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BOUNDEDLY RATIONAL DYNAMIC PROGRAMMING: SOME PRELIMINARY RESULTS

Xavier Gabaix

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I thank David Laibson for a great many conversations about bounded rationality. For useful comments I also thank seminar participants at the AEA meetings, Chicago, U. Penn, and Yale. This note summarizes some elements of Gabaix (2011, 2012), which remain the primary references for this research. The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

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ABSTRACT

A key open question in economics is the practical, portable modeling of bounded rationality. In this short note, I report ongoing progress that is more fully developed elsewhere. I present some results from a new model in which the decision-maker builds a simplified representation of the world. The model allows to model boundedly rational dynamic programming in a parsimonious and quite tractable way. I illustrate the approach via a boundedly rational version of the consumption-saving life cycle problem. The consumer can pay attention to the variables such as the interest rate and his income, or replace them, in his mental model, by their average values. Endogenously, the consumer pays little attention to interest rate but pays keen attention to his income. One consequence of this is that Euler equations will be biased, and the intertemporal elasticity of substitution will be biased toward 0, in a manner that is quantitatively important.

Xavier Gabaix
New York University
Finance Department
Stern School of Business
44 West 4th Street, 9th floor
New York, NY 10012
and NBER
xgabaix@stern.nyu.edu

A key open question in economics is the practical, portable modeling of bounded rationality. In this short note, I report ongoing progress that is more fully developed elsewhere (Gabaix 2011, 2012).

The approach has at its core a model with boundedly rational (BR) features in which the decision-maker (DM) builds a simplified representation of the world. This representation is "sparse," i.e., uses few parameters that are non-zero, or differ from the usual state of affairs.[1] The DM may imperfectly maximize, based again on a penalty related to sparsity. Sparsity is formulated so as to lead to well-behaved, convex maximization problems.

The model is a tractable algorithm that can be used to inject bounded rationality into the basic models of economics. For instance, it delivers a way to model boundedly rational dynamic programming. Given its tractability, it can be embedded in equilibrium, e.g., in market equilibria with rational firms and boundedly rational consumers, and in general equilibrium. To illustrate the sparse framework, this paper uses the model to solve a canonical consumption-savings problem.

1 Bounded Rationality in a 2-Period Problem

Consider for concreteness the following decision problem with just two periods: the DM's value function is:

$$V(c, \widehat{r}_t, \widehat{y}_t) = u(c) + \beta v\left((1 + \overline{r} + \widehat{r}_t)(w - c) + \overline{y} + \widehat{y}_t\right),$$

and he wishes to $\max_c V(c, \widehat{r}_t, \widehat{y}_t)$. That is, the consumer starts from an initial wealth w, and picks his consumption c in order to maximize his utility, given that next period's consumption will be next period's income, $y_t = \overline{y} + \widehat{y}_t$, plus today's savings, $w - c$, compounded by the interest rate, $r_t = \overline{r} + \widehat{r}_t$. Here $\overline{r}$ is the average value of the interest rate (I take the default value to be the average), and $\widehat{r}_t$ is the (mean-zero) deviation of the interest rate from its average; the same holds for $\overline{y}$, the average income, and $\widehat{y}_t$, the deviation of income from its average.

A rational consumer will do: $\max_c V(c, \widehat{r}_t, \widehat{y}_t)$. What will a BR consumer do? Using

[1] The meaning of "sparse" is that of a sparse vector or matrix. For instance, a vector in $\theta \in \mathbb{R}^{10,000}$ with only a few non-zero elements is sparse.

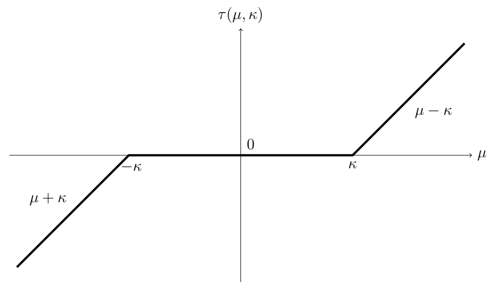

Figure 1: The anchoring-and-adjustment function τ

a mix of psychological and economic reasoning, I propose in Gabaix (2011) a reasonably systematic way of handling that. The DM trades off the cost of having an imperfect decision against the benefits of saving on "thinking costs." This leads to an algorithm that boils down to the following procedure in our consumption-investment case.

