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#### THE HOUSING CYCLE AND PROSPECTS FOR TECHNICAL PROGRESS

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The Housing Cycle and Prospects for Technical Progress Casey B. Mulligan NBER Working Paper No. 15971 May 2010 JEL No. E32,O33

#### ABSTRACT

Information technology has already transformed some areas of our lives, and has the prospect for transforming other sectors. This paper is about economic behaviors that anticipate technical progress, and how they may describe the housing price and construction boom of 2000-2006 and the bust thereafter. Specifically, I note that only a minority of housing output remains as an operating surplus for the structures' owners. It follows the prospect of productivity shocks to the mortgage and real estate industries have the potential to both move housing prices and non-residential consumption in the same direction, and that demand impulses are magnified in their effects on housing prices. A bust occurs when those impulses are realized later, or in a lesser magnitude, than originally anticipated. This view has testable implications for vacancy rates, net operating surplus, aggregate consumption patterns, net investment rates, and non-residential construction – all of which confirm the theory.

Casey B. Mulligan University of Chicago Department of Economics 1126 East 59th Street Chicago, IL 60637 and NBER c-mulligan@uchicago.edu Information technology has dramatically advanced over the years, and it is unreasonable to assume that today it will suddenly cease. But what types of production will be most transformed by future advances, when will the gains will be realized, and in what magnitude? This paper suggests that housing is one sector to be significantly impacted by technical progress, and that some part of the housing boom and bust 2000-2006 was a rational and efficient anticipation of such events.

The prospects for technical progress are exciting, but they are inherently uncertain. We do not yet know how to cure cancer, or how to make a cheap personal computer run at 100 Ghz. Nevertheless, we would be ill-advised to make investment decisions today under the assumption that lifetimes will remain constant, or that personal computers will always run at the same speed. Some of the inputs that are complementary with future technologies – those that are inelastically supplied in the short run – should begin to be accumulated today, even though the progress on which they rely has not yet occurred.

If the new technologies are realized earlier – or in greater magnitude – than originally anticipated, then the complementary inputs will enjoy a favorable and abnormal return. If the new technologies are realized later, or in lesser magnitude, then the complementary inputs will be under-utilized for a while and thereby earn an unfavorable return.<sup>2</sup> This paper modifies the existing economic theory of housing investment to include these ideas.<sup>3</sup> I conclude that at least part of the housing boom

<sup>&</sup>lt;sup>2</sup> The widely discussed "IT productivity paradox" acknowledges at least part of this process – that sometimes the gains from information technology investments are realized later, or in lesser magnitudes, than originally anticipated.

<sup>&</sup>lt;sup>3</sup> My model has similarities with models in the literature designed to jointly explain the employment cycle, stock market fluctuations, and news about aggregate productivity. See Beaudry and Portier (2006) and Christiano et al, (2008).

occurred because technology was expected to advance in ways that would make existing housing structures more productive.

When a bust occurs as a consequence of bringing the future into better (and less exuberant) focus, the boom time construction was, in hindsight, too much and too soon, but nonetheless the housing stock might not return to its pre-boom trend. As long as there was at least partial truth in the original anticipations – i.e., at least some productivity advance really did occur – then part of the construction boom was necessary and efficient, even with the benefit of hindsight. Housing prices must crash during the bust, but net housing investment may continue to be positive during the bust period. A "bubble" – that is, an increase in housing prices that occurred for no fundamental reason – would not have these properties. Thus, a bust period that keeps the housing stock and housing prices above their pre-boom trends means that housing fundamentals are in fact better, and thereby rejects the pure bubble theory.

Figure 1 shows the basic price and investment patterns of housing boom and bust to be explained. The left scale shows two monthly measures of the average value of housing properties, known as the "Case-Shiller Home Price Index" and the "OFHEO Home Price Index" divided by the PCE deflator and normalized to 100 in the first month shown (January 2000).<sup>4</sup> The right scale shows residential construction spending, expressed as a percentage of personal income. The bottom series measures tenant-occupied housing construction and the top series adds owner-occupied construction. The Figure shows that real residential construction grew as housing values did through 2005. Sometime between late 2005 and early 2006, both values and construction spending peaked and then declined through early or mid 2009. Since then, construction spending increased a bit, as did one of the measures of real home values (both value measures agree that the 2009 change was significantly less than the changes during 2007 and 2008). The Figure also shows that the large majority of the amount and changes in residential construction spending are for owner-occupied structures.

Figure 2 displays quantity indices (2000 = 100) for investment in three types of structures: owner-occupied residential, tenant-occupied residential, and non-residential.

<sup>&</sup>lt;sup>4</sup> Himmelberg et al (2005, p. 69) point out that same-property comparisons such as the two indices shown may exaggerate housing price appreciation because properties are improved over time, especially during the boom. Thus, I refer to them as indices of property values, rather than indices of housing prices.

The three types of structures investment stayed in about the same proportions 1993-2000. Both types of residential investment increased significantly 2001-2005, and thereafter fell back to pre-boom amounts. In contrast, investment in non-residential structures fell after 2000, and increased significantly 2005-2008. Because the boom increased both types of residential investment and reduced non-residential structures investment, the question addressed in this paper is why housing prices and construction increased 2000-2005, without particular attention to the distinction between owner-occupied and tenantoccupied housing.

The time patterns for housing prices, investment (both residential and nonresidential), and income flows qualitatively match my stochastic technology model, and are unlike market responses to a short run shift in housing demand. Thus, the bulk of the housing boom appears to be a response to expectations about the future, and the housing bust a revision of those expectations. Some of those expectations are related to technical change, although it is also possible that additional expectations related to the anticipation of government bailouts, and even irrational exuberance, added to – or even multiplied – the effects of prospects for technical progress.

Section I explains how much of housing output goes for banking services, real estate brokerage, and management, as opposed to the owner of the structure. Structures have a leveraged claim on housing output because they are less elastically supplied than are the housing sector's other inputs. Moreover, prospects for technical progress in the provision of the elastically supplied inputs should increase both housing prices and housing construction, especially for types of houses that are relatively intensive in the elastically supplied inputs. Section II amends the familiar "q-theory" model of housing investment with an elastically supplied intermediate goods sector. I measure time series for the net operating surplus and vacancy rates emphasized by the theory, and show how they suggest that optimism about the future fueled the housing boom, rather than tastes, technologies, or subsidies during the boom. Section III therefore looks at the market dynamics that would be an efficient response to good news about the future. It shows the expected size of the housing service demand shift – that is, the amount that the housing stock is expected to increase in the long run – that is implied by a given amount of short run housing price elevation. It shows the quantitative expectations about tastes and

technology that are required to generate large housing price increases in the short run and housing stock increases in the long run, and how to distinguish the theory from alternative theories of the housing cycle, leaving future research to answer the question of whether such expectations were reasonable. Section IV points out that real housing prices are still higher than they were before the housing boom, despite the boom's apparent legacy of overbuilding, which suggests that some of the boom's optimism still remains three years later.

#### I. Resource Use in the Housing Industry

#### I.A. National Accounting for Housing Output

A physical housing structure has value because it helps provide people with flows of shelter, privacy, and convenience. However, landlords and homeowners know that housing services flows are not produced with structures alone: also important are intermediate inputs of brokerage, management and banking services that can match and maintain the physical structures with the families who live in them and the investors who build them. As shown in the national accounts, banking, real estate brokerages, and others outside the construction industry annually supply hundreds of billions of dollars worth of the intermediate inputs to the housing industry.

Figure 3 shows the claims on housing output net of depreciation for 2006, as reported by Mayerhauser and Reinsdorf (2007).<sup>5</sup> Almost a third of housing output goes to intermediate goods and services. Another six percent goes to labor (largely management).<sup>6</sup> Thus, the entire rent paid by occupants does not go to the investor in the structure – much of it goes to the suppliers of other resources.

#### I.B. Prospects for Technical Progress

Some optimism about the future costs of intermediate inputs was warranted because applied information technology was rapidly advancing during the 2000s. Much

<sup>&</sup>lt;sup>5</sup> Figure 4 shows data for the tenant-occupied housing units only, because the labor of owner-occupants is not included in the national accounts. The results are similar for owner-occupied housing units, except that labor would be zero and the other items proportionally larger.

<sup>&</sup>lt;sup>6</sup> Henceforth, I do not conceptually distinguish intermediate inputs from compensation of employees.

of the banking, real estate, and property management value added relates to information. Bankers screen borrowers and value heterogeneous collateral. Hall and Woodward (2008) claim "Recent years have seen great improvements in data, especially the introduction of credit scores, which gave lenders new powers to forecast mortgage defaults and to adjust interest rates offered to prospective borrowers. In 1990, credit scores were rare; by 1996, they were standard." Perhaps lenders also expected to use information technology to better monitor and collect on loans, and therefore put subprime lending programs in place.<sup>7</sup>

Real estate brokers match heterogeneous families to heterogeneous properties; among other things, they help reduce vacancy rates. Real estate brokers might also expect to benefit from technical progress: Virtual Office Web sites and their technological descendants might significantly reduce the brokerage resources needed to match homes with the persons who value them most.

It is natural to model information-technology progress like this as sometimes having productivity effects only with a lag. After all, information technology investments in a number of other industries did little to enhance measured productivity in the short run, but nonetheless were thought to add to the investing firms' value.<sup>8</sup> The Department of Justice alleged in 2005 that information technology was reducing costs in the real estate brokerage business, but that anti-competitive practices up to that date had so far prevented many of those gains from being passed on to the consumer.<sup>9</sup> If true, but the anti-competitive practices would eventually be outweighed by either regulation or sufficiently vigorous entry, this would be another reason for optimism about the future costs of intermediate inputs (even while the current costs were not particularly low).

Residential real estate, mortgage, and lease transactions are small as compared to those in the business sector, so the former ought to be a lot more sensitive to transactions and administrative costs. Thus, both real progress and the prospect of additional progress should motivate more construction in the residential sector (especially units that, absent

<sup>&</sup>lt;sup>7</sup> Himmelberg et al (2005, p. 88) note the emergence of cost-reducing innovations in the mortgage market, although they do not consider that housing boom prices might have embedded some optimism about further progress in this direction.

<sup>&</sup>lt;sup>8</sup> Bharadwaj, Bharadwaj, and Konsynski (1999).

<sup>&</sup>lt;sup>9</sup> United States Department of Justice (2005).

technical progress, would be intensive in the costs of brokerage, management, etc.), and may even crowd out non-residential construction.

