INTERNATIONAL RECESSIONS

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ABSTRACT

The 2008-2009 crisis was characterized by an unprecedented degree of international synchronization as all major industrialized countries experienced large macroeconomic contractions around the date of Lehman bankruptcy. At the same time countries also experienced large and synchronized tightening of credit conditions. We present a two-country model with financial market frictions where a credit tightening can emerge as a self-fulling equilibrium caused by pessimistic but fully rational expectations. As a result of the credit tightening, countries experience large and endogenously synchronized declines in asset prices and economic activity (international recessions). The model suggests that these recessions are more severe if they happen after a prolonged period of credit expansion.

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1 Introduction

One striking feature of the 2007/2009 recession is that in the midst of it (the quarter after the Lehman bankruptcy) all major industrialized countries experienced extraordinarily large and synchronized contractions in real and financial aggregates. The main objective of this paper is to provide an explanation for these two features of the crisis: the international dimension (both real and financial) and its depth.

Our hypothesis is that multiple self-fulfilling equilibria in credit markets can explain these features. We will make our point by developing a two-country incomplete-markets model where firm managers use credit to finance hiring and to pay dividends to owners. If managers are up against a credit constraint, equilibrium employment is affected by the shadow cost of credit, which in turn is affected by the tightness of credit constraints.

Our first contribution is to show that if the two countries are financially integrated, the shadow cost of credit is equalized across countries. Hence, an 'exogenous' tightening of credit constraints affects economic employment and activity in both countries, regardless of where the tightening originates. This result suggests a transmission channel for credit shocks but does not deal with the more fundamental question of what causes a credit shock. Our second contribution is to provide an 'endogenous' mechanism for the credit tightening. More specifically, we show that tighter/looser credit constraints can emerge endogenously as multiple self-fulling equilibria.

In 'bad' equilibria, markets expect low resale prices for the assets of defaulting firms. Because of this, firms face low credit capacity and are liquidity constrained. Then, the fact that firms are liquidity constrained implies that the resale price of their assets is low since there are no firms that have the ability to purchase the assets of liquidated firms. This rationalizes, ex-post, the ex-ante expectation of low prices leading to bad equilibria. These equilibria are characterized by depressed economic activity, financial intermediation and asset prices. On the other hand, in 'good' equilibria, markets expect high resale prices of firms' assets which allows for higher debt. As a result of the high borrowing capacity, firms are unconstrained. This implies that ex-post there are firms with the required liquidity to purchase the assets of liquidated firms, which in turn keeps prices high. This rationalizes, ex-post, the ex-ante expectations of high prices leading to equilibria with sustained levels of economic activity, financial intermediation and asset prices.

An important difference between exogenous and endogenous credit shocks is that, in finan-

cially integrated economies, the former imply co-movement in real activity but not necessarily in financial intermediation, while the latter always imply co-movement in both real activity and financial intermediation. The reason is because endogenous credit shocks are determined by the resale price of firms's assets. In a financially integrated economy the resale price is common across countries. Hence, credit shocks are also common across countries and generate co-movement in all variables, real and financial. Since we document that during the 2008 crisis there has been strong co-movement in both real and financial variables, the theory of endogenous credit shocks seems better suited to explain recent events. Modeling the shocks as an endogenous process has also important policy implications. It suggests that changes in structural features of the economy, such us financial integration or the public provision of liquidity, can change the volatility and the international correlation of shocks, which usually are taken as exogenous.

Our third contribution relates to the depth of the crisis. We argue that 'ordinary' credit shocks, that is, shocks that would cause mild contraction under normal circumstances, can indeed generate 'extra-ordinary' recessions if they happen after a long period of credit expansion. In order to illustrate this, we study a version of the model with occasionally binding constraints and show that there exists an equilibrium path in which credit constraints are not binding for a long period of time. During this period both economies undergo a persistent expansion in economic activity (gradual) and in credit (rapid). However, if constraints become binding after this long expansionary phase, firms are forced to implement a large de-leveraging which causes a sharp contraction in real macroeconomic variables. This happens even if agents fully anticipate the possibility of the reversal. We argue that this asymmetry between the expansion phase—with fast growing credit growth and mild real growth—and the contraction phase—with sharp collapse in both economic activity and credit—captures well the macroeconomic developments of advanced economies during the recent cycle and during other episodes of financial crises (see for example Reinhart and Rogoff (2009)).

One important observation concerning the international dimension of the recent crisis is that, although real GDP experienced similar contractions in the US and in the rest of the G7 countries, employment was hit particularly hard in the US but not in the remaining G7 countries (see Ohanian (2010)). As a consequence of this, labor productivity soared in the US but declined in the rest of the G7 countries. Our baseline model with integrated credit markets and symmetric labor markets fails to explain this cross country difference. However, in the final section of the paper we show that this is not necessarily a problem with credit shocks

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but it is likely to derive from cross-country differences in the structure of the real economies and, more specifically, in the labor markets. We show this by introducing a stylized labor market asymmetry (more flexibility in the US and less flexibility in other G7 countries). With this extension credit shocks have the potential to explain the similar responses of GDP and financial markets and the asymmetric responses of employment and labor productivity.

Our paper is related to the vast literature (both empirical and theoretical) studying the sources of macroeconomic co-movement and international transmission of shocks. Usually co-movement is explained as the result of synchronized disturbances (global or common shocks, see for example Crucini, Kose and Otrok (2011)) and/or as the result of country-specific shocks that spill to other countries (international transmission of country specific shocks). In this paper we show that credit shocks generate co-movement for both reasons: *exogenous* credit shocks spill-over from one country to the other, and *endogenous* credit shocks will appear to the econometrician like a common-shock or a global factor. This finding is consistent with the empirical results of Helbling, Huidrom, Kose & Otrok (2011) according to which credit market shocks matter in explaining global business cycles, especially during the 2009 global recession. Recent contributions that analyze directly the strong international co-movement during the 2007-2009 crisis include Dedola & Lombardo (2010), Devereux & Yetman (2010), Devereux & Sutherland (2011), and Kollmann, Enders & Müller (2011). All these studies focus on the international transmission of shocks in models with financial market frictions but do not consider the possibility of endogenously generated credit shocks.

The role of credit shocks for macroeconomic fluctuations has been investigated recently but in closed economies.[1] Furthermore, while in most of the recent contributions credit shocks are purely exogenous, in our paper we provide a micro foundation which is based on self-fulling expectations. Our theory is in line with the idea of liquidity crises resulting from multiple equilibria outcomes as discussed in Lucas and Stokey (2011). It shares some similarities with the multiple equilibria property of the model studied in Kocherlakota (2009).

In our model the multiplicity of equilibria derives from 'occasionally binding' constraints. This is another important difference between our paper and other studies that investigate the macroeconomic impact of financial shocks. Most of these contributions limit the analysis to equilibria with always binding constraints and the quantitative properties are studied using linear approximation techniques. In our model, instead, borrowing constraints are only occa-

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[1]Examples are Christiano, Motto and Rostagno (2009), Gertler and Karadi (2009), Gertler and Kiyotaki (2009), Goldberg (2010), Jermann and Quadrini (2011), Khan and Thomas (2010), Lorenzoni and Guerrieri (2010).

sionally binding and this is important to generate the asymmetry between long and gradual credit driven booms and sharp credit driven recessions. Mendoza (2010) also studies an economy with occasionally binding constraints but does not investigate the importance of financial shocks. Furthermore, by focusing on a small open economy, the paper does not address the issue of international co-movement which is central to our study. Occasionally binding constraints are also central to Brunnermeier and Sannikov (2010) but the analysis is limited to productivity shocks and in a closed economy.

The paper is organized as follows. In Section 2 we discuss the macroeconomic and financial evidence about the recent crisis. We then present the theoretical framework gradually, starting in Section 3 with a version of the model where capital is fixed and credit shocks are exogenous. Section 4 makes the credit shocks endogenous and describes the conditions for multiple equilibria. Section 5 adds capital accumulation and Section 6 presents the quantitative results. In Section 7 we extend the model by allowing for cross-country heterogeneity in domestic labor markets. Section 8 concludes.

2   Macroeconomic evidence

In this section we first present some facts about international co-movement in several variables during the 2007-2009 crisis and then some evidence on the relation between credit and employment for different countries.

2.1   International co-movement

Figure 1 plots the GDP dynamics for the G7 countries in six of the most recent US recessionary episodes. In each panel we plot the percent deviations of GDP for each country from the level of GDP in the quarter preceding the start of the US recession (based on the NBER business cycle dating committee). Comparison of the bottom right panel of the figure with the other panels suggests how the 2007-2009 period, and in particular the period following the Lehman crisis (shaded area), stands out both in terms of depth and in terms of macroeconomic synchronization among the G7 countries. In none of the previous recessions the GDP fell so much and in all countries.

To make the point of increasing international co-movement a bit more generally, Figure 2 plots the average correlation of 10 years rolling windows of quarterly GDP growth between all G7 countries. The figure also plots 2 standard deviation bands. The dates in the graph
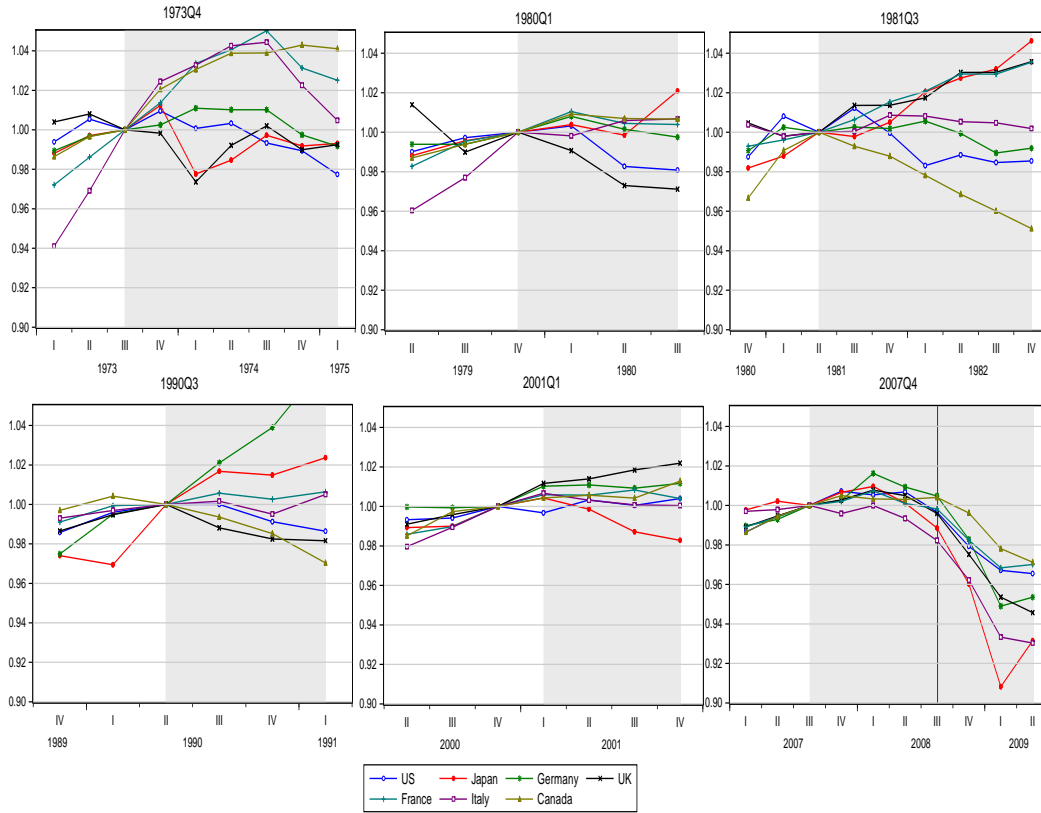
Figure 1: The dynamics of GDP during the six most recent recessions in the G7 countries.

correspond to the end points of the window used to compute the correlation. The figure shows that during the last two quarters of 2008 (the vertical line marks the third quarter), the average correlation jumped from 0.3 to 0.7 and the sample standard deviation of the correlations fell from 0.19 to 0.09. This confirms that the 2007-2009 stands out in the post-war era as a period of extraordinarily high co-movement for all developed countries. See also Imbs (2010).

The high degree of international co-movement between US and other major countries is also observed in other real and financial variables. Figure 3 plots GDP, consumption, investment and employment in the period 2005-2010 for the US and an aggregate of the other countries in the G7 group (from now on G6). The figure highlights that GDP, consumption and investment were all hit hard in both the US and the G6 countries. This is especially noticeable after the third quarter of 2008, marked by the vertical line. Employment also declined in US and abroad
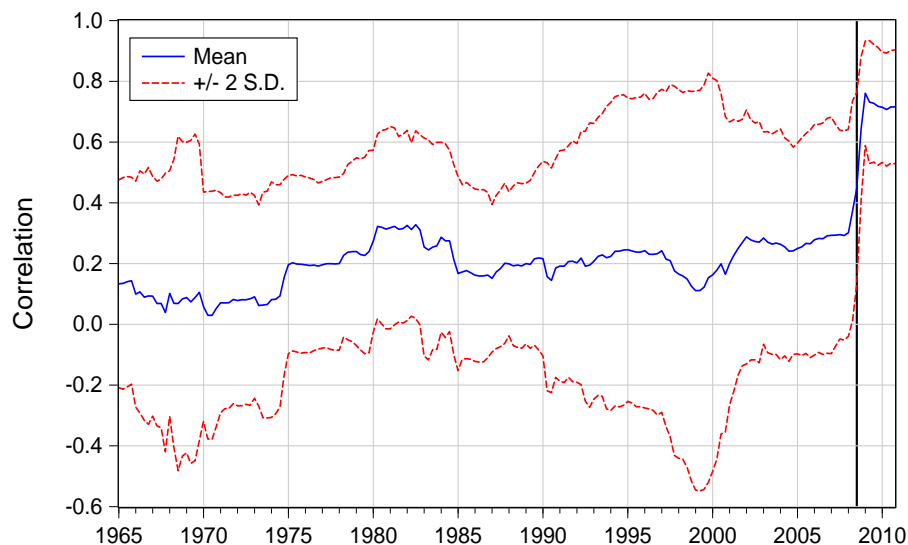
Figure 2: Rolling correlations of quarterly GDP growth among G7 countries.

even though the US decline is much larger than the decline in the G6.[2].

