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ABSTRACT

Motivated by recent empirical work, this paper formalizes a theory of competitive savings - an arms race in household savings for mating competition that is made more fierce by an increase in the male-to-female ratio in the pre-marital cohort. Relative to the empirical work, the theory can clarify a number of important questions: What determines the strength of the savings response by males (or households with a son)? Can women (or households with a daughter) dis-save? What are the conditions under which aggregate savings would go up in response to a higher sex ratio? This theory can potentially help to understand the savings patterns in China, India, Vietnam, Singapore, Hong Kong, and other economies that have experienced a dramatic increase in the pre-marital sex ratio.

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1 Introduction

Recent empirical work suggests that one explanation for the rapid rise in the household savings rate in China, India, Singapore, Vietnam and several other economies is an arms race in savings for competition for marriage partners triggered by a rise in the pre-marital sex ratio, a phenomenon that has erupted vigorously since the beginning of the 21st century (Wei and Zhang, 2011). This source of incremental savings - dubbed as the competitive saving motive - is distinct from the precautionary saving motive or savings for life-cycle reasons, the relatively more standard explanations for household savings. The competitive saving motive can be quantitatively important. It is estimated by Wei and Zhang (2011) to account for half of the observed increase in the Chinese household savings rate in recent years. Without taking this into account, one would not have a complete picture of the underlying causes for the global current account imbalances, and might be prone to write incorrect prescriptions for the problem.

Because the existing empirical work is not accompanied by a formal theory, it leaves many important questions unanswered. For example, what determines the strength of the competitive savings motive by males (when there is a relative surplus of males)? What is the effect of a higher sex ratio on the aggregate savings given the potential that females may under-save? The goal of this paper is to develop a formal theory of the competitive saving motive that can clarify these questions. With the theory, we can also assess welfare implications of the competitive saving motive.

We construct a simple overlapping generations (OLG) model with two sexes and a desire to marry. To focus on the macroeconomic implications of sex ratio imbalances, we intentionally shut down channels such as the usual precautionary savings motive, habit formation, culture, and financial development. Because it is an OLG model, there are still life-cycle considerations, which, however, do not lead to current account imbalances on their own.

Under reasonable conditions, we show that men respond to a rise in the sex ratio by raising their savings rates. Moreover, the increment in their savings is always enough to offset any decrease in women's savings. As a result, the aggregate savings rises with the sex ratio. We also discuss a number of extensions that aim to allow for additional realism: (a) incorporate parental savings for children, (c) introduce intra-household bargaining, (c) consider an OLG structure in which each generation lives for 50 periods and makes savings decisions in multiple periods, and (d) allow for income inequality. In each case, under reasonably general conditions, both the aggregate savings rate and the current account rise in response to a rise in the sex ratio. (Some of the extensions are reported in online appendices.)

To check if the model can deliver an effect that is economically significant, we employ quantitative calibrations. In the benchmark case, for a small open economy, as the sex ratio rises from 1 to 1.15, the economy-wide savings rate and the current account will both rise by more than 6% of GDP. We also consider a case of two large economies, whose relative sizes and income levels are calibrated to mimic China and the United States. The synthetic United States is assumed to always have a balanced sex ratio, while the synthetic China experiences a significant rise in the sex ratio. The rise in China's sex ratio produces a rise in its current account surplus, and a corresponding rise in the current account deficit for the United States. The magnitudes of the current account imbalances in the simulations (about 4.4% of GDP for China and -1.5% of GDP for the United States when China's sex ratio rises from 1 to 1.15) are around one-half of the actual current account imbalances observed in the data. While the sex ratio imbalance is not the only factor affecting the global current account imbalances in recent years, it could be one of the significant, and yet thus far unrecognized, factors. (This extension is also reported in an online appendix.)

A desire to enhance one's prospects in the marriage market through a higher level of wealth could be a motive for savings even in countries with a balanced sex ratio. But such a motive is not as easy to detect when the competition is modest. When the sex ratio gets out of balance, obtaining a marriage partner becomes much less assured. A host of behaviors that are motivated by a desire to succeed in the marriage market may become magnified. But sex ratio imbalances so far have not been investigated by macroeconomists. This may be a serious omission. A sex ratio imbalance at birth and in the marriage age cohort is a common demographic feature in many economies, especially in Asia, such as Korea, India, Vietnam, Singapore, Taiwan and Hong Kong, in addition to China. In many economies, parents have a preference for a son over a daughter. This used to lead to large families, not necessarily an unbalanced sex ratio. However, in the last three decades, as the technology to detect the gender of a fetus (Ultrasound B) has become less expensive and more widely available, many more parents engage in selective abortions in favor of a son, resulting in an increasing relative surplus of men. The spread of technology started in the early 1980s and accelerated quickly afterwards. 1985 was the first year in which half of the hospitals in China had acquired at least one Ultrasound B machine. By the early 1990s, all county-level hospitals had at least one such machine (Ebenstein, Li, and Meng, 2010). The strict family planning policy in China, introduced in the early 1980s, has induced Chinese parents to engage in sex-selective abortions more aggressively than their counterparts in other countries. The sex ratio at birth in China rose from 106 boys per hundred girls in 1980 to 122 boys per hundred girls in 1997 (see Wei and Zhang, 2011, for more detail). It may not be a coincidence that the Chinese current account surplus started to garner international attention around 2002 just when the first cohort born after the implementation of the strict family planning policy was entering the marriage market.

In the benchmark model and numerical examples, we assume an exogenous sex ratio. While the sex ratio is endogenous in the long-run as parental preference evolves, the assumption of an exogeous sex ratio can be defended on two grounds. First, the technology that enables the rapid rise in the sex ratio has only become inexpensive and widely accessible in developing countries within the last 25 years or so. As a result, it is reasonable to think that the rising sex ratio affects only the relatively young cohorts' savings decisions, but not those who have passed half of their working careers. Second, data suggests that if the preference for a son has a mean-reverting property, it must be a very slow-moving process. Almost all countries that have a skewed sex ratio today have exhibited a gradual climb over the last decade or two. This suggests that a systematic reversal of the sex ratio is unlikely to happen in most economies in the short run. In any case, we also consider endogenous sex ratios in an extension and find that all qualitative results still hold.

To see if the theoretical predicion has any support in the data, we check if a country's savings rate is systematically linked to its sex ratio. After controlling for the effects on the savings rate from income, the share of working age people in the population (i.e., a proxy for the life cycle theory), the ratio of private bank credit to GDP (a proxy for financial development), and social security expenditure as a share of GDP (a proxy for the precautionary savings motive), we find that the sex ratio, the savings rate, and the current account as a share of GDP are strongly positively correlated.

There are three bodies of work that are related to the current paper. First, the literature on status goods, positional goods, and social norms (e.g., Cole, Mailath and Postlewaite, 1992, Corneo and Jeanne, 1999, Hopkins and Kornienko, 2004 and 2009) has offered many useful insights. One key point is that when wealth can improve one's social status (including improving one's standing in the marriage market), in addition to affording a greater amount of consumption goods, there is an extra incentive to save. This element is in our model as well. However, all existing theories on status goods feature a balanced sex ratio. Yet, an unbalanced sex ratio presents some non-trivial challenges. In particular, while a rise in the sex ratio is an unfavorable shock to men (or parents with sons), it is a favorable shock to women (or parents with daughters). Could the latter group strategically reduce their savings so as to completely offset whatever increments in savings men or parents with sons may have? In other words, the impact on aggregate savings appears ambiguous. Our model will address this question. In any case, the literature on status goods has no discernible impact in policy circles. For example, while there are voluminous documents produced by the International Monetary Fund or speeches by U.S. officials on China's high savings rate and large current account surplus, no single paper or speech thus far has pointed to a possible connection with its high sex ratio imbalance.

A second related literature is the economics of family, which is too vast to be summarized here comprehensively. One interesting insight of this literature is that a married couple's consumption has a partial public goods feature (Browning, Bourguignon and Chiappori, 1994; Donni, 2006). We make use of this feature in our model as well. None of the papers in this literature explores the general equilibrium implications for aggregate savings from a change in the sex ratio.

The third literature examines empirically the causes of a rise in the sex ratio. The key insight is that the proximate cause responsible for a majority of the recent rise in the sex ratio imbalance is sex-selective abortions, which have been made increasingly possible by the spread of Ultrasound B machines. There are two deeper causes for parental willingness to disproportionately abort female fetuses. The first is the parental preference for sons, which in part has to do with the relatively inferior economic status of women. When the economic status of women improves, sex-selective abortions appear to decline (Qian, 2008). The second is either something that leads parents to voluntarily choose to have fewer children than earlier generations, or a government policy that limits the number of children a couple can have. In regions of China where the family planning policy is less strictly enforced, there is also less sex ratio imbalance (Wei and Zhang, 2009). Bhaskar (2011) examines parental sex selections and their welfare consequences.

The rest of the paper is organized as follows: in Section 2, we provide some suggestive data patterns that motivate the theory. In Section 3, we present a benchmark model that delivers the main mechanism. In Section 4, we consider an extension that allows for parental savings and endogenous sex ratios. In section 5, we also calibrate the model to see if the sex ratio imbalance can produce changes in the aggregate savings rate and current account whose magnitudes are economically significant. Finally, in section 6, we offer concluding remarks and discuss possible future research.

2 Some Data Patterns

To motivate the theory, we discuss two types of empirical approaches that allow us to check for plausibility and empirical importance of the theory. First, we provide some cross-country evidence on the relationships between a country's sex ratio and its savings rate, and between the sex ratio and its current account. Second, we review household-level evidence from China on the association between sex ratios and savings rates.

2.1 Cross country data patterns

We run a panel regression of the savings rate on the sex ratio and other control variables including country and year fixed effects. To be precise, the specification equation is the following:

$$savings.rate_{it} = \beta_0 + \beta_1 \cdot |\text{sex ratio}_{it} - 1| + \beta_2 \cdot |\text{sex ratio}_{it} - 1| \cdot LowSR_{it} + \beta_3 Z_{it} + \varepsilon_{it}$$

where $savings.rate_{it}$ is the ratio of country i's national savings to its GDP in year t. The sex ratio is defined as the male to female ratio for the age group of 10-24 (from the United Nations Population Division). In our theory, men and women are symmetric. The savings rate is predicted to be higher with either a surplus of men or a surplus of women. For this reason, our key regressor is $|\text{sex ratio}_{it} - 1|$. Our key coefficient is β_1. If our story is right, this coefficient should be positive.

In reality, the ability to tolerate singlehood could be different between men and women, and the desire to compete in the marriage market by raising savings could also be different. That is, for a given rise in the gender imbalance, the savings response could be different, depending on whether an economy has too many men or too many women. For this reason, we create a dummy LowSR to indicate cases in which there is a surplus of young women in the marriage market, or when sex $\text{ratio}_{it} < 1$. We add as a second regressor an interaction term between this dummy and the extent of gender ratio imbalance, $|\text{sex ratio}_{it} - 1| \cdot LowSR_{it}$. The coefficient before this regressor, β_2, tells us if the aggregate savings response to a higher sex ratio depends on whether the gender ratio imbalance is in the direction of excess men or excess women.

Since our theory is about savings in the private sector, we control for the government deficit in the regression. Our choice of other control variables is guided by the life-cycle theory, precautionary saving theory, and financial development theory. We therefore include in Z_i log per capita GDP, dependency ratio (a proxy for life-cycle theory), private credit to GDP ratio (a proxy for financial development), and both country and year fixed effects. Unfortunately, we are not able to obtain a panel data on social security enrollment or social security coverage; we therefore assume that the country fixed effects capture the absence or presence of a social security system and the generosity of the system across countries.

Government deficit data are obtained from the IMF's World Economic Outlook (WEO) database. Current account, GDP, the share of working age in the population and private credit to GDP ratio are obtained from the World Bank's WDI database. The intertemporal theory predicts that a country's current account should be sensitive to temporary shocks. The sex ratio data is from the United Nations' Population Division.

The first four columns of Table 1 report a set of savings regressions with a progressively expanding set of control variables. In each regression, we have a positive and statistically significant coefficient on the sex ratio: as the sex ratio becomes more unbalanced, the savings rate tends to go up. Using column (4) of Table 1, we can illustrate the magnitude of the estimates: a rise in the sex ratio from 1.00 to 1.10 is associated with a rise in the savings rate by 5.8 percent of GDP (=58.5% x 0.10).

Because β_2 is not statistically significant, we cannot reject the null that the aggregate savings response to a higher sex ratio is the same regardless of whether the surplus gender is male or female. However, because there are relatively few observations with the sex ratio less than one. This coefficient is not precisely estimated, and we need to exercise caution in interpreting it. Interestingly, the coefficient on the dummy for excess females itself is positive and significant. This suggests that the savings rate for such countries tend to be higher than sample average. We do not have a good explanation for this phenomenon, but it could reflect other differences between surplus female and surplus male countries that are not related to the sex ratio per se.

We comment briefly on other control variables. We find that a higher income is associated with a higher savings rate, which is a quite typical finding in the literature. A higher government deficit is associated with a lower savings rate. The financial development index (private credit as % of GDP) sometimes has a negative and significant sign; the negative sign is consistent with Caballero, Farhi, and Gourinchas (2008) and Ju and Wei (2010 and 2011). The dependence ratio is statistically insignificant, which suggests that life cycle considerations may not play a strong role in explaining cross country variations in the savings rate.

Some countries report savings rates in excess of 80% of GDP and are likely outliers. To ensure such observations do not drive the data pattern, we exclude these observations and re-do the regressions and report them in Columns 5-8 of Table 1. This turns out to make little difference for the basic results.

We also examine the relationship between a country's current account (as % of GDP) and its sex ratio, and report the results in Table 2. The coefficients on the sex ratio in all the regressions are positive and statistically significant. This means that the current account tends to be higher in countries with a higher sex ratio. To illustrate the economic magnitude of the estimates, we use the last column in Table 2: A rise in the sex ratio from 1.00 to 1.10 is projected to be associated with a rise in the current account by 2.2% of GDP (=22.27% x 0.10).

There are important caveats with the empirical patterns. First, in spite of our best efforts, there may still be potential control variables that are missing from our list. Second, the sex ratio can be endogenous and/or measured with errors. This would normally call for an instrumental variable approach. At this point, we are not able to come up with convincing instrumental variables in a cross-country context. For these reasons, it is important to review some micro-evidence from within China.

2.2 Cross-household and cross-region evidence from China

The sex ratio for the Chinese pre-marital cohort increased from being basically balanced in 1990 to about 115 young men per 100 young women in 2007. Its household savings rate (out of disposable income) almost doubled from 16% to 30% during the same period. While China is not the only economy with a high sex ratio (and a high savings rate), it is the one with the most extreme sex ratio imbalance at the moment, and, because of its size, its savings rate and current account attract the most international attention. For this reason, it is useful to highlight a few empirical patterns documented in Wei and Zhang (2011) that are most relevant for the current paper.

First, let us start with Chinese households' self-reported reasons for savings. A survey of rural households (Chinese household income project in 2002) asked households why they save. There were seven possible categories for saving rationale in the questionnaire: (1) children's wedding, (2) children's education, (3) bequest to children, (4) building a house, (5) (own) retirement, (6) medical expenses, and (7) others. The first three reasons could be grouped under the heading of "savings directly for children." If we just focus on families with an unmarried child, one sees a stunning difference between families with a son versus those with a daughter. 29.8% of families with a son list savings for their child's wedding as either the most or the second most important reason for savings, versus 18.3% of families with a daughter who do the same. Overall, 92.2% of son-families list one of the top three reasons as their primary reasons for savings, which is 5.8 percentage points higher than the percent of the daughter families who say the same. In comparison, 45.5% of daugher-families and 37.3% of son-families say their most or the second most important reason for savings is their own retirement. (Note that the sum of the percentage of households that list various reasons as the most or the second most important reason for savings can be more than 100% since a given household could list one category as the most important reason for savings, and another category as the second most important reason for savings.)

Second, we now summarize the relationship between household savings rates (out of disposable income) and local sex ratios (at the county or city level), holding constant other determinants of savings rate (household income, household head's age, gender, ethnicity, and educational level, and children's age, and whether there is a family member that has a major illness). What is most revealing for our theory is not just a direct comparison in the savings rates between son-families and daughter-families, but the effect of an interaction term between having an unmarried son and living in a region with a high local sex ratio. This exercise is interesting in China because the migration rate for the purpose of marriage is low (about 92% of marriages take place between a man and a woman from the same county). When focusing on families with a son in rural areas, Wei and Zhang report that these families' savings rate tends to be higher in regions with a more skewed sex ratio. In comparison, the savings rate by families with a daughter appears to be uncorrelated with the local sex ratio. Across Chinese cities, the savings rates by both son-families and daughter families tend to rise with the local sex ratio. These patterns are consistent with our model that allows for intra-family bargaining. When women (or their parents) are concerned with erosion of bargaining power within a family, they may not reduce their savings rate in response to a higher sex ratio. When the effect of intra-family bargaining dominates, the savings rate by daughter-families could rise in response to a rise in the sex ratio.

Third, across Chinese provinces, Wei and Zhang report a strong positive correlation between local savings rates and local sex ratios, controlling for the age structure of local population, per capita income, the share of employment in state-owned firms in the local labor force, and the share of local labor force enrolled in social security. To go from correlation to causality, Wei and Zhang employ variations in the local enforcement of family planning policy (including monetary penalties for violating birth quotas) as instruments for the sex ratio. The 2SLS estimation confirms the basic finding: regions with a higher sex ratio are also likely to have a higher household savings rate. Based on the 2SLS estimates, 40-60% of the rise in the household savings rate from 1990 to 2007 can be attributed to the observed rise in the sex ratio for the pre-marital age cohort during the period.

Overall, the evidence from within China is consistent with the theoretical predictions.

3 The Benchmark Model

We construct an overlapping generations model with two sexes. Both men and women live two periods: young and old. An individual (of either sex) receives an exogenous endowment in the first period and nothing in the second period. She or he consumes a part of the endowment in the first period and saves the rest for the second period.

A marriage can only take place between a man and a women in the same generation and at the beginning of their second period. Once married, the husband and the wife pool their first-period savings together and consume an identical amount in the second period. The second period consumption within a marriage has a partial public good feature. In other words, the husband and the wife can each consume more than half of their combined second period income - the exact proportion is an exogenous parameter to be explained below. Everyone is endowed with an ability to give his/her spouse some emotional utility (or "love" or "happiness"). This emotional utility is a random variable in the first period with a common and known distribution across all members of the same sex, and its value is realized and becomes public information when the individual enters the marriage market.

Each generation is characterized by an exogenous ratio of men to women $\phi(\geq 1)$. All men are identical *ex ante*, and all women are identical *ex ante*. Men and women are symmetric in all aspects except that the sex ratio may be unbalanced.

We describe the equilibrium in this economy in six steps. First, we start with a representative woman's optimization, followed by a representative man's optimization problem. Second, we describe how the marriage market works. Third, we perform comparative statics, in particular, on how the savings rates change in response to a rise in the sex ratio. Fourth, we consider a small open economy with production and discuss the current account response to a change in the sex ratio. Fifth, we solve for a two-country model in which the global interest rate is endogenous. Sixth, we use numerical calibrations to see if the model can deliver current account responses that are economically significant.

3.1 A Representative Woman's Optimization Problem

A representative woman makes her consumption/saving decisions in her first period, taking as given the choices made by men and all other women. If she is not married, her second-period consumption is

$$c_{2w,n} = Rs^w y^w$$

where R, y^w and s^w are the gross interest rate, her endowment, and savings rate, respectively.

If she is married (at the beginning of the second period), her second-period consumption is

$$c_{2w} = \kappa \left(Rs^w y^w + Rs^m y^m \right)$$

where y^m and s^m are her husband's endowment and savings rate, respectively. κ ($\frac{1}{2} \leq \kappa \leq 1$) represents the notion that consumption within a marriage is a public good with congestion. As an example, if a couple buys a car, both spouses can use it. When $\kappa = \frac{1}{2}$, the husband and the wife only consume private goods. In contrast, when $\kappa = 1$, all the consumption is a public good with no congestion[1].

She chooses her savings rate to maximize the following objective function:

$$V^w = \max_{s^w} u(c_{1w}) + \beta E \left[u(c_{2w}) + \eta^m \right]$$

[1] By assuming the same κ for the wife and the husband, we abstract from a discussion of bargaining within a household. In an extension later in the paper, we allow κ to be gender specifc, and to be a function of the sex ratio and the relative wealth levels of the two spouses, along the lines of Chiappori (1988 and 1992) and Browning and Chiappori (1998). This tends to make the response of the aggregate savings stronger to a given rise in the sex ratio.

subject to the budget constraints that

$$\begin{aligned} c_{1w} &= (1-s^w)y^w & (3.1) \\ c_{2w} &= \begin{cases} \kappa\left(Rs^w y^w + Rs^m y^m\right) & \text{if married} \\ Rs^w y^w & \text{otherwise} \end{cases} & (3.2) \end{aligned}$$

where V^w is her value function, and E is the expectation operator. η^m is the emotional utility (or "love") she obtains from her husband, which is a random variable with a distribution function F^m. Utility function $u(\cdot)$ satisfies the standard properties that $u' > 0$, and $u'' < 0$. The exact value of emotional utility is revealed at the beginning of the second period and becomes common knowledge at that time. Bhaskar (2011) also introduces a similar "love" variable.

3.2 A Representative Man's Optimization Problem

A representative man has a similar optimization problem as the representative woman. In particular, if he is not married, his second-period consumption is

$$c_{2m,n} = Rs^m y^m$$

If he is married, his second-period consumption is

$$c_{2m} = \kappa\left(Rs^w y^w + Rs^m y^m\right)$$

He chooses his savings rate to maximize the following value function:

$$V^m = \max_{s^m} u(c_{1m}) + \beta E\left[u(c_{2m}) + \eta^w\right]$$

subject to the budget constraints that

$$\begin{aligned} c_{1m} &= (1-s^m)y^m & (3.3) \\ c_{2m} &= \begin{cases} \kappa\left(Rs^w y^w + Rs^m y^m\right) & \text{if married} \\ Rs^m y^m & \text{otherwise} \end{cases} & (3.4) \end{aligned}$$

where V^m is his value function. η^w is the emotional utility he obtains from his wife, which is drawn from a distribution function F^w. We assume η^w and η^m are independent.

3.3 The Marriage Market[2]

In the marriage market, every woman (or man) ranks all members of the opposite sex by a combination of two criteria: (1) the level of wealth (which is determined solely by the first-period savings), and (2) the size of "love" he/she can obtain from his/her spouse. The weights on the two criteria are implied by the utility functions specified earlier. More precisely, woman i prefers a higher ranked man to a lower ranked one, where the rank on man j is given by $u(c_{2w,i,j})+\eta_j^m$. Symmetrically, man j assigns a rank to woman i based on the utility he can obtain from her $u(c_{2m,j,i})+\eta_i^w$. (To ensure that the preference is strict for men and women, when there is a tie in terms of the above criteria, we break the tie by assuming that a woman prefers j if $j < j'$ and a man does the same.) Note that "love" is not in the eyes of the beholder in the sense that every woman (man) has the same ranking over men (women).

The marriage market is assumed to follow the Gale-Shapley algorithm, which produces a unique and stable equilibrium of matching (Gale and Shapley, 1962; and Roth and Sotomayor, 1990). The algorithm specifies the following: (1) Each man proposes in the first round to his most preferred choice of woman. Each woman holds the proposal from her most preferred suitor and rejects the rest. (2) Any man who is rejected in round k-1 makes a new proposal in round k to his most preferred woman among those who have not yet rejected him. Each available women in round k "holds" the proposal from her most preferred man and rejects the rest. (3) The procedure repeats itself until no further proposals are made.[3]

With many women and men in the marriage market, all women (and all men) approximately form a continuum and each individual has a measure close to zero. Let I^w and I^m denote the continuum formed by women and men, respectively. We normalize I^w and let $I^w = (0, 1)$. Since the sex ratio is ϕ, the set of men $I^m = (0, \phi)$. Men and women are ordered in such a way that a higher value means a higher ranking by members of the opposite sex.

In equilibrium, there exists a unique mapping (π^w) for women in the marriage market.

$$\pi^w : I^w \rightarrow I^m$$

That is, woman i ($i \in I^w$) is mapped to man j ($j \in I^m$), given all the initial wealth and emotional utility draws. This implies a mapping from a combination (s_i^w, η_i^w) to another combination (s_j^m, η_j^m). In other words, for woman i, given all her rivals' (s_{-i}^w, η_{-i}^w) and all men's (s^m, η^m), the type of husband j she can marry depends on her (s_i^w, η_i^w). Before she enters the marriage market, she knows only the distribution of her own type but not the exact value. As a result, the type of her future husband (s_j^m,

[2] We use the word "market" informally here. The pairing of husbands and wives in this model is in fact not done through prices.

[3] If only women can propose and men respond with deferred acceptance, the same matching outcomes will emerge. What we have to rule out is that both men and women can propose, in which case, one cannot prove that the matching is unique.

η_j^m) is also a random variable. Woman i's second period expected utility is

$$\int \max\left[\ u\left(c_{2w,i,j}\right) + \eta^m_{\pi^w\left(i|s_i^w,\eta_i^w,s_{-i}^w,\eta_{-i}^w,s^m,\eta^m\right)},\quad u\left(Rs_i^w y_i^w\right)\ \right] dF^w\left(\eta_i^w\right)$$
$$= \int_{\bar{\pi}_i^w} \left[u\left(c_{2w,i,j}\right) + \eta^m_{\pi^w\left(i|s_i^w,\eta_i^w,s_{-i}^w,\eta_{-i}^w,s^m,\eta^m\right)}\right] dF^w\left(\eta_i^w\right) + \int^{\bar{\pi}_i^w} u\left(Rs_i^w y_i^w\right) dF^w\left(\eta_i^w\right)$$

where $\bar{\pi}_i^w$ is her threshold ranking on men such that she is indifferent between marriage or not. Any lower-ranked man, or any man with $\pi_i^w < \bar{\pi}_i^w$, won't be chosen by her.

Since we assume there are (weakly) fewer women than men, we expand the set I^w to $\widetilde{I}^w$ so that $\widetilde{I}^w = (0, \phi)$. In the expanded set, women in the marriage market start from value $\phi - 1$ to ϕ. The measure for women in the marriage market remains one. In equilibrium, there exists a unique mapping for men in the marriage market:

$$\pi^m : I^m \to \widetilde{I}^w$$

where π^m maps man j ($j \in I^m$) to woman i ($i \in I^w$). Those men who are matched with a low value $i < \phi - 1$ in set $\widetilde{I}^w$ will not be married. In that case, $\eta^w_{\pi^m(j)} = 0$ and $c_{2m,j,i} = Rs_j^m y_j^m$. In general, man j's second period expected utility is

$$\int \max\left[\ u\left(c_{2m,j,i}\right) + \eta^w_{\pi^m\left(j|s_j^m,\eta_j^m,s_{-j}^m,\eta_{-j}^m,s^w,\eta^w\right)},\quad u\left(Rs_j^m y_j^m\right)\ \right] dF^m\left(\eta_j^m\right)$$
$$= \int_{\bar{\pi}_j^m} \left[u\left(c_{2m,j,i}\right) + \eta^w_{\pi^m\left(j|s_j^m,\eta_j^m,s_{-j}^m,\eta_{-j}^m,s^w,\eta^w\right)}\right] dF^m\left(\eta_j^m\right) + \int^{\bar{\pi}_j^m} u\left(Rs_j^m y_j^m\right) dF^m\left(\eta_j^m\right)$$

where $\bar{\pi}_j^m$ is his threshold ranking on all women. Any woman with a poorer rank, $\pi_j^m < \bar{\pi}_j^m$, will not be chosen by him.

