NBER WORKING PAPER SERIES

HOMEWORK IN MONETARY ECONOMICS:
INFLATION, HOME PRODUCTION, AND THE PRODUCTION OF HOMES

S. Boragan Aruoba
Morris A. Davis
Randall Wright

Working Paper 18276
http://www.nber.org/papers/w18276

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 2012

We thank many friends and colleagues for helpful input, especially Chrisophe Andre, Jeff Campbell, Larry Christiano, Marcello Veracierto, Yu Zhu, and Gwen Eudey. Aruoba and Wright also thank the NSF, Davis and Wright thank the Lincoln Institute of Land Policy, and Wright thanks the Ray Zemon Chair in Liquid Assets at the Wisconsin School of Business for research support. The usual disclaimer applies. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Homework in Monetary Economics: Inflation, Home Production, and the Production of Homes
S. Boragan Aruoba, Morris A. Davis, and Randall Wright
NBER Working Paper No. 18276
August 2012
JEL No. E41,E52,R21

ABSTRACT

We study models incorporating money, household production, and investment in housing. Inflation, as a tax on market activity, encourages substitution into household production, and thus investment in household capital. Hence, inflation increases the (appropriately deflated) value of the housing stock. This is documented in various data sources. A calibrated model accounts for a fifth to a half of the observed relationships. While this leaves much to be explained, it demonstrates the channel is economically relevant. We also show models with home production imply higher costs of inflation than models without it, especially when home and market goods are close substitutes.

S. Boragan Aruoba
Department of Economics
University of Maryland
3105 Tydings Hall
College Park, MD 20742-7211
aruoba@econ.umd.edu

Morris A. Davis
University of Wisconsin
mdavis@bus.wisc.edu

Randall Wright
Department of Finance and Department of Economics
University of Wisconsin - Madison
Grainger Hall
975 University Ave
Madison, WI 53706
and NBER
rwright@bus.wisc.edu

It is sometimes heard that housing is a good hedge against inflation. Is this true? And, if so, one might ask *where and when* – i.e., in what countries and time periods – and of course, one might want to know *why*? To address these issues we think that one needs to look carefully at the data, build a rigorous theoretical model, and apply it quantitatively. This paper presents the results of our efforts in that endeavor.

Using various data sources, mainly for the U.S., although we also look at other countries, we find that appropriately deflated values of the housing stock are positively related to both inflation and nominal interest rates. While one may be able to think of different ways to interpret this fact, we pursue the following idea. In any reasonable monetary model, inflation is a tax on market activity as long as cash is used in at least some market transactions. Thus, inflation leads agents to substitute out of market and into nonmarket activity, and in particular into household production – e.g., when inflation is higher, people go out less and eat more meals at home, given that going out is cash intensive. Of course not all market activity uses cash, but it uses cash more than household production, since home-produced goods (like home-cooked meals) are generally not even traded, let alone traded using currency. Hence, inflation increases demand for inputs to household production, including time, and also capital, such as appliances and housing itself. In this way, inflation increases the value of the housing stock – perhaps mostly through prices in the short run, and more in quantities as supply catches up, but either way it increases.

We formalize this reasoning by integrating the literature on the microfoundations of monetary exchange, the literature incorporating household production into macro, and the literature that models housing explicitly.[1] This is motivated by much previous work showing that incorporating home production into otherwise standard nonmonetary macro models has

[1]For surveys on the microfoundations of monetary economics, see Nosal and Rocheteau (2011) or Williamson and Wright (2010); our setup is closest to Aruoba et al. (2011), which is a generalization of Lagos and Wright (2005). As an independent contribution to monetary economics, this is the first paper we know that builds an explicit retail sector into the framework. On home production, there are surveys by Greenwood et al. (1995) and Gronau (1997), but we review many of the more recent contributions below. We also review some of the literature on housing; as a preview, we follow Davis and Heathcote (2005).

significant effects.[2] We use our setup to formalize the economic intuition described above, to organize the empirical findings, and to measure the effects of inflation. The last application is natural, since it is well known that home production models can give different quantitative answers, compared to models without home production, to other policy questions. We want to see if it makes a difference for a classic monetary policy question: what is the cost of anticipated inflation?

In our theory, housing capital is valued as an input to household production, like market capital is valued as an input to market production in standard macro. We first prove several analytic results, then ask how well calibrated versions of the model capture the empirical findings. The model accounts for a sizable fraction, but not all, of the relationship between appropriately deflated measures of value of the housing stock, on the one hand, and inflation or nominal interest rates, on the other, while matching several additional observations of interest. To be precise, we can account for between a fifth to a half of the key relationships, depending on details. While this leaves a significant amount to be explained by other channels, it demonstrates that the effect we isolate is economically relevant. We also show that models with home production imply a higher cost of inflation than models without it, especially when home and market goods are relatively close substitutes,

In terms of other work, several papers study the relation between stock markets and inflation, which is similar in that equity and housing are both assets (see Geromichalos et al. 2007 for references). Other papers analyze house prices and inflation explicitly through

[2] By way of example, Benhabib et al. (1991) and Greenwood and Hercowicz (1991) put the ideas of Becker (1965,1988) into dynamic general equilibrium, and show models with home production do a better job than otherwise similar models matching key business-cycle moments. It has been shown that such models also account better for consumption (Baxter and Jerrmann 1999; Baxter 2010; Aguiar and Hurst 2005,2007*a*), investment (Gomme et al. 2001; Fisher 1997,2007), female labor-force participation (Greenwood et al. 2005; House et al 2008; Albanesi and Olivetti 2009), and labor supply generally, including retirement and other life-cycle issues (Rios-Rull 1993; Rupert et al. 2000; Gomme et al. 2004; Aguiar and Hurst 2007*b*; Ngai and Pissarides 2008; Rogerson and Wallenius 2009). Home production models give different answers to certain policy questions, such as the impact of taxation (McGrattan et al. 1997; Rogerson 2009). They also do a better job accounting for international differences in income, and provide a different perspective on growth and development (Einarsson and Marquis 1997; Parente et al. 2000).

the effects on mortgages, including Kearl (1979), Follain (1982) and Poterba (1991). These are complementary ideas, but we focus on different channels. Brunnermeier and Juiliard (2008) in particular assume "money illusion" where agents confound nominal and real interest rates, which may ring true, but we want to see how far we can get with rational agents. An innovative recent paper by Burnside et al. (2011) also studies housing markets with departures from standard rational expectations assumptions, but again we want to see how far we can get without this. He et al. (2011) show how housing can bear a liquidity premium, because home equity can be used to collateralize loans. We are after different issues, and abstract from home equity loans. As we remark below, this suggests concentrating on data prior to 2000, before the use of home equity loans rose dramatically, but we still show data pre- and post-2000. Similar in spirit is work by Piazzesi and Schneider (2010), who study how portfolios adjust when inflation taxes returns to financial assets but not housing. In terms of differentiating our product, we incorporate household production explicitly, allowing us to take advantage of many theoretical and empirical results in home production research.[3]

Section 1 discusses the data. Section 2 presents the theoretical framework, and Section 3 defines equilibrium. Section 4 presents a simplified version of the model and derives analytic results. Section 5 presents the numerical analysis. Section 6 concludes.

1 Data

Here we make the case that the value of home capital, scaled by either nominal output or the money supply, is positively related to inflation and nominal interest rates. We obviously have to scale by something, to correct for purely nominal increases in value, and we emphasize that our theory is about more than the idea that inflation makes prices go up. Our model

[3]Also, we mention here that although our model has frictional goods markets, housing is traded in frictionsless markets, just like capital is traded in standard growth theory. The growing body of research on frictional housing markets includes Wheaton (1990), Albrecht et al. (2007), Caplin and Leahy (2008), Coulson and Fisher (2009), Ngai and Tenreyro (2009), Novy-Marx (2009), Piazzesi and Schneider (2009), and Head and Lloyd-Ellis (2010).

predicts the value of home capital over nominal output, and of home capital over the money supply, both increase with inflation or nominal interest rates. We first show this is true in the U.S., then examine other countries. We consider several measures of the value of home capital, each defined by an estimate of housing wealth (price times quantity), plus the stock of durable goods from the Bureau of Economic Analysis (BEA). We add durables since we think of home capital more broadly than just housing, even if housing is the biggest component – i.e., around 75%. We describe the different approaches to measuring housing wealth in some detail here, but more information on data sources is given in the Appendix.

1.1 United States

Our first estimate of housing wealth uses data from Davis and Heathcote (2007), DH for short. This is a relatively short sample, starting in 1975, but provides a very accurate measure. Figure 1 shows time series and color-coded scatter plots for home capital over nominal output vs inflation and vs the nominal interest rate, as well as home capital over the money supply vs inflation and vs the nominal rate. Our measure of output is GDP minus rents paid to housing services, as in related studies on home production. Our measure of money is the M1S series that adjusts M1 for the practice of banks since the 1990s of "sweeping" checkable deposits into overnight money market accounts (Cynamon et al. 2006). For inflation we use the appropriately adjusted GDP deflator, and for the nominal rate here we use T-bills. We consider both inflation and interest rates, even though the Fisher Equation predicts they should move together in theory, because they do not cohere perfectly in practice. Figure 1 also reports correlations, and semi-elasticities computed from regressing of the log of home capital over GDP or home capital over M1S on inflation or interest rates. The black line shows the predicted relationship 1975-1999, and the blue line shows it 2000-2009.

It is apparent that something unusual – the housing bubble – happens in the 2000s relative to the previous 25 years. Unlike some of the work mentioned in the Introduction, we do not have much to say in this paper about the post-2000 U.S. housing boom and bust, and there may well have been something going on that our model is not designed to capture (see, e.g., Burnside et al. 2011, Mian and Sufi 2011, He et al. 2011 and references therein). Therefore it may be reasonable to concentrate more on "normal" times, by ending the sample at 1999, but we do not want to hide anything, and so we still show what happened after 2000. From 1975 to 1999, home capital over GDP and home capital over M1S are obviously positively correlated with both inflation and interest rates. This is also true after 2000, but that decade simply looks different. Over the whole sample, 1975-2009, there is not much of a correlation, but *not* because there is no relationship – rather, because the positive relationship rotates or shifts, depending on whether we look at inflation or interest rates.

Our second measure of housing wealth uses data from the Flow of Funds Accounts (FFA). An advantage of FAA data is that they go back to 1952, although as is common in macro we prefer to start in 1955, post Korean war. In any case, we believe FFA data do not provide the best estimates of the house values in 1975-1990, which is a key period for our analysis because it has a lot of variation in inflation. This is because the housing capital gains implied by the FFA data in this period do not align well with capital gains computed using standard price indexes. Unlike the DH data, where capital gains are taken directly from price indexes, for this period FFA data spline together estimates from various surveys. With this caveat in mind we show the results in Figure 2. There little correlation between home capital over GDP and inflation prior to 2000, although the relationship is positive after 2000, and it is positive in both periods for home capital over GDP and interest rates. The correlation between home capital over M1S and either inflation or interest rates is also positive in both periods, and again one can see a rotation or shift post 2000. So, even if we are less confident in this data, the basic pattern is similar.

Our third measure is based on the replacement cost of structures estimated by the BEA, as used in, e.g., McGrattan et al. (1997). A disadvantage of these data is that they do not include the value of land, which prior to 1970 was between 10 to 20 percent and is currently about a third of aggregate housing value (Davis and Heathcote 2007). The advantage is that they are constructed according to well-documented methods, and go back to the 1930s. It is not clear, however, what to make of data that far back, where the situation is confounded by the Great Depression, as well as some swings in inflation having to do with price controls and wars. Given these caveats, Figure 3 shows the BEA data.[4] In this case the regression lines come from splitting the sample between 1930-1954 and 1955-1999 (although we show the data to 2009). For home capital over GDP vs inflation, the relationship in the latter (earlier) period is slightly positive (negative) For home capital over GDP vs interest rates, the correlation is positive in both subsamples. For home capital over M1S vs inflation the relationship in the latter (earlier) period is strongly positive (slightly negative). For home capital over M1S vs interest rates, the relationship between home capital over M1S and interest rates is remarkably strong in both periods, with correlation coefficients around 0.9.

Our fourth and final measure of housing wealth is computed directly from the Decennial Censuses of Housing (DCH). The advantages are that these numbers are very accurate, and go back to 1930, although of course we have only one observation every 10 years, so we interpolate between data points. In Figure 4, the regression lines in the scatters are from splitting the sample between 1930-1954 and 1955-1999. For home capital over GDP vs inflation, again the relationship in the latter (earlier) period is slightly positive (negative). For home capital over GDP vs interest rates it is always positive. For home capital over M1S vs inflation the relationship is positive (slightly negative) in the latter (earlier) period. For home capital vs the nominal interest rate the relationship is again remarkably strong in both periods.[5] We are mostly interested in what happens post 1955, and do not have a lot

[4]Since here we go back to the 1930s, the interest rate uses AAA corporate bonds. If we use these instead of T-bill rates for Figures 1 and 2, there is little change before 2000, although there is some change after 2000 (which is not too surprising given so few post-2000 observations).