Figure 4 plots the dynamics of some selected financial variables. The top left panel reports the growth rate of stock prices for the US and for the G6 and it documents the massive and synchronous decline in stock prices that took place during the crisis.[3] The other panels plot different measures of credit conditions in the business sector. The top right panel reports the growth in total net debt for the non-financial businesses sector.[4]

Indicators of credit market conditions based on credit volumes have been criticized as they do not take into account that a credit crunch might induce firms to draw on existing credit lines, so creating a distress that does not (immediately) show up in quantities. See, for example, Gao and Yun (2009). For this reason the bottom left panel reports a different indicator of credit market conditions. The indicator is not based on volumes of credit but on

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[2]This feature of the crisis has been emphasized by Ohanian (2010). In the last part of the paper we will consider a possible explanation for the heterogeneous behavior of the labor market

[3]Stock prices for the US are the MSCI BARRA US stock market index, while stock prices in the G6 countries are computed using the MSCI BARRA EAFE+Canada index which is an average of stock prices in advanced economies except the US.

[4]The series for the US real debt is from the Flows of Funds Accounts and for the whole nonfinancial business sector. The series for the G6 is the sum of net debt (in constant PPP dollars) of the corporate non-financial sector for the Euro Area, Japan and Canada. Debt is defined as credit markets instruments minus liquid assets i.e. the sum of foreign deposits, checkable deposits and currency, time and savings deposits, money market funds, securities RPs, commercial paper, treasury securities, agency and GSE backed securities, municipal securities and mutual fund shares

Note: Data for GDP, private consumtion, gross fixed capital formation are from OECD Quarterly National Accounts in PPP constant dollars.
Data for employment are from OECD Main Economic Indicators. All series are normalized to 1 in the fourth quarter of 2007.
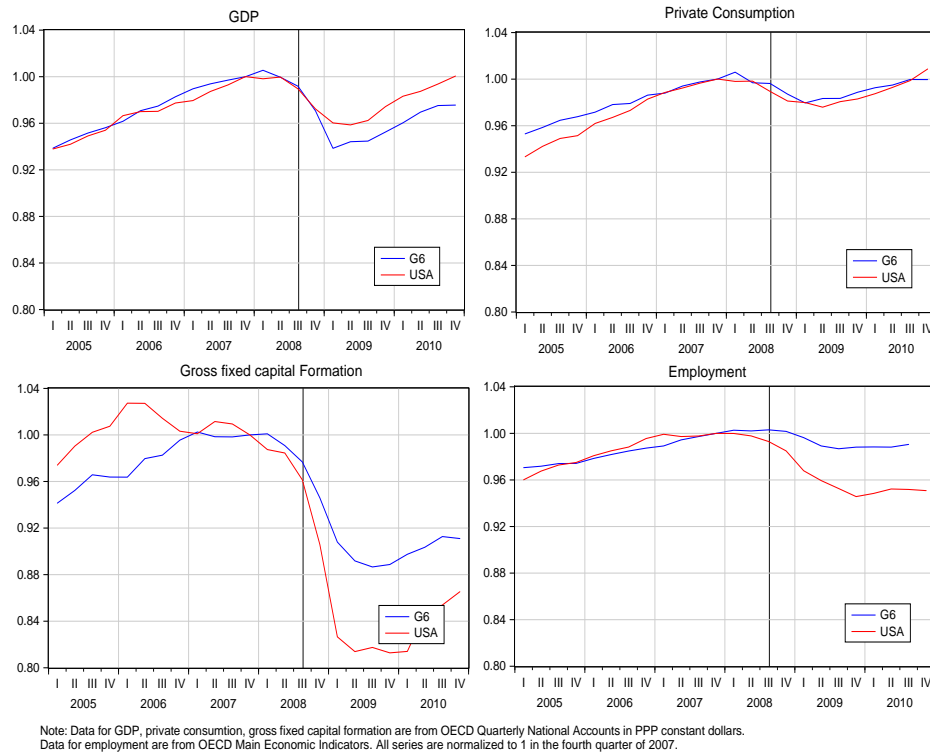
Figure 3: GDP, Consumption, Investment and Employment in US and G6: 2005-2010

opinion surveys of senior loan officers of banks. The plotted index is the percentage of banks that relaxed the standards to approve commercial and industrial loans minus the percentage of banks that tightened the standards. Therefore, a negative number represents an overall tightening of credit.[5]

As can be seen from the third panel of Figure 4, the index shows a credit tightening that starts before the decline in credit growth. To take both types of evidence into account, the bottom right panel constructs a credit index that is a simple average of the two previous measures, where each measure is normalized by its own standard deviation. The key lesson we learn from Figure 4 is that, right around 2008, credit conditions moved from strongly loose/expansionary to strongly tight/contractionary both in the US and in the G6 countries.

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[5]The series for the US is collected by the Federal Reserve Board (Senior Loan Officers Opinions Survey). The series for the G6 is based on similar surveys collected by the European Central Bank (ECB Bank Lending Survey), Bank of Japan (Senior Loan Officer Opinion Survey) and Bank of Canada (Senior Loan Officers Opinions Survey). The index for the G6 is a weighted average of the indexes in the three areas with weights proportional to the size of their debt. This index is typical reported with the inverted sign (tightening credit standards instead of relaxing credit standards).

This evidence will be particularly important in the second part of the paper as it allows us to identify more precisely the nature of the crisis.
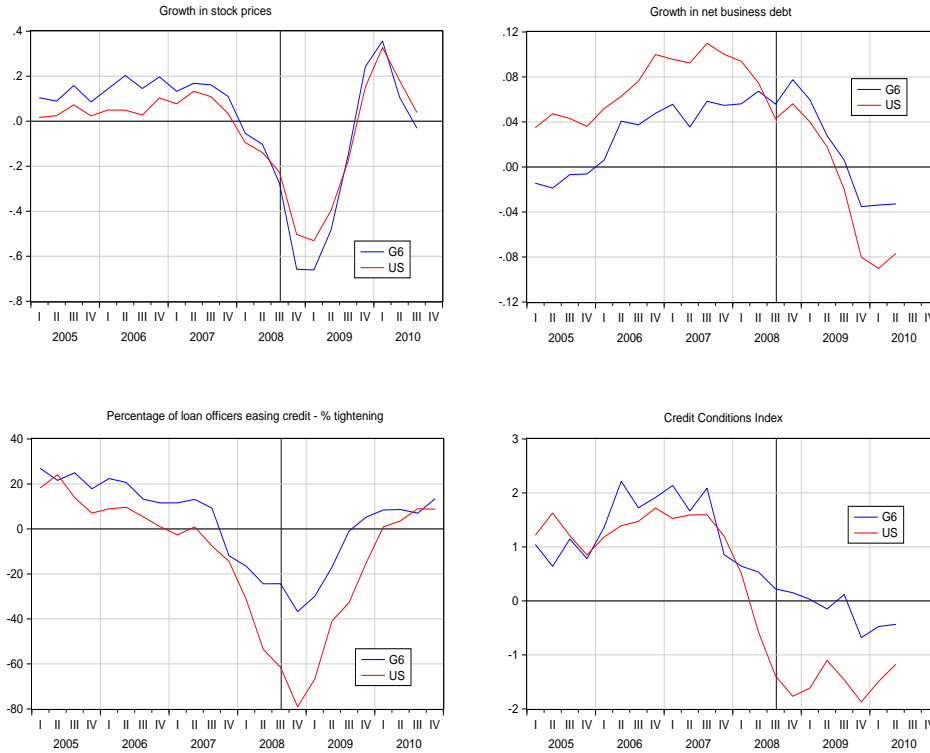
Figure 4: GDP, Stock markets and credit conditions in US and G6: 2005-2010

2.2   Domestic co-movement between credit and employment

As discussed in the introduction, our main hypothesis is that tight credit affects economic activity and especially employment. Here we provide some empirical support for this idea by plotting the growth rates of employment, GDP and business credit during the crisis in US and in the G6. Figure 5 shows that in the quarters following the Lehman crisis (shaded area) both credit and employment slow-down significantly in US and in the G6. Interestingly, GDP also declines initially but recovers more quickly than employment and credit. For example, in the first quarter of 2009, credit and employment are still depressed (experiencing negative growth) in US and in the G6. However, GDP has already recovered (experiencing positive growth) in both countries. We view this evidence as consistent with our basic hypothesis: tight credit reduces employment and as employment fall labor productivity increases so that the decline

in GDP is not as severe as the decline in employment.

Figure 5: Domestic co-comovement of credit, employment and GDP: 2005-2010
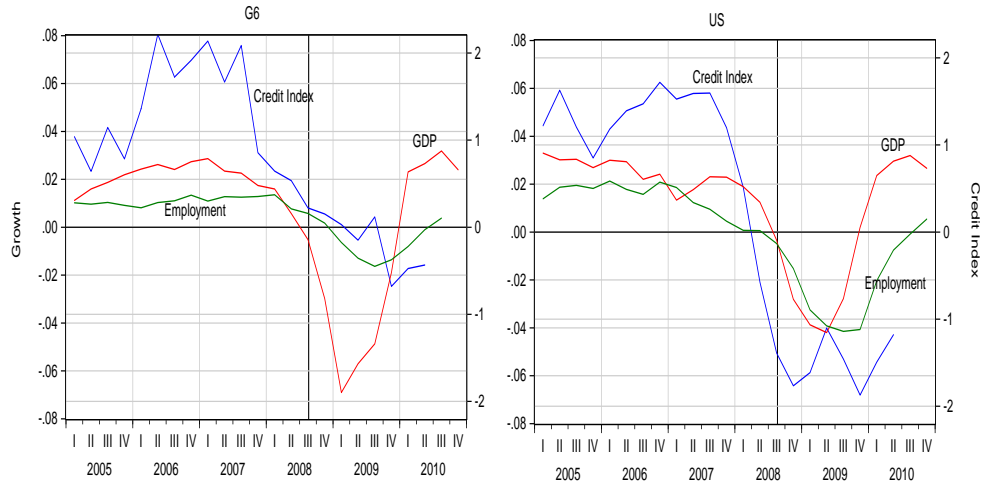
A final observation relates to the asymmetry between real and financial variables in the expansion phase before the crisis and the collapse during the crisis. Figure 5 shows that debt experienced rapid growth (about 6% per year in US and 4% per year in the G6) in the years preceding the crisis, while the growth in real variables has been moderate (GDP grew at about 2% per year both in US and the G6). In the crisis period, instead, all variables, real and financial, contracted sharply. This feature is not unique to the 2007-2009 crisis. Several authors have observed that many historical episodes of credit booms are not associated with much faster growth in real economic activity. However, when the credit booms reach a sudden stop, their reversals are often associated with sharp macroeconomic contractions. See for example Reinhart and Rogoff (2009), Classens, Kose and Terrones (2011), and Jordà, Schularick, and Taylor (2011).

The facts presented in this section—high international co-movement in real and financial variables during the crisis, the large employment (for US) and stock markets collapse, and the asymmetry between the pre-crisis phase and post crisis phase—cannot be easily explained with a standard work-horse international business cycle model. In the next sections we propose a theoretical framework with credit disturbances that we believe is useful for understanding the empirical facts.

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3 The model with fixed capital and exogenous credit shocks

We start with a simple model without capital accumulation and with exogenous credit shocks. This allows us to provide the intuitions for some of the key results of the paper analytically. After the presentation of the simple model it will be easy to add capital accumulation.

There are two types of atomistic agents, investors and workers. A key difference between these two types of agents is the availability of different investment opportunities. Due to the assumption of markets segmentation only investors have access to the ownership of firms while workers can only save in the form of bonds. Investors discount the future at rate β while the discount factor of workers is $\delta > \beta$. The different discounting between the owners of firms (investors) and workers implies that firms borrow from workers subject to the enforcement constraints describe below.

To facilitate the presentation we first describe the closed-economy version of the model. Once we have characterized the autarkic equilibrium, it will be easy to extend it to the environment with international mobility of capital.

3.1 Investors and firms

Investors have lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$. They are the owners of firms and derive income only from dividends. Denoting by d_t the dividends paid by firms, the effective discount factor for investors is $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$. This is also the discount factor used by firms since they maximize shareholders' wealth. As we will see, fluctuations in the effective discount rate play a central role in the analysis of this paper.

Firms operate the production function $F(h_t) = \bar{k} h_t^\nu$, where $\bar{k}$ is a fixed input of capital and h_t is the variable input of labor. The parameter ν is smaller than 1 implying decreasing returns to scale in the variable input. In this version of the model without capital accumulation we can think of $\bar{k}$ as a normalizing constant.

Firms start the period with intertemporal debt b_t. Before producing they choose the labor input h_t, the dividends d_t, and the next period debt b_{t+1}. The budget constraint is

$$b_t + w_t h_t + d_t = F(h_t) + \frac{b_{t+1}}{R_t}, \tag{1}$$

where R_t is the gross interest rate.

