^*_{2L}, \tilde{P}^{2L})}{\partial \tilde{P}}} = \frac{q_L s'(a^*_{1L}) - c'(e^*(a^*_{1L}, \tilde{P}^{1L})) \frac{\partial e^*(a^*_{1L}, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1L}, \tilde{P}^{1L})) \frac{\partial e^*(a^*_{1L}, \tilde{P}^{1L})}{\partial \tilde{P}}}.$$

Moreover, as $H^1(a) \succeq_{LR} H^2(a)$,

$$\frac{h^1(a)}{h^1(a^*_{1L})} \geq \frac{h^2(a)}{h^2(a^*_{2L})} \text{ for any } a > a^*_{1L} = a^*_{2L}.$$

Thus,

$$\int_{a^*_{1L} \leq a < a^*_{1H}} c'(e^*(a, \tilde{P}^{1L})) \frac{\partial e^*(a, \tilde{P}^{1L})}{\partial \tilde{P}} \frac{h^1(a)}{h^1(a^*_{1L})} da > \int_{a^*_{2L} \leq a < a^*_{2H}} c'(e^*(a, \tilde{P}^{2L})) \frac{\partial e^*(a, \tilde{P}^{2L})}{\partial \tilde{P}} \frac{h^2(a)}{h^2(a^*_{2L})} da,$$

implying that

$$-\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a^*_{iL} \leq a < a^*_{iH}} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) > 0$$

when evaluated at the non-discrimination quota ($\mu = 1$).

Consider then the first component of the derivative, which is given by

$$-\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a^*_{iH}} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a).$$

>From the previous section, we have that

$$\begin{aligned}\frac{\partial \tilde{P}^{iH}}{\partial \mu} &= \frac{\left[\triangle q s'(a^*_{iH}) - c'(e^*(a^*_{iH}, \tilde{P}^{iH})) \frac{\partial e^*(a^*_{iH}, \tilde{P}^{iH})}{\partial a} + c'(e^*(a^*_{iH}, \tilde{P}^{iL})) \frac{\partial e^*(a^*_{iH}, \tilde{P}^{iL})}{\partial a} \right] \frac{\partial a^*_{iH}}{\partial \mu}}{c'(e^*(a^*_{iH}, \tilde{P}^{iH})) \frac{\partial e^*(a^*_{iH}, \tilde{P}^{iH})}{\partial \tilde{P}}} \\ &\quad + \frac{c'(e^*(a^*_{iH}, \tilde{P}^{iL})) \frac{\partial e^*(a^*_{iH}, \tilde{P}^{iL})}{\partial \tilde{P}} \frac{\partial \tilde{P}^{iL}}{\partial \mu}}{c'(e^*(a^*_{iH}, \tilde{P}^{iH})) \frac{\partial e^*(a^*_{iH}, \tilde{P}^{iH})}{\partial \tilde{P}}}.\end{aligned}$$

This means that at the non-discrimination quotas,

$$\begin{aligned}
\gamma_1 \frac{\partial \tilde{P}^{1H}}{\partial \mu} &= -\frac{\triangle q s'(a^*_{1H}) - c'(e^*(a^*_{1H}, \tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H}, \tilde{P}^{iH})}{\partial a} + c'(e^*(a^*_{1H}, \tilde{P}^{1L}))\frac{\partial e^*(a^*_{1H}, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1H}, \tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H}, \tilde{P}^{1H})}{\partial \tilde{P}}} \frac{\theta^*_H \alpha_H}{h^1(a^*_{1H})} \\
&\quad - \frac{c'(e^*(a^*_{1H}, \tilde{P}^{1L}))\frac{\partial e^*(a^*_{1H}, \tilde{P}^{1L})}{\partial \tilde{P}} \left(\frac{q_L s'(a^*_{1L}) - c'(e^*(a^*_{1L}, \tilde{P}^{1L}))\frac{\partial e^*(a^*_{1L}, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1L}, \tilde{P}^{1L}))\frac{\partial e^*(a^*_{1L}, \tilde{P}^{1L})}{\partial \tilde{P}}} \right)}{c'(e^*(a^*_{1H}, \tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H}, \tilde{P}^{1H})}{\partial \tilde{P}}} \frac{\theta^*_H \alpha_H + \theta^*_L \alpha_L}{h^1(a^*_{1L})}, \\
\gamma_2 \frac{\partial \tilde{P}^{2H}}{\partial \mu} &= \frac{\triangle q s'(a^*_{2H}) - c'(e^*(a^*_{2H}, \tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H}, \tilde{P}^{2H})}{\partial a} + c'(e^*(a^*_{2H}, \tilde{P}^{2L}))\frac{\partial e^*(a^*_{2H}, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2H}, \tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H}, \tilde{P}^{2H})}{\partial \tilde{P}}} \frac{\theta^*_H \alpha_H}{h^2(a^*_{2H})} \\
&\quad + \frac{c'(e^*(a^*_{2H}, \tilde{P}^{iL}))\frac{\partial e^*(a^*_{2H}, \tilde{P}^{iL})}{\partial \tilde{P}} \left(\frac{q_L s'(a^*_{2L}) - c'(e^*(a^*_{2L}, \tilde{P}^{2L}))\frac{\partial e^*(a^*_{2L}, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2L}, \tilde{P}^{2L}))\frac{\partial e^*(a^*_{2L}, \tilde{P}^{2L})}{\partial \tilde{P}}} \right)}{c'(e^*(a^*_{2H}, \tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H}, \tilde{P}^{2H})}{\partial \tilde{P}}} \frac{\theta^*_H \alpha_H + \theta^*_L \alpha_L}{h^2(a^*_{2L})}.
\end{aligned}$$

