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ABSTRACT

By postulating a simple stochastic process for the firm's cash flows in which the drift and the variance of the process depend on the investment policy of the firm, we develop a theoretical model, determine the optimal investment policy and, given this policy, calculate the ratio of the current value of the firm and the current cash flow which we call the "cash flow multiplier". The main contribution of the paper, however, is empirical. Using a very extensive data set comprised of more than 13,000 fims over 44 years we examine the determinants of the cash flow multiplier using as explanatory variables macro and firm specific variables suggested by the theoretical model. We find strong support for the variables suggested by the model. Perhaps the most interesting aspect of the paper is the formulation of a parsimonious empirical asset pricing model, based on the fundamental discounted cash flow approach but using current macroeconomic variables and firm specific variables easily observable for its implementation. We obtain valuation equations that could potentially form part of a new valuation framework which does not require estimating future cash flows nor risk adjusted discount rates.

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1 Introduction

It is a well established idea in Financial Economics that the current value of a firm should be the discounted expected future cash flows, and that these future cash flows should depend on the investment policy of the firm. In this paper we give content to this idea by postulating a simple stochastic process for the firm's cash flows (before investment) in which the drift and the variance of the process depend on the investment policy of the firm. This framework allows us to estimate the optimal investment policy and, given this policy, determine the ratio of the current value of the firm and the current cash flow which we call the "cash flow multiplier".

We develop a valuation model which in its simplest form has only one exogenous state variable (the risk free short interest rate) and quantitatively shows how the cash flow multiplier is (negatively) related to the discount rate and (positively related) to optimal investment. We are then able to show that the cash flow multiplier has two components: the first part corresponds to the situation with no (zero) investments and the second to the real option to invest optimally in the future.

Furthermore, firms in certain industrial sectors require more investment because obsolescence in the sector is faster, or because the sector is more competitive. In our theoretical model the drift of the cash flow process (without investments) can proxy for this phenomenon. The smaller (or more negative) is this drift without investments, the more investment will be required to keep or increase the level of cash flows. This would imply that, even though the cash flow multiplier for a give firm is positively related to the proportion of its cash flow invested, the multiplier should be negatively related to the average investment proportion of the industry to which it belongs since it would be a more intensive investment industry. We find evidence of this in the data.

Using a very extensive data set comprised of more than 13,000 firms over 44 years we examine the determinants of the cash flow multiplier, using as explanatory variables the variables suggested by the theoretical model. We find strong support for the variables suggested by the model. In the empirical analysis we include four macro variables that affect all firms at a given point in time and three firm specific variables. The macro variables affect the discount rate and include the short rate interest rate, the slope of the term structure of interest rates, a credit spread (spread of BBB bonds over Treasuries) and the volatility of the S&P500 index. Increases in all of these variables have a positive effect on the discount rate, and therefore should have a negative effect on the cash flow multiplier. As firm specific variables we include a measure of liquidity, size and the proportion of cash flows invested. We would expect that more liquid firms and larger firms have higher cash flow multipliers. The fraction of cash flow invested is a variable that comes directly from our theoretical model and should, if investment is optimal, be positively related to the cash flow multiplier. In addition to these explanatory variables, in most of the regressions we also include dummy variables to take into account firm and/or industry fixed effects.

Our paper is related to several strands of the literature. In a series of papers, Ang and Liu (2001, 2004, and 2007) address theoretical issues related to our paper. For example, Ang and Liu (2001) derive a model that relates firm value to accounting data under stochastic interest rates, heteroskedasticity and adjustments for risk aversion. Ang and Liu (2004) develop a model that consistently values cash flows with changing riskfree rates, predictable risk premiums, and conditional betas in the context of a conditional CAPM. Finally, Ang and Liu (2007) show theoretically that a given dividend process and any of the variables – expected return, return volatility, and the price-dividend ratio – determines the other two. Although they do not model investment decisions explicitly, they derive a partial differential equation for the price-dividend ratio that is also satisfied by the cash flow multiplier in our paper given that the corresponding firm invests optimally.

Berk, Green, and Naik (1999) develop a model that values the firm as the sum of the present value of its current cash flows and its growth options and is thus similar in spirit to the theoretical model in our paper. But their main interest is to study the dynamics for conditional expected returns. In a related paper, Carlson, Fisher, and Giammarino (2004) derive two theoretical models that relate endogenous firm investment to expected return. Titman, Wei, and Xie (2004) document a negative relation between abnormal capital investments and future stock returns. Similarly, Anderson and Garcia-Feijoo (2006) find in an empirical study over the time period from 1976 to 1999 that growth in capital expenditures explains returns to portfolios and the cross section of future stock returns.

Our theoretical findings concerning the value of a firm's option to invest are related to the real options literature that started with the papers by Brennan and Schwartz (1985) and McDonald and Siegel (1986). More recently, Grenadier (2002) and Aguerrevere (2009) have shown that in competitive markets the value of the option to invest can decrease substantially.

Our paper also contributes to an extensive literature on multiples. For instance, Baker and Ruback (1999) study how to estimate industry multiples and how to choose a measure of financial performance as a basis of substitutability. They find that EBITDA is a better single basis of substitutability than EBIT or revenue. They analyze the valuation properties of a comprehensive list of multiples and also examine related issues such as the variation in performance across industries and over time. Liu and Thomas (2002) analyze the valuation properties of a comprehensive list of multiples. They also examine related issues such as the variation in performance across industries and over time. This analysis is extended by Liu and Thomas (2007). Bhojraj and Ng (2007) examine the relative importance of industry and country membership in explaining cross-sectional variation in firm multiples. These papers, however, do not include macroeconomic variables in the analysis.

To summarize, this paper makes contributions in several dimensions. In the theoretical front we develop a discounted cash flow valuation model that takes into account optimal investment and how this investment affects future cash flows. Using a data set covering 44 years from 1962 to 2005 and including over 13,000 firms we then regress the cash flow multiplier onto a set of explanatory variables, which include both macroeconomic variables as well as variables related to individual firms, and obtain results that are broadly consistent with the theoretical model. All the explanatory variables related to the discount rate - the short term interest rate, the slope of the term structure, the spread of BBB bonds over Treasuries, and the volatility of the S&P500 - have the correct sign and most of them are significantly negative. The proportion of cash flow invested is always highly significant and positive as predicted by the model.

Since the cash flow multiplier is simply the ratio of the current value of the firm and the current cash flow, perhaps the most interesting contribution of the paper is the formulation of a parsimonious empirical asset pricing model based on the fundamental discounted cash flow approach but using current macroeconomic variables and firm specific variables easily observable for its implementation. We obtain valuation equations using a very large sample of firms over a very long period of time. These could potentially form part of a new valuation framework based on discounted cash flows which does not require estimating future expected cash flows nor risk adjusted discount rates.

The paper proceeds as follows. Section 2 develops the theoretical model. Section 3 solves for the optimal investment and the optimal cash flow multiplier. Section 4 illustrates the implications of the model taking IBM as an example. Section 5 describes the data used in the empirical testing and Section 6 presents the results of the panel regressions. Section 7 reports results of several robustness checks on the basic results. Section 8 provides insights into the value of the option to invest using the data available. Finally, Section 9 concludes. Some of the proofs and technical details, including an extension to two state variables, are given in the appendices.

2 Model

We develop a parsimonious model where firm value naturally arises as the present value of the firm's free cash flows before investments. Investment decisions are made endogenously affecting the expected growth rate and the volatility of the cash flow stream. The value of the firm is defined as the net present value of the free cash flows. This implies that the firm value is given by

$$V(c,x) = \max_{\pi} \mathrm{E}\Big[\int_0^\infty e^{-\int_0^s R_u\,du}(C_s - I_s)ds\Big], \tag{2.1}$$

where C denotes the firm's free cash flows before investments and $I = \pi C$ denotes the dollar amount of the cash flow that is invested. The variable π stands for the percentage of the cash flow that the firm invests. The variable x denotes the initial value of a state process X that captures economic variables that impact the firm value, such as interest rates. The risk-adjusted discount rate R is assumed to be a function of this state process, i.e. with a slight abuse of notation $R = R(X)$. For the moment, suppose for simplicity that X is equal to the default-free short interest rate and that the risk-adjusted discount rate is linear in the short rate, i.e.

$$R = \varphi + \psi r \tag{2.2}$$

with φ and ψ being constants. A simple model that would fit into this framework is the following: Suppose that the risk-adjusted discount rate is given by $R = r + \beta\lambda$, where λ is the risk premium and β is the firm's beta that is constant. If the default-free interest rate predicts the risk premium, then the premium could be linear in the interest rate, $\lambda = \overline{\lambda} + \lambda^r r$, with constants $\overline{\lambda}$ and λ^r such that $\varphi = \beta\overline{\lambda}$ and $\psi = 1 + \beta\lambda^r$ in our parametrization (2.2). We assume the cash flow to follow the dynamics[1]

$$dC = C[\mu(\pi, X)dt + \sigma(\pi, X)dW], \quad C(0) = c,$$

where expected growth rate and volatility, μ and σ, are functions of the state process and the percentage of the firm's cash flow reinvested. The process W is a Brownian motion. This specification implies the following result:

Proposition 2.1 (Linearity of Firm Value). *Firm value is linear in the cash flow, i.e.*

$$V(c,x) = f(x)c, \tag{2.3}$$

where $f(x) = V(1,x)$.

[1] This builds on the ideas of Merton (1974), Duffie and Lando (2001), and Goldstein, Ju, and Leland (2001), among others, who use lognormal models in which the firm cannot control for investment.

Notice that V/c is the firm-value-cash-flow ratio (for short: cash flow multiplier), which is similar to, but not the same as the price-dividend ratio. In our model, f is equal to the cash flow multiplier which will be central in our further analysis. One can think of the firm-value-cash-flow ratio as the multiplier by which the current cash flow is multiplied to obtain the current firm value. In the literature on the dividend-discount model and its generalization, usually this multiplier is assumed to be beyond the control of the firm and thus to be exogenously given. In contrast, we explicitly model the firm's opportunity to change its risk-return tradeoff by allowing the firm to control the expected growth rate and the volatility of the cash flow stream by its investment policy. To illustrate our approach and unless otherwise stated, we use the following specification of these parameters

$$\mu(\pi, x) = \mu_0(x) + \mu_1\sqrt{\pi} + \mu_2\pi, \qquad \sigma(\pi) = \sigma_0 + \sigma_1\sqrt{\pi} + \sigma_2\pi,$$

where all coefficients except for μ_0 are constants and μ_0 is a linear function of the state process, $\mu_0(x) = \overline{\mu}_0 + \widehat{\mu}_0 x$. The function μ_0 characterizes the expected growth rate if the firm does not invest at all ($\pi = 0$). If no investment implies that future cash flows decrease, we expect μ_0 on average to be negative. We also expect μ_0 to depend on the industry and the environment in which the firm operates. The parameters μ_1 and μ_2 capture the firm's impact on its future growth rate when the firm invests part of its cash flows. Since the second derivative of the expected growth rate with respect to π is $-\mu_1\pi^{-1.5}/4$, the coefficient μ_1 captures the curvature of the growth rate with respect to the firm's investments. If μ_1 is positive, then the expected growth rate is concave implying decreasing returns to investment. Finally, to avoid explosion of the model, μ_2 is assumed to be negative.[2] This implies that there is a point beyond which additional investments are not beneficial any more since the growth rate then decreases in π. Furthermore, we allow the investment decisions to have an impact on the riskiness of the firms cash flow stream and thus the volatility σ can depend on π as well. Clearly, if a firm invests in a new product, then we would expect this investment to increase both the firm's expected growth rate, but also the volatility of its cash flow stream. Figure 1 illustrates two possible forms of the drift when the investment proportion is varied between zero and one. The drift starts below zero and then increases until it reaches its peak. For the lower curve the peak is reached around $\pi = 0.7$, whereas for the upper curve the peak is reached for some π that is greater than one. We emphasize that the peak is in general not equal to the optimal expected growth rate; the firm chooses an expected growth rate that is smaller than the maximum. This is because investments are not for free, but consume some of the firms cash flows. The actual

[2] Loosely speaking, this ensures that a transversality condition is satisfied.

optimal investment strategy thus depends on the tradeoff between additional expected growth and the fraction of the cash flows that must be spent to achieve this growth. Therefore, the steepness of the expected growth rate as a function of the investment strategy π is crucial.