[5]We remark that the relationship between housing and interest rates is generally more clear than the relationship between housing and inflation. One reason may be that nominal rates better reflect perceptions of long-term inflation than actual reported inflation rates, especially in periods with price controls.

to say about the Depression, major wars or price controls. Once we move past those events there is a clear relationship between the variables in question.

To make one more point, Figure 5 shows Shiller's (2005) house price index (HPI) series, focusing on post-1955. Over this period, HPI declines secularly relative to GDP and M1S, rendering correlations uninformative. More importantly, Table 1 reconciles decade-by-decade changes in HPI and housing wealth. Column 1 reports HPI growth; Column 2 reports growth in the average price of housing units; Column 3 reports growth in the number of units; and Column 4 reports growth in nominal GDP. The percentage change in the value of housing is approximately Column 2 plus Column 3. Columns 1 and 2 show average prices increase faster than the HPI in every decade. This is because Shiller's data holds housing quality constant, and the gap between Columns 1 and 2 reflects quality improvement. Also, Shiller's data do not track changes in the number of units, which increased rapidly over the period (Column 3). To sum up, Columns 2 and 3 show the change in housing wealth between 1950 and 2000 is 528 percent, about the same the 517 percent change in nominal GDP. The HPI increased only 284 percent, about half the change in housing wealth. Given this, the HPI is less useful for our purposes than it might be for others. Also, this may explain why people who focus on Shiller's data have not noticed the facts we document in the other sources.

To reiterate, we are interested in the value of home capital scaled by either GDP or M1S, and we want to know how this is related to inflation and interest rates, because our theory makes strong predictions about that. We are less interested in house prices, or house prices relative to general consumer prices, because as we show theory makes less definitive predictions about that. Several U.S. data sets indicate that appropriately scaled values of home capital are positively related to inflation and interest rates. Although the exact semi-elasticities vary, the pattern is usually quite clear, and when it is less clear, like in the earlier half of the last century, this may be due to events like the Depression, major wars and price controls.

1.2 Other Countries

Data on housing wealth prior to 1990, and certainly prior to 1980, are virtually nonexistent outside the U.S. However, various estimates of prices have been constructed for several countries. As we said, we prefer price times quantity, but for most countries prices are all we have. Figure 6 presents house prices over GDP vs inflation for 16 countries, where now we do not distinguish the 2000s since the boom and bust is less apparent in these economies. We have direct source data for Belgium, France, Ireland, Switzerland and the UK. For the remaining countries, we use 1971-2009 data from the Bank for International Settlements (BIS) (see Andre 2010). We are less sure about these data, since we do not know the original sources, but we checked price growth from the BIS against source data for the five economies mentioned above, and they align well, giving us some confidence. The bottom line is that there is a positive relationship in 13 out of the 16 countries.

To conclude, virtually every source of information we could find – and we made an effort to cast our net widely – points to similar conclusions. There is a positive relationship between house prices over GDP or over the money supply, on the one hand, and inflation or nominal interest rates, on the other hand. This is clear at least for the US, and we did what we could for other countries. The rest of the paper presents one way of accounting for this evidence.[6]

2 Model

Time is discrete and continues forever. Each period has two subperiods, with a distinct set of markets, as discussed below. This temporal structure is natural, since it allows households to adjust home production after realizing the outcome of their stochastic retail market experience. There is a $[0, 1]$ continuum of homogeneous households, a $[0, n]$ continuum of retail firms, and a set of production firms, the cardinality of which does not matter due to

[6] Jeff Campbell asked whether one should interpret these findings in terms of a *moment* – a repeated pattern in the time series – or an *event* – one big rise and fall in postwar inflation, and suggested the evidence may be even more compelling from the latter point of view. We are agnostic.

constant returns. There is also a government that controls the money supply M and levies a lump sum tax T. In the first subperiod's market, production firms hire labor and capital from households at nominal factor prices w and r, to make output x. Households purchase x for direct consumption, and for investment in home and market capital. Retailers purchase x and transform it into a different good q, to sell to households in the market that convenes in the second subperiod.

Households discount at rate $\rho = 1/\beta - 1 > 0$ across periods, but not across subperiods, without loss of generality. Within-period utility is

$$u(c_x, c_m, c_n) - A_m \ell_m - A_n \ell_n,$$

where c_x, c_m and c_n are consumption in the first market, consumption in the second (retail) market, and nonmarket (home) consumption, while ℓ_m and ℓ_n are market and nonmarket labor hours. Having utility linear in labor keeps the analysis tractable. For some of what we do, we shut down c_x so that there are only two goods, but it is useful to bring c_x back for quantitative work, as it is important for matching velocity observations. Also, in the quantitative work we consider two specific functional forms, but for now we keep a general specification $u(\cdot)$, satisfying the usual monotonicity and convexity assumptions.

There are two nominal assets: money m, which is is used as a medium of exchange; and a bond b, the role of which is merely to compute an interest rate – i.e., b need not be traded in equilibrium, but we can still price it. There are three real assets: market capital k_m, residential structures k_s, and land k_l. As is standard, k_m is an input to the market production function $f(\ell_m, k_m)$. There is also a nonmarket production function $g(\ell_n, k_n)$, where k_n is nonmarket capital, which in the theoretical discussion we call housing even though in the empirical analysis we add consumer durables. Housing combines residential structures and land according to $k_n = h(k_s, k_l)$. One could alternatively say that home production has three inputs, labor, structures and land, but for our purposes we want housing to be an explicit function of k_s and k_l. Market capital and structures depreciate at rates δ_m and δ_s, while land does not depreciate and it's fixed supply is normalized to 1. The technologies $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ are strictly increasing, concave and homogeneous of degree 1.

While the first-subperiod market is frictionless, the retail market has search-type frictions. There are two distinct locations, one where agents have access to record keeping, and another where they do not. The measures of households that get to trade in the first and second locations are σ_1 and σ_2, so $\sigma_0 = 1 - \sigma_1 - \sigma_2$ is the measure that do not get to trade at all. To remember the notation, the j in σ_j refers to the number of payment instruments available: with probability σ_1 there is one, money; with probability σ_2 there two, money and credit; and probability σ_0 there are none. Credit means acceptance by retailers of households' promises, to be honored in first-subperiod next period. When record keeping is not available, credit is impossible and trade requires currency.[7] We assume the same measure of retailers and households trade in each location, and since the ratio of retailers to households is n, a given retailer trades in the credit market with probability σ_2/n, and in the money market with probability σ_1/n. We assume that all agents are price takers in the retail markets.[8]

2.1 Households' Problems

At the start of each period, a household's state variable is a portfolio $\mathbf{z} = (m, b, k_m, k_s, k_l)$, plus outstanding debt from the previous period, d. Given quasi-linear utility, without loss in generality we assume all debt is paid off in the first subperiod. Let $W(\mathbf{z}, d)$ and $V(\hat{\mathbf{z}}, c_x)$ be the value functions in the first and second subperiods. The first-subperiod problem is

$$\begin{aligned} W(\mathbf{z}, d) &= \max\{-A_m \ell_m + V(\hat{\mathbf{z}}, c_x)\} \\ \text{st } w\ell_m + rk_m + \Omega &= p_x c_x + \hat{m} + p_b \hat{b} + p_x \hat{k}_m + p_x \hat{k}_s + p_l \hat{k}_l, \end{aligned}$$

[7] Since Kocherlakota (1998), we know it is imperfect record keeping that generates an essential role for a medium of exchange. For more on why currency, and not, say, bonds or claims to capital, is used in this capacity, see Lester et al. (2011) and references therein (although this is not a solved problem).

[8] One can say they meet multilaterally in these markets, on islands, as in the Lucas-Prescott labor-search model, instead of bilaterally, as in the Mortensen-Pissarides search-and-bargaining model.

where p_x, p_l and p_b are the prices of x, k_l and b, respectively, and we define net wealth, after a lump sum tax T, by[9]

$$\Omega = m + b + p_x(1-\delta_m)k_m + p_x(1-\delta_s)k_s + p_l k_l - d - T$$

Eliminating ℓ_m using the budget equation, we reduce the problem to

$$W(\mathbf{z}, d) = \frac{A_m}{w}\Omega + \max\left\{-p_x c_x \frac{A_m}{w} - \frac{A_m}{w}(\hat{m} + p_b\hat{b} + p_x\hat{k}_m + p_x\hat{k}_s + p_l\hat{k}_l) + V(\hat{\mathbf{z}}, c_x)\right\}. \quad (1)$$

Notice W is linear in Ω, with slope A_m/w, and the choice of $\hat{\mathbf{z}}$ is independent of Ω. For each asset $z = m, b...$ the FOC is $\partial V/\partial z = A_m p_z/w$, where for m the price is $p_m = 1$, for b it is p_b etc. A similar condition holds for c_x which is carried into the second subperiod to be combined with c_m and c_n. The envelope conditions are

$$\partial W/\partial m = \partial W/\partial b = -\partial W/\partial d = A_m/w \quad (2)$$

$$\partial W/\partial k_m = [r + (1-\delta_m)p_x]A_m/w \quad (3)$$

$$\partial W/\partial k_s = (1-\delta_s)p_x A_m/w \quad (4)$$

$$\partial W/\partial k_l = p_l A_m/w. \quad (5)$$

In the second subperiod, three events may occur for households: with probability σ_0 they have no opportunity to trade in the retail market; with probability σ_1 they have an opportunity to trade using money; and with probability σ_2 they have an opportunity to trade using money or credit. Conditional on each event, the value function is denoted $V^j(\hat{\mathbf{z}}, c_x)$, $j = 0, 1, 2$, and $V(\hat{\mathbf{z}}, c_x) = \Sigma_j \sigma_j V^j(\hat{\mathbf{z}}, c_x)$. In what follows, we use two standard results: First, in the money-only retail market households cash out – i.e., they bring no more money than they plan to spend in the event that credit is unavailable. Second, when credit is available, all agents are indifferent between using credit or cash, so we assume they use credit only.

[9] Since x, k_m and k_s are the same physical good, they have the same price, p_x. Also, notice households choose market work ℓ_m in the first subperiod, but home work ℓ_n is not decided until the second. Finally, one can imagine that households build their own homes using purchases of k_s and k_l, but as we discuss below, it is equivalent to have them buy houses from competitive construction firms.

The second-subperiod problem is to choose consumption and home work, given what happened in the retail market. For a household with no retail opportunity, $c_m^0 = 0$,

$$V^0(\hat{\mathbf{z}}, c_x) = \max\left\{u\left(c_x, 0, c_n^0\right) - A_n \ell_n^0 + \beta W(\hat{\mathbf{z}})\right\} \text{ st } c_n^0 = g(\ell_n^0, \hat{k}_n),$$

where $\hat{k}_n = h(\hat{k}_s, \hat{k}_l)$. Then (c_n^0, ℓ_n^0) satisfies the constraint and the FOC for ℓ_n^0, given by $A_n = u_3(c_x, 0, c_n^0)\, g_1(\ell_n^0, \hat{k}_n)$. For a household with a money-only retail opportunity, $c_m^1 = \hat{m}/p_1$ where p_1 is the price, and

$$V^1(\hat{\mathbf{z}}, c_x) = \max\left\{u\left(c_x, \hat{m}/p_1, c_n^1\right) - A_n \ell_n^1 + \beta\left[W(\hat{\mathbf{z}}) - \hat{m}A_m/w'\right]\right\} \text{ st } c_n^1 = g(\ell_n^1, \hat{k}_n),$$

where from (2) the cost of c_m^1 in terms of next period's value function is $\hat{m}A_m/w'$ and w' is next period's wage. The solution satisfies the constraint and $A_n = u_3(c_x, \hat{m}/p_1, c_n^1)\, g_1(\ell_n^1, \hat{k}_n)$. And finally, for a household with a credit retail opportunity,

$$V^2(\hat{\mathbf{z}}, c_x) = \max\left\{u\left(c_x, c_m^2, c_n^2\right) - A_n \ell_n^2 + \beta\left[W(\hat{\mathbf{z}}) - p_2 c_m^2 A_m/w'\right]\right\} \text{ st } c_n^2 = g(\ell_n^2, \hat{k}_n),$$

where p_2 is the price. In this case the solution satisfies the FOC's $u_2(c_x, c_m^2, c_n^2) = p_2 \beta A_m/w'$ and $u_3(c_x, c_m^2, c_n^2) g_1(\ell_n^2, \hat{k}_n) = A_n$.

