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Charles Mason
Andrew Plantinga

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ABSTRACT

Governments contracting with private agents for the provision of an impure public good must contend with agents who would potentially supply the good absent any payments. This additionality problem is centrally important to the use of carbon offsets to mitigate climate change. We analyze optimal contracts for forest carbon, an important offset category. A novel national-scale simulation of the contracts is conducted that uses econometric results derived from micro data. For a 50 million acre increase in forest area, annual government expenditures with optimal contracts are found to be about $4 billion lower compared to costs with a uniform subsidy.

Charles Mason
Department of Economics and Finance
University of Wyoming
1000 E. University Avenue
Laramie, WY 82071-3985
bambuzlr@uwyo.edu

Andrew Plantinga
Department of Agricultural
& Resource Economics
Oregon State University
Rm #212B Ballard Extension Hall
Corvallis, OR 97331-4501
Plantinga@oregonstate.edu

1 Introduction

Governments regularly pay private agents to supply public goods. Prominent examples include road construction, prisons, and national security. Studies in the contracting literature examine the principal's problem of minimizing the costs of procuring goods when agents have private information about their costs (Laffont and Tirole, 1993; Lewis, 1996; Rogerson, 2003; Montero, 2005). In this paper, we study an important case involving cost-based procurement of an impure public good. An impure public good has both private and public characteristics (Kotchen, 2006; Cornes and Sandler, 1994). The private good feature of impure public goods has the potential to raise the government's cost substantially if there are many agents who supply the good primarily for its private attributes. In addition to the usual complication of low-cost agents masquerading as high-cost agents, the government must contend with agents who would potentially supply the good in the absence of any payments. The latter problem, referred to as additionality, is a central issue in current international efforts to limit climate change.[1] We analyze the government's problem of contracting for carbon offsets, which in many important cases are jointly produced with private goods.

As we enter the second decade of the 21st century there is an emerging consensus that carbon emissions must be limited. But there is also a strong sense that cutting emissions can be very costly, particularly if conventional sources of energy are heavily taxed or abandoned. One way to reduce the costs of carbon reduction that has received a great deal of attention is offsets. The idea is that countries could meet emissions targets by substituting lower-cost

[1] Additionality also arises with government subsidies for R&D and job creation (Picard, 2001; Gorg and Strobl, 2007).

offsets for reductions in emissions from energy production. Carbon sequestration in forests is one promising type of offset. Numerous studies have found that forest sequestration can be used to offset a substantial share of carbon emissions at costs that are similar to or lower than those associated with energy-based mitigation approaches (Richards and Stokes, 2004; van Kooten et al., 2004; Stavins and Richards, 2005). Other offset categories include carbon storage in agricultural soils and the transfer of clean energy technologies to developing countries not subject to emissions targets. Some types of forest and energy offsets are allowed under the Clean Development Mechanism (CDM) of the Kyoto Protocol, and there is interest in expanding their use under future agreements.[2]

The additionality of offsets is important in two respects. First, governments will want to avoid paying for non-additional offsets in order to limit their expenditures.[3] The government costs associated with purchases of offsets could be enormous. For example, in the U.S. an average of 1.3 million acres was deforested annually between 1982 and 1997 (USDA, 2000). While this produced significant carbon emissions that might be avoided at reasonable social cost, one must consider that the area of (non-federal) forest in the U.S. is approximately 400 million acres. If the government were to implement a subsidy for avoided deforestation and apply it uniformly across all forested acres, then in the extreme case it would subsidize all forest land when less than 1% of the area would have been deforested.[4] Second, to legit-

[2] See, e.g., UNFCCC (2005). In principle, all carbon sources and sinks could be included under an emissions control policy, such as a cap-and-trade program. There are practical obstacles to this approach in the case of offsets from forests and agricultural lands due to the large and diverse population of landowners and apparent political obstacles–as suggested by the Kyoto Protocol negotiations–in the case of developing countries.

[3] One could argue that government expenditures are welfare-neutral, and thus economically irrelevant, because government costs are simply transfer payments from one set of agents in the economy to another. We have two responses. First, governments are commonly concerned with the budgetary effects of policies, as witnessed, for example, by debates over the size of the recent federal stimulus package in the U.S. Second, there are standard economic arguments that public funds have opportunity costs and, thus, reduce net social benefits.

[4] The Conservation Reserve Program is an example of a land conservation policy in the U.S. that, in its

imately use offsets to meet emissions reduction targets, countries must be able to verify that offsets are additional; procedures that fail to clearly identify the increment of sequestration produced have been soundly criticized (Richards and Andersson, 2001).

In this paper, we describe a contracting scheme for carbon offsets and investigate its performance empirically with a national-scale simulation. Our theoretical model adapts a standard principal-agent framework (Laffont and Tirole, 1993; Salanié, 2005) to the problem at hand. The principal's objective is to maximize expected net benefits from forestation,[5] where forestation benefits are tied to an exogenously determined carbon price and costs are defined in terms of government expenditures. The problem is one of moral hazard: the principal is assumed to know the distribution over landowners' opportunity costs, but not the realization for any particular individual.[6] In the maximization problem we investigate, the optimality conditions induce a set of optimal contracts, one for each type of agent. Each contract has two ingredients: a per-unit payment, and a lump-sum transfer (from the agent to the government). The essential feature of the contract scheme is that it induces agents to truthfully reveal their type (i.e., their opportunity costs). This enables the government to identify *ex post* how much additional forest is contributed by each landowner. Further, the government is able to

initial phase, relied on uniform subsidies (Smith, 1995). Of course, the government can avoid large expenditures by levying taxes instead of paying subsidies. We regard this option as politically unviable, especially in the context of private landowners in the U.S.

[5] Conversion of non-forest lands to forest is referred to as afforestation, while maintenance of land in forest is avoided deforestation. We use the term forestation to refer collectively to these activities.

[6] Accordingly, it cannot identify the amount of land the landowner would place in forest, with or without a payment. van Benthem and Kerr (2010) consider a similar framework to ours, but do not consider the potential for the principal to reduce information rents by the use of two-part contracts; instead, they focus on the role of landowner size on contract efficiency. Using a variation of the law of large numbers, they argue that efficiency increases with the size of the landowner's holdings. Montero (2008) considers a problem of adverse selection, in which the government wishes to buy a certain amount of offsets from a group of firms whose costs are private information. In his model, once a firm commits to a certain amount of offsets its actions can be perfectly monitored. In this framework, Montero constructs an auction mechanism that induces firms to perfectly reveal their cost curves.

minimize *ex ante* its expenditures on forestation.

We then present a national-scale simulation of the optimal contracts based on individual-level data. The analysis draws on the econometric model of land-use change developed by Lubowski et al. (2006). We use the model to estimate marginal costs distributions for forestation and, with these estimates, compute the optimal contract menus using the theoretical results. We compare costs (both government costs and private opportunity costs) under the contracting approach to the costs of a uniform subsidy offered to all landowners. The results show that government costs are considerably lower under the optimal contracts than under the uniform subsidy. However, because the optimal contract scheme sets different subsidies for different agents, it violates the equi-marginal principle. This social cost inefficiency turns out to be small in relation to the reduction in government outlays associated with the optimal contract scheme. Thus, the contract scheme will be preferable at the social level as well, so long as there is a modest cost of social funds.

