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ABSTRACT

We develop a model in which, in order to provide managerial incentives, it is optimal to have costly bankruptcy. If benevolent governments can commit to their policies, it is optimal not to interfere with private contracts. Such policies are time inconsistent in the sense that, without commitment, governments have incentives to bail out firms by buying up the debt of distressed firms and renegotiating their contracts with managers. From an ex ante perspective, however, such bailouts are costly because they worsen incentives and thereby reduce welfare. We show that regulation in the form of limits on the debt-to-value ratio of firms mitigates the time-inconsistency problem by eliminating the incentives of governments to undertake bailouts. In terms of the cyclical properties of regulation, we show that regulation should be tightest in aggregate states in which resources lost to bankruptcy in the equilibrium without a government are largest.

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Recent experience has shown that governments can and will intervene during financial crises. During such crises, many firms are faced with the prospect of costly bankruptcy and liquidation. In order to minimize these costs, governments intervene and bail out debt holders. Anticipations of such interventions, however, alter the incentives for firms and financial intermediaries ex ante and by doing so reduce ex ante welfare. If the costs of these altered incentives exceed the benefits of intervention during crises, governments would refrain from bailouts if they had the power to commit themselves. In practice, however, governments do not seem to have this power. The lack of commitment creates a time-inconsistency problem in that the outcomes are worse without commitment than they are with commitment. Here, we ask how optimal regulation should be designed to mitigate this problem.

To answer this question, we develop an infinitely repeated model that highlights the time-inconsistency problem. In our model, debt contracts between firms and investors are optimal. Such contracts lead some distressed firms to declare bankruptcy when their productivity is sufficiently low. The desire to avoid the associated costs of bankruptcy provides incentives for a benevolent government to bail out distressed firms. Anticipation of bailouts leads managers and investors to design contracts under which the managers exert inefficiently low levels of effort. We show that ex ante regulation in the form of limits on the debt-to-value ratio of firms mitigates the time-inconsistency problem by eliminating the incentives of a government to bail out distressed firms. The regulation, we also show, should vary over the business cycle.

We begin by analyzing a one-period model without a government. The technology in the model is as follows. Firms produce output using the effort of managers and the funds of investors. The productivity of the firm has two idiosyncratic stochastic components: a public one and a private one. The public component, referred to as the *health* status of the firm, is the average level of productivity, which can be either high or low, indicating whether a firm is *healthy* or *distressed*. Higher effort by the managers increases the likelihood that the firm is healthy, that is, that average productivity is high. The private component is the firm's level of productivity relative to the average.

In the one-period model, managers and investors design optimal contracts intended to induce effort and share output in the face of three key frictions. First, the effort of the manager is privately observed by the manager. Second, although the public component of productivity is costlessly observed by both the manager and the investors, only the manager costlessly observes the private component of productivity. Investors can observe the private component only by putting the firm through bankruptcy. We assume that bankruptcy is costly in that it reduces the productivity of firms proportionately.[1] (This feature implies that our model has costly state verification, as in Townsend (1979), except that we have proportional rather than fixed costs of verification.) Third, the manager and the investors cannot commit to the terms of their contracts; that is, if the manager and investors agree to renegotiate, they can renegotiate the terms of a contract after the manager chooses effort and the public component of productivity has been realized. This lack of commitment implies that any contract must be immune to renegotiation.

We show that costly bankruptcy, together with lack of commitment to the contract, implies that the optimal contracts between firms and investors are *debt* contracts. These contracts specify a fixed payment to investors contingent on the public component as long as the firm can meet the fixed payment and bankruptcy otherwise. In the event of bankruptcy, the investors receive all of the proportionately reduced output of the firm. The optimal contracting problem is, thus, to find the debt contracts with payoffs contingent on the public component, namely, the health of the firms, which provide the best incentives for the managers to exert effort. The resulting competitive equilibrium is *efficient* in that a planner, confronted with the same information and renegotiation frictions, would choose the same outcomes.

We introduce a benevolent government, in the form of a *bailout authority*, into the one-period model in order to investigate the incentives of the government to bail out distressed firms and show that lack of commitment by this authority leads to a time-inconsistency problem. If this authority could commit to its policies, it would not intervene because the competitive equilibrium is efficient.

Without such commitment, the bailout authority's policies are very different. Consider an ex post situation in which many distressed firms are about to undergo costly bankruptcies.

[1] We think of this output reduction as arising from a variety of sources, including replacing incumbent managers with new managers with fewer firm-specific skills. Alternatively, the reduction in output could arise because bankruptcy leads specialized forms of capital to be sold for less suitable uses.

The bailout authority has a strong incentive to bail out such distressed firms. We think of such bailouts as effectively a voluntary renegotiation among managers, investors, and the authority. This renegotiation must respect the same kinds of constraints as the private renegotiations, with one important difference. Unlike private agents, the bailout authority can levy taxes on healthy firms and use these resources to make payments to distressed firms in order to induce them to renegotiate their contracts. The availability of tax revenues from healthy firms effectively relaxes the participation constraints relative to those in private renegotiations.

The relaxation of participation constraints implies that the bailout authority has stronger incentives to renegotiate the contracts of distressed firms than do private agents. These incentives to renegotiate contracts can induce the bailout authority to levy taxes on healthy firms and use the revenues to bail out distressed firms. Indeed, since intervention has no ex post costs in the one-period model, the bailout authority bails out all distressed firms and cancels all bankruptcies. Anticipation of such policies reduces the incentives of managers to exert effort and leads to a time-inconsistency problem in that outcome incentives without commitment are inefficient, whereas those with commitment are efficient.

In the one-period model, since there are no ex post costs to intervention, all debt holders are bailed out and no bankruptcies occur in equilibrium. In practice, we observe partial but not complete bailouts. In order to generate such outcomes, we extend the model to an infinitely repeated version of the one-period model. In this extension, reputational considerations can impose ex post costs on intervention by affecting private agents' beliefs about future policies so that the model allows for partial but not complete bailouts. To see how ex posts costs can arise, suppose, for example, that an unexpectedly large bailout today leads private agents to expect that all distressed firms will be bailed out in the future. Such expectations imply that a bailout authority contemplating an unexpectedly large bailout may be deterred from doing so, because the current gain from reducing bankruptcy may be outweighed by future losses from reduced effort. We show that such logic implies that equilibrium outcomes must satisfy a *sustainability constraint*, which captures the idea that current gains from policy deviations must outweigh future losses induced by changes in private expectations from such deviations. We show that if the discount factor is not too high, the sustainability constraint binds, and the bailout authority bails out some but not all distressed firms in equilibrium. In this sense, our model is consistent with the view that, absent regulation, bailouts are bound to occur.

We go on to show that optimal regulation can entirely eliminate the incentives of the bailout authority to bail out distressed firms. Such regulation consists of limits on debt-to-value ratios for firms. These limits reduce the amount of resources at risk of bankruptcy and thereby reduce the benefits of current bailouts. By improving future incentives, these limits also raise the cost of current bailouts.

Regulation is needed because of an externality created by the policies of the bailout authority without commitment. Any individual firm does not internalize the costs that it imposes on other firms from increasing its debt level. These costs arise because when all firms raise their debt levels, the bailout authority is more tempted to intervene ex post. Such ex post intervention implies that healthy firms have to pay taxes, and incentive problems worsen. Regulation mitigates the externality created by lack of commitment on the part of the bailout authority but does not fully eliminate it. Under optimal regulation, welfare is higher than without any regulation at all, but lower than it would be if the bailout authority could commit to its policies.

We then ask how regulatory policy should vary over the business cycle. To answer this question, we introduce aggregate shocks into our model. The general principle we derive is that regulation should be tighter when, absent intervention, the lost resources arising from bankruptcy are larger. The implications for the cyclicality of policy depend on the detailed specification of how the shocks affect outcomes. We provide one specification in which the lost resources are highest in recessions, so that *countercyclical regulation* is optimal; that is, the optimal ex ante debt limits become tighter during recessions. We provide another specification in which the lost resources are highest in booms so that *procyclical regulation* is optimal; that is, the optimal ex ante debt limits become tighter during booms.

Related Literature. Our analysis is motivated by the work of Stern and Feldman (2004), who argue that regulation may be desirable because of lack of commitment.

A recent and growing literature has analyzed the role of lack of commitment in bailout policy. An early example of the time-inconsistency problem in regulating financial institutions is the work of Mailath and Mester (1994). They consider an environment in which a bank chooses the risk level of its investments. In their model, a regulator must decide whether to close a bank and pay off depositors. They show that the optimal policy is time inconsistent. Similarly, Acharya and Yorulmazer (2007) analyze bailout policy in a banking model and show that adding a bailout authority induces banks to take on correlated risks.

Much of the more recent literature has emphasized the result that lack of commitment in bailout policy can lead to multiple Markov equilibria. (See, for example, the work of Schneider and Tornell (2004), Ennis and Keister (2009), and Farhi and Tirole (2012).) The multiplicity of equilibria in this work is reminiscent of the kind of multiplicity that arises in government policy games with a time-inconsistency problem. (See, for example, the work of Calvo (1988).) In this literature, if banks expect to be bailed out, for example, they take actions, such as adopting risky financial structures, that then make a government bailout optimal. If instead banks do not expect to be bailed out, then they do not take such actions, and bailouts are not optimal.

Our work here builds on this literature by emphasizing the time-inconsistency problem associated with bailout policy, but differs from most of the literature by focusing on designing ex ante policies that mitigate the time-inconsistency problem. The work most closely related to ours is that of Farhi and Tirole (2012). In their model, under no commitment and no ex ante regulation, the economy has multiple equilibria, one of which coincides with the commitment equilibrium, namely, the *Ramsey* equilibrium, which, by definition, is the best achievable one. This equilibrium features no government intervention and, in particular, no bailouts. Farhi and Tirole show that appropriate ex ante regulation reduces the set of no-commitment equilibria to a single one, the Ramsey equilibrium—the good one. In this sense, without ex ante regulation, the economy faces a *fragility problem*: perhaps the good equilibrium will happen, or perhaps one of many other not-so-good equilibria will. Ex ante policy fixes this fragility problem, and the good equilibrium always occurs.

Our model is different. In it, when there is no commitment and the bailout authority is sufficiently impatient, even the best equilibrium has bailouts and is strictly worse than the Ramsey equilibrium. In this sense, the economy without commitment has an incentive problem rather than a fragility problem. We show that appropriate ex ante regulation can fix the incentive problem, and the best equilibrium then has no bailouts and dominates any no-commitment equilibrium. Overall, we think of our work as complementary to that of Farhi and Tirole (2012): in both studies, ex ante regulation is beneficial because of a time-inconsistency problem, but for different reasons.

In related work, Keister (2012) considers an environment in which it is efficient for the government to provide transfers to intermediaries in distressed states financed by reductions in government expenditures. In his environment, without a regulatory system, intermediaries anticipate receiving these transfers and become illiquid. A regulatory system that taxes such transfers makes the economy less fragile by reducing the set of parameters for which the economy has multiple equilibria. That regulatory system can also improve welfare by correcting a subtle externality that arises in his setup because private agents cannot commit to contracts. More generally, a burgeoning recent literature gives a prominent role to regulation as the way to correct subtle externalities arising either from lack of commitment by private agents or from hidden trading. (See, for example, the work of Lorenzoni (2008); Farhi, Golosov, and Tsyvinski (2009); Bianchi and Mendoza (2010); and Bianchi (2011).) In contrast, in our work, a subtle externality arises because of lack of commitment by the government.

Finally, a recent literature has also examined the quantitative effects of policy interventions like bailouts on the risk-taking decisions of financial institutions. (See, for example, the work of Gertler, Kiyotaki, and Queralto (2012).)

1. The One-Period Economy with Only Private Agents

We begin with a one-period version of our benchmark economy with only private agents. We show that in this economy, optimal contracts take a specific form, called *debt contracts.* Some bankruptcies occur, but the competitive equilibrium is efficient.

Consider a one-period model in which decisions are made in two stages: a first stage at the beginning of the period and a second stage at the end. The economy has two types of agents, called *managers* and *investors,* both of whom are risk neutral and consume at the end of the period. The economy has a measure 1 of managers and a measure 1 of investors.

The technology requires two inputs in the first stage: an effort level e of managers and an investment of 1 unit of goods per manager. (Later on we extend this model to allow for heterogeneity and variability in the scale of investment.) This technology transforms these inputs into capital goods. The capital goods can then be used to make consumption goods. The effort level e of managers is unobserved by investors.