We assume that the density functions of η^m and η^w are continuously differentiable. Since all men (women) in the marriage market have identical problems, they make the same savings decisions. In equilibrium, a *positive assortative matching* emerges for those men and women who are matched. In other words, there exists a mapping M from η^w to η^m such that

$$\begin{aligned} 1 - F^w(\eta^w) &= \phi\left(1 - F^m\left(M(\eta^w)\right)\right) \\ &\Leftrightarrow \\ M(\eta^w) &= \left(F^m\right)^{-1}\left(\frac{F^w(\eta^w)}{\phi} + \frac{\phi - 1}{\phi}\right) \end{aligned}$$

For simplicity, we assume that η^w and η^m are drawn from the same distribution, $F^w = F^m = F$. The lowest possible value of emotional utility $\eta^{\min}$ is assumed to be sufficiently small (and can be negative) such that any person with a low realized value of emotional utility may not succeed in getting married. Define $\bar{\eta}^w$ and $\bar{\eta}^m$ as the threshold values of emotional utility for women and men, respectively, such that only those with emotional utilities higher than the threshold value will get married. In other words,

$$\bar{\eta}^w = \max\left\{u_{2m,n} - u_{2m}, M^{-1}\left(\bar{\eta}^m\right)\right\} \text{ and } \bar{\eta}^m = \max\left\{u_{2w,n} - u_{2w}, M\left(\bar{\eta}^w\right)\right\} \tag{3.5}$$

For woman i, given all her rivals' and men's savings decisions and η^w, her second period utility is

$$\delta_i^w u(\kappa\left(Rs_i^w y^w + Rs^m y^m\right)) + \left(1-\delta_i^w\right) u(Rs^w y^w) + \int_{\tilde{\eta}_i^w \geq \bar{\eta}^w} M(\tilde{\eta}_i^w) dF(\eta_i^w)$$

where $\tilde{\eta}_i^w = u(\kappa\left(Rs_i^w y^w + Rs^m y^m\right)) - u(\kappa\left(Rs^w y^w + Rs^m y^m\right)) + \eta_i^w$. δ_i^w is the probability that she will get married,

$$\begin{aligned}\delta_i^w &= \Pr\left(u(\kappa\left(Rs_i^w y^w + Rs^m y^m\right)) - u(\kappa\left(Rs^w y^w + Rs^m y^m\right)) + \eta_i^w \geq \bar{\eta}^w \middle| Rs^w y^w, Rs^m y^m\right) \\ &= 1 - F\left(\bar{\eta}^w - u(\kappa\left(Rs_i^w y^w + Rs^m y^m\right)) + u(\kappa\left(Rs^w y^w + Rs^m y^m\right))\right)\end{aligned} \tag{3.6}$$

Due to symmetry (i.e., all women are identical ex ante), we drop sub-index i for women in subsequent discussions. Given men's savings decisions, the first order condition for her optimization problem is

$$-u_{1w}' y^w + \beta\left[\begin{array}{c}\delta^w u_{2w}' \frac{\partial c_{2w}}{\partial s^w} + \left(1-\delta^w\right) u_{2w,n}' R y^w + \frac{\partial \int_{\tilde{\eta}^w \geq \bar{\eta}^w} M(\tilde{\eta}^w) dF(\eta^w)}{\partial s^w} \\ + \frac{\partial \delta^w}{\partial s^w}\left(u_{2w} - u_{2w,n}\right)\end{array}\right] = 0 \tag{3.7}$$

where

$$\begin{aligned}\frac{\partial \int_{\tilde{\eta}^w \geq \bar{\eta}^w} M(\tilde{\eta}^w) dF(\eta^w)}{\partial s^w} &= \kappa u_{2w}' R y^w \left[\int_{\bar{\eta}^w} M'(\eta^w) dF(\eta^w) + M(\bar{\eta}^w) f(\bar{\eta}^w)\right] \\ \frac{\partial \delta^w}{\partial s^w} &= f\left(\bar{\eta}^w\right) \kappa u_{2w}' R y^w\end{aligned}$$

Similarly, a representative man's second-period utility, given his rivals' and all women's savings decisions, is

$$\delta_j^m u\left(\kappa\left(Rs^w y^w + Rs_j^m y^m\right)\right) + (1-\delta_j^m) u(Rs_j^m y^m) + \int_{\tilde{\eta}_j^m \geq \bar{\eta}^m} M^{-1}\left(\tilde{\eta}_j^m\right) dF(\eta_j^m)$$

where $\tilde{\eta}_j^m = u\left(\kappa\left(Rs^w y^w + Rs_j^m y^m\right)\right) - u(\kappa\left(Rs^w y^w + Rs^m y^m\right)) + \eta_j^m$ and δ_j^m is his probability of marriage.

$$\begin{aligned}\delta_j^m &= \Pr\left(u\left(\kappa\left(Rs^w y^w + Rs_j^m y^m\right)\right) - u(\kappa\left(Rs^w y^w + Rs^m y^m\right) + \eta_j^m \geq \bar{\eta}^m \middle| Rs^w y^w, Rs^m y^m\right) \\ &= 1 - F\left(\bar{\eta}^m - u\left(\kappa\left(Rs^w y^w + Rs_j^m y^m\right)\right)\right) + u(\kappa\left(Rs^w y^w + Rs^m y^m\right))\end{aligned} \tag{3.8}$$

The first order condition for his optimization problem is

$$-u'_{1m}y^m + \beta \left[\begin{array}{c} \delta^m u'_{2m}\frac{\partial c_{2m}}{\partial s^m} + \frac{\partial \int_{\tilde{\eta}^m \geq \bar{\eta}^m} M^{-1}(\tilde{\eta}^m) dF(\eta^m)}{\partial s^m} + (1-\delta^m)\, u'_{2m,n} R y^m \\ +\frac{\partial \delta^m}{\partial s^m}\left(u_{2m} - u_{2m,n}\right) \end{array} \right] = 0 \qquad (3.9)$$

where

$$\begin{aligned} \frac{\partial \int_{\tilde{\eta}^m \geq \bar{\eta}^w} M^{-1}(\tilde{\eta}^m) dF(\eta^m)}{\partial s^m} &= \kappa u'_{2m} R y^m \left[\int_{\bar{\eta}^m} \frac{\partial M^{-1}(\eta^m)}{\partial \eta^m} dF(\eta^m) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m) \right] \\ \frac{\partial \delta^m}{\partial s^m} &= f\left(\bar{\eta}^m\right) \kappa u'_{2m} R y^m \end{aligned}$$

In the rest of the paper, we assume that the average value of emotional utility $E\eta$ is sufficiently high such that a representative man, ex ante, always prefers marriage to being single. For simplicity, we also assume $\beta R = 1$ througout the paper (except for the large country case).

3.4 Equilibrium Savings Rates

In the benchmark, we assume that all women and men automatically enter the marriage market (We will later consider an extension in which agents decide whether or not to enter the marriage market). An equilibrium is defined as a collecton of savings rates by men and women that solve their respective optimalization problems, taking all other men and women's decisions as given.

Definition 1 *An equilibrium is* $\{ s^w, s^m | \, y^w, y^m, F^w, F^m \}$ *that satisfies the following conditions:*

$$s_i^w = \arg\max \left(V_i^w \middle| \, s_{-i}^w, s^m, y^w, y^m, F^w, F^m \right)$$

and

$$s_j^m = \arg\max \left(V_j^m \middle| \, s^w, s_{-j}^m, y^w, y^m, F^w, F^m \right)$$

where i *and* j *stand for a representative woman and man, respectively, and* $-i$ *and* $-j$ *represent all women other than* i *and all men other than* j*, respectively.* $s^w = \left(s_i^w, s_{-i}^w\right)$ *and* $s^m = \left(s_j^m, s_{-j}^m\right)$ *are the sets of women's and men's savings rates respectively.*

To simplify the discussion, we assume that the population growth rate is zero, and women and men receive the same first period income ($y^w = y^m = y$). Before period t, the economy has a balanced sex ratio. In this case, $s^w = s^m = s$, and s can be obtained from solving the set of first order conditions (3.7) or (3.9):

$$-u'_{1w} + 2\left(1 - F\left(\bar{\eta}\right)\right) \kappa u'_2 + F\left(\bar{\eta}\right) u'_{2n} = 0 \qquad (3.10)$$

and

$$\bar{\eta} = u_{2n} - u_2$$

where we use the fact that at $\phi = 1$, $M(\eta) = \eta$.

The first key proposition concerns the effect of a rise in the sex ratio on the aggregate savings rate. The thought experiment assumes that people in the old cohort have made their savings decision when the sex ratio is balanced. When the sex ratio rises, any change in the aggregate savings is driven by a change in the savings by the young cohort. This simplifying assumption is motivated by the reality: A rise in the sex ratio in almost all economies is a recent phenomenon, since large-scale sex-selective abortions are a recent phenomenon. More precisely, the diagnostic sonography used for prenatal checkups became gradually more affordable to people in countries that now have a high sex ratio only since the early 1980s. (The strict version of the Chinese family planning policy, another contributor to the spread of sex-selective abortions, was also put in place in the early 1980s.) For this reason, the savings pattern for the currently old was largely decided when there was no severe sex ratio imbalance.

In what follows, whenever we say a man (or woman), we mean a young man (or woman), unless otherwise specified. We first state the proposition formally, and then explain the intuition behind the key parts of the proposition. A detailed proof is provided in Appendix A.

Proposition 1 *Assume emotional utility η^w and η^m are drawn from an independent and identical uniform distribution $[\eta^{\min}, \eta^{\max}]$ with a sufficiently low $\eta^{\min}$* [4] *and the mean $E\eta \geq 0$. If $u(c) = \ln(c)$, then, as the sex ratio rises, (1) the savings rate of the representative man goes up, but the change in women's savings is ambiguous; (2) however, the economy-wide savings rate increases unambiguously.*

Proof. See Appendix A.1. ■

A few remarks are in order. First, it is perhaps not surprising that the representative man raises his savings rate in response to a rise in the sex ratio since the need to compete in the marriage market becomes greater. Why is the impact of a higher sex ratio on a representative woman's savings rate ambiguous? The answer is that a higher sex ratio produces two offsetting effects for her. On the one hand, as she anticipates more savings from her future husband, she can free-ride and does not need to sacrifice her first-period consumption as much as she otherwise would have to. On the other hand, precisely because men have increased their savings rate in the first period in response to a higher sex ratio, they will be more reluctant to share their wealth with a woman with both a low savings rate and a low emotional utility. The last point raises the probability that low-savings women may not get married. Since the representative women also prefers marriage than spinsterhood, she may raise her savings rate to improve her chance in the marriage market. Because the two effects go in the opposite directions, the net effect of a higher sex ratio on a representative woman's savings is ambiguous.

Second, why does the aggregate savings rate rise unambiguously in response to a rise in the sex ratio even when women reduce their savings? The answer comes from both an intensive margin and

[4] This assumption greatly simplifies the proof. Relaxing the assumption will not change our qualitative result when the sex ratio is sufficiently unbalanced.

an extensive margin. On the intensive margin, the increment in the representative man's savings can be shown to be greater than the reduction in the representative woman's savings. Heuristically, the representative man raises his savings rate for two separate reasons: in addition to improving his relative standing in the marriage market, he wants to smooth his consumption over the two periods and would raise his savings rate to make up for the lower savings rate by his future wife. The more his future wife is expected to cut down her savings, the more he would have to raise his own savings to compensate. This ensures that his incremental savings is more than enough to offset any reduction in his future wife's savings. On the extensive margin, a rise in the sex ratio implies a change in the mix of the population with relatively more higher-saving men and relatively fewer lower-saving women. While both margins contribute to a rise in the aggregate savings rate, we can verify in calibrations that the intensive margin is quantitatively more important.

Third, we use log utility function because its simplicity allows us to prove Proposition 1 relatively easily. While log utility is one of many possible choices for a utility function, it also turns out to have interesting empirical support. Using datasets across individuals with different income levels in different countries, Stevenson and Wolfers (2008) show that self-reported happiness rises approximately linearly in log income. They conclude that the "true" utility function should be close to a log utility function. We interpretate the evidence as suggesting that log utility is a reasonable choice for our baseline case.

3.5 Mixed-strategy equilibrium

In this section, we extend our benchmark model by allowing men and women to choose to enter and exit the marriage market. Formally, this is a mixed-strategy game in which the representative woman chooses the probability of entering the marriage market ρ^w, a savings rate if she decides to enter, and a separate savings rate if she decides to abstain from the marriage market.

Conditional on deciding to enter the marriage market, she has the same optimization problem as in the previous section. However, she can also choose to be single, and conditional on such a choice, her life-time utility is

$$V_n^w = \max_{s_n^w} u(c_{1w,n}) + \beta u(c_{2w,n})$$

where V_n^w denotes the value function of a representative woman who is single throughout her life.

Her overall optimization problem when she is young is

$$\max_{\rho^w, s^w, s_n^w} \rho^w V^w + (1 - \rho^w) V_n^w$$

Obviously, she would choose $\rho^w = 1$ if and only if $V^w > V_n^w$.

Similarly, a representative man chooses the probability of entering the marriage market ρ^m as well as two potentially separate savings rates. His overall optimization problem is

$$\max_{\rho^m, s^m, s_n^m} \rho^m V^m + (1-\rho^m) V_n^m$$

where V_n^m denotes the value function of a representative man who is single throughout his life. Obviously, the representative man decides to enter the marriage market with probability one if and only if the expected utility of doing so is greater than otherwise, or $V^m > V_n^m$.

Now we can re-define the equilibrium as following:

Definition 2 *An equilibrium is* $\{s^w, s^m, s_n^w, s_n^m, \rho^w, \rho^m | y^w, y^m, F^w, F^m\}$ *that satisfies the following conditions:*

$$\left(s_i^w, s_{n,i}^w, \rho_i^w\right) = \arg\max \left(\rho_i^w V_i^w + (1-\rho_i^w) V_{n,i}^w \middle| s_{-i}^w, s^m, s_{n,-i}^w, s_n^m, \rho_{-i}^w, \rho^m, y^w, y^m, F^w, F^m\right)$$

and

$$\left(s_j^m, s_{n,j}^m, \rho_j^m\right) = \arg\max \left(\rho_j^m V_j^m + (1-\rho_j^m) V_{n,j}^m \middle| s^w, s_{-j}^m, s_n^w, s_{n,-j}^m, \rho^w, \rho_{-j}^m, y^w, y^m, F^w, F^m\right)$$

where i and j stand for a representative woman and man, respectively, and $-i$ and $-j$ represent all women other than i and all men other than j, respectively. $s^w = \left(s_i^w, s_{-i}^w, s_{n,i}^w, s_{n,-i}^w\right)$ and $s^m = \left(s_j^m, s_{-j}^m, s_{n,j}^m, s_{n,-j}^m\right)$ are the sets of women's and men's savings rates respectively. $\rho^w = \left(\rho_i^w, \rho_{-i}^w\right)$ and $\rho^m = \left(\rho_j^m, \rho_{-j}^m\right)$ are the sets of women's and men's probabilities of entering the marriage market respectively.

We can show a more general proposition:

Proposition 2 *Under the same assumptions as those in Proposition 1, there exists a threshold value $\phi_1 > 1$ that satisfies $V^m = V_n^m$.*

(i) For $\phi < \phi_1$, both women and men choose to enter the marriage market with probability one. In addition, as the sex ratio rises, a representative man increases his savings rate while the change in the savings rate of a representative woman is ambiguous. However, the economy-wide savings rate increases unambiguously.

(ii) For $\phi \geq \phi_1$, as the sex ratio rises, a representative man chooses a positive probability of being single while a representative woman still chooses to enter the marriage market with probability one. The effect on the aggregate savings rate is ambiguous.

Proof. See Appendix A.2. ■

Three remarks are in order. First, for $\phi < \phi_1$, as the sex ratio rises, men endure a welfare loss while the effect on women's welfare is ambiguous. Men lose because (i) they face a lower probability of marriage, and (ii) the reductions in their first-period consumption do not in the end alter their probability of marriage. In comparison, women face two opposing effects. On the one hand, they may gain both from an ability to free ride on their future husbands' higher savings rates and from an improved chance to marry a man with a higher level of emotional utility. On the other hand, precisely because men have raised their savings, they become more choosy in their selection of a mate as sharing their higher savings rate with a low-type woman may be worse than being single. As a result, women ex ante may face a rising risk of not getting married. The net effect of a higher sex ratio on women's welfare is ambiguous.

Second, after the sex ratio reaching and then going beyond the threshold ϕ_1, with a savings rate already very high, some men would find it better to skip the marriage market (or equivalently, the representative man would assign a positive probability for not entering the marriage market). Otherwise, they would have to share their high savings rate with a low-type woman, resulting in a lower level of welfare than being single. From women's point of view, however, as long as the mean level of emotional utility is high enough, they always achieve a higher level of welfare by choosing to enter the marriage market. In this case, the sex ratio in the marriage market is always equal to ϕ_1. Both men and women who choose to enter the marriage market will keep their savings rates constant. The rest of men choose another (constant) savings rate to maximize their utilities, but it is ambiguous whether the life-time bachelors' savings rate is lower than women's savings rate. Therefore, the effect of a rise in the sex ratio on the aggregate savings rate is ambiguous.

Third, the log utility assumption greatly simplifies the proof. More general functional forms for utility can also yield the same results if the mean of the emotional utility is sufficiently large such that (i) the condition in Proposition 1 holds

$$E(\eta) \geq \frac{R\kappa u_2'}{2}\sqrt{\frac{\max\left(0, \kappa u_2'\left(u_{2w,n}' + u_{2m,n}'\right) - u_{2w,n}' u_{2m,n}'\right)}{u_{1m}'' u_{1w}''}}$$

and (ii), at the balanced sex ratio, all women and men enter the marriage market.

3.6 A Production Economy

To analyze how the sex ratio affects a country's current account imbalance, we need to compare economy-wide savings with investments. We make the same assumptions as those in Proposition 1, and introduce a production sector. We assume perfect competition for both the final good market and the factor markets. The production function is Cobb-Douglas:

$$Q_t = \zeta K_t^{\alpha} L_t^{1-\alpha} \tag{3.11}$$

where K_t is the capital stock and L_t is the labor input. α is the share of capital input to total output and ζ is the total factor productivity (TFP). Everyone in the economy inelastically supplies one unit of labor and earns the same income[5].

A representative firm maximizes the profit

$$\max_{K_t, L_t} Q_t - R_t K_t - W_t L_t$$

The capital return and the wage rate are determined by

$$R_t = \frac{\partial Q_t}{\partial K_t} = \alpha\zeta\left(\frac{1}{K_t}\right)^{1-\alpha} \tag{3.12}$$

$$W_t = \frac{\partial Q_t}{\partial L_t} = (1-\alpha)\,\zeta K_t^{\alpha} \tag{3.13}$$

where we normalize the aggregate labor supply in the economy to be 1, i.e., $L_t = 1$.

For simplicity, we assume no tax or government expenditure; then $y_t = W_t$ where y_t is the corresponding first period disposable income in the endowment economy. We also assume complete depreciation in each period. The aggregate capital supply in period $t+1$ is predetermined by the aggregate savings in period t

$$K_{t+1}^s = \frac{\phi}{1+\phi} s_t^m W_t + \frac{1}{1+\phi} s_t^w W_t \tag{3.14}$$

3.7 Current Account in a Small Open Economy

In a small open economy, we assume that capital can flow freely among countries and the gross interest rate R is exogenously determined by the rest of the world. By (3.12) and (3.13), the wage rate is also a constant, and the aggregate investment in the economy is

$$K_t^d = \frac{\alpha W_t}{(1-\alpha) R_t} \tag{3.15}$$

Substituting (3.12) and (3.15) into the production function, we have

$$Q_t = \frac{W_t}{1-\alpha}$$

The current account in period t equals the increase in net foreign assets,

$$\Delta NFA_t = Q_t + (R-1)\cdot NFA_{t-1} - C_{1t} - C_{2t} - K_{t+1}^d$$

where $(R-1)\cdot NFA_{t-1}$ is the factor income from abroad. C_{1t} and C_{2t} represent the aggregate consumptions by young and old people respectively. Then

$$\Delta NFA_t = \frac{\phi}{1+\phi} s_t^m W_t + \frac{1}{1+\phi} s_t^w W_t - NFA_{t-1} - K_{t+1}^d$$

[5] Allowing men and women to earn different wages (with a fixed proportional gap) would not change our results.

We define the economy-wide savings rate as the aggregate private savings to GDP ratio; then

$$s_t^P = \frac{Q_t + (R-1)\cdot NFA_{t-1} - C_{1t} - C_{2t}}{Q_t} \tag{3.16}$$

We assume that the country has a balanced sex ratio in period $t-1$, and the sex ratio in the young cohort in period t, rises from one to $\phi(>1)$. Then the ratio of the current account to GDP is

$$\begin{aligned} ca_t &= \frac{Q_t + (R-1)\cdot NFA_{t-1} - C_{1t} - C_{2t} - K_{t+1}^d}{Q_t} \\ &= (1-\alpha)\left(\frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w - s_{t-1}\right) \end{aligned} \tag{3.17}$$

where the second equality holds because[6]

$$NFA_{t-1} = s_{t-1}W_{t-1} - K_t^d$$

where s_{t-1} is the savings rate by the cohort born in period $t-1$. Since the sex ratio is balanced at that time, both the women and the men will have the same savings rate.

Since the wage rate is constant in the small open economy, we can show that a country's current account rises as its sex ratio rises (up to a point).

Proposition 3 *In a small open economy with production, both the economy-wide savings rate and the current account would rise in response to a rise in the sex ratio.*

Proof. See Appendix A.3. ■

In this two-period model, the rise in the current account lasts for only one period in response to a one-time permanent rise in the sex ratio. From the second period onwards, the increase in old people's dis-savings completely offset the increment in young people's savings, and the economy achieves a new equilibrium in which the current account goes back to zero**.** In the calibration section, we consider a (more realistic) multi-period OLG model and generate longer lasting savings and current account responses to the same one-time rise in the sex ratio.

The assumption of an exogenous interest rate holds only for a small open economy. But some of the countries that motivate this study are large. An increase in the savings rate in such economies could lower the world interest rate, which could alter investment and savings decisions in all countries. We examine the large country case in the next subsection.

[6] In overlapping generations models, net foreign asset is equal to the difference between the savings by the young cohort and the domestic investment demand.

4 Parental savings and endogenous sex ratios

In this section, we make the sex ratio for any cohort to be an endogenous choice of their parents. We introduce parental savings for children, which is a part of the economy-wide household savings. To incorporate these features, we consider an OLG model in which every cohort lives two periods (young and old). Everyone works and earns labor income in the first period. If one gets married, the marriage takes place at the beginning of the second period, and the couple produces a single child right away.

As noted in Wei and Zhang (2011), widespread sex selective abortions are a relatively recent phenomenon because the inexpensive technology (especially Ultrasound B machines) employed to detect the gender of a fetus became available only since the 1980s. For example, 1985 was the first year in which half of the county-level hospitables in China acquired at least one ultrasound B machine (Li and Zheng, 2009). Therefore, the first cohort born with a severe sex ratio imbalance was entering the marriage market only after the start of the 21st century. To capture this feature of the data, we assume in the model that sex-selective abortions are not technologically feasible in periods before t_0 so that the sex ratio is always balanced. Starting from period t_0, parents can directly choose a sex ratio ϕ_t for the next cohort. As a result, parents in period t have a son with probability of $\frac{\phi_t}{1+\phi_t}$, and a daughter with probability of $\frac{1}{1+\phi_t}$.

Parents can save and invest in a risk-free bond for their child, and that savings potentially depends on the gender of their child. For simplicity, we assume that parents do not invest for girls[7]. Let τ_t be the rate of the parental savings for their son. In period $t+1$, parents transfer the bond revenue to their son. If their son gets married in period $t+1$, the son and his wife will share the transfer which yields a utility of $\chi \ln \left(\kappa R \tau_t y^P_{m,t}\right)$ to both of them,[8] where $y^P_{m,t}$ represents the wealth of the parents with a son in period t. If the son fails to get married in period $t+1$, parental transfers will yield a utility of $\chi \ln \left(R \tau_t y^P_{m,t}\right)$ to the young man.

For simplicity, we assume that the parents' cohort dies when their grandchild's cohort is born. Parents derive emotional utility from both their child and their grand-child[9]. For a representative young woman who enters the marriage market

$$V^w_t = \max_{s^w_t} \left\{ u(c^w_{1t}) + \beta E\left[u(c^w_{2,t+1}) + \chi \ln\left(T_{t+1}\right)\right] + \beta \delta^w_t E\left[\eta^m + \frac{\phi_{t+1}\eta^s}{1+\phi_{t+1}} + \frac{\eta^d}{1+\phi_{t+1}}\right] + \beta \delta^w_t \Lambda_{t+2} \right\}$$

where

$$\Lambda_{t+2} = \theta^s \frac{\phi_{t+1}\delta^m_{t+1}}{1+\phi_{t+1}} \left(\frac{\phi_{t+2}\eta^s}{1+\phi_{t+2}} + \frac{\eta^d}{1+\phi_{t+2}}\right) + \theta^d \frac{\delta^w_{t+1}}{1+\phi_{t+1}} \left(\frac{\phi_{t+2}\eta^s}{1+\phi_{t+2}} + \frac{\eta^d}{1+\phi_{t+2}}\right)$$

[7] This assumption is consistent with the historical evidence described by Botticini and Siow (2003).

[8] In this case, the parental transfers are assumed to be spent on a partial public consumption good within the marriage. Similar to our benchmark model, κ is a congestion index.

[9] If we allow parents to obtain emotional utility from their daughter-in-law or son-in-law, the results remain the same qualitatively. If we make a more general assumption by allowing parents to derive their altruistic utility from their child's utility, the model would be harder to sovle but the results are likely to be the same qualitatively.

represents the expected emotional utility obtained by the woman if she has a grandchild at the end of her life. δ_t^w is the probability that the woman gets married. $\delta_{t+1}^w(\delta_{t+1}^m)$ is the probability that her daughter (son) gets married. η^s and η^d are the emotional utilities each parent obtains from having a son and a daughter, respectively. θ^s and θ^d $(\theta^d < \theta^s < 1)$ are the parameters representing the degree of the emotional utilities from her son's and daughter's child, respectively. T_{t+1} is the transfer from the parents to their child.

For a representative young man,

$$V_t^m = \max_{s_t^m} \left\{ u(c_{1t}^m) + \beta E \left[u(c_{2,t+1}^m) + \chi \ln (T_{t+1}) \right] + \beta \delta_t^m E \left[\eta^w + \frac{\phi_{t+1}\eta^s}{1+\phi_{t+1}} + \frac{\eta^d}{1+\phi_{t+1}} \right] + \beta \delta_t^m \Lambda_{t+2} \right\}$$

Let c_{1t}^w and s_t^w denote the representative woman's first-period consumption and savings rate, respectively. Naturally,

$$c_{1t}^w = (1 - s_t^w)\, y_t^w$$

If she fails to get married, her second-period consumption is

$$c_{2,t+1}^{w,n} = R s_t^w y_t^w$$

If she gets married, her second-period consumption is

$$c_{2,t+1}^{w,i} = \kappa \left(1 - \tau_t^i\right) \left(R s_t^w y_t^w + R s_t^m y_t^m\right)$$

where s_t^m and y_t^m are the first period savings rate and income by a man, respectively. i (=w or m) stands for the child's gender.

Assume that $\eta^d (< \eta^s)$ is sufficiently large such that a woman (man) would not choose to be single if she/he can be matched with someone. One sufficient condition is

$$\eta^d \geq (1+\chi) \ln \kappa^{-1} - \eta^{\min}$$

For simplicity, we also assume that men do not observe women's wealth in the marriage market (but women do observe men's wealth). An important consequence of this assumption is that men rank women only by women's emotional utility. Young women and young men are assumed to earn the same first period income, $y_t^w = y_t^m = y$. Furthermore, η^w and η^m are assumed to be drawn from the same uniform distribution. With log utility, $u(c) = \ln c$, the optimization condition for the representative woman is

$$-\frac{1}{1-s_t^w} + \beta \left[\frac{1 - F(\bar{\eta}_t^w)}{s_t^w + s_t^m} + \frac{F(\bar{\eta}_t^w)}{s_t^w} \right] = 0 \tag{4.1}$$

where $\bar{\eta}^w$, similarly defined as in the benchmark model, is the lowest type of women who can get married. In this extension, $\bar{\eta}_t^w = \eta^{\min}$ and $\bar{\eta}_t^m = M\left(\eta^{\min}\right)$.

