

NBER WORKING PAPER SERIES

SHOULD SOCIAL SECURITY
BE MEANS TESTED?

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Working Paper No. 1775

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 1985

The research reported here is part of the NBER's research program in Taxation and project in Government Budgets. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

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ABSTRACT

The provision of social security benefits to retirees distorts the saving decisions of workers who are rational enough to save for their future. Since the implicit rate of return in an unfunded social security program is less than the marginal product of capital, the resulting decline in saving causes a welfare loss. It has been suggested that this welfare loss could be reduced, while still protecting those who lack the foresight to save for their retirement (the "myopes" and "partial myopes" of the paper), by replacing the current universal social security program with a means-tested program that pays benefits only to the "myopic" individuals who have little or no other retirement income or assets.

The present paper evaluates this suggestion with the help of an explicit steady-state welfare comparison of the optimal universal and optimal means-tested programs. The relative welfare levels depend on characteristics of the economy (the growth rates of population and real wages and the productivity of capital) and of the population (the frequency and degree of myopia with respect to saving for retirement).

The analysis shows that, although a means tested program is generally superior, it does not always dominate the best alternative universal program. A universal program can be preferable under conditions which imply that the optimal means-tested program would induce rational savers to stop saving. The analysis also implies that overall welfare can be increased by using different social security programs for different groups of workers if the working population as a whole can be divided into two or more subgroups with different mixes of myopes, partial myopes and rational life-cycle savers.

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SHOULD SOCIAL SECURITY BENEFITS BE MEANS-TESTED?

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Every society must solve the problem of supporting those individuals who become too old to work but have not made adequate provision for their own old age by saving when they were young. At the present time, the major industrial countries of the world have responded to this problem by creating social security programs that tax the working population and use the proceeds to provide a "universal" benefit to all retirees regardless of their financial condition.

This universal provision of social security benefits distorts the saving decisions of those workers who are rational enough to plan for the future.² Since the implicit rate of return that taxpayers get on their contributions to an unfunded social security program is less than the return available on

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²For a discussion of the effect of promised retirement benefits on the savings of employees, see Feldstein (1974, 1982a). Barro (1974, 1978) has discussed an interesting special case in which social security would have no effect on private saving. His conclusion requires that each individual acts to maximize a utility function that has as one of its arguments the utility of his children, and that the initial optimum behavior involves a positive amount of bequest. Of course, if everyone exhibited such rationality, there would be no need or justification for social security retirement benefits. The Barro conditions imply that social security does not change aggregate savings but some individuals may save more while others save less. Every individual's savings will be unchanged only if all individuals in each current and future generation are alike. [Footnote continued on next page]

savings invested in real capital,³ the individual who substitutes social security tax contributions for private savings suffers a welfare loss equal to the present value of the difference between the social security benefits and the amount of future income that the displaced savings would have earned.⁴ The basic problem of designing a social security program is to set the level of benefits (and the conditions for receiving benefits) in the way that best balances the desirability of protecting those who would otherwise make inadequate provisions for their old age against the cost of reduced saving by those who would otherwise save in a rational way.⁵

Feldstein (1974) showed that the provision of social security benefits that are conditional on retirement can increase personal saving if the effect on saving of the induced increase in retirement outweighs the asset substitution effect. Sheshinski and Weiss (1981), Abel (1984) and Hubbard (1984) examine the implications of uncertain mortality and the absence of perfect annuity markets for the effect of social security on saving.

Although there is a wide range of empirical estimates of the effect of social security on private saving, the bulk of the evidence appears to this author to support the conclusion that increases in social security benefits reduce private saving. Studies supporting this conclusion include Blinder, Gordon and Wise (1983) and Diamond and Hausman (1984). Those who find little or no effect of social security on savings include Lesnoy and Leamer (1982) and Munnell (1975).

³Paul Samuelson's (1958) classic article showed that the implicit rate of return in an unfunded social security program is the rate of growth of aggregate wages. Samuelson considered the special case in which real wages per worker are constant, making the implicit rate of return on social security equal to the rate of growth of population.

⁴See Feldstein (1982b). There is a further source of welfare loss to the rational individual if the social security program distorts labor supply either during working years (because the reward to working is reduced by the social security tax to an extent that is not compensated by the present value of future benefits) or at an age when retirement is possible (because potential benefits are reduced in whole or in part if the individual continues to work.) See Gustman and Steinmeier (1983) and Danziger et al. (1981) for recent discussions of these issues. The present analysis abstracts from these issues by assuming that the quantity of labor supplied and the age of retirement are both fixed.

⁵Feldstein(1985a) derives the optimal level of social security benefits in a universal social security program for two alternative specifications of

Milton Friedman (1972) and others have suggested that the common system of universal eligibility for social security retirement benefits be replaced by a means-tested program that pays benefits only to those who lack assets or private pension income with which to finance adequate post-retirement consumption. Proponents of this change argue that limiting benefits only to those in financial need would reduce the size of the program and therefore the distortionary effect of the tax that is used to pay for it. It is also argued that, since rational savers would receive no benefits, their saving would be influenced only by the presumably modest amount of tax that they pay to finance the means-tested benefits. Despite the potential magnitude of the welfare gain, the proposal for a means-tested program has not previously been the subject of theoretical or empirical analysis.

Some social security specialists oppose switching to a means-tested program because, they argue, there is a stigma attached to accepting means-tested benefits which is undesirable in itself and which discourages eligible individuals from applying for benefits. In addition, eligible individuals may fail to receive benefits because they do not understand that they are entitled to benefits.⁶

In assessing these arguments it is of course difficult to know how much value to place on avoiding the stigma per se that accompanies means-tested benefits. It is in principle easier to evaluate the welfare cost of individuals' failure to apply for means-tested benefits because of such

imperfect foresight. For evidence on the significance of inadequate retirement savings, see Diamond (1977) and Kotlikoff et al. (1982).

⁶See Cohen (1972). For more general discussions of social security policy and proposals for reform, see Boskin (1977), Feldstein (1975, 1977, 1985b), and Munnell (1977).

stigma or because of ignorance of eligibility. Moreover, experience with other means-tested and conditional programs suggests that the utilization rate of means-tested retirement benefits by eligible individuals would increase over time and could be raised by education and advertising. The potential gains to rational savers of switching the program to a means-tested basis makes it worthwhile to look beyond the stigma issue and to evaluate more formally the choice between universal and means-tested programs. That is the purpose of the present paper.

The analysis shows that there may be a strong case for a means-tested program but that it is more ambiguous than the casual analysis of Friedman and others would suggest. Although a means-tested program may be smaller in total size than a universal program, it does not necessarily produce greater social welfare even when the implicit return on social security taxes is substantially lower than the return available on private saving. The analysis in this paper shows two reasons why an optimal program of universal unconditional benefits would, under certain conditions, provide a higher level of social welfare than an optimal program of benefits conditioned by a means-test.

The principal reason that a universal program may be superior is that a means-tested program with benefits set at the optimal level may induce some utility-maximizing workers to save nothing. Although their resulting consumption in retirement would then be less than they would have chosen without a social security program, the utility value of the extra consumption during working years more than offsets the reduced consumption during retirement. For these individuals, the means-tested program distorts savings

and reduces individual utility by more than a universal program. If that group is large enough in the population, the universal program may be more desirable than a means-tested program.

It is also possible for the universal program to be superior even if the benefits in the alternative means-tested program are set at a level that does not induce any utility-maximizing workers to stop saving. This can occur if, in order to avoid inducing the utility-maximizers to stop saving, the level of the means-tested benefits has to be set substantially lower than would otherwise be optimal. In this case, switching from a universal program to a means-tested program reduces the welfare of those who receive the constrained means-tested benefits by more than it increases the welfare of the nonrecipients.

In general, the choice between a means-tested program and a universal program depends on the parameters of the economy (in particular the growth rates of income and population and the rate of return on real investments) and on the character and extent of economic shortsightedness among the working population. The nature of this dependence is examined in the present paper. The specific results in the very simple models examined here are of course only suggestive but they do indicate some important qualitative properties that may be robust and suggest a framework for a more realistic detailed analysis.

The fact that the optimal choice between a means-tested program and a universal program depends on the character of the working population has an important general implication for the design of social security programs. It implies that, if the working population can be subdivided into groups that

differ in the relevant parameters, it may be optimal to have a means-tested program for some groups and a universal program for others. The specific implications of this are discussed in the final section of the present paper.

1. The Optimal Program in a Two Class Society

It is useful to begin the analysis with the simple case in which workers are either fully rational life-cycle utility maximizers (or "cyclers" for short) or completely myopic individuals who always consume their entire net income ("myopes"). In this case, a means-tested program will produce higher social welfare than a universal program unless the level of means-tested benefits must be reduced substantially below the unconstrained optimum in order to prevent the cyclers from becoming non-savers. Such a constraint may be binding because allowing the cyclers to become non-savers would imply that the means-tested program was no longer selective but provided benefits to all retirees. The present section shows conditions under which such a constraint causes the universal program to be optimal. It also develops the basic structure of the analysis that is then extended in the next section to deal with a more heterogeneous population in which an optimal means-tested program will induce workers with limited myopia to stop saving.

The analysis is set in an overlapping generations model of the type developed by Samuelson (1958) and Diamond (1965). Individuals live for two periods, working in the first and being fully retired in the second. At time t there are L_t workers and L_{t-1} retirees. The population grows at rate n per period, implying that $L_t = (1+n)L_{t-1}$. Real wages per worker grow at rate g ; thus $w_t = (1+g)w_{t-1}$. The rate of return on capital is r per period.