We believe ours is the first paper to combine modern contract theory with a careful application of econometric results based on a concrete example.[7] As such, we are able to obtain meaningful estimates of the costs associated with employing optimal contracts, to both principal and agents, as well as the costs under perfect information (i.e., the first-best outcome) and under a common per-unit subsidy that yields the identical level of forestation. These estimates

[7] While one can find papers that apply modern contracting theory to specific problems, these applications generally use casual empirical evidence to flesh out particular examples, rather than incorporating detailed econometric evidence into the problem (e.g., Bourgeon et al. (1995). One can also find empirical studies of existing contracts (e.g., Babcock et al. (1997) and Parkhurst and Shogren (2008)). As a general rule, these researchers do not have information about the distribution over agents' marginal costs, and so cannot investigate the relation between observed behavior and contracting theory. Nemes et al. (2008) conduct an experimental analysis of an offset problem; as with most experimental papers, they analyze behavior by financially motivated subjects, rather than naturally observed behavior.

allow us to articulate the potential welfare gains from the use of sophisticated contracts that are based on defensible, empirically-based parameter values. Importantly, our results show these potential welfare gains are significant, in a variety of geographical contexts.

The paper proceeds with a description of the theoretical model in section 2. In section 3, we present a national-scale simulation of the optimal contracts based on individual-level data. Section 4 presents the results from this simulation analysis; there we show that government costs are considerably lower under the optimal contracts than under the uniform subsidy, and are much closer to first-best costs. Further discussion and conclusions are provided in section 5.

2 Theoretical statement of the problem

We suppose there is a governmental agency, which we term the "principal," that is interested in having land placed in forest. Each unit of land placed in forest generates an amount of sequestration, which yields a benefit P^c to the principal. This induced benefit can naturally be thought of as a value of marginal product, which depends on the price of carbon (which can either be explicit, if a formal carbon market exists, or implicit, as with an emissions trading scheme) and the marginal product of forest land in a sequestration production function.[8] For expositional concreteness, we will refer to P^c as the 'value of marginal product' (VMP) in the pursuant discussion, with the understanding that this VMP is defined with respect to the production of sequestration services. The land that may be placed in forest is managed by a private entity, whom we call the "agent." In practice, the principal will interact with a number

[8] This interpretation implicitly assumes that each acre of land stores the same quantity of carbon.

of agents; in our description of the model we focus on the interaction with a canonical agent. Agents are characterized by their *type* θ, which is private information. Each agent would place an amount of land in forest absent any subsidy; we will interpret the agent's type as this amount. The set of possible types is $\Theta = [\underline{\theta}, \overline{\theta}]$. An agent's type determines his opportunity cost of placing an amount of land x in forest; since the agent would willingly place θ in forest at price 0 we assume this opportunity cost is an increasing, convex function of the additional amount of land placed in forest, $c(x-\theta)$. With such a specification both total and marginal costs are decreasing in type: $\partial c/\partial\theta = -c'(x-\theta) < 0, \partial^2 c/\partial x\partial\theta = -c''(x-\theta) < 0$. Let the probability distribution over θ be $f(\theta)$ and the cumulative distribution function be $F(\theta)$; we assume these distribution functions are continuous in θ, and that the hazard rate $h(\theta) = \frac{f(\theta)}{1-F(\theta)}$ is increasing in θ.[9] We assume the principal is unable to observe θ *ex ante* but does know the distribution over θ.

The principal's goal is to maximize expected net returns

$$\Omega \equiv \int_{\underline{\theta}}^{\overline{\theta}} \left\{[P^c - p(\theta)]x(\theta) + T(\theta)\right\} f(\theta)\mathrm{d}\theta,$$

where $x(\theta)$ is the amount of land an agent of type θ places in forest based on the contract scheme adopted by the principal. (Equivalently, one can regard the principal's goal as minimizing expected net outlays.) To maximize Ω, the principal offers the agent a menu of contracts of the form $\left\{p(\theta), T(\theta)\right\}$, where $p(\theta)$ is interpreted as a per-unit subsidy and $T(\theta)$ is interpreted as a transfer from the agent to the principal.[10]

[9] The hazard rate is the conditional probability an agent's type belongs to the interval $[\theta, \theta + d\theta]$, given the agent is known to be of type θ or larger. The condition $h'(\theta) > 0$ holds for many distributions, including the one we employ in the empirical application we discuss below.

[10] One can think of this transfer as reflecting a sort of 'clawback,' or that the payment $p(\theta)$ only applies to

Denote the profit earned by a type θ agent who chooses the contract intended for a type $\hat{\theta}$ agent by

$$\Pi(\hat{\theta},\theta) = p(\hat{\theta})x(\hat{\theta},\theta) - c(x(\hat{\theta},\theta)-\theta) - T(\hat{\theta}).$$

The incentive constraint requires that Π is maximized at $\hat{\theta}=\theta$:

$$0 = \partial\Pi(\theta,\theta)/\partial\hat{\theta} = p'(\theta)x(\theta,\theta) + [p(\theta) - c'(x(\theta,\theta)-\theta)(\partial x/\partial\hat{\theta}) - T'(\theta).$$

As the agent's choice of x is optimal it must satisfy $c'(x(\hat{\theta},\theta)-\theta) = p(\hat{\theta})$; it follows that the incentive constraint can be written as

$$T'(\theta) = p'(\theta)x(\theta,\theta). \tag{1}$$

Associated with this problem, an agent of type θ may earn *information rents*

$$\nu(\theta) = p(\theta)x - c(x-\theta) - T(\theta). \tag{2}$$

Information rents change with θ as follows:

$$\begin{aligned} \nu'(\theta) &= [p - c'(x-\theta)]\partial x/\partial\theta + c'(x-\theta) + [p'(\theta)x - T'(\theta)] \\ &= c'(x-\theta), \end{aligned} \tag{3}$$

units of land after some $z(\theta)$. In this way, the principal would retain the amount $p(\theta)z(\theta)$, which would represent the transfer component $T(\theta)$ in the contract. Alternatively, one can think of the transfer as a fraction b of revenue that depends on the agent's type; in this case $T(\theta) = p(\theta)x(\theta)b(\theta)$.

where the first square-bracketed term vanishes by the optimality of x and the second square-bracketed term vanishes because of the incentive compatibility constraint. Intuitively, an agent can guarantee a positive payoff by imitating a lower type agent; this guaranteed payoff is the agent's information rent. Equation (3) tells us that the information rent is larger for larger type agents. Because the principal wishes to minimize expected net outlays, the net outlays paid to each agent type are equal to that agent type's information rent, and the schedule of information rents is indexed by the rent paid to the lowest-type agent, it follows that the optimal menu of contracts sets the lowest type agent's information rents to zero: $\nu(\underline{\theta}) = 0$ (Salanié, 2005).

Substituting the expression in (2) into the principal's objective functional, we have

$$\Omega \equiv \int_{\underline{\theta}}^{\overline{\theta}} \left\{ P^c x(\theta) - [\nu(\theta) + c(x-\theta)] \right\} f(\theta) \mathrm{d}\theta.$$

Applying integration by parts to the component of the integrand involving $-\nu(\theta)f(\theta)$, and noting that $\nu(\theta)(1-F(\theta)) = 0$ at both $\theta = \overline{\theta}$ (since $F(\overline{\theta}) = 1$) and $\theta = \underline{\theta}$ (since $\nu(\underline{\theta}) = 0$), we get

$$\Omega = \int_{\underline{\theta}}^{\overline{\theta}} \left\{ [P^c x(\theta) - c(x-\theta)] f(\theta) - \nu'(\theta)[1-F(\theta)] \right\} \mathrm{d}\theta. \tag{4}$$

Maximizing Ω at any given value of θ, and taking note of equation (3), yields the first-order condition for an interior solution:

$$\begin{aligned} 0 &= \left[P^c - c'(x-\theta)\right] f(\theta) - \frac{\partial \nu'(\theta)}{\partial x}[1-F(\theta)] \\ &= \left[P^c - c'(x-\theta)\right] f(\theta) - c''(x-\theta)[1-F(\theta)]. \end{aligned} \tag{5}$$

In addition, the optimal contract scheme must satisfy the participation constraint — that no agent can earn negative profits at his choice. The solution to equation (5), combined with the condition that $\nu(\underline{\theta}) = 0$ and the participation constraint, yields the second-best value of land placed in forest as a function of θ.

