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ABSTRACT

This paper analyzes the economics of the private equity fund compensation. We build a novel model to estimate the expected revenue to fund managers as a function of their investor contracts. In particular, we evaluate the present value of the fair-value test (FVT) carried interest scheme, which is one of the most common profit-sharing arrangements observed in practice. We extend the simulation model developed in Metrick and Yasuda (2010a) and compare the relative values of the FVT carry scheme to other benchmark carry schemes. We find that the FVT carry scheme is substantially more valuable to the fund managers than other commonly observed (and more conservative) carry schemes, largely due to the early timing of carry compensation that frequently occurs under the FVT scheme. Interestingly, conditional on having an FVT carry scheme, fund managers' incremental gains from inflating the reported values of the funds' un- exited portfolio companies would be negligible.

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1. Introduction

Private equity funds are typically organized as limited partnerships, with private equity firms serving as general partners (GPs) of the funds and investors providing capital as limited partners (LPs). These partnerships usually last for ten years, and partnership agreements (investor contracts) signed at the funds' inceptions clearly define the expected GP compensation. Since the payments to GPs can account for a significant portion of the total cash flows of the fund, the fund fee structure is a critical determinant of the expected net fund returns that the LPs receive. Metrick and Yasuda (2010a) estimate the expected present value of the compensation to GPs as a function of the fee structure specified in investor contracts, but do not consider the fair-value test (FVT) scheme, which is a commonly used carried interest scheme in practice.[1] In this paper, we evaluate the present value of the FVT carried interest scheme by extending the simulation model developed in Metrick and Yasuda (2010a), and compare the relative values of the FVT carry scheme to other benchmark carry schemes.

The FVT carried interest scheme allows early carry payments before the fund's carry basis has been returned to investors if certain conditions are met. The FVT scheme is almost always accompanied by a clawback provision (see Section 2.6 for definitions); thus, the final *nominal* amount of carry for the fund's lifetime is unchanged whether the fund uses an FVT scheme or a more conservative carry timing scheme, holding all other fund terms (such as carry % level) equal. In other words, the main impact of the FVT scheme derives from the time value of money.

The conditions for the FVT scheme are twofold. First, upon any exit, the cost bases of all exited or written-off companies to date must be returned to LPs before any distribution to GPs.

[1] DowJones (2007).

In addition, the distribution to GPs is made only if the sum of the fair values of all un-exited (i.e., remaining) companies under management at the time of the exit equals or exceeds a threshold value, defined as a multiple of the total cost bases of un-exited investments with the most typical multiple being 1.2 (120%). The fair values of remaining investments cannot be easily marked to market since these private equity investments are illiquid by nature; in practice, estimate values that are reported by GPs are used. Since GPs are thought to possess an information advantage over LPs as insiders, the information asymmetry between them gives rise to a potential agency problem when GPs use self-reported portfolio values to calculate their carried interest. We investigate whether GPs are tempted to inflate the portfolio values of un-exited companies by examining the effects of inflated values on the expected PV of GP compensation.

In our analyses, we extend the model employed in Metrick and Yasuda (2010a) by mapping the exit timing and exit values of portfolio investments *as well as* the interim values of un-exited investments into the timing and amount of GP carry according to the FVT carry scheme. We obtain detailed information on the terms and conditions for fair-value tests used in practice from a large anonymous investor who also provided other information for the analyses in Metrick and Yasuda (2010a). We match the parameter values of our FVT model to the values most commonly used in these actual funds. We then compare the expected GP compensation of the fund with an FVT carry scheme to those of two other benchmark funds.

Our findings generally indicate that the FVT carry scheme is substantially more valuable to the fund managers than other commonly observed (and more conservative) carry schemes, but interestingly, conditional on having an FVT carry scheme, fund managers' incremental gains from inflating the reported values of the funds' un-exited portfolio companies would be negligible.

The remainder of the paper is organized as follows. In section 2, we describe a model of private equity fund compensation in a risk-neutral pricing framework. In section 3, we report the model outputs as a function of various input values. We conclude in section 4.

2. A Model of Private Equity Fund Compensation

Payments to GPs running private equity funds consist of management fees and carried interest for VC funds; for BO funds, there are additional fees called transaction fees and monitoring fees. While management fees are based on the cost bases of fund portfolio investments (and/or the fund size), the amount of carried interest (= carry) received by GPs is based in general on the timing and exit values of portfolio companies and thus is sensitive to fund performance. In the FVT carry scheme, the timing and amount of GP carry also depend on the interim values of un-exited portfolio companies.