It will simplify notation and interpretation in the analysis that follows if we define $x = (1+n)(1+g)/(1+r)$, the ratio of "one plus the growth rate of aggregate wages" to "one plus the rate of return on capital." Since the growth rate of aggregate wages is the implicit rate of return on social security, x measures the efficiency of social security "saving" relative to savings invested in real capital.

A fraction μ of workers are myopes who always consume their entire earnings during their working period. The remaining $1-\mu$ choose a saving level during their working years that maximizes lifetime utility, $u(C_{1t}, C_{2,t+1})$ where C_{1t} is consumption in period t of workers and $C_{2,t+1}$ is consumption of retirees in period $t+1$. To be able to derive explicit values of consumption and utility, I shall assume that the lifetime utility function is loglinear: $u(C_{1t}, C_{2,t+1}) = \ln C_{1t} + \ln C_{2,t+1}$.

1.1 A Universal Social Security Program

When there is a universal social security program, the government levies a tax at rate θ on wages and uses the proceeds to finance concurrent benefits of b_t^u to each retiree. The budget constraint of a universal social security program is

$$(1) \quad \theta w_t L_t = b_t^u L_{t-1}$$

or

$$(2) \quad b_t^u = (1+n)\theta w_t.$$

Myopes consume $C_{1t} = (1-\theta)w_t$ during their working years and $C_{2,t+1} = b_{t+1}^u$ when they are retired. Cyclers choose C_{1t} to maximize lifetime utility subject to the personal budget constraint $C_{2,t+1} = [(1-\theta)w_t - C_{1t}](1+r) + b_{t+1}^u$. Optimal first period consumption is therefore $C_{1t}^* = 0.5[(1-\theta)w_t + b_{t+1}^u/(1+r)]$

and the corresponding second period consumption is $C_{2,t+1}^* = 0.5[(1-\theta)w_t(1+r) + b_{t+1}^u]$. Using the government's budget constraint (equation 2) to write $b_{t+1}^u = (1+n)\theta w_{t+1} = (1+n)(1+g)\theta w_t$ and recalling that $x = (1+n)(1+g)/(1+r)$ is the efficiency of social security savings, yields $C_{1t}^* = 0.5[(1-\theta) + \theta x]w_t$ and $C_{2,t+1}^* = 0.5[(1-\theta) + \theta x](1+r)w_t$.

Total utility at time t is the sum of the utilities of the L_t workers and the L_{t-1} retirees. With a program of universal social security benefits, total utility at time t is:

$$(3) \quad W_t^u = L_t [\mu \ln(1-\theta)w_t + (1-\mu) \ln 0.5(1-\theta + \theta x)w_t] \\ + L_{t-1} [\mu \ln b_{t+1}^u + (1-\mu) \ln 0.5(1-\theta + \theta x)(1+r)w_{t-1}]$$

After substituting $b_{t+1}^u = (1+n)\theta w_t$ and $w_{t-1} = w_t/(1+g)$, it is possible to factor out L_{t-1} and $\ln w_t$ and write

$$(4) \quad W_t^u = L_{t-1} [(1+n) \mu \ln(1-\theta) + (1+n)(1-\mu) \ln 0.5(1-\theta + \theta x) \\ + \mu \ln(1+n)\theta + (1-\mu) \ln 0.5(1-\theta + \theta x)(1+r)(1+g)^{-1}] \\ + L_{t-1}(2+n) \ln w_t.$$

Since the terms including θ are in the square brackets and do not change from period to period, the value of θ that maximizes W_t^u is the same as the value that maximizes total utility in any other period.⁷ I shall assume that the government wishes to choose θ to maximize this total utility.⁸

⁷Except for the initial period when the program is started. In that period, retirees get windfall benefits that they never paid for. This is discussed in Feldstein (1985a, pp. 310-314).

⁸The government might instead choose θ to maximize the discounted value of utility in all years including the initial period. Ignoring the initial period is equivalent to assuming that the discount rate that the government applies in aggregating utilities is relatively low so that the initial period

The first order condition for the value θ^* that maximizes total utility is⁹

$$(5) \quad \frac{\mu}{\theta^*} - \frac{(1+n)\mu}{1-\theta^*} - \frac{(2+n)(1-\mu)(1-x)}{1-(1-x)\theta^*} = 0$$

The qualitative implications of this condition are easily established by totally differentiating equation 5. It is easily shown in this way that $d\theta^*/d\mu > 0$, $d\theta^*/dx > 0$ and $d\theta^*/dn < 0$. Before interpreting these it is useful to derive also the effects of the parameters on the optimal value of the per worker benefits. Since $b_t^{u^*} = \theta^*(1+n)w_t$, it follows immediately that $db^{u^*}/d\mu > 0$ and $db^{u^*}/dx > 0$. The sign of db^{u^*}/dn is ambiguous and depends on the value of the other parameters.

It is easy to interpret these properties. Since an increase in the fraction of myopes increases the number of individuals who gain from a higher level of the universal benefit and decreases the number who lose, it raises the optimal benefit level ($db^{u^*}/d\mu > 0$) and therefore the corresponding tax rate ($d\theta^*/d\mu > 0$). An increase in the relative efficiency of social security in comparison to real investment, i.e., a rise in the ratio of the implicit rate of return on social security to the rate of return on real capital, reduces the cost to the cyclers of the compulsory social security program and therefore raises the optimal level of both benefits ($db^{u^*}/dx > 0$) and taxes ($d\theta^*/d\mu > 0$). Finally, since an increase in the relative number of workers

gets little weight or that the sum does not converge so that it is only meaningful to maximize the utility of a representative year. Feldstein (1985a) develops the discounted value and discusses these issues more explicitly. If the government's utility discount rate is less than $(1+g)(1+n)-1$, the discounted sum of utilities does not converge and the government must maximize the representative value of W_t .

⁹The second order condition is satisfied for all feasible parameter values.

permits a given level of benefits to be achieved with a lower tax rate, $d\theta^*/d(1+n) < 0$.

Table 1 presents specific numerical values of θ^* and of the resulting level of welfare¹⁰ for a range of values of μ and x . To limit the dimension of the table, the calculations are done for a single value of n based on U.S. experience. The value of n represents the rate of labor force growth for a generation, which I will take to be 30 years. Since the U.S. population grew at an annual rate of 1.4 percent for the three decades beginning in 1950, I shall take $(1+n) = (1.014)^{30} = 1.52$.

The value of $x = (1+g)(1+n)/(1+r)$ that corresponds to U.S. experience during these years to $x = 0.115$. This reflects the annual labor force growth of 1.4 percent, an annual rate of growth of real compensation per hour of 2.2 percent and a pretax real rate of return on U.S. nonfinancial corporate capital of 11.4 percent.¹¹ Together these imply $x = [(1.014)(1.022)/(1.114)]^{30} = 0.115$. Reducing the population growth rate to zero and the rate of real compensation growth to 0.32 percent (the average for the most recent decade, 1974-84) cuts the relative efficiency of social security to $x = 0.043$. In contrast, maintaining the values of g and n but cutting the projected real rate of return in half (from 0.114 to 0.057) raises the relative efficiency of social security to $x = 0.553$. The tabulations in Table 1 are presented for these three values and for selected values of μ between 0.1 and 1.0.

¹⁰Except for the factor L_{t-1} and the term $(2+n) \ln w_t$ that does not depend on θ . The values in the table¹ are actually $[w_t^u - (2+n) \ln w_t]/L_{t-1}$.

¹¹The derivation of the 11.4 percent estimate is discussed in Feldstein, Poterba and Dicks-Mireaux (1983).

Table 1

Optimal Tax Rates and Total Utility for Universal
and Means-Tested Programs in a Two-Class Society

Relative Efficiency of Social Security (x)	Relative Frequency of Myopes (μ)	Optimal Tax Rates		Total Utility ^a		Saving Constraint Binding
		Universal	Means- Tested	Universal	Means- Tested	
		θ_u^*	θ_u^*	\hat{w}_u	\hat{w}_m	
		(1)	(2)	(3)	(4)	(5)
0.115	0.10	0.044	0.040	0.384	0.603	no
0.115	0.20	0.088	0.079	0.074	0.375	no
0.115	0.50	0.214	0.198	-0.592	-0.282	no
0.115	0.80	0.328	0.317	-1.046	-0.894	no
0.115	0.90	0.363	0.357	-1.166	-1.087	no
0.043	0.10	0.041	0.040	1.262	1.488	no
0.043	0.20	0.082	0.079	0.848	1.162	no
0.043	0.50	0.204	0.198	-0.123	0.210	no
0.043	0.80	0.321	0.317	-0.866	-0.698	no
0.043	0.90	0.359	0.357	-1.078	-0.989	no
0.553	0.10	0.082	0.040	-0.968	-0.811	no
0.553	0.20	0.151	0.079	-1.073	-0.881	no
0.553	0.50	0.291	0.184	-1.213	-1.069	yes
0.553	0.80	0.366	0.266	-1.260	-1.225	yes
0.553	0.80	0.366	0.266	-1.260	-1.225	yes
0.553	0.90	0.383	0.289	-1.268*	-1.271	yes

^aThe number actually reported (\hat{w}) is not total utility (w) but $w - (2+n) \ln w / L_{t-1}$.

The optimal tax rates shown in column 1 increase with the relative frequency of myopes. With the historic value of $x = 0.115$, the optimal tax rate increase from $\theta_U^* = 0.044$ when only 10 percent of the population are myopes to $\theta_U^* = 0.363$ when 90 percent of the population are myopes. In the limiting case in which everyone is myopic, $\theta_U^* = (2+n)^{-1} = 0.396$; this corresponds to the tax rate that distributes income equally between workers and myopes. Although a lower efficiency of social security reduces the optimal value of θ_U^* , the effect is not very substantial.