The amount of capital goods produced in the second stage stochastically depends on the effort level e of the manager and two idiosyncratic shocks. One of these shocks, denoted by A_s, $s \in \{H, L\}$, is publicly observed at no cost. This shock determines the average level of productivity and is called the health status. We refer to A_H as the *healthy state* and A_L as the *distressed state*. These states satisfy $A_H > A_L$. We also assume that $A_L < 1$, which will ensure that the full information efficient level of effort cannot be sustained in equilibrium.

The other idiosyncratic shock, denoted by ε, is privately observed by the manager and is made public only if the firm declares bankruptcy, as described below. We assume that ε has density $h(\varepsilon)$ and distribution $H(\varepsilon)$ with mean 1 and support $[\underline{\varepsilon}, \bar{\varepsilon}]$. The idiosyncratic shocks A_s and ε are realized after the effort level is chosen and are independently and identically distributed across firms. Given the state s and the shock ε, $A_s\varepsilon$ units of capital goods are produced.

Given effort level e, with probability $p_H(e)$ the healthy state is realized, and with complementary probability $p_L(e)$ the distressed state is realized. We assume that $p_H(e)$ is an increasing, strictly concave function of e. Thus, higher effort levels increase the probability of the high productivity level, but do so at a diminishing rate. Notice that since $p_H(e)$ is increasing, this technology satisfies the monotone likelihood ratio property, and since $p_H(e)$ is strictly concave, it satisfies the convexity of distribution function property.[2] These assumptions guarantee that the first-order approach is valid. (For details on the first-order analysis, see Rogerson (1985).)

We imagine that production takes place by firms. In each of these firms, managers perform two tasks: in the first stage, they exert effort e that, together with funds from investors, produces capital goods, and in the second stage, they transform capital goods into final consumption goods.

After a manager has completed the first task and a certain amount of capital has been

[2]Recall that the monotone likelihood ratio property is that if $e > \hat{e}$, then $p_H(e)/p_H(\hat{e}) > [1 - p_H(e)]/[1 - p_H(\hat{e})]$, whereas the convexity of distribution property is that the cumulative distribution function induced by $p_H(e)$, namely, $1 - p_H(e)$, has a strictly positive second derivative.

produced, the firm can choose to continue the project under the incumbent manager, or it can declare bankruptcy. If it continues, then the project produces one unit of output for every unit of capital, so that the firm's output is $A_s\varepsilon$ for $s \in \{H, L\}$, and the idiosyncratic shock ε is observed only by the manager. If the firm declares bankruptcy, then the incumbent manager is removed, the firm is monitored, and the idiosyncratic shock ε becomes publicly known. The replacement manager is less efficient and produces consumption goods from the given capital $A_s\varepsilon$ according to $RA_s\varepsilon$, where $R < 1$. We let $\delta_s(\varepsilon) = 0$ denote that the firm declares bankruptcy with health state s and shock ε and let $\delta_s(\varepsilon) = 1$ denote no bankruptcy. Note that we assume that monitoring is deterministic. (For analyses with stochastic monitoring, see the work of Townsend (1979) and Mookherjee and Png (1989).)

We think of replacement managers as being chosen from the pool of managers who have been replaced due to bankruptcy and randomly assigned to manage capital in a firm that has undergone bankruptcy. We think of incumbent managers as having developed specialized expertise in particular firms and, therefore, as being more productive than replacement managers who have not developed specialized expertise.

Managers have no endowments of goods but do have the specialized skills needed to operate the technology. Investors have ω units of endowments but do not have these specialized skills. Investors choose how much to invest in the technology and can store the rest of their endowments at a one-for-one rate. (The only role of storage is to pin down the opportunity costs of funds to be 1.) We assume that $\omega > 1$, so that some amount of the endowment is always stored. This assumption guarantees that the rate of return to investing is 1. We also assume that the technology is sufficiently attractive so that it is always active; thus, the investors invest one unit of their endowments in the technology and store $\omega - 1$ units. (Notice that here we follow a long tradition in financial economics, including the work of Diamond and Dybvig (1983), of consolidating banks, financial markets, and households into one entity called investors.)

Let $c_s(\varepsilon)$ denote the consumption of the managers when the health state is $s \in \{H, L\}$ and the idiosyncratic shock is ε. Let B_s denote the *bankruptcy set*, namely, the set of idiosyncratic shocks ε such that the firms declare bankruptcy when the health state is $s \in \{H, L\}$. The complementary set N_s in which no bankruptcy occurs is then implicitly defined by B_s.

Managers are risk neutral over consumption and have preferences given by

$$(1) \quad \sum_s p_s(e) \int c_s(\varepsilon)\ dH(\varepsilon) - e,$$

where the consumption of the managers must satisfy a *nonnegativity constraint*:

$$(2) \quad c_s(\varepsilon) \geq 0.$$

Let $d_s(\varepsilon)$ denote the payments the firm makes to the investors when the shocks are s and ε. Investors invest 1 unit of their endowment with the managers and store $\omega - 1$ units, so their utility is given by

$$(3) \quad \sum_s p_s(e) \int d_s(\varepsilon)\ dH(\varepsilon) + \omega - 1.$$

When the firm does not declare bankruptcy, the consumption level of the managers and the payments to the investors must satisfy

$$(4) \quad c_s(\varepsilon) + d_s(\varepsilon) = A_s\varepsilon,$$

and when the firm does declare bankruptcy, the payments must satisfy

$$(5) \quad c_s(\varepsilon) + d_s(\varepsilon) = RA_s\varepsilon.$$

The total consumption of investors $c_s^I(\varepsilon)$ is the sum of the payments $d_s(\varepsilon)$ from the production technology and $\omega - 1$ from storage. Thus, the *overall resource constraint* in this economy is given by

$$(6) \quad \sum_s p_s(e) \left[\int c_s(\varepsilon)\ dH(\varepsilon) + \int c_s^I(\varepsilon)\ dH(\varepsilon) \right]$$

$$\leq \sum_s p_s(e) \left[\int_{\delta_s(\varepsilon)=1} A_s\varepsilon\ dH(\varepsilon) + \int_{\delta_s(\varepsilon)=0} RA_s\varepsilon\ dH(\varepsilon) \right] + \omega - 1.$$

An allocation, or a *contract*, consists of $x = \{c_s(\varepsilon), d_s(\varepsilon), \delta_s(\varepsilon)\}$. The timing is as follows. The investors and managers first agree to a contract, and then the managers choose the effort level e given the contract. After the effort level is chosen, the health status of each firm s is publicly realized. Investors and managers then renegotiate the contract. Finally, the idiosyncratic shocks ε are realized, and the bankruptcy decisions are made according to the contract.

To be part of a competitive equilibrium (CE), a contract has to satisfy various conditions. One is that any contract must be *incentive compatible*; that is, a manager must prefer to report the idiosyncratic shock ε truthfully rather than misreport it. A manager with a shock ε in the nonbankruptcy set must not have an incentive to misreport any other shock $\hat{\varepsilon}$ in this nonbankruptcy set, so that

$$(7) \quad c_s(\varepsilon) = A_s\varepsilon - d_s(\varepsilon) \geq A_s\varepsilon - d_s(\hat{\varepsilon}) \text{ for all } \varepsilon \in N_s, \hat{\varepsilon} \in N_s.$$

This constraint implies that for all $\varepsilon \in N_s$, payments to investors $d_s(\varepsilon)$ are constant in the nonbankruptcy set at some level, denoted d_s. Also, a manager with a shock ε in the bankruptcy set must not have an incentive to misreport any $\hat{\varepsilon}$ in the nonbankruptcy set, so that

$$(8) \quad c_s(\varepsilon) = RA_s\varepsilon - d_s(\varepsilon) \geq A_s\varepsilon - d_s \text{ for all } \varepsilon \in B_s, \hat{\varepsilon} \in N_s,$$

where we have imposed that $d_s(\hat{\varepsilon})$ is constant in nonbankruptcy sets. We will say that a contract is *incentive feasible* if it is incentive compatible, in that it satisfies (7) and (8), and feasible, in that it satisfies the resource constraints (4) and (5) and the nonnegativity constraint (2).

We also require that neither managers nor investors have an incentive to renegotiate the contract. Before renegotiation begins, a particular contract x has been agreed to, effort e has been chosen, and a health shock s has been realized. We say that a contract x is *immune to renegotiation* given e at s if it is incentive feasible and no alternative incentive feasible contract exists that makes the managers and the investors strictly better off at s. Specifically, an alternative contract $\hat{x} = \{\hat{c}_s(\varepsilon), \hat{d}_s(\varepsilon), \hat{\delta}_s(\varepsilon)\}$ cannot exist that satisfies the resource and incentive constraints (4)–(8) and makes both the manager and the investors better off:

$$(9)\quad \int \hat{c}_s(\varepsilon)\ dH(\varepsilon) \geq \int c_s(\varepsilon)\ dH(\varepsilon)$$

$$(10)\quad \int \hat{d}_s(\varepsilon)\ dH(\varepsilon) \geq \int d_s(\varepsilon)\ dH(\varepsilon)$$

with at least one of the two inequalities strict. Let C_s and D_s denote the values of the right sides of (9) and (10), namely, the expected consumption values of the manager and the expected payments to investors, and let $U_s = (C_s, D_s)$.

We now turn to the ex ante optimal contract in our economy. We think of managers as offering contracts $x = \{c_s(\varepsilon), d_s(\varepsilon), \delta_s(\varepsilon)\}$ and an intended level of effort e to potential investors.[3] Such investors will accept the contract as long as the expected rate of return on their investment is at least 1. Thus, any contract must satisfy the *participation constraint*

$$(11)\quad \sum_s p_s(e) \int d_s(\varepsilon)\ dH(\varepsilon) \geq 1$$

as well as the resource constraints (4) and (5). The contract must also give the manager incentive to exert the intended level of effort e and thus satisfy

$$(12)\quad e \in \arg\max_e \sum_s p_s(e) \int c_s(\varepsilon)\ dH(\varepsilon) - e.$$

Since all contracts can be renegotiated after the manager has chosen effort, when defining an equilibrium it suffices to consider contracts that are immune to renegotiation.

A *competitive equilibrium* in this static economy consists of a contract x and an effort level e such that (x, e) maximize the manager's utility (1) subject to the restrictions that the contract satisfies both the participation constraint, (11), and the manager's incentive constraint, (12), and is immune to renegotiation. Note that in this definition, the requirement that contracts be immune to renegotiation incorporates incentive feasibility, so we do not need to have incentive feasibility as a separate constraint.

[3]Here, we abstract from contracts with randomized effort. For analysis of such contracts, see the work of Fudenberg and Tirole (1990).

Here, we have defined a competitive equilibrium by having managers offer contracts to investors. An alternative way of setting up the equilibrium is to have investors offer contracts to managers—contracts that maximize expected profits subject to the incentive constraints, feasibility constraints, and participation constraints on managers. By duality, the two definitions are equivalent.

We now turn to the efficiency of a competitive equilibrium. A contract x and an effort level e are *efficient* if the contract is immune to renegotiation, the incentive constraint on the manager's effort is satisfied, and no alternative pair (x', e') exists that is immune to renegotiation, satisfies the manager's effort incentive constraint, and has higher utility levels for the manager and the investor, with at least one being strictly higher. The following proposition is immediate.

Proposition 1. The competitive equilibrium is efficient.

We now turn to characterizing the competitive equilibrium. Consider a contract with $U_s = (C_s, D_s)$ defined by the right sides of (9) and (10). We begin by showing that a contract is immune to renegotiation if and only if it has a simple form, which we refer to as a *debt contract*. This form has two key features. First, the contract specifies a cutoff level ε_s^* that depends on U_s such that the firm continues for $\varepsilon_s > \varepsilon_s^*$ and declares bankruptcy for $\varepsilon_s \leq \varepsilon_s^*$. Second, the payments to investors are constant in the nonbankruptcy set, bankruptcy occurs when the firm is unable to make this constant payment, and investors receive all the profits of the firm in bankruptcy.

