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ASSET PRICE VOLATILITY, BUBBLES AND PROCESS SWITCHING

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ABSTRACT

Evidence of excess volatilities of asset prices compared with those of market fundamentals is often attributed to speculative bubbles. This study examines the sense in which speculative bubbles could in theory lead to excess volatility, but it demonstrates that some of the variance bounds evidence reported to date precludes bubbles as a reason why asset prices might violate such bounds. The findings must represent some other model misspecification or market inefficiency. One important misspecification occurs when the researcher incorrectly specifies the time series properties of market fundamentals. A bubble-free example economy characterized by a potential switch in government policies produces paths of asset prices that would appear, to an unwary researcher, to contain bubbles.

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The introduction and use of variance bounds tests by financial economists interested in asset pricing and market efficiency has generated considerable controversy. The first tests postulated a simple model in which market efficiency required assets to have a constant expected real rate of return. The apparent strong rejection of this hypothesis in the work of Shiller (1981a, 1981b) and particularly Grossman and Shiller (1981) was followed quickly by a number of different responses. The statistical properties of the tests in small samples and the time series assumptions of the data were criticized.[1] Substantial resources have also been devoted to complications of the model that allow time variation in discount rates and risk premiums, while remaining within the representative agent pardigm.[2] Others have taken the performance of the simple model and the excess volatility of asset prices described in the tests to be indicative of market inefficiency.

One particular type of market inefficiency that receives much attention in these discussions is that asset markets may be characterized by speculative bubbles. Representative of these statements is Ackley's (1983, p. 13) discussion of Shiller's (1981a, 1981b) findings, in which he states, "But, surely, it is possible that speculative price bubbles, upward or downward, ..., supply part of the explanation." Similarly, in Fischer's (1984, p. 500) discussion of Shiller (1984), he states, "Backing up that empirical evidence was the development, by Shiller and others, of the theory of speculative bubbles, providing a reason that prices could fluctuate excessively without smart investors being able to profit from knowing they were living in a bubble." Similar statements have been made by others in discussions of stock price volatility and in discussions of the volatility of foreign exchange rates.[3]

In this paper we examine whether some of the variance bounds tests reported to date can be evidence for the hypothesis that asset prices contain speculative bubbles. The speculative bubbles discussed here are of the type studied by Flood and Garber (1980) and Blanchard and Watson (1982). The variance bounds tests we discuss are of the types that were conducted by Shiller (1981a, 1981b, 1982, 1984) and Mankiw, Romer and Shapiro (1985) and that were discussed by Grossman and Shiller (1981). We demonstrate the sense in which the existence of bubbles can in theory lead to excess volatility of asset prices relative to the volatility of market fundamentals, but we explain why certain variants of variance bounds tests preclude bubbles as a reason why asset prices might violate such bounds. This result, without its formal demonstration, is mentioned by Mankiw, Romer and Shapiro (1985, p. 681) who state, "the inequalities ... hold even if there are bubbles." Since many researchers have mentioned bubbles as a possible reason for the failure of the simple rational expectations model in variance bounds tests, we thought it worthwhile to elaborate on the remark in Mankiw, Romer and Shapiro (1985).

The issue turns on how one measures the inherently unobservable construct that Shiller (1981a) denoted the ex post rational price. If one uses the sample's terminal market price to construct a measurable counterpart to ex post rational price, as is done by Shiller (1982, 1984), Grossman and Shiller (1981) and Mankiw, Romer and Shapiro (1985), failure of a variance bounds test cannot be attributed to the existence of speculative bubbles. The reason is that use of the sample market price effectively builds bubbles into the null hypothesis. Rejection of the null must consequently be due to other sources. Potential explanations include small sample properties of the tests, general misspecification of the model and failure of the data to satisfy the ergodicity assumption implicit in the use of the statistics.[4]

In bubble research one particularly important misspecification of the model occurs when the researcher incorrectly specifies the time series properties of market fundamentals. The second purpose of this paper is to explain, in terms of a simple model economy, how anticipated changes in market fundamentals may produce asset price paths that would appear to an empirical researcher, who is unaware of the potential change, to be characterized by bubbles even though the economy is bubble free. The example economy is described by a potential change in government policies that we label a process switch.

