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ABSTRACT

An accurate global algorithm is critical for quantifying the dynamics of the Diamond-Mortensen-Pissarides model. Loglinearization understates the mean and volatility of unemployment, overstates the unemployment-vacancy correlation, and ignores impulse responses that are an order of magnitude larger in recessions than in booms. Although improving on loglinearization, the second-order perturbation in logs also induces large errors. We demonstrate these insights in the context of Hagedorn and Manovskii (2008). Once solved accurately, their small surplus calibration fails to explain the Shimer (2005) puzzle. While the volatility of labor market tightness is close to the data, the unemployment volatility is too high.

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1 Introduction

Since the foundational work of Diamond (1982), Mortensen (1982), and Pissarides (1985), the search model of unemployment has become the dominant framework for the economic analysis of the labor market. Shimer (2005) argues that the unemployment volatility in the baseline Diamond-Mortensen-Pissarides (DMP) model is too low relative to that in the data. A large subsequent literature has developed to address this volatility puzzle.[1] For the most part, this literature has relied on loglinearization to quantify the properties of the DMP model.

Our key insight is that an accurate global approximation algorithm is critical for characterizing the dynamics of the DMP model. We use the Hagedorn and Manovskii (2008) calibration of the DMP model as the prime example. Hagedorn and Manovskii argue that under their calibration, the DMP model produces realistic labor market volatilities. In particular, the volatility of labor market tightness is 0.292 in the model, which is close to 0.259 in the data. The unemployment volatility is 0.145 in the model, close to 0.125 in the data.

However, using an accurate projection algorithm and their parameter values, we calculate the unemployment volatility to be 0.258, which is about twice as large as 0.125 in the data. The unemployment-vacancy correlation is also lower in magnitude, -0.567, relative to -0.724 from loglinearization. The volatility of labor market tightness is not much affected, 0.268. Also, the stochastic mean of the unemployment rate using the projection solution, 6.17%, is almost one percentage point higher than its deterministic steady state rate, 5.28%. These results cast doubt on the validity of calibration that relies exclusively on steady state relations.

[1] For example, Hall (2005) shows how wage stickiness, which satisfies the condition that no worker-firm pair has any unexploited opportunity for mutual gain, helps increase labor market volatilities. Mortensen and Nagypál (2007) and Pissarides (2009) show that fixed matching costs can explain the volatility puzzle. Gertler and Trigari (2009) propose a staggered wage bargaining framework to generate wage rigidity. Hagedorn and Manovskii (2008) show that a calibration with small profits and a low bargaining power for workers can produce realistic volatilities in the DMP model. Finally, Hall and Milgrom (2008) replace the Nash bargaining wage with a credible bargaining wage, and show that the change helps explain the volatility puzzle.

What drives the differences between loglinearization and projection? The unemployment dynamics in the DMP model with the Hagedorn-Manovskii (2008) calibration are highly nonlinear. In recessions unemployment rises rapidly, whereas in booms unemployment declines only gradually. The projection algorithm fully captures the nonlinearity. In contrast, by focusing on local dynamics around the deterministic steady state, loglinearization ignores completely the unemployment spikes in recessions. Because of this omission, loglinearization underestimates the volatility of unemployment but overestimates its correlation with vacancies.[2]

Since Merz (1995) and Andolfatto (1996), the DMP model has been embedded into the dynamic stochastic general equilibrium framework to study business cycles. While the Hagedorn and Manovskii (2008) calibration implies a quarterly persistence of 0.88 for the log productivity process, business cycle studies typically calibrate the persistence to be 0.95 (e.g., Cooley and Prescott (1995)). We show that the higher persistence induces even larger differences between loglinearization and projectio, even when the unconditional volatility of the log productivity is reduced. Intuitively, because job creation is forward looking, a low realization of productivity reduces job creation more if the low productivity is expected to last longer. As such, the higher persistence strengthens the nonlinear dynamics in unemployment, thereby exacerbating the deviation of loglinearization from the accurate projection solution.

Going beyond loglinearization, we also study whether the second-order perturbation in logs can deliver quantitatively accurate results. The answer is not affirmative. Under the Hagedorn-Manovskii (2008) calibration, we show that although improving on log-

[2]In addition, impulse responses are stronger in recessions than in booms under the projection solution. In response to a negative one-standard-deviation shock to log labor productivity, the unemployment rate increases only by 0.03% if the economy starts at the 95 percentile of the model's stationary distribution of employment and productivity. In contrast, the unemployment rate jumps by 0.85% (about 30 times larger in magnitude) if the economy instead starts at the 5 percentile of the bivariate distribution. However, the nonlinearity in impulse responses is entirely missed by loglinearization.

linearization, the second-order perturbation still deviates greatly from the projection solution. For instance, the unemployment volatility is 0.165, which, although higher than 0.133 from loglinearization, is still substantially lower than 0.258 from projection. Similarly, the unemployment-vacancy correlation is -0.789, which, although lower in magnitude than -0.848 from loglinearization, is still substantially higher than -0.567 from projection.[3]

Following Den Haan (2010), we perform accuracy tests on the different numerical algorithms. The results show that the projection algorithm provides an extremely accurate solution, whereas loglinearization and the second-order perturbation in logs induce large approximation errors. In particular, dynamic Euler equation errors in consumption are in the order of 10^{-14} for the projection algorithm, but are over 100% for the other two algorithms.

Our work suggests that many prior conclusions in the macro labor literature might need to be reexamined with an accurate global solution. Although the Hagedorn-Manovskii (2008) calibration seems extreme in its implications on the magnitude of profits, it provides a parsimonious setting with reasonable labor market volatilities in loglinear approximations. As such, their calibration offers a natural starting point for our analysis. Many other economic mechanisms have been proposed to generate realistic fluctuations in labor market tightness. Our insights should apply as a consequence of nonlinear dynamics in unemployment.

More broadly, our insights apply to a wide body of academic and policy research because the DMP model has been adopted throughout macroeconomics. Recent examples include Merz (1995) and Andolfatto (1996) in the business cycle literature, Gertler and Trigari (2009) in the New Keynesian literature, Blanchard and Gali (2010) on monetary policy, and

[3]We have also experimented with the third-order perturbation solution in logs. However, as discussed in Den Haan and Wind (2012), high-order perturbation solutions often have the problem of explosive simulated paths. In our practice, we do find the third-order perturbation produce explosive sample paths. Because pruning explosive paths (a standard way of dealing with this problem) seems ad hoc, we do not pursue high-order perturbation solutions.

Monacelli, Perotti, and Trigari (2010) on fiscal policy. Characterizing dynamics accurately is crucial for deriving quantitative implications in these areas. The ability to perform policy counterfactuals requires a macroeconomic model to be estimated (e.g., Smets and Wouter (2003, 2007)). However, inferences can be biased if the model's solution does not adequately account for its nonlinear dynamics (e.g., Fernandez-Villaverde and Rubio-Ramirez (2007)).

We build on the projection algorithm of Kuehn, Petrosky-Nadeau, and Zhang (2013), who embed the DMP model into an equilibrium asset pricing framework to study the impact of labor market frictions on aggregate asset prices. Petrosky-Nadeau and Zhang (2013) adopt a version of the DMP model augmented with credible bargaining and fixed matching costs to explain the tail probabilities of unemployment crises. Our work differs because we focus on how different solution algorithms can impact on the second moments of the labor market (that are at the center of the contemporary macro labor literature).

The rest of the paper is organized as follows. Section 2 sets up the DMP model. Section 3 describes the projection algorithm. Section 4 shows how the labor market moments based on loglinearization deviate quantitatively from those based on the projection algorithm. Section 5 reports a battery of accuracy tests. Finally, Section 6 concludes.

2 The DMP Model

The model is populated by a representative household and a representative firm that uses labor as the single productive input. Following Merz (1995), we use the representative family construct, which implies perfect consumption insurance. The household has a continuum with a unit mass of members who are, at any point in time, either employed or unemployed. The fractions of employed and unemployed workers are representative of the population at large. The household pools the income of all the members together before choosing per capita consumption and asset holdings. The household is risk neutral with a time discount factor β.

The representative firm posts a number of job vacancies, V_t, to attract unemployed workers, U_t. Vacancies are filled via a constant returns to scale matching function, $G(U_t, V_t)$:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^\iota + V_t^\iota)^{1/\iota}}, \tag{1}$$

in which $\iota > 0$ is a constant parameter. This matching function, from Den Haan, Ramey, and Watson (2000), implies that matching probabilities fall between zero and one.

Define $\theta_t \equiv V_t/U_t$ as the vacancy-unemployment (V/U) ratio. The probability for an unemployed worker to find a job per unit of time (the job finding rate), $f(\theta_t)$, is:

$$f(\theta_t) \equiv \frac{G(U_t, V_t)}{U_t} = \frac{1}{\left(1 + \theta_t^{-\iota}\right)^{1/\iota}}. \tag{2}$$

The probability for a vacancy to be filled per unit of time (the vacancy filling rate), $q(\theta_t)$, is:

$$q(\theta_t) \equiv \frac{G(U_t, V_t)}{V_t} = \frac{1}{(1 + \theta_t^\iota)^{1/\iota}}. \tag{3}$$

An increase in the scarcity of unemployed workers relative to vacancies makes it harder to fill a vacancy, $q'(\theta_t) < 0$. As such, θ_t is labor market tightness from the firm's perspective.