Specifically, if the expected payment D_s to the investors is sufficiently small, in that $D_s \leq A_s\underline{\varepsilon}$, then the contract has no bankruptcy. Payments to the investors are then given by

(13) $d_s(\varepsilon) = d_s$ for all ε,

where $d_s \leq A_s\underline{\varepsilon}$, and the manager's consumption is given by $c_s(\varepsilon) = A_s\varepsilon - d_s$. If this expected payment to investors is sufficiently large, in that $D_s > A_s\underline{\varepsilon}$, then a cutoff ε_s^* exists such that the contract has bankruptcy for $\varepsilon \leq \varepsilon_s^*$ and the payments to the investors are given by

$$(14)\quad d_s(\varepsilon) = \left\{ \begin{array}{c} d_s = A_s\varepsilon_s^* \text{ for } \varepsilon > \varepsilon_s^*, \\ RA_s\varepsilon \text{ for } \varepsilon \leq \varepsilon_s^* \end{array} \right\},$$

and the consumption of the manager is given by $c_s(\varepsilon) = A_s\varepsilon - d_s$ for $\varepsilon > \varepsilon_s^*$ and $c_s(\varepsilon) = 0$ for $\varepsilon \leq \varepsilon_s^*$.

Proposition 2. A contract is immune to renegotiation if and only if it is a debt contract, in that it has the form given in (13) and (14), where $\varepsilon_s^* = \varepsilon_s^*(U_s)$ is the cutoff for bankruptcy.

The proof of this proposition is in the Appendix and is similar to that of Townsend (1979). For such debt contracts, we refer to d_s as the *face value of the debt*, which is the constant amount that investors are paid outside of bankruptcy.

Under the assumption that the first-order approach is valid, the contracting problem in this economy is given by

$$(15)\quad U^{CE} = \max_{e,\varepsilon_H^*,\varepsilon_L^*,d_s} \sum_s p_s(e) \left[\int_{\varepsilon_s^*}^{\bar{\varepsilon}} (A_s\varepsilon - d_s)\ dH(\varepsilon) \right] - e$$

subject to

$$(16)\quad p_H'(e) \left[\int_{\varepsilon_H^*}^{\bar{\varepsilon}} (A_H\varepsilon - d_H)\ dH(\varepsilon) - \int_{\varepsilon_L^*}^{\bar{\varepsilon}} (A_L\varepsilon - d_L)\ dH(\varepsilon) \right] = 1$$

$$(17)\quad \sum_s p_s(e) \left[RA_s \int_{\underline{\varepsilon}}^{\varepsilon_s^*} \varepsilon\ dH(\varepsilon) + \int_{\varepsilon_s^*}^{\bar{\varepsilon}} d_s\ dH(\varepsilon) \right] \geq 1$$

$$(18)\quad d_s \leq A_s\varepsilon_s^*$$

$$(19)\quad \varepsilon_s^* \geq \underline{\varepsilon}.$$

We have written the constraint (18) as an inequality so that the same contracting problem covers the cases of $d_s \leq A_s\varepsilon^*$ with $\varepsilon^* = \underline{\varepsilon}$ and $d_s = A_s\varepsilon_s^*$ with $\varepsilon_s^* > \underline{\varepsilon}$. The reason (18) covers the latter case is that, as one can show in the solution to this problem, when there is bankruptcy, $\varepsilon_s^* > \underline{\varepsilon}$, and thus $d_s = A_s\varepsilon_s^*$. Let the associated contract and effort levels be denoted X^{CE} and E^{CE}.

Clearly, for some economies, the solution to the contracting problem has no bankruptcy. The analysis of this case is trivial. Hereafter, we focus on the interesting case in which the optimal contract has bankruptcy. Note that this assumption implies that the manager's incentive constraint is binding.

Next, we show that firms that are distressed have higher levels of bankruptcy than when they are healthy, as long as the distribution of idiosyncratic shocks ε satisfies a *monotonicity condition*:

$$(20) \quad \frac{\varepsilon h(\varepsilon)}{1 - H(\varepsilon)} \text{is increasing in } \varepsilon.$$

Proposition 3. Under the monotonicity condition (20), the competitive equilibrium has $\varepsilon_L^* > \varepsilon_H^*$.

Proof. Suppose first that $\varepsilon_H^* > \underline{\varepsilon}$. The first-order conditions of (15) are then given by

$$-[p_H(e)+\phi p_H'(e)]A_H[1-H(\varepsilon_H^*)]+\lambda p_H(e)\left[(R-1)A_H\varepsilon_H^*\right]h(\varepsilon_H^*)+\lambda p_H(e)A_H[1-H(\varepsilon_H^*)]=0$$

$$-[p_L(e)-\phi p_H'(e)]A_L[1-H(\varepsilon_L^*)]+\lambda p_L(e)\left[(R-1)A_L\varepsilon_L^*\right]h(\varepsilon_L^*)+\lambda p_L(e)A_L[1-H(\varepsilon_L^*)]+\nu_L=0,$$

where ϕ, λ, and ν_L are the multipliers on (16), (17), and (19). We can manipulate these first-order conditions to imply that

$$\frac{\varepsilon_L^* h(\varepsilon_L^*)}{1 - H(\varepsilon_L^*)} > \frac{\varepsilon_H^* h(\varepsilon_H^*)}{1 - H(\varepsilon_H^*)},$$

which by our monotonicity condition (20) implies that $\varepsilon_L^* > \varepsilon_H^*$.

Suppose next that $\varepsilon_H^* = \underline{\varepsilon}$. We have assumed that the solution to the contracting problem always has some bankruptcy. Thus, since $\varepsilon_H^* = \underline{\varepsilon}$, we must have $\varepsilon_L^* > \underline{\varepsilon}$. So $\varepsilon_L^* > \varepsilon_H^*$. *Q.E.D.*

In what follows, we focus on economies in which there is no bankruptcy when the idiosyncratic state $s = H$. It will be clear that all our results continue to hold in the more general case in which bankruptcy occurs with both $s = L$ and $s = H$.

2. Adding a Bailout Authority

Here, we introduce a benevolent government in the form of a bailout authority. The bailout authority can intervene by buying debt from investors and then renegotiating the terms of the outstanding debt with the managers. It can finance these purchases by levying taxes on payments to all investors. The bailout authority is confronted with the same informational constraints as the private agents.

Suppose first that the bailout authority chooses its policies at the beginning of the period and can commit to them. As we have shown in Proposition 1, since the competitive equilibrium is efficient, it follows that a bailout authority with commitment will choose not to intervene.

Suppose next that the bailout authority cannot commit to its policies. We model this lack of commitment by having the bailout authority choose its policies after the manager's effort choice has been made and all the shocks have been realized. We will show that the bailout authority intervenes so as to stop all bankruptcy. The intervention lowers welfare relative to the economy without such an authority. In this sense, the equilibrium here is inefficient. The difference between the bailout authority's policy with and without commitment implies that the bailout authority faces a time-inconsistency problem.

Formally, the timing in the period is that in the first stage, each firm chooses a contract x. Next, each manager chooses an effort level e. Then the health shock s for each firm is realized. After that, the private agents renegotiate the contract. The bailout authority observes the contracts and the health state of each firm and uses the optimal decision rules of managers to infer their effort level. The bailout authority then chooses its policy π, which has three parts: a debt purchase policy, a renegotiation policy, and a tax policy. Finally, the idiosyncratic shocks ε are realized.

In what follows, we focus on symmetric equilibria in which all agents have the same decision rules. In order to ensure that the contracting problem is well defined, we need to describe how the debt purchase policy depends on the individual levels of debt at firms, both at the equilibrium values and for any deviations by individual firms.

Since there is no bankruptcy in the healthy state, the bailout authority will intervene, if at all, only in the distressed state. To develop the bailout authority's *debt purchase policy* in the distressed state, consider a firm with a face value of debt given by $d_L = A_L\varepsilon_L^*$. Without intervention by the bailout authority, the firm will declare bankruptcy for $\varepsilon < \varepsilon_L^*$, so this debt has market value

$$(21)\quad M(d_L) = RA_L \int_{\underline{\varepsilon}}^{d_L/A_L} \varepsilon \, dH(\varepsilon) + d_L[1 - H(d_L/A_L)].$$

Here, $M(d_L)$, the *market value function*, is given by the right side of (21), where we have used $d_L = A_L\varepsilon_L^*$. Given the face value of debt d_L of an individual contract, the bailout authority offers to buy either all of this debt or none of it. We denote the decision to buy the debt by $z(d_L) = 1$ and a decision not to buy the debt by $z(d_L) = 0$. If the bailout authority decides to buy the debt, it offers investors a payment of $Q(d_L)$. The investors will accept this offer as long as the payment exceeds the market value of the debt, in that $Q(d_L) \geq M(d_L)$. For convenience, we assume that the bailout authority pays the minimum amount to investors that induces them to accept the offer, so that $Q(d_L) = M(d_L)$.

Consider, next, the bailout authority's *renegotiation policy*. If the bailout authority buys the outstanding debt, it then renegotiates the terms of the debt contract with managers. Given the information assumptions and our earlier results, we can restrict attention to new debt contracts of the form (13) and (14), with a bankruptcy cutoff ε_L^g and a face value $d_L^g = A_L\varepsilon_L^g$, where the superscript g distinguishes these government-chosen cutoffs and face values from the privately chosen ones. The manager will accept the bailout authority's renegotiated offer if and only if the new offer has a lower face value than the old contract, which from (21) implies that $d_L^g \leq d_L$.

So far we have described the bailout authority's decisions with respect to each individual contract. These are summarized by a schedule for buying debt $z(\cdot)$ and new debt contracts with face value $d_L^g(\cdot)$ and associated cutoffs $\varepsilon_L^g(\cdot) = d_L^g(\cdot)/A_L$ that depend on the level of debt of the individual contract. Here, the manager and investors associated with an individual contract confront a schedule that depends on their individual choices.

Consider, finally, the bailout authority's *tax policy*. Let (D_H, D_L) denote the representative levels of debt of a representative firm and E the level of effort of a representative manager. Let $z_L = z(D_L)$ and $d_L^g = d_L^g(D_L)$ be the associated purchase and renegotiation policies. The bailout authority finances its expenditures with a uniform tax τ on all payments by firms to investors, so that the revenues that the bailout authority collects are

$$\tau\big[p_H(E)D_H + p_L(E)\left[z_L Q(D_L) + (1-z_L)M(D_L)\right]\big],$$

which, using $Q(D_L) = M(D_L)$, we can simplify to $\tau\left[p_H(E)D_H + p_L(E)Q(D_L)\right]$. Hence, we can write the bailout authority's *budget constraint* for such a history as

$$\text{(22)}\quad p_L(E)z_L\left[RA_L\int_{\underline{\varepsilon}}^{d_L^g/A_L}\varepsilon\, dH(\varepsilon) + d_L^g\left(1 - H\left(\frac{d_L^g}{A_L}\right)\right)\right] + \tau\left[p_H(E)D_H + p_L(E)Q(D_L)\right]$$

$$= z_L p_L(E)Q(D_L).$$

The left side of (22) is the revenues of the bailout authority, which come from the payments on the renegotiated debt contracts plus the taxes collected from investors. The right side of (22) is the expenditures of the bailout authority to purchase the debt of distressed firms.

The objective function of the bailout authority is the sum of the utilities of the manager and the investors from the contract (which, of course, equals the aggregate output from the contract minus the disutility of effort of the manager) and is given by

$$\text{(23)}\quad U(X,E,\pi) = p_H(E)A_H + p_L(E)\left[z_L Y_L\left(\frac{d_L^g}{A_L}\right) + (1-z_L)Y_L\left(\frac{D_L}{A_L}\right)\right] - E,$$

where $Y_L(\tilde{\varepsilon}) = RA_L\int_{\underline{\varepsilon}}^{\tilde{\varepsilon}}\varepsilon\, dH(\varepsilon) + A_L\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}}\varepsilon\, dH(\varepsilon)$ denotes output in the distressed state for any cutoff level of bankruptcy $\tilde{\varepsilon}$.

Next we develop the strategy of the bailout authority. In order to solve their contracting problem, managers need to forecast the bailout authority's policy both when these managers choose the representative contract and when they deviate from this choice. Thus, the purchase and renegotiation policies of the bailout authority must be specified for each possible level of debt implied by a contract. These considerations lead us to specify a *strategy* for the bailout authority as a collection $\pi(X) = \big(z(\cdot|X), d_L^g(\cdot|X), \tau(X)\big)$ for each representative contract that the bailout authority may face. Here, $z(\cdot|X)$ and $d_L^g(\cdot|X)$ are functions of individual debt levels.