Our presentation is in the next two sections. In section II we describe a common asset pricing model and show how it responds to variance bounds tests when bubbles are present. In Section III we develop our example economy and explore possible process switches as explanations of bubble type phenomena. Section IV contains some concluding remarks.

II. Variance Bounds Tests of An Asset Pricing Model

Most variance bounds tests examine present value relations that are derived from a representative consumer's optimization problem. If a_t is the real dividend of an asset at time t and z_t is the real price ex-dividend on the asset at time t, a typical first order condition of a representative agent requires

$$(1) \qquad p_t = \rho E_t(p_{t+1} + d_{t+1})$$

where $p_t \equiv U'(c_t)z_t$, $d_t \equiv U'(c_t)a_t$, $U'(c_t)$ is the marginal utility of consumption at time t, ρ is the fixed discount factor of the agent and $E_t(\cdot)$ is the conditional expectation operator based on all time t information. Equation (1) requires that the utility of the real value sacrificed by the individual in purchasing the asset be equal to the conditional expectation of the utility of the real value of the benefit from holding and selling the asset.

Equation (1) has the form of a linear difference equation that arises in many rational expectations models. Hence, a solution that depends only on market fundamentals can be written as

$$(2) \qquad p_t^f = \sum_{i=1}^{\infty} \lambda_i E_t(d_{t+i}),$$

and substitution of (2) into (1) with equality of p_t^f and p_t requires $\lambda_i = \rho^i$.

Notice that if (1) is postulated as the entire model, an additional arbitrary element, b_t, can be added to the market fundamentals solution to provide an alternative solution,

$$(3) \qquad p_t = \sum_{i=1}^{\infty} \rho^i E_t(d_{t+i}) + b_t.$$

The model requires only that the sequence of b_t's possess the property that

$$(4) \qquad E_t(b_{t+i}) = \rho^{-i} b_t, \qquad i = 1,2,\cdots,$$

since with this property the solution (3) satisfies (1). The time series is termed a rational bubble according to Flood and Garber (1980), since it satisfies the Euler equation (1). Absence of bubbles requires that each element of the sequence is zero.[5] The time series property of bubbles described by (4) assures that bubbles cannot be a reason that the Euler equation (1) is deemed to be misspecified in an econometric investigation such as Hansen and Singleton (1982).

Since we are interested in how various variance bounds tests perform in the presence of speculative bubbles, we take (3) as our representation of equilibrium asset price with no additional restrictions placed on the b_t sequence other than those imposed by (4).[6]

The basic insights of the variance bounds tests are that the variance of an actual variable must be greater than or equal to the variance of its conditional expectation and that this latter variance must be greater than or equal to the variance of a forecast based on a subset of the information used by agents. In order to see how the existence of bubbles could lead in theory to a violation of variance bounds, consider the ex post rational price. The variance bounds literature defines the theoretical ex post rational price to be the price that would prevail if everyone knew the future market fundamentals with certainty and there were no bubbles. Therefore, the ex post rational price is

$$(5) \qquad p_t^* = \sum_{i=1}^{\infty} \rho^i d_{t+i}.$$

The theoretical relation that is the foundation of many variance bounds tests is obtained by subtracting (3) from (5) and rearranging terms:

$$(6) \qquad p_t^* = p_t + u_t - b_t ,$$

where $u_t \equiv \sum_{i=1}^{\infty} \rho^i [d_{t+i} - E_t(d_{t+i})]$ is the deviation of the present value of dividends from its expected value based on time t information. By construction, u_t is uncorrelated with p_t and b_t, but p_t and b_t may be correlated with each other.

Notice in (4) that since $\rho^{-1} > 1$, a rational stochastic bubble is nonstationary. Consequently, its unconditional moments are undefined. For this reason we address our arguments to variances and covariances of the innovations of processes, which are well-defined.