It is routine to differentiate $V(\hat{\mathbf{z}}, c_x) = \Sigma_j \sigma_j V^j(\hat{\mathbf{z}}, c_x)$ and insert the results into the FOC from (1) to get the Euler equations

$$\hat{m} \;:\; A_m/w = \sigma_1 u_2\left(c_x, c_m^1, c_n^1\right)/p_1 + (1-\sigma_1)\,\beta A_m/w' \tag{6}$$

$$\hat{b} \;:\; p_b A_m/w = \beta A_m/w' \tag{7}$$

$$k_m \;:\; p_x A_m/w = \beta\left[r' + (1-\delta_m)\, p_x'\right] A_m/w' \tag{8}$$

$$k_s \;:\; p_x A_m/w = h_1(\hat{k}_s, \hat{k}_l)\mathbb{E}(u_2 g_2) + \beta(1-\delta_s)\, p_x' A_m/w' \tag{9}$$

$$k_l \;:\; p_l A_m/w = h_2(\hat{k}_s, \hat{k}_l)\mathbb{E}(u_2 g_2) + \beta p_l' A_m/w', \tag{10}$$

$$c_x \;:\; p_x A_m/w = \mathbb{E}(u_1) \tag{11}$$

where $\mathbb{E}(u_1) = \Sigma_j \sigma_j u_1(c_x, c_m^j, c_n^j)$ and $\mathbb{E}(u_2 g_2) = \Sigma_j \sigma_j u_3(c_x, c_m^j, c_n^j) g_2(\ell_n^j, k_n)$. These all have simple economic interpretations – e.g., (6) has the marginal cost of a dollar on the LHS, and the marginal benefit on the RHS, where said dollar is spent in the retail market with probability σ_1 and carried into the next period with probability $1-\sigma_1$. Also, (7) implies

$$p_b = \beta w/w' = 1/(1+\rho)(1+\pi), \tag{12}$$

where $\pi = w'/w - 1$ is inflation. The real interest rate on a loan between the first subperiod at t and $t+1$ is ρ, and if we define the nominal rate i via the Fisher Equation $1+i = (1+\rho)(1+\pi)$, (12) says $p_b = 1/(1+i)$ Agents are happy with any b iff this holds.

2.2 Firms' Problems

The representative production firm maximizes profit by hiring labor and capital,

$$\Pi^P = \max\{p_x f(L,K) - wL - rK\}, \tag{13}$$

where L and K are aggregates. The solution satisfies $w = p_x f_1(L,K)$ and $r = p_x f_2(L,K)$, and $\Pi^P = 0$ by constant returns.

Retail firms purchase x and convert it into c_m, where for simplicity the conversion is one to one. They have an opportunity to sell it for money or credit in the retail market with probabilities σ_1/n and σ_2/n, resp. Because we assume that unsold inventory fully depreciates when the retail market closes, venders supply their wares inelastically, so $c_m^1 = c_m^2 = x$ (consumption is the same in the cash and credit markets). Therefore, expected retail profit is

$$\Pi^R = \max\left\{-p_x x + \frac{\sigma_1}{n} p_b p_1 x + \frac{\sigma_2}{n} p_b p_2 x\right\}. \tag{14}$$

The first term on the RHS of (14) is the cost of inventories; the second is expected revenue from cash sales, discounted by $p_b = 1/(1+i)$ since this can only be dispersed next period; and the third is expected revenue from credit sales discounted the same way. The FOC is

$$p_x = \frac{\sigma_1}{n} p_b p_1 + \frac{\sigma_2}{n} p_b p_2. \tag{15}$$

Retailers are happy with any x as long as (15) holds, and $\Pi^R = 0$. To pay for x, in the first market, retailers issue bonds that households are happy to buy given $p_b = 1/(1+i)$ (although nothing interesting would change if, say, retailers instead issue equity).

2.3 Policy

Government controls the supply of money, which grows at rate μ. This can be implemented either using the lump sum tax/transfer T, or by spending new money on x (with quasi-linear utility, nothing depends on this except ℓ_m). Without loss of generality, we balance the budget each period. We focus on stationary outcomes where all real variables are constant, implying $\pi = \mu$. Hence, it is equivalent to use the money growth rate or inflation rate as a policy instrument. Since $1 + i = (1 + \rho)(1 + \pi)$, it is also equivalent to use the nominal interest rate i. Policy does not determine the real interest rate across periods; that is pinned down by the preference parameter ρ.

3 Equilibrium

We first show how to express all prices $(w, r, p_x, p_l, p_b, p_1, p_2)$ in terms of the allocation. To begin with factor prices, we already know $w/p_x = f_1\left(\bar{\ell}_m, k_m\right)$ and $r/p_x = f_1\left(\bar{\ell}_m, k_m\right)$, where $\bar{\ell}_m$ is aggregate market hours and equilibrium requires $k_m = K$ and $\bar{\ell}_m = L$. Note that for each household $\ell_m = \ell_m(\Omega)$ depends on their wealth at the start of the period, which differs according to their recent retail experience, but all we need to characterize macro equilibrium is $\bar{\ell}_m = \int \ell_m(\Omega)$. In terms of land, (10) implies it is priced by its capitalized value as an input to housing, which is itself an input to home production,

$$p_l = (1-\beta)^{-1} h_2(k_s, k_l)\, \mathbb{E}(u_2 g_2)\, w/A_m.$$

Notice on the RHS we divide by A_m and multiply by the wage w, to convert utility into time and time into money. For bond prices, we already know $p_b = 1/(1+i)$ where i is effectively set by policy. For retail prices, in the cash market $p_1 c_m^1 = \hat{m} = (1+\mu) M$, and in the credit market we have the FOC $u_2(c_x, c_m^2, c_n^2) = p_2 \beta A_m / w'$. Hence, $p_1 = (1+\mu) M / c_m^1$ and $p_2 = (1+i) u_2(c_x, c_m^2, c_n^2) w / A_m$. Finally, from the retailers' FOC (15) we get the price level in the first subperiod,

$$p_x = \frac{\sigma_1 p_1 + \sigma_2 p_2}{n(1+i)}.$$

This gives all prices as functions of the allocation. In fact, we can pare down the description of an allocation as follows. In terms of land, structures and housing, since $k_l = 1$ and $k_n = h(k_s, 1)$, we need only keep track of k_s. And in terms of consumption, $c_m^1 = c_m^2 = x$ and $c_n^j = g[\ell_n^j, h(k_s, 1)]$. So we can fully describe an allocation by retail inventories, market and home capital, and market and home work.

Definition 1 *A steady state equilibrium is given by* $(c_x, x, k_m, k_s, \bar{\ell}_m, \ell_n^j)$ *satisfying: the FOC for home work conditional on retail experience*

$$A_n = u_3[c_x, c_m^j, g(\ell_n^j, k_n)] g_1(\ell_n^j, k_n),\ j = 0, 1, 2, \tag{16}$$

where it is understood that $c_m^0 = 0$, $c_m^1 = c_m^2 = x$ *and* $k_n = h(k_s, 1)$*; steady-state versions of the investment Euler equations (8)-(9)*

$$\begin{aligned} \rho + \delta_m &= f_2\left(\bar{\ell}_m, k_m\right) && (17) \\ \rho + \delta_s &= f_1\left(\bar{\ell}_m, k_m\right) h_1(k_s, 1)\, \mathbb{E}(u_2 g_2)(1+\rho)/A_m; && (18) \end{aligned}$$

a simplified version of the money Euler equation (6)

$$A_m(\sigma_1 + i) = (1+i)\sigma_1 u_2\left(c_x, x, c_n^1\right) w / p_1, \tag{19}$$

where w *and* p_1 *are as described above; plus (11) and aggregate feasibility*

$$\begin{aligned} A_m &= f_1\left(\bar{\ell}_m, k_m\right) \mathbb{E}(u_1) && (20) \\ f\left(\bar{\ell}_m, k_m\right) &= c_x + nx + \delta_k k_m + \delta_s k_s. && (21) \end{aligned}$$

Given $(c_x, x, k_m, k_s, \bar{\ell}_m, \ell_n^j)$, we can solve for prices as discussed above, plus some new variables. The average retail price and markup are[10]

$$p_c = \frac{\sigma_1 p_1 + \sigma_2 p_2}{\sigma_1 + \sigma_2} \text{ and } \frac{p_c}{p_x} = \frac{n(1+i)}{\sigma_1 + \sigma_2}. \tag{22}$$

Nominal GDP is $(\sigma_1 p_1 + \sigma_2 p_2) x + p_x (\delta_m k_m + \delta_s k_s + c_x)$. With p_x as a price index, real GDP is

$$y = (\sigma_1 p_1 + \sigma_2 p_2) x / p_x + \delta_m k_m + \delta_s k_s + c_x. \tag{23}$$

Velocity is $v = p_x y / M$, and a standard notion of money demand is $1/v = M/p_x y$ (see, e.g., Lucas 2000). Finally, consider the price of a house p_n and value of the stock $p_n k_n$, the empirical objects discussed earlier. If we introduce competitive home builders, the allocation is the same as if individuals build their own houses; yet we can still imagine builders combining land and structures according to household specifications to deliver $k_n = h(k_s, k_l)$. The profit from this activity is

$$\Pi^B = p_n h(k_s, k_l) - p_x k_s - p_l k_l, \tag{24}$$

which is 0 in equilibrium. Hence, $p_n k_n = p_x k_s + p_l k_l$ and $p_n = (p_x k_s + p_l k_l) / h(k_s, k_l)$.

4 Simple Model: Analytic Results

The general framework is complicated, because we are interested in some complicated issues, but we can simplify it to convey the intuition. First, eliminate investment decisions by fixing market capital and structures at K_m and K_s, with $\delta_m = \delta_s = 0$, so housing is fixed at $K_n = h(K_s, 1)$. Then to ease the presentation, for now, simplify household production by setting $c_n = k_n$. Also, rather than trading k_s and k_l, households now trade k_n directly at price p_n, and trade k_m at price p_m. Also set $\sigma_2 = 0$, so there is no retail credit, and ignore bonds, so $\mathbf{z} = (m, k_m, k_n)$. Finally, ignore $c_x = 0$ by writing utility as $u(c_m, c_n)$. Then the

[10] Note $p_c > p_x$ (a positive markup) due to search and inflation. This is one reason we are content to use Walrasian pricing in this exercise, although we can imagine bringing back bargaining in future work.

household's problem reduces to

$$
\begin{aligned}
W(\mathbf{z}) &= \frac{A_m}{w}\Omega + \max\left\{-\frac{A_m}{w}(\hat{m} + p_m\hat{k}_m + p_n\hat{k}_n) + V(\hat{\mathbf{z}})\right\} \\
V(\hat{\mathbf{z}}) &= \sigma_1\left[u(\hat{m}/p_c, \hat{k}_n) - \beta\hat{m}A_m/w'\right] + (1-\sigma_1)\,u(0,\hat{k}_n) + \beta W(\hat{\mathbf{z}}).
\end{aligned}
$$

The Euler equations in this special case are

$$\hat{m} \;:\; A_m/w = \sigma_1 u_1(\hat{m}/p_c, \hat{k}_n)/p_c + (1-\sigma_1)\,\beta A_m/w' \tag{25}$$

$$\hat{k}_m \;:\; p_m A_m/w = \beta\,(r' + p_m')\,A_m/w' \tag{26}$$

$$\hat{k}_n \;:\; p_n A_m/w = \mathbb{E}\,(u_2) + \beta p_n' A_m/w'. \tag{27}$$

In this stripped-down specification, equilibrium is summarized by $(x, \bar{\ell}_m)$ satisfying (19)-(21), or, if one prefers, one equation in aggregate employment

$$A_m\,(\sigma_1 + i)\,n = \sigma_1^2 u_1\left[f(\bar{\ell}_m, K_m)/n, K_n\right] f_1\left(\bar{\ell}_m, K_m\right). \tag{28}$$

Now (26)-(27) deliver prices p_m and p_n. The relative price of housing is

$$\frac{p_n}{p_c} = \frac{\sigma_1\mathbb{E}u_2\,(x, k_n)\,f_1\left(\bar{\ell}_m, k_m\right)}{A_m\,(1-\beta)\,n\,(1+i)} \text{ or } \frac{p_n}{p_x} = \frac{\mathbb{E}u_2 f_1\left(\bar{\ell}_m, k_m\right)}{A_m\,(1-\beta)},$$

depending on whether we deflate by the retail or wholesale price index.