For a representative man, similar to the benchmark model, the optimal condition is

$$-\frac{1}{1-s_t^m}+\beta\left[\begin{array}{c}\left(\left(1+\phi_t\right)\left(1-F\left(\bar{\eta}_t^m\right)\right)+\eta^{\min} f(\bar{\eta}_t^m)\right)\frac{1}{s_t^w+s_t^m}+F\left(\bar{\eta}_t^m\right)\frac{1}{s_t^m}\\ +\frac{f\left(\bar{\eta}_t^m\right)}{s_t^w+s_t^m}\left(U_{t+1}^P+\chi\ln\kappa-u_{2,n,t+1}^m\right)\end{array}\right]=0 \tag{4.2}$$

where $u_{2,t+1}^{w,m}$ and $u_{2,t+1}^{w,w}$ stand for the utilities obtained from consumption when the representative woman has a son and a daughter, respectively. U_{t+1}^P denotes the expected utility of parents

$$U_{t+1}^P=\frac{\phi_{t+1}\left(u_{2,t+1}^{w,m}+\eta^s+\theta^s\delta_{t+1}^m\left(\frac{\phi_{t+2}\eta^s}{1+\phi_{t+2}}+\frac{\eta^d}{1+\phi_{t+2}}\right)\right)}{1+\phi_{t+1}}+\frac{u_{2,t+1}^{w,w}+\eta^d+\theta^d\delta_{t+1}^w\left(\frac{\phi_{t+2}\eta^s}{1+\phi_{t+2}}+\frac{\eta^d}{1+\phi_{t+2}}\right)}{1+\phi_{t+1}}$$

Parents optimally choose how much to save for (and transfer to) their sons. For a representative couple i, given all other households' choices $\tau_{-i,t}$, and all young people's choices s_t^w and s_t^m, the probability that their son can get married is

$$\delta_{it}^m=\Pr\left(\eta_{it}^m+\chi\ln\left(\frac{\tau_{it}}{\tau_{-i,t}}\right)\geq\bar{\eta}_t^m\,\middle|\,\tau_{-i,t},s_t^w,s_t^m\right)$$

The optimization problem for parents at time t is

$$\max_{\tau_t} u(c_{2,m,t})+\theta^s\delta_t^m\left(\frac{\phi_{t+1}\eta^s}{1+\phi_{t+1}}+\frac{\eta^d}{1+\phi_{t+1}}\right)$$

where we drop the sub-index i due to the symmetry. The first order condition is

$$-\frac{1}{1-\tau_t}+\chi\theta^s\frac{f\left(\bar{\eta}_t^m\right)}{\tau_t}\left(\frac{\phi_{t+1}\eta^s}{1+\phi_{t+1}}+\frac{\eta^d}{1+\phi_{t+1}}\right)=0 \tag{4.3}$$

Parents also choose the sex ratio (although they don't directly choose the gender of the child) to maximize their utility U_t^P. The first order condition on the sex ratio ϕ_t chosen by parents in period t is

$$\left(u_{2,t}^{w,w}+\theta^d\delta_t^w\left(\frac{\phi_{t+1}\eta^s}{1+\phi_{t+1}}+\frac{\eta^d}{1+\phi_{t+1}}\right)+\eta^d\right)-\left(u_{2,t}^{w,m}+\theta^s\delta_t^m\left(\frac{\phi_{t+1}\eta^s}{1+\phi_{t+1}}+\frac{\eta^d}{1+\phi_{t+1}}\right)+\eta^s\right)=0 \tag{4.4}$$

We assume that parents favor sons, $\eta^s>\eta^d$; furthermore, the difference $\eta^s-\eta^d$ is sufficiently large such that, when $\phi=1$ in period t_0-1,

$$\left(u_{2,t_0-1}^{w,w}+\frac{\theta^d}{2}\left(\eta^s+\eta^d\right)+\eta^d\right)-\left(u_{2,t_0-1}^{w,m}+\frac{\theta^s}{2}\left(\eta^s+\eta^d\right)+\eta^s\right)<0 \tag{4.5}$$

We also make the Darwinian assumption that $E\eta^m$ and $E\eta^w$ are sufficiently large so that marriage is strongly attractive. Totally differentiating (4.1), (4.2), and (4.3), we have the following proposition:

Proposition 4 *Assume that the sex ratio becomes a choice variable from period t_0 onwards. Under the same assumptions as those in Proposition 1 and Proposition 3, if (4.5) holds, we can show that*

(i) $\phi_t > 1$ $(t \geq t_0)$*;*

(ii) In period t_0, both young men and parents with a son have higher savings rates, but the savings rates by young women and parents with a daughter decline. In a small open economy, the aggregate savings rate rises and the country runs a current account surplus in period t_0.

Proof. See Appendix A.4. ■

Note that the assumption that parents do not save for their daughter can be relaxed. Since daughters are assumed to bring a lower utility than boys to their parents if the sex ratio is balanced, parents will choose a higher sex ratio (more boys than girls) in period t_0. Then all women will get married even if parents with a daughter do not save for their child. As a result, parents with a daughter optimally choose zero savings for their daughter.

5 Numerical Examples

Are the actual sex ratios observed in the data capable of generating a current account response whose magnitude is economically significant? We answer this question in this section by quantitative calibrations of the model. We start with a small open economy and allow endogenous entry/exit to the marriage market. Then we move on to two cases of a large economy. We also consider two extensions that would add some more realism to the model. First, we discuss potential intra-family bargaining between husband and wife, with their relative bargaining power depending in part on their relative savings rate. Second, we extend the benchmark two-period model to a multi-period model.

5.1 The Small Open Economy

Assume that the utility function is of the log form

$$u(c) = \ln(c)$$

In the calibrations for a small open economy, we fix $R = \beta^{-1}$. (In the large country case, the interest rate is to be endogenously determined.)

The emotional utility η needs to follow a continuously differential distribution. We assume a normal distribution which might be more realistic than the uniform distribution used in the analytical model (for which the uniform distribution assumption is analytically more convenient). We choose the mean and the standard deviation of emotional utility by matching them with the empirical moments implied by the estimates in Blanchflower and Oswald (2004). To be precise, here is what we do to calibrate the mean. Note first that, within the model, we can compute the incremental income needed for a man to be indifferent between being married and being forever single when the sex ratio is balanced:

$$u\left(Rs^{m}y+my\right)=u\left(\kappa R\left(s^{m}+s^{w}\right)y\right)+E\left(\eta\right)$$

where $m \cdot y$ is the additional compensation paid to a life-time bachelor. Regressing a measure of subjective well-being on income and marital status (and other determinants of happiness) in the United States during 1972-1998, Blanchflower and Oswald (2004) estimate that, on average, a lasting marriage is equivalent to augmenting one's income by \$100,000 (in 1990 dollars) per year every year. Since the average annual income per working person was about \$48,000 in that period, a sustained marriage is worth twice the average income. We therefore choose $m = 2.08(\simeq \frac{100{,}000}{48{,}000})$ as the benchmark. This implies that the mean value of emotional utility is:

$$E\left(\eta\right)=\ln\left(\frac{Rs^{m}+m}{\kappa R\left(s^{m}+s^{w}\right)}\right)$$

where s^w and s^m are solved for the case when $\phi = 1$. We will vary the value of m in the robustness checks.

We calibrate the standard deviation of emotional utility to match the standard errors for the coefficient on the marriage status dummy in the happiness regressions. Since the t-statistic for the marriage status dummy is around 20, the implied standard deviation of emotional utility, $\sigma = \frac{E\eta}{t.stat}$, is about 0.05. As a robustness check, we will also consider $\sigma = 0.1$.

For other parameters, whenever relevant data are available, we assign values that are consistent with the data, and they are summarized in the following table.

Choice of Parameter Values

Parameters	*Benchmark*	*Source and robustness checks*
Discount factor	$\beta = 0.7$	$\beta = R^{-1}$. Song et al (2011) suggests that the annual gross deposit rate in China takes value around 1.0175. As we take 20 years as one period, we set $\beta = (1/1.0175)^{20} \simeq 0.7$
Share of capital input	$\alpha = 0.4$	From China's input-output table in year 2007
Congestion index	$\kappa = 0.8$	$\kappa = 0.7, 0.9$ in the robustness checks.
Love, standard deviation	$\sigma = 0.05$	$\sigma = 0.1$ in the robustness checks
Love, mean	$m = 2.08$	$m = 1$ in the robustness checks

Figure 1 plots the aggregate savings rate as a function of the sex ratio.When the sex ratio goes up from 1 to 1.15, the savings rate would go up by 6.2 percentage points (=0.342-0.280).[10] As the sex ratio continues to rise, the savings rate continues to rise but, after a certain point (i.e., after the sex

[10] In Online Appendix Tables 1a, 1b and 1c, we also report the calibration results for individual savings when the sex ratio changes from 1 to 1.5.

ratio approaches 1.4), it starts to decline. This is because the sex ratio has exceeded the threshold ϕ_1 in Proposition 2; some men quit the marriage market and choose a lower savings rate, which drives down the economy-wide savings rate. Note, since no economy in the real world has a sex ratio exceeding 1.4, we may not have an opportunity to observe the declining portion of the savings curve in the data.

For sensitivity analyses, we consider different combinations of parameter values involving $\kappa = 0.7, 0.8$ and 0.9, $m = 2.08$ and 0.5, and $\sigma = 0.05$, and 0.1. There are a few noteworthy patterns. First, the economy-wide savings rate always rises in response to a rise in the sex ratio (up to a relatively high threshold value of the sex ratio). Second, the response in the economy-wide savings rate is not sensitive to changes in parameter κ. Third, when the mean value of emotional utility becomes higher (e.g., comparing $m = 2.08$ to $m = 1$), both the economy-wide savings rate and the current account respond more strongly to a given rise in the sex ratio. This is intuitive since men have a stronger desire to compete for a marriage partner. Fourth, as the dispersion for emotional utility becomes smaller, the economy-wide savings rate and the current account respond more strongly to a rise in the sex ratio. The reason is similar to before: when men are more similar in terms of the amount of "love" they can offer to women, the need to compete on the basis of wealth also rises.

5.2 Multi-period Model Calibrations

We now extend our benchmark model to a setting in which every cohort lives for 50 periods. Everyone works in the first 30 periods, and retires in the remaining 20 periods. If one gets married, the marriage take place in the τth period. We have not been able to solve the problem that allows for parental savings for their child in the 50-period setup. Instead, we study a case in which men and women save for themselves. However, as we recognize the quantitative importance of parental savings in the data, we choose $\tau = 10$ as our benchmark case so the timing of the marriage is somewhere between the typical number of working years by parents when their child gets married and the typical number of working years by children themselves when they get married. Besides the base case of $\tau = 10$, we also examine the case of $\tau = 20$ as a sensitivity check. Generally speaking, the greater the value of τ, the stronger is the aggregate savings response to a given rise in the sex ratio.

We consider the following experiment: at time 0, the sex ratios in all existing generations are one. Starting from period 1, the sex ratio in all newly-born generations becomes $\phi(> 1)$, which is not anticipated in previous periods. A representative woman's optimization problem is

$$\max \sum_{t=1}^{\tau-1} \beta^{t-1} u(c_t^w) + E_1 \left[\sum_{t=\tau}^{50} \beta^{t-1} \left(u(c_t^w) + \eta^m \right) \right]$$

For $t < \tau$, when the woman is still single, the intertemporal budget constraint is

$$A_{t+1} = R \left(A_t + y_t^w - c_t^w \right)$$

where A_t is her wealth level at the beginning of period t. After marriage ($t \geq \tau$), her family budget constraint becomes

$$A_{t+1}^H = \begin{cases} R\left(A_t^H + y_t^H - c_t^H/\kappa\right) & \text{if } t \leq 30 \\ R\left(A_t^H - c_t^H/\kappa\right) & \text{if } t > 30 \end{cases}$$

where A_t^H is the level of family wealth (held jointly by the wife and the husband) at the beginning of period t. c_t^H is the public good consumption by both spouses, which takes the same form as in the two period OLG model. The optimization problem for a represenative man is similar.

On the production side, we assume the same production function as in the benchmark except that we now assume an annual capital depreciation rate equal to 0.1.

Given the increase in the number of periods in a lifetime, we need to adjust some parameters in the calibrations. Following Song, Storesletten, and Zilibotti (2011), we take 1.0175 as the annual gross interest rate in China. The subjective discount factor is set at $\beta = 1/R$. All other parameters are the same as in the previous 2-period OLG model.

In this multi-period OLG setup, earning profiles may also have implications on the aggregate savings rate. The Chinese data suggest an interesting (and maybe peculiar) feature about a typical worker's life-time earnings profile. Using data from urban household surveys, Song and Yang (2011) document that a typical worker in China faces a fairly flat lifetime (real) earnings profile (although the starting salary of each successive cohort tends to rise fast). Within a given cohort, we also assume a flat earnings profile over time. Since we do not consider an exogenous growth in productivity, we do not feature a steady rise in income from one cohort to the next.

In Figure 2, we trace out the evolution of the aggregate savings and the current account when the sex ratio rises from 1 to 1.10 (and when the marriage is assumed to always take place in the 10th period). Over a period of nine years, both the economy-wide savings rate and current account rise by about 3.4% of GDP. Starting from the 10th period, the increased dis-savings by the old generation start to partially offset the increase in young generation's savings. Both aggregate savings rate and the current account begin falling. In the 50th period, the aggregate savings rate and the current account converge to the new equilibrium.

As a robustness check, consider the case in which marriages always take place in the 20th period. In this case, after the same rise in the sex ratio, the economy-wide savings rate and the current account would rise by about 5.7% of GDP in nineteen years. On the other hand, if marriages take place in the 5th period, the aggregate savings rate and current account would rise by about 1.7% of GDP (we do not report the corresponding figures to save space). Because in the real world, savings response come from both the young cohort and their parents, we think setting the timing of marriages in the 10th period is reasonable, as it represents a weighted average of the number of working years by the young cohort and their parents.

In all these experiments, we see clearly that, a higher sex ratio can generate responses in both the aggregate savings rate and the current account that are both sizable and long-lasting. For instance, in Figure 2, as the sex ratio rises from 1 to 1.10, the current account surplus will stay above 2% of GDP for more than 20 years (from the 6th to 29th period) after the shock.

5.3 Additional Extensions

We also study several additional extensions to the benchmark model. First, we consider a world with two large countries. If the two countries are identical in every respect except that Country 2 has a more unbalanced sex ratio than Country 1, we show analytically that Country 2 then runs a current account surplus while Country 1 runs a current account deficit. In a calibration, we consider a version in which the two countries mimic the United States and China in some important ways. In particular, while Country 1 (United States) always has a balanced sex ratio, we vary the sex ratio in Country 2 (China) from 1 to 1.5. When the Chinese sex ratio reaches 1.15, it runs a current account surplus on the order of 4.4% of its GDP. At the same time, the United States runs a current account deficit of 1.5% of its GDP. This calibration suggests that a rise in the sex ratio in a large economy may induce other countries to run a current account deficit (even though they have a balanced sex ratio). This analysis is presented in Online Appendix B.

Second, we analyze welfare implications from a rise in the sex ratio. With log utility function, we show that men's welfare under a decentralized equilibrium (our benchmark case) relative to the central planner's economy declines as the sex ratio increases. In comparison, women's relative welfare increases as the sex ratio goes up. With transferable utility, the social welfare (the sum of all men 's and women's welfare) goes down as the sex ratio rises. The details are reported in Online Appendix C.

Third, we incorporate intra-household bargaining between wives and husbands into the model. If the bargaining powers of the two parties depend on their initial relative wealth, women's savings rate in our simulations decline much more slowly as the sex ratio rises. If the sensitivity of the bargaining power to the relative wealth is high enough, and/or private consumption is important in women's second period consumption bundle, women may even raise their savings as the sex ratio goes up. For the aggregate savings rate, it responds more strongly to a rise in the sex ratio than in the benchmark case. This is reported in Online Appendix D.

Fourth, recognizing that, even out of a precautionary (or hedging) saving motive, the savings rate can go up in response to a higher sex ratio, we compare the relative quantitative importance of the competitive and precautionary saving motives. Changes in the savings rate due to a pure hedging or precautionary motive can be ascertained by making one's savings decision to be private information. If savings cannot be observed, it cannot used as a competitive weapon in the marriage market. As a result, the competitive saving motive is shut down. Numerical simulations show that the incremental savings due to a pure hedging motive is very small relative to the competitive saving motive. This means that when the sex ratio rises, much of the action in the savings response comes from the competitive motive. A detailed comparison is presented in Online Appendix E.

Fifth, as the sex ratio rises, raising savings rate may not be the only response in the real world. In particular, young man may increase human capital accumulation (or parents may invest more in their son's human capital). We incorporate this aspect into the model, and show both theoretically and numerically that a higher sex ratio still generates a significant increase in the savings rate. Intuitively, if both physical wealth and human capital can enhance a man's status in the marriage market, the optimal allocation in the first period must be such that marginal gains from additional savings and from additional education expenditure are equal. As both yield a diminishing returns, an interior solution emerges in which the man chooses to raise both the savings rate and human capital accumulation. The derivations and simulations can be found in Online Appendix F.

Sixth, since a man's savings/consumption choice depends on the elasticity of intertemporal substitution, we check for the robustness our results with a more general utility function (CRRA) in our numerical exercises. We find that, as the sex ratio rises, the aggregate savings rate and current account always go up. As the elasticity of intertemporal substitution becomes lower, the responses of the savings rate and current account become weaker but still economically significant. This is reported in Online Appendix G.

Finally, we investigate how income inequality may affect the savings and current account responses to sex ratio imbalances. Through simulaitons of a multi-period model, we find that greater inequality generally raises the savings and current account responses to a given rise in the sex ratio. There are two reasons for this result. For high income men, a rise in the income gap among women motivates men to try harder to be matched with a high-income woman. While the behavior of a low-income woman exhibits some non-monoticity, she also eventually raises her savings rate in response to a higher sex ratio. The details can be found in Online Appendix H.

6 Concluding Remarks and Future Research

This paper builds a theoretical model to analyze whether and how a rise in the sex ratio may trigger a competitive race in the savings rate by men (or households with sons). Generally speaking, men raise their savings rate in order to improve their relative standing in the marriage market. If we don't consider intra-household bargaining, women may respond by reducing their savings rate because they may free ride on the increased savings from their future husbands. If we consider intra-household bargaining, then the women's response becomes ambiguous because they also have an incentive to raise their savings rate in order to protect their bargaining power within a family. In any case, the aggregate savings always rises unambiguously in response to a rise in the sex ratio, as long as the sex ratio is below some threshold. We argue conceptually and through calibrations that the threshold is higher than the sex ratios in all real economies.

When the country with an unbalanced sex ratio is large, this could have global ramifications.

In particular, as the sex ratio rises, the world interest rate becomes lower. Other countries with a balanced sex ratio could be induced to run a current account deficit. Calibration results suggest that the sex ratio effect could potentially explain about half of China's current account surplus and the U.S. current account deficit. In other words, the effect is economically significant.

The theory can be extended in a number of directions. Initial results in several extensions are presented in seven online appendices. In addition, while the model focuses on the responses of savings and current account to a rise in the sex ratio, one may extend it to study effects on entrepreneurship and economic growth.

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Table 1: Savings rate vs sex ratio, 1990-2010

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Full sample				Excluding outliers			
|sex ratio(i,t) – 1|	55.24***	57.02***	58.47***	58.51***	54.87***	56.66***	58.07***	58.11***
	(12.13)	(12.00)	(11.94)	(11.93)	(12.01)	(11.87)	(11.81)	(11.79)
| sex ratio(i,t)– 1|*dmy(if sex ratio(i,t)<1)	9.71	20.49	25.65	38.31	10.94	21.99	27.27	42.08
	(30.44)	(30.40)	(30.30)	(30.80)	(30.13)	(30.07)	(29.96)	(30.45)
dmy(if sex ratio(i,t)<1)	2.78***	3.27***	3.00***	3.06***	2.78***	3.27***	2.99***	3.06***
	(0.859)	(0.858)	(0.881)	(0.880)	(0.851)	(0.849)	(0.871)	(0.870)
ln(real GDP per capita)	5.30***	6.68***	6.60***	16.67***	5.55***	6.97***	6.92***	18.70***
	(1.24)	(1.25)	(1.26)	(4.70)	(1.23)	(1.23)	(1.25)	(4.65)
Fiscal deficit, % of GDP	-0.477***	-0.415***	-0.422***	-0.425***	-0.467***	-0.404***	-0.411***	-0.414***
	(0.032)	(0.032)	(0.032)	(0.032)	(0.031)	(0.032)	(0.032)	(0.032)
Credit to private sector, % of GDP		-0.035***	-0.037***	-0.032***		-0.036***	-0.038***	-0.032***
		(0.008)	(0.008)	(0.008)		(0.008)	(0.008)	(0.008)
Dependency ratio			-0.014	0.002			-0.008	0.010
			(0.034)	(0.035)			(0.034)	(0.034)
ln(real GDP per capita) square				-0.704**				-0.822**
				(0.316)				(0.313)
Time fixed effect	Y	Y	Y	Y	Y	Y	Y	Y
Country fixed effect	Y	Y	Y	Y	Y	Y	Y	Y
Observations	2,376	2,344	2,323	2,323	2,374	2,342	2,321	2,321
R-squared	0.16	0.15	0.16	0.16	0.16	0.15	0.16	0.16
Number of countries	159	159	158	158	159	159	158	158

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

- We use the sex ratio in age group 10-24. The data can be obtained from United Nations' Population Division (http://esa.un.org/unpd/wpp/Excel-Data/population.htm).
- |sex ratio(i,t) – 1| is the absolute difference between country i's sex ratio at time t and one.
- In Columns (5) to (8), we exclude the observations with extreme savings rate (with absolute value greater than 80% of GDP).

Table 2: CA/GDP vs sex ratio, 1990-2010

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Full sample				Excluding outliers			
|sex ratio(i,t) – 1|	26.93**	29.18**	30.33**	29.87**	19.13*	21.32**	22.60**	22.27**
	(12.86)	(12.69)	(12.71)	(12.70)	(10.65)	(10.48)	(10.48)	(10.47)
| sex ratio(i,t)– 1|*dmy(if sex ratio(i,t)<1)	19.13	32.09	36.46	24.54	35.24	46.01*	49.73*	37.80
	(32.92)	(32.75)	(32.85)	(33.36)	(26.91)	(26.69)	(26.73)	(27.14)
dmy(if sex ratio(i,t)<1)	0.744	1.07	0.655	0.578	2.28***	2.58***	2.21***	2.14***
	(0.949)	(0.943)	(0.973)	(0.973)	(0.781)	(0.774)	(0.797)	(0.797)
ln(real GDP per capita)	1.16	3.50**	4.14***	-6.26	1.78	3.84***	4.43***	-5.88
	(1.39)	(1.40)	(1.42)	(5.35)	(1.144)	(1.15)	(1.16)	(4.39)
Fiscal deficit, % of GDP	-0.189***	-0.136***	-0.139***	-0.137***	-0.136***	-0.098***	-0.103***	-0.100***
	(0.034)	(0.034)	(0.034)	(0.034)	(0.029)	(0.029)	(0.029)	(0.029)
Credit to private sector, % of GDP		-0.056***	-0.060***	-0.066***		-0.051***	-0.056***	-0.061***
		(0.009)	(0.009)	(0.009)		(0.007)	(0.007)	(0.008)
Dependency ratio			0.099***	0.084**			0.096***	0.080**
			(0.038)	(0.039)			(0.031)	(0.032)
ln(real GDP per capita) square				0.723**				0.717**
				(0.358)				(0.294)
Time fixed effect	Y	Y	Y	Y	Y	Y	Y	Y
Country fixed effect	Y	Y	Y	Y	Y	Y	Y	Y
Observations	2,318	2,293	2,273	2,273	2,268	2,243	2,223	2,223
R-squared	0.04	0.05	0.06	0.06	0.05	0.07	0.08	0.08
Number of countries	162	162	161	161	160	160	159	159

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

- We use the sex ratio in age group 10-24. The data can be obtained from United Nations' Population Division (http://esa.un.org/unpd/wpp/Excel-Data/population.htm).
- |sex ratio(i,t) – 1| is the absolute difference between country i's sex ratio at time t and one.
- In Columns (5) to (8), we exclude the observations with extreme current account (with absolute value greater than 30% of GDP)

Figures and Tables

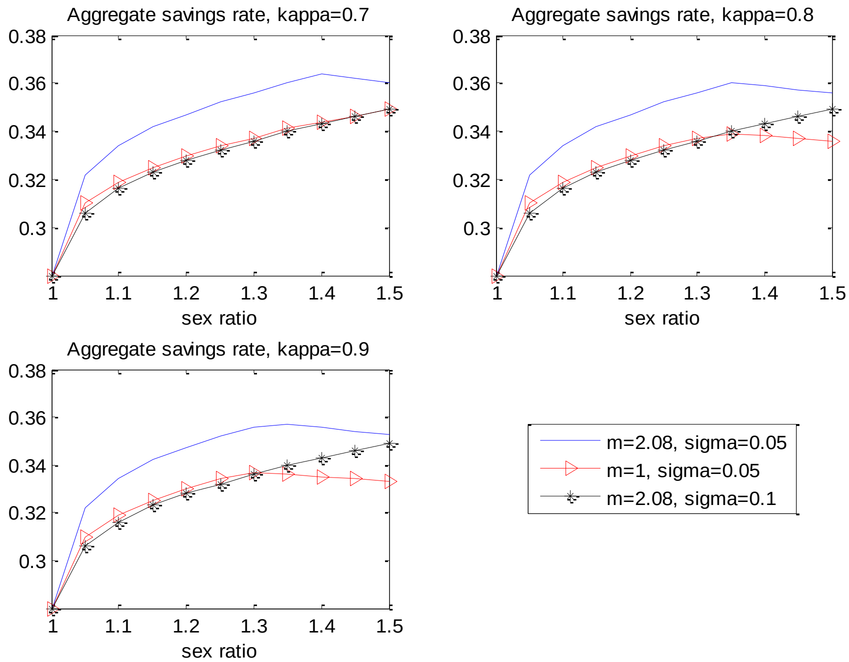

Figure 1: Economy-wide savings rate vs sex ratio

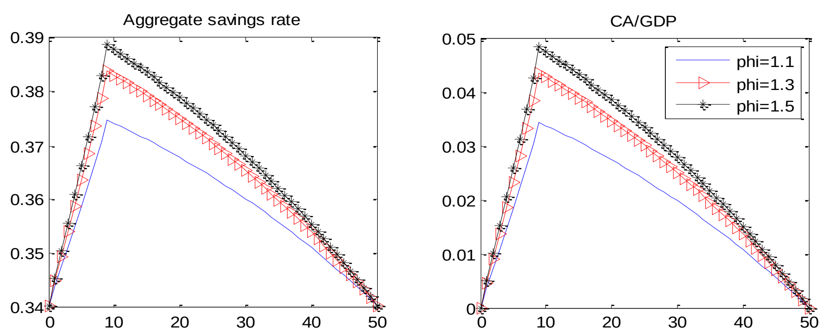

Figure 2: 50-period calibrations, τ=10, σ=0.05

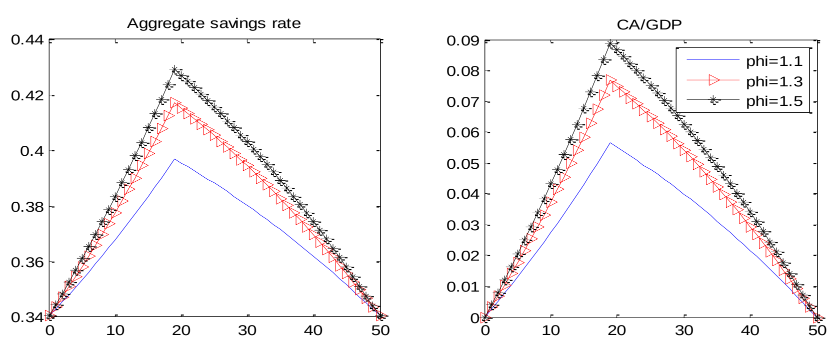

Figure 3: 50-period calibrations, τ=20, σ=0.05

Online Appendices to "A Theory of the Competitive Saving Motive"

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A Proofs

A.1 Proof of Proposition 1

Proof. The first order conditions for a woman and a man, respectively, are:

$$-u'_{1w} + \left[\begin{array}{c} \kappa u'_{2w} \left(\delta^w + \left[\frac{1}{\phi} \left(1 - F\left(\bar{\eta}^w\right)\right) + \bar{\eta}^m f(\bar{\eta}^w) \right] \right) + \left(1 - \delta^w\right) u'_{2w,n} \\ + f\left(\bar{\eta}^w\right) \kappa u'_{2w} \left(u_{2w} - u_{2w,n}\right) \end{array} \right] = 0 \tag{A.1}$$

$$-u'_{1m} + \left[\begin{array}{c} \kappa u'_{2m} \left(\delta^m + \left[\phi \left(1 - F\left(\bar{\eta}^m\right)\right) + \bar{\eta}^w f(\bar{\eta}^m) \right] \right) + \left(1 - \delta^m\right) u'_{2m,n} \\ + f\left(\bar{\eta}^m\right) \kappa u'_{2m} \left(u_{2m} - u_{2m,n}\right) \end{array} \right] = 0 \tag{A.2}$$

We show by contradiction that $\bar{\eta}^w = u_{2m,n} - u_{2m}$ and $\bar{\eta}^m = M(\bar{\eta}^w)$ hold for $\phi \geq 1$. Suppose not, then

$$\bar{\eta}^m > M(\bar{\eta}^w) \geq \bar{\eta}^w$$

where the second inequality holds because $\phi \geq 1$. Then we have

$$\bar{\eta}^m = u\left(Rs^w y\right) - u\left(\kappa(Rs^w y + Rs^m y)\right) > \bar{\eta}^w \geq u\left(Rs^m y\right) - u\left(\kappa(Rs^w y + Rs^m y)\right)$$

and hence, $s^w > s^m$.