Let the innovation in X_t from time t - n be $[X_t - E_{t-n}(X_t)]$. Then the innovation variance and covariance operators are defined by

$$V_n(X_t) = E\{[X_t - E_{t-n}(X_t)]^2\}$$

and

$$C_n(X_t, Y_t) = E\{[X_t - E_{t-n}(X_t)][Y_t - E_{t-n}(Y_t)]\}$$

where $E(\cdot)$ denotes the unconditional mathematical expectation. In what follows we treat n as a finite positive integer.

Applying the innovation variance operator to both sides of (6) yields

$$(7) \qquad V_n(p_t^*) = V_n(p_t) + V_n(u_t) + V_n(b_t) - 2C_n(p_t, b_t),$$

which follows from the conditional orthogonality of u_t to p_t and b_t.

Suppose that a researcher had errorless measurements of p_t^* and p_t over a long time series and could develop very good estimates of $V_n(p_t^*)$ and $V_n(p_t)$.[7] Assume also that the researcher knows that (1) is not rejected by the data. Since the innovation variances of u_t and b_t in (7) are strictly nonnegative, a finding of $V_n(p_t) > V_n(p_t^*)$ could be rationalized, within the framework of the model, by $C_n(p_t, b_t) > 0$.

In typical presentations of variance bounds tests, the stochastic bubble is excluded from (6) because absence of bubbles is intended to be part of the joint null hypothesis. A theoretical variance bound derived from (7) in the absence of bubbles is $V_n(p_t^*) > V_n(p_t)$. It is easy to construct examples in which a sufficiently large innovation variance in b_t causes this variance bound to be violated. Consider the situation in which the innovations in b_t are orthogonal to the innovations in market fundamentals. In this case, $C_n(p_t, b_t) = V_n(b_t)$. Therefore, the right-hand side of (7) reduces to $V_n(p_t) + V_n(u_t) - V_n(b_t)$, and a sufficiently large innovation variance in the bubble could cause $V(p_t^*) < V(p_t)$.[8]

We imagine that theoretical exercises similar to the above have spawned the popular argument that the failure of variance bounds tests can be due to speculative bubbles. Analogously, it has been argued that failure to reject the variance bounds inequalities is due to exclusion from the sample of time periods containing bubbles.[9] We now demonstrate that this theoretical intuition is not correct in all situations. The practical implementation of many variance bounds tests precludes rational stochastic bubbles, per se, as the explanation for the failure of the test.

The difference between the theoretical exercise described above and its practical implementation arises, of course, in the constuction of an observable counterpart to p_t^*. In practice it is impossible to measure the ex post rational price because it depends on the infinite future. Researchers therefore typically measure a related variable denoted $\hat{p}_t$. Since actual price and dividend data are available for a sample of observations on $t = 0,1,\ldots,T$, researchers use

$$\hat{p}_t \equiv \sum_{i=1}^{T-t} \rho^i d_{t+i} + \rho^{T-t} p_T, \qquad t = 0,\cdots,T-1, \tag{8}$$

in place of p_t^*.[10] Notice from (5) and (8) that

$$(9a) \qquad \hat{p}_t = p_t^* - \rho^{T-t} p_T^* + \rho^{T-t} p_T,$$

which implies from (6) that

$$(9b) \qquad \hat{p}_t = p_t^* + \rho^{T-t}(b_T - u_T).$$

Since u_T is the innovation in the present value of dividends between time T and the infinite future, it is uncorrelated with all elements of the time T information set which includes time t information. Since b_T depends on the evolution of the stochastic bubble between t and T, it is not orthogonal to time t information.

Notice what happens when (9b) is solved for p_t^* and the result is substituted into (6). After slight rearrangement, one obtains

$$(10) \qquad \hat{p}_t = p_t + w_t,$$

where

$$(11) \qquad w_t \equiv (u_t - \rho^{T-t} u_T) + (\rho^{T-t} b_T - b_t).$$

Equation (10) is the empirical counterpart of (6) and forms the basis of the usual variance bounds tests. The only important difference between our version of (10) and that of previous researchers is that we have explicitly allowed for rational stochastic bubbles in our derivation.

