LIMITED CAPITAL MARKET PARTICIPATION AND HUMAN CAPITAL RISK

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ABSTRACT

The non-tradability of human capital is often cited for the failure of traditional asset pricing theory to explain agents' portfolio holdings. In this paper we argue that the opposite might be true --- traditional models might not be able to explain agent portfolio holdings because they do not explicitly account for the fact that human capital does trade (in the form of labor contracts). We derive wages endogenously as part of a dynamic equilibrium in a production economy. Risk is shared in labor markets because firms write bilateral labor contracts that insure workers, allowing agents to achieve a Pareto optimal allocation even when the span of asset markets is restricted to just stocks and bonds. Capital markets facilitate this risk sharing because it is there that firms offload the labor market risk they assumed from workers. In effect, by investing in capital markets investors provide insurance to wage earners who then optimally choose not to participate in capital markets. The model can produce some of the most important stylized facts in asset pricing: (1) limited asset market participation, (2) the seemingly high equity risk premium, (3) the very large disparity in the volatility of consumption and the volatility of asset prices, and (4) the time dependent correlation between consumption growth and asset returns.

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A commonly held view amongst financial economists is that a significant fraction of wealth consists of non-tradable assets, most notably human capital wealth. Indeed, this hypothesis is often used to explain why one of the key predictions of the CAPM does not hold, that all agents hold the same portfolio of risky assets. Because investors should use the capital markets to diversify as much risk as possible, and because non-tradable human capital exposure varies across individuals, investors should optimally choose to hold different portfolios of risky assets. Although this explanation certainly has the potential to explain the cross sectional variation in portfolio holdings, it also necessarily implies wide stock market participation. However, the fact is that the majority of people do not participate in the capital markets. Not only do these individuals appear to eschew the opportunity to partially hedge their human capital exposure, the hedging of human capital risk does not appear to be a primary motivator for the minority of people who do participate in capital markets. Instead, the anecdotal evidence suggests that rather than a desire to hedge, what motivates most investors is a willingness take on *additional* risk because they find the risk return tradeoff attractive.[1] The object of this paper is to put forward a plausible explanation for these two characteristics of investor behavior.

The most commonly cited explanation for why most people do not participate in capital markets is barriers to entry, although in economies such as the United States it is difficult to accept that significant economic barriers to entry exist that prevent people from participating. Instead, most researchers cite educational barriers to entry because research has shown that education level is strongly correlated with participation.[2] But the problem with this explanation for limited stock market participation is that it does not address the question of *why* the educational barriers exist at all. After all, we see wide participation in arguably more complicated financial products such as mortgages, auto leases and insurance. In these cases the educational barriers to entry were removed by the motivation to make profits — firms invested considerable resources in educating people so they could sell these products. Given the welfare gain to hedging non-tradable human capital, why does a similar economic motivation to educate consumers to hold stocks apparently not exist?

Market incompleteness may potentially offer an explanation for limited stock market participation. For instance, the asset span might be so "narrow" that the stock market offers little opportunity for Pareto improving trades. Although rarely cited explicitly, this explanation is implicit in the literature on non-traded wealth. But, for this explanation to hold water, one must also then account for why the asset span does not endogenously expand. In fact, the span of traded assets has changed only marginally in recent years, despite the explosion in the number of new assets. More importantly, one would not naturally expect incompleteness to result in non-participation. Indeed, the low correlation between human capital and stock mar-

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[1] For example, considerable resources are devoted to advising people on how to find high return investments whereas advice on investments with good hedging characteristics is largely non-existent.

[2] Mankiw and Zeldes (1991) document the relation between education and participation and Hong, Kubik, and Stein (2004) document that non-formal education, such as social interaction, is also correlated with participation.

ket returns documented in Lustig and Van Nieuwerburgh (2008) would suggest that despite the incompleteness, the stock market offers diversification benefits which would imply wide participation. Thus, market incompletness appears to be an unlikely explanation for limited stock market participation.

If frictions, like barriers to entry and market incompleteness, are not preventing agents from participating, then they must be choosing not to participate. One possibility is that agents' initial endowments are naturally so close to a Pareto optimal allocation that there is little reason to engage in further trade. But considering the heterogeneity in actual endowments, this explanation seems implausible. A more plausible possibility is that some agents are able to achieve their Pareto optimal allocation through trade in other markets and therefore trading in stock markets provides no incremental benefit.