#### I.C. A Model of the Production and Distribution of Housing Output

Although intermediate inputs appear to be necessary, presumably they are poor substitutes for the structures themselves: one cannot significantly increase the effective square footage of his house by paying his banker or broker to work harder. I therefore model the production of housing services s(t) during period t as a Leontief function of the net stock h(t) of housing structures and the amount x(t) of intermediate inputs supplied:

$$s(t) = \min\{h(t), x(t)\}\tag{1}$$

I let  $p_x$  denote the purchase price of the intermediate inputs and  $\rho$  denote the rental rate of housing structures. Perfect competition in the market for housing services will mean that the rental rate *r* for housing services will equal the total cost of producing each unit of *s*:

$$r(t) = \rho(t) + p_x(t) \tag{2}$$

 $\rho(t)$  is the housing sector's operating surplus rate (gross of depreciation) because it is the difference between the rental rate for housing services and the rental rate for (non-structure) resources needed to provide those resources. Figure 3 shows that, after subtracting depreciation,  $\rho(t)$  is less than half of the total rent paid for housing services, so the often-cited result that "housing prices are the present value of rents received for housing services" will prove to be a poor approximation in my model.<sup>10</sup>

Moreover, equation (2) says that even 100 percent equity owners of housing structures have a leveraged claim on housing output: an increase in *r* holding  $p_x$  constant increases the ratio  $\rho/r$ . For example, the net operating surplus share shown in Figure 3 implies that a 10 percent increase in the housing rental rate *r* would increase net operating surplus by more than 20 percent. This leverage is not an consequence of financial

<sup>&</sup>lt;sup>10</sup>  $\rho(t)$  could be considered the rental rate for housing structures, which is only part of the cost of supplying housing. To avoid confusion, hereafter I reserve the term rent for the payments for housing services, and describe  $\rho(t)$  as the gross operating surplus rate.

arrangements – it does not exist because there may be multiple claims on the income paid to structures. Structures have a leveraged claim on housing output because they are less elastically supplied than are the housing sector's other inputs.<sup>11</sup>

The housing output in Figure 3 pertains to occupied structures; vacant housing produces no output. Conceptually, I treat this additional cost – a structure owner's risk of having no occupant to pay rent – as a cost like the intermediate inputs. With this convention, r(t) must be interpreted as the period t rent earned on occupied housing and  $p_x(t)$  as the cost of the intermediate inputs as well as the foregone rent on vacant housing.<sup>12</sup> The vacancy cost could be added to the intermediate goods slice of Figure 3, thereby decreasing the fraction of gross housing rent that goes to structure's owners net operating surplus.<sup>13</sup>

Thus, my model of housing prices and construction explicitly acknowledges the importance of intermediate inputs, and considers how: (a) the existence of intermediate inputs affects comparative statics with respect to other parameters and (b) anticipation of progress in the intermediate goods sector could elevate housing prices.

<sup>&</sup>lt;sup>11</sup> Many of the formal housing price and construction models do not recognize claims on housing output aside from structures' rent and (occasionally) property taxes. See, for example, Poterba (1984), Topel and Rosen (1988), Himmelberg et al (2005), Glaeser et al. (2008), and Iacoviello and Neri (2008).

<sup>&</sup>lt;sup>12</sup> Alternatively, the foregone rent could be moved to the left hand side of equation (2), which would then say that the average gross rent earned (including the zero rent earned by vacant units) equals the surplus rate for the structure owner plus the price of the intermediates x.

<sup>&</sup>lt;sup>13</sup> However, an offsetting adjustment would be to add to the net operating surplus slice of the Figure, due to the fact that 2006 (the year for which Mayerhauser and Reinsdorf (2007) gave a detailed breakdown of the claims on housing output) had a relatively low operating surplus.

#### **II.** Analytical Framework and the Role of Anticipation

I assume that there exists a representative consumer,<sup>14</sup> with preferences over continuous-time stochastic processes for housing services s(t) and other consumption items c(t). Given the near constancy of the housing service share of personal consumption expenditures, it is both realistic and convenient to consider a logarithmic utility function:<sup>15</sup>

$$U = E\left[\int_{0}^{\infty} e^{-\theta t} \left[\ln c(t) + \eta(t)\ln s(t)\right]dt\right]$$
(3)

where  $\theta$  is a constant preference parameter and  $\eta(t)$  is a stochastic process for preference parameters reflecting the state of aggregate housing service demand.<sup>16</sup> Expectations are with respect to realization of time paths for tastes and technology (more details below).

In order to abstract from the details of wage setting and labor supply, I assume that the economy is endowed with a given stream w(t) of labor product. The labor product together with the production made possible by nonresidential capital k(t) is available for four types of spending: housing structure investment, housing intermediates, non-residential investment, and other consumption items.

$$w(t) + Ak(t) = h(t)Sf(I(t)/h(t)) + B(t)x(t) + \dot{k}(t) + c(t)$$
  
$$\dot{h}(t) = I(t) - \delta h(t)$$
(4)

where A is the (constant) marginal product of non-residential capital net of depreciation and B(t) is the time t marginal rate of transformation between housing intermediates and consumption goods. Dots denote derivatives with respect to time.

<sup>&</sup>lt;sup>14</sup> This assumption facilitates discussion of efficiency and aggregate wealth effects, but is unnecessary for the other results.

<sup>&</sup>lt;sup>15</sup> See also Davis and Ortalo-Magne (2010).

<sup>&</sup>lt;sup>16</sup> A straightforward embellishment of the model would have multiple housing stocks that differ by location. If preferences were shifting from one region to another, then such a model would imply that the growing regions (in terms of housing preference) have fundamental prices that are above the national average.

I(t) is gross residential investment. *Shf* is the cost of that investment, inclusive of installation or "adjustment" costs, with f', f'' > 0. *Shf* is homogenous of degree one in h and I, which implies that housing investment is infinitely elastically supplied in the long run.<sup>17</sup> The convexity of f reflects the imperfect short run supply of resources to construction activities.

*S* is an exogenous shifter of the cost or productivity of investment activity. In principle, *Sf'* would be reflected in the measured price of housing investment relative to the price of consumption goods. In practice, changes in relative shadow prices of investment may not be fully reflected in price measures because the latter do not adequately account for waiting times and other convex costs of adjusting capital stocks. I assume that adjustment costs in excess of those reflected in measured investment prices are recorded in the national accounts as spending on housing intermediate goods. I normalize  $f'(\delta) = 1$ .

Integration of the non-residential capital accumulation equation (plus a transversality condition for nonresidential capital) helps describe the intertemporal production set for this economy:

$$\int_{0}^{\infty} e^{-At} \Big[ c(t) + B(t)x(t) + h(t)Sf(I(t)/h(t)) \Big] dt \le k_0 + \int_{0}^{\infty} e^{-At}w(t)dt$$
(5)

In words, the present value (using the net marginal product of non-residential capital to discount) of spending on non-housing consumption, housing intermediates, and housing investment costs must (for each realizations of the time paths for tastes, technology and subsidies) equal the initial value of non-residential capital plus the present value labor of income. Henceforth, I assume that the rate of time preference  $\theta$  equals the net marginal product *A* of non-residential capital. This implies that the efficient non-housing

<sup>&</sup>lt;sup>17</sup> Some previous studies, such as Poterba (1984), Topel and Rosen (1988), and Glaeser at al (2008), assume that investment costs are not homogeneous, so that housing is less-that-perfectly elastically supplied even in the long run. Because quantitative predictions for housing prices depend on the magnitude of the supply response and I want to be conservative as to the amount of price elevation that can derive from fundamentals, I follow Summers (1981) and Hayashi (1982) in assuming homogeneity.

consumption path is constant over time, except for jumps at moments in which information arrives about tastes, technology, or subsidies.

I characterize fundamental determinants of housing prices in two steps. First, I calculate the allocation of housing and other goods over time, alternatively assuming either that (a) the allocation is efficient or (b) that the only distortion in the system appears through the intermediaries' price (more on this below). Second, I use the marginal conditions from that allocation to calculate the time path of operating surplus for a hypothetical owner of one unit of housing. After characterizing fundamental housing prices, I relate housing price comparative statics to aggregate wealth effects.

#### II.A. Housing Prices and Housing Investment

Assume perfect foresight, for the moment. The system of differential equations below helps characterize both the efficient allocation and allocations in which the supply of housing intermediates is distorted (but all other margins are efficient):

$$\dot{h}(t) / h(t) = \varphi(q(t) / S) - \delta$$

$$\dot{q}(t) = \left[A + \delta - \varphi(q(t) / S)\right]q(t) + B(t)(1 - \sigma(t)) + Sf\left(\varphi(q(t) / S)\right) - \frac{\eta(t)c}{h(t)}$$

$$q(t) = \int_{t}^{\infty} e^{-(A+\delta)(\tau-t)} \left[\frac{\eta(\tau)c}{h(\tau)} - B(\tau)\left(1 - \sigma(\tau)\right) - Sf\left(\frac{I(\tau)}{h(\tau)}\right) + \frac{I(\tau)}{h(\tau)}Sf'\left(\frac{I(\tau)}{h(\tau)}\right)\right]d\tau$$

$$f'(\varphi(q)) = q$$
(6)

where  $\sigma(t)$  is the date *t* subsidy rate for housing intermediates (equal to zero for the efficient allocation).  $\varphi$  is "housing investment supply curve": the inverse of the marginal investment cost function *f'*. For the moment, *q* is just a co-state variable defined for analytical convenience. Once paths for *h* and *q* have been calculated, a path for housing investment is calculated as  $I(t) = \varphi(q(t)/S)$ .

It is tempting, but incorrect, to interpret A as "the interest rate." A is the annual opportunity cost of residential investment expenditure: the marginal product of non-residential capital net of depreciation.<sup>18</sup> Subsidies, liquidity considerations, and capital

 $<sup>^{18}</sup>$  Non-residential capital income taxes are not included in my model, but as an investor's opportunity cost *A* would properly be measured net of taxes.

market frictions may cause *A* to differ from, say, mortgage rates. Some of those considerations are captured by the model's "subsidy" parameter  $\sigma(t)$ .<sup>19</sup> As shown below, comparative statics with respect to the subsidy parameter also aid understanding of comparative statics with respect to the technology parameter *B* because the latter can be decomposed into a substitution effect represented by a change in  $\sigma$  ( $\sigma$  affects non-housing consumption *c* only through the time path for the housing stock – see the resource constraint (5)) and a pure wealth effect represented by offsetting changes in  $\sigma$  and *B*.