Note that at the non-discrimination quota:

$$\begin{aligned}
& \theta^*_H \alpha_H \frac{\triangle q s'(a^*_{1H}) - c'(e^*(a^*_{1H}, \tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H}, \tilde{P}^{iH})}{\partial a} + c'(e^*(a^*_{1H}, \tilde{P}^{1L}))\frac{\partial e^*(a^*_{1H}, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1H}, \tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H}, \tilde{P}^{1H})}{\partial \tilde{P}}} \\
= \quad & \theta^*_H \alpha_H \frac{\triangle q s'(a^*_{2H}) - c'(e^*(a^*_{2H}, \tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H}, \tilde{P}^{2H})}{\partial a} + c'(e^*(a^*_{2H}, \tilde{P}^{2L}))\frac{\partial e^*(a^*_{2H}, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2H}, \tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H}, \tilde{P}^{2H})}{\partial \tilde{P}}} \equiv A.
\end{aligned}$$

Moreover, at the non-disrimination quota:

$$\begin{aligned}
& (\theta^*_H \alpha_H + \theta^*_L \alpha_L) \frac{c'(e^*(a^*_{1H}, \tilde{P}^{1L}))\frac{\partial e^*(a^*_{1H}, \tilde{P}^{1L})}{\partial \tilde{P}} \left(\frac{q_L s'(a^*_{1L}) - c'(e^*(a^*_{1L}, \tilde{P}^{1L}))\frac{\partial e^*(a^*_{1L}, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1L}, \tilde{P}^{1L}))\frac{\partial e^*(a^*_{1L}, \tilde{P}^{1L})}{\partial \tilde{P}}} \right)}{c'(e^*(a^*_{1H}, \tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H}, \tilde{P}^{1H})}{\partial \tilde{P}}} \\
= \quad & (\theta^*_H \alpha_H + \theta^*_L \alpha_L) \frac{c'(e^*(a^*_{2H}, \tilde{P}^{iL}))\frac{\partial e^*(a^*_{2H}, \tilde{P}^{iL})}{\partial \tilde{P}} \left(\frac{q_L s'(a^*_{2L}) - c'(e^*(a^*_{2L}, \tilde{P}^{2L}))\frac{\partial e^*(a^*_{2L}, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2L}, \tilde{P}^{2L}))\frac{\partial e^*(a^*_{2L}, \tilde{P}^{2L})}{\partial \tilde{P}}} \right)}{c'(e^*(a^*_{2H}, \tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H}, \tilde{P}^{2H})}{\partial \tilde{P}}} \equiv B.
\end{aligned}$$

Thus, at the non-disrimination quota:

$$\begin{aligned}
\gamma_1 \frac{\partial \tilde{P}^{1H}}{\partial \mu} &= -\frac{A}{h^1(a^*_H)} - \frac{B}{h^1(a^*_L)}, \\
\gamma_2 \frac{\partial \tilde{P}^{2H}}{\partial \mu} &= \frac{A}{h^2(a^*_H)} + \frac{B}{h^2(a^*_L)}.
\end{aligned}$$