Under the log utility assumption, $u(c) = \ln c$, we have $\kappa u'_{2m} < u'_{2m,n}$ and $\kappa u'_{2w} < u'_{2w,n}$. Then

$$\begin{aligned} u'_{1w} &= \kappa u'_{2w} \left(\delta^w + \left[\frac{1}{\phi} \left(1 - F\left(\bar{\eta}^w\right)\right) + M(\bar{\eta}^w) f(\bar{\eta}^w) \right] \right) + \left(1 - \delta^w\right) u'_{2w,n} + f\left(\bar{\eta}^w\right) \kappa u'_{2w} \left(u_{2w} - u_{2w,n}\right) \\ &< \kappa u'_{2w} \left(\delta^m + \left[\frac{1}{\phi} \left(1 - F\left(\bar{\eta}^w\right)\right) + M(\bar{\eta}^w) f(\bar{\eta}^w) \right] \right) + \left(1 - \delta^m\right) u'_{2w,n} + f\left(\bar{\eta}^w\right) \kappa u'_{2w} \left(u_{2w} - u_{2w,n}\right) \\ &< \kappa u'_{2m} \left(\delta^m + \left[\phi \left(1 - F\left(\bar{\eta}^m\right)\right) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m) \right] \right) + \left(1 - \delta^m\right) u'_{2m,n} + f\left(\bar{\eta}^m\right) \kappa u'_{2m} \left(u_{2m} - u_{2m,n}\right) = u'_{1m} \end{aligned}$$

where the first inequality holds because

$$\delta^m = 1 - F(\bar{\eta}^m) < 1 - F(\bar{\eta}^w) = \delta^w$$

and $\kappa u'_{2m} < u'_{2m,n}$. The second inequality holds because (i)

$$\begin{aligned}
& \frac{1}{\phi}(1 - F(\bar{\eta}^w)) + M(\bar{\eta}^w) f(\bar{\eta}^w) \\
= \;& 1 - F(M(\bar{\eta}^w)) + M(\bar{\eta}^w) f(\bar{\eta}^w) = \frac{\eta^{\max}}{\eta^{\max} - \eta^{\min}} \\
= \;& 1 - F(\bar{\eta}^w) + \bar{\eta}^w f(\bar{\eta}^w) = \phi(1 - F(M(\bar{\eta}^w))) + \bar{\eta}^w f(\bar{\eta}^m) \\
\leq \;& \phi(1 - F(M(\bar{\eta}^w))) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m)
\end{aligned}$$

by using the uniform distribution assumption; (ii) $s^w > s^m$ under our conjecture, and (iii),

$$u_{2m} - u_{2m,n} > u_{2w} - u_{2w,n}$$

The inequality $u'_{1w} < u'_{1m}$ derived above implies

$$s^m > s^w$$

Contradiction! Therefore, we have $\bar{\eta}^m = M(\bar{\eta}^w)$ and $s^m \geq s^w$[1] for $\phi \geq 1$.

Substituting the expressions of $\bar{\eta}^w$ and $\bar{\eta}^m$ into (A.1) and (A.2), and totally differentiating the system and re-arrange the matrix, we obtain

$$\Omega \cdot \mathbf{ds} = \mathbf{dz} \tag{A.3}$$

where

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}, \; \mathbf{ds} = \begin{pmatrix} ds^w \\ ds^m \end{pmatrix} \text{ and } \mathbf{dz} = \begin{pmatrix} 0 \\ A \end{pmatrix}$$

[1] We can $s^m \geq s^w$ for $\phi \geq 1$ by contradiction. Suppose not, then $s^w > s^m$, following the same steps as showing $\bar{\eta}^m = M(\bar{\eta}^w)$, we find a contradiction.

$$\Omega_{11} = u''_{1w}y + Ry\left[\begin{array}{c}\kappa^2 u''_{2w}\left(\left(1+\frac{1}{\phi}\right)(1-F(\bar{\eta}^w))\right) + (1-\delta^w)u''_{2w,n} \\ +f(\bar{\eta}^w)\kappa^2 u''_{2w}(u_{2w}+M(\bar{\eta}^w)-u_{2w,n}) + 2f(\bar{\eta}^w)\kappa u'_{2w}\left(\kappa u'_{2w}-u'_{2w,n}\right)\end{array}\right]$$

$$\Omega_{12} = Ry\left[\begin{array}{c}\kappa^2 u''_{2w}\left(\left(1+\frac{1}{\phi}\right)(1-F(\bar{\eta}^w))\right) + f(\bar{\eta}^w)\kappa^2 u''_{2w}(u_{2w}+M(\bar{\eta}^w)-u_{2w,n}) \\ +f(\bar{\eta}^w)\kappa^2 u'^2_{2w} + f(\bar{\eta}^w)\left(u'_{2m,n}-\kappa u'_{2m}\right)\left(u'_{2w,n}-\kappa u'_{2w}\right)\end{array}\right]$$

$$\Omega_{21} = Ry\left[\kappa^2 u''_{2m}((1+\phi)(1-F(M(\bar{\eta}^w)))) + \frac{f(\bar{\eta}^w)\kappa u'_{2m}}{\phi}\left(\kappa u'_{2m}-u'_{2m,n}\right) + f(\bar{\eta}^w)(\kappa u'_{2m})^2\right]$$

$$\Omega_{22} = u''_{1m}y + Ry\left[\begin{array}{c}\kappa^2 u''_{2m}((1+\phi)(1-F(M(\bar{\eta}^w)))) + (1-\delta^m)u''_{2m,n} \\ -\frac{1}{\phi}f(\bar{\eta}^w)\left(\kappa u'_{2m}-u'_{2m,n}\right)^2 + f(\bar{\eta}^w)\left(\kappa u'_{2m}-u'_{2m,n}\right)\kappa u'_{2m}\end{array}\right]$$

and

$$A = \frac{1}{\phi^2}[1-F(\bar{\eta}^w)]\left(\kappa u'_{2m}-u'_{2m,n}\right) < 0$$

It is easy to show that

$$\det(\Omega) = positve.terms - \frac{Ryf(\bar{\eta}^w)\left(\kappa u'_{2m}-u'_{2m,n}\right)^2\Omega_{11}}{\phi} + u''_{1m}u''_{1w}y^2 + \frac{(\kappa u'_2)^2 R^2 y^2\left[u'_{2w,n}u'_{2m,n} - \kappa u'_2\left(u'_{2w,n}+u'_{2m,n}\right)\right]}{(\eta^{\max}-\eta^{\min})^2}$$

With log utility, we can obtain

$$\frac{(\kappa u'_2)^2 R^2 y^2\left[u'_{2w,n}u'_{2m,n} - \kappa u'_2\left(u'_{2w,n}+u'_{2m,n}\right)\right]}{(\eta^{\max}-\eta^{\min})^2} = 0$$

and hence $\det(\Omega) > 0$.

The derivatives of the savings rates with respect to the sex ratio are as following:

$$\frac{ds^m}{d\phi} = \frac{A\Omega_{11}}{\det(\Omega)} > 0$$

and

$$\begin{aligned}\frac{ds_t^w}{d\phi} &= -\frac{A\left(\left(1+\frac{1}{\phi}\right)(1-F(\bar{\eta}^w)) + f(\bar{\eta}^w)\left(M(\bar{\eta}^w) + \log\left(\kappa\left(1+\frac{s_t^m}{s_t^w}\right)\right) - 2\right)\right)}{Ry\,(s_t^m+s_t^w)^2\det(\Omega)} \\ &= -\frac{A\left(2\eta^{\max} + 2\log\kappa + \log\left(2+\frac{s_t^m}{s_t^w}+\frac{s_t^w}{s_t^m}\right) - 2\right)}{Ry\,(\eta^{\max}-\eta^{\min})(s_t^m+s_t^w)^2\det(\Omega)}\end{aligned}$$

where the first equality holds because we use the log utility function. The sign of $\frac{ds_t^w}{d\phi}$ is ambiguous.[2]

However, the aggregate savings rate by the young cohort, $s_t^{young} = \frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$, rises as the

[2] A sufficient condition for $\frac{ds_t^w}{d\phi} < 0$ is $\eta^{\max} > 1$.

sex ratio becoms more unbalanced.

$$\begin{aligned}\frac{ds_t^{young}}{d\phi} &= \frac{s_t^m - s_t^w}{(1+\phi)^2} + \frac{\phi-1}{1+\phi}\frac{ds_t^m}{d\phi} + \frac{1}{1+\phi}\left(\frac{ds_t^m}{d\phi} + \frac{ds_t^w}{d\phi}\right) \\ &= \frac{s_t^m - s_t^w}{(1+\phi)^2} + \frac{\phi-1}{1+\phi}\frac{ds_t^m}{d\phi} + \frac{A\left(u_{1w}''y + Ry\begin{bmatrix} (1-\delta^w)\,u_{2w,n}'' \\ -f(\bar\eta^w)\left(\kappa u_2'\left(u_{2w,n}' - u_{2m,n}'\right) + u_{2w,n}'u_{2m,n}'\right)\end{bmatrix}\right)}{(1+\phi)\det(\Omega)}\end{aligned}$$

where all the terms on the right hand side are positive and hence, $\frac{ds_t^{young}}{d\phi} > 0$. Therefore, the aggregate savings rate of the young cohort increases as the sex ratio rises. Since the (dis-)savings rate of the old cohort is fixed, an increase in the savings rate by the young cohort translates into an increase in the economy-wide savings rate.

Remarks: In this proof, we have assumed a uniform distribution for emotional utility η^i $(i = w, m)$. We note that many other distributions can give us the same results as long as they satisfy three sufficient conditions:[3]

$$\frac{\partial \int f(M(\eta^w))d\eta^w}{\partial\phi} \geq 0, \quad \frac{\partial\left[\frac{1}{\phi}\int \frac{f(\eta^w)}{f(M(\eta^w))}dF(\eta^w) + M(\bar\eta^w)f(\bar\eta^w)\right]}{\partial\phi} \leq 0$$

and $f(\bar\eta^w)$ is small enough.

The first sufficient condition is equivalent to

$$\int \frac{f'(M(\eta^w))}{f(M(\eta^w))}\frac{1-F(\eta^w)}{\phi^2}d\eta^w \geq 0$$

and the second one is equivalent to

$$\frac{1}{\phi^2}\int\left[\frac{f(\eta^w)}{f(M(\eta^w))} - \frac{f(\bar\eta^w)}{f(M(\bar\eta^w))}\right]dF(\eta^w) + \frac{1}{\phi}\int \frac{f(\eta^w)f'(M(\eta^w))}{f^3(M(\eta^w))}\frac{1-F(\eta^w)}{\phi^2}d\eta^w \geq 0$$

■

A.2 Proof of Proposition 2

Proof. Under the log utility assumption, at the balanced sex ratio, (3.10) becomes

$$-\frac{1}{1-s} + \frac{\beta}{s} = 0 \tag{A.4}$$

where we use the logic that since men and women are symmetric at $\phi = 1$, they would choose the same savings rate. Notice that (A.4) is the same first order condition for a lifetime bachelor. Even if some men or women choose to be single, they would choose the same savings rate s.

For a representative woman, at the balanced sex ratio, if she chooses to enter the marriage market,

[3] We use normal distributions in the calibration section, which gives the same qualitative result.

with probability $F(\bar{\eta})$ she will be married and receive welfare

$$\begin{aligned} V^w &= \ln((1-s)y) + \beta(1-F(\bar{\eta}))\ln(\kappa R(2s)y) + \beta F(\bar{\eta})\ln(Rsy) + (1-F(\bar{\eta}))E[\eta|\eta^w \geq \bar{\eta}] \\ &\geq \ln((1-s)y) + \beta\ln(Rsy) = V_n^w \end{aligned}$$

where the inequality holds because $\kappa > 1/2$ and $E[\eta|\eta^w \geq \bar{\eta}] \geq 0$. Therefore, entering the marriage market is a dominant strategy for all women. Since men and women are symmetric at $\phi = 1$, all men and all women will enter the marriage market with probability one.

As in Proposition 1,

$$\frac{ds^m}{d\phi} > 0 \text{ and } \frac{ds^m}{d\phi} + \frac{ds^w}{d\phi} > 0$$

we can show that

$$\begin{aligned} \frac{\partial V^m}{\partial\phi} &= y\left(-u'_{1m} + \kappa\delta^m u'_{2m} + (1-\delta^m)u'_{2m,n}\right)\frac{ds^m}{d\phi} + \delta^m y\kappa u'_{2m}\frac{ds^w}{d\phi} - \beta\int_{M(\bar{\eta}^w)}[1-F(\eta)]\,d\eta \\ &< -\beta\int_{M(\bar{\eta}^w)}[1-F(\eta)]\,d\eta - (\phi-1)\,\delta^m y\kappa u'_{2w}\frac{ds^m}{d\phi} < 0 \end{aligned} \tag{A.5}$$

where the first equality in (A.5) holds because

$$\begin{aligned} \frac{\partial\delta^m}{\partial\phi}(u_{2m} - u_{2m,n}) &= -\frac{1-F(\bar{\eta}^w)}{\phi^2}(u_{2m} - u_{2m,n}) \\ &\quad -\frac{Ryf(\bar{\eta}^w)}{\phi}\left[u'_{2m,n}\frac{ds^m}{d\phi} - \kappa u'_{2m}\left(\frac{ds^m}{d\phi} + \frac{ds^w}{d\phi}\right)\right](u_{2m} - u_{2m,n}) \\ \frac{d\left(\int_{M(\bar{\eta}^w)} M^{-1}(\eta^m)dF(\eta^m)\right)}{d\phi} &= -\int_{M(\bar{\eta}^w)}[1-F(\eta)]\,d\eta - \frac{\bar{\eta}^w(1-F(\bar{\eta}^w))}{\phi^2} \\ &\quad -\frac{Ry\bar{\eta}^w f(\bar{\eta}^w)}{\phi}\left[u'_{2m,n}\frac{ds^m}{d\phi} - \kappa u'_{2m}\left(\frac{ds^m}{d\phi} + \frac{ds^w}{d\phi}\right)\right] \end{aligned}$$

and the first inequality in (A.5) holds because

$$\frac{ds^w}{d\phi} \leq \frac{ds^m}{d\phi}$$

In summary, men lose as the sex ratio rises.

Now consider women's welfare. Given the equilibrium s^m and s^w under a sex ratio of ϕ, if a woman deviates from the equilibrium choice s^w, for instance, by choosing a savings rate $s^{w\prime} = s^m$, then she would receive a lower lifetime utility $V^{w\prime}$ ($\leq V^w$). But with $s^{w\prime} = s^m \geq s^w$, this woman is more likely to get married and is also more likely to marry a better man. Then

$$\begin{aligned} V^{w\prime} &= u_{1w'} + \beta \left[\delta' u_{2w'} + \left(1-\delta'\right) u_{2w',n} + \int_{\bar{\eta}^w} M\left(\eta^w + u_{2w'} - u_{2w}\right) dF\left(\eta^w\right) \right] \\ &\geq u_{1w'} + \beta \left[\left(1 - F(\bar{\eta}^w)\right) u_{2w'} + F\left(\bar{\eta}^w\right) u_{2w',n} + \int_{\bar{\eta}^w} M\left(\eta^w\right) dF\left(\eta^w\right) \right] \\ &= u_{1m} + \beta \left[\left(1 - F(\bar{\eta}^w)\right) u_{2m} + F\left(\bar{\eta}^w\right) u_{2m,n} + \int_{\bar{\eta}^w} M\left(\eta^w\right) dF\left(\eta^w\right) \right] \\ &\geq u_{1m} + \beta \left[\left(1 - F(M\left(\bar{\eta}^w\right))\right) u_{2m} + F\left(M\left(\bar{\eta}^w\right)\right) u_{2m,n} + \int_{M(\bar{\eta}^w)} M^{-1}\left(\eta^m\right) dF\left(\eta^m\right) \right] = V^m \end{aligned}$$

where $u_{1w'}$, $u_{2w'}$ and $u_{2w',n}$ denote the first period consumption-led utility, the second period consumption-led utilty when she is married, and the second period utility when she fails to be matched with any man, respectively. u_{2w} is the second period consumption-led utility for all other women who get married. The first inequality holds because the woman faces a greater possibility of getting married and because she will receive a higher expected emotional utility from her husband. The second inequality holds because, women are more likely than men to both get married and to receive higher emotional utilities from their spouses.

Therefore, for $\phi \geq 1$, women always achieve higher welfare than men, $V^w \geq V^m$.

If the sex ratio ϕ approaches infinity in the marriage market, given his rivals' choices, if a representative man chooses to stay in the marriage market, he will follow the first order condition (A.2) and achieve an approximate lifetime utility of $u_{1m} + \beta u_{2m,n}$. If he chooses to be single, his lifetime utility is $u_1 + \beta u_2$. The first order condition in this case is

$$-u_1' + u_2' = 0 \tag{A.6}$$

The two savings decisions, in the marriage market and being single, will be different since he would follow two different first order conditions. Then

$$V_n^m = \max u_1 + \beta u_2 > u_{1m} + \beta u_{2m,n} \to V^m$$

when $\phi \to \infty$. The representative man will then choose to be single which violates the assumption that, for all ϕs, entering the marriage market is the dominant strategy for all men. Therefore, there must be a threshold ϕ_1 such that for $\phi \geq \phi_1$, $V_n^m = V^m$.

For $\phi \geq \phi_1$, with probability $\frac{\phi_1}{\phi}$, a representative man chooses to enter the marriage market, and with probability $1 - \frac{\phi_1}{\phi}$, he remains single. For a representative woman, since she earns the same first period income as a representative man, we can show that

$$V_n^w = V_n^m = V^m < V^w$$

Therefore, the representative woman would choose to enter the marriage market with probability one.

We now turn to the aggregate savings rate in the young cohort. Similar to what we have shown in Proposition 1 that for $\phi < \phi_1$, as the sex ratio rises, the aggregate savings rate in the young cohort will rise. For $\phi \geq \phi_1$, as the sex ratio rises, some men begin to quit the marriage market and choose a different savings rate according to (A.6). Comparing (A.2) with (A.6), it is not possible to determine whether $s^m > s_n^m$ or not. This means that the effect on the aggregate savings rate for a higher sex ratio beyond the threshold is ambiguous. ■

A.3 Proof of Proposition 3

Proof. We rewrite the the economy-wide savings rate as following

$$s_t^P = (1-\alpha)\left(\frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w - s^{young}\right) + \frac{\alpha}{R}$$

As we have shown in Proposition 1, $\frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$ strictly increases in ϕ since men will save at a higher rate than women. s^P then is an increasing function of ϕ. By the expression of the current account to GDP ratio, this is also the condition that the current account is an increasing function of the sex ratio. Therefore, the economy-wide savings rate and the current account rise as the sex ratio becomes more unbalanced. ■

A.4 Proof of Proposition 4

Proof. We can rewrite the first order conditions (4.1), (4.2), (4.3), and (4.4) as equations on variables s_t^w, s_t^m, τ_t, ϕ_t and young people's labor income in each period. By (4.1) and (4.2), the optimal choices of s_t^w and s_t^m by young women and young men do not depend on their parents' wealth. Similarly, by (4.3), the parents' optimal choice of τ_t does not depend on their own wealth either. Under the same assumption as in Proposition 3, in a small open economy, the wage rate is constant. This implies that s_t^w, s_t^m, τ_t and ϕ_t will be constants from period t_0 onwards.

By the assumption that the sex ratio only becomes a choice variable from $t = t_0$ onwards, parents in all previous periods take as given that the sex ratio is balanced. That is, $\phi_t = 1$ for $t \leq t_0$. They make optimal decisions on savings for themselves and savings for children by solving the first order conditions, (4.1), (4.2), (4.3) and (4.4). In the initial equilibrium when parents are not able to choose the probability of having a son, equation (4.4) becomes

$$\left(u_{2,t_0-1}^{w,w} + \frac{\theta^d}{2}\left(\eta^s + \eta^d\right) + \eta^d\right) - \left(u_{2,t_0-1}^{w,m} + \frac{\theta^s}{2}\left(\eta^s + \eta^d\right) + \eta^s\right) = \Delta_{t_0-1}$$

where Δ_{t_0-1} is negative by assumption. The savings responses to a shock in period t_0 that allows parents to choose the sex ratio is equivalent to the savings responses to a permanent increase in Δ (from Δ_{t_0-1} to zero).

Since parents and young people will choose an optimal (but constant) savings rate from $t = t_0$, we drop the time index for the rest of the proof. We pursue the rest of the proof in two steps: (i) we first show that $\phi > 1$ from period t_0 onwards and then (ii) we show that both s^m and τ^m would rise while s^w would decrease. The aggregate savings rate and the current account will rise.

(i) We wish to show by contradiction that parents choose a higher sex ratio in period t_0 than in period $t_0 - 1$.

Suppose not, then parents choose $\phi \leq \phi_{t_0-1}$. In this case, all young men will get married since period t_0, i.e., $\delta^m = 1$ and $\delta^w \leq 1$. Parents with a son do not have any incentive to save for their son since they can do nothing to raise their son's probability of marriage, hence $\tau = 0$. By (4.4),

$$u_2^w - u_2^m = \left(\theta^s \delta^m \left(\frac{\phi\eta^s}{1+\phi} + \frac{\eta^d}{1+\phi}\right) + \eta^s\right) - \left(\theta^d \delta^w \left(\frac{\phi\eta^s}{1+\phi} + \frac{\eta^d}{1+\phi}\right) + \eta^d\right) > 0$$

where u_2^w and u_2^m denote the consumption-led utility obtained by parents when they have a daughter and a son, respectively. However, since all parents make zero savings for their child,

$$u_2^w - u_2^m = 0$$

Contradiction! Therefore, the sex ratio must rise in period t_0.

(ii) Given that $\phi > 1$ from period t_0 onwards, all women will get married, i.e., $\bar{\eta}^w = \eta^{\min}$. By (4.1), we can obtain

$$s^w = \frac{\beta}{1+\beta} - \frac{s^m}{1+\beta} \tag{A.7}$$

Then (4.2) becomes

$$-\frac{1}{1-s^m} + \beta \left[\begin{array}{c} \left(1 + 1/\phi + \eta^{\min} f(\bar{\eta}^m)\right) \frac{1+\beta}{\beta(1+s^m)} + \frac{1-1/\phi}{s_t^m} \\ +f\left(\bar{\eta}^m\right) \frac{1+\beta}{\beta(1+s^m)} \left(U^P + \chi \ln \kappa\right) - f\left(\bar{\eta}^m\right) \frac{1+\beta}{\beta(1+s^m)} \ln\left(Rs^m y\right) \end{array} \right] = 0 \tag{A.8}$$

where $\bar{\eta}^m = M\left(\eta^{\min}\right)$. Notice that parents select ϕ to maximize U^P and at time t_0, the balanced sex ratio ($\phi_{t_0-1} = 1$) is not optimal since the first order condition (4.4) does not hold. This means that the optimal sex ratio ϕ yields a larger value of U^P than that in period $t_0 - 1$. Then,

$$\begin{aligned} \frac{1}{1-s^m} &= \beta \left[\begin{array}{c} \left(1 + 1/\phi + \eta^{\min} f(\bar{\eta}^m)\right) \frac{1+\beta}{\beta(1+s^m)} + \frac{1-1/\phi}{s^m} \\ +f\left(\bar{\eta}^m\right) \frac{1+\beta}{\beta(1+s^m)} \left(U^P + \chi \ln \kappa\right) - f\left(\bar{\eta}^m\right) \frac{1+\beta}{\beta(1+s^m)} \ln\left(Rs^m y\right) \end{array} \right] \\ &> \beta \left[\begin{array}{c} \left(1 + 1/\phi_{t_0-1} + \eta^{\min} f(\bar{\eta}^m)\right) \frac{1+\beta}{\beta(1+s^m)} + \frac{1-1/\phi_{t_0-1}}{s_t^m} \\ +f\left(\bar{\eta}^m\right) \frac{1+\beta}{\beta(1+s^m)} \left(U_{t_0-1}^P + \chi \ln \kappa\right) - f\left(\bar{\eta}^m\right) \frac{1+\beta}{\beta(1+s^m)} \ln\left(Rs^m y\right) \end{array} \right] \end{aligned}$$

which implies

$$-\frac{1}{1-s^m}+\beta\left[\begin{array}{c}\left(1+1/\phi_{t_0-1}+\eta^{\min}f(\bar{\eta}^m)\right)\frac{1+\beta}{\beta(1+s^m)}+\frac{1-1/\phi_{t_0-1}}{s^m}\\ +f\left(\bar{\eta}^m\right)\frac{1+\beta}{\beta(1+s^m)}\left(U^P_{t_0-1}+\chi\ln\kappa\right)-f\left(\bar{\eta}^m\right)\frac{1+\beta}{\beta(1+s^m)}\ln\left(Rs^m y\right)\end{array}\right]<0 \tag{A.9}$$

It is easy to show that, given ϕ, term

$$-\frac{1}{1-s^m}+\beta\left[\begin{array}{c}\left(1+1/\phi+\eta^{\min}f(\bar{\eta}^m)\right)\frac{1+\beta}{\beta(1+s^m)}+\frac{1-1/\phi}{s^m}\\ +f\left(\bar{\eta}^m\right)\frac{1+\beta}{\beta(1+s^m)}\left(U^P+\chi\ln\kappa\right)-f\left(\bar{\eta}^m\right)\frac{1+\beta}{\beta(1+s^m)}\ln\left(Rs^m y\right)\end{array}\right]$$

is decreasing in s^m. Then by (A.8) at ϕ_{t_0-1} and (A.9), we can obtain

$$s^m > s^m_{t_0-1}$$

From period t_0 onwards, young men will raise their savings rate. Young women will reduce their savings by (A.7), however,

$$s^w+s^m=\frac{\beta\left(1+s^m\right)}{1+\beta}$$

rises.

By (4.3),

$$-\frac{1}{1-\tau}+\chi\theta^s\frac{f\left(\bar{\eta}^m\right)}{\tau}\left(\frac{\phi\eta^s}{1+\phi}+\frac{\eta^d}{1+\phi}\right)=0$$

it is easy to show that

$$\frac{d\tau}{d\phi}=\frac{\chi\theta^s f\left(\bar{\eta}^m\right)}{(1+\phi)^2}\frac{\eta^s-\eta^d}{\left(\frac{1}{1-\tau}\right)^2+\frac{\theta^s f(\bar{\eta}^m)}{\tau^2}\left(\frac{\phi\eta^s}{1+\phi}+\frac{\eta^d}{1+\phi}\right)}>0$$

Parents will increase their savings for their sons.