Because the agent will choose x so that $p = c'$, the envelope theorem implies $c'' = 1/x'(p)$. Combining this observation with equation (5) and rearranging, we have

$$\left[P^c - p(\theta)\right] x'(p(\theta)) = \frac{1 - F(\theta)}{f(\theta)}. \tag{6}$$

The left-hand side of eq. (6) represents the marginal gain associated with increasing p slightly (thereby increasing the amount of land in forest). The opportunity cost of increasing $p(\theta)$ slightly is measured by the inverse of the hazard rate, the right-hand side of eq. (6); this derives from the fact that all higher θ types would then be tempted to misrepresent themselves. To prevent such opportunistic behavior, the principal would then be obliged to increase price for all these higher values of θ. Eq. (6) can be manipulated to produce an explicit rule for determining the optimal per-unit subsidy paid to a type θ agent:[11]

$$p(\theta) = P^c - \frac{1}{h(\theta) x'(p(\theta))}. \tag{7}$$

[11] Depending on the particular functional relation $x(p)$ and the hazard rate $h(\theta)$, the right-hand side of eq. (7) could be negative. Negative values of $p(\theta)$ would in turn imply negative operating profits, and thereby force the principal to pay the agent lump-sum subsidies. Since such a scheme would result in non-negative combined payments to this type of agent, while inducing less land in forest than would obtain at a price of zero, a contract scheme that includes negative prices is apparently infeasible. With this interpretation, there would be a cutoff value $\tilde{\theta}$, where agents with smaller values of θ opt out of the contracting scheme; the participation constraint then implies the boundary condition $\nu(\tilde{\theta}) = 0$.

Noting that $F(\overline{\theta}) = 1$, we must have either $p(\overline{\theta}) = P^c$ or $x'(p(\overline{\theta})) = 0$; we would typically expect the former branch to apply. Moreover, because information rents increase with θ, we expect $p'(\theta) > 0$ and $T'(\theta) > 0$ for interior solutions.[12]

The contracting scheme produces important information about the additional amount of forest provided by agents (and hence the associated sequestration services). As a by-product of the contracting scheme, and in return for paying information rents, the principal learns the amount of land that would have been placed in forest in the absence of any financial incentives; comparing this to the amount placed in land under the contracting scheme defines the additional amount. By contrast,were the principal to offer some simpler form of incentive, such as the common per-acre subsidy we consider below, they would only know the *expected* amount of additional forest.

The optimal contract scheme for a problem with two agent types is illustrated in Figure 1. In this diagram, one type of agent has a low θ ($= \theta_0$; we term this agent 'low type'), and one type has a high θ ($= \theta_1$; we term this agent 'high type'). The types of agent are equally likely, and aside from differences in their θ's the agents' costs are the same (hence, their marginal cost curves are parallel). Under the optimal contract scheme, the principal offers the agent the choice of prices p_H and p_L; the transfer payments T_L, T_H are set so that a type i agent would choose the price $p_i, i = L, H$. At the price p_L, the low type would choose x_L*. Since the optimal contract scheme sets the transfer payment T_L so as to extract all the low type's surplus, it must equal the low type's operating profit, area A_1. At the price p_H, the high type would choose

[12] For the interior solution to obtain, the second-order condition $\Gamma = -x'(p) + (P^c - p)x''(p) < 0$ must also be satisfied. In light of eq. (6), Γ is proportional to $-h'(\theta)$; the assumption that the hazard rate h is increasing in θ then ensures the second-order condition holds.

x_H*. The incentive compatibility constraint implies the transfer payment associated with the high price, T_H, renders this type of agent indifferent between 'telling the truth' (choosing the high price) and 'lying' (choosing the low price). Accordingly, the high type winds up with profits equivalent to the level he would obtain were he to misrepresent himself as a low type. In that event he would choose x_{HL}, and earn operating profit equal to the sum of the areas A_1 and A_2. From this operating profit he would have to forfeit the transfer payment described above, which would then leave him with a net profit equal to the parallelogram A_2—this corresponds to his information rent. Total expected government expenditures under the optimal contract scheme are thus equal to these information rents, plus the costs born by a high type at x_H*, multiplied by $\frac{1}{2}$ (the probability of observing a high type), plus the costs born by a low type at x_L*, multiplied by $\frac{1}{2}$.

By contrast, the same expected level of acreage could be induced if all agents were offered the price $p_a = \frac{p_L+p_H}{2}$ (the expected price). At that price, low types would select x_L^{**} and high types would select x_H^{**}. Since the government knows that all agents would offer at least θ_0 without payment, it seems reasonable that the price p_a only be offered for levels above x_0. Accordingly, expected government expenditures would be

$$\frac{p_a(x_L^{**}-x_0)}{2}+\frac{p_a(x_H^{**}-x_0)}{2}=p_a\left[\frac{x_L^{**}+x_H^{**}}{2}-x_0\right].$$

It is straightforward, though tedious, to verify that the government's expected costs are smaller under the optimal contract scheme. This cost savings comes at the expense of somewhat higher private costs, in that these costs would be lower at the common price of P_a (because of the equi-marginal principle). Nevertheless, so long as the social cost of funds is sufficiently large, the imputed benefits accruing from limiting government expenditures will outweigh the welfare cost attributable to asymmetric costs at the margin.

Before moving on to the empirical analysis, we briefly discuss the first-best arrangement. If the principal could identify any agent's true costs, its optimal scheme would be to offer each agent a common price of P^c, and then extract the agent's surplus profit by setting the transfer payment equal to profits. In that case, no agent earns any information rents.

3 Empirical Analysis of Carbon Sequestration Contracts

We conduct a national-level simulation of the carbon sequestration contracts. Two key ingredients for the simulation are the θ distributions and the forest response functions $x(p)$. These are derived using the econometric model of land use developed by Lubowski et al. (2006) to derive the supply function for carbon sequestration in forests. These authors estimate a discrete-choice model of private land-use decisions using parcel-level data. The random utility framework is naturally suited to our principal-agent problem. Landowners are assumed to allocate their land to the use that maximizes utility, where utility has a deterministic component, observed by all, and a random component. The landowner observes the realization of the random variable, but the researcher only knows its distribution. Thus, the random utility model assumes asymmetric information between the researcher and the landowner. We adopt the same information structure for our principal-agent problem, where the agent has perfect information and the principal knows only the distribution of the random component of landowner utility. We interpret a landholder as having one unit of land, and so the amount of an agent's land in forest also equals the fraction of the agent's land in forest. The amount of forest at a larger geographical level, such as a state, can then be found by aggregating over all agents within the cohort. As we treat all agents within a particular state and land class as *ex ante* identical, multiplying the predicted share of an agent's land in forest by the total amount of land in the land class for that state will yield the expected amount of land allocated to forest.