In this paper, we argue that what traditional models of investment ignore are other markets on which people share risk. Building on this insight, we identify the labor market as one such market and posit that the unwillingness of some individuals to use capital markets is a consequence of the fact that they are able to share enough risk through their wage contracts so that the capital markets offer little incremental benefit. A Pareto optimal allocation can therefore be achieved even with the "narrow" asset span we observe in actual stock markets implying that limited stock market participation is an efficient *equilibrium* outcome.

We focus on labor markets because they are an ideal place to share risk. The structure of most firms has historically been built around long-term tailored labor contracts between the firm and its workers. Indeed, viewed this way, one might wonder why all risk cannot be optimally shared in labor markets. The reason is that long-term labor contracts are not necessarily efficient for all employees — some employees are better off retaining the flexibility to switch jobs. Because of this labor market mobility, to achieve efficient risk sharing, asset markets are also required.

Our results call into question one of the basic assumptions in asset pricing — that because asset markets do not span labor risk, human capital is not traded and so most wealth is non-tradable. In our equilibrium, the labor market has an important role in sharing risk, effectively allowing firms to insure workers. That implies that the owners of the firm are the ultimate insurers of this human capital risk. Consequently, shareholders must be compensated for taking on this additional risk, so the risk premium for holding equity is considerably higher than the risk premium for holding pure consumption risk. Indeed, by parameterizing our model we are able to show that it is not difficult to reproduce the observed level of the equity risk premium with realistic values of risk aversion, the risk free rate, consumption growth and consumption volatility.

By recognizing the important role of labor contracts in the riskiness of equity, we demonstrate why, in an otherwise standard neoclassical model of asset prices, the volatility of equity prices might bear little apparent relation to the volatility of consumption. One of the most important empirical facts that the standard neoclassical model of asset prices appears to be unable to

explain is the seeming disconnect between the volatility of consumption and asset prices. Not only is consumption volatility significantly lower than the volatility of asset prices, but the two series behave manifestly differently. For example, average quarterly volatility of the S&P 500 index is 68% higher during recessions (as identified by the NBER).[3] Yet, the concomitant increase in consumption volatility is much smaller if it exists at all — over the period 1947-2009, the point estimate of the volatility of (seasonally adjusted) quarterly GDP growth in NBER recessions is only 11% higher than in expansions.[4] As we will show, for a reasonable choice of parameter values, our model can quantitatively match this stylized fact. We can also explain a related empirical finding: that equity wealth is considerably more volatile and human capital wealth is less volatile than total wealth.[5] Because human capital wealth is traditionally measured using *wage income*, that is, the income that results once risk sharing has already taken place in the labor market, traditional measures underestimate the volatility of human capital wealth.

Finally, the model implies a time varying (and a low unconditional) correlation between consumption growth and equity returns in line with what has been observed empirically. For example, Duffee (2005) shows that the conditional correlation between consumption growth and equity returns is almost zero when the equity wealth-to-consumption ratio is low, whereas the correlation is high when the equity wealth-to-consumption ratio is high. This is exactly what our model predicts and, further, it suggests that the degree of heterogeneity in labor productivity should be a powerful instrument in a conditional asset pricing test. Our model also implies a low unconditional correlation between labor income and equity returns, in line with Fama and Schwert (1977).

The paper is organized as follows. In the next section we provide a brief literature review. In sections 2-4 we introduce the model and derive several theoretical implications. Section 5 provides a simple calibration to realistic parameters, and section 6 makes some concluding remarks. All proofs are left to the appendix.

1 Background

The idea that one role of the firm is to insure its workers' human capital risk dates at least as far back as Knight (1921). Knight takes as a primitive that the job of worker and manager entails taking on different risks and notes that entrepreneurs bear most of the risk. Using this idea Kihlstrom and Laffont (1979) endogenizes who, in a general equilibrium, becomes an entrepreneur and who becomes a worker. Less risk averse agents choose to be entrepreneurs who then optimally insure workers. However, the wage contract in that paper is exogenously

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[3]It is 21.4% in recessions and 12.7% at other times. Quarterly volatility is defined to be the standard deviation of daily returns of the S&P 500 index over the quarter over the period 1962-2009. This difference is highly statistically significant.

[4]Using quarterly data published by the BEA, the volatility of GDP growth in recessions is 4.66% while is is 4.19% at other times over the 1947-2009 time period.

[5]See, for example, Lustig, Van Nieuwerburgh, and Verdelhan (2007).

imposed rather than an endogenous response to the desire to optimally share risk and so the resulting equilibrium is not Pareto efficient.