In this section, we describe a risk-neutral valuation method for the estimation of the PV of carry starting with the determination of the initial investment value of a portfolio company. We then specify the dynamics of the company value during the holding period, the stochastic exit time point, and the values of exited *and* all other un-exited investments in the fund portfolio at every exit time point. We finally apply various functions that correspond to specific profit sharing rules by mapping the exit (and interim) values of portfolio companies to the amount of GP carry.

2.1 Risk-neutral valuation

The estimation of the present value of GP carry for a VC/PE fund is complicated because appropriate discount rates are hard to estimate empirically. Since investments are illiquid and individual project returns are not fully realized until the end of the fund life—usually ten years—it is not easy to measure risk ("beta") at the fund level using standard time-series correlations with the market and other factor returns. Many of the studies that employ fund-level cash flow data make an effective assumption that market beta for the asset class is equal to one.[2] In this analysis, we take a risk-neutral valuation approach and build a simulation model to overcome this data problem while matching parameter values of the simulation model to those that are supported by empirical evidence wherever estimates are available.[3]

2.2 Initial investment values

Since GPs receive a stream of semi-fixed compensation through management fees and these fees come out of committed capital, the investment capital that can be used for investments is always less than the committed capital that is provided by LPs. Since a minimum necessary condition for any type of equilibrium should state that at least the committed capital be returned to LPs in expectation, GPs must somehow create values to reconcile the gap between the investment capital and the committed capital. For example, the value creation may come from the possibility that GPs make a lucrative purchase at a low price, and/or from the possibility that GPs has a special skill to improve the value of the firm over time.

We assume a fixed amount of value creation by GPs in each investment, following Metrick and Yasuda (2010a). We set this value such that a fund with \$100 of committed capital would have a total initial value of investments at \$106.71. This number is chosen so that the expected value to LPs is exactly equal to committed capital for our benchmark VC fund (Fund I as described in Section 2.6). That is, for every \$100 in committed capital, the LPs pay some

[2] See Section 4.1 of Metrick and Yasuda (2011) and the citations therein.
[3] See Section 2.2 of Metrick and Yasuda (2010a) for more detailed discussions.

amount in management fees and the GPs then create value (after which the portfolio is worth \$106.71 in present value) and take out another expected amount in carried interest, after which exactly \$100 in expectation is left over for the LPs. Then, given this initial investment value, we simulate the value paths for individual investments by assuming stochastic processes as described in the following section.

2.3 The dynamics of the value of a portfolio company

Let X_t^i be the market value of portfolio company i at time *t*. It is assumed to follow a Gaussian diffusion process in a risk neutral world of the following form:

$$\frac{dX_t^i}{X_t^i} = rdt + \sigma\left(\sqrt{1-\rho^2}\,dW_t^i + \rho dW_t^F\right) , \tag{1}$$

where r is the risk-free rate and σ is the volatility of the investment. Note that W_t^i and W_t^F are standard Brownian motions, which are mutually independent where W_t^i is specific to portfolio company *i* and W_t^F is common across portfolio companies. By assuming the diffusion process as such, ρ captures the correlation between the values of a portfolio company and the common factor. We further assume that W_t^i and W_t^j $(i \neq j)$ are uncorrelated so that $corr\left(d\ln X_t^i, d\ln X_t^j\right) = \rho^2$.[4]

It is important to note that the process is not for the intrinsic value of a company, but for its market value. The intrinsic value of an illiquid asset is generally different from its market value that would be appraised once it becomes tradable. However, the carry distributions to GPs

[4] This correlation structure in a stochastic process is widely used in credit risk management and commonly known as one-factor Gaussian copula (see, e.g., Briys and De Varenne (1997), Duffie and Singleton (2003), Hull (2007), and Schonbucher (2003)).

could occur only when a fund makes any exit after which the exited company becomes less illiquid. For this reason, we assume that the proceeds from an exited company at any exit are equivalent to its market value while ignoring some frictions.[5] It is also important to note that this assumption makes our risk-neutral valuation method consistent. Although the interim values of un-exited companies under management might not be close to the market values, the interim values are not correlated with the exited values in our model, so the assumption of the market value for un-exited companies is not inconsistent with the risk-neutral valuation.

2.4 Random investment duration and random exit time

Random investment duration

Let d_i be the investment duration for portfolio company *i*. We specify d_i as a random variable that follows an exponential distribution with the instantaneous hazard rate of λ as follows:

$$f(d_i) = \lambda e^{-\lambda d_i} \quad (d_i \geq 0) \tag{2}$$

This distributional assumption is based on the observation that, in practice, neither LPs nor GPs control the exit timing; rather, exit opportunities arrive more or less exogenously. Furthermore, d_i is assumed to be independent of the company value. While this second assumption is certainly false, it is computationally expensive to handle these correlations on large portfolios, and in robustness checks using small portfolios we have not found any clear pattern between correlation structures and expected carried interest.