The representative firm incurs costs in posting vacancies with the unit cost per vacancy, κ_t, specified as in Hagedorn and Manovskii (2008):

$$\kappa_t = \kappa_K X_t + \kappa_W X_t^\xi, \tag{4}$$

in which κ_K, κ_W, and ξ are positive parameters. Once matched, jobs are destroyed at a constant rate of s per period. Employment, N_t, evolves as:

$$N_{t+1} = (1 - s)N_t + q(\theta_t)V_t, \tag{5}$$

in which $q(\theta_t)V_t$ is the number of new hires. The population is normalized to be unity, $U_t = 1 - N_t$. As such, N_t and U_t are also the rates of employment and unemployment, respectively.

The firm takes aggregate labor productivity, X_t, as given. We specify $x_t \equiv \log(X_t)$ as:

$$x_{t+1} = \rho x_t + \sigma \epsilon_{t+1}, \tag{6}$$

in which $\rho \in (0, 1)$ is the persistence, $\sigma > 0$ is the conditional volatility, and ϵ_{t+1} is an independently and identically distributed (i.i.d.) standard normal shock. The firm uses labor to produce output, Y_t, with a constant returns to scale production technology,

$$Y_t = X_t N_t. \tag{7}$$

The dividends to the firm's shareholders are given by:

$$D_t = X_t N_t - W_t N_t - \kappa_t V_t, \tag{8}$$

in which W_t is the wage rate. Taking $q(\theta_t)$ and W_t as given, the firm posts an optimal number of job vacancies to maximize the cum-dividend market value of equity, denoted S_t:

$$S_t \equiv \max_{\{V_{t+\triangle t}, N_{t+\triangle t+1}\}_{\triangle t=0}^{\infty}} E_t \left[\sum_{\triangle t=0}^{\infty} \beta^{\triangle t} \left[X_{t+\triangle t} N_{t+\triangle t} - W_{t+\triangle t} N_{t+\triangle t} - \kappa_{t+\triangle t} V_{t+\triangle t} \right] \right], \tag{9}$$

subject to equation (5) and a nonnegativity constraint on vacancies:

$$V_t \geq 0. \tag{10}$$

Because $q(\theta_t) > 0$, this constraint is equivalent to $q(\theta_t)V_t \geq 0$. As such, the only source of job destruction in the model is the exogenous separation of employed workers from the firm.

Let λ_t denote the multiplier on the constraint $q(\theta_t)V_t \geq 0$. From the first-order conditions with respect to V_t and N_{t+1}, we obtain the intertemporal job creation condition:

$$\frac{\kappa_t}{q(\theta_t)} - \lambda_t = E_t\left[\beta\left[X_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_{t+1}}{q(\theta_{t+1})} - \lambda_{t+1}\right]\right]\right]. \tag{11}$$

Intuitively, the marginal costs of hiring at time t (with the nonnegativity constraint accounted for) equal the marginal value of employment to the firm, which in turn equals the marginal benefit of hiring at period $t+1$, discounted to t with the discount factor, β. The marginal benefit at $t+1$ includes the marginal product of labor, X_{t+1}, net of the wage rate, W_{t+1}, plus the marginal value of employment, which equals the marginal costs of hiring at $t+1$, net of separation. Finally, the optimal vacancy policy also satisfies the Kuhn-Tucker conditions:

$$q(\theta_t)V_t \geq 0, \quad \lambda_t \geq 0, \quad \text{and} \quad \lambda_t q(\theta_t)V_t = 0. \tag{12}$$

The wage rate is from the sharing rule per the outcome of a generalized Nash bargaining process between the employed workers and the firm. Let $\eta \in (0,1)$ be the workers' relative bargaining weight and b the workers' value of unemployment activities. The wage rate is:

$$W_t = \eta\left(X_t + \kappa_t\theta_t\right) + (1-\eta)b. \tag{13}$$

Let C_t denote consumption. In equilibrium, the goods market clearing condition says:

$$C_t + \kappa_t V_t = X_t N_t. \tag{14}$$

3 Solution Algorithm

We approximate the model solution with a projection algorithm. The state space of the model consists of employment and log productivity, (N_t, x_t). The goal is to solve for the optimal vacancy function, $V_t = V(N_t, x_t)$, and the multiplier function, $\lambda_t = \lambda(N_t, x_t)$ from the functional equation (the intertemporal job creation condition):

$$\frac{\kappa_t}{q(\theta_t)} - \lambda(N_t, x_t) = E_t\left[\beta\left[X_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_{t+1}}{q(\theta_{t+1})} - \lambda(N_{t+1}, x_{t+1})\right]\right]\right]. \quad (15)$$

$V(N_t, x_t)$ and $\lambda(N_t, x_t)$ must also satisfy the Kuhn-Tucker condition (12). We work with the job creation condition because the competitive equilibrium is in general not Pareto optimal.

The standard projection method would approximate $V(N_t, x_t)$ and $\lambda(N_t, x_t)$ directly to solve the job creation condition, while obeying the Kuhn-Tucker condition. However, with the vacancy nonnegativity constraint, these functions are not smooth, causing problems in the approximation with smooth functions. To deal with this issue, we follow Christiano and Fisher (2000) to approximate the conditional expectation in the right-hand side of equation (15) as $\mathcal{E}_t \equiv \mathcal{E}(N_t, x_t)$. A mapping from $\mathcal{E}_t$ to policy and multiplier functions then eliminates the need to parameterize the multiplier function separately. Specifically, after obtaining $\mathcal{E}_t$, we first calculate $\tilde{q}(\theta_t) \equiv \kappa_t/\mathcal{E}_t$. If $\tilde{q}(\theta_t) < 1$, the nonnegativity constraint is not binding, we set $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$, and then solve $\theta_t = q^{-1}(\tilde{q}(\theta_t))$, in which $q^{-1}(\cdot)$ is the inverse function of $q(\cdot)$ from equation (3). We then set $V_t = \theta_t(1 - N_t)$. If $\tilde{q}(\theta_t) \geq 1$, the constraint is binding, we set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_t - \mathcal{E}_t$.

We approximate the log productivity process, x_t, in equation (6) based on the discrete state space method of Rouwenhorst (1995). Kopecky and Suen (2010) show that the Rouwenhorst method is more accurate than other methods in approximating highly persistent autoregressive processes. We use 17 grid points to cover the values of x_t, which are precisely within four unconditional standard deviations from the unconditional mean of zero. For the N_t grid, we set the minimum value to be 0.01 and the maximum 0.99 (N_t never hits one of the bounds in simulations). For each grid point of x_t, we use cubic splines with 45 basis functions on the N_t space to approximate $\mathcal{E}(N_t, x_t)$. We use extensively the approximation toolkit in the Miranda and Fackler (2002) CompEcon Toolbox in MATLAB. To obtain an initial guess, we use the model's loglinear solution.

We implement the loglinear solution (and the second-order perturbation in logs) using the software of Dynare (e.g., Adjemian et al. (2011)). Because this solution method is well known, we do not discuss the details. Instead, we report our Dynare program in Appendix A. Two comments on the implementation are in order. First, in the loglinear solution, we ignore the nonnegativity constraint of vacancy by setting the multiplier, λ_t, to be zero for all t. Doing so is consistent with the common practice in the literature. Second, following Den Haan's (2011) recommendation, we substitute out as many variables as we can, and use only a minimum number of equations in the Dynare program to minimize approximation errors. In particular, we use only three equations (the employment accumulation equation, the job creation condition, and the law of motion for log productivity) with three primitive variables (employment, log productivity, and consumption).

4 Quantitative Results

In this section we quantify how the results based on the projection solution deviate from those based on the (commonly adopted) loglinear solution.

4.1 The Second Moments of the Labor Market

We first report an updated set of second moments in the data as in Shimer (2005, Table 1) and Hagedorn and Manovskii (2008, Table 3). We obtain seasonally adjusted monthly unemployment (thousands of persons 16 years of age and older) from Bureau of Labor Statistics (BLS), and seasonally adjusted help wanted advertising index from the Conference Board. The sample is from January 1951 to June 2006 (666 months). (The Conference Board switched from help wanted advertising index to help wanted online index in June 2006. The two indexes are not directly comparable. We follow the common practice in the literature in using the longer time series before the switch.) We take quarterly averages of the monthly series to obtain quarterly observations. The average labor productivity is seasonally adjusted real average output per job in the nonfarm business sector from BLS.

It is customary to report all variables in log-deviations from the HP-trend with a smoothing parameter of 1,600. In contrast, we detrend all variables as the HP-filtered cyclical component of proportional deviations from the mean with the same smoothing parameter. We do not take logs because vacancies can be zero in simulations from the projection solution when the vacancy nonnegativity constraint is binding. In the data, the two detrending methods yield quantitatively similar results. Table 1 reports the data moments. The standard deviations of unemployment, vacancy, and the labor market tightness are 0.119, 0.134, and 0.255, respectively. Unemployment and vacancy have a correlation of -0.913, indicating a downward-sloping Beveridge curve. These moments are very close to those reported in Hagedorn and Manovskii (2008, Table 3).