The payments of wages, $w_t h_t$, dividends, d_t, and current debt net of the new issue, $b_t - b_{t+1}/R_t$, are made before the realization of revenues. This implies that the firm faces a cash

flow mismatch during the period. The cash needed at the beginning of the period is $w_t h_t + d_t + b_t - b_{t+1}/R_t$. To cover the cash flow mismatch, the firm contracts the intra-period loan $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t$ which is repaid at the end of the period, after the realization of revenues. From the budget constraint, equation (1), we can also see that the intra-period loan is equal to the revenue $F(h_t)$.

Debt contracts are not perfectly enforceable as the firm can default. Default takes place at the end of the period before repaying the intra-period loan. At this stage the firm holds the revenues $F(h_t)$ which are equal to the intra-period loan l_t. These are liquid funds that can be easily diverted in the event of default. Default gives the lender the right to liquidate the firm's assets. But after the diversion of $l_t = F(h_t)$, the only remaining asset is the physical capital $\bar{k}$. Suppose that the liquidation value of capital is $\xi_t \bar{k}$, where ξ_t is stochastic. Since default arises at the end of the period, the total liabilities of the firm are $l_t + b_{t+1}/R_t$. To ensure that the firm does not default, the total liabilities are subject to the enforcement constraint

$$\xi_t \bar{k} \geq l_t + \frac{b_{t+1}}{R_t}. \tag{2}$$

A formal derivation of this constraint is provided in Appendix A and it is based on similar assumptions as in Hart and Moore (1994).

Fluctuations in ξ_t affect the ability to borrow and, as we will see, they generate pro-cyclical movements in real and financial variables.[6] Our goal is to derive the variable ξ_t endogenously from liquidity considerations. As we will describe below, fluctuations in this variable are induced by self-fulfilling expectations leading to multiple equilibria. For the moment, however, we treat ξ_t as an exogenous stochastic variable. Once we have characterized the equilibrium with an exogenous ξ_t, we will make it endogenous in Section 4.

To illustrate the role played by fluctuations in ξ_t, consider a pre-shock equilibrium in which the enforcement constraint is binding. Starting from this equilibrium, suppose that ξ_t decreases. In response to the decline in ξ_t the firm is forced to reduce either the dividends and/or the input of labor. To see this, let's start with the case in which the firm is unwilling to change the input of labor. This implies that the intra-period loan $l_t = F(h_t)$ also does not change. Thus, the only way to satisfy the enforcement constraint, equation (2), is by reducing the intertemporal debt b_{t+1}. We can then see from the budget constraint, equation (1), that the reduction in b_{t+1} requires a reduction in dividends. Thus, the firm is forced to substitute

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[6]Eisfeldt and Rampini (2006) provide some evidence that the liquidity of capital ξ_t must be procyclical to match the amount of capital reallocation observed in the data.

debt with equities.

Alternatively the firm could keep the dividend payments unchanged and reduce the intra-period loan $l_t = F(h_t)$. This would also ensure that the enforcement constraint is satisfied but it requires the reduction in the input of labor. Therefore, after a reduction in ξ_t, the firm faces a trade-off: paying lower dividends or cutting employment. The optimal choice depends on the relative cost of changing these two margins which, as we will see, depends on the stochastic discount factor for investors $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$.

Firm's problem: The optimization problem of the firm can be written recursively as

$$V(\mathbf{s}; b) \quad = \quad \max_{d, h, b'} \left\{ d + E m' V(\mathbf{s}'; b') \right\} \tag{3}$$

subject to:

$$b + d = F(h) - wh + \frac{b'}{R} \tag{4}$$

$$\xi \bar{k} \geq F(h) + \frac{b_{t+1}}{R_t}, \tag{5}$$

where $\mathbf{s}$ are the aggregate states, including the shock ξ, and the prime denotes the next period variable. The enforcement constraint takes into account that the intra-period loan is equal to the firm's output, that is, $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t = F(h_t)$.

In solving this problem the firm takes as given all prices and the first order conditions are

$$F_h(h) = \frac{w}{1 - \mu}, \tag{6}$$

$$R E m' = 1 - \mu, \tag{7}$$

where μ is the Lagrange multiplier for the enforcement constraint. These conditions are derived under the assumption that dividends are always positive, which will be the case if the investors' utility satisfies $u_c(0) = \infty$. The detailed derivation is in Appendix B.

We can see from condition (6) that there is a wedge in the demand for labor if the enforcement constraint is binding ($\mu > 0$). This derives from the fact that the input of labor needs

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to be financed and part of the financing has to come from equity (through lower payment of dividends). As long as the cost of equity, $1/Em'$, is greater than the cost of debt, R, expanding the input of labor is costly in the margin because the firm needs to substitute debt with equity. It is then the equity premium $1/Em' - R$ that determines the labor wedge as can be seen from condition (7).[7] The wedge is strictly increasing in μ and disappears when $\mu = 0$, that is, when the enforcement constraint is not binding.

Some partial equilibrium properties: The characterization of the firm's problem in partial equilibrium provides helpful insights about the property of the model once extended to a general equilibrium set-up. For partial equilibrium we mean the allocation achieved when the interest rate and the wage rate are both exogenously given and constant.

Under these conditions, equation (7) shows that μ decreases with the expected discount factor Em'. A decrease in ξ makes the enforcement constraint tighter. Because firms reduce the payment of dividends, the investors's consumption has to decrease. This induces a decline in the discount factor $m' = \beta u_c(d')/u_c(d)$ and an increase in the multiplier μ according to condition (7). From condition (6) we can then see that the demand for labor declines.

Intuitively, when the credit conditions become tighter, firms need to rely more on equity financing and less on debt. This requires investors to cut consumption (dividends) which is costly since they have concave utility. Because of this, in the short-term firms do not raise enough equity needed to keep the pre-shock production scale and cut employment. If investors' utility were linear (risk-neutrality), the discount factor would be equal to $Em' = \beta$ and the credit shock would not affect employment. This also requires that the interest rate does not change, which is the case in the partial equilibrium considered here. In the general equilibrium, of course, prices do change. In particular, movements in the demand of credit and labor affect the interest rate R and the wage rate w. To derive the aggregate effects we need to close the model and characterize the general equilibrium.

3.2   Closing the model and general equilibrium

There is a representative household/worker with lifetime utility $E_0 \sum_{t=0}^{\infty} \delta^t U(c_t, h_t)$, where c_t is consumption, h_t is labor and δ is the intertemporal discount factor. It will be convenient to

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[7]Notice that we are using the term 'equity premium' to denote the differential between the expected shareholders' return and the interest rate on bonds. Since shareholders and bondholders are different agents, the equity premium is not only determined by the cost of risk (risk premium).

assume that the period-utility takes the form

$$U(c_t, h_t) = \log(c_t) - \alpha \frac{h_t^{1+\frac{1}{\eta}}}{1 + \frac{1}{\eta}}.$$

The worker's budget constraint is

$$w_t h_t + b_t = c_t + \frac{b_{t+1}}{R_t},$$

and the first order conditions for labor, h_t, and next period bonds, b_{t+1}, are

$$U_h(c_t, h_t) + w_t U_c(c_t, h_t) = 0, \tag{8}$$

$$\delta R_t E_t \left\{ \frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} \right\} = 1. \tag{9}$$

We can now define a competitive general equilibrium. The aggregate states, denoted by $\mathbf{s}$, are given by the credit conditions ξ and the aggregate stock of bonds B.

Definition 3.1 (Recursive equilibrium) *A recursive competitive equilibrium is defined by a set of functions for (i) workers' policies $h_w(\mathbf{s})$, $c_w(\mathbf{s})$, $b_w(\mathbf{s})$; (ii) firms' policies $h(\mathbf{s}; b)$, $d(\mathbf{s}; b)$, $b(\mathbf{s}; b)$; (iii) firms' value $V(\mathbf{s}; b)$; (iv) aggregate prices $w(\mathbf{s})$, $R(\mathbf{s})$, $m(\mathbf{s}')$; (v) law of motion for the aggregate states $\mathbf{s}' = \Psi(\mathbf{s})$. Such that: (i) household's policies satisfy the optimality conditions (8)-(9); (ii) firms' policies are optimal and $V(\mathbf{s}; b)$ satisfies the Bellman's equation (3); (iii) the wage and interest rates are the clearing prices in the markets for labor and bonds, and the discount factor for firms is $m(\mathbf{s}') = \beta u_c(d_{t+1})/u_c(d_t)$; (iv) the law of motion $\Psi(\mathbf{s})$ is consistent with the aggregation of individual decisions and the stochastic processes for z and ξ.*

To illustrate the main properties of the model we look at some special cases. Consider first the economy without uncertainty, that is, ξ is constant. In this economy the enforcement constraint binds in a steady state equilibrium. To see this, consider the first order condition for the bond, equation (9), which in a steady state becomes $\delta R = 1$. Using this condition to eliminate R in (7) and taking into account that in a steady state $Em' = \beta$, we get $\mu = 1 - \beta/\delta > 0$ (since $\delta > \beta$). Firms want to borrow as much as possible because the cost of borrowing—the interest rate—is smaller than their discount rate.

With uncertainty, however, the enforcement constraint may be binding only occasionally.

In particular, after a large and unexpected decline in ξ. In this case firms will be forced to cut dividends inducing a change in the discount factor Em'. Furthermore, the change in the demand for credit impacts on the equilibrium interest rate. Using condition (7) we can see that these changes affect the multiplier μ, which in turn impacts on the demand for labor (see equation (6)). On the other hand, an increase in ξ may leave the enforcement constraint non-binding without direct effects on the demand of labor. Therefore, the responses to credit shocks could be highly asymmetric: negative shocks induce large falls in employment and output while the impacts of positive shocks is moderate.

3.3   Capital mobility

Let's consider now two countries, domestic and foreign, with the same size, preferences and technology as described in the previous section. Although we consider the case with only two symmetric countries, the model can be easily extended to any number of countries and with different degrees of heterogeneity. For the moment we continue to assume that ξ_t is an exogenous stochastic variable, specific to each country.

Once we allow for cross-country capital mobility, we have to specify what agents can do in an integrated financial market. We continue to assume that there is market segmentation in the ownership of firms, that is, workers are unable to purchase shares of firms. However, in addition to domestic bonds they can purchase foreign financial assets as specified below. Furthermore, investors are now able to purchase shares of foreign firms.

Investors/firms: Because firms are subject to country specific shocks, investors would gain from diversifying the cross-country ownership of shares. Therefore, in an economy that is financially integrated, investors choose to own the worldwide portfolio of shares and we have a representative 'worldwide' investor.[8] Because domestic and foreign firms are owned by the same representative shareholder, they will use the same discount factor $m_{t+1} = \beta u_c(d_{t+1} + d^*_{t+1})/u_c(d_t + d^*_t)$, where investors' consumption is the sum of dividends paid by domestic firms, d_t, plus the dividends paid by foreign firms, d^*_t. From now on we will use the star superscript to denote variables pertaining to the foreign country.

Besides the common discount factor, firms continue to solve problem (3) and the first order conditions are given by equations (6) and (7). Let's focus on condition (7), which we rewrite

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[8]A perfect diversification of portfolios is optimal because investors' utility depends only on consumption. If investors derived utility also from leisure, a perfect diversification would not be necessarily optimal.

here for both countries,

$$R_t Em_{t+1} = 1 - \mu_t,$$
$$R_t^* Em_{t+1}^* = 1 - \mu_t^*.$$

Since the discount factor is common to domestic and foreign firms, that is, $Em_{t+1} = Em_{t+1}^*$, and the interest rate is equalized across countries, $R_t = R_t^*$, the above conditions imply that the lagrange multipliers will also be equalized, that is, $\mu_t = \mu_t^*$. Therefore, independently of which country is hit by a shock, if the enforcement constraint is binding for domestic firms, it will also be binding for foreign firms. This also implies that the labor wedges are equalized across countries. In fact, condition (6) is still the optimality condition for the choice of labor in both countries, that is,

$$F_h(h_t) = w_t \left(\frac{1}{1 - \mu_t} \right),$$
$$F_h(h_t^*) = w_t^* \left(\frac{1}{1 - \mu_t^*} \right).$$

This property is crucial for understanding the cross-country impact of a financial shock as we will describe below. Later we will also consider an extension of the model where the labor wedge may respond differently in the two countries.

Households/workers: Although workers are still prevented from accessing the market for the ownership of firms, with capital mobility they can engage in international financial transactions with foreign workers. In addition to holding bonds issued by domestic firms, domestic workers can buy state-contingent claims from foreign workers. We still assume that firms borrow from domestic workers but they cannot sign state contingent contracts with workers. The assumption that firms borrow only from domestic workers is without loss of generality: whether they borrow domestically or in the foreign market is irrelevant in an integrated capital market. The unavailability of state-contingent claims between firms and workers is essential to retain market incompleteness.

Denote by $n_{t+1}(\mathbf{s}_{t+1})$ the units of consumption goods received at time $t + 1$ by domestic workers if the aggregate states are $\mathbf{s}_{t+1}$. These are worldwide states, and therefore, they include the aggregate states of both countries as will be made precise below. Of course, in equilibrium, the consumption units received by workers in the domestic country must be equal to the

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consumption units paid by workers in the foreign country, that is, $n_{t+1}(\mathbf{s}_{t+1}) + n^*_{t+1}(\mathbf{s}_{t+1}) = 0$. This must be satisfied for all possible realizations of the aggregate states $\mathbf{s}_{t+1}$.