First, the consumer knows what to do under a "default model" where $(\widehat{r}_t, \widehat{y}_t) = (0,0)$, i.e., all variables are at their average values. Then, the consumer has cognitive access to $\partial c/\partial r = -V_{cr}/V_{cc}$ at the default model, i.e., by how much consumption should change if the interest rate goes up by a small amount. It may seem a bit strange that the consumer might know so much, but this assumption captures parsimoniously the fact that people do have a sense that some quantities (e.g., their income) matter a lot, while others (e.g., the volatility of the 1-year interest rate and, perhaps, that interest rate itself) do not matter very much.

Step 1. Replace the interest rate $\widehat{r}_t$ (to be more precise, the deviation of the interest rate from its average) by its truncated version: the interest rate perceived by a BR agent is (very shortly I will motivate and explain this particular formula):

$$\widehat{r}_t^{BR} = \tau\left(1, \frac{\kappa\sigma_c}{\frac{\partial c}{\partial r}\sigma_r}\right)\widehat{r}_t, \tag{1}$$

where $\partial c/\partial r$ is taken at the default model, and the truncation function $\tau(\mu, \kappa') = (|\mu| - |\kappa'|)_+ \, sign(\mu)$ is represented in Figure 1. $\widehat{r}_t^{BR}$ is the deviation of the interest rate from its default perceived by a BR consumer.

Likewise, the perceived income innovation is: $\widehat{y}_t^{BR} = \tau\left(1, \frac{\kappa\sigma_c}{\frac{\partial c}{\partial y}\sigma_y}\right)\widehat{y}_t$.

Step 2: Then, the BR agent does $\max_c V\left(c, \widehat{r}_t^{BR}, \widehat{y}_t^{BR}\right)$.

Step 2 is unproblematic: given the perceived interest rate and income, the DM optimizes consumption. The nerve of the model is in Step 1. To interpret rule (1), note that it implies:

"Replace the interest rate by 0 if taking the interest rate into account changes consumption by less than κ standard deviations, i.e., if $\left|\frac{\partial c}{\partial r}\sigma_r\right| < \kappa\sigma_c$."

That means: on average, a one-standard-deviation change in the interest rate makes the BR agent change his consumption by only $\frac{\partial c}{\partial r}\frac{\sigma_r}{\sigma_c}$ standard deviations of consumption. If that ratio is small enough (I calibrate the model to $\kappa = 0.3$, so that features which account for less than $\kappa^2 = 9\%$ of the variance are eliminated), then replace the interest rate by 0.[2]

The penalty for lack of sparsity, κ, is akin to an index of bounded rationality: if $\kappa = 0$, the agent is fully rational.

Take the case where $\left|\frac{\partial c}{\partial r}\frac{\sigma_r}{\sigma_c}\right| < \kappa$, so that $\widehat{r}_t^{BR} = 0$ and the DM proceeds as if the interest rate was the average interest rate $\overline{r}$ rather than the true interest rate r_t. We have the picture of a sensible agent: he does not pay attention to the interest rate all the time, he saves (so he is not "myopic" in the sense of heavily discounting the future), but he does not obsess about smoothing his consumption given all wrinkles to the interest rate. This agent is arguably more sensible and realistic than the traditional agent (below I will offer some empirical evidence for that intuition).

Here, we use the "average values" for the interest rate and income shocks. In a one-shot problem, we would use the above rule, replacing $|\sigma_r|$ by $|\widehat{r}_t|$, so that instead of (1) we obtain $\widehat{r}_t^{BR} = \tau\left(\widehat{r}_t, \frac{\kappa\sigma_c}{\frac{\partial c}{\partial r}}\right)$. Then, the rule becomes: "Replace the interest rate by 0 iff taking it into account makes consumption change by less than κ standard deviations." Indeed, the agent does not respond to the interest rate at all if $\left|\frac{\partial c}{\partial r} \times \widehat{r}_t^{BR}\right| < \kappa\sigma_c$. Thus, most of the time, the agent will not take the wrinkles of the interest rate into account, but will pay attention to changes in the interest rate only when changes are very large (e.g., if there is a large, one-time discount of, say, cars).