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[5] A majority of exits are made through IPOs or sales to other companies. While the proceeds from an exit may be different from the market value, for the purpose of our analysis we ignore these differences. In the case of IPOs, the difference may come from the total direct costs (see Lee, Lochhead, Ritter, and Zhao (1996)) and the underpricing of IPOs.

For the baseline model, we use the exit rate of 20% (an inverse of 5) since the average holding period for early VC investments is about 5 years.[6]

Random exit time

Private equity funds may invest in a portfolio company at any time during the fund's investment period, which typically lasts for 5 years starting from its inception. Denote the time point of an investment in a portfolio company i by s_i. In our simulation model, s_i is deterministic and is set to match the average investment pace that is empirically observed in the data (see Section 2.7 for details). Then, the exit time point t_i for portfolio company i is the sum of s_i and d_i, which is again a random variable.

2.5 Fair value and exit value

The fair value of a managed portfolio is the sum of the fair values of individual portfolio companies under management. At time t, portfolio company i is under management if and only if $s_i \le t < t_i$. Given the diffusion process (1), the fair value of portfolio company i (FV_t^i) follows a log-normal distribution:

For $i \in \{i \mid s_i \le t < t_i\}$,

$$FV_t^i = X_t^i \text{ where } \ln X_t^i \sim N\left(\ln X_{S_i}^i + \left(r - \frac{\sigma^2}{2}\right)(t - s_i), \sigma^2 (t - s_i)\right). \quad (3)$$

Similarly, the exit value of portfolio company i ($EV_{t_i}^i$) at its exit t_i follows a log-normal distribution:

6 See Metrick and Yasuda (2010b).

For $i \in \{i \mid t = t_i\}$,

$$EV_{t_i}^i = X_{t_i}^i \quad \text{where} \quad \ln X_{t_i}^i \sim N\left(\ln X_{S_i}^i + \left(r - \frac{\sigma^2}{2}\right)(t_i - s_i), \sigma^2(t_i - s_i)\right). \tag{4}$$

2.6 Mapping to carry amount

In our analysis, we evaluate the PV of carry for a fund with the FVT carry scheme along with two other funds with commonly used carry rules (benchmark funds) and compare them. Following Metrick and Yasuda (2010a), we choose the most typical carry scheme for VC and BO funds, respectively (Fund I), and also the simple carry scheme that does not allow early carry timing (Fund II) as the benchmark funds. Fund III is the FVT fund, as described below:

Fund I: The fund with no or 8% hurdle, contributed capital returned first, with clawback

The carry rule for this fund requires that, upon any exit, LPs must have received cumulative exit distributions equal to the contributed capital (= cost bases of all investments made to date + cumulative management fees paid to date), plus any hurdle return (if any), before any distribution of carried interests to GPs is allowed. Note that the calculation of contributed capital is at the aggregate fund level, not at the deal level. Thus this amount starts small at the beginning of a fund's life, and eventually converges to the committed capital at the end of its life for a fully invested fund. This carry rule with no hurdle is the most popular carry rule for VC funds; for BO funds, this carry rule with an 8% hurdle rate with a (100%) catch-up is most commonly employed.[7] The main reason for its popularity is that it allows GPs opportunities to earn carry

[7] A hurdle rate (also known as preferred return) is quite popular among BO funds; it is less popular among VC funds. The catch-up feature is almost always present in funds with hurdle rate. This feature allows GPs to receive

early on in the fund's life. However, since the contributed capital changes over time, it is possible for a fund with this carry rule to overpay carry to GPs (for example, if the fund does well early on and then falters). The clawback provision requires GPs in such instances to return the overpaid portion of their carry at the end of the fund life.

Fund II: The fund with committed capital returned first

The fund with this carry rule requires that, upon any exit, LPs must have received cumulative exit distributions equal to the committed capital before any distribution of carried interest to GPs is allowed. Note that, this carry rule employs a stricter notion of "fund profitability" than Fund I, and as a result carry timing is delayed. It rules out possibilities of carry overpayment to GPs by ensuring that the LPs get paid the entire carry basis (= committed capital) before any carry is distributed to GPs. While it is the least GP-friendly type of carry rule, it is found in about a quarter of the VC fund data used in Metrick and Yasuda (2010a), and is a useful benchmark case.