[INSERT FIGURE 1 ABOUT HERE]

3 Solving for the Optimal Cash-Flow Multiplier

The firm's decision problem (2.1) is a dynamic optimization problem that can be solved using stochastic control methods. This is the first goal of this section. We assume that the state of the economy is characterized by the short rate that has Vasicek dynamics[3]

$$dr = (\theta - \kappa r)dt + \eta dW_r, \tag{3.4}$$

where W_r is a Brownian motion that is correlated with the Brownian motion W that drives cash flows, i.e. $d < W, W_r >= \rho dt$ with constant correlation ρ. As motivated in the previous section, the expected growth rate, the volatility of the cash flow stream, and the risk-adjusted interest rate are assumed to be $\mu(\pi, r) = \overline{\mu}_0 + \widehat{\mu}_0 r + \mu_1\sqrt{\pi} + \mu_2\pi$, $\sigma(\pi) = \sigma_0 + \sigma_1\sqrt{\pi} + \sigma_2\pi$, and $R = \varphi + \psi r$. In the Appendix, it is shown that the cash flow multiplier satisfies the following Hamilton-Jacobi-Bellman equation

$$\begin{aligned} 0 \;=\; & \max_{\pi}\{(\overline{\mu}_0 + \widehat{\mu}_0 r + \mu_1\sqrt{\pi} + \mu_2\pi)f + 1 - \pi - (\varphi + \psi r)f \\ & +(\theta - \kappa r)f_r + 0.5\eta^2 f_{rr} + \rho\eta(\sigma_0 + \sigma_1\sqrt{\pi} + \sigma_2\pi)f_r\}. \end{aligned} \tag{3.5}$$

Under the assumption that the Bellman equation is concave in π, which follows if $\mu_1 > 0$, the optimal investment strategy of the firm is given by

$$\pi^* = \left(\frac{\mu_1 f + \rho\eta\sigma_1 f_r}{2(1 - \mu_2 f - \rho\eta\sigma_2 f_r)}\right)^2. \tag{3.6}$$

First, notice that this optimal strategy does not depend on the second derivative with respect to the interest rate. Second, it does not depend on the first derivative if either the correlation between the cash flow stream and the short rate is zero, $\rho = 0$, or if the firm's investment decision has no impact on the volatility of the cash flow stream, $\sigma_1 = \sigma_2 = 0$. To get some intuition about the optimal investment, assume that we are in one of these two cases. Then an upper bound on the investment strategy is $\pi^{max} = (0.5\mu_1/\mu_2)^2$, which obtains if the cash

[3] A generalization to two state variables can be found in the Appendix.

flow multiplier f goes to infinity. On the other hand, if the cash flow multiplier is zero, then the firm's optimal investment is zero. If investments were free, then the optimal investment strategy would be equal to the upper bound π^{max}. We have however assumed that the firm has to spend a fraction of its cash flow, π, if it chooses to invest. This is the reason why there is a one present in the denominator of (3.6), which implies that the optimal investment strategy is smaller than the upper bound, $\pi^* < \pi^{max}$. How close the optimal strategy is to π^{max} depends on the trade off between additional expected growth rate - modeled by μ_1 and μ_2 - and the necessary expenditures to achieve it.

Substituting the optimal investment level back into the Bellman equation (3.5) leads to a differential equation for the cash flow multiplier

$$0 = (\widehat{\varphi} + \widehat{\psi} r) f + 1 + (\theta + \rho\eta\sigma_0 - \kappa r) f_r + 0.5\eta^2 f_{rr} + \frac{(\mu_1 f + \rho\eta\sigma_1 f_r)^2}{4(1 - \mu_2 f - \rho\sigma_2\eta f_r)}, \tag{3.7}$$

where $\widehat{\varphi} = \overline{\mu}_0 - \varphi$ and $\widehat{\psi} = \widehat{\mu}_0 - \psi$ are constants. The presence of the last ratio in this equation is crucial. It can be easily shown that this ratio disappears if the firm does not invest. In this case, the cash flow multiplier has the explicit solution

$$f(r) = \int_0^\infty \widehat{\mathrm{E}}\left[e^{\int_0^s \widehat{\varphi} + \widehat{\psi} r_u \, du}\right] ds = \int_0^\infty e^{A(s) - B(s) r} \, ds \tag{3.8}$$

with A and B being deterministic functions of time. The expected value $\widehat{\mathrm{E}}[\cdot]$ is taken under the measure under which the short rate has the dynamics

$$dr = (\theta + \rho\eta\sigma_0 - \kappa r) dt + \eta d\widehat{W}$$

with $\widehat{W}$ being a Brownian motion under this measure. If the firm is however investing optimally, then the last term in (3.7) can be thought of as an additional cash flow that the firm is able to generate by doing so. From a real option perspective, this fraction can be interpreted as the firm's option to invest optimally at a particular time t in the future. Brealey, Myers, and Allen (2010) call this the net present value of growth opportunities. The present value of this continuous series of options is given by

$$\mathcal{O}(r; f) = \int_0^\infty \widehat{\mathrm{E}}\left[e^{\int_0^s \widehat{\varphi} + \widehat{\psi} r_u \, du} \frac{(\mu_1 f(r_s) + \rho\eta\sigma_1 f_r(r_s))^2}{4(1 - \mu_2 f(r_s) - \rho\sigma_2\eta f_r(r_s))}\right] ds \tag{3.9}$$

such that the optimal cash flow multiplier becomes the sum of (3.8) and (3.9), i.e.

$$f(r) = \int_0^\infty e^{A(s) - B(s) r} \, ds + \mathcal{O}(r; f). \tag{3.10}$$

We have added a second argument in the definition of $\mathcal{O}$ to emphasize that it depends on f. The firm has a series of options to invest and the net present value of these options is positive, $\mathcal{O} \geq 0$. Otherwise, the firm would decide to refrain from investing. Clearly, the option value $\mathcal{O}$ is not explicit since it depends on the optimal cash flow multiplier f which is unknown and a part of the solution.[4] Nevertheless, at least the first part of the representation (3.10) is explicitly known and equal to the solution without investing.

To gain further insights, let us assume for the moment that the interest rate r is constant. In this case, it makes sense to simplify notations by setting $\widehat{\mu}_0 = 0$, $\varphi = \lambda = const$, and $\psi = 1$. This implies that $\mu_0 = \overline{\mu}_0 = const$ and $\widehat{\varphi} + \widehat{\psi} r = \mu_0 - r - \lambda = const$. The risk-adjusted interest rate is the sum of the short rate and a risk premium, i.e. $R = r + \lambda$. Furthermore, the optimal cash flow multiplier f is a constant and (3.10) simplifies into

$$f = \int_0^\infty e^{(\mu_0 - r - \lambda)s}\, ds + \underbrace{\int_0^\infty e^{(\mu_0 - r - \lambda)s} \frac{(\mu_1 f)^2}{4(1 - \mu_2 f)}\, ds}_{=\mathcal{O}(f)}, \tag{3.11}$$

where the transversality condition $\mu_0 - r - \lambda < 0$ is assumed to hold. Then we obtain the following proposition.

Proposition 3.1 (Cash Flow Multiplier under Constant State Process). *If $\mu_1^2/4 - \mu_2(\mu_0 - r - \lambda) < 0$, then the optimal cash flow multiplier is uniquely given as the positive root of the quadratic equation*

$$0 = \left[\mu_1^2/4 - \mu_2(\mu_0 - r - \lambda)\right] f^2 + (\mu_0 - r - \lambda - \mu_2) f + 1. \tag{3.12}$$

Notice that in the special case when the firm has no control over the expected growth rate of its cash flow stream ($\mu_1 = \mu_2 = 0$), relation (3.12) becomes a linear equation with solution $f = 1/(r + \lambda - \mu_0)$. This is a version of the Gordon growth model. Furthermore, due to the transversality condition, a necessary requirement for the condition of Proposition 3.1 to hold is $\mu_2 < 0$.

To make the implications of Proposition 3.1 as clear as possible, let us consider a numerical example. Similar to the previous section, we choose $\mu_0 = -0.03$, $\mu_1 = 0.1$ and $\mu_2 = -0.03$. Besides, we assume that $r = 0.04$ and $\lambda = 0.03$ such that the risk-adjusted interest rate is $R = 0.07$. The positive root of (3.12) which is the cash flow multiplier equals 13.06. If the firm suboptimally decides not to invest, then the option value in (3.11) is zero and the cash flow multiplier is 10. Therefore, the option value equals $\mathcal{O} = 3.06$. Put differently, the opportunity

[4] From a mathematical point of view, (3.10) is a fixed point problem for f.

to invest increases the cash flow multiplier by 30 percent. Let us consider a second example where all parameters are the same as in the first example except for μ_0 which is assumed to be -0.05. As discussed in the introduction, one reason for this lower value might be that the industry requires more investments. The cash flow multiplier resulting from optimal investing is now 9.91, whereas the cash flow multiplier without investing is 8.33. Therefore, the option value becomes $\mathcal{O} = 1.58$ or 18% of the optimal cash flow multiplier. This suggests that in an investment intensive industry the real option to invest loses value both in absolute as well as in relative terms. In fact, we are able to show that this is in general true.

Theorem 3.2 (Value of the Option to Invest)**.** *If, in addition to the assumption of Proposition 3.1, condition $\mu_0 - r - \lambda - \mu_2 < 0$ holds, then the optimal cash flow multiplier f, the option value $\mathcal{O}$, and the ratio $\mathcal{O}/f$ are increasing in μ_0.*

Remark. The requirement $\mu_0 - r - \lambda - \mu_2 < 0$ is a bit stronger than the transversality condition since $\mu_2 < 0$. Nevertheless, it is satisfied for reasonable parametrizations of the model.

Put differently, the previous theorem says that the option's absolute and relative values decrease if μ_0 becomes more negative. This result puts some of the classical results on real options into perspective and it is related to Grenadier (2002) and Aguerrevere (2009): If the firm is forced to invest for instance because competitors do the same, then the option to invest loses (part of) its value. Hence, the cash flow multiplier decreases.

We now study the more general case of stochastic interest rates. Then, the presence of the fraction in (3.7) turns the differential equation into a highly nonlinear equation, which makes solving the equation more challenging.[5] Nevertheless, we are able to provide an explicit power series representation in the following theorem.

Theorem 3.3 (Optimal Cash Flow Multiplier under Stochastic Interest Rates)**.** *The cash flow multiplier has the following series representation*

$$f(r) = \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} a_i^{(n)} \left(r - \frac{\theta}{\kappa} \right)^i \eta^n \tag{3.13}$$

where the coefficients $a_i^{(n)}$ are given by an explicit recursion the can be found in Appendix B.

The main insight of Theorem 3.3 that leads to an explicit recursion for the coefficients is to

[5] At least this is so if there are two state variables since in this case the equation is a non-linear partial differential equation, a case treated in the Appendix.

expand f in two directions, r and η. If one expands f only in terms of r, i.e.

$$f(r) = \sum_{i=0}^{\infty} \widehat{a}_i \left(r - \frac{\theta}{\kappa} \right)^i ,$$

then we also obtain a recursion for the coefficients $\widehat{a}_i$. However, the first two coefficients of this recursion, $\widehat{a}_0$ and $\widehat{a}_1$, are not determined. The coefficients of both representations are related as follows.

$$\widehat{a}_i = \sum_{n=0}^{\infty} \eta^n a_i^{(n)}. \tag{3.14}$$

Therefore, one can calculate first the coefficients $a_0^{(n)}$ and $a_1^{(n)}$ with the algorithm provided in Appendix B and then apply relation (3.14) to obtain $\widehat{a}_0$ and $\widehat{a}_1$. With these starting values one can then directly compute $\widehat{a}_i$ using the above mentioned algorithm.[6] Not surprisingly, in all our numerical experiments both algorithms lead to the same results.

Furthermore, it is convenient to use an expansion at zero for the volatility η and at the short rate's mean reversion level θ/κ. To see the advantage, we substitute $\eta = 0$ and $r = \theta/\kappa$ into the expansion (3.13) to obtain $f = a_0^{(0)}$. Now, recall that this choice is equivalent to assuming that the short rate is constant and equal to the mean reversion level θ/κ. Therefore, as shown in Appendix B, the coefficient $a_0^{(0)}$ satisfies the same quadratic equation (3.12) as the cash flow multiplier if we assume constant interest rates. Consequently, our expansion (3.13) is an expansion around the cash flow multiplier for constant interest rates.

4 Numerical Example

In this section, we illustrate the implications of our model by considering one particular firm, IBM. We study its cash flow multiplier over the time period from 1962 to 2005, where the relevant information about IBM comes from Compustat. Since this is intended to be an introductory example, we postpone a detailed description of the data for our empirical study to Section 5. To start with, we choose the parameters of the riskfree short rate process (3.4) to be $\kappa = 0.1$, $\eta = 0.015$, and $\theta = 0.005$. This implies that the mean reversion level is $\theta/\kappa = 0.05$, which is close to the sample average of the one-month Fama-French riskfree rate as reported by CRSP. The reason for not formally estimating θ and κ over the period from 1962 to 2005 is that there are regime shifts during this period like the spike in 1979 that cannot be well calibrated by

[6]The algorithm is available from the authors upon request. Since it does not provide additional insights it is not reported here.

a one-factor model. We define firm value as the sum of book value plus the difference between market value and book value of equity minus deferred taxes. Free cash flows are defined as EBIDTA minus taxes.[7] For IBM we have 44 observations of the cash flow multiplier observed on December 31 of the particular year. Figure 2 plots these observations against the realizations of the Fama-French one-month risk free rate. Obviously, both variables are negatively related. This figure also depicts the least-square fit of our model.[8] This calibration is so similar to the calibration of an exponential model that they are hard to distinguish in the figure.