Consider the competitive equilibria that support the allocations described above. That is, the time paths for housing service rents r(t), rental rates of housing structures  $\rho(t)$ , housing investment prices  $p_I(t)$ , and housing intermediate prices  $p_x(t)$  that would cause consumers of housing to demand the available housing, owners of housing structures to allow their structure to be occupied, and suppliers of housing intermediates to receive the marginal cost of supplying intermediates and incurring the adjustment costs. In this case, housing services would rent at the rate r(t) that covered the structure's gross operating surplus  $\rho(t)$  and the payment of intermediates  $p_x(t)$  (the equation (2) above).<sup>20</sup> The housing service rental rate r(t) would also be equal to consumers' marginal rate of substitution between housing services and other consumption, which, given the Cobb-Douglas utility function, is  $\eta(t)c/h(t)$ .

$$p_{x}(t) = B(t)(1 - \sigma(t)) + \left[ f\left(\frac{I(t)}{h(t)}\right) - \frac{I(t)}{h(t)} f'\left(\frac{I(t)}{h(t)}\right) \right] S$$

$$r(t) = \frac{\eta(t)c}{h(t)} = \rho(t) + p_{x}(t)$$

$$p_{I}(t) = q(t)$$
(7)

where the term in square brackets is the effect of the housing *stock* on average adjustment costs.

<sup>&</sup>lt;sup>19</sup> For more detailed studies of the determinants of  $\sigma(t)$ , see Hendershott and Slemrod (1983), Mayer (1993), Himmelberg et al (2005).

<sup>&</sup>lt;sup>20</sup> Whether suppliers of housing intermediates earn positive, negative, or zero profits depends on the level of the adjustment cost function, which does not affect any of the comparative statics in this paper except where noted below.

The competitive equilibrium interpretation of the co-state variable q(t) is as the date *t* purchase price of a unit of housing, because it is equal to both the price of new housing and the present discounted value of the structure's rental rates received by each unit of existing housing (recognizing that, due to depreciation, a unit housing at time *t* will be worth less than a unit of housing after date *t*).

$$q(t) = \int_{t}^{\infty} e^{-(A+\delta)(\tau-t)} \rho(\tau) d\tau$$

$$\rho(\tau) = r(\tau) - p_{x}(\tau) = \frac{\eta(\tau)c}{h(\tau)} - B(\tau)(1-\sigma) - \left[ f\left(\frac{I(\tau)}{h(\tau)}\right) - \frac{I(\tau)}{h(\tau)} f'\left(\frac{I(\tau)}{h(\tau)}\right) \right] S$$
(8)

Thus, the top equation (6) says that housing investment is a monotone function of housing prices, where the shape and position of that function depends on the supply curve for housing investment. This theoretical result is well known (Poterba, 1984; Topel and Rosen, 1988). Interestingly, the supply curve for housing investment was stable enough during the housing boom and bust 2000-2008 to make this monotone relationship obvious in the data, as in Figure 1.

#### **II.B.** Current Versus Future Fundamentals

Equation (8) shows that equilibrium housing prices and investment depend on current and expected future tastes, technologies, and subsidy rates. A greater taste for housing (either in the present or in the future), more productive intermediates (either in the present or in the future), or greater subsidies (either in the present or in the future) are associated with higher prices (in the present) for housing structures.

The flow  $\rho(t) - \delta$  of net operating surplus per unit stock in the housing sector can be used to gauge the relative importance of current versus future fundamentals for explaining the current housing price. Future fundamentals that raise price affect the current net operating surplus  $\rho(t) - \delta$  only through the current stock h(t), and thus would tend to *reduce*  $\rho(t)$ . High current housing demand, currently productive intermediate inputs, or a high current subsidy rate each *increase*  $\rho(t)$ .<sup>21</sup>

Figure 4 displays the housing sector's aggregate annual net operating surplus per dollar of housing stock at the beginning of each year, for the years 1990-2008.<sup>22</sup> Note that the housing stock is measured in real terms – not in terms of replacement cost – so that the low rates during the boom are due a high real quantity of housing, not a high market value of that housing.<sup>23</sup> The second series uses the 1990-2000 linear trend housing stock as the denominator. It also falls as housing boomed, but less than then series using the actual stock: part (but not all) of the reduced net operating surplus rate is due to an increase of the stock above trend.<sup>24</sup>

The net operating surplus depends on rental income, which is the product of the rental rate for an occupied property and the fraction of properties that are occupied. Thus, the low net operating surplus rates since 2002 shown in Figure 4 might reflect either low rental rates for occupied properties or high vacancy rates, or both. Both possibilities are consistent with the hypothesis that expectations about the future fueled the construction of housing that was not highly demanded at the time of construction, but Figure 5 helps to gauge the relative importance of rental and vacancy rates for pulling down the net operating surplus. Figure 5 displays an index (year 2000 = 1) of the ratio of real housing consumed to the real stock in place at the beginning of the year. Because the BEA calculates real housing consumption according to structures occupied (by either tenants or owners), this ratio changes because the vacancy rate changes. The other two series in Figure 5 are indexes of the occupancy rate (one minus the vacancy rate) from the Census Bureau.

<sup>&</sup>lt;sup>21</sup> A high current portfolio demand for housing (that is, an investor's demand to hold housing as an asset in his portfolio, regardless of whether he lives in it) would also tend to reduce  $\rho(t)$  as a greater number of investor-landlords compete for a constant number of tenants.

<sup>&</sup>lt;sup>22</sup> The year 2005 value is interpolated because its net operating surplus is very low due to the extraordinary "depreciation" during Hurricane Katrina.

<sup>&</sup>lt;sup>23</sup> Net operating surplus is deflated with the deflator for Personal Consumption Expenditures. Davis, Lehnert, and Martin's (2008) "rent-price" ratio, calculated from the Decennial Census of Housing, is related to my net operating surplus rate except that they measure the stock as a market value, and (aside from monthly utilities) do not deduct any of the intermediate inputs from gross housing rents.

<sup>&</sup>lt;sup>24</sup> Because owner-occupied housing output is imputed from rental rates of tenant-occupied housing, a shift in demand toward owner occupied could reduce rental rates on tenant-occupied housing and thereby give the false impression of reduced rental rates for the housing sector overall. However, we can at least conclude that tenant-occupied demand was low during this period of relatively high construction in the tenant-occupied sector (see Figure 2).

Clearly occupancy rates were low during the boom: 4 percent lower according to the BEA measure, and 1-2 percent lower by the other two. The low occupancy rates during the housing boom suggest that a number of homes were built with characteristics (such as location) not particularly desired by the population at the time, but may have been anticipated to serve future demands.<sup>25</sup> To the degree that occupancy rates fell less than three percent, they are not enough by themselves to explain the 10+ percent drop in net operating surplus shown in Figure 4 because the elasticity of net operating surplus with respect to occupancy rate is about 3.<sup>26</sup>

In summary, low occupancy rates and low income flows to structures owners since 2000 suggest that the housing boom was fueled by expectations about the future, and not demand or supply conditions during the boom itself. The anticipation of favorable future fundamentals (to the extent that the future is not so distant than current housing will already be fully depreciated) raises current housing prices and creates housing investment even before those favorable fundamentals are realized. As the stock accumulates leading up to that realization, housing service rents fall and/or property vacancy rates increase.

#### **III. Market Dynamics in Response to Future Fundamentals**

#### III.A. Information Flows

A bust following a boom could occur because changes in technology or demand happened later that originally anticipated, or in lesser magnitude. The theory of dynamic programming suggests that these two possibilities have a lot in common, so for concreteness I model only the latter.

Figure 6 shows how I model the parameters shifts that might describe a housing boom and bust. Prior to time zero, market participants believe that the taste, technology, and subsidy parameters  $\eta(t)$  and  $B(t)[1-\sigma(t)]$  will remained forever constant. At time zero

 $<sup>^{25}</sup>$  The 2000-2007 increase in the real quantity consumed is 78% of the increase in the total real stock available.

<sup>&</sup>lt;sup>26</sup> If intermediate inputs were elastically supplied in the short run, unoccupied structures use intermediate inputs too, and net operating surplus were about a third of gross housing output (Figure 4 shows that net operating surplus is about half of *net* housing output), then a one percent decline in the housing occupancy rate would reduce the net operating surplus rate by three percent.

(circa 2002), they are informed that these parameters will shift once and for all at a specific date T > 0 (circa 2006). Between time zero and time T, they know that the time T shift will be in the direction of more demand and more productivity, but they do not know the magnitude. With probability  $\pi$ , the shifts will be large: in the amounts  $\Delta_{\eta}^{H}$  and  $\Delta_{B}^{H}$ , respectively. With probability 1- $\pi$ , the shifts will be small: in the amounts  $\Delta_{\eta}^{L}$  and  $\Delta_{B}^{L}$ , respectively.

As I prove below, the time interval (0,T) will be a "boom" period of high and rising housing prices and rates of housing investment. Housing prices will jump at time *T*, with the direction of the jump determined by how the uncertainty was resolved. With probability 1- $\pi$ , housing price will jump down and housing investment rates will be discretely lower than they were during the prior boom – a "bust."