The first result is that, as usual, inflation reduces market hours and consumption: $\partial\bar{\ell}_m/\partial i = -n^2 A_m/D < 0$ and $\partial x/\partial i = -nA_m f_1/D < 0$, where $D = -\sigma_1^2\,(nu_1 f_{11} + f_1^2 u_{11})$. Then the effect of i on the relative price of housing is

$$\frac{\partial}{\partial i}\left(\frac{p_n}{p_c}\right) = \frac{\sigma_1^3 f_1^3\,(u_{11}\mathbb{E}u_2 - u_1\mathbb{E}u_{21}) - (1-\sigma_1)\,\sigma_1 A_m n\,(nf_{11}\mathbb{E}u_2 + f_1^2\mathbb{E}u_{21})}{A_m\,(1-\beta)\,n\,(1+i)^2\,D}.$$

The first term in the numerator is negative if k_n is a normal good, while the second is positive as long as c_m and c_n are substitutes, $u_{21} < 0$, which we take to be the relevant case.[11] If $\sigma_1 = 1$ (no search frictions) the second term vanishes and p_n/p_c falls with i, which can be

[11] As pointed out below in the discussion on calibration, using different methods and data, various empirical studies find substitution elasticities in a range around 2.0.

understood as follows: When i increases c_m falls, and households are worse off; if k_n is normal they demand less given the relative price, so p_n/p_c must fall to clear the market. But when $\sigma_1 < 1$ there is second effect that goes the other way, and the net result is ambiguous. By contrast, deflating by p_x rather than p_c, as long as c_m and c_n are substitutes[12]

$$\frac{\partial}{\partial i}\left(\frac{p_n}{p_x}\right) = \frac{-n\left(nf_{11}\mathbb{E}u_2 + f_1^2\mathbb{E}u_{21}\right)}{(1-\beta)\,D} > 0.$$

We can also deflate by M to again get an unambiguous result

$$\frac{\partial}{\partial i}\left(\frac{p_n}{M}\right) = -\frac{\beta\sigma_1\left[c_m f_1^2\mathbb{E}u_{21} + \left(c_m n f_{11} - f_1^2\right)\mathbb{E}u_2\right]}{(1-\beta)\,c_m^2 D} > 0,$$

when home and market goods are substitutes. Inflation also increases p_n deflated by $p_x x$, nominal GDP defined using producer prices. Alternatively, defining nominal GDP using consumer prices, one can show

$$\begin{aligned}\frac{\partial}{\partial i}\left(\frac{p_n}{p_c c_m}\right) &= \tilde{D}\sigma_1^3 f_1^3\mathbb{E}u_2\left(c_m u_{11} + u_1\right) - \tilde{D}\sigma_1^3 f_1^3 u_1\mathbb{E}u_{21}c_m \\ &\quad -\tilde{D}\left(1-\sigma_1\right)\sigma_1 A n\left[\left(nf_{11}c_m - f_1^2\right)\mathbb{E}u_2 + c_m f_1^2\mathbb{E}u_{21}\right],\end{aligned}$$

where $\tilde{D} > 0$. The second and third terms on the RHS are positive if home and market goods are substitutes, while the first term is positive iff $-c_m u_{11} < u_1$. Hence, if households are not too risk averse, inflation also increases p_n deflated by nominal GDP defined using consumer prices.

Of course, with K_n fixed, all the results about p_n can be restated as effects on the value of housing wealth $p_n K_n$. We summarize all this as follows:

Proposition 2 *In the simplified model described by (28),*

1. $\frac{\partial}{\partial i}\left(\frac{p_n}{p_c}\right) < 0$ *if* k_n *is normal and* $\sigma = 1$*; if* $\sigma < 1$ *the sign is ambiguous.*
2. $\frac{\partial}{\partial i}\left(\frac{p_n}{p_x}\right) > 0$ *iff* c_m *and* c_n *are substitutes,* $u_{12} < 0$.

[12]Heuristically, since the retail markup $p_c/p_x = (1+i)\,n/\sigma_1$ increases lockstep with i, even if p_n deflated by consumer prices fall with inflation, p_n deflated by producer prices goes up.

3. $\frac{\partial}{\partial i}\left(\frac{p_n}{M}\right) > 0$ *if* $u_{12} < 0$.

4. $\frac{\partial}{\partial i}\left(\frac{p_n}{p_x x}\right) > 0$ *if* $u_{12} < 0$.

5. $\frac{\partial}{\partial i}\left(\frac{p_n}{p_c c_m}\right) > 0$ *if* $u_{12} < 0$ *and* $-c_m u_{11}/u_1$ *is not too big.*

We are most interested in the effects of i on housing wealth $p_n k_n$, relative to either M or GDP, as discussed above. The idea is that inflation affects the economy in general, and housing in particular, by creating an incentive to move economic activity out of the market and into the home. We have shown how these effects work qualitatively, and later we consider their magnitude. Before the quantitative work, however, we mention that the analytic results can be generalized to some degree, and sketch the extension to a general home technology $c_n = g(\ell_n, k_n)$. For this we set $n = \sigma_1 = 1$ and still abstract from investment.

Generalizing (28), we now get two equations in time use (ℓ_m, ℓ_n)

$$\begin{aligned} (1+i) A_m &= u_1\left[f\left(\bar{\ell}_m, K_m\right), g\left(\ell_n, K_n\right)\right] f_1\left(\bar{\ell}_m, K_m\right) && (29) \\ A_n &= u_2\left[f\left(\bar{\ell}_m, K_m\right), g\left(\ell_n, K_n\right)\right] g_1\left(\ell_n, K_n\right). && (30) \end{aligned}$$

These conditions exhibit a nice symmetry between market and home time use, only broken by exactly the channel we want to highlight: the former is taxed, while the latter is not taxed, by inflation. It is routine to derive

$$\frac{\partial \ell_m}{\partial i} = \frac{A_m\left(g_{11}u_2 + g_1^2 u_{22}\right)}{\hat{D}} \text{ and } \frac{\partial \ell_n}{\partial i} = \frac{-A_m f_1 g_1 u_{21}}{\hat{D}},$$

where $\hat{D} > 0$. Thus, ℓ_m and c_m decrease with i, while ℓ_n and c_n increase iff c_m and c_n are substitutes. As before, the effect of i on p_n/p_c is ambiguous, but

$$\frac{\partial}{\partial i}\left(\frac{p_n}{p_x}\right) = \frac{u_{21} f_1^2 u_2\left(g_2 g_{11} - g_1 g_{21}\right) + u_2 g_2 f_{11}\left(g_{11}u_2 + g_1^2 u_{22}\right)}{(1-\beta)\hat{D}} > 0$$

in the natural case where c_m and c_n are substitutes while ℓ_n and k_n are compliments. We can also show p_n/M increases with i, and other results.

We like the fact that this theory generates such clean analytic predictions, at least with capital fixed. Even with endogenous investment, the model is still fairly tractable. Continuing to set $\sigma_1 = 1$, with endogenous investment we have

$$\begin{aligned}
(1+i)\, A_m &= u_1 \left[c_m, g\left(\ell_n, k_n\right)\right] f_1\left(\ell_m, k_m\right) \\
A_n &= u_2 \left[c_m, g\left(\ell_n, k_n\right)\right] g_1\left(\ell_n, k_n\right) \\
\rho + \delta_m &= f_2\left(\ell_m, k_m\right) \\
\rho + \delta_n &= f_1\left(\ell_m, k_m\right) u_2\left[x, g\left(\ell_n, k_n\right)\right] g_2\left(\ell_n, k_n\right) / A_m
\end{aligned}$$

where c_m satisfies the obvious feasibility condition. If so inclined, one can also assume k_m is endogenous but k_n fixed (or vice-versa) by dropping the final (penultimate) equation. While some analytic results are available, at this stage computational methods are appropriate – especially since we want the magnitudes, not just signs, of the relevant effects.

5 General Model: Quantitative Results

We use standard technologies: $f\left(\ell_m, k_m\right) = \ell_m^{\chi_m} k_m^{1-\chi_m}$, $g\left(\ell_n, k_n\right) = \ell_n^{\chi_n} k_n^{1-\chi_n}$ and $h\left(k_s, k_l\right) = k_s^{\chi_h} k_l^{1-\chi_h}$. We have three goods, $\left(c_x, c_m, c_n\right)$ and, as we mentioned above, we cannot drop c_x if we want to match standard observations on velocity (something that obviously never came up in nonmonetary home production models). Hence, we consider two specifications, both of which seem *a priori* reasonable. In Calibration 1, c_x and c_m are perfect substitutes and we identify them collectively as market consumption:[13]

$$u\left(c_x, c_m, c_n\right) = \log\left[\left(c_m + Bc_x\right)^{\omega} + c_n^{\omega}\right]^{\frac{1}{\omega}}$$

Here $1/(1-\omega)$ is the EOS between market and home goods. In Calibration 2, we use:

$$u\left(c_x, c_m, c_n\right) = B \log\left(c_x\right) + (1/\omega) \log\left(c_m^{\omega} + c_n^{\omega}\right)$$

[13] There is implicitly a constant multiplying c_n, but we can set it to 1 without loss of generality by a choice of units.

Here ω no longer directly captures the EOS we want to target, so we average (using expenditure shares) the EOS between c_x and c_n, which is 1, and between c_m and c_n, which is $1/(1-\omega)$.

We use long-run observations to calibrate as many parameters as we can, and use properties of the empirical money demand curve, including its elasticity, for some others. Unless otherwise noted, our targets are computed using US data, for the period 1975 to 1999 (since we are not attempting to explain the housing bubble). The length of a period is a quarter, although the results are robust to this choice. The average annual inflation rate and 3-month T-bill rate are 4.13% and 6.80% in the sample, which yield $\rho = 0.0066$ and $\mu = 0.0103$. We set $\chi_h = 0.73$ to match the value of residential structures plus durables relative to the value of housing capital, and set $\delta_m = \delta_s = 0.015$ for a 6% annual depreciation rate on both k_m and k_s, all obtained using data discussed in Section 2. We set $\sigma_2 = 0.5\sigma_1$, so that there are half as many retail trades with credit as there are with money, based on data from the recent Bank of Canada Methods of Payment Survey.[14]

Since a key parameter is ω, as it governs the EOS between market and home goods, we discuss this in some more detail. In their original home production macro model, Benhabib et. al. (1991) argue for an EOS of 5. Estimates by Rupert et al. (1995) using PSID data, however, yield numbers closer to 1.8 or 2. Estimates by McGrattan et al. (1995) using aggregate time series and a DSGE model yield values between 1.5 and 1.8. Aguiar and Hurst (2007a), using scanner and time-use data, estimate an EOS of 1.8 Chang and Schorfheide (2003) using aggregate data get a slightly higher estimate of 2.3. While there is obviously no unique definitive number, there is a consensus on a reasonable range, and as a compromise we use an EOS of 1.8, the median of these estimates. In Calibration 1 this immediately implies $\omega = 0.44$, while in Calibration 2 we need to calibrate ω jointly with the remaining

[14]See Arango and Welte (2012). To give a little more detail, the probabilities of using cash in any transaction for single and married people are 55% and 48%, resp., and the probabilities of using credit cards are 20% and 25%, resp., making σ_2 close to $\sigma_1/2$.

parameters to match the EOS target.

At this point, for Calibration 1 the parameters $(A_m, A_n, B, \chi_m, \chi_n, n, \sigma_1)$, and for Calibration 2 these plus ω, are calibrated to match the following targets. As is standard in the literature (Greenwood et al. 1995) households work on average $\bar{\ell}_m = 33\%$ of their discretionary time in the market and $\bar{\ell}_n = 25\%$ in the home, while market capital over annual output is $k_m/y = 2.07$ and household capital over annual output is $k_n/y = 1.96$. For the retail markup we target 30% based on data from the Annual Retail Trade Survey.[15] Finally, we match the level and slope of money demand, targeting an average annual velocity of 5.76 and a semi-elasticity of with respect to i of 2.56%, obtained using a standard log-linear regression. While the parameters are calibrated jointly, heuristically A_m and A_n match the targets for hours, χ_m and χ_n match the capital-output ratios, n matches the markup, B matches the level of velocity and σ_1 matches its elasticity.

The top panel of Table 2 report calibration results. In Calibration 2 ω is 0.83, which implies a large EOS between c_m and c_n (coincidently, very nearly the value used in Benhabib et. al. 1991), but maintains the target EOS between overall market and nonmarket consumption of 1.8. In terms of the retail market, for Calibration 1 (2 is similar), the measure of retailers is $n = 0.22$, and the measure of households that trade each period is $\sigma_1 + \sigma_2 = 0.17$. Therefore $(\sigma_1 + \sigma_2)/n$, or 78%, is the fraction of retail inventories sold each period, while 17% of households make a purchase in this market each period. Defining market expenditure as $p_x c_x + \sigma_1 p_1 c_m^1 + \sigma_2 p_2 c_m^2$, about 15% of total market consumption is purchased in the frictional retail market, while the rest is c_x; this is largely driven by average velocity in the data. To understand the importance of home production, we compute the percentage increase in market consumption required to compensate for the loss if home consumption set to 0. For households that are able to trade in the retail market, each period, market

[15] We first saw this data discussed in Faig and Jerez (2005). To give more detail, at the low end (Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations), markups range from 17% to 21%; at the high end (Specialty Foods, Clothing, Footware and Furniture), they range from 42% to 44%. We pick 30%, right in the middle.

consumption has to increase by 77% to compensate for no home production. For households that cannot trade in the retail market, this number is 152%. Hence, both retail and home consumption are economically important.