As a result, at the non-disrimination quota:

$$\begin{aligned}
& -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a^*_{iH}} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\
= \quad & \left(\frac{A}{h^1(a^*_H)} + \frac{B}{h^1(a^*_L)} \right) \int_{a \geq a^*_H} c'(e^*(a, \tilde{P}^H)) \frac{\partial e^*(a, \tilde{P}^H)}{\partial \tilde{P}} h^1(a) da \\
& - \left(\frac{A}{h^2(a^*_H)} + \frac{B}{h^2(a^*_L)} \right) \int_{a \geq a^*_H} c'(e^*(a, \tilde{P}^H)) \frac{\partial e^*(a, \tilde{P}^H)}{\partial \tilde{P}} h^2(a) da.
\end{aligned}$$

Taking into account the stochastic order of the distributions of total ability, it is straightforward to see that

$$
\begin{aligned}
A\int_{a\geq a_H^*} c'(e^*(a,\tilde{P}^H))\frac{\partial e^*(a,\tilde{P}^H)}{\partial \tilde{P}}\frac{h^1(a)}{h^1(a_H^*)}da &> A\int_{a\geq a_H^*} c'(e^*(a,\tilde{P}^H))\frac{\partial e^*(a,\tilde{P}^H)}{\partial \tilde{P}}\frac{h^2(a)}{h^2(a_H^*)}da,\\
B\int_{a\geq a_H^*} c'(e^*(a,\tilde{P}^H))\frac{\partial e^*(a,\tilde{P}^H)}{\partial \tilde{P}}\frac{h^1(a)}{h^1(a_L^*)}da &> B\int_{a\geq a_H^*} c'(e^*(a,\tilde{P}^H))\frac{\partial e^*(a,\tilde{P}^H)}{\partial \tilde{P}}\frac{h^2(a)}{h^2(a_L^*)}da.
\end{aligned}
$$

In other words, at the non-discrimination quota:

$$
-\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu}\int_{a\geq a_{iH}^*} c'(e^*(a,\tilde{P}^{iH}))\frac{\partial e^*(a,\tilde{P}^{iH})}{\partial \tilde{P}}dH^i(a) > 0.
$$

To sum up, we have shown that the derivative of social welfare with respect to μ evaluated at the non-discrimination quota is positive. This means that discriminating in favor of group 1 locally increases the social welfare. This results is the same as that in the case of one university. That is, the effort effect works in favor of the advantaged group.

$$
EE_{\mu=1} > 0.
$$

This makes sense as weaker students need to put in more effort to get in and this effort is wasteful. So discriminating against the less advantaged group raises welfare. Next, we explore the role of the selection effect.

The Selection Effect

Recall that the social welfare when $\beta < 1$ is given by

$$
\begin{aligned}
W &= \sum_i \gamma_i \int_{a_N+a_A\geq a_{iH}^*}\Big(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^{iH})) - F\Big)\,dH_N(a_N)dH_A^i(a_A)\\
&\quad+\sum_i \gamma_i \int_{a_{iL}^*\leq a_N+a_A< a_{iH}^*}\Big(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^{iL})) - F\Big)\,dH_N(a_N)dH_A^i(a_A)\\
&= \sum_i \gamma_i \int_{a_N+a_A\geq a_{iH}^*}\Big(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^{iH})) - T_H\Big)\,dH_N(a_N)dH_A^i(a_A)\\
&\quad+\sum_i \gamma_i \int_{a_{iL}^*\leq a_N+a_A< a_{iH}^*}\Big(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^{iL})) - T_L\Big)\,dH_N(a_N)dH_A^i(a_A)\\
&\quad+\alpha_H(T_H-F)+\alpha_L(T_L-F).
\end{aligned}
$$

The latter can be written as follows:

$$
\begin{aligned}
W &= \sum_i \gamma_i \int_0^{a^A_{\max,i}}\left(\int_{a_{iH}^*-a_A}^{a^N_{\max}}\Big(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^{iH})) - T_H\Big)\,dH_N(a_N)\right)dH_A^i(a_A)\\
&\quad+\sum_i \gamma_i \int_0^{a^A_{\max,i}}\left(\int_{a_{iL}^*-a_A}^{a_{iH}^*-a_A}\Big(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^{iL})) - T_L\Big)\,dH_N(a_N)\right)dH_A^i(a_A)\\
&\quad+\alpha_H(T_H-F)+\alpha_L(T_L-F).
\end{aligned}
$$