1.2 A Means-Tested Social Security Program

If the social security program is means tested and the level of taxes and benefits is set in a way that causes cyclers to save, benefits are paid only to retired myopes and the budget constraint linking the tax rate (θ) to the level of means-tested benefits per beneficiary (b^m) is

$$(6) \quad b_t^m = \mu^{-1} \theta (1+n) w_t$$

since only a fraction μ of retirees receives benefits. The behavior of the myopes is the same under a means-tested program as under the universal program. They consume their entire disposable income while working ($C_{1t} = (1-\theta)w_t$) and depend exclusively on the benefits to finance consumption during retirement ($C_{2,t+1} = b_{t+1}$).

If the cyclers save, the level of means-tested benefits is irrelevant and their behavior is affected only by the tax. They maximize $u(C_{1t}, C_{2,t+1}) = \ln C_{1t} + \ln[(1-\theta)w_t - C_{1t}](1+r)$ and therefore choose $C_{1t}^* = 0.5(1-\theta)w_t$ and $C_{2,t+1}^* = 0.5(1-\theta)w_t(1+r)$.

Cyclers will save if the utility that results from this combination of

$C_{1,t}^*$, $C_{2,t+1}^*$ is greater than the utility of consuming their entire disposable income while they are working and relying on the benefits to finance their consumption. That is, cyclers choose to save if and only if

$$(7) \quad \ln 0.5(1-\theta)w_t + \ln 0.5(1-\theta)w_t(1+r) > \ln(1-\theta)w_t + \ln b_{t+1}$$

Since $b_t = \mu^{-1}(1+n)\theta w_t$ and $w_t = (1+g)w_{t-1}$, this inequality condition can be written

$$(8) \quad \ln 0.5(1-\theta) + \ln 0.5(1-\theta)(1+r) > \ln(1-\theta) + \ln(1+n)(1+g)\theta - \ln \mu$$

After rearranging terms, this implies that cyclers save if and only if

$$(9) \quad \ln\left(\frac{1-\theta}{\theta}\right) > \ln x + 2 \ln 2 - \ln \mu$$

This inequality condition is important because it indicates the maximum value at which θ can be set in a means-tested program:

$$(10) \quad \theta_{\max} = \frac{\mu}{\mu+4x}$$

This maximum tax rate implies a maximum value for the ratio of the benefits paid to myopes (and therefore their level of retirement consumption) to the retirement consumption of cyclers. Since $b_t = \mu^{-1}\theta(1+n)w_t$ and $C_{2,t}^* = 0.5(1-\theta)w_{t-1}(1+r)$, the ratio of benefits to the retirement consumption of cyclers is:

$$(11) \quad \frac{b_t}{C_{2,t}^*} = \frac{\mu^{-1}\theta(1+n)w_t}{0.5(1-\theta)w_{t-1}(1+r)} \\ = \frac{2\theta x}{(1-\theta)\mu}$$

Substituting θ_{\max} from equation 10 yields

$$(12) \quad \frac{b_t}{C_{2,t}^*} = 0.5.$$

Thus the maximum level of benefits consistent with continued saving by the cyclers is one-half of the level of consumption that the cyclers would obtain by their optimal saving. While this simple result reflects the particular loglinear utility, it illustrates how the level of benefits may have to be constrained significantly in a means-tested program in order to keep cyclers saving.

The value of θ_{\max} increases with the relative number of myopes but less than proportionately, implying that the maximum benefits in a means-tested program varies inversely with the number of myopes. The value of θ also varies inversely with x , implying that the maximum tax is reduced as the benefits produced by any level of tax increases. For example, with $x = 0.115$, θ_{\max} rises from $\theta_{\max} = 0.30$ at $\mu = 0.2$ to $\theta_{\max} = 0.52$ at $\mu = 0.5$ and $\theta_{\max} = 0.63$ at $\mu = 0.8$. Similarly, with $\mu = 0.2$, θ_{\max} rises from $\theta_{\max} = 0.083$ at $x = 0.553$ to $\theta_{\max} = 0.30$ at $x = 0.115$ and $\theta_{\max} = 0.538$ at $x = 0.043$.

If the value θ^* that maximizes total utility on the assumption that cyclers save exceeds θ_{\max} , the value θ^* is irrelevant and the feasible optimum value for a means-tested program is θ_{\max} .¹²

¹²A means-tested program in which cyclers do not save is, ex post, a universal program. Such a program is dominated by an ex ante universal program in which cyclers do some saving.

The analysis now proceeds by deriving, for different parameter combinations, the optimal tax value (θ^*) on the assumption that cyclers save. If $\theta^* \leq \theta_{\max}$, the total utility is calculated at θ^* ; if $\theta^* > \theta_{\max}$, total utility is calculated at θ_{\max} . These total utility values for a means-tested program can then be compared to the total utility values for the universal program to decide whether the means-tested program or the universal program provides the higher level of total utility.

Total utility at time t under the means-tested program can be written as the sum of the utilities of the L_t workers and L_{t-1} retirees as:¹³

$$(13) \quad W_t^m = L_t [\mu \ln(1-\theta)w_t + (1-\mu) \ln 0.5(1-\theta)w_t] \\ + L_{t-1} [\mu \ln b_t^m + (1-\mu) \ln 0.5(1-\theta)(1+r)w_{t-1}].$$

After substituting $b_t^m = \mu^{-1}(1+n)\theta w_t$ and $w_{t-1} = w_t/(1+g)$, total utility can be written as

$$(14) \quad W_t^m = L_{t-1} [\mu \ln \theta + (2+n-\mu) \ln (1-\theta) - (2+n)(1-\mu) \ln 2 \\ + \ln (1+n) - \mu \ln \mu - (1-\mu) \ln x] + L_{t-1} (2+n) \ln w_t.$$

¹³This assumes that cyclers save.

Once again a constant value of θ maximizes w_t^m in each period. The first order condition for the means-tested program implies

$$(15) \quad \theta^* = \frac{\mu}{2+n}$$

and, since $\beta_m^* = b_t^{m^*}/w_t = \mu^{-1}\theta(1+n)$,

$$(16) \quad \beta_m^* = \frac{1+n}{2+n}$$

This result is very striking. It implies that the optimal benefit-wage ratio depends only on the rate of population growth and is independent of the frequency of myopes in the population and the relative efficiency of social security and real investment.

This striking conclusion is easily explained. A means-tested social security program is equivalent to a redistribution of income from all workers to those retirees who have no private assets. As such, it is a problem in optimal income redistribution. With no distorting effect of the tax or the benefit on labor supply or saving,¹⁴ the tax should be used to redistribute income until the marginal utility of a dollar of additional income for the working generation equals the marginal utility of a dollar to the retired myopes. The relative efficiency of private saving and social security at converting present to future income (x) is irrelevant. And while the relative number of myopes influences the tax rate required to support the benefits, it does not alter the optimal level of benefits (β^* is independent of μ).

¹⁴The tax reduces the saving of the cyclers by reducing their disposable income but there is no substitution effect distortion.

Substituting this value of θ^* into equation 14 yields an expression for the optimal level of total utility in an unconstrained means-tested program:

$$(17) \quad W_t^{m*} = L_{t-1} [(2+n-\mu) \ln (2+n-\mu) - (2+n) \ln (2+n) \\ - (2+n)(1-\mu) \ln 2 + \ln(1+n) - (1-\mu) \ln x + (2+n) \ln w_t]$$

Of course, if $\theta^* = \mu/(2+n) > \theta_{\max}$ of equation 10, the feasible maximum value of W_t^m is lower than W_t^{m*} and must be calculated by substituting θ_{\max} into equation 14. Since $\theta_{\max} = \mu/(\mu+4x)$, this saving constraint $\theta_{\max} < \theta^*$ will be binding unless $\mu/(\mu+4x) < \mu/(2+n)$. Equivalently, the unconstrained $\theta^* = \mu/(2+n)$ must be replaced by $\theta_{\max} = \mu/(\mu+4x)$ whenever $\mu > 2+n-4x$. With $n = 0.52$, the constraint is binding only for high values of μ or x . For example, with $x = 0.553$, the constraint is binding for $\mu \geq 0.31$. With $x \geq 0.63$, the constraint is always binding. While with $x < 0.38$, the constraint is never binding.

When the constraint is not binding, total utility for the means tested program is always greater than total utility for the universal program. Although this apparently cannot be shown analytically because the optimal value of θ for the universal program is only defined implicitly by equation 5, it can be shown by calculating the optimal values of θ^u and θ^m and the resulting total utility measure numerically for all conceivable values of μ and x [$0 \leq \mu \leq 1.0$ and $0 < x \leq 1.0$] and for values of n corresponding to annual population growth rates between zero and three percent ($0 \leq n \leq 1.43$).

Even when the constraint on the optimal means-tested tax rate is binding ($\theta^* > \theta_{\max}$), the total utility for the means tested program may still be

higher than total utility for the universal program. The universal program provides a higher level of total utility only when the constraint on the level of benefits given to the myopes in the means-tested program depresses their utility below what they would get in an optimal universal program by more than enough to outweigh the greater adverse effects on the cyclers of a universal program.