5.1 Positive Results

We now ask how well the model accounts for the facts discussed in Section 1 – the relationship between the inflation or nominal interest rate, on the one hand, and the value of home capital over nominal output or over the money supply, on the other. Here we denote the value of home capital over nominal output and over the money supply by H/Y and H/M. These ratios are tightly connected to inverse velocity, Y/M, by the relationship $H/M = H/Y \times Y/M$. Since the average value of H/Y and Y/M are calibration targets, the average value of H/M matches the data, by construction. The only elasticity we target is the semi-elasticity of Y/M with respect to the nominal rate, denoted $\xi_{Y/M}$. The model generates endogenously a semi-elasticity for H/Y, denoted $\xi_{H/Y}$, and the elasticity of H/M then follows from $\xi_{H/M} = \xi_{H/Y} + \xi_{Y/M}$. We are mainly interested in seeing how the model accounts for $\xi_{H/Y}$.[16]

We first compute the steady state for different values of i in the data between 1975-1999. This takes every year to be a steady state of the model with a particular i, which is obviously not strictly correct, due to the persistence of shocks and transitions. As we think the theory is more relevant for medium- to longer-run observations, this may not be such a bad approximation; in any case, studying shocks or transitional dynamics is beyond the scope of this project. Then, among other statistics, for a particular variable of interest X, we compute the relevant elasticity by regressing $\log(X_t)$ on a constant and i. Table 2 reports the results and Figure 7 shows Y/M, H/Y and H/M vs. i for Calibrations 1 and 2, as well as the data. In Calibration 1 we get $\xi_{H/Y} = 0.23$ and in Calibration 2 we get $\xi_{H/Y} = 0.55$, or

[16] One reason for this is something pointed out by Marcelo Veracierto: even if $p_n k_n$ does not move at all, as long as real balances fall with i, we get $\xi_{H/M}$ right, at least qualitatively, by construction. It therefore seems more interesting, and challenging, to focus on $\xi_{H/Y}$.

21% and 50% of the semi-elasticities in the data, respectively.[17] It obviously matters which specification one uses: different ways a disaggregating market consumption into c_x and c_m are simply not equivalent, even when they are roughly similar in terms of their predictions for the EOS. More work can be done to uncover which is a better specification.

Given that we tend to think the theory is mainly relevant at medium-run frequencies, and less relevant for business cycles, we do not expect to match all the fluctuations in the data. But as Figure 7 shows, we capture the main long-run patterns fairly well. These findings are robust. Changing the level of risk aversion from 1 to 1/2 or 2, e.g., has little effect. Nor does increasing the calibration frequency to monthly or even weekly. This is important, to us, because it is one of the reasons for modeling retail as a frictional market. With a typical cash-in-advance model, where households spend all their money each period, there is no way to match velocity when the period is short. With our frictional retail market, we can shorten the period length and keep everything else basically the same by scaling the arrival rates. As regards the EOS target, varying this in the range 1.5 to 2.3 yields roughly similar results, although as the EOS increases the fraction of $\xi_{H/Y}$ accounted for by the model increases. In fact, if in Calibration 2 we use an EOS of 5, we can account for basically 100% of $\xi_{H/Y}$. But an EOS of 5 is too high, given the body of empirical work. We prefer to conclude that for a reasonable EOS we can account for $20\% - 50\%$ of the relationship.

5.2 Normative Results

We are also interested in how inflation affects welfare. There are two reasons for revisiting this classic issue. First, it is well known that models with relatively explicit microfoundations for money can generate bigger costs of inflation than reduced-form models. A standard exercise is to compute the cost of 10% inflation as the amount of consumption households would be willing to sacrifice to go from 10% to the Friedman rule, which means an inflation

[17] When we consider $\xi_{H/M}$, the model accounts between 78% and 87% of the elasticity in the data, but again we prefer to focus on $\xi_{H/Y}$.

rate consistent with a 0 nominal interest rate. A consensus estimate from reduced-form analyses is that the answer is around 1/2 of 1%, while in various versions of the model in Lagos and Wright (2005) the number can easily be closer to 5%, an order of magnitude higher (see Aruoba et al. 2011 for references). Second, it is well known that incorporating home production into otherwise standard models can have a big quantitative impact on fiscal policy, so why not see if this is also the case for monetary policy?[18]

Figure 8 shows how the equilibrium changes with inflation. Results are shown for the two benchmark calibrations, as well as a version of the model with home production shut down. Since inflation is a tax on money holdings, higher inflation reduces real money balances and c_m^1. Since $c_m^1 = c_m^2 = x$, this also reduces demand by retailers for inventories, employment and investment. In the model with no home production, market activity falls with inflation, and this creates a loss in welfare. In the household production model households can make up for some of the lost market consumption by increasing home consumption. In the end, the cost of inflation is larger when home production is an option: going from 10% inflation to the Friedman rule implies a welfare gain of 0.4% without home production, 0.53% in Calibration 1, and 0.66% in Calibration 2.

Hence inflation is more costly with home production, especially when home and market goods are better substitutes. As in standard public finance, a household's optimal behavior in response to an increase in inflation is to reduce market and increase nonmarket activity. Depending on the degree of substitutability, individual welfare may be minimally affected by this. But if all households do the same thing, aggregate demand for home capital and p_n/p_x rise – e.g., by about 0.15% as inflation goes up from the Friedman rule to 10% in Calibration

[18] In fact, the main reason for the big welfare effects in Lagos-Wright models is that they use bargaining, rather competitive pricing. Bargaining power can be calibrated to match the markup, which would be 0 with competitive pricing. With a markup calibrated to the data, bargaining introduces what is sometimes called a hold-up problem in money demand, which, in our notation, reduces c_m below the efficient level even at the Friedman rule. In this paper, even with competitive pricing we get a markup. Although it may be worth incorporating noncompetitive pricing in the future, here we ask how home production affects the cost of inflation without the complication of hold-up problems.

1. Due to these general equilibrium effects, the increase in home consumption does not completely make up for the loss in market consumption. In Calibration 2 c_n responds significantly more, due to the higher substitutability between c_m and c_n, and households increase their demand for housing so much that p_n/p_x grows by about three times as much as in Calibration 1. While there is room for more work on this important topic, we want to simply make one point: microfoundations matter for quantitative policy issues, not just for aesthetics, and this is true for both the microfoundations of monetary economics and the microfoundations of household behavior.

6 Conclusion

Many people these days seem interested in housing. Our view is that housing is household capital, and it only makes sense to study this by specifying how this capital enters the home production function, exactly as we study investment in market capital by specifying how it enters into the market production function. We began by asking if housing is a good hedge against inflation. The answer is yes, in this sense: when inflation is high, and hence one's money is worth less, the value of the housing stock is high. In terms of hard data, we documented that appropriately-deflated values of the housing stock, measured using various sources, go up with inflation or nominal interest rates. In terms of theory, we proved analytically the model is consistent with this finding. Numerical work shows the model accounts for a reasonable portion of the key observations, between 20% and 50%, depending on which version one uses. Clearly it matters which utility function one uses, and this is worth further study. In any case, although we can account for a relevant fraction of the observations under study, there a lot left to be explained, and hence more to be done in future work.

REFERENCES

1. Aguiar, M. and E. Hurst, 2005, "Consumption versus Expenditure," *Journal of Political Economy* 113, 919-48.

2. Aguiar, M. and E. Hurst, 2007a, "Life-Cycle Prices and Production," *American Economic Review* 97, 1533-59.

3. Aguiar, M. and E. Hurst, 2007b, "Measuring Trends in Leisure: The Allocation of Time over Five Decades," *Quarterly Journal of Economics* 122, 969-1006.

4. Arango, C. and A. Welte, 2012, "The Bank of Canada's 2009 Methods of Payment Survey: Methodology and Key Results," Bank of Canada Working Paper.

5. Albanesi, S. and C. Olivetti, 2009, "Home Production, Market Production and the Gender Wage Gap: Incentives and Expectations," *Review of Economic Dynamics* 12, 80-107.

6. Albrecht, J., A. Anderson, E. Smith and S. Vroman, 2007, "Opportunistic Matching In The Housing Market," *International Economic Review* 48, 641-64.

7. Andre, C., 2010, "A Bird's Eye View of OECD Housing Markets," OECD Economics Department Working Paper 746.

8. Aruoba, S.B., C.J. Waller and R. Wright, 2011, "Money and Capital," *Journal of Monetary Economics* 58, 98-116.

9. Baxter, M. and U.J. Jermann, 1999, "Household Production and the Excess Sensitivity of Consumption to Current Income," *American Economic Review* 89, 902-20.

10. Baxter, M., 2010, "Detecting Household Production," manuscript, BU.

11. Becker, G.S., 1965, "A Theory of the Allocation of Time," *Economic Journal* 75, 493-508.

12. Becker, G.S., 1988, "Family Economics and Macro Behavior," *American Economic Review* 78, 1-13.

13. Benhabib, J., R. Rogerson and R. Wright, 1991, "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy* 99, 1166-87.

14. Brunnermeier, M.K. and C. Julliard, 2008, "Money Illusion and Housing Frenzies," *Review of Financial Studies* 21, 135-80.

15. Burnside, M. Eichenbaum, and S. Rebelo, 2011, "Understanding Booms and Busts in Housing Markets," manuscript, Duke.

16. Caplin, A. and J. Leahy, 2008, "Trading Frictions and House Price Dynamics," manuscript, NYU.

17. Chang, Y. and F. Schorfheide, 2003, "Labor-Supply Shifts and Economic Fluctuations," *Journal of Monetary Economics* 50, 1751-1768.

18. Coulson, N.E. and L.M. Fisher, 2009, "Housing Tenure and Labor Market Impacts: The Search Goes On," *Journal of Urban Economics* 65, 252-64.

19. Cynamon, B.Z., D.H. Dutkowsky and B.E. Jones, 2006, "Redefining the Monetary Agggregates: A Clean Sweep," *Eastern Economic Journal* 32, 661-72.

20. Davis, M.A. and J. Heathcote, 2005, "Housing And The Business Cycle," *International Economic Review* 46, 751-84.

21. Davis, M.A. and J. Heathcote, 2007, "The Price and Quantity of Land in the United States," *Journal of Monetary Economics* 54, 2595-620.

22. Einarsson, T. and M.H. Marquis, 1997, "Home Production with Endogenous Growth," *Journal of Monetary Economics* 39, 551-69.

23. Faig, M. and B. Jerez, 2005, "A Theory of Commerce," *Journal of Economic Theory* 122, 60-99.

24. Fisher, J., 1997, "Relative Prices, Complementarities and Comovement among Components of Aggregate Expenditures," *Journal of Monetary Economics* 39, 449-74.

25. Fisher, J., 2007, "Why Does Household Investment Lead Business Investment over the Business Cycle?" *Journal of Political Economy* 115, 141-68.

26. Follain, J.R., 1982, "Does Inflation Affect Real Behavior: The Case of Housing," *Southern Economic Journal* 48, 570-82.

27. Geromichalos, A., J.M. Licari and J. Suarez-Lledo, 2007, "Asset Prices and Monetary Policy," *Review of Economic Dynamics* 10, 761-79.

28. Gomme, P., F. Kydland and P. Rupert, 2001, "Home Production Meets Time to Build," *Journal of Political Economy* 109, 1115-31.

29. Gomme, P., P. Rupert, R. Rogerson and R. Wright, 2004, "The Business Cycle and the Life Cycle," *NBER Macroeconomics Annual* 19.

30. Greenwood, J. and Z. Hercowicz, 1991, "The Allocation of Capital and Time over the Business Cycle," *Journal of Political Economy* 99, 1188-214.

31. Greenwood, J., R. Rogerson and R. Wright, 1995, "Household Production in Real Business Cycle Theory," in *Frontiers of Business Cycle Theory*, ed. T. Cooley, Princeton.

32. Greenwood, J., A. Seshadri and M. Yorukoglu, 2005, “Engines of Liberation,” *Review of Economic Studies* 72, 109-33.

33. Gronau, R., 1997, “The Theory of Home Production: The Past Ten Years,” *Journal of Labor Economics* 15, 197-205.

34. He, C., R. Wright and Y. Zu, 2011, “Housing Price Bubbles,” manuscript, Wisconsin.

35. Head, A. and H. Lloyd-Ellis, 2010, “Housing Liquidity, Mobility and the Labour Market,” manuscript, Queen’s.

36. House, C., J. Laitner and D. Stolyarov, 2008, “Valuing Lost Home Production Of Dual Earner Couples,” *International Economic Review* 49, 701-36.

37. Kearl, J.R., 1979, “Inflation, Mortgages, and Housing,” *Journal of Political Economy* 87, 1115-38.

38. Kocherlakota, N., 1998, “Money is Memory,” *Journal of Economic Theory* 81, 232-51.

39. Lagos, R. and R. Wright, 2005 “A Unified Framework for Monetary Theory and Policy Analysis” *Journal of Political Economy* 113, 463-84.

40. Lester, B., A. Postlewaite and R. Wright, 2011, “Information, Liquidity, Asset Prices and Monetary Policy,” *Review of Economic Studies*, in press.

41. Lucas, R.E., 2000, “Inflation and Welfare,” *Econometrica* 68, 247-74.

42. McGrattan, E.R., R. Rogerson and R. Wright, 1997, “An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy,” *International Economic Review* 38, 267-90.