With these information flows, each of the endogenous variables follows one of two possible time paths, depending on the information realized at time *T*. I let superscripts denote the realization: for example,  $h^L(t)$  denotes the time path for the capital stock after time *T* when the low parameter shifts were realized. The evolution of the housing price *q* prior to time *T* is described by:

$$if \ t < T:$$

$$\dot{q}(t) = \left[A + \delta - \varphi(q(t) / S)\right]q(t) + B(1 - \sigma) + Sf\left(\varphi(q(t) / S)\right) - \frac{\eta c}{h(t)}$$

$$q(t) = \int_{t}^{T} e^{-(A + \delta)(\tau - t)} \left[\frac{\eta c}{h(\tau)} - B(1 - \sigma) - Sf\left(\frac{I(\tau)}{h(\tau)}\right) + \frac{I(\tau)}{h(\tau)}Sf'\left(\frac{I(\tau)}{h(\tau)}\right)\right] d\tau + e^{-(A + \delta)(T - t)}q(T)$$
(9)

After time *T*, the new tastes, technology, and subsidy rate determine the evolution of *q*:

$$if \ t > T, \ j = L, H:$$

$$\dot{q}^{j}(t) = \left[A + \delta - \varphi\left(q^{j}(t) / S\right)\right]q^{j}(t) + B(1 - \sigma) - \Delta_{B}^{j} + Sf\left(\varphi\left(q^{j}(t) / S\right)\right) - \frac{(\eta + \Delta_{\eta}^{j})c^{j}}{h^{j}(t)} \quad (10)$$

$$q^{j}(t) = \int_{t}^{\infty} e^{-(A + \delta)(\tau - t)} \left[\frac{(\eta + \Delta_{\eta}^{j})c^{j}}{h(\tau)} - B\left(1 - \sigma\right) + \Delta_{B}^{j} - Sf\left(\frac{I^{j}(\tau)}{h^{j}(\tau)}\right) + \frac{I^{j}(\tau)}{h^{j}(\tau)}Sf'\left(\frac{I^{j}(\tau)}{h^{j}(\tau)}\right)\right] d\tau$$

Rational expectations requires that consumption and housing prices before and after time T fit together according to the probability of each post-T outcome:

$$q(T) = q^{L}(T) + \pi \frac{c}{c^{H}} \left[ q^{H}(T) - q^{L}(T) \right]$$

$$1 = (1 - \pi) \frac{c}{c^{L}} + \pi \frac{c}{c^{H}}$$
(11)

At this point, it is straight-forward to add possible effects of both irrational exuberance, and government bailouts of the mortgage market, by interpreting the parameter  $\pi$  as exceeding the objective probability of realizing the favorable fundamentals.<sup>27</sup>

#### IV.B. The Case of Inelastic Supply

The less elastic is the supply of housing investment, the more that information about fundamentals affects housing prices rather than housing investment (Glaeser, Gyourko, and Saiz, 2008). As we see from Figure 1, housing investment did change over time – supply was not completely inelastic – so the assumption (for the sake of argument) that housing supply was expected to be forever fixed can be used to obtain an upper bound on the amount by which anticipated progress elevates housing prices and a lower bound on the amount prices would fall if and when bad news about that progress arrived. Moreover, the fixed supply case offers closed form pricing formulas, which (according to the numerical results shown below) can, with a small adjustment, help quantify price effects in the elastic supply case. After working out the fixed supply case in this subsection, I then present some calculations of how market outcomes would differ with elastic supply, and with some degree of anticipation of the technical change.

The inelastic supply case is described by:

$$f(I/h) = \begin{cases} I/h & \text{if } I/h \le \delta \\ \infty & \text{if } I/h > \delta \end{cases}$$
(12)

<sup>&</sup>lt;sup>27</sup> Suppose, for example, that the government were expected to absorb a fraction  $\tau \in [0,1]$  of the capital losses experienced when housing prices jump down, but shares in none of the capital gains. Then the rational expectations pricing formula would be exactly (11) with  $\pi = \pi'/[1-(1-\pi')\tau] \ge \pi'$ , where  $\pi'$  is the objective probability of realizing the favorable fundamentals.

As long as investment does not add to the net stock, each dollar of investment is a dollar of foregone consumption (that is, no adjustment cost). Additions to the net stock are very costly.<sup>28</sup>

Assuming that the housing stock began at a value h(0) that was not too high, the stock will optimally remain constant over time. Assuming further that non-housing consumption is constant,<sup>29</sup> the time *t* value of the housing stock is:

$$q(t) = \frac{\eta c / h(0) - \left[ (1 - \sigma)B + \delta S \right]}{A} + \frac{e^{-A(T - t)}}{A} \left[ \frac{\eta c}{h(0)} \frac{E(\Delta_{\eta})}{\eta} + E(\Delta_{B}) \right] \quad t < T$$
(13)

where *E* denotes the expectations operator with respect to the probability  $\pi$ . To calculate housing values after time *T*, replace the expectations in equation (13) with their realized values and replace *T* with *t*.

The first term in equation (13) shows the relationship between housing prices and the quantity of housing when tastes and technology are constant over time, and is therefore the long run demand for housing structures. In particular, both the demand for housing services (parameterized by  $\eta$ ) and the technology for housing intermediates (parameterized by (1- $\sigma$ )*B*) affect the demand for housing structures.

Three price changes calculated from equation (13) might be compared with data from the housing cycle: (a) the time zero proportional price impact of the information that  $\Delta_{\eta}$  and  $\Delta_{B}$  are different from zero, (b) the level of prices q(T) at the end of the boom period, and (c) the size of the log price drop at the end of the boom period, conditional on realizing the smaller amount of progress. The first of the three are obtained by taking the ratio of the second term in equation (13) to the first term:

<sup>&</sup>lt;sup>28</sup> With both depreciation and fixed supply, there is a gap between the value of a structure's cash flows and the cost of replacement. Here I assume that homeowners own the right to replace their housing at constant cost *S*, so a permanent \$1 increase in the operating surplus is worth 1/A to the homeowner. In this case, there is no housing that sells for *S* in the marketplace. This assumption is irrelevant in the case of endogenous supply, because supply adjusts so that new and (depreciation adjusted) existing housing sell for the same price.

<sup>&</sup>lt;sup>29</sup> As seen from the intertemporal production set (5), expectations of the taste and subsidy rate parameters have no effect on non-housing consumption because (in the fixed supply case) they do not affect the path of the housing stock. Only real productivity affects consumption, in which case the price impacts would be even larger because of the wealth effect on demand (see also Appendix III).

$$\frac{q(0) - q(0)\big|_{\Delta_{\eta} = \Delta_{B} = 0}}{q(0)\big|_{\Delta_{\eta} = \Delta_{B} = 0}} = e^{-AT} \frac{cE(\Delta_{\eta}) + h(0)E(\Delta_{B})}{\eta c - [(1 - \sigma)B + \delta S]h(0)}$$
(14)

where the numerator is the expected change in the net operating surplus at date *T*, and the denominator is the net operating surplus at date t < 0. Equation (14) shows how much housing prices jump (with fixed supply) when it is learned at time zero that time *T* will deliver some progress (amount unknown). In order to see how much prices are elevated by time *T* compared to what they were prior to time zero, just evaluate (14) at T = 0.

Equation (14) shows how information about future fundamentals increases housing prices even before those fundamentals are realized (as measured by the net operating surplus per unit housing). Moreover, the positive derivative of that function with respect to t shows that information causes prices to appreciate (prior to date T) even if no new information arrives in the interim and current fundamentals are unchanged. This is a basic implication of rational expectations: the market anticipates housing price appreciation in order to make structures' owners willing to retain ownership through out the interval (0,T), despite the fact that net operating surplus is low during those years as compared to the net operating surplus anticipated after date T. Thus, a rational market that anticipates improvements in net operating surplus must have housing prices that are (a) high relative to current net operating surplus and (b) appreciate over time prior to any realized improvement in net operating surplus.

If the small amount of progress (rather than the large amount) is realized at time *T*, then house prices will drop:

$$\frac{q(T) - q^L(T)}{q(T)} = \pi \frac{(\Delta_\eta^H - \Delta_\eta^L)c + (\Delta_B^H - \Delta_B^L)h(0)}{\eta c - [(1 - \sigma)B + \delta S]h(0)}$$
(15)

Perhaps most interesting for today is the housing price percentage change from before the boom started (the first term in the equation (14)'s sum) and just after the bust occurs:

$$\frac{\Delta_{\eta}^{L}c + \Delta_{B}^{L}h(0)}{\eta c - \left[(1 - \sigma)B + \delta S\right]h(0)} \ge 0$$
(16)

The fixed supply case says that  $q^{L}(T)$  does not fall below its pre-boom value, because fundamentals are not any worse than before. As I show below, this is not necessarily true with endogenous supply, because the anticipation of the possibility of best outcome (state *H*) may lead to the accumulation of housing before time *T* beyond what would ever be accumulated if state *L* had been known with certainty.

The magnitudes of the price changes shown in equations (14), (15), and (16) can be large. An anticipated change in demand by itself ( $\Delta_B = 0$ ) has a greater than proportional effect on housing prices (unless that change is sufficiently far in the future) because the owners of residential structures have a leveraged claim on the value of housing output  $\eta c$ .

Table 1 offers some impressions of the possible size of a housing cycle caused by prospects for technical progress and housing demand increases. The first several rows of Table display information about each of the relevant model parameters, and the columns vary according to what is assumed about those parameters. One of the key parameters is the pre-boom magnitude of spending on intermediates: Figure 3 shows that net housing output is 47% net operating surplus and 36% intermediates (inclusive of spending on labor), so compensation of intermediates is 43% of the operating surplus gross of that compensation and 77% of net operating surplus (0.77 = 36/43).

The last rows of the table show model outcomes: the expected shift in housing structure demand and the peak price elevation. The housing structure demand shift is defined to be the permanent increase in the log housing stock that would be consistent with a housing price that was constant and equal to *S*. The peak price elevation is defined to be the amount the equilibrium housing price exceeds *S* at the moment before market participants learn that they overestimated the amount of progress that would occur. Under fixed supply, this price peak elevation is equation (14) evaluated at T = 0, and is decomposed into preference and technology pieces as shown in the formula. The Table's last row offers a back-of-the-envelope estimate of the price peak with an endogenous

supply response by taking 52 percent of the fixed supply case (that is, the price elevation shown in the row above).

The first column of the Table shows the maximum possible housing price cycle that derived from services-demand-adjusted technical progress alone, because it assumes not only fixed supply but also that the progress is perfectly certain to eliminate all spending on intermediate inputs. The estimate is services-demand-adjusted because it holds fixed non-housing consumption c and the pre-subsidy technology B and assumes that  $\sigma$  is expected to go to one with certainty.<sup>30</sup> In this case, the derived housing structures demand has shifted 0.41 log points and the housing price peak is 77 percent above what prices were before the progress was anticipated: see the second-to-last row of the table. The second column shows the case in which elimination of intermediates spending is accompanied by a 20% housing services demand increase – one can interpret this as a rough estimate of the wealth effect of a change in B that is not present with a change in the subsidy rate – and the peak housing price with fixed supply 122 percent (structures demand shifts 0.59 log points). With elastic supply, the peak housing price elevation would be about 63 percent (see the last row of the second column). Of course, technical progress would not be so dramatic as to fully eliminate intermediates spending and would not be rationally expected to occur with perfect certainty, so the model suggests that prospects for technical progress alone (even including a large wealth effect that feeds back into housing demand) would make a housing cycle with a price peak significantly below 63 percent.

The remaining columns of the Table acknowledge uncertainty in two ways. First, one possible outcome is a much smaller change in the housing subsidy or technology parameters ( $\sigma = 0.25$  rather than  $\sigma = 1$ ); the probability  $\pi$  of the larger change is assumed to be 0.67, rather than the probability one assumed in the first two columns. Second, housing market participants are assumed to bear only part (namely, half – see the Table's second row indicating the "crash subsidy rate") of the housing capital loss that will occur in the state that the technological progress is not realized, which has the effect of distorting the objective probability of favorable progress.<sup>31</sup> The third "acknowledge

<sup>&</sup>lt;sup>30</sup> Recall from above that the technology parameter *B* has a wealth effect that the subsidy rate does not.