The budget constraint of a worker in the domestic country is

$$w_t h_t + b_t + n_t = c_t + \frac{b_{t+1}}{R_t} + \int_{\mathbf{s}_{t+1}} n_{t+1}(\mathbf{s}_{t+1}) q(\mathbf{s}_{t+1})/R_t,$$

where $q_t(\mathbf{s}_{t+1})/R_t$ is the unit price of the contingent claims.

Given the specification of the utility function, the first order conditions for the choice of labor, h_t, next period bonds, b_{t+1}, and foreign claims, $n_{t+1}(\mathbf{s}_{t+1})$, are

$$\alpha h_t^\gamma c_t = w_t, \tag{10}$$

$$\delta R_t E_t \left(\frac{c_t}{c_{t+1}} \right) = 1, \tag{11}$$

$$\delta R_t \left(\frac{c_t}{c_{t+1}(\mathbf{s}_{t+1})} \right) p(\mathbf{s}_{t+1}) = q(\mathbf{s}_{t+1}), \qquad \text{for all } \mathbf{s}_{t+1}, \tag{12}$$

where $p(\mathbf{s}_{t+1})$ is the probability (or probability density) of the aggregate states in the next period for the world economy.

Since in equilibrium the prices and probabilities of the contingencies are the same for domestic and foreign workers, condition (12) implies that

$$\frac{c_t}{c_t^*} = \frac{c_{t+1}(\mathbf{s}_{t+1})}{c_{t+1}^*(\mathbf{s}_{t+1})} = \chi. \tag{13}$$

Therefore, the ratio of consumption of domestic and foreign workers remains constant over time. This is a well known property in environments with a full set of state-contingent claims. In our environment the constancy of the consumption ratio is among workers (and among investors) but not between workers and investors because of the assumption of market segmentation.

Before continuing we would like to clarify that the assumption of contingent claims among workers is not essential for the results of the paper. We could simply assume that workers can engage in international non-contingent lending and borrowing only. Or equivalently, that firms can engage in international borrowing. However, the availability of contingent claims greatly simplifies the characterization of the equilibrium because it allows us to reduce the number of 'sufficient' state variables. This property will be convenient once we extend the model with

capital accumulation.

Aggregate states and equilibrium: We can now define the equilibrium for the open-economy version of the economy. The aggregate states $\mathbf{s}$ are given by the variables ξ and ξ^*, the financial liabilities of firms, B_t and B_t^*, and the net foreign asset position of the domestic country, N_t. Since in equilibrium the net foreign asset position of the domestic country is the negative of the foreign position, once we know B_t, B_t^* and N_t we also know the total wealth of domestic workers, $B_t + N_t$, and foreign workers, $B_t^* - N_t$. Therefore, $\mathbf{s}_t = (\xi, \xi^*, B_t, B_t^*, N_t)$.

Definition 3.2 (Recursive equilibrium) *A recursive competitive equilibrium is defined by a set of functions for: (i) households' policies $h_w(\mathbf{s})$, $c_w(\mathbf{s})$, $b_w(\mathbf{s})$, $n_w(\mathbf{s}; \mathbf{s}')$, $h_w^*(\mathbf{s})$, $c_w^*(\mathbf{s})$, $b_w^*(\mathbf{s})$, $n_w^*(\mathbf{s}; \mathbf{s}')$; (ii) firms' policies $h(\mathbf{s}; b)$, $d(\mathbf{s}; b)$, $b(\mathbf{s}; b)$, $h^*(\mathbf{s}; b)$, $d^*(\mathbf{s}; b)$, $b^*(\mathbf{s}; b)$; (iii) firms' values $V(\mathbf{s}; b)$ and $V^*(\mathbf{s}; b)$; (iv) aggregate prices $w(\mathbf{s})$, $w^*(\mathbf{s})$, $R(\mathbf{s})$, $m(\mathbf{s}, \mathbf{s}')$, $q(\mathbf{s}; \mathbf{s}')$; (v) law of motion for the aggregate states $\mathbf{s}' = \Psi(\mathbf{s})$. Such that: (i) household's policies satisfy the optimality conditions (8)-(12); (ii) firms' policies are optimal and satisfy the Bellman's equation (3) for both countries; (iii) the wages clear the labor markets; the interest rates and the price for contingent claims clear the worldwide financial markets; the discount rate used by firms satisfies $m(\mathbf{s}, \mathbf{s}') = \beta u_c(d_{t+1} + d_{t+1}^*)/u_c(d_t + d_t^*)$; (iv) the law of motion $\Psi(\mathbf{s})$ is consistent with the aggregation of individual decisions and the stochastic process for ξ and ξ^*.*

The only difference with respect to the equilibrium in the closed economy is that there is the additional market for foreign claims and the discount factor for firms is given by the worldwide representative investor. The market clearing condition for the foreign claims is $N(\mathbf{s}') + N^*(\mathbf{s}') = 0$. This is in addition to the clearing conditions for the domestic bond markets (lending to firms).

Although the general definition of the recursive equilibrium is based on the set of state variables $\mathbf{s}_t = (\xi_t, \xi_t^*, B_t, B_t^*, N_t)$, we can use some of the properties derived above and characterize the equilibrium with a smaller set of states. Let $W_t = B_t + B_t^*$ be the worldwide wealth of households/workers. This is the sum of bonds issued by domestic firms, B_t, and foreign firms, B_t^*. Then using the fact that the consumption ratio of domestic and foreign workers is constant at χ and the employment policy of firms does not depend on the individual debt, the recursive equilibrium can be characterized using the state variables $\mathbf{s}_t = (\xi_t, \xi_t^*, W_t)$. The assumption of cross-country risk-sharing among workers and investors (but not between workers and investors) allows us to reduce the number of 'endogenous' states to only one variable.

Intuitively, by knowing W_t, we know the worldwide liability of firms, but not the distribution between domestic and foreign firms. However, to characterize the firms' policies, we only need to know the worldwide debt, which is equal to W_t. Since investors own an internationally diversified portfolio of shares, effectively there is only one representative global investor. It is as if there is a representative firm with two productive units: one unit located in the domestic country and the other in the foreign country. Since both units have a common owner, it does not matter how the debt is distributed between the two units. What matters from the perspective of the investor is the total debt and the total payment of dividends.[9]

Total workers' wealth is also a sufficient statistic for the characterization of the workers' policies since the consumption ratio between domestic and foreign households remains constant at χ. This property limits the computational complexity of the model, making feasible the use of non-linear approximation methods. We will come back to this point after the description of the general model with capital accumulation.

We are not ready to state the following proposition about the impact of a financial shock.

Proposition 3.1 *An unexpected change in ξ_t (domestic credit shock) has the same impact on employment and output of domestic and foreign countries.*

Proof 3.1 *We have already shown that the Lagrange multiplier μ_t is common for domestic and foreign firms. If the wage ratio in the two countries does not change, the first order conditions imply that all firms choose the same employment. To complete the proof we have to show that the cross-country wage ratio stays constant. Because firms in both countries have the same demand for labor and the ratio of workers' consumption remains constant, the first order condition for the supply of labor from workers implies that the wage ratio between the two countries does not change.*

Therefore, independently of whether a credit shock hits the domestic or foreign markets, both countries experience the same macroeconomic consequences. However, although exogenous credit shocks can explain co-movement in GDP and other real variables, there are two problems with this approach. The first is that treating shocks as exogenous does not help us

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[9]This is similar to the problem solved by a multinational firms that faces demand uncertainty in different countries as studied in Goldberg and Kolstad (1995). There is also some similarity with the problem faced by multinational banks that own subsidiaries in different countries. Cetorelli and Goldberg (2010) provide evidence that multinational banks do reallocate financial resources internally in response to country specific shocks.

understanding the causes of these shocks and the desirability of policy interventions. The second and more specific problem is that, even if a country-specific exogenous shock can generate co-movement in real macroeconomic variables, it also induces financial flows that tend to move in opposite directions. To show this, consider an initial equilibrium in which the enforcement constraints are not binding in either countries. Starting from this equilibrium suppose that the domestic economy is hit by a credit contraction (reduction in ξ_t but not in ξ_t^*) inducing binding enforcement constraints in both countries. Since ξ_t is lower only in the domestic country, the outstanding debt of domestic firms contracts but the debt of foreign firms increases. Foreign firms increase their debt to pay more dividends to shareholders in order to compensate the reduction in dividends paid by domestic firms. Therefore, the model with 'exogenous' credit shocks generates negative cross-country co-movement in debt.

This feature of the model is inconsistent with the data in figure 4 showing a high degree of cross-country co-movement also in credit variables. However, as we will see in the next section, once we make fluctuations in ξ_t and ξ_t^* endogenous, the model also generates a high degree of co-movement in financial flows, introducing a second source of real macroeconomic synchronization.

4   Endogenous credit shocks

After illustrating how a credit shock propagates to the real sector of the economy, we now provide a micro foundation for endogenous fluctuations in ξ_t. We proceed first with the closed-economy model and then we extend it to a two-country set-up.

Financial autarky: Suppose that in case of liquidation, physical capital $\bar{k}$ can be sold either to households or firms. In the first case one unit of capital is transformed in $\underline{\xi}$ units of consumption. Alternatively, the capital can be sold to other firms for productive uses. In this case one unit of capital can be transformed in $\overline{\xi}$ units of reinstalled capital. The reallocation in other firms is more efficient than its transformation in consumption goods, that is, $\underline{\xi} < \overline{\xi} \leq 1$. However, in order for non-defaulting firms to purchase additional capital, they need liquid funds. In this sense our model shares some features of the model studied in Kiyotaki and Moore (2008). Since all firms face the enforcement constraint

$$\xi_t \bar{k} \geq \frac{b_{t+1}}{R_t} + y_t, \tag{14}$$

a non-defaulting firm can purchase additional capital only if the firm has previously chosen not to borrow up to the limit, that is, constraint (14) is not binding. Therefore, if at the beginning of the period firms choose not to borrow up to the limit, ex-post there will be firms that have the ability to purchase the capital of a defaulting firm. In this case the market price of the liquidated capital is $\bar{\xi}$. On the other hand, if at the beginning of the period all firms choose to borrow up to the limit, ex-post there will not be any firm with liquidity. Then the capital of a defaulting firm can only be sold to households and the price is $\underline{\xi}$.

Since the value of liquidated capital depends on the financial choices of firms, which in turn depends on the expected liquidation value, the model could generate multiple self-fulfilling equilibria.

Suppose that the expected liquidation price is $\xi_t = \underline{\xi}$. The low price makes the enforcement constraint (14) tighter, which may induce firms to borrow up to the limit in order to contain the cut in dividends and/or employment. Then, if all firms borrow up to the limit, ex-post there will not be any firm that has liquidity to purchase the capital of a defaulting firm. Thus, the ex-post liquidation price is $\underline{\xi}$, fulfilling the market expectation.

Now suppose that the expected liquidation price is $\bar{\xi}$. Because the enforcement constraint (14) is not tight in the current period but could become tighter in the future, firms may choose not to borrow up to the limit. But then, in case of liquidation, there will be firms capable of purchasing the liquidated capital and the market price is $\bar{\xi}$. So also in this case we have that the ex-ante expectation of a high liquidation price is fulfilled by the firms' borrowing choice.

Whether the two equilibria described above are possible depends on the particular states of the economy. Three cases are possible:

1. The liquidation price is $\underline{\xi}$ with probability 1. This arises if we are in a state in which firms choose to borrow up to the limit independently of the expectation over ξ_t.

2. The liquidation price is $\bar{\xi}$ with probability 1. This arises if we are in a state in which firms do not borrow up to the limit independently of the expectation over ξ_t.

3. The liquidation price is $\underline{\xi}$ with some probability $p \in (0, 1)$. This arises if we are in a state in which firms choose to borrow up to the limit when the expectation for the liquidation value is $\xi_t = \underline{\xi}$ but they do not borrow up to the limit when the expectation for the liquidation price is $\xi_t = \bar{\xi}$.

The third case is the most interesting because it generates multiple sunspot equilibria, and therefore, potential fluctuations in ξ_t. In this case the low liquidation price $\underline{\xi}$ could arise with

any probability p. In general we can denote by $p_t(\mathbf{s}_t)$ the probability of $\xi_t = \underline{\xi}$. Besides the fact that the probability distribution of ξ_t could be time variant, the properties of the model characterized in previous sections do not change.

Financial integration: As in the closed economy, different values of ξ_t are associated to self-fulling expectations. If countries are in financial autarky, ξ_t could be different from ξ_t^*. We want to show now, however, that ξ_t cannot be different from ξ_t^* once the two countries become financially integrated.

As we have seen in the previous section, if the enforcement constraint is binding in one country, it must also be binding in the other country, that is, $\mu_t = \mu_t^* > 0$. This eliminates equilibria where $\xi_t = \underline{\xi}$ and $\xi_t^* = \bar{\xi}$. We state this property formally in the next proposition.

Proposition 4.1 *In equilibria with integrated financial markets, ξ_t is always equal to ξ_t^**

Proof 4.1 *Suppose that the equilibrium is characterized by $\xi_t = \underline{\xi}$ and $\xi_t^* = \bar{\xi}$. In order to have $\xi_t = \underline{\xi}$ we need that $\mu_t > 0$ and to have $\xi_t^* = \bar{\xi}$ we need that $\mu_t^* = 0$. But in an equilibrium with integrated financial markets μ_t is always equal to μ_t^*. Therefore, this cannot be an equilibrium. Using the same argument we can exclude the possibility of an equilibrium with $\xi_t = \bar{\xi}$ and $\xi_t^* = \underline{\xi}$. Thus, the only possible equilibria are those with $\xi_t = \xi_t^*$.*

Therefore, financial integration implies perfect cross-country co-movement in ξ_t, which introduces a second channel of real macroeconomic synchronization: not only a change in one country ξ affects the real sector of the other country but movements in ξ become perfectly correlated across countries. This also implies international co-movement in financial flows. This is a key theoretical result of this paper and suggests that the Lehman default could be interpreted as the trigger that switched the world economy from an equilibrium with globally loose credit to an equilibrium with tight credit and shortage of liquidity, causing widespread contraction in economic and financial activities.