To solve and simulate from the model, we use exactly the same parameter values from the weekly calibration in Hagedorn and Manovskii (2008) reported in Table 2. With these

Table 1 : The Second Moments of the Labor Market in the Data

Seasonally adjusted monthly unemployment (U, thousands of persons 16 years of age and older) is from the Bureau of Labor Statistics. The seasonally adjusted help wanted advertising index, V, is from the Conference Board. The sample is from January 1951 to June 2006 (666 months). Both U and V are converted to quarterly averages of monthly series. We define the labor market tightness as $\theta = V/U$. The average labor productivity, X, is (quarterly) seasonally adjusted real average output per job in the nonfarm business sector from the Bureau of Labor Statistics. All variables are in HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600.

	U	V	θ	X
Standard deviation	0.119	0.134	0.255	0.012
Autocorrelation	0.902	0.922	0.889	0.761
Correlation matrix		−0.913	−0.801	−0.224
			0.865	0.388
				0.299

parameter values set, we first reach the model's stationary distribution by simulating the economy for $100 \times 12 \times 4$ weekly periods from the initial condition of zero for log productivity and 0.947 for employment (the deterministic steady state). From the stationary distribution, we repeatedly simulate 5,000 artificial samples, each with 666×4 weekly periods. We take the quarterly averages of the weekly unemployment, vacancy, and labor productivity to obtain 222 quarterly observations, and then calculate the model moments for each sample and report the cross-simulation averages.

Table 3 reports the labor market moments from the model under its projection and loglinear solutions. Panel A is borrowed from Table 4 in Hagedorn and Manovskii (2008). Panel B is our replication of their results using the Dynare codes reported in Appendix A. Although the moments are not identical, Panel B largely replicates the results in Panel A. (Following Hagedorn and Manovskii, we detrend all the variables in Panel B as log-deviations from the HP-trend.) The standard deviation for the labor market tightness is 0.284, which is close

Table 2 : Parameter Values in the Weekly Hagedorn-Manovskii (2008) Calibration

Notation	Parameter	Value
β	Time discount factor	$0.99^{1/12}$
ρ	Aggregate productivity persistence	0.9895
σ	Conditional volatility of productivity shocks	0.0034
η	Workers' bargaining weight	0.052
b	The value of unemployment activities	0.955
s	Job separation rate	0.0081
ι	Elasticity of the matching function	0.407
κ_K	The capital cost of vacancy posting	0.474
κ_W	The labor cost of vacancy posting	0.11
ξ	The exponential parameter in the labor cost	0.449

to 0.292 reported by Hagedorn and Manovskii. The standard deviations for unemployment and vacancy are also close. In addition, the unemployment-vacancy correlation is -0.776, which is close to -0.724 reported in their Table 4.

Because vacancy can be zero (albeit very infrequently) in simulations based on the projection solution, we cannot take logs. As such, we detrend the variables as HP-filtered proportional deviations from the mean. Panel C reports the results using this detrending method with the loglinear solution. Comparing Panels B and C shows that adjusting the detrending method does not (materially) impact the results. The standard deviation of the labor market tightness becomes 0.328, which is slightly higher than 0.284 with the log-deviations. The unemployment volatility is 0.133, which is close to 0.147 with the log-deviations. However, the unemployment-vacancy correlation is -0.848, which is somewhat higher in magnitude than -0.776 with the log-deviations.

Panel D reports the key message of the paper. The quantitative results based on the projection solution diverge from those based on loglinearization in two quantitatively important ways. First, the standard deviation of unemployment is 0.258, which is almost twice as

Table 3 : The Second Moments of the Labor Market in the Model

Panel A is from Hagedorn and Manovskii (2008, Table 4). We simulate 5,000 artificial samples from the loglinear solution (Panels B and C) and the projection solution (Panel D) to the model, with 666×4 weekly observations in each sample. We take the quarterly averages of weekly unemployment U, vacancy, V, and labor productivity, X, to convert to 222 quarterly observations. $\theta = V/U$ denotes labor market tightness. We implement the same procedures as in Table 1 on these simulated (quarterly) series, and report the cross-simulation averages. All the variables in Panel B are in log-deviations from the HP-trend with a smoothing parameter of 1,600. The variables in Panels C and D are in HP-filtered proportional deviations from the mean with the same smoothing parameter.

	U	V	θ	X		U	V	θ	X
	Panel A: Hagedorn-Manovskii					Panel B: Loglinearization (log-deviations)			
Standard deviation	0.145	0.169	0.292	0.013		0.147	0.155	0.284	0.013
Autocorrelation	0.830	0.575	0.751	0.765		0.827	0.674	0.787	0.760
Correlation matrix		−0.724	−0.916	−0.892	U		−0.776	−0.938	−0.901
			0.940	0.904	V			0.945	0.965
				0.967	θ				0.990
	Panel C: Loglinearization					Panel D: Projection			
Standard deviation	0.133	0.144	0.328	0.013		0.258	0.175	0.268	0.013
Autocorrelation	0.831	0.681	0.783	0.760		0.826	0.583	0.759	0.760
Correlation matrix		−0.848	−0.863	−0.927	U		−0.567	−0.662	−0.699
			0.857	0.985	V			0.889	0.907
				0.889	θ				0.997

large as that from loglinearization, 0.133. Second, the unemployment-vacancy correlation is −0.567 in the projection solution, and is substantially lower in magnitude than −0.848 in the loglinear solution. As such, relative to the accurate solution, the loglinear solution understates the unemployment volatility but overstates the unemployment-vacancy correlation.

It is also clear from Panel D that once solved accurately, the Hagedorn-Manovskii calibration fails to explain the Shimer (2005) puzzle. Although the standard deviation of the labor market tightness in the projection solution is 0.268, which is close to 0.255 in the data (see Table 1), the unemployment volatility of 0.258 is more than twice as large as that in the data, 0.119. In addition, although the projection solution implies a downward sloping Beveridge curve, the unemployment-vacancy correlation of -0.567 is substantially smaller in magnitude than -0.913 in the data.[4]

4.2 Nonlinear Unemployment Dynamics

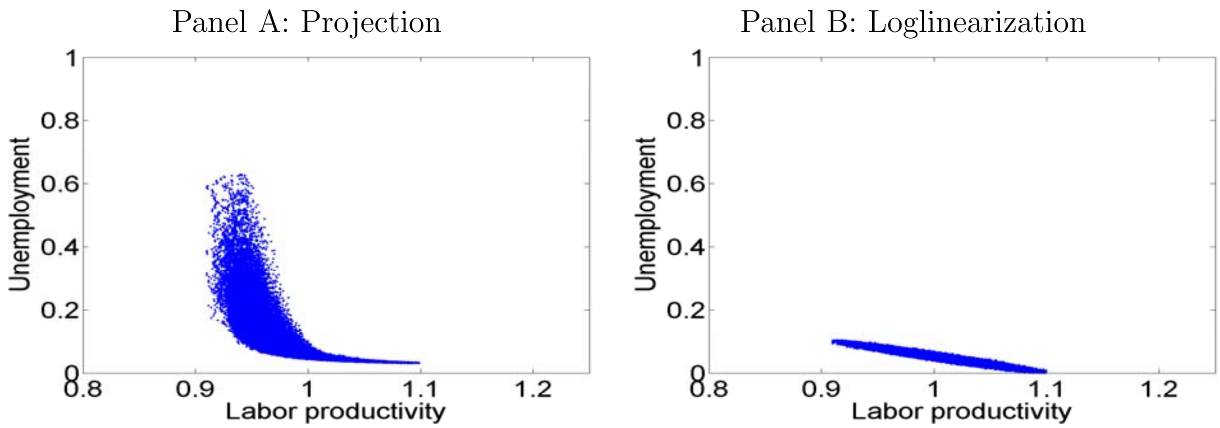
Why do the loglinearization results differ greatly from the projection results? The crux is that, given realistic fluctuations in labor market tightness, the unemployment dynamics in the DMP model are highly nonlinear. In recessions unemployment rises rapidly, whereas in booms unemployment declines only gradually. The distribution of unemployment is highly skewed with a long right tail. The projection solution fully captures the nonlinear dynamics of unemployment. In contrast, by focusing only on local dynamics around the deterministic steady state, loglinearization ignores the large unemployment dynamics in recessions, thereby understating its volatility but overstating its correlation with vacancies.

Figure 1 plots unemployment against labor productivity using one million weekly periods simulated from the model's stationary distribution. From Panel A, the unemployment-productivity relation is highly nonlinear. When the labor productivity is above its mean, unemployment falls gradually, whereas when the productivity is below its mean, unemployment rises drastically. The unemployment rate can reach above 60% in simulations. The correlation between unemployment and labor productivity is -0.699. From Panel B, not surprisingly, the loglinear solution misses entirely the nonlinear dynamics. The unemployment-

[4]It might be possible that under the projection algorithm, the Hagedorn-Manovskii (2008) calibration strategy can lead to parameter values different from $b = 0.955$ and $\eta = 0.052$. Hagedorn and Manovskii (p. 1699) state that the values for b and η are chosen "to match the data on the average value for labor market tightness $\theta = 0.634$ and the elasticity of wages with respect to productivity" of 0.449. In simulations from the projection algorithm, the average labor market tightness is 0.654, and the elasticity of wages with respect to productivity is 0.477. As such, their calibration strategy gives rise to parameter values that are relatively close to $b = 0.955$ and $\eta = 0.052$ under the globally accurate solution.