The truncation rule embodies the idea that a DM who seeks "sparsity" (uncluttering his mind from lots of small things) should sensibly drop relatively unimportant features: if they account for less than κ standard deviations of the actions, they are dropped entirely. In addition, if the features are larger than that cutoff, they are still dampened: in Figure 1, $\tau(\mu, \kappa)$ is below the 45 degree line (for positive μ; in general, $|\tau(\mu, \kappa)| < |\mu|$). This reflects Kahneman and Tversky's "anchoring-and-adjustment" process, in which there is an anchor in the default model, and then a partial adjustment toward the truth. This feature could be abandoned, using the "hard-thresholding" function $\tau^H(\mu, \kappa) = \mu 1_{|\mu|>|\kappa|}$ instead:

[2]The main paper provides a microfoundation based on the welfare loss from a suboptimal answer.

if $|\mu| > \kappa$, then this is the 45 degree line, $\tau^H(\mu, \kappa) = \mu$. However, the above function τ has the advantage of yielding continuous demand curves, which are likely in practice. For many cases, the smooth adjustment makes more empirical sense than the "all-or-nothing" adjustment, which predicts discontinuities that we are unlikely to see empirically.

I hope that the reader got a sense of the intuition for the model in a (quasi-)static context. Let us now see how to proceed in more dynamic contexts.

2 Bounded Rationality in an Infinite-Horizon Problem

One important payoff from the framework is that it allows for boundedly rational dynamic programming (BRDP). This is important because many models in macroeconomics and finance take the form of dynamic programming (Ljungqvist and Sargent 2004). The outcome will be a model that is often *simpler* than the traditional model, because agents pay attention to fewer things and, in particular, do not react to all future variables.

In addition, it is well-known that an important conceptual and practical problem when dealing with dynamic programming is the curse of dimensionality. Strictly speaking, there are perhaps over 1,000 state variables that should matter in our decisions, but solving dynamic-programming problems with more than a few state variables (let alone 1,000 state variables) is extremely hard in practice because of the combinatorial explosion of the problem's complexity. Even the most powerful computers cannot handle such complexity and solve the problems exactly. Given that, how would a boundedly rational agent proceed?

I illustrate the approach in a canonical consumption-investment problem. The agent has utility $\mathbb{E}\sum_{t=0}^{\infty}\beta^t c_t^{1-\gamma}/(1-\gamma)$. We assume he has solved the life-cycle problem in a simple model, where the interest rate is constant at $\overline{r}$ (for simplicity, assume here that $\overline{R} \equiv 1 + \overline{r} = \beta^{-1}$) and his income is constant at $\overline{y}$: his wealth w_t evolves according to $w_{t+1} = (1+\overline{r})(w_t - c_t) + \overline{y}$ (that is, wealth at $t+1$ is savings at t, $w_t - c_t$, invested at rate $\overline{r}$, plus current income, $\overline{y}$). Then, the optimal consumption is $c^d(w_t) = (\overline{r}w_t + \overline{y})/\overline{R}$, and the value function is $V^d(w_t) = A(\overline{r}w_t + \overline{y})^{1-\gamma}$ for a constant A.

Now, the agent is told that the world is more complicated: the interest rate is actually $\overline{r} + \widehat{r}_t$ and his income is $\overline{y} + \widehat{y}_t$, where $\widehat{r}_t$ and $\widehat{y}_t$ are deviations of the interest rate and income from their trends, respectively, and follow AR(1)s: $y_{t+1} = \rho_y\widehat{y}_t + \varepsilon_{t+1}^y$ and $r_{t+1} = \rho_r\widehat{r}_t + \varepsilon_{t+1}^r$, where $\varepsilon_{t+1}^{\cdot}$ are mean-0 disturbances. Hence, wealth follows:

$$w_{t+1} = (1 + \overline{r} + \widehat{r}_t)(w_t - c_t) + \overline{y} + \widehat{y}_t.$$

What will be the consumption function $c(w_t, \widehat{y}_t, \widehat{r}_t)$ of a BR agent? It is difficult, because this is a dynamic-programming problem with 3 state variables, and has no closed forms. Under the previous approach, one might think that one should solve for the value function $V(w_t, \widehat{y}_t, \widehat{r}_t)$; but that would be a very difficult task in general: DP with 3 or more (and in practice perhaps 20) state variables is very difficult. However, we obviate this difficulty by using the following algorithm.