Also in the case of financial integration the probability of $\xi_t = \underline{\xi}$ can be expressed as a function of the aggregate states, that is, $p(\mathbf{s}_t)$. Now, however, one of the two equilibria can be induced by changes in expectations in one of the two countries. For simplicity suppose that in states with multiple equilibria the domestic country expects $\xi_t = \underline{\xi}$ with probability $\bar{p}$. The same for the foreign country. Based on this assumption we have that $p(\mathbf{s}_t)$ can take three values, that is, $p(\mathbf{s}_t) \in \{0, 2\bar{p}(1-\bar{p}) + \bar{p}^2, \bar{p}^2, 1\}$. The probability is zero when firms choose not

to borrow up to the limit ($\mu_t = \mu_t^* = 0$) even if the expectation is $\xi_t = \xi_t^* = \underline{\xi}$. The probability is $2\bar{p}(1 - \bar{p}) + \bar{p}^2$ if firms choose to borrow up to the limit ($\mu_t = \mu_t^* > 0$) when either ξ_t or ξ_t^* are equal to $\underline{\xi}$. The probability is $\bar{p}^2$ if firms choose to borrow up to the limit ($\mu_t = \mu_t^* > 0$) only if both ξ_t and ξ_t^* are equal to $\underline{\xi}$. Finally, the probability is 1 if firms choose to borrow up to the limit ($\mu_t = \mu_t^* > 0$) independently of the values of ξ_t and ξ_t^*.

The general definition of equilibrium is analogous to the definition provided for the model with exogenous ξ_t. We simply need to add the probability function $p(\mathbf{s}_t)$ which must be consistent with the optimal decisions of firms as described here.

5 Model with capital accumulation

We now relax the assumption that the input of capital is fixed. This introduces additional state variables that increase the computational complexity of the model. Since the enforcement constraint is only occasionally binding, we need to use global approximation techniques. Unfortunately, these techniques are computationally intensive and become quickly impractical when we have a large numbers of state variables. Therefore, in order to reduce the sufficient set of state variables, we will make some special assumptions about the production technology.

Investors-firms: The production function takes the form

$$y_t = (K_t + K_t^*)^{1-\theta} k_t^\theta h_t^\nu,$$

where K_t is the 'aggregate' capital in the domestic country and K_t^* in the foreign country, k_t is the 'individual' input of capital and h_t is the 'individual' input of labor. We assume that $\theta + \nu < 1$.

The dependence of the production function from the worldwide stock of capital, $K_t + K_t^*$, captures positive externalities. The purpose of the externalities is to have constant returns in reproducible factors (AK technology), without loosing the competitive structure of the model, that is, each producer runs a production technology with non-increasing returns. As we will see, the AK structure simplifies the computation of the equilibrium and this is the only reason we make this assumption.

Given i_t the flow of investment, the stock of capital evolves according to

$$k_{t+1} = (1 - \tau)k_t + \Upsilon\left(\frac{i_t}{k_t}\right) k_t,$$

where τ is the depreciation rate and the function $\Upsilon(.)$ is strictly increasing and concave, capturing adjustment costs in investment. The assumption of capital adjustment costs is common in international macro models and it is made to prevent excessive volatility of investments.

The budget constraint of the firm is

$$b_t + d_t + i_t = (K_t + K_t^*)^{1-\theta} k_t^\theta h_t^\nu - w_t h_t + \frac{b_{t+1}}{R_t},$$

and the enforcement constraint

$$\xi_t k_{t+1} \geq (K_t + K_t^*)^{1-\theta} k_t^\theta h_t^\nu + \frac{b_{t+1}}{R_t}.$$

We will now take advantage of the AK structure and normalize the model by the worldwide stock of capital $K_t + K_t^*$. Using the tilde sign to denote normalized variables, we can rewrite the budget constraint, law of motion for capital and enforcement constraint as

$$\tilde{b}_t + \tilde{d}_t + \tilde{i}_t = F(\tilde{k}_t, h_t) - \tilde{w}_t h_t + \frac{g_t \tilde{b}_{t+1}}{R_t}, \tag{15}$$

$$g_t \tilde{k}_{t+1} = (1 - \tau)\tilde{k}_t + \Upsilon\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right)\tilde{k}_t, \tag{16}$$

$$\xi_t g_t \tilde{k}_{t+1} \geq F(\tilde{k}_t, h_t) + \frac{g_t \tilde{b}_{t+1}}{R_t}. \tag{17}$$

We have denoted by $F(\tilde{k}_t, h_t) = \tilde{k}_t^\theta h_t^\nu$ the normalized production function with $\tilde{k}_t = k_t/(K_t + K_t^*)$ the normalized individual capital. The variable $g_t = (K_{t+1} + K_{t+1}^*)/(K_t + K_t^*)$ is the gross growth rate of worldwide capital. We will denote by $s_t = K_t/(K_t + K_t^*)$ the aggregate share of capital owned by domestic firms. Since in equilibrium $k_t = K_t$, we also have that $\tilde{k}_t = s_t$.

As in the model without capital accumulation, investors hold an internationally diversified portfolio of shares, and firms use the common discount factor $m_{t+1} = \beta[(d_{t+1} + d_{t+1}^*)/(d_t + d_t^*)]^{-\sigma}$. In terms of normalized variables, the discount factor can be rewritten as

$$m_{t+1} = g_t^{-\sigma}\beta\left(\frac{\tilde{d}_{t+1} + \tilde{d}_{t+1}^*}{\tilde{d}_t + \tilde{d}_t^*}\right)^{-\sigma} = g_t^{-\sigma}\tilde{m}_{t+1}.$$

The optimization problem solved by an individual firm can be rewritten as

$$\tilde{V}(\tilde{\mathbf{s}}; \tilde{k}, \tilde{b}) = \max_{\tilde{d}, \tilde{h}, \tilde{i}, \tilde{b}'} \left\{ \tilde{d} + g^{1-\sigma} E\tilde{m}' \tilde{V}(\tilde{\mathbf{s}}'; \tilde{k}', \tilde{b}') \right\} \tag{18}$$

subject to (15), (16), (17),

where $\tilde{V}$ is the firm's value normalized by aggregate worldwide capital $K + K^*$, and $\tilde{\mathbf{s}}$ denotes the normalized aggregate states as specified below.

We can now see the analytical convenience of having the capital externality. Thanks to the AK structure, we can write the firm's value function as $V_t = (K_t + K_t^*) \cdot \tilde{V}_t$ and rescale the problem of the firm by worldwide capital. By doing so, we do not need to keep track of the aggregate stock of capital as a state variable. Of course, because we are looking at a general equilibrium, we also need to make sure that the supply of labor does not grow over time. This will be the case with the workers' utility function specified earlier.

Appendix C derives the first order conditions for the firm. After imposing the equilibrium conditions $k_t = K_t$ and $\tilde{k}_t = s_t$, the first order conditions can be written as

$$F_h(s_t, h_t) = \frac{\tilde{w}_t}{1 - \mu_t}, \tag{19}$$

$$g_t^{-\sigma} R_t E\tilde{m}_{t+1} = 1 - \mu_t, \tag{20}$$

$$Q_t \Upsilon'\left(\tilde{i}_t\right) = 1, \tag{21}$$

$$Q_t = \xi_t \mu_t + \bar{g}_t^{-\sigma} E\tilde{m}_{t+1} \Bigg\{ (1 - \mu_{t+1}) F_k(s_{t+1}, h_{t+1}) - \tilde{i}_{t+1}$$
$$+ \left[1 - \tau + \Upsilon\left(\tilde{i}_{t+1}\right) \right] Q_{t+1} \Bigg\}. \tag{22}$$

Here μ_t is the Lagrange multiplier associated with the enforcement constraint and Q_t is the Lagrange multiplier associated with the law of motion for the stock of capital (Tobin's q). We can verify that capital does not enter these equations, which validates the conjecture that the optimal policies are independent of the stock of capital.

Notice that the property that the Lagrange multipliers and the labor wedge $1/(1 - \mu_t)$ are

equalized across countries also applies to this extended model. In fact, from condition (20) we can see that the common discount factor and the equalization of the interest rates across countries imply $\mu_t = \mu_t^*$. Therefore, if the enforcement constraint is binding in one country, it must also be binding in the other.

Aggregate states and equilibrium: Denote by $\tilde{W}_t = \tilde{B}_t s_t + \tilde{B}_t^*(1 - s_t)$ the normalized worldwide wealth of households/workers. Thanks to the AK technology and the normalization described above, we only need to keep track of two 'endogenous' state variables: $\tilde{W}_t$ and s_t. Therefore, compared to the simpler model considered earlier, the introduction of capital accumulation adds only one state variable, that is, the share of worldwide capital owned by domestic firms, s_t.[10] By having only two endogenous state variables, it becomes manageable to solve the model numerically using global approximation methods. Appendix D reports the list of equilibrium conditions and describes the computational procedure.

6 Quantitative analysis

This section studies the properties of the model quantitatively. We think of country 1 as the US and country 2 as representative of the other countries in the group of the seven largest industrialized economies, that is, Canada, Japan, France, Germany, Italy and the United Kingdom. We refer to this group as G6 countries.

The discount factor for workers, δ, and the discount factor for investors, β, are set to target an average interest rate of 1.6 percent and an average return on equity of 7 percent. In the deterministic steady state the interest rate is equal to $1/\delta - 1$ and the return on equity is equal to $1/\beta - 1$. In the stochastic economy the relations between the intertemporal discount factors and the average returns are more complex. Therefore, to choose δ and β we follow an iterative procedure where we fix the parameters, solve the model, and check whether the average returns match the targets. The required values are $\delta = 0.996$ and $\beta = 0.984$. Therefore, there is a 1 percent difference between the two discount factors. This is smaller than the equity premium, $5.4\%/4 = 1.35\%$. The difference is the 'risk' premium.

The utility function takes the form $U(c, h) = \ln(c) - \alpha h^{1+1/\eta}/(1 + 1/\eta)$ where η is the Frisch elasticity of labor supply. We set the elasticity to 0.75 which is between the micro and macro estimates. The parameter α is set so that working hours are 0.3 on average.

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[10]This additional state is necessary because of the adjustment cost in investment. In absence of adjustment costs, we could also ignore s_t.

Next we parameterize the production function. The parameter ν is chosen to have a steady state labor income share of 0.7. Without uncertainty, the fraction of output going to workers in the form of wages is $\nu\beta/\delta$.[11] Given the values of δ and β, we choose ν so that this fraction is equal to 0.7. Of course, in the stochastic economy the average labor share is not exactly 0.7 but the difference is small. Next we set the return to scale for an individual firm to $\theta + \nu = 0.9$. Given the value of ν we derive the value of $\theta = 0.9 - \nu$.

The stock of capital evolves according to $k' = (1 - \tau)k + \Upsilon(i/k)k$ with

$$\Upsilon\left(\frac{i}{k}\right) = \frac{\phi_1}{1 - \zeta}\left(\frac{i}{k}\right)^{1-\zeta} + \phi_2.$$

This functional form is widely used in the literature (see, for example, Jermann (1998)). The parameters ϕ_1 and ϕ_2 are chosen so that in the deterministic steady state $Q = 1$ and $I = \tau K$. This requires $\phi_1 = \tau^\zeta$ and $\phi_2 = -\zeta\tau/(1 - \zeta)$. Therefore, we need to calibrate two parameters, τ and ζ. The first is the depreciation rate which we set to $\tau = 0.02$. The second determines the sensitivity of the adjustment cost and we set it to $\zeta = 0.5$.

At this point we are left with the calibration of the shock process. The variable ξ takes only two values. In addition to the choice of these two values we have to pin down $\bar{p}$, that is, the probability with which each country forms pessimistic expectations ($\xi = \underline{\xi}$) in states with multiple equilibria. We choose $\underline{\xi}$, $\bar{\xi}$ and $\bar{p}$ to match three targets: (i) the average leverage (debt over capital) which we set to 0.5; (ii) the standard deviation of debt over output; (iii) the frequencies of crisis, which we set to about 4%.[12] The full list of parameters are reported in Table 1.

Appendix D describes the computational procedure which is based on the discretization of the state space. The endogenous states $\tilde{b}_t$ and s_t are each discretized on a grid with eleven points. Values outside the grids are determined through bi-linear interpolation.

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[11]From the first order condition of labor, equation (6), we derive $wh/F(z, k, h) = \nu(1 - \mu)$, which provides an expression for the labor share. We now use condition (7) to derive an expression for μ. Taking into account that in a deterministic steady state $m' = \beta$ and $R = 1/\delta$, this condition becomes $\beta/\delta = 1 - \mu$. Substituting in the labor share $\nu(1 - \mu)$, we get the expression reported in the main text.

[12]Although the three parameters are chosen jointly, we can identify the primary parameter that affects each of the three targets. The average leverage is mostly determined by the average ξ. The standard deviation of debt is mostly determined by the difference between $\bar{\xi}$ and $\underline{\xi}$. The frequency of crisis is mostly determined by $\bar{p}$.