43. Mian, A., and A. Sufi, 2011, “House Prices, Home Equity-Based Borrowing, and the US Household Leverage Crisis,” *American Economic Review* 101, 2132–56.

44. Ngai, L.R. and S. Tenreyro, 2009, “Hot and Cold Seasons in the Housing Market,” manuscript, LSE.

45. Ngai, L.R. and C.A. Pissarides, 2008, “Trends in Hours and Economic Growth,” *Review of Economic Dynamics* 11, 239-56.

46. Nosal, E., and G. Rocheteau, 2011, *Money, Payments, and Liquidity*, MIT Press.

47. Novy-Marx, R., 2009, “Hot and Cold Markets,” *Real Estate Economics* 37, 1-22.

48. Parente, S., R. Rogerson and R.Wright, 2000, “Homework in Development Economics: Household Production and the Wealth of Nations,” *Journal of Political Economy* 108, 680-7.

49. Piazzesi, M. and M. Schneider, 2009, "Momentum Traders in the Housing Market: Survey Evidence and a Search Model," *AER Papers & Proceedings* 99, 406-11.

50. Piazzesi, M. and M. Schneider, 2010, "Inflation and the Price of Real Assets," manuscript, Stanford.

51. Poterba, J. M., 1991, "House Price Dynamics: The Role of Tax Policy and Demography," *Brookings Papers on Economic Activity* 22, 143-203.

52. Ríos-Rull, J.-V., 1993, "Working in the Market, Working at Home, and the Acquisition of Skills: A General-Equilibrium Approach," *American Economic Review* 83, 893-907.

53. Rogerson, R., 2009, "Market Work, Home Work, and Taxes: A Cross-Country Analysis," *Review of International Economics* 17, 588-601.

54. Rogerson, R. and J. Wallenius, 2009, "Micro and Macro Elasticities in a Life Cycle Model with Taxes," *Journal of Economic Theory* 144, 2277-92.

55. Rupert, P., R. Rogerson and R. Wright, 1995, "Estimating Substitution Elasticities in Household Production Models," *Economic Theory* 6, 179-93.

56. Rupert, P., R. Rogerson and R. Wright, 2000, "Homework in Labor Economics: Household Production and Intertemporal Substitution," *Journal of Monetary Economics* 46, 557-79.

57. Shiller, R., 2005, *Irrational Exuberance*, Princeton University Press.

58. Wheaton, W.C., 1990, "Vacancy, Search, and Prices in a Housing Market Matching Model," *Journal of Political Economy* 98, 1270-92.

59. Williamson, S. and R. Wright, 2010, "New Monetarist Economics: Models," *Handbook of Monetary Economics*, eds. B. Friedman and M. Woodford, Elsevier.

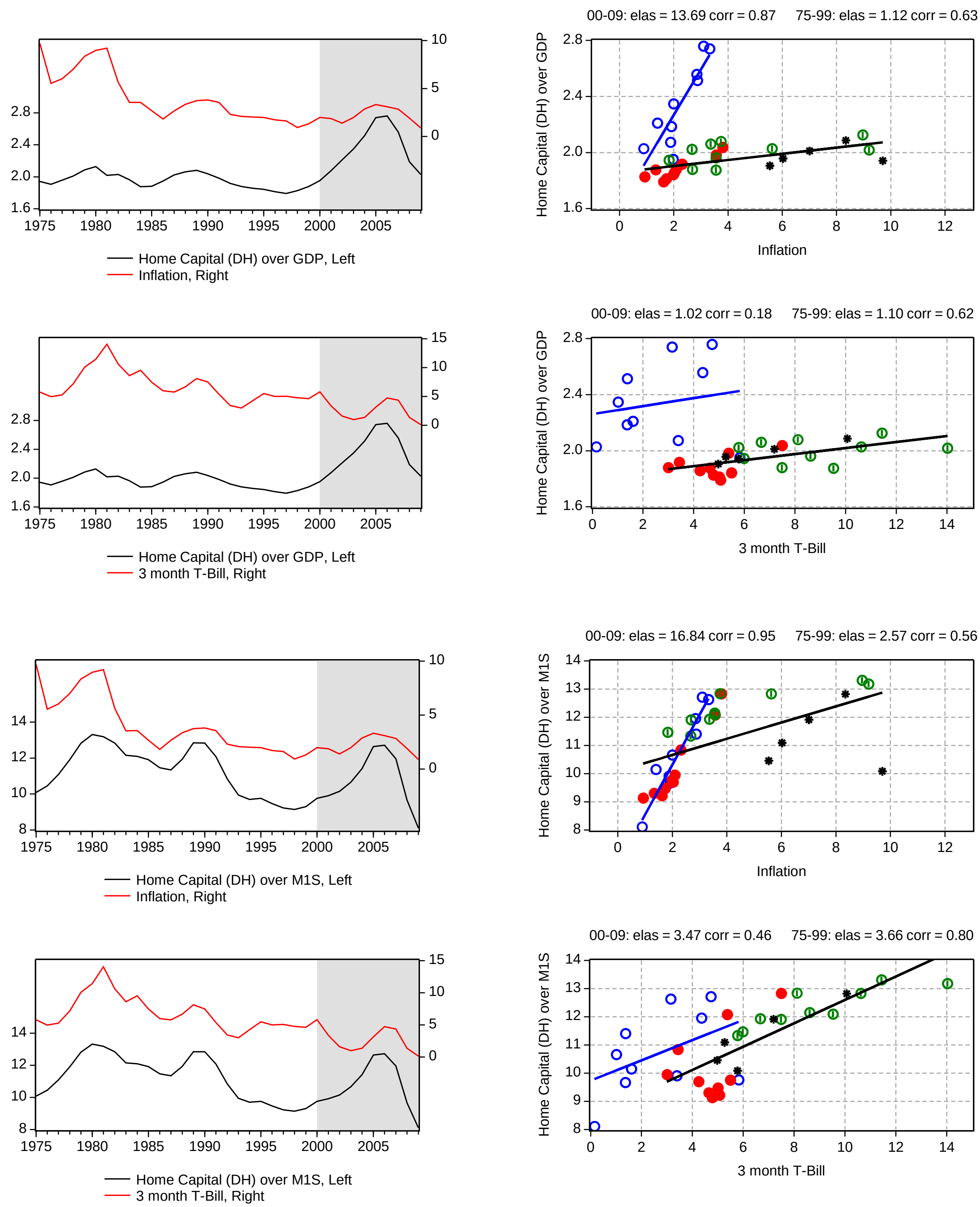
Table 1: Comparison of Shiller and Decennial Census Data

Decade	Total Percent Growth			
	Shiller HPI	Avg. Price (DCH)	Housing Units (DCH)	Nominal GDP
	(1)	(2)	(3)	(4)
1950-1960	30.1	45.2	31.1	74.9
1960-1970	28.9	49.1	24.7	97.8
1970-1980	118.3	188.6	27.3	166.6
1980-1990	71.2	93.4	12.8	106.1
1990-2000	35.3	41.2	14.9	71.5

Table 2: Results

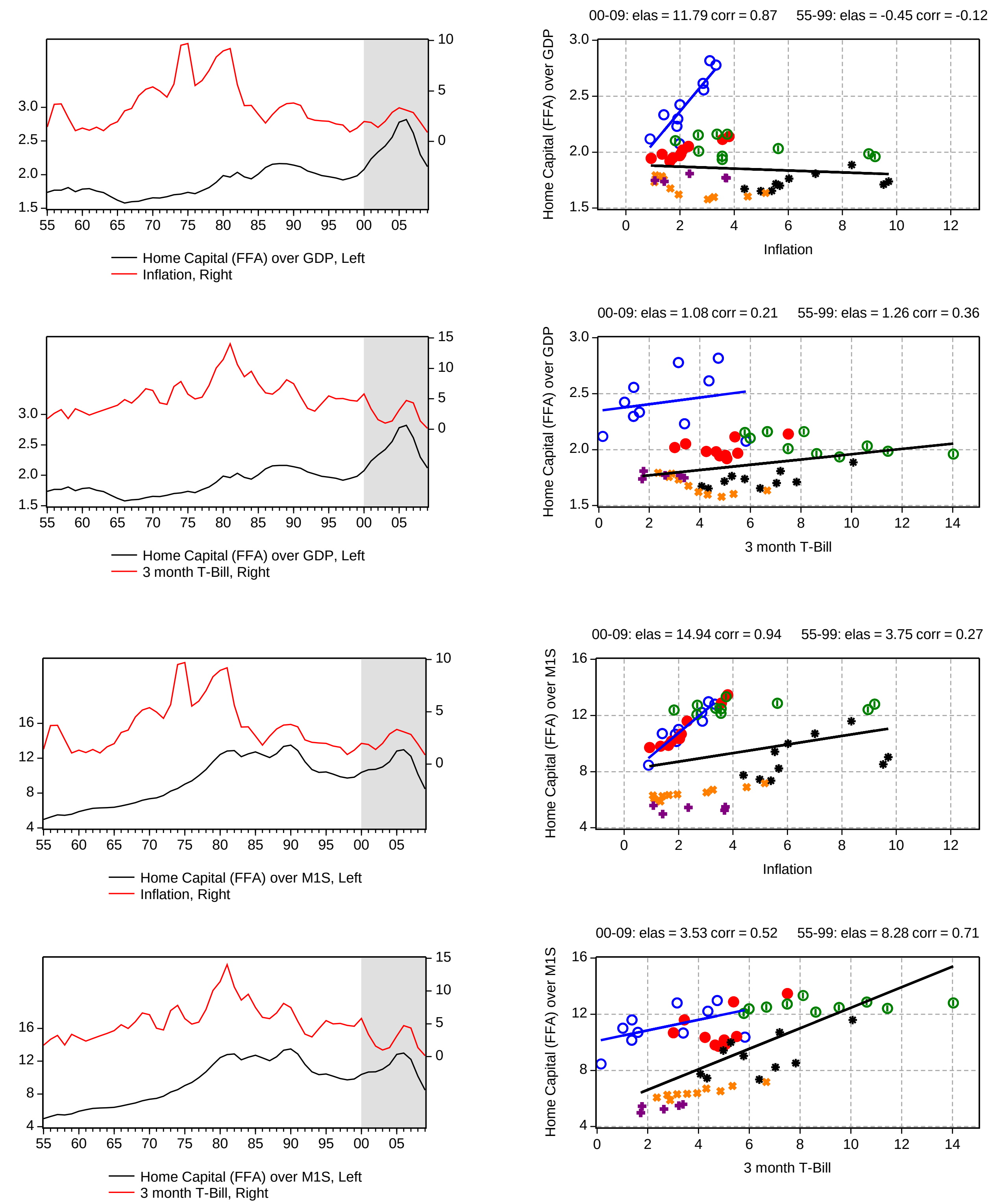
	Calibration 1	**Calibration 2**
Calibrated Parameters		
A_m	1.27	2.14
A_n	2.09	3.15
B	0.33	0.59
χ_m	0.82	0.82
χ_n	0.91	0.90
σ_1	0.11	0.12
σ_2	0.06	0.06
n	0.22	0.24
ω	0.44	0.83
Key Implications		
$\xi_{H/Y}$ (Data: 1.10%)	0.23	0.55
% accounted for	21	50
$\xi_{H/M}$ (Data: 3.66%)	2.79	3.11
% accounted for	78	87
Welfare Gain of Going from 10% Inflation to FR		
% of market cons.	0.53	0.66