Fund III: The fund with a fair-value test, with clawback

The fund with this carry rule requires that, upon any exit, (i) LPs must have first received the cost basis of all exited (and written off) companies to date plus prorated management fees and (ii) the fair-value test (FVT) is met before any distribution of carried interest to GPs is allowed. The fair-value test requires that the fair value (= estimated, reported value) of the remaining fund portfolio companies equals or exceeds a preset threshold amount. The threshold amount is

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disproportionate amounts of exit distributions after the fund distributes the required hurdle returns to LPs until GPs "catch up" with LPs. With the catch-up feature in place, the hurdle return affects carry timing but not the final carry amount as long as the overall fund return is equal to or above the hurdle rate; if the fund return is below the hurdle rate, then the carry amount is also affected. See Metrick Yasuda (2010a, b) for more detailed explanations and examples.

calculated as a fixed percentage of the cost basis of all un-exited investments, and a typical percentage is in the neighborhood of 120%. If the first criterion is met but there is a small deficit between the fair value of the remaining fund portfolio and the threshold value, the remaining exit value can be used to pay down the deficit so that the FVT is met, and any leftover exit value can then be split between GPs and LPs according to the carry level (e.g., 20:80 for a 20% carry). Like Fund I, this type of carry rule is designed to allow GPs to earn carry early in the fund's life. Consequently, this carry rule is also susceptible to potential carry overpayment. If the GPs are found to be overpaid carry at the end of the fund's life, the clawback provision requires that GPs return the overpaid portion of the carry payment to LPs.

Note that, according to a survey on fees and carried interest (DowJones (2007)), the majority of respondent funds require the return of only a portion of contributed capital before carry kicks in, suggesting that the first part of the FVT scheme is commonly practiced. Furthermore, the second part of the FVT scheme (the fair-value test) is also employed by about a fifth of the survey respondent funds (21.2% of VC funds and 14.0% of BO funds). However, the same survey also indicates that there are concerns among LPs that "GPs who tie the timing of carried interest to [fair-value] tests might have an incentive to report higher valuations than other GPs." To the best of our knowledge, the effects of having this type of carry rule on (i) the value of GP compensation and (ii) GP incentives to inflate the value of unexited company portfolios have not been examined before. Our paper sheds light on both of these questions.

2.7 Simulation

Assessing the present value of a GP carry scheme is analogous to pricing a basket call option.

Although a basket option can be priced approximately in a closed form,[8] the evaluation of a GP carry scheme is more complicated because (1) the number of assets in the portfolio changes over time, and (2) the strike price also fluctuates during the fund life for some of the carry schemes. Thus, we use the Monte Carlo simulation method and compute the PV of carry numerically. To analyze the GP carry as a function of the value paths of portfolio companies, we further parameterize the baseline model as follows:

(1) The fund makes a predetermined number of investments with equal sizes. PE funds often have covenant provisions that prohibit GPs from investing a large portion of the fund's capital in a single investment, thereby ensuring that the fund capital is diversified across investments (Metrick and Yasuda (2010b)). The number of investments in a fund is set to match the median value of the fund sample used in Metrick and Yasuda (2010a) — 25 for VC funds and 11 for BO funds.

(2) Investments are made at the beginning of each calendar year during the first 5 years. The investment pace is set to match the empirically observed average pace used in Metrick and Yasuda (2010a) — 8, 6, 7, 3, 1 for VC funds, and 3, 3, 3, 1, 1 for BO funds.

(3) The annual exit probability (20%) is set to match the inverse of the average holding period (5 years), as in Metrick and Yasuda (2010a).

(4) Any remaining investments not yet exited are (forced to be) liquidated at the end of the 12th year from its inception. This cutoff date is based on the observation that a fund commonly lasts for 10 years and there is frequently a provision in the fund partnership agreement that allows up to two consecutive one-year extensions on the fund's life

[8] A basket option is an option on a portfolio of assets with a predetermined strike price. A basket option can be priced only approximately in a closed form. See Gentle (1993) and Huynh (1994) for lognormal approximations and Milevsky and Posner (1998) for a reciprocal Gamma approximation.

subject to LP approval.

(5) For the BO fund model, we extend the VC fund model with additional structures pertaining to (i) leverage, (ii) transaction fees, and (iii) monitoring fees. Each individual BO fund investment is leveraged with 2:1 leverage ratio; thus, the transaction price for each investment is three times the equity investment in the firm by the BO fund. Entry transaction fees are charged to the portfolio company at the time of the initial investment by the BO fund and the fees are then split 50:50 between LPs and GPs. We set the entry transaction fees to match the empirical average of 1.37% of firm value, as in Metrick and Yasuda (2010a). Monitoring fees are set to be 2% of EBITDA, or 0.4% of firm value per year for a firm with an EBITDA multiple of five, with a five-year contract. These fees are assessed (as 0.4%*5 years = 2% of firm value) at exit, and then split 80:20 between LPs and GPs. Note that leverage has direct impacts on the transaction and monitoring fees, since these fees are charged as percentages of the total firm value, as opposed to just the equity value. Furthermore, both transaction fees and monitoring fees paid to LPs are used to pay down the carry basis; thus, these fees affect the timing and amounts of carry for BO funds, and thus are integral parts of our simulation model.