Step A (Taylor expansion around the simple, default model with just one state variable). We observe that a rational agent would consume, for small disturbances $\widehat{y}_t$ and $\widehat{r}_t$:

$$\begin{aligned}\ln c^{\mathrm{Rat}}(w_t, \widehat{y}_t, \widehat{r}_t) &= \ln c^d(w_t) + b_y \widehat{y}_t + b_r \widehat{r}_t \\ &\quad + \text{2nd-order terms.}\end{aligned} \tag{2}$$

Importantly, the terms b_y, b_r are easy to derive by a local expansion of the simple, one-dimensional value function $V^d(w_t)$ (i.e., without solving for the full function $V(w_t, \widehat{y}_t, \widehat{r}_t)$). Indeed, by perturbation arguments, which would be too long to describe here but are of methodological interest,[3] one finds:

$$b_y = \frac{\overline{r}}{\overline{R}\left(\overline{R} - \rho_y\right) c_t^d}, \qquad b_r = \frac{\overline{r}\left(\frac{w_t}{c_t^d} - 1\right) - 1/\gamma}{\overline{R} - \rho_r}. \tag{3}$$

Then, we assume that the BR agent somehow has cognitive access to b_r and b_y: while it

[3] The sketch is the following. First, we write the value function with the full problem:

$$V(w, r) = \max_c u(c) + \beta \mathbb{E} V(w', r')$$

$$w' = \left(\overline{R} + r\right)(w - c) + Y, \qquad r' = \rho_r r + \varepsilon^r.$$

Then, we take the partial derivative w.r.t. r, and then take the total derivative w.r.t. w, $\frac{D}{Dw} f = \frac{\partial}{\partial w} f + \frac{\overline{r}}{\overline{R}} \frac{\partial}{\partial c} f$ (which includes the indirect effect of w on c). After some algebra, one arrives at $V_{w,r} = \frac{V_w^d}{\overline{R}} \frac{1 - \gamma \overline{r}\left(\frac{w}{c} - 1\right)}{\overline{R} - \rho_r}$, the key term in the expansion of the value function, from which one can derive $\frac{\partial c}{\partial r}$.

may seem counterintuitive, this merely represents that the BR agent senses that, for instance, the interest rate is not a very important decision for his consumption ($|b_r|$ is small).

Step B (Simplification of the reaction function). The DM does a BR truncation of (2), according to formula (1). Hence, we obtain the following.

Proposition 1 *A BR agent has the following consumption policy:*

$$\ln c_t^{BR} = \ln c^d(w_t) + b_y^{BR}\widehat{y}_t + b_r^{BR}\widehat{r}_t, \tag{4}$$

where (for $x = y, r$) $b_x^{BR} := \tau\left(b_x, \frac{\kappa\sigma_{\ln c}}{\sigma_x}\right)$ and b_x are in (3).

Formula (4) shows a "feature-by-feature" truncation. It is useful because it embodies in a compact way the policy of a BR agent in a quite complicated world. Note that the agent can do that without solving the 3-dimensional (and potentially 21-dimensional, say, if there are 20 state variables besides wealth) problem. Only local expansions and truncations are necessary.

In this manner, we arrive at a quite simple way to do BRDP. There is just one continuously-tunable parameter, κ. When $\kappa = 0$, the agent is (to the leading order) the traditional rational agent. When κ is large enough, the agent is fully BR, and does not react to any variable. Hence, we have a simple, smooth way to parametrize the agent, from very BR to (essentially) fully rational.

2.1 Application: Insensitivity to the Interest Rates and Low Measured Intertemporal Elasticity of Substitution

To get a feel for the effects, consider a calibration with: $\gamma = 1$, $r = 5\%$, $\overline{w} = 2\overline{c}$, $\overline{c} = 1$, $\sigma_r = 0.8\%$, $\sigma_y = 0.2\overline{c}$, $\rho_y = 0.95$, $\sigma_{\ln c} = 5\%$, and $\rho_r = 0.7$ with yearly units: as income shocks are persistent, they are important to the consumer's welfare. Then, Figure 2 shows the impact of a change in the interest rate and income on consumption. Consider the left panel, b_r^{BR}. If the cost of rationality is $\kappa = 0$, then the agent reacts like the rational agent: if interest rates go up by 1%, then consumption falls by 2.8% (the agent saves more). However, for a sparsity parameter $\kappa \simeq 0.45$, the agent essentially does not respond to interest rates. Psychologically, he thinks "the interest rate is too unimportant, so let me not take it into account." Hence, the agent does not react much to the interest rate, but will react more

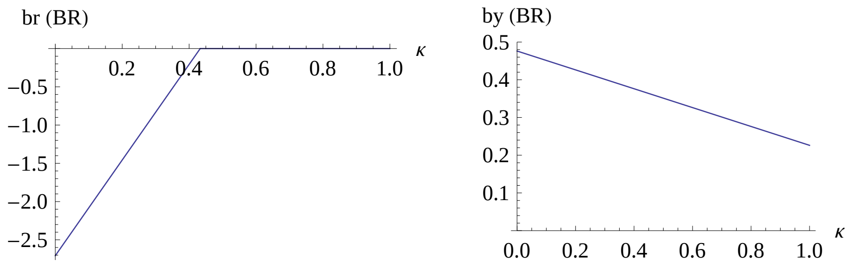

Figure 2: Impact of a change in the interest rate (resp. income) on consumption, as the function of weight on sparsity, κ. $\kappa = 0$ is the rational-agent model.

