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ABSTRACT

We develop a new theory of the firm where asset owners sometimes want to change partners ex-post. The model identifies a fundamental trade-off between (i) a "displacement externality" under non-integration, where a partner leaves a relationship even though the benefit is worth less than the loss to the displaced partner, and (ii), a "retention externality" under integration, where a partner inefficiently retains the other. Renegotiation cannot eliminate these inefficiencies when agents are wealth constrained. When there is more asset specificity, displacement externalities matter more and retention externality less, so that integration becomes more attractive. Our model also predicts that integration always provides stronger incentives for specific investments, and that wealthy owners actually want to commit to ex-post wealth constraints. Our analysis differs from the received theories of the firm because of our emphasis on dynamic partner changes.

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1 Introduction

The central questions in the theory of the firm are why firms exist, why assets are sometimes held under some form of common ownership, and why economic transactions are not always organized via markets. In this paper we explore the idea that integration (such as through joint asset ownership) may guarantee continued access to critical trading partners. We develop a model of asset ownership that is based on a dynamic trade-off between the commitment to a trading relationship (integration) versus the flexibility of seeking new relationships (non-integration).

We consider an environment in which trading partners cannot take each other for granted, and fear being displaced by alternative partners. In the presence of wealth constraints there can be ex-post inefficiencies, where one partner's gain from leaving the relationship does not outweigh the loss to the partner left behind. Joint asset ownership protects against such inefficient partner displacements. Then, why does not every trading relationship involve integration? This is because under joint ownership one partner can prevent the other from leaving the relationship, even though leaving would be jointly more efficient.

Our theory focuses on the dynamics of how asset owners match and possibly rematch with each other. We show that integration generates a commitment that makes it more difficult, but not impossible, to change trading partners. We explain how joint asset ownership endogenously raises the costs of leaving a relationship, and how this can be beneficial under some circumstances and detrimental under others. We also introduce the idea that integration is a function of the uncertainty that partners face about how the match with the current partner compares against possible future matches.

Our focus on partner changes is fundamentally different from the property rights perspective, associated with the seminal work of Grossman, Hart and Moore (see Grossman and Hart (1986), Hart and Moore (1990), Hart (1995)). In their models, alternative partners create an outside option that is never exercised in equilibrium. That is, property rights models always assume that the current partner is and will remain the best possible match. Our model introduces uncertainty about future matches, and shows how ex-post inefficiencies, such as displacement and retention externalities, can affect the ex-ante integration decision. Our main insights also do not rely on incentives for specific investments.

Our model is also related to the transaction cost theories associated with Williamson (1975, 1979, 1985), who explicitly worries about ex-post inefficiencies. However, Williamson too focuses mostly on the current match, and worries mainly about opportunism and haggling costs. We focus on different and potentially more important ex-post inefficiencies, related to the inefficient displacement or retention of trading partners. Interestingly, a central prediction of our theory is that asset specificity favor integration, although the reason is different than in standard transaction cost economics.

In our model there are two symmetric owner-managers with co-specialized assets. Ex-ante they determine the optimal asset allocation. The base model deliberately ignores any specific investments, although a model extension adds them back in. At some ex-post date uncertainty is resolved about whether the original partners have found more attractive outside partners or not. At that stage the original partners can stay together or part their ways.

We allow for the possibility of ex-post inefficiencies in the process of switching partners. One possibility is that a partner gets displaced even though staying together would be jointly efficient – we will call this a displacement externality. Another possibility is that partners stay together even though separating would be jointly efficient – we will call this a retention externality. We endogenously derive these two new ex-post inefficiencies on the basis of two important assumptions. First, we assume that all parties are wealth constrained – we discuss this below. Second, we allow for a team moral hazard problem, where joint production requires the partners to provide non-contractible inputs such as private efforts. The combination of wealth constraints and private efforts generate a concave utility frontier where partners cannot freely transfer utility to each other without affecting incentives and joint efficiency.

We first show that whenever the two partners have symmetric outside options, the ownership structure is irrelevant. Both parties either agree to stay together, or they agree to part ways, but there is no disagreement. Asset ownership only becomes relevant when the two partners have asymmetric outside options.

Consider individual asset ownership and let us call the partners A and B. Suppose A found an alternative partner (call him C) but B did not, so that A wants to leave B for C. There can be what we call a *displacement externality*, where A's gain from leaving is worth less than B's loss from being displaced. Obviously the two partners would like to renegotiate. Without a wealth constraint this would be straightforward and renegotiation would ensure joint utility maximization. With a wealth constraint, however, the two partners have to renegotiate along a concave utility frontier. The weaker partner (B) may offer to give up some share of the profit to the stronger partner (A), in order to convince him not to leave. There are two possible outcomes in this renegotiation game. Either B's best offer is not good enough to retain A, and despite being jointly inefficient, displacement occurs in equilibrium. Or A accepts a higher profit share and stays. The threat of displacement is then not realized in equilibrium, but the unequal profit shares distort team incentives. The outcome after renegotiation is individually optimal, but fails to maximize the sum of utilities.

Joint asset ownership can give rise to the opposite problem. If A cannot leave without B's consent, there can be a *retention externality*. This occurs when A's benefit from leaving is higher than B's benefit from retaining A. Without a wealth constraint, renegotiation would again achieve ex-post efficiency. With a wealth constraint, however, the only way that A can compensate B is to give him a stake in his new partnership with C. This requires a negotiation among three parties. Again we obtain two possible bargaining outcomes. Either there simply is no offer such that B is willing to let go of A, and A still finds it attractive to leave. In equilibrium B retains A, even though it is jointly inefficient. Or it is possible to structure some buyout deal, but the outcome does not maximize A's and B's joint utility. This is because B receives a share of profits generated by A and C, without actually contributing any productive inputs. The retention externality matters, irrespective of whether inefficient retention occurs, or whether it gets renegotiated into some other outcome that still does not maximize the sum of utilities.

The optimal ex-ante allocation of asset ownership depends on the desirability of changing partners. The main result of our base model is that joint asset ownership is optimal when displacement externalities loom large, whereas individual asset ownership is preferred when retention externalities matter more. The relative importance of displacement and retention externalities depends on how good the original match between partners is. The greater the asset specificity, the greater the displacement externality, and also the smaller the retention externality. A higher asset specificity therefore favors joint asset ownership. Our theory thus delivers one of the key predictions from the transaction cost theory, without referring to the typical ex-post problems emphasized by Williamson.

Our base model deliberately omits specific investments. This allows us to generate a set of predictions about optimal asset ownership that are orthogonal to the standard concerns of the property rights theory. Once we put relation-specific investments back into the model, we find that joint asset ownership always provides stronger incentives for specific investments. The key intuition is that joint asset ownership is efficient when the internal match is good, but can cause retention inefficiencies when the internal match is poor. By contrast, individual asset ownership is efficient when the internal match is poor, but can cause displacement inefficiencies when the internal match is good. Consequently joint asset ownership increases the utility gap between the good and the bad match, whereas individual asset ownership actually narrows the gap.

Our base model critically depends on binding wealth constraints. We submit that this is actually a realistic assumption for many owner-managed firms. Moskowitz and Vissing-Jørgensen (2002) provide empirical evidence that on average 82 percent of the wealth of owner-managers is tied up in their businesses. This implies that for a large number of owner-manager the available wealth for transfer payments is low to non-existent. Empirical plausibility apart, we provide a three-part defence of our base assumption. First, while it is true that large amounts of wealth can always solve all the ex-post inefficiencies, we show that having a small amount of wealth sometimes doesn't change the ex-post game at all. For example, a small transfer payment is simply not enough to retain a partner who has a clear preference for leaving. Second, we obtain the surprising result that wealthy owners would *always* want to commit ex-ante to some kind of ex-post wealth-constraint, either partial or complete. Having wealth turns out to be a two-edged sword. On the one hand, wealth helps the partners to mitigate ex-post inefficiencies. On the other hand, it weakens ex-ante incentives for specific investments, precisely because it allows partners to mitigate ex-post inefficiencies when the quality of their match turns out to be poor. Third, we show that wealth constraints are only one way of generating ex-post inefficiencies. The main trade-off of our paper also remains valid in a model where there is sufficient wealth, but where there are costs associated with making transfer payments.

Any new theory about the boundaries of firms stands on the shoulder of giants. The economic theory of the firm is dominated by three main schools of thought: transaction cost economics – associated mainly with the work of Williamson; the property rights perspective – associated mainly with the work of Grossman, Hart and Moore; and the incentive theory perspective – associated in particular with the work of Holmström, Milgrom and Roberts (see Holmström and Milgrom (1994), Holmström and Roberts (1998), Milgrom and Roberts (1990)). Our theory does not fit squarely into one of these schools. Instead it borrows a little from each, but then focuses on the dynamics of partner displacement and retention – issues that do not feature prominently in any of the existing schools of thought.

Our theory makes predictions about asset specificity that are consistent with the predictions of transaction cost economics. There is strong empirical support that asset specificity is associated with integration – see Lafontaine and Slade (2007) for a comprehensive survey of the empirical literature. Our theory provides a fresh interpretation as to why asset specificity leads to integration. Transaction cost economics tends to provide verbal arguments about opportunism and ex-post price haggling as the main sources of inefficiencies under non-integration. However, more formal theories have often dismissed these explanations, because rational agents should be able to anticipate any distributional concerns ex-ante, and thus to resolve ex-post inefficiencies. Some recent exceptions are Bajari and Tadelis (2001), Tadelis (2002), Matouschek (2004), and Casas-Arce and Kittsteiner (2011), who develop formal models with costly ex-post adjustments. In our model it is the lack of transferable utility that endogenously creates ex-post inefficiencies. Asset specificity increases the cost of displacement externalities, and also lowers the cost of retention externalities – both of which favor integration. Our theory therefore combines the very definition of asset specificity, namely that an asset is worth more within than outside a specific trading relationship, with the possibility that in equilibrium partners do go outside the relationship, in order to generate a fresh insight as to why specific assets favor integration.

In the property rights theory it is not the level of asset specificity, but the marginal incentive to increase asset specificity through specific investments, that determines the optimal asset allocation (see Whinston (2003)). Our model differs from the property rights approach in several key aspects: (i) it does not rely on specific investments, (ii) it allows for ex-post inefficiencies, (iii) it allows for partner switching in equilibrium, and (iv), it allows partners to contractually specify prices ex-ante.[4] Moreover, once we add specific investments to our base model we obtain a clear-cut prediction: incentives for specific investments are always stronger under joint than under individual asset ownership.[5]

Our approach clearly differs from the incentive-based theories of the firm, which focus on risk-aversion and multi-tasking. However, we borrow from these incentive-based theories by incorporating the moral-hazard-in-teams problem (Holmström, 1982) into our production function. Together with the assumption of wealth-constrained agents (see also Sappington (1983)), this allows us to endogenously derive a concave utility frontier which generates the ex-post inefficiencies.

Our model does not address differences between integration and relational contracting. In fact, in our model all the outcomes under joint asset ownership can also be achieved using binding long-term contracts. An explanation of relational contracts would require modeling reputations in a repeated game environment. Interestingly, the main theories of relational contracts, such as Baker, Gibbons and Murphy (2002), focus on a given set of partners, and do not consider how in equilibrium agents may change their trading partners.[6]

Our model is not the first to consider wealth constraints. Aghion and Bolton (1992) introduce wealth constraints into a financial contracting model. In their model there are fixed non-transferable private benefits that can lead to ex-post inefficient decisions, depending on the allocation of control rights. The main difference to our model is that Aghion and Bolton look

[4]The non-contractibility of prices is crucial for the property rights theory. We assume that prices are contractible at all times. However, our model does have some contractual incompleteness concerning interim information that allows partners to update their profitability forecasts. If these updates were verifiable, then the optimal allocation of assets becomes state-contingent. Even then the underlying trade-off between displacement and retention externalities remains valid.

[5]Another difference worth mentioning is that in the standard property rights model, joint asset ownership is never optimal, and integration always consists of one agent owning both assets. Cai (2003) derives a model with both specific and general investments, and shows that joint asset ownership becomes optimal when the two types of investments are substitutes. Halonen (2002) provides conditions under which joint asset ownership is optimal in a repeated game framework; see also Blonski and Spagnolo (2003). Our model provides an alternative reason for the optimality of joint asset ownership: to prevent the dissolution of otherwise efficient partnerships.

[6]One interesting exception is Board (2011), who considers relational contracting between a principal and an agent, where the latter can be replaced in every period depending on changing production requirements. However, Board does not address issues of asset ownership.

at a model with a single asset, and focus on the inherently asymmetric relationship between an investor and entrepreneur. Our model is closer to the Grossman, Hart and Moore set-up with two assets and two productive agents. In our model we also do not rely on fixed private benefits, but instead consider renegotiation of ownership shares in the presence of moral hazard and wealth constraints. In a related vein, Aghion and Tirole (1994) consider the importance of wealth constraints in a model with a single asset. They focus on intellectual property as an asset, and ask whether it should be owned by the developer or user of the innovation. None of these models considers the possibility of switching trading partners at the ex-post stage.

There is a prior literature that looks at ex-post inefficiencies.[7] Of historic interest, in addition to their seminal 1986 paper, Grossman and Hart published a less known book chapter in 1987 with a model where there are ex-post inefficiencies and no specific investments (Grossman and Hart, 1987). More recently, Hart (2009) examines asset ownership in a model of reference points with ex-post irrational and inefficient behaviors. Dessein (2012) outlines a model with inefficient ex-post decisions, leading to a trade-off between adaptation and coordination. Aghion et al. (2012) provide a model with ex-post asymmetric information, showing how the ex-ante asset allocation plays a role over and above any contractual arrangements.

Our paper is loosely related to the large literature on vertical foreclosure. Aghion and Bolton (1987) examine how a seller can lock buyers into long-term contracts to reduce the threat of entry from a competing seller. Bolton and Whinston (1993) use a property-rights approach to study how concerns about supply assurances can motivate vertical integration. Segal and Whinston (2000) further examine how exclusive contracts may (or indeed may not) affect specific investments. Matouschek and Ramezzana (2007) use a search model with sellers and buyers to study how exclusive contracts can reduce ex-post inefficiencies arising from price haggling.[8] One important difference to the literature on exclusive contracts is that our model focuses on a set-up with two symmetric partners, who are both concerned about retaining their trading partner.

Our paper is also related to parts of the literature on partnerships. Hellmann and Thiele (2012) consider the formation of entrepreneurial teams. De Frutos and Kittsteiner (2008) examine the efficient dissolution of partnerships.

The remainder of this paper is structured as follows. The next section features a simplified and curiously romanticized version of our model that helps to bring out many of the key ideas of the main model. Section 3 then introduces our main model. Section 4 examines how partners make choices about staying versus leaving a relationship. Section 5 identifies the optimal allo-

[7] Gibbons (2005) identifies them broadly as adaptation-based theories of the firm; Segal and Whinston classify them as theories with imperfect bargaining.

[8] See also Jing and Winter (2011) for an overview over the recent literature on exclusionary contracts.

cation of control rights over critical assets. We introduce specific investments in Section 6. In Section 7 we consider the role of wealth in a partnership, and ask whether partners can benefit from limited wealth. In Section 8 we discuss robustness properties of our model. Section 9 summarizes our main results, and explores avenues for future theoretical and empirical work. All proofs are in the Appendix.

2 A Simplified Model

This self-contained section introduces a simplified version of our model which quickly establishes some of its key intuitions. The more technically-minded reader may go straight to Section 3 where we introduce our main model.

Our theory on partner matching bears uncanny resemblances to marriage decisions. To make the simplified model more memorable, let us tell the story in terms of a romantic couple that is considering getting married.

Alice and Bob are dirt poor, love each other, and derive a non-transferable utility θ from being together (irrespective whether married or not). Unfortunately Alice is not sure if Bob is really the love of her life. She assigns a probability q to the possibility that at a later point she could meet the man of her dreams, let us call him Charles. Being with Charles would give Alice a utility λ, where $\lambda > \theta$. Bob too has his doubts; he still hankers after his childhood sweetheart Dora. He thinks there is a probability q that she will come back to him, which would give him a utility λ.

Should Alice and Bob get married? We assume that once Alice and Bob are married, they can only divorce each other by mutual consent. Suppose at some future date Alice met Charles and Dora actually wanted to marry Bob too. Alice and Bob would mutually agree to a divorce, and both live happily thereafter. However, suppose that Alice meets Charles, but that Dora continues to reject Bob. Without marriage, Alice can simply leave Bob. With marriage, however, she needs Bob to agree to a divorce. Agreeing to a divorce gives Bob zero utility, but if he refuses, then Alice stays which gives him θ – obviously Alice and Bob are rational economic agents without feelings of remorse or jealousy. If married, Bob refuses to get divorced, and since Alice is poor, she also cannot bribe Bob into changing his mind. It follows that marriage matters for the case where only one of the two partners has an outside option. (The same is obviously also true if Alice does not meet Charles, and Dora wants Bob).