Figure 1: DH based Home Capital



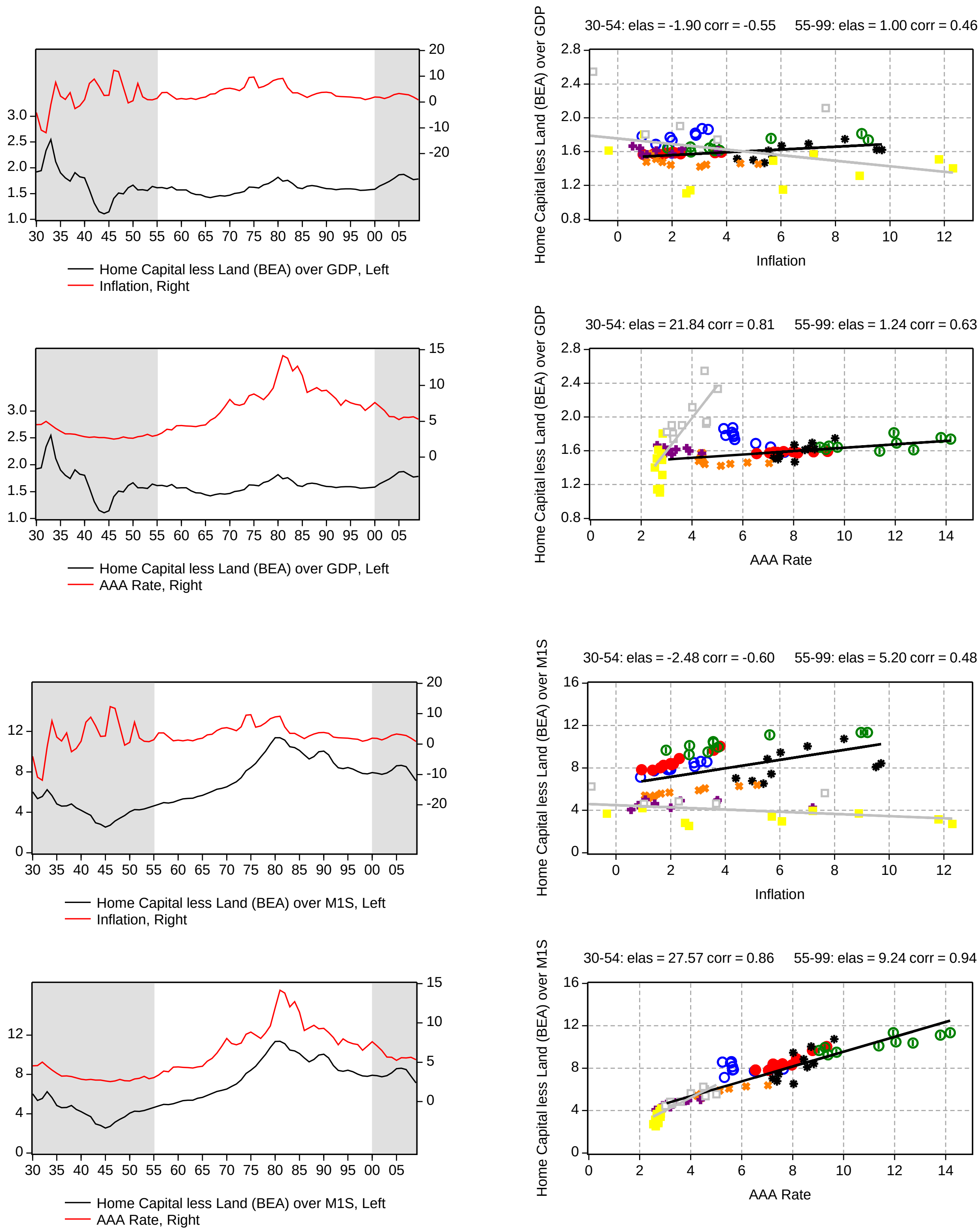
blue 00-09; red 90-99; green 80-89; black 75-79

Figure 2: FFA based Home Capital



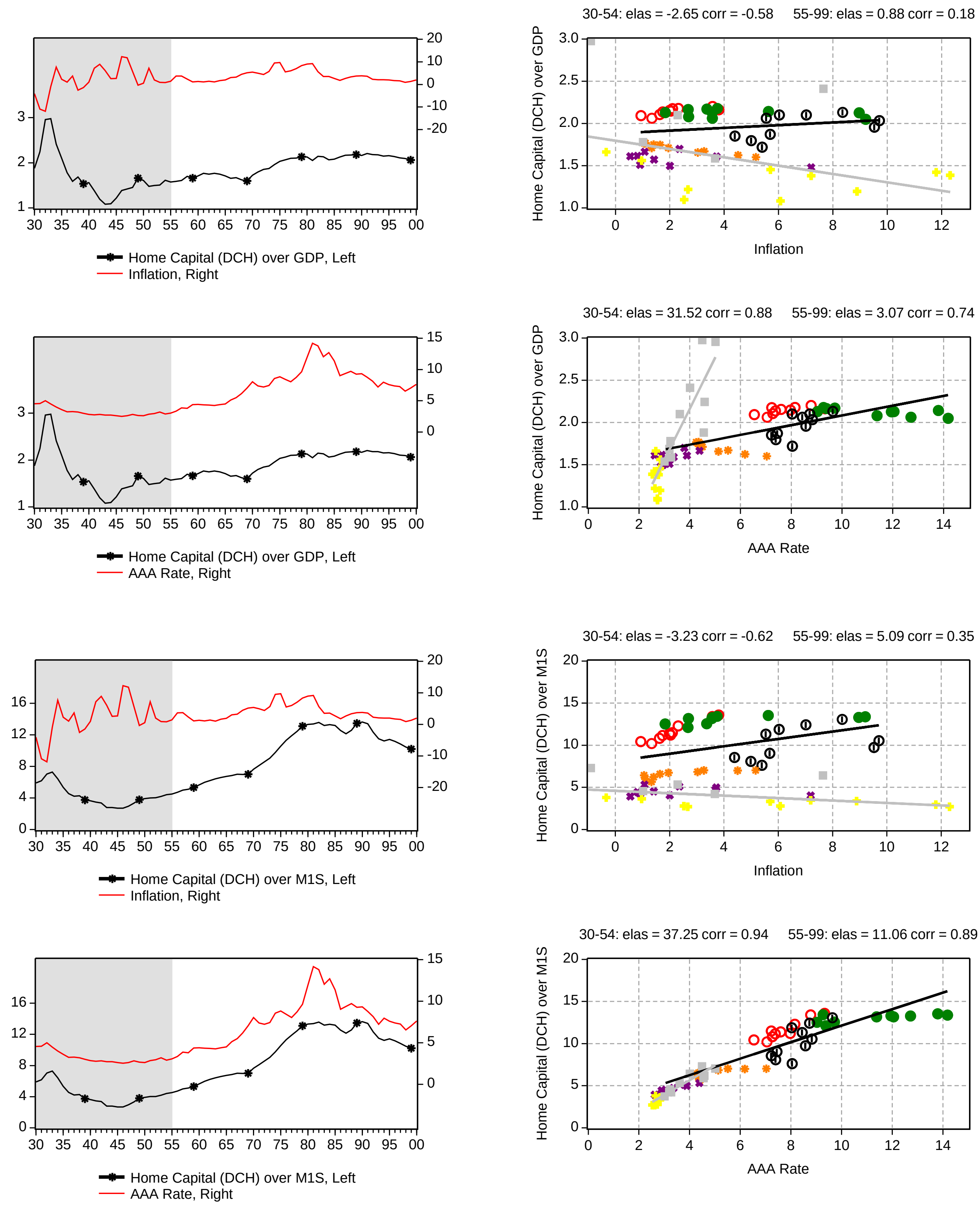
blue 00-09; red 90-99; green 80-89; black 70-79; orange 60-69; purple 55-59

Figure 3: BEA based Home Capital



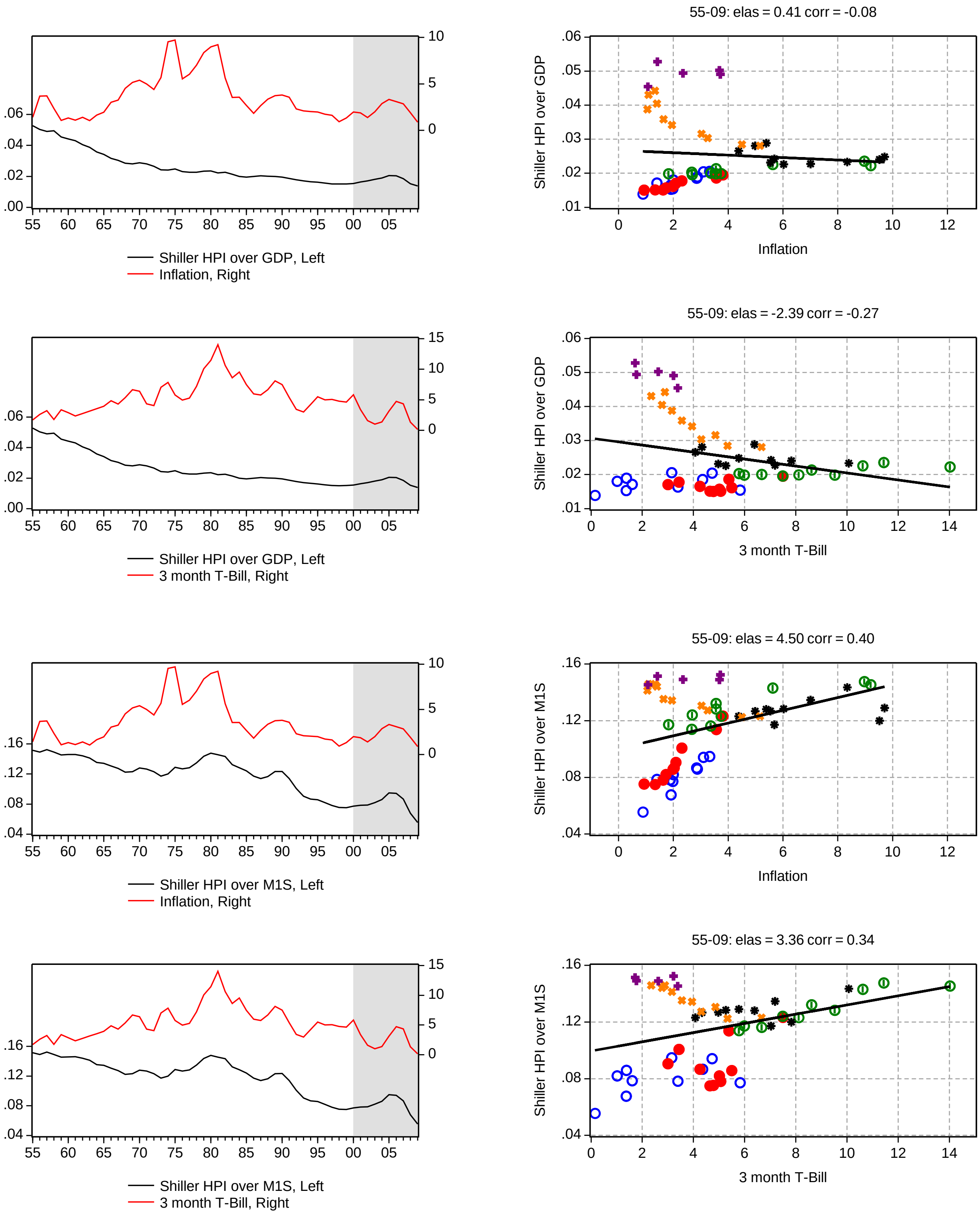
blue 00-09; red 90-99; green 80-89; black 70-79; orange 60-69; purple 50-59; yellow 40-49; gray 30-39

Figure 4: DCH based Home Capital



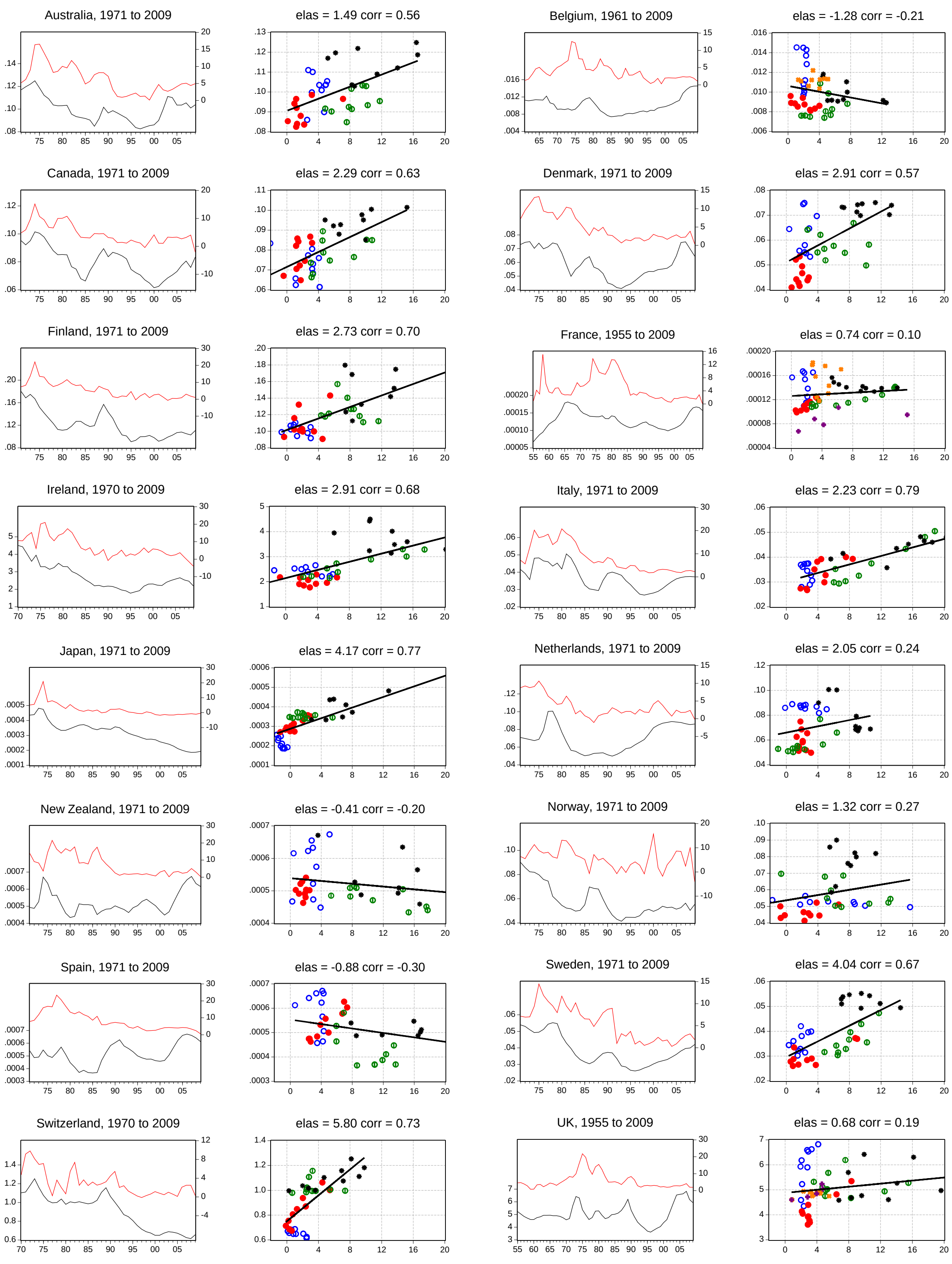
red 90-00; green 80-89; black 70-79; orange 60-69; purple 50-59; yellow 40-49; gray 30-39