As shown in the previous analysis, if we normalize the measure of young people in period t_0-1 to be one, then the economy-wide savings rate

$$s^P_{t_0}=\frac{Q_{t_0}+(R-1)\cdot NFA_{t_0-1}-C_{1,t_0}-C_{2,t_0}}{Q_{t_0}}$$

where

$$NFA_{t_0-1}=\left(s^w_{t_0-1}+s^m_{t_0-1}\right)W_{t_0-1}+\tau^m_{t_0-1}R\left(s^w_{t_0-1}+s^m_{t_0-1}\right)W_{t_0-1}-K^d_t$$

Then

$$\begin{aligned}s^P_{t_0} &= \frac{Q_{t_0}+(R-1)\cdot NFA_{t_0-1}-C_{1,t_0}-C_{2,t_0}}{Q_{t_0}}\\ &= (1-\alpha)\left(\frac{s^w}{1+\phi}+\frac{\phi s^m}{1+\phi}+R\left(s^w_{t_0-1}+s^m_{t_0-1}\right)\left(\frac{\phi\tau}{1+\phi}\right)\right)\end{aligned}$$

By (4.1) and (4.2),

$$\begin{aligned}
\frac{1}{1-s^w} &= \frac{\beta}{s^w+s^m} < \beta\left(\frac{1}{\phi}\frac{1}{s^w+s^m}+\frac{1-1/\phi}{s_t^m}\right) \\
&< \beta\left[\begin{array}{c} \left(1+1/\phi+\eta^{\min} f(\bar{\eta}^m)\right)\frac{1}{s^w+s^m}+\frac{1-1/\phi}{s_t^m} \\ +f\left(\bar{\eta}^m\right)\frac{1}{s^w+s^m}\left(U^P+\chi\ln\kappa\right)-f\left(\bar{\eta}^m\right)\frac{1+\beta}{\beta(1+s^m)}\ln\left(Rs^m y\right)\end{array}\right] = \frac{1}{1-s^m}
\end{aligned}$$

Therefore

$$s^m > s^w$$

If the sex ratio rises in period t_0, s^m, τ^m and s^w+s^m all rise. Then the aggregate savings rate rises since

$$\begin{aligned}
s_{t_0}^P - s_{t_0-1}^P &= (1-\alpha)\left[\begin{array}{c} \left(\frac{s^w+s^m-\left(s_{t_0-1}^w+s_{t_0-1}^m\right)}{1+\phi_{t_0-1}}+\frac{\left(\phi_{t_0-1}-1\right)\left(s^m-s_{t_0-1}^m\right)}{1+\phi_{t_0-1}}+R\left(s_{t_0-1}^w+s_{t_0-1}^m\right)\left(\frac{\phi_{t_0}\left(\tau-\tau_{t_0-1}\right)}{1+\phi_{t_0-1}}\right)\right) \\ +\left(s^m-s^w\right)\left(\frac{\phi}{1+\phi}-\frac{\phi_{t_0-1}}{1+\phi_{t_0-1}}\right)+R\left(s_{t_0-1}^w+s_{t_0-1}^m\right)\tau\left(\frac{\phi}{1+\phi}-\frac{\phi_{t_0-1}}{1+\phi_{t_0-1}}\right)\end{array}\right] \\
&> 0
\end{aligned}$$

Similarly, the current account

$$\begin{aligned}
ca_{t_0} &= \frac{Q_{t_0}+(R-1)\cdot NFA_{t_0-1}-C_{1,t_0}-C_{2,t_0}-K_{t_0+1}^d}{Q_{t_0}} \\
&= (1-\alpha)\left(\frac{s^w}{1+\phi}+\frac{\phi s^m}{1+\phi}+R\left(s_{t_0-1}^w+s_{t_0-1}^m\right)\left(\frac{\phi\tau^m}{1+\phi}\right)\right)-\frac{\alpha}{R}
\end{aligned}$$

will rise. ■

B Two large countries

Consider a world consisting of only two countries. The two countries are identical in every respect except for their sex ratios in period t (they both have balanced sex ratios in period $t-1$). Country 1's sex ratio ϕ^1 is smaller than Country 2's sex ratio ϕ^2. There are no barriers to either goods trade or capital flows (although labor is not mobile internationally). We can show the following result:

Proposition 1 *Country 1 (with a more balanced sex ratio) runs a current account deficit while Country 2 runs a current account surplus.*

Proof. Since capital can flow freely internationally, the interest rates are equal in both countries. By (3.12) and (3.13), the wage rates are also equal in the two countries.

Given the same wage rates, the households in the two countries have the same first period income. By Proposition 1, Country 2 will have a higher savings rate than Country 1. On the other hand, in equilibrium, given a constant R, the investments in both countries are the same, and the world capital market always clears. Therefore, Country 2 runs a current account surplus and Country 1 runs a current account deficit. ■

To see the intuition, let us fix $\phi^1 = 1$ (i.e., Country 1 has a balanced sex ratio). If Country 2 were to have a balanced sex ratio, the current account must be zero for both countries since they are identical in every respect. In other words, within each country, the investment must be equal to the aggregate savings. However, the sex ratio imbalance in Country 2 causes it to have a higher aggregate savings for a given world interest rate. This depresses the world interest rate. The lower interest rate raises the investment level in both economies (and reduces the savings rate a little bit). This must imply that the desired investment level in Country 1 is now greater than its desired savings rate. As a result, capital flows from Country 2 to Country 1. That is, Country 1 runs a current account deficit, and Country 2 a surplus.

Numerically, we study two cases to see how a rise in the sex ratio of Country 2 will affect the current account positions. In the first case, we assume that the two countries are identical in every respect except for their sex ratios. While Country 1 always has a balanced sex ratio ($\phi^1 = 1$), we vary the sex ratio in Country 2 from 1 to 1.5. Appendix Table 2 reports the calibration results. Appendix Figure 1 traces out the current account responses in both countries as Country 2's sex ratio increases. The most important result is that a rise in Country 2's sex ratio first triggers a rise in its current account surplus and a rise in Country 1's current account deficit. After Country 2's sex ratio exceeds threshold ϕ_1, a further rise in Country 2's sex ratio induces a decline in both its current account surplus and Country 1's deficit.

We have also done robustness checks by varying the values of κ, m, and σ. Based on the same reasoning as in the small open economy case, for larger κ, m, or smaller σ, a given increase in Country 2's sex ratio results in a greater current account imbalance in the two countries.

In the second case, we attempt to let Countries 1 and 2 mimic the United States and China, respectively. In particular, we assume that $L_1 = 1/5 \cdot L_2$ to match the fact that the U.S. population is around 1/5 that of China. In addition, we choose the TFP parameter in Country 1, ζ_1, to match the fact that the U.S. per capita GDP was about 15 times the Chinese level around 2000 when the sex ratio in China for the marriage age cohort was not yet seriously out of balance. The remaining parameters are set to be the same as before. We let the sex ratio in the United States be always balanced, and vary the Chinese sex ratio from 1 to 1.5.

Appendix Tables 3a, 3b and 3c report the benchmark result and robustness checks. Appendix Figure 2 plots the calibration results. Qualitatively, they look similar to the first large-country experiment. Quantitatively, Country 2's (China) current account response (as a share of GDP) becomes stronger. With China's sex ratio at 1.15 (and $\kappa = 0.8$), it runs a current account surplus on the order of 4.4% of its GDP, and at the same time, the United States runs a current account deficit of 1.5% of

GDP. This resembles one third to a half of the real world pattern in which the U.S. deficit is about 4-6% of GDP, whereas the Chinese surplus is on the order of 7-10% of GDP just before the global financial crisis. In other words, a rise in the Chinese sex ratio, while it does not provide a complete explanation, could be a significant contributor to the global current account imbalances. If the sex ratio rises beyond threshold ϕ_1, the Chinese surplus can begin to decline.

To summarize, the calibrations suggest that a rise in the sex ratio (when the sex ratio takes some reasonable values) could produce an economically significant increase in the aggregate savings rate that results in a current account surplus. If the country is large enough, this could induce other countries to run a current account deficit even if they have a balanced sex ratio.

C Welfare

There are two sources of market failure in the model economy. On one hand, a part of the savings in the competitive equilibrium is motivated by a desire to out-save one's competitors in the marriage market. The increment in savings, while individually rational, is not useful in the aggregate, since when everyone raises the savings rate by the same amount, the ultimate marriage market outcome is not affected by the increase in savings. In this sense, the competitive equilibrium produces too much savings. On the other hand, because the savings contribute to a pubic good in a marriage (an individual's savings raises the utility of his/her partner), but an individual in the first period does not take this into account, he/she may under-save relative to the social optimum. Note that these sources of market failure exist even with a balanced sex ratio. They also have opposite effects on the aggregate savings rate. (In calibrations, we find that these two effects cancel each other out when the sex ratio is balanced.) As the sex ratio rises, the importance of the over-saving effect also increases, which gives rise to our first proposition.

We now consider what a welfare-maximizing central planner would do. The central planner gives equal weight to each man and women. He assigns the marriage matching outcomes and chooses women's and men's savings rates to maximize the following social welfare function,

$$\max\ U = \frac{1}{1+\phi}U^w + \frac{\phi}{1+\phi}U^m$$

The first order conditions are

$$-u'_{1w} + [1 - F(\bar{\eta}^w) + \phi\,(1 - F(M\,(\bar{\eta}^w)))]\,\kappa u'_{2w} + F(\bar{\eta}^w)u'_{2w,n} \quad = \quad 0 \qquad \text{(C.1)}$$

$$-u'_{1m} + \left[1 - F(M\,(\bar{\eta}^w)) + \frac{1}{\phi}\,(1 - F(\bar{\eta}^w)\right]\kappa u'_{2m} + F(M\,(\bar{\eta}^w))u'_{2m,n} \quad = \quad 0 \qquad \text{(C.2)}$$

Comparing (C.1), (C.2) to (A.1) and (A.2), in general, it is not obvious whether women or men will save at a higher rate in a decentralized equilibrium than that under central planning due to the two opposing sources of market failure. However, when $\phi = 1$, since women and men have the same optimal conditions, by (3.10), women and men will save the same amount in a competitive equilibrium as in the central planned economy.

As a thought experiment, one may also consider what the central planner would do if she can choose the sex ratio (in addition to the savings rates) to maximize the social welfare. The first order condition with respect to ϕ is

$$\frac{U^m - U^w}{(1+\phi)^2} = 0 \tag{C.3}$$

The only sex ratio that satisfies (C.3) is $\phi = 1$. In other words, the central planner would have chosen a balanced sex ratio. Deviations from a balanced sex ratio represent welfare losses.

In calibrations with a log utility function, we show in Appendix Table 4 that men's welfare under a decentralized equilibrium relative to the central planner's economy declines as the sex ratio increases. In comparision, women's relative welfare increases as the sex ratio goes up. The social welfare (the sum of all men's and women's welfare) goes down as the sex ratio rises.[4] Appendix Figures 3, 3a and 3b trace out the savings rates for men (the upper left panel), women (the upper right panel), the economy as a whole (the lower left panel) and the welfare (the lower right panel). With a log-utility function, the optimal savings rates chosen for men and women by the planner do not depend on the sex ratio and intra-household bargaining powers.[5] When the sex ratio is balanced, the savings rates by women, men and the economy as a whole are the same as those under the planner's economy. With unbalanced sex ratios, men's (decentralized) savings rates overshoot the socially optimally level, and the extent of excessive savings rises with the sex ratio. Women's savings rates follow an opposite pattern. The economy-wide savings rate follows a pattern that is qualitatively similar to the men's savings rate. In particular, the economy in a decentralized equilibrium tends to save too much relative to the social optimum, and the excess savings rises with the sex ratio. In the lower right panel, we can see that the welfare levels for both men and the economy as a whole decline as the sex ratio increases, while the welfare for women rises with the sex ratio.

D Endogenous Intra-household Bargaining

One problem in the benchmark calibration is that, as the sex ratio rises, women's savings rates decline very quickly. In this extension, we incorporate intra-household bargaining between wives and husbands into the model. The goal is to show that, when allowing intra-household bargaining, the women's savings rate declines much more slowly.

We assume that everyone consumes two goods in the second period, a public good (e.g., a house)

[4] The results are similar if we change the utility function to a CRRA form.
[5] This feature does not hold when we use the CRRA utility function.

and a private good. The aggregate second period consumption index is

$$c_{2i} = \frac{z_i^{\gamma} h^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \quad i = w, m \tag{D.1}$$

where z_w and z_m are private goods consumption by women and men, respectively, and h is the public good consumption. γ is the share of private expenditure in the second period consumption index.

A representative household maximizes the weighted sum of the utilities of the husband and the wife. Let μ denote the weight on the wife's utility, which represents her bargaining power in the family. Then the household's optimization problem is

$$\max_{h, z_w, z_m} \ \mu u(c_{2w}) + (1-\mu) u(c_{2m})$$

with the resource constraint

$$z_w + z_m + h = R s^w y^w + R s^m y^m \tag{D.2}$$

If we assume $u(c) = \ln c$, solving the household's maximization problem, we have

$$\begin{aligned} c_{2w} &= \mu^{\gamma} \left(R s^w y^w + R s^m y^m \right) \\ c_{2m} &= (1-\mu)^{\gamma} \left(R s^w y^w + R s^m y^m \right) \end{aligned}$$

If $\mu = \frac{1}{2}$, this is the case in our benchmark model and $\frac{1}{2} < \kappa = 2^{-\gamma} < 1$.

More generally, similar to Browning et al. (1994), μ is a function of the sex ratio ϕ, the relative wealth and other characteristics of the household members. For simplicity, we assume that the intra-household bargaining power depends only on the relative wealth of household members. In particular, we assume that the wife's bargaining power within a family is

$$\mu = \frac{(s^w)^{\varepsilon}}{(s^w)^{\varepsilon} + (s^m)^{\varepsilon}}$$

and the husband's bargaining power is $1 - \mu$. ε is the parameter that governs the sensitivity of bargaining power to relative wealth. A larger ε means that household bargaining power will respond to the relative wealth more strongly.

We take the same values for other parameters as in the benchmark. Appendix Table 5 reports the calibration results, and Appendix Figures 4a and 4b plot the saving rates. Relative to the case of no intra-household bargaining, women now reduce their savings rates more slowly as the sex ratio rises and may even raise their savings when (i) the sensitivity of bargaining power to relative wealth is high (large ε) and (ii) private consumption is important in her second period consumption bundle (large γ). Since there is no big change in men's response to the rise in the sex ratio, the economy-wide savings rate responds more strongly to a rise in the sex ratio than the benchmark case. For $\sigma = 0.05$, $\varepsilon = 0$ and $\gamma = 0.5$, as the sex ratio rises from 1 to 1.15, the current account to GDP ratio rises by

6.2%. For $\sigma = 0.05$, $\varepsilon = 0.5$ and $\gamma = 0.5$, as the sex ratio rises from 1 to 1.15, the current account to GDP ratio rises by 11.2%. As the sex ratio keeps rising (and exceeds the threshold ϕ_1, which is around 1.35 in this case), some men quit the marriage market and the aggregate savings rate declines.

In Appendix Table 6, we re-calibrate the case of two asymmetric countries (the United States versus China) by allowing for intra-household bargaining. The corresponding current account responses and the welfare changes are plotted in Appendix Figure 5. Relative to the benchmark case of no intra-household bargaining, the responses of the aggregate savings and the current account to higher sex ratios are similar. However, the savings response by women is more realistic. For example, for $\sigma = 0.05$, $\varepsilon = 0$ and $\gamma = 0.5$, we can find in Appendix Table 6 that a rise in the sex ratio in country 2 (China) from 1 to 1.15 generates a current account surplus of 4.4% of GDP in China and a deficit of 1.5% of GDP in the U.S. For $\sigma = 0.05$, $\varepsilon = 0.5$ and $\gamma = 0.5$, the same rise in the sex ratio in China can generate a very large current account surplus of 8.2% of GDP in China and a deficit of 2.7% of GDP in the U.S.

In the right panel of Appendix Figure 5, we trace out the economy-wide welfare in a decentralized equilibrium for a given sex ratio relative to the welfare under a balanced sex ratio. The country that experiences a rise in the sex ratio (e.g., China) clearly suffers from an ever-deteriorating welfare. The country with a balanced sex ratio (e.g., the United States) also experiences a welfare loss (but with a much smaller magnitude compared to the country with an unbalanced sex ratio). Intuitively, a rise in the Chinese sex ratio depresses the global interest rate, but this produces two effects with opposite signs in the United States. On one hand, the lower cost of capital boosts the real wage in the United States, which is positive for Americans. On the other hand, the lower interest rate also implies a lower interest income for a given amount of savings, which is negative for Americans. In this numerical example, the second effect dominates the first effect, which yields a welfare loss to Americans.

We note, however, that the quantitative effect of a rise in the Chinese sex ratio on U.S. welfare is very small. The Chinese lose the most from a rise in the sex ratio. From the right panel of Appendix Figure 5, if the Chinese sex ratio reaches 1.15, Americans have a utility loss that is equivalent to a reduction in consumption by 0.4%. In contrast, the Chinese suffer a welfare loss that is equivalent to a decline in consumption by 32.7%. The large Chinese welfare loss comes principally from a rise in the incidence of involuntary bachelorhood.

We also calibrate a multi-period model with endogenous intra-household bargaining. Appendix Figures 6 and 7 report the results. The responses to a higher sex ratio are stronger. When $\varepsilon = \gamma = 0.5$, and $\sigma = 0.05$, if the marriage takes place in the 10th period, a rise in the sex ratio from 1 to 1.1 would produce a rise in both the economy-wide savings rate and current account by about 4% of GDP. If the marriage takes place in the 20th period instead, then the same sex ratio increase would generate a bigger increase in the savings rate and current account surplus (by about 7% of GDP).

E Competitive versus Precautionary Savings Motives

As in the previous analysis, women and men can benefit from marriage through two channels. First, by pooling their resources and consuming a partial public good in a marriage, wives and husbands can free ride on their spouses. Second, married people can augment happiness by deriving emotional utility from having a partner. In other words, if a young man fails in getting married, he loses some future consumption as well as the emotional utility. This means the rise in the savings rate by young men come from a combination of a competitive savings motive (to compete more effectively in the mating market) and a hedging motive (to hedge against the event that he may be single in the second period). Which of the two effects is the more important one when the sex ratio rises? In this appendix, we provide an answer.

Based on the same assumptions in Proposition 1, we can write down the first order condition for a young man

$$-u'_{1m} + \left[\delta^m \kappa u'_{2m} + (1-\delta^m)\, u'_{2m,n} + CS\right] = 0$$

where the term CS represents the marginal gain from the likely marriage when the young man saves more in the first period,

$$CS = \kappa u'_{2m} \left(\phi \left(1 - F\left(\bar{\eta}^m\right)\right) + \bar{\eta}^w f(\bar{\eta}^m) f\left(\bar{\eta}^m\right)\left(u_{2m} - u_{2m,n}\right)\right)$$

We can examine the pure hedging motive for saving by shutting down the competitive saving motive. For example, if one's savings rate is private information, and no one else, especially members of the opposite sex, cannot see, then there is no way to enhance (or hurt) one's competitive position in the marriage market by adjusting one's savings rate. Then, the only reason one may raise savings rate is to prepare for the increased likelihood that he will be unmarried in the second period. This is equivalent to setting $CS = 0$. We call a rise in the savings after a higher sex ratio when CS=0 as a hedging-induced precautionary savings response. A rise in the savings rate in the case without the restriction of CS=0 reflects the sum of the precautionary savings and the competitive savings.

When $CS = 0$ (i.e., no competitive savings), the first order conditions for women and men are, respectively,

$$\begin{aligned} -u'_{1w} + \left[\delta^w \kappa u'_{2w} + (1-\delta^w)\, u'_{2w,n}\right] &= 0 \\ -u'_{1m} + \left[\delta^m \kappa u'_{2m} + (1-\delta^m)\, u'_{2m,n}\right] &= 0 \end{aligned}$$

where

$$\delta^w = 1 - F\left(\bar{\eta}^w\right) \quad \text{and} \quad \delta^m = 1 - F\left(\bar{\eta}^m\right)$$

$\bar{\eta}^w$ and $\bar{\eta}^m$ are similary defined as in the benchmark.

We calibrate the model by using the benchmark parameters: $\beta = 0.7$, $\alpha = 0.4$, $\kappa = 0.8$, $\sigma = 0.05$

and $m = 2.08$. Appendix Figure 8 shows the comparison between precautionary savings response and the total savings response in our benchmark. We can clearly see that, if we shut down the competitive savings motive, the changes in men's, women's and the aggregate savings are much smaller than the benchmark case. For instance, as the sex ratio rises from 1 to 1.10, the aggregate savings rate goes up by less than 1 percent if there is only precautionary savings. In compariosn, when the competitive savings motive is also present, the economy-wide savings rate rises by 5.4%. This means that the competitive savings motive plays a quantitatively large role in understanding the savings response to a higher sex ratio.

F A Model with Educational Expenditure

When the sex ratio rises, raising savings rate may not be the only response in the real world. In particular, households could alter efforts in accumulating human capital. If we allow this channel in the model, how would this affect the savings response? It is tempted to think that if accumulation of human capital requires increases in educational expenditure, and this may dampen the savings response to a given rise in sex ratio. In this appendix, we extend the benchmark model by allowing human capital accumulation. We show that our basic results continue to hold at least qualitatively.

We modify our two-period benchmark model by allowing for education input in the first period (which enhances one's productivity). A young person i's labor productivity ξ depends on two inputs: educational expenditure T_e^i, and effort e^i. The labor productivity $\xi\left(T_e^i, e^i\right)$ increases in each of its argument. For simplicity, we assume that the first period income for individual i is

$$\xi\left(T_e^i, e^i\right) y = \min\left[aT_e, bey\right]$$

where y is the effective wage. By this assumption, effort and educational expenditure are complements.

Assume that everyone enters the marriage market, then the optimization problem for a representative woman becomes

$$\max_{s^w, T_e^w} u(c_{1w}) + v(1-e^w) + \beta E\left[u(c_{2w}) + \eta^m\right]$$

subject to the budget constraints that

$$\begin{aligned} c_{1w} &= (1-s^w)\xi^w y - \lambda^w y \\ c_{2w} &= \begin{cases} \kappa\left(Rs^w\xi^w y + Rs^m\xi^m y\right) & \text{if married} \\ Rs^w\xi^w y & \text{otherwise} \end{cases} \end{aligned}$$

where $\lambda^w y$ represents the educational expenditure (in terms of effective wage) in the young woman's first period consumption basket. $v(\cdot)$ represents the utility obtained from leisure, which is denoted by $1-e^w$.

Similarly, for a representative young man

$$\max_{s^m, T_e^m} u(c_{1m}) + v(1-e^m) + \beta E\left[u(c_{2m}) + \eta^m\right]$$

subject to the budget constraints that

$$\begin{aligned} c_{1m} &= (1-s^m)\xi^m y - \lambda^m y \\ c_{2m} &= \begin{cases} \kappa\left(Rs^w\xi^w y + Rs^m\xi^m y\right) & \text{if married} \\ Rs^m\xi^m y & \text{otherwise} \end{cases} \end{aligned}$$

Now we can show the following proposition.

Proposition 2 *Under the assumptions in Proposition 1, if $\eta^{\max} \geq 1$, given the same aggregate production function as in (3.11), we can show that as the sex ratio rises, (1) the savings rate of a representative man goes up, while the savings rate of a representative woman declines; (2) however, the economy-wide savings rate increases unambiguously.*

Proof. In equilibrium, we have

$$a\lambda^i y = be^i y \Longrightarrow \lambda^i = \frac{b}{a}e^i,\ i = w, m \tag{F.1}$$

Under the same assumptions in Proposition 1 and following the same steps as in Appendix A, we can show that the first order condition with respect to s^w and e^w are

$$\begin{aligned} -\frac{1}{1-s^w-1/a} + \beta\xi^w\left[\frac{\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right) + f\left(\bar{\eta}^w\right)\left(u_{2w} + \bar{\eta}^m - u_{2w,n}\right)}{s^w\xi^w + s^m\xi^m} + \frac{F\left(\bar{\eta}^w\right)}{s^w\xi^w}\right] &= 0 \qquad \text{(F.2)} \\ \frac{1}{e^w} - v'\left(1-e^w\right) + \beta bs^w\left[\frac{\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right) + f\left(\bar{\eta}^w\right)\left(u_{2w} + \bar{\eta}^m - u_{2w,n}\right)}{s^w\xi^w + s^m\xi^m} + \frac{F\left(\bar{\eta}^w\right)}{s^w\xi^w}\right] &= 0 \end{aligned}$$

respectively. By (F.2), we can re-write the second condition as

$$v'\left(1-e^w\right)e^w = \frac{1-1/a}{1-s^w-1/a} \tag{F.3}$$

Similarly, for a representative young man, the first order conditions are

$$-\frac{1}{1-s^m} + \beta\xi^m\left[\frac{(1+\phi)\left(1-F\left(\bar{\eta}^m\right)\right) + f\left(\bar{\eta}^m\right)\left(u_{2m} + \bar{\eta}^w - u_{2m,n}\right)}{s^w\xi^w + s^m\xi} + \frac{F\left(\bar{\eta}^m\right)}{s^m\xi^m}\right] = 0 \tag{F.4}$$

and

$$v'\left(1-e^w\right)e^w = \frac{1-1/a}{1-s^w-1/a} \tag{F.5}$$

By (F.3) and (F.5), we can show that

$$\frac{de^i}{ds^i} = \frac{\left(v^{i\prime}e^i\right)^2}{v^{i\prime} - v^{i\prime\prime}e^i}\frac{1}{1-1/a} > 0,\ i = w, m \tag{F.6}$$

Similar to the proof of Proposition 1, we can show that $\bar{\eta}^m = M(\bar{\eta}^w)$ and $s^m\xi^m \geq s^w\xi^w$ for $\phi \geq 1$. Since at $\phi = 1$, women and men are symmetric, and hence $s^m = s^w$ and $\xi^m = \xi^w$. For $\phi \geq 1$, by (F.6), $s^m\xi^m \geq s^w\xi^m$ means $s^m \geq s^w$ and $\xi^m \geq \xi^m$.