[INSERT FIGURE 2 ABOUT HERE]

Given these results, we run a simple OLS regression of the logarithm of the cash flow multiplier on the riskfree rate. The slope of the fitted line is -0.096 and the R^2 is about 0.33, which reflects the strong relation between both variables. In the sequel, we will extend this analysis by running panel regressions and controlling for other explanatory variables. The following section describes the panel data that we use.

5 Data

Our sample period covers 44 years ranging from 1962 to 2005. The data comes from several sources. The first is the combined annual, research, and full coverage 2006 Standard and Poor's Compustat industrial files. The sample is selected by first deleting any firm-year observations with missing data. The only exception is deferred taxes (Compustat Item74) that we set to zero if it is missing. The reason is that deferred taxes are typically an insignificant part of firm value compared to the book and market value of the assets (Item6, Item25, Item199, and Item60) and we would have lost around 10% of the observations if we had deleted them. To check the robustness of this assumption, we run our benchmark regression (1) that is reported in Table 4 excluding all observations where deferred taxes are missing. As expected, the results are virtually unchanged. Furthermore, in our benchmark regressions we have only included a firm if its fiscal year ends in December 31. This is by far the largest group. The second largest group consists of firms with fiscal year ending in June, but the number of observations is almost ten times smaller than for the firms with fiscal year ending in December. The main reason for initially looking at these firms is that we include a liquidity variable measuring share turnover

[7] Therefore, firm value is given by Item6 + Item199 $\times$ Item25 - Item60 - Item74 and free cash flows are given by Item13 - Item16. Recall that our free cash flows definition excludes investments.

[8] The calibrated values are $\hat{\phi} = -0.1047$, $\hat{\psi} = -1.4735$, $\mu_1 = 0.0280$, $\mu_2 = -0.0011$, $\sigma_0 = 1.2400$, $\sigma_2 = -2.3509$, and $\rho = -0.5689$ by applying the results of Theorem 3.3.

that Compustat only reports for the calendar year. We also report regression results where we include all firms independent of their fiscal year end (and exclude share turnover) obtaining very similar results.

In our analysis we use two definitions of cash flow multipliers. In the first, and conceptually more appropriate one, we use free cash flow (before investment) defined as the difference between EBITDA and taxes (Item13 minus Item16). In the second one we use gross profits (Item12 minus Item41) as a proxy for free cash flows. For the firms with fiscal year ending in December we have 113,972 observations for the first definition and 115,118 observations for the second definition.

Since our theoretical model and empirical analysis does not allow for negative cash flows, we need to trim the top and bottom 21% of the data set in the first case and 7% in the second case. The reason for including gross profits as a proxy for free cash flows is precisely to avoid deleting so many observations, and since the correlation between the two cash flow measures is very high (0.9), we do not expect the differences to be large. Anyhow, we also run regressions on subsamples where the cutoff quantiles are smaller. In all cases, the results are similar as we will show in our extensive robustness checks.[9] To summarize, after trimming we end up with unbalanced panels of firms over the period from 1962 to 2005 with 66,102 observations stemming from 8,043 firms in the first case and 99,002 observations stemming from 10,949 firms in the second case. If all firms independent of their fiscal year are included, we have 119,270 observations stemming from 13,682 firms in the first case and 177,197 observations stemming from 17,926 firms in the second case.

The one-month Fama-French riskfree rate was obtained from CRSP. The Treasury yields and the corporate bond yields are from Global Finance Data. The slope of the Treasury yield curve is defined as the difference between the 15y Treasury yield and the riskfree rate. The 15y Treasury yield is obtained by linearly interpolating between the 10y and 20y yield. We use this maturity since it can be matched against the industrial BBB 15y corporate bond yields reported by S&P to calculate the 15y yield spread between corporate bonds and Treasury bonds. Finally, we calculate the historical volatility of the stock market from the value weighted S&P 500 index as reported in CRSP. We use the version without dividends, but the volatilities obtained from the version with dividends are almost identical. Therefore, our results are robust with respect to this choice. We include the last 250 trading days to compute the volatility. Figure 3 depicts the sample paths of the four macro variables measured at the last trading day in December of the particular year. Table 1 presents the corresponding summary statistics. All variables

[9]See, e.g., Tables 8 and 12.

are annualized and quoted in percentages. For instance, the average riskfree rate over the period from 1962 to 2005 was 5.192%. The maximum of 12.528% was reached in 1980 and the minimum of 0.827% in 2003. There are four years where the slope went negative, namely 1968, 1973, 1980, and 2000. These are times that either fall into an NBER recession period or are close to such a period. The yield spread reached its maximum of 3.445% in 1990 and its minimum of 0.415% in 1964. Finally, the average annualized historical volatility is about 14%. Since a year has about 250 trading days, we multiply the daily volatility by the square root of 250 to obtain the annual volatility.

[INSERT FIGURE 3 AND TABLE 1 ABOUT HERE]

In the empirical analysis to follow, we regress the logarithm of the cash flow multiplier on several variables. The first three are closely related to the term structure and include the riskfree rate, the slope of the Treasury term structure, and the spread of BBB rated bonds over Treasury bonds. Holding the other variables fixed, an increase in either of these variables increases the discount rate at which free cash flows are discounted. Since in our model the discount rate is negatively related with the multiplier, we expect to observe negative relations between the multiplier and these variables. We also include the historical volatility of the the S&P 500 as an explanatory variable measuring aggregate equity market risk. We expect to observe a negative relationship between volatility and the multiplier.

Besides, as the multiplier is a measure of how firms are valued by the market, we also control for liquidity effects that might affect the valuation of a firm's equity. We proxy liquidity by the annual share turnover that is defined as the ratio between the number of common stocks traded in the calendar year (Compustat Item28) and the number of common shared outstanding (Compustat Item25). The distribution of this ratio is highly skewed with a mean of 12.45 and a median of 0.37 since there are a few observations with very high turnovers. To correct for skewness we use the logarithm of turnover in our regressions which leads to almost identical values of median and mean. In general, a high value of this ratio reflects that the corresponding company's shares are frequently traded. Therefore, it is a measure for the liquidity of the company's shares. If investors are willing to pay a liquidity premium, then we expect the cash flow multiplier to be positively related with this variable. Alternatively, this variable could also capture idiosyncratic events happening to a particular firm in a particular year. To control for size effects, we include the logarithm of the market capitalization as an explanatory variable.

The market capitalization of a firm is defined as the product of the number of shares outstanding and the price per share (Compustat Item25 and Item199). Finally, the first-order condition (3.6) of our model suggests that the multiplier increases with the proportion of the cash flows invested. To test this prediction empirically, we add a proxy for this variable to the set of our explanatory variables. We measure the investment proportion by the ratio of the annual capital expenditures (Compustat Item128) over the free cash flows. Table 2 presents the summary statistics of the firms specific variables and Table 3 summarizes the correlations between all firm specific and all macro variables. Note that the highest correlation in the table is between volatility and the BBB spread (0.51); both of them represent some measure of global risk. The corresponding statistics for the gross profits as a proxy for cash flows will be presented in the section discussing our robustness checks.

[INSERT TABLES 2, 3 ABOUT HERE]

6 Panel Regression Results

In this section we examine the determinants of the cash flow multiplier by running several panel regressions that use all the information contained in the time-series. The residuals of the cross-sectional regressions are likely to be serially correlated. Furthermore, as we will demonstrate later on, there might be cross-sectional dependance as well. To overcome these potential problems, we correct our t-statistics using the approach outlined in Driscoll and Kraay (1998). They assume an error structure that is heteroscedastic, autocorrelated up to some lag, and possibly correlated between the units.[10] The resulting standard errors are heteroscedasticity consistent as well as robust to very general forms of cross-sectional and temporal dependence. In our robustness checks, we will discuss this point in more detail.

[INSERT TABLE 4 ABOUT HERE]

Our benchmark result (1) is a fixed-effects regression presented in Table 4. As postulated in the previous section, the riskfree rate, the slope of the term structure, and volatility have significantly negative impacts on the cash flow multiplier. Additionally, the fraction of the cash flows invested, π, and market capitalization are significantly positively related with the multiplier. Interestingly, credit spread is insignificant and the same is true for the turnover variable, although both have the expected signs. The results for turnover are ambiguous, though. If we include the stock's turnover instead of the logarithm of turnover in our regression, then turnover is significant at all levels and the significance levels of the other variables are not affected. This result is driven by about 60 large realizations with turnovers of more than 40 that also cause turnover to be highly skewed.

[10] In our regressions, the maximum lag is three.

Given the magnitude of the coefficient a one standard deviation positive move in the riskfree rate (volatity, slope) implies a 6% (3%, 1%) lower cash flow multiplier relative to its mean value. On the other hand, a one standard deviation positive move in π increases the cash flow multiplier by 12%. Thus, the effects of these variables are both statistically and economically significant.

Columns (2) and (3) in Table 4 report results when we run regressions either with dummies for the 48 Fama-French industries or with two dummies for regulated, financial, or public service firms (one dummy for Fama-French industries 31 and 48 as well as one dummy for Fama-French industries 44-47). The significance levels of the significant coefficients remain the same in both regressions. However, the logarithm of turnover now becomes highly significant as well. One explanation for this might be that a fixed-effects regression is similar to a regression with dummies for each firm. Since regressions (2) and (3) aggregate these firm dummies, our results suggest that turnover may also pick up some of the firms' idiosyncratic effects. In order to test for the presence of subject-specific fixed effects, we performed a robust version of the Hausman test.[11] The null hypothesis of no fixed-effects is rejected at all levels suggesting that there are fixed effects in the data. This is the reason why we have chosen regression (1) to be our benchmark regression. Unless otherwise stated, in the sequel we thus report the results of fixed-effects regressions.

Finally, we consider regressions where we exclude some of the explanatory variables. The results are reported in Table 5. Regression (4) shows that the spread variable becomes significant at the 1% level if volatility is disregarded. On the other hand, the significance level of volatility increases to the 1% level if the spread variable is disregarded. This suggests that both variables measure similar effects. Since volatility is significant in the benchmark regression (1), whereas spread is not, volatility has more explanatory power in our sample. This is also documented by magnitude of the R^2s of regressions (4) and (5). Additionally, regressions (6) and (7) report the results if the logarithm of turnover is excluded. It can be seen that the coefficients and significance levels of the other variables are not affected.

[INSERT TABLE 5 ABOUT HERE]

7 Robustness Checks

In this section, we report the results of several checks on the basic results. The tests consider standard errors, endogeneity issues, inclusion of firms with fiscal year different from the calendar

[11] See, e.g., Wooldridge (2002), p. 288ff.

year, and exclusion of firms with few observations. In Subsection 7.1, we go through these points for our free cash flow definition. We also briefly discuss the investment definition that we use. In Subsection 7.2, we present some empirical evidence when we proxy free cash flows by gross profits. This allows us to include a much larger sample and to check our results against varying the cash flow definition.

7.1 Free Cash Flows

We first compare the standard errors of regression (2) with the standard errors that obtain if we form clusters by firm and year (regression (8)) or by firm only (regression (9)).[12] The results are reported in Table 6. Notice that the point estimates for the first two regressions are exactly the same. Besides, the standard errors are similar leading to the same significance levels of the coefficients. Clustering by firm only however leads to overly optimistic standard errors. The same would be true if we run a fixed-effects regression clustering by firm only. To get an idea whether it is appropriate to use Driscoll-Kray errors, we performed Pesaran's test of cross sectional independence on a subsample of firms with at least 30 observations. This test rejects independence at all significance levels, which suggest that Driscoll-Kray standard errors are more appropriate.

[INSERT TABLE 6 ABOUT HERE]

The next issue we consider is the possibility of endogeneity. One may argue, for example, that high cash flow multipliers may lead to more investment activity. A straightforward way to address this is to consider the relation between cash-flow multiplier and one-year lagged investment proportion, π. Results from doing so appear in Table 7. It turns out that the significance of π is preserved in this alternative specification, even though, understandably, the point estimate of the coefficient declines. The same is true for the logarithm of market capitalization. Column (13) reports the results if we exclude both variables from the set of explanatory variables. It can be seen that the estimates of the other variables are hardly affected. Interestingly, in this case the logarithm of turnover becomes significant.

[INSERT TABLE 7 ABOUT HERE]

We also run regressions on subsamples with more observations per firm. The reason for this is that for these firms the number of disregarded observations are much smaller. The results

[12] See, e.g., Pedersen (2009) and the references therein.

of the panel regressions when we only include firms that have at least 10, 20, 30 and 40 full observations are presented in Table 8. It can be seen that the cutoff quantiles are decreasing, whereas the average market capitalization is increasing as we require more observations per firm. Nevertheless, our main results remain unchanged. The riskfree rate and π are still significant at all levels. The only differences are that slope becomes more significant for larger firms, whereas volatility is only significant at the 10% level in regression (15) and not significant in regressions (16) and (17).

Furthermore, our proxy for investments are capital expenditures (Compustat Item128) that do not include R&D expenses (Compustat Item46). The main reason for using this proxy is that we would have lost almost 60% of our observations since Item46 is often missing in Compustat. To check whether including R&D expenses changes our results, we run our benchmark regression (1) with the ratio of R&D expenses over free cash flows as an additional explanatory variable. This regression is based on 27,959 observations coming from 3992 firms. Both investment ratios are highly significant and have positive coefficients. The levels of the other variables are not affected except for slope which is not significant any more. However, this cannot be attributed to the inclusion of the R&D ratio, but to the smaller sample size since slope is also insignificant when we run the same regression on the subsample without the R&D ratio.