Explicit numerical comparison shows that the universal program is optimal only for high values of x and relatively high values of μ . Moreover, the greater the relative efficiency of social security (i.e., the higher the value of x), the lower is the relative frequency of myopes in the population at which the universal program becomes optimal. For example, although the constraint on θ can be binding at $x = 0.4$, it is only binding if $\mu > 0.92$ and, even then it is never optimal to use a universal program.¹⁵ When $x = 0.6$, the constraint on θ is binding for μ as low as 0.12 but the universal program is only optimal at values of μ greater than 0.22. Figure 1 shows the combinations of x and μ values at which the universal program is optimal (the upper triangle) and the wider set of x and μ values at which the constraint on θ is binding but the means-tested program remains optimal (including the upper triangle and the area between the two lines). The area at the bottom shows the range of x and μ values for which the constraint is not binding and the means-tested program is optimal.

Note that with the historic measure of social security efficiency

¹⁵This calculation assumes the historic rate of population growth of 1.4 percent a year.

($x = 0.115$), the universal program is never optimal. This conclusion remains true for all rates of population growth. For the combinations of μ and x reported in Table 1, the universal program is optimal only for $x = 0.553$ and $\mu = 0.9$. Recall that $x = 0.553$ corresponds to cutting the real rate of return on capital in half. But even with this extreme assumption, the universal program is optimal only if myopes constitute nearly all of the population.

This conclusion about the general dominance of the means-tested program depends of course on the simplified characterization of the population as either pure cyclers or pure myopes. In the richer model of the next section, there is more scope for a universal program to dominate.

2. The Optimal Program When Benefits Distort Saving

A social security program that pays means-tested benefits to only a fraction of retirees cannot distort saving in an economy with only two types of individuals. With only two types of individuals, the level of taxes and benefits can be set in a way that achieves perfect separation between myopes and cyclers. Myopes save nothing and depend on benefits while cyclers save in a way that is influenced by the level of potential benefits. Under these circumstances, a means-tested program will be optimal unless achieving separation requires a substantial restraint on the level of benefits.

In an economy with more than two types of individuals, a separation that does not distort saving is not always a characteristic of an optimal means-tested program. The analysis in this section shows that the optimal level of means-tested benefits may cause some individuals who are neither completely myopic nor perfectly rational life-cycle utility maximizers to stop

saving and rely exclusively on social security benefits. For these individuals, the reduction in saving is greater than it would be with a universal program. If they are sufficiently important in the population, the total utility of a means-tested program may be lower than the total utility of a universal program. The current analysis examines the characterization of the economy and of the population that influences this choice.

The analysis of the previous section can be extended to deal with this more general case by introducing a third class of individuals who are partially myopic, that is, who give too little weight to their future utility when they make savings decisions during their working years. More specifically, I will assume that a fraction Π of the population acts during their working years to maximize $\ln C_1 + \lambda \ln C_2$ with $\lambda < 1$ even though their true lifetime utility is given by $\ln C_1 + \ln C_2$. In Pigou's () words, these partial myopes have a "faulty telescopic faculty" that causes them to give too little weight to future utility.

As before, it is useful to begin by analyzing the effects of a universal social security program and calculating the total utility level that results from an optimal universal program. The optimal means-tested program can then be derived and the resulting total utility level compared with that of the universal program.

2.1 An Optimal Universal Social Security Program

The budget constraint of a universal social security program is the same in the current three-class economy as it was in the simple economy of the previous section:

$$(18) \quad b_t = \theta(1+n)w_t.$$

The fundamental difference between the previous section and the current one lies in the behavior of the partial myopes. The partial myopes choose their first period consumption by maximizing $\ln C_{1t} + \lambda \ln C_{2,t+1}$ subject to the personal budget constraint $C_{2,t+1} = [(1-\theta)w_t - C_{1t}](1+r) + b_{t+1}$. This implies first period consumption of

$$(19) \quad C_{1t}^* = (1+\lambda)^{-1} [(1-\theta)w_t + b_{t+1}/(1+r)]$$

Using the government budget constraint (18) to eliminate b_t and recalling that $x = (1+g)(1+n)/(1+r)$ measures the efficiency of social security, the first-period consumption of the partial myopes can be written:

$$(20) \quad C_{1t}^* = (1+\lambda)^{-1} [1-\theta + \theta x] w_t.$$

Their consumption during retirement can be written:

$$(21) \quad \begin{aligned} C_{2,t+1}^* &= [(1-\theta)w_t - C_{1t}^*](1+r) + b_{t+1} \\ &= \frac{\lambda}{1+\lambda} (1+r)w_t(1-\theta + \theta x) \end{aligned}$$

Although the partial myopes choose their consumption levels by maximizing $\ln C_{1t} + \lambda \ln C_{2,t+1}$, the proper social valuation of the total utility of the consumption of the partial myopes living at time t is $\Pi L_t \ln C_{1t}^* + \Pi L_{t-1} \ln C_{2t}^*$ where Π is the fraction of partial myopes in the population. The consumption behavior and utility level of the pure myopes and the pure cyclers is the same as in the previous section. Total utility at time t with the universal social security program can therefore be written

$$\begin{aligned}
 (22) \quad w_t &= L_t [\mu \ln(1-\theta)w_t + \Pi \ln(1+\lambda) (1-\theta + \theta x)w_t \\
 &+ (1-\mu-\Pi) \ln 0.5 (1-\theta+\theta x)w_t] + L_{t-1} [\mu \ln \theta(1+n)w_t \\
 &+ \Pi \ln(\frac{\lambda}{1+\lambda})(1+r)(1-\theta+\theta x)w_{t-1} + (1-\mu-\Pi) \ln 0.5(1+r)(1-\theta+\theta x)w_{t-1}]
 \end{aligned}$$

After substituting $w_t = (1+g)w_{t-1}$ and $L_t = (1+n)L_{t-1}$, it is possible to factor out L_{t-1} and $\ln w_t$ and write

$$\begin{aligned}
 (23) \quad w_t^u &= L_{t-1} [(1+n)\mu \ln(1-\theta) + (1+n) \Pi \ln(1+\lambda)^{-1}(1-\theta+\theta x) \\
 &+ (1+n)(1-\mu-\Pi) \ln 0.5(1-\theta+\theta x) + \mu \ln \theta(1+n) \\
 &+ \Pi \ln(\frac{\lambda}{1+\lambda})(1+n) x^{-1}(1-\theta+\theta x) + (1-\mu-\Pi) \ln 0.5(1+n) x^{-1} (1-\theta+\theta x)] \\
 &+ L_{t-1}(2+n) \ln w_t
 \end{aligned}$$

This in turn can be simplified somewhat by collecting like terms to yield:

$$\begin{aligned}
 (24) \quad w_t^u &= L_{t-1} [(1+n) \mu \ln(1-\theta) + \mu \ln \theta \\
 &- (1-\mu) \ln x + (2+n)(1-\mu) \ln(1-\theta+\theta x) \\
 &+ \ln(1+n) - (2+n)(1-\mu-\Pi) \ln 2 \\
 &- (2+n) \Pi \ln(1+\lambda) + \Pi \ln \lambda] + L_{t-1}(2+n) \ln w_t.
 \end{aligned}$$

When there are no partial myopes ($\Pi=0$), equation 24 is equivalent to equation 4 of the previous section.

Consider now the value of θ that maximizes total utility with this universal program. Since the terms in equation 24 that include θ do not involve either Π or λ , it follows immediately that the optimal value of θ is

independent of the relative number of partial myopes or of their degree of myopia. It can be verified directly by differentiating W_t^U with respect to θ that the optimal tax rate in the universal program satisfies

$$(25) \quad \frac{\mu}{\theta^*} - \frac{(1+n)\mu}{1-\theta^*} - \frac{(2+n)(1-\mu)(1-x)}{1-(1-x)\theta^*} = 0.$$

This is exactly the same as the optimum condition of equation 5 for an economy with no partial-myopes.¹⁶

The values of θ^* corresponding to different values of μ and χ that are presented in Table 1 can therefore be used to evaluate the total utility level given by equation 24. These maximum values for the total utility achievable with an optimal universal social security program can then be compared with the corresponding maximum values that can be attained with a means-tested program. Before presenting the total utility levels implied by equation 24, it is therefore useful to analyze the means-tested program.

2.2 A Means-Tested Program With Partial Myopes

A universal program of social security benefits reduces the saving of all individuals who save to finance future consumption, including both pure life-cyclers and partial myopes.¹⁷ In contrast, a means-tested program reduces saving only if it induces individuals to cut their saving in order to qualify

¹⁶This simplifying property is a result of the loglinear nature of the utility function. It follows directly from equations 20 and 21 that changes in θ alter utility in a way that does not depend on the value of λ . The effect of θ on utility is therefore the same for partial-myopes and for pure life-cyclers. Therefore only the relative number of pure myopes is relevant to determining the optimal value of θ .

¹⁷This is in addition to the reduction in saving caused by the reduction in disposable income that results from the tax. The discussion in the remainder of the paragraph also ignores this tax effect and focuses on the substitution of benefits for private wealth accumulation.

for the means-tested benefit. In order to analyze this possibility, I will assume that benefits are provided only to individuals who do not save at all.¹⁸ Interest then focuses on the possibility that the partial-myopes choose to stop saving and rely only on the benefits to finance retirement consumption. Since the level of benefits that induces such substitution may not be high enough to induce full life cyclers to stop saving, the result may be a means-tested program in which only some retirees receive benefits but in which a substantial number of people substitute social security benefits for private saving.

The analysis that follows shows that the means-tested program may be preferred to a universal program even when partial myopes are induced to become nonsavers. A means-tested program is preferable to a universal program under a wide range of parameter values. But the possibility of inducing partial myopes to become nonsavers does increase the range of economic parameter values in which a universal program is preferred to a means-tested program. The implications of this are discussed more fully in the concluding section of the paper. But first the formal results must be derived and analyzed.