Application of the innovation variance operator to (10) gives

$$(12) \qquad V_n(\hat{p}_t) = V_n(p_t) + V_n(w_t) + 2C_n(p_t, w_t).$$

The important point concerning (12) is that the innovation covariance between p_t and w_t is zero. To understand why, consider the nature of the composite disturbance term, w_t. First, as noted above, both u_t and u_T are uncorrelated with p_t, since p_t is in the time t information set which is a subset of the T information set. Second, and most important, the combined term $\rho^{T-t}b_T - b_t$ is uncorrelated with the time t information set even though each term separately is not orthogonal to time t information. This is true because $E_t(b_T) = \rho^{-(T-t)}b_t$ which follows from (4). Hence, $E_t(\rho^{T-t}b_T - b_t) = 0$, and $C_n(p_t, w_t) = 0$.

Therefore, (12) takes the form

$$(13) \qquad V_n(\hat{p}_t) = V_n(p_t) + V_n(w_t),$$

from which it follows that

$$(14) \qquad V_n(\hat{p}_t) \geq V_n(p_t),$$

by the nonnegativity of $V_n(w_t)$. The important point is that (14) is derived in the presence of rational stochastic bubbles. If the variables $\hat{p}_t$ and p_t are actually used, as they have been in several tests, and the inequality in (14) is found to be violated, one cannot conclude that stochastic bubbles are an explanation of model misspecification.

Of course, research that does not use the terminal price p_T to construct $\hat{p}_t$ does not discriminate among possible reasons for rejection of the null hypothesis. The research which does use the actual terminal market price therefore incorporates one of the alternative hypotheses into the null hypothesis. While this may make it more difficult to reject the null hypothesis, a reversal of the inequality in (14) cannot be attributed to rational stochastic bubbles.

IIa. The Mankiw, Romer, Shapiro Test

Flavin (1983) and Kleidon (1985b) argued that estimation of the sample variance by subtraction of the sample mean rather than the population mean produces small sample bias in variance bounds tests. In an effort to develop an unbiased test Mankiw, Romer and Shapiro (1985) consider a "naive forecast" of stock price defined by

$$(15) \qquad p_t^0 = \sum_{i=1}^{\infty} \rho^i F_t(d_{t+i})$$

where $F_t(d_{t+i})$ is the naive forecast of dividends at time t + i based on some information available at time t.

Consider the following identity:

$$(16) \qquad p_t^* - p_t^0 = (p_t^* - p_t) + (p_t - p_t^0).$$

In order to avoid sample means, Mankiw, Romer and Shapiro (1985) work with the conditional second moments of (16). Substitute from (6) into (16) and take conditional second moments to derive

$$(17)\quad E_{t-n}(p_t^* - p_t^0)^2 = E_{t-n}(p_t^* - p_t)^2 + E_{t-n}(p_t - p_t^0)^2 - 2E_{t-n}[b_t(p_t - p_t^0)].$$

The last term in (17) appears because $p_t^* - p_t = u_t - b_t$, and although u_t is orthogonal to $(p_t - p_t^0)$, b_t is not. Notice, therefore, that the two inequalities derived by Mankiw, Romer and Shapiro (1985) in the absence of bubbles,

$$(18a)\qquad E_{t-n}(p_t^* - p_t^0) > E_{t-n}(p_t^* - p_t)^2$$

and

$$(18b)\qquad E_{t-n}(p_t^* - p_t^0)^2 > E_{t-n}(p_t - p_t^0)^2,$$

need not hold in theory in the presence of bubbles. As in the case discussed above, from (17) we see that if $E_{t-n}[b_t(p_t - p_t^0)]^2 > 0$, bubbles would be one of the reasons that the theoretical construct p_t^* could fail a second moment test.