The papers that first recognized the importance of endogenizing the wage contract, and therefore the ones most closely related to our paper, are Dreze (1989) and Danthine and Donaldson (2002). Like us, Dreze (1989) considers the interaction between a labor and capital market in general equilibrium and focuses on efficient risk sharing. Our point of departure is how we model production — Dreze does not consider the implication of productive heterogeneity. Consequently, there is no natural reason (beyond differences in risk aversion and wealth) for some workers to insure other workers in Dreze's model. Hence, the model does not explain limited capital market participation or focus on the return to bear labor risk.

Danthine and Donaldson (2002), like us, explicitly model both labor and financial markets with agent heterogeneity. Their model features investors and workers, but, importantly, Danthine and Donaldson (2002) do not allow workers to invest or investors to work. In that model investors are endowed with wealth rather than productivity and hence have a precautionary reason to save which they do by investing in firms. Because this motive is missing in our model, prices must adjust to induce some workers to invest. This is a key difference between the two models and is responsible for the stark differences in some of the models' implications. Because workers have to be induced, in our model, to take on the additional risks of equity, we get a large equity premium while the equity risk premium in Danthine and Donaldson (2002) is small.[6]

Our paper also contributes to the large literature, that started with Mayers (1972), studying the effect of non-tradable wealth in financial markets. The main results in that literature are that investors should no longer hold the same portfolio of risky assets and the single factor pricing relation must be adjusted. Although Fama and Schwert (1977) finds little evidence supporting Mayer's model, both Campbell (1996) and Jagannathan and Wang (1997) find that adding a measure of human capital risk significantly increases the explanatory power of the CAPM. Santos and Veronesi (2006) find that the labor income to consumption ratio has predictive power for stock returns and can help explain risk premia in the cross section. Because wage contracts provide insurance for human capital risk, our model implies that wages (the typical measure of human capital used in the literature) should have explanatory power for stock returns.

Because most of the theoretical predictions of the neoclassical asset pricing model rely on effectively complete markets, initially researchers were tempted to attribute the failure of those models to market incompleteness. However, Telmer (1993) and Heaton and Lucas (1996) convincingly argue that market incompleteness cannot account for important puzzles such as the apparently high equity risk premium. As we show in this paper, quite the opposite intuition might be true. The failure of the models might stem from the fact that agents actually share risk more completely than is supposed in the literature. If labor markets effectively share risk, then

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[6]Danthine and Donaldson (2002) can only get a large premium by introducing market frictions that restrict risk sharing.

because equity holders are the ultimate insurers of human capital risk, they will demand a high risk premium. As we will demonstrate, our results are consistent with the findings in Mankiw and Zeldes (1991) and Brav, Constantinides, and Geczy (2002) in that those who choose not to participate are less wealthy, less educated and more reliant on wage income as their source of wealth. Furthermore, consistent with the anecdotal evidence, the primary motivation for investing in capital markets is the attractive risk return tradeoff offered, not a desire to hedge human capital risk.

2 Model

Like any source of risk, human capital risk has both an idiosyncratic component and a systematic component. Although the idiosyncratic component is likely to be large, especially early in a person's career, we will focus exclusively on the systematic component because we are interested in the implications of how agents share risk in the economy. Idiosyncratic risk, by its very nature, can be diversified away, so there is little reason for any agent to hold this risk in a complete market equilibrium. Consequently, the risk sharing implications of sharing idiosyncratic risk are well understood.[7]

Given our objective to study how systemic risk is shared in the economy, our model must include heterogeneous agents. An important source of individual heterogeneity in the economy is worker flexibility: Some workers only have access to a single production technology while others can choose between production technologies. Building on this insight we model productivity as follows. Our economy consists of a continuum of workers that produce a single, perishable, consumption good. Workers produce goods using a productive technology that has a common component and an individual component we term a worker's *production technology.*

Formally, there is a closed set of production technologies (industries), $\mathcal{P} \subset [0,1] \times [0,\overline{K}]$, for some $\overline{K} > 0$. Each *inflexible* agent only has access to a single production technology in this set, (b, f), and produces $A_t(b + fs)$ of a consumption good, where $A_t \equiv A_0 e^{rt}$ is the non-stochastic[8] part of production common to all agents, and s is a state variable that captures the current state of the economy. *Flexible* agents have access to every production technology in $\mathcal{P}$. The production technology set has the properties that $(0, \overline{K}) \in \mathcal{P}$, $(1, 0) \in \mathcal{P}$, $(b, \overline{K}) \in \mathcal{P} \Rightarrow b = 0$ and $(1, f) \in \mathcal{P} \Rightarrow f = 0$.