Figure 1 : The Unemployment-labor Productivity Relation

From the model's stationary distribution under the projection solution or loglinearization, we simulate one million weekly periods, and plot unemployment against labor productivity.



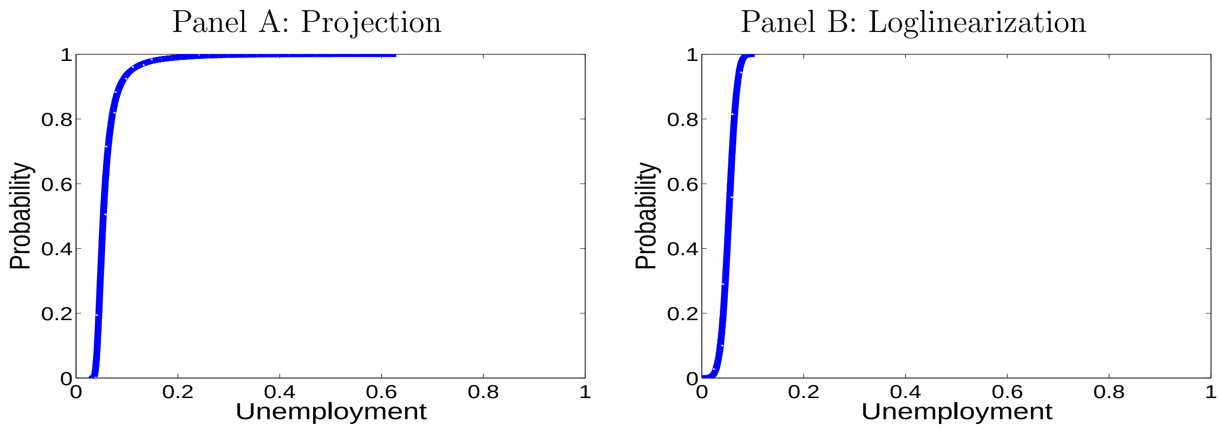
productivity correlation in simulations is nearly perfect, −0.96. The maximum unemployment rate in simulations is only 10.38%.

Figure 2 further illustrates the nonlinear dynamics in the Hagedorn-Manovskii calibration. We plot the empirical cumulative distribution function for unemployment based on simulations of one million weekly periods from the model's stationary distribution. Panel A reports the results from the projection solution. Unemployment is positively skewed with a long right tail. The mean unemployment rate is 6.17%, the median is 5.35%, and the skewness is 5.43. The 2.5 percentile, 3.81%, is close to the median, whereas the 97.5 percentile is far away, 13.94%. The 1 and 99 percentiles are 3.66% and 19.83%, respectively. As noted, the unemployment rate can reach above 60%.

From Panel B, the nonlinear dynamics are missed by the loglinear solution. The mean unemployment rate is 5.28%, which is close to the median of 5.29%. The skewness is almost zero, −0.05. The 2.5 and 97.5 percentiles, 2.72% and 7.78%, respectively, are distributed symmetrically around the median. Incidentally, the stochastic mean of the unemployment

Figure 2 : Empirical Cumulative Distribution Functions of Unemployment

Results are based on one million weeks simulated from the model's stationary distributions under the projection solution and the loglinear solution.



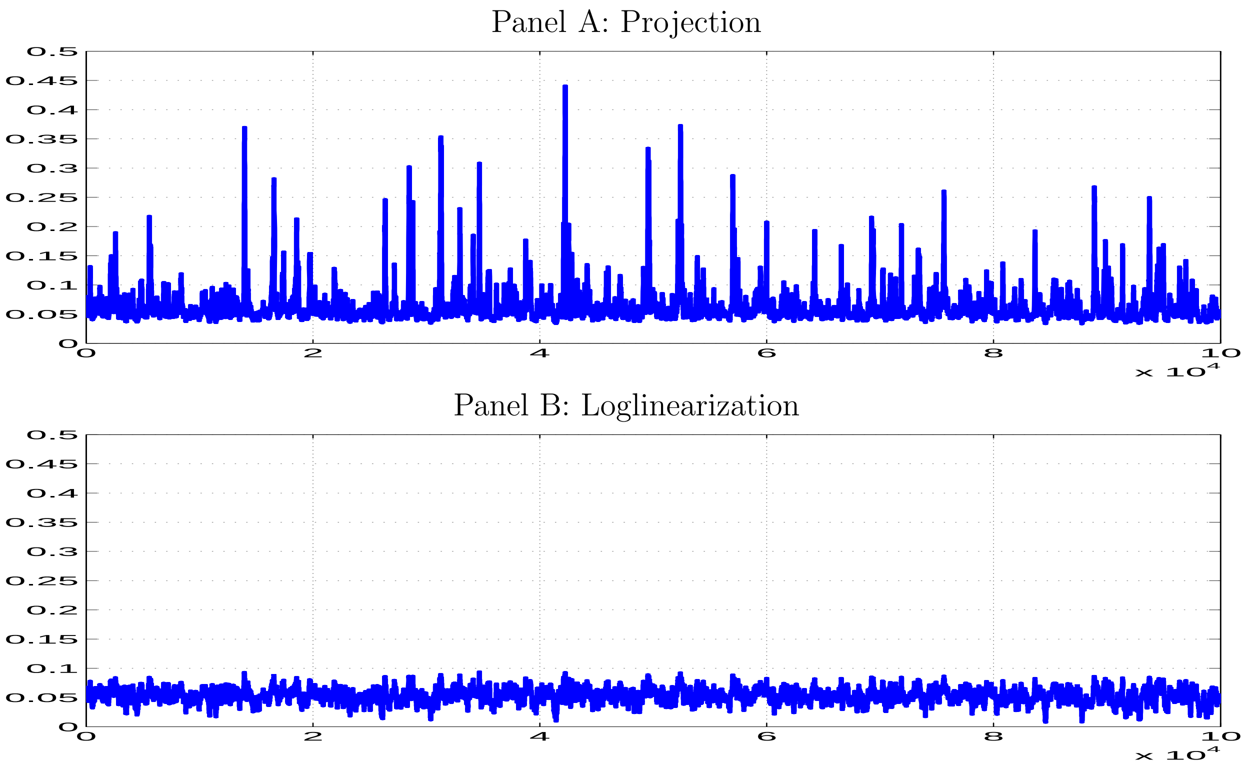
rate using the projection solution, 6.17%, is almost one percentage point higher than its deterministic steady state rate, 5.28%. This result casts doubt on the validity of calibration that relies exclusively on steady state relations.

In Figure 3, we contrast a simulated sample path of unemployment from the projection solution with that from the loglinear solution. The labor productivity process underlying the two sample paths is identical. As such, the only difference is the solution method. From Panel A, the unemployment rate from the projection solution spikes up frequently to levels above 10%. In contrast, no such spikes are visible in Panel B, which is based on loglinearization.

The nonlinear dynamics also appear in impulse response functions. In particular, the magnitude of the impulse responses depends on the initial point of simulations. We calculate the impulse responses from three different starting points: bad, median, and good economies. The bad economy is the 5 percentile of the model's bivariate stationary distribution of employment and log productivity, the median economy is the median, and the good economy is the 95 percentile. We compute the responses to a one-standard-deviation shock

Figure 3 : Simulated Sample Paths of the Unemployment Rate with a Fixed Shock Process

This figure plots two unemployment series (100,000 weeks), one for each solution. The underlying labor productivity process is identical across the two series.

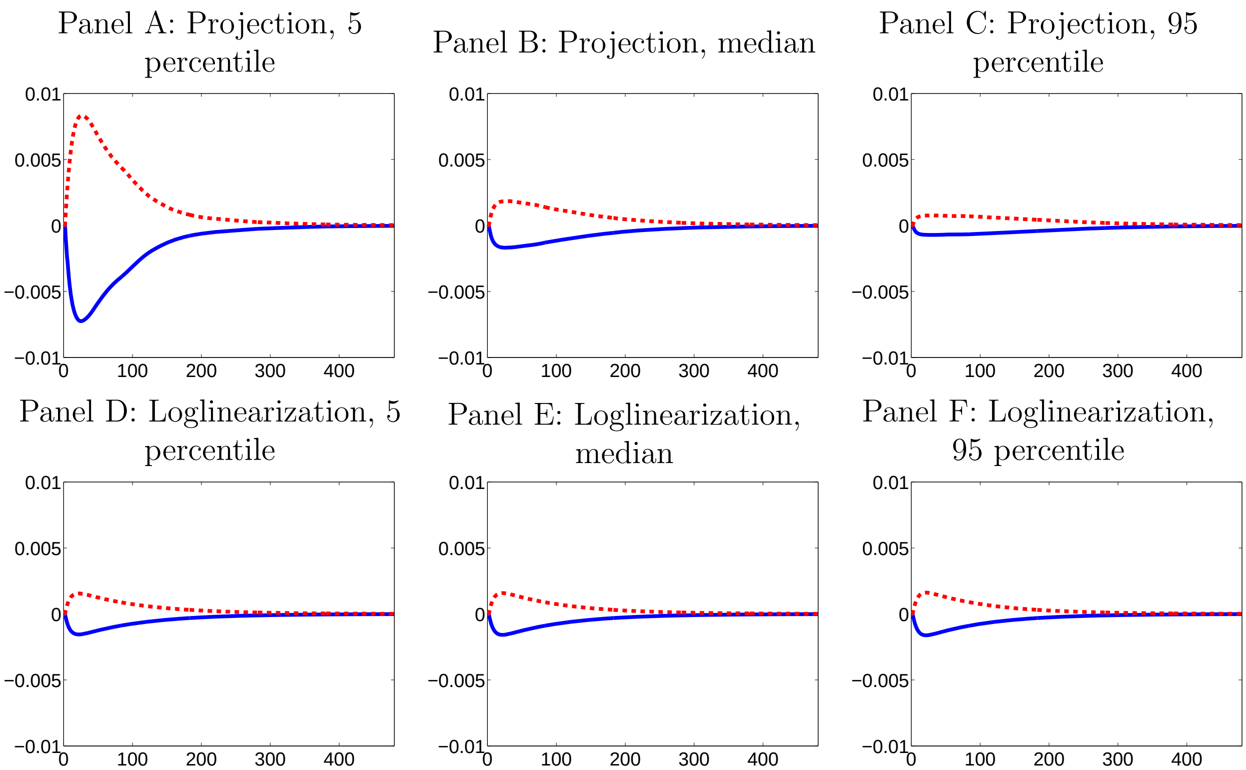


to log productivity, both positive and negative.