In the Lubowski et al. model, utility is represented by net returns, a measure of the variable profits per acre from each land use. Land parcels begin in one of six uses (cropland, pasture, forest, urban, Conservation Reserve Program, range) and are assumed to be allocated to the use generating the highest level of net returns. Because of the random component of net returns, land-use decisions are a probabilistic phenomenon from the perspective of the principal. Estimation of the model yields land-use transition probabilities of the following form:

$$\hat{P}_{ijkt} = f(\mathbf{X}_{c(i)t}, \mathbf{Y}_i; \hat{\boldsymbol{\beta}}_{jk}), \tag{8}$$

where i indexes parcels, j indexes the starting use, k indexes the ending use, and t indexes time. Thus, equation (8) signifies the probability that parcel i changes from use j to use k during the time period beginning in t.[13] Probabilities are a logistic function f of observable variables $\mathbf{X}_{c(i)t}$ and $\mathbf{Y}_i$, where $c(i)$ is a function that maps from parcel i to the county in which it is located. Thus, $\mathbf{X}_{c(i)t}$ is a vector of county-level variables (specifically, average county-level net returns to each use) and $\mathbf{Y}_i$ is a vector of parcel-level variables (specifically, measures of

[13] Lubowski et al. (2006) estimate the land-use model with data from the National Resources Inventory (NRI), a panel survey of land use in the U.S. conducted at 5-year intervals over the 1982 to 1997. Due to the structure of the data, the probabilities correspond to land-use changes over a 5-year period.

plot-level land quality that are used to scale county average returns and to proxy for conversion costs). $\hat{\boldsymbol{\beta}}_{jk}$ is a vector of estimated parameters specific to the j-to-k transition. We denote the estimated covariance matrix for the full set of parameters as $\hat{\mathbf{V}}$.

According to the sampling scheme underlying the land-use data, each parcel i represents a certain number of acres A_i. If parcel i is initially in use j, this corresponds to A_{ij} acres in this use. If k indexes an alternative use, the expected amount of A_{ij} allocated to use k by the end of the 5-year time period beginning in t is $A_{ij} \times \hat{P}_{ijkt}$. More generally, if $\mathbf{A}_{it}$ is a vector of acres by use in period t and $\hat{\mathbf{P}}_{it}$ is a 6×6 matrix of transition probabilities, then

$$\mathbf{A}_{it+N} = \mathbf{A}_{it} \times \hat{\mathbf{P}}_{it}^{N} \tag{9}$$

gives the expected acres in each use N periods in the future. Because forests require several decades or more to grow to maturity, our application is necessarily concerned with land-use change over long periods of time. We adopt a planning horizon of 100 years (i.e., N = 20 periods).

Land-use allocations many years in the future are subject to great uncertainty. We represent this uncertainty using the distributions on the estimated parameters from the econometric model. Using $\hat{\mathbf{V}}$, we randomly draw 1000 parameter vectors $\hat{\boldsymbol{\beta}}_{jk}^{m}$, where m indexes draws. By substitution of $\hat{\boldsymbol{\beta}}_{jk}^{m}$ into equation (8) for all j and k we obtain the matrix of transition probabilities $\hat{\mathbf{P}}_{it}^{m}$. We then apply the Markov chain in equation (9) to the vector of initial acres, $\mathbf{A}_{i0}$, to obtain the expected number of acres in 20 periods (i.e., 100 years), $\hat{\mathbf{A}}_{i20}^{m}$, for each parcel i and draw m.

To ease the burden of computing the optimal contracts, and to correspond to a scale at which carbon contracts might actually be implemented, we aggregate parcels within states or groups of states. States with little private forest (North Dakota, South Dakota, Nebraska, Kansas, Nevada, and Arizona, and the western portions of Oklahoma and Texas) are dropped and small states are combined (the southern New England states, the northern New England states, and the mid-Atlantic states of New Jersey, Delaware, and Maryland). As well, based on climatic similarities, we reconfigure Oregon and Washington as the western and eastern portions of these states, and combine the eastern portions of Oklahoma and Texas. Henceforth, the term "state" will be used to refer to one of the thirty-five states, groups of states, or portions of states considered in the analysis. Within a state, differences in the quality of land produce differences in the opportunity costs of forestation (Stavins and Jaffe, 1990). In the econometric model, this heterogeneity is captured by $\mathbf{Y}_i = (Y_{1i}, Y_{2i}, Y_{3i}, Y_{4i})$, where Y_{qi} is a dummy variable indicating if parcel i is in land quality class $q = 1, \ldots 4$. Land quality classes are ordered from highest-quality ($q = 1$) to lowest-quality ($q = 4$). Within each state, we aggregate only parcels of the same quality. Thus, we implement the contract scheme separately for $4 \times 35 = 140$ state and land quality combinations.

Following the aggregation, we have 1000 replicates of $\mathbf{A}^m_{sq20}$, for each state s and land quality class q. The element of $\mathbf{A}^m_{sq20}$ that is of central importance to us is the one corresponding to the area of forest 100 years in the future. This variable, when expressed as a percentage of the maximum possible forest area, defines θ^m_{sq}, the share of land in state s and quality class q that is allocated to forest in the future in the absence of any carbon sequestration incentives.[14] The maximum possible forest area is defined as the total area of private land in crops, pasture, forest, and range.[15] This measure excludes urban land, which we assume cannot be converted back to undeveloped uses, and federal lands, which are managed by U.S. government agencies. Our measure of maximum forest area likely overstates the potential forest area in those Rocky Mountain states where tree growth is limited by climatic conditions, especially moisture availability. We return to this issue below.

Consistent with the theoretical model, we assume that θ is distributed logistic.[16] For each state and land quality class, we fit the logistic function,

$$F_{sq}(\theta) = \frac{1}{1 + e^{-\phi_{0sq} - \phi_{1sq}\theta}}, \tag{10}$$

to the 1000 replicates of θ^m_{sq} over the interval $[\underline{\theta}_{sq}, \overline{\theta}_{sq}]$, where $\underline{\theta}_{sq}$ (respectively, $\overline{\theta}_{sq}$) is the minimum (respectively, maximum) value of θ^m_{sq} in the list of 1000 replicates.[17] Across states, the average values of $\underline{\theta}_{sq}$ and $\overline{\theta}_{sq}$ are 0.01 and 0.44, respectively, for the highest-quality lands ($q = 1$). For lowest-quality lands ($q = 4$), the average values of $\underline{\theta}_{sq}$ and $\overline{\theta}_{sq}$ are 0.43 and 0.70, respectively. Agriculture tends to be more profitable than forestry on higher quality lands and, thus, relatively small shares of high quality land are forested in the absence of government payments. Forestry tends to be more profitable on lower quality lands and, in most states, a

[14] The unobserved component of landowner utility implies there is a strictly positive probability that each land parcel will be put into forest.

[15] The NRI does not identify lands owned by states and counties and, thus, they are included in our measure of maximum forest area. These public lands do not represent a large share of the land base in any state and, in many cases, are forested.

[16] The estimates of θ produced from the econometric model are 20th-order polynomial functions of logistic probabilities. We approximate the true distribution with a simple logistic in order to simplify the computation of the optimal contracts.

[17] The estimated values of $\underline{\theta}_{sq}$ and $\overline{\theta}_{sq}$ are reported in Table A.1 in the appendix; the estimates of ϕ_{0sq} and ϕ_{1sq} are available from the authors upon request.

large share of low quality land is forested without incentives.