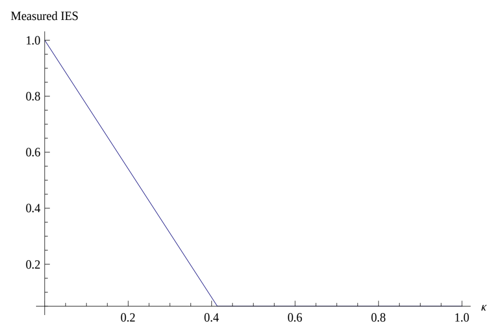

Figure 3: Measured IES $\widehat{\psi}$ if the consumer is boundedly rational with sparsity cost κ.

to a change in income (right panel of Figure 2), which is more important: the sensitivity of consumption to income remains high even for a high cognitive friction κ. Note that this "feature-by-feature" selective attention could not be rationalized by just a fixed cost to consumption, which is not feature-dependent.

The same reasoning holds in every period. The above describes a practical way to do BR dynamic programming. In some cases, this is simpler than the rational way (as the agent does not need to solve for the equilibrium), and this may also be more sensible.

Let us sketch a consequence. For many finance applications (e.g., Bansal and Yaron 2004, Barro 2009, Gabaix forth.), a high intertemporal elasticity of substitution (IES, denoted $\psi = 1/\gamma$) is important ($\psi > 1$). However, micro studies point to an IES less than 1 (e.g., Hall 1988). I show how this may be due to the way econometricians proceed, by fitting the Euler equation, which yields $\ln c_{t+1} - \ln c_t = \frac{\widehat{\psi}}{\overline{R}} r_t +$ constant, where $\widehat{\psi}$ is the measured IES. If the consumer "underreacts to the interest rate," the measured IES will be biased towards 0. Using the above model, we can more precisely calculate that if consumers are boundedly rational (in the sense laid out above), the estimated IES will be : $\widehat{\psi} = \overline{r}\left(w_t/c_t^d - 1\right) - b_r^{BR}\overline{R}\left(\overline{R} - \rho_R\right)$. This is a point prediction that goes beyond Chetty (forth.)'s prediction of an interval bound. Hence we obtain:

Proposition 2 *An econometrician fitting an Euler equation even though the agent is BR will estimate a biased IES (intertemporal elasticity of substitution):*

$$\widehat{\psi} = \psi - \overline{R}\left(\overline{R} - \rho_R\right)\left(b_r^{BR} - b_r\right)$$

where $\widehat{\psi}$ *is the estimated IES,* ψ *the true IES,* $b_r^{BR} - b_r$ *is the difference between the BR agent's and the traditional rational agent's interest-rate sensitivity of consumption.*

The above calibration yields Figure 3, which plots the measured IES $\widehat{\psi}$ if the consumer is BR with sparsity cost κ. If $\kappa = 0$, the consumer is the traditional, frictionless rational agent. We see that as κ increases, the IES is more and more biased. Hence, BR may explain why while macro-finance studies require a high IES, microeconomic studies find a low IES.[4]

2.2 Application: Source-dependent Marginal Propensity to Consume

The agent has initial wealth w, future income y, he can consume c at time 1, and invest the savings at a rate R. Hence, the problem is as follows. Given an initial wealth w, solve $\max_c V = u(c) + \mathbb{E}\left[v\left(y + R(w - c)\right)\right]$, where income is $y = y_* + \sum_{i=1}^n y_i$: there are n sources of income y_i with mean 0. Let us study the solution of this problem with the algorithm. The DM observes the income sources sparsely: he uses the model $y(m) = y_* + \sum_{i=1}^n m_i y_i$, with m_i to be determined. Applying the model, we obtain (assuming exponential utility with absolute risk aversion γ for simplicity)