Consider, next, the problem of a manager confronting such policies. The manager chooses individual effort e given the individual contract $x = (d_H, d_L)$, taking as given the representative contract X and the associated policies of the bailout authority $\pi(X)$, where, suppressing the dependence on X, we write $\pi = \big(z(d_L), d_L^g(d_L), \tau\big)$ to solve

$$(24) \quad e \in \arg\max_e p_H(e)c_H + p_L(e)c_L - e,$$

where $c_H = A_H - d_H$ and

$$(25) \quad c_L = A_L \left[z(d_L) \int_{d_L^g(d_L)/A_L}^{\bar{\varepsilon}} \left[\varepsilon - \frac{d_L^g(d_L)}{A_L} \right] \, dH(\varepsilon) + [1 - z(d_L)] \int_{d_L/A_L}^{\bar{\varepsilon}} \left(\varepsilon - \frac{d_L}{A_L} \right) \, dH(\varepsilon) \right].$$

We denote the solution as $e(x, \pi)$.

Consider, finally, the contracting problem. In terms of the debt levels, if the contract specifies a debt level d_L such that $z(d_L) = 1$, then the bailout authority will buy the firm's debt from investors at a price of $Q(d_L)$ and then renegotiate the contract with managers so that the firms end up having debt with the bailout authority with face value $d_L^g(d_L) = A_L \varepsilon_L^g(d_L)$. If the contract specifies a debt level d_L such that $z(d_L) = 0$, then the bailout authority will not buy the debt, and the original contract will be implemented. The bailout authority also taxes all the payments made to investors by any firm at rate τ. The private contracting problem thus reduces to a simple problem that we refer to as the *static contracting problem.* This problem is given by

$$(26) \quad \max_{\{e, c_H, c_L, d_L, d_H\}} p_H(e)c_H + p_L(e)c_L - e$$

subject to (24) and the participation constraint of investors

$$(27) \quad I(x, e, \pi) = (1 - \tau)\big(p_H(e)d_H + p_L(e)\big[z(d_L)Q(d_L) + [1 - z(d_L)]M(d_L)\big]\big) \geq 1,$$

where $I(x, e, \pi)$ is the expected investment income of the investor.

A *bailout equilibrium* consists of a contract X, an effort function $e(x, \pi)$, and a bailout strategy $\pi(X)$, such that (i) given the policy, the contract X solves (26); (ii) for every individual contract x, the effort of the manager solves the manager's problem (24); and (iii) for every representative contract X and inferred effort level $e\big(X, \pi(X)\big)$, the policy π maximizes the bailout authority's objective (23) subject to its budget constraint (22).

We now turn to characterizing the outcomes of a bailout equilibrium. Consider the bailout authority's problem given that some contract X and some effort level E have already been chosen. The bailout authority's objective function is maximized by buying all the debt in the distressed state by setting $z = 1$, then renegotiating with the managers so as to eliminate all bankruptcies by setting $\varepsilon_L^g = \underline{\varepsilon}$, and setting the tax rate τ to satisfy the budget constraint (22).

We have shown that in the representative contract, the bailout authority will purchase all the debt. Technically, the bailout authority is indifferent to whether or not it purchases the debt of a set of measure zero firms that deviate from the representative contract. If, because of trembles, a positive measure (no matter how small) of firms have debt levels different from the representative level, then it is strictly optimal for the bailout authority to buy the debt of such firms as well and eliminate all bankruptcy. Given these observations, we make the natural assumption that the bailout authority buys the debt of all firms regardless of their debt levels. Let $\pi^s(X, E)$ denote the associated policy function and

$$(28) \quad U^s(X, E) = U\big(X, E, \pi^s(X, E)\big)$$

denote the associated payoff for the bailout authority. We refer to $\pi^s(X, E)$ as a *full bailout strategy* and $U^s(X, E)$ as the *full bailout payoff*. (Note for later that in the dynamic version of the model, the payoff from the best one-shot deviation of the bailout authority is given by (28).)

In the static model, the investors and managers will choose their individual contract x and effort level e anticipating that the bailout authority will set the policy $\pi^s(X, E)$. Let (X^s, E^s, π^s) denote the resulting equilibrium outcomes, which we refer to as the *static outcomes*. We have already established that part of the bailout authority's policy is given by $z(d_L) = 1$, $Q(d_L) = M(d_L)$, $\varepsilon_L^g = \underline{\varepsilon}$. Given this policy, the debt level in the distressed state d_L appears only in the participation constraint (27) in the private contracting problem and does so by affecting $Q(d_L) = M(d_L)$. From the participation constraint, we know that choosing debt in the distressed state d_L to maximize the receipts $Q(d_L)$ from the bailout authority allows the payments in the healthy state d_H to be minimized. Doing so increases the payments to the manager in the healthy state $c_H = A_H - d_H$ and hence provides incentives for greater effort e, thereby raising utility. Hence, the equilibrium level of debt D_L in the distressed state solves $D_{\max} = \arg\max_{d_L} M(d_L)$, and the effort level e and the payment d_H in the healthy state solve

$$(29) \quad \max_{\{e, d_H\}} p_H(e)(A_H - d_H) + p_L(e)A_L(1 - \underline{\varepsilon}) - e$$

subject to

$$(30) \quad p_H'(e)\left[(A_H - d_H) - A_L(1 - \underline{\varepsilon})\right] = 1$$

$$(31) \quad (1 - \tau)\left[p_H(e)d_H + p_L(e)D_{\max}\right] \geq 1.$$

Let U^s denote the sum of the manager's payoff and the investor's payoff from the static contract.

Now we can formally state our result for this static economy with a bailout authority that has no ability to commit to a policy.

Proposition 4. The full bailout equilibrium (π^s, x^s, e^s) has the form described above and is inefficient.

Proof. We have already established the first part of the proposition. Inefficiency follows because the bailout equilibrium outcomes differ from the competitive equilibrium, which is efficient from Proposition 1. *Q.E.D.*

Thus far we have considered bailout policies in which the bailout authority directly buys all the debt of distressed firms and renegotiates contracts with managers. These policies can be interpreted as ones in which the bailout authority provides funds to distressed banks under the implicit or explicit condition that these funds be used to renegotiate the terms of bank loans. To do so, consider a slight variant of our model in which households invest their endowments with banks, which then provide funds to firms. Suppose that banks do not hold a completely diversified portfolio. Some banks will then be confronted with situations in which a large fraction of their funds have been lent to distressed firms. Such banks may well be threatened with the possibility of default. The bailout authority will then find it optimal to bail out such banks under the condition that the banks renegotiate the terms of their loans with the distressed firms. In this sense, our model is consistent with bailouts, in practice, being primarily directed at banks and similar financial institutions.

3. The Dynamic Model

Now consider extending the static model above to a dynamic infinite horizon model with a bailout authority. In the one-period model without commitment, the equilibrium has no bankruptcy because bailouts have no ex post costs. Here, we develop a dynamic contracting model without commitment by the bailout authority in which these costs do arise because of reputational considerations, which make the nature of future contracts depend on whether bailouts have occurred in the past.

Our dynamic model is an infinite repetition of a modified version of our static model. The infinite repetition allows for trigger strategies in which contracts depend on the history of past bailouts and, in this sense, allows for reputational considerations. We start by showing that bailouts occur in equilibrium if the bailout authority is not too patient. We show that for intermediate levels of patience by the bailout authority, outcomes better than the full bailout equilibrium but worse than the efficient outcome can be sustained in each period using reputational considerations.

We then add a regulatory authority to the economy that can limit the amount of debt that each firm takes on. We show that by setting these limits sufficiently low, the regulatory authority can eliminate the incentives of the bailout authority to bail out firms. Here, the optimal regulation mitigates the time-inconsistency problem but does not eliminate it: when the bailout authority is sufficiently impatient, this regulation raises welfare relative to any bailout equilibrium, but does not raise it all the way to the optimal outcome with commitment.

A. Bailouts

Each period of the dynamic model is identical to the one period in the static model. Recall that the timing within each period t is that the managers and the investors agree to a contract, the managers choose effort, shocks are realized, and then the bailout authority chooses its policies. Since the dynamic model is an infinite repetition of the static model, no physical state variables link the periods. The only links between periods are strategic ones in which the bailout authority forecasts the responses of private agents in the future to its current actions. To capture these strategic links, we specify the histories faced by agents when they choose actions.

Setup and Definition of Bailout Equilibrium. Specifically, in the first stage of the period, investors choose a contract x_t. The representative contract is X_t. Next the manager chooses effort e_t, and then the idiosyncratic shocks (s_t, ε_t) for each firm are realized. Finally, the bailout authority chooses its policy π_t.

We make an anonymity assumption that prevents long-term contracts between managers and investors so that we can focus attention on the dynamic incentive problem of the bailout authority. To do so, we assume that managers are anonymous in the sense that their identities cannot be recorded from period to period. Hence, current contracts cannot be conditioned on the past track record of individual managers. We assume that past aggregates, including the policies of the bailout authority, are observable. These assumptions imply that the only intertemporal link is the behavior of the bailout authority.

Given these informational assumptions, we now recursively describe how histories relevant for actions evolve in our dynamic model. (Technically, we focus attention on perfect, public equilibria.) Let H_t be the history at the beginning of period t. Let $H_{Bt} = (H_t, X_t)$ denote the history faced by the bailout authority, and let $H_{t+1} = (H_{Bt}, \pi_t)$.

We now describe the strategies of all the agents. The strategy for the contract is denoted by $x_t(H_t)$; the strategy for the effort level of an individual manager, by $e_t(H_t, x_t)$; and the strategy for the bailout authority, by $\pi_t(H_{Bt})$. The effort level of the representative manager is then given by $e_t(H_t, X_t)$.

The payoffs of the bailout authority given a history H_{Bt} are the sum of its period payoffs and continuation values and are given by

$$(32)\quad U(X_t, E_t, \pi_t) + \beta V_{t+1}(H_{t+1}),$$

where the period payoff $U(X_t, E_t, \pi_t)$ is given by (23) and the continuation payoff $V_{t+1}(H_{t+1})$ is given by the present expected value of period payoffs for the bailout authority starting from period $t+1$ induced by the strategies, and $\beta < 1$ is the discount factor.

The payoffs of individual investors in period t are $I(x_t, e_t, \pi_t) + \beta V^I_{t+1}(H_{t+1})$, where the period payoff $I(x_t, e_t, \pi_t)$ is given by (27), and the payoffs of the manager are

$$(33)\quad p_H(e_t)c_{Ht} + p_L(e_t)c_{Lt} - e_t + \beta V^M_{t+1}(H_{t+1}),$$

where $c_{Ht} = A_H - d_{Ht}$, c_{Lt} is given by (25), and the continuation payoffs $V^I_{t+1}(H_{t+1})$ and $V^M_{t+1}(H_{t+1})$ are given by the present expected value of period payoffs starting from period $t+1$ induced by the strategies.

Given a history H_{t-1}, we say that a contract x_t and associated effort level e_t solve the dynamic contracting problem if they maximize the payoff of the manager (33) subject to the incentive constraint

$$(34)\quad e \in \arg\max_{e} p_H(e_t)c_{Ht} + p_L(e_t)c_{Lt} - e_t + \beta V^M_{t+1}(H_{t+1})$$

and the participation constraint for the investor, namely,

$$(35)\quad I(x_t, e_t, \pi_t) \geq 1,$$

where $c_{Ht} = A_H - d_{Ht}$ and c_{Lt} is given by (25). Given a history H_{t-1} and an arbitrary contract, the manager's problem is to maximize (34), where $c_{Ht} = A_H - d_{Ht}$ and c_{Lt} is given by (25).

Here, a *bailout equilibrium* is a collection of strategies $\{x_t(H_t), e_t(H_t, x_t), \pi_t(H_{Bt})\}$ for private agents and the bailout authority such that (i) given the history H_t, the contract $x_t(H_t)$ solves the dynamic contracting problem; (ii) for every contract x_t, the effort $e_t(H_t, x_t)$

maximizes the manager's payoffs; and (*iii*) given the strategies of the private agents, the policy $\pi_t(H_{Bt})$ maximizes the payoff for the bailout authority (32) subject to its budget constraint.

The outcomes associated with a bailout equilibrium, then, are sequences $\{X_t, E_t, \pi_t\}$ and associated continuation utilities for the bailout authority $\{V_t\}$, where

$$(36) \quad V_t = U(X_t, E_t, \pi_t) + \beta V_{t+1}.$$

Characterization and Implementation of Bailout Equilibrium. We characterize the bailout outcomes in this dynamic model in two steps. We first show that in any bailout equilibrium, given the policies of the bailout authority, the private outcomes are part of a bailout equilibrium if and only if they solve the static contracting problem in each period given the policies of the bailout authority. Second, we show that the policies of the bailout authority are part of a bailout equilibrium if and only if they satisfy a sustainability constraint.