Figure 4 reports impulse response functions of unemployment. From Panels A to C, the responses from the projection solution are clearly stronger in recessions than those in booms. In response to a one-standard-deviation negative shock to log productivity, the unemployment rate shoots up by 0.85% if the economy starts at the 5 percentile of the bivariate distribution of employment and productivity. In contrast, the response is only 0.03% if the economy instead starts at the 95 percentile. Panels D to F show that such asymmetry is absent under loglinearization. The response in unemployment to a negative shock is around 0.125%, regardless of the initial condition.

Figure 4 : Nonlinear Impulse Response Functions for The Unemployment Rate

Impulse response functions are from three different initial points: the 5, 50, and 95 percentile of the model's bivariate distribution of employment and log productivity. The responses are averaged across 1,000 simulations, each with 480 weeks. The blue solid (red broken) lines are the responses to a positive (negative) one-standard-deviation shock to log productivity.

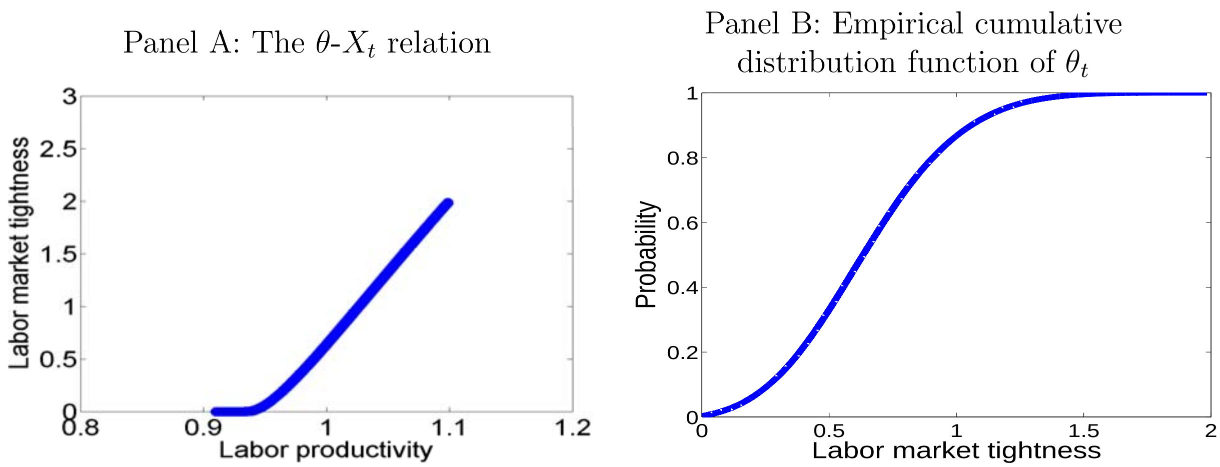


Intuition

The nonlinear dynamics are driven by a combination of matching frictions and small and volatile firm profits due to high and inelastic wages. (The high and inelastic wages are the key in the calibration proposed by Hagedorn and Manovskii (2008) to achieve large responses in the labor market tightness to small changes in labor productivity.) In particular, the matching frictions induce a congestion externality in the labor market. In booms, many vacancies compete for a small pool of unemployed workers. An extra vacancy can cause a large drop in the vacancy filling rate, raising the marginal costs of hiring. As such, hiring

Figure 5 : The Labor Market Tightness-Productivity Relation and Empirical Cumulative Distribution Function of Labor Market Tightness

From the model's stationary distributions under the projection solution, we simulate one million weekly periods, and plot labor market tightness, θ_t, against labor productivity, X_t, as well as the empirical cumulative distribution function of θ_t.



increases only gradually. In recessions, many unemployed workers compete for a small pool of vacancies. An extra vacancy is quickly filled, and hardly reduces the vacancy filling rate. As such, the marginal costs of hiring hardly decline in recessions (downward rigidity), and a given decline in vacancies leads to a disproportionately large drop in hiring.

In response to a negative productivity shock, profits drop disproportionately more. Also, the downward rigidity of the marginal costs of hiring exacerbates the impact of falling profits on the incentives of hiring, stifling job creation flows. Consequently, as job openings become scarce, unemployment spikes up, giving rise to large unemployment dynamics in recessions. In contrast, in response to a positive shock, the congestion externality causes the marginal costs of hiring to increase rapidly. The increasing marginal costs of hiring mitigate the impact of rising profits on the incentives of hiring, slowing down job creation flows. As such, unemployment declines only gradually in booms.

In Table 3, the volatility of labor market tightness, θ_t, from loglinearization is not far from that implied by the projection algorithm. The intuition is that $\theta_t = V_t/U_t$ exhibits nonlinear dynamics that are not nearly as strong as those for unemployment. Panel A of Figure 5 plots θ_t against productivity in simulations. The principal source of nonlinearity is the nonnegativity constraint on vacancies. When the constraint is binding when productivity is very low, θ_t is bounded below from zero (because V_t is bounded below from zero). When the constraint is not binding, θ_t is largely linear in productivity. Panel B shows further that the empirical distribution of θ_t is mostly symmetric, in contrast to the long right tail in unemployment (see Panel A of Figure 2).

4.3 The Impact of Persistence in Labor Productivity

A higher persistence of the log productivity gives rise to even larger quantitative differences between loglinearization and projection. This result holds even when the unconditional standard deviation of productivity is lower. In particular, we recalibrate the model by setting the persistence of log productivity to 0.9957 and the conditional volatility to 0.0021. These weekly values correspond to the quarterly values of 0.95 and 0.007, respectively, which are typically used in business cycle studies (e.g., Cooley and Prescott (1995)). From Table 4, the unconditional volatility of labor productivity under the high persistence calibration, 0.0085, is lower than that under the Hagedorn-Manovskii calibration, 0.0131.

Despite the lower productivity volatility, the quantitative differences between loglinearization and projection are larger when labor productivity is more persistent. The standard deviation of unemployment from projection, 0.291, is almost three times as large as that from loglinearization, 0.108. The unemployment-vacancy correlation is -0.496 in the projection solution, in contrast to -0.861 in the loglinear solution. While the standard deviation of

Table 4 : Labor Market Moments, High Persistence in Labor Productivity

We simulate 1,000 samples from the stationary distribution of with the high persistence calibration, in which $\rho = 0.9957$ and $\sigma = 0.0021$, with 666×4 weekly observations in each sample. We take the quarterly averages of weekly unemployment, U, vacancy, V, and labor productivity, X, to convert to 222 quarterly observations. We implement the same procedures as in Table 1, and report the cross-simulation averages for all the moments. All the variables are in HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600.

	U	V	θ	X		U	V	θ	X
	Panel A: Loglinearization					Panel B: Projection			
Standard deviation	0.108	0.115	0.291	0.009		0.291	0.158	0.216	0.009
Autocorrelation	0.848	0.712	0.807	0.785		0.857	0.613	0.783	0.784
Correlation matrix		−0.861	−0.818	−0.934	U		−0.496	−0.542	−0.593
			0.817	0.986	V			0.841	0.873
				0.842	θ				0.992

labor market tightness from loglinearization is (relatively) close to that from projection with the Hagedorn-Manovskii calibration, 0.328 versus 0.268 (see Table 3), the gap widens with the high persistence, 0.291 versus 0.216.

Intuitively, job creation is a forward looking decision in which the persistence of labor productivity plays a determining role. A low realization reduces job creation more if it is expected to last a longer time. As a result, as illustrated in Figure 6, the nonlinear dynamics for the Hagedorn-Manovskii model are more severe under the high persistence calibration. In particular, the unemployment rate can reach above 90%, which is substantially higher than 60% under their original calibration. The unemployment-productivity correlation in the simulated data is -0.614, which is lower in magnitude than -0.699 under the original calibration. Because higher persistence induces stronger nonlinear dynamics, loglinearization deviates further from the projection solution.