Table 1: List of parameters

Discount factor for households/workers, δ	0.996
Discount factor for entrepreneurs, β	0.986
Utility parameter, α	16.293
Production technology, θ	0.200
Production technology, ν	0.700
Depreciation rate, τ	0.020
Capital adjustment cost, ζ	0.050
Low liquidation value, $\underline{\xi}$	0.550
High liquidation value, $\bar{\bar{\xi}}$	0.650
Frequency of low liquidation value, $\bar{p}$	0.200

6.1 Results

Our first result follows simply by noticing that proposition 4.1 extends to this more general environment so that with endogenous credit disturbances credit markets conditions (changes in ξ_t and ξ_t^*) are perfectly correlated. Since countries are symmetric, with credit shocks alone all variables (real and financial) are perfectly correlated across countries. Hence a large credit shock can generate very strong co-movements in real and financial variables like the ones documented in Section 2.

In presenting additional results we outline four main properties: (i) the asymmetric response to shocks; (ii) the counter-cyclicality of labor productivity in response to credit shocks; (iii) the severity of crisis that arrive after long periods of credit and macroeconomic booms; (iv) the importance of credit shocks for the volatility of labor and asset prices.

Asymmetry: Figure 6 plots the impulse responses to a credit expansion and a credit contraction. We report only the responses for one country since for the other countries they are equal. A credit expansion is generated starting from the limiting equilibrium in which the economy converges after a long series of draws $\xi_t = \underline{\xi}$. From this equilibrium we consider a sequence of draws $\xi_t = \bar{\xi}$ starting at $t = 1$. Therefore, a credit expansion is generated by a permanent switch from $\underline{\xi}$ to $\bar{\xi}$. Similarly, the impulse responses to a credit contraction are generated starting from the limiting equilibrium in which the economy converges after a long series of draws $\xi_t = \bar{\xi}$. Starting at $t = 1$ the economy experiences a sequence of draws $\xi_t = \underline{\xi}$.

Two remarks are in order. First, the impulse responses take place in a range of states that admit multiple equilibria. Therefore, the draws of ξ_t are possible equilibrium outcomes. Second, agents do not know in advance the actual draws of ξ_t. Therefore, they take into

account the uncertainty induced by the stochastic distribution of ξ_t.
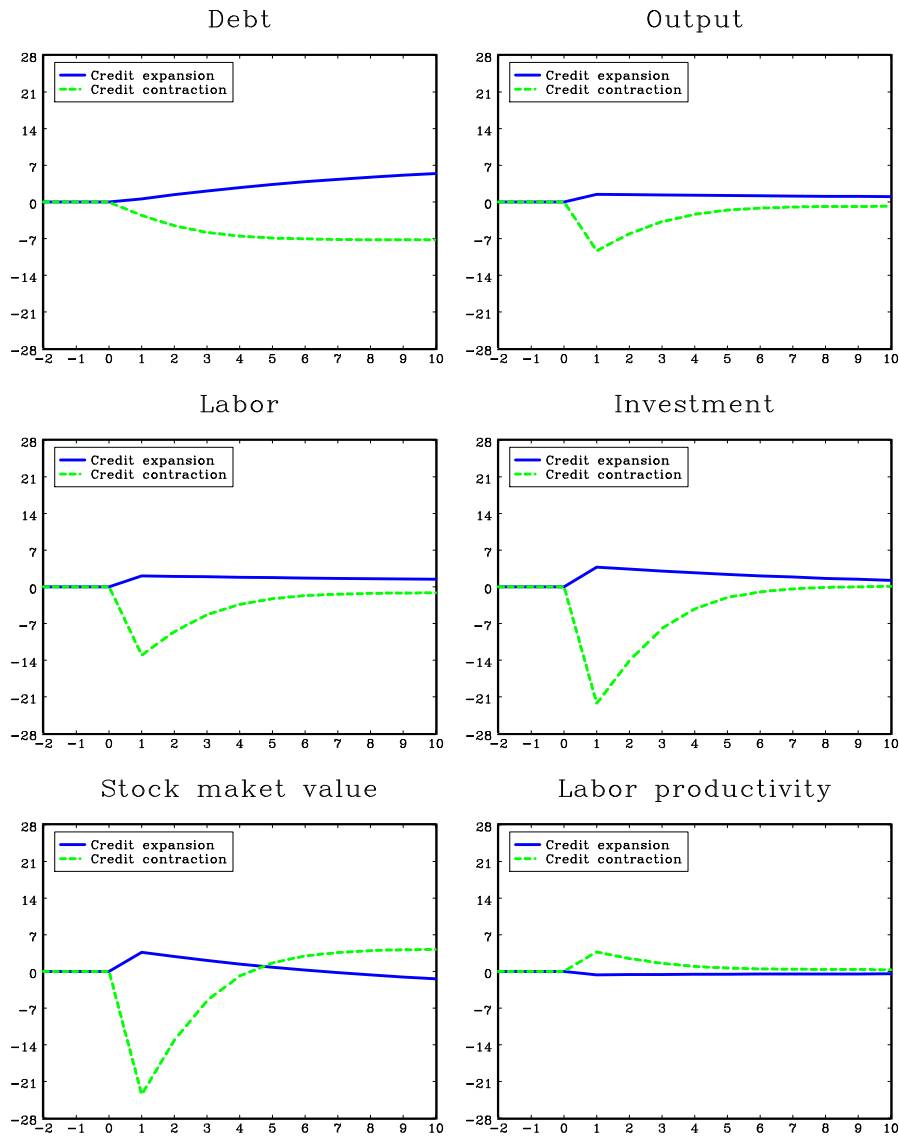
Figure 6: Impulse responses to credit expansions and contractions.

In response to the credit expansion we see a gradual increase in the stock of debt and a persistent expansion in labor and output. The magnitude of the macroeconomic expansion, however, is not large at impact. The macroeconomic expansion induced by the credit boom arises through the following mechanism. At impact the firm becomes unconstrained which eliminates the labor wedge. In addition to that and after the initial period, there is a second

mechanism. As firms take on more debt, they pay more dividends, increasing the discount factor m'. Thanks to the lower discounting, firms invest more. At the same time, the higher borrowing from firms increases the equilibrium interest rate which in turn increases labor supply and output. The response to a credit contraction displays a different pattern. The stock of debt declines more quickly and the response of labor, output and investment are much larger at impact. Therefore, the model generates a strong asymmetry in the responses to credit expansions and contractions.

The intuition for the asymmetry is best understood starting from a situation in which the enforcement constraint is not binding. If the constraint gets relaxed, the Lagrange multiplier cannot fall below zero and the expansionary effect on unemployment is mild (only through the general equilibrium discussed above). Instead, if the constraints get tighter, the Lagrange multiplier goes from 0 to being positive and that causes a fall in employment and output (through equation 19). As we discussed in section 2, this asymmetry is consistent with the macroeconomic dynamics observed during the period 2005-2010.

Countercyclical labor productivity: The last panel of Figure 6 plots the impulse responses of labor productivity, that is, the ratio between output and hours. As in the previous figure we see an asymmetry between credit expansions and credit contractions. More importantly, a credit expansion causes a decline in labor productivity while a credit contraction generates an increase in labor productivity. This is important for capturing the counter-cyclical dynamics of the US labor productivity during the recent crisis as documented in section 2.

Credit booms and severity of recessions: Figure 7 plots the impulse responses to credit expansions that later revert back to the pre-expansion levels. A credit boom is generated as described above. Starting from an equilibrium to which the economy converges after a long series of $\xi_t = \underline{\xi}$, we assume that at time 1 the economy experiences a switch to $\xi_t = \overline{\xi}$ (credit expansion). The value of ξ stays at the higher value for several periods and then it reverts back to $\underline{\xi}$ permanently. We consider credit booms with duration of 4 quarters (left panels) and 20 quarters (right panels).

The key finding is that the macroeconomic impact of the credit contraction increases with the duration of the credit expansion. After a protracted credit boom, the economy accumulates large leverages. When the credit reversal arrives, the required de-leveraging is more severe. This forces firms to implement larger hiring cuts and generates a stronger macroeconomic
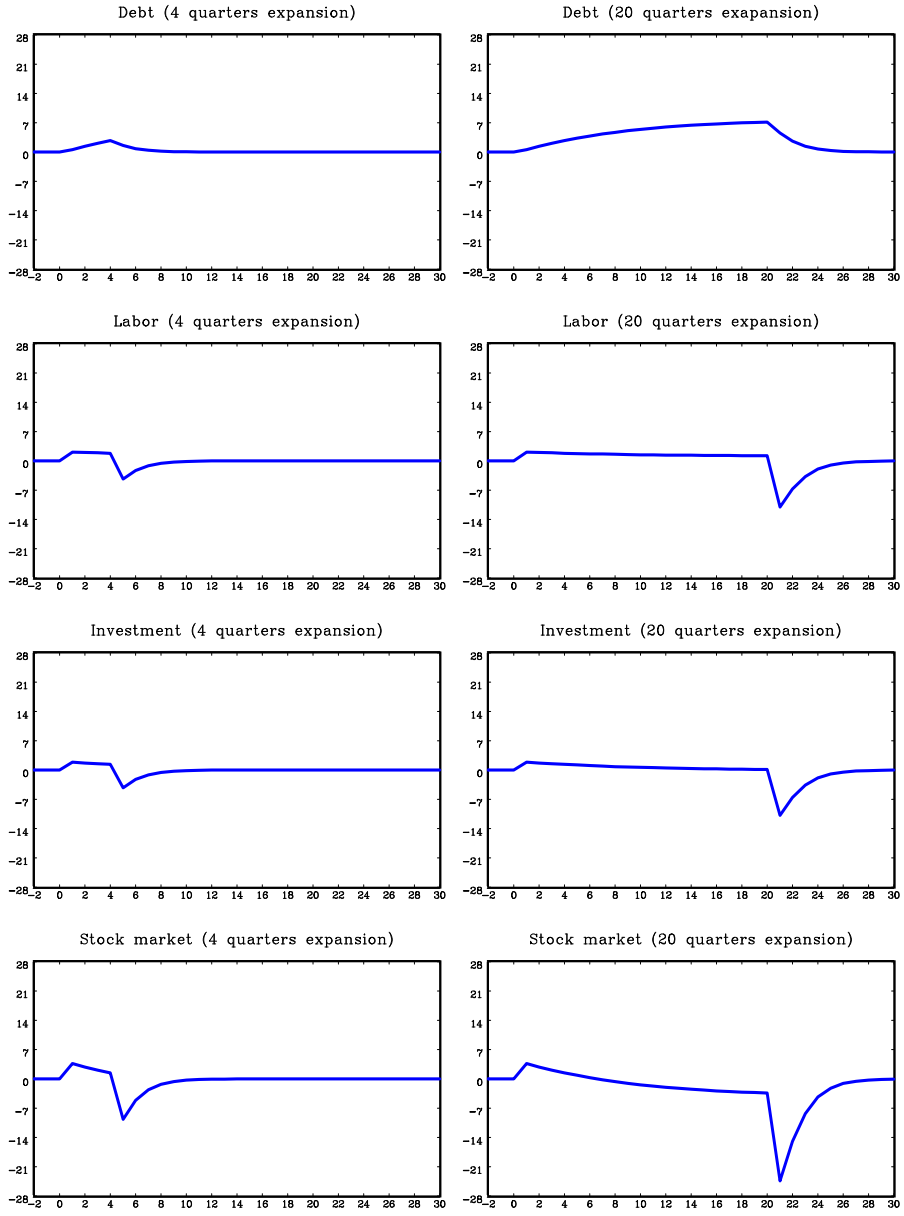
Figure 7: Duration of credit expansions and severity of contractions.

contraction. In this way the model captures why recessions that arise after long periods of financial and macroeconomic expansions tend to be associated with more severe contractions.

Volatility of labor and asset prices: The first column of Table 2 reports the standard deviations of various variables. The statistics are computed after detrending the simulated series with a band-pass filter that preserves cycles of 1.5-8 years (Baxter and King (1999)).

Table 2: Business cycle statistics of key variables from detrended simulated series.

	Productivity shocks only	Credit shocks only	Both shocks
Standard deviations			
Output	0.88	0.76	1.16
Consumption	0.68	0.44	0.77
Labor	1.26	0.26	1.26
Investment	2.27	0.77	2.36
Tobin's q	1.14	0.38	1.18
Stock market value	2.46	0.54	2.45
Interest rate	0.48	0.25	0.48
Return on equity	5.82	0.37	5.82
Expected returns (% annualized)			
Interest rate	1.40	1.56	1.40
Return on equity	6.96	5.62	6.96
Equity risk premium	1.56	0.06	1.56
Nonbinding constraints, %	96.44	99.99	96.04

Notes: The standard deviations for the returns on stocks and bonds are calculated on unfiltered data.

Two properties are especially noticeable. First, the model can generate high volatility of labor, bringing the model closer to the US data for the crisis where employment fell even more than output (see figure 5). The reason is simply that credit shocks cause, through the Lagrange multiplier on the enforcement constraints, autonomous movements in employment that, due to decreasing returns, drive smaller movements in output. Second, credit shocks also generate a high volatility of asset prices. In particular, the stock market value (equity value of firms) is almost three times more volatile than output. This can also be seen in the bottom panel of Figure 6 which plots the impulse responses of the market value of equity to a credit expansion and contraction. The reason for the high asset prices volatility is mainly that credit shocks can sharply change the stochastic discount factor of investors (see equation 19) who hold shares

of firms. Hence, large fluctuations in stock prices emerge in equilibrium. This suggests that credit shocks can contribute to explaining at least part of the large volatility of stock prices we have observed during the crisis (see figure 4).

As a result of the higher volatility of asset prices and of the discount factor of investors, the model can also generate a non-negligible equity risk-premium.[13] This is about 1.56 percent yearly. We also observe that the volatility of equity returns is quite high in the model but the volatility of the interest rate is small.