Similar to the proof of Proposition 2, total differentiate equations (F.2) and (F.4), we can obtain

$$\Omega^E \cdot \left[\begin{array}{cc} ds^w & ds^m \end{array}\right]^T = \left[\begin{array}{cc} z_1 & z_2 \end{array}\right]^T d\phi$$

where

$$\begin{aligned}
\Omega_{11}^E &= \xi^w\left\{u_{1w}'' + \beta R^2\left[\begin{array}{c} \kappa^2 u_{2w}''\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right) + \left(1-\delta^w\right)u_{2w,n}'' \\ +f\left(\bar{\eta}^w\right)\kappa^2 u_{2w}''\left(u_{2w} + M(\bar{\eta}^w) - u_{2w,n}\right) \\ +2f\left(\bar{\eta}^w\right)\kappa u_{2w}'\left(\kappa u_{2w}' - u_{2w,n}'\right) \end{array}\right]\left(1 + \frac{s^w}{\xi^w}\frac{d\xi^w}{ds^w}\right)\right\} \\
\Omega_{12}^E &= \beta R^2\xi^m\left[\begin{array}{c} \kappa^2 u_{2w}''\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right) \\ +f\left(\bar{\eta}^w\right)\kappa^2 u_{2w}''\left(u_{2w} + M(\bar{\eta}^w) - u_{2w,n}\right) \\ +f\left(\bar{\eta}^w\right)\kappa^2 u_{2w}'^2 + f(\bar{\eta}^w)\left(u_{2m,n}' - \kappa u_{2m}'\right)\left(u_{2w,n}' - \kappa u_{2w}'\right) \end{array}\right]\left(1 + \frac{s^m}{\xi^m}\frac{d\xi^m}{ds^m}\right) \\
\Omega_{21}^E &= \beta R^2\xi^w\left[\begin{array}{c} \kappa^2 u_{2m}''\left(\left(1+\phi\right)\left(1-F\left(M\left(\bar{\eta}^w\right)\right)\right)\right) \\ +f\left(\bar{\eta}^w\right)\kappa u_{2m}'\left(\left(1+\frac{1}{\phi}\right)\kappa u_{2m}' - \frac{1}{\phi}u_{2m,n}'\right) \end{array}\right]\left(1 + \frac{s^w}{\xi^w}\frac{d\xi^w}{ds^w}\right) \\
\Omega_{22}^E &= \xi^m\left\{u_{1m}'' + \beta R^2\left[\begin{array}{c} \kappa^2 u_{2m}''\left(\left(1+\phi\right)\left(1-F\left(M\left(\bar{\eta}^w\right)\right)\right)\right) + \left(1-\delta^m\right)u_{2m,n}'' \\ +f\left(\bar{\eta}^w\right)\left(\kappa u_{2m}' - u_{2m,n}'\right)\left(\left(1+\frac{1}{\phi}\right)\kappa u_{2m}' - \frac{1}{\phi}u_{2m,n}'\right) \end{array}\right]\left(1 + \frac{s^m}{\xi^m}\frac{d\xi^m}{ds^m}\right)\right\}
\end{aligned}$$

and

$$z_1 = 0,\ z_2 = \frac{\left[1 - F(\bar{\eta}^w)\right]\left(\kappa u_{2m}' - u_{2m,n}'\right)}{\phi^2}$$

Similar to the proof of Proposition 1, we can show that

$$\det\left(\Omega^E\right) > 0$$

Then

$$\frac{ds_t^m}{d\phi} = \frac{z_2\Omega_{11}^E}{\det\left(\Omega^E\right)} > 0 \ \text{ and } \ \frac{ds_t^w}{d\phi} = -\frac{z_2\Omega_{12}^E}{\det\left(\Omega^E\right)}$$

By the definition of $\bar{\eta}^w$ and $M(\bar{\eta}^w)$, it is easy to show that

$$u_{2w} + M(\bar{\eta}^w) - u_{2w,n} = u_{2w} + M(\bar{\eta}^w) - u_{2m,n} + u_{2m,n} - u_{2w,n} = M(\bar{\eta}^w) - \bar{\eta}^w + u_{2m,n} - u_{2w,n}$$

By the definition of $\bar{\eta}^w$,

$$\bar{\eta}^w = \log(s^m \xi^m) - \log(\kappa(s^m \xi^m + s^w \xi^w))$$

Similar to the proof of Proposition 2, under the log utilty assumption, we can show that

$$\frac{ds_t^w}{d\phi} < \frac{z_2 \beta R^2 \left(2\eta^{\max} + 2\log\kappa + \log\left(2 + \frac{s_t^m}{s^w} + \frac{s_t^w}{s^m}\right) - 2\right)\xi^m}{(\eta^{\max} - \eta^{\min})(s^m\xi^m + s^w\xi^w)^2 \det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}} < 0$$

where the last inequality holds because

$$2\eta^{\max} + 2\log\kappa + \log\left(2 + \frac{s_t^m}{s_t^w} + \frac{s_t^w}{s_t^m}\right) - 2 \geq 2\log\kappa + \log\left(2 + \frac{s_t^m}{s_t^w} + \frac{s_t^w}{s_t^m}\right) \geq 2\log(2\kappa) > 0$$

By (F.6), we have

$$\frac{d\xi^m}{d\phi} > 0 \text{ and } \frac{d\xi^w}{d\phi} < 0$$

The aggregate savings rate in the young cohort is

$$s^{young} = \frac{\frac{\phi}{1+\phi}s^m\xi^m + \frac{1}{1+\phi}s^w\xi^w}{\frac{\phi}{1+\phi}\xi^m + \frac{1}{1+\phi}\xi^w} = \frac{\frac{\phi}{1+\phi}\xi^m}{\frac{\phi}{1+\phi}\xi^m + \frac{1}{1+\phi}\xi^w}s^m + \frac{\frac{1}{1+\phi}\xi^w}{\frac{\phi}{1+\phi}\xi^m + \frac{1}{1+\phi}\xi^w}s^w$$

then

$$\frac{ds^{young}}{d\phi} = \frac{\frac{\phi}{1+\phi}\xi^m}{\frac{\phi}{1+\phi}\xi^m + \frac{1}{1+\phi}\xi^w}\frac{ds^m}{d\phi} + \frac{\frac{1}{1+\phi}\xi^w}{\frac{\phi}{1+\phi}\xi^m + \frac{1}{1+\phi}\xi^w}\frac{ds^w}{d\phi} + \frac{d\left(\frac{\frac{\phi}{1+\phi}\xi^m}{\frac{\phi}{1+\phi}\xi^m + \frac{1}{1+\phi}\xi^w}\right)}{d\phi}(s^m - s^w)$$

The sum of the first two terms on the right hand side yields

$$\begin{aligned}
& \frac{\frac{\phi}{1+\phi}\xi^m}{\frac{\phi}{1+\phi}\xi^m + \frac{1}{1+\phi}\xi^w}\frac{ds^m}{d\phi} + \frac{\frac{1}{1+\phi}\xi^w}{\frac{\phi}{1+\phi}\xi^m + \frac{1}{1+\phi}\xi^w}\frac{ds^w}{d\phi} > \frac{\frac{1}{1+\phi}}{\frac{\phi}{1+\phi}\xi^m + \frac{1}{1+\phi}\xi^w}\left(\xi^m\frac{ds^m}{d\phi} + \xi^w\frac{ds^w}{d\phi}\right) \\
= \quad & \frac{\frac{1}{1+\phi}}{\frac{\phi}{1+\phi}\xi^m + \frac{1}{1+\phi}\xi^w}\frac{z_2}{\det(\Omega^E)}(\xi^m\Omega_{11} - \xi^w\Omega_{12}) \\
> \quad & \frac{\frac{1}{1+\phi}\xi^m\xi^w}{\frac{\phi}{1+\phi}\xi^m + \frac{1}{1+\phi}\xi^w}\frac{z_2}{\det(\Omega^E)}\begin{pmatrix} u''_{1w,t} + \beta R^2\kappa^2 u''_{2w}\left(\left(1+\frac{1}{\phi}\right)(1 - F(\bar{\eta}^w))\right)\frac{s^w}{\xi^w}\frac{d\xi^w}{ds^w} \\ -\beta R^2\left[\kappa^2 u''_{2w}\left(\left(1+\frac{1}{\phi}\right)(1 - F(\bar{\eta}^w))\right)\right]\frac{s^m}{\xi^m}\frac{d\xi^m}{ds^m} \end{pmatrix}
\end{aligned}$$

$$
\begin{aligned}
> & \frac{\frac{1}{1+\phi}\xi^m\xi^w}{\frac{\phi}{1+\phi}\xi^m+\frac{1}{1+\phi}\xi^w}\frac{z_2}{\det\left(\Omega^E\right)}\left(-\frac{u'_{1w}}{(1-s^w-1/a)\xi^w}+\frac{\beta R\kappa u'_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)}{s^m\xi^m+s^w\xi^w}\frac{s^m}{\xi^m}\frac{d\xi^m}{ds^m}\right) \\
> & \frac{\frac{1}{1-s^m}\frac{1}{1+\phi}\xi^m\xi^w}{\frac{\phi}{1+\phi}\xi^m+\frac{1}{1+\phi}\xi^w}\frac{z_2}{\det\left(\Omega^E\right)}\left(\begin{array}{c}-\frac{(1-s^m-1/a)\xi^m}{(1-s^w-1/a)\xi^w}\frac{s^m\xi^m+s^w\xi^w}{s^m\xi^m}u'_{1w} \\ +\beta R\kappa u'_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)\end{array}\right)
\end{aligned}
$$

where the last inequality holds because

$$
\frac{s^m}{\xi^m}\frac{d\xi^m}{ds^m}=\frac{s^m}{1-s^m-1/a}\frac{1}{1-\frac{v^{m\prime\prime}\xi^m}{v^{m\prime}}}<\frac{s^m}{1-s^m-1/a}
$$

We can show that by (F.2) and (F.4)

$$
\begin{aligned}
\frac{(1-s^m-1/a)\xi^m}{(1-s^w-1/a)\xi^w}\frac{s^m\xi^m+s^w\xi^w}{s^m\xi^m} &= \frac{\left(\begin{array}{c}\kappa u'_{2w}\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)+\left(1-\delta^w\right)u'_{2w,n} \\ +f\left(\bar{\eta}^w\right)\kappa u'_{2w}\left(u_{2w}+M(\bar{\eta}^w)-u_{2w,n}\right)\end{array}\right)}{\kappa u'_{2m}\left(1+\phi\right)\left(1-F\left(\bar{\eta}^m\right)\right)+\left(1-\delta^m\right)u'_{2m,n}}\frac{s^m\xi^m+s^w\xi^w}{s^m\xi^m} \\
&> \frac{\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)+F\left(\bar{\eta}^w\right)\frac{s^m\xi^m+s^w\xi^w}{s^m\xi^m}+f\left(\bar{\eta}^w\right)\left(M(\bar{\eta}^w)-\bar{\eta}^w\right)}{\frac{s^m\xi^m}{s^m\xi^m+s^w\xi^w}\left(1+\phi\right)\left(1-F\left(\bar{\eta}^m\right)\right)+\left(1-\delta^m\right)} \\
&> \frac{1+\frac{1}{\phi}\left(1-F\left(\bar{\eta}^w\right)\right)+F\left(\bar{\eta}^m\right)-F\left(\bar{\eta}^w\right)}{\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)+F\left(\bar{\eta}^m\right)}=1
\end{aligned}
$$

and

$$
-u'_{1w}+\beta R\kappa u'_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)<0
$$

then

$$
\frac{\frac{\phi}{1+\phi}\xi^m}{\frac{\phi}{1+\phi}\xi^m+\frac{1}{1+\phi}\xi^w}\frac{ds^m}{d\phi}+\frac{\frac{1}{1+\phi}\xi^w}{\frac{\phi}{1+\phi}\xi^m+\frac{1}{1+\phi}\xi^w}\frac{ds^w}{d\phi}>0
$$

Since $\frac{d\xi^m}{d\phi}>0$ and $\frac{d\xi^w}{d\phi}<0$, we can show that

$$
\frac{d\left(\frac{\frac{\phi}{1+\phi}\xi^m}{\frac{\phi}{1+\phi}\xi^m+\frac{1}{1+\phi}\xi^w}\right)}{d\phi}=\frac{\xi^m\xi^w+\phi\left(\xi^w\frac{d\xi^m}{d\phi}-\xi^m\frac{dL_t^w}{d\phi}\right)}{\left(\phi\xi^m+\xi^w\right)^2}>0
$$

Then

$$
\frac{ds_t^{young}}{d\phi}>0
$$

The aggregate savings rate in this economy is

$$\begin{aligned} s_t^P &= \frac{Q_t + (R-1)\cdot NFA_{t-1} - C_{1t} - C_{2t}}{Q_t} \\ &= (1-\alpha)\frac{\frac{\phi}{1+\phi}s^m\xi^m + \frac{1}{1+\phi}s^w\xi^w}{\frac{\phi}{1+\phi}\xi^m + \frac{1}{1+\phi}\xi^w} = (1-\alpha)\, s^{young} \end{aligned}$$

Therefore, as the sex ratio rises, aggregate savings rate goes up. ■

Some remarks are in order. First, as the sex ratio rises, men will raise their educational effort, and by (F.1), also their educational expenditure. In this case, the increase in educational expenditure does not reduce their savings. To see the reason, we first note that both savings and educational investment can enhance men's status in the marriage market. The optimal allocation in the first period must be such that the marginal gains from additional savings and from additional education expenditure are equal. Since both yield diminishing returns in the marriage market, we expect both instruments to be deployed by men and therefore both to rise as the sex ratio rises. Qualitatively, both men's savings rate and aggregate savings rate always go up as the sex ratio rises even in the presence of educational expenditure.

Second, the assumption that the educational expenditure and the costly effort are complements can be relaxed. If there exists some substitution effect between the two, but the utility function on leisure is not that concave, men may find that increasing the effort can lead to higher returns in the marriage market. In this case, they are more likely to raise the effort rather than to increase the educational expenditure. In this case, men may allocate more of their first period income into savings. Quantitatively, a positive substitution between the two may generate a bigger response in the savings.

We now present a numerical example. We consider a similar multi-period model as in the benchmark except that we allow productivity to be augmented by educational expenditure and effort. We assume in this experiment that individual i's labor productivity is

$$\xi_t^i = \frac{\left(X_t^i\right)^\rho}{\rho},\ \rho \in (0,1) \tag{F.7}$$

where X_t^i is a composite input of educational expenditure and effort,

$$X_t^i = \left[a\left(T_{e,t}^i\right)^{\frac{\psi-1}{\psi}} + (1-a)\left(e_t^i\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}$$

Parameter ρ is smaller than one; this means that the marginal gain in labor productivity is decreasing as the individual exerts additional effort or spends additional income on education.

A representative woman's optimization problem is

$$\max \sum_{t=1}^{\tau-1} \beta^{t-1} \left[u(c_t^w) + v(1-e_t^w)\right] + E_1 \left[\sum_{t=\tau}^{50} \beta^{t-1} \left(u(c_t^w) + v(1-e_t^w) + \eta^m\right)\right]$$

For $t < \tau$, when the woman is still single, the intertemporal budget constraint is

$$A_{t+1} = R\left(A_t + \xi_t^w y_t - c_t^w\right)$$

where A_t is her wealth level at the beginning of period t. After marriage ($t \geq \tau$), her family budget constraint becomes

$$A_{t+1}^H = \begin{cases} R\left(A_t^H + \xi_t^w y_t^H - c_t^H/\kappa\right) & \text{if } t \leq 30 \\ R\left(A_t^H - c_t^H/\kappa\right) & \text{if } t > 30 \end{cases}$$

where A_t^H is the level of family wealth (held jointly by the wife and the husband) at the beginning of period t. c_t^H is the public good consumption by both spouses, which takes the same form as in the two period OLG model. The optimization problem for a represenative man is similar. $v(1-e_t^w)$ represents the utility from leisure. We assume that

$$v\left(1-e_t^w\right) = \begin{cases} B\ln\left(1-e_t^w\right) & \text{if } t \leq 30 \\ 0 & \text{if } t > 30 \end{cases}$$

A representative man's optimization problem is similar. For simplicity, we assume that, there exist lower bounds on the education inputs, $\underline{T_e}$ and $\underline{e}$.[6] In the calibration, we set $\bar{T}_e$ and $\bar{e}$ to be the equilibrium choices by women and men when the sex ratio is one. We set $\underline{e} = 0.1$ in the benchmark, which means that learning time represents at least 10 percent of the total time endowment.[7]

We vary the values of parameters ρ, a and ψ in (F.7) to study the responses of the aggregate savings and current account. We set $\rho = 0.5$ in the benchmark and examine the case of $\rho = 0.1$ in the robustness checks. Parameter a represents the weight of educational expenditure in the aggregate human capital investment X_t. We set $a = 0.7$ in the benchmark, and examine the case of $a = 0.4$ (educational expenditure is less important) in robustness checks. Parameter ψ represents the elasticity of substitution between educational expenditure and effort. We consider two cases in the calibration: $\psi = 0.5$ (the two inputs are gross complements) and $\psi = 2$ (they are gross substitutes). For all other parameters, we take the values in the benchmark model.

Consider a rise in the sex ratio in the first period from one to 1.1. Appendix Figures 9a, 9b, 10a and 10b report the calibration results. Similar to the benchmark, both the aggregate savings rate and the current account rise first and persist for a number of periods. The adjustments in the aggregate

[6] We verify in unreported calibrations that, relaxing this assumption does not dramatically change the quantitative results.

[7] Moderate changes in the value of $\underline{e}$ only generate moderate changes in the quantitative results.

savings and the current account are quantitatively sizable. For instance, if $\tau = 10$ (marriage happens in the 10th period), in all experiments, the current account surplus reaches more than 2.8 percent of GDP by the 9th period. From the 10th period onwards, both the aggregate savings rate and the current account start to fall progressively until reaching zero. In all experiments, the current account surplus lasts for a long time. For instance, if $\tau = 10$, the current account remains in excess of 2 percent of GDP for more than twenty years under each combination of the parameters.

G CRRA utility

In the benchmark model, we show that, as the sex ratio rises, the increase in a representative man's savings will dominate any reduction in a representative woman's savings. The reason is, in addition to improving his relative standing in the marriage market, the representative man also wants to smooth his consumption over the two periods and would raise his savings rate to compensate for the lower savings rate by his future wife. This ensures that his incremental savings is more than enough to offset any reduction in his future wife's savings. How willing the representative man is to smooth his consumption depends on the elasticity of intertemporal substitution. If this elasticity is low, men may be reluctant to sacrifice his first period consumption to make up for the lower savings by women. As a result, the change in aggregate savings might not be quantitatively large. In this section, we consider a more general utility function (CRRA utility) and examine whether different choices of elasticities of intertemporal substitution will affect the quantitative results.

We assume that the utility takes the form $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$, where $1/\theta$ is the elasticity of intertemporal substitution. We make a small change in the emotional utilities: the emotional utilities from marriage in this case are $\eta^w \left(\frac{y^{1-\theta}-1}{1-\theta}\right)$ and $\eta^m \left(\frac{y^{1-\theta}-1}{1-\theta}\right)$, where y represents the labor income in the model[8]. In a two-period small-open economy example, we choose the same parameter values as in the benchmark: $\beta = 0.7$, $\alpha = 0.4$, $\kappa = 0.8$, $\sigma = 0.05$ and $m = 2.08$. Then, we vary the choices of θ to see how the responses in the aggregate savings and the current account would change.

Appendix Figure 11 reports the results. In all experiments, we can see that, as the sex ratio rises, both the aggregate savings rate and the current account will rise. As the elasticity of intertemporal substitution becomes lower (higher θ), the responses of the aggregate savings rate and the current account to the change in the sex ratio are weaker. However, they are still economically sizable. For instance, as the sex ratio rises from 1 to 1.15 (the current level of sex ratio in China), the current account will rise by around 2 percent when $\theta = 2$.

[8] With this assumption, income does not affect the optimal savings choices. This allows us to do the normalization on income as in the benchmark calibration.

H Income Heterogeneity

We consider an extension in which people differ in their incomes. For simplicity, we assume that there are two income levels: a high level of y^H and a low level of $y < y^H$. Assume young women and young men have identical income distributions, and let ω denote the fraction of the young males (or females) that are in the high-income group. Then, in a 50-period model that is otherwise similar to the benchmark, we study the savings rate and current account responses when we vary the degree of income inequality.

We need to make an assumption to address a technical difficulty: in general, the matching between men and women is not unique as a man (woman) may be indifferent between a partner with a high income but low emotional utility and someone with a low income but high emotional utility. To simplify our discussion, we assume that the highest possible value of emotional utility is not high enough such that every high-income man is ranked higher than all low-income men by a representative woman. We also assume that the lowest possible value of the emotional utility is sufficiently high such that everyone prefers to get married. Formally, we assume that η is drawn from a truncated normal distribution $[E\eta - 3\sigma, E\eta + 3\sigma]$.

With these assumptions, there is no hope for a low-income man to be matched with a high-income woman when there is a relative shortage of brides. There exist four types of feasible match patterns: a high-income man with a high-income woman, a high-income man (with a relatively low emotional utility) with a low-income woman, a low-income man with a low-income woman, and a low-income man (with a relatively low emotional utility) who stays single. Let $M^{ij}(\eta^w)$ denote the mapping from women to men, where i and j denote the income types of women and men, respectively.

As in the benchmark model, it is easy to show that the optimization problem for a person with income y is equivalent to

$$\max_{c_1,c_2} u(c_1) + \hat{\beta}\left[u(c_2) + \hat{\eta}\right]$$

subject to the life-time budget constraint

$$c_2 = \left[\frac{1-R^{\tau_1}}{1-R}(y-c_1) + \frac{1-R^{\tau_1-31}}{1-R^{-1}}y\right]\left(\frac{1-R^{\tau_1-51}}{1-R^{-1}}\right)^{-1}$$

where τ_1 is the marriage age as defined in the benchmark. c_1 and c_2 represents the consumption before and after the marriage age, respectively. $\hat{\beta}$ is an increasing function of discount factor β, and $\hat{\eta}$ is an increasing function of η and β.

Now we compute the optimal conditions for the four types of agents: a high-income man, a low-income man, a high-income woman, and a low-income woman, respectively. They are given by the following equations:

$$
\begin{aligned}
u_{1m}^{H\prime} + \hat{\beta} \begin{bmatrix} u_{2m}^{HH\prime} \frac{\partial c_{2m}^{HH}}{\partial c_{1m}^{H}} \left(\delta_m^{HH} + \int \left(M^{HH}\right)^{-1\prime} (\tilde{\eta}^{m,HH}) dF(\eta^m) \right) \\ + u_{2m}^{HL\prime} \frac{\partial c_{2m}^{HL}}{\partial c_{1m}^{H}} \left(\left(1 - \delta_m^{HH}\right) + \int \left(M^{LH}\right)^{-1\prime} (\tilde{\eta}^{m,LH}) dF(\eta^m) \right) \end{bmatrix} &= 0 \\
u_{1m}^{L\prime} + \hat{\beta} \left[u_{2m}^{LL\prime} \frac{\partial c_{2m}^{LL}}{\partial c_{1m}^{L}} \left(\delta_m^{LL} + \int \left(M^{LL}\right)^{-1\prime} (\tilde{\eta}^{m,L}) dF(\eta^m) \right) + \left(1 - \delta_m^{LL}\right) u_{2m,n}^{L\prime} \frac{\partial c_{2m,n}^{L}}{\partial c_{1m}^{H}} \right] &= 0 \\
u_{1w}^{H\prime} + \hat{\beta} u_{2w}^{HH\prime} \frac{\partial c_{2w}^{HH}}{\partial c_{1w}^{H}} \left[1 + \int M^{HH\prime} (\tilde{\eta}^{w,H}) dF(\eta^w) \right] &= 0 \\
u_{1w}^{L\prime} + \hat{\beta} \begin{bmatrix} u_{2w}^{LH\prime} \frac{\partial c_{2w}^{LH}}{\partial c_{1w}^{L}} \left(\delta_w^{LH} + \int M^{LH\prime} (\tilde{\eta}^{w,L}) dF(\eta^m) \right) \\ + u_{2w}^{LL\prime} \frac{\partial c_{2w}^{LL}}{\partial c_{1w}^{L}} \left(\left(1 - \delta_w^{LH}\right) + \int M^{LL\prime} (\tilde{\eta}^{w,L}) dF(\eta^m) \right) \end{bmatrix} &= 0
\end{aligned}
$$

where δ_m^{HH}, δ_m^{LL} and δ_w^{LH} represent the probabilities that a high income man is getting married with a high income woman, a low income man is getting married with a low income woman, and a low income woman is getting married with a high income man, respectively. For the matching functions,

$$
\begin{aligned}
M^{HH}(\eta^w) &= (F)^{-1} \left(\frac{F(\eta^w)}{\phi} + \frac{\phi - 1}{\phi} \right) \\
M^{LH}(\eta^w) &= (F)^{-1} \left(F\left(\bar{\eta}^{m,H}\right) - \frac{(1-\omega)\left(1 - F(\eta^w)\right)}{\phi\omega} \right) \\
M^{LL}(\eta^w) &= (F)^{-1} \left(1 - \frac{F\left(\bar{\eta}^{w,L}\right) - F(\eta^w)}{\phi} \right)
\end{aligned}
$$

where $\bar{\eta}^{m,H}$ and $\eta^{w,L}$ represent the two thresholds above which a high-income man can get married with a high-income woman, and a low-income woman can get married with a high-income man, respectively.

To simulate the model, we set the values of parameters β, α, κ, m and σ to be the same as in the benchmark. We choose $\tau = 10$ and fix the share of high-income people to be 20% of the population.[9] With this setup, the ratio of incomes for the rich and the poor, y^H/y, represents the degree of income inequality (which can be easily converted into a Gini coefficient). We will vary the degree of income inequality to investigate its effect on the strength of the competitive saving motive. Mindful that the Gini index in China is about 0.47 in recent years (changed from a low value of 0.29 in 1981), we report results in four cases that correspond to four values of the Gini coefficient: 0, 0.20, 0.47, and 0.60.

In Appendix Figures 12, we report the responses of the aggregate savings rate and the current account, respectively, to a sudden increase in the sex ratio from 1 to 1.2. As we can see, the existence of income heterogeneity does not weaken the quantitative responses. Instead, as the Gini coefficient rises, a given increase in the sex ratio leads to stronger responses by the aggregate savings rate and current account. For instance, if the Gini coefficient is 0.47, both the aggregate savings rate and current account would rise by more than 6% of GDP after nine years.

[9] In unreported numerical calculations, we verify that different values of ω do not change the qualitative results.

To see what is going on, we now look into the response patterns by individual type, which are reported in Appendix Figure 13.[10] For a high-income man, as the income inequality rises, he raises his savings rate more aggressively as the sex ratio becomes more unbalanced. Here is the intuition. With a positive probability, a high-income man will marry a low-income woman. If the income gap becomes larger, marrying a low-income woman yields a greater losss of utility. As a result, the competitive saving motive becomes stronger among high-income men; they raise their savings rate by a larger amount for a given rise in the sex ratio. For high income women, since they can always get married with a high-income man, they respond by reducing their savings rate.

For a low-income man, since he cannot hope to get married with a high-income woman, he does not suffer much additional utility loss from an increase in the income gap and does not need to modify his behavior much.[11] His savings rate does not differ much as the income gap rises. Interestingly, Appendix Figure 13 shows that, under a higher degree of income inequality, a low-income woman first reduces her savings as the sex ratio rises, before raising the savings rate again as the sex ratio become more unbalanced. Why such a non-monotonic response? To understand this, we note that there exist two main forces driving the savings decision of a low-income woman. On the one hand, to free ride on her future husband, she may lower her savings. On the other hand, as she aspires to marry a high-income man, which has been made more desirable by a rising gap in income, she would want to save more. When the sex ratio imbalance is moderate, the free-riding incentive dominates as the chance of marrying a high-income man is low anyway. However, as the sex ratio becomes sufficiently high, the chance for a low-income woman to be married with a high-income man improves sufficiently. In this case, the competitive savings motive dominatew the free-riding motive. The shifting balance of the two motives generates a non-linear response by a low-income woman.

In the current set of calibrations, a rise in income inequality leads to a strengthening of the overall savings response to a higher sex ratio. We note, however, that there can be scenarios in which the savings response may be weakened. In particular, if the configurations of the rich and poor and the sex ratio are such that almost no poor man can get married, a rise in income inequality could reduce the probability of marriage for a poor man (since he is less able to catch up with the rich men). In this case, in response to a higher sex ratio, a representative poor man may choose to give up the competition in the marriage market and save less. While this is theoretically possible, it may not be realistic in the data. Even in China, the country currently with the highest sex ratio for the pre-marital age cohort, most poor men still think they have a chance to get married if they try hard enough. Therefore, in empirically plausible settings, a rise in income inequality tends to reinforce the savings response to a higher sex ratio.

[10] Notice that there is no income difference when Gini index equals 0. In this case, we draw the savings curve only in the low-income boxes.

[11] As we analyze below, with greater income inequality, low-income women may raise their savings as the sex ratio rises. This indirectly raises the incentive for a low-income man to get married. However, the effect is small quantitatively.