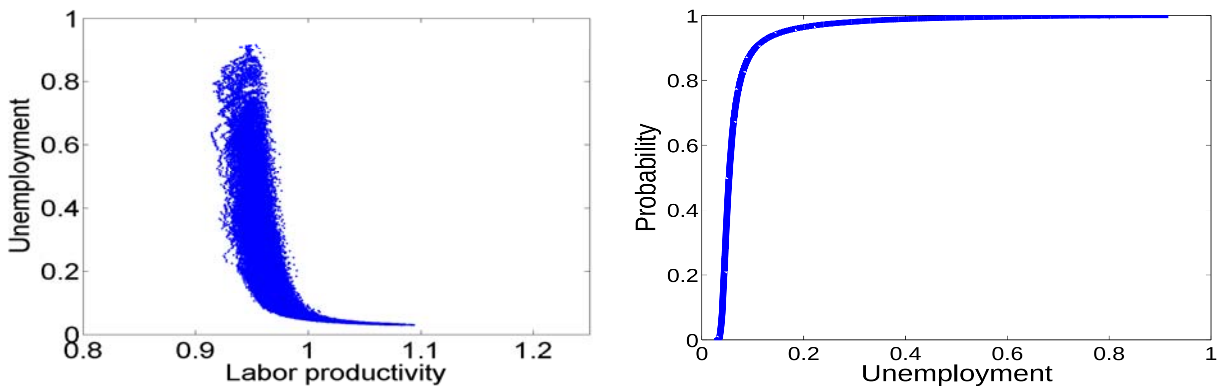
Figure 7 shows further why loglinearization should not be used to solve the search model

Figure 6 : The Unemployment-labor Productivity Relation and Empirical Cumulative Distribution Function of Unemployment, High Persistence in Labor Productivity

Results are based on one million weeks simulated from the projection solution to the Hagedorn-Manovskii model with the high persistence calibration for labor productivity.

Panel A: The unemployment-labor productivity relation

Panel B: Empirical cumulative distribution function of unemployment

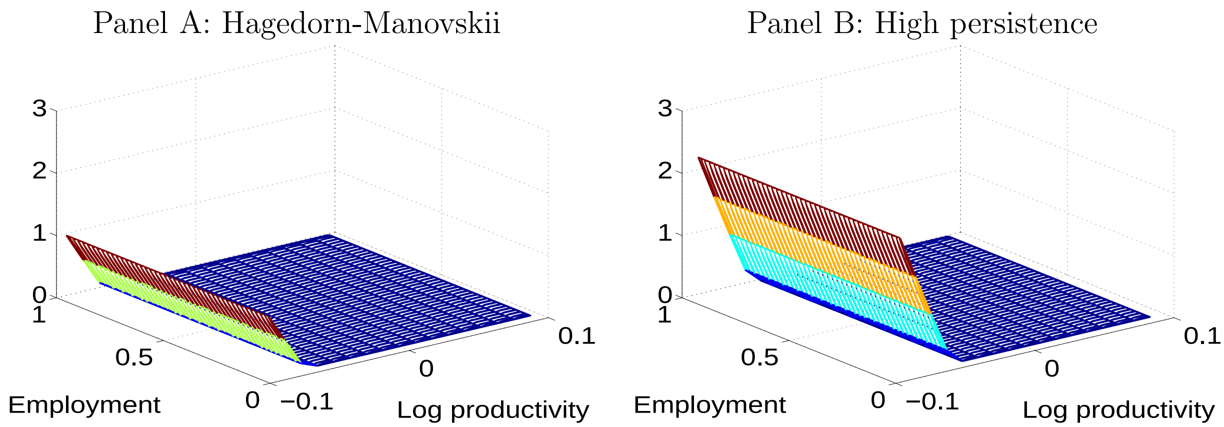


with small profits, especially when productivity is calibrated as in business cycle studies. In general, perturbation methods cannot handle the nonnegativity constraint of vacancies. However, the constraint is binding more frequently when the persistence of labor productivity increases. Figure 7 plots the Lagrangian multiplier for the constraint on the state space. From Panel A, with the original Hagedorn-Manovskii calibration, the multiplier is positive when productivity is low. In simulations, the frequency of the constraint binding is 0.176%. Panel B shows that the high persistence calibration increases the multiplier. In simulations, the frequency of the constraint binding is 1.024%.

4.4 Second-order Perturbation in Logs

Loglinearization is the first-order perturbation solution in logs. Can the second-order perturbation in logs eliminate the quantitative differences between projection and loglinearization? The answer is not affirmative. The second-order perturbation in logs continues to induce

Figure 7 : The Lagrangian Multiplier for the Vacancy Nonnegativity Constraint, the Hagedorn-Manovskii Model



large approximation errors. The second moments of the labor market continue to deviate significantly from those based on the projection solution.

Panel A of Table 5 shows that although improving on loglinearization, the labor market moments from the second-order perturbation still deviate significantly from those based on the projection solution. Relative to loglinearization, the unemployment volatility from the second-order perturbation increases slightly from 0.133 to 0.165. However, the 0.165 figure still deviates significantly from 0.258 from the projection solution. Similarly, the unemployment-vacancy correlation drops in magnitude from −0.848 to −0.789, once we move from loglinearization to the second-order perturbation. However, the −0.789 figure still deviates significantly from −0.567 from the projection solution. Panel B shows that the deviation is further exacerbated with the high persistence calibration.

Figure 8 shows that unlike loglinearization, the second-order perturbation captures some nonlinear unemployment dynamics. However, the nonlinearity is not nearly as strong as that captured by the projection solution. These results explain why, although improving on log-

Table 5 : Second Moments of the Labor Market, Second-order Perturbation in Logs

We simulate 1,000 samples from the second-order perturbation (in logs) solution to the Hagedorn-Manovskii model, with 666×4 weekly observations in each sample. Panel A uses the original Hagedorn-Manovskii calibration, in which $\rho = 0.9895$ and $\sigma = 0.0034$, and Panel B uses the high persistence calibration, in which $\rho = 0.9957$ and $\sigma = 0.0021$. We take the quarterly averages of weekly unemployment, U, vacancy, V, and labor productivity, X, to convert to quarterly observations. We implement the same procedures as in Table 1, and report the cross-simulation averages for all the moments. All the variables are in HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600.

	U	V	θ	X		U	V	θ	X
	Panel A: Hagedorn-Manovskii					Panel B: High persistence			
Standard deviation	0.165	0.178	0.263	0.013		0.147	0.155	0.212	0.009
Autocorrelation	0.832	0.704	0.788	0.760		0.856	0.741	0.808	0.785
Correlation matrix		−0.789	−0.790	−0.792	U		−0.755	−0.714	−0.742
			0.945	0.973	V			0.933	0.961
				0.993	θ				0.989

linearization, the second-order perturbation produces second moments of the labor market that are closer to those from loglinearization than those from the projection solution.

5 Accuracy Tests

Finally, we report a battery of accuracy tests to show that projection provides an extremely accurate approximation, whereas perturbations admit very large errors.

To evaluate the accuracy of a given algorithm, we follow Den Haan (2010) in calculating Euler equation errors, both static and dynamic.[5] The static Euler equation error is defined as:

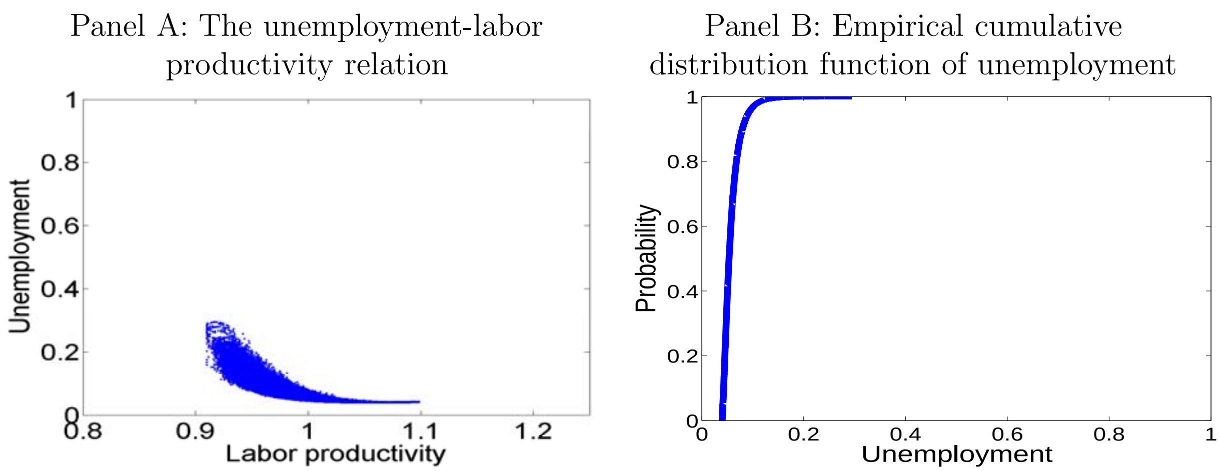
$$e_t^s \equiv E_t\left[\beta\left[X_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_{t+1}}{q(\theta_{t+1})} - \lambda_{t+1}\right]\right]\right] - \left(\frac{\kappa_t}{q(\theta_t)} - \lambda_t\right). \tag{16}$$

If a solution method is accurate, e_t^s should be close to zero at all points in the state space. We calculate the static Euler equation error on the employment-log productivity grid. To

[5]We thank Wouter Den Haan for recommending these accuracy tests in a private conversation.