Tracing the utility through all the possible permutations of who can partner with whom, we find that the expected utility from marriage is given by

$$EU(marriage) = (1-q)^2\theta + 2q(1-q)\theta + q^2\lambda,$$

whereas the utility of remaining unmarried is

$$EU(no - marriage) = (1-q)^2\theta + q(1-q)\lambda + q^2\lambda.$$

Thus we find that

$$EU(marriage) - EU(no - marriage) = q(1-q)(2\theta - \lambda).$$

This says that marriage is optimal whenever $2\theta > \lambda$. In more romantic terms, Alice marries Bob if she loves him more than half as much as Charles.[9] Why is marrying Bob a good idea if Alice prefers Charles over Bob? The answer is that Alice trades off the loss of flexibility – she will need Bob's permission to be with Charles – against the benefit of having Bob's commitment – she retains Bob in case she does not meet Charles.

Things get a little more complicated if Alice and Bob are uncertain about how much they really love each other. Suppose that they envision two possible scenarios, one where their love remains strong, so that they get a utility θ_H with $\lambda < 2\theta_H$, and one where their love fizzles out, such that they get θ_L with $\lambda > 2\theta_L$. If they knew that their love remains strong, they should marry; if they knew that it will fizzle out, they should not. The uncertainty raises the possibility of two types of inefficiencies. First, if they do not get married, Alice may leave Bob even though they love each other. This happens for $\theta_H < \lambda < 2\theta_H$, where Alice loves Charles more than Bob, but not twice as much. The break-up is inefficient since the loss to Bob (θ_H) is greater than the gain to Alice ($\lambda - \theta_H$). We will call this a *displacement externality*. Second, if they get married and their love fizzles out, then there is a possibility of a *retention externality*. For example, Bob may refuse to divorce Alice even though it would be jointly efficient, namely when $\lambda > 2\theta_L$.

Our results depend on Alice and Bob being dirt poor. If they had sufficient wealth, they could always buy themselves out of trouble. In fact, it is easy to show that marriage is irrelevant with sufficient wealth.[10] However, having a small amount of wealth may not change anything. For example, if Bob wants to retain Alice (without marriage), he needs a minimum wealth of $\underline{w} = \lambda - \theta$; otherwise even his best offer will not suffice to keep Alice. Moreover, if Alice and Bob have sufficient wealth, but there are costs to making transfer payments – such as a tax on transfers – then the key trade-off comes back. In the Appendix we show that as soon as there are

[9] Interestingly, this result does not depend on the value of q, provided that there is some uncertainty ($0 < q < 1$).

[10] If Alice and Bob are married, and Alice wants a divorce that is jointly efficient (i.e., $\lambda > 2\theta_L$), then there exists a transfer payment from Alice to Bob $t_{AB} \in [\theta_L, \lambda - \theta_L]$ such that both are better off with such a divorce settlement. And if Alice and Bob are not married, and Alice wants to inefficiently leave Bob for Charles (i.e., $\lambda < 2\theta_H$), then there exists a transfer payment from Bob to Alice $t_{BA} \in [\lambda - \theta_H, \theta_H]$ such that both are better off staying together.

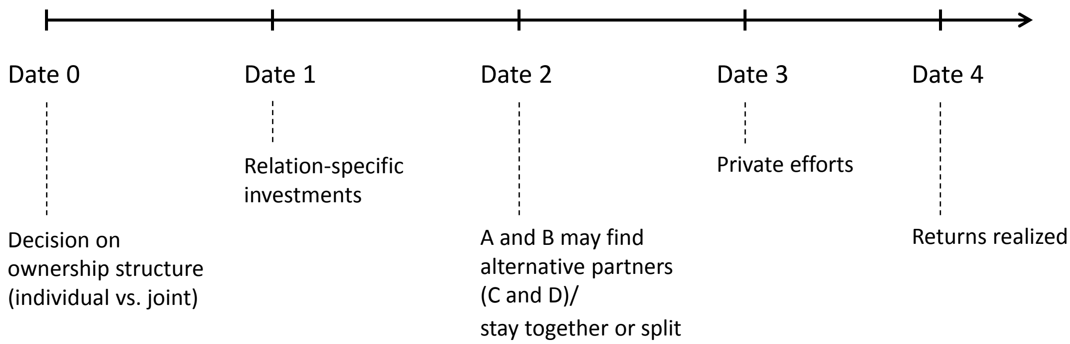

Figure 1: Timeline Base Model

any costs of making transfers, inefficient leaving (under no-marriage) and inefficient retention (under marriage) can occur in equilibrium. And once again, it is optimal for Alice and Bob to get married whenever $2\theta > \lambda$.

Our model hardly provides the most nuanced description of the marriage decision.[11] The purpose of this simplified model is to convey some key intuitions of our main model in the simplest possible way. One may ask why we even need the main model? First, our marriage model takes utilities as given, but in our theory of the firm we want to model the underlying value-creation process. Second, the simplified model is based on fully non-transferable utilities (i.e., private benefits); for our theory of the firm we want to allow for transfer prices and ownership shares that, even though imperfect, still transfer some utility across partners. Third, we want to look at a wider array of contractual structures and renegotiation possibilities than in the simplified model. Hence we now turn to the fully specified theory model.

3 The Base Model

Consider an initial match of two risk-neutral economic agents A and B, called *partners*. For example, B can be an upstream firm selling an input to A as downstream firm, which A needs to manufacture an end product. The value of their initial outside options is normalized to zero. Each partner initially owns a co-specialized asset that only he can operate.

The two partners face constraints on any ex-post transfer payments (non-transferable utility). We model this by assuming that both partners have zero wealth. We relax this assumption in Section 7, where we show that a wealth constraint can arise endogenously in our framework.

[11] Our objective here is to illustrate the key intuitions for our main model, not to provide a fully-fledged marriage model. For more sophisticated models of the marriage decision see, e.g., Brien, Lillard and Stern (2006) and Matouschek and Rasul (2008).

There are five dates; see Figure 1 for a graphical overview. At date 0, both partners decide on an optimal ownership structure for both assets. While we consider all ownership structures, the key decision will be whether partners keep individual asset ownership, or whether they agree on joint asset ownership. As discussed in Section 8.1, we can also think of this difference in terms of short-term versus long term contracts. At date 1, both partners can make relation-specific investments to improve the value of joint production. However, to show that our base results do not depend on specific investments, we deliberately shut down this part of the model until Section 6. At date 2, A and B may find alternative partners. They then decide whether to stay with their original partner, or to leave and form a new partnership. Partners may also renegotiate any division of surplus. At date 3, partners exert private effort to produce output. Finally, at date 4, all returns are realized.

Suppose A and B want to engage in joint production at date 3. This requires the use of both A's and B's assets, as well as the partners' private efforts, which we denote e_A and e_B. A partner's disutility of effort is $c(e_i)$, $i \in \{A, B\}$, with $c'(e_i) > 0$, $c''(e_i) > 0$, and $c(0) = c'(0) = 0$. Production either generates a joint profit at date 4 (success), or no profit at all (failure). The success probability is given by $\mu(e_A e_B)$, which is increasing and concave in its argument $e_A e_B$, with $\mu(0) = 0$. Thus, partner efforts are complementary, and success requires that both partners apply strictly positive efforts (i.e., e_A, $e_B > 0$).

In case of success, the realized profit at date 4 is denoted by y. We assume that y is verifiable, and that it has a distribution $\Omega_{in}(y)$ over some interval $y \in [\underline{y}, \overline{y}]$ with $0 \leq \underline{y} < \overline{y} \leq \infty$. We denote the expected value by $\pi = \int_{\underline{y}}^{\overline{y}} y d\Omega_{in}(y)$, and refer to it as the *inside prospect*. That is, π measures the expected profit in case of success, and therefore reflects the quality of the match between A and B. In the base model we assume that there is no learning about the distribution of y until its realization at date 4. In Section 5.2 we add learning, where there is updating on the expected value π at date 2.

The realized profit y at date 4 can be divided between the two partners according to any sharing rule where A obtains αy and B receives βy, with $\alpha + \beta = 1$. Depending on the ownership structure this sharing rule can be implemented in different ways. Under joint asset ownership, we think of α and β as a division of ownership shares from the jointly owned venture. Under individual asset ownership there are no ownership shares, so the division of surplus comes from some transfer price – we provide a more formal discussion in Section 8.1. Because of the wealth constraint, α and β are always non-negative.[12]

[12]In principle it is possible to make α and β functions of y. In the case of joint asset ownership, it is easy to verify that for any division of surplus with variable α and β, there exists an equivalent division of surplus with a constant α and β. W.l.o.g we can therefore focus on constant α's and β's. In the case of individual asset ownership, α and β depend on how transfer prices are specified, i.e., how they depend on the realization of y. to keep our

The initial partners A and B can break away from their original partnership and match with alternative partners. Specifically we assume that at date 2, A finds an alternative partner, called C, with probability $q > 0$. We assume symmetry so that B discovers an alternative partner, called D, with the same probability q.[13] The outside option of each alternative partner, C and D, is normalized to zero. Alternative partners have the same disutility of effort, and the same probability of success. However, in an alternative partnership the profits y in case of success have a distribution $\Omega_{out}(y)$. We denote the expected value by $\sigma = \int y d\Omega_{out}(y)$, which we refer to as the *outside prospect*.[14]

Ownership defines control rights over the productive assets. We assume that A and B initially have full rights of control over their respective assets. We refer to this scenario as *individual asset ownership*. In this case there is no need to write a contract at date 0; A and B simply wait until date 2 to see whether in fact they want to partner up.[15] If they do, they negotiate a transfer price which determines their profit shares (α, β) at that time. Alternatively, the partners can agree at date 0 to share control rights over both assets, which we refer to as *joint asset ownership*. This requires that A and B negotiate the ownership shares α and β at date 0. Ownership matters because it affects the ability of a partner to leave: Under individual asset ownership, a partner with a superior outside option can always leave without the consent of the other. Under joint asset ownership, the two partners share control rights over both assets, so that leaving requires consent of the other partner.

At date 0, the two partners A and B determine asset ownership. Bargaining can also occur at date 2, where it may involve two or more parties. Because of the wealth constraint, we need a bargaining solution for games with non-transferable utilities. We adopt the bargaining protocol of Hart and Mas-Colell (1996), where in each round one member at the bargaining table is selected at random to make a proposal, and where there is a small probability that a partner whose proposal was rejected, is permanently eliminated from the bargaining. This is a multi-player generalization of the breakdown game by Binmore, Rubinstein, and Wolinsky (1986). This bargaining protocol generates the Maschler-Owen consistent NTU value, which is a generalization of the Shapley value for games with non-transferable utility (Maschler and Owen, 1992).

notation simple we focus on the case of constant α's and β's. This is w.l.o.g since all that matters is the expected profit share at date 2.

[13]Recall that A and B have complementary assets which both are needed for production. This excludes the possibility of A partnering with D, or B partnering with C.

[14]An alternative interpretation is that A and B always find alternative partners, but that the outside prospect is then $\sigma_H(= \sigma)$ with probability q, and $\sigma_L = 0$ with probability $1 - q$.

[15]This specific aspect is due to our simplification that no production occurs between the dates 0 and 2. If the two partners were to produce at the earlier stage, then they would write a short-term contract for the early production period, but retain the right to leave the relationship over the long term.

For bilateral bargaining games, the Maschler-Owen consistent NTU value reduces to the Nash bargaining solution.[16] We assume that the only members at the bargaining table are those who have the control rights to affect a decision. This means that under individual asset ownership, bargaining takes place between the two partners who want to engage in joint production. Under joint asset ownership, however, leaving requires the consent of the other partner. A new partner (C or D) therefore has to engage in trilateral bargaining with both of the original partners (A and B). In Section 8.3 we discuss alternative bargaining protocols.

4 Optimal Partner Changes

In this section we determine whether, for a given ownership structure, a partner remains in the relationship, or switches to a new partner. We first assume that the inside prospect π is constant; we relax this assumption later in Section 5.2. Moreover, we ignore relation-specific investments until Section 6. We only focus on two key ownership structures: individual versus joint asset ownership. Section 8.1 shows why we can limit ourselves to those two ownership structures.

We proceed in four steps. First, we consider the joint production of A and B in the absence of any outside partners. Second, we examine the case where A and B have symmetric outside options, i.e., either both or none of them found an alternative partner at date 2. For both types of asset ownership we identify (i) the optimal choice of A and B between staying and leaving, and (ii), the equilibrium division of profits. We then repeat this analysis for the case where A and B have asymmetric outside options, i.e., only A or only B found an alternative partner. Finally we identify the optimal asset ownership at date 0 when the presence of alternative partners is still unknown to both A and B.

4.1 Joint Production

We first examine the joint production process for A and B, explaining how profit shares affect incentives, success probabilities, and partner utilities. The analysis is analogous for joint production with a new partner (C or D) except that the inside prospect π is to be replaced by the outside prospect σ.

[16] For a more extensive discussion of this, see also Hart (2004).

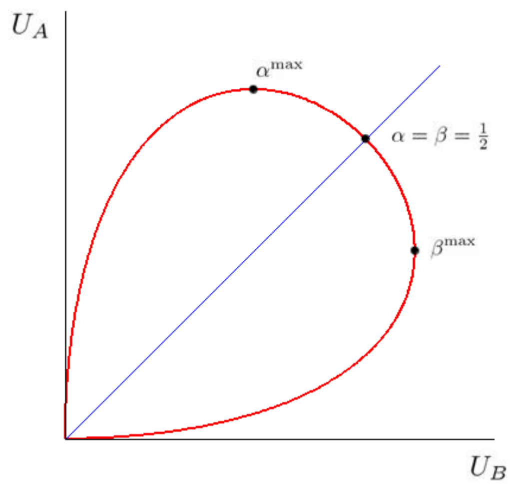

Figure 2: Utility-possibility Frontier for Joint Production

In our model there is team production. Because of the binary nature of outcomes (success or failure), there is no possibility for budget breaking as in Holmström (1982). Partners choose their respective efforts e_A and e_B to maximize their expected utilities:

$$U_A(\alpha; \pi) \;=\; \alpha\mu(e_A e_B)\pi - c(e_A)$$

$$U_B(\beta; \pi) \;=\; \beta\mu(e_A e_B)\pi - c(e_B).$$

The optimal effort levels, denoted $e_A(\alpha)$ and $e_B(\beta)$, are characterized by the first-order conditions

$$\alpha\mu'(e_A e_B)e_B\pi \;=\; c'(e_A) \tag{1}$$

$$\beta\mu'(e_A e_B)e_A\pi \;=\; c'(e_B). \tag{2}$$

It is straightforward to show that $de_i^*/d\pi > 0$, $i = A, B$. That is, a higher inside prospect π motivates both partners to exert more effort.

The division of joint profit, as reflected by α and β, affects the effort of each partner, and thus their expected utilities. The next lemma lays the foundation for our analysis by identifying the division of profits that maximizes total surplus, and the division preferred by an individual partner. For this we define $V \equiv U_A + U_B$ as the total surplus from joint production. Moreover, we denote α^{max} and β^{max} as the individually optimal profit shares for A and B, respectively.

Lemma 1 *The joint surplus V is maximized when $\alpha = \beta = 1/2$. By contrast, the individually optimal profit shares α^{max} and β^{max} satisfy $1/2 < \alpha^{max} = \beta^{max} < 1$.*

Figure 2 illustrates the utility-possibility frontier for joint production for different profit shares α and $\beta = 1 - \alpha$. The frontier is backward bending because every partner relies on the productive effort of his co-partner. If one partner exerts no effort (which occurs when $\alpha \in \{0, 1\}$), joint production never succeeds ($\mu(0) = 0$), and A and B both get a zero utility. We can also see from Figure 2 that the total surplus is maximized when each (symmetric) partner gets exactly half of the expected profit π ($\alpha = \beta = 1/2$). However, each partner prefers to get more than half of the expected profit (as $\alpha^{max} = \beta^{max} > 1/2$). We also note that any bargaining outcome is always located on the downward sloping part of the utility-possibility frontier. Thus, any equilibrium division of profit, denoted by α^* and $\beta^* = 1 - \alpha^*$, satisfies $\alpha^* \in [1 - \beta^{max}, \alpha^{max}]$.

Under joint asset ownership, shares are negotiated upfront. At date 0 the two partners A and B are perfectly symmetric. It is easy to see that the optimal ownership shares satisfy $\alpha^* = \beta^* = 1/2$; we formally prove this in the Appendix. Under individual asset ownership, we assume that profit shares are also negotiated upfront, but that they can be renegotiated at date 2.[17] Because of symmetry, equilibrium profit shares naturally satisfy $\alpha^* = \beta^* = 1/2$.

4.2 Symmetric Outside Options

Suppose that neither A nor B found an alternative partner at date 2, which occurs with probability $(1-q)^2$. Joint production is then the only option, regardless of the asset ownership structure. Because of symmetry, both partners share the profits equally: $\alpha^* = \beta^* = 1/2$.