Under these assumptions, we make 10,000 Monte Carlo simulations and obtain the average of GP carry.

3. Model Outputs

3.1 Baseline Results

We first report the results of our baseline model, using the parameter values as described in the first vertical panel of Table 1. The parameter values for the baseline VC (BO) model are: 20% exit probability, 20% carry level, $100 carry basis (= committed capital), 90% (60%) total volatilities, 50% (20%) pairwise correlation, and 120% fair-value threshold level (plus 2:1 leverage ratio for the BO model). Table 2 presents the simulation results. Panel A presents the PV of carry per $100 of committed capital for VC funds; Panel B presents the results for BO funds. For the VC fund with a 120% fair-value test and clawback, the PV of carry is $9.42. This compares quite favorably to the two benchmark VC funds: the fund with contributed capital returned first, with clawback, has the expected PV of carry of $8.67, while the fund with committed capital returned first has the PV of carry of only $8.61. Thus, our baseline model analysis indicates that the FVT scheme is quite GP friendly for VC funds.

The BO model results are qualitatively similar. The PV of carry for the BO fund with a 120% FVT is $6.18. The low value in comparison to the VC fund is due to the differences in underlying parameter values, in particular the lower volatility (60% vs. 90%) for individual investments and also the lower pairwise correlation (20% vs. 50%) between investments. More importantly, the relative GP friendliness of the FVT scheme remains unchanged: the BO fund with contributed capital plus 8% hurdle returned first, with clawback, has the expected PV of carry of $5.04, which is a lot lower than the FVT fund expected carry of $6.18. The 8% hurdle delays the carry timing, which hurts the PV of GP carry and makes this fund term less GP friendly than the other two. The BO fund with committed capital returned first has the expected PV of carry of $5.53, which is also significantly smaller than the FVT fund carry.

3.2 Effect of early timing advantage of the FVT scheme on PV of carry

The baseline analysis indicates that, while the FVT scheme is the most GP friendly for both the VC and BO funds, it particularly favors GPs of funds with high-volatility investment portfolios, such as early-stage VC funds. This is because high volatility makes it more likely that GPs earn early carry; conditional on getting carry early, the high volatility also makes such carry larger in expectation. Note, however, that the FVT fund we examine in the analysis above, as well as virtually all funds observed in practice, come with the clawback provision, so that the nominal amount of carry net of clawback at the end of the fund's life is the same across all three funds. The clawback provision requires GPs to return any excess carry payment at the end of fund life, when all three funds are required to have returned to LPs exit distributions equal to committed capital (because the contributed capital converges to committed capital for fully-invested, completed funds). Thus, the difference in PV of carry across three funds derives entirely from the time value of money, or the discount rate. In other words, the FVT scheme is GP friendly because of its carry timing advantage, not because it entitles GPs to more carry in expectation.

To illustrate this point, we simulate and present the VC model results with different values of risk-free rate in Table 3. For the two funds with early carry possibilities and clawback, we further break down the results into (i) the PV of carry before clawback, (ii) the clawback amount, and (iii) the PV of carry after clawback. With 0% risk-free rate (as shown in the last column), the amounts of GP carries net of clawback are identical across the three funds ($8.49), since the excess early carry is exactly offset by the clawback amount. Note that the FVT fund has a clawback amount ($1.48) that is ten times as large as the fund with contributed capital returned first ($0.14). When the risk-free rate (which is the discount rate in the risk-neutral world) is positive, this large early carry gives the FVT carry scheme (Fund III) a larger PV of carry than the other two. Furthermore, while the fund with the contributed capital returned first (Fund II) also earns a larger PV of carry than Fund I when the risk-free rate is positive, the impact of increasing risk-free rates is more pronounced for the FVT fund. Thus the present value of GP carry in Fund III is more sensitive to risk-free rate increases than that in Fund I.

3.3 Effect of inflation of un-exited investment values

Next we investigate whether GPs are tempted to inflate the portfolio values of un-exited companies. As mentioned earlier, there are concerns among LPs that "GPs who tie the timing of carried interest to [fair-value] tests might have an incentive to report higher valuations than other GPs." Yet it is also plausible that having some kind of FVT is better than not requiring any FVT threshold at all (effectively setting the FVT threshold at 0%) even in the presence of asymmetric information. Table 4 examines the effects of inflated values of un-exited portfolios on the GP compensation. The benchmark case of 100% is the case in which the market values are accurately appraised and self-reported by GPs. Relative to the benchmark case, the present value of GP carry before clawback increases slightly with the inflated level of 125% from $10.25 to $10.40, and increases further but only marginally with the level of 150% from $10.40 to $10.48. However, note that the clawback condition kicks in and offsets much of these increases. As expected, the amount of clawback is bigger with a higher inflated level for un-exited investments. With this offset, the PV of carry net of clawback is affected only moderately when GPs inflate the value of their un-exited investments, where the minor increases (from $9.42 at 100% to $9.51 at 150%) come from the time value of early carry. These increases amount to less 1% of the total PV of carry. The results are qualitatively similar for BO funds (presented in Panel B), though we note that the increases are proportionately larger; the increase from $6.18 (at 100%) to $6.34 (150%) represents 2.6% of the total PV of carry ($6.18). Thus, we find that, conditional on having the FVT carry scheme, GPs make only negligible amounts of gains by inflating the values of their un-exited portfolios, suggesting that the LPs' concerns are not warranted.