Figure 8 : The Unemployment-labor Productivity Relation and Empirical Cumulative Distribution Function of Unemployment, Second-order Perturbation in Logs

Results are based on simulations of one million weekly periods from the Hagedorn-Manovskii model under the second-order perturbation solution in logs.



compute the conditional expectation in equation (16), we discretize x_t via the Rouwenhorst (1995) method, and calculate the conditional expectation via matrix multiplication.[6]

To calculate interpretable Euler equation errors per Den Haan (2010), we compute two consumption values on each grid point. The first consumption value, C_t, is computed using a given numerical solution method. The second consumption value, $C_t^\star$, is the implied value from an accurately calculated conditional expectation, which is the right-hand side of the

[6] We have experimented with Gauss-Hermite quadrature in the continuous state space of x_t when calculating the conditional expectation. However, when x_t is extreme, implementing this method involves large extrapolation errors in the policy functions. For example, suppose we want to calculate the expectation conditional on the minimum value of the x_t grid, which is precisely four unconditional standard deviations below the mean of zero. Applying the Gauss-Hermite quadrature requires us to put another grid of x_t around this extreme x_t value, which serves as the mean of the second grid. As such, the lower half of the second grid lies outside the original x_t grid, on which we solve the policy functions. Extrapolating the policy functions outside the original x_t grid induces errors that can be large. However, these errors are (virtually) irrelevant for the quantitative results in simulations because the original grid is large enough to cover x_t values within four unconditional standard deviations from the unconditional mean.

Euler equation. We then calculate the interpretable Euler equation error as:

$$e^s_{Ct} \equiv \left| \frac{C_t - C^\star_t}{(C_t + C^\star_t)/2} \right|. \tag{17}$$

We scale e^s_{Ct} with $(C_t+C^\star_t)/2$, instead of $C^\star_t$, because $C^\star_t$ can be small, leading e^s_{Ct} to explode.

Figure 9 reports Euler equation errors. From Panels A and B, the projection algorithm offers an extremely accurate solution to the model. In particular, e^s_t is in the magnitude of 10^{-13}, and e^s_{Ct} is in the magnitude of 10^{-11}. In contrast, Panels C and D show that the loglinear solution admits very large approximation errors. From Panel C, e^s_t is in the magnitude of 10%. From Panel D, even when employment is around the deterministic steady state, e^s_{Ct} is in the magnitude of 10%. When the economy moves away from the steady state, e^s_{Ct} increases further. When employment is very low, the errors can be more than 100%.

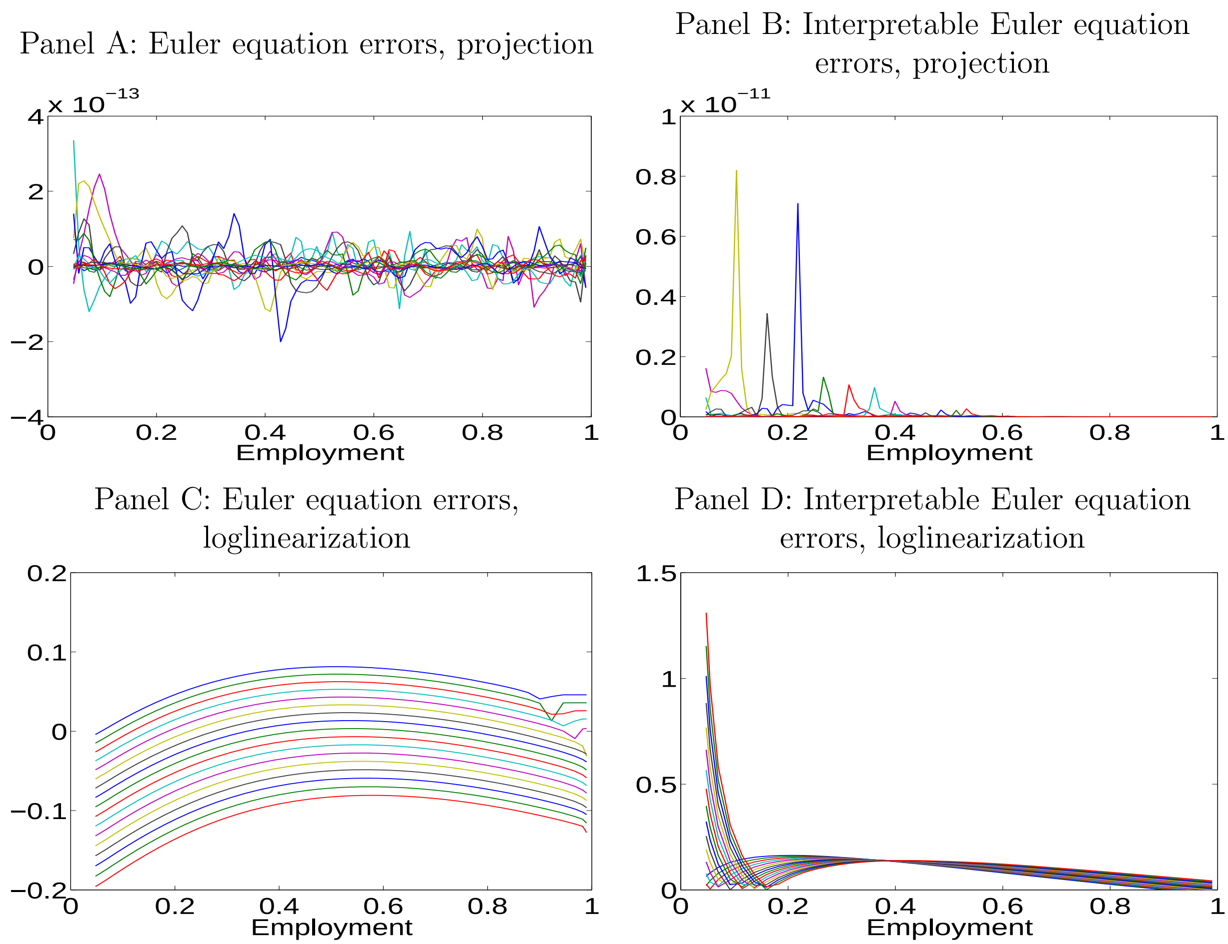
The static and the interpretable Euler equation errors are only one-period ahead forecast errors, which ignore the accumulation of small errors. To allow for this possibility, we follow Den Haan (2010) to calculate dynamic Euler equation errors. We simulate two series for endogenous variables C_t and N_t. First, we use a given numerical solution in simulations. Second, we simulate alternative series using the following steps in each period: (i) Use a given numerical solution to calculate the conditional expectation accurately; (ii) use this conditional expectation to calculate implied consumption value, $C^\star_t$; and (iii) compute employment, $N^\star_t$, from this implied consumption value and the aggregate resource constraint. The dynamic Euler equation errors are defined as:

$$e^d_{Ct} \equiv \frac{C_t - C^\star_t}{(C_t + C^\star_t)/2} \qquad \text{and} \qquad e^d_{Nt} \equiv \frac{N_t - N^\star_t}{(N_t + N^\star_t)/2}. \tag{18}$$

Figure 10 shows that the difference in accuracy between projection and loglinearization is

Figure 9 : Euler Equation Errors, Projection versus Loglinearization

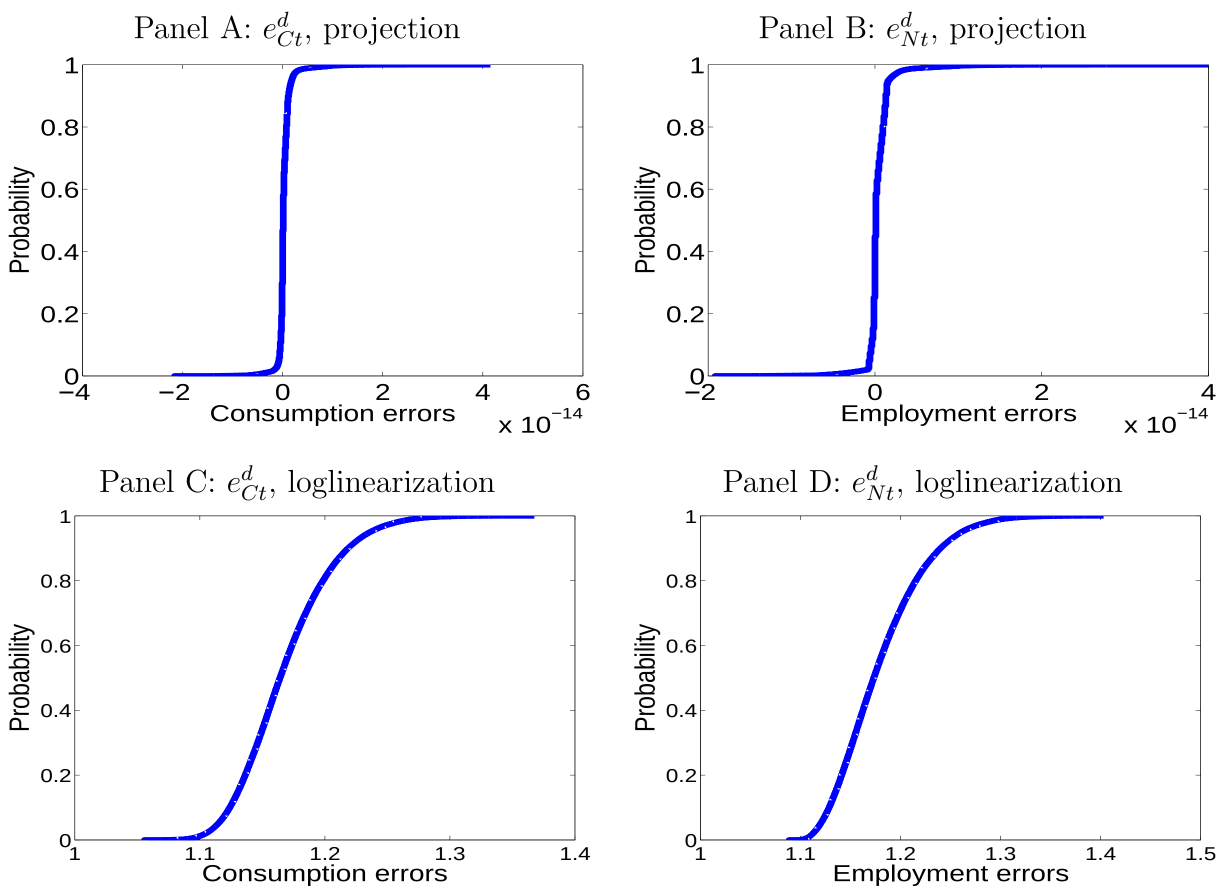
Euler equation errors are defined in equation (16) and interpretable errors in equation (17). Each line in a panel corresponds to one out of 17 values of log productivity on the grid.