The next step in the analysis is to derive the response function $x(p)$. We assume this function is a quadratic:

$$x(p) = \alpha + \delta_0 p + \delta_1 p^2. \tag{11}$$

The intercept of this function represents the average amount of forest land that would be allocated at a price of zero (i.e., the mean of θ), which is induced by the distribution over θ. To estimate the coefficients δ_0 and δ_1 for each land quality category in each state, we follow Stavins (1999) and modify the transition probabilities in equation (8) by introducing an annual per-acre subsidy or tax p.[18] This yields:

$$\hat{P}_{ijkt} = f(\mathbf{X}_{c(i)t} + \mathbf{p}, \mathbf{Y}_i; \hat{\boldsymbol{\beta}}_{jk}), \tag{12}$$

for all i, j, k. As placing land in forest represents the third possible use, $\mathbf{p} = (0,0,p,0,0,0)$ if $k = 3 \neq j$ (land goes into forest), $\mathbf{p} = (-p,-p,0,-p,-p,-p)$ if $j = 3 \neq k$ (land goes out of forest), and $\mathbf{p} = (0,0,0,0,0,0)$ otherwise. Thus, transitions into forest are subsidized by p and transitions out of forest are taxed by p. Using, again, randomly drawn parameter vectors, we repeat the above procedure and predict future land use for each state and land quality class and for subsidy/tax values $p = 25, 50, ..., 250$. This yields multiple observations of the share of the potential forested area that is forested after 100 years, at different values of p and for each state and land quality class. Sets of observations of x and p are used to estimate the price

[18] In principle, p could emerge from a variety of policies, including the optimal contracting scheme we outlined in the previous section. For the purposes of our deriving the $x(p)$ function, however, the source of this tax or subsidy is not important. All that matters is that its presence changes the payoff from forestation, relative to other options.

coefficients of $x(p)$ for each state and land quality class.[19]

4 Simulation Results

By combining equations (10) - (11) with the results from section 2, we are able to simulate the optimal menu of contracts for each land class within each state for a range of VMPs.[20] The simulated contracts can then be used to calculate the expected amount of land in forest, by state and land class, and the associated cost to the principal. We can also use information regarding the distribution over θ and the $x(p)$ function to determine the constant subsidy that would induce the same amount of land in forest.[21] This allows us to compare the expected costs under the optimal contract with the constant subsidy scheme.

Table 1 presents the results of the empirical analysis of carbon contracts for a VMP of \$100.[22] The second and third columns report the maximum forest area for each state and the increase in forest area under the contracts.[23] Across all states, approximately 52 million acres of forest would be added or conserved, which represents about 5.3% of the maximum potential forest. The largest gains, in absolute and percentage terms, are predicted in Rocky Mountain

[19] The estimation results are presented in Table A.2 in the appendix. While one would expect a positive relationship between the net return to a use and the probability that land will be converted to or remain in that use, the transition probabilities are a function of the net returns to all land uses. This can result in complicated cross-effects when subsidy/tax incentives are introduced (Lubowski, 2002) and produce a negative coefficient on price. In our application, this counterintuitive outcome arose in a small number of cases. These cases are omitted from the analysis, and left blank in Table A.2.

[20] Details of the derivation are contained in the Appendix.

[21] Part of the simulation exercise is the determination of the smallest value of θ at which an agent would just be willing to participate. In our applications, this cutoff value was larger than $\underline{\theta}$ in all but a small number of cases.

[22] Here and below, we express the VMP values and all cost estimates in annual terms. Lubowski et al. (2006) find that a per-acre payment of \$100 translates into a carbon price of approximately \$50 per ton of carbon.

[23] Results disaggregated by land quality classes and for other VMPs are available from the authors upon request.

states such as Utah, Montana, and Wyoming. As discussed above, these states contain arid regions where forestation is limited by moisture availability. As a check on our results, we compared our predictions of forest area under the contracts with state-level estimates of the area of private land that can support forests. These estimates are based on fine-scale measures of moisture availability (annual deficit and evapotranspiration) that determine the dominant potential vegetation type (e.g., coniferous forests, tallgrass prairie) (Stephenson, 1998). In all cases, our predictions were within the limits imposed by climatic conditions. Relatively large increases in forest area are also predicted for some states in the Midwest, such as eastern Oklahoma and Texas and Minnesota, and small increases are predicted for most eastern states. The general pattern is that larger forest gains occur where there is abundant land with low opportunity costs, such as rangelands in the Rocky Mountain region.

As in the example presented in Figure 1, the next two columns of Table 1 report the government expenditures under the contracts compared to a subsidy that pays landowners the same amount per acre. At a given VMP, the increase in forest area tends to be larger with the subsidy than with the contracts. Thus, to make the expenditures comparable, we set forest area equal to the area under the contracts and calculate the corresponding subsidy that would yield this area using the estimated forest response functions in Table A.2. The government expenditures under the contracts are calculated by summing the payments to landowners across types and subtracting the transfer payments they make back to the government. To compute the costs under the subsidy policy, we multiply the per-acre subsidy by the forest area provided by each landowner type net of the area that the highest cost type would provide for free.

Our key finding is that government expenditures are dramatically lower under the contracts than under a subsidy policy. At a carbon price of \$100, the optimal contracts increase forest area by 52 million acres nationwide, at an annual cost to the government of \$3.09 billion. To achieve the same increase in forest area with a uniformly-applied subsidy, the government would have to spend over twice as much, about \$6.86 billion. On a state-by-state basis, there is considerable variation in relative expenditures. Although subsidies are always more expensive for the government than contracts, they differ by a factor of 2 or less in many states (e.g., Indiana and Iowa) but by a factor of 5 or more in South Carolina, Georgia, and elsewhere. These differences can be traced back to the slopes of the forest response functions in Table A.2.

While government expenditures under the contracts are roughly 45% of the level that would obtain under a uniform subsidy, one naturally wonders about the difference in private opportunity costs under the two policies. Because the uniform subsidy equates the marginal costs of forestation across agents (i.e., satisfies the equi-marginal principle), we know that it will minimize the private opportunity costs associated with the increases in forest area. In contrast, under the contracts prices vary by type. A comparison of these costs for each state is given in the last columns of Table 1. Annual private opportunity costs are about \$2.02 billion on a national scale under the contracts, compared to \$1.77 billion under the subsidy policy.[24] In contrast to government expenditures, there is less variation among states in the differences between private opportunity costs (western Oregon and Washington is an exception). In most states, opportunity costs differ by less than a factor of 2.

[24] Thus, the increase in private opportunity costs corresponds to 6.6% of the governmental cost savings. Thus, if the cost of public funds exceeds \$.07 per dollar raised the use of the optimal contracting scheme is socially advantageous, as well as cost-saving from the governmental agency's perspective.

To get a sense for the robustness of these results to variations in the VMP, we repeat the cost calculations for values ranging from \$25 to \$150; the results are summarized in Figure 2. As the VMP increases, it is optimal to convert more land to forest and so private and government costs increase under the contract and subsidy policies. However, the relative magnitudes of the costs remain roughly the same. The ratio of government costs under subsidies to government costs under contracts is about 3 at a VMP of \$25, falling to 2.2 at a VMP of \$150. Similarly, the ratio of private costs under subsidies to private costs under contracts is about 0.8 at a VMP of \$25, rising to 0.9 at a carbon price of \$150. These results show that the optimal contracting scheme generates substantial cost savings over a wide range of VMP.