<sup>&</sup>lt;sup>31</sup> See footnote 27.

technology uncertainty" column shows that the net effect of these two forces is for housing prices to peak at 110 percent (57 percent with elastic supply), rather than 122 percent. In other words, fixed supply and a 67% chance of eliminating all intermediate inputs means that prices peak at 110 percent the moment before market participants learn that they over estimated the amount of progress that would occur.

"Imperfect progress" is modeled in the fourth column by assuming that the bestcase subsidy rate is  $\sigma = 0.75$ , rather than  $\sigma = 1$ . In other words, the best case progress has an elimination of 75% of intermediates expenditure, rather than 100%. This puts the peak price elevation at 95% with fixed supply and about 49% with elastic supply.

The fifth "acknowledge demand uncertainty" column assumes that the low state has not only less technological progress, but also no demand shift. Of all of the columns, it is probably the best indicator of how much realistic expectations about technical progress could elevate housing prices: it puts the peak price elevation at 86% with fixed supply and about 45% with elastic supply.

There has been much discussion of how anticipation of possible government bailouts of AIG, mortgage lenders, and underwater mortgage borrowers contributed to the housing boom. The final column of the Table suggests that those bailouts might explain significantly less than half of the price boom, because with a zero crash subsidy rate the peak price elevation is still 75%.

The Table shows how the prospect of a significant reduction in the cost of housing intermediates, and/or a modest increase the demand for housing services can significantly shift the derived demand for housing structures and thereby significantly elevate prices. It follows that the derived demand for housing structures has to shift significantly in order for housing prices to rise significantly. Figure 7's green series shows the equilibrium relationship between peak price elevation and the amount of the structures demand shift, assuming for the moment that the shift derives from expected changes in technology, not tastes. To give the reader the opportunity to consider smaller derived demand shifts than shown in Table 1, the horizontal axis is limited to the range [0,0.5] and some of the smaller shift-price elevation pairs from Table 1 are displayed as

boxes in the Figure.<sup>32</sup> The green series shows how, with fixed supply, even a derived demand shift as small as 10 percent could elevate housing prices more than 20 percent.

#### III.C. Supply Responses: Unanticipated Fundamentals

At the other extreme of fixed supply is the model with no adjustment costs even in the short run (f(I/h) = I/h). In this case, the housing stock is unchanged until date *T*, at which time it jumps immediately to its new steady state.<sup>33</sup> Neither net operating surplus nor housing prices are impacted by either the anticipation or realization of changes in fundamentals.

The actual impact on net operating surplus and housing prices depends on where between these two extremes lies the actual path for the housing stock. This is easiest to see in the case  $(\Delta_{\eta}^{\ H} = \Delta_{\eta}^{\ L} \text{ and } \Delta_{B}^{\ H} = \Delta_{B}^{\ L})$  in which the future values of the fundamentals are fully revealed at time zero. Figure 8 shows three cases: inelastic supply (infinite marginal adjustment cost), no adjustment costs, and finite adjustment costs. The last case is based on the integral (9) calculating housing prices, and shows how supply responses mitigate the housing price impact of future fundamentals. The positive boom time (before date *T*) impact on the housing stock means a negative near term impact on housing service rents and therefore the net operating surplus rate. This reduces price impacts relative to the fixed supply case. The impact on the path of the housing stock after date *T* is positive relative to the fixed supply case and negative relative to the perfectly elastic supply case. This proves that net operating surplus at each date is less than it would be with fixed supply, so that the price impact is less than it would be with fixed supply. The price impact has to be positive net investment to be optimal.

In order to say more about the price impact, more has to be shown about the optimal path of the housing stock that solves the dynamical system (6). When model parameters are known to be constant over time and non-housing consumption is held constant, optimal housing prices are defined to be those on the one dimensional stable

<sup>&</sup>lt;sup>32</sup> Some of the boxes are not on the green curve because they reflect housing service demand shifts as well as intermediates cost reductions.

<sup>&</sup>lt;sup>33</sup> The size of the jump depends on whether the fundamentals change a large amount (state H) or a small amount (state L).

manifold (a.k.a., "stable arm" or "saddle path") of that system. Those allocations below that manifold (in the [h,q] plane) involve zero housing in finite time, which is not efficient because of the Inada condition on the utility function. Those above the stable manifold violate the transversality condition of optimization.<sup>34</sup>

When new fundamentals are realized immediately (T = 0), the terms B(t),  $\eta(t)$ ,  $\sigma(t)$ , and S(t) are constant over time  $t \ge 0$ . The system is stationary with a single steady state:

$$\frac{(\eta + \Delta_{\eta})c}{h_{ss}} = \left[A + f\left(\delta\right)\right]S + B(1 - \sigma) - \Delta_{B} \equiv r_{ss}$$

$$q_{ss} = S$$
(17)

Figure 9 is a phase diagram in the [h,q] plane showing the dynamics of the stationary system for initial values of h and q that do not coincide with  $(h_{ss},q_{ss})$ . The optimal path is the stable manifold of this system – the union of the only two time paths (one from the left and the other from the right) that converge to the steady state. If the fundamentals suddenly and permanently changed at time 0 in a way that left h(0) less than the new steady state, the housing price q would jump up to the point on the stable manifold that corresponded to h(0), and then the system would move along that manifold toward the steady state.<sup>35</sup> Thus, quantifying the slope of the stable manifold is essential for quantifying price impacts of changes in fundamentals.

Appendix I shows how to globally calculate the stable manifold to any desired degree of precision. A closed form price impact formula is available when the changes in fundamentals are small and the housing stock began in the steady state (with respect to the previous fundamentals),

$$dq = \lambda \left[ d \ln \eta - \frac{d[B(1-\sigma)]}{r_{ss}} \right]$$

$$\lambda \equiv \frac{S}{2\varphi'(1)} \left[ \sqrt{A^2 + 4r_{ss}\varphi'(1)/S} - A \right]$$
(18)

<sup>&</sup>lt;sup>34</sup> More below on violation of the transversality condition of optimization. Given my application of the model to the housing boom, for simplicity I do not include an non-negative gross investment condition. If I did, other arguments may be needed to rule out paths that perpetually have zero gross investment.

<sup>&</sup>lt;sup>35</sup> See Summers (1981, Figure 2) or Poterba (1984, Figure I).

 $\varphi'$  is the inverse of the slope of the marginal adjustment cost function;  $S/\varphi'$  is the convexity of adjustment costs.  $d\ln\eta$  and  $d[B(1-\sigma)]$  denote small permanent parameter changes that are effective immediately at time zero.

Assuming that T = 0, the price responses in the presence of adjustment costs (quantified by the parameter  $\lambda$  evaluated at some finite  $\varphi'$ ) can be compared to price responses with fixed supply merely by examining the relationship between the parameter  $\lambda$  and the degree of adjustment cost convexity  $S/\varphi'$ , holding fixed A,  $\delta$ , and  $r_{ss}$ . This amounts to holding fixed the steady state while changing the slope of the stable manifold in Figure 9. For this purpose, numerical values are assumed for the marginal product of non-residential capital, the depreciation rate, housing expenditure share, and the relative importance of intermediate inputs as shown in Table 2. Figure 10's blue curve shows the result: the more convex are adjustment costs (measured on the left vertical axis), the larger is the price response.

Adjustment cost convexity  $S/\phi'$  can also be expressed in terms of an elasticity of response of investment intensity I/h with respect to the housing price q, which has been estimated in the literature, because the elasticity is  $\phi'/\delta$  in the model's steady state.<sup>36</sup> Figure 10's red curve shows the unsurprising result that the housing price impact falls with the degree to which housing investment responds to prices. Figure 10 also includes a green tick mark on its right scale to indicate the median elasticity among Topel and Rosen's (1988) empirical estimates of 1.575 (see their Table 3), which corresponds to a price impact of an immediate and unanticipated permanent shock that is 52 percent of the fixed supply case. I have truncated Figure 10's right vertical scale at 4.0, which closely corresponds with the largest elasticity estimate displayed by Topel and Rosen; the red curve shows how an elasticity as large as 4 corresponds to a price impact as small as 37 percent of the fixed supply case. In other words, the Topel and Rosen estimates suggests that the price impact is at least 37 percent, and reasonably estimated as 52 percent, of the fixed supply case. This is the basis for Table 1's assumption for converting fixed supply price impacts into elastic supply price impacts using a multiplicative factor of 0.52.

<sup>&</sup>lt;sup>36</sup> Recall that  $\delta$  is the annual housing depreciation rate.

The stable manifold is a kind of long run "derived demand curve" for housing structures because, as long as the model parameters are expected to be constant, it indicates a permanent, downward sloping, relationship between the housing stock and housing prices. Moreover, as long as the model parameters remain constant, the system's dynamics involve moving along that "demand curve" curve toward the point at which housing prices are at their long run value q = S, which is independent of preferences, adjustment cost convexity, and the cost of housing intermediates. Thus, the amount of price elevation indicates a particular sized "derived demand shift" in the quantity dimension and thereby indicates market expectations about the future of the housing stock according to the shape of the stable manifold. Specifically, once the system parameters have reached their long run values, the time *t* anticipated value for the long run housing stock  $h_{ss}$  can be approximated by inverting (19):<sup>37</sup>

$$\frac{q(t)}{S} - 1 \doteq \frac{\overline{\lambda}}{S} \ln \frac{h_{ss}}{h(t)}$$

$$\overline{\lambda} \equiv \frac{S}{2\varphi'(1)} \left[ \sqrt{A^2 + 4\overline{r}} \varphi'(1) / S - A \right]$$

$$\overline{r} \equiv \left[ (\delta + A)S + (1 - \sigma)B \right] \frac{1 - \frac{h(t)}{h_{ss}}}{\ln \frac{h_{ss}}{h(t)}}$$
(19)

Figure 7 displays this relationship under two alterative assumptions about the amount of adjustment cost convexity: (blue series)  $S/\phi' = 25.4$  years, and thereby consistent with a short run housing investment elasticity of 1.575 at the steady state (see Appendix II), and (red series) the limit of infinite marginal adjustment costs  $(f'' \rightarrow \infty)$ , in which case the  $a(t) = (1 - \sigma)B + \delta S + AS \begin{bmatrix} h(t) \end{bmatrix}$ 

stable manifold approaches 
$$\frac{q(t)}{S} - 1 = \frac{(1 - \sigma)B + \delta S + AS}{AS} \left[ 1 - \frac{h(t)}{h_{ss}} \right]$$
, which coincides

<sup>&</sup>lt;sup>37</sup> This is an approximation (increasingly accurate as  $h(t) \rightarrow h_{ss}$ ) because the actual stable manifold for the system with constant marginal adjustment cost is nonlinear in the [ln *h*,*q*] plane, and equation (19) approximates the price elevation per unit shift in the stable manifold with the slope from equation (18) evaluated at a housing stock between h(t) and  $h_{ss}$ .

with the "fixed supply" case considered in Table 1.<sup>38</sup> For the purpose of drawing the curves in Figure 7, I assume that the convex adjustment cost technology does not create economic rents in the steady state (i.e., that  $f(\delta) = \delta$ ) and that the long run housing stock exceeds the current value because of a change in technology, rather than tastes.<sup>39</sup> Based on elastic supply, the Figure's blue series shows that the fixed supply model exaggerates the price impact of derived demand shifts (consistent with the 52 percent adjustment used in the last row of Table 1), and that any given percentage shift of derived housing demand (in the quantity dimension) creates peak price elevation of about the same number of percentage points.