As mentioned earlier most of the firms in our sample have their fiscal year ending December 31, but there are firms with fiscal year ending before that date. This may raise some concerns since our benchmark regression (1) only includes firms with fiscal year ending in December. To deal with this issue, Table 9 reports regression results when we include all firms independent of their fiscal year. This leads to more than 119,000 observations coming from around 13,700 firms (compared to 66,102 coming from 8,043 firms). In these regressions, we need to exclude turnover since we have only annual data for this variable. Nevertheless, our results are very similar to the benchmark regression. Slope is still significant, albeit at the 10% level only. To summarize, all the above mentioned regressions confirm that our results are very robust to different specifications of the regressions, data and estimation procedures.

[INSERT TABLES 8 and 9 ABOUT HERE]

7.2 Gross Profits

In this subsection, we report the results when we proxy free cash flow by gross profits. Since gross profits are more likely to be positive, this approach allows us to include more observations in our sample. It can be justified by the large correlation between free cash flows and gross profits, which is about 90%. Running a simple regression of gross profits on free cash flows leads to a regression coefficient of about 1.6 that is significant at all levels. The R^2 of this regression is 80%. This suggests that gross profits are a good proxy for free cash flows. Tables 10 and 11 present the summary statistics and the correlations for this alternative approach. From Table 11 note that the sign of the correlation between the cash flow multiplier and the spread BBBGov changes. This spread is however insignificant in our regressions as long as Vol250 is included.

[INSERT TABLES and 10 and 11 ABOUT HERE]

To simplify comparisons between the regression results on free cash flows and gross profits, we use the convention that regressions with respect to the same explanatory variables have the same numbers. To distinguish both cases, the numbers of the regressions for gross profits have primes. Column (1') of Table 12 presents the benchmark regression for gross profits. Notice that this regression is now based on 90,492 observations stemming from 10,197 firms. The significance levels of the coefficients are very similar to the levels of regression (1). The main difference is that, although slope has still a negative sign, it is not significant. In fact, its p-value is 19%. Regressions (6') and (7') exhibit similar patterns as before since both spread and volatility alone are significant at the 1% level. Besides, column (12') reports the results when we lag π and the logarithm of market capitalization. Both remain significant. Interestingly, the logarithm of turnover is highly significant, which was not the case in regression (12). This shows again the ambivalent nature of turnover that we have already discussed before. The same is true for regression (13') where we disregard π and market capitalization completely. For this regression, slope is significant at the 10% level.

Tables 13 and 14 report the results when we either consider subsamples of firms with more observations or include firms independent of their fiscal year. Since we now proxy free cash flows by gross profits, the percentages of disregarded firms in Table 13 are much smaller than in Table 8 as can be seen from the cutoff quantiles that are smaller. The patterns are however the same. In particular, the relevance of slope increases for bigger firms, whereas volatility becomes insignificant. Actually, slope is already significant at the 10% level in regression (14'), whereas volatility is only borderline significant at this level in (15').

Finally, the regressions reported in Table 14 rely on the largest sample studied in this paper. The number of observation is 177,197 coming from 17,929 firms. Nevertheless, our main results are confirmed.

[INSERT TABLES and 12, 13, 14 ABOUT HERE]

8 Value of the Option to Invest

We have shown that the cash flow multiplier consists of two parts (see, e.g., (3.10)): Whereas the first part is exogenous, the second part is endogenous and captures the firm's real option to invest, the so-called net present value of growth opportunities. Besides, Theorem 3.2 proves that the option value is decreasing with μ_0. This parameter equals the expected cash flow growth if the firm does not invest at all. We expect μ_0 to be on average smaller when the firm operates in an industry that is more investment intensive. Investment intensity is measured by the average fraction of cash flows that is reinvested, i.e. by the average π of a particular industry. To test this hypothesis, we run regressions where this average is included as an additional explanatory variable. We have already seen that the cash flow multiplier increases with π. Following our line of argument, the opposite should be true for the mean of the industry. There are two ways of calculating an industry mean. Firstly, one can calculate the mean over the whole sample period leading to a constant. Secondly, one can compute the mean for every year of the sample period, which provides us with 48 time series of means for the 48 Fama-French industries. In the first case, it clearly makes no sense to include firm dummies or fixed effects since otherwise the coefficients of the average π cannot be identified. But also in the second case dummies would absorb a lot of the variability that we expect to be captured by the industry means of π. For this reason, we run four pooled regressions without dummies and report the results in Table 15. Columns (22) and (23) are based on the same set of observations as our benchmark regression (1), whereas regressions (24) and (25) include all firms independent of their fiscal year such as in regression (19). The variable Av_pi denotes the average π of the corresponding industry over the whole sample period of 44 years. In contrast, Av_pi_annual denotes the average π of the corresponding industry calculated every year leading to 48 time series. It can be seen that in all regressions the coefficients on the average π are significantly negative, which supports our line of argument above. In the last two regressions, volatility is significant at the 10% level only, whereas slope is not significant any more. The significance levels of the other coefficients do not change. Finally, we compare the results of regression (22) with the results of regression (2) where we included industry dummies for the Fama-French industries. Figure 4 plots the values of the dummies that are significant at a 5% level against the average πs of the corresponding industries, Av_pi. The relation is strongly negative showing that the dummies are related to the investment intensities of the industries. The result does not change when we also include the dummies that are insignificant.

[INSERT TABLE 15 and FIGURE 4 ABOUT HERE]

9 Conclusion

We develop a simple discounted cash flow valuation model with optimal investment that provides the basis for an extensive empirical analysis. The dependent variable in the valuation exercise is the cash flow multiplier, defined as the ratio of current firm value and current cash flows (before investment). The explanatory variables include macro variables such as interest rates, credit spreads and equity market volatility, and firm specific variables such as liquidity, size and the proportion of cash flows reinvested in the firm. In addition we include dummy variables to take into account firm and/or industry fixed effects. The panel regression results indicate that the explanatory variables have the correct sign and for the most part are highly significant. In addition, we perform extensive robustness checks to deal with econometric and data issues such as different estimation of standard errors, endogeneity issues, inclusion of firms with fiscal year different from calendar year, exclusion of firms with few observations, and different definitions of the cash flow multiplier. In all cases the main results of the analysis stand. Since the cash flow multiplier depends on observable and relatively easily obtainable variables, the approach taken in this paper can be considered as an alternative valuation framework. Even though it is based on a discounted cash flow model it does not require the estimation of expected future cash flow and an appropriate risk adjusted discount rate. Potentially then, the approach could be used to value non-traded firms and to determine under and over priced firms.

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A Proofs

Proof of Proposition 2.1. We set $Y_t = C_t/c$ such that $Y_0 = 1$ and $dY = dC$. Then, problem (2.1) can be rewritten as

$$V(c,x) = \max_\pi \mathrm{E}\Big[\int_0^\infty e^{-\int_0^s r_u\,du}(cY_s - \pi_s cY_s)ds\Big] = c\max_\pi \mathrm{E}\Big[\int_0^\infty e^{-\int_0^s r_u\,du}(Y_s - \pi_s Y_s)ds\Big] = cV(1,x).$$

This implies $V_c(c,x) = V(1) = const$ and $V_{cc}(c,x) = 0$, which shows that V is linear in c. □

Proof of the Hamilton-Jacobi-Bellman equation (3.5). The firm value satisfies the Hamilton-Jacobi-Bellman equation

$$0 = \max_\pi\{(\mu c V_c + c - \pi c - rV + \alpha(x)cV_x + 0.5\eta^2(x)cV_{xx} + \eta(x)c\sigma(\pi,x)\rho V_{cx}\}.$$

Applying the separation (2.3) yields (3.5). □

Proof of Proposition 3.1. The equation (3.12) follows from (3.11). By Vieta's formulas, the two solutions, f_1 and f_2, satisfy

$$f_1 f_2 = \frac{1}{\mu_1^2/4 - \mu_2(\mu_0 - r - \lambda)}.$$

implying that there exists a unique positive cash flow multiplier if $\mu_1^2/4 - \mu_2(\mu_0 - r - \lambda) < 0$, i.e. if the parabola defined in (3.12) has a maximum. □

Proof of Theorem 3.2. We set $K = \mu_1^2/4 - \mu_2(\mu_0 - r - \lambda)$. By assumption of Proposition 3.1, K is negative. Due to the transversality condition, this implies that $\mu_2 < 0$. We interpret (3.12) as the implicit definition of f as a function of μ_0. For this reason, we interpret the right-hand side of (3.12) as a function F of f and μ_0. Then

$$f' = \frac{df}{d\mu_0} = -\frac{\partial F/\partial \mu_0}{\partial F/\partial f} = \frac{-\mu_2 f^2 + f}{2Kf + \mu_0 - r - \lambda - \mu_2} > 0,$$

since, by assumption, $\mu_0 - r - \lambda - \mu_2 < 0$. Next recall that

$$\mathcal{O} = \frac{-1}{\mu_0 - r - \lambda}\frac{\mu_1^2 f^2}{4(1-\mu_2 f)}.$$

Therefore,

$$\mathcal{O}' = \frac{d\mathcal{O}}{d\mu_0} = \frac{1}{(\mu_0 - r - \lambda)^2}\frac{\mu_1^2 f^2}{4(1-\mu_2 f)} + \frac{-1}{\mu_0 - r - \lambda}\frac{\mu_1^2}{4}\frac{ff'(2-\mu_2 f)}{(1-\mu_2 f)^2} > 0.$$

Finally,

$$\frac{\mathcal{O}}{f} = \frac{\mathcal{O}}{\frac{-1}{\mu_0 - r - \lambda} + \mathcal{O}}.$$

Consequently,

$$\frac{d}{d\mu_0}\frac{\mathcal{O}}{f} = \frac{-1}{\mu_0 - r - \lambda}\frac{\mathcal{O}' - \frac{1}{\mu_0 - r - \lambda}\mathcal{O}}{f^2} > 0,$$

since

$$\mathcal{O}' - \frac{1}{\mu_0 - r - \lambda}\mathcal{O} = \frac{-1}{\mu_0 - r - \lambda}\frac{\mu_1^2}{4}\frac{ff'(2 - \mu_2 f)}{(1 - \mu_2 f)^2} > 0.$$

This completes the proof of Theorem 3.2. □

B Series Expansion of Theorem 3.3

We firstly provide a representation of the coefficients in the series expansion (3.13) and then prove Theorem 3.3). We define $\widehat{\theta} = \theta/\kappa$, $\widetilde{\varphi} = \widehat{\varphi} + \widehat{\widehat{\psi}\theta}$, and

$$H_{i,j}^{k,\nu} = \widetilde{a}_i^{(k)} b_j^{(\nu)} + 0.25 c_i^{(k)} c_j^{(\nu)},$$

where

$$\begin{aligned}
\widetilde{a}_i^{(\nu)} &= \mathbf{1}_{\{\nu=i=0\}} - \mu_2 a_i^{(\nu)} - \rho\sigma_2(i+1)a_{i+1}^{(\nu-1)}, \\
b_i^{(\nu)} &= \mathbf{1}_{\{\nu=i=0\}} + \widetilde{\varphi} a_i^{(\nu)} + \widehat{\psi} a_{i-1}^{(\nu)} - \kappa i a_i^{(\nu)} + \rho\sigma_0(i+1)a_{i+1}^{(\nu-1)} + 0.5(i+2)(i+1)a_{i+2}^{(\nu-2)} \\
c_i^{(\nu)} &= \mu_1 a_i^{(\nu)} + \rho\sigma_1(i+1)a_{i+1}^{(\nu-1)}.
\end{aligned}$$

Then the coefficients are given by the following explicit recursion

$$\begin{aligned}
a_0^{(n)} &= -\frac{\sum_{k=1}^{n-1} H_{0,0}^{k,n-k} + R_0^{(n)}}{D_0}, \\
a_m^{(n)} &= -\frac{\sum_{(i,k)\in\mathcal{I}} H_{i,m-i}^{k,n-k} + R_m^{(n)}}{D_m},
\end{aligned} \tag{B.15}$$

where

$$\begin{aligned}
R_m^{(n)} &= (1 - \mu_2 a_0^{(0)})\left[\mathbf{1}_{\{m=n=0\}} + \widehat{\psi} a_{m-1}^{(n)} + \rho\sigma_0(m+1)a_{m+1}^{(n-1)} + 0.5(m+2)(m+1)a_{m+2}^{(n-2)}\right] \\
&\quad + (1 + \widetilde{\varphi} a_0^{(0)})\left[\mathbf{1}_{\{m=n=0\}} - \rho\sigma_2(m+1)a_{m+1}^{(n-1)}\right] + 0.5\mu_1\rho\sigma_1(m+1)a_0^{(0)}a_{m+1}^{(n-1)}, \\
D_m &= (1 - \mu_2 a_0^{(0)})(\widetilde{\varphi} - m\kappa) - \mu_2(1 + \widetilde{\varphi} a_0^{(0)}) + 0.5\mu_1^2 a_0^{(0)}
\end{aligned}$$

and $\mathcal{I} = \{0, 1, \ldots, m-1, m\} \times \{0, 1, \ldots, n-1, n\} \setminus \{(0,0), (m,n)\}$ is an index set.[13]
We emphasize that this recursion is explicit and all equations (B.15) do not involve $a_m^{(n)}$ on the right-hand side. The only exception is the equation for $a_0^{(0)}$ where $a_0^{(0)}$ appears on the left- and right-hand side. This leads to the following quadratic equation.

$$0 = (0.25\mu_1^2 - \mu_2\widetilde{\varphi})(a_0^{(0)})^2 + (\widetilde{\varphi} - \mu_2)a_0^{(0)} + 1.$$

For reasonable parametrizations, numerical experiments suggest that this equation has one positive and one negative root.

Proof of Theorem 3.3. We firstly multiply equation (3.7) by $4(1 - \mu_2 f - \rho\sigma_2\eta f_r)$. Then we substitute the representation (3.13) into the resulting equation. This leaves us with several products of power series. Expanding these products and rearranging, we can rewrite equation (3.7) as follows:

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left\{\sum_{k=0}^{n}\sum_{j=0}^{m}\widetilde{a}_j^{(k)}b_{m-j}^{(n-k)} + 0.25c_j^{(k)}c_{m-j}^{(n-k)}\right\}(r-\widehat{\theta})^m\eta^n = 0.$$

Since the representation of a power series is unique, we conclude that $\{\ldots\} = 0$ for all $(n,m) \in I\!N_0 \times I\!N_0$. This gives a series of equations for the coefficients $a_m^{(n)}$. Solving these equations yields (B.15). $\square$

C Two State Processes

We consider an economy that is driven by two state processes Y and Z that have Vasicek dynamics

$$\begin{aligned} dY &= (\theta_Y - \kappa_Y Y)dt + \eta_Y dW_Y, \\ dZ &= (\theta_Z - \kappa_Z Z)dt + \eta_Z dW_Z. \end{aligned}$$

The cash flow process of the firm is given by

$$dC = C[\mu(\pi, Y, Z)dt + \sigma(\pi)dW], \quad C(0) = c,$$

where

$$\begin{aligned} \mu(\pi, Y, Z) &= \overline{\mu}_0 + \mu_0^Y Y + \mu_0^Z Z + \widehat{\mu}_0 YZ + \mu_1\sqrt{\pi} + \mu_2\pi, \\ \sigma(\pi) &= \sigma(\pi) = \sigma_0 + \sigma_1\sqrt{\pi} + \sigma_2\pi \end{aligned}$$

[13]Therefore, the difference $\sum_{k=0}^{n}\sum_{i=0}^{m}\ldots$ has two more elements than $\sum_{(i,k)\in\mathcal{I}}\ldots$, namely the elements with indices $(i,k) = (0,0)$ and $(i,k) = (m,n)$.

with constants $\overline{\mu}_0$, μ_0^Y, μ_0^Z, $\widehat{\mu}_0$, μ_1, and μ_2. The processes W, W_Y, and W_Z are correlated Brownian motions with the constant correlations ρ_{YC}, ρ_{ZC}, and ρ_{YZ}. The firm value reads

$$V(c,y,z) = \max_{\pi} \mathrm{E}\Big[\int_0^\infty e^{-\int_0^s R_u\,du}(C_s - I_s)ds\Big],$$

where, with a slight abuse of notation, the risk-adjusted discount rate R is of the form

$$R(Y,Z) = \overline{r} + r^Y Y + r^Z Z + \widehat{r} YZ$$

with constants $\overline{r}$, r^Y, r^Z, and $\widehat{r}$. This specification gives us some flexibility and allows for several possible interpretations. For instance, assume that Y is the default-free interest rate. Then, one could choose R to be of the form

$$R = Y + \beta\lambda,$$

where β is the firm's beta and λ is the risk premium. If the default-free interest rate predicts the risk premium, then one can set

$$\lambda = \overline{\lambda} + \lambda^Y Y.$$

Then Z could model a stochastic beta of the firm. In this case,

$$R = Y + Z(\overline{\lambda} + \lambda^Y Y) = Y + \overline{\lambda} Z + \lambda^Y YZ$$

or in our above notation

$$\overline{r} = 0, \quad r^Y = 1, \quad r^Z = \overline{\lambda}, \quad \widehat{r} = \lambda^Y.$$

Alternatively, one could assume the beta of the firm to be constant and identify Z with the risk premium. Then,

$$R = Y + \beta Z,$$

or in our notation above

$$\overline{r} = 0, \quad r^Y = 1, \quad r^Z = \beta = const, \quad \widehat{r} = 0.$$

The Bellman equation for this problem reads

$$\begin{aligned} 0 \;=\; & \max_{\pi} \Big\{ \mu(y,z,\pi)cV_c + 0.5\sigma^2(\pi)cV_{cc} - R(y,z)V + c - \pi c + (\theta_Y - \kappa_Y y)V_y + 0.5\eta_Y^2 V_{yy} \\ & + (\theta_Z - \kappa_Z z)V_z + 0.5\eta_Z^2 V_{zz} + \eta_Y \eta_Z \rho_{YZ} V_{yz} + \eta_Y \sigma(\pi) c \rho_{YC} V_{yc} + \eta_Z \sigma(\pi) c \rho_{ZC} V_{zc} \Big\}. \end{aligned}$$

We conjecture the following form of the firm value

$$V(c,y,z) = cf(y,z)$$

and obtain

$$\begin{aligned} 0 &= \max_{\pi}\Big\{\mu(y,z,\pi)f - R(y,z)f + 1 - \pi + (\theta_Y - \kappa_Y y)f_y + 0.5\eta_Y^2 f_{yy} \\ &\quad + (\theta_Z - \kappa_Z z)f_z + 0.5\eta_Z^2 f_{zz} + \eta_Y\eta_Z\rho_{YZ}f_{yz} + \eta_Y\sigma(\pi)\rho_{YC}f_y + \eta_Z\sigma(\pi)\rho_{ZC}f_z\Big\}. \end{aligned}$$

Notice that the term involving V_{cc} drops out since the firm value is linear in the current cash flow. The first-order condition for the optimal investment proportion reads

$$\pi^* = \left(\frac{\mu_1 f + \eta_Y\sigma_1\rho_{YC}f_y + \eta_Z\sigma_1\rho_{ZC}f_z}{2(1 - \mu_2 f - \eta_Y\sigma_2\rho_{YC}f_y - \eta_Z\sigma_2\rho_{ZC}f_z)}\right)^2.$$

Substituting back into the Bellman equation leads to a non-linear second-order partial differential equation for f:

$$\begin{aligned} 0 &= (\overline{\alpha} + \alpha_Y y + \alpha_Z z + \widehat{\alpha}yz)f + 1 + (\theta_Y + \eta_Y\sigma_0\rho_{YC} - \kappa_Y y)f_y + 0.5\eta_Y^2 f_{yy} \\ &\quad + (\theta_Z + \eta_Z\sigma_0\rho_{ZC} - \kappa_Z z)f_z + 0.5\eta_Z^2 f_{zz} + \eta_Y\eta_Z\rho_{YZ}f_{yz} \\ &\quad + 0.25\frac{(\mu_1 f + \eta_Y\sigma_1\rho_{YC}f_y + \eta_Z\sigma_1\rho_{ZC}f_z)^2}{1 - \mu_2 f - \eta_Y\sigma_2\rho_{YC}f_y - \eta_Z\sigma_2\rho_{ZC}f_z}, \end{aligned} \tag{C.16}$$

where $\overline{\alpha} = \overline{\mu}_0 - \overline{r}$, $\alpha_Y = \mu_0^Y - r^Y$, $\alpha_Z = \mu_0^Z - r^Z$, and $\widehat{\alpha} = \widehat{\mu}_0 - \widehat{r}$. We solve this equation in two steps. Firstly, we expand f in terms of η_Y and η_Z in the following way

$$f(y,z) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} A^{n,m}(y,z)(\eta_Y)^n(\eta_Z)^m. \tag{C.17}$$

This leads to the following result.

Proposition C.1 (PDEs for $A^{n,m}$)**.** *The functions $A^{n,m}$ satisfy the following series of partial differential equations*

$$\sum_{p=0}^{i}\sum_{q=0}^{k}\Big\{\widetilde{A}^{p,q}(y,z)B^{i-p,k-q}(y,z) + 0.25C^{p,q}(y,z)C^{i-p,k-q}(y,z)\Big\} = 0, \quad (i,k) \in I\!N_0 \times I\!N_0, \tag{C.18}$$

where

$$\begin{aligned} \widetilde{A}^{n,m} &= \mathbf{1}_{\{n=m=0\}} - \mu_2 A^{n,m} - \sigma_2\rho_{YC}A_y^{n-1,m} - \sigma_2\rho_{ZC}A_z^{n,m-1}, \\ B^{n,m} &= \mathbf{1}_{\{n=m=0\}} + (\overline{\alpha} + \alpha_Y y + \alpha_Z z + \widehat{\alpha}yz)A^{n,m} + (\theta_Y - \kappa_Y y)A_y^{n,m} + \sigma_0\rho_{YC}A_y^{n-1,m} \\ &\quad + 0.5A_{yy}^{n-2,m} + (\theta_Z - \kappa_Z z)A_z^{n,m} + \sigma_0\rho_{ZC}A_z^{n,m-1} + 0.5A_{zz}^{n,m-2} + \rho_{YZ}A_{yz}^{n-1,m-1}, \\ C^{n,m} &= \mu_1 A^{n,m} + \sigma_1\rho_{YC}A_y^{n-1,m} + \sigma_1\rho_{ZC}A_z^{n,m-1}, \end{aligned}$$

with the convention that coefficients with negative indices are zero.

Proof. Substituting (C.17) into (C.16) and long calculations yield (C.18). $\square$

We now expand $A^{n,m}$ in terms of the state variables y and z centered at the mean reversion levels $\widehat{\theta}_Y = \theta_Y/\kappa_Y$ and $\widehat{\theta}_Z = \theta_Z/\kappa_Z$, i.e.

$$A^{n,m}(y,z) = \sum_{\nu=0}^{\infty}\sum_{\ell=0}^{\infty} a_{\nu,\ell}^{n,m}\left(y-\widehat{\theta}_Y\right)^{\nu}\left(z-\widehat{\theta}_Z\right)^{\ell}.$$

This leads to the following representation of f:

$$\begin{aligned}
f(y,z) &= \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} A^{n,m}(y,z)(\eta_Y)^n(\eta_Z)^m \\
&= \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{\nu=0}^{\infty}\sum_{\ell=0}^{\infty} a_{\nu,\ell}^{n,m}\left(y-\widehat{\theta}_Y\right)^{\nu}\left(z-\widehat{\theta}_Z\right)^{\ell}(\eta_Y)^n(\eta_Z)^m \\
&= \sum_{\nu=0}^{\infty}\sum_{\ell=0}^{\infty}\underbrace{\left(\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(\eta_Y)^n(\eta_Z)^m a_{\nu,\ell}^{n,m}\right)}_{=\widehat{a}_{\nu,\ell}}\left(y-\widehat{\theta}_Y\right)^{\nu}\left(z-\widehat{\theta}_Z\right)^{\ell}
\end{aligned}$$

To derive this representation, the following lemma firstly provides the expansions of the functions $\widetilde{A}^{n,m}$, $B^{n,m}$, and $C^{n,m}$.

Lemma C.2. *For $(n,m) \in I\!N_0 \times I\!N_0$ we obtain*

$$\begin{aligned}
\widetilde{A}^{n,m} &= \sum_{\nu=0}^{\infty}\sum_{\ell=0}^{\infty}\widetilde{a}_{\nu,\ell}^{n,m}\left(y-\widehat{\theta}_Y\right)^{\nu}\left(z-\widehat{\theta}_Z\right)^{\ell}, \quad B^{n,m} = \sum_{\nu=0}^{\infty}\sum_{\ell=0}^{\infty} b_{\nu,\ell}^{n,m}\left(y-\widehat{\theta}_Y\right)^{\nu}\left(z-\widehat{\theta}_Z\right)^{\ell}, \\
C^{n,m} &= \sum_{\nu=0}^{\infty}\sum_{\ell=0}^{\infty} c_{\nu,\ell}^{n,m}\left(y-\widehat{\theta}_Y\right)^{\nu}\left(z-\widehat{\theta}_Z\right)^{\ell}.
\end{aligned}$$

with[14]

$$\begin{aligned}
\widetilde{a}_{\nu,\ell}^{n,m} &= \mathbf{1}_{\{n,m,\nu,\ell=0\}} - \mu_2 a_{\nu,\ell}^{n,m} - \sigma_2\rho_{YC}(\nu+1)a_{\nu+1,\ell}^{n-1,m} - \sigma_2\rho_{ZC}(\ell+1)a_{\nu,\ell+1}^{n,m-1}, \\
b_{\nu,\ell}^{n,m} &= \mathbf{1}_{\{n,m,\nu,\ell=0\}} + (\overline{\overline{\alpha}} - \nu\kappa_Y - \ell\kappa_Z)a_{\nu,\ell}^{n,m} + \widehat{b}_{\nu,\ell}^{n,m}, \\
c_{\nu,\ell}^{n,m} &= \mu_1 a_{\nu,\ell}^{n,m} + \sigma_1\rho_{YC}(\nu+1)a_{\nu+1,\ell}^{n-1,m} + \sigma_1\rho_{ZC}(\ell+1)a_{\nu,\ell+1}^{n,m-1},
\end{aligned}$$

where

$$\begin{aligned}
\overline{\overline{\alpha}} &= \overline{\alpha} + \alpha_Y\widehat{\theta}_Y + \alpha_Z\widehat{\theta}_Z + \widehat{\alpha}\widehat{\theta}_Y\widehat{\theta}_Z, \\
\widehat{b}_{\nu,\ell}^{n,m} &= (\alpha_Y + \widehat{\alpha}\widehat{\theta}_Z)a_{\nu-1,\ell}^{n,m} + (\alpha_Z + \widehat{\alpha}\widehat{\theta}_Y)a_{\nu,\ell-1}^{n,m} + \widehat{\alpha}a_{\nu-1,\ell-1}^{n,m} + \sigma_0\rho_{YC}(\nu+1)a_{\nu+1,\ell}^{n-1,m} \\
&\quad +0.5(\nu+2)(\nu+1)a_{\nu+2,\ell}^{n-2,m} + \sigma_0\rho_{ZC}(\ell+1)a_{\nu,\ell+1}^{n,m-1} + 0.5(\ell+2)(\ell+1)a_{\nu,\ell+2}^{n,m-2} \\
&\quad +\rho_{YZ}(\nu+1)(\ell+1)a_{\nu+1,\ell+1}^{n-1,m-1}
\end{aligned}$$

[14] $\mathbf{1}_{\{n,m,\nu,\ell=0\}}$ is one if $n=m=\nu=\ell=0$ and zero otherwise.

Remark. Splitting up $b^{n,m}_{\nu,\ell}$ into a term involving $a^{n,m}_{\nu,\ell}$ and into the term $\widehat{b}^{n,m}_{\nu,\ell}$ will be useful later on. This is because $\widehat{b}^{n,m}_{\nu,\ell}$ involves lower order coefficients only that are known when one calculates $a^{n,m}_{\nu,\ell}$ with the help of a recursion that we will provide below.

Combining our results above, we can rewrite the Bellman equation (C.16) in the following way:

$$0=\sum_{\nu=0}^{\infty}\sum_{\ell=0}^{\infty}\sum_{N=0}^{\infty}\sum_{M=0}^{\infty}\Lambda^{N,M}_{\nu,\ell}\left(y-\widehat{\theta}_Y\right)^{\nu}\left(z-\widehat{\theta}_Z\right)^{\ell}(\eta_Y)^N(\eta_Z)^M,$$

where

$$\Lambda^{N,M}_{\nu,\ell}=\sum_{p=0}^{\nu}\sum_{q=0}^{\ell}\sum_{n=0}^{N}\sum_{m=0}^{M}\underbrace{\widetilde{a}^{n,m}_{p,q}b^{N-n,M-m}_{\nu-p,\ell-q}+0.25c^{n,m}_{p,q}c^{N-n,M-m}_{\nu-p,\ell-q}}_{=H^{n,m,N-m,M-m}_{p,q,\nu-p,\ell-q}}.$$

Since the representation of a power series is unique, we obtain that for all combinations $(\nu,\ell,N,M)\in I\!N_0\times I\!N_0\times I\!N_0\times I\!N_0$

$$\Lambda^{N,M}_{\nu,\ell}=0.$$

We thus obtain the following result.

Proposition C.3 (Recursion for $a^{N,M}_{\nu,\ell}$)**.** *The coefficients are given by the following recursion*

$$\begin{aligned}a^{N,M}_{0,0}&=-\frac{\sum_{(n,m)\in\mathcal{K}}H^{n,m,N-m,M-m}_{0,0,0,0}+R^{N,M}_{0,0}}{D_{0,0}},\\ a^{N,M}_{\nu,\ell}&=-\frac{\sum_{(p,q,n,m)\in\mathcal{I}}H^{n,m,N-m,M-m}_{p,q,\nu-p,\ell-q}+R^{N,M}_{\nu,\ell}}{D_{\nu,\ell}},\end{aligned}\tag{C.19}$$

where

$$\begin{aligned}R^{N,M}_{\nu,\ell}&=(1-\mu_2a^{0,0}_{0,0})\left[\mathbf{1}_{\{N,M,\nu,\ell=0\}}+\widehat{b}^{n,m}_{\nu,\ell}\right]\\&\quad+(1+\overline{\overline{\alpha}}a^{0,0}_{0,0})\left[\mathbf{1}_{\{N,M,\nu,\ell=0\}}-\sigma_2\rho_{YC}(\nu+1)a^{N-1,M}_{\nu+1,\ell}-\sigma_2\rho_{ZC}(\ell+1)a^{N,M-1}_{\nu,\ell+1}\right]\\&\quad+0.5\mu_1\sigma_1a^{0,0}_{0,0}\left[\rho_{YC}(\nu+1)a^{N-1,M}_{\nu+1,\ell}+\rho_{ZC}(\ell+1)a^{N,M-1}_{\nu,\ell+1}\right],\\ D_{\nu,\ell}&=(1-\mu_2a^{0,0}_{0,0})(\overline{\overline{\alpha}}-\nu\kappa_Y-\ell\kappa_Z)-\mu_2(1+\overline{\overline{\alpha}}a^{0,0}_{0,0})+0.5\mu_1^2a^{0,0}_{0,0}\end{aligned}$$

and $\mathcal{K}=\{0,1,\ldots,N-1,N\}\times\{0,1,\ldots,M-1,M\}\setminus\{(0,0),(N,M)\}$ *as well as* $\mathcal{J}=\{0,1,\ldots,\nu-1,\nu\}\times\{0,1,\ldots,\ell-1,\ell\}\times\{0,1,\ldots,N-1,N\}\times\{0,1,\ldots,M-1,M\}\setminus\{(0,0,0,0),(\nu,\ell,N,M)\}$ *are index sets.*[15]

[15] Therefore, the sum $\sum_{n=0}^{N}\sum_{m=0}^{M}\cdots$ has two more elements than $\sum_{(n,m)\in\mathcal{K}}\cdots$, namely the elements with indices $(n,m)=(0,0)$ and $(n,m)=(M,N)$. The same property holds for the index set $\mathcal{I}$ accordingly.

We emphasize that this recursion is explicit. Although the previous proposition is also valid for $a^{0,0}_{\nu,\ell}$, we summarize the corresponding results in a separate corollary. In particular, the equation for $a^{0,0}_{0,0}$ is special because in this case $a^{0,0}_{0,0}$ appears on both sides of equation (C.19). This is the only equation of the recursion that is non-linear.

Corollary C.4 (Representation of $A^{0,0}$)**.** *The coefficient $a^{0,0}_{0,0}$ satisfies the quadratic equation*

$$0 = (0.25\mu_1^2 - \mu_2\overline{\overline{\alpha}})(a^{0,0}_{0,0})^2 + (\overline{\overline{\alpha}} - \mu_2)a^{0,0}_{0,0} + 1. \tag{C.20}$$

The subsequent coefficients can be calculated from the explicit representation

$$a^{0,0}_{\nu,\ell} = -\frac{\sum_{(p,q)\in\mathcal{J}} H^{0,0,0,0}_{p,q,\nu-p,\ell-q} + R^{0,0}_{\nu,\ell}}{D_{\nu,\ell}} \tag{C.21}$$

where

$$R^{0,0}_{\nu,\ell} = (1-\mu_2 a^{0,0}_{0,0})\Big[\mathbf{1}_{\{\nu,\ell=0\}} + (\alpha_Y + \widehat{\alpha}\widehat{\theta}_Z)a^{0,0}_{\nu-1,\ell} + (\alpha_Z + \widehat{\alpha}\widehat{\theta}_Y)a^{0,0}_{\nu,\ell-1} + \widehat{\alpha}a^{0,0}_{\nu-1,\ell-1}\Big] + (1+\overline{\overline{\alpha}}a^{0,0}_{0,0})\mathbf{1}_{\{\nu,\ell=0\}}$$

and $\mathcal{J} = \{0,1,\dots,\nu-1,\nu\} \times \{0,1,\dots,\ell-1,\ell\} \setminus \{(0,0),(\nu,\ell)\}$ is an index set.

Notice that (C.20) becomes (3.12) if the state processes are constant.

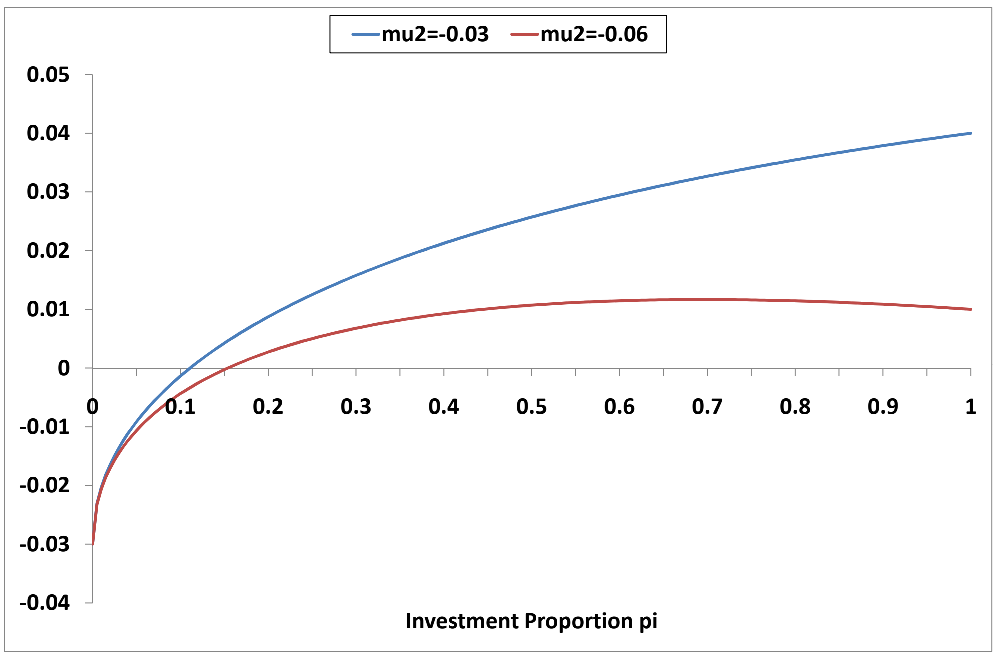

Figure 1: The figure illustrates two different forms of the expected growth rate. In both cases, it is assumed that $\mu_0 = -0.03$ and $\mu_1 = 0.1$. For the upper curve, we have $\mu_2 = -0.03$ and for the lower one $\mu_2 = -0.06$.

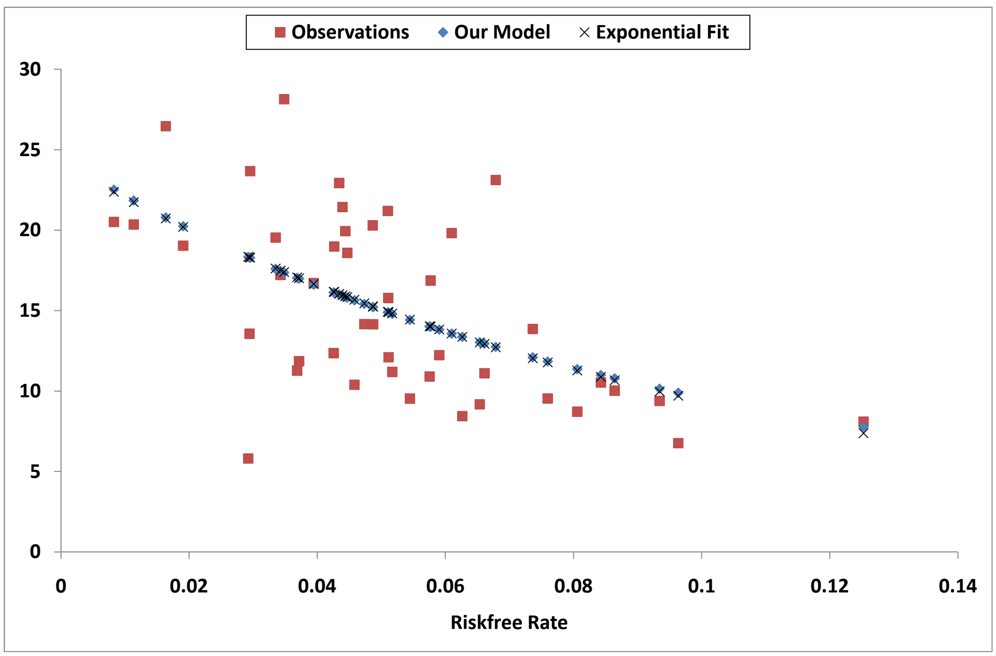

Figure 2: This figure depicts 44 observations of IBM's cash flow multiplier over the period from 1962 to 2005, as a function of risk free rate. It also shows the fit of our model and an exponential fit. As can be seen, both calibrations are so similar that they are hard to distinguish in the figure.

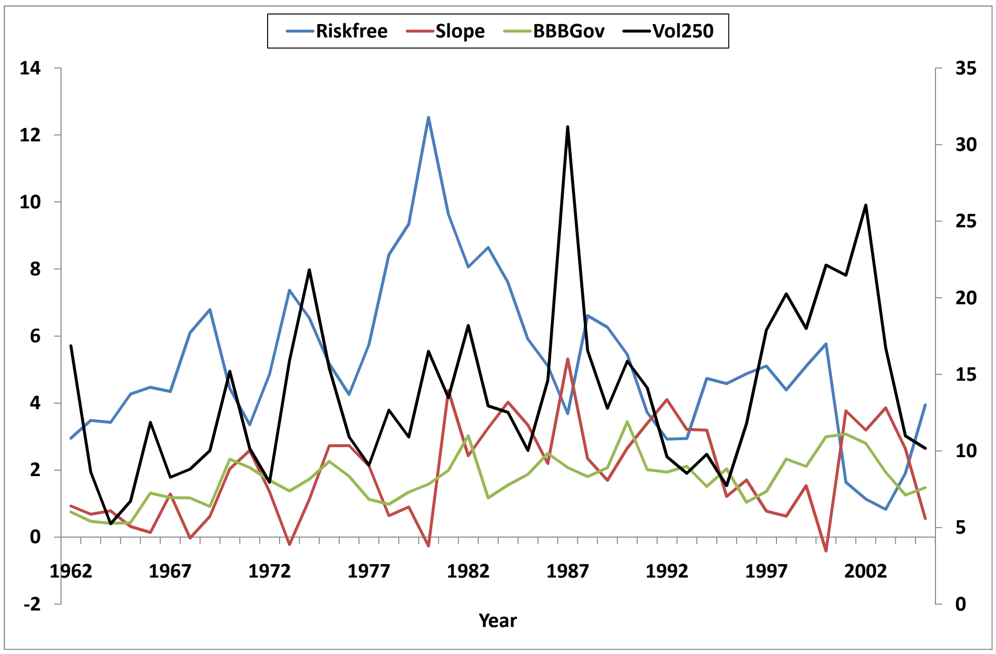

Figure 3: This figure depicts the sample paths of the four macro variables that we use in our regressions. Riskfree denotes the one-month Fama-French riskfree rate. Slope denotes the difference between the 15y yield on Treasury bonds and the riskfree rate. BBBGov denotes the spread between the 15y yield on BBB corporate bonds and the Treasury bonds. Vol250 denotes the annualized historical volatility of the S&P-500 calculated using index values of the last 250 trading days. The left y-axis applies to the first three time series, whereas the right one applies to the volatility.

	Mean	Std. Dev.	Min.	Max.	Median
Riskfree	5.192	2.343	0.827	12.528	4.871
Slope	1.943	1.428	-0.414	5.318	1.871
BBBGov	1.738	0.71	0.415	3.445	1.766
Vol250	13.781	5.366	5.253	31.161	12.727

Table 1: This table provides summary statistics for the macro variables. Riskfree denotes the one-month Fama-French riskfree rate. Slope denotes the difference between the 15y yield on Treasury bonds and the riskfree rate. BBBGov denotes the spread between the 15y yield on BBB corporate bonds and the Treasury bonds. Vol250 denotes the annualized historical volatility of the S&P-500 calculated using index values of the last 250 trading days.

	Mean	Std. Dev.	Min.	Max.	Median
Ratio	10.933	3.588	2.727	18.956	10.570
Pi	0.675	0.718	-0.749	63.652	0.516
Log_turnover	-1.095	1.288	-14.453	11.582	-1.001
Log_market_cap	4.932	2.236	-6.043	12.75	4.857

Table 2: This table provides summary statistics when the cash flow multipliers are calculated using free cash flows. To shorten notation, ratio stands for cash flow multiplier. Pi denotes the investment proportion given by the ratio between the annual capital expenditures and the free cash flows. Log_turnover is defined as the logarithm of the ratio between the number of common stocks traded in the calendar year and the number of common shared outstanding. Log_market_cap denotes the logarithm of the market capitalization which is defined as the product of the number of shares outstanding and the price per share. The statistics are based on 66102 observations. There are only 7 for observations where Pi < 0.

	Log_ratio	Riskfree	Slope	BBBGov	Vol250	Pi	Log_turnover	Log_market_cap
Log_ratio	1.000							
Riskfree	-0.172	1.000						
Slope	0.029	-0.357	1.000					
BBBGov	-0.010	-0.255	0.241	1.000				
Vol250	-0.069	-0.126	0.104	0.511	1.000			
Pi	0.044	0.123	-0.038	-0.041	-0.024	1.000		
Log_turnover	0.132	-0.118	0.038	0.070	0.047	-0.025	1.000	
Log_market_cap	0.246	-0.164	0.038	0.032	0.004	0.000	0.151	1.000

Table 3: This table reports the correlations between the relevant variables when cash flow multipliers are calculated using free cash flows. The correlations are calculated on the basis of 66102 observations.

	(1)	(2)	(3)
Riskfree	-0.026***	-0.024***	-0.023***
	(-4.95)	(-6.61)	(-6.46)
Slope	-0.012*	-0.007	-0.007
	(-2.28)	(-1.18)	(-1.17)
BBBGov	-0.019	-0.011	-0.009
	(-1.21)	(-0.62)	(-0.50)
Vol250	-0.006*	-0.006*	-0.006*
	(-2.18)	(-2.19)	(-2.17)
Pi	0.123***	0.071***	0.038***
	(7.10)	(7.17)	(5.06)
Log_turnover	0.007	0.018***	0.025***
	(1.24)	(3.30)	(4.57)
Log_market_cap	0.081***	0.040***	0.034***
	(9.80)	(8.75)	(7.20)
Intercept	2.141***	2.405***	2.392***
	(23.17)	(47.34)	(39.64)
R^2	0.1859	0.1783	0.1134
Fixed effects	yes	no	no
FF industry dummies	no	yes	no
Bank-utility dummies	no	no	yes

Table 4: The table reports the results of panel regressions with Driscoll-Kraay errors that correct for a variety of dependencies including spatial dependencies. The first regression is a pooled regression with fixed effects. The second one is a pooled regression with dummies for the 48 Fama-French industries. The third one is a pooled regression with two dummy variables, one for financial industry firms (Fama-French industries 44-47) and one for public service firms (Fama-French industries 31 and 48). All regressions are based on 66102 observations stemming from 8043 firms. The t-statistics are reported in brackets. The significance levels correspond to the following p-values: $*p < 0.05, **p < 0.01, ***p < 0.001$.

	(1)	(4)	(5)	(6)	(7)
Riskfree	-0.026***	-0.027***	-0.025***	-0.027***	-0.025***
	(-4.95)	(-4.72)	(-4.82)	(-4.73)	(-4.83)
Slope	-0.012*	-0.013*	-0.013*	-0.013*	-0.013*
	(-2.28)	(-2.10)	(-2.24)	(-2.11)	(-2.24)
BBBGov	-0.019	-0.041**		-0.041**	
	(-1.21)	(-2.67)		(-2.66)	
Vol250	-0.006*		-0.007**		-0.007**
	(-2.18)		(-2.73)		(-2.69)
Pi	0.123***	0.123***	0.123***	0.123***	0.123***
	(-7.10)	(-7.07)	(-7.15)	(-7.09)	(7.17)
Log_turnover	0.007	0.006	0.006		
	(-1.24)	(-0.97)	(-1.21)		
Log_market_cap	0.081***	0.082***	0.080***	0.084***	0.082***
	(-9.80)	(-9.67)	(-9.70)	(-8.34)	(8.64)
Intercept	2.141***	2.097***	2.120***	2.083***	2.104***
	(-23.17)	(-23.78)	(-23.92)	(-21.05)	(21.78)
R^2	0.1859	0.1776	0.1846	0.1773	0.1843

Table 5: The table reports the results of panel regressions when some of the explanatory variables are excluded. All regressions are fixed effects regressions with Driscoll-Kraay errors. The first regression corresponds to the first regression that is reported in Table 4. The reported R^2s are the within R^2s of the fixed effect regressions. As in Table 4, all regressions are based on 66102 observations stemming from 8043 firms. The t-statistics are reported in brackets. The significance levels correspond to the following p-values: $*p < 0.05, **p < 0.01, ***p < 0.001$.

	(2)	(8)	(9)
Riskfree	-0.024***	-0.024***	-0.025***
	(-6.61)	(-5.99)	(-24.37)
Slope	-0.007	-0.007	-0.011***
	(-1.18)	(-1.11)	(-9.73)
BBBGov	-0.011	-0.011	-0.017***
	(-0.62)	(-0.59)	(-8.31)
Vol250	-0.006*	-0.006*	-0.006***
	(-2.19)	(-2.23)	(-23.62)
Pi	0.071***	0.071***	0.110***
	(7.17)	(7.38)	(6.83)
Log_turnover	0.018***	0.018***	0.013***
	(3.30)	(4.54)	(7.48)
Log_market_cap	0.040***	0.040***	0.062***
	(8.75)	(12.83)	(39.84)
Intercept	2.405***	2.405***	2.322***
	(47.34)	(37.26)	(44.03)

Table 6: The table reports two additional regressions that we run as robustness checks for the standard errors. The first regression corresponds to the second regression that is reported in Table 4. All regression include industry dummies for the 48 Fama-French industries. Panel regression (2) calculates the standard errors according to Driscoll and Kraay (1998). Panel regression (8) computes standard errors by clustering by year and firm (see, e.g., Pedersen (2009) and the references therein). Panel regression (9) clusters by firm only. As in Table 4 and 5, all regressions are based on 66102 observations stemming from 8043 firms. The t-statistics are reported in brackets. The significance levels correspond to the following p-values: $*p < 0.05, **p < 0.01, ***p < 0.001$.

	(1)	(10)	(11)	(12)	(13)
Riskfree	-0.026***	-0.029***	-0.022***	-0.023***	-0.027***
	(-4.95)	(-4.81)	(-3.75)	(-3.90)	(-5.00)
Slope	-0.012*	-0.010	-0.011	-0.009	-0.012*
	(-2.28)	(-1.73)	(-1.75)	(-1.40)	(-2.06)
BBBGov	-0.019	-0.016	-0.014	-0.016	-0.016
	(-1.21)	(-0.83)	(-0.78)	(-0.76)	(-0.81)
Vol250	-0.006*	-0.007*	-0.006*	-0.007*	-0.006*
	(-2.18)	(-2.26)	(-2.23)	(-2.31)	(-2.11)
Pi	0.123***	0.146***			
	(7.10)	(29.24)			
Lag_pi			0.0003***	0.0003***	
			(3.62)	(3.47)	
Log_turnover	0.007	0.014*	0.007	0.010	0.035**
	(1.24)	(2.17)	(1.20)	(1.50)	(3.04)
Log_market_cap	0.081***		0.079***		
	(9.80)		(9.11)		
Lag_log_market_cap		0.058***		0.061***	
		(5.53)		(5.92)	
Intercept	2.141***	2.268***	2.197***	2.318***	2.663***
	(23.17)	(20.73)	(21.16)	(20.98)	(40.24)
R^2	0.1859	0.1675	0.1283	0.1001	0.0634

Table 7: The table reports three additional regressions that address the issue of endogeneity. We use the one-year lagged logarithm of the market capitalization and/or the one-year lagged percentage of the cash flow invested. The first regression corresponds to the first regression that is reported in Table 4. All regressions are fixed effect regressions with Driscoll-Kraay errors. Since we are losing one year of observations and because of missing observations, regression (10) is based on 62048 observations stemming from 7525 firms and regressions (11) and (12) are based on 61353 observations stemming from 7469 firms. Regression (13) disregards π and market capitalization. The number of observations is the same as for (1). The t-statistics are reported in brackets. The significance levels correspond to the following p-values: $*p < 0.05, **p < 0.01, ***p < 0.001$.

	(1)	(14)	(15)	(16)	(17)
Riskfree	-0.026***	-0.030***	-0.035***	-0.043***	-0.062***
	(-4.95)	(-5.25)	(-5.80)	(-6.73)	(-9.53)
Slope	-0.012*	-0.015*	-0.020**	-0.028***	-0.045***
	(-2.28)	(-2.35)	(-2.81)	(-3.60)	(-4.58)
BBBGov	-0.019	-0.017	-0.016	-0.014	-0.013
	(-1.21)	(-1.02)	(-0.86)	(-0.71)	(-0.58)
Vol250	-0.006*	-0.006*	-0.005	-0.005	-0.005
	(-2.18)	(-2.28)	(-1.79)	(-1.48)	(-1.45)
Pi	0.123***	0.158***	0.176***	0.193***	0.208***
	(7.10)	(29.07)	(33.46)	(30.80)	(12.69)
Log_turnover	0.007	0.006	0.007	-0.010	-0.011
	(1.24)	(0.98)	(0.74)	(-0.86)	(-0.76)
Log_market_cap	0.081***	0.081***	0.077***	0.085***	0.077***
	(9.80)	(8.76)	(6.68)	(6.11)	(5.09)
Intercept	2.141***	2.183***	2.229***	2.178***	2.308***
	(23.17)	(20.68)	(17.79)	(15.04)	(16.09)
R^2	0.1859	0.1988	0.2146	0.2428	0.2893
# ob. included	66102	56520	35023	20493	6686
# firms included	8043	3835	1379	615	166
Cutoff quantile	21	14	7	4	2
Av. market cap.	1507.145	1787.128	2550.220	3702.969	7324.377

Table 8: The table reports panel regressions when we only include firms that have at least 0, 10, 20, 30, 40 full observations. We report the quantiles of observations that are disregarded at the upper and lower tail. The t-statistics are reported in brackets. The significance levels correspond to the following p-values: $*p < 0.05, **p < 0.01, ***p < 0.001$.

	(18)	(19)	(20)	(21)
Riskfree	-0.022***	-0.024***	-0.022***	-0.018***
	(-4.37)	(-4.03)	(-4.04)	(-4.38)
Slope	-0.012	-0.013	-0.013	
	(-1.80)	(-1.72)	(-1.89)	
BBBGov	-0.019	-0.039*		
	(-1.16)	(-2.39)		
Vol250	-0.006*		-0.007*	-0.007**
	(-1.96)		(-2.39)	(-2.59)
Pi	0.130***	0.130***	0.130***	0.130***
	(11.43)	(11.35)	(11.56)	(11.49)
Log_market_cap	0.084***	0.084***	0.083***	0.082***
	(9.28)	(8.82)	(9.19)	(8.90)
Intercept	2.142***	2.098***	2.121***	2.079***
	(21.32)	(21.99)	(20.92)	(22.16)
R^2	0.1744	0.1658	0.1732	0.1700

Table 9: The table reports panel regressions when we include all firms with arbitrary fiscal years. This leads to 119270 observations stemming from 13682 firms. The t-statistics are reported in brackets. The significance levels correspond to the following p-values: $*p < 0.05, **p < 0.01, ***p < 0.001$.

	Mean	Std. Dev.	Min.	Max.	Median
Ratio_gp	6.498	5	0.654	25.949	4.900
Pi_gp	0.325	0.586	-10.654	28.821	0.150
Log_turnover	-0.955	1.33	-14.453	11.865	-0.854
Log_market_cap	4.736	2.317	-11.043	13.073	4.662

Table 10: This table provides summary statistics when the cash flow multipliers are calculated using gross profits. To emphasize this, the index gp is added. The statistics of the corresponding firm specific explanatory variables are reported as well. The statistics are based on 99002 observations. There are only 12 for observations where Pi_gp < 0.

	Log_ratio_gp	Riskfree	Slope	BBBGov	Vol250	Pi_gp	Log_turnover	Log_market_cap
Log_ratio_gp	1.000							
Riskfree	-0.150	1.000						
Slope	0.023	-0.370	1.000					
BBBGov	0.018	-0.245	0.203	1.000				
Vol250	-0.015	-0.148	0.098	0.541	1.000			
Pi_gp	0.271	0.096	-0.011	-0.009	-0.003	1.000		
Log_turnover	0.094	-0.143	0.003	0.058	0.055	-0.039	1.000	
Log_market_cap	0.325	-0.137	-0.006	-0.014	-0.019	0.040	0.164	1.000

Table 11: This table reports the correlations between the relevant variables if the cash flow multipliers are calculated using gross profits. The correlations are calculated on the basis of 99002 observations. The macro variables have slightly different correlations since the correlations are calculated on different samples.

	(1')	(6')	(7')	(12')	(13')
Riskfree	-0.021**	-0.022**	-0.020**	-0.025***	-0.027***
	(-2.94)	(-2.73)	(-2.78)	(-3.97)	(-4.75)
Slope	-0.012	-0.013	-0.013	-0.012	-0.015
	(-1.31)	(-1.39)	(-1.31)	(-1.44)	(-1.80)
BBBGov	-0.022	-0.051**		-0.021	-0.023
	(-1.13)	(-2.81)		(-0.88)	(-0.91)
Vol250	-0.007*		-0.008**	-0.008*	-0.008*
	(-2.36)		(-2.86)	(-2.54)	(-2.39)
Pi_gp	0.220***	0.220***	0.220***		
	(10.28)	(10.24)	(10.33)		
Lag_pi_gp				0.002***	
				(3.85)	
Log_turnover	0.001			0.034***	0.060***
	(0.09)			(3.89)	(4.55)
Log_market_cap	0.147***	0.148***	0.147***		
	(12.65)	(12.75)	(12.08)		
Lag_log_market_cap				0.085***	
				(6.46)	
Intercept	1.112***	1.065***	1.085***	1.523***	1.989***
	(10.15)	(9.63)	(9.45)	(12.35)	(25.84)
R^2	0.1898	0.1846	0.1890	0.0836	0.0392

Table 12: The table reports the panel regressions when free cash flows are proxied by gross profits. The numbers of the regression correspond to the numbers of regressions reported in Tables 4-7. The regressions (1'), (6'), (7'), and (13') are based on 99002 observations stemming from 10949 firms. Since we are losing one year of observations and because of missing observations, regression (12') is based on 90492 observations stemming from 10197 firms. The t-statistics are reported in brackets. The significance levels correspond to the following p-values: $*p < 0.05, **p < 0.01, ***p < 0.001$.

	(1')	(14')	(15')	(16')	(17')
Riskfree	-0.021**	-0.026***	-0.036***	-0.047***	-0.066***
	(-2.94)	(-3.95)	(-5.73)	(-7.71)	(-10.18)
Slope	-0.012	-0.016	-0.025**	-0.037***	-0.060***
	(-1.31)	(-1.77)	(-3.07)	(-4.33)	(-5.46)
BBBGov	-0.022	-0.022	-0.016	-0.011	-0.016
	(-1.13)	(-1.07)	(-0.75)	(-0.53)	(-0.66)
Vol250	-0.007*	-0.007*	-0.005	-0.005	-0.003
	(-2.36)	(-2.31)	(-1.62)	(-1.33)	(-0.92)
Pi_gp	0.220***	0.268***	0.329***	0.369***	0.480***
	(10.28)	(11.23)	(14.47)	(22.55)	(13.45)
Log_turnover	0.001	0.008	0.014	0.003	-0.013
	(0.09)	(0.91)	(1.27)	(0.23)	(-0.84)
Log_market_cap	0.147***	0.128***	0.119***	0.126***	0.142***
	(12.65)	(11.21)	(9.27)	(8.30)	(8.05)
Intercept	1.112***	1.183***	1.185***	1.100***	0.911***
	(10.15)	(10.44)	(9.18)	(7.55)	(5.44)
R^2	0.1898	0.1924	0.2300	0.2890	0.3856
# ob. included	99002	71864	39939	21892	6865
# firms included	10949	4211	1430	617	167
Cutoff quantile	7	5	2	1	1
Av. market cap.	1550.778	1824.736	2560.66	3657.959	7228.487

Table 13: The table parallels the results of Table 8 if the cash flow multipliers are calculated using gross profits. We report the quantile of observations that is disregarded at the upper and lower tail. Obviously, there are much smaller now, i.e. there are more observations included compared to Table 8. The t-statistics are reported in brackets. The significance levels correspond to the following p-values: $*p < 0.05, **p < 0.01, ***p < 0.001$.

	(18')	(19')	(20')	(21')
Riskfree	-0.019*	-0.020*	-0.018*	-0.013*
	(-2.40)	(-2.30)	(-2.18)	(-2.22)
Slope	-0.015	-0.016	-0.016	
	(-1.44)	(-1.47)	(-1.45)	
BBBGov	-0.022	-0.046*		
	(-1.08)	(-2.41)		
Vol250	-0.007*		-0.008*	-0.008**
	(-2.12)		(-2.52)	(-2.68)
Pi_gp	0.248***	0.248***	0.248***	0.248***
	(14.80)	(14.76)	(14.86)	(14.62)
Log_market_cap	0.143***	0.144***	0.143***	0.143***
	(13.50)	(13.90)	(13.47)	(12.60)
Intercept	1.056***	1.009***	1.028***	0.973***
	(8.35)	(8.55)	(8.09)	(9.07)
R^2	0.1839	0.1786	0.1831	0.1811

Table 14: The table reports panel regressions when we include all firms with arbitrary fiscal years. Cash flow multipliers are calculated using gross profits. We have 177197 observations stemming from 17926 firms. The t-statistics are reported in brackets. The significance levels correspond to the following p-values: $*p < 0.05, **p < 0.01, ***p < 0.001$.

	(22)	(23)	(24)	(25)
Riskfree	-0.023***	-0.016***	-0.018***	-0.013***
	(-6.89)	(-3.78)	(-5.86)	(-3.54)
Slope	-0.005	-0.005	-0.005	-0.005
	(-0.91)	(-0.78)	(-0.83)	(-0.70)
BBBGov	-0.007	-0.008	-0.003	-0.006
	(-0.39)	(-0.43)	(-0.16)	(-0.36)
Vol250	-0.006*	-0.006*	-0.004	-0.004
	(-2.21)	(-2.15)	(-1.72)	(-1.85)
Pi	0.071***	0.068***	0.062***	0.061***
	(7.01)	(6.77)	(9.24)	(9.09)
Log_turnover	0.021***	0.021***		
	(3.88)	(4.15)		
Log_market_cap	0.039***	0.035***	0.036***	0.033***
	(7.68)	(6.78)	(8.75)	(7.90)
Av_pi	-0.338***		-0.308***	
	(-22.08)		(-21.79)	
Av_pi_annual		-0.223***		-0.217***
		(-10.14)		(-11.23)
Intercept	2.575***	2.481***	2.518***	2.452***
	(40.61)	(37.79)	(37.62)	(36.97)
R^2	0.147	0.127	0.119	0.105

Table 15: The table reports regression results if we include the average πs of the Fama-French industries as additional explanatory variables. These are pooled OLS regressions with Driscoll-Kraay erros where we neither include fixed effects nor Fama-French industry dummies. The variable Av_pi denotes the average π of the corresponding industry over the whole sample period of 44 years. In contrast, Av_pi_annual denotes the average π of the corresponding industry calculated every year, i.e. these are 48 time series. Regressions (22) and (23) are based upon the same observations as our benchmark regression (1), whereas regressions (24) and (25) are based upon the same observations as regression (19) that includes all firms independent of their fiscal years. Therefore, the last two regression do not include the turnover variable. The t-statistics are reported in brackets. The significance levels correspond to the following p-values: $*p < 0.05, **p < 0.01, ***p < 0.001$.

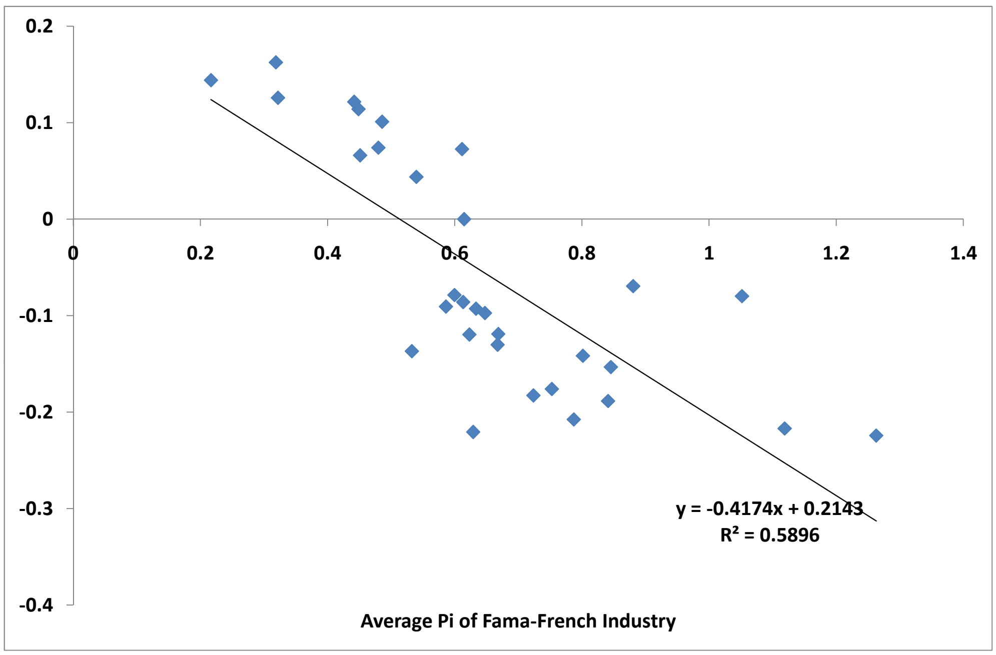

Figure 4: This figure plots the significant industry dummies of regression (2) against the means of π for the 48 Fama-French industries. It is based on 66102 observations stemming from 8043 firms.