In the simpler case in which individuals are either pure myopes or pure cyclers, I derived the maximum value of θ that was consistent with continued saving by the cyclers. In the current context, two separate sets of conditions must be derived in order to evaluate total utility. In the first case, the value of θ is such that both the cyclers and the partial-myopes

¹⁸If the means-tested program provided full benefits to partial-myopes, there would be no basis for analyzing the substitution of benefits for private saving. A more complex means-tested program that provided partial benefits to individuals with private assets but with less than a 100 percent effective tax rate would be worth analyzing.

continue to save. In the second case, the value of θ is such that only the cyclers continue to save and the partial myopes are induced to substitute benefits for private wealth accumulation. For each set of parameter values (μ , x , n , λ and Π), both cases must be evaluated separately and the corresponding values of total utility calculated. The higher utility value indicates the optimum means-tested program. This total utility value can then be compared with the value of the corresponding universal program.

Consider first the conditions under which both cyclers and partial myopes save. Cyclers choose to save if and only if the utility that results from saving exceeds the utility of consuming all disposable income and relying on benefits to finance retirement consumption. This condition is exactly the same as equation 7 of section 1:

$$(26) \quad \ln 0.5(1-\theta)w_t + \ln 0.5(1-\theta)w_t(1+r) > \\ \ln(1-\theta)w_t + \ln b_{t+1}.$$

Since in this case benefits are paid only to myopes, the government's budget constraint implies $b_t = \mu^{-1}\theta(1+n)w_t$ and cyclers save if

$$(27) \quad \ln 0.5(1-\theta)w_t + \ln 0.5(1-\theta)w_t(1+r) > \\ \ln(1-\theta)w_t + \ln \mu^{-1}\theta(1+n)(1+g)w_t.$$

Solving this implies that cyclers will save only if θ is less than

$$(28) \quad \theta_{\max}^C = \frac{\mu}{\mu+4x},$$

just as in the previous section. This maximum value of θ is denoted θ_{\max}^C to emphasize that it is the highest value of θ consistent with saving by the cyclers.

The maximum value of θ that is consistent with continued saving by the partial myopes can be derived in a similar way. Partial myopes save only if the resulting lifetime utility (as they perceive it, with weight λ given to second period utility) exceeds the lifetime utility that results if all disposable income is consumed. If the partial myopes do reject the benefits and save, their consumption is chosen to maximize $\ln C_{1t} + \lambda \ln C_{2,t+1}$ subject to $C_{2,t+1} = ((1-\theta)w_t - C_{1t})(1+r)$. This implies $C_{1t} = (1+\lambda)^{-1}(1-\theta)w_t$ and therefore that the perceived lifetime utility is $\ln (1+\lambda)^{-1}(1-\theta)w_t + \lambda \ln(1-(1+\lambda)^{-1})(1-\theta)w_t(1+r)$. The partial myopes therefore save if and only if

$$(29) \quad \ln(1+\lambda)^{-1}(1-\theta)w_t + \lambda \ln(\lambda/(1+\lambda))(1-\theta)w_t(1+r) > \ln(1-\theta)w_t + \lambda \ln b_{t+1}.$$

Note again that, with partial myopes saving, benefits are paid only to pure myopes and therefore $b_t = \mu^{-1}\theta(1+n)w_t$. Substituting for b_{t+1} and factoring out w_t yields:

$$(30) \quad \ln(1+\lambda)^{-1}(1-\theta) + \lambda \ln(\lambda/(1+\lambda))(1-\theta) > \ln(1-\theta) + \lambda \ln \mu^{-1} \theta x.$$

Rearranging terms and solving implies the important condition that partial myopes save¹⁹ if and only if θ is less than

$$(31) \quad \theta_{\max}^{\lambda} = \frac{\lambda\mu}{\lambda\mu + x(1+\lambda)^{\lambda}/\lambda},$$

where the superscript λ on θ_{\max}^{λ} indicates that this is the maximum value of θ consistent with continued saving by partial myopes.²⁰ Differentiating this

¹⁹This is conditional on saving by pure cyclers as well. As I note below, pure cyclers always save if it is optimal for partial myopes to save.

²⁰It is easy to see (and perhaps reassuring) that with $\lambda=1$ this is equivalent to the value of θ_{\max} for pure cyclers presented in equation 28.

expression with respect to λ shows that θ_{\max} rises monotonically with λ for $0 < \lambda \leq 1$. This means that $\theta < \theta_{\max}^{\lambda}$ implies $\theta < \theta_{\max}^C$; if the value of θ is low enough to keep partial-myopes saving, it is also low enough to keep cyclers saving.

Before looking at the second case in which partial-myopes do not save, it is interesting to see the way in which the presence of partial myopes reduces the maximum value of θ (and the corresponding value of benefits) that is consistent with leaving saving unchanged.²¹ For example, with the value of $x = 0.115$ corresponding to U.S. experience of the past three decades, the maximum tax rate consistent with unchanged saving drops from $\theta_{\max}^C = 0.30$ when $\mu = 0.2$ to $\theta_{\max}^{\lambda} = 0.11$ when partial myopes have $\lambda = 0.5$ and to only $\theta_{\max}^{\lambda} = 0.06$ when partial myopes have $\lambda = 0.2$. The relative reductions are only slightly smaller when the fraction of myopes is larger. For example, when half of the population are myopes ($\mu = 0.5$), $\theta_{\max}^C = 0.52$ but declines to $\theta_{\max}^{\lambda} = 0.39$ when partial myopes have $\lambda = 0.5$ and $\theta_{\max}^{\lambda} = 0.23$ when $\lambda = 0.2$. Thus the maximum value of θ consistent with unchanged saving drops substantially when there are partial myopes whose initial level of saving is relatively low.

I turn therefore to the second case in which the level of θ is high enough to cause partial-myopes not to save but not so high that cyclers cease saving. The values of θ that define this range are not the values of θ_{\max}^C and θ_{\max}^{λ} that have just been derived because they were derived using the government constraint that holds when only the pure myopes are nonsavers.

²¹Unchanged saving refers to no substitution of benefits for private saving. The tax per se will always alter saving.

Since the current case corresponds to providing benefits to partial myopes as well as pure myopes, the government budget constraint becomes $b_t = (\mu + \Pi)^{-1} \theta(1+n)w_t$. It then follows directly that the values of θ that define the current range can be obtained by substituting $\mu + \Pi$ for μ in the previously derived expressions for θ_{\max}^c and θ_{\max}^λ .

More specifically, the cyclers will continue to save if and only if

$$(32) \quad \theta < \frac{\mu + \Pi}{\mu + \Pi + 4x} .$$

Similarly, the partial myopes will not save only if

$$(33) \quad \theta > \frac{\lambda(\mu + \Pi)}{\lambda(\mu + \Pi) + x(1+\lambda)^{(1+\lambda)/\lambda}} .$$

Since the right hand side of 33 is an increasing function of λ for $0 < \lambda \leq 1$, it follows that there exists a range of values of θ that satisfies both inequalities:

$$(34) \quad \frac{\mu + \Pi}{\mu + \Pi + 4x} > \theta > \frac{(\mu + \Pi)\lambda}{(\mu + \Pi)\lambda + x(1+\lambda)^{(1+\lambda)/\lambda}} .$$

Expression 34 is the condition for the cyclers to save while the partial myopes do not.

The total utility equation depends on whether the maximum value of θ corresponds to the first or second case. Consider first the total utility equation if only the pure myopes do not save. Total utility is then given by

$$\begin{aligned}
 (35) \quad w_t^m &= L_t [\mu \ln(1-\theta)w_t + \Pi \ln(1+\lambda)^{-1}(1-\theta)w_t \\
 &+ (1-\mu-\Pi) \ln 0.5(1-\theta)w_t] + L_{t-1} [\mu \ln b_t \\
 &+ \Pi \ln(\frac{\lambda}{1+\lambda})(1-\theta)w_{t-1}(1+r) + (1-\mu-\Pi) \ln 0.5(1-\theta)w_{t-1}(1+r)]
 \end{aligned}$$

Using $w_t = (1+g) w_{t-1}$, $L_t = (1+n)L_{t-1}$ and $x = (1+g)(1+n)/(1+r)$, equation 35 can be rewritten as:

$$\begin{aligned}
 (36) \quad w_t^m &= L_{t-1} [(1+n) \mu \ln(1-\theta) + (1+n)\Pi \ln(1+\lambda)^{-1}(1-\theta) \\
 &+ (1+n)(1-\mu-\Pi) \ln 0.5(1-\theta) + \mu \ln \mu^{-1} \theta(1+n) \\
 &+ \Pi \ln(\frac{\lambda}{1+\lambda})(1-\theta)(1+n)x^{-1} + (1-\mu-\Pi) \ln 0.5(1-\theta)(1+n)x^{-1}] \\
 &+ L_{t-1}(2+n) \ln w_t.
 \end{aligned}$$

The first order condition for the optimal value of θ in the means-tested program (conditional on that value also being consistent with saving by both cyclers and partial myopes) is²²

$$(37) \quad \theta^* = \frac{\mu}{2+n}$$

Note that this is exactly the same value of the optimal tax rate for the means tested program that was obtained for the simpler structure in which there were no partial myopes. This is not surprising since, in the current case, the partial myopes continue to save and are therefore unaffected by the level of

²²The analysis that follows shows that the cyclers save if the partial myopes save.

benefits.