Now suppose that A and B each found an alternative partner, which occurs with probability q^2. A and B can either stay together, or split in order to form new partnerships with C and D. If A and B decide to stay together, symmetry implies $\alpha^* = \beta^* = 1/2$. The expected utility of partner $i = A, B$ is then given by

$$U_i(\pi) = \frac{1}{2}\mu(e_A^* e_B^*)\pi - c(e_i^*). \tag{3}$$

We henceforth suppress the subscript of U whenever the expected utilities of A and B are identical. Alternatively, A and B can decide to match with C and D respectively. Under individual asset ownership they can do so directly; under joint asset ownership they first need to dissolve their partnership. Each of them bargains with his alternative partner, C or D, over the division of the expected profit σ from the new partnership. Recall that the alternative partners C and D both have zero outside options. The same applies to A and B during the bargaining. This is because once A and B approach their new partners, they expect to close a deal with that

[17] Because there is no production between the dates 0 and 2, we could also assume that profit shares are only negotiated at date 2. This alternative interpretation generates identical results.

new partner, and are therefore no longer available as a fall-back option.[18] The Nash bargaining solution then suggests that a partner gets exactly half of the profits from his new partnership. We denote the profit shares for A and B in their new partnerships by $\widehat{\alpha} = \widehat{\beta} = 1/2$.[19] The expected utility of partner $i = A, B$ then becomes

$$U(\sigma) = \frac{1}{2}\mu(e_i^* e_k^*)\sigma - c(e_i^*), \quad k \in \{C, D\}. \tag{4}$$

Using the Envelope Theorem one can show that $dU(\pi)/d\pi > 0$ and $dU(\sigma)/d\sigma > 0$. Moreover, we can infer from (3) and (4) that $U(\pi) = U(\sigma)$ when $\pi = \sigma$. Thus, A and B stay together as partners as long as $\pi \geq \sigma$. Otherwise they dissolve their partnership, and match with their alternative partners C and D. And because leaving is mutually beneficial for $\sigma > \pi$, it is irrelevant whether they agreed on individual or joint asset ownership at date 0. We summarize these observations in the following lemma:

Lemma 2 *Suppose A and B each found an alternative partner at date 2. They then stay together with $\alpha^* = \beta^* = 1/2$ (joint production) if $\pi \geq \sigma$; otherwise they dissolve their partnership.*

4.3 Asymmetric Outside Options

The most interesting scenario arises when only one partner, say A, found an alternative partner, C. This occurs with probability $q(1-q)$. The case where only B found an alternative partner is symmetric and also occurs with probability $q(1-q)$. The allocation of control rights over the two assets then matters as it defines A's freedom to leave the partnership with B. We discuss the implications of individual and joint asset ownership separately.

4.3.1 Individual Asset Ownership

Suppose A and B agreed on individual asset ownership at date 0, and only A found an alternative partner, C. Clearly, A can then unilaterally take his asset and form a new partnership with C without B's consent. In the bargaining game between A and C, A's outside option is to go back to B, who does not have an alternative partner to bargain with. According to the Hart and

[18] Technically, under the Hart and Mas-Colell protocol, there is an ε probability that the bargaining fails. Thus, with probability ε, A has the fall-back option of going back to B, and vice-versa. The Hart and Mas-Colell protocol then assumes that $\varepsilon \to 0$, implying that A's and B's outside options converge to zero.

[19] Throughout the paper we use an asterisk (*) to indicate equilibrium profit shares under joint production (A and B match); a hat ($\widehat{\ }$) indicates the equilibrium profit shares in alternative matches (either A and C match, or B and D match).

Mas-Colell bargaining protocol, this outside option would only be realized if the bargaining between A and C breaks down, so that A loses C as a potential trading partner. In this case, both A and B would have zero outside options, so that they split the equity in half. The outside option of A when bargaining with his alternative partner C is thus given by $U(\pi)$. Let $\widehat{\alpha}_I$ denote the equilibrium profit share for A, where the subscript 'I' indicates individual asset ownership. Using Nash bargaining, $\widehat{\alpha}_I$ maximizes $[U_A(\widehat{\alpha}_I;\sigma) - U(\pi)]^{1/2}[U_C(1-\widehat{\alpha}_I;\sigma)]^{1/2}$. It is easy to see that for any $U(\pi) > 0$ we have $\widehat{\alpha}_I \in (1/2, 1)$, with $d\widehat{\alpha}_I/d\pi \geq 0$. The expected utility of A then becomes

$$U_A(\widehat{\alpha}_I;\sigma) = \widehat{\alpha}_I \mu(\widehat{e}_A\widehat{e}_C)\sigma - c(\widehat{e}_A). \tag{5}$$

If A displaces B with C in this manner, B's expected utility becomes $U_B = 0$. This is clearly smaller than his expected utility $U_B(\pi)$ under joint production with A. Thus, A imposes a *displacement externality* on B when displacing him with the alternative partner C.

Displacing partner B for C may not always be in the best interest of A. He could also stay but use his better outside option to renegotiate a higher profit share, denoted by α_I^*. A would then only use his outside option of matching with C if the bargaining with B breaks down. Again, according to the Hart and Mas-Colell bargaining protocol, A would then lose B as a potential trading partner. In that case, A and C would both have zero outside options, and they would split the surplus in half. The outside option of A when renegotiating the profit shares with B is therefore given by $U(\sigma)$. Using Nash bargaining, A's new profit share α_I^* maximizes $[U_A(\alpha_I^*;\pi) - U(\sigma)]^{1/2}[U_B(1-\alpha_I^*;\pi)]^{1/2}$. The renegotiation at date 2 under individual asset ownership thus leads to the following expected utilities for A and B:

$$U_A(\alpha_I^*;\pi) = \alpha_I^*\mu(e_A^*e_B^*)\pi - c(e_A^*) \tag{6}$$

$$U_B(\alpha_I^*;\pi) = (1-\alpha_I^*)\mu(e_A^*e_B^*)\pi - c(e_B^*). \tag{7}$$

Again, for any $U(\sigma) > 0$ we have $\alpha_I^* \in (1/2, 1)$, with $d\alpha_I^*/d\sigma \geq 0$. Relative to the equal division of profits with $\alpha = \beta = 1/2$, this outcome is more favorable to A, and less favorable to B. Most importantly, it is jointly inefficient since joint surplus is maximized at $\alpha = \beta = 1/2$. And because both partners have zero wealth in the base model, it is also impossible for them to eliminate this inefficiency through side-payments.

Our concept of 'joint efficiency' looks at whether A and B maximize the sum of their ex-ante utilities. This differs from the concept of 'social efficiency', which would also take into account the utilities of the alternative partners C and D. We briefly discuss in Section 5.1 how our main results based on joint efficiency compare to social efficiency.

We can now identify the optimal choice of the partner with the better outside option between (i) switching to his alternative partner, and (ii), staying but negotiating a higher than equal profit share.

Lemma 3 *Consider individual asset ownership and suppose that A and B have asymmetric outside options. Then, there exists a threshold $\widehat{\sigma}_I(\pi) = \pi$ such that the partner with the better outside option leaves if $\sigma > \widehat{\sigma}_I(\pi)$. Otherwise, if $\sigma \leq \widehat{\sigma}_I(\pi)$, he stays but renegotiates his share on the joint profit π.*

Joint production between A and B is the outcome under individual asset ownership with asymmetric outside options, whenever the outside prospect σ is sufficiently low ($\sigma \leq \widehat{\sigma}_I(\pi)$). The partner with the better outside option then renegotiates the division of surplus, which is optimal from a selfish perspective but compromises the efficiency of joint production. Displacement, on the other hand, occurs whenever the alternative partnership is sufficiently attractive ($\sigma > \widehat{\sigma}_I(\pi)$). This imposes a displacement externality on the partner without outside option.

4.3.2 Joint Asset Ownership

Now consider joint asset ownership, where both partners share control rights over their two assets. This has two implications: First, joint ownership prevents Pareto-inefficient renegotiation between A and B, so that $\alpha^* = \beta^* = 1/2$ as long as they both prefer to stay together. Second, leaving requires the permission from the other partner, which will necessitate an appropriate compensation.

Suppose again that only A found an alternative partner (C) at date 2, and wants to leave B. Without adequate compensation B refuses to let go of A. The question is whether B can structure some deal to buy himself free. Because he has no wealth, A can offer B only a share on the future return σ from his new partnership with C. Productive effort is then only applied by A and C, so that B is a shareholder who does not add any value. We define $\widehat{\alpha}_J$ and $\widehat{\beta}_J$ as the equilibrium shares on the return σ for A and B, respectively. The equilibrium share for C is denoted by $\widehat{\gamma}_J$, where efficiency requires $\widehat{\gamma}_J = 1 - \widehat{\alpha}_J - \widehat{\beta}_J$. We provide a complete characterization of $\widehat{\alpha}_J$, $\widehat{\beta}_J$, and $\widehat{\gamma}_J$ in the Appendix, using the Maschler-Owen consistent NTU value. Leaving under joint asset ownership leads to the following expected utilities for A and B, assuming that only A found his alternative partner C:

$$U_A(\widehat{\alpha}_J; \sigma) = \widehat{\alpha}_J \mu(\widehat{e}_A \widehat{e}_C)\sigma - c(\widehat{e}_A) \tag{8}$$

$$U_B(\widehat{\beta}_J; \sigma) = \widehat{\beta}_J \mu(\widehat{e}_A \widehat{e}_C)\sigma. \tag{9}$$

We can now characterize the equilibrium outcome under joint asset ownership with asymmetric outside options.

Lemma 4 *Consider joint asset ownership and suppose that A and B have asymmetric outside options. Then, there exists a threshold $\widehat{\sigma}_J(\pi)$, with $\widehat{\sigma}_J(\pi) > \pi$, such that the partner with the better outside option leaves with consent if $\sigma > \widehat{\sigma}_J(\pi)$. Otherwise, if $\sigma \leq \widehat{\sigma}_J(\pi)$, he stays with $\alpha^* = \beta^* = 1/2$.*

Joint production between A and B is the equilibrium outcome as long as the outside prospect σ is sufficiently low ($\sigma \leq \widehat{\sigma}_J(\pi)$). Both partners then split everything in half ($\alpha^* = \beta^* = 1/2$), so that total surplus is maximized. In contrast, both partners agree to break up whenever the alternative partnership is attractive enough ($\sigma > \widehat{\sigma}_J(\pi)$). The partner without outside option then gets a stake in the new partnership in exchange for relinquishing his control rights over the two assets.

Overall we note that while joint asset ownership prevents inefficient displacement, it can lead to a retention externality. This occurs whenever the partner without outside option refuses to let go of the partner with outside option, even if leaving maximizes their joint utility. Renegotiation requires that the productive partner, say A, gives up part of the future returns from his joint production with C, in order to compensate B. This buyout arrangement impairs effort incentives, and thus lowers the expected payoff of A's partnership with C. Thus, the profit share $\widehat{\beta}_J$ offered to B may not suffice to buy his consent, so that A is forced to stay despite leaving being jointly efficient. And because both partners have zero wealth, A cannot eliminate this inefficiency by making a direct payment to B.

5 Optimal Asset Ownership

In this section we identify the partners' optimal decision at date 0 whether to retain full control over their assets (individual asset ownership), or to share their control rights (joint asset ownership). We first stick to our base model where the inside prospect π remains constant between the dates 0 and 2. We then relax this assumption and allow for some learning at date 2.

5.1 Constant Inside Prospect

Suppose the inside prospect π does not change between the dates 0 and 2. We first state the expected utility functions under the two ownership structures. Consider individual asset ownership. For convenience we adjust our notation for the asymmetric case as follows: For joint production, we now denote the profit share of the partner with the outside option by α_I^+, and the profit share of the partner without an outside option by α_I^-, where $\alpha_I^- = 1 - \alpha_I^+$. In case the partner with the outside option leaves, his equilibrium profit share is then denoted by $\widehat{\alpha}_I$. The expected utility of a partner at date 0 is thus given by:

$$EU_I(\pi,\sigma) = q^2 \max\{U(\pi), U(\sigma)\} + (1-q)^2 U(\pi) + q(1-q)V_I(\pi,\sigma), \tag{10}$$

where

$$V_I(\pi,\sigma) = \begin{cases} U(\widehat{\alpha}_I;\sigma) & \text{if } \sigma > \widehat{\sigma}_I(\pi) \\ U(\alpha_I^+;\pi) + U(\alpha_I^-;\pi) & \text{if } \sigma \leq \widehat{\sigma}_I(\pi). \end{cases}$$

Note that $V_I(\pi,\sigma)$ is the total expected utility of a partner in case of asymmetric outside options. Due to symmetry, $V_I(\pi,\sigma)$ is also the joint surplus when the two partners have asymmetric outside options.

Now consider joint asset ownership. For convenience we adopt the following notation for the asymmetric case: When the original partnership is dissolved, the partner with the outside option gets the profit share $\widehat{\alpha}_J$ in the new partnership, while the partner without an outside option gets $\widehat{\beta}_J$ as compensation. The expected utility of a partner at date 0 is then given by

$$EU_J(\pi,\sigma) = q^2 \max\{U(\pi), U(\sigma)\} + (1-q)^2 U(\pi) + q(1-q)V_J(\pi,\sigma), \tag{11}$$

where

$$V_J(\pi,\sigma) = \begin{cases} U(\widehat{\alpha}_J;\sigma) + U(\widehat{\beta}_J;\sigma) & \text{if } \sigma > \widehat{\sigma}_J(\pi) \\ 2U(\pi) & \text{if } \sigma \leq \widehat{\sigma}_J(\pi) \end{cases}$$

is the total expected utility of a partner in case of asymmetric outside options.

When comparing the expected utilities $EU_I(\cdot)$ and $EU_J(\cdot)$ under individual and joint asset ownership, we immediately see that the expected payoffs are the same in case of symmetric outside options. In equilibrium there is joint production with an equal split of profits, unless both partners strictly benefit from dissolving their partnership (case of two alternative partners and $\sigma > \pi$).

For asymmetric outside options, however, the allocation of control rights matters. Individual ownership provides flexibility to dissolve an inefficient partnership, but can also lead to an inefficient partner displacement. Joint ownership, on the other hand, protects a partner without outside option from opportunism, but hampers the dissolution of inefficient partnerships, thus creating a retention externality.

The next proposition identifies the optimal asset ownership that the two partners agree on at date 0.

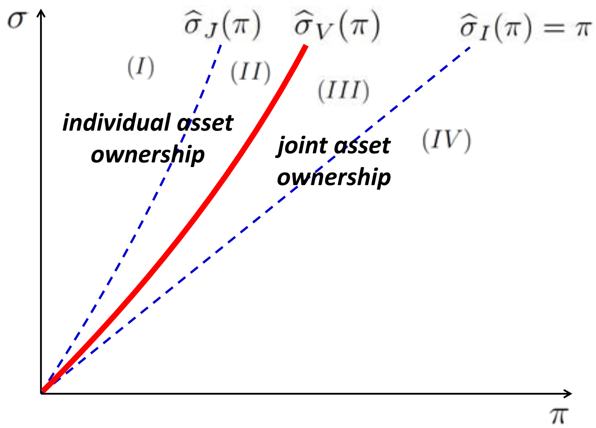

Figure 3: Optimal Asset Ownership

Proposition 1 *For a given inside prospect π,- there exists a threshold $\widehat{\sigma}_V(\pi)$ such that the two partners A and B choose joint asset ownership at date 0 if $\sigma \leq \widehat{\sigma}_V(\pi)$, and individual asset ownership if $\sigma > \widehat{\sigma}_V(\pi)$. The threshold $\widehat{\sigma}_V(\pi)$ satisfies $\widehat{\sigma}_I(\pi) < \widehat{\sigma}_V(\pi) < \widehat{\sigma}_J(\pi)$, and is increasing in π.*

Figure 3 illustrates the insights from Proposition 1. As long as the outside prospect σ is sufficiently low ($\sigma \leq \widehat{\sigma}_V(\pi)$), the two partners choose joint asset ownership at date 0. Otherwise they prefer individual asset ownership in order to retain the flexibility to switch to a superior outside partner ($\sigma > \widehat{\sigma}_V(\pi)$).[20]

The rationale behind Proposition 1 becomes clear when comparing the outcomes under joint and individual asset ownership for the different parameter regions in Figure 3. This figure presumes that A and B have asymmetric outside options. Region (I) contains the highest outside prospect σ (on the vertical axis), relative to the inside prospect π (on the horizontal axis). Under individual asset ownership, the partner with outside option would always leave, which is jointly efficient. Under joint asset ownership, however, the partner with outside option has to buy himself free, which compromises effort incentives in his new partnership. Region (II) still contains a high outside prospect, so that leaving under individual asset ownership is jointly efficient. Under joint asset ownership, however, there are not enough expected profits with the outside prospect to warrant a buyout. As a consequence the partner without outside option inefficiently retains the other. Thus, regions (I) and (II) both suffer from a retention

[20]Recall that only the ex-post inefficiencies associated with asymmetric outside options matter for the optimal ownership structure. Ex-ante A and B know that they will have asymmetric outside options at date 2 with probability $2q(1-q)$. Thus, asset ownership matters as long $0 < q < 1$, so that outside options will be asymmetric with a strictly positive probability. However, the specific value of q is then irrelevant (as long as $0 < q < 1$), because both A and B have the same chance of finding an alternative partner.

externality under joint asset ownership. The difference is that in region (II) inefficient retention occurs in equilibrium, whereas in region (I) it leads to renegotiation that ultimately results in an inefficient buyout. Obviously, these retention inefficiencies can be avoided with individual asset ownership.

Regions (III) and (IV) both have a sufficiently low outside prospect so that A and B's joint utility is maximized when they remain together. Under individual asset ownership there is a displacement externality. In region (III), the outside prospect is sufficiently attractive so that the partner with outside option simply leaves without renegotiation. In region (IV), the partner with outside option merely uses his opportunity to switch as a bargaining chip. Both of these outcomes are ex-post inefficient from a joint perspective. These inefficiencies can be avoided with joint asset ownership, where the two partners always remain together without renegotiation.