3.4 Effect of other parameter values

We examine the effects of altering various parameter values on the present value of GP compensation across the three funds to investigate whether these effects are more or less substantial for Fund III that applies the FVT. We examine the effects on carry of perturbing six (seven for BO) model parameter values — exit probabilities, carry levels, carry basis, total volatilities of companies, pairwise correlations between portfolio companies, and fair-value threshold levels (plus leverage for BO funds).

Table 5 presents the effects of altering parameter values on PV of carry for the three funds. The parameter values for the baseline VC (BO) model are: 20% exit probability, 20% carry level, $100 carry basis (= committed capital), 90% (60%) total volatility, 50% (20%) pairwise correlation, and 120% fair-value threshold level (plus 2:1 leverage for BO funds). We perturb these values as described in the second vertical panel of Table 1. Panel A of Table 5 presents the results for VC funds; Panel B presents the results for BO funds. The exit probabilities have negative effects on the present values of GP carry in a concave way across the three levels of 10%, 20%, and 30%. The carry levels have positive effects in a concave way across the three levels of 20%, 25%, and 30%, but the concavity is quite negligible. When the carry basis changes to the investment capital of $82 ($88 for BO funds),[9] the magnitude and the percentage of the increases in the value is the largest for Fund III, and the smallest for Fund II.

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[9] The investment capital level is set to match the empirically observed average fund terms, as in Metrick and Yasuda (2010a). The BO fund has larger investment capital than the VC fund because BO funds on average charge lower management fees.

As expected, increases in either the volatility of an individual company or the pairwise correlation lead to higher compensation to GPs. However, given the levels of GP carry, the effects of either volatility or pairwise correlation are about the same across the three funds for VC; for the BO, the impact is larger for Fund I, because this fund has to meet the 8% hurdle rate. Also as expected, the FVT threshold level has a negative effect on the present value of GP carry. Finally, the leverage has a negative effect on the PV of carry for BO funds. Though this may be surprising, note that the total expected compensation to GPs, which includes transaction fees and monitoring fees, rise with higher leverage.[10] Both the transaction fees and monitoring fees are assessed on the total firm value, which become larger relative to the size of the BO fund's equity investment when leverage is higher. Entry transaction fees reduce the initial value of equity investments, while the exit monitoring fees reduce the amount of exit value to be split between LPs and GPs. Both of these effects reduce the amount of carry that GPs receive in expectation, while sharply increasing the transaction and monitoring fees that GPs and LPs share. The impact of leverage on PV of carry is quantitatively similar across the three funds examined.

4. Conclusion

This paper analyzes the economics of the private equity fund compensation. We evaluate the effect of using a fair-value test GP carry scheme on the present value of GP carried interest relative to other carry schemes. We find that, while the use of the fair-value test has a significantly positive effect on the PV of carry relative to other commonly used carry schemes, GPs gain only a marginal increase in their expected PV of carry by reporting inflated values for the un-exited (and therefore illiquid) investments remaining in their fund portfolios. Our findings

[10] See Metrick and Yasuda (2010a), Figure 4 (p.2334).

suggest that the fair-value test scheme is a favorable compensation scheme for GPs, but should not induce GPs to significantly misreport portfolio values.

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Table 1. Parameter values for the simulation model