The dynamics of A_t are meant to capture overall economic growth and allows us to model

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[7]Although it's not the focus of the paper, Harris and Holmström (1982) makes it clear how agents share idiosyncratic labor risk. The paper shows that most, but not all, of this risk can be removed by the labor contract. Under the optimal labor contract firms insure all agents against negative realizations of idiosyncratic labor risk but agents remain exposed to some positive realizations. Of course, the owners of these firms do not have to expose themselves to this labor risk because by holding a large portfolio of firms, they can diversify the risk away.

[8]It is straightforward to extend our analysis to allow stochastic growth in A_t, as long as innovations in A_t are independent of innovations in s.

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recessions as a relative drop in productivity. The stochastic process s is a diffusion process on $\mathbb{R}_+$ that summarizes the state of the world:

$$ds \;=\; \mu(s)\,dt + \sigma(s)\,d\omega.$$

We will model s as a mean-reverting square root process,

$$ds = \theta(\bar{s} - s)\,dt + \sigma\sqrt{s}\,d\omega, \tag{1}$$

where the condition $2\theta\bar{s} > \sigma^2$ ensures positivity. The mean-reversion introduces a business cycle interpretation and is useful for calibrations, although, as we shall see, much of the theory goes through for general $\mu(s)$ and $\sigma(s)$ so long as μ and σ are smooth, σ is strictly positive, and the growth conditions $|\mu(s)| \le c_1(1+s)$, $\sigma(s) \le c_2(1+s)$ are satisfied for finite constants, c_1 and c_2. It is natural to define a recession as states for which $s < \bar{s}$, whereas an expansion is when $s > \bar{s}$.

Let the inflexible agents be indexed by $i \in \mathcal{I} = [0, \alpha]$, where $0 < \alpha < 1$, with agent i working in industry (b_i, f_i), where we assume that b_i and f_i are measurable functions that are nondegenerate in the sense that it is neither the case that the full mass of agents work in industry $(0, \overline{K})$, nor in industry $(1, 0)$. Then the total productivity of all inflexible agents in the economy is

$$A_t K_I(s) = A_0 e^{rt} K_I(s),$$

where

$$K_I(s) \equiv \int_{i \in \mathcal{I}} (b_i + f_i s)\,di = \bar{b} + \bar{f}s. \tag{2}$$

Note that $0 < \bar{b} < 1$ and $0 < \bar{f} < \overline{K}$.

The rest of the agents in the economy are *flexible* agents, comprising mass $1 - \alpha$, $i \in \mathcal{F} = (1 - \alpha, 1]$. Because these agents have access to any production technology in $\mathcal{P}$ and are free to move between production technologies at any point in time, for a given s, it is optimal for them to work in an industry (b^*, f^*), that solves

$$(b^*, f^*) = \arg\max_{(b,f) \in \mathcal{P}} b + fs, \tag{3}$$

leading to the optimal productivity of flexible agents

$$A_t K_F(s) = A_0 e^{rt} K_F(s)$$

where

$$K_F(s) \equiv b^*(s) + f^*(s)s. \tag{4}$$

Notice that, at any point in time, all flexible agents choose to work in industries that generate

the same output. Furthermore, we have

Lemma 1 *The optimal production function of flexible agents satisfies:*

(a) $K_F(0) = 1$,

(b) $\lim_{s \to \infty} \frac{K_F(s)}{s} = \overline{K}$,

(c) $K_F(s)$ *is a convex function of s.*

We next assume that flexible agents can work part time in different industries, i.e., if $(b_1, f_1) \in \mathcal{P}$ and $(b_2, f_2) \in \mathcal{P}$, then $(\lambda b_1 + (1 - \lambda)b_2, \lambda f_1 + (1 - \lambda)f_2) \in \mathcal{P}$ for all $\lambda \in [0, 1]$. This implies that for all $b \in [0, 1]$ there is a $(b, f) \in \mathcal{P}$. Now, flexible agents will only consider production technologies on the efficient frontier, $(b, f(b))$, where $f(b) \stackrel{\text{def}}{=} \max_{(b,f) \in \mathcal{P}} \{f : (b, f) \in \mathcal{P}\}$, and it follows immediately that f is a strictly decreasing, concave function defined on $b \in [0, 1]$, such that $f(0) = \overline{K}$ and $f(1) = 0$. Going forward, we make the additional technical assumptions that f is strictly concave, twice continuously differentiable, and that $f'(0) = 0$, and $f'(1) = -\infty$. Under these assumptions it is easy to show that

Lemma 2 $K_F(s)$ *is a twice continuously differentiable, strictly convex function, such that* $K_F'(0) = 0$ *and* $\lim_{s \to \infty} K_F'(s) = \overline{K}$.