even starker in dynamic Euler equation errors. We simulate one million weekly periods from the model's stationary distribution under a given solution, and report the empirical cumulative distribution functions of e^d_{Ct} and e^d_{Nt}. From Panels A and B, the dynamic errors from the projection solution are extremely small (in the magnitude of 10^{-14}). In contrast, Panels C and D report very large dynamic errors for the loglinear solution. For the consumption errors, the mean is 116.83%, and the median is 116.41%. For the employment errors, the mean is 117.98%, and the median is 117.34%.

Figure 10 : Dynamic Euler Equation Errors, Projection versus Loglinearization

Dynamic Euler equation errors, e^d_{Ct} and e^d_{Nt}, are defined in equation (18). We simulate one million weeks from the model's stationary distribution, and then plot the empirical cumulative distribution functions for e^d_{Ct} and e^d_{Nt}.



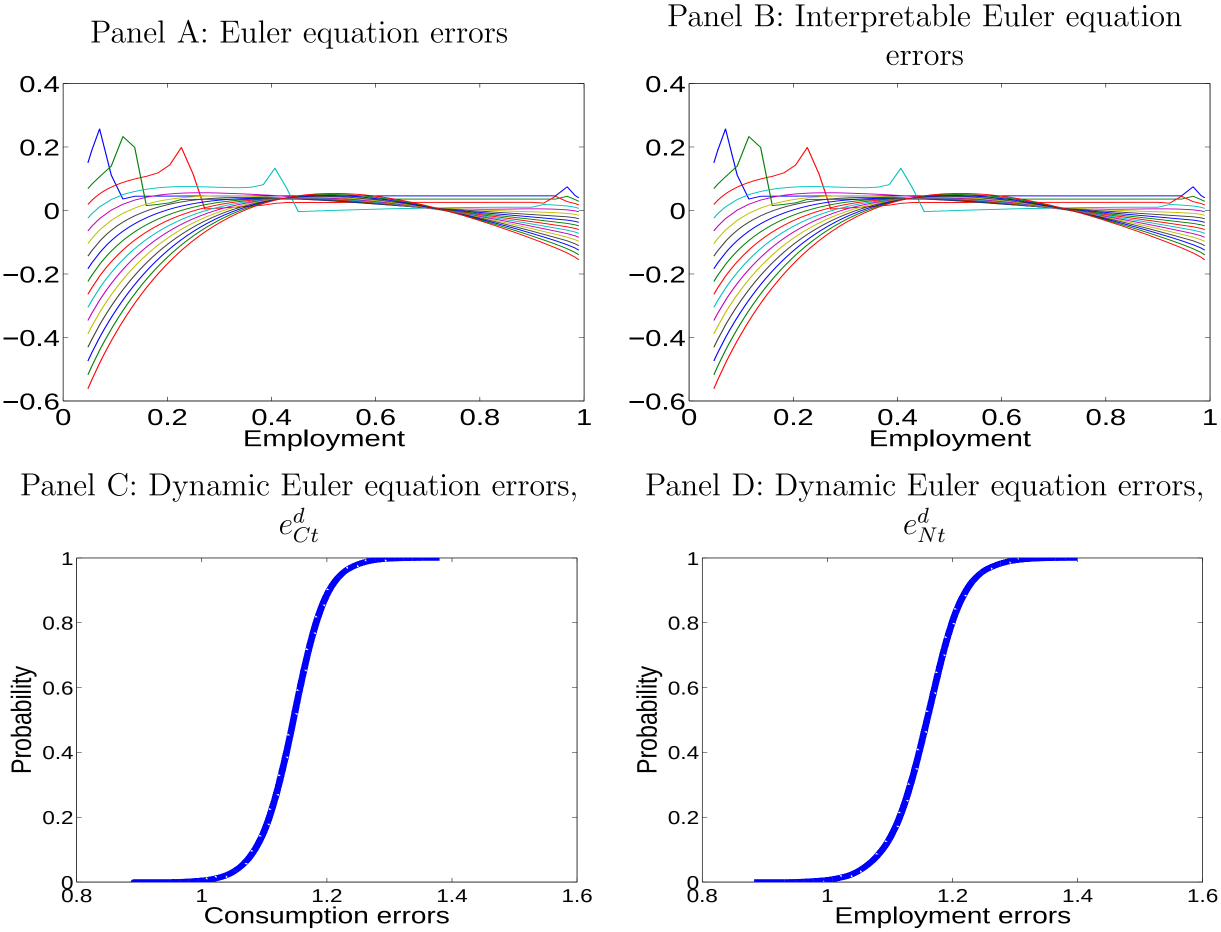
Finally, Figure 11 shows that the second-order perturbation in logs continues to be inaccurate. The accuracy seems to even deteriorate relative to loglinearization in some cases. From Panels A and B, when employment drops below 0.20, the second-order perturbation is very inaccurate, with Euler equation errors above 20%. From Panels C and D, the dynamic Euler equation errors are large. For the consumption errors, the mean is 114.54%, and the median is 114.64%. For the employment errors, the mean is 115.72%, and the median is 115.92%.

6 Conclusion

An accurate global approximation algorithm is critical for quantifying the dynamics of the Diamond-Mortensen-Pissarides search model of unemployment. In the context of the Hagedorn-Manovskii (2008) calibration, we show that the standard loglinear solution understates the volatility of unemployment (by about 50%), overstates the unemployment-vacancy correlation, and ignores impulse responses that are an order of magnitude larger in recessions than in booms. Although improving on loglinearization, the second-order perturbation in logs continues to induce large errors. Our results have broad implications for the macro labor literature as well as macroeconomic studies that are built on the DMP model.

Figure 11 : Approximation Errors, Second-order Perturbation in Logs

Euler equation errors are defined in equation (16) and interpretable errors are in equation (17). Each line in Panels A and B corresponds to one out of 17 values of log productivity on the grid. Dynamic Euler equation errors, e^d_{Ct} and e^d_{Nt}, are defined in equation (18). For a given solution algorithm, we simulate one million weeks from the model's stationary distribution, and then plot the empirical cumulative distribution functions for e^d_{Ct} and e^d_{Nt}.



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A The Dynare Code for Solving the DMP Model with Lower-order Perturbations

```
// HM_loglinear.mod: the log-linear solution to the Hagedorn-Manovskii model

var N, x, C;
predetermined_variables N;
varexo e;

parameters bet, rhox, stdx, eta, b, s, iota, kappa_K, kappa_W, xi;
bet     = 0.99^(1/12);
rhox    = 0.9895;
stdx    = 0.0034;
eta     = 0.052;
b       = 0.955;
s       = 0.0081;
iota    = 0.407;
kappa_K = 0.474;
kappa_W = 0.11;
xi      = 0.449;

model;
# kappa_t = kappa_K*exp(x) + kappa_W*(exp(x)^xi);
# V       = (exp(x)*exp(N) - exp(C))/kappa_t;
# theta   = V/(1 - exp(N));
# q       = (1 + theta^iota)^(-1/iota);
# kappa_p = kappa_K*exp(x(+1)) + kappa_W*(exp(x(+1))^xi);
# V_p     = ( exp(x(+1))*exp(N(+1)) - exp(C(+1)) )/kappa_p;
# theta_p = V_p/(1 - exp(N(+1)));
# q_p     = (1 + theta_p^iota)^(-1/iota);
```

```
# W_p     = eta*(exp(x(+1)) + kappa_p*theta_p) + (1 - eta)*b;

exp(N(+1)) = (1 - s)*exp(N) + q*V;
kappa_t/q  = bet*( exp(x(+1)) - W_p + (1 - s)*kappa_p/q_p );
x          = rhox*x(-1) + e;
end;

initval;
N = log(1 - 0.1);
x = 0;
e = 0;
end;

steady;
check;

shocks;
var e = stdx^2;
end;

stoch_simul (order = 1, nocorr, nomoments, IRF = 0);
```

For the second-order perturbation solution in logs, we change the last command to:

```
stoch_simul (order = 2, nocorr, nomoments, IRF = 0);
```