This table describes (i) the default parameter values used in the baseline simulation model and (ii) variations considered for sensitivity analysis. Panel A presents the parameter values chosen for the VC model; Panel B presents the values for the BO model. In the baseline model, a VC (BO) fund makes 25 (11) investments of equal sizes at the pace of 8, 6, 7, 3, and 1 (3, 3, 3, 1, and 1) investment(s) at the beginning of each of the first 5 years, respectively. The investment pace follows the empirically observed average investment pace as discussed in Metrick and Yasuda (2010a). From the time of the investment, each portfolio company is assumed to have the instantaneous hazard rate (= death rate, or exit probability) of 20%, independently with respect to any other portfolio companies. The market value of portfolio company *i* at time *t*, X_t^i, is assumed to follow $\frac{dX_t^i}{X_t^i} = rdt + \sigma\left(\sqrt{1-\rho^2}dW_t^i + \rho dW_t^F\right)$, where the default risk-free rate (*r*) is 5%, the volatility (σ) is 90% (60% for BO), and the pairwise correlation (= ρ^2) is 50% (20% for BO). For a given carry scheme, the default carry level is 20%, the carry basis is \$100, the threshold level for the fair-value test is 120%, and the reported value of un-exited investments is 100% of the actual value (that is privately observed/assessed by GPs). For the baseline BO model, the leverage ratio of 2:1 is also assumed. While the carry level and basis determine the nominal amount of carry that GPs are entitled to, the fair-value threshold level and the ratio of reported to actual values of un-exited investments determine the carry timing.

	Baseline model	Variation considered
Panel A: Venture Capital Funds		
Exit probability	20%	10%, 30%
Carry level	20%	25%, 30%
Carry basis	$100	$82 (investment capital)
Total volatility of an investment	90%	60%, 120%
Pairwise correlation	50%	30%, 70%
Fair-value threshold level	120%	112%, 125%, 130%
Inflated value of un-exited investments	100%	125%, 150%
Panel B: Buyout Funds		
Exit probability	20%	10%, 30%
Carry level	20%	25%, 30%
Carry basis	$100	$88 (investment capital)
Total volatility of an investment	60%	30%, 90%
Pairwise correlation	20%	10%, 50%
Fair-value threshold level	120%	112%, 125%, 130%
Inflated value of un-exited investments	100%	125%, 150%
Leverage	2:1	1:1, 4:1

Table 2. Baseline model results

This table presents the simulation results of calculating the expected PV of carry for the three representative funds. Panel A presents the PV of carry per $100 of committed capital for VC funds; Panel B presents the results for BO funds. The parameter values for the baseline VC (BO) model are: 20% exit probability, 20% carry level, $100 carry basis (= committed capital), 90% (60%) total volatilities, 50% (20%) pairwise correlation, and 120% fair-value threshold level (plus 2:1 leverage ratio for BO). For the VC model, the most common fund (Fund I) requires that, upon any exit, LPs must have received cumulative exit distributions equal to the contributed capital (= cost bases of all investments made to date + cumulative management fees paid to date) before any distribution of carried interests to GPs is allowed. For the BO model, the most common fund (Fund I) requires that, upon any exit, LPs must have received cumulative exit distributions equal to the contributed capital (= cost bases of all investments made to date + cumulative management fees paid to date), plus 8% hurdle return, before any distribution of carried interests to GPs is allowed. GPs then catch up with LPs with the catch-up rate of 100%. For both the VC and BO model, the "no early carry" fund (Fund II) requires that, upon any exit, LPs must have received cumulative exit distributions equal to the committed capital before any distribution of carried interests to GPs is allowed. For both the VC and BO model, the FVT fund (Fund III) requires that, upon any exit, (i) LPs must have first received the cost bases of all exited (and written off) companies to date plus prorated management fees and (ii) the fair-value test (FVT) is met before any distribution of carried interests to GPs is allowed. The fair-value test requires that the fair value (= estimated, reported value) of the remaining fund portfolio companies equals or exceeds 120% of the cost bases of all un-exited investments. If the first criterion is met but there is a small deficit between the fair value of the remaining fund portfolio and the threshold value, the remaining exit value can be used to pay down the deficit so that the FVT is met, and any leftover exit value can then be split 20:80 between GPs and LPs.

Most comon (Fund I)	No early carry (Fund II)	FVT (Fund III)
Panel A: Venture Capital Funds		
$8.67	$8.61	$9.42
Panel B: Buyout Funds		
$5.04	$5.60	$6.18

Table 3. The Effect of Carry Timing Rules on PV of Carry

This table presents the simulation results for the PVs of carried interest (in $, per $100 of committed capital) as functions of carry timing rules and the level of the risk-free rate. PVs of GP carry are calculated for three different fund terms: "Fund I: with no hurdle, contributed capital returned first with clawback" is a fund whose VC GPs are entitled to carry after returning the contributed capital to LP, subject to clawback. "Fund II: with no early carry" is a fund whose GPs must return all of carry basis before they are entitled to carry, thus ruling out any necessity for clawback. "Fund III: with a 120% threshold fair-value test, with clawback" is a fund whose GPs are entitled to carry after returning the cost basis of all exited (or written-off) investments and meeting the 120% fair-value test criteria for un-exited investments. The risk-free rates vary from 0% to 5% by 1% increments.