Lemma 2 ensures that $K_F(s)$ is a diffusion process (which is, of course, also true of $K_I(s)$). The total output in the economy at time t is:

$$A_t K_{tot}(s_t) = A_0 e^{rt} K_{tot}(s_t), \tag{5}$$

where

$$K_{tot}(s) \equiv \alpha K_I(s) + (1 - \alpha) K_F(s), \tag{6}$$

implying that $K_{tot}(s)$ is also a diffusion process.

Figure 1 plots a production function for a flexible and inflexible worker. When the two workers work in the same industry they are equally productive. At all other times the flexible agent will be more productive than the inflexible agent. The next lemma adds the additional observation that the flexible worker's mobility implies that flexible workers will move into safer jobs in bad times and riskier jobs in good times so that flexible workers will have a natural advantage in providing insurance to inflexible workers.

Lemma 3 *The following results hold for the volatility of the agents' productivity:*
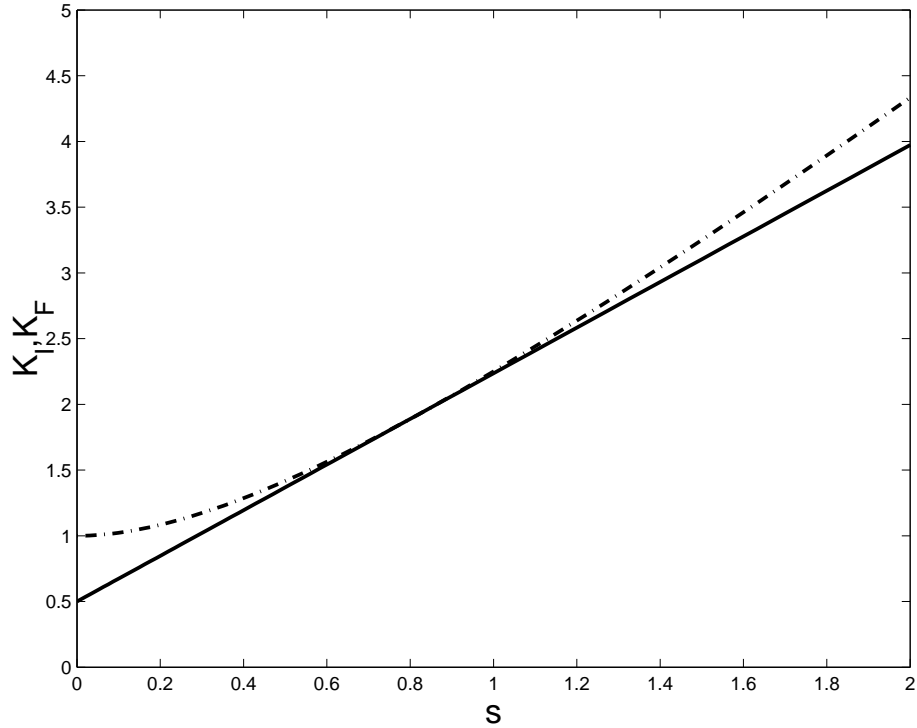
Figure 1: **Optimal production technology functions of flexible and inflexible agents,** $K_F(s)$ **and** $K_I(s)$: In this example, all inflexible agents work in the same industry, so K_I and K_F intersect at the point where flexible agents choose the same productivity as inflexible agents have. If there is dispersion among inflexible agents, K_I lies strictly below K_F, since f is strictly concave.

(a) *For low s, the volatility of the flexible agent's productivity is lower than that of the inflexible agent:*

$$Vol\left(\frac{dK_F}{K_F}\right) < Vol\left(\frac{dK_I}{K_I}\right).$$

(b) *For high s, the volatility of the flexible agent's productivity is higher than that of the inflexible agent,*

$$Vol\left(\frac{dK_F}{K_F}\right) > Vol\left(\frac{dK_I}{K_I}\right).$$

Workers and firms are organized as follows. A worker can choose either to work for himself and produce the consumption good, or he can choose to "sell" his production to a firm and earn a wage instead. Firms are specialized — they can only employ workers with a single production technology. Consequently, if a flexible worker decides to switch production technologies, then if she is working for a firm, she must quit her job and find a job in the industry she wishes to switch into (or simply work for herself). The fact that firms are specialized and so workers

must switch employers in order to switch industries is a crucial assumption in our model that we contend reflects reality.[9]

Workers are also owners — they are free to invest in firms through the capital markets and consume any dividend payments. In equilibrium, markets must clear — all firms must attract enough investment capital to fulfill their wage obligations.