The analysis focuses on maximizing joint efficiency, which is precisely what the two partners try to achieve. However, one may also ask how the outcomes rank in term of social efficiency, when also accounting for the utilities of alternative trading partners. Because of the moral-hazard-in-teams problem at date 3, we note that there will always be deviations from the first-best outcome. Let us therefore accept that any outcome has some inefficient effort incentives, and focus instead on the question of socially efficient partner choices. In regions (I) and (II) of Figure 3, the alternative partnership is more efficient than the original one, so switching is socially efficient. Recall that the original partners (A and B) also prefer individual asset ownership in these two regions, which results in partner switching. Thus, total surplus is also maximized in regions (I) and (II). In region (IV) the alternative partnership is less efficient, and A and B prefer joint asset ownership. Partner switching does then not occur in equilibrium, so that the outcome is again socially efficient.

The interesting case concerns region (III) where $\sigma \in (\pi, \widehat{\sigma}_J)$. Since $\sigma > \pi$, the alternative partnership is more efficient than the original one, so that switching is socially efficient. This would occur in equilibrium under individual asset ownership. However, A and B actually prefer joint asset ownership in this region, and they stay together. In region (III) we therefore find a divergence between the jointly optimal partner choice of staying together, versus the socially efficient choice of switching partners.

5.2 Learning about the Inside Prospect

So far we considered a simple setting where the inside prospect π remained constant between date 0 and date 2. We now extend our base model by allowing for some learning about the distribution of potential profits y. This introduces the notion that A and B learn not only about possible outside opportunities, but also about the strength of their existing relationship. We consider the simplest possible extension where at date 2 the expected profit (or insight prospect) π can take on two values: $\pi \in \{\pi_L, \pi_H\}$, with $\pi_H > \pi_L > 0$. The inside prospect π will be high with probability p ($\pi = \pi_H$), and low with probability $1 - p$ ($\pi = \pi_L$). We treat the probability p as exogenous for now, but endogenize it in Section 6.[21]

For our subsequent analysis it is useful to express the condition from Proposition 1 in terms of the inside prospect π: With asymmetric outside options, dissolving the partnership between A and B is jointly efficient if $\pi < \widehat{\pi}_V(\sigma)$; otherwise joint production with an equal split of profits maximizes joint surplus. We then need to consider three scenarios: (i) $\widehat{\pi}_V(\sigma) \leq \pi_L < \pi_H$, (ii) $\pi_L < \pi_H < \widehat{\pi}_V(\sigma)$, and (iii), $\pi_L < \widehat{\pi}_V(\sigma) \leq \pi_H$. In scenario (i) joint production with an equal split of profits is always jointly optimal; A and B therefore choose joint asset ownership at date 0. In scenario (ii) dissolving the partnership is efficient as soon as either A or B (or both) found an alternative partner; A and B thus choose individual asset ownership at date 0 to facilitate efficient leaving. In scenario (iii) individual asset ownership is optimal if the inside prospect π turns out to be low ($\pi = \pi_L < \widehat{\pi}_V(\sigma)$), but joint asset ownership is optimal if the inside prospect turns out to be high ($\pi = \pi_H \geq \widehat{\pi}_V(\sigma)$). From now on we focus on the most interesting third scenario where $\pi_L < \widehat{\pi}_V(\sigma) \leq \pi_H$.

If the internal learning process is based on verifiable signals so that an ex-ante contract can distinguish between $\pi = \pi_L$ and $\pi = \pi_H$, then a contingent ownership structure is optimal. Specifically, A and B would write a contingent contract which stipulates individual asset ownership at date 2 whenever $\pi = \pi_L < \widehat{\pi}_V(\sigma)$, and joint asset ownership whenever $\pi = \pi_H \geq \widehat{\pi}_V(\sigma)$.[22]

The assumption that the inside prospect $\pi \in \{\pi_L, \pi_H\}$ is verifiable is arguably too strong. Assuming that π is non-verifiable seems more plausible, given that learning about the inside prospect is specific to the collaboration of the two partners. It is not based on objective past performance, but instead, is based on expectations about the benefits of a future joint production. Obviously there is also the theoretical possibility of using mechanism design ex-ante, which might allow the two partners to reveal the observable state through a cleverly designed revelation game, in the spirit of Moore and Repullo (1988) or Maskin and Tirole (1999). We do not pursue this approach for several reasons. From an applied perspective, these mechanisms seem somewhat remote from the contracts used in the 'real world'. From a purely theoretical

[21] We assume that there is no updating at date 1 and date 3. Allowing for further updating would considerably complicate the analysis without generating much additional insight.

[22] Note that such a contingent contract does not require the outside prospect σ to be verifiable at date 2. As shown, the ownership structure only matters in case of asymmetric outside options. Depending on the inside prospect π, the optimal contingent contract stipulates the allocation of control rights which then maximizes the joint surplus.

perspective, the mechanism of Maskin and Tirole relies on risk-aversion and the ability to implement large punishments. In our model we have risk-neutral players. Most importantly, we have a wealth constraint that prevents large punishments that are central to these implementation games. Moreover, the recent work by Aghion et al. (2012) shows that subgame perfect implementation games are not robust to even small deviations from the common knowledge assumption.

For the remainder of this paper we assume that the inside prospect $\pi \in \{\pi_L, \pi_H\}$ is observable but not verifiable at date 2. Without verifiability of π at date 2, partners cannot rely on contingent contracts. This implies that the ex-ante decision over asset ownership now involves a trade-off between the flexibility of individual asset ownership versus the commitment value of joint asset ownership.

The expected utility of a partner at date 0 under individual asset ownership ($EU_I(p)$) and joint asset ownership ($EU_J(p)$) is given by

$$EU_k(p) = pEU_k(\pi_H, \sigma) + (1-p)EU_k(\pi_L, \sigma), \quad k = I, J$$

where $EU_I(\pi, \sigma)$ and $EU_J(\pi, \sigma)$ are defined by (10) and (11), respectively. Clearly, both partners A and B agree on joint asset ownership at date 0 whenever $EU_J(p) \geq EU_I(p)$. For a comparison we note that $\Delta_H \equiv V_J(\pi_H, \sigma) - V_I(\pi_H, \sigma) > 0$ and $\Delta_L \equiv V_J(\pi_L, \sigma) - V_I(\pi_L, \sigma) < 0$ for $\pi_L < \widehat{\pi}_V(\sigma) \leq \pi_H$. Simple algebra leads to the following proposition.

Proposition 2 *Define*

$$\widehat{p} = \frac{-\Delta_L}{-\Delta_L + \Delta_H}.$$

Then, A and B choose joint asset ownership at date 0 whenever $p \geq \widehat{p}$; otherwise they choose individual asset ownership.

The intuition behind Proposition 2 is that both partners choose joint asset ownership if the inside prospect π is likely to be high ($p \geq \widehat{p}$), because preserving the partnership is likely to be valuable. Otherwise they choose individual asset ownership in order to retain the flexibility to dissolve a likely inefficient partnership ($p < \widehat{p}$). Thus, the threshold $\widehat{p}$ balances (i) the risk of preserving inefficient partnerships (joint ownership with $\pi = \pi_L$), and (ii), the risk of compromising otherwise efficient partnerships (individual ownership with $\pi = \pi_H$).

In the model with non-verifiable learning about the inside prospect, the ex-ante optimal allocation of control rights can lead to ex-post inefficiencies. Under individual asset ownership there can be a displacement inefficiency, associated with regions (III) and (IV) in Figure 3. Under joint ownership there can be a retention inefficiency, associated with regions (I) and (II).

6 Relation-specific Investments

In this section we extend our model by allowing both partners A and B to make relation-specific investments at date 1. We then examine how the allocation of control rights over critical assets affects the partners' incentives to invest in their relationship.

Suppose that A and B can invest in their relationship at date 1 to improve the distribution of potential profits y. For simplicity we focus on improvements that make a high inside prospect π_H more likely. Specifically we assume that $p = p(r_A, r_B)$, where p is concave increasing in the partners' relation-specific investments r_A and r_B. Specific investments are non-contractible, and impose convex private costs $\psi(r_i)$, $i = A, B$, with $\psi(0) = \psi'(0) = 0$. To ensure interior solutions we assume that $p(0,0) = 0$ and $\partial p(\cdot)/\partial r_i|_{r_i=0} = \infty$, $i = A, B$. We also assume that the cross-partial is not too negative: $\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa$, where $\kappa > 0$. This ensures that the reaction functions of both partners are well-behaved.[23]

Consider individual asset ownership. At date 1 partner $i = A, B$ chooses his specific investment r_i to maximize his expected utility:[24]

$$\begin{aligned} EU_I(r_i, r_j) &= p(r_i, r_j)\left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)V_I(\pi_H, \sigma)\right] \\ &\quad + (1 - p(r_i, r_j))\left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_I(\pi_L, \sigma)\right] - \psi(r_i), \end{aligned}$$

where $j \in \{A, B\}$ and $j \neq i$. The equilibrium investment levels $r^*_{A(I)}$ and $r^*_{B(I)}$ under individual asset ownership are then characterized by the first-order conditions:

$$\frac{\partial p(r_A, r_B)}{\partial r_i}\Phi_I = \psi'(r_i), \quad i = A, B,$$

where

$$\begin{aligned} \Phi_I &= \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)V_I(\pi_H, \sigma)\right] \\ &\quad - \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_I(\pi_L, \sigma)\right]. \end{aligned}$$

Because A and B are symmetric at date 1, their investment levels $r^*_{A(I)}$ and $r^*_{B(I)}$ must be also symmetric in equilibrium. We define $r^*_I \equiv r^*_{A(I)} = r^*_{B(I)}$ as the equilibrium relation-specific investment of a partner under individual asset ownership.

[23] A sufficient and intuitive assumption is that specific investments r_A and r_B are (weak) strategic complements, so that $\partial^2 p(\cdot)/(\partial r_A \partial r_B) \geq 0$.

[24] Note that $U(\sigma) > U(\pi_L)$ when A and B each found an alternative partner; thus, $\max\{U(\pi_L), U(\sigma)\} = U(\sigma)$.

Now consider joint asset ownership. The expected utility of partner $i = A, B$ at date 1 is then given by

$$\begin{aligned} EU_J(p_i, p_j) &= p(r_i, r_j)\left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)V_J(\pi_H, \sigma)\right] \\ &\quad + (1 - p(r_i, r_j))\left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_J(\pi_L, \sigma)\right] - \psi(r_i). \end{aligned}$$

The following first-order conditions define the equilibrium investment levels $r^*_{A(J)}$ and $r^*_{B(J)}$ under joint asset ownership:

$$\frac{\partial p(r_A, r_B)}{\partial r_i}\Phi_J = \psi'(r_i) \quad i = A, B,$$

where

$$\begin{aligned} \Phi_J &= \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)V_J(\pi_H, \sigma)\right] \\ &\quad - \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_J(\pi_L, \sigma)\right]. \end{aligned}$$

Again, the Nash equilibrium is symmetric; we thus define $r^*_J \equiv r^*_{A(J)} = r^*_{B(J)}$ as the equilibrium relation-specific investment of a partner under joint asset ownership.

The next proposition compares the partners' incentives to make specific investments under individual versus joint asset ownership.

Proposition 3 *Joint asset ownership provides greater incentives for relation-specific investments, i.e.,* $r^*_J > r^*_I$.

The key intuition for this result is as follows. Incentives for specific investments stem from the difference in the utility levels associated with a low versus high inside prospect. With joint asset ownership, the partner combination is always efficient when the inside prospect is high, but leads to inefficient retention when the inside prospect is low. The latter inefficiency of joint asset ownership widens the difference in utilities between a low and a high inside prospect. With individual asset ownership, the partner combination is always efficient when the inside prospect is low, but causes displacement problems when the inside prospect is high. The latter inefficiency of individual asset ownership narrows the difference in utilities between a low and a high inside prospect. We therefore find that joint asset ownership provides stronger incentives for specific investments, precisely because the inefficiency then arises when the partners have failed to develop a strong internal relationship.

7 The Role of Wealth

An important feature of our model is that partners define ownership ex-ante, knowing that there will be some ex-post inefficiencies. And partners cannot smooth out these ex-post inefficiencies because of our assumption that they are perfectly wealth-constrained. We now augment our analysis by allowing both partners A and B to have some initial wealth $w \equiv w_A = w_B > 0$. In Section 7.1 we examine how the initial wealth w affects the partners' payoffs under individual versus joint asset ownership (date 2). In Section 7.2 we derive the effect of wealth on partners' incentives to invest in their relationship (date 1). In Section 7.3 we characterize the optimal asset allocation for different levels of wealth (date 0), as well as the optimal level of wealth that partners would want to commit to. Section 7.4 briefly discusses alternative sources of wealth for transfers.

7.1 Renegotiation among Wealthy Partners

Consider date 2. We showed in Section 4.2 that the joint surplus $V = U_A + U_B$ is always maximized when A and B have symmetric outside options. The presence of initial wealth w is then irrelevant for joint efficiency. It is only if the partners have asymmetric outside options that their initial wealth matters. With individual asset ownership, there can be a displacement externality when the inside prospect is high ($\pi = \pi_H$). And with joint asset ownership, there can be a retention externality when the inside prospect is low ($\pi = \pi_L$). The question is whether and how initial wealth affects these inefficient ex-post outcomes.

It is easy to see that with unlimited wealth all the ex-post inefficiencies can be completely eliminated. In the Appendix we characterize the minimum amount of wealth that is required to fully eliminate any ex-post inefficiencies (see Proof of Proposition 4). We denote this amount by $\overline{w}_I$ for individual asset ownership, and by $\overline{w}_J$ for joint asset ownership. It is easy to show that for any $w \geq \max\{\overline{w}_I, \overline{w}_J\}$ asset ownership is irrelevant; this is because the equilibrium outcomes are always the same. Moreover, from an ex-ante perspective, the expected transfer payments always cancel out each other because of symmetry.

Next we ask what minimum wealth is required to affect the renegotiation outcomes? Consider first individual asset ownership. We define $\underline{w}_I$ as the lower bound above which wealth actually changes the renegotiation outcome. Using the insights from Lemma 3 we can characterize $\underline{w}_I$ as follows:[25]

Lemma 5 *Consider individual asset ownership with a high inside prospect ($\pi = \pi_H$).*

[25] The threshold $\widehat{\pi}_I(\sigma)$ in the next lemma follows from expressing the condition in Lemma 3 in terms of the inside prospect π.

(*i*) *For* $\pi_H < \widehat{\pi}_I(\sigma)$*, renegotiation without wealth leads to inefficient displacement. In this case* $\underline{w}_I > 0$*, so that a small amount of wealth does not affect the renegotiation outcome at all.*

(*ii*) *For* $\pi_H \geq \widehat{\pi}_I(\sigma)$*, renegotiation without wealth leads to no displacement but unequal profit shares. In this case* $\underline{w}_I = 0$*, so that any small amount of wealth changes the renegotiation outcome.*

Likewise, we define $\underline{w}_J$ as the minimum amount of wealth that is required under joint asset ownership to change the renegotiation outcome. Using Lemma 4 we get the following characterization of $\underline{w}_J$:[26]

Lemma 6 *Consider joint asset ownership with a low inside prospect (*$\pi = \pi_L$*).*

(*i*) *For* $\pi_L > \widehat{\pi}_J(\sigma)$*, renegotiation without wealth leads to inefficient retention. In this case* $\underline{w}_J > 0$*, so that a small amount of wealth does not affect the renegotiation outcome at all.*

(*ii*) *For* $\pi_L \leq \widehat{\pi}_J(\sigma)$*, renegotiation without wealth leads to an inefficient buyout. In this case* $\underline{w}_J = 0$*, so that any small amount of wealth changes the renegotiation outcome.*

The two lemmas generate the surprising insight that sometimes having a small amount of wealth makes no difference at all. In fact, this is true whenever renegotiation does not occur in the absence of any wealth, corresponding to regions (II) and (III) in Figure 3. To get the intuition, consider the case of displacement under individual asset ownership, represented by region (III) in Figure 3. The partner with the outside option, say A, has a strict preference for working with C, rather than accepting B's most generous retention offer ($\alpha = \alpha^{max}$). Sweetening the retention offer with a small transfer payment is therefore not enough to win over A. However, if renegotiation occurs in the absence of wealth (as in region (IV)), then we have a problem of unbalanced incentives. B can then use his wealth to pay A in order to retain a greater profit share. This in turn improves the incentive balance. The intuition is similar for joint asset ownership. In the case where retention occurs in equilibrium (region (II) in Figure 3), a small amount of wealth is not enough for A to persuade B to let him go. However, if renegotiation results in a buyout (region (I)), then A uses his wealth to make a transfer payment to B, thereby preserving his profit share in the new partnership with C.

[26] Note that the threshold $\widehat{\pi}_J(\sigma)$ follows directly from the condition in Lemma 4.