Proposition 3 *Time-1 consumption is:* $c = \frac{1}{1+R}(Rw + \delta/\gamma - \gamma\sigma_\varepsilon^2/2 + y_* + \sum_i m_i y_i)$, $m_i = \tau(1, \frac{\kappa^m \sigma_{c2}}{\sigma_{y_i}})$. *The marginal propensity to consume (MPC) at time 1 out of income source i is:*

$$MPC_i^{BR} = MPC_i^{Rat} \cdot m_i, \tag{5}$$

[4] This is in the spirit of Gabaix and Laibson (2002)'s analysis of the biases in the estimation of the coefficient of risk aversion with inattentive agents, in a different context and a more tractable model. See also Fuster, Laibson and Mendel (2010) for a model where agents' use of simplified models leads to departures from the standard aggregate model.

where $MPC_i^{BR} = (\partial c/\partial y_i)^{BR}$ *is the MPC under the BR model, and* $MPC_i^{Rat} = (\partial c/\partial y_i)^{Rat}$ *is the MPC under the traditional rational-actor model. Hence, in the BR model, unlike in the traditional model, the marginal propensity to consume is source-dependent.*

Different income sources have different marginal propensities to consume – this is reminiscent of Thaler (1985)'s mental accounts. Equation (5) makes another prediction, namely that consumers pay more attention to sources of income that usually have large consequences, i.e., have a high σ_{y_i}. Slightly extending the model, it is plausible that a shock to the stock market does not affect the agent's disposable income much – hence, there will be little sensitivity to it: the MPC out of wage income will be higher than the MPC to consume out of portfolio income.

This model shares similarities with models of inattention based on a fixed cost of observing information. Those models are rich and relatively complex (they necessitate many periods, or either many agents or complex, non-linear boundaries for the multidimensional s, S rules, or signal extraction as in Sims 2003), whereas the present model is simpler and can be applied with one or several periods. As a result, the present model, with an equation like (5), lends itself more directly to empirical evaluation. Some interesting "low-complexity" models include Bordalo, Gennaioli, and Shleifer (2011) and Koszegi and Szeidl (2011). A distinctive feature of the model presented in this note is its ability to handle continuous choices (e.g., a consumption level) rather than the discrete choice between distinct goods.

3 Conclusion

I sketched here a practical way to do boundedly rational dynamic programming. It is portable and to the first order has just one free continuous parameter, κ, the penalty for lack of sparsity, which can also be interpreted as a cost of complexity. The model is easy to solve, and avoids the curse of dimensionality. In ongoing research, I further develop this framework in order to extend it to contexts of agents playing games and paying attention to equilibrium market prices, and to study how bounded rationality can sometimes materially affect the conclusions drawn from the traditional model. Hence, we may be one step closer to having a tractable, parsimonious enrichment of the traditional rational model that allows us to explore when bounded rationality matters in economic situations.

REFERENCES

Bansal, Ravi, and Amir Yaron. 2004. "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." *Journal of Finance*, 59: 1481–1509.

Barro, Robert. 2009. "Rare Disasters, Asset Prices, and Welfare Costs." *American Economic Review* 99: 243–64.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer. 2011. "Salience and Consumer Choice." Harvard University Working Paper.

Chetty, Raj. 2012. "Bounds on Elasticities with Optimization Frictions: A Synthesis of Micro and Macro Evidence on Labor Supply." *Econometrica,* forthcoming.

Fuster, Andreas, David Laibson, and Brock Mendel. 2010. "Natural Expectations and Macroeconomic Fluctuations," *Journal of Economic Perspectives*, 24: 67-84.

Gabaix, Xavier. 2011. "A Sparsity-Based Model of Bounded Rationality." NBER Working Paper 16911.

Gabaix, Xavier. 2012. "Boundedly Rational Dynamic Programming." Unpublished.

Gabaix, Xavier. 2012. "Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance," *Quarterly Journal of Economics*, forthcoming.

Gabaix, Xavier, and David Laibson. 2002. "The $6D$ bias and the Equity Premium Puzzle." *NBER Macroeconomics Annual*, 16: 257–312.

Gabaix, Xavier, and David Laibson. 2006. "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets." *Quarterly Journal of Economics*, 121: 505–40.

Hall, Robert E. 1988. "Intertemporal Substitution in Consumption." *Journal of Political Economy* 96: 339–57.

Koszegi, Botond, and Adam Szeidl. 2011. "A Model of Focusing in Economic Choice." Unpublished.

Sims, Christopher. 2003. "Implications of Rational Inattention," *Journal of Monetary Economics*, 50: 665–690.

Thaler, Richard H. 1985. "Mental Accounting and Consumer Choice," *Marketing Science*, 4: 199-214.