#### III.D. Supply Responses: Anticipated Fundamentals

Consider now the case that the date *T* change in fundamentals is anticipated as of date 0 < T, still holding non-housing consumption *c* constant. Prior to date zero, the system (9) (together with the housing stock evolution equation) looks like Figure 9, except that the steady state and dynamics are determined by the "old" taste and technology parameters. After date *T*, the system (10) (together with the housing stock evolution equation) looks like Figure 9 with the steady state and dynamics determined by the "new" taste and technology parameters and the economy moving along the stable manifold toward the steady state that corresponds to the new parameters. The new steady state has more housing, and is therefore shown in Figure 11a to the right of the steady state *h*<sub>ss</sub> corresponding to the old parameters.

Figure 11a illustrates with red to indicate dynamics according to the old parameters, and black to indicate dynamics according to the new parameters.<sup>40</sup> During the interval  $t \in (0,T)$ , the dynamics correspond to the "old" parameters, but instead of converging to the steady state along the "old" stable manifold, the economy is following those dynamics that leave it on the "new" stable manifold exactly at date *T*. The peak housing price occurs exactly at date *T* (see the allocation  $C_T$  shown in Figure 11a)

<sup>&</sup>lt;sup>38</sup> As  $f'' \rightarrow \infty$ , the stable manifold does not approach vertical – although movements along it become infinitely slow – because a finite but persistently elevated housing price is enough to equate the finite demand for housing to the current supply.

<sup>&</sup>lt;sup>39</sup> The relationship between price elevation and the long run demand shift is slightly different if prices are elevated because of a change in tastes.

<sup>&</sup>lt;sup>40</sup> See Summers (1981, Figure 5) for a related analysis of an "announcement effect."

because, among dates with the new fundamentals, date T is the date with the smallest housing stock. As with the fixed supply case, prices must be rising during the interval (0,T) in order to make home owners willingly own housing during a time interval when prices are high but fundamentals are not enhancing the net operating surplus.

The duration *T* of time that the new fundamentals are anticipated reduces the size of the time zero price jump. In the "unanticipated fundamentals" case considered above, the time interval is zero and the system at time zero jumps to the point  $C_0$  on the stable manifold in Figure 11a. When the time interval *T* is longer, more housing is accumulated in anticipation of the fundamentals. With more housing in place by time *T* when the new fundamentals are realized, the housing price can be lower. Comparing the allocations  $C_0$ and  $C_T$  in Figure 11a, we see that, as long as supply can respond, the unanticipated fundamentals case has the larger peak housing price.

Figure 11a illustrates the system when the new fundamentals are anticipated with perfect certainty. To characterize the system with two possible states (*L* or *H*) to be realized at date *T*, first consider the steady state and stable manifold corresponding to each of them, as shown in Figure 11b. Regardless of whether the change in fundamentals is small or large, the new steady state has more housing, and is therefore shown to the right of the steady state  $h_{ss}$  corresponding to the old parameters. If nonhousing consumption were not affected by the new fundamentals, then the housing price just before date *T* must be the expected value of the price at date *T*, where that price will be on one of the two stable manifolds, depending on the state that is realized.<sup>41</sup> In other words, the dynamics on (0,*T*) are exactly as in Figure 11a, except that the system at date *T* is on the vertical average of two stable manifolds, rather than on the single stable manifold shown in Figure 11a.

After date *T*, the system (10) (together with the housing stock evolution equation) has its steady state and dynamics determined by the "new" taste and technology parameters and the economy moves along the stable manifold toward the steady state that corresponds to the realized state (*L* or *H*).<sup>42</sup> When the low state *L* is realized, the peak

<sup>&</sup>lt;sup>41</sup> More generally, the housing price just before date T satisfies the Euler equation (11).

<sup>&</sup>lt;sup>42</sup> The model's equilibrium dynamics can be calculated recursively. For the case of fixed non-housing consumption, first calculate the two stable manifolds (one for each possible realization of the parameters) that describe the dynamics after time T. A locus of time T destinations for the system (9) is calculated by

housing price occurs just prior to time T. For the reasons explained above, the amount of the peak price elevation decreases with the short run elasticity of housing supply and with the duration T of time that the possibility of new fundamentals are anticipated.

Figures 12a and 12b show the time paths for housing prices. Housing prices jump up at time zero and gradually increase over time until time T. If the smaller parameter change is realized at time T, then housing prices jump down. If the time T housing stock is still below the steady state corresponding to the realized parameters (the case shown in Figure 11), then the price path is as shown in Figure 12a: prices gradually fall as the housing stock approaches the new steady state from below. Otherwise the price path is as shown in Figure 12b, with prices below pre-boom levels and gradually rising as the housing stock approaches the new steady state from above.

Gross housing investment follows the same qualitative pattern as do housing prices. As shown in Figure 13a, the housing stock therefore accelerates prior to date T and decelerates (but still grows) after date T. Figure 13b shows the case in which the time T housing stock exceeded that consistent with the fundamentals that were ultimately realized, in which case the housing stock declines after time T.

For an anticipation period of five years, Figure 7's red series displays the model's peak price elevation (which occurs immediately before date T = 5) as a function of the size of the expected long run derived demand shift (equivalently, the amount by which the long run log housing stock will exceed the time zero log stock) assumed to derive from prospects for technological change, rather than tastes. For comparison, the blue series shows the relationship when the same permanent change in fundamentals occurs at date T = 0, and was unanticipated.<sup>43</sup> The blue and red series are close, because the Topel-Rosen short run supply elasticity estimates imply that the housing stock cannot respond enough in five years to put the system on the stable manifold at a point that is

 $5\overline{\lambda}$ 

<sup>43</sup> The red schedule is calculated from the blue with the adjustment factor  $e^{-\overline{\varphi'(1)/5}}$ , which indicates the amount the gap  $\ln h_{ss}/h(0)$  closes during 5 years (see equation (19)). It follows that schedules corresponding to anticipation periods of length  $T \in (0,5)$  would be in between the red and blue schedules in Figure 7.

averaging (in the price dimension) the two manifolds with weights  $\pi$  and  $1-\pi$ . Then choose the point on that locus so that following the old parameter dynamics backwards leaves the housing stock at  $h_{ss}$  in exactly T units of time.

significantly below the point that would have been attained if the parameter changes had been unanticipated.

Using the parameter values shown in the second to last column of Table 1, adjustment cost convexity of 25.4 years (see Appendix II), and an anticipation period of 3 years, Figure 14 displays a quantitative phase diagram for the system (9) and (10).<sup>44</sup> The stable manifolds for the H-state and L-state are shown as dotted and dashed black schedules, respectively, and their probability weighted average is shown as the blue schedule. The time path on the interval [0,*T*] is shown as a red curve, with housing prices jumping up at time zero when the possibility of new fundamentals becomes known, and then rising continuously for 3 years and jumping down at time *T* = 3 when it becomes known that the state is *L*. The system then follows the L-state stable manifold toward the L-state steady state (shown in the Figure as a black box).

Simulated gross housing investment, the housing stock, and housing price are graphed against calendar time in Figure 15, assuming that time zero corresponds with the middle of 2003 and time *T* corresponds with the middle of 2006.<sup>45</sup> In percentage terms, housing investment booms more (about 60 percent above its pre-boom value) than housing prices do (35-40 percent above their pre-boom value), which is a consequence of an assumed value for adjustment cost convexity that is sufficiently small. Housing prices peak in mid 2006 at more than 40 percent above their pre-boom values, and then quickly fall to about 5 percent above their pre-boom values. By the end of 2009 (2012), housing prices are still 4.5 (3.7) percent above their pre-boom values. Because the L-state stock is assumed to be sufficiently large compared to what can be accumulated over three years, the housing stock continues to increase (albeit slowly) even after 2006 when it is known that the H-state will not be realized.

#### IV. If the Optimism were Gone, then Real Housing Prices would be Lower

As long as the fundamentals are different (in the direction of a higher steady state) after time T than they were before the boom, the housing stock will always remain

<sup>&</sup>lt;sup>44</sup> Figures 14 and 15 are based on numerical solutions to the system, assuming that the adjustment cost technology is quadratic and creates no economic rents in the steady state (algrebraically, the assumptions are: f'' is constant and  $f(\delta) = \delta$ ). Thus, neither approximation (18) nor (19) is needed.

<sup>&</sup>lt;sup>45</sup> Empirically, the housing boom does not obviously begin discretely in a specific month. However, Figure 1 suggests that the period of heaviest (detrended) construction activity is July 2003 through March 2006.

discretely above its pre-boom levels even if housing prices do not. This is a basic difference between the rational expectations approach and "bubble" explanations of the housing cycle. The "bubble" theory by definition assumes that fundamentals are unchanged, which means that the housing construction boom was completely wasteful because the housing stock would ultimately return to its pre-boom level. The rational expectations approach says that, even with the benefit of hindsight, some of the housing construction was efficient because the long run housing stock is higher.<sup>46</sup>

Whether these two approaches can be distinguished in the short run depends on the amount of the supply response that had occurred by date T. Figure 13a shows the case in which the supply response was small: at all dates it has positive net investment and housing prices above their steady state level, even after it is known that fundamentals would change less than expected. In this case, it would be clear even in the short run that the housing stock would not return to what it was before the possibility of new fundamentals were anticipated because the housing stock would continue to increase.<sup>47</sup> However, as shown in Figure 13b, it is possible that supply had responded enough by date T that it exceeded its steady state value corresponding to the fundamentals that were ultimately realized. In this case, housing prices after date T would be lower than their steady state value S.