When we explore the selection effect only, by definition we look at the effect via the cutoffs directly and not via the performance cutoffs. Thus, the selection effect is as follows:

$$\begin{aligned}
SE &= -\sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(q^H s(a_{iH}^* - a_A + \beta a_A) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) - T_H\right) h_N(a_{iH}^* - a_A) dH_A^i(a_A) \\
&\quad + \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(q^L s(a_{iH}^* - a_A + \beta a_A) - c(e^*(a_{iH}^*, \tilde{P}^{iL})) - T_L\right) h_N(a_{iH}^* - a_A) dH_A^i(a_A) \\
&\quad - \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(q^L s(a_{iL}^* - a_A + \beta a_A) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L\right) h_N(a_{iL}^* - a_A) dH_A^i(a_A) \\
&= -\sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \begin{pmatrix} \triangle q s(a_{iH}^* - a_A + \beta a_A) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) \\ + c(e^*(a_{iH}^*, \tilde{P}^{iL})) - \triangle T \end{pmatrix} h_N(a_{iH}^* - a_A) dH_A^i(a_A) \\
&\quad - \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(q^L s(a_{iL}^* - a_A + \beta a_A) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L\right) h_N(a_{iL}^* - a_A) dH_A^i(a_A).
\end{aligned}$$

Taking into account the equilibrium conditions, we know that

$$\begin{aligned}
\triangle q s(a_{iH}^*) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) + c(e^*(a_{iH}^*, \tilde{P}^{iL})) - \triangle T &= 0 \\
q^L s(a_{iL}^*) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L &= 0
\end{aligned}$$

so that have

$$\begin{aligned}
& \triangle q s(a_{iH}^* - a_A + \beta a_A) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) + c(e^*(a_{iH}^*, \tilde{P}^{iL})) - \triangle T \\
= \; & \triangle q \left(s(a_{iH}^* - a_A + \beta a_A) - s(a_{iH}^*)\right), \text{ and} \\
& q^L s(a_{iL}^* - a_A + \beta a_A) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L \\
= \; & q^L \left(s(a_{iL}^* - a_A + \beta a_A) - s(a_{iL}^*)\right).
\end{aligned}$$

Hence,

$$\begin{aligned}
SE &= -\triangle q \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(s(a_{iH}^* - a_A + \beta a_A) - s(a_{iH}^*)\right) h_N(a_{iH}^* - a_A) dH_A^i(a_A) \qquad (40) \\
&\quad - q^L \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(s(a_{iL}^* - a_A + \beta a_A) - s(a_{iL}^*)\right) h_N(a_{iL}^* - a_A) dH_A^i(a_A).
\end{aligned}$$

Note that in the case of one university we have only the second component of the above expression. However, we can apply the technique developed for the case with one university to both components, as they have similar functional forms.

Consider, for instance, the first term in the above expression given by:

$$SE_1 = -\triangle q \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(s(a_{iH}^* - a_A + \beta a_A) - s(a_{iH}^*)\right) h_N(a_{iH}^* - a_A) dH_A^i(a_A).$$

Recall that

$$\frac{\partial a_{1H}^*}{\partial \mu} = -\frac{\theta_H^* \alpha_H}{\gamma_1 h(a_{1H}^*)} < 0, \; \frac{\partial a_{2H}^*}{\partial \mu} = \frac{\theta_H^* \alpha_H}{\gamma_2 h(a_{2H}^*)} > 0.$$

Hence, SE_1 (evaluated at the non-discrimination quota) can be written as follows:

$$\begin{aligned} SE_1|_{\theta=\theta^*} &= \triangle q\alpha_H\theta_H^*\int_0^{a^A_{\max,1}}\left(s(a_H^*-a_A+\beta a_A)-s(a_H^*)\right)\frac{h_N(a_H^*-a_A)h_A^1(a_A)}{h(a_H^*)}da_A \\ &\quad -\triangle q\alpha_H\theta_H^*\int_0^{a^A_{\max,2}}\left(s(a_H^*-a_A+\beta a_A)-s(a_H^*)\right)\frac{h_N(a_H^*-a_A)h_A^2(a_A)}{h(a_H^*)}da_A. \end{aligned}$$

As in the benchmark case, we consider the following density functions:

$$\tilde{h}_A^i(a_A)\equiv\frac{h_A^i(a_A)h_N(a_H^*-a_A)}{\int_0^{a^A_{\max,i}}h_N(a_H^*-y)h_A^i(y)dy}.$$