Figure 5: Shiller HPI



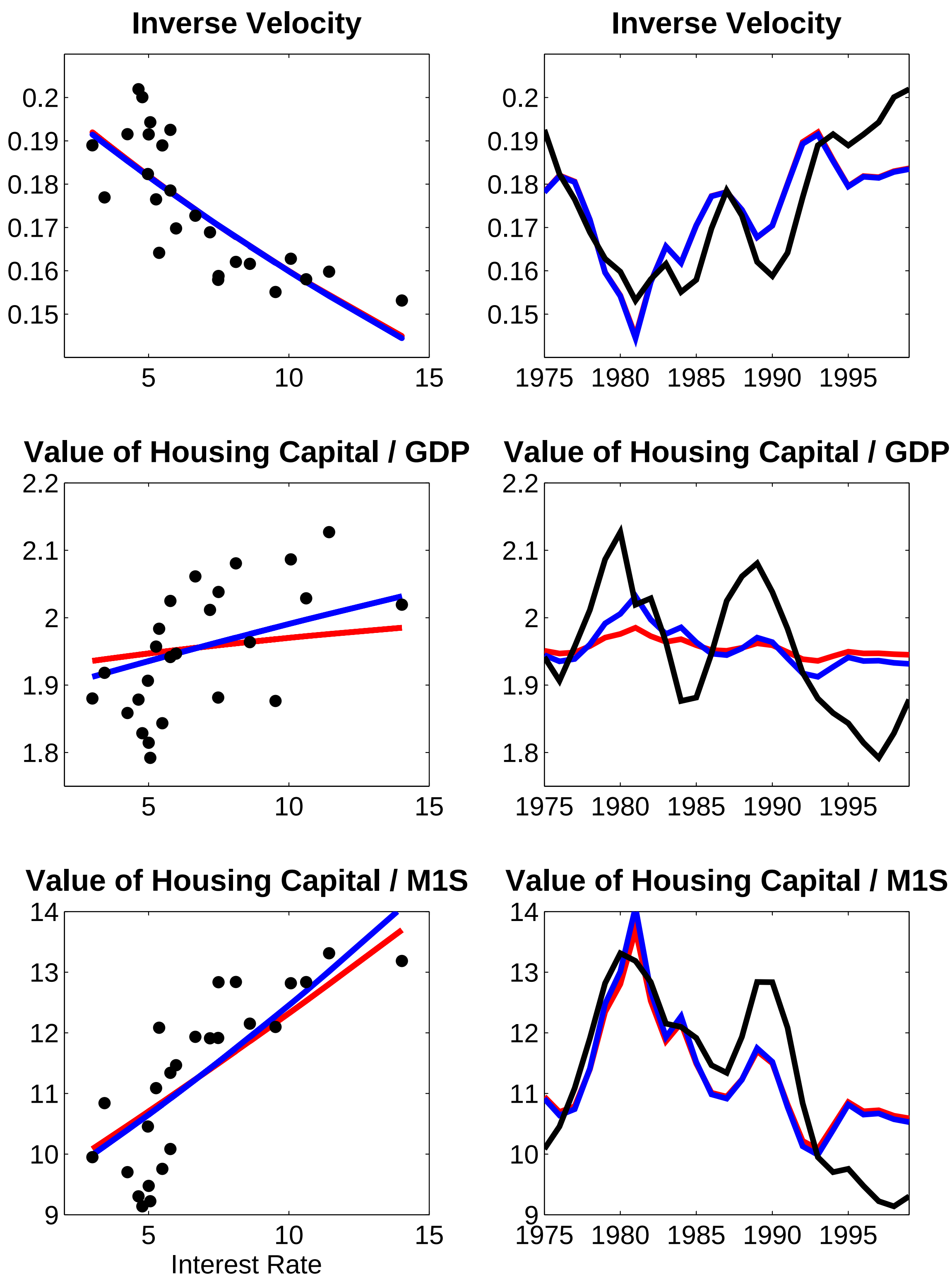
blue 00-09; red 90-00; green 80-89; black 70-79; orange 60-69; purple 55-59

Figure 6: International HPI



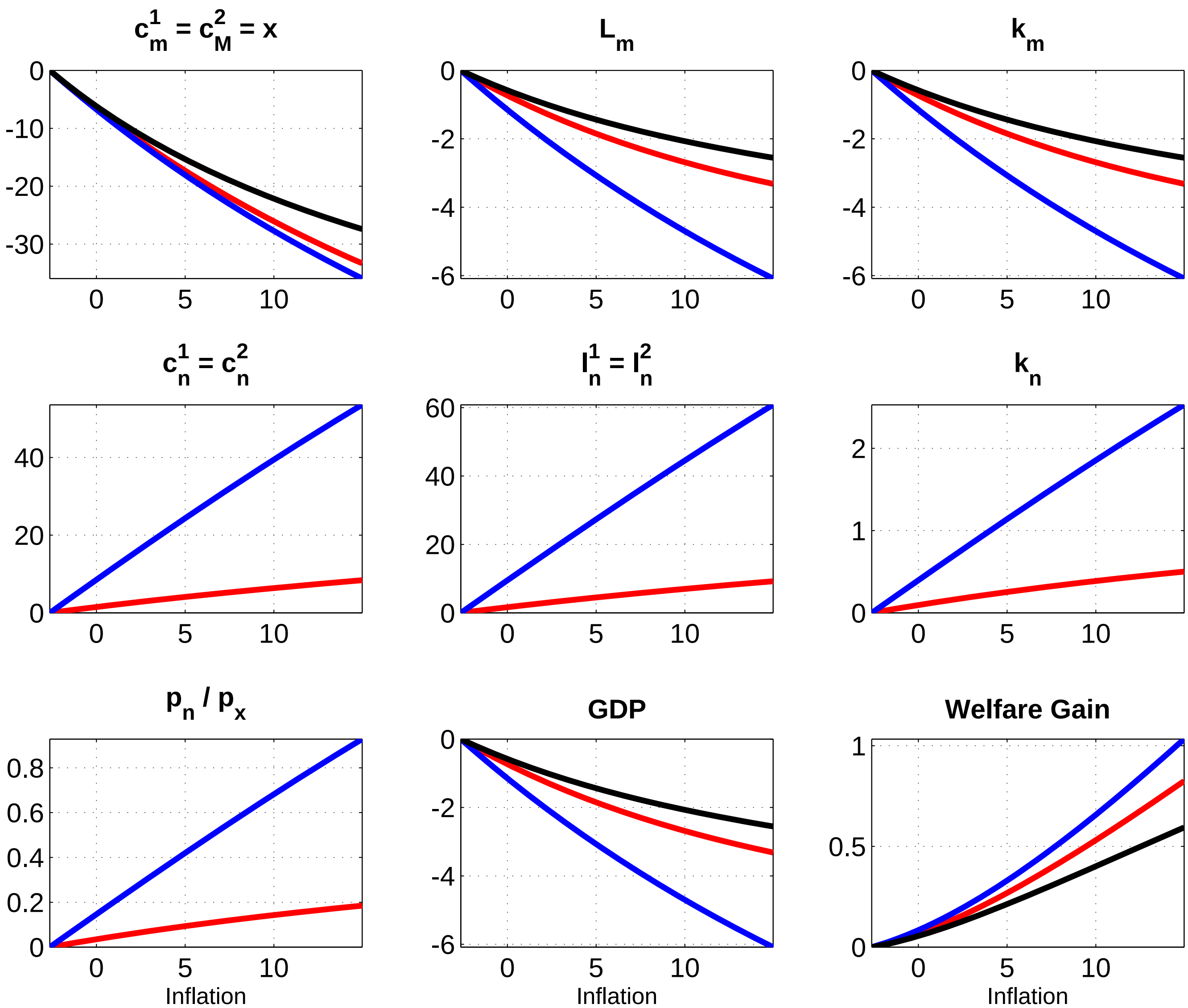
blue 00-09; red 90-00; green 80-89; black 70-79; orange 60-69; purple 55-59

Figure 7: Results from the Model - Key Objects



Notes: Red line: calibration 1, blue line: calibration 2, black: data.

Figure 8: Results from the Model - Effect of Inflation



Notes: Red: Calibration 1, blue: Calibration 2, black: no home production. All quantities are relative to the value for 0% interest rate, expressed in percentage units except for welfare which shows the welfare gain of reducing the interest rate from the shown level to 0%, expressed in percentage of consumption.

Appendix (Not for Publication) : US Data sources

Nominal Stock of Durable Goods	Fixed Asset Tables Table 1.1, line 13 Bureau of Economic Analysis
Davis-Heathcote based Housing Wealth	Davis and Heathcote (2007) available at: http://www.lincolninst.edu/subcenters/land-values/price-and-quantity.asp
Flow of Funds based Housing Wealth	Flow of Funds Tables Table B.100 line 4 plus Table B.10 line 4 Board of Governors of the Federal Reserve
BEA based Housing Wealth	Fixed Asset Tables Table 1.1 line 2 less Table 1.1 line 7 Bureau of Economic Analysis
DCH based Housing Wealth	Appendix of Davis and Heathcote (2007) Tables 5-12 available at: http://morris.marginalq.com/landdata_files/2006-11-Davis-Heathcote-Land.appendix.pdf
Nominal GDP	National Income and Product Accounts Table 1.1.5 line 1 less Table 2.3.5 line 15 Bureau of Economic Analysis
M1	1959 forward: Cynamon et. al. (2006) "M1S" file available at: http://www.sweepmeasures.com/data.html Prior to 1959: U.S. Bureau of the Census (1960, Series X-267)
Inflation	National Income and Product Accounts Computed from Price Index corresponding to GDP Based on: Table 1.1.4 line 1 and Table 2.3.4 line 15 Bureau of Economic Analysis
3-Month T-Bill	Monthly data from January 1934 forward available at: http://www.federalreserve.gov/releases/h15/data.htm Board of Governors of the Federal Reserve
AAA rate	Monthly data from January 1919 forward available at: http://www.federalreserve.gov/releases/h15/data.htm Board of Governors of the Federal Reserve
Shiller HPI	Shiller (2005) available at: http://www.econ.yale.edu/~shiller/data.htm

Appendix (Not for Publication): International Data sources (House Prices)

Belgium	Stadim price index of houses, average price From stadim.be, available at: http://www.stadim.be/uploads/pdf/indexen2011/WOH-E2010.pdf Annual, 1960 -
France	Ministere de l'Ecologie, du Developpement durable, des Transports et du Logement Available at: http://www.cgedd.developpement-durable.gouv.fr/rubrique.php3?id_rubrique=137 Annual, 1936 -
Ireland	From Environment, Community and Local Government Average house (including apartments) prices, Second-hand houses, Annual, Whole country (1974 -) and Dublin (1970-1973)
Switzerland	Swiss National Bank Monthly Statistical Bulletin, $O4_3$ Real Estate price indices – total Switzerland, Single family homes Annual, 1970 -
United Kingdom	Nationwide UK house prices since 1952 All Houses (UK) Index Quarterly, 1952:Q4 -
All other countries	Bank of International Settlements database Annual, 1971-2009

- Data for GDP for all countries comes from the IFS.
- Data for inflation (GDP deflator) for all countries except England and France comes from the World Bank World Development Indicators, which can be downloaded from http://data.worldbank.org/data-catalog/world-development-indicators . These data begin in 1961, explaining the use of different data for England and France
- The data for inflation in England are derived from the Retail Price Index (RPI) all items, as published by the Office for National Statistics table RP02.
- The data for inflation for France are from the same source as for house prices: http://www.cgedd.developpement-durable.gouv.fr/rubrique.php3?id_rubrique=137