This optimal θ^* will be consistent with the result that $\theta^* < \theta_{\max}^\lambda$ of equation 31 if and only if

$$(38) \quad \frac{\mu}{2+n} < \frac{\lambda\mu}{\lambda\mu + x(1+\lambda)^{(1+\lambda)/\lambda}}$$

or

$$(39) \quad x\lambda^{-1}(1+\lambda)^{(1+\lambda)/\lambda} < 2+n-\mu$$

For example, with the historic values of $x = 0.115$ and $1+n = 1.52$, this inequality will be satisfied for all values of $\lambda \geq 0.23$ regardless of the relative frequency of the pure myopes (μ). Thus unless partial myopes give very little weight to future utility, the unconstrained optimal tax rate for the means-tested program is consistent with continued saving by partial myopes. At very low values of λ , the opposite is true; with $\lambda < 0.13$, the inequality cannot be satisfied for any value of $\mu < 1$. Thus if partial myopes put little enough weight on future utility, it may not be possible to have partial myopes saving at the optimum value of θ^* .

Consider now the optimum value of θ in the second case in which only the cyclers save. In this case, the total utility is given by

$$(40) \quad W_t^m = L_t [(\mu+\Pi) \ln(1-\theta)w_t + (1-\mu-\Pi) \ln 0.5(1-\theta)w_t] \\ + L_{t-1} [(\mu+\Pi) \ln b_t + (1-\mu-\Pi) \ln 0.5(1-\theta)w_{t-1}(1+r)]$$

or, since in this case $b_t = (\mu+\Pi)^{-1} \theta(1+n)w_t$,

$$(41) \quad W_t^m = L_{t-1} [(1+n)(\mu+\Pi) \ln(1-\theta) + (1+n)(1-\mu-\Pi) \ln 0.5(1-\theta) \\ + (\mu+\Pi) \ln(\mu+\Pi)^{-1} \theta(1+n) + (1-\mu-\Pi) \ln 0.5(1-\theta)(1+n)x^{-1}] \\ + L_{t-1}(2+n) \ln w_t.$$

The first order condition for the optimum tax rate in the means-tested program when partial myopes behave like pure myopes is

$$(42) \quad \theta^* = \frac{\mu + \Pi}{2 + n}$$

This is exactly analogous to the previously derived optimum tax rates for programs except that $\mu + \Pi$ now replaces μ because partial myopes behave like pure myopes.

This optimal value of θ^* will be consistent with the limits of equation 34 on θ that are required for cyclers to save while partial myopes do not save if:

$$(43) \quad \frac{\mu + \Pi}{\mu + \Pi + 4x} > \frac{\mu + \Pi}{2 + n} > \frac{(\mu + \Pi)\lambda}{(u + \Pi)\lambda + x(1 + \lambda)^{(1 + \lambda)/\lambda}}$$

or

$$(44) \quad \mu + \Pi + x\lambda^{-1}(1 + \lambda)^{(1 + \lambda)/\lambda} > 2 + n > \mu + \Pi + 4x$$

The second of these inequalities is satisfied for the historic values of $n = 0.52$ and $x = 0.115$ for all possible values of $\mu + \Pi$. The first inequality is satisfied for low values of λ but not for high values. More precisely, with the historic values, the first inequality is satisfied for $\lambda < 0.13$ but is never satisfied for $\lambda \geq 0.23$. Thus the double inequality that defines this case in which partial myopes do not save at the optimal θ^* is satisfied only for a rather narrow range of very low values of λ if x and n take their historic values. But with a higher value of x , the range of λ values consistent with nonsaving by the partial myopes is much broader. For example, with $x = 0.553$ (and n continuing at 0.52), the first of the inequalities is satisfied for

values ranging to more than $\lambda = 0.8$. The double inequality is satisfied by values between $\lambda = 0$ and $\lambda = 0.6$ for very low values of $\mu + \Pi$ but is satisfied for $0 < \lambda \leq 0.5$ even at $\mu + \Pi = 0.5$.

Finally, it is possible that the value of θ derived on the assumption that only cyclers save will in fact cause even cyclers to want to stop saving. Since this limit on θ is given by equation 32 and the optimal value of θ is given by 42, we see that the constraint on θ^* will be binding only if

$$(45) \quad \frac{\mu + \Pi}{2 + n} \geq \frac{\mu + \Pi}{\mu + \Pi + 4x}$$

or

$$(46) \quad \mu + \Pi + 4x \geq 2 + n$$

Although this can never be binding for the historic values of $x = 0.115$ and $n = 0.52$, the constraint could be binding if social security were more efficient. When $x = 0.5$, the constraint is binding if $\mu + \Pi > n$. So even low values of μ and Π could then make it necessary to constrain $\theta = (\mu + \Pi) / (\mu + \Pi + 4x)$ instead of allowing it to take its unconstrained maximum value.

To summarize, there are three possible values of θ^* and the corresponding total utility values that must be examined in assessing the means-tested program. In the first case, only myopes do not save, $\theta^* = \mu / (2 + n)$ and the total utility is obtained by substituting this value into equation 36. In the second case, both the myopes and the partial myopes do not save, $\theta^* = (\mu + \Pi) / (2 + n)$ and total utility is obtained by substituting the value into equation 41. Finally, if both myopes and partial myopes do not save and θ must be set equal to the maximum value consistent with saving by the cyclers

$\theta = (\mu + \Pi)(\mu + \Pi + 4x)$, the total utility is obtained by substituting this value into equation 41.

Table 2 compares the universal and means-tested programs for a wide range of the parameter values. The top half of the table corresponds to the historic experience ($x = 0.115$ and $n = 0.52$) while the bottom half corresponds to a relatively much more efficient social security program ($x = 0.553$ and $n = 0.52$). Calculations are presented for values of μ and Π ranging from 0.1 to 0.5 and for $\lambda = 0.1$, $\lambda = 0.2$ and $\lambda = 0.5$.

For each set of parameter values, the table shows six things. Columns 1 and 2 show the optimal tax rate for a universal program (θ_u^*) and the resulting level of total welfare²³ (\hat{W}_u). The same two statistics are presented for means tested programs in columns 3 and 4. An asterisk next to the higher utility value indicates the type of program that is optimal. Column 5 indicates for the means-tested programs whether the partial myopes save. The final column indicates whether the tax rate in the means-tested program had to be limited to assure that the cyclers continue to save.

Over a wide range of parameter values, a means-tested program is preferable to any universal program. With the historical values of social security efficiency ($x = 0.115$) and population growth ($n = 0.52$), the universal program is optimal only for low values of λ and μ . Even when partial myopes give even as little weight to future utility as $\lambda = 0.2$, the means-tested program is always optimal. But when the partial myopes weight future utility at only $\lambda = 0.1$, their implied saving rate is so low that an

²³Again, this is after rescaling by L_{t-1} and ignoring the term $(2+n)lnw_t$ that enters all measures equally.

Table 2

Optimal Universal and Means-Tested Programs
when Benefits Distort Saving

Frequency of		Utility	Universal Programs		Means-Tested Programs		Do Partial Myopes Save?	Saving Con- strain ($\theta_c < \theta^*$)
Myopes (μ)	Partial Myopes (Π)	Weight by Myopes (λ)	Optimal Tax * (θ_u)	Welfare ^a \hat{w}_u	Optimal Tax * (θ_m)	Welfare ^a \hat{w}_m		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

Historic Relative Efficiency of Social Security ($x = 0.115$)

0.1	0.1	0.1	0.044	0.304	0.079	0.375*	no	no
0.1	0.2	0.1	0.044	0.225*	0.119	0.151	no	no
0.1	0.5	0.1	0.044	-0.014*	0.238	-0.492	no	no
0.2	0.1	0.1	0.088	-0.006	0.119	0.151*	no	no
0.2	0.2	0.1	0.088	-0.085	0.159	-0.068*	no	no
0.2	0.5	0.1	0.088	-0.324*	0.278	-0.696	no	no
0.5	0.1	0.1	0.214	-0.781	0.238	-0.492*	no	no
0.5	0.2	0.1	0.214	-0.751	0.278	-0.696*	no	no
0.1	0.1	0.2	0.044	0.352	0.040	0.571*	yes	no
0.1	0.2	0.2	0.044	0.319	0.040	0.538*	yes	no
0.1	0.5	0.2	0.044	0.223	0.040	0.442*	yes	no
0.2	0.1	0.2	0.088	0.042	0.079	0.343*	yes	no
0.2	0.2	0.2	0.088	0.010	0.079	0.310*	yes	no
0.2	0.5	0.2	0.088	-0.087	0.079	0.214*	yes	no
0.5	0.1	0.2	0.214	-0.624	0.198	-0.314*	yes	no
0.5	0.2	0.2	0.214	-0.656	0.198	-0.347*	yes	no
0.1	0.1	0.5	0.044	0.387	0.040	0.606*	yes	no
0.1	0.2	0.5	0.044	0.890	0.040	0.609*	yes	no
0.1	0.5	0.5	0.044	0.400	0.040	0.619*	yes	no
0.2	0.1	0.5	0.088	0.077	0.079	0.378*	yes	no
0.2	0.2	0.5	0.088	0.080	0.079	0.381*	yes	no
0.2	0.5	0.5	0.088	0.090	0.079	0.391*	yes	no
0.5	0.1	0.5	0.214	-0.589	0.198	-0.279*	yes	no
0.5	0.2	0.5	0.214	-0.585	0.198	-0.276*	yes	no

Higher Relative Earnings of Social Security ($x = 0.553$)