Let $\tilde{H}_A^i(a_A)$ be its associated distribution function. As $H_A^1(a_A)\succeq_{LR}H_A^2(a_A)$,

$$\frac{\tilde{h}_A^1(a_A)}{\tilde{h}_A^1(x)}=\frac{h_A^1(a_A)h_N(a_H^*-a_A)}{h_A^1(x)h_N(a_H^*-x)}\geq\frac{h_A^2(a_A)h_N(a_H^*-a_A)}{h_A^2(x)h_N(a_H^*-x)}=\frac{\tilde{h}_A^2(a_A)}{\tilde{h}_A^2(x)}\text{ for any }a_A,\,x:\ a_A\geq x.$$

That is, $\tilde{H}_A^1(a_A)\succeq_{LR}\tilde{H}_A^2(a_A)$ implying $\tilde{H}_A^1(a_A)\succeq_1\tilde{H}_A^2(a_A)$.

Then, SE_1 can be rewritten in the following way:

$$\begin{aligned} SE_1|_{\theta=\theta^*} &= -\triangle q\alpha_H\theta_H^*\int_0^{a^A_{\max,1}}\left(s(a_H^*)-s(a_H^*-(1-\beta)\,a_A)\right)d\tilde{H}_A^1(a_A) \\ &\quad +\triangle q\alpha_H\theta_H^*\int_0^{a^A_{\max,2}}\left(s(a_H^*)-s(a_H^*-(1-\beta)\,a_A)\right)d\tilde{H}_A^2(a_A). \end{aligned}$$

Equivalently, as $H_A^1(a_A)\succeq_{LR}H_A^2(a_A)$ implies $a^A_{\max,1}\geq a^A_{\max,2}$

$$\begin{aligned} SE_1|_{\theta=\theta^*} &= -\triangle q\alpha_H\theta_H^*\int_{a^A_{\max,2}}^{a^A_{\max,1}}\left(s(a_H^*)-s(a_H^*-(1-\beta)\,a_A)\right)d\tilde{H}_A^1(a_A) \\ &\quad -\triangle q\alpha_H\theta_H^*\int_0^{a^A_{\max,2}}\left(s(a_H^*)-s(a_H^*-(1-\beta)\,a_A)\right)d\left(\tilde{H}_A^1(a_A)-\tilde{H}_A^2(a_A)\right). \end{aligned}$$

Integrating the second term above by parts implies that

$$\begin{aligned} SE_1|_{\theta=\theta^*} &= -\triangle q\alpha_H\theta_H^*\int_{a^A_{\max,2}}^{a^A_{\max,1}}\left(s(a_H^*)-s(a_H^*-(1-\beta)\,a_A)\right)d\tilde{H}_A^1(a_A) \\ &\quad +\triangle q\alpha_H\theta_H^*\left(s(a_H^*)-s(a_H^*-(1-\beta)\,a^A_{\max,2})\right)\left(1-\tilde{H}_A^1(a^A_{\max,2})\right) \\ &\quad -\triangle q\alpha_H\theta_H^*\,(1-\beta)\int_0^{a^A_{\max,2}}\left(\tilde{H}_A^2(a_A)-\tilde{H}_A^1(a_A)\right)s'(a_H^*-(1-\beta)\,a_A)da_A \\ &< 0, \end{aligned}$$

as $s'(\cdot)$ is positive and $\tilde{H}_A^2(a_A)\geq\tilde{H}_A^1(a_A)$ for any a_A (recall that $\tilde{H}_A^1(a_A)\succeq_1\tilde{H}_A^2(a_A)$) and, moreover,

$$\int_{a^A_{\max,2}}^{a^A_{\max,1}}\left(s(a_H^*)-s(a_H^*-(1-\beta)\,a_A)\right)d\tilde{H}_A^1(a_A)>\left(s(a_H^*)-s(a_H^*-(1-\beta)\,a^A_{\max,2})\right)\left(1-\tilde{H}_A^1(a^A_{\max,2})\right).$$

Similarly, we can show that the second term in (40) evaluated at the non-discrimination quota:

$$SE_2|_{\theta=\theta^*}=-q^L\sum_i\gamma_i\frac{\partial a_{iL}^*}{\partial\mu}\int_0^{a^A_{\max,i}}\left(s(a_L^*-a_A+\beta a_A)-s(a_L^*)\right)h_N(a_L^*-a_A)dH_A^i(a_A),$$

is negative as well (the proof is exactly the same as that for SE_1). As a result, we can show that the selection effect evaluated at the non-discrimination quotas is negative, suggesting that we need to give quotas to the disadvantaged group. Moreover, if the groups are symmetric, then the selection effect is equal to zero at the non-discrimination quotas.