If, *ex poste*, the expectations of new fundamentals were completely unfounded, the steady states values  $h_{ss}$  and  $h_{ss}^{L}$  would coincide. In this case, net investment must be negative and housing prices below *S* after date *T*. These two housing market indicators are at their lowest immediately after date *T*, with amounts determined by the gap between  $h_T$  and  $h_{ss}$  and the short run elasticity of housing supply. The percentage price depression immediately after date *T* must be less than the percentage by which date *T* net operating surplus was depressed relative to the net operating surplus that had prevailed before the

<sup>&</sup>lt;sup>46</sup> In an economy with constant growth rates of population or other determinants of housing demand, the question would be whether the housing stock stays above the previous trend, even after the housing price bust.

<sup>&</sup>lt;sup>47</sup> I have assumed that time *T* is both the time of information arrival and a time of change in the fundamentals. If instead the information arrived at *T* and the fundamentals changed at some later date  $T_2$ , then housing prices could be: (a) rising after they crashed at date *T*, (b) below their previous peak, and (c) at all dates be higher than they were before the boom. In other words, the entire time interval  $[0,T_2]$  would be a construction boom, but part way through the boom the time *T* information revealed that the early boom was (in hindsight) a bit too exuberant.

housing boom, because housing rents and the net operating surplus will gradually increase after date *T* as housing supply is reduced. For the same reasons cited above, Topel and Rosen's estimates imply that the supply responses cause prices to be 48 percent closer to the steady state than they would be without supply responses (recall the 52 percent adjustment factor determined from Figure 10). The net operating surplus at the end of the housing boom, as a ratio to the real housing stock, was about 7 percent below what it was before,<sup>48</sup> so with the 52 percent supply-adjustment factor, housing bust real housing prices would be depressed 3 or 4 percent below what they were before the housing boom (0.04 = 0.52\*0.07) if fundamentals had not changed in the direction of more housing. Conversely, if fundamentals had changed in the direction of more housing, the housing bust should depress real housing prices less than 3 percent below their pre-boom levels, if at all.

Perhaps that's why proponents of the "bubble" theory of the 2000s housing cycle were forecasting in the first half of 2009 that housing prices would fall further (Shiller 2009); real housing prices then were still higher than they were before the boom. However, Table 3 shows that, even by the end of 2009, various indicators still show real housing values to be as high or higher than they were at the turn of the century.<sup>49</sup> Moreover, as shown in Figure 1, the real housing values ceased their declines at levels that were no where near the 3-4 percent depression that would be consistent with depressed net operating surplus and no change in housing fundamentals, which suggests either that some of the housing boom's optimism was justified, or that a significant amount of unfounded optimism still remains. On the other hand, Figures 1 and 2 suggest that housing construction activity did drop below what it was in the late 1990s: more time

<sup>&</sup>lt;sup>48</sup> Figure 5 shows a net operating surplus rate that falls about 25 percent, rather than 7, because Figure 5's rate is calculated as a ratio to replacement cost (the real stock at the end of the boom was about 4 percent above trend, whereas the real stock adjusted for the rising cost of replacement was almost 20 percent above trend due to the fact that housing prices were high during the boom).

<sup>&</sup>lt;sup>49</sup> The various national measures disagree much more on the amount by which real housing prices increased 2000-2006, with a range from 20% (Census Bureau quality-adjusted new homes) to 73% (Radar-Logic). A significant part of the disagreement relates to the weighting of the various regions to form the national composite, which is why my Table 3 also reports unweighted averages of the regions, and real price changes for the median region.

is needed to determine whether housing construction will remain depressed long enough to return the housing stock to its pre-boom trend.<sup>50</sup>

#### V. Conclusions

The housing boom that lasted through 2006 featured high and rising real housing prices, high rates of real investment in both owner-occupied and tenant-occupied housing, and low rates of real investment in non-residential structures. Housing sector net operating surplus – that minority part of housing service rents that accrues to homeowners and their creditors – was low relative to the housing stock during the boom, as were occupancy rates. More recently, real housing prices are much lower, although probably still above their pre-boom levels. Housing investment is now below, and non-residential structures investment above, pre-boom levels.

It sometimes suggested that the housing boom derived largely from low interest rates, and public policies promoting homeownership, in the mid 2000s. But low interest rates by themselves should push up all kinds of structures investment, not just residential structures. And policies to encourage homeownership should have depressed construction of rental housing units, when in fact that construction boomed too. Something else must have changed, and perhaps in a way that interacted with public policies regarding interest rates and home ownership.

That "something else" can be broadly characterized as "optimism" (perhaps rational, perhaps not) about the future because the housing boom featured rising housing prices and low net operating surplus. The facts that the housing boom ended with a large

$$\frac{\dot{q}(t)}{q(t)} = \left[A - \frac{\rho(t) - \delta}{q(t)}\right] + \delta \left[\frac{q(t) - 1}{q(t)}\right]$$

 $<sup>^{50}</sup>$  If real housing prices are now (after the housing crash) still higher than the steady state, and housing's net operating surplus relative to replacement cost is back to normal (as suggested by Mulligan and Threinen's (2010) quarterly estimates of net operating surplus through 2009 Q4), then the model predicts that housing prices are expected to be slightly higher in the future. The expected housing price appreciation is small, though, as seen from equation (8) as rearranged below. The first square bracket term is the deviation of the net operating surplus (as a ratio to replacement cost) from its steady state value *A*, which Mulligan and Threinen find to be about 0.3 percent per year. With a depreciation rate of 2.5 percent per year and housing prices currently elevated no more than 10 percent, the second square bracket term is 0.2 percent per year or less, for a combined expected appreciation rate of no more than 0.5 percent per year.

stock of housing yet real housing prices did not fall to pre-boom levels suggests that some of that optimism still remains in the marketplace.

One achievement of this paper is to quantify the expected size of the shift in the long run demand for housing structures that would be consistent with any given peak housing price elevation. For example, Figure 7 reports that real housing prices can be elevated more than 40 percent if the derived demand for housing structures were soon expected to shift by 0.40 log points in the quantity dimension, or 0.46 log points if that shift were anticipated 5 years in advance. To some readers, the stock of housing cannot be reasonably expected to increase that much, even in the long run, so that this result by itself confirms their suspicion that the housing cycle was indeed a "bubble": a housing boom that was divorced from reasonable expectations about the economic fundamentals.

If the housing cycle is to be linked to fundamentals, then it must be explained why housing structures' demand would shift so much. Another contribution of this paper is to show how, even in the absence of financial leverage, housing structures are a naturally and significantly leveraged claim on the value of housing services because some of the payments for those services must be used for depreciation, spending on intermediate goods, and property taxes, with only the residual remaining for the owners of structures. Thus, an expected 40 percent increase in real housing service expenditure could elevate housing prices by more than 40 percent in the short run.

Perhaps housing demand was expected to increase as a growing fraction of the population chose to work from home, entertain at home, and otherwise demand more housing. But this paper also points out that reduced production costs in the mortgage and real estate sectors – "housing intermediates" – could ultimately make existing housing more productive. Table 1's first and second columns show that the derived demand for structures is significantly reduced by the necessity of spending resources on housing intermediates, and thereby that the long run housing stock would be significantly increased if some of those costs could be eliminated. I showed, for example, that a 2/3 probability of a modest housing service demand increase combined with a 75 percent reduction in the costs of housing intermediates would elevate prices by 40 percent or more. An important step for future research is to determine whether such expectations

were quantitatively reasonable, and to monitor actual technical progress in the housing intermediates sectors.

The prospects for technical progress are inherently uncertain, so that their effects might be magnified by "irrational exuberance," subsidies, or both. For example, market participants may focus too much on the rosy technology scenario in which technical progress is realized rather quickly, or in an especially large magnitude. Implicit government guarantees and reasonable prospects for government bailouts may also cause market participants to rationally downweight the less favorable states of nature.<sup>51</sup>

Flood and Hodrick (1990, p. 99) found that "when agents expect the future to be somewhat different than history" price movements induced by fundamentals may look like bubbles to a researcher who is not fully aware of those fundamentals. In this regard, it is difficult to distinguish bubbles from rational expectations, and thus difficult to know whether the housing cycle of the 2000s is something that should have been "fixed." However, I have strengthened this application of rational expectations theory to assume that agents' expectations for progress are at least partly correct *ex poste* – that is, that some progress actually does occur (even if it were smaller or later than originally anticipated) – in which case the two theories are different in terms of the housing stock path after the bust. Perhaps the recession and transitory credit market interruptions are obstacles to immediate execution of this test, but one day we will know whether the housing stock (relative to trend) will ultimately remain above what it was before the housing boom, or instead that the construction boom of the 2000s was completely unnecessary.

<sup>&</sup>lt;sup>51</sup> Specifically, the prospect for technical progress raises the conditional variance of housing prices, thereby raising the market value of government bailouts and guarantees that pay out according to the amount that housing prices fall. From this perspective, it is not historically low home mortgage rates that stimulated home buying, but rather the failure of mortgage rates (or other mortgage costs) to rise to reflect the value of the mortgage default option, which increased as a result of the greater uncertainty associated with prospects for technical change.

#### VI. Appendix I: Calculation of the Stable Manifold

The stable manifold is the solution to the dual boundary value stationary dynamical system:

$$\begin{split} \dot{H}(t) &= \varphi(q(t) / S) - \delta \\ \dot{q}(t) &= \left[A + \delta - \varphi(q(t) / S)\right] q(t) + B(1 - \sigma) + Sf(\varphi(q(t) / S)) - \eta(t)ce^{-H(t)} \\ \lim_{t \to \infty} q(t) &= S \quad , \quad \lim_{t \to \infty} H(t) = \ln h_{ss} \quad , \quad H(0) \text{ given} \end{split}$$

where H denotes the log of the housing stock.

As shown by Mulligan (1991), the stable manifold can be represented as a single function q(H) that solves a single differential equation with a single boundary condition:

$$q'(H) = \frac{\left[A + \delta - \varphi(q \mid S)\right]q + B(1 - \sigma) + Sf(\varphi(q \mid S)) - \eta c e^{-H}}{\varphi(q \mid S) - \delta}$$

$$q_{ss} = q'(\ln h_{ss})$$

$$\lim_{H \to \ln h_{ss}} q'(H) = \frac{S}{2\varphi'(1)} \left[A - \sqrt{A^2 + 4r_{ss}\varphi'(1) \mid S}\right]$$

$$r_{ss} = \eta c \mid h_{ss}$$

q(H) is readily calculated numerically by integrating q'(H) away from the steady state point.