Appendix Table 1a: Savings rates vs sex ratios, small country

K=0.8	m=2.08, σ=0.05				m=1, σ=0.05				m=2.08, σ=0.1			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.00	0.412	0.412	0.280	0.000	0.412	0.412	0.280	0.000	0.412	0.412	0.280	0.000
1.05	0.675	0.281	0.322	0.042	0.604	0.311	0.310	0.030	0.586	0.319	0.306	0.026
1.10	0.741	0.240	0.334	0.054	0.660	0.275	0.319	0.039	0.646	0.282	0.316	0.036
1.15	0.774	0.216	0.342	0.062	0.691	0.253	0.325	0.045	0.679	0.258	0.323	0.043
1.20	0.795	0.199	0.347	0.067	0.709	0.237	0.330	0.050	0.701	0.241	0.328	0.048
1.25	0.808	0.187	0.352	0.072	0.722	0.225	0.334	0.054	0.715	0.229	0.332	0.052
1.30	0.818	0.177	0.356	0.076	0.732	0.216	0.337	0.057	0.726	0.219	0.336	0.056
1.35	0.825	0.169	0.360	0.080	0.736	0.210	0.339	0.059	0.734	0.210	0.340	0.060
1.40	0.825	0.169	0.359	0.079	0.736	0.210	0.338	0.058	0.740	0.203	0.343	0.063
1.45	0.825	0.169	0.357	0.077	0.736	0.210	0.337	0.057	0.745	0.197	0.346	0.066
1.50	0.825	0.169	0.356	0.076	0.736	0.210	0.336	0.056	0.749	0.192	0.349	0.069

sm—men's savings rate, sw—women's savings rate, st—economy-wide savings rate, ca—current account to GDP ratio

Appendix Table 1b: Savings rates vs sex ratios, small country

K=0.7	m=2.08, σ=0.05				m=1, σ=0.05				m=2.08, σ=0.1			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.00	0.412	0.412	0.280	0.000	0.412	0.412	0.280	0.000	0.412	0.412	0.280	0.000
1.05	0.675	0.281	0.322	0.042	0.604	0.311	0.310	0.030	0.586	0.319	0.306	0.026
1.10	0.741	0.240	0.334	0.054	0.660	0.275	0.319	0.039	0.646	0.282	0.316	0.036
1.15	0.774	0.216	0.342	0.062	0.691	0.253	0.325	0.045	0.679	0.258	0.323	0.043
1.20	0.795	0.199	0.347	0.067	0.709	0.237	0.330	0.050	0.701	0.241	0.328	0.048
1.25	0.808	0.187	0.352	0.072	0.722	0.225	0.334	0.054	0.715	0.229	0.332	0.052
1.30	0.818	0.177	0.356	0.076	0.732	0.216	0.337	0.057	0.726	0.219	0.336	0.056
1.35	0.825	0.169	0.360	0.080	0.738	0.208	0.341	0.061	0.734	0.210	0.340	0.060
1.40	0.830	0.162	0.364	0.084	0.743	0.202	0.344	0.064	0.740	0.203	0.343	0.063
1.45	0.830	0.162	0.362	0.082	0.747	0.196	0.346	0.066	0.745	0.197	0.346	0.066
1.50	0.830	0.162	0.360	0.080	0.750	0.191	0.349	0.069	0.749	0.192	0.349	0.069

sm—men's savings rate, sw—women's savings rate, st—economy-wide savings rate, ca—current account to GDP ratio

Appendix Table 1c: Savings rates vs sex ratios, small country

K=0.9	m=2.08, σ=0.05				m=1, σ=0.05				m=2.08, σ=0.1			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.00	0.412	0.412	0.280	0.000	0.412	0.412	0.280	0.000	0.412	0.412	0.280	0.000
1.05	0.675	0.281	0.322	0.042	0.604	0.311	0.310	0.030	0.586	0.319	0.306	0.026
1.10	0.741	0.240	0.334	0.054	0.660	0.275	0.319	0.039	0.646	0.282	0.316	0.036
1.15	0.774	0.216	0.342	0.062	0.691	0.253	0.325	0.045	0.679	0.258	0.323	0.043
1.20	0.795	0.199	0.347	0.067	0.709	0.237	0.330	0.050	0.701	0.241	0.328	0.048
1.25	0.808	0.187	0.352	0.072	0.722	0.225	0.334	0.054	0.715	0.229	0.332	0.052
1.30	0.818	0.177	0.356	0.076	0.731	0.217	0.337	0.057	0.726	0.219	0.336	0.056
1.35	0.821	0.173	0.357	0.077	0.731	0.217	0.336	0.056	0.734	0.210	0.340	0.060
1.40	0.821	0.173	0.356	0.076	0.731	0.217	0.335	0.055	0.740	0.203	0.343	0.063
1.45	0.821	0.173	0.354	0.074	0.731	0.217	0.334	0.054	0.745	0.197	0.346	0.066
1.50	0.821	0.173	0.353	0.073	0.731	0.217	0.333	0.053	0.749	0.192	0.349	0.069

sm—men's savings rate, sw—women's savings rate, st—economy-wide savings rate, ca—current account to GDP ratio

Appendix Table 2: CA/GDP vs country 2's sex ratio, two large countries, differing in the sex ratios

K=0.8	m=2.08, σ=0.05				m=1, σ=0.05				m=2.08, σ=0.1			
	(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)			
1.00	0.247	0.247	0.000	0.000	0.247	0.247	0.000	0.000	0.247	0.247	0.000	0.000
1.05	0.247	0.287	-0.020	0.020	0.247	0.274	-0.014	0.014	0.247	0.272	-0.012	0.012
1.10	0.247	0.299	-0.026	0.026	0.247	0.283	-0.018	0.018	0.247	0.281	-0.017	0.017
1.15	0.247	0.306	-0.030	0.030	0.247	0.289	-0.021	0.021	0.247	0.288	-0.020	0.020
1.20	0.247	0.312	-0.032	0.032	0.247	0.293	-0.023	0.023	0.247	0.293	-0.023	0.023
1.25	0.247	0.317	-0.035	0.035	0.247	0.297	-0.025	0.025	0.247	0.297	-0.025	0.025
1.30	0.247	0.321	-0.037	0.037	0.247	0.300	-0.027	0.027	0.247	0.300	-0.027	0.027
1.35	0.247	0.321	-0.037	0.037	0.247	0.302	-0.028	0.028	0.247	0.304	-0.028	0.028
1.40	0.247	0.309	-0.031	0.031	0.247	0.291	-0.022	0.022	0.247	0.307	-0.030	0.030
1.45	0.247	0.298	-0.025	0.025	0.247	0.280	-0.016	0.016	0.247	0.310	-0.031	0.031
1.50	0.247	0.287	-0.020	0.020	0.247	0.269	-0.011	0.011	0.247	0.312	-0.033	0.033

s1—country 1's economy-wide savings rate, s2—country 2's economy-wide savings rate,
ca1—country 1's current account to GDP ratio, ca2—country 2's current account to GDP ratio

Appendix Table 3a: CA/GDP vs country 2's sex ratio, two large countries: US and China

K=0.8	m=2.08, σ=0.05				m=1, σ=0.05				m=2.08, σ=0.1			
	(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)			
1.00	0.247	0.247	0.000	0.000	0.247	0.247	0.000	0.000	0.247	0.247	0.000	0.000
1.05	0.247	0.287	-0.010	0.030	0.247	0.274	-0.007	0.021	0.247	0.272	-0.006	0.019
1.10	0.247	0.299	-0.013	0.039	0.247	0.283	-0.009	0.027	0.247	0.281	-0.009	0.026
1.15	0.247	0.306	-0.015	0.044	0.247	0.289	-0.010	0.031	0.247	0.288	-0.010	0.030
1.20	0.247	0.312	-0.016	0.049	0.247	0.293	-0.012	0.035	0.247	0.293	-0.011	0.034
1.25	0.247	0.317	-0.017	0.052	0.247	0.297	-0.013	0.038	0.247	0.297	-0.012	0.037
1.30	0.247	0.321	-0.018	0.055	0.247	0.300	-0.013	0.040	0.247	0.300	-0.013	0.040
1.35	0.247	0.321	-0.019	0.056	0.247	0.302	-0.014	0.041	0.247	0.304	-0.014	0.043
1.40	0.247	0.309	-0.016	0.047	0.247	0.291	-0.011	0.033	0.247	0.307	-0.015	0.045
1.45	0.247	0.298	-0.013	0.038	0.247	0.280	-0.008	0.024	0.247	0.310	-0.016	0.047
1.50	0.247	0.287	-0.010	0.030	0.247	0.269	-0.005	0.016	0.247	0.312	-0.016	0.049

Appendix Table 3b: CA/GDP vs country 2's sex ratio, two large countries: US and China

K=0.7	m=2.08, σ=0.05				m=1, σ=0.05				m=2.08, σ=0.1			
	(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)			
1.00	0.247	0.247	0.000	0.000	0.247	0.247	0.000	0.000	0.247	0.247	0.000	0.000
1.05	0.247	0.287	-0.010	0.030	0.247	0.274	-0.007	0.021	0.247	0.272	-0.006	0.019
1.10	0.247	0.299	-0.013	0.039	0.247	0.283	-0.009	0.027	0.247	0.281	-0.009	0.026
1.15	0.247	0.306	-0.015	0.044	0.247	0.289	-0.010	0.031	0.247	0.288	-0.010	0.030
1.20	0.247	0.312	-0.016	0.049	0.247	0.293	-0.012	0.035	0.247	0.293	-0.011	0.034
1.25	0.247	0.317	-0.017	0.052	0.247	0.297	-0.013	0.038	0.247	0.297	-0.012	0.037
1.30	0.247	0.321	-0.018	0.055	0.247	0.300	-0.013	0.040	0.247	0.300	-0.013	0.040
1.35	0.247	0.325	-0.019	0.058	0.247	0.303	-0.014	0.042	0.247	0.304	-0.014	0.043
1.40	0.247	0.323	-0.019	0.057	0.247	0.306	-0.015	0.044	0.247	0.307	-0.015	0.045
1.45	0.247	0.312	-0.016	0.049	0.247	0.309	-0.015	0.046	0.247	0.310	-0.016	0.047
1.50	0.247	0.300	-0.013	0.040	0.247	0.311	-0.016	0.048	0.247	0.312	-0.016	0.049

s1—country 1's economy-wide savings rate, s2—country 2's economy-wide savings rate,
ca1—country 1's current account to GDP ratio, ca2—country 2's current account to GDP ratio

Appendix Table 3c: CA/GDP vs country 2's sex ratio, two large countries: US and China

K=0.9	m=2.08, σ=0.05				m=1, σ=0.05				m=2.08, σ=0.1			
	(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)			
1.00	0.247	0.247	0.000	0.000	0.247	0.247	0.000	0.000	0.247	0.247	0.000	0.000
1.05	0.247	0.287	-0.010	0.030	0.247	0.274	-0.007	0.021	0.247	0.272	-0.006	0.019
1.10	0.247	0.299	-0.013	0.039	0.247	0.283	-0.009	0.027	0.247	0.281	-0.009	0.026
1.15	0.247	0.306	-0.015	0.044	0.247	0.289	-0.010	0.031	0.247	0.288	-0.010	0.030
1.20	0.247	0.312	-0.016	0.049	0.247	0.293	-0.012	0.035	0.247	0.293	-0.011	0.034
1.25	0.247	0.317	-0.017	0.052	0.247	0.297	-0.013	0.038	0.247	0.297	-0.012	0.037
1.30	0.247	0.321	-0.018	0.055	0.247	0.300	-0.013	0.040	0.247	0.300	-0.013	0.040
1.35	0.247	0.312	-0.016	0.049	0.247	0.289	-0.011	0.032	0.247	0.304	-0.014	0.043
1.40	0.247	0.301	-0.013	0.040	0.247	0.278	-0.008	0.023	0.247	0.307	-0.015	0.045
1.45	0.247	0.290	-0.011	0.032	0.247	0.267	-0.005	0.015	0.247	0.310	-0.016	0.047
1.50	0.247	0.279	-0.008	0.024	0.247	0.257	-0.002	0.007	0.247	0.312	-0.016	0.049

s1—country 1's economy-wide savings rate, s2—country 2's economy-wide savings rate, ca1—country 1's current account to GDP ratio, ca2—country 2's current account to GDP ratio

Appendix Table 4: The planner's economy vs the decentralized economy, small country

	planner's economy			K =0.7						K =0.8						K =0.9					
	(sm, sw, st)			(sm, Δw(m), sw, Δw(w), st, Δw)						(sm, Δw(m), sw, Δw(w), st, Δw)						(sm, Δw(m), sw, Δw(w), st, Δw)					
1.00	0.412	0.412	0.280	0.412	0.000	0.412	0.000	0.280	0.000	0.412	0.000	0.412	0.000	0.280	0.000	0.412	0.000	0.412	0.000	0.280	0.000
1.05	0.412	0.412	0.280	0.675	-0.484	0.281	0.267	0.322	-0.118	0.675	-0.489	0.281	0.272	0.322	-0.118	0.675	-0.493	0.281	0.275	0.322	-0.118
1.10	0.412	0.412	0.280	0.741	-0.686	0.240	0.306	0.334	-0.213	0.741	-0.694	0.240	0.314	0.334	-0.214	0.741	-0.702	0.240	0.321	0.334	-0.215
1.15	0.412	0.412	0.280	0.774	-0.810	0.216	0.312	0.342	-0.288	0.774	-0.822	0.216	0.324	0.342	-0.289	0.774	-0.833	0.216	0.333	0.342	-0.290
1.20	0.412	0.412	0.280	0.795	-0.894	0.199	0.307	0.347	-0.348	0.795	-0.910	0.199	0.321	0.347	-0.350	0.795	-0.923	0.199	0.334	0.347	-0.352
1.25	0.412	0.412	0.280	0.808	-0.955	0.187	0.296	0.352	-0.399	0.808	-0.974	0.187	0.314	0.352	-0.402	0.808	-0.990	0.187	0.329	0.352	-0.404
1.30	0.412	0.412	0.280	0.818	-1.000	0.177	0.283	0.356	-0.442	0.818	-1.021	0.177	0.303	0.356	-0.445	0.818	-1.041	0.177	0.321	0.356	-0.449
1.35	0.412	0.412	0.280	0.825	-1.034	0.169	0.269	0.360	-0.479	0.825	-1.058	0.169	0.292	0.360	-0.484	0.821	-1.058	0.173	0.317	0.357	-0.473
1.40	0.412	0.412	0.280	0.830	-1.058	0.162	0.255	0.364	-0.511	0.825	-1.058	0.169	0.292	0.359	-0.496	0.821	-1.058	0.173	0.317	0.356	-0.485
1.45	0.412	0.412	0.280	0.830	-1.058	0.162	0.255	0.362	-0.522	0.825	-1.058	0.169	0.292	0.357	-0.507	0.821	-1.058	0.173	0.317	0.354	-0.497
1.50	0.412	0.412	0.280	0.830	-1.058	0.162	0.255	0.360	-0.533	0.825	-1.058	0.169	0.292	0.356	-0.518	0.821	-1.058	0.173	0.317	0.353	-0.508

sm—men's savings rate, sw—women's savings rate, st—economy-wide savings rate, Δw(m)—men's welfare gain under the decentralized economy compared to that under the planner's economy, Δw(w)—women's welfare gain under the decentralized economy compared to that under the planner's economy, Δw—social welfare gain under the decentralized economy compared to that under the planner's economy

Appendix Table 5: Savings rates vs sex ratios, endogenous intra-household bargaining, small country, σ =0.05

γ=0.5	ε=0				ε =0.5				ε =1			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.00	0.412	0.412	0.280	0.000	0.412	0.412	0.280	0.000	0.412	0.412	0.280	0.000
1.05	0.675	0.281	0.322	0.042	0.702	0.402	0.366	0.086	0.724	0.482	0.397	0.117
1.10	0.741	0.240	0.334	0.054	0.766	0.382	0.383	0.103	0.784	0.470	0.414	0.134
1.15	0.774	0.216	0.342	0.062	0.798	0.370	0.392	0.112	0.814	0.463	0.423	0.143
1.20	0.795	0.199	0.347	0.067	0.818	0.362	0.399	0.119	0.832	0.457	0.430	0.150
1.25	0.808	0.187	0.352	0.072	0.831	0.356	0.405	0.125	0.844	0.452	0.435	0.155
1.30	0.818	0.177	0.356	0.076	0.840	0.350	0.409	0.129	0.864	0.449	0.440	0.160
1.35	0.825	0.169	0.360	0.080	0.853	0.346	0.414	0.134	0.864	0.449	0.437	0.157
1.40	0.830	0.163	0.363	0.083	0.853	0.346	0.412	0.132	0.864	0.449	0.434	0.154
1.45	0.830	0.163	0.362	0.082	0.853	0.346	0.409	0.129	0.864	0.449	0.430	0.150
1.50	0.830	0.163	0.360	0.080	0.853	0.346	0.406	0.126	0.864	0.449	0.427	0.147

γ=0.3	ε=0				ε =0.5				ε =1			
1.00	0.412	0.412	0.280	0.000	0.412	0.412	0.280	0.000	0.412	0.412	0.280	0.000
1.05	0.675	0.281	0.322	0.042	0.690	0.365	0.352	0.072	0.702	0.429	0.374	0.094
1.10	0.741	0.240	0.334	0.054	0.755	0.341	0.368	0.088	0.765	0.413	0.391	0.111
1.15	0.774	0.216	0.342	0.062	0.788	0.327	0.377	0.097	0.797	0.403	0.401	0.121
1.20	0.795	0.199	0.347	0.067	0.808	0.317	0.384	0.104	0.816	0.396	0.408	0.128
1.25	0.808	0.187	0.352	0.072	0.822	0.310	0.389	0.109	0.829	0.391	0.414	0.134
1.30	0.818	0.177	0.356	0.076	0.844	0.303	0.396	0.116	0.853	0.387	0.419	0.139
1.35	0.824	0.169	0.360	0.080	0.844	0.303	0.394	0.114	0.853	0.387	0.416	0.136
1.40	0.824	0.169	0.358	0.078	0.844	0.303	0.391	0.111	0.853	0.387	0.414	0.134
1.45	0.824	0.169	0.357	0.077	0.844	0.303	0.389	0.109	0.853	0.387	0.411	0.131
1.50	0.824	0.169	0.355	0.075	0.844	0.303	0.387	0.107	0.853	0.387	0.408	0.128

γ=0.7	ε=0				ε =0.5				ε =1			
1.00	0.412	0.412	0.280	0.000	0.412	0.412	0.280	0.000	0.412	0.412	0.280	0.000
1.05	0.675	0.281	0.322	0.042	0.714	0.433	0.379	0.099	0.745	0.522	0.415	0.135
1.10	0.741	0.240	0.334	0.054	0.777	0.415	0.396	0.116	0.802	0.513	0.432	0.152
1.15	0.774	0.216	0.342	0.062	0.808	0.405	0.405	0.125	0.830	0.507	0.441	0.161
1.20	0.795	0.199	0.347	0.067	0.827	0.398	0.412	0.132	0.847	0.502	0.447	0.167
1.25	0.808	0.187	0.352	0.072	0.839	0.392	0.417	0.137	0.858	0.498	0.452	0.172
1.30	0.818	0.177	0.356	0.076	0.848	0.387	0.422	0.142	0.872	0.495	0.456	0.176
1.35	0.825	0.169	0.360	0.080	0.855	0.383	0.425	0.145	0.872	0.495	0.452	0.172
1.40	0.830	0.162	0.364	0.084	0.860	0.379	0.429	0.149	0.872	0.495	0.448	0.168
1.45	0.834	0.156	0.367	0.087	0.862	0.378	0.429	0.149	0.872	0.495	0.445	0.165
1.50	0.836	0.154	0.368	0.088	0.862	0.378	0.426	0.146	0.872	0.495	0.442	0.162

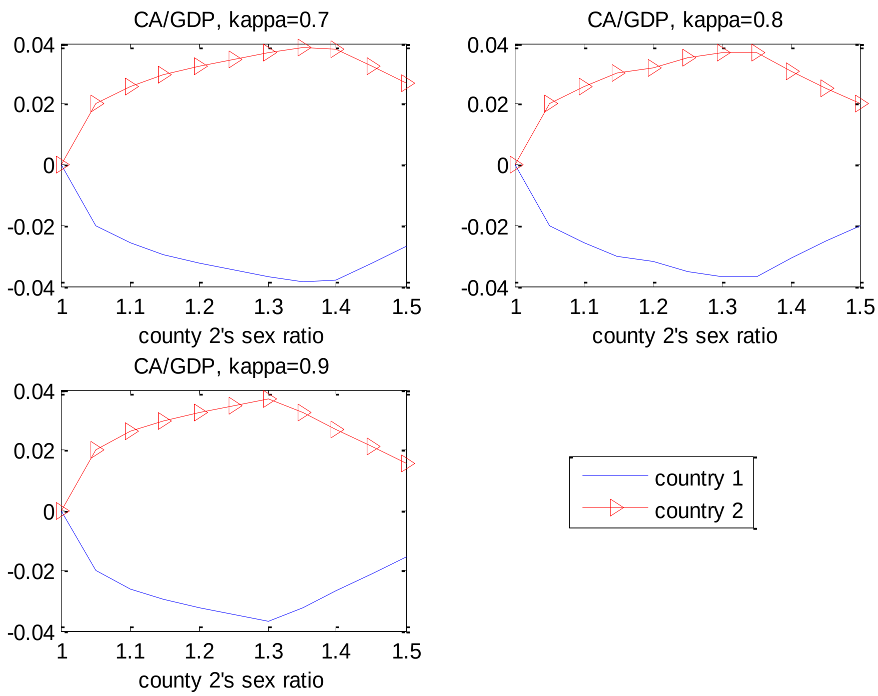
sm—men's savings rate, sw—women's savings rate, st—economy-wide savings rate, ca—current account to GDP ratio

Appendix Table 6: Two large countries, endogenous intra-household bargaining, China vs US, σ =0.05

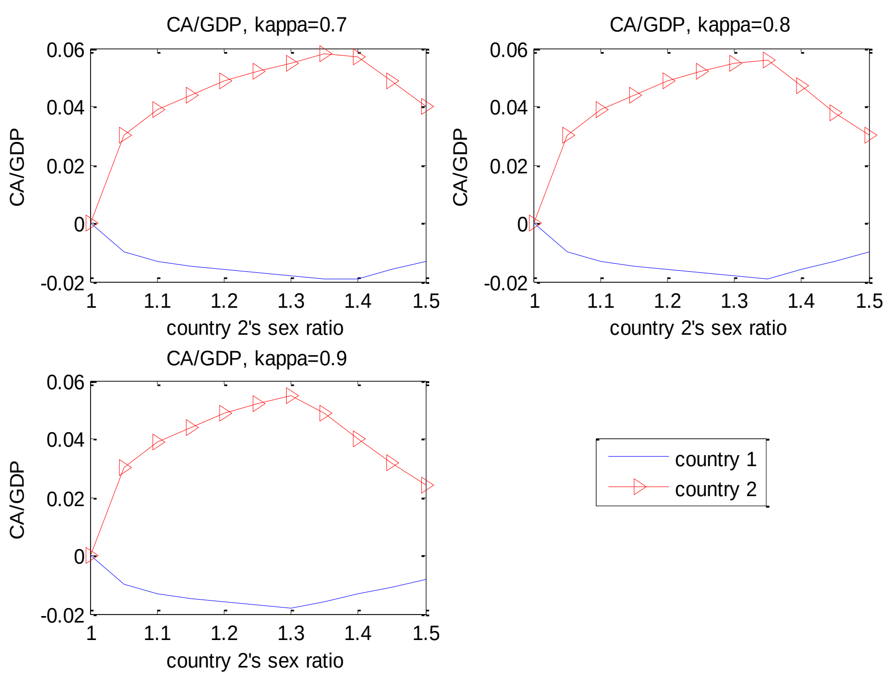
γ=0.5	ε=0						ε =0.5						ε =1					
	(s1, s2, ca1, ca2, dw1, dw2)						(s1, s2, ca1, ca2, dw1, dw2)						(s1, s2, ca1, ca2, dw1, dw2)					
1.00	0.247	0.247	0.000	0.000	0.000	0.000	0.247	0.247	0.000	0.000	0.000	0.000	0.247	0.247	0.000	0.000	0.000	0.000
1.05	0.247	0.287	-0.010	0.030	-0.001	-0.108	0.247	0.331	-0.021	0.063	-0.003	-0.148	0.247	0.361	-0.029	0.086	-0.004	-0.200
1.10	0.247	0.299	-0.013	0.039	-0.002	-0.197	0.247	0.347	-0.025	0.075	-0.003	-0.249	0.247	0.378	-0.033	0.098	-0.004	-0.311
1.15	0.247	0.306	-0.015	0.044	-0.002	-0.266	0.247	0.357	-0.027	0.082	-0.004	-0.327	0.247	0.388	-0.035	0.105	-0.004	-0.394
1.20	0.247	0.312	-0.016	0.049	-0.002	-0.323	0.247	0.364	-0.029	0.087	-0.004	-0.390	0.247	0.394	-0.037	0.110	-0.005	-0.459
1.25	0.247	0.317	-0.017	0.052	-0.002	-0.370	0.247	0.369	-0.030	0.091	-0.004	-0.442	0.247	0.399	-0.038	0.114	-0.005	-0.513
1.30	0.247	0.321	-0.018	0.055	-0.002	-0.410	0.247	0.374	-0.032	0.095	-0.004	-0.486	0.247	0.392	-0.036	0.108	-0.005	-0.573
1.35	0.247	0.325	-0.019	0.058	-0.003	-0.446	0.247	0.368	-0.030	0.091	-0.004	-0.529	0.247	0.378	-0.033	0.098	-0.005	-0.578
1.40	0.247	0.322	-0.019	0.056	-0.003	-0.468	0.247	0.355	-0.027	0.081	-0.004	-0.536	0.247	0.365	-0.029	0.088	-0.005	-0.584
1.45	0.247	0.310	-0.016	0.048	-0.003	-0.475	0.247	0.343	-0.024	0.072	-0.004	-0.542	0.247	0.352	-0.026	0.079	-0.005	-0.589
1.50	0.247	0.299	-0.013	0.039	-0.002	-0.483	0.247	0.331	-0.021	0.063	-0.004	-0.548	0.247	0.340	-0.023	0.070	-0.004	-0.594
γ=0.3	ε=0						ε =0.5						ε =1					
1.00	0.247	0.247	0.000	0.000	0.000	0.000	0.247	0.247	0.000	0.000	0.000	0.000	0.247	0.247	0.000	0.000	0.000	0.000
1.05	0.247	0.287	-0.010	0.030	-0.001	-0.108	0.247	0.316	-0.017	0.052	-0.002	-0.129	0.247	0.339	-0.023	0.069	-0.003	-0.157
1.10	0.247	0.299	-0.013	0.039	-0.002	-0.198	0.247	0.332	-0.021	0.064	-0.003	-0.226	0.247	0.356	-0.027	0.081	-0.003	-0.260
1.15	0.247	0.306	-0.015	0.044	-0.002	-0.267	0.247	0.341	-0.024	0.071	-0.003	-0.301	0.247	0.365	-0.030	0.089	-0.004	-0.338
1.20	0.247	0.312	-0.016	0.049	-0.002	-0.325	0.247	0.348	-0.025	0.076	-0.003	-0.362	0.247	0.372	-0.031	0.094	-0.004	-0.401
1.25	0.247	0.317	-0.017	0.052	-0.002	-0.373	0.247	0.354	-0.027	0.080	-0.003	-0.413	0.247	0.383	-0.034	0.102	-0.004	-0.512
1.30	0.247	0.321	-0.018	0.055	-0.002	-0.414	0.247	0.350	-0.026	0.077	-0.004	-0.475	0.247	0.369	-0.030	0.091	-0.004	-0.519
1.35	0.247	0.320	-0.018	0.055	-0.002	-0.442	0.247	0.337	-0.023	0.068	-0.003	-0.483	0.247	0.356	-0.027	0.082	-0.004	-0.526
1.40	0.247	0.308	-0.015	0.046	-0.002	-0.451	0.247	0.325	-0.020	0.059	-0.003	-0.490	0.247	0.343	-0.024	0.072	-0.004	-0.532
1.45	0.247	0.297	-0.012	0.037	-0.002	-0.459	0.247	0.314	-0.017	0.050	-0.003	-0.497	0.247	0.331	-0.021	0.063	-0.004	-0.539
1.50	0.247	0.286	-0.010	0.029	-0.002	-0.466	0.247	0.302	-0.014	0.041	-0.003	-0.504	0.247	0.320	-0.018	0.054	-0.004	-0.545
γ=0.7	ε=0						ε =0.5						ε =1					
1.00	0.247	0.247	0.000	0.000	0.000	0.000	0.247	0.247	0.000	0.000	0.000	0.000	0.247	0.247	0.000	0.000	0.000	0.000
1.05	0.247	0.287	-0.010	0.030	-0.001	-0.108	0.247	0.344	-0.024	0.072	-0.003	-0.169	0.247	0.380	-0.033	0.099	-0.004	-0.246
1.10	0.247	0.299	-0.013	0.039	-0.002	-0.196	0.247	0.360	-0.028	0.085	-0.004	-0.275	0.247	0.396	-0.037	0.112	-0.005	-0.364
1.15	0.247	0.306	-0.015	0.044	-0.002	-0.265	0.247	0.370	-0.031	0.092	-0.004	-0.355	0.247	0.405	-0.040	0.119	-0.005	-0.450
1.20	0.247	0.312	-0.016	0.049	-0.002	-0.321	0.247	0.376	-0.032	0.097	-0.004	-0.419	0.247	0.412	-0.041	0.123	-0.005	-0.518
1.25	0.247	0.317	-0.017	0.052	-0.002	-0.367	0.247	0.382	-0.034	0.101	-0.004	-0.471	0.247	0.416	-0.042	0.127	-0.005	-0.573
1.30	0.247	0.321	-0.018	0.055	-0.002	-0.407	0.247	0.386	-0.035	0.104	-0.004	-0.516	0.247	0.408	-0.040	0.121	-0.005	-0.618
1.35	0.247	0.325	-0.019	0.058	-0.003	-0.441	0.247	0.390	-0.036	0.107	-0.004	-0.554	0.247	0.394	-0.037	0.110	-0.005	-0.623
1.40	0.247	0.328	-0.020	0.061	-0.003	-0.471	0.247	0.389	-0.036	0.107	-0.005	-0.584	0.247	0.381	-0.033	0.100	-0.005	-0.628
1.45	0.247	0.332	-0.021	0.063	-0.003	-0.497	0.247	0.376	-0.032	0.097	-0.004	-0.590	0.247	0.368	-0.030	0.091	-0.005	-0.632
1.50	0.247	0.323	-0.019	0.057	-0.003	-0.507	0.247	0.364	-0.029	0.088	-0.004	-0.595	0.247	0.356	-0.027	0.081	-0.005	-0.636

s1—country 1's economy-wide savings rate, s2—country 2's economy-wide savings rate, ca1—country 1's current account to GDP ratio, ca2—country 2's current account to GDP ratio, dw1—welfare change in country 1 in units of consumption goods relative to the case Φ=1, dw1—welfare change in country 2 in units of consumption goods relative to the case Φ=1