Now consider substitution for p_t^* in (16) from (9b):

$$(19)\qquad \hat{p}_t - p_t^0 = (\hat{p}_t - p_t) + (p_t - p_t^0).$$

Since p_t^* appears on both sides of (16), the term $\rho^{T-t}(b_T - u_T)$ in (9b) does not appear in (19). From (10), notice that $\hat{p}_t - p_t$ on the right-hand side of (19) is uncorrelated with information at time t. Therefore, since $p_t - p_t^0$ is in the time t information set,

$$(20)\qquad E_{t-n}(\hat{p}_t - p_t^0)^2 = E_{t-n}(\hat{p}_t - p_t)^2 + E_{t-n}(p_t - p_t^0)^2 ,$$

which provides two empirical counterparts to (18a) and (18b):

$$(21a) \qquad E_{t-n}(\hat{p}_t - p_t^0)^2 \geq E_{t-n}(\hat{p}_t - p_t)^2$$

and

$$(21b) \qquad E_{t-n}(\hat{p}_t - p_t^0)^2 \geq E_{t-n}(p_t - p_t^0)^2 .$$

Since (21a) and (21b) were derived under the hypothesis that rational stochastic bubbles are present in the data, rejection of the hypotheses expressed in (21a) and (21b) cannot be attributed to the presence of stochastic bubbles. Similarly, failure to reject the hypotheses cannot, as claimed by Shiller (1985) in his comments on Mankiw, Romer and Shapiro (1985), be attributed simply to the failure to include speculative bubbles episodes in the sample data.[11]

III. A Cautionary Note in Tests for Bubbles

This section presents an example economy illustrating an important point. All bubbles tests are conditional on the researcher having the correct specification of the model including the nature of market fundamentals. Flood and Garber (1980), Blanchard (1979) and Tirole (1985) all define the bubble as what is left over after market fundamentals have been removed from the price. Since neither bubbles nor market fundamentals are directly observable, one can never be sure that market fundamentals have been specified appropriately.[12] The example serves to illustrate the point in the context of the model of the previous section.

The Euler equation (1) may be rewritten as

$$(22) \qquad U'(c_t)z_t = E_t[\rho U'(c_{t+1})(z_{t+1} + a_{t+1})].$$

Consider an economy with a_{t+1} constant at $\bar{a}$ and for simplicity assume that aggregate output is not storable and is constant at $\bar{y}$. With population normalized to one, per capita consumption is simply $c_t = \bar{y}$ in equilibrium. We first solve the model without uncertainty assuming the absence of bubbles. The solution for price is

$$(23) \qquad z_t = \rho\bar{a}/(1-\rho), \quad t = 0,1,\cdots.$$

Now consider the solution if everyone knows that a government will come into existence at T. Assume that the government will institute a tax system to finance its expenditures and that the service flow of government goods enters the utility function separably from the utility of private goods. Assume also that the government will take g units of the consumption goods each period, which lowers equilibrium consumption to $c = \bar{y} - g$ in each period from T onwards.

The advent of the government sector raises the marginal utility of private consumption in period T and thereafter. We parameterize this change by writing $U'(\bar{y} - g) = (1 + \alpha)U'(\bar{y})$. Suppose further that the government finance system taxes all income flows but not capital gains at the rate θ, so that $g = \theta\bar{y}$. After-tax income from owning an asset is therefore $(1-\theta)\bar{a}$.

To determine the price of the asset in periods before T, consider first what price must hold after the advent of the government. The first order condition at T is

$$(24) \qquad U'(\bar{y}-g)z_T = \rho U'(\bar{y}-g)[z_{T+1}+(1-\theta)\bar{a}],$$

which has the no-bubbles solution

$$(25) \qquad z_t = \rho(1-\theta)\bar{a}/(1-\rho), \; t = T,T+1,\cdots.$$

The price of the asset before T can now be determined by recognizing that agents know at T-1 that dividends will be taxed and that the marginal utility of private consumption will rise at T. Price must obey

$$(26) \qquad U'(\bar{y})z_{T-1} = \rho U'(\bar{y}-g)(z_T+(1-\theta)\bar{a}),$$

which, from (25), implies $z_{T-1} = (1+\alpha)z_T$. Price falls from T-1 to T to offset the increase in marginal utility and the incidence of taxation. Treating z_{T-1} as a terminal price, we solve for asset prices in periods before time T-1.