5 Conclusion

In this paper, we describe a contracting scheme to encourage carbon offsets from forestation at minimal cost to the government, subject to the landholders having private information about their opportunity costs of placing land in forest. The optimal contract scheme typically leaves some rents in landowners' hands, and so these contracts are second-best in nature. But, assuming that the government is concerned with the budgetary effects of the policy, the contracts are a much cheaper approach to inducing the expansion or maintenance of forests than a simple, constant per-unit subsidy. In a national-scale simulation, we find that for given increases in forest area government expenditures with contracting are about 40% of those with subsidies. In absolute terms, contracts lower expenditures by almost \$4 billion per year when applied to 50 million acres of land. Since the contracting scheme does not satisfy the equi-marginal principle, private opportunity costs are necessarily higher than under the uniform subsidy. However, we find that this difference is small (about $250 million for a 50 million acre increase) relative to the reductions in expenditures. Thus, the contracting scheme is preferable from society's perspective provided there is a modest cost of government funds—in our simulations, at least $.07 per dollar raised, considerably less than values that are often suggested.

Carbon offsets are a promising approach to reducing the global costs of climate change mitigation, but additionality presents significant obstacles. Besides the potential impacts on government costs, it must be possible to verify that offsets represent true incremental adjustments so they do not compromise international efforts to achieve real reductions in greenhouse gas emissions. The contracting scheme that we analyze allows governments to identify ex post the amount of additional forest provided by each landowner and, thus, to make credible claims for offset credits. In this regard, our contracting scheme offers an important advantage over a subsidy policy. Knowledge of the distribution over marginal costs allows one to determine ex ante the expected increments in forest under either policy. However, only the contracts identify the actual increments in forest ex post because they induce truth telling by landowners. Implementation of the subsidy policy does not reveal additional information about agent types.

While we analyze optimal contracts when the government purchases offsets, one can imagine cases in which private agents would be the buyers. For example, if energy producers are subject to emissions restrictions, they may be allowed to substitute lower-cost offsets for emissions reductions. In this case, the regulating agency will want to ensure that the offsets are additional since non-additional offsets would effectively relax the emissions controls. In this setting, non-additional offsets would be inexpensive and, thus, particularly appealing to emissions sources. Our contracting scheme is a remedy for this problem. The regulating agency could require private buyers of offsets to structure contracts following our design. Conditional on their purchasing only additional offsets, private buyers would want to implement our scheme because it minimizes their costs.

Figure 1: Figure 1: Optimal contracts with two types of agents

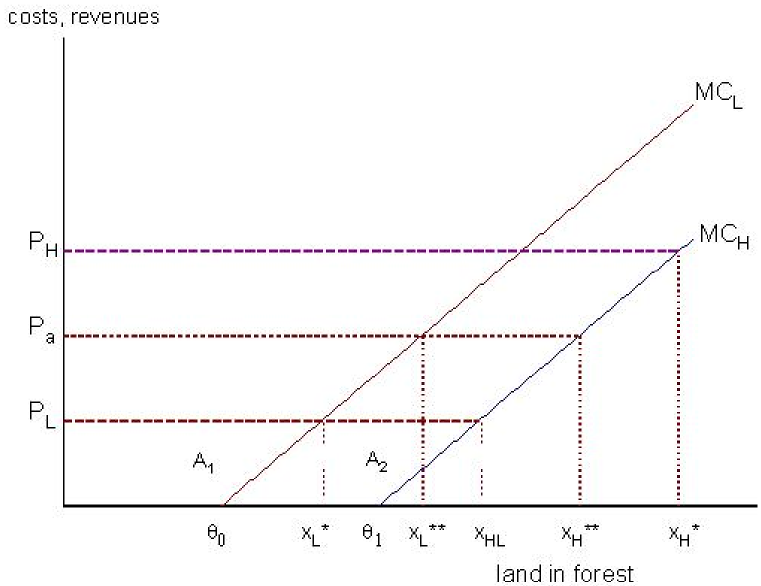

Figure 2: The cost of carbon offsets with contracts and subsidies

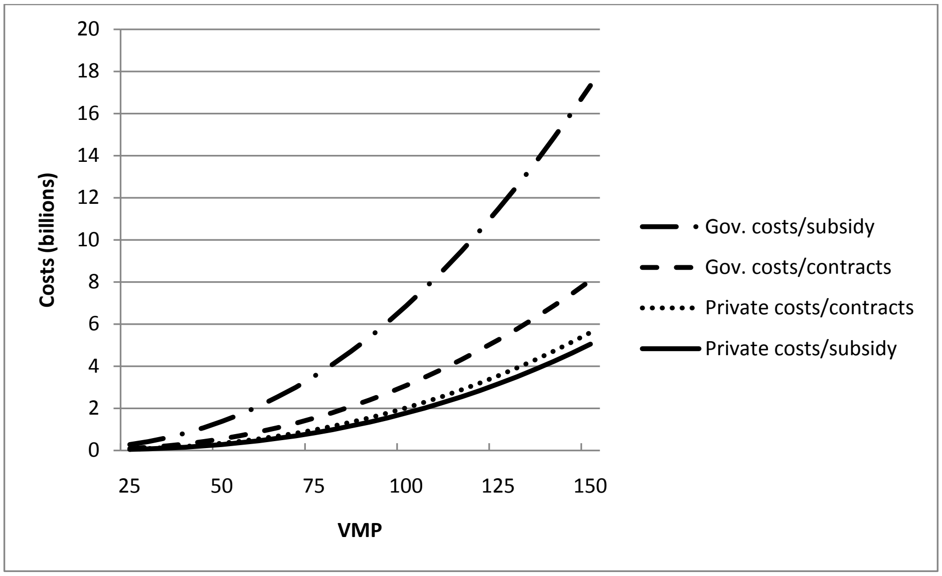

Table 1. State-level results for a VMP of $100/acre						
	Maximum	Increase in	Government costs		Private costs	
State	forest area	forest	Contracts	Subsidy	Contracts	Subsidy
	1000 acres		million dollars		million dollars	
AL	28338	181	12.0	59.5	4.2	1.6
AR	28255	467	32.7	109.3	11.9	6.0
CA	43061	3697	214.7	592.7	128.7	104.9
CO	39886	5136	303.2	610.3	240.4	221.6
CT, MA, RI	5648	49	3.4	15.5	1.0	0.5
DE, MD, NJ	7746	117	8.0	19.2	2.5	1.3
FL	22868	553	36.6	153.8	18.4	12.3
GA	29776	64	4.2	30.7	1.4	0.4
ID	18065	1015	60.9	142.4	38.4	31.9
IL	31023	443	33.0	72.7	7.8	3.3
IN	19395	607	42.1	74.8	13.7	8.0
IA	32803	1478	95.1	158.9	40.1	29.4
KY	21863	283	20.1	61.0	6.1	3.0
LA	21688	240	15.6	62.4	6.8	3.7
ME, NH, VT	27512	166	11.0	46.8	3.7	2.1
MI	27248	193	13.9	48.5	3.8	1.4
MN	42640	1262	84.5	163.0	31.6	19.8
MS	26039	279	19.4	68.3	6.3	2.7
MO	38724	1358	90.0	239.5	41.6	28.8
MT	63515	11894	643.4	1279.9	555.0	531.0
NM	48029	3924	274.7	427.2	121.3	112.1
NY	25895	274	19.1	47.5	6.1	3.2
NC	23768	80	5.4	26.7	1.8	1.0
OH	21038	844	54.7	110.4	22.5	15.3
Eastern OK & TX	55222	3507	221.5	556.0	136.0	109.0
PA	22884	68	4.5	35.0	1.5	0.5
SC	15221	31	2.0	13.7	0.7	0.3
TN	22049	391	26.4	77.6	9.8	5.3
UT	15206	2752	153.8	299.0	123.7	115.8
VA	19299	114	7.8	37.1	2.6	1.0
WV	12972	125	8.6	30.6	2.7	1.5
WI	28717	487	33.5	90.2	11.6	5.6
WY	31873	6504	361.4	684.8	310.5	298.4
Eastern OR & WA	36904	2953	168.8	410.1	107.3	90.3
Western OR & WA	18788	7	0.4	8.0	0.2	0.0
All states	973961	51542	3086.2	6862.9	2022	1773