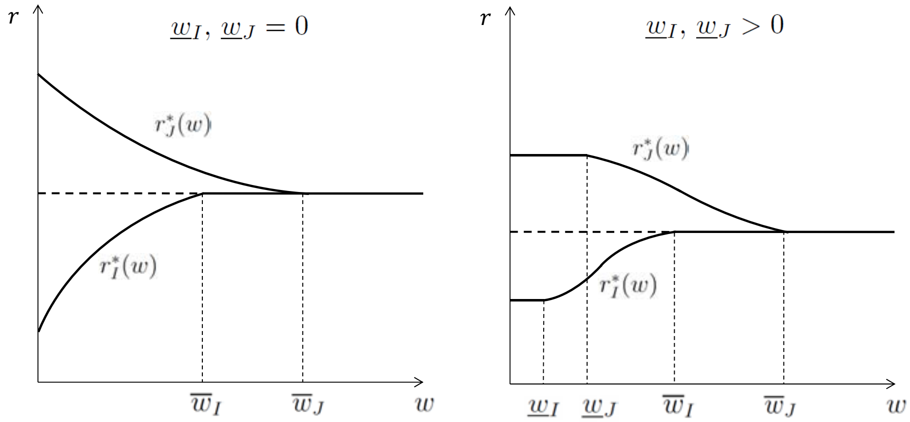

Figure 4: Wealth and Relation-specific Investments

7.2 Wealth and Specific Investments

The previous section shows that for $w \in (\underline{w}_I, \overline{w}_I)$ and/or $w \in (\underline{w}_J, \overline{w}_J)$ wealth can be used to obtain a more efficient renegotiation outcome ex-post. In a model without specific investments, more wealth always improves the joint efficiency. The interesting question is whether this result carries forward to the model with specific investments. For this we first examine how wealth affects the partners' incentives to make relation-specific investments at date 1.

Proposition 4

(i) *Under individual asset ownership, relation-specific investments r_I^* are increasing in the partners' wealth w. Formally, $dr_I^*(w)/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$.*

(ii) *Under joint asset ownership, relation-specific investments r_J^* are decreasing in w. Formally, $dr_J^*(w)/dw < 0$ for $\underline{w}_J \leq w < \overline{w}_J$.*

(iii) *For sufficiently high wealth w, relation-specific investments are identical and constant under individual and joint asset ownership. Formally, $r_I^*(w) = r_J^*(w)$ for all $w \geq \max\{\overline{w}_I, \overline{w}_J\}$.*

Figure 4 illustrates the insights from Proposition 4. If both partners have zero wealth, we know from Lemma 3 that joint control over critical assets provides stronger incentives to invest in the relationship ($r_J^*(0) > r_I^*(0)$). The question is what happens to specific investments when both partners have some initial wealth $w > 0$? Under individual asset ownership, wealth allows the partners to smooth out ex-post inefficiencies in the good state $\pi = \pi_H$. This improves the marginal benefit of specific investments, so that $r_I^*(w)$ is increasing in w. Under joint asset ownership, wealth helps the partners to correct ex-post inefficiencies in the bad state $\pi = \pi_L$. This makes the difference between the bad and the good state relatively smaller, and therefore compromises the partners' incentives to make relation-specific investments. Thus, $r_J^*(w)$ decreases in w.

The partners can eliminate all ex-post inefficiencies for sufficiently high wealth levels ($w \geq \max\{\overline{w}_I, \overline{w}_J\}$). That is, with enough wealth, they can always dissolve their partnership in the bad state (π_L), so that $V(\pi_L) = U(\sigma)$; and they can always agree on staying together with an equal split of profits in the good state (π_H), so that $V(\pi_H) = 2U(\pi)$. The allocation of control rights is then irrelevant, and the marginal incentives for specific investments are the same. This explains why $r_I^*(w) = r_J^*(w)$ for $w \geq \max\{\overline{w}_I, \overline{w}_J\}$.

Figure 4 provides another interesting insight: Relation-specific investments are maximized under joint asset ownership with zero wealth. This is an important and surprising result. Under joint asset ownership, wealth allows the partners to mitigate ex-post inefficiencies in the bad state $\pi = \pi_L$. However, this compromises the marginal benefit of specific investments at date 1. Thus, incentives for specific investments are maximized under joint asset ownership when partners have no wealth at all.

7.3 Optimal Asset Ownership with Wealth

The previous two sections identified two distinct facets of wealth. On the one hand, having wealth allows the partners to mitigate potential inefficiencies arising from asymmetric outside options; and doing so is always optimal ex-post. On the other hand, having wealth affects the partners' incentives for relation-specific investments. While more wealth leads to more specific investments under individual asset ownership, it compromises incentives under joint asset ownership. We now complete our analysis by asking two related questions. First we ask how the optimal asset ownership decision depends on the initial level of wealth. Second we ask whether partners would want to commit ex-ante to limit the use of their wealth ex-post.

To answer these questions we first consider individual asset ownership, and examine how the presence of wealth affects the partners' expected utilities at date 0.[27]

Lemma 7 *Under individual asset ownership, the expected utility of a partner at date* 0*, denoted by* $EU_I(w)$*, has three distinct segments:*

(i) *For* $w < \underline{w}_I$*,* $EU_I(w)$ *is constant in* w*.*

(ii) *For* $\underline{w}_I \leq w < \overline{w}_I$*,* $EU_I(w)$ *is strictly increasing in* w*.*

[27]The expected utility is obviously increasing in wealth itself, so we focus on the expected utility from the productive activities, net of initial wealth. This expected utility still depends on wealth, since wealth affects both incentives and ex-post payoffs (asymmetric outside options).

(iii) *For $w \geq \overline{w}_I$, $EU_I(w)$ is constant in w.*

Under individual asset ownership the ex-post efficiency effect of wealth and the incentive effect of wealth both go in the same direction. More wealth improves the renegotiation outcome. Because the inefficiencies are associated with a high inside prospect, this also improves ex-post incentives. The expected utility $EU_I(w)$ is therefore increasing in wealth w in the range $w \in [\underline{w}_I, \overline{w}_I)$, and constant everywhere else.

We now turn to joint asset ownership. For the next lemma we define w_J^* as the wealth level which maximizes the expected utility of a partner at date 0.

Lemma 8 *Under joint asset ownership, the expected utility of a partner at date 0, denoted by $EU_J(w)$, has the following properties:*

(i) *For $w < \underline{w}_J$, $EU_J(w)$ is constant in w.*

(ii) *For $\underline{w}_J \leq w < \overline{w}_J$, there exists a threshold $\widehat{\pi}_H$ such that $w_J^* = \underline{w}_J$ for all $\pi_H \geq \widehat{\pi}_H$, and $w_J^* \in (\underline{w}_J, \overline{w}_J)$ for all $\pi_H < \widehat{\pi}_H$.*

If $w_J^* = \underline{w}_J$, then $EU_J(w)$ is strictly decreasing in w for $w \in [\underline{w}_J, \overline{w}_J)$.

If $w_J^* > \underline{w}_J$, then $EU_J(w)$ is strictly increasing in w for $w \in [\underline{w}_J, w_J^*)$, and strictly decreasing in w for $w \in (w_J^*, \overline{w}_J)$.

(iii) *For $w \geq \overline{w}_J$, $EU_J(w)$ is constant in w.*

Lemma 8 shows that a partner's expected utility under joint asset ownership is not necessarily monotone in wealth. This is because wealth has two opposite effects: It allows two partners with asymmetric outside options to improve their ex-post payoffs in the bad state $\pi = \pi_L$. However, this concurrently compromises the partners' ex-ante incentives to invest in their relationship (see Proposition 4). Which effect dominates then depends on the importance of relation-specific investments, as reflected by the parameter π_H. For sufficiently high values of π_H ($\pi_H \geq \widehat{\pi}_H$), the incentive effect always dominates. In this case the expected utility $EU_J(w)$ is decreasing in w, and reaches its maximum at zero wealth.[28] For lower values of π_H ($\pi_H < \widehat{\pi}_H$), the incentive effect does not always dominate. In the Appendix we show that the expected utility $EU_J(w)$ then first increases in wealth, and then decreases.

The key insight from Lemmas 7 and 8 is that the effect of wealth on the partners' expected utilities depends on assets ownership. Under individual asset ownership, having wealth (up to $\overline{w}_I$) is always good, because it mitigates ex-post inefficiencies as well as improves incentives

[28] If $\underline{w}_J > 0$, there is a range $[0, \underline{w}_J]$ where $EU_J(w)$ is maximized.

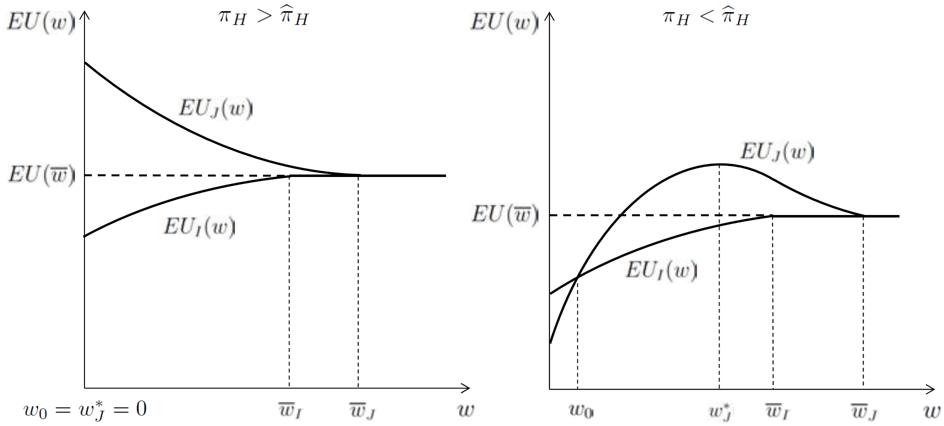

Figure 5: Wealth and Expected Utility under Individual and Joint Asset Ownership

for specific investments. Under joint asset ownership, however, having more wealth reduces specific investments ex-ante.

We now identify a condition so that partners prefer joint asset ownership at date 0. For parsimony we define $\overline{w} \equiv \max\{\overline{w}_I, \overline{w}_J\}$.

Proposition 5 *There always exists a critical wealth level w_0, with $w_0 \in [0, w_J^*]$, such that the partners strictly prefer joint over individual asset ownership for all $w \in (w_0, \overline{w})$.*

Figure 5 compares the expected utility levels under individual versus joint asset ownership, using the insights from Lemmas 7 and 8, and Proposition 5. Consider first the left graph where $\pi_H > \widehat{\pi}_H$. If the partners have sufficient wealth ($w \geq \overline{w}$), they can eliminate all ex-post inefficiencies in case of asymmetric outside options. The specific ownership structure is then irrelevant (i.e., $EU_I(w) = EU_J(w)$ for $w \geq \overline{w}$). For $w < \overline{w}$, however, there is a divergence between individual and joint asset ownership. In fact, there always exists a region where joint asset ownership is preferred to individual asset ownership. This region extends all the way down to w_0. In some cases we have $w_0 = 0$, so that joint asset ownership is optimal for all levels of wealth. In other cases we have $w_0 > 0$, so that joint asset ownership is only optimal for intermediate levels of wealth; see the right graph of Figure 5.

Our final proposition considers what wealthy partners would like to do if they can commit ex-ante to a wealth constraint ex-post, i.e., if they can commit to limiting the amounts that can be used for ex-post transfer payments.

Proposition 6 *If partners with unlimited wealth can commit to limiting the wealth available for ex-post transfer payments, then they always choose joint asset ownership and commit to being wealth constrained at $w = w_J^*$.*

This proposition follows immediately from the above, and can be seen directly off Figure 5. The maximum of the expected utilities, $EU^{\max} = \max\{EU_I(w), EU_J(w)\}$, is always reached at $EU_J(w_J^*)$. This implies that the combination of joint asset ownership with a wealth constraint at w_J^* achieves the best trade-off between ex-ante incentives for specific investments and ex-post efficiency. Interestingly, in the case of $\pi_H \geq \widehat{\pi}_H$, we even have $w_J^* = 0$. The optimal arrangement for wealthy partners is then joint asset ownership with the commitment to a zero wealth constraint ex-post.

7.4 Alternative Sources of Wealth

Our analysis showed the various effects of initial wealth on specific investments and ex-post payoffs. We now briefly discuss the use of alternative sources of wealth that can be used for transfer payments.

First we note that wealth is only required at date 2. For simplicity there is no production between dates 0 and 2. If we allowed for it, any surplus (losses) generated between date 0 and date 2 would need to be added (subtracted) to our measure of wealth w.

A second source of wealth concerns a potential liquidation value of the assets themselves. In our model partners cannot liquidate their assets at date 2. However, it is conceivable that the assets have some residual value at date 4, even in the case of failure. The liquidation value of the assets can then be harnessed at date 2 by obtaining some securitized debt financing, where an outside investor receives a fixed and safe claim on the liquidation value. In the Appendix we briefly outline how the partners need to structure such debt claims; see also Hellmann and Thiele (2012) for a more extensive discussion in a related model.

One may also ask whether, over and beyond any securitized liquidation value, it is possible to relax the wealth constraint by bringing in outside investors? The answer is no! Outside investors cannot fix the partners' wealth constraint, because they themselves have to take profit share in return for their investments. These profit shares generate a distortion that is very similar to the very distortions they are trying to solve. To see this, suppose an outside investor provides funding in exchange for a profit share τ. Under individual asset ownership, outside capital can be used to retain the partner with the better outside option. However, giving the outside investor a profit share $\tau > 0$ weakens the partners' effort incentives (as $\alpha + \beta < 1$), and therefore diminishes their joint surplus. In equilibrium the partner receiving the cash would in fact want to use it to buy back profit shares. Similarly, outside capital can be used to buy out a partner under joint asset ownership. However, the outside investor would get exactly the same stake in the new partnership as the original partner that needs to be bought out. This essentially means replacing one unproductive partner with another. Overall we note that using outside capital to address ex-post inefficiencies is not a substitute for wealth; it either does not improve ex-post efficiency at all (joint asset ownership), or even impairs the total surplus from a partnership (individual asset ownership).

8 Robustness

8.1 Alternative Ownership Structures

So far we focused on individual and joint asset ownership as the only possible ownership structures. We now briefly explain why we can safely ignore all other ownership structures.

The main alternative ownership structure is full asset ownership in the hands of one of the two partners. This ownership structure plays a large role in the property rights literature. With ex-ante symmetric partners, it does not matter which partner owns both assets; w.l.o.g. we assume it is A. It is easy to verify that whenever A finds an alternative partner and B does not, then the model behaves just like under individual asset ownership. And if B finds an alternative partner and A does not, then the model behaves just like under joint asset ownership. From Proposition 1 we know that either individual or joint asset ownership is optimal; thus, mixing individual and joint ownership is never optimal.

In fact, asymmetric asset ownership creates additional inefficiencies. If both A and B find alternative partners, then A can hold up B before releasing B's asset. Thus, B would have to share some of the profits from his new partnership with D, which is clearly inefficient. At the ex-ante stage, B would also not relinquish his asset for free. In fact, A would have to give B a larger profit share ex-ante, which would lead to further inefficiencies. Asymmetric ownership is therefore never optimal.

From Proposition 2 it is immediate that randomization among the symmetric ownership structures is suboptimal (except for the special case of $p = \widehat{p}$ where it is a matter of indifference). Giving ownership to outsider is not optimal in our model either, since owner-managers want to retain maximal effort incentives. Moreover, the type of outside ownership suggested by Gans (2005) is not an equilibrium, because we allow asset owners to coordinate on joint asset ownership. Finally, because there are no sequential investments in our model, there is no role for options on ownership as in Nödelke and Schmidt (1998).

8.2 Joint Ownership versus Long-term Contracts

Can long-term contracts be used instead of joint asset ownership? Long-term contracts are often associated with reputational concerns in infinitely repeated games, which we do not consider in our model. However, even in our finite horizon model we can look at long-term contracts. Indeed, our main trade-off between individual versus joint asset ownership can also be recast as a trade-off between short-term versus long-term contracts.

To see this, suppose that the two partners retain individual asset ownership at date 0, but commit to a contract that specifies the price at which they transact at date 4. If the contract is only structured as an option without commitment, then it obviously has no effect at all. However, if the contract is binding, then the two partners face a very similar renegotiation game as under joint asset ownership: If they want to switch partners, they cannot do so without the consent of their original trading partner.

The only difference between a long-term contract and joint asset ownership concerns the division of surplus. Under joint asset ownership, the two partners get a constant fraction of the profits, as defined by α and β. With a long-term contract the partners agree on a pre-specified price (or pricing formula) that determines the division of surplus. What matters for the model is not the actual distribution at date 4, but the expected distribution at dates 2 and 3. Consider the easiest example where a seller offers a single good to a buyer. Let $\widetilde{v}$ be the value of the good for the buyer at date 4, and $\widetilde{c}$ be the cost of the seller. The joint profit is then given by $y = \widetilde{v} - \widetilde{c}$. We conveniently denote the joint distribution of $\widetilde{v}$ and $\widetilde{c}$ by $\Omega_{vc}(\widetilde{v}, \widetilde{c})$, so that $\int y d\Omega_{in}(y) = \int (\widetilde{v} - \widetilde{c}) d\Omega_{vc}(\widetilde{v}, \widetilde{c})$. Let λ be the transfer price specified in the long-term contract. This price can be made contingent on verifiable information, in this case on the realizations of $\widetilde{v}$ and $\widetilde{c}$ at date 4.