Finally, we assume that all agents are infinitely lived, with constant relative risk-aversion (CRRA), risk-aversion coefficient $\gamma > 0$, and expected utility of consumption

$$U_i(t) = E_t \left[\int_t^\infty e^{-\rho(s-t)} u(c_s) \, ds \right]. \tag{7}$$

Here,

$$u(c) = \begin{cases} \log(c), & \gamma = 1, \\ \frac{c^{1-\gamma}}{1-\gamma}, & \gamma \neq 1. \end{cases} \tag{8}$$

Throughout the paper, we assume an infinite horizon economy. In our numerical calcululations we use a finite horizon set-up, where the horizon is large enough to get convergence to the solution of the infinite horizon economy (i.e., the steady state solution). An advantage of this approach is that it leads to a unique solution, allowing us to avoid the nontrivial issue of defining transversality conditions to rule out bubble solutions in an economy with nonlinear dynamics.

3 Complete Markets Equilibrium

We begin by deriving the complete market Pareto optimal equilibrium and then explain how this equilibrium can implemented. Because $A_t K_{tot}$ maximizes total output, any Pareto optimal equilibrium must have this output. Under the complete markets assumption a representative agent with utility u_r exists such that the solution to the representative agent problem is identical to the solution of the multi-agent problem. Moreover, all agents have CRRA utility functions with the same γ, so u_r is also of the CRRA form, with the same γ. Thus, in a complete market equilibrium, the value of any asset generating instantaneous consumption flow $\delta(s_t, t)dt$, is

$$
\begin{aligned}
P(s_t) &= \frac{1}{u_r'(A_t K_{tot}(s_t))} E \left[\int_t^\infty e^{-\rho(\tau-t)} u_r'(A_\tau K_{tot}(s_\tau)) \delta(s_\tau, t) d\tau \right] \\
&= K_{tot}(s_t)^\gamma E \left[\int_t^\infty e^{-\hat{\rho}(\tau-t)} K_{tot}(s_\tau)^{-\gamma} \delta(s_\tau, t) d\tau \right],
\end{aligned} \tag{9}
$$

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[9]For example, Moscarini and Thomsson (2007) document that 63% of workers who changed occupations also changed employers.

where $\hat{\rho} = \rho + \gamma r$. Hence, the total value of human capital of all agents of each type (their total wealth) at time $t = 0$ is:

$$W_I \equiv \alpha A_0 K_{tot}(s_0)^\gamma E\left[\int_0^\infty e^{-(\hat{\rho}-r)\tau} K_{tot}(s_\tau)^{-\gamma} K_I(s_\tau)d\tau\right], \tag{10}$$

$$W_F \equiv (1-\alpha) A_0 K_{tot}(s_0)^\gamma E\left[\int_0^\infty e^{-(\hat{\rho}-r)\tau} K_{tot}(s_\tau)^{-\gamma} K_F(s_\tau)d\tau\right]. \tag{11}$$

Any Pareto optimal equilibrium features perfect risk sharing — all agents' consumption across states have the same ordinal ranking. Moreover, because of the CRRA assumptions, it is well known that a stronger result applies in our equilibrium — all agents' ratio of consumption across any two states is the same. In other words, every agent consumes the same fraction of total output in every state:

$$c_I(s,t) = \eta \ A_t K_{tot}(s) = \eta \ A_t(\alpha K_I(s) + (1-\alpha)K_F(s)) \tag{12}$$

$$c_F(s,t) = (1-\eta) \ A_t K_{tot}(s) = (1-\eta) \ A_t(\alpha K_I(s) + (1-\alpha)K_F(s)) \tag{13}$$

where c_I and c_F if the aggregate consumption of all the inflexible and flexible agents respectively and η is the fraction of the total output consumed by all the inflexible agents. Given that the agents can trade their human capital, from the budget constraint at time 0 it follows that,

$$\eta = \frac{W_I}{W_I + W_F}. \tag{14}$$

We can also view η as a function of the initial state, $\eta(s_0)$. The following properties of η follow immediately:

Lemma 4 *The function $\eta(s_0)$, defined by (14) satisfies the following conditions*

(i) η *is twice continuously differentiable.*

(ii) $\eta \in \left[\min\left\{\frac{1}{1+\frac{1-\alpha}{\alpha}\frac{1}{\underline{b}}}, \frac{1}{1+\frac{1-\alpha}{\alpha}\frac{\overline{K}}{\overline{f}}}\right\}, \alpha\right].$

(iii) η *is increasing for small s_0 and decreasing for large s_0.*

(iv) $\lim_{s_0\to\infty} \eta(s_0) = \frac{1}{1+\frac{1-\alpha}{\alpha}\frac{\overline{K}}{\overline{f}}}.$

In words, because inflexible agents can never be less productive than flexible agents, the equilibrium fraction of consumption inflexible agents consume is bounded above by α, the fraction of the economy they comprise. It is bounded below by the inflexible consumption in the state that their share of productivity is minimized, that is either when $s = 0$ or ∞. Because inflexible agents share of productivity is lower both for low and high values of the state variable, η has

an interior maximum. It therefore follows that if we define $s^* \overset{\text{def}}{=} \min\{\arg\max_{s_0} \eta(s_0)\}$, then $0 < s^* < \infty$. The wealth share of the inflexible agent is thus maximized at a finite value, s^*, and we denote the wealth share at this point by $\eta^* \equiv \eta(s^*)$. Finally, as the state variable goes to infinity, both types of agents productivity converge to a linear function of the state variable and, in addition, the value of insurance becomes negligible, so the equilibrium converges to one with no risk sharing — each agent consumes what he produces.

To solve explicitly for the equilibrium requires computing the expectation in (11), which can be done using standard techniques from dynamic programming:

Proposition 1 *The price, $P(s,t)$, of an asset that pays dividends $\delta(s,t)$ satisfies the PDE*

$$P_t + \left(\mu(s) - \gamma R(s)\sigma(s)^2\right) s P_s + \frac{\sigma(s)^2}{2} P_{ss}$$
$$- \left(\hat{\rho} + \gamma\mu(s)R(s) - \frac{\sigma(s)^2}{2}\gamma(\gamma+1)R(s)^2 + \frac{\sigma(s)^2}{2}\gamma T(s)\right) P + \delta(s,t) = 0 \qquad (15)$$

where

$$R(s) = \frac{K'_{tot}(s)}{K_{tot}(s)}, \qquad and \qquad T(s) = \frac{K''_{tot}(s)}{K_{tot}(s)}. \qquad (16)$$

An immediate implication of this proposition is that the instantaneous risk free interest rate is captured by the term in front of P:[10]

$$r_s \equiv \hat{\rho} + \gamma\mu(s)R(s) - \frac{\sigma(s)^2}{2}\gamma(\gamma+1)R(s)^2 + \frac{\sigma(s)^2}{2}\gamma T(s). \qquad (17)$$

In general, we will need to solve (15) numerically, which may be nontrivial because it is defined over the whole of the positive real line, $s \in \mathbb{R}_+$. It is also not *a priori* clear what the boundary conditions are either at $s = 0$, or at $s = \infty$ where P_s may become unbounded. We can avoid these issues by making the transformation, $z \overset{\text{def}}{=} \frac{s}{s+1}$ to get:

Proposition 2 *The price, $P(s,t)$, of an asset that pays dividends $\delta(s,t)$, where $\delta(s,t) \leq ce^{rt}K_{tot}(s)^\gamma$ for some positive constant c, and $t < T$ is*

$$P(s,t) = K_{tot}(s)^\gamma Q\left(\frac{s}{s+1}, t\right),$$

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[10]Heuristically, a risk-free zero coupon bond with maturity at dt will have a price that is almost independent of P, so P_s and P_{ss} are close to zero. Therefore, the local dynamics are $P_t - r_s P = 0$, i.e., $\frac{dP}{P} = -r_s dt$ so the short term discount rate is indeed r_s.

where $Q : [0,1] \times [0,T] \to \mathbb{R}_+$ solves the PDE

$$Q_t + (1-z)^2 \left(\mu \left(\frac{z}{1-z} \right) - \sigma \left(\frac{z}{1-z} \right)^2 z \right) Q_z + \frac{1}{2} (1-z)^4 \sigma \left(\frac{z}{1-z} \right)^2 Q_{zz} - \hat{\rho} Q$$
$$+ \delta \left(\frac{z}{1-z}, t \right) K_{tot} \left(\frac{z}{1-z} \right)^{-\gamma} = 0, \quad (18)$$

and $Q(z,T) = 0$.