0.1	0.1	0.1	0.082	-1.048	0.079	-0.881*	no	no
0.1	0.2	0.1	0.082	-1.127	0.119	-0.948*	no	no
0.1	0.5	0.1	0.082	-1.366	0.213	-1.124*	no	yes
0.2	0.1	0.1	0.151	-1.153	0.119	-0.948*	no	no
0.2	0.2	0.1	0.151	-1.233	0.153	-1.010*	no	yes
0.2	0.5	0.1	0.151	-1.472	0.240	-1.176*	no	yes
0.5	0.1	0.1	0.291	-1.292	0.213	-1.124*	no	yes
0.5	0.2	0.1	0.291	-1.372	0.240	-1.176*	no	yes

Table 2 Continued

Frequency of		Utility Weight by Myopes (λ)	Universal Programs Optimal Welfare ^a		Means-Tested Programs Optimal Welfare ^a		Do Partial Myopes Save?	Saving Constraint ($\theta_c < \theta^*$)
Myopes (μ)	Partial Myopes (Π)		Tax * (θ_u)	(\hat{W}_u)	Tax * (θ_{m0})	(\hat{W}_m)		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0.1	0.1	0.2	0.082	-1.000	0.079	-0.881*	no	no
0.1	0.2	0.2	0.082	-1.032	0.119	-0.948*	no	no
0.1	0.5	0.2	0.082	-1.129	0.213	-1.124*	no	yes
0.2	0.1	0.2	0.151	-1.106	0.119	-0.948*	no	no
0.2	0.2	0.2	0.151	-1.138	0.153	-1.010*	no	yes
0.2	0.5	0.2	0.151	-1.235	0.240	-1.176*	no	yes
0.5	0.1	0.2	0.291	-1.245	0.213	-1.124*	no	yes
0.5	0.2	0.2	0.291	-1.277	0.240	-1.176*	no	yes
0.1	0.1	0.5	0.082	-0.965	0.079	-0.881*	no	no
0.1	0.2	0.5	0.082	-0.962	0.119	-0.948*	no	no
0.1	0.5	0.5	0.082	-0.952*	0.213	-1.124	no	yes
0.2	0.1	0.5	0.151	-1.070	0.119	-0.948*	no	no
0.2	0.2	0.5	0.151	-1.067	0.153	-1.010*	no	yes
0.2	0.5	0.5	0.151	-1.058*	0.240	-1.176	no	yes
0.5	0.1	0.5	0.291	-1.209	0.213	-1.124*	no	yes
0.5	0.2	0.5	0.291	-1.206	0.240	-1.176*	no	yes

^aThe number actually reported (\hat{W}) is not total utility (W) but $W - (2+n) \ln w / L_{t-1}$.

optimal means tested program would cause partial myopes to stop saving. This is not a sufficient condition to make a universal program optimal but, if the partial myopes are frequent enough in the population, it does become important enough to outweigh the adverse effects of a universal program on the pure cyclers. Thus the universal program dominates when $\mu = 0.1$ and $\Pi = 0.2$, or $\mu = 0.1$ and $\Pi = 0.5$ and similarly when $\mu = 0.2$ and $\Pi = 0.5$.

Two things should be noted about these three combinations. First, it follows from the structure of the problem that if the universal program is optimal for $\mu = 0.1$, $\Pi = 0.2$ and for $\mu = 0.1$ and $\Pi = 0.5$, then the universal program is also optimal for all values of $\Pi \geq 0.2$ if $\mu = 0.1$. Note also that since the means tested program is preferable at $\mu = 0.1$ and $\Pi = 0.1$, there is some value of Π between 0.1 and 0.2 at which the universal program becomes optimal. Thus, for low enough λ and μ , the universal program is optimal as long as at least a modest fraction of the population are partial myopes.

Second, at $\mu = 0.2$ the universal program is not optimal at $\Pi = 0.2$ but becomes optimal at some value of $0.2 < \Pi < 0.5$. When there are more myopes, the optimal means-tested program becomes relatively more expensive and thus keeps the universal program optimal until the distortion of saving by the partial myopes becomes a more important problem, i.e., until Π reaches a higher level. For the same reason, when $\mu = 0.5$ the mean-tested program is optimal at $\Pi = 0.2$ and remains optimal until $\Pi \geq 0$.

When myopes put a higher value on future utility ($\lambda = 0.2$ and $\lambda = 0.5$), the partial myopes continue to save with the unconstrained optimal means-tested program. Under these conditions, the means-tested program dominates the universal program.

When social security is more efficient relative to private saving, two things happen. First, under an optimal means-tested program, the partial myopes will in general choose to stop saving and depend on the social security benefits. Note that, with $x = 0.553$, the partial myopes do not save at any of the parameter combinations. Second, although the means-tested program causes partial myopes to stop saving, the result is a smaller welfare loss when social security is relatively efficient. As a result, the means-tested program remains preferable at moderate values of λ .

As λ rises, the reduction in the saving of partial myopes caused by a means-tested program becomes large. In addition, with a high value of $\lambda = 0.5$ and with $\Pi = 0.5$, it is necessary to constrain the means-tested tax rate to avoid causing the cyclers to stop saving as well. In these cases, where $\theta_c < \theta^*$ for the means-tested program, the universal program is optimal.

The optimal tax rates in columns 1 and 3 show that an optimal means-tested program is not in general smaller than an optimal universal program. Since the tax base is the same for both types of programs, $\theta_m^* > \theta_u^*$ implies that total taxes and therefore total benefits are higher under the means-tested program. With the historic value of $x = 0.115$, the optimal means-tested program is larger than the corresponding universal program with low values of λ (because it is then optimal to induce the partial myopes to stop saving) but is smaller than the universal program when λ is not very low. With a higher value of $x = 0.553$, the relative size of the universal and means-tested programs depends on the values of Π and μ and no simple generalization is possible. It cannot be said that a means-tested program is better because it is smaller, although table 2 indicates that, whenever the

universal program is preferable, it is also smaller than the corresponding optimal means-tested program.

3. A Concluding Comment

The analysis of this paper shows that a means-tested social security program may be preferable to a universal program under a wide range of economic conditions. But the analysis also shows that the optimal type of program and the optimal level of benefits in a universal program depend on the parameters that describe the population and the economy. For any combination of Π and λ , an increase in the relative frequency of myopes raises the optimal tax in the means-tested program. Similarly, if the means-tested program is optimal but causes partial myopes to stop saving, the optimal tax rate increases with the combined frequency of myopes and partial myopes.

This suggests that overall welfare can be increased if the working population as a whole can be divided into two subgroups with a different type of program provided to each group. Note that this does not require that the subgroups be homogeneous -- all cyclers or all myopes -- but only that the groups have different mixes of myopes, partial myopes and cyclers. Moreover, although it would obviously be best to know the characteristics of each subgroup with precision, a welfare improvement could be achieved even with imperfect information.

An obvious criteria for grouping individuals is income. Low income individuals are more likely to be myopes or to be partial myopes who give low weight to future consumption. If so, the low income group might be an appropriate candidate for a universal program.

All of the analysis in this paper has reflected the fact that the utility

characteristics that influence the saving behavior of each individual are unobservable. It is possible, however, to observe saving. An individual who saves, especially in the form of a pension or special retirement account, is certainly not a myope in the sense of this paper. The analysis presented here implies that any individual who demonstrates sufficient saving should be eligible for a means-tested program even if a universal program would be optimal for that individual's population subgroup.

Finally, it should be noted that, within the population for whom a means-tested program is optimal, there is no way to increase efficiency by varying the program among subgroups (unless the means-tested benefits would cause cyclers to become nonsavers). To see this, note that the optimal level of means-tested benefits does not depend on the values of λ , μ , or Π ; the optimal tax rate varies only because differences in μ (and in Π if partial myopes stop saving) changes the tax rate required to finance a given level of benefits. To see this, recall that $\theta^* = \mu/(2+n)$ for the means-tested program if partial myopes save (equation 37) and $\theta^* = (\mu+\Pi)/(2+n)$ if only cyclers save (equation 42). But benefits per retiree are $b_t = (\mu+\Pi)^{-1}(1+n)\theta w_t$ if only cyclers save. The benefit to wage ratio is therefore always $b_t/w_t = (1+n)/(2+n)$. Creating separate subgroups would not cause the optimal benefit/wage ratio to vary among subgroups. Since redistributing the tax burden would not provide any overall welfare change, there is no gain from disaggregation within the means-tested population.

Cambridge, Massachusetts

October 1985

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Table 1

Optimal Tax Rates and Total Utility for Universal
and Means-Tested Programs in a Two-Class Society

Relative Efficiency of Social Security (x)	Relative Frequency of Myopes (μ)	Optimal Tax Rates		Total Utility*		Saving Constraint Binding
		Universal θ_u^*	Means- Tested θ_u^*	Universal \hat{w}_u	Means- Tested \hat{w}_m	
		(1)	(2)	(3)	(4)	(5)
0.115	0.10	0.044	0.040	0.384	0.603	no
0.115	0.20	0.088	0.079	0.074	0.375	no
0.115	0.50	0.214	0.198	-0.592	-0.282	no
0.115	0.80	0.328	0.317	-1.046	-0.894	no
0.115	0.90	0.363	0.357	-1.166	-1.087	no
0.043	0.10	0.041	0.040	1.262	1.488	no
0.043	0.20	0.082	0.079	0.848	1.162	no
0.043	0.50	0.204	0.198	-0.123	0.210	no
0.043	0.80	0.321	0.317	-0.866	-0.698	no
0.043	0.90	0.359	0.357	-1.078	-0.989	no
0.553	0.10	0.082	0.040	-0.968	-0.811	no
0.553	0.20	0.151	0.079	-1.073	-0.881	no
0.553	0.50	0.291	0.184	-1.213	-1.069	yes
0.553	0.80	0.366	0.266	-1.260	-1.225	yes
0.553	0.80	0.366	0.266	-1.260	-1.225	yes
0.553	0.90	0.383	0.289	-1.268*	-1.271	yes

*The number actually reported (\hat{w}) is not total utility (W) but $W - (2+n) \ln w / L_{t-1}$.