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6 Appendix

In our empirical analysis, we assume that θ is logistically distributed over the interval $[\underline{\theta},\overline{\theta}]$. Accordingly,

$$F(\theta) = K_0 + \frac{K_1}{1+e^{-\phi_0-\phi_1\theta}},$$

where the constants K_0 and K_1 are chosen so as to ensure $F(\underline{\theta}) = 0, F(\overline{\theta}) = 1$. That is,

$$\begin{aligned} K_0 + \frac{K_1}{1+e^{-\phi_0-\phi_1\underline{\theta}}} &= 0, \\ K_0 + \frac{K_1}{1+e^{-\phi_0-\phi_1\overline{\theta}}} &= 1. \end{aligned}$$

With this specification,

$$f(\theta) = F' = \frac{K_1(\phi_0+\phi_1\theta)}{[1+e^{-\phi_0-\phi_1\theta}]^2}.$$

Accordingly, we have

$$\begin{aligned} \frac{1-F(\theta)}{f(\theta)} &= \left(\frac{1+e^{-\phi_0-\phi_1\theta}}{\phi_0+\phi_1\theta}\right) \times \left[1+\frac{1-K_0}{K_1}[1+e^{-\phi_0-\phi_1\theta}]-1\right] \\ &= \left(\frac{1+e^{-\phi_0-\phi_1\theta}}{\phi_0+\phi_1\theta}\right) \times \left[\frac{1+e^{-\phi_0-\phi_1\theta}}{1+e^{-\phi_0-\phi_1\overline{\theta}}-1}\right] \equiv \lambda(\theta). \end{aligned}$$

The assumed quadratic relation $x(p) = \alpha + \delta_0 p + \delta_1 p^2$ implies $x'(p) = \delta_0 + 2\delta_1 p$. Inserting this expression into equation (6) yields the quadratic relation

$$\left(P^c - p\right)(\delta_0 + 2\delta_1 p) = \lambda(\theta). \tag{13}$$

The optimal set of prices $p(\theta)$ solves equation (13). Assuming $\delta_0 + 2\delta_1 p > 0$, as is the case in the numerical results discussed below, the solution requires $p < P^c$.[25] Since negative prices would yield smaller amounts of forest, at non-negative cost, than could be obtained without spending anything, we restrict attention to the interval $0 \leq p(\theta) \leq P^c$.[26]

In general, there will be two solutions to the quadratic equation in (13):

$$p(\theta) = -\frac{1}{4\delta_1}(\delta_0 - 2\delta_1 P^c) \pm \frac{1}{4\delta_1}\sqrt{(\delta_0 - 2\delta_1 P^c)^2 + 8\delta_1(\delta_0 P^c - \lambda(\theta))} \tag{14}$$

The optimal price will be the smallest positive solution, which is induced by the positive branch from equation (14).[27] Taking the positive branch and simplifying, we find the rule for determining the optimal price (when an interior solution applies):

$$p(\theta) = \frac{1}{2}[P^c - \frac{\delta_0}{2\delta_1}] + \frac{\sqrt{(\delta_0 + 2\delta_1 P^c)^2 - 8\delta_1\lambda(\theta)}}{4\delta_1}. \tag{15}$$

If $\delta_1 < 0$ this rule will apply for all values of θ. But if $\delta_1 > 0$, for sufficiently large values of θ no rational solution will exist. In such a case, there is no positive price that satisfies the optimality condition, and so the appropriate choice is $p(\theta) = 0$.

The solution is completed by deriving the optimal transfer function $T(\theta)$, which is induced by equation (1). Differentiating the right-side of equation (15) with respect to θ, and

[25] Note that $\delta_0 + 2\delta_1 p = x'(p)$, which we would expect to be non-negative. This restriction needs to hold for all prices under consideration. If $\delta_1 > 0$ then the condition $\delta_0 + 2\delta_1 p > 0$ for all non-negative prices; if $\delta_1 < 0$ then the condition may be reduced to $\delta_0 + 2\delta_1 P^c > 0$.

[26] As noted in footnote 11, requiring $p \geq 0$ implies a cutoff value $\tilde{\theta}$ that is implicitly defined by $p(\tilde{\theta}) = 0$.

[27] The radical term on the right-hand side of equation (14) is positive, so if $\delta_1 < 0$ the expression following the $\pm$ sign is negative, and hence must be added to the first term. But if $\delta_1 > 0$ the first term in equation (14) is negative, so the solution must entail adding the second expression. In either case, the solution requires using the positive branch.

making use of equation (15), we obtain

$$p'(\theta) = \frac{\lambda'(\theta)}{2\delta_1 \left[P^c - 2p(\theta)\right] - \delta_0}. \tag{16}$$

Then, combining equations (11) and (16), we obtain the differential equation that describes the optimal transfer function:

$$T'(\theta) = -\frac{\alpha + \delta_0 p(\theta) + \delta_1 p(\theta)^2}{\delta_0 + \delta_1 [2p(\theta) - P^c]}. \tag{17}$$

The final step is to apply the boundary condition, $\nu(\underline{\theta}) = 0$. To use this condition, one needs to know the costs associated with placing an increment e in forest. By definition, $x = \theta + e$, so $e = \delta_0 p + \delta_1 p^2$. Inverting this relation yields $p(e)$:

$$p(e) = -\frac{\delta_0}{2\delta_1} + \frac{\sqrt{\delta_0^2 + 4\delta_1 e}}{2\delta_1}, \tag{18}$$

where we take the positive root so as to obtain the smallest positive price.[28] Since the agent will choose e so as to maximize profit, we know $c'(e) = p$. Thus, integrating the right-hand side of equation (18) will yield costs

$$c(e) = -\frac{\delta_0 e}{2\delta_1} + \frac{(\delta_0^2 + 4\delta_1 e)^{3/2}}{12\delta_1^2} - \frac{\delta_0^3}{12\delta_1^2}. \tag{19}$$

Using equations (15), (11) and (19), one can calculate the operating profit for a type $\underline{\theta}$ firm;

[28] If $\delta_1 > 0$ the intercept is negative while the second fraction is positive, so we must add the second fraction. If $\delta_1 < 0$ the intercept is positive but the second fraction is negative, so again we want to add the second fraction.

setting the transfer $T(\underline{\theta})$ equal to that operating profit solves the boundary condition $\nu(\underline{\theta}) = 0$.

It is then straightforward to numerically solve the differential equation (17).