The proof of Proposition 2 follows along identical lines as the analysis in Parlour, Stanton, and Walden (2009), where a similar transformation is made and it is shown that no boundary conditions are needed at $z = 0$ and $z = 1$, and is therefore omitted. Without loss of generality, we assume that $A_0 = 1$ going forward, since all variables are homogeneous of degree zero or one in A_0. All the numerical solutions in this paper were derived by solving (18).

4 Implementation

To gain insight into how actual markets, which are far from complete, share risk, it is important that we model these markets realistically. Hence, we restrict agents' and firms' ability to write and trade contracts in the following ways:

Restriction 1

(i) *Binding contracts cannot be written directly between agents.*

(ii) *Firms may enter into binding contracts with agents subject to the following restrictions: (1) Limited liability may not be violated. (2) Workers and equity holders cannot be required to make payments,*

(iii) *Banks may enter into short term debt contracts with agents and firms, paying an interest rate r_s.*

These restrictions reflect the practical limitations of markets. Because individualized binding contracts cannot trade in anonymous markets a matching mechanism does not exist that would allow for widespread use of bilateral contracts as a risk sharing device. Perhaps because there are far fewer firms than agents in the economy, so it is easier to match firms and agents, we do observe binding bilateral labor contracts written between agents and firms. However, even these contracts are limited. Both equity and labor contracts are one sided in the sense that typically firms commit to make payments to agents. Agents very rarely commit to make payments to firms and courts rarely enforce such contracts. The only condition under which agents can enter a contract that commits them to make payments is if they take a loan from a bank. Both firms

and agents can either borrow or lend from a bank subject to the condition that in equilibrium the supply of loans must equal the amount of deposits. Thus, the span of traded assets consists of debt and equity. As we will see, there is no default in equilibrium so the interest rate banks pay is the risk free rate.

We also impose the following restriction on the industries in which firms operate:

Restriction 2 *Firms are restricted to operate in only one industry. That is, all workers in a firm must have the same b and f.*

In reality, most firms operate in a single industry. Although conglomerates do exist, even these firms typically operate in only a few industries. Our results would not change if we allowed firms to operate in finitely many industries. What we cannot allow is a firm that operates in *every* industry.

We assume that there is a (very) small cost to dynamic trading in capital markets:

Restriction 3 *Dynamic trading in equity markets imposes a utility cost of $\epsilon = 0^+$ per unit time.*

This restriction captures transaction costs of active trading, as well as the utility cost of designating time and effort to active portfolio rebalancing strategies. The condition implies that an equilibrium outcome that does not require active portfolio trading in asset markets dominates an equilibrium that is identical in real terms, but that *does* require active portfolio trading. We do not impose any transaction costs of switching jobs, although it can be argued that such costs are also present, and in fact may be higher than the costs of dynamic trading in asset markets. The reason we do not impose these costs is they would not change our results qualitatively, although the analysis would be far less tractable.[11]

We are now ready to describe how the complete markets equilibrium can be implemented under these restrictions. At first glance it might appear as if asset markets are unnecessary. After all, we allow firms to write bilateral contracts with agents, so by serving as an intermediary, firms can effectively allow agents to write bilateral contracts between themselves. For example, firms could hire both types of workers, pool their production and reallocate it by paying wages equal to a constant fraction of the total. However, such contracts alone cannot implement the Pareto optimal equilibrium. The reason is that in such an equilibrium, although risk is efficiently shared conditional on production, total production is not maximized — flexible workers must switch industries in order to maximize their production. But the only way for the firm to pool production and reallocate it would be to extract a commitment of lifetime employment from

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[11]The key point here is that, even with small friction costs of job-switching, it will still be optimal for the flexible agent to switch industries (and for the inflexible agent to stay in the same industry), since the productivity loss of not switching is significant. The only difference is that he will switch industries less often when the friction is present.

flexible workers. Such a commitment is suboptimal. Because of the need for worker mobility, both labor and asset markets are required to implement the complete markets equilibrium.

To achieve the complete market equilibrium all inflexible agents sign a binding employment contract with firms in the industry of their specialty that commits both parties to lifetime employment.[12] Agents give up all their productivity and in return receive a wage equal to their Pareto optimal equilibrium allocation, