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SENTIMENTS AND AGGREGATE DEMAND FLUCTUATIONS

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ABSTRACT

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Sentiments and Aggregate Demand Fluctuations*

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Abstract

We formalize the Keynesian insight that aggregate demand driven by sentiments can generate output fluctuations under rational expectations. When production decisions must be made under imperfect information about aggregate demand, optimal decisions based on sentiments can generate stochastic self-fulfilling rational expectations equilibria in standard economies without aggregate shocks, externalities, persistent informational frictions, or even any strategic complementarity. Our general equilibrium model is deliberately simple, but could serve as a benchmark for more complicated equilibrium models with additional features.

Keywords: Keynesian Self-fulfilling Equilibria, Sentiments, Sunspots

1 Introduction

We formalize the Keynesian insight that sentiments about aggregate demand can generate output and employment fluctuations in a rational expectations framework. In our benchmark model each firm must make a production decision before demand is realized, based on noisy signals about what its demand will be. The signals, based on initial inquiries, advance sales, early orders, market research and public forecasts about the state of the economy, provide imperfect information about firm-level demand and aggregate demand. After production decisions are made, demand is realized and prices adjust to clear the market. In this set-up firms have signal extraction problems that can lead to multiple equilibria and endogenous fluctuations in aggregate output.

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Self-fulfilling stochastic equilibria in our model are not based on randomizations over fundamental certainty equilibria. Since at such equilibria firms make their production decisions based on the correctly anticipated distribution of aggregate demand and their own idiosyncratic demand shocks, these self-fulfilling stochastic equilibria are consistent with rational expectations. Furthermore we obtain such equilibria even though we have strategic substitutability in firms' actions: the optimal production of each firm is a declining function of other firms' total output.

Our model is similar to that of Angeletos and Lao (2011) in that sentiments can drive output and that our self-fulfilling equilibria are not based on randomizations over fundamental certainty equilibria. The fundamental certainty equilibrium not driven by sentiments is in fact unique in our model. Our informational structure is also simple: trades take place in centralized markets rather than bilaterally through random matching, and at the end of the period all trading history is public knowledge. Informational asymmetries exist only within the period as firms decide on how much to produce on the basis of the signals they receive at the beginning of the period.

Our benchmark model is of course also related to the Lucas (1972) island model and its signal extraction problem. However unlike Lucas (1972) we obtain multiple rational expectations equilibria. In the absence of aggregate shocks, we get a unique rational expectations equilibrium in which output and aggregate demand are constant, and firms receive signals that reveal their idiosyncratic demand shocks. This is our fundamental certainty equilibrium. If however agents believe that their signals contain "information" about changes in aggregate demand, and that the signals attach sufficient (more than one half) weight to this information, then all firms will adjust their production in response. Furthermore there will exist an equilibrium belief about the distribution of aggregate demand that is self-fulfilling: if firms use this distribution in making optimal decisions then indeed this distribution of output will be realized over time.[1] So we obtain an additional rational expectations equilibrium that, in contrast to the fundamental certainty equilibrium, will exhibit aggregate fluctuations in output and employment despite the lack of any fundamental aggregate shocks. We characterize this self-fulfilling equilibrium and show that its mean output is lower than the output under the certainty equilibrium.

It may also be interesting to contrast the results of our model with those obtained under global games (see, for example, Morris and Shin, 1998). In global games multiple coordination equilibria can become unique once agents receive a small, noisy, private signal about an economic fundamental. By contrast in our model we start with a unique equilibrium that has constant output, but when we introduce perceived private uncertainty about aggregate demand, an endogenous variable, we obtain additional equilibria with stochastic output. These additional equilibria, however, disappear when the weight given to aggregate demand uncertainty in the noisy signal becomes small. Our paper is related to others in the global games literature where endogenous variables

[1] This may be interpreted as a correlated equilibrium. See footnote 4.

provide further information about the underlying fundamentals. Hellwig, Mukherji and Tsyvinki (2006) introduce an endogenous public signal, the market clearing interest rate for bonds, into a currency crisis model. The agents can then condition their demand for domestic bonds vs. foreign currency both on the endogenous interest rate and their private signal about the central bank's commitment to the peg. Angeletos and Werning (2011) introduce a publicly traded asset into a similar model. When the public signal is exogenous, agents rely heavily on the private signal if its relative precision is high. Since private signals are dispersed, multiple coordination equilibria can be ruled out. However, if the public signal, the price of the asset, is endogenous and its precision is related to that of the private signal, coordination is facilitated and the uniqueness of the equilibrium can be lost. In both cases the endogenous price helps disseminate information about the underlying fundamental to restore the multiplicity of coordination equilibria. Angeletos, Hellwig and Pavan (2006) also examine how policy choices can reveal information about fundamentals in global games (about the type of policy maker in this instance), and destroy the uniqueness of the equilibrium that would be obtained when policy choices are uninformative.[2] In our model, however, the mechanism for the multiplicity of equilibria arises directly through the effect of perceived aggregate demand uncertainty on the optimal output of firms. There exists a distribution of the perceived uncertainty which generates a self-fulfilling stochastic rational expectations equilibrium in addition to the constant output certainty equilibrium.[3]

In the sections below we describe first the benchmark model and derive the various equilibria. Section 5 introduces more general signal structures. Since we abstract from capital accumulation and avoid the persistence of informational rigidities, each period is independent of the past. We show in section 6 that we can obtain persistence in output fluctuations using a variety of mechanisms, including Markov sunspots across equilibria, multiple islands, productivity shocks, and time-varying parameters. Section 7 extends our main result to more general settings. In particular, Section 7.1 provides a more abstract version of the model that captures the main forces responsible for the existence of self-fulfilling stochastic equilibria. In section 7.2 we introduce a related competitive-market model where firms must make investment decisions before observing their idiosyncratic productivity shocks and the aggregate capital stock that determines the market rate of return. In section 7.3, we modify a canonical price setting model with imperfect information. We show that in all such models there exist self-fulfilling stochastic rational expectations equilibria in addition to a unique certainty equilibrium. Finally in section 8 we conclude.

[2] See Atkeson (2001) for an early discussion of endogenous information and multiplicity in models of global games, and Amador and Weil (2012) for a study of the welfare effects of endogenous public market signals in a microfounded monetary model.

[3] For a related model where endogenous market signals can generate multiple equilibria in a microfounded monetary model, see Gaballo (2012).

2 The Benchmark Model

The model has a representative household, a representative final goods producer, and a continuum of monopolistic intermediate-goods producers indexed by $j \in [0, 1]$. The intermediate-goods producers decide on how much to produce based on their observation of a noisy signal reflecting initial inquiries, advance sales, early orders and market research. Formally the signal is $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)z_t$ where ε_{jt} is an idiosyncratic demand shock to their own good j and z_t is the stochastic component of aggregate demand in the self-fulfilling equilibrium. In section 5 below, we generalize the signal structure to introduce a firm-specific *iid* noise to s_{jt}, and then a second noisy public signal on aggregate demand based on public forecasts of the economy.

2.1 Households

A representative household maximizes utility

$$\max E_0 \sum \beta^t [\log(C_t) - \psi N_t] \tag{1}$$

The budget constraint for the household is

$$C_t \leq \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t}, \tag{2}$$

where W_t denotes real wage and Π_t aggregate profit income from firms, all measured in final goods. Denoting Λ_t as the Lagrangian multiplier for the budget constraint, the first-order conditions imply

$$\Lambda_t = \frac{1}{C_t} = \psi \frac{P_t}{W_t}, \tag{3}$$

or

$$C_t = \frac{1}{\psi} \frac{W_t}{P_t}. \tag{4}$$

2.2 Final Goods Producers

The final goods firm produces output according to

$$Y_t = \left[\int \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{5}$$

where $\theta > 1$.The final goods producer maximizes profit

$$\max P_t \left[\int \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} - \int P_{jt} Y_{jt} dj. \tag{6}$$

The exponential $\frac{1}{\theta}$ on the idiosyncratic shock is just a normalization to simplify expressions later on. The first-order condition with respect to input Y_{jt} is

$$0 = P_t \left[\int \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} \epsilon^{\frac{1}{\theta}} Y_{jt}^{-\frac{1}{\theta}} - P_{jt}. \tag{7}$$

this implies

$$\frac{P_{jt}}{P_t} = Y_{jt}^{-\frac{1}{\theta}} (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} \tag{8}$$

$$Y_{jt} = \left(\frac{P_t}{P_{jt}} \right)^{\theta} \epsilon_{jt} Y_t. \tag{9}$$

Substituting the last equation into the production function and rearranging gives

$$P_t^{1-\theta} = \int \epsilon_{jt} P_{jt}^{1-\theta} dj. \tag{10}$$

2.3 Intermediate Goods Producers

Each intermediate goods firm produces good j to meet its demand Y_{jt} without perfect knowledge about either ϵ_{jt} or the aggregate demand Y_t which could also be random. Instead, as in the Lucas island model, they infer their demand from a signal s_{jt},

$$s_{jt} = \lambda \log \epsilon_{jt} + (1-\lambda) \log Y_t, \tag{11}$$

where λ reflects the weights assigned by firms to the idiosyncratic and aggregate components of demand. The signal is based on early orders, initial inquiries, advance sales, and market research. On the basis of its signal, the firm chooses its production to maximize profits.

An intermediate goods producer j has the production function

$$Y_{jt} = A N_{jt}. \tag{12}$$

So the firm maximizes expected nominal profits $\Pi_{jt} = P_{jt} Y_{jt} - \frac{W_t}{A} Y_{jt}$ by solving

$$\max_{Y_{jt}} E_t \left[P_t Y_{jt}^{1-\frac{1}{\theta}} (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A} Y_{jt} | s_{jt} \right]. \tag{13}$$

The first order condition for quantity Y_{jt} is given by

$$\left(1 - \frac{1}{\theta}\right) Y_{jt}^{-\frac{1}{\theta}} E_t \left[P_t (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} | s_{jt} \right] = \frac{1}{A} E_t \left[W_t | s_{jt} \right] |. \tag{14}$$

Using equation (4), we impose the equilibrium condition

$$C_t = Y_t = \frac{1}{\psi}\frac{W_t}{P_t}, \tag{15}$$

or

$$P_t = \frac{1}{\psi}\frac{W_t}{Y_t}. \tag{16}$$

Since we can either normalize the aggregate final good price P_t or the aggregate nominal wage to 1, for simplicity we choose to set $W_t = 1$. Equation (14) then becomes

$$Y_{jt} = \left\{ \left(1 - \frac{1}{\theta}\right) \frac{A}{\psi} E_t \left[(\epsilon_{jt})^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | s_{jt} \right] \right\}^{\theta}. \tag{17}$$

The final aggregate output is

$$Y_t^{1-\frac{1}{\theta}} = \int \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{1-\frac{1}{\theta}} dj \tag{18}$$

Note from (17) that the optimal firm output in equilibrium declines with aggregate output since $1 - \frac{1}{\theta} < 0$, which implies that we have strategic substitutability. Despite this, we will show that the rational expectations equilibrium is not unique.

An equilibrium consists of equations (17), (18), (4) and the final goods market clearing condition

$$C_t = Y_t = \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t}, \tag{19}$$

such that the quantities and prices are all consistent with each other.

3 The Certainty Equilibrium

There exists a fundamental certainty equilibrium in this economy, defined as the allocation with $Y_t = Y^*$ and $P_t = P^*$. Under this certainty equilibrium with constant aggregate demand information is perfect and the signal fully reveals the firm's own demand. Equation (17) becomes

$$Y_{jt}^{\frac{1}{\theta}} = \left(1 - \frac{1}{\theta}\right) \frac{A}{\psi} \epsilon_{jt}^{\frac{1}{\theta}} Y_t^{\frac{1-\theta}{\theta}}, \tag{20}$$

or if we use $P_t = \frac{1}{\psi}\frac{1}{Y_t}$, this equation is identical to

$$P_{jt} = \left(\frac{\theta}{\theta - 1}\right) \frac{1}{A} = \bar{P} \tag{21}$$

Substituting into equation (10) gives

$$P_t = \left(\frac{\theta}{\theta - 1}\right)\frac{1}{A}\left[\int \epsilon_{jt} dj\right]^{\frac{1}{1-\theta}}. \tag{22}$$

Hence, equation (4) implies

$$C^* = Y^* = \frac{A}{\psi}\left(1 - \frac{1}{\theta}\right)\left[\int \epsilon_{jt} dj\right]^{\frac{1}{\theta-1}}. \tag{23}$$

Since aggregate output is constant in the certainty equilibrium, the firms know their idiosyncratic shocks from their signals. Then if their demand curve shifts by ε_{jt} units, their optimal output will change in such a way as to leave their prices invariant, so all firms will charge the same price. (To see this substitute 8 into (14).) Substituting equation (20) into equation (18) gives

$$\begin{aligned} Y_t^{1-\frac{1}{\theta}} &= \int \epsilon_{jt}^{\frac{1}{\theta}}\left(\left(1 - \frac{1}{\theta}\right)\frac{A}{\psi}\epsilon_{jt}^{\frac{1}{\theta}} Y_t^{\frac{1-\theta}{\theta}}\right)^{\theta-1} dj \\ &= \left(\left(1 - \frac{1}{\theta}\right)\frac{A}{\psi} Y_t^{\frac{1-\theta}{\theta}}\right)^{\theta-1}\int \epsilon_{jt} dj. \end{aligned} \tag{24}$$

If without loss of generality we normalize $\left(1 - \frac{1}{\theta}\right)\frac{A}{\psi} = 1$ we have

$$Y_t^{\frac{\theta-1}{\theta}} = Y_t^{(1-\theta)\frac{\theta-1}{\theta}}\int \epsilon_{jt} dj, \tag{25}$$

or

$$\log Y_t = \frac{1}{\theta - 1}\log E\exp(\varepsilon_{jt}), \tag{26}$$

where $\varepsilon_{jt} \equiv \log \epsilon_{jt}$ has zero mean and variance σ_ε^2. Therefore, under the assumption of log normal distribution,

$$\log Y_t = \frac{1}{2(\theta - 1)}\sigma_\varepsilon^2 = \bar{\phi}_0, \tag{27}$$

which is an alternative way of expressing equation (23).

4 A Self-fulfilling Equilibrium

We conjecture that there exists another equilibrium, such that aggregate output is not a constant. In particular we assume that

$$\log Y_t = \phi_0 + z_t, \tag{28}$$

where z_t is a normally distributed random variable with zero mean and variance σ_z^2. The noisy signal received by each firm is

$$s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)z_t. \tag{29}$$

so that with fluctuations in aggregate output, the firm's signal is no longer fully revealing[4].

We may view z_t as a sentiment held by agents about aggregate demand, as perceived through their signals s_{jt}. We will show that in our self-fulfilling equilibrium the distribution of the sentiments $\{z_t\}$ assumed by the firms will be consistent with the realized distribution of aggregate output $\{Y_t\}$ given by equation (28).[5]

Proposition 1 *If $\lambda \in \left(0, \frac{1}{2}\right)$, there exists a self-fulfilling rational expectations equilibrium with stochastic aggregate output Y_t. Furthermore* $\log Y_t$ *is normally distributed with mean $\phi_0 = \frac{(1-\lambda)+(\theta-1)\lambda}{\theta(1-\lambda)}\bar{\phi}_0 < \bar{\phi}_0$ and variance $\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2$.*

Proof. See the Appendix. ■

If firms believe that their signals contain information about changes in aggregate demand in addition to the firm-level demand shocks, then these beliefs will partially coordinate their output responses, up or down, and sustain self-fulfilling fluctuations consistent with their beliefs about the distribution of output. Both the variance of the sentiment shock σ_z^2 and λ affect the firms' optimal output responses through their signal extraction problems. Given λ and the variance of the idiosyncratic shock σ_ε^2, for markets to clear for all possible realizations of the aggregate demand sentiment z_t, the variance σ_z^2 has to be precisely pinned down, as indicated in Proposition 1.[6] If however agents perceive that the signal comes with a low weight on aggregate as opposed to idiosyncratic demand, that is if $\lambda \in [0.5, 1]$ and the covariance of the signal with aggregate demand is low, then a positive variance σ_z^2 that will clear the markets for every z_t does not exist.

[4] Note that here we define the signal as the weighted sum of the idiosyncratic shock and the innovation to aggregate demand. The mean of the log of aggregate demand will be absorbed by the constant ϕ_0 in equation (28) and incorporated into output decisions of firms. So $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)y_t$ is equivalent to $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)z_t$ as ϕ_0 is common knowledge.

[5] Under an alternative approach, firms could simply start with the belief that their signals are correlated. They would choose optimal outputs based on their signals and their beliefs about the distribution (that is σ_z^2) of aggregate output. In a self-fulfilling equilibrium, the variance of aggregate output σ_z^2 would be restricted as in Propositions 1, or later below in Propositions 3 or 4, to ensure that the goods market clears. With this restriction, the distribution of shocks z_t and ϵ_{jt} (and also the distribution of the noise v_{jt} in section 5 below) would induce a distribution on the signals s_{jt}.Given their signals s_{jt} the firms would optimally choose y_{jt}. We can interpret the self-fulfilling equilibrium as a *correlated equilibrium* that follows from the optimal output choices of firms given the market clearing variance of aggregate output σ_z^2 and the distribution of signals induced by the shocks. Such equilibria are typically defined for finite games with a finite number of agents and discrete strategy sets, but for an extention to continuous games see Hart and Schmeidler (1989) and more recently Stein, Parillo, and Ozdaglar (2008). We thank Martin Schneider for alerting us to this point.

[6] To see this look at equations (A.10), (A.11) and (A.20) in the proof of the Proposition in the Appendix.

Since $Y_{jt} = \epsilon_{jt} Y_t^{1-\sigma}$ and $\sigma > 1$, in equilibrium firm-level outputs as well as markups depend negatively on aggregate output because intermediate goods are substitutes.[7] Hence, if we ignore firm-specific demand shocks, the certainty or fundamental equilibrium in the model will be unique as a result of this strategic substitutability. However, the optimal supply of the firm's output positively depends on firm-level demand shocks. Consequently, if firms cannot distinguish firm-level shocks from aggregate demand, informational strategic complementarities can arise, giving rise to self-fulfilling equilibria.

The optimal output of an intermediate goods firm declines with σ_z^2 as the firm attributes more of the signal to an aggregate demand shock. In the self-fulfilling equilibrium, σ_z^2 is determined at a value that will clear markets for all z. In particular, note that the mean output ϕ_0 in the self-fulfilling equilibrium will be lower than the output $\bar{\phi}_0$ under the certainty equilibrium, and the mean markup will be higher.

If $\lambda = 1$ and the signal attaches no weight to sentiments regarding aggregate demand, the signal fully reveals the idiosyncratic shock and the only equilibrium is the certainty equilibrium with constant aggregate output. When $\lambda = 1$ each firm produces according to its own demand shocks, and the aggregate output is constant. If $\lambda = 0$ on the other hand, then again the certainty equilibrium is the unique equilibrium. In this case firms do not know their own demand but they do know the aggregate demand. Hence each firm produces a constant quantity related to aggregate demand based on the expected value of their own demand shock. This is equivalent to setting the idiosyncratic shocks to a constant so that the certainty equilibrium is the only equilibrium.

5 More General Signal Structures

5.1 Imperfect signals with firm-specific noise

So far we have assumed that firms can get an initial signal for the overall demand for their product, but cannot disaggregate it into its components arising from idiosyncratic and aggregate demand. Since the signals are based on early and initial demand indications for each of the firms, they may well contain an additional firm-specific noise component. Suppose then that the signal takes the slightly more general form,

$$s_{jt} = v_{jt} + \lambda \varepsilon_{jt} + (1 - \lambda) z_t, \tag{30}$$

where v_{jt} is a pure firm-specific iid noise with zero mean and variance σ_v^2.

As before we define

$$\log Y_t = y_t = \phi_0 + z_t$$

[7] The marginal cost is proportional to aggregate output in the Dixit-Stiglitz model. Hence, the markup is inversely related to the aggregate demand. Both the marginal cost and firms' markups are constant whenever the aggregate demand is constant.

and we also set $\mu = \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\sigma_v^2+\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\sigma_z^2}$. In this setup, both the certainty equilibrium and the self-fulfilling equilibrium will be different from those of the benchmark setting of Proposition 1. We first state the result for the certainty equilibrium.

Proposition 2 *Under the signal given by (30) there is a constant certainty equilibrium,* $y_t = \tilde{\phi}_0$, *given by*

$$\begin{aligned} \tilde{\phi}_0 &= \frac{1}{2}\left[\left(\frac{\theta + \theta\mu\lambda(\theta-1) + (\theta\mu\lambda(\theta-1))^2}{\theta^2(\theta-1)}\right)\sigma_\varepsilon^2 + (\theta-1)(\theta\mu)^2\sigma_v^2\right] \\ &= \bar{\phi}_0\left(\frac{\theta + \theta\mu\lambda(\theta-1) + (\theta\mu\lambda(\theta-1))^2}{\theta^2}\right) + (\theta-1)(\theta\mu)^2\sigma_v^2 \end{aligned} \tag{31}$$

Proof. *See the Appendix.* ■

Note that if $\sigma_v^2 = 0$, then $\mu = \frac{1}{\theta\lambda}$ and $\phi_0 = \bar{\phi}_0$, so the solution reduces the previous benchmark case given by (27). The self-fulfilling equilibrium is given by the following Proposition.

Proposition 3 *Let* $\lambda < \frac{1}{2}$, *and* $\sigma_v^2 < \lambda(1-2\lambda)\sigma_\varepsilon^2$. *In addition to the certainty equilibrium given in Equation (31), there also exists a self-fulfilling rational expectations equilibrium with stochastic aggregate output,* $\log Y_t$, *that has a mean*

$$\phi_0 = \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\lambda)}{\theta(1-\lambda)}\frac{1}{(\theta-1)}\right)\sigma_\varepsilon^2 - \frac{(\theta-1)\sigma_v^2}{2\theta^2(1-\lambda)^2}$$

and a variance $\sigma_y^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2\theta}\sigma_v^2$.

Proof. See the Appendix. ■

Notice that if either $\lambda \geq \frac{1}{2}$, or $\sigma_v^2 > \lambda(1-2\lambda)\sigma_\varepsilon^2$, then $\sigma_z^2 < 0$, suggesting that the only equilibrium is $z_t = 0$. Hence, to have a self-fulfilling expectations equilibrium, we require $\lambda \in \left(0, \frac{1}{2}\right)$ and $\sigma_v^2 < \lambda(1-2\lambda)\sigma_\varepsilon^2$. This pins down the equilibrium value of $\sigma_z^2 > 0$, the variance of z_t or aggregate output as a function of σ_ε^2 and σ_v^2. Note that introducing the extra noise v_{jt} into the signal makes output in the self-fulfilling equilibrium less volatile. This is in contrast to the previous case where the signal was $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)z_t$, and the variance of output was $\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2$. The reason for the smaller volatility of output when $\sigma_v^2 > 0$ is that the signal now is more noisy, and firms attribute a smaller fraction of the signal to demand fluctuations. Note however that this requires the additional restriction that the variance of the extra noise cannot be too big, $\sigma_v^2 < \lambda(1-2\lambda)\sigma_\varepsilon^2$, to ensure that $\sigma_z^2 > 0$.

5.2 Multiple Sources of Signals

The government and public forecasting agencies as well as news media often release their own forecasts of the aggregate economy. Such public information may influence and coordinate output decisions of firms and affect the equilibria. Suppose firms receive two independent signals, s_{jt} and s_{pt}. The firm-specific signal s_{jt} is based on a firm's' own preliminary information about its demand and is identical to that in equation (30). The public signal in the case of the self-fulfilling equilibrium is

$$s_{pt} = z_t + e_t \tag{32}$$

where we can interpret e_t as common noise in the public forecast of aggregate demand with mean 0 and variance σ_e^2.

We also assume that $\sigma_e^2 = \gamma\sigma_z^2$, where $\gamma > 0$. This assumption states that the variance of the forecast error of the public signal for aggregate demand is proportional to the variance of z, or equilibrium output. Then in the certainty equilibrium where output is constant over time, the public forecast of output is correct and constant as well.[8]

Proposition 4 *If $\lambda < \frac{1}{2}$, and $\sigma_v^2 < \lambda(1-2\lambda)\sigma_\varepsilon^2$, then there exists a self-fulfilling rational expectations equilibrium with stochastic aggregate output* $\log Y_t = y_t = z_t + \eta e_t + \phi_0 \equiv \hat{z}_t + \phi_0$, *which has mean* $\phi_0 = \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\lambda)}{\theta(1-\lambda)}\frac{1}{(\theta-1)}\right)\sigma_\varepsilon^2 - \frac{(\theta-1)\sigma_v^2}{2\theta^2(1-\lambda)^2}$ *and variance* $\sigma_y^2 = \sigma_{\hat{z}}^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2\theta}\sigma_v^2 > 0$ *with* $\eta = -\frac{\sigma_z^2}{\sigma_e^2} = -\frac{1}{\gamma}$. *In addition, there is a certainty equilibrium with constant output identical to that given in Proposition 2 with* $\sigma_z^2 = \gamma\sigma_e^2 = 0$.

Proof. See the Appendix. ■

As shown in the proof of Proposition 4, when $\sigma_{\hat{z}}^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2\theta}\sigma_v^2$, the optimal weight that firms place on the public signal is zero. Nevertheless aggregate output is stochastic, and driven by the volatility of $\hat{z}_t \equiv z_t + \eta e_t$.

It is easy to see that the certainty equilibrium of Proposition 2 with $\sigma_z^2 = 0$ also applies Proposition 4 since we also have $\sigma_e^2 = \gamma\sigma_z^2 = 0$, i.e. the public signal also becomes a constant. We can then directly apply Proposition 2 to find the equilibrium output (see the proof in Appendix A4).

6 Persistence

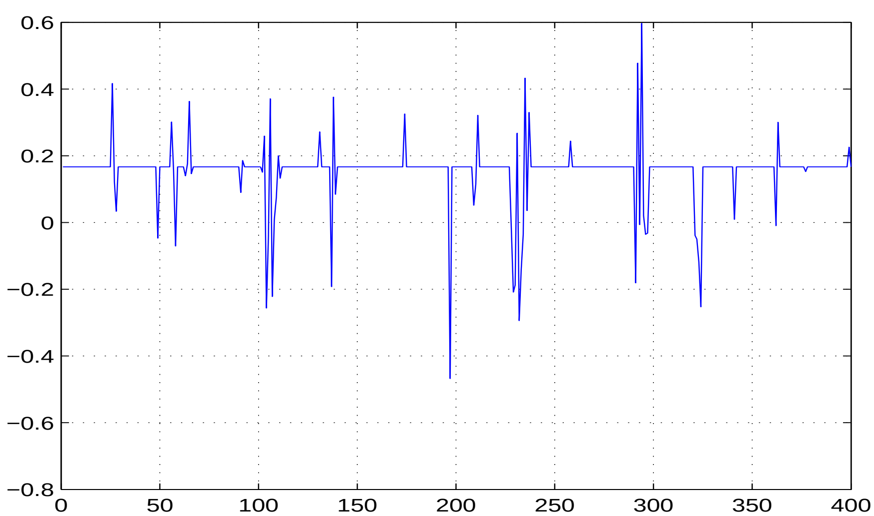
We have shown that imperfect information can lead to self-fulfilling fluctuations. By construction, these fluctuations are *iid* across time. We now extend our baseline model to allow for persistent

[8] See also the proof of Proposition 4 in the Appendix.

fluctuations in various ways. First, we show that it is possible to construct Markov sunspot equilibria. Depending on the sunspot, the agents coordinate on either the certainty equilibrium or on the stochastic equilibrium. Second, we consider a Multiple-Island Economy, in which a fraction α_t of islands are in the certainty equilibrium while the rest are in the uncertainty equilibrium. The total output of all islands hence fluctuates persistently if α_t are hit by persistent shocks. Third we introduce productivity shocks that can be persistent, and we show that output in a self-fulfilling equilibrium inherits the stochastic properties of the productivity shock. Finally, we introduce time variation into the parameters of the signal, or more precisely, into the weights λ and $(1-\lambda)$ entering the sum of the idiosyncratic and the aggregate demand shocks. Such variations generate an expected time variation or GARCH behavior in the equilibrium variance of output. In all these cases, the persistence of output fluctuations does not require persistent imperfection information across periods.

6.1 Markov Sunspot Equilibria

We now construct a persistent sunspot equilibrium with Markov transitions between the certainty equilibrium and the self-fulfilling stochastic equilibrium. To construct such an equilibrium, we introduce a sunspot $S_t = 1$ or 0. We have the transition probabilities $\Pr(S_t = 1|S_{t-1} = 1) = \rho$ and $\Pr(S_t = 0|S_{t-1} = 0) = \xi\rho$. Then the stationary distribution is $\Pr(S_\infty = 1) = \frac{1-\xi\rho}{1-\rho+1-\xi\rho}$, and $\Pr(S_\infty = 0) = \frac{1-\rho}{1-\rho+1-\xi\rho}$. The agents observe the sunspots first and if $S_t = 1$, the coordinate on the certainty equilibrium but if $S_t = 0$, they coordinate on the uncertainty equilibrium.



Simulated Path of Output Over Time

A simulated path of aggregate output is presented in Figure 1. We set $\theta = 4$, $\sigma_{\varepsilon}^2 = 1$, $\rho = 0.95$, and $\xi = 0.7$. For these parameters the certainty equilibrium is $\log Y_t = \frac{1}{2(\theta-1)}\sigma_{\varepsilon}^2 = \bar{\phi}_0 = 0.1667$. Figure 1 illustrates how the economy alternates between periods of calm (the certainty equilibrium) and high volatility (the self-fulfilling stochastic equilibrium). Of course the frequency and duration of volatile periods can be adjusted by changing the parameters of the transition matrix, ρ and ξ. Because the self-fulfilling stochastic equilibrium has a lower mean output and employment, an econometrician who does not believe in sunspots may incorrectly infer from the fluctuations that the economy has been hit by a permanent negative technology shock with a higher variance. Conversely, because the certainty equilibrium has a higher mean and a lower variance, an econometrician may incorrectly infer that the economy has entered a period of "great moderation" under "good luck".

6.2 Multiple-Island Economy

Now consider an economy with a continuum of identical islands of measure 1. The households on each island have utility functions as in equation (1). Let us index each island by $i \in [0, 1]$. Each island may have one of two possible equilibria, a certainty or a self-fulfilling equilibrium, namely $\log Y_t^i = \bar{\phi}_0$, or $\log Y_t^i = \phi_0 + z_t^i$, where z_t^i is normally distributed variable with mean zero and variance $\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_{\varepsilon}^2$. The aggregate economy is

$$Y_t = \int_0^1 Y_t^i di = E\exp(\log Y_t^i)di. \tag{33}$$

To construct aggregate sunspot fluctuations in output, let a fraction $1 - \alpha_t$ of islands have the self-fulfilling equilibria in each period. So we have

$$Y_t = \alpha_t \exp(\bar{\phi}_0) + (1 - \alpha_t)\exp(\hat{\phi}_0), \tag{34}$$

where

$$\hat{\phi}_0 = \phi_0 + \frac{1}{2}\sigma_z^2 = \frac{1}{2}\frac{(1-\lambda)^2 + (\theta-1)\,\lambda(2-3\lambda)}{\theta(1-\lambda)^2(\theta-1)}\sigma_{\varepsilon}^2. \tag{35}$$

We now show that $\hat{\phi}_0$ is no larger than $\bar{\phi}_0$. Notice that the term $\hat{\phi}_0 \leq \bar{\phi}_0$ is equivalent to

$$\frac{(1-\lambda)^2 + (\theta-1)\,\lambda(2-3\lambda)}{\theta(1-\lambda)^2(\theta-1)} \leq \frac{1}{\theta-1}, \tag{36}$$

or

$$\lambda(2-3\lambda) \leq (1-\lambda)^2. \tag{37}$$

Rearranging terms, the above inequality can be written as:

$$1-4\lambda+4\lambda^2=(1-2\lambda)^2\geq 0, \tag{38}$$

This inequality holds strictly if $\lambda\neq\frac{1}{2}$. Since α_t can be any stochastic sunspot process, it can generate persistence in output according to equation (34). An increase in α_t allows more islands to coordinate on the certainty equilibrium, which reduces overall uncertainty and results in a higher aggregate output.

6.3 Productivity Shocks

If productivity A_t is a stochastic process that firms can observe before making production decisions, in the self fulfilling equilibrium of our benchmark model we can express output as

$$\log Y_t=\phi_0+z_t+\log A_t. \tag{39}$$

Using equation (17) but now setting $\left(1-\frac{1}{\theta}\right)\frac{1}{\psi}$ instead of $\left(1-\frac{1}{\theta}\right)\frac{A_t}{\psi}$ to unity, market clearing requires the sum of log outputs of firms to equal aggregate log output for every z_t, as in equation (A.10). However, as easily computed, the term $\log A_t$ now cancels out, so equilibrium is consistent with any stochastic process for $\log A_t$. Since preferences are logarithmic in consumption and linear in leisure, and consumption is equal to output, optimal labor supply is constant and independent of A_t. Therefore $\log Y_t$ inherits the stochastic properties of A_t and exhibits the same persistence as $\log A_t$ in the self-fulfilling equilibrium. Similarly, in the fundamental equilibrium $\log Y_t$ also includes $\log A_t$ as an additive term with a coefficient of 1, but as before, does not depend on $z_{t.}$

6.4 Time-varying λ

Suppose λ is time varying and follows the AR(1) process:

$$\lambda_t=\rho\lambda_{t-1}+(1-\rho)\,\bar{\lambda}+v_t, \tag{40}$$

where $\bar{\lambda}=0.25$ and the *iid* shock v_t has support $\left(-\left(1-\rho\right)\bar{\lambda},(1-\rho)\bar{\lambda}\right)$. It can be shown that λ_t is bounded between 0 and $\frac{1}{2}$. In this economy, all aggregate variables in the self-fulfilling equilibrium are subject to additional shocks so that the variance of equilibrium aggregate demand is time-varying across periods, with

$$\sigma_z^2(t)=\frac{\lambda_t\,(1-2\lambda_t)}{(1-\lambda_t)^2\theta}\sigma_\varepsilon^2, \tag{41}$$

leading to GARCH behavior even though σ_ε^2 is constant. This fits the observed behavior of unemployment and inflation (where GARCH behavior was first discovered). The time variability of λ

may reflect changes in how the signal captures the idiosyncratic and perceived aggregate demand shocks over time. Note that λ enters the "constant" term in Propositions 1-4, which in equilibrium would also vary across periods.

7 General Models

So far we have relied on a particular general equilibrium model to generate self-fulfilling stochastic equilibria. In this section, we show that such equilibria can exist in more general settings. We first use a more abstract model to illustrate this point. We then construct two additional examples to show that our approach to constructing self-fulfilling stochastic equilibria can be applied in other economic environments, where an individual's action depends on his/her expectation of a mix of their idiosyncratic shock and the aggregate of actions.

7.1 A More Abstract Model

To illustrate the forces at work that produce the self-fulfilling stochastic equilibrium, we can abstract from the household and production side of our model. Let us assume for simplicity that the economy is log-linear, so optimal log output (or investment, price, labor, etc.) of firms is given by the rule

$$y_{jt} = E_t\{[\beta_0\varepsilon_{jt} + \beta y_t] \, |s_{jt}\}. \tag{42}$$

This log-linear specification allows us to avoid any constant term in the equilibrium output, so we can maintain a zero mean for y_t. The coefficient β can be either negative or positive, so we can have either strategic substitutability or strategic complementarity in firms' actions. The signal s_{jt} is given by

$$s_{jt} = v_{jt} + \lambda\varepsilon_{jt} + (1-\lambda)\, y_t, \tag{43}$$

where both the exogenous noise v_{jt} and the idiosyncratic demand shock ε_{jt} are *iid* and normally distributed with a zero mean. Market clearing then requires

$$y_t = \int y_{jt} dj. \tag{44}$$

In the certainty equilibrium y_t is constant, so equation (42) yields

$$y_{jt} = \beta y_t + \frac{\lambda\beta_0\sigma_\varepsilon^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2} \left(v_{jt} + \lambda\varepsilon_{jt}\right). \tag{45}$$

Substituting the above solution into equation (44) and integrating give

$$y_t = \int y_{jt} dj = \beta y_t \tag{46}$$

So unless $\beta = 1$, in which case there is a continuum of certainty equilibria, the unique certainty equilibrium is given by $y_t = 0$.

In the self-fulfilling stochastic equilibrium, assume that y_t is normally distributed with zero mean and variance σ_z^2. Based on the simple response function given by equation (42), signal extraction implies

$$y_{jt} = \frac{\lambda\beta_0\sigma_\varepsilon^2 + (1-\lambda)\beta\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}\left[v_{jt} + \lambda\varepsilon_{jt} + (1-\lambda)\, y_t\right]. \tag{47}$$

Then market clearing requires

$$y_t = \int y_{jt}dj = \frac{\lambda\beta_0\sigma_\varepsilon^2 + (1-\lambda)\beta\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(1-\lambda)y_t. \tag{48}$$

Since this relationship has to hold for every realization of y_t, we need

$$\frac{\lambda\beta_0\sigma_\varepsilon^2 + (1-\lambda)\beta\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(1-\lambda) = 1, \tag{49}$$

which implies

$$\sigma_z^2 = \frac{\lambda(\beta_0 - (1+\beta_0)\lambda)\sigma_\varepsilon^2 - \sigma_v^2}{(1-\lambda)^2(1-\beta)}. \tag{50}$$

Thus, σ_z^2 is pinned down uniquely and it defines the self-fulfilling equilibrium. Note that if $\beta < 1$, then a necessary condition for σ_z^2 to be positive is $\lambda \in \left(0, \frac{\beta_0}{1+\beta_0}\right)$. If $\beta_0 = 1$, this restriction becomes $\lambda \in (0, 0.5)$, as in Propositions 1, 3 or 4. In particular, under the usual Dixit-Stiglitz specification with strategic substitutability across intermediate goods, we have $\beta = (1-\theta) < 0$. Note however that if $\beta > 1$, which may correspond to a special model with externalities, σ_z^2 will be positive if $\lambda \in \left(\frac{\beta_0}{1+\beta_0}, 1\right)$. If $\beta > 1$, firm output will respond more than proportionately to aggregate demand, a situation that may in some sense be unstable. However, in the self-fulfilling equilibrium where firms respond to the imperfect signal of aggregate demand, this more than proportionate response is moderated if the signal is weakly related to aggregate demand, that is if $\lambda \in \left(\frac{\beta_0}{1+\beta_0}, 1\right)$.[9]

7.2 A Competitive Model with Investment

There is a continuum of firms indexed by j. Each period, after observing a firm specific idiosyncratic productivity shock ε_{jt} and the market real wage W_t, firms hire labor. They maximize profits

$$\max_{N_{jt}}\left\{(\epsilon_{jt}K_{jt})^\alpha N_{jt}^{1-\alpha} - W_t N_{jt}\right\} \tag{51}$$

[9] In the knife-edge case where $\beta = 1$ we have a continuum of certainty equilibria since any y_t satisfies (46). Similarly if $\beta = 1$, $\lambda = \frac{\beta_0}{1+\beta_0}$ and $\sigma_v^2 = 0$, there is a continuum of self-fulfilling stochastic equilibria since any σ_z^2 satisfies (49).

where N_{jt} is labor and K_{jt} capital for firm j. Employment then is

$$N_{jt} = \left(\frac{1-\alpha}{W_t}\right)^{\frac{1}{\alpha}} \epsilon_{jt} K_{jt}, \tag{52}$$

and the revenue net of labor cost is

$$R_t \epsilon_{jt} K_{jt} = (\epsilon_{jt} K_{jt})^{\alpha} N_{jt}^{1-\alpha} - W_t N_{jt} = \alpha \left(\frac{1-\alpha}{W_t}\right)^{\frac{1-\alpha}{\alpha}} \epsilon_{jt} K_{jt}, \tag{53}$$

where R_t is the competitive market rate of return. The firms' investment decisions are made before observing ϵ_{jt} or R_t. Firm j has a convex cost function $\frac{A}{1+\gamma} K_{jt}^{1+\gamma}$ and solves

$$\max E_{jt} \left(R_t \epsilon_{jt} K_{jt}\right) |S_{jt} - \frac{A}{1+\gamma} K_{jt}^{1+\gamma}, \tag{54}$$

where S_{jt} is a signal received by firm j about the market rate of return for investment. Suppose $\gamma = 1$ (generalizing γ is straightforward) so firm j invests

$$K_{jt} = \frac{E_{jt} \left(R_t \epsilon_{jt}\right) |S_{jt}}{A}. \tag{55}$$

Define

$$K_t = \int_0^1 \epsilon_{jt} K_{jt} dj \tag{56}$$

Aggregate labor supply is $\bar{N} = 1$, so integrating (52) the labor market equilibrium requires

$$\left(\frac{1-\alpha}{W_t}\right)^{\frac{1}{\alpha}} K_t = \bar{N} = 1, \tag{57}$$

and the competitive market return on capital is

$$R_t = \alpha K_t^{\alpha - 1}. \tag{58}$$

Equilibrium is characterized by two equations. First, we have equation (56). Second, using (55) and (58) and setting $\frac{\alpha}{A} = 1$ without loss of generality, we have

$$K_{jt} = E_{jt} \left(K_t^{\alpha-1} \epsilon_{jt}\right) |S_{jt}. \tag{59}$$

To complete the model, we assume that the signal S_{jt} is noisy and is defined by the log-linear weighted sum: $s_{jt} \equiv \log S_{jt} = \lambda \log \epsilon_{jt} + (1-\lambda) \log K_t$.

Certainty Equilibrium The unique certainty equilibrium can be solved as $K_{jt} = K_t^{\alpha-1}\epsilon_{jt}$ and

$$K_t = K_t^{\alpha-1}\int_0^1 \epsilon_{jt}^2 dj, \tag{60}$$

or $K_t = \left[\int_0^1 \epsilon_{jt}^2 dj\right]^{\frac{1}{2-\alpha}}$. For example, if we assume, without loss of generality, that $\varepsilon_{jt} = \log\epsilon_{jt}$ is distributed normally with mean $-\sigma_\varepsilon^2$ and variance σ_ε^2, then $\int_0^1 \epsilon_{jt}^2 dj = 1$ and $K_t = 1$.

Uncertainty Equilibrium

Proposition 5 *Under the signal* $\log s_{jt} = \lambda\log\epsilon_{jt} + (1-\lambda)\log K_t$*, there exists another equilibrium such that* $k_t \equiv \log K_t = \bar{k} + z_t$ *where* z_t *represents investor sentiment and is normally distributed with zero mean and variance* σ_z^2*. In addition,* $\bar{k} = \frac{-\alpha(1-2\lambda)\lambda}{2(2-\alpha)(1-\lambda)^2}\sigma_\varepsilon^2 < 0$ *and* $\sigma_z^2 = \frac{\lambda(1-2\lambda)\sigma_\varepsilon^2}{(1-\lambda)^2(2-\alpha)}$.

Proof. *See the Appendix.* ■

Note in this example that, if we ignore the constant term $\bar{k}$, we can rewrite equation (59) in a log-linear form as

$$k_{jt} = E_{jt}[\varepsilon_{jt} + (\alpha-1)k_t|s_{jt}], \tag{61}$$

and rewrite equation (56) approximately as

$$k_t = \int_0^1 (\varepsilon_{jt} + k_{jt})dj = \int_0^1 k_{jt}dj. \tag{62}$$

This is just a special case of the model in section 7.1 with $\beta = \alpha - 1 < 0$, $\beta_0 = 1$ and $\sigma_v^2 = 0$. We can apply equation (50) directly to obtain σ_z^2. Again, as in our baseline model, the investment level in the uncertainty equilibrium is lower than that in the certainty equilibrium.

7.3 A Monetary Business Cycle Model with Imperfect Information

Our second example is based on Hellwig (2008). We introduce an idiosyncratic cost shock into the firm's problem in the Hellwig model. There is a continuum of firms indexed by $j \in [0,1]$. Each firm sets its (log-) price p_{jt} equal to its expectation of a target price p_{jt}^*, i.e. we have $p_{jt} = E_{jt}p_{jt}^*$. The target price depends on the aggregate price $p_t = \int p_{jt}dj$ and its unit cost of production. The unit cost in turn depends on an idiosyncratic cost shock ε_{jt} and aggregate output y_t (aggregate output y_t affects the common wage rate). Hence we have

$$p_{jt} = E_{jt}(p_t + \eta y_t + \varepsilon_{jt}), \tag{63}$$

where $\eta > 0$. To complete the model, we add a quantity equation as Hellwig (2008) did, so $y_t + p_t = m_t$. Suppose there is no monetary shock, we then have $y_t = -p_t$. We also assume that the firm receives a mixed signal $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)p_t + v_{jt}$ regarding its own cost shock and the aggregate price (average of other firms' pricing decisions). We then have

$$p_{jt} = E_{jt}\{[(1-\eta)p_t + \varepsilon_{jt}]|s_{jt}\}. \tag{64}$$

and

$$p_t = \int p_{jt}dj. \tag{65}$$

If we assume that ε_{jt} are normally distributed with mean 0 and variance σ_ε^2, the certainty equilibrium is $p_t = 0$. However, there also exists a stochastic equilibrium where p_t is normally distributed with mean 0 and variance

$$\sigma_z^2 = \frac{\lambda(1-2\lambda)\sigma_\varepsilon^2 - \sigma_v^2}{(1-\lambda)^2(1+\eta)}, \tag{66}$$

in accordance with equation (50).

8 Conclusion

We often talk about the microfoundations of the macroeconomy, but seldom discuss the macrofoundations of the microeconomy (as Keynes did in the past). In reality, the outcome of individual agents' optimal plans often depend crucially on macroeconomic conditions over which agents can have significant influence collectively but little influence individually. When agents' optimal decisions must be conditioned on their expectations of such macroeconomic conditions, such expectations can be self-fulfilling when synchronized.

We explored the Keynesian insight that changing sentiments or expectations about aggregate demand can generate self-fulfilling output fluctuations under rational expectations when information is noisy and imperfect. If production (or investment) decisions must be made in advance under uncertain demand (or rates of return), optimal decisions based on sentiments can generate self-fulfilling stochastic equilibria in simple production economies without externalities, persistent informational frictions, or aggregate shocks to fundamentals. Our results hold even though there is strategic substitutability rather than strategic complenentarity across the actions of agents (e.g., the optimal output of a firm declines with aggregate output and the optimal investment of each firm declines with the aggregate capital stock). Although our settings are very simple, the basic logic of our argument may be applied more generally, to richer and more complicated DSGE models.

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A Appendix

A. 1 Proof of Proposition 1

Proof. Equation (17) gives the optimal output of firms contingent on their signals s_{jt}, since it is derived using equations (3) and (4), it already embodies the market clearing for labor and consumption. It can be written as:

$$\begin{aligned} y_{jt} &\equiv \log Y_{jt} = \theta \log E_t \left[(\epsilon_{jt})^{\frac{1}{\theta}} Y_t^{\frac{1-\theta}{\theta}} | s_{jt} \right] && \text{(A.1)} \\ &= \theta \log E_t \left[\exp(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t) + \frac{1-\theta}{\theta}\phi_0 | s_{jt} \right] \\ &= \theta \frac{1-\theta}{\theta}\phi_0 + \theta \log E_t \left[\exp(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t) | s_{jt} \right] \\ &= (1-\theta)\,\phi_0 + \theta \log E_t \left[\exp(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t) | s_{jt} \right] \end{aligned}$$

In order to calculate the conditional expectation we first note that the conditional distribution of $\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t$ is still a normal distribution. Note also then that $E_t\left[\exp(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t)|s_{jt}\right]$ is the moment generating function of the normal random variable $(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t)|s_{jt}$ so that

$$E_t \left[\exp(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t) | s_{jt} \right] = \exp\left(E\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t | s_{jt} \right) + \frac{1}{2} var \left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t | s_{jt} \right) \right) \tag{A.2}$$

We have,

$$\begin{aligned} E(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t | s_{jt}] &= \frac{cov\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t, s_{jt}\right)}{var\left(s_{jt}\right)} s_{jt} && \text{(A.3)} \\ &= \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} (\lambda\varepsilon_{jt} + (1-\lambda) z_t). \end{aligned}$$

Denote the conditional variance by

$$\Omega_s = var(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t | s_{jt}]. \tag{A.4}$$

Since $\frac{1}{\theta}\varepsilon_{jt}$, $\frac{1-\theta}{\theta} z_t$ are Gaussian, the conditional variance Ω_s will not depend on observations s_{jt} and will be given by

$$\Omega_s = var\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t \right) - \frac{\left(cov\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t, s_{jt}\right)\right)^2}{var\left(\lambda\varepsilon_{jt} + (1-\lambda)\, z_t\right)} \tag{A.5}$$

We then have

$$
\begin{aligned}
y_{jt} &= (1-\theta)\phi_0 + \theta\frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(\lambda\varepsilon_{jt} + (1-\lambda)z_t) + \frac{\theta}{2}\Omega_s && \text{(A.6)}\\
&\equiv \varphi_0 + \theta\mu(\lambda\varepsilon_{jt} + (1-\lambda)z_t) && \text{(A.7)}
\end{aligned}
$$

where

$$
\begin{aligned}
\mu &= \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} && \text{(A.8)}\\
\varphi_0 &= (1-\theta)\phi_0 + \frac{\theta}{2}\Omega_s && \text{(A.9)}
\end{aligned}
$$

Now for equilibrium to hold we need aggregate demand to equal the output of the final good produced using the intermediate goods supplied. Markets will clear if, from equation (18), we have for each z_t,

$$
\begin{aligned}
(\phi_0 + z_t)(1-\frac{1}{\theta}) &= \log\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{1-\frac{1}{\theta}} dj && \text{(A.10)}\\
&= \log E\exp[(\frac{1}{\theta}\varepsilon_t + (1-\frac{1}{\theta})\,(\varphi_0 + \theta\mu(\lambda\varepsilon_t + (1-\lambda)z_t))]\\
&= (1-\frac{1}{\theta})\varphi_0 + \frac{1}{2}[\frac{1}{\theta} + (1-\frac{1}{\theta})\theta\mu\lambda]^2\sigma_\varepsilon^2\\
&\quad +\theta\mu(1-\lambda)z_t(1-\frac{1}{\theta})
\end{aligned}
$$

Matching the coefficients yields two constraints:

$$
\theta\mu = \frac{1}{1-\lambda}, \tag{A.11}
$$

and

$$
(1-\frac{1}{\theta})\phi_0 = (1-\frac{1}{\theta})\varphi_0 + \frac{1}{2}[\frac{1}{\theta} + (1-\frac{1}{\theta})\theta\mu\lambda]^2\sigma_\varepsilon^2 \tag{A.12}
$$

$$
(\frac{\theta-1}{\theta})\phi_0 = (\frac{\theta-1}{\theta})\varphi_0 + \frac{1}{2}[\frac{1}{\theta} + (\frac{\theta-1}{\theta})\theta\mu\lambda]^2\sigma_\varepsilon^2 \tag{A.13}
$$

$$
\begin{aligned}
\phi_0 &= \varphi_0 + \frac{\theta}{\theta-1}\frac{1}{2}[\frac{1}{\theta} + (\frac{\theta-1}{\theta})\theta\mu\lambda]^2\sigma_\varepsilon^2 && \text{(A.14)}\\
\phi_0 &= \varphi_0 + \frac{1}{2}[\frac{1}{\theta-1} + \theta\mu\lambda]^2\sigma_\varepsilon^2 && \text{(A.15)}
\end{aligned}
$$

Since $\theta\mu = \frac{1}{1-\lambda}$ we can solve for σ_z^2 so this equality holds. We have

$$\theta\mu = \theta\frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} = \frac{1}{1-\lambda} \tag{A.16}$$

$$(1-\lambda)\left(\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2\right) = \lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2 \tag{A.17}$$

or

$$\begin{aligned}
\left[-(1-\lambda)^2 + (1-\theta)(1-\lambda)^2\right]\sigma_z^2 &= \left[\lambda^2 - (1-\lambda)\lambda\right]\sigma_\varepsilon^2 && \text{(A.18)}\\
-(1-\lambda)^2\theta\sigma_z^2 &= \lambda(2\lambda-1)\sigma_\varepsilon^2 && \text{(A.19)}\\
\sigma_z^2 &= \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 \quad if\ \lambda < \frac{1}{2} && \text{(A.20)}
\end{aligned}$$

so we assume $0 < \lambda < \frac{1}{2}$.[10] Now we consider the two constants ϕ_0 and φ_0. First we have, using (A.16),

$$\begin{aligned}
\Omega_s &= var(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta}z_t | s_{jt}] && \text{(A.21)}\\
&= \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2 + \left(\frac{1-\theta}{\theta}\right)^2\sigma_z^2 - (\mu)\left[\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2\right]\\
&= \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2 + \left(\frac{1-\theta}{\theta}\right)^2\sigma_z^2 - \left(\frac{1}{\theta}\frac{1}{1-\lambda}\right)\left[\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2\right]\\
&= \left[\left(\frac{1}{\theta}\right)^2 - \frac{1}{1-\lambda}\frac{1}{\theta^2}\lambda\right]\sigma_\varepsilon^2 + \left[\left(\frac{1-\theta}{\theta}\right)^2 + \frac{\theta-1}{\theta^2}\right]\sigma_z^2
\end{aligned}$$

[10] Note that if we define $Q = (1-\lambda)\theta\mu$, then $\lim_{\sigma_z^2\to\infty} Q = 1-\theta < 0$, and if $\lambda < \frac{1}{2}$, $\lim_{\sigma_z^2\to 0} Q = \frac{1-\lambda}{\lambda} > 1$, $\frac{dQ}{d\sigma_z^2} < 0$ and $\frac{dQ}{d\sigma_\varepsilon^2} > 0$.

The second line follows from equations (A.5) and (A.11). Since $\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2$ from (A.20), we have

$$\begin{aligned}
\Omega_s &= \left[\left(\frac{1}{\theta}\right)^2 - \frac{1}{1-\lambda}\frac{1}{\theta^2}\lambda\right]\sigma_\varepsilon^2 + \left[\left(\frac{1-\theta}{\theta}\right)^2 + \frac{\theta-1}{\theta^2}\right]\sigma_z^2 \qquad (A.22) \\
&= \left[\left(\frac{1}{\theta}\right)^2 - \frac{1}{1-\lambda}\frac{1}{\theta^2}\lambda\right]\sigma_\varepsilon^2 + \left(\frac{1-\theta}{\theta}\right)\left[\left(\frac{1-\theta}{\theta}\right) - \frac{1}{\theta}\right]\sigma_z^2 \\
&= \frac{1}{\theta^2}\frac{1-2\lambda}{1-\lambda}\sigma_\varepsilon^2 + \frac{1+\theta^2-2\theta+\theta-1}{\theta^2}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 \\
&= \frac{1}{\theta^2}\frac{1-2\lambda}{1-\lambda}\sigma_\varepsilon^2 + \frac{\theta-1}{\theta^2}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2}\sigma_\varepsilon^2 \\
&= \frac{(1-\lambda)(1-2\lambda) + (\theta-1)\lambda(1-2\lambda)}{\theta^2(1-\lambda)^2}\sigma_\varepsilon^2
\end{aligned}$$

Then, from equation (A.9),

$$\begin{aligned}
\varphi_0 &= (1-\theta)\phi_0 + \frac{\theta}{2}\Omega_s \qquad (A.23) \\
&= (1-\theta)\phi_0 + \frac{1}{2\theta}\left[\frac{(1-\lambda)^2 - \lambda(1-\lambda) + (\theta-1)\lambda(1-2\lambda)}{(1-\lambda)^2}\right]\sigma_\varepsilon^2,
\end{aligned}$$

From equation (A.15) we have,

$$\phi_0 = \varphi_0 + \frac{\theta}{\theta-1}\frac{1}{2}[\frac{1}{\theta} + (1-\frac{1}{\theta})\theta\mu\lambda]^2\sigma_\varepsilon^2. \qquad (A.24)$$

Combining these implies

$$\phi_0 = (1-\theta)\phi_0 + \frac{\theta}{2}\Omega_s + \frac{\theta}{\theta-1}\frac{1}{2}[\frac{1}{\theta} + (1-\frac{1}{\theta})\theta\mu\lambda]^2\sigma_\varepsilon^2,$$

or

$$\begin{aligned}
\phi_0 &= \frac{1}{2}\Omega_s + \frac{1}{\theta-1}\frac{1}{2}[\frac{1}{\theta} + (1-\frac{1}{\theta})\theta\mu\lambda]^2\sigma_\varepsilon^2 \qquad (A.25) \\
&= \frac{1}{2}\Omega_s + \frac{1}{\theta-1}\frac{1}{2}[\frac{1}{\theta} + (1-\frac{1}{\theta})\frac{\lambda}{1-\lambda}]^2\sigma_\varepsilon^2
\end{aligned}$$

Simplifying further gives,

$$\begin{aligned}
\phi_0 &= \frac{1}{2}\frac{(1-\lambda)(1-2\lambda) + (\theta-1)\lambda(1-2\lambda)}{\theta^2(1-\lambda)^2}\sigma_\varepsilon^2 + \qquad (A.26) \\
&\quad \frac{1}{\theta-1}\frac{1}{2}[\frac{1}{\theta} + (1-\frac{1}{\theta})\frac{\lambda}{1-\lambda}]^2\sigma_\varepsilon^2
\end{aligned}$$

$$
\begin{aligned}
\phi_0 &= \frac{1}{2}\left[\frac{(1-\lambda)+(\theta-1)\,\lambda}{\theta^2(1-\lambda)^2}(1-2\lambda)+\frac{1}{\theta-1}\frac{((\theta-1)\lambda+(1-\lambda))^2}{\theta^2(1-\lambda)^2}\right]\sigma_\varepsilon^2 \qquad \text{(A.27)}\\
&= \frac{1}{2}\left[\left(\frac{(1-\lambda)+(\theta-1)\,\lambda}{\theta^2(1-\lambda)^2}\right)\frac{(1-2\lambda)(\theta-1)+(\theta-1)\lambda+(1-\lambda)}{\theta-1}\right]\sigma_\varepsilon^2\\
&= \frac{1}{2}\left[\frac{(1-\lambda)+(\theta-1)\,\lambda}{\theta^2(1-\lambda)^2}\frac{(1-\lambda)\theta}{\theta-1}\right]\sigma_\varepsilon^2\\
&= \frac{1}{2}\frac{(1-\lambda)+(\theta-1)\,\lambda}{\theta(1-\lambda)}\frac{1}{\theta-1}\sigma_\varepsilon^2\\
&= \bar{\phi}_0\frac{(1-\lambda)+(\theta-1)\,\lambda}{\theta(1-\lambda)}
\end{aligned}
$$

Therefore the outputs of intermediate goods firms, conditioned on signals $s_{jt}=\lambda\varepsilon_{jt}+(1-\lambda)z_t$, are given by,

$$
y_{jt}\equiv\varphi_0+\theta\mu(\lambda\varepsilon_{jt}+(1-\lambda)z_t). \qquad \text{(A.28)}
$$

They constitute a market clearing stochastic rational expectations equilibrium. Now to show that the mean of the self-fulfilling stochastic equilibrium exceeds that of the certainty equilibrium, note that

$$
\begin{aligned}
\frac{(1-\lambda)+(\theta-1)\,\lambda}{\theta(1-\lambda)}-1 &= \frac{(1-\lambda)+(\theta-1)\,\lambda-\theta(1-\lambda)}{\theta(1-\lambda)} \qquad \text{(A.29)}\\
&= \frac{(1-\lambda)(1-\theta)+(\theta-1)\,\lambda}{\theta(1-\lambda)}\\
&= \frac{(1-2\lambda)(1-\theta)}{\theta(1-\lambda)}<0
\end{aligned}
$$

for $\lambda<\frac{1}{2}$. This implies that $\phi_0<\bar{\phi}$, and the mean output of the self-fulfilling equilibrium is lower than that of the certainty equilibrium. ■

A. 2 Proofs of Propositions 2 and 3

We start with the proof of the self-fulfilling equilibrium, Proposition 3, and give the proof of Proposition 2 for the certainty equilibrium later below.

1. **The Self-Fulfillling Equilibrium**: Let $s_{jt}=v_{jt}+\lambda\varepsilon_{jt}+(1-\lambda)\,z_t$. Firms conjecture that output is equal to

$$
\log Y_t=y_t=\phi_0+z_t, \qquad \text{(A.30)}
$$

as before where ϕ_0, and σ_z^2 are constants to be determined. As in the previous case, the optimal output of a firm can be written as

$$
\begin{aligned}
y_{jt} &\equiv \log Y_{jt} = \theta \log E_t \left[\left(\epsilon_{jt}\right)^{\frac{1}{\theta}} Y_t^{\frac{1-\theta}{\theta}} | s_{jt} \right] \qquad \text{(A.31)} \\
&= \left(1-\theta\right) \phi_0 + \theta \log E_t \left[\exp\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t\right) | s_{jt} \right]
\end{aligned}
$$

Note that

$$
E_t \left[\exp\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t\right) | s_{jt} \right] = \exp\left(E\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t | s_{jt}\right) + \frac{1}{2} var\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t | s_{jt}\right)\right) \tag{A.32}
$$

where

$$
\begin{aligned}
E\left[\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t\right) | s_{jt}\right] &= \frac{cov\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t, s_{jt}\right)}{var\left(s_{jt}\right)} s_{jt} \qquad \text{(A.33)} \\
&= \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}\left(1-\lambda\right)\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + \left(1-\lambda\right)^2\sigma_z^2} \left(v_{jt} + \lambda\varepsilon_{jt} + \left(1-\lambda\right) z_t\right).
\end{aligned}
$$

Denote the conditional variance by

$$
\Omega_s = var\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t | s_{jt}\right]. \tag{A.34}
$$

Since $\frac{1}{\theta}\varepsilon_{jt}$, $\frac{1-\theta}{\theta} z_t$ are Gaussian, the conditional variance Ω_s will not depend on the observed s_{jt} and will be given by

$$
\Omega_s = var\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t\right) - \frac{\left(cov\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t, s_{jt}\right)\right)^2}{var\left(v_{jt} + \lambda\varepsilon_{jt} + \left(1-\lambda\right) z_t\right)} \tag{A.35}
$$

We then have

$$
\begin{aligned}
y_{jt} &= (1-\theta)\phi_0 + \theta \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}\left(1-\lambda\right)\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + \left(1-\lambda\right)^2\sigma_z^2} \left(v_{jt} + \lambda\varepsilon_{jt} + \left(1-\lambda\right) z_t\right) + \frac{\theta}{2}\Omega_s \qquad \text{(A.36)} \\
&\equiv \varphi_0 + \theta\mu\left(v_{jt} + \lambda\varepsilon_{jt} + \left(1-\lambda\right) z_t\right) \qquad \text{(A.37)}
\end{aligned}
$$

where now

$$
\mu = \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}\left(1-\lambda\right)\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + \left(1-\lambda\right)^2\sigma_z^2} \tag{A.38}
$$

$$
\varphi_0 = (1-\theta)\phi_0 + \frac{\theta}{2}\Omega_s \tag{A.39}
$$

Now for equilibrium to hold we need aggregate demand to equal the output of the final good produced using the intermediate goods supplied. Markets will clear if, from equation (18), we have for each z_t,

$$\begin{aligned}\left(1-\frac{1}{\theta}\right)(\phi_0+z_t) &= \log\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{1-\frac{1}{\theta}}dj \qquad (A.40)\\ &= \log E\exp\left[\frac{1}{\theta}\varepsilon_t+\left(1-\frac{1}{\theta}\right)[\varphi_0+\theta\mu(v_{jt}+\lambda\varepsilon_{jt}+(1-\lambda)z_t)]\right]\\ &= \left(1-\frac{1}{\theta}\right)\varphi_0+\left[\left(1-\frac{1}{\theta}\right)\theta\mu(1-\lambda)\right]z_t\\ &\quad+\frac{1}{2}\left[\frac{1}{\theta}+\left(1-\frac{1}{\theta}\right)\theta\mu\lambda\right]^2\sigma_\varepsilon^2+\frac{1}{2}\left[\left(1-\frac{1}{\theta}\right)\theta\mu\right]^2\sigma_v^2\end{aligned}$$

Matching the coefficients yields two constraints: If $\mu\neq 0$, then

$$\theta\mu=\frac{1}{1-\lambda}, \qquad (A.41)$$

and

$$\left(1-\frac{1}{\theta}\right)\phi_0=\left(1-\frac{1}{\theta}\right)\varphi_0+\frac{1}{2}\left[\frac{1}{\theta}+\left(1-\frac{1}{\theta}\right)\theta\mu\lambda\right]^2\sigma_\varepsilon^2+\frac{1}{2}\left[\left(1-\frac{1}{\theta}\right)\theta\mu\right]^2\sigma_v^2 \qquad (A.42)$$

$$\begin{aligned}\phi_0 &= \varphi_0+\frac{\theta}{\theta-1}\frac{1}{2}\left[\frac{1}{\theta}+\left(1-\frac{1}{\theta}\right)\theta\mu\lambda\right]^2\sigma_\varepsilon^2+\frac{1}{2}\frac{\theta}{\theta-1}\left[\left(1-\frac{1}{\theta}\right)\theta\mu\right]^2\sigma_v^2 \qquad (A.43)\\ \phi_0 &= \varphi_0+\left(\frac{\theta-1}{\theta}\left(\frac{\theta}{\theta-1}\right)^2\right)\frac{1}{2}\left[\frac{1}{\theta}+\left(\frac{\theta-1}{\theta}\right)\theta\mu\lambda\right]^2\sigma_\varepsilon^2+\frac{1}{2}\frac{\theta}{\theta-1}\left[\left(1-\frac{1}{\theta}\right)\frac{1}{1-\lambda}\right]^2\sigma_v^2 \qquad (A.44)\\ \phi_0 &= \varphi_0+\frac{\theta-1}{\theta}\frac{1}{2}\left[\frac{1}{\theta-1}+\theta\mu\lambda\right]^2\sigma_\varepsilon^2+\frac{1}{2}\left(\frac{\theta-1}{\theta}\right)(\theta\mu)^2\sigma_v^2 \qquad (A.45)\\ \phi_0 &= \varphi_0+\frac{\theta-1}{\theta}\frac{1}{2}\left(\left[\frac{1}{\theta-1}+\theta\mu\lambda\right]^2\sigma_\varepsilon^2+(\theta\mu)^2\sigma_v^2\right) \qquad (A.46)\end{aligned}$$

Notice $\theta\mu=\frac{1}{1-\lambda}$ (when $\mu\neq 0$) implies

$$\theta\mu=\theta\frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2+\frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\sigma_v^2+\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\sigma_z^2}=\frac{1}{1-\lambda} \qquad (A.47)$$

$$\sigma_v^2+\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\sigma_z^2=\lambda(1-\lambda)\sigma_\varepsilon^2+(1-\theta)(1-\lambda)^2\sigma_z^2 \qquad (A.48)$$

or we have

$$\left[-(1-\lambda)^2+(1-\theta)(1-\lambda)^2\right]\sigma_z^2 = \left[\lambda^2-(1-\lambda)\lambda\right]\sigma_\varepsilon^2+\sigma_v^2 \tag{A.49}$$

$$-(1-\lambda)^2\theta\sigma_z^2 = \lambda(2\lambda-1)\sigma_\varepsilon^2+\sigma_v^2 \tag{A.50}$$

$$\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2-\frac{1}{(1-\lambda)^2\theta}\sigma_v^2. \tag{A.51}$$

Notice that if either $\lambda \geq \frac{1}{2}$, or $\sigma_v^2 > \lambda(1-2\lambda)\sigma_\varepsilon^2$, then $\sigma_z^2 < 0$, suggesting that the only equilibrium is $z = 0$. Hence, to have a self-fulfilling expectations equilibrium, we require $\lambda \in \left(0, \frac{1}{2}\right)$ and $\sigma_v^2 < \lambda(1-2\lambda)\sigma_\varepsilon^2$. This pins down σ_z^2, the variance of z or of output as a function of σ_ε^2 and σ_v^2. Note that introducing the noise v_{jt} into the signal makes output in the self-fulfilling equilibrium less noisy: we had $\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2$ in the previous case where the signal was $s_{jt} = \lambda\varepsilon_{jt}+(1-\lambda)z_t$. The reason is that the signal is now more noisy, and firms attribute a smaller fraction of the signal to demand fluctuations.

Now we consider the two constants ϕ_0 and φ_0. First we have, using (A.16),

$$\begin{aligned}
\Omega_s &= var(\frac{1}{\theta}\varepsilon_{jt}+\frac{1-\theta}{\theta}z_t|s_{jt}] \qquad \text{(A.52)}\\
&= \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2+\left(\frac{1-\theta}{\theta}\right)^2\sigma_z^2-(\mu)\left[\frac{1}{\theta}\lambda\sigma_\varepsilon^2+\frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2\right]\\
&= \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2+\left(\frac{1-\theta}{\theta}\right)^2\sigma_z^2-\left(\frac{1}{\theta}\frac{1}{1-\lambda}\right)\left[\frac{1}{\theta}\lambda\sigma_\varepsilon^2+\frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2\right]\\
&= \left[\left(\frac{1}{\theta}\right)^2-\frac{1}{1-\lambda}\frac{1}{\theta^2}\lambda\right]\sigma_\varepsilon^2+\left[\left(\frac{1-\theta}{\theta}\right)^2+\frac{\theta-1}{\theta^2}\right]\sigma_z^2
\end{aligned}$$

The second line follows from equations (A.5) and (A.11). Since $\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2\theta}\sigma_v^2$ from

(A.20), we have

$$\begin{aligned}
\Omega_s &= \left[\left(\frac{1}{\theta}\right)^2 - \frac{1}{1-\lambda}\frac{1}{\theta^2}\lambda\right]\sigma_\varepsilon^2 + \left[\left(\frac{1-\theta}{\theta}\right)^2 + \frac{\theta-1}{\theta^2}\right]\sigma_z^2 && \text{(A.53)} \\
&= \left[\left(\frac{1}{\theta}\right)^2 - \frac{1}{1-\lambda}\frac{1}{\theta^2}\lambda\right]\sigma_\varepsilon^2 + \left(\frac{1-\theta}{\theta}\right)\left[\left(\frac{1-\theta}{\theta}\right) - \frac{1}{\theta}\right]\sigma_z^2 \\
&= \frac{1}{\theta^2}\frac{1-2\lambda}{1-\lambda}\sigma_\varepsilon^2 + \left(\frac{\theta-1}{\theta}\right)\left(\frac{\lambda\,(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2\theta}\sigma_v^2\right) \\
&= \frac{1}{\theta^2}\frac{1-2\lambda}{1-\lambda}\sigma_\varepsilon^2 + \frac{\theta-1}{\theta^2}\frac{\lambda\,(1-2\lambda)}{(1-\lambda)^2}\sigma_\varepsilon^2 - \frac{\theta-1}{\theta^2}\frac{1}{(1-\lambda)^2}\sigma_v^2 \\
&= \frac{(1-\lambda)(1-2\lambda) + (\theta-1)\,\lambda(1-2\lambda)}{\theta^2(1-\lambda)^2}\sigma_\varepsilon^2 - \frac{\theta-1}{\theta^2}\frac{1}{(1-\lambda)^2}\sigma_v^2 \\
&= \frac{(1-\lambda+(\theta-1)\,\lambda)(1-2\lambda)\sigma_\varepsilon^2 - (\theta-1)\,\sigma_v^2}{\theta^2(1-\lambda)^2} && \text{(A.54)}
\end{aligned}$$

Then, from equation (A.39),

$$\begin{aligned}
\varphi_0 &= (1-\theta)\phi_0 + \frac{\theta}{2}\Omega_s && \text{(A.55)} \\
&= (1-\theta)\phi_0 + \frac{1}{2\theta}\frac{(1-\lambda+(\theta-1)\,\lambda)(1-2\lambda)\sigma_\varepsilon^2 - (\theta-1)\,\sigma_v^2}{(1-\lambda)^2},
\end{aligned}$$

From equation (A.46) we have,

$$\phi_0 = \varphi_0 + \frac{\theta-1}{\theta}\frac{1}{2}\left(\left[\frac{1}{\theta-1} + \theta\mu\lambda\right]^2\sigma_\varepsilon^2 + (\theta\mu)^2\,\sigma_v^2\right) \tag{A.56}$$

Combining these implies

$$\begin{aligned}
\phi_0 &= (1-\theta)\phi_0 + \frac{1}{2\theta}\frac{(1-\lambda+(\theta-1)\,\lambda)(1-2\lambda)\sigma_\varepsilon^2 - (\theta-1)\,\sigma_v^2}{(1-\lambda)^2} \\
&\quad + \frac{\theta}{\theta-1}\frac{1}{2}[\frac{1}{\theta} + (1-\frac{1}{\theta})\theta\mu\lambda]^2\sigma_\varepsilon^2,
\end{aligned}$$

or

$$\begin{aligned}
\phi_0 &= \frac{1}{2}\frac{(1-\lambda+(\theta-1)\,\lambda)(1-2\lambda)\sigma_\varepsilon^2 - (\theta-1)\,\sigma_v^2}{\theta^2(1-\lambda)^2} \\
&\quad + \frac{1}{2}\frac{1}{\theta-1}[\frac{1}{\theta} + (1-\frac{1}{\theta})\theta\mu\lambda]^2\sigma_\varepsilon^2,
\end{aligned}$$

Simplifying further gives,

$$
\begin{aligned}
\phi_0 &= \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\,\lambda)(1-2\lambda)}{\theta^2(1-\lambda)^2}+\frac{1}{(\theta-1)}[\frac{(1-\lambda)+(\theta-1)\,\lambda}{\theta\,(1-\lambda)}]^2\right)\sigma_\varepsilon^2-\frac{(\theta-1)\,\sigma_v^2}{2\theta^2(1-\lambda)^2} \\
&= \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\,\lambda)}{\theta^2(1-\lambda)^2}\frac{(1-2\lambda)\,(\theta-1)+(1-\lambda)+(\theta-1)\,\lambda}{(\theta-1)}\right)\sigma_\varepsilon^2-\frac{(\theta-1)\,\sigma_v^2}{2\theta^2(1-\lambda)^2} \\
&= \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\,\lambda)}{\theta^2(1-\lambda)^2}\frac{(1-\lambda)\,(\theta-1)+(1-\lambda)}{(\theta-1)}\right)\sigma_\varepsilon^2-\frac{(\theta-1)\,\sigma_v^2}{2\theta^2(1-\lambda)^2} \\
&= \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\,\lambda)}{\theta^2(1-\lambda)^2}\frac{(1-\lambda)\,\theta}{(\theta-1)}\right)\sigma_\varepsilon^2-\frac{(\theta-1)\,\sigma_v^2}{2\theta^2(1-\lambda)^2} \\
&= \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\,\lambda)}{\theta(1-\lambda)}\frac{1}{(\theta-1)}\right)\sigma_\varepsilon^2-\frac{(\theta-1)\,\sigma_v^2}{2\theta^2(1-\lambda)^2} \\
&= \bar{\phi}_0\frac{(1-\lambda)+(\theta-1)\,\lambda}{\theta(1-\lambda)}-\frac{1}{2}\frac{(\theta-1)\,\sigma_v^2}{\theta^2(1-\lambda)^2}
\end{aligned}
$$

Therefore the outputs of intermediate goods firms, conditioned on signals $s_{jt}=v_{jt}+\lambda\varepsilon_{jt}+(1-\lambda)z_t$, are given by

$$y_{jt}\equiv\varphi_0+\theta\mu(v_{jt}+\lambda\varepsilon_{jt}+(1-\lambda)z_t). \tag{A.57}$$

They constitute a market clearing stochastic rational expectations equilibrium. Note also that $\phi_0<\bar{\phi}_0$ since $\frac{(1-\lambda)+(\theta-1)\lambda}{\theta(1-\lambda)}$ as in the proof of Proposition 1. We now turn to the case of the certainty equilibrium.

2. The Certainty Equilibrium: Now firms again take aggregate output as constant so $z_t=0$ and $\log Y_t=y_t=\phi_0$, but the signal $s_{jt}=\lambda\varepsilon_{jt}+v_{jt}$ gives them imperfect information on their idiosyncratic shock. We can compute the new certainty equilibrium by setting $z_t=\sigma_z^2=0$, we have

$$\mu = \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2}{\sigma_v^2+\lambda^2\sigma_\varepsilon^2} \tag{A.58}$$

$$\Omega_s = var(\frac{1}{\theta}\varepsilon_{jt}|s_{jt}] \tag{A.59}$$

$$= \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2-\mu\frac{1}{\theta}\lambda\sigma_\varepsilon^2 \tag{A.60}$$

$$= \left(\frac{1}{\theta}\right)^2(1-\mu\theta\lambda)\,\sigma_\varepsilon^2$$

$$
\begin{aligned}
\varphi_0 &= (1-\theta)\phi_0 + \frac{\theta}{2}\Omega_s = (1-\theta)\phi_0 + \frac{\theta}{2}\left(\frac{1}{\theta}\right)^2 (1-\mu\theta\lambda)\,\sigma_\varepsilon^2 \\
\phi_0 &= \varphi_0 + \frac{\theta-1}{\theta}\frac{1}{2}\left(\left[\frac{1}{\theta-1}+\theta\mu\lambda\right]^2 \sigma_\varepsilon^2 + (\theta\mu)^2\,\sigma_v^2\right)
\end{aligned}
$$

so that

$$
\begin{aligned}
\phi_0 &= (1-\theta)\phi_0 + \frac{\theta}{2}\left(\frac{1}{\theta}\right)^2 (1-\mu\theta\lambda)\,\sigma_\varepsilon^2 + \frac{\theta-1}{\theta}\frac{1}{2}\left(\left[\frac{1}{\theta-1}+\theta\mu\lambda\right]^2 \sigma_\varepsilon^2 + (\theta\mu)^2\,\sigma_v^2\right) \\
\phi_0 &= \frac{1}{2}\left(\frac{1}{\theta}\right)^2 (1-\mu\theta\lambda)\,\sigma_\varepsilon^2 + \frac{(\theta-1)}{\theta^2}\frac{1}{2}\left(\left[\frac{1}{\theta-1}+\theta\mu\lambda\right]^2 \sigma_\varepsilon^2 + (\theta\mu)^2\,\sigma_v^2\right) \\
&= \frac{1}{2}\left(\frac{1}{\theta}\right)^2\left[\left((1-\mu\theta\lambda) + (\theta-1)\left[\frac{1}{\theta-1}+\theta\mu\lambda\right]^2\right)\sigma_\varepsilon^2 + (\theta-1)\,(\theta\mu)^2\,\sigma_v^2\right] \\
&= \frac{1}{2}\left(\frac{1}{\theta}\right)^2\left[\left((1-\mu\theta\lambda) + \frac{(1+\theta\mu\lambda\,(\theta-1))^2}{\theta-1}\right)\sigma_\varepsilon^2 + (\theta-1)\,(\theta\mu)^2\,\sigma_v^2\right] \\
&= \frac{1}{2}\left(\frac{1}{\theta}\right)^2\left[\left(\frac{(1-\mu\theta\lambda)\,(\theta-1) + (1+\theta\mu\lambda\,(\theta-1))^2}{\theta-1}\right)\sigma_\varepsilon^2 + (\theta-1)\,(\theta\mu)^2\,\sigma_v^2\right] \\
&= \frac{1}{2}\left(\frac{1}{\theta}\right)^2\left[\left(\frac{(\theta-1) - \mu\theta\lambda\,(\theta-1) + 1 + 2\theta\mu\lambda\,(\theta-1) + (\theta\mu\lambda\,(\theta-1))^2}{(\theta-1)}\right)\sigma_\varepsilon^2 + (\theta-1)\,(\theta\mu)^2\,\sigma_v^2\right] \\
&= \frac{1}{2}\left[\left(\frac{\theta + \theta\mu\lambda\,(\theta-1) + (\theta\mu\lambda\,(\theta-1))^2}{\theta^2\,(\theta-1)}\right)\sigma_\varepsilon^2 + (\theta-1)\,(\theta\mu)^2\,\sigma_v^2\right] \\
&= \bar{\phi}_0\left(\frac{\theta + \theta\mu\lambda\,(\theta-1) + (\theta\mu\lambda\,(\theta-1))^2}{\theta^2}\right) + \frac{1}{2}\,(\theta-1)\,(\theta\mu)^2\,\sigma_v^2
\end{aligned}
$$

Note that if $\sigma_v^2 = 0$, then $\mu = \frac{1}{\theta\lambda}$ and $\phi_0 = \bar{\phi}_0$.

A. 3 Proof of Proposition 4

In our previous case, output was equal to $y_t = z_t + \phi_0$. Now the agent receives two signals. The first is $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)y_t + v_{jt}$, which is equivalent to $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)z_t + v_{jt}$ as ϕ_0 is common knowledge. The second signal is $s_{pt} = z_t + e_t$, where we can interpret e_t as common noise in the public forecast of aggregate demand. Conjecture that output is equal to

$$
\log Y_t = y_t = \phi_0 + z_t + \eta e_t, \tag{A.61}
$$

where ϕ_0, σ_z^2 and η are constants to be determined. In that case,

$$cov(s_{pt}, y_t) = \sigma_z^2 + \eta\sigma_e^2. \tag{A.62}$$

(Note that if $\eta = -\frac{\sigma_z^2}{\sigma_e^2}$, then this covariance term becomes zero.) The agent has two signals. The private signal is

$$s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)[z_t + \eta e_t] + v_{jt} \tag{A.63}$$

and the public signal is

$$s_{pt} = z_t + e_t \tag{A.64}$$

so we have

$$y_{jt} \equiv (1-\theta)\,\phi_0 + \theta \log E_t\left[\exp(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta}(z_t + \eta e_t))|s_{jt}, s_{pt}\right] \tag{A.65}$$

Since the random variables are assumed normal, we can write

$$y_{jt} \equiv (1-\theta)\,\phi_0 + \frac{\theta}{2}\Omega_s + \theta[\beta_0 s_{jt} + \beta_1 s_{pt}] \tag{A.66}$$

where Ω_s is the conditional variance of $x_{jt} = \frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta}(z_t + \eta e_t)$ based on s_{jt} and s_{pt}. Market clearing implies

$$Y_t^{1-\frac{1}{\theta}} = \int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{1-\frac{1}{\theta}} dj, \tag{A.67}$$

so taking logs and equating the stochastic elements on the left and right, we must have

$$\begin{aligned} \frac{z_t + \eta e_t}{\theta} &= [\beta_0 \int s_{jt} dj + \beta_1 s_{pt}], \qquad \text{(A.68)} \\ &= \beta_0(1-\lambda)(z_t + \eta e_t) + \beta_1(z_t + e_t) \end{aligned}$$

which requires

$$\frac{1}{\theta} = \beta_0(1-\lambda) + \beta_1 \tag{A.69}$$

$$\frac{\eta}{\theta} = \beta_0(1-\lambda)\eta + \beta_1 \tag{A.70}$$

If $\beta_1 = 0$ these two equations collapse to $\frac{1}{\theta} = \beta_0(1-\lambda)$.