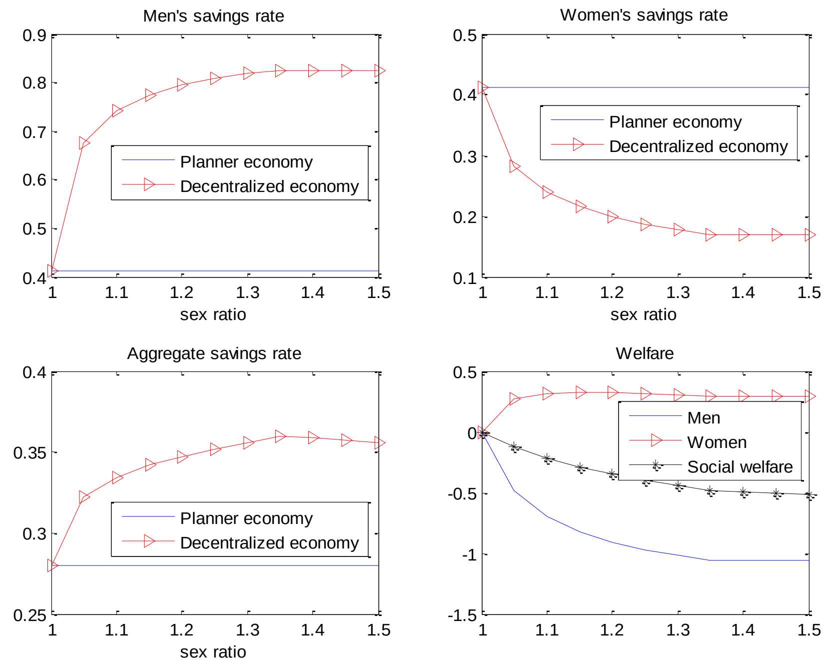
Figures and Tables



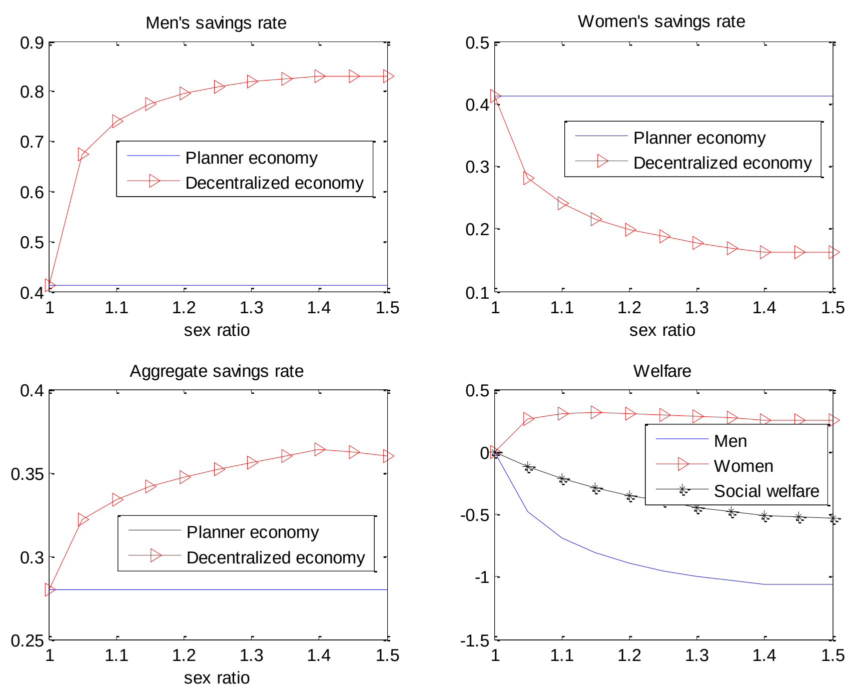
Appendix Figure 1: Two large countries, differing only in the sex ratios, σ=0.05



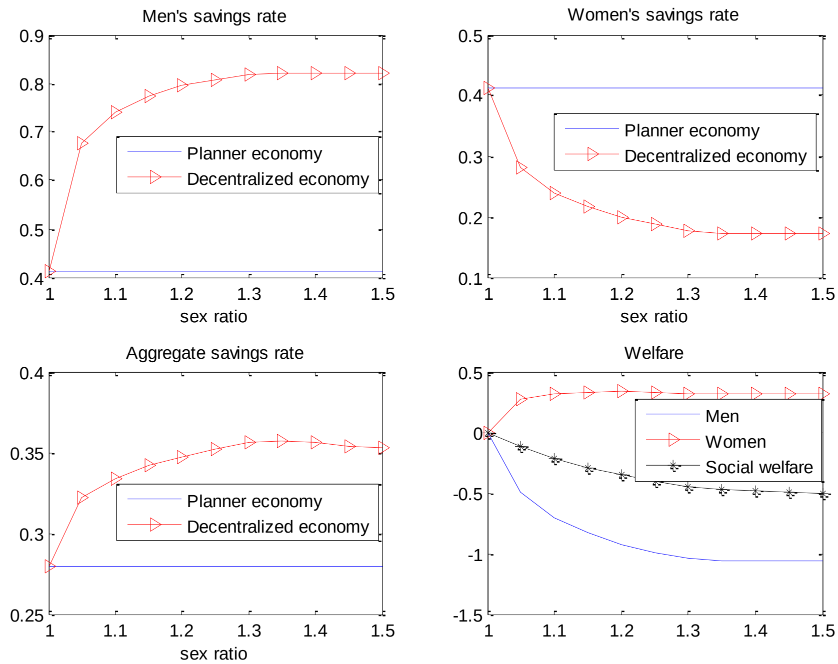
Appendix Figure 2: Two large countries, (GDP per capita)$_1$=15*(GDP per capita)$_2$, σ=0.05



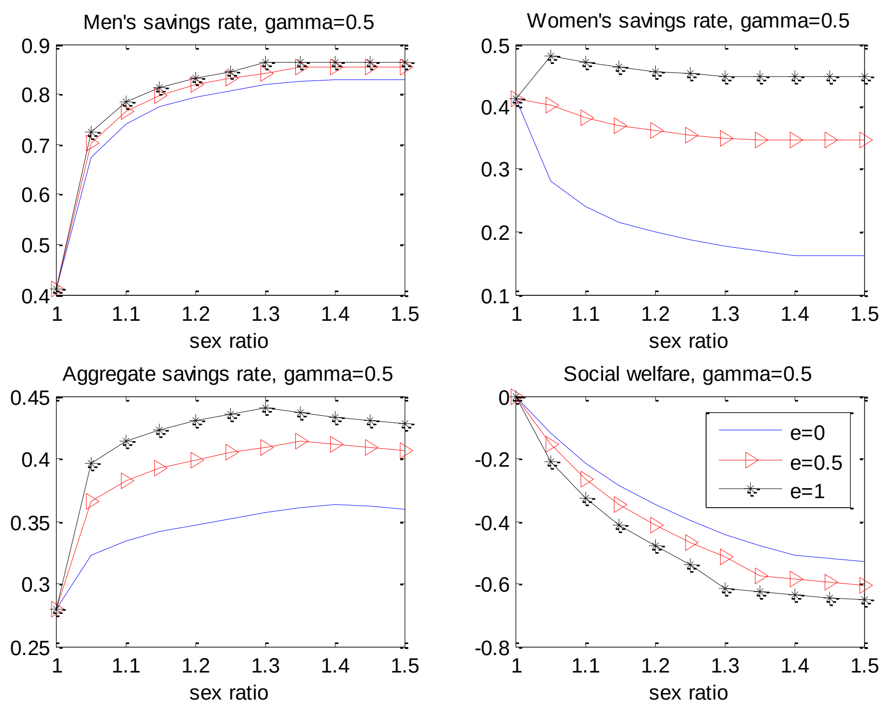
Appendix Figure 3: The planner's economy vs the decentralized economy, K=0.8, σ=0.05



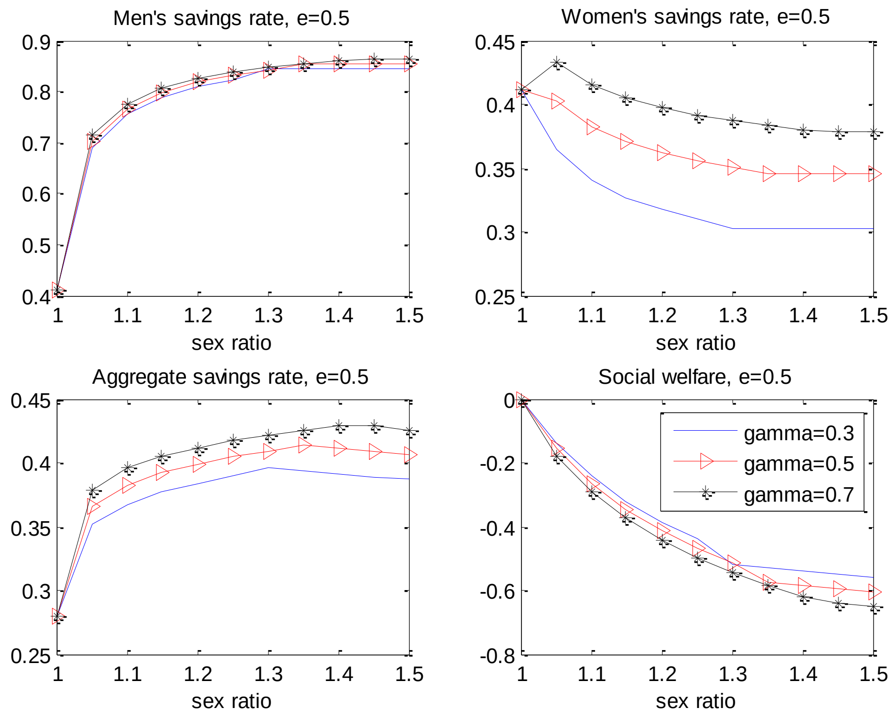
Appendix Figure 3a: The planner's economy vs the decentralized economy, K=0.7, σ=0.05



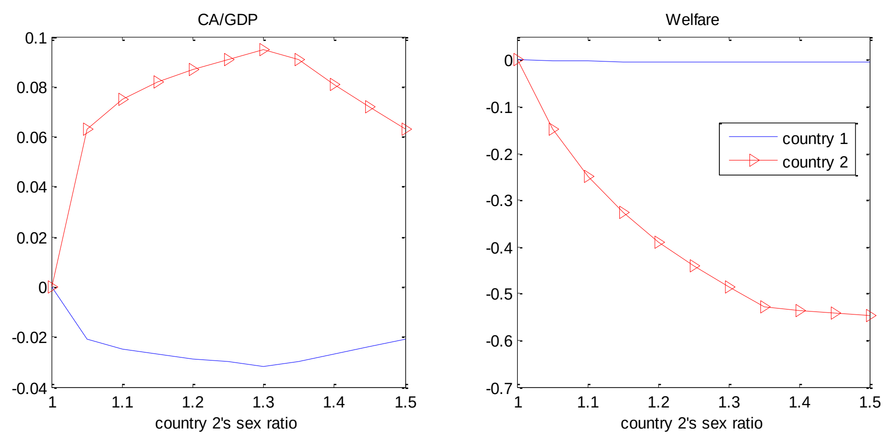
Appendix Figure 3b: The planner's economy vs the decentralized economy, K=0.9, σ=0.05



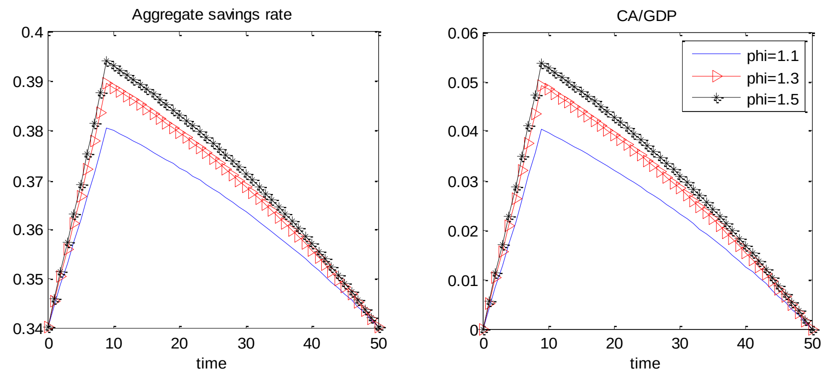
Appendix Figure 4a: Savings rates vs sex ratios, endogenous intra-household bargaining, σ=0.05



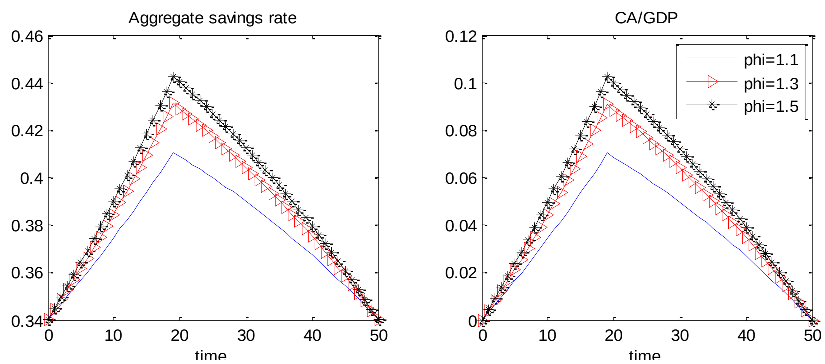
Appendix Figure 4b: Savings rates vs sex ratios, endogenous intra-household bargaining, σ=0.05



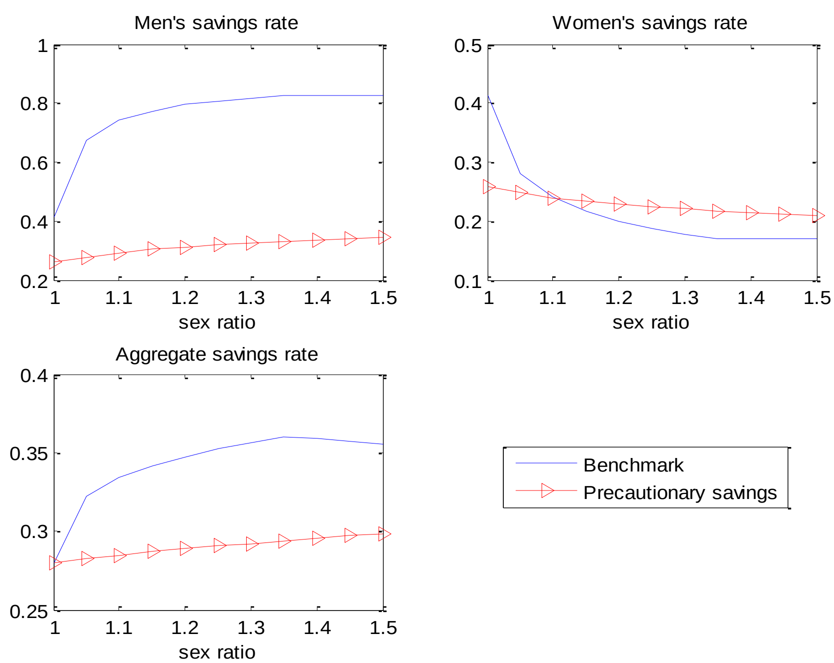
Appendix Figure 5: Two large countries, endogenous bargaining power, welfare loss in units of consumption goods relative to the case of Φ=1, (GDP per capita)$_1$=15*(GDP per capita)$_2$



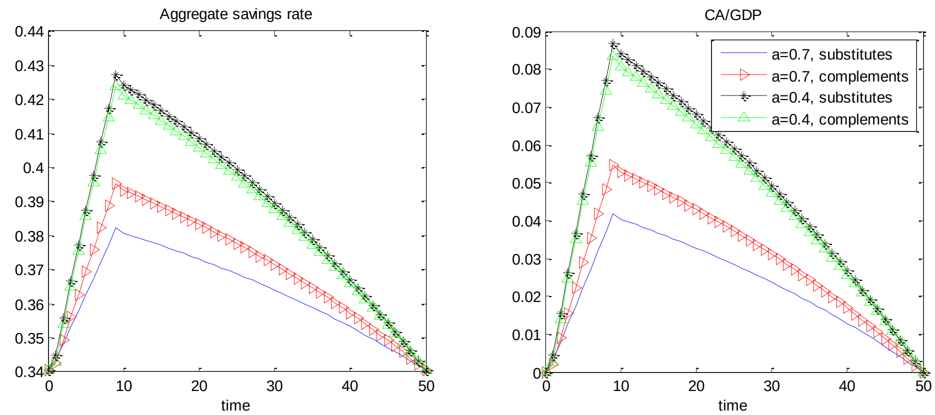
Appendix Figure 6: 50-period calibrations, τ=10, endogenous intra-household bargaining, σ=0.05



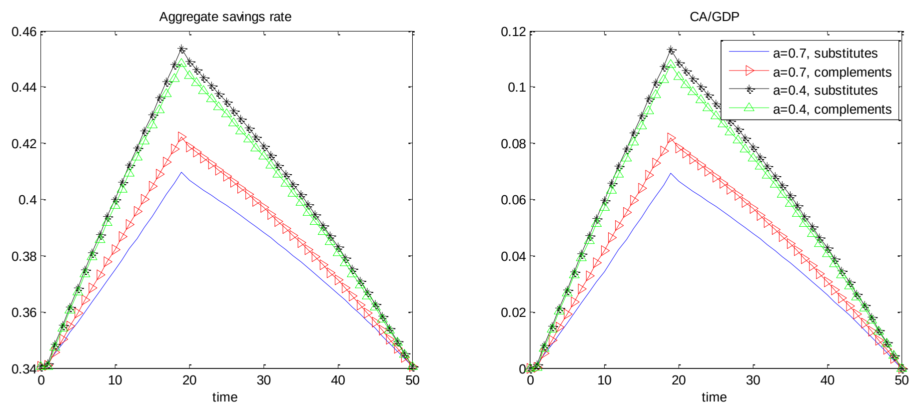
Appendix Figure 7: 50-period calibrations, τ=20, endogenous intra-household bargaining, σ=0.05



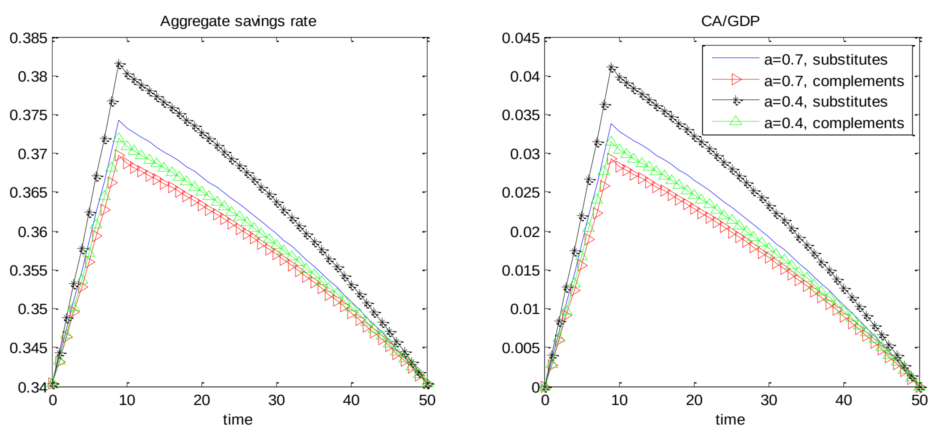
Appendix Figure 8: Benchmark vs Precautionary savings



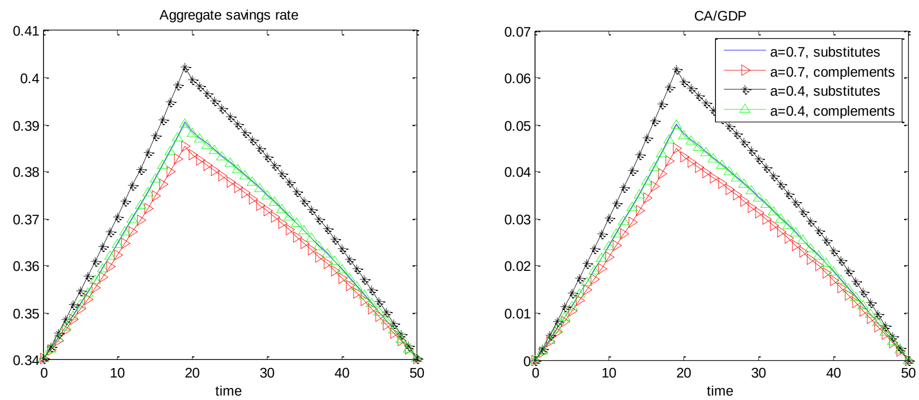
Appendix Figure 9a: 50-period calibration with education, τ=10, ρ=0.5



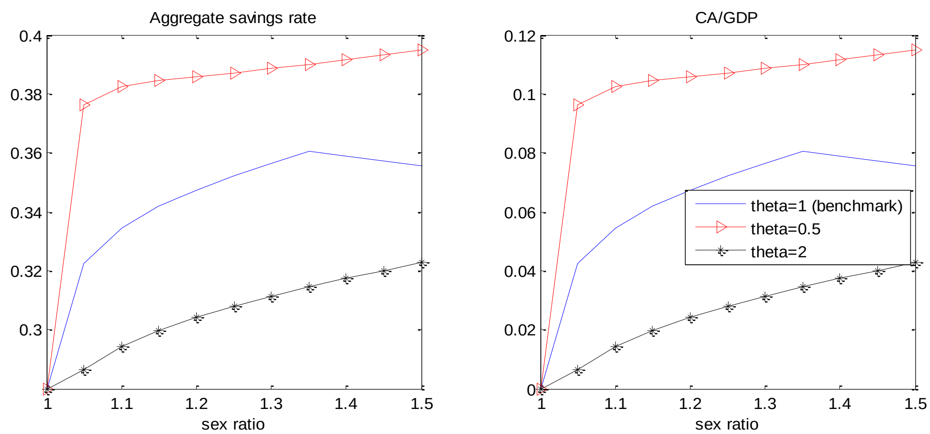
Appendix Figure 9b: 50-period calibration with education, τ=20, ρ=0.5



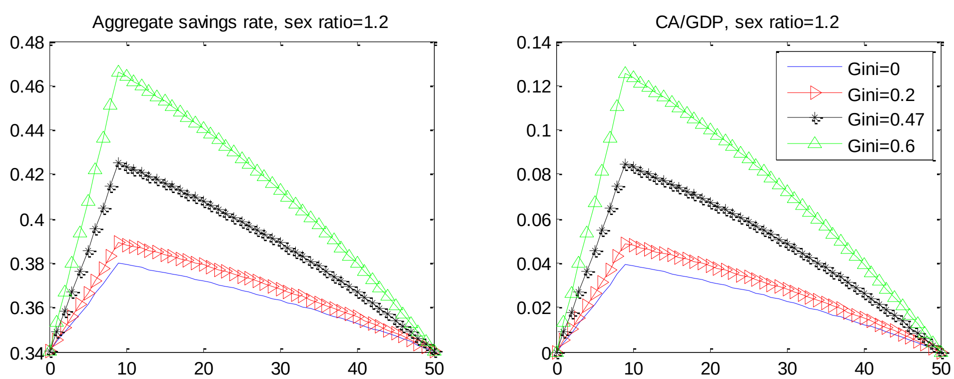
Appendix Figure 10a: 50-period calibration with education, τ=10, ρ=0.1



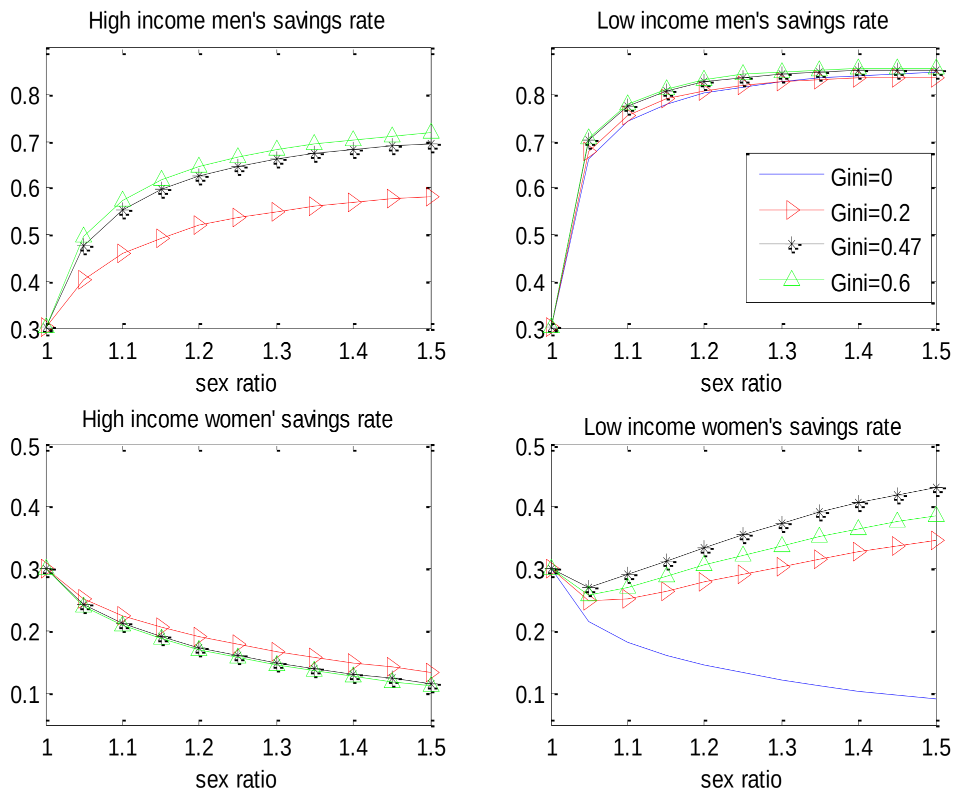
Appendix Figure 10b: 50-period calibration with education, τ=20, ρ=0.1



Appendix Figure 11: 2-period calibration, CRRA utility, σ=0.05



Appendix Figure 12: Multi-period calibration, heterogenous income, τ=10



Appendix Figure 13: Individual savings rate vs sex ratio, τ=10