The solution for asset price is

$$(27) \qquad z_t = A\rho^{-t} + \rho\bar{a}/(1-\rho), \quad t = 0,1,\cdots,T-1,$$

where

$$(28) \qquad A = \left[(1-\theta)(1+\alpha)-1\right]\left[\rho\bar{a}/(1-\rho)\right]\rho^{(T-1)}.$$

The price path prior to T-1 will rise, fall, or remain constant depending on the sign of A which is governed by whether $(1-\theta)(1+\alpha)$ is greater than, less than, or equal to one.[13]

Now suppose agents are not sure that a government will be installed at T. Assume that their uncertainty can be represented by a time-invariant probability π that the government will begin operation at T with corresponding probability $(1-\pi)$ that the government will not begin operation then or any other time in the future.

With this change the constant term in (28) becomes

$$(29) \qquad A' = \left[\pi+(1-\pi)(1+\alpha)(1-\theta)-1\right]\left[\rho\bar{a}/(1-\rho)\right]\rho^{T-1}.$$

If the transition probability is not constant but moves through time, a solution for the price path is

$$(30) \qquad z_t = \left[\pi_t+(1-\pi_t)(1+\alpha)(1-\theta)-1\right]\left[\rho\bar{a}/(1-\rho)\right]\rho^{T-t-1}$$

$$+ \rho\bar{a}/(1-\rho), \qquad t = 0,1,\cdots,T-1,$$

which may vary stochastically through time in addition to its deterministic movements.

The important point about these simple examples is that they illustrate situations in which an expected future event produces a price path which, if compared to the path of market fundamentals prior to the future event, appears to be characterized by a bubble. Examination of (27), (28), (29) and (30) indicates that asset prices correspond to the no-bubbles with constant fundamentals price in (23) plus something else. The additional element must fulfill the bubble property given in equation (4) since it is in the homogenous part of the solution. The homogenous part will be present in price either if there are bubbles or if the no bubbles system needs to position itself in advance of a future switch in forcing processes. The last example as well as more complex versions would generate stochastic price paths that would appear to contain stochastic bubbles that satisfy (4) even though the examples are bubble free.

The econometric problem arises because the investigator never knows precisely what information is used by economic agents. Consider a naive investigator who examined data from periods surrounding the possible institution of the above government policies in a situation in which the policies were not instituted. If A' in (29) is nonzero, the unwary researcher who treated data for a_t as market fundamentals would conclude that the price path prior to T contained a bubble that burst. The example, of course, was bubble free. The problem is that a_t does not capture all of the market fundamentals. The potential taxation and government spending programs also are part of fundamentals.

The point of this section was to provide a cautionary note to the interpretation of bubbles research. Empirical bubbles tests must be interpreted either conditionally assuming an investigator has correctly modeled market fundamentals or as joint tests for bubbles and possibilities of misspecification of market fundamentals. Perhaps the latter interpretation is more attractive to some researchers. It is interesting nevertheless to inquire whether bubble-type processes characterize the data, and if so, what misspecification of market fundamentals might be behind such a finding.

IV. Concluding Remarks

Speculative bubbles are possible in some theoretical models and are precluded in others. Whether they are important phenomena in actual economic data is an open question presumably susceptible to scientific investigation.

One point of this paper is that the implementation of variance bounds tests often precludes rational speculative bubbles as a reason for rejection of the null hypothesis in such tests. Construction of an observable counterpart of ex post rational price, $\hat{p}_t$, by employing an actual terminal market price, builds any rational bubbles into the empirical analysis. Therefore, bubbles must be unrelated to findings that the volatility of p_t is greater than that of $\hat{p}_t$.

Of course, this is not an indictment of variance bounds tests. These tests play an important role in the econometric analysis of financial markets. We have observed the controversy surrounding these tests and their empirical implementation. Our purpose is simply to clarify these tests and their empirical implementation on one particular issue.