Productivity shock: Before closing this section we would also like to report how the model performs if instead of credit shocks we consider standard productivity shocks. Since a large body of literature in international macroeconomics has developed from the International Real Business Cycle, this seems a natural exercise.

To add productivity shocks we specify the production function as $y_t = z_t(K_t + K_t^*)^{1-\theta}k_t^\theta h_t^\nu$, where z_t denotes the stochastic level of productivity. The variable z_t is country-specific and follows a first order Markov process. After constructing Solow residuals series for the US and for the aggregate of the G6 countries, we estimate the dynamic system

$$\begin{pmatrix} \log(z_{t+1}^{US}) \\ \log(z_{t+1}^{G6}) \end{pmatrix} = \begin{bmatrix} \rho_z & \psi_z \\ \psi_z & \rho_z \end{bmatrix} \begin{pmatrix} \log(z_t^{US}) \\ \log(z_t^{G6}) \end{pmatrix} + \begin{pmatrix} \epsilon_{t+1}^{US} \\ \epsilon_{t+1}^{G6} \end{pmatrix}.$$

The log series are linearly detrended and ϵ_{t+1}^{US} and ϵ_{t+1}^{G6} are mean zero white noises with standard deviation σ_z^{US} and σ_z^{G6} respectively. The estimation returns $\rho_z = 0.98$, $\psi_z = -0.008$, $\sigma_z^{US} = 0.0059$, $\sigma_z^{G6} = 0.0065$. The correlation between residuals is 0.15.

The estimation shows that there is very low cross-country comovement between the Solow residuals of the two countries and they have very similar standard deviations. Therefore, the process for the productivity variables can be well approximated by symmetric and independent first order autoregressive processes with autocorrelation parameter $\rho = 0.98$ and standard deviation of residuals $\sigma = 0.0062$.

The second column of Table 2 reports the business cycle properties with only productivity

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[13]We should be careful in defining the equity risk-premium. Since bond holders (workers) have a higher discounting than equity holders (investors), the difference between the expected return on equity (for investors) and on bonds (for workers) is not the risk premium. In fact, even in absence of risk, the return on equity will be higher than the return on bonds. Given the calibration of $\delta = 0.996$ and $\beta = 0.986$, the return differential in absence of risk would be about 4 percent yearly. Given this, we define the equity risk-premium as the difference between the return differential between equity and bonds and the difference in discount rates between investors and workers.

shocks. As we can see, the model generates much lower volatilities of hours and asset prices. It is also worth nothing that the enforcement constraint is basically never binding. Because of this, the labor wedge is (almost) always zero, which explains why labor and asset prices are not very volatile. However, productivity shocks generate larger movements in investment. When we consider both shocks (third column), the model generates reasonable business cycle statistics in all macroeconomic variables.

7   Global financial crisis with heterogeneous labor markets

In Section 2 we pointed out that the dynamics of employment during the recent crisis is different between US and other G7 countries, a fact first shown by Ohanian (2010). This point is also illustrated using the 'labor wedge', that is, the difference between the marginal rate of substitution in consumption and leisure and the marginal product of labor. Formally, this is defined as $U_h(c_t, h_t)/U_c(c_t, h_t) - F_h(k_t, h_t)$, where U_h and U_c are the marginal utilities of leisure and consumption respectively and F_h is the marginal product of labor. With CES utility and Cobb-Douglas production function, the labor wedge is equal to

$$Wedge = \frac{\phi c_t}{1 - h_t} - (1 - \theta)\frac{y_t}{h_t}. \tag{23}$$

Using this formula, Ohanian and Raffo (2011) find that while in the US the labor wedge dropped dramatically during the recent crisis, the average wedge in other G7 counties experienced a modest drop. In few countries like Germany it even increased. The goal of this section is to show that the different behavior of the labor market can be reconciled with the view of a global financial crisis when the structure of the labor market differs across countries.

In order to show this point we extend our model by adding two elements: variable labor utilization and heterogeneous labor rigidities. The role of variable labor utilization is to allow for a more powerful mechanism for endogenous fluctuations in measured labor productivity. The role of labor rigidities is to allow for a different response of labor utilization and measured labor input to shocks. By further assuming that labor rigidities differ across countries, the model can generate heterogeneous responses of macroeconomic and labor market variables. Ohanian and Raffo (2011) refer to indicators from the OECD Employment Outlook (2008) and report that the US is the country with the most flexible labor market. On the other hand, many of the countries in continental Europe and Japan are placed at the opposite end in the scale of labor market flexibility.

Let's start with labor utilization. The production function is specified as $F(K_t, k_t, n_t)$, where n_t is the 'effective' input of labor. This results from the combination of (measured) hours, h_t, and (unmeasured) utilization, e_t, according to the function

$$n_t = \left[h_t^{\frac{\varrho-1}{\varrho}} + e_t^{\frac{\varrho-1}{\varrho}} \right]^{\frac{\varrho}{\varrho-1}}.$$

The parameter ϱ is the elasticity of substitution between hours spent in the working place and the actual utilization of labor. When $\varrho = 1$ we have $n_t = h_t \cdot e_t$, which is used often in the literature. The cost of utilization comes from workers disutility. Given the utility function $U(c_t, h_t + e_t)$, workers face higher disutility not only when they spent more hours in the working place but also when their services are utilized more intensively. An implication of this specification is that the utilization cost is equal to the wage rate w_t, and the total cost of labor for the firm is $(h_t + e_t)w_t$.

So far the addition of labor utilization is inconsequential for the properties of the model. Given the CES aggregation and the fact that the wage rate is the price for both h_t and u_t, firms always choose $e_t = h_t$. Thus, we can simply focus on h_t as in the original model and abstract from utilization. This equivalence no longer holds once we add labor market rigidities on working hours h_t. Some authors interpret labor market rigidities as constraining the extensive margin (employment) rather than the intensive margin (per-worker hours). However, since the model does not distinguish the extensive from the intensive margin, we interpret labor market rigidities as restricting total hours h_t. More specifically, firms incur the convex cost

$$\kappa(h_t - \bar{h})^2 w_t,$$

where $\bar{h}$ is exogenous.

Ideally, we would like to use a more standard adjustment cost. For example something like $\kappa(h_t - h_{t-1})^2 w_t$. This alternative formulation, however, would introduce an additional state variable, h_{t-1}, which increases the computational complexity of the model. To avoid this, we specify the cost as deviation from a fixed target. The multiplication by the wage rate is motivated by economic and technical considerations. From an economic point of view it is likely that the direct cost of labor, which depends on the wage, also affects the cost of changing employment. An example is severance payments. From a technical point of view the presence of the wage allows us to apply the same normalization procedure used in the version of the

model with capital accumulation.

The key parameter is κ. With a positive value of κ, the response of utilization e_t to shocks is bigger than the response of total hours h_t. This generates a decline in measured TFP and, potentially, a decline in measured labor productivity y_t/h_t. These effects increase with the value of κ (labor market rigidity). Therefore, if in our model the first country (the US) is characterized by lower labor market rigidities than the second country (the other G7), the model could generate very different responses of labor markets to a financial shock. This will also be reflected in the responses of the labor wedge.

7.1   Simulation results

We describe here only the calibration of the parameters that need to be re-calibrated or were not present in the baseline model. We start with the elasticity of substitution between hours and utilization, the parameter ϱ, which we set to 5. This value implies a high degree of substitutability between hours and utilization. The utility parameter α is chosen to have average working hours of 0.33 in the equilibrium without labor rigidities.

At this point we are left with the parameters $\bar{h}$, κ^1 for country 1 and κ^2 for country 2. Given the values of κ^1 and κ^2, we could choose $\bar{h}$ to have the desired differential in average employment between the two countries. We choose total hours in the US to be 5 percent higher than in other G7 countries. However, this is not important for the quantitative properties of the model. The important parameters are κ^1 and κ^2. Unfortunately we are not aware of statistics that can be used directly to pin down these two parameters. Because of this we take a more pragmatic approach. We pick the values of $\kappa^1 = 0.3$ and $\kappa^2 = 1.5$ so that the model generates heterogeneous drops in labor wedges after a negative credit shock similar to the drops observed during the recent crisis. Of course, the relevance of the exercise is only to show that the model 'could' generate the heterogeneous responses of the labor market observed in the US and in the G6 countries. The exercise is helpful in clarifying that the idea of a global financial crisis as a driver of the recent recession cannot be written down by the observation of cross country heterogeneity in labor market dynamics.

Figure 8 plots the impulse responses of several variables to a permanent credit contraction. The impulse responses are constructed using the same methodology as in Figure 6. As can be seen from the figure, the responses of investment and output are very similar in the two countries. However, the responses of hours and the labor wedge are significantly smaller in

country 2.[14] We also observe strong heterogeneity in the response of labor productivity which falls only slightly in country 1 but experiences a large drop in country 2. Therefore, the model could replicate the different dynamics of the labor market between the US and other G6 countries even if the dynamics of other macroeconomic variables are similar.

Before closing, we would like to make some remarks about the concept of labor rigidities. These are typically interpreted as the consequence of institutional factors such as regulations and union power. Here, instead, we would like to give a broader interpretation. For example, it is well known that labor market rigidities are different across sectors. To the extent that in certain countries the crisis has impacted sectors with greater labor market flexibility, we may observe larger declines in employment and hours. For instance, the construction sector is typically characterized by flexible hiring because of its cyclicality. Then, countries that experience large contractions in the real estate sector are also likely to experience large drops in employment. This is the case for Spain, a country where the real estate sector experienced an abnormal boom before the crisis. In this sense, a country like Spain could be considered a country with a flexible labor market, simply because the sectors with greater labor market flexibility were hit harder by the crisis.

8 Conclusion

We have documented that the recent financial crisis has been characterized by an historically high degree of international synchronization in real and financial variables. We have proposed a theoretical framework in which endogenous credit booms and credit crises can result from self-fulfilling expectations. These episodes affect the real sector of the economy through a credit channel: booms enhance the borrowing capacity of firms and in the general equilibrium they lead to (mildly) higher employment and production. Crises curtail borrowing capacity and they lead to sharp contractions in real activity and financial prices.

When countries are financially integrated, self-fulfilling credit booms/crises also generate large spillovers to the real and financial sectors of other countries. There are two channels

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[14]In our model, the labor wedge is slightly different from equation (23) because the production function is not constant returns. Furthermore, there is labor utilization and c_t is only the consumption of workers, not aggregate consumption. However, we measure the labor wedge as if the true model was the standard RBC since this is the way it has been measured in the literature. After simulating the model and generating the series for c_t, h_t and y_t, we compute the wedge by plugging the series in equation (23). The values of the parameters are the same values used in Ohanian and Raffo (2011), that is, $\beta = 0.99$, $\theta = 0.36$, $\delta = 0.0175$, $g = 0.005$ and ϕ is chosen so to have steady hours of 0.33.

Hours worked

Labor productivity

Labor wedge

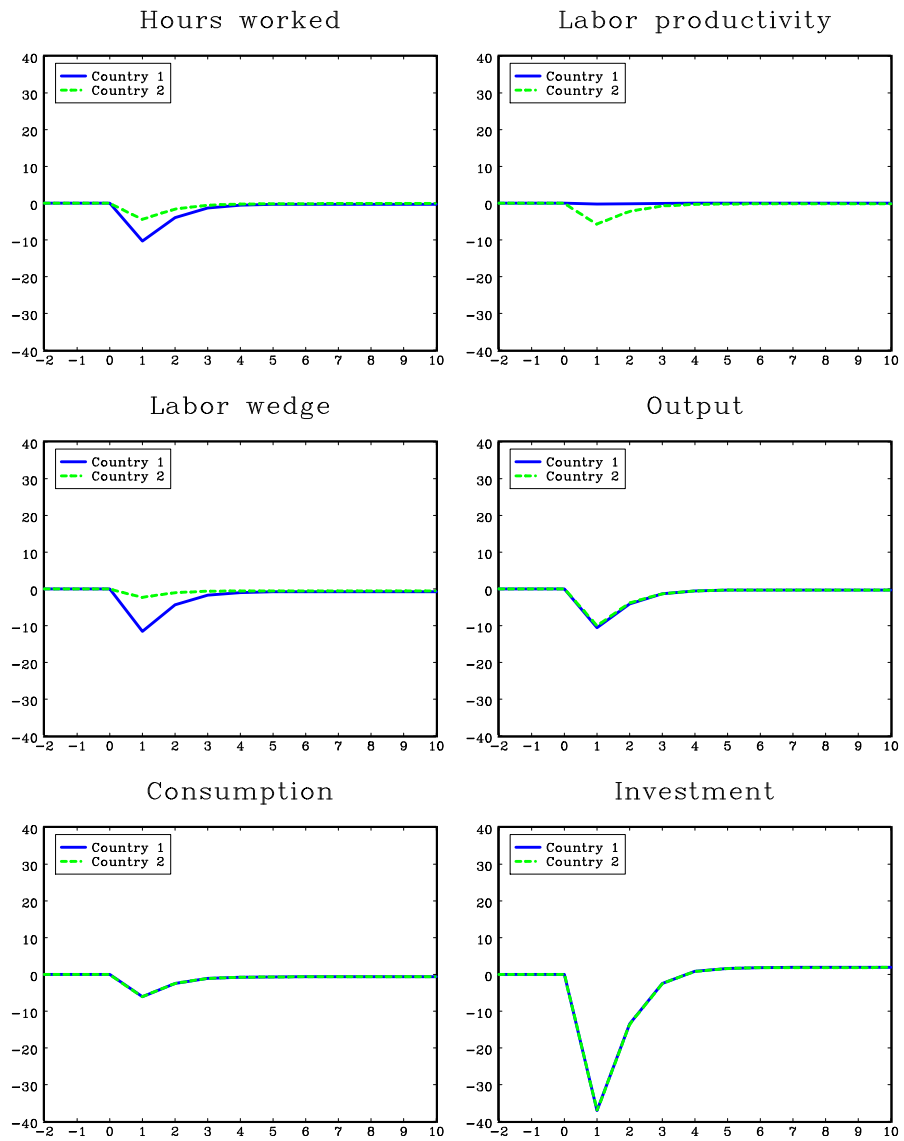## Output

Consumption

Investment

Figure 8: Impulse responses to a credit contraction with asymmetric labor markets.

of international transmission. The first is through the cost of capital which in an integrated financial market is equalized across countries. The second channel is based on the endogenous nature of credit market conditions. These conditions change when the economy switches from one self-fulfilling equilibrium to another self-fulfilling equilibrium. But in an integrated world market the shift in one country can only arise if the shift takes place also in the other. Therefore, changing financial market conditions are highly synchronized when financial markets are internationally integrated.