Consider the private outcomes. Given our anonymity assumption, the continuation payoffs for the manager and investors are independent of current actions. Therefore, the dynamic version of the private contracting problem coincides with the static version (26).

In our infinite horizon model, we focus attention on equilibria that can be supported by trigger-type strategies that specify reversion to outcomes that are no worse than the static outcomes. This set of equilibrium outcomes is analogous to the set of equilibrium outcomes in repeated games that are supported by the one-shot Nash equilibria. (Of course, following the work of Abreu (1988), we could use more sophisticated strategies that support a larger set of equilibria. The results are similar, but the analysis is more cumbersome.) Specifically, we focus on equilibria in which for every history, even those after deviations by the bailout authority from a given policy plan, the continuation values of the bailout authority satisfy

$$(37) \quad V_{t+1}(H_{t+1}) \geq \frac{U^s}{1-\beta}.$$

This condition restricts the severity of the trigger strategies to be no worse than that of the strategies implicit in the infinite reversion to the static equilibrium.

To set up our sustainability constraint in this model, we need to define the best one-shot deviation for the bailout authority when it is faced with some arbitrary history. To that end, consider a period t in which a contract X_t and effort level E_t have been chosen. The statically best policy for the bailout authority is to buy all the debt in the distressed state and then renegotiate with the managers so as to eliminate all bankruptcies by setting $\varepsilon^g_{Lt} = \underline{\varepsilon}$. That is, the statically best policy is the full bailout policy defined earlier. The *sustainability constraint* of the bailout authority is then given by

$$(38) \quad U(X_t, E_t, \pi_t) + \beta V_{t+1} \geq U^s(X_t, E_t) + \frac{\beta}{1-\beta} U^s,$$

where V_{t+1} is given by (36).

Proposition 5. Under (37), a set of outcomes (X_t, E_t, π_t) are the outcomes of a bailout equilibrium in a dynamic model if and only if (i) the outcomes solve the one-period contracting problem (26) and (ii) the outcomes satisfy (38).

Proof. Suppose first that the outcomes (X_t, E_t, π_t) are the outcomes of a bailout equilibrium. Since the contracting problem is static, these outcomes must solve the one-period contracting problem. Next, we show that under (37), they must satisfy the sustainability constraint. To see why, suppose by way of contradiction that in equilibrium these outcomes violate (38). Then the authority, by setting the bankruptcy set to be empty in the current period, obtains current payoffs equal to the first term on the right side of (38), and under (37), its continuation payoff is at least as large as the last term. Thus, outcomes that violate (38) contradict optimality by the bailout authority.

Suppose, next, that a set of candidate equilibrium outcomes $(\hat{X}_t, \hat{E}_t, \hat{\pi}_t)$ with associated histories $\hat{H}_t$ and $\hat{H}_{Bt}$ satisfy (i) and (ii) of Proposition 5. We will construct revert-to-static strategies that support these outcomes as an equilibrium. For private agents, these strategies specify that if the history $H_t = \hat{H}_t$, then the contract X_t equals the desired one $\hat{X}_t$; otherwise, the contract X_t equals the static contract X^s. For the bailout authority, these strategies specify that if $H_{Bt} = \hat{H}_{Bt}$, then the policies equal the desired ones $\hat{\pi}_t$; otherwise, they equal the full bailout policy $\pi^s(X_t, E_t)$.

Now consider the bailout authority. If there has been no deviation from these specified outcomes in or before period t, in that $H_{Bt} = \hat{H}_{Bt}$, then the payoffs associated with choosing the desired policy $\hat{\pi}_t$ are given by the left side of (38). The payoffs associated with any deviation are smaller than the right side of (38) because the first term on the right side represents the best one-shot deviation. The inequality in (38) guarantees that the desired policies are indeed optimal. If there has been a deviation in or before t, so that $H_{Bt} \neq \hat{H}_{Bt}$, then the continuation payoffs of the bailout authority are independent of the current policy. Hence, the bailout authority's optimal choice is the statically optimal full bailout policy.

Clearly, the private agent's strategies are optimal by construction. $Q.E.D.$

We now show that if the bailout authority is sufficiently impatient, the equilibrium has bailouts. To do so, we show that if the discount factor is above a critical level, then the efficient outcome is sustainable, and if the discount factor is below this critical level, then, under a sufficient condition, the equilibrium necessarily has bailouts. That is, at the equilibrium, the tax rate τ is positive, the bailout authority buys debt in the distressed state, and it renegotiates the terms of the contract of the manager in order to have less bankruptcy. The critical level of the discount factor $\bar{\beta}$ is defined as the discount factor such that the sustainability constraint holds with equality at the competitive equilibrium outcome; that is,

$$(39)\quad \frac{U^{CE}}{1-\beta} = U^s(X^{CE}, E^{CE}) + \frac{\beta}{1-\beta}U^s.$$

Clearly, $\bar{\beta} < 1$ since $U^s(X^{CE}, E^{CE}) > U^{CE} > U^s$.

We now turn to developing a condition on the relationship between effort and the probability of a healthy state that we will use in the next proposition. For convenience, we will change variables by letting $v(p_H)$ be defined as the effort level e that leads to a probability of the healthy state of p_H; that is, $v(p) = p_H^{-1}(p)$. Consider the following condition on the cost of effort $v(p)$, namely, that

$$(40)\quad \frac{p^2 v''(p)}{1-p} > 1$$

for all $p \geq p_s$, where p_s denotes the probability of the healthy state in the full bailout equilibrium. We then have the following proposition, which is proved in the Appendix.

Proposition 6. If $\beta \geq \bar{\beta}$, then the efficient outcome is sustainable; and if $\beta < \bar{\beta}$, with (40), then any equilibrium allocation has bailouts, so that $\tau > 0$, and at the equilibrium level of debt $D_L, z(D_L) = 1$ and $d_L^g(D_L) < D_L$.

We briefly discuss some intuition for the contradiction argument used to prove this proposition. When $\beta < \bar{\beta}$, how could it be that there are no bailouts in equilibrium, yet an individual firm is not at the competitive outcome? It must be that if such a firm deviated to, say, the competitive outcome, then the bailout authority would buy up all the debt, renegotiate the contract to have fewer bankruptcies, and, in so doing, make this firm worse off. Such a policy of the bailout authority has two effects on the value of the firm. Bailing out the firm by buying up all the debt at face value in a distressed state tends to raise the value of the firm. Renegotiating the contract to have fewer bankruptcies worsens the manager's incentives to supply effort and thereby tends to reduce the value of the firm. Our condition (40) guarantees that the bailout effect is larger than the incentive effect, so that such a deviation is always profitable. Hence, there must be bailouts in equilibrium.

We now argue that the policies of the bailout authority create an externality. The reason is that when some firms increase their debt levels, the bailout authority has stronger incentives to intervene and levy higher taxes to support bailouts. These higher taxes impose costs on all other firms. An individual firm does not internalize the costs that it imposes on other firms from increasing its debt level. This externality creates a role for regulation.

B. Optimal Regulation

We now consider adding a benevolent governmental regulatory authority that can limit the amount of debt that each firm takes on. We will show that by setting these limits appropriately low, the regulator can eliminate the incentives of the bailout authority to bail out firms ex post. As discussed above, such optimal regulation mitigates the time-inconsistency problem arising from the inability of the bailout authority to commit to not bailing out firms but does not eliminate it.

Specifically, the regulator can specify maximum debt levels $\bar{d}_{Ht}$ and $\bar{d}_{Lt}$ that no firm can exceed. Let $\bar{d}_t = (\bar{d}_{Ht}, \bar{d}_{Lt})$ denote the policy of the regulator. In terms of the timing, in each period t, the regulator moves first and chooses its policies at the beginning of the period. Private agents then agree to contracts, and the rest of the timing is as before.

A simple way to set up the *regulatory equilibrium* is to treat the regulator's policies

$\{\bar{d}_t\}_{t=0}^{\infty}$ as an exogenously given sequence. Given this sequence and given the bailout policies, the contracting problem is the same as in the definition of the bailout equilibrium, with the additional constraint that

$$(41)\quad d_{Ht} \leq \bar{d}_{Ht} \text{ and } d_{Lt} \leq \bar{d}_{Lt}.$$

Given the regulatory policies $\{\bar{d}_t\}_{t=0}^{\infty}$, a *regulatory equilibrium* is a collection of strategies $\{x_t(H_t), e_t(H_t, x_t), \pi_t(H_{Bt})\}$ for private agents and the bailout authority such that (i) given the history H_t, the contract $x_t(H_t)$ solves the contracting problem, and for every contract x_t, the effort $e_t(H_t, x_t)$ maximizes the manager's payoffs; and (ii) given the strategies of the private agents, the policy $\pi_t(H_{Bt})$ maximizes the payoffs for the bailout authority (32).

Next we show that the optimal regulatory outcome solves a simple programming problem. This problem is to solve

$$(42)\quad \max_{d_H, d_L, e} p_H(e)(A_H - d_H) + p_L(e) \int_{d_L/A_L}^{\bar{\varepsilon}} (A_L\varepsilon - d_L)\ dH(\varepsilon) - e$$

subject to the manager's incentive constraint

$$(43)\quad e \in \arg\max_e p_H(e)(A_H - d_H) + p_L(e) \int_{d_L/A_L}^{\bar{\varepsilon}} (A_L\varepsilon - d_L)\ dH(\varepsilon) - e,$$

the participation constraint

$$(44)\quad p_H(e)d_H + p_L(e) \left[R \int_{\underline{\varepsilon}}^{d_L/A_L} A_L\varepsilon\ dH(\varepsilon) + d_L \left(1 - H\left(\frac{d_L}{A_L}\right)\right)\right] \geq 1,$$

and the sustainability constraint

$$(45)\quad \frac{U}{1-\beta} \geq U^s(x, e) + \frac{\beta}{1-\beta} U^s,$$

where $U = p_H(e)A_H + p_L(e) \int_{d_L/A_L}^{\bar{\varepsilon}} A_L\varepsilon\ dH(\varepsilon) - e$. Let U^R denote the sum of manager utility and investor utility resulting from the solution to the regulatory problem, and let $(\bar{d}^R, e^R)$ denote the associated allocations. We will show that these allocations are outcomes of a regulatory equilibrium. Notice that if $\beta \geq \bar{\beta}$, then from Proposition 6, we know that the regulatory outcome coincides with the efficient outcome. If $\beta < \bar{\beta}$, then clearly $\bar{d}_L^R < d_L^{CE}$. Notice that if β is sufficiently small, then only the full bailout equilibrium solves the regulatory problem.

Next we show that the outcomes that solve the regulatory problem (42) can be decentralized as an equilibrium by the appropriate choice of debt limits. The basic idea is that the sustainability constraint (45) can be replaced by debt limits. With such replacement, the regulatory problem reduces to a private contracting problem with debt limits. Formally, we now argue this:

Proposition 7. The outcomes of the best regulatory equilibrium solve (42), and in that equilibrium, the bailout authority does not intervene. Moreover, for $\beta < \bar{\beta}$, ex ante regulations strictly improve welfare relative to any bailout equilibrium.

Proof. Clearly the constraints on (42) are all necessary conditions for any regulatory equilibrium. Hence, U^R is at least as large as the utility in any regulatory equilibrium.

We now construct a regulatory equilibrium that achieves U^R. In this equilibrium, the regulatory policy is to set debt limits equal to $\bar{d}^R$. For all histories H_t along the equilibrium path, the bailout policy is to set $\tau_t = 0$ and to set $z(d_{Lt}) = 0$ for all d_{Lt}. For any histories not on the equilibrium path, the bailout policy is the full bailout policy.

Given these policies, the private contracting problem is identical to the regulatory problem except that the sustainability constraint (45) is replaced by

$$d_H \leq \bar{d}_H^R \text{ and } d_L \leq \bar{d}_L^R.$$

Since $\bar{d}_L^R \leq d_L^{CE}$—that is, the regulatory debt limits are below the debt levels in the competitive equilibrium—the solution to the private contracting problem with such debt limits sets the debt levels at the maximum possible levels. Thus, the effort level of the manager in the best regulatory equilibrium is given by e^R.

Next, the bailout authority has no incentive to intervene in the best regulatory equilibrium outcomes because its payoff if it does not intervene is the left side of (45), and its payoff if it does intervene is the right side of (45), which is lower.