Table 2

Optimal Universal and Means-Tested Programs
when Benefits Distort Saving

Frequency of		Utility	Universal Programs		Means-Tested Programs		Do Partial Myopes Save?	Saving Con- strain ($\theta_c < \theta^*$)
Myopes (μ)	Partial Myopes (Π)	Weight by Myopes (λ)	Optimal Tax * (θ_U)	Welfare ¹ (\hat{W}_U)	Optimal Tax * (θ_m)	Welfare ¹ (\hat{W}_m)		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

Historic Relative Efficiency of Social Security ($x = 0.115$)

0.1	0.1	0.1	0.044	0.304	0.079	0.375*	no	no
0.1	0.2	0.1	0.044	0.225*	0.119	0.151	no	no
0.1	0.5	0.1	0.044	-0.014*	0.238	-0.492	no	no
0.2	0.1	0.1	0.088	-0.006	0.119	0.151*	no	no
0.2	0.2	0.1	0.088	-0.085	0.159	-0.068*	no	no
0.2	0.5	0.1	0.088	-0.324*	0.278	-0.696	no	no
0.5	0.1	0.1	0.214	-0.781	0.238	-0.492*	no	no
0.5	0.2	0.1	0.214	-0.751	0.278	-0.696*	no	no
0.1	0.1	0.2	0.044	0.352	0.040	0.571*	yes	no
0.1	0.2	0.2	0.044	0.319	0.040	0.538*	yes	no
0.1	0.5	0.2	0.044	0.223	0.040	0.442*	yes	no
0.2	0.1	0.2	0.088	0.042	0.079	0.343*	yes	no
0.2	0.2	0.2	0.088	0.010	0.079	0.310*	yes	no
0.2	0.5	0.2	0.088	-0.087	0.079	0.214*	yes	no
0.5	0.1	0.2	0.214	-0.624	0.198	-0.314*	yes	no
0.5	0.2	0.2	0.214	-0.656	0.198	-0.347*	yes	no
0.1	0.1	0.5	0.044	0.387	0.040	0.606*	yes	no
0.1	0.2	0.5	0.044	0.890	0.040	0.609*	yes	no
0.1	0.5	0.5	0.044	0.400	0.040	0.619*	yes	no
0.2	0.1	0.5	0.088	0.077	0.079	0.378*	yes	no
0.2	0.2	0.5	0.088	0.080	0.079	0.381*	yes	no
0.2	0.5	0.5	0.088	0.090	0.079	0.391*	yes	no
0.5	0.1	0.5	0.214	-0.589	0.198	-0.279*	yes	no
0.5	0.2	0.5	0.214	-0.585	0.198	-0.276*	yes	no

Higher Relative Earnings of Social Security ($x = 0.553$)

0.1	0.1	0.1	0.082	-1.048	0.079	-0.881*	no	no
0.1	0.2	0.1	0.082	-1.127	0.119	-0.948*	no	no
0.1	0.5	0.1	0.082	-1.366	0.213	-1.124*	no	yes
0.2	0.1	0.1	0.151	-1.153	0.119	-0.948*	no	no
0.2	0.2	0.1	0.151	-1.233	0.153	-1.010*	no	yes
0.2	0.5	0.1	0.151	-1.472	0.240	-1.176*	no	yes
0.5	0.1	0.1	0.291	-1.292	0.213	-1.124*	no	yes
0.5	0.2	0.1	0.291	-1.372	0.240	-1.176*	no	yes

Figure 1

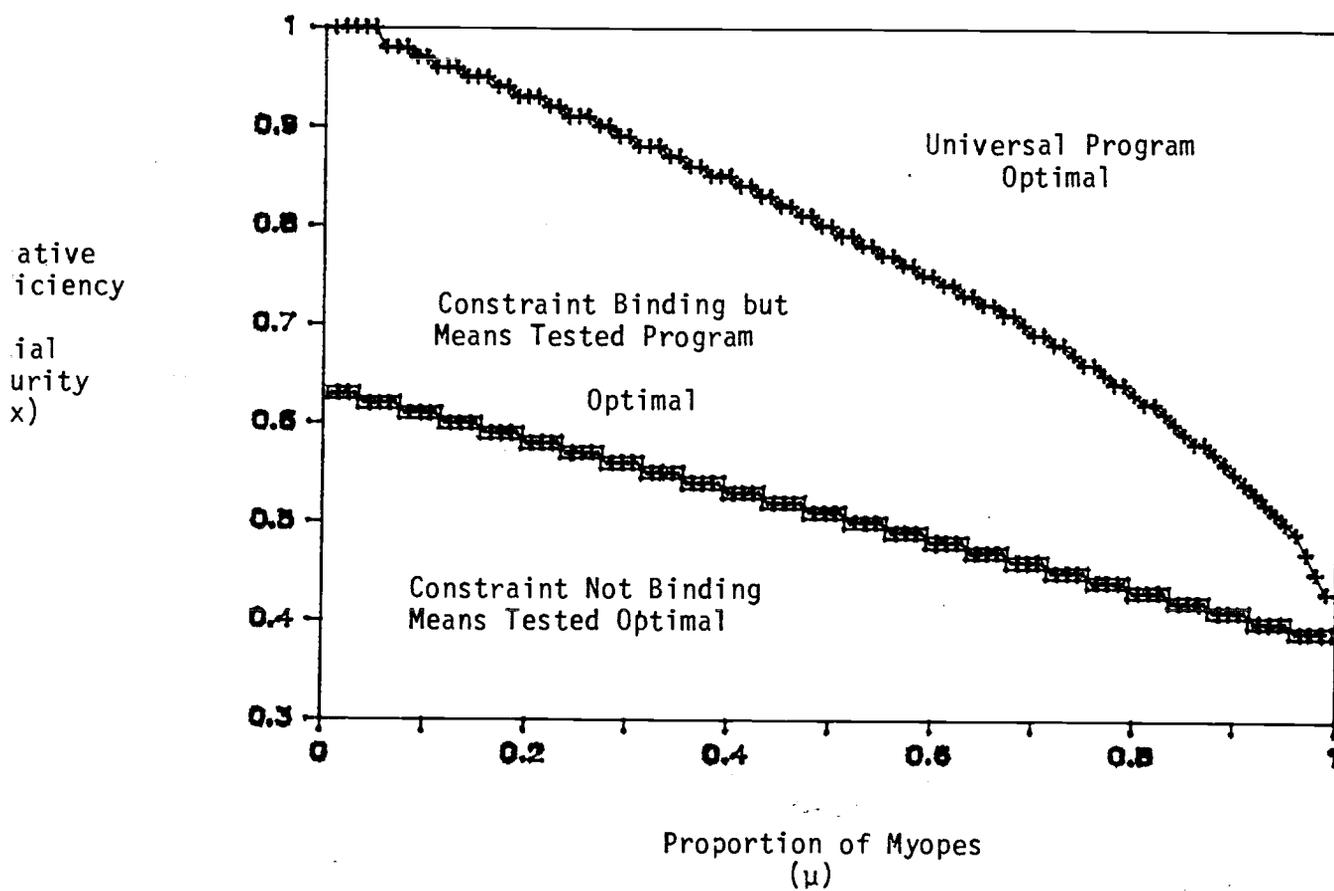


Table 2 Continued

Frequency of		Utility	Universal Programs		Means-Tested Programs		Do Partial Myopes Save?	Saving Con- straint ($\theta_c < \theta^*$)
Myopes (μ)	Partial Myopes (Π)	Weight by Myopes (λ)	Optimal Tax * (θ_u)	Welfare ¹ \hat{W}_u	Optimal Tax * (θ_{m0})	Welfare ¹ \hat{W}_m		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0.1	0.1	0.2	0.082	-1.000	0.079	-0.881*	no	no
0.1	0.2	0.2	0.082	-1.032	0.119	-0.948*	no	no
0.1	0.5	0.2	0.082	-1.129	0.213	-1.124*	no	yes
0.2	0.1	0.2	0.151	-1.106	0.119	-0.948*	no	no
0.2	0.2	0.2	0.151	-1.138	0.153	-1.010*	no	yes
0.2	0.5	0.2	0.151	-1.235	0.240	-1.176*	no	yes
0.5	0.1	0.2	0.291	-1.245	0.213	-1.124*	no	yes
0.5	0.2	0.2	0.291	-1.277	0.240	-1.176*	no	yes
0.1	0.1	0.5	0.082	-0.965	0.079	-0.881*	no	no
0.1	0.2	0.5	0.082	-0.962	0.119	-0.948*	no	no
0.1	0.5	0.5	0.082	-0.952*	0.213	-1.124	no	yes
0.2	0.1	0.5	0.151	-1.070	0.119	-0.948*	no	no
0.2	0.2	0.5	0.151	-1.067	0.153	-1.010*	no	yes
0.2	0.5	0.5	0.151	-1.058*	0.240	-1.176	no	yes
0.5	0.1	0.5	0.291	-1.209	0.213	-1.124*	no	yes
0.5	0.2	0.5	0.291	-1.206	0.240	-1.176*	no	yes