In the model without learning, it is easy to see that for every α there exists a unique transfer price λ that leads to the same expected utilities at date 2 as joint asset ownership.[29] Alternatively, it is possible to define a pricing schedule that actually achieves the identical utilities at date 4. The flexible pricing schedule must then satisfy $\alpha y = \widetilde{v} - \lambda$, which can always be achieved with $\lambda = (1 - \alpha)\widetilde{v} + \alpha\widetilde{c}$. By construction this pricing schedule continues to implement the same outcomes as joint asset ownership, even in the model with interim learning about the inside prospect. We thus find that long-term contracts can always be structured such that they generate the same ex-ante utilities as joint asset ownership. Our model is therefore consistent with an interpretation of the integration decision as either joint asset ownership or long-term contracts.

8.3 Alternative Bargaining Protocols

Our model uses the bargaining protocol of Hart and Mas-Colell (1996), which generates the Maschler-Owen consistent NTU value. A natural question is whether our results are sensitive

[29] Specifically, we have $\alpha \int (\widetilde{v} - \widetilde{c}) d\Omega_{vc}(\widetilde{v}, \widetilde{c}) = \int (\widetilde{v} - \lambda) d\Omega_{vc}(\widetilde{v}, \widetilde{c})$, or equivalently, $\lambda = \int \widetilde{v} d\Omega_{vc}(\widetilde{v}, \widetilde{c}) - \alpha \int (\widetilde{v} - \widetilde{c}) d\Omega_{vc}(\widetilde{v}, \widetilde{c})$.

to alternative bargaining solutions. We now explain that alternative bargaining solutions may generate different levels of utility, but they do not affect the boundaries between the four key regions in Figure 3. Consequently they do not change the basic logic of how partners make optimal asset ownership decisions.

If both partners have zero outside options, they are perfectly symmetric. Any reasonable bargaining solution then suggests an equal split of surplus. Similarly, if A and B both found alternative partners, then we have two pairs of symmetric partners. Again we note that an equal split of surplus is the most reasonable bargaining outcome. Alternative bargaining protocols therefore only matter for the case of asymmetric outside options. We distinguish between the bargaining games under individual versus joint asset ownership.

Consider first the bargaining game under joint asset ownership, where A wants to leave B to partner with C. Because the agreement of all three parties is required, any reasonable bargaining involves trilateral bargaining. While there may be many bargaining protocols that affect the distribution of rents between the three parties, the key insight is that the critical threshold $\widehat{\sigma}_J(\pi)$ from Lemma 4 does not depend on the specific distribution of these rents. This threshold only depends on the feasibility of obtaining an agreement between A, B and C that satisfies all three participation constraints. Specifically, at $\sigma = \widehat{\sigma}_J(\pi)$ both A and B are indifferent between dissolving their partnership and staying together (each getting $U(\pi)$), while C receives the minimum equity stake $\gamma = 1 - \alpha^{\max}$. For any $\sigma < \widehat{\sigma}_J(\pi)$ it is impossible to get a tripartite agreement, and for any $\sigma \geq \widehat{\sigma}_J(\pi)$ it is always possible get such an agreement. As a consequence, the specific bargaining protocol actually does not matter for the partners' decision to stay together or to do a buyout.

Under individual asset ownership, we know from Lemma 3 that there exists a critical threshold $\widehat{\sigma}_I(\pi) = \pi$, such that A leaves B whenever $\sigma > \pi$, and stays whenever $\sigma \leq \pi$. Again we argue that reasonable alternative bargaining protocols may generate different utilities, but the critical threshold remains unaffected. One important restriction of the bargaining protocol by Hart and Mas-Colell (1996) is that at any point in time only one party can make an offer. Consider relaxing this assumption, and suppose that there can be simultaneous offers. In particular assume that the unique partner (A) can hold an auction for offers from the non-unique partners (B and C). Such an auction game results in a standard Bertrand pricing. It is easy to show that these Bertrand offers are more favorable to A than the bargaining outcome under the Hart and Mas-Colell protocol. However, since the auction is always won by the player with the highest valuation, it continues to be true that A teams up with B whenever $\sigma \leq \pi$, and with C whenever $\sigma > \pi$. Again we find that the critical threshold $\widehat{\sigma}_I(\pi) = \pi$ remains unaffected by the specific bargaining protocol.

8.4 Costly Transfers Payments

For our main model we derive ex-post inefficiencies on the basis of binding wealth constraints. In this section we briefly sketch a model where partners do not face wealth constraints and cannot commit to limiting their wealth, but where any ex-post transfer payments are costly. We show that our central trade-off between displacement and extension externalities continue to hold.

Suppose that for any transfer of wealth, T, there is a cost τT. That is, if one partner pays T, the other only gets $(1-\tau)T$. The simplest interpretation is that transfer payments are taxed at a rate τ. One can also think of financial intermediation costs, or transaction costs more broadly. For simplicity we focus on costs that are linear in T, but it is easy to extend our analysis to the case of non-linear costs (including fixed costs).

In our model the ownership decision at date 0 is solely driven by the ex-post inefficiencies associated with asymmetric outside options. We therefore focus on the asymmetric case, and assume that A found an alternative partner at date 2. Consider the case of individual asset ownership.[30] If A leaves B, then he gets the utility $U(\sigma)$ from his new relationship with C. If A stays and renegotiates with B, he gets some shares $\alpha(\tau)$, as well as a transfer payment $T_{BA}(\tau)$ from B. We are interested in the cutoff level $\widehat{\sigma}_I(\tau,\pi)$, below which it is possible for A and B to come to an agreement, and A therefore stays.

Figure 6 provides a graphical analysis for how $\widehat{\sigma}_I(\tau,\pi)$ depends on τ. In the renegotiation, A is asking B for a more favorable deal than the status quo of $\alpha=\beta=1/2$. We can see from Figure 6 how costly transfer payments affect the utility frontier between A and B. Giving A a higher utility can initially be done by increasing his profit share α. A then receives a higher share, up to some $\alpha^*(\tau) > 1/2$ where the marginal rate of substitution satisfies $dU_A/dU_B = -(1-\tau)$. Beyond this point is would be inefficient to offer an even higher share α, since the incentive costs exceed the costs of making transfer payments. B then prefers to transfer more utility to A through costly transfer payments, so that the utility frontier has a slope of $-(1-\tau)$. The maximum transfer $T_{BA}^{\max}$ that B is willing to offer, satisfies $U_B(1-\alpha^*(\tau);\pi) - T_{BA}^{\max} = 0$. This gives A the utility $U_A(\alpha^*(\tau);\pi) + (1-\tau)T_{BA}^{\max}$, or equivalently, $U_A(\alpha^*(\tau);\pi) + (1-\tau)U_B(1-\alpha^*(\tau);\pi)$. The cutoff $\widehat{\sigma}_I(\tau,\pi)$, below which it is possible for A and B to reach an agreement, therefore satisfies

$$U_A(\sigma) = U_A(\alpha^*(\tau);\pi) + (1-\tau)U_B(1-\alpha^*(\tau);\pi).$$

For $\tau = 1$ the model reverts back to our main model with perfect wealth constraints. The highest utility that A can get from staying with B is then given by $U_A(\alpha^{\max};\pi)$, and the cutoff

[30] The case of joint asset ownership follows a similar logic.

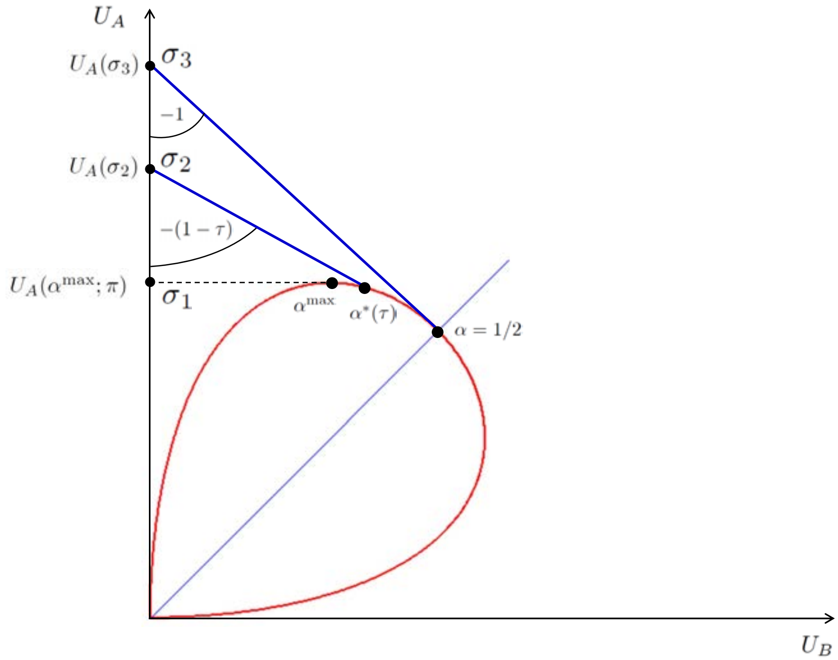

Figure 6: Utility-possibility Frontier for Joint Ownership with Ex-post Transfers

$\widehat{\sigma}_I(\tau = 1, \pi)$ satisfies $U_A(\sigma) = U_A(\alpha^{\max}; \pi)$ as before. This point is represented by σ_1 in Figure 6.

For $\tau = 0$ we have a model with perfectly efficient ex-post wealth transfers. The highest utility that A can get from staying with B is then given by $U_A(\alpha^*(\tau); \pi) + U_B(1 - \alpha^*(\tau); \pi)$, where $\alpha^*(\tau = 0) = 1/2$. And the cutoff $\widehat{\sigma}_I(\tau = 0, \pi)$ satisfies $U_A(\sigma) = V(\alpha = 1/2; \pi)$; the cutoff is thus equivalent to the efficient level, i.e., $\widehat{\sigma}_I(\tau = 0, \pi) = \widehat{\sigma}_V(\pi)$. This point is represented by σ_3 in Figure 6.

The most important insight is that for any $\tau \in (0, 1)$, the maximum utility that A can get from B lies somewhere between the two extremes σ_1 and σ_3. In Figure 6 we denote this point by σ_2. Relative to the model with perfect wealth constraints, the range where renegotiation succeeds is increased from σ_1 to σ_2. However, relative to the model with perfect wealth transfers, there is a region between σ_2 and σ_3 where the joint utility would be maximized if A stayed with B, but transfer costs prevent renegotiation so that A leaves. Put differently, as long as transfer payments are costly, there is a region $\sigma \in (\sigma_2, \sigma_3)$ where inefficient displacement occurs. Moreover, even though displacement does not occur for $\sigma < \sigma_2$, there are still some inefficiencies whenever $\tau > 0$. This is because the profit share for A remains above the efficient level $\alpha = 1/2$, and because B may need to make some costly transfers in equilibrium.

Overall we note that even a small cost of making ex-post transfers is enough to generate the trade-off between displacement externalities (individual ownership) and retention externalities (joint ownership). Thus, while we consider binding wealth constraints a natural choice for the base model, our key insights do not depend on this assumption. In fact, the trade-off between the benefit of retaining flexibility under individual asset ownership versus the benefit of commitment under joint asset ownership, carries over to a model with unlimited wealth but costly transfer payments.

9 Conclusion

This paper develops a new theory of the firm, based on how partners dynamically switch in and out of relationships. The model identifies a fundamental trade-off between two ex-post inefficiencies. Under non-integration (i.e., individual asset ownership) there can be a *displacement externality*, where a partner may leave, even though the benefit is worth less than the loss to the displaced partner. Under integration (i.e., joint asset ownership) there can be a *retention externality*, where a partner may hold on to the other, even though the benefit to the departing partner would exceed the loss to the remaining partner. These inefficiencies arise endogenously in our model with wealth-constrained agents and team incentive problems, and are robust to renegotiations. Optimal firm boundaries are determined by the relative importance of displacement and retention externalities: The greater the asset specificity in a given relationship, the greater the displacement and the smaller the retention externality, and hence the more attractive is integration.

Our main model does not rely on relation-specific investments; but when added to the model, we find that joint asset ownership always provides stronger incentives for specific investments. Moreover, when relaxing the assumption of binding wealth constraints, we find that wealth has two distinct effects. Ex-post, wealth mitigates the displacement and retention externalities. However, ex-ante wealth reduces incentives for specific investments. A surprising result is that wealthy owners would always want to commit ex-ante to limiting ex-post transfer payments.

Our model draws on insights from several of the leading theories of the firm, including transaction costs, property rights, and incentive-based theories. However, it differs in terms of focusing on the dynamics of how partners change relationships. This allows us to provide a fresh perspective on some of the existing explanations for integration. In particular, our model generates the traditional prediction from transaction cost economics that asset specificity favors integration. However, while the verbal arguments of Williamson typically require different explanations for the advantages of integration and non-integration, we provide a unified framework where one underlying force generates the trade-off. Moreover, while transaction cost economics typically relies on problems of ex-post opportunism and price haggling, we focus instead on the problems of displacement and retention as the main sources of ex-post inefficiencies. This allows us to go directly to the core of asset specificity, namely that the primary concern of the owner of a specific asset is to be not displaced by another.

Our model also allows us to derive some new empirical predictions about the boundaries of the firm. Most of the theoretical work has focused on comparative statics, and consequently most of the empirical work has focused on cross-sectional determinants of the integration decision. Our theory suggests an empirical research agenda about the time-series properties of integration.

For a given level of asset specificity, our model predicts that dynamic partner switching is more common under individual than under joint asset ownership. The same applies to short-term versus long-term contracting. It seems highly intuitive that switching to an outside supplier or outside buyer is more rare in a vertically integrated setting, so we would be surprised if this prediction was not found in the data. The interesting point here is actually that this simple prediction cannot be obtained from theories with ex-post efficiency, such as the property rights theories. This is because in those models all efficient partner changes always occur in equilibrium, irrespective of asset ownership.

Our model predicts that non-integration (or short-term contracting) is more common in environments with considerable uncertainty about what the best trading partners are. Previous empirical work typically focused on general measures of uncertainty, often with mixed results (see Lafontaine and Slade, 2007). We contend that these measures fail to distinguish between uncertainty about production and demand, versus uncertainty about partner choices. Moreover, our theory suggests that the dynamics of partner choices also depends on the internal match quality, i.e., the interim learning about how well partners are suited for each other.

Our analysis suggests avenues for further theoretical work. We focus on team incentives and wealth constraints as source of ex-post inefficiencies; but there may be other sources of inefficiencies, such as asymmetric information (Aghion et al., 2012). Future research may examine how alternative ex-post inefficiencies affect the dynamic properties of firm boundaries. For analytical tractability we impose ex-ante symmetry of partners, but it would be interesting to explore the role of ex-ante asymmetries. Moreover, in this paper we chose the simplest possible dynamic specification where partners have at most one opportunity to switch partners. A worthwhile future research agenda is to extend the model to an infinite horizon. This would allow for a more comprehensive analysis of how asset ownership affects the timing and frequency of partner changes. Such a model might also look at the process of wealth accumulation, and at how partners dynamically trade-off ex-post benefits versus ex-ante costs of having wealth. Overall we believe that looking at the dynamics of asset ownership constitutes a new and promising direction in the theory of the firm.

Appendix

Marriage Example with Imperfect Ex-post Transfers.

Suppose that Alice and Bob both have sufficient wealth, but for any ex-post transfer T there is a cost τT. That is, if the sender pays T, the receiver only gets $(1-\tau)T$. As shown, the marriage decision is only relevant in the case of asymmetric outside options, which we now focus on. Suppose that Alice found Charles, but Dora continues to reject Bob. Recall that staying together is then jointly efficient if $\lambda \leq \lambda_V \equiv 2\theta$.

Consider the case without marriage. If Alice leaves Bob for Charles, she gets λ, and Bob gets 0. To retain Alice, Bob can offer her the transfer T_{BA}; Alice then gets $\theta + (1-\tau)T_{BA}$, while Bob gets $\theta - T_{BA}$. Thus, the maximum transfer that Bob is willing to offer is $T_{BA}^{max} = \theta$. Alice then gets the utility $U_A(\tau) = \theta + (1-\tau)T_{BA}^{max} = (2-\tau)\theta$. Thus, Alice stays with Bob if $U_A(\tau) \geq \lambda$, or equivalently, $\lambda \leq \lambda_{No-M}(\tau) \equiv (2-\tau)\theta$. We can immediately see that $\lambda_{No-M}(\tau) < \lambda_V$ for $\tau > 0$. Thus, for $\lambda \in (\lambda_{No-M}(\tau), \lambda_V)$ Alice will leave Bob even though it is jointly inefficient.

Now consider the case of marriage. If Alice stays with Bob each gets θ. However, Alice can also buy herself out by offering Bob the transfer T_{AB}. Alice would then get $\lambda - T_{AB}$, while Bob gets $(1-\tau)T_{AB}$. The maximum transfer T_{AB}^{max} that Alice is willing to offer Bob satisfies $\lambda - T_{AB}^{max} = \theta$, so that $T_{AB}^{max} = \lambda - \theta$. Bob would then get $U_B(\tau) = (1-\tau)T_{AB}^{max} = (1-\tau)(\lambda-\theta)$. Thus, Bob would only let Alice go if $U_B(\tau) > \theta$, or equivalently, $\lambda > \lambda_M(\tau) \equiv \theta(2-\tau)/(1-\tau)$. Note that $\tau = 0$ implies $\lambda_M(\tau = 0) = \lambda_V$. Moreover, it is straightforward to show that $d\lambda_M(\tau)/d\tau > 0$ for $\tau \in (0,1)$. Thus, $\lambda_V < \lambda_M(\tau)$ for $\eta > 0$. This implies that for $\lambda \in (\lambda_V, \lambda_M(\tau))$, Alice stays with Bob even though leaving would be jointly efficient.