Finally, we show that for $\beta < \bar{\beta}$, the best regulatory equilibrium yields higher welfare than any bailout equilibrium. We begin by noting that, trivially, any bailout outcome can be implemented as a regulatory outcome in which the debt limits are set to be greater than or equal to debt levels in the bailout equilibrium. Thus, welfare in the best regulatory equilibrium is at least as high as that in any bailout equilibrium.

We now show that for $\beta < \bar{\beta}$, regulatory equilibrium welfare is strictly higher. To see this, notice that the best regulatory outcome has $\tau = 0$, whereas, from Proposition 6, we know that any bailout equilibrium has $\tau > 0$. Thus, welfare is higher in the equilibrium with regulation. *Q.E.D.*

In all versions of our model, firms are all the same size, so optimal regulation takes the form of a uniform cap on the debt of all such firms. In the data, of course, firms are of different sizes. It is straightforward to extend our model to allow for heterogeneity in firm size. In our benchmark model, the technology required a fixed size of investment, 1, that was the same for all firms, together with e units of managerial effort. Suppose now that firms differ in the scale of required investment, $k \in [\underline{k}, \bar{k}]$. Here, a firm's type k is simply the scale of the required investment. The technology for a firm of type k is that an investment of k units, together with managerial effort ke, produces $A_s \varepsilon k$ for $s \in \{H, L\}$ with probability $p_s(e)$ in the event of no bankruptcy and $RA_s \varepsilon k$ in the event of bankruptcy. Notice that this technology has constant returns to scale: producing twice as much output with a given probability, for example, requires twice as much investment and twice as much effort. In this variant, the regulatory problem is identical to (42) up to the scale factor k. The optimal regulation can thus be interpreted as a cap on the amount of debt relative to the size of investment, which can be thought of as a cap on the debt-to-value ratio of each firm.

4. Optimal Regulation with Aggregate Shocks

So far we have considered an economy without aggregate shocks and have shown that ex ante regulation in the form of debt limits can improve outcomes. An important policy question is how such regulation should vary over the course of the business cycle. To answer this question, we extend our model to have cyclical fluctuations by introducing aggregate shocks. Our main result is that regulation should be tighter when, absent intervention, the lost resources arising from bankruptcy are larger. Translating this general result into specific implications for the cyclicality of policy, however, depends on the detailed specification of how the shocks affect outcomes. To illustrate this dependence, we provide one specification in which the lost resources are highest in recessions, so that *countercyclical regulation* is optimal; that is, the optimal ex ante debt limits become tighter during recessions. We provide another specification in which the lost resources are highest in booms, so that *procyclical regulation* is optimal; that is, the optimal ex ante debt limits become tighter during booms.

A. Theory: Regulate Most When Bailout Incentives Greatest

Consider extending our dynamic economy to incorporate aggregate shocks. The aggregate shock S is i.i.d. over time and can take on a finite set of values, each with probability $\mu(S)$. In each period, the aggregate shock is realized before all other decisions, and the rest of the timing is then the same as before. This shock can affect the probability of the healthy idiosyncratic states, now given by $p_H(e, S)$, as well as the productivities in the healthy and distressed states, now given by $A_H(S)$ and $A_L(S)$.

In this economy, *regulatory equilibrium* is defined analogously to that in the dynamic economy without aggregate shocks. Given an aggregate shock S, the regulatory problem is

$$(46) \quad \max_{d_H, d_L, e} p_H(e, S)(A_H(S) - d_H) + p_L(e, S) \int_{d_L/A_L(S)}^{\bar{\varepsilon}} [A_L(S)\varepsilon - d_L] \; dH(\varepsilon) - e$$

subject to the manager's incentive constraint

$$(47) \quad e \in \arg\max_{e} p_H(e, S)(A_H(S) - d_H) + p_L(e, S) \int_{d_L/A_L(S)}^{\bar{\varepsilon}} (A_L(S)\varepsilon - d_L) \; dH(\varepsilon) - e,$$

the investor's participation constraint

$$(48) \quad p_H(e, S)d_H + p_L(e, S) \left[R \int_{\underline{\varepsilon}}^{d_L/A_L(S)} A_L(S)\varepsilon \; dH(\varepsilon) + d_L \left[1 - H\left(\frac{d_L}{A_L(S)} \right) \right] \right] \geq 1,$$

and the sustainability constraint

$$(49) \quad U(S) + \frac{\beta \sum_{S'} \mu(S')U(S')}{1-\beta} \geq U^s(x, e, S) + \frac{\beta \sum_{S'} \mu(S')U^s(S')}{1-\beta},$$

where $U(S)$ denotes the sum of manager utility and investor utility in state S and $U^s(S)$ denotes this same sum in the static contracting problem analogous to (26). Rearranging the sustainability constraint and using the definitions of $U(S)$ and $U^s(S)$, we obtain that

$$(50)\quad \frac{\beta \sum_{S'} \mu(S')[U(S') - U^s(S')]}{1-\beta} \geq p_L(e,S)(1-R)A_L(S) \int_{\underline{\varepsilon}}^{d_L/A_L(S)} \varepsilon \, dH(\varepsilon).$$

The left side of (50) is a constant independent of S, and the right side of (50) equals the lost resources due to bankruptcy. Let U^* denote the value of the left side at this optimum. This value is the *dynamic gain* from sticking with the prescribed policy.

Now consider the competitive equilibrium without regulation or bailouts. The associated contracting problem here is to maximize (46) subject to the manager's incentive constraint (47) and the investor's participation constraint (48). Let $e^{CE}(S)$, $d_H^{CE}(S)$, and $d_L^{CE}(S)$ denote the solution to this problem. Let $G^{CE}(S)$ denote the *static gain* from eliminating bankruptcy in state S, namely,

$$(51)\quad G^{CE}(S) = p_L\big(e^{CE}(S), S\big)(1-R)A_L(S) \int_{\underline{\varepsilon}}^{d_L^{CE}(S)/A_L(S)} \varepsilon \, dH(\varepsilon).$$

We then have this:

Proposition 8. If $G^{CE}(S) > U^*$, so that the static gain from eliminating bankruptcy exceeds the dynamic gain from sticking with the policy, it is then optimal to have ex ante regulation in state S. If the inequality is reversed, regulation in state S is not optimal.

Proof. Notice that the contracting problem is a relaxed version of the regulatory problem. If the solution to the relaxed problem violates the sustainability constraint, then regulation is necessary in order to support the solution to the regulatory problem as a private equilibrium. If instead the solution to this relaxed problem is feasible for the regulatory problem, then it is also optimal for that problem; thus, no regulation is necessary to support the solution to the regulatory problem as a private equilibrium. *Q.E.D.*

The logic that underlies Proposition 8 is general: regulation is desirable in states for which the bailout authority has the greatest incentive to intervene. These incentives are largest in states for which the competitive equilibrium has the largest costs of bankruptcy.

B. Business Cycle Implications

To relate this general result to the design of optimal regulation over the business cycle, we need to impose more structure on how aggregate shocks affect the probability of success and productivity. In principle, optimal regulation could be pro- or countercyclical, depending on how aggregate shocks affect the competitive equilibrium.

We illustrate this point with two specifications for the aggregate shocks: in one, optimal regulation is countercyclical; in the other, it is procyclical.

Countercyclical Regulation. In the first example, we generate booms and recessions by changing only the probabilities of healthy and distressed states, with mean productivities constant across states. In particular, in a boom a given level of effort leads to a higher probability of any firm becoming healthy and a correspondingly lower probability of becoming distressed. Hence, for a given level of effort, in booms a higher fraction of firms are healthy, and in recessions a higher fraction of firms are distressed.

Specifically, we suppose that

$$p_H(e, S) = p_H(e) + \gamma(S), \tag{52}$$

where $p_H(e) = \psi e^\alpha$ and γ is an increasing function of S, with the understanding that $\gamma(S)$ and $p_H(e)$ are such that for relevant levels of effort, $p_H(e, S)$ lies in $[0, 1]$. Note that with this additive formulation, the marginal effect of effort on the healthy outcome is independent of the aggregate state. We set $A_H(S) = A_H$ and $A_L(S) = A_L$. We also assume that the idiosyncratic shock ε satisfies the monotonicity condition (20). In the Appendix, we show that there is a cutoff state S^* such that for $S > S^*$, no regulation is needed and for $S \leq S^*$, regulation is needed. Since it is optimal to set limits on debt only for sufficiently low realizations of S, which we interpret as sufficiently severe recessions, this result implies that optimal regulation is countercyclical. The proof of the following proposition is in the Appendix.

Proposition 9. Under the monotonicity condition (20) and with $p_H(e, S)$ given by (52), if A_H is sufficiently high and α and ψ are sufficiently small, then optimal regulation is countercyclical.

Procyclical Regulation. We now consider a specification in which a shock affects the scale of investment. In booms the scale of investment is high, and in recessions it is small.

Specifically, consider a variant of our model in which the technology is changed in two ways. First, each project requires $k(S)$ units of investment rather than one unit at the beginning of the period and delivers $A_s k(S)$ units in each idiosyncratic state s. We assume that $k(S)$ is increasing in the aggregate state S. Second, for the project to succeed with probability $p_H(e)$, the required effort cost of the manager is $k(S)e$ units of utility. (In this sense, managing larger projects takes more effort.) With this specification, the contracting problem in a competitive equilibrium becomes

$$(53)\quad \max_{d_H, d_L, e} p_H(e)(A_H k(S) - d_H) + p_L(e) \int_{d_L/A_L k(S)}^{\bar{\varepsilon}} [A_L k(S)\varepsilon - d_L]\ dH(\varepsilon) - k(S)e$$

subject to the manager's incentive constraint

$$(54)\quad e \in \arg\max_{e} p_H(e)[A_H k(S) - d_H] + p_L(e) \int_{d_L/A_L k(S)}^{\bar{\varepsilon}} [A_L k(S)\varepsilon - d_L]\ dH(\varepsilon) - k(S)e$$

and the resource constraint

$$(55)\quad p_H(e) d_H + p_L(e) \left[R \int_{\underline{\varepsilon}}^{d_L/A_L k(S)} A_L k(S)\varepsilon\ dH(\varepsilon) + d_L \left(1 - H\left(\frac{d_L}{A_L k(S)}\right)\right)\right] \geq 1.$$

This problem is homogeneous of degree 1 in $k(S)$ in the sense that if $\tilde{e}, \tilde{d}_L, \tilde{d}_H$ solve the problem for $k(S) = 1$, then $\tilde{e}, \tilde{d}_L k(S), \tilde{d}_H k(S)$ solve it for any value of $k(S)$. Thus, the static gains from eliminating bankruptcy are simply proportional to the scale of investment:

$$(56)\quad G^{CE}(S) = k(S) \left[p_L(\tilde{e})(1 - R) A_L \int_{\underline{\varepsilon}}^{\tilde{d}_L/A_L} \varepsilon\ dH(\varepsilon) \right].$$

In this model, booms correspond to high S states in which the scale of investment is high; recessions, to low S states in which this scale is low. Since $G^{CE}(S)$ is increasing in S, optimal regulation is now procyclical; limits on debt are needed in boom states and not in recession states. We then have the following proposition:

Proposition 10. In the model with a variable scale of investment, optimal regulation is procyclical.

5. Conclusion

We have shown that if governments alter private contracts during crises, then the prospect of bailouts reduces ex ante welfare. But the prescription that governments not bail out distressed firms is unrealistic. Benevolent governments simply do not have the power to commit themselves to such a prescription. A pragmatic approach to policy dictates that we take as given the incentives of governments to undertake bailouts and so design ex ante regulation to minimize the ex ante costs of these ex post bailouts. We have shown here that such ex ante regulation can, in fact, eliminate the incentives of governments to undertake costly bailouts and so improve welfare. We have also shown that regulation should be tightest when the resources lost to bankruptcy without bailouts would be largest.

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6. Appendix: Proofs of Some Propositions

Here we provide the proofs for some of our propositions.

A. Proving Proposition 2

We begin with a lemma that is helpful in proving Proposition 2.

Lemma 1. There exist cutoffs $\varepsilon_s^*(U_s)$ such that the optimal contract has this form: continue if $\varepsilon > \varepsilon_s^*(U_s)$ and declare bankruptcy otherwise.

Proof. Suppose by way of contradiction that a contract that is immune to renegotiation has this form: there is a nonbankruptcy region $N = (\varepsilon_1, \varepsilon_2)$ and a bankruptcy region to the right of it, namely, $M = (\varepsilon_2, \varepsilon_3)$, where $\varepsilon_1 < \varepsilon_2 < \varepsilon_3$.