#### VII. Appendix II: The Short Run Supply Elasticity and Adjustment Cost Convexity

Topel and Rosen (1988) examined the national time series relationship between housing construction activity and the quality-adjusted prices of new homes, as measured by the Census Bureau. Their Table 3 reports elasticities of short run housing investment with respect to housing price increases ranging from 0.72 to 3.94 (median of 1.575), depending on the expected duration of the price increase and the details of their empirical specification.

My model predicts that date *t* gross housing investment is a stationary function  $\varphi(q(t)/S)$  of the contemporaneous housing price q(t). Moreover, that relationship is linear when the adjustment cost technology *f* is quadratic: gross housing investment is proportional to (q(t)/S - 1) with a constant coefficient  $\varphi'$ . In the steady state, the elasticity of housing investment with respect to q(t) is  $\varphi'/\delta$ .<sup>52</sup>

In order for this elasticity to be 1.575 with a depreciation rate of 2.5% per year, adjustment cost convexity  $S/\varphi'$  would have to be 25.4 years, which is the value used in sections III.C and III.D of the paper.

<sup>52</sup> The proof is:

$$\dot{h}(t) + \delta h(t) = \varphi(q(t) / S) h(t)$$

$$\ln[\dot{h}(t) + \delta h(t)] = \ln \varphi(q(t) / S) + \ln h(t)$$

$$\frac{\partial \ln[\dot{h}(t) + \delta h(t)]}{\partial q(t)} q(t) = \frac{\varphi'(q(t) / S)}{\varphi(q(t) / S)} \frac{q(t)}{S} \rightarrow \frac{\varphi'}{\delta}$$

#### VIII. Appendix III: Anticipated Technical Change: Evidence from Consumption

Consumption is constant over time unless news arrives about the nature of the intertemporal production set (5), re-arranged below to solve for the optimal amount of consumption.

$$c = Ak_0 + A \int_0^\infty e^{-At} \left[ w(t) - B(t)h(t) - h(t)Sf(I(t) / h(t)) \right] dt$$
(20)

Expectations of the taste and subsidy rate parameters have no effect on consumption except through the path of the housing stock. Thus, if it were learned that future housing demand or housing subsidies would be high, that would immediately reduce non-housing consumption and lead to a path of increasing housing construction. Conversely, news of low housing demand or housing subsidies would immediately increase non-housing consumption and lead to a path of decreasing housing construction.

Good news about the productivity 1/B of the housing sector's intermediate inputs increases housing investment, but it also likely increases non-housing consumption. Consider, for example, the inelastic supply case. The intertemporal production set (20) shows that good news about B frees resources for non-housing consumption in the present value amount  $h(0)\Delta_B e^{-AT}/A$  (recall from above that  $\Delta_B$  is the magnitude of the anticipated reduction in B).<sup>53</sup> If we measure the market value of the initial housing stock as h(0)q(0)/S, recall from equation (13) that the same good news increases the market value of the housing stock by  $h(0)\Delta_B e^{-(A+\delta)T}/(A+\delta)$ . Thus the consumption impact of the news is greater than the housing value impact of the news to the degree that housing depreciates.

At the other extreme - no adjustment costs - good news about future B has no effect on housing prices (and therefore no effect on the market value of housing) but increases non-housing consumption.<sup>54</sup> Thus, the present value of non-housing

<sup>&</sup>lt;sup>53</sup> Iacoviello and Neri (2008) connect housing prices and consumption by another mechanism: high housing prices alleviate liquidity constraints. <sup>54</sup> To prove this, use the intertemporal production set (20) and the steady state equation (17) to solve for the

consumption impact as a ratio to the amount of consumption prior to the news.

consumption increases more than does the market value of housing, as it does in the fixed supply case.

Mulligan and Threinen (2008, Table 1) used the Case-Shiller and OFHEO price indices to prepare alternative calculations of the reduction in the value of housing wealth since 2006. They found \$8.3 trillion and \$4.5 trillion, respectively. Based on the assumption that each dollar of lost housing wealth was a dollar in reduced non-housing consumption (which, in their approach, included leisure time), they calculated that each \$5 trillion of lost housing wealth would reduce non-housing consumption by 2 percent. The q-theory of investment presented in this paper suggests that the consumption impact would be even larger if the news were entirely about housing intermediates' productivity, because the change in housing wealth from this shock does not fully measure the expansion of the intertemporal production set. The consumption impact could be zero (or even in the other direction) if the news were about other fundamentals.<sup>55</sup> In other words, consumption behavior is an indicator of the type of shocks causing the housing boom and bust.

Figure 16 shows quarterly nonhousing consumption, decomposed into durables and nondurables + services. Both types of consumption expenditure are deflated with the same deflator – population times the PCE deflator, expressed as a deviation from a trend from 2 percent per year, and measured on the right scale. In order to indicate the housing boom, the red series displays construction spending, deflated in the same way and measured on the left scale. Non-housing consumption rises significantly at the end of the 1990s – arguably the years when market participants became optimistic about technical progress in mortgages, etc. Consumption fell sharply in 2007 and 2008. Case et al. (2005) and Campbell and Cocco (2007) also conclude from U.S. data that housing prices affect aggregate consumption.

<sup>&</sup>lt;sup>55</sup> An intermediate case – the consumption impact is less than the housing wealth change – is also possible when the price effects of technical progress are accompanied, or reinforced by, distorted price of the mortgage put option.

### Table 1. Price Peak with Fixed Supply, as a Function of Model Parameters

el	iminate	combine with	acknowledge	acknowledge			
interm	ediates	20% demand	technology	imperfect	demand	no crash	
	w.p. 1	increase	uncertainty	progress	uncertainty	subsidy	
annual depreciation rate	0.025	0.025	0.025	0.025	0.025	0.025	
annual opportunity cost	0.05	0.05	0.05	0.05	0.05	0.05	
high state probability	1	1	0.67	0.67	0.67	0.67	
crash subsidy rate	0	0	0.5	0.5	0.5	0	
intermediaries as share of							
gross operating surplus	0.43	0.43	0.43	0.43	0.43	0.43	
net operating surplus	0.77	0.77	0.77	0.77	0.77	0.77	
flow subsidy rate							
initial	0	0	0	0	0	0	
high	1	1	1	0.75	0.75	0.75	
low	0.25	0.25	0.25	0.25	0.25	0.25	
housing preference							
initial	0.176	0.176	0.176	0.176	0.176	0.176	
high	0.176	0.212	0.212	0.212	0.212	0.212	
low	0.176	0.212	0.212	0.212	0.176	0.176	
		Outcome	S				
Expected dmd shift (log quantity dimension)	0.41	0.59	0.53	0.43	0.40	0.35	
peak price elevation							
preference term	0.00	0.45	0.45	0.45	0.36	0.30	
technology term	<u>0.77</u>	<u>0.77</u>	<u>0.65</u>	<u>0.50</u>	<u>0.50</u>	<u>0.45</u>	
combined	0.77	1.22	1.10	0.95	0.86	0.75	
peak price elevation with elastic							
supply (estimated as 52% of above)	0.40	0.63	0.57	0.49	0.45	0.39	

### Table 2. Calibration of Housing Market Parameters

parameter	symbol	value	units (of value)
opportunity cost (marginal non-residential product)	A	5	%/year
housing depreciation rate	δ	2.5	%/year
intermediate expenditure per unit opportunity cost	B(1-σ)/A	36/47	share
housing expenditure share	η/(1+η)	15	%

Sources: National Accounts and Annual Population Growth

### Table 3. Real Housing Values Were Not Low at the End of 2009

1999:IV - 2009:IV appreciation in various housing price indices, adjusted for inflation

Aggegrate MSAs or	Source:					
				Quality-Adjusted		
Regions by:	Case-Shiller	Radar-Logic	OFHEO	New Homes		
Weighted Composite	12%	25%	17%	1%		
Unweighted Avg	2%	5%	19%	NA		
Median	0%	4%	16%	NA		

#### <u>Notes</u>

inflation-adjusted according to the deflator for Personal Consumption Expenditures MSA = "Metropolitan Statistical Area" Case-Shiller and Radar-Logic sample 20 (25) MSAs, respectively OFHEO samples 50 states, and aggregates to 9 regions Quality adjusted new home price index from Census Bureau Radar Logic index is measured in 2000:I, rather than 1999:IV

Fig 1. Real Housing Construction and Average Property Values



Fig 2. Quantity Indexes for Investment in Structures







Fig 4. Housing Sector Net Operating Surplus per Unit Stock

(2005 interpolated, stocks valued at replacement cost)

Fig 5. Occupancy Rates, 1985-2009



Fig 6. Possible Paths for Tastes and Technologies





## Fig 8. Possible Housing Stock Paths

The Figure shows three housing stock time paths corresponding to three alternative assumptions about the costs of investment.



### Fig 9. Phase Diagram for the Stationary System

The Figure shows the stationary system's steady state, dynamics, and stable manifold.





## Fig 11a. Phase Diagram for Anticipated Changes: One State

The Figure shows the system's dynamics and stable manifold. The dynamics shown by the red arrows correspond to the "old" taste and technology parameters. When the new parameters are first anticipated at date 0, price jumps up. The economy then follows the red path, reaching the end exactly at time T when the new parameters take effect. The new stable manifold (shown as a black path) describes dynamics thereafter.



## Fig 11b. Phase Diagram for Anticipated Changes: Two States

The Figure shows the system's dynamics and stable manifold. The dynamics shown by the red arrows correspond to the "old" taste and technology parameters. When the new parameters are first anticipated at date 0, price jumps up. The economy then follows the red path, reaching the end exactly at time *T* when the new parameters take effect. The new stable manifolds (shown as black paths, one for each possible realization of the parameters) describes dynamics thereafter.



Fig 12a. Time Path for Housing Prices: Small Supply Response



Fig 12b. Time Path for Housing Prices: Large Supply Response



Fig 13a. Time Path for the Housing Stock: Small Supply Response



Fig 13b. Time Path for the Housing Stock: Large Supply Response



# Fig 14. Quantitative Stock and Price Trajectories (cut $(1-\sigma)B$ in 1/2, 3/4 and raise exp share 0, .025)







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