Finally we note that $\lambda_{No-M}(\tau) < \lambda_V < \lambda_M(\tau)$. For $\lambda \leq \lambda_V = 2\theta$ we know that staying together is then jointly optimal for Alice and Bob in case they have asymmetric outside options. In fact, staying is then the equilibrium outcome when Alice and Bob are married (as $\lambda_V < \lambda_M(\tau)$). Without marriage, either inefficient leaving can occur in equilibrium ($\lambda_{No-M}(\tau) < \lambda \leq \lambda_V$), or staying together requires costly transfer payments ($\lambda \leq \lambda_{No-M}(\tau)$). For $\lambda > \lambda_V = 2\theta$, leaving the partner is jointly optimal in case of asymmetric outside options. This is then indeed the equilibrium outcome when Alice and Bob are not married ($\lambda_{No-M}(\tau) < \lambda_V$). With marriage, either leaving requires costly transfer payments ($\lambda > \lambda_M(\tau)$), or one partner inefficiently retains the other ($\lambda_V < \lambda \leq \lambda_M(\tau)$). Thus, marriage is optimal whenever $2\theta > \lambda$; and for $2\theta \leq \lambda$ Alice and Bob are better off without marriage.

Proof of Lemma 1.

Recall that A and B implement the effort levels $e_A^*(\alpha)$ and $e_B^*(\beta)$ as defined by (1) and (2). Using $\beta = 1 - \alpha$, the joint surplus is given by

$$V(\alpha;\pi) = \mu(e_A^*(\alpha)e_B^*(\alpha))\pi - c(e_A^*(\alpha)) - c(e_B^*(\alpha)).$$

The jointly optimal profit share α^J satisfies the first-order condition

$$\pi\mu'(e_A^*e_B^*)\left[\frac{de_A^*}{d\alpha}e_B^* + \frac{de_B^*}{d\alpha}e_A^*\right] = c'(e_A^*)\frac{de_A^*}{d\alpha} + c'(e_B^*)\frac{de_B^*}{d\alpha}. \tag{12}$$

Symmetry implies $de_A^*/d\alpha = -de_B^*/d\alpha$ and $e_A^* = e_B^*$ at $\alpha = 1/2$. Thus, (12) is satisfied for $\alpha = \beta = 1/2$. The solution $\alpha^J = \beta^J = 1/2$ is also unique due to convexity of $c(e_i)$, $i = A, B$. Thus,

$$\left.\frac{dU_A}{d\alpha}\right|_{\alpha=1/2} = -\left.\frac{dU_B}{d\alpha}\right|_{\alpha=1/2} > 0.$$

Moreover, $e_A^*(0) = e_B^*(1) = 0$. This implies $1/2 < \alpha^{max} = \beta^{max} < 1$.

Now consider the bargaining at date 0, and suppose that partner A gets the profit share $\alpha > 1/2$ with probability $1/2$, and $1 - \alpha < 1/2$ otherwise. A's expected utility at date 0 is then given by $U_A(\alpha;\pi)/2 + U_A(1-\alpha;\pi)/2$. However, from the above we note that A's expected utility is maximized when $\alpha = 1/2$ (symmetric for B). Thus, at date 0 both partners A and B agree on splitting the expected joint surplus in half: $\alpha = \beta = 1/2$. □

Proof of Lemma 3.

Under individual asset ownership, partner A is indifferent between staying (and renegotiating his profit share α) and leaving, if

$$U_A(\alpha_I^*;\pi) = U_A(\widehat{\alpha}_I;\sigma). \tag{13}$$

Recall that B and C both have zero outside options. The bargaining protocol à la Hart and Mas-Colell (1996) then implies that (13) is satisfied for $\pi = \sigma$. Thus, $\sigma \leq \pi$ implies $U_A(\alpha_I^*;\pi) \geq U_A(\widehat{\alpha}_I;\sigma)$. For $\sigma > \pi$ the opposite holds: $U_A(\alpha_I^*;\pi) < U_A(\widehat{\alpha}_I;\sigma)$. Finally we define $\widehat{\sigma}_I(\pi) = \pi$ as the threshold above which A is better off leaving B ($\sigma > \widehat{\sigma}_I(\pi)$). □

Profit Shares under Joint Ownership with Asymmetric Outside Options.

W.l.o.g. suppose that only A found an alternative partner, C. We denote a coalition by S, with $S \subset 3$. Let $\kappa = (\kappa_A, \kappa_B, \kappa_C)$ be a vector which measures the rate at which utility can be transferred. Moreover, $\eta_T \in V(T)$ denotes the payoff vector for the subcoalition T.

According to Hart (2004), the Maschler-Owen consistent NTU value can be derived by the following procedure: First, for all $i \in S$, let the payoff vector $\mathbf{z} \in \mathbb{R}^S$ satisfy

$$\kappa_i z_i = \frac{1}{|S|}\left[v_\kappa(S) - \sum_{j \in S \setminus i} \kappa_j \eta_{S\setminus i}(j) + \sum_{j \in S \setminus i} \kappa_i \eta_{S\setminus j}(i)\right],$$

where the maximum possible value $v_\kappa(S)$ is defined by

$$v_\kappa(S) = \sup\left\{\sum_{i \in S} \kappa_i U_i : (U_i)_{i \in S} \in V(S)\right\}.$$

Second, if $\mathbf{z}$ is feasible, then the payoff vector is given by $\eta_S = \mathbf{z}$.

The coalition functions for our setting are as follows:

$$\begin{aligned}
V_{\{A\}} &= V_{\{B\}} = V_{\{C\}} = 0 \\
V_{\{A,B\}} &= \left\{(U_A(\alpha;\pi), U_B(\beta;\pi)) \in \mathbb{R}^{\{A,B\}} : \alpha + \beta \leq 1; \alpha, \beta \geq 0\right\} \\
V_{\{A,C\}} &= \{0,0\} \\
V_{\{B,C\}} &= \{0,0\} \\
V_{\{A,B,C\}} &= \left\{(U_A(\alpha;\sigma), U_B(\beta;\sigma), U_C(\gamma;\sigma)) \in \mathbb{R}^{\{A,B,C\}} : \alpha + \beta + \gamma \leq 1; \alpha, \beta, \gamma \geq 0\right\},
\end{aligned}$$

where $V_{\{A,C\}} = \{0,0\}$ follows from the fact that A cannot leave without B's consent under joint ownership. Note that the bargaining outcome must satisfy $\alpha^* \in (0, \alpha^{\max})$ and $\beta^* \in (0, \beta^{\max})$ for the (A,B) coalition, and $\alpha^* \in (0, \alpha^{\max})$, $\beta^* \in (0, \beta^{\max})$, and $\gamma^* \in (0, \gamma^{\max})$ for the grand coalition (A,B,C). Thus, $dU_A/d\alpha > 0$, $dU_B/d\beta > 0$, and $dU_C/d\gamma > 0$ for the relevant values of α, β, and γ. This implies that the inverse of each utility function exists. We define $\alpha(U_A) \equiv U_A^{-1}(\alpha)$, $\beta(U_B) \equiv U_B^{-1}(\beta)$, and $\gamma(U_C) \equiv U_C^{-1}(\gamma)$. Pareto efficiency then requires

$$\begin{aligned}
\alpha(U_A) + \beta(U_B) &= 1 \quad \text{for } \overline{V}_{\{A,B\}} \\
\alpha(U_A) + \beta(U_B) + \gamma(U_C) &= 1 \quad \text{for } \overline{V}_{\{A,B,C\}}.
\end{aligned}$$

The payoffs for the single-player coalitions are given by $\eta_1(A) = \eta_1(B) = \eta_1(C) = 0$. For the two-player coalitions, the equilibrium payoffs satisfy the Nash bargaining solution. Due to symmetry, the payoffs are given by

$$\begin{aligned}
\eta_2(A,B) &= (U(\pi), U(\pi)) \\
\eta_2(A,C) &= (0,0) \\
\eta_2(B,C) &= (0,0).
\end{aligned}$$

It remains to derive the payoff vector $\eta_3(A, B, C)$ for the hyperplane game. For a vector $\mathbf{z} = (z_A, z_B, z_C)$ the equation of the hyperplane is

$$\alpha'(U_A)z_A + \beta'(U_B)z_B + \gamma'(U_C)z_C = r, \tag{14}$$

where

$$r = \alpha'(U_A)U_A + \beta'(U_B)U_B + \gamma'(U_C)U_C. \tag{15}$$

Using the payoffs for the two-player coalitions, we can now define the equilibrium payoffs for the grand coalition (A, B, C):

$$\begin{aligned}
\eta_3(A) &= U_A(\alpha; \sigma) = \frac{1}{3}\left[U(\pi) + z_A\right] \\
\eta_3(B) &= U_B(\beta; \sigma) = \frac{1}{3}\left[U(\pi) + z_B\right] \\
\eta_3(C) &= U_C(\gamma; \sigma) = \frac{1}{3} z_C,
\end{aligned}$$

where, using (15),

$$\begin{aligned}
z_A &= \frac{1}{\alpha'(U_A)}\left[r - \beta'(U_B)\cdot 0 - \gamma'(U_C)\cdot 0\right] = \frac{r}{\alpha'(U_A)} \\
z_B &= \frac{1}{\beta'(U_B)}\left[r - \alpha'(U_A)\cdot 0 - \gamma'(U_C)\cdot 0\right] = \frac{r}{\beta'(U_B)} \\
z_C &= \frac{1}{\gamma'(U_C)}\left[r - \alpha'(U_A)U(\pi) - \beta'(U_B)U(\pi)\right].
\end{aligned}$$

Using the Inverse Function Theorem we get $\alpha'(U_A) = (dU_A/d\alpha)^{-1}$, $\beta'(U_B) = (dU_B/d\beta)^{-1}$, and $\gamma'(U_C) = (dU_C/d\gamma)^{-1}$. The equations for the fixed point for the grand coalition are thus given by

$$U_A(\alpha; \sigma) = \frac{1}{3}\left[U(\pi) + r\frac{dU_A(\alpha; \sigma)}{d\alpha}\right] \tag{16}$$

$$U_B(\beta; \sigma) = \frac{1}{3}\left[U(\pi) + r\frac{dU_B(\beta; \sigma)}{d\beta}\right] \tag{17}$$

$$U_C(\gamma; \sigma) = \frac{1}{3}\frac{dU_C(\gamma; \sigma)}{d\gamma}\left[r - U(\pi)\left[\left(\frac{dU_A(\alpha; \sigma)}{d\alpha}\right)^{-1} + \left(\frac{dU_B(\beta; \sigma)}{d\beta}\right)^{-1}\right]\right], \tag{18}$$

where, using (15),

$$r = U_A(\alpha; \sigma)\left(\frac{dU_A(\alpha; \sigma)}{d\alpha}\right)^{-1} + U_B(\beta; \sigma)\left(\frac{dU_B(\beta; \sigma)}{d\beta}\right)^{-1} + U_C(\gamma; \sigma)\left(\frac{dU_C(\gamma; \sigma)}{d\gamma}\right)^{-1}.$$

The equilibrium payoff vector $\eta_3(A, B, C) = (\widehat{U}_A(\alpha; \sigma), \widehat{U}_B(\beta; \sigma), \widehat{U}_C(\gamma; \sigma))$ thus satisfies the system of three equations, (16), (17), and (18), which also defines the equilibrium profit shares $\widehat{\alpha}_J$, $\widehat{\beta}_J$, and $\widehat{\gamma}_J$.

Proof of Lemma 4.

Under joint asset ownership, partner A stays with an equal split of profits if

$$U_A(\pi) \geq U_A(\widehat{\alpha}_J; \sigma). \tag{19}$$

Note that (19) is satisfied with strict inequality when $\pi > 0$ and $\sigma = 0$. Moreover, $dU_A(\widehat{\alpha}_J; \sigma)/d\sigma > 0$, with $\lim_{\sigma \to \infty} U_A(\widehat{\alpha}_J; \sigma) = \infty > U_A(\pi)$ for any finite π. Thus, there exists a threshold $\widehat{\sigma}_J(\pi)$ such that (19) is satisfied for $\sigma \leq \widehat{\sigma}_J(\pi)$. Now consider the case where both A and B found alternative partners (symmetric outside options). They then stay together if $U(\pi) \geq U(\sigma)$, which is equivalent to $\pi \geq \sigma$. Recall that $U(\sigma) > U_A(\widehat{\alpha}_J; \sigma)$ for all $\sigma > 0$ because $\widehat{\beta}_J > 0$ and $e_B^* = 0$ in case of asymmetric outside options. Thus, $\widehat{\sigma}_J(\pi) > \pi$. □

Proof of Proposition 1.

Recall that the ownership structure is irrelevant for the partners' joint surplus in case of symmetric outside options. We can thus focus on the case with asymmetric outside options. Moreover, note that maximizing a partner's expected utility at date 0 is equivalent to maximizing the joint surplus of A and B.

W.l.o.g. suppose that only A found an alternative partner at date 2 (the case where only B found an alternative partner is symmetric). According to Lemma 1 the joint surplus for the $A-B$ match is maximized when $\alpha = \beta = 1/2$. The joint surplus is then given by $2U(\pi)$. When A leaves, the joint surplus of A and B is maximized when B, as unproductive partner, does not get a stake in the new $A-C$ partnership. The joint surplus is then given by $U_A(\widehat{\alpha}; \sigma)$. Thus, staying together (with $\alpha = \beta = 1/2$) and dissolving the partnership are both jointly efficient if

$$2U(\pi) = U_A(\widehat{\alpha}; \sigma). \tag{20}$$

Note that $dU_A(\widehat{\alpha}; \sigma)/d\sigma > 0$, with $U_A(\widehat{\alpha}; 0) = 0$ and $\lim_{\sigma \to \infty} U_A(\widehat{\alpha}; \sigma) = \infty > 2U(\pi)$ for any finite π. Thus, there exists a threshold $\widehat{\sigma}_V(\pi)$ such that $2U(\pi) \geq U_A(\widehat{\alpha}; \sigma)$ for $\sigma \leq \widehat{\sigma}_V(\pi)$.

Next, we identify the optimal ownership structure which guarantees the jointly efficient outcome in case of asymmetric outside options. Suppose $\sigma > \widehat{\sigma}_V(\pi)$, so that dissolving the partnership is jointly optimal. Under individual asset ownership, A would leave if $\sigma > \widehat{\sigma}_I(\pi)$, where $\widehat{\sigma}_I(\pi)$ is defined by

$$U_A(\alpha_I^*; \pi) = U_A(\widehat{\alpha}_I; \sigma). \tag{21}$$

Note that $2U(\pi) > U_A(\alpha_I^*;\pi)$, whereas the right-hand sides of (20) and (21) are identical. Thus, $\widehat{\sigma}_I(\pi) < \widehat{\sigma}_V(\pi)$, which implies that individual asset ownership always ensures the dissolution of the partnership for $\sigma > \widehat{\sigma}_V(\pi)$ in case of asymmetric outside options. Now suppose $\sigma \leq \widehat{\sigma}_V(\pi)$, so that staying together with $\alpha = \beta = 1/2$ is jointly optimal in case of asymmetric outside options. Under joint asset ownership, A stays (with $\alpha = \beta = 1/2$) if $\sigma \leq \widehat{\sigma}_J(\pi)$. Recall that $\widehat{\sigma}_J(\pi)$ is defined by

$$U(\pi) = U_A(\widehat{\alpha}_J;\sigma). \tag{22}$$

To show that $\widehat{\sigma}_J(\pi) > \widehat{\sigma}_V(\pi)$, we define $\widehat{\sigma}_J^V(\pi)$ as the value of σ under joint asset ownership where staying together (with $\alpha = \beta = 1/2$) and dissolving the partnership (with $\widehat{\beta} > 0$) lead to the same joint surplus:

$$2U(\pi) = U_A(\widehat{\alpha}_J;\sigma) + U_B(\widehat{\beta}_J;\sigma). \tag{23}$$

Note that $U_A(\widehat{\alpha};\sigma) > U_A(\widehat{\alpha}_J;\sigma) + U_B(\widehat{\beta}_J;\sigma)$, whereas the left-hand sides of (20) and (23) are identical. Thus, $\widehat{\sigma}_J^V(\pi) > \widehat{\sigma}_V(\pi)$. Moreover, we can write (23) as

$$U(\pi) + \underbrace{U(\pi) - U_B(\widehat{\beta}_J;\sigma)}_{\equiv\chi} = U_A(\widehat{\alpha}_J;\sigma), \tag{24}$$

where, according to the Maschler-Owen consistent NTU value, $\chi < 0$. Thus, the left-hand side of (24) is smaller than the left-hand side of (22), while their right-hand sides are identical. Hence, $\widehat{\sigma}_J(\pi) > \widehat{\sigma}_J^V(\pi)$. This in turn implies that $\widehat{\sigma}_J(\pi) > \widehat{\sigma}_V(\pi)$. Consequently, joint asset ownership always preserves the partnership with $\alpha = \beta = 1/2$ for $\sigma \leq \widehat{\sigma}_V(\pi)$. □

Proof of Proposition 3.