This study does not exclude the possibility that other sources of business cycle fluctuations also generate international co-movement in real variables. Our interest in changing credit market conditions as a source of business cycle is motivated by their ability to generate large cross-country co-movements in the real sector of the economy together with the large international co-movements in the flows of financing and asset prices.

Appendix

A   Debt renegotiation

The enforcement constraint is derived from the following assumptions. Default arises at the end of the period before repaying the intra-temporal loan l_t. In case of default the lender can confiscate the firm and sell the physical capital at price ξ_t but it cannot confiscate the liquidity $l_t = F(h_t)$.

Define first the value of the firm recursively as

$$V_t(b_t) = d_t + E_t m_{t+1} V_{t+1}(b_{t+1}),$$

where m_{t+1} is the discount factor, taken as given by an individual firm. Since default takes place at the end of the period, after the dividends, the value of not defaulting is $E_t m_{t+1} V_{t+1}(b_{t+1})$. We now derive the value of defaulting.

In the event of default the parties negotiate a repayment τ_t to the lender. If they reach an agrement, the firm continues operation and its value is $E_t m_{t+1} V_{t+1}(b_{t+1}) + l_t - \tau_t$. What this says is that the firm retains its continuation value $E_t m_{t+1} V_{t+1}(b_{t+1})$ plus the liquidity net of the bargained payment τ_t. Without an agrement the firm retains only the divertible liquidity l_t (threat value). The net value of an agrement is the difference between the value of renegotiation and the threat value, that is

$$E_t m_{t+1} V_{t+1}(b_{t+1}) - \tau_t. \tag{24}$$

Let's consider now the lender. With an agrement the lender gets $\tau_t + b_{t+1}/R_t$. The intertemporal debt is discounted since it will be repaid next period. Without an agrement the lender receives the liquidation value of physical capital, $\xi_t \bar{k}$ (threat value). Thus, the net value of renegotiation is

$$\tau_t + \frac{b_{t+1}}{R_t} - \xi_t \bar{k}. \tag{25}$$

The net surplus is the sum of the net values for the firm, (24), and the lender, (25), that is

$$S_t(b_{t+1}) = E_t m_{t+1} V_{t+1}(b_{t+1}) + \frac{b_{t+1}}{R_t} - \xi_t \bar{k} \tag{26}$$

Under the assumption that the firm has all the bargaining power, the value of defaulting is $l_t + S_t(b_{t+1})$.

Incentive compatibility requires that the value of not defaulting is not smaller than the value of default, that is

$$E_t m_{t+1} V_{t+1}(b_{t+1}) \geq l_t + S_t(b_{t+1}).$$

Substituting the definition of the net renegotiation surplus $S_t(b_{t+1})$, equation (26) and rearranging we obtain the enforcement constraint $\xi_t \bar{k} = l_t + \frac{b_{t+1}}{R_t}$.

B First order conditions

Consider the optimization problem (3) and let λ and μ be the Lagrange multipliers associate with the two constraints. Taking derivatives we get:

$$d: \quad 1 - \lambda = 0$$

$$h: \quad \lambda[F_h(h) - w] - \mu F_h(h) = 0$$

$$b': \quad Em'V_{b'}(\mathbf{s}';b') + \frac{\lambda}{R} - \frac{\mu}{R} = 0$$

The envelope condition is:

$$V_b(\mathbf{s};b) \quad = \quad -\lambda$$

The above conditions can be re-arranged as in (6) and (7).

C First order conditions for the model with capital

Differentiating the firm's problem (18) with respect to h_t, $\tilde{b}_{t+1}$, $\tilde{i}_t$, $\tilde{k}_{t+1}$, we get:

$$F_h(z_t, \tilde{k}_t, h_t) = \frac{\tilde{w}_t}{1 - \mu_t} \tag{27}$$

$$\frac{1 - \mu_t}{R_t} + g_t^{-\sigma} E\tilde{m}_{t+1} \tilde{V}_b(\tilde{\mathbf{s}}_{t+1}; \tilde{k}_{t+1}, \tilde{b}_{t+1}) = 0 \tag{28}$$

$$Q_t \Upsilon'\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right) = 1 \tag{29}$$

$$Q_t = \xi_t \mu_t + g_t^{-\sigma} E\tilde{m}_{t+1} \tilde{V}_k(\tilde{\mathbf{s}}_{t+1}; \tilde{k}_{t+1}, \tilde{b}_{t+1}) \tag{30}$$

where μ_t is the lagrange multiplier associated with the enforcement constraint and Q_t is the lagrange multiplier associated with the law of motion of capital (Tobin's q). The multiplier associated with the budget constraint is 1. For the foreign country we have the same conditions but with country specific variables denoted with the start superscript.

The envelope conditions are:

$$\tilde{V}_b(\tilde{\mathbf{s}}_t; \tilde{k}_t, \tilde{b}_t) = -1 \tag{31}$$

$$\tilde{V}_k = (1 - \mu_t)F_k(z_t, \tilde{k}_t, h_t) + \left[1 - \tau + \Upsilon\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right) - \Upsilon'\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right)\frac{\tilde{i}_t}{\tilde{k}_t}\right]Q_t \tag{32}$$

Substituting the envelope conditions and imposing the equilibrium conditions $k_t = K_t$ and $\tilde{k}_t = s_t$, we obtain (19)-(22).

D   Dynamic system and solution approach

We will use the bar sign to denote aggregate worldwide variables normalized by the worldwide stock of capital. For example, $\bar{d}_t$ is the normalized worldwide dividend, defined as

$$\bar{d}_t = \frac{d_t + d_t^*}{K_t + K_t^*} \equiv \tilde{d}_t + \tilde{d}_t^*.$$

The full list of equilibrium conditions are:

$$1 = \delta g_t^{-1} R_t E_t \left(\frac{\bar{c}_{t+1}}{\bar{c}_t} \right)^{-1} \tag{33}$$

$$\tilde{c}_t^* = \chi \tilde{c}_t \tag{34}$$

$$\tilde{w}_t h_t + \tilde{w}_t^* h_t^* + \bar{b}_t = \bar{c}_t + \frac{g_t \bar{b}_{t+1}}{R_t} \tag{35}$$

$$\bar{b}_t + \bar{d}_t + \bar{i}_t = F(z_t, s_t, h_t) + F(z_t^*, s_t^*, h_t^*) - \tilde{w}_t h_t - \tilde{w}_t^* h_t^* + \frac{\bar{g}_t \bar{b}_{t+1}}{R_t} \tag{36}$$

$$g_t(\xi_t s_{t+1} + \xi_t^* s_{t+1}^*) \geq \frac{g_t \bar{b}_{t+1}}{R_t} + F(z_t, s_t, h_t) + F(z_t^*, s_t^*, h_t^*) \tag{37}$$

$$(1 - \mu_t)\bar{d}_t^{-\sigma} = \beta g_t^{-\sigma} R_t E \bar{d}_{t+1}^{-\sigma} \tag{38}$$

$$\alpha h_t^\gamma = \frac{\tilde{w}_t}{\tilde{c}_t} \tag{39}$$

$$\alpha (h_t^*)^\gamma = \frac{\tilde{w}_t^*}{\tilde{c}_t^*} \tag{40}$$

$$g_t s_{t+1} = (1 - \tau)s_t + \Upsilon \left(\frac{\tilde{i}_t}{s_t} \right) s_t \tag{41}$$

$$g_t s_{t+1}^* = (1 - \tau)s_t^* + \Upsilon \left(\frac{\tilde{i}_t^*}{s_t^*} \right) s_t^* \tag{42}$$

$$F_h(z_t, s_t, h_t) = \frac{\tilde{w}_t}{1 - \mu_t} \tag{43}$$

$$F_h(z_t^*, s_t^*, h_t^*) = \frac{\tilde{w}_t^*}{1 - \mu_t} \tag{44}$$

$$Q_t \Upsilon' \left(\frac{\tilde{i}_t}{s_t} \right) = 1 \tag{45}$$

$$Q_t^* \Upsilon' \left(\frac{\tilde{i}_t^*}{s_t^*} \right) = 1 \tag{46}$$

$$Q_t = \xi_t \mu_t + \beta g_t^{-\sigma} E \left(\frac{\bar{d}_{t+1}}{\bar{d}_t} \right)^{-\sigma} \left\{ (1 - \mu_{t+1})F_k(z_{t+1}, s_{t+1}, h_{t+1}) - \frac{\tilde{i}_{t+1}}{s_{t+1}} + \left[1 - \tau + \Upsilon \left(\frac{\tilde{i}_{t+1}}{s_{t+1}} \right) \right] Q_{t+1} \right\} \tag{47}$$

$$Q_t^* = \xi_t^* \mu_t + \beta g_t^{-\sigma} E \left(\frac{\bar{d}_{t+1}}{\bar{d}_t} \right)^{-\sigma} \left\{ (1 - \mu_{t+1})F_k(z_{t+1}^*, s_{t+1}^*, h_{t+1}^*) - \frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} + \left[1 - \tau + \Upsilon \left(\frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} \right) \right] Q_{t+1}^* \right\} \tag{48}$$

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Equations (33)-(48) form a dynamic system composed of 16 equations. Given the states $z_t, \xi_t, z_t^*, \xi_t^*, \bar{b}_t, s_t$, the unknown variables are h_t, h_t^*, c_t, c_t^*, w_t, w_t^*, i_t, i_t^*, Q_t, Q_t^*, g_t, μ_t, R_t, $\bar{d}_t$, $\bar{b}_{t+1}$, s_{t+1}. Therefore, we have a dynamic system of 16 equations in 16 unknowns.

The computational procedure is based on the approximation of four functions:

$$\Gamma_1(\mathbf{s}_{t+1}) = \bar{c}_{t+1}^{-1}$$

$$\Gamma_2(\mathbf{s}_{t+1}) = \bar{d}_{t+1}^{-\sigma}$$

$$\Gamma_3(\mathbf{s}_{t+1}) = \bar{d}_{t+1}^{-\sigma} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}, s_{t+1}, h_{t+1}) - \frac{\tilde{i}_{t+1}}{s_{t+1}} + \left[1 - \tau + \Upsilon\left(\frac{\tilde{i}_{t+1}}{s_{t+1}}\right) \right] Q_{t+1} \right\}$$

$$\Gamma_4(\mathbf{s}_{t+1}) = \bar{d}_{t+1}^{-\sigma} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}^*, s_{t+1}^*, h_{t+1}^*) - \frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} + \left[1 - \tau + \Upsilon\left(\frac{\tilde{i}_{t+1}^*}{s_{t+1}^*}\right) \right] Q_{t+1}^* \right\}.$$

In addition to these four function, we need to guess the function $p(\mathbf{s}_{t+1})$, that is, the probability of $\xi_{t+1} = \underline{\xi}$. This is necessary to compute the next period expectation.

The procedure starts with a guess for the values of the approximated functions $\Gamma_1(\mathbf{s}_{t+1})$, $\Gamma_2(\mathbf{s}_{t+1})$, $\Gamma_3(\mathbf{s}_{t+1})$, $\Gamma_4(\mathbf{s}_{t+1})$. We first form a two dimensional grid for the endogenous states $\bar{b}$ and s. Then for each realization of the exogenous shocks—$z_t, \xi_t, z_t^*, \xi_t^*$—we guess the values taken by the above functions over the grid points. Values outside the grid are obtained through bi-linear interpolation. Next we guess $p(\mathbf{s}_{t+1})$ for each grid point. Once we know the approximated functions and probabilities for ξ_{t+1}, we can solve for the 16 unknowns of the system (33)-(48) at each grid point and for each possible values of $z_t, \xi_t, z_t^*, \xi_t^*$. In finding the solutions we check whether the enforcement constraint is binding ($\mu_t > 0$) or not binding ($\mu_t = 0$). We then use the solutions found at each grip point to update the guesses for the four functions $\Gamma_1(\mathbf{s}_{t+1})$, $\Gamma_2(\mathbf{s}_{t+1})$, $\Gamma_3(\mathbf{s}_{t+1})$, $\Gamma_4(\mathbf{s}_{t+1})$ and the probabilities $p(\mathbf{s}_{t+1})$. To update these probabilities we need to check whether multiple equilibria are feasible at any possible states. Effectively we check this on the grid points of the states. We keep iterating until the guesses for $\Gamma_1(\mathbf{s}_{t+1})$, $\Gamma_2(\mathbf{s}_{t+1})$, $\Gamma_3(\mathbf{s}_{t+1})$, $\Gamma_4(\mathbf{s}_{t+1})$ and $p(\mathbf{s}_{t+1})$ at each grid point for the states are equal to the values obtained by solving the dynamic system.

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