	Risk-Free Rate					
	5%	4%	3%	2%	1%	0%
Fund I: with no hurdle, contributed capital returned first with clawback						
Present value of GP carry before clawback	8.77	8.75	8.72	8.69	8.66	8.63
Present value of the clawback	0.10	0.10	0.11	0.12	0.13	0.14
Present value of GP carry (net of clawback)	8.67	8.64	8.61	8.57	8.53	8.49
Fund II: with no early carry						
Present value of GP carry	8.61	8.59	8.57	8.55	8.52	8.49
Fund III: with a 120% threshold fair-value test, with clawback						
Present value of GP carry before clawback	10.25	10.21	10.15	10.09	10.03	9.97
Present value of the clawback	0.84	0.94	1.05	1.18	1.32	1.48
Present value of GP carry (net of clawback)	9.42	9.27	9.10	8.91	8.71	8.49

Table 4. The Effect of Inflated (Reported) Values of Un-exited Investments on the PVs of Carry

This table presents the simulation results for the PVs of GP carry as a function of the ratio of reported to actual values (that are privately observed/assessed by GPs) of un-exited investments. The actual portfolio values of un-exited investments are generated by the stochastic process as described in Equation (1). For the baseline model, the reported value is 100% of the actual value (no inflation). For the results in the last two columns, the reported values are assumed to be inflated by 25% and 50%, respectively, from the actual (privately observed) values.

	Inflation Level of Un-exited Investments		
	100%	125%	150%
Panel A: venture capital funds			
Fund III: with a fair-value test, with clawback			
Present value of GP carry before clawback	10.25	10.40	10.48
Present value of the clawback	0.84	0.93	0.98
Present value of GP carry (net of clawback)	9.42	9.48	9.51
Panel B: buyout funds			
Fund III: with a fair-value test, with clawback			
Present value of GP carry before clawback	6.61	6.85	7.01
Present value of the clawback	0.43	0.57	0.67
Present value of GP carry (net of clawback)	6.18	6.28	6.34

Table 5. Sensitivity Analysis

This table presents the effects of altering the parameter values of the simulation model on the estimated PV of carry. Fund I for the VC model (BO model) is a fund with no hurdle (8% hurdle), contributed capital returned first with clawback. Fund II is a fund with committed capital returned first. Fund III is a fund with a fair-value test and with clawback. The baseline model refers to the model results reported in Table 2. "10% exit probability" refers to an altered model that is the same as the baseline model, except that the exit probability is set to 10% (instead of 20%). "30% exit probability" is similarly defined. "25% carry level" refers to an altered model that is the same as the baseline model except that the carry level is set to 25%. "30% carry level" is similarly defined. "Investment capital basis" refers to an altered model that is the same as the baseline model except that the carry basis is investment capital (which is set to $82 ($88 for BO) per $100 of committed capital). "60% volatility" refers to an altered model that is the same as the baseline model except that the annual volatility of individual investments is set to 60%. "120% volatility" is similarly defined. "30% pairwise correlation" is an altered model that is the same as the baseline model except that the pairwise correlation between individual investments is set to 30%. "70% pairwise correlation" is similarly defined. "112% fair-value test threshold" is an altered model that is the same as the baseline model except that the threshold level for the fair-value test is set to 112%. "125% fair-value test threshold" and "130% fair-value test threshold" are similarly defined.

	Most comon (Fund I)	No early carry (Fund II)	FVT (Fund III)
Panel A: Venture Capital Funds			
Baseline model	8.67	8.61	9.42
10% exit probability	11.43	11.41	12.17
30% exit probability	7.43	7.32	8.06
25% carry level	10.84	10.76	11.77
30% carry level	13.00	12.92	14.13
Investment capital basis	9.74	9.65	10.60
60% volatility	6.78	6.76	7.55
120% volatility	9.55	9.45	10.24
30% pairwise correlation	7.93	7.88	8.66
70% pairwise correlation	9.42	9.35	10.16
112% fair-value threshold level			9.44
125% fair-value threshold level			9.41
130% fair-value threshold level			9.39

	Most comon (Fund I)	No early carry (Fund II)	FVT (Fund III)
Panel B: Buyout Funds			
Baseline model	5.04	5.60	6.18
10% exit probability	6.50	7.46	8.04
30% exit probability	4.03	4.41	4.90
25% carry level	6.27	7.00	7.72
30% carry level	7.49	8.40	9.27
Investment capital basis	5.98	6.49	7.23
30% volatility	2.65	4.01	4.54
90% volatility	7.20	7.40	7.98
10% pairwise correlation	4.84	5.46	6.01
50% pairwise correlation	5.66	6.11	6.74
112% fair-value threshold level			6.21
125% fair-value threshold level			6.16
130% fair-value threshold level			6.14
1:1 leverage	5.16	5.73	6.31
4:1 leverage	4.80	5.35	5.91