We first develop a simple inequality that will be useful in our argument. Note that since M is part of the bankruptcy region from (8) and $R < 1$, it follows that

(57) $d_s(\varepsilon) < d_s$ for all $\varepsilon \in M$.

Now consider an alternative contract, denoted by $\{\hat{c}_s(\varepsilon), \hat{d}_s(\varepsilon), \hat{\delta}_s(\varepsilon)\}$. In terms of bankruptcy, this contract is the same as the original contract except that it turns M from a bankruptcy region to a nonbankruptcy region. In terms of payments to investors, it reduces the payments everywhere except the region M by a constant amount a and raises payments in region M so as to give the investors the same expected payments as in the original contract.

Finally, the manager's new consumption is defined residually from the resource constraint. Of course, since this manager is paying the same expected amount to the investor but reaps the benefit $(1-R)A_s\varepsilon$ for all $\varepsilon \in M$, this manager's expected utility increases.

More formally, let $\hat{\delta}_s(\varepsilon) = 1$ for $\varepsilon \in M$ and coincide with $\delta_s(\varepsilon)$ for all other realizations of the idiosyncratic shocks. Let $\hat{d}_s(\varepsilon) = d_s - a$ for $\varepsilon \in M$, and for other ε, let $\hat{d}_s(\varepsilon) = d_s(\varepsilon) - a$, where the constant a is chosen so that the payment to the investors is the same as in the original contract:

$$(58)\quad D_s = \int_{N_s} d_s \, dH(\varepsilon) + \int_{B_s} d_s(\varepsilon) \, dH(\varepsilon) =$$

$$\int_{N_s} (d_s - a) \, dH(\varepsilon) + \int_{M} (d_s - a) \, dH(\varepsilon) + \int_{B_s/M} [d_s(\varepsilon) - a] \, dH(\varepsilon).$$

Subtracting the left side from the right side of the second equality in (58) gives $a = \int_M [d_s - d_s(\varepsilon)] \, dH(\varepsilon)$, which we know from (57) is strictly positive. The consumption of the managers in the original contract is given by

$$C_s = \int_{N_s} [A_s\varepsilon - d_s] \, dH(\varepsilon) + \int_{B_s} [RA_s\varepsilon - d_s(\varepsilon)] \, dH(\varepsilon),$$

and in the alternative contract their consumption is given by

$$(59)\quad \int_{N_s} [A_s\varepsilon - d_s + a] \, dH(\varepsilon) + \int_{M} [A_s\varepsilon - d_s + a] \, dH(\varepsilon) + \int_{B_s/M} [RA_s\varepsilon - d_s(\varepsilon) + a] \, dH(\varepsilon),$$

which we know, from (58), equals $C_s + \int_M (1-R)A_s\varepsilon \, dH(\varepsilon)$.

Under this alternative contract, the consumption of the managers satisfies the nonnegativity constraint. To see this, note that in all states but those in M, we have simply added a positive number a to the managers' consumption. To argue that consumption is positive for states in M, we note that under our contradiction hypothesis, the set N is to the left of M. Since the consumption of the managers in the alternative contract $A_s\varepsilon - d_s + a$ satisfies nonnegativity for any $\varepsilon \in N$, this same expression clearly satisfies nonnegativity for the region M, which has larger idiosyncratic shocks.

This alternative contract is clearly incentive compatible. For all states besides those in M, we have subtracted off a constant from the repayments of the managers so that the incentive constraints are automatically satisfied. We have switched M to a nonbankruptcy region, and the only incentive constraint that applies in this region is that the repayments are constant, which is satisfied by construction. Thus, we have established a contradiction. *Q.E.D.*

We now characterize the payments in the optimal contract. We let ε_s^* be shorthand for $\varepsilon_s^*(U_s)$. Any contract that is immune to renegotiation must maximize, say, the payoffs to the manager subject to the constraint that investors receive at least D_s. Furthermore, Lemma 1 implies that any contract that is immune to renegotiation must be of the form $c_s(\varepsilon) = A_s\varepsilon - d_s$ for $\varepsilon \geq \varepsilon_s^*$. Nonnegativity then implies that

(60) $d_s \leq A_s\varepsilon_s^*$ for $\varepsilon > \varepsilon_s^*$.

Incentive compatibility requires that

(61) $c_s(\varepsilon) = RA_s\varepsilon - d_s(\varepsilon) \geq A_s\varepsilon - d_s$ for $\varepsilon \leq \varepsilon_s^*$,

and nonnegativity requires that

(62) $d_s(\varepsilon) \leq RA_s\varepsilon$ for $\varepsilon \leq \varepsilon_s^*$.

Therefore, any contract that is immune to renegotiation must solve

$$\max_{\varepsilon_s^*, d_s(\varepsilon), d_s} \int_{\underline{\varepsilon}}^{\varepsilon_s^*} [RA_s\varepsilon - d_s(\varepsilon)]\ dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} (A_s\varepsilon - d_s)\ dH(\varepsilon)$$

subject to (60), (61), (62), and

(63) $$\int_{\underline{\varepsilon}}^{\varepsilon_s^*} d_s(\varepsilon) dH(\varepsilon) + d_s[1 - H(\varepsilon_s^*)] \geq D_s.$$

The solution to this problem depends on the size of the debt D_s owed to investors. If the debt is low enough, then there is no default, and managers pay a constant amount less than $A_s\underline{\varepsilon}$, whereas if the debt is higher, then there is default and payments are as we said. Finally, if D_s

is too large, then this problem does not have a solution because there is a maximal amount of expected payments D_s that can be raised by any contract that satisfies the constraints on this problem.

Proposition 2. A contract is immune to renegotiation if and only if it is a debt contract.

Proof. It is immediate that a debt contract is immune to renegotiation. We now show that if a contract is immune to renegotiation, it must be a debt contract. Consider the case that $D_s > A_s\underline{\varepsilon}$. First, from (60) and (63), we know that in order to generate payments of D_s to the investors, some bankruptcy is required, so that $\varepsilon_s^* > \underline{\varepsilon}$. We now show that $d_s = A_s\varepsilon_s^*$. The argument is by contradiction. Since $d_s \leq A_s\varepsilon_s^*$, we need only show that $d_s < A_s\varepsilon_s^*$ leads to a contradiction. Recall from (57) that $d_s(\varepsilon) < d_s$ for $\varepsilon \leq \varepsilon_s^*$.

We will construct an alternative contract that satisfies (60)–(63) and raises the payoffs to the manager. This alternative contract has a bankruptcy region $[\underline{\varepsilon}, \hat{\varepsilon}]$, where $A_s\hat{\varepsilon} = d_s$, so that $\hat{\varepsilon} < \varepsilon^*$. In this contract, set $\hat{d}_s(\varepsilon) = d_s(\varepsilon) - a$, where a is constructed so that it satisfies (63). Hence, a satisfies

$$(64)\quad D_s = \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} d_s(\varepsilon)\ dH(\varepsilon) + \int_{\hat{\varepsilon}}^{\varepsilon^*} d_s(\varepsilon)\ dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} d_s\ dH(\varepsilon) =$$

$$\int_{\underline{\varepsilon}}^{\hat{\varepsilon}} [d_s(\varepsilon) - a]\ dH(\varepsilon) + \int_{\hat{\varepsilon}}^{\varepsilon^*} (d_s - a)\ dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} (d_s - a)\ dH(\varepsilon).$$

Hence, $a = \int_{\hat{\varepsilon}}^{\varepsilon^*} [d_s - d_s(\varepsilon)]\ dH(\varepsilon)$, which (61) indicates is strictly positive. This alternative contract also satisfies (60)–(62) because we have simply reduced d_s and $d_s(\varepsilon)$ by a.

We now show that in the alternative contract, the expected consumption of managers is higher than in the original contract. The consumption of the managers in the original contract is given by

$$C_s = \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} [RA_s\varepsilon - d_s(\varepsilon)]\ dH(\varepsilon) + \int_{\hat{\varepsilon}}^{\varepsilon^*} [RA_s\varepsilon - d_s(\varepsilon)]\ dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} (A_s\varepsilon - d_s)\ dH(\varepsilon),$$

and in the alternative contract it is given by

$$(65)\quad \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} [RA_s\varepsilon - d_s(\varepsilon) + a]\; dH(\varepsilon) + \int_{\hat{\varepsilon}}^{\varepsilon^*} (A_s\varepsilon - d_s + a)\; dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} (A_s\varepsilon - d_s + a)\; dH(\varepsilon),$$

which, using (64), equals $C_s + \int_{\hat{\varepsilon}}^{\varepsilon^*}(1-R)A_s\varepsilon\; dH(\varepsilon)$. Since $R < 1$, the managers' expected payoff is strictly higher. Hence, we have proved the desired result for the case that $D_s > A_s\underline{\varepsilon}$.

Consider, next, the case that $D_s \leq A_s\underline{\varepsilon}$. Clearly, it is feasible to have no bankruptcy and repay the investors D_s. Since bankruptcy simply wastes resources, it is optimal to set $\varepsilon_s^* = \underline{\varepsilon}$, and from (60), $d_s \leq A_s\underline{\varepsilon}$.

We now show that the managers' and the investors' consumption has the desired form in the bankruptcy region. Suppose, by way of contradiction, that $\int_{\underline{\varepsilon}}^{\varepsilon^*} c_s(\varepsilon)dH(\varepsilon) > 0$. Consider an alternative contract that leaves the bankruptcy set as well as the expected consumption of managers and investors unchanged. This contract reduces the managers' consumption in the bankruptcy set to zero and raises the managers' consumption in the nonbankruptcy interval by an amount that leaves overall expected consumption the same. Since the bankruptcy region is unchanged, this alternative contract gives the same expected payoffs to the investors as the original contract but has the property that $A_s\varepsilon_s^* > d_s$. From the first step, however, we know that any such contract is strictly dominated by the optimal contract. This gives us a contradiction. *Q.E.D.*

B. Proving Proposition 6

Proposition 6. If $\beta \geq \bar{\beta}$, then the efficient outcome is sustainable, and if $\beta < \bar{\beta}$, with (40), then any equilibrium allocation has bailouts, so that $\tau > 0$, and at the equilibrium level of debt $D_L, z(D_L) = 1$ and $d_L^g(D_L) < D_L$.

Proof. If $\beta \geq \bar{\beta}$, then the result follows from Proposition 5.

Consider $\beta < \bar{\beta}$. By way of contradiction, suppose that the equilibrium has no bailouts along the equilibrium outcome path. Let U be the utility of a manager associated with such an equilibrium. Since the efficient outcome U^{CE} is not sustainable, this purported equilibrium has $U < U^{CE}$. We will construct a deviation for an individual firm that leads to utility $\hat{U}$ with $\hat{U} \geq U^{CE} > U$, which will contradict that the original outcomes were equilibrium outcomes.

The deviation is to choose the debt level in the distressed state $\hat{d}_L$ equal to that in the competitive equilibrium, namely, d_L^{CE}. For this deviation, the bailout authority's policy is either to not alter the bankruptcy cutoff or to alter it. Either way, we will show a contradiction.

Suppose first that the government's strategy is to not alter the bankruptcy cutoff (either by not buying the debt or by buying it and then not renegotiating the terms of the debt). The deviation contract implements the competitive equilibrium so that $\hat{U} = U^{CE}$ and hence improves on that original allocation. We have established a contradiction.

Suppose next that the government's strategy is to buy this debt at its market value $M(d_L^{CE})$ and renegotiate the bankruptcy cutoff from ε_L^{CE} to some lower level ε_L^g. We will show that under (40), the value of utility under the deviation satisfies $\hat{U} > U^{CE}$. Since $U^{CE} > U$, the purported equilibrium level of utility, we have a contradiction.

We turn now to showing that $\hat{U} > U^{CE}$. To see this, consider some arbitrary ε_L^g. Let the probability of the healthy state p and d_H associated with ε_L^g be given by the solutions to the participation constraint

$$(66)\quad pd_H + (1-p)M^{CE} = 1$$

and the manager's effort incentive constraint

$$(67)\quad A_H - d_H - A_L \int_{\varepsilon_L^g}^{\bar{\varepsilon}} (\varepsilon - \varepsilon_L^g)\, dH(\varepsilon) - v'(p) = 0,$$

where M^{CE} is the market value of debt in the competitive equilibrium. The manager's utility associated with the cutoff ε_L^g in the distressed state is given by

$$(68)\quad U(\varepsilon_L^g) = p\,[A_H - d_H] + (1-p)A_L \int_{\varepsilon_L^g}^{\bar{\varepsilon}} (\varepsilon - \varepsilon_L^g)\, dH(\varepsilon) - v(p).$$

We will show that for all $\varepsilon_L^g < \varepsilon_L^{CE}$, $U(\varepsilon_L^g) > U(\varepsilon_L^{CE})$, which, since $U(\varepsilon_L^{CE}) > U$, yields the desired contradiction.