We first explore the self-fulfilling equilibrium with stochastic output where $\beta_1 = 0$. Note that the optimal solutions for β_0 and β_1 must satisfy

$$Ex_{jt}s_{jt} - \beta_0\sigma_{s_{jt}}^2 - \beta_1 cov(s_{jt}, s_{pt}) = 0, \tag{A.71}$$

$$Ex_{jt}s_{pt} - \beta_0 cov(s_{jt}, s_{pt}) - \beta_1\sigma_{s_{pt}}^2 = 0. \tag{A.72}$$

From (A.71),

$$\beta_0 = \frac{\lambda \frac{1}{\theta}\sigma_\varepsilon^2 + (1-\lambda)\frac{1-\theta}{\theta}(\sigma_z^2 + \eta^2\sigma_e^2)}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2(\sigma_z^2 + \eta^2\sigma_e^2) + \sigma_v^2} = \frac{1}{\theta}\frac{1}{1-\lambda}, \tag{A.73}$$

which yields

$$\sigma_z^2 + \eta^2\sigma_e^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2}\sigma_v^2 \tag{A.74}$$

Again, as in Proposition 3, if either $\lambda \geq \frac{1}{2}$, or $\sigma_v^2 > \lambda(1-2\lambda)\sigma_\varepsilon^2$, then $\sigma_z^2 + \eta^2\sigma_e^2 \leq 0$ and there is only the certainty equilibrium.

Now we need to determine η. Notice that

$$Ex_{jt}s_{pt} = E\left[(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta}(z_t + \eta e_t)) \times (z_t + e_t)\right] = \frac{1-\theta}{\theta}(\sigma_z^2 + \eta\sigma_e^2). \tag{A.75}$$

and

$$\begin{aligned} cov(s_{jt}, s_{pt}) &= E(\lambda\varepsilon_{jt} + (1-\lambda)(z_t + \eta e_t)) \times (z_t + e_t) \\ &= (1-\lambda)(\sigma_z^2 + \eta\sigma_e^2) \end{aligned} \tag{A.76}$$

If $\beta_0 \neq 0$ in this case we have

$$\sigma_z^2 + \eta\sigma_e^2 = 0, \tag{A.77}$$

or

$$\eta = -\frac{\sigma_z^2}{\sigma_e^2} \tag{A.78}$$

and (A.72) is satisfied. By our assumption $\sigma_e^2 = \gamma\sigma_z^2$ we have $\eta = -\frac{1}{\gamma}$. Suppose that $\lambda < \frac{1}{2}$. We have to find out whether it is possible to have a rational expectation equilibrium satisfying $\sigma_z^2 > 0$. Note from (A.74) that

$$\sigma_z^2 + \eta^2\sigma_e^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2}\sigma_v^2 \tag{A.79}$$

Substituting η into the expression we then have

$$\begin{aligned} \left(\sigma_e^2\right)^2\sigma_z^2 + \left(\sigma_z^2\right)^2 &= \left(\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2}\sigma_v^2\right)\left(\sigma_e^2\right)^2 \\ \left(\sigma_e^2\right)^{-2}\left(\sigma_z^2\right)^2 + \sigma_z^2 &= \left(\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2}\sigma_v^2\right) \end{aligned} \tag{A.80}$$

Using the relationship between σ_e^2 and σ_z^2 we have

$$\frac{1+\gamma}{\gamma}\sigma_z^2 = \left(\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2}\sigma_v^2\right)$$

Notice that the above equation has an unique solution for $\sigma_z^2 > 0$:

$$\sigma_z^2 = \frac{\gamma}{1+\gamma}\left(\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2}\sigma_v^2\right) \tag{A.81}$$

If γ approaches zero, σ_z^2 also approaches to zero. However, since $\sigma_e^2 = \gamma\sigma_z^2$ and $\eta = -\frac{1}{\gamma}$, the variance of output is given by

$$\sigma_y^2 = \frac{1+\gamma}{\gamma}\sigma_z^2 = \left(\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2}\sigma_v^2\right), \tag{A.82}$$

which is not affected and the uncertainty equilibrium will continue to exist.

Finally, since the public signal is not informative at all, the firm's effective signal is only the private one. We can redefine

$$\hat{z}_t = z_t + \eta e_t = z_t - \frac{1}{\gamma}e_t \tag{A.83}$$

which then has variance

$$\sigma_{\hat{z}}^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2}\sigma_v^2 \tag{A.84}$$

where we again use $\sigma_e^2 = \gamma\sigma_z^2$ and $\eta = -\frac{1}{\gamma}$ to derive (A.84). So output will be as in Proposition 3,

$$y_t = z_t + \eta e_t + \phi_0 = \hat{z}_t + \phi_0, \tag{A.85}$$

where the constant term is $\phi_0 = \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\lambda)}{\theta(1-\lambda)}\frac{1}{(\theta-1)}\right)\sigma_\varepsilon^2 - \frac{(\theta-1)\sigma_v^2}{2\theta^2(1-\lambda)^2}$. With z_t redefined as $\hat{z}_t$, the property of output fluctuations is not affected.

We now turn to the certainty equilibrium. From (A.69) and (A.70), if $\beta_1 \neq 0$, we must have $\eta = 1$. Namely aggregate output will be

$$y_t = \phi_0 + z_t + e_t, \tag{A.86}$$

If the public signal is still as $s_{pt} = z_t + e_t$ it fully reveals aggregate demand y_t. The private signal would now be $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)[z_t + e_t] + v_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)[(y_t - \phi_0)] + v_{jt}$ where by construction $y_t - \phi_0$ will be known. If we define $\hat{z}_t = z_t + e_t$, and attempt to define an equilibrium analogous to the certainty equilibrium of Proposition 2 , with the difference that the aggregate demand shock

$\hat{z}_t = z_t + e_t$ is not taken as zero but is perfectly observed each period prior to the production decision, we reach a contradiction. Setting $z_t = 0$, the "constant" term ϕ_0 can be defined to include e_t and solved as in Proposition 2 as a function of time-invariant parameters of the model. However this will contradict the randomness of e_t unless $e_t = 0$ for all t. The certainty equilibrium of Proposition 2 with constant output is not compatible with a time-varying public forecast of aggregate demand since firms would forecast the constant output. The public signal $s_{pt} = z_t + e_t$ would be observed in the self-fulfilling equilibrium, but in the certainty equilibrium the public forecast of aggregate output would be a constant, and identical to the equilibrium in Proposition 2. If on the other hand we use our assumption that the variance of the forecast error of the public signal is proportional to the variance of z, that is if $\sigma_e^2 = \gamma\sigma_z^2$, then we can recover the certainty equilibrium of Proposition 2 where output is constant: for this equilibrium we would have $z_t = e_t = 0$ for all t.■

A. 4 Proof of Proposition 5

Notice according to equation (59), we have

$$\begin{aligned} k_{jt} &= \log K_{jt} = (\alpha-1)\bar{k} + \log E_{jt} \exp\{[(\alpha-1)z_t + \varepsilon_{jt}] \mid \lambda\varepsilon_{jt} + (1-\lambda)z_t\} \\ &= (\alpha-1)\bar{k} + \frac{(\alpha-1)(1-\lambda)\sigma_z^2 + \lambda\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)\sigma_z^2}[\lambda\varepsilon_{jt} + (1-\lambda)z_t] + \frac{1}{2}\Omega_s \\ &\equiv \hat{k} + \mu s_{jt} \end{aligned} \tag{A.87}$$

where Ω_s is the conditional variance of $(\alpha-1)z_t + \varepsilon_{jt}$ based on s_{jt}. Aggregate capital is

$$\begin{aligned} \log K_t &= \bar{k} + z_t = \log E \exp\{\varepsilon_{jt} + k_{jt}\} \\ &= \log E \exp\{\varepsilon_{jt} + \hat{k} + \mu\lambda\varepsilon_{jt} + \mu(1-\lambda)z_t\} \\ &= \hat{k} - \sigma_\varepsilon^2(1+\mu\lambda) + \frac{1}{2}\sigma_\varepsilon^2(1+\mu\lambda)^2 + \mu(1-\lambda)z_t \\ &= (\alpha-1)\bar{k} + \frac{1}{2}\Omega_s - \sigma_\varepsilon^2(1+\mu\lambda) + \frac{1}{2}\sigma_\varepsilon^2(1+\mu\lambda)^2 + \mu(1-\lambda)z_t \end{aligned} \tag{A.88}$$

This will hold for all realizations of z_t if

$$\mu = \frac{1}{1-\lambda}, \tag{A.89}$$

and

$$\bar{k} = (\alpha-1)\bar{k} + \frac{1}{2}\Omega_s - \sigma_\varepsilon^2(1+\mu\lambda) + \frac{1}{2}\sigma_\varepsilon^2(1+\mu\lambda)^2 \tag{A.90}$$

Solving (A.89) gives

$$(\alpha-1)(1-\lambda)^2\sigma_z^2+\lambda(1-\lambda)\sigma_\varepsilon^2=\lambda^2\sigma_\varepsilon^2+(1-\lambda)\sigma_z^2 \tag{A.91}$$

or

$$\sigma_z^2=\frac{\lambda(1-2\lambda)\sigma_\varepsilon^2}{(1-\lambda)^2(2-\alpha)}. \tag{A.92}$$

And the constant term is

$$\bar{k}=\frac{\frac{1}{2}\Omega_s+\sigma_\varepsilon^2\frac{2\lambda-1}{2(1-\lambda)^2}}{2-\alpha}. \tag{A.93}$$

Finally

$$\begin{aligned}\Omega_s &= (\alpha-1)^2\sigma_z^2+\sigma_\varepsilon^2-\frac{1}{1-\lambda}\left[(\alpha-1)(1-\lambda)\sigma_z^2+\lambda\sigma_\varepsilon^2\right]\\ &= (1-\alpha)(2-\alpha)\sigma_z^2+\frac{1-2\lambda}{1-\lambda}\sigma_\varepsilon^2\\ &= \frac{(1-\alpha)\lambda(1-2\lambda)\sigma_\varepsilon^2}{(1-\lambda)^2}+\frac{1-2\lambda}{1-\lambda}\sigma_\varepsilon^2\end{aligned}$$

hence we have

$$\begin{aligned}\bar{k} &= \frac{1}{2(2-\alpha)}[\frac{(1-\alpha)\lambda(1-2\lambda)\sigma_\varepsilon^2}{(1-\lambda)^2}+\frac{1-2\lambda}{1-\lambda}\sigma_\varepsilon^2+\sigma_\varepsilon^2\frac{2\lambda-1}{(1-\lambda)^2}]\\ &= \frac{-\alpha(1-2\lambda)\lambda}{2(2-\alpha)(1-\lambda)^2}. \qquad (A.94)\end{aligned}$$

Hence the uncertainty equilibrium has a lower mean than the certainty equilibrium. ■