The second point of the paper is to provide a cautionary note to the empirical analysis of bubbles. West (1984, 1985) has designed and implemented theoretically correct bubbles tests. West's methodology requires an unrejected Euler equation (1) and a forecasting equation for market fundamentals. Interpretation of the results of such tests requires consideration of potential changes in the market fundamentals that agents may be forecasting. Much work has been devoted to attempts to find an unrejected Euler equation, but success has been elusive. Less effort has gone into understanding how process switching can affect asset pricing tests. Both aspects are critical to our understanding of the economics of asset price volatility.

Footnotes

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1. See, in particular, Flavin (1983), Kleidon (1985a, 1985b) and Marsh and Merton (1984).

2. Eichenbaum and Hansen (1984) and Dunn and Singleton (1985) explore alternatives that allow considerable variation in the representative agent's intertemporal marginal rate of substitution.

3. Dornbusch (1982) argues that the flexible exchange rate system has not worked well and suggests that speculative bubbles may be one of the culprits. See Meese (1986) for a test of speculative bubbles in the foreign exchange market.

4. Marsh and Merton (1984) use the sample average price as the terminal price in constructing their counter example to Shiller's derived variance bounds.

5. Satisfaction of a transversality condition such as $\lim_{h\to\infty} \rho^h E_t(p_{t+h}) = 0$ requires the absence of bubbles. Tests for speculative bubbles can consequently be thought of as tests of this model's transversality condition.

6. The restrictions that (4) places on the b_t process are not very severe. The time series process can take many possible forms including bubble innovations that are conditionally heteroscedastic.

7. In order to simplify our argument we abstract from the sampling distribution of the sample statistics and regard them as precise estimates of their population counterparts.

8. Geweke (1980) notes that in linear environments a variance bounds test is not always powerful at detecting deviations from the theory. Since $p_t = p_t^f + b_t$ from (2) and (3), and because $p_t^* = p_t^f + u_t$ from (5) and the definition of u_t, $V_n(p_t^*) - V_n(p_t) = V_n(u_t) - V_n(b_t) - 2C_n(p_t^f, b_t)$. When $V_n(u_t) > V_n(b_t) + 2C_n(p_t^f, b_t)$, the variance bound test is unable to detect bubbles even though they are present in the data. Geweke demonstrates that an alternative regression test is more powerful. Frankel and Stock (1983) reach a similar conclusion but argue that variance bounds test may be more powerful against nonlinearities in the misspecification.

9. Shiller (1985, p. 689), in his discussion of Mankiw, Romer and Shapiro (1985) argues that one reason one of their tests does not find excess volatility is "that the major 'speculative bubbles' in this century of data, that of the 1920's and that of the 1950's, are given less weight."

10. The variable $\hat{p}_t$ is used by Shiller (1982), Grossman and Shiller (1981), and Mankiw, Romer and Shapiro (1985). Earlier, Shiller (1981a, 1981b) used the average market price over the sample period as the terminal price in (8). See note 6. Kleidon (1985a) and LeRoy (1984) demonstrate that use of $\hat{p}_t$ produces a smooth series compared to p_t even in situations in which the data are constructed to satisfy (1). The series look smooth because for small k, $V(p_t^* - p_{t-k}^*) < V(p_t - p_{t-k})$ with the results being quite dramatic for highly autocorrelated dividend series. Hence, the graphs in Grossman and Shiller (1981) are quite misleading.

11. In fairness to Shiller, his (1984) work clearly indicates that he rejects the notion of rational speculative bubbles discussed in this paper, although Fischer (1984) argues that Shiller's fads and fashions may ultimately prove to be the same thing as speculative bubbles.

12. This point was emphasized by Flood and Garber (1980, pp. 749-50) and has been reiterated by Hamilton and Whiteman (1985).

13. If $U(c_t) = c_t^{1-\beta}/(1-\beta)$, $\beta > 0$, then $(1+\alpha)(1-\theta)^{\beta} = 1$ and the type of price path followed until date T-1 depends on the relationship of β to unity. If $\beta > 1$, prices rise prior to T while they fall if $\beta < 1$. For quadratic utility the relationship of $(1+\alpha)(1-\theta)$ to unity will depend on the scale of the economy and ratios of the utility function parameters.

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