Table A.1. Lower and upper bounds of the θ distribution, by state and land quality category								
	Land quality 1		Land quality 2		Land quality 3		Land quality 4	
State	lower	upper	lower	upper	lower	upper	lower	upper
AL	0.010	0.485	0.209	0.658	0.600	0.856	0.528	0.812
AR	0.007	0.349	0.155	0.481	0.539	0.768	0.568	0.786
CA	0.010	0.331	0.114	0.391	0.313	0.594	0.095	0.430
CO	0.009	0.212	0.082	0.204	0.089	0.181	0.203	0.324
CT, MA, RI	0.004	0.406	0.139	0.630	0.539	0.831	0.526	0.786
DE, MD, NJ	0.008	0.322	0.159	0.564	0.467	0.817	0.368	0.795
FL	0.013	0.643	0.180	0.504	0.496	0.757	0.343	0.631
GA	0.018	0.688	0.261	0.742	0.583	0.876	0.413	0.812
ID	0.015	0.597	0.197	0.500	0.239	0.603	0.265	0.557
IL	0.019	0.554	0.236	0.586	0.510	0.811	0.553	0.784
IN	0.015	0.414	0.222	0.528	0.538	0.794	0.589	0.795
IA	0.012	0.315	0.175	0.438	0.414	0.714	0.492	0.722
KY	0.014	0.470	0.231	0.552	0.579	0.813	0.562	0.807
LA	0.010	0.464	0.172	0.523	0.588	0.850	0.436	0.709
ME, NH, VT	0.003	0.574	0.172	0.714	0.617	0.878	0.647	0.839
MI	0.014	0.485	0.228	0.613	0.643	0.875	0.606	0.812
MN	0.011	0.318	0.170	0.490	0.543	0.813	0.573	0.790
MS	0.008	0.444	0.178	0.536	0.579	0.807	0.591	0.812
MO	0.008	0.241	0.136	0.356	0.474	0.674	0.555	0.749
MT	0.007	0.149	0.069	0.138	0.173	0.245	0.180	0.247
NM	0.013	0.613	0.193	0.590	0.100	0.674	0.151	0.690
NY	0.005	0.358	0.152	0.575	0.597	0.836	0.612	0.808
NC	0.015	0.703	0.248	0.693	0.608	0.852	0.522	0.799
OH	0.012	0.287	0.187	0.486	0.585	0.820	0.500	0.807
Eastern OK & TX	0.011	0.377	0.151	0.384	0.362	0.503	0.386	0.547
PA	0.015	0.653	0.269	0.672	0.607	0.888	0.441	0.813
SC	0.016	0.704	0.242	0.730	0.588	0.875	0.446	0.804
TN	0.009	0.334	0.175	0.516	0.541	0.738	0.593	0.799
UT	0.010	0.374	0.108	0.318	0.131	0.311	0.133	0.261
VA	0.008	0.503	0.184	0.604	0.578	0.821	0.543	0.791
WV	0.003	0.295	0.121	0.577	0.447	0.791	0.629	0.874
WI	0.012	0.370	0.209	0.545	0.610	0.830	0.526	0.800
WY	0.010	0.247	0.073	0.187	0.081	0.165	0.103	0.163
Eastern OR & WA	0.011	0.635	0.213	0.565	0.363	0.662	0.147	0.579
Western OR & WA	0.002	0.441	0.159	0.684	0.358	0.920	0.129	0.830
Average	0.010	0.439	0.176	0.522	0.459	0.721	0.427	0.696

Table A.2. Price coefficients for the forest response function x(p), by state and land quality category

	Land quality 1		Land quality 2		Land quality 3		Land quality 4	
State	price	price squared	price	price squared	price	price squared	price	price squared
AL	0.00126	-0.00000267	0.00011	0.00000149	0.00023	-0.000000085	0.00065	-0.00000161
AR	0.00186	-0.00000380	0.00029	0.00000252	0.00026	0.000000427	0.00065	-0.00000137
CA	0.00106	-0.00000371	0.00050	0.00000153	0.00156	-0.000001630	0.00305	-0.00000610
CO	0.00205	-0.00000450	0.00080	0.00000314	0.00108	0.000005750	0.00269	-0.00000127
CT, MA, RI	0.00125	-0.00000264	0.00011	0.00000117	0.00029	-0.000000414	0.00059	-0.00000140
DE, MD, NJ	0.00161	-0.00000436	0.00030	0.00000129	0.00040	-0.000000620	0.00054	-0.00000110
FL	0.00075	-0.00000186	0.00043	0.00000241	0.00041	0.000000903	0.00173	-0.00000282
GA	0.00089	-0.00000243	0.00012	0.00000038	0.00028	-0.000000123	0.00072	-0.00000174
ID	0.00119	-0.00000365	0.00004	0.00000222	0.00099	0.000003730	0.00247	-0.00000282
IL	0.00153	-0.00000337			0.00036	0.000000549	0.00056	-0.00000106
IN	0.00184	-0.00000379			0.00035	0.000000480	0.00055	-0.00000125
IA	0.00208	-0.00000428			0.00053	0.000000768	0.00066	-0.00000091
KY	0.00170	-0.00000336			0.00028	0.000000645	0.00055	-0.00000123
LA	0.00139	-0.00000301	0.00028	0.00000212	0.00026	-0.000000151	0.00151	-0.00000319
ME, NH, VT	0.00099	-0.00000144	0.00010	0.00000074	0.00032	-0.000000936	0.00059	-0.00000165
MI	0.00149	-0.00000320	0.00003	0.00000196	0.00025	-0.000000334	0.00057	-0.00000143
MN	0.00190	-0.00000389	0.00014	0.00000246	0.00029	-0.000000035	0.00058	-0.00000135
MS	0.00151	-0.00000291	0.00017	0.00000241	0.00030	-0.000000101	0.00061	-0.00000149
MO	0.00214	-0.00000443	0.00022	0.00000334	0.00032	0.000001340	0.00080	-0.00000150
MT	0.00229	-0.00000532	0.00179	0.00000074	0.00129	0.000004080	0.00365	-0.00000371
NM	0.00109	-0.00000298	0.00012	0.00000015	0.00095	0.000006180	0.00266	0.00000012
NY	0.00162	-0.00000304	0.00013	0.00000179	0.00027	-0.000000432	0.00057	-0.00000146
NC	0.00088	-0.00000238			0.00022	0.000000471	0.00067	-0.00000150
OH	0.00213	-0.00000481	0.00018	0.00000299	0.00026	0.000000292	0.00054	-0.00000121
Eastern OK & TX	0.00167	-0.00000367	0.00080	0.00000216	0.00082	0.000003060	0.00221	-0.00000322
PA	0.00089	-0.00000249			0.00024	-0.000000060	0.00057	-0.00000133
SC	0.00070	-0.00000194	0.00006	0.00000054	0.00027	-0.000000135	0.00074	-0.00000179
TN	0.00180	-0.00000349	0.00014	0.00000256	0.00033	0.000000447	0.00058	-0.00000137
UT	0.00141	-0.00000338	0.00019	0.00000340	0.00087	0.000005470	0.00306	-0.00000092
VA	0.00120	-0.00000246	0.00011	0.00000184	0.00026	0.000000169	0.00058	-0.00000129
WV	0.00172	-0.00000346	0.00019	0.00000138	0.00032	-0.000000657	0.00052	-0.00000137
WI	0.00184	-0.00000385	0.00015	0.00000266	0.00029	0.000000102	0.00056	-0.00000126
WY	0.00205	-0.00000443	0.00150	0.00000257	0.00118	0.000006530	0.00365	-0.00000179
Eastern OR & WA	0.00056	-0.00000231			0.00140	0.000000057	0.00364	-0.00000690
Western OR & WA	0.00036	-0.00000053	0.00007	0.00000048	0.00024	-0.000000404	0.00045	-0.00000112