To see this, totally differentiate the participation constraint (66) and the manager's incentive constraint (67) holding M^{CE} fixed. Differentiating the participation constraint gives that

$$(69)\quad \frac{\partial p}{\partial \varepsilon_L^g} = \left(\frac{p}{M^{CE} - d_H}\right) \frac{\partial d_H}{\partial \varepsilon_L^g}.$$

Differentiating the manager's incentive constraint and substituting for (69) gives that

$$(70)\quad \frac{\partial d_H}{\partial \varepsilon_L^g} = \frac{pv''(p)}{M^{CE} - d_H + pv''(p)} \left[A_L(1 - H(\varepsilon_L^g)\right].$$

Next, differentiating the utility function (68) and substituting for $\partial d_H / \partial \varepsilon_L^g$ from (70) and using (66) to substitute for $M^{CE} - d_H$ gives that $\partial U / \partial \varepsilon_L^g$ has the same sign as

$$(71)\quad -\left[\frac{p^3 v''(p)}{p^2 v''(p) - (1 - M^{CE})}\right] - (1 - p).$$

Since $M^{CE} < 1$, if we can show that $p > p_s$, then inequality (40) implies that $\partial U / \partial \varepsilon_L^g < 0$; and since $\varepsilon_L^g < \varepsilon_L^{CE}$, the utility of this deviation $U(\varepsilon_L^g) > U(\varepsilon_L^{CE})$, and we will have established the desired contradiction.

We now show that $p > p_s$. To see this, note that p_s solves the analogs of (66) and (67) with M^{CE} replaced by $A_L \underline{\varepsilon}$ and ε_L^g replaced by $\underline{\varepsilon}$. Since M^{CE} is larger than $A_L \underline{\varepsilon}$ in this deviation, the revenues in the distressed state are higher than those in the full bailout equilibrium. Thus, it is possible to increase the manager's consumption in the healthy state, and since $\varepsilon_L^g \geq \underline{\varepsilon}$, the manager's consumption in the distressed state in the deviation is lower than in the full bailout equilibrium. Both of these forces lead the manager to exert effort, so that $p > p_s$. *Q.E.D.*

C. Proving Proposition 9

Proposition 9. Under the monotonicity condition (20) and with $p_H(e, S)$ given by (52), if A_H is sufficiently high and α is sufficiently small, then optimal regulation is countercyclical.

We establish Proposition 9 using a series of lemmas. Given our setup, establishing this proposition amounts to conducting a comparative statics exercise on the optimal contract when varying γ. We will show that the static gain from eliminating bankruptcy is higher when γ is lower. Proposition 8 then implies that regulation is optimal if and only if γ is low enough.

We conduct this comparative static exercise by comparing outcomes in two states: a *recession state* with success probability of $p_H(e)$, so that γ is normalized to zero, and a *boom state* with probability $\hat{p}_H(e) = p_H(e) + \hat{\gamma}$, where $\hat{\gamma} > 0$. Let d_H, d_L, e and $\hat{d}_H, \hat{d}_L, \hat{e}$ denote the solutions to the private contracting problem for the two economies, and let U and $\hat{U}$ denote the corresponding utilities of the manager. Throughout we will use the first-order conditions for d_L, d_H, and e in the competitive equilibrium, namely,

$$(72) \quad -p_H - \phi p_H' + \lambda p_H = 0$$

$$(73) \quad -(1-p_H)(1-H) + \phi p_H'(1-H) + \lambda(1-p_H)\left[1 - H - (1-R)\frac{d_L}{A_L}h\right] = 0$$

$$(74) \quad \begin{array}{l} \phi p_H'' \left[A_H - d_H - \int_{d_L/A_L}^{\bar{\varepsilon}} (A_L\varepsilon - d_L)\, dH(\varepsilon)\right] \\ +\lambda p_H' \left[d_H - R\int_{\underline{\varepsilon}}^{d_L/A_L} A_L\varepsilon\, dH(\varepsilon) - d_L\left(1 - H\left(\frac{d_L}{A_L}\right)\right)\right] = 0, \end{array}$$

where ϕ is the multiplier on the manager's incentive constraint and λ is the multiplier on the investor participation constraint. It will be convenient to let $c(d_L) = \int_{d_L/A_L}^{\bar{\varepsilon}} (A_L\varepsilon - d_L)\, dH(\varepsilon)$, $R(d_L) = R\int_{\underline{\varepsilon}}^{d_L/A_L} A_L\varepsilon\, dH(\varepsilon) + d_L\left(1 - H\left(\frac{d_L}{A_L}\right)\right)$, and $\kappa = A_H - d_H - c(d_L)$. Note for later that $R(d_L)$ is increasing in d_L and $c(d_L)$ is decreasing in d_L.

Throughout we will make use of some simple results. The first is that

$$(75) \quad d_H - R(d_L) > 0.$$

This result follows by rewriting the investor participation constraint

$$p_H(e)\left[d_H - R(d_L)\right] + R(d_L) = 1$$

and noting that $R(d_L) < 1$ because $A_L < 1$.

We first show that under the solution to the optimal contract, the probability of a healthy state is higher in the boom state than in the recession state.

Lemma 2. Under the monotonicity condition (20), $\hat{p}_H(\hat{e}) > p_H(e)$.

Proof. We begin by proving that the multiplier on the participation constraint for the boom state $\hat{\lambda}$ is less than the corresponding multiplier λ for the recession state. We do so by showing that the constraint set is more relaxed in the boom state than in the recession state. To see this result, note that the contract in the boom state solves

$$(76) \quad \max \left(p_H(e) + \hat{\gamma}\right)\kappa + c(d_L) - e$$

subject to

$$p_H'(e)\kappa = 1$$

$$[p_H(e) + \hat{\gamma}]\ d_H + [1 - p_H(e) - \hat{\gamma}]R(d_L) \geq 1,$$

with solution $(\hat{d}_H, \hat{d}_L, \hat{e})$. The problem (76) can be rewritten as one of choosing d_H, d_L, and e to solve the following problem, taking as given $\hat{\kappa}, \hat{d}_H$, and $\hat{d}_L$:

$$(77) \quad \max p_H(e)\kappa + c(d_L) - e + \hat{\gamma}\hat{\kappa}$$

subject to

$$p_H'(e)\kappa = 1$$

$$p_H(e)d_H + [1 - p_H(e)]R(d_L) + \hat{\gamma}\left[\hat{d}_H - R(\hat{d}_L)\right] \geq 1.$$

Since any solution to (76) is feasible for (77) and any solution to (77) is feasible for (76), the solutions coincide. Note the similarity between (77) and the contracting problem in the recession state. The differences are that the investor participation constraint is relaxed, which follows from (75), and that a constant has been added to the maximand. Since the investor participation constraint has been relaxed and the constant in the maximand does not affect the multiplier, we have that $\hat{\lambda} < \lambda$.

Now we can combine (72) and (73) to obtain

$$(78)\quad \lambda\left[1-(1-p_H)(1-R)\frac{d_L}{A_L}\frac{h}{1-H}\right]=1.$$

Since $\hat{\lambda}<\lambda$, we know that

$$(79)\quad (1-\hat{p}_H)\left(\frac{\hat{d}_L}{A_L}\right)\left(\frac{\hat{h}}{1-\hat{H}}\right)<(1-p_H)\left(\frac{d_L}{A_L}\right)\left(\frac{h}{1-H}\right).$$

Now assume by way of contradiction that $\hat{p}_H(\hat{e})<p_H(e)$. Then, under (20), inequality (79) implies that $\hat{d}_L<d_L$.

We next show that utility maximization implies that $\hat{d}_L>d_L$, which gives us a contradiction. Note, for later, that $\hat{p}_H(\hat{e})=p_H(\hat{e})+\hat{\gamma}<p_H(e)$ implies that $\hat{e}<e$. Next we define an intermediate allocation (U^*,d_H^*,d_L^*,e^*) from the contract in the recession state (d_H,d_L,e) by letting $d_L^*=d_L$ but decreasing d_H^* from d_H, so that at the intermediate allocation, the investor gets the same payment as in the recession state. That is,

$$[p_H(e)+\hat{\gamma}]\ =d_H^*+[1-p_H(e)-\hat{\gamma}]R(d_L)=p_H(e)d_H+[1-p_H(e)]R(d_L).$$

Clearly, since $d_H^*<d_H$,

$$\kappa^*=A_H-d_H^*-c(d_L)>\kappa=A_H-d_H-c(d_L).$$

Then from the manager's incentive constraint, $p_H'(e^*)\kappa^*=1$ implies that $e^*>e$.

Since $\hat{U}$ is the optimal allocation, it follows that

$$\hat{U}\geq U^*\geq U.$$

In what follows, it is useful to decompose the utility of the manager into two parts. To do so, let the utility of the manager under any contract be given by $W(\kappa)+c(d_L)$, where $W(\kappa)=p_H(e)\kappa-e$, where e solves $p_H'(e)\kappa=1$. Note that $W'(\kappa)=p_H>0$. Under the contradiction hypothesis, $\hat{e} < e < e^*$, so that $W(\hat{\kappa}) < W(\kappa) < W(\kappa^*)$ and

$$\hat{U} = W(\hat{\kappa}) + \hat{\gamma}\hat{\kappa} + c(\hat{d}_L) \geq W(\kappa^*) + \hat{\gamma}\kappa^* + c(d_L^*).$$

Since $W(\hat{\kappa}) < W(\kappa^*)$ and $\hat{\kappa} < \kappa^*$, it follows that $c(\hat{d}_L) > c(d_L^*) = c(d_L)$. Since c is an increasing function of d_L, this implies that $\hat{d}_L > d_L$, which is the desired contradiction. *Q.E.D.*

With this lemma, we can now establish the following proposition:

Proposition 9. Under (20) and (52), if A_H is sufficiently high and α is sufficiently small, then optimal regulation is countercyclical.

Proof. From Proposition 8 it suffices to prove that lost resources L in the recession state are greater than lost resources $\hat{L}$ in the boom state. Clearly, if $\hat{d}_L < d_L$, then from the earlier lemma the proposition follows. Suppose by way of contradiction that $\hat{d}_L > d_L$. We first show that $\hat{e} > e$ under this hypothesis. To see this, note from the investor participation constraint that

$$\hat{p}_H(\hat{e})\left[\hat{d}_H - R(\hat{d}_L)\right] + R(\hat{d}_L) = p_H(e)\left[d_H - R(d_L)\right] + R(d_L).$$

Since $R(d_L)$ is increasing in d_L under our contradiction hypothesis, $R(\hat{d}_L) > R(d_L)$. This fact, together with the result from Lemma 2 that $\hat{p}_H(\hat{e}) > p_H(e)$, implies that

$$\hat{d}_H - R(\hat{d}_L) < d_H - R(d_L).$$

Simple algebra establishes that

$$\hat{\kappa} = A_H - \hat{d}_H - c(\hat{d}_L) > A_H - d_H - c(d_L) = \kappa.$$

From the manager's incentive constraint, it then follows that $\hat{e} > e$.

Finally, we can manipulate (72)–(74) to obtain

$$(80) \quad -(1 - p_H)\left(\frac{p_H}{p'_H}\right)^2\left(\frac{p''_H}{p'_H}\right) = \frac{1 - R(d_L)}{(1 - R)\frac{d_L}{A_L}\frac{h}{1-H}}.$$

Note that if we use our functional form assumption, we can evaluate the left side of (80) at $\hat{p}_H = p_H + \hat{\gamma}$ and write it as

$$f(e, \hat{\gamma}) = (1 - \alpha)(1 - p_H - \hat{\gamma}) \left(\frac{p_H + \hat{\gamma}}{p'_H} \right)^2 \frac{1}{e}.$$

It is easy to show that if $p_H < 2/3$, then f is increasing in $\hat{\gamma}$, and if $\alpha < 1/2$, then f is increasing in e.

Thus, the left side of (80) is greater in the boom state than in the recession state. Note that since $R(d_L)$ is increasing in d_L and we have assumed monotonicity, the right side of (80) is decreasing in d_L. Under our contradiction hypothesis, the right side of (80) is less in the boom state than in the recession state. This result contradicts that (80) holds as an equality, which is a necessary condition for optimality. *Q.E.D.*