We define

$$\begin{aligned} F &\equiv \frac{\partial p(r_A, r_B)}{\partial r_A}\Phi - \psi'(r_A) = 0 \\ G &\equiv \frac{\partial p(r_A, r_B)}{\partial r_B}\Phi - \psi'(r_B) = 0, \end{aligned}$$

where $\Phi \in \{\Phi_I, \Phi_J\}$. Applying Cramer's Rule we get

$$\frac{dr_A^*}{d\Phi} = \frac{\det(X_1)}{\det(X_2)}$$

where

$$X_1 = \begin{pmatrix} -\frac{\partial F}{\partial \Phi} & \frac{\partial F}{\partial r_B} \\ -\frac{\partial G}{\partial \Phi} & \frac{\partial G}{\partial r_B} \end{pmatrix} \qquad X_2 = \begin{pmatrix} \frac{\partial F}{\partial r_A} & \frac{\partial F}{\partial r_B} \\ \frac{\partial G}{\partial r_A} & \frac{\partial G}{\partial r_B} \end{pmatrix}.$$

Because $U_i(\cdot)$, $i = A, B$, is concave, X_2 must be negative definite, so that $\det(X_2) > 0$. Thus, $dr_A^*/d\Phi > 0$ if

$$\det(X_1) = -\frac{\partial F}{\partial \Phi}\frac{\partial G}{\partial r_B} + \frac{\partial G}{\partial \Phi}\frac{\partial F}{\partial r_B} > 0.$$

The second-order condition for r_B^* implies $dG/dr_B < 0$. Moreover,

$$\frac{\partial F}{\partial \Phi} = \frac{\partial p(r_A, r_B)}{\partial r_A} > 0 \qquad \frac{\partial G}{\partial \Phi} = \frac{\partial p(r_A, r_B)}{\partial r_B} > 0,$$

and

$$\frac{\partial F}{\partial r_B} = \frac{\partial^2 p}{\partial r_A \partial r_B}\Phi.$$

Thus, $dr_A^*/d\Phi > 0$ for $\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa$, where $-\kappa$ is the lower bound of the cross-partial satisfying $\det(X_1) = 0$. Symmetry implies $dr_B^*/d\Phi > 0$.

Finally, $\Phi_J > \Phi_I$ is equivalent to

$$V_J(\pi_H, \sigma) - V_I(\pi_H, \sigma) > V_J(\pi_L, \sigma) - V_I(\pi_L, \sigma).$$

Recall that $V_J(\pi_H, \sigma) > V_I(\pi_H, \sigma)$ and $V_J(\pi_L, \sigma) < V_I(\pi_L, \sigma)$. Thus, $\Phi_J > \Phi_I$. This, in conjunction with $dr_A^*/d\Phi > 0$ and $dr_B^*/d\Phi > 0$, implies that $r_J^* > r_I^*$. □

Proof of Proposition 4.

With wealth w the equilibrium levels of relation-specific investments $r^*_{A(k)}(w)$ and $r^*_{B(k)}(w)$, $k \in \{I, J\}$, are characterized by

$$\begin{aligned} F &\equiv \frac{\partial p(r_A, r_B)}{\partial r_A}\Phi_k(w) - \psi'(r_A) = 0 \\ G &\equiv \frac{\partial p(r_A, r_B)}{\partial r_B}\Phi_k(w) - \psi'(r_B) = 0. \end{aligned}$$

where, using $V_I(\pi_L, \sigma) = U(\sigma)$ and $V_J(\pi_H, \sigma) = 2U(\pi_H)$,

$$\begin{aligned} \Phi_I &= \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)V_I(w, \pi_H, \sigma)\right] \\ &\quad - \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)U(\sigma)\right] \\ \Phi_J &= \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)2U(\pi_H)\right] \\ &\quad - \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_J(w, \pi_L, \sigma)\right]. \end{aligned}$$

By applying Cramer's Rule we get

$$\frac{dr^*_{A(k)}(w)}{dw} = \frac{\det(X_1)}{\det(X_2)},$$

where

$$X_1 = \begin{pmatrix} -\frac{\partial F}{\partial w} & \frac{\partial F}{\partial r_B} \\ -\frac{\partial G}{\partial w} & \frac{\partial G}{\partial r_B} \end{pmatrix} \qquad X_2 = \begin{pmatrix} \frac{\partial F}{\partial r_A} & \frac{\partial F}{\partial r_B} \\ \frac{\partial G}{\partial r_A} & \frac{\partial G}{\partial r_B} \end{pmatrix}.$$

Because $U_i(\cdot)$, $i = A, B$, is concave, X_2 must be negative definite, so that $\det(X_2) > 0$. Thus, $dr^*_{A(k)}(w)/dw > 0$ if

$$\det(X_1) = -\frac{\partial F}{\partial w}\frac{\partial G}{\partial r_B} + \frac{\partial G}{\partial w}\frac{\partial F}{\partial r_B} > 0.$$

The second-order condition for $r^*_{B(k)}(w)$ implies $\partial G/\partial r_B < 0$. Moreover,

$$\frac{\partial F}{\partial r_B} = \frac{\partial^2 p(\cdot)}{\partial r_A \partial r_B}\Phi_k(w)$$

and

$$\frac{\partial F}{\partial w} = \frac{\partial p(\cdot)}{\partial r_A}\frac{d\Phi_k(w)}{dw} \qquad \frac{\partial G}{\partial w} = \frac{\partial p(\cdot)}{\partial r_B}\frac{d\Phi_k(w)}{dw},$$

where

$$\begin{aligned} \frac{d\Phi_I(w)}{dw} &= q(1-q)\frac{dV_I(w, \pi_H, \sigma)}{dw} \\ \frac{d\Phi_J(w)}{dw} &= -q(1-q)\frac{dV_J(w, \pi_L, \sigma)}{dw}. \end{aligned}$$

For individual asset ownership, recall that $dV_I(w, \pi_H, \sigma)/dw > 0$ for $w \geq \underline{w}_I$, which implies that $\partial F/\partial w > 0$ and $\partial G/\partial w > 0$ for $w \geq \underline{w}_I$. Thus, $dr^*_{A(I)}(w)/dw > 0$ for $w \geq \underline{w}_I$ and $\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa$, where κ is the lower bound of the cross-partial satisfying $\det(X_1) = 0$. Symmetry implies $dr^*_{A(I)}(w)/dw = dr^*_{B(I)}(w)/dw$. For joint asset ownership, recall that $dV_J(w, \pi_L, \sigma)/dw > 0$ for $w \geq \underline{w}_J$, so that $\partial F/\partial w < 0$ and $\partial G/\partial w < 0$ for $w \geq \underline{w}_J$. Thus, $dr^*_{A(J)}(w)/dw < 0$ for $w \geq \underline{w}_J$ and $\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa$. Due to symmetry, $dr^*_{A(J)}(w)/dw = dr^*_{B(J)}(w)/dw$.

Finally recall that $V_I(w, \pi_H, \sigma)$ (individual ownership) and $V_J(w, \pi_L, \sigma)$ are both (weakly) increasing in w. We define $\overline{w}_I$ as the minimum value of w which satisfies $V_I(w, \pi_H, \sigma) = 2U(\pi_H)$ (individual ownership), and $\overline{w}_J$ as the minimum value of w which satisfies $V_J(w, \pi_L, \sigma) = U(\sigma)$ (joint ownership). Because $V_I(w, \pi_H, \sigma) = 2U(\pi_H)$ for $w \geq \overline{w}_I$, and $V_J(w, \pi_L, \sigma) = U(\sigma)$ for $w \geq \overline{w}_J$, we have $\Phi_I(w) = \Phi_J(w)$ for $w \geq \max\{\overline{w}_I, \overline{w}_J\}$. This implies $r^*_I(w) = r^*_J(w)$ for $w \geq \max\{\overline{w}_I, \overline{w}_J\}$. □

Proof of Lemma 7.

Under individual asset ownership the expected utility of partner A at date 0 is given by

$$\begin{aligned} EU^A_I(p^*, w) &= p^*\left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)V_I(w, \pi_H, \sigma)\right] \\ &\quad + (1-p^*)\left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_I(\pi_L, \sigma)\right] - \psi(r^*_{A(I)}), \end{aligned}$$

with $p^* \equiv p(r^*_{A(I)}, r^*_{B(I)})$ and $V_I(\pi_L, \sigma) = U(\sigma)$. The expected utility of partner B is symmetric. Applying the Envelope Theorem we get

$$\frac{dEU_I^A(p^*, w)}{dw} = \frac{\partial EU_I^A(p^*, w)}{\partial r_{B(I)}} \frac{dr^*_{B(I)}}{dw} + p^* q(1-q) \frac{\partial V_I(w, \pi_H, \sigma)}{\partial w}.$$

Note that $\partial EU_I^A(\cdot)/\partial r_{B(I)} > 0$. We need to consider three cases: (i) $w \leq \underline{w}_I$, (ii) $w > \overline{w}_I$; and (iii), $\underline{w}_I < w \leq \overline{w}_I$. For the first two cases we know that $dr^*_{B(I)}/dw = 0$ and $\partial V_I/\partial w = 0$; thus, $dEU_I^A(p^*, w)/dw = 0$. For $\underline{w}_I < w \leq \overline{w}_I$ we know that $dr^*_{B(I)}/dw > 0$ and $\partial V_I/\partial w > 0$; thus, $dEU_I^A(p^*, w)/dw > 0$. This also implies that $EU_I^A(p^*, w)$ is maximized for $w \geq \overline{w}_I$. □

Proof of Lemma 8.

Under joint asset ownership the expected utility of partner A at date 0 is given by

$$\begin{aligned} EU_J^A(p^*, w) &= p^* \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) V_J(\pi_H, \sigma)\right] \\ &\quad + (1-p^*) \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) V_J(w, \pi_L, \sigma)\right] - \psi(r^*_{A(I)}), \end{aligned}$$

with $p^* = p(r^*_{A(J)}, r^*_{B(J)})$ and $V_J(\pi_H, \sigma) = 2U(\pi_H)$. The expected utility of partner B is symmetric. Applying the Envelope Theorem yields

$$\begin{aligned} \frac{dEU_J^A(p^*, w)}{dw} &= \frac{\partial EU_J^A(p^*, w)}{\partial r_{B(J)}} \frac{dr^*_{B(J)}}{dw} + (1-p^*) q(1-q) \frac{\partial V_J(w, \pi_L, \sigma)}{\partial w} \\ &= \underbrace{\Phi_J(w) \frac{\partial p(\cdot)}{\partial r_{B(J)}} \frac{dr^*_{B(J)}}{dw}}_{\equiv \psi_1} + \underbrace{(1-p^*) q(1-q) \frac{\partial V_J(w, \pi_L, \sigma)}{\partial w}}_{\equiv \psi_2}, \end{aligned}$$

where

$$\begin{aligned} \Phi_J(w) &= \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) 2U(\pi_H)\right] \\ &\quad - \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) V_J(w, \pi_L, \sigma)\right] > 0. \end{aligned}$$

By definition, $\partial p(\cdot)/\partial r_{B(J)} > 0$. Moreover, recall from Proposition 4 that $dr^*_{B(J)}/dw < 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Thus, $\psi_1 < 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Furthermore, $\partial V_J(w, \pi_L, \sigma)/\partial w > 0$ for $\underline{w}_J \leq w < \overline{w}_J$, so that $\psi_2 > 0$ for $\underline{w}_J \leq w < \overline{w}_J$. We define w_J^* as the wealth level which satisfies $dEU_J^A(p^*, w)/dw = 0$ for $\underline{w}_J \leq w < \overline{w}_J$, and thus maximizes A's expected utility at date 0. Note that $\underline{w}_J \leq w_J^* < \overline{w}_J$ because $dr^*_{B(J)}/dw = 0$ and $\partial V_J(w, \pi_L, \sigma)/\partial w = 0$ for $w < \underline{w}_J$ and $w \geq \overline{w}_J$. To summarize, (i) $dEU_J^A(\cdot)/dw = 0$ for $w \leq \underline{w}_J$, $w \geq \overline{w}_J$, and $w = w_J^*$ (as $\psi_1 + \psi_2 = 0$), (ii) $dEU_J^A(\cdot)/dw > 0$ for $\underline{w}_J < w < w_J^*$ (as $\psi_1 + \psi_2 > 0$); and (iii), $dEU_J^A(\cdot)/dw < 0$ for $w_J^* < w < \overline{w}_J$ (as $\psi_1 + \psi_2 < 0$).

Finally note that $\lim_{\pi_H \to \infty} \Phi_J(w) = \infty$ as $dU(\pi_H)/d\pi_H > 0$ with $\lim_{\pi_H \to \infty} U(\pi_H) = \infty$. This implies that $\lim_{\pi_H \to \infty} \psi_1 = -\infty$ for $\underline{w}_J \leq w < \overline{w}_J$, while $\sup(\psi_2) < \infty$. Thus, there exists a threshold $\widehat{\pi}_H$ such that $dEU_J^A(\cdot)/dw < 0$ for all $\pi_H \geq \widehat{\pi}_H$ and $w \in (\underline{w}_J, \overline{w}_J)$, which implies a corner solution with $w_J^* \leq \underline{w}_J$. $\square$

Proof of Proposition 5.

Suppose $w \geq \overline{w} = \max\{\overline{w}_I, \overline{w}_J\}$. Under individual asset ownership, $V_I(w, \pi_H, \sigma) = 2U(\pi_H)$ for $w \geq \overline{w}_I$. Under joint asset ownership, $V_J(w, \pi_L, \sigma) = V_I(\pi_L, \sigma) = U(\widehat{\alpha}_I; \sigma)$ for $w \geq \overline{w}_J$. Moreover, recall from Proposition 4 that $r_I^*(w) = r_J^*(w)$ for all $w \geq \overline{w}$. Thus, $EU_I(r_I^*, w) = EU_J(r_J^*, w)$ for $w \geq \overline{w}$.

Next, recall from Lemma 7 that $dEU_I(\cdot)/dw > 0$ for $\underline{w}_I < w \leq \overline{w}_I$, where $EU_I(\cdot)$ is maximized for $w \geq \overline{w}_I$. Moreover, we know from Lemma 8 that $dEU_J(\cdot)/dw > 0$ for $\underline{w}_J < w < w_J^*$, and $dEU_J(\cdot)/dw < 0$ for $w_J^* < w \leq \overline{w}_J$, where $EU_J(\cdot)$ is maximized when $w = w_J^*$. This implies that $EU_J(\cdot) > EU_I(\cdot)$ for $w \in [w_J^*, \overline{w})$.

Finally we examine whether $EU_I(\cdot) > EU_J(\cdot)$ for some $w < w_J^*$. Suppose $\pi_H \to \pi_L$. We can then immediately see that $r_I^*(w) = r_J^*(w) = 0$, and hence, $EU_I(\cdot) > EU_J(\cdot)$. We define w_0 as the critical wealth level so that $EU_J(\cdot) > EU_I(\cdot)$ for $w \in [w_0, \overline{w})$. Note that $w_0 < w_J^*$ because $EU_J(\cdot) > EU_I(\cdot)$ for $w_J^* \leq w < \overline{w}$. Moreover, $w_0 \geq 0$ because, when π_H is sufficiently high, $EU_J(\cdot) > EU_I(\cdot)$ even for $w = 0$. Thus, joint asset ownership is strictly optimal for $w_0 \leq w < \overline{w}$, with $w_0 \in [0, w_J^*]$. According to Lemma 8, the optimal wealth level is then $w_J^* \in [0, \overline{w})$, with $w_J^* \leq \underline{w}_J$ for all $\pi_H \geq \widehat{\pi}_H$. $\square$

Liquidation Value of Assets and Debt Claims.

Suppose the partnership between A and B generates a minimum payoff $\underline{\pi} > 0$, with $\underline{\pi} < \pi_L < \pi_H$. The payoff $\underline{\pi}$ reflects the liquidation value of any tangible or intangible assets other than the partners' two original assets (the ones required for joint production). The liquidation value $\underline{\pi}$ accrues to the partners whenever they choose to dissolve their partnership, or their joint production fails (which occurs with probability $1 - \mu(\cdot)$). This allows the partners to define debt claims at date 0 or date 2, which we denote d_A and d_B. One can easily show that a partner's expected utility is linear in his debt claim d_i, $i \in \{A, B\}$. Thus, a positive liquidation value enables two wealth-constrained partners to transfer utility without affecting their effort incentives.

Ex-ante symmetry implies $d_A^* = d_B^* = \underline{\pi}/2$. However, when do A and B agree on changing the division of $\underline{\pi}$? Only if they have asymmetric outside options! Suppose that only A found an alternative partner, and consider individual asset ownership. B can then offer A up to $d_B = \underline{\pi}/2$ in order to (i) prevent A from leaving, and (ii), retain a higher profit share for himself (which strengthens B's effort incentives). Under joint asset ownership, A can relinquish his debt claim $d_A = \underline{\pi}/2$ to buy out B, instead of giving B a stake in his new partnership with C (which improves A's effort incentives). All this implies that a positive liquidation value acts like wealth in our model, and thus helps the partners to mitigate ex-post inefficiencies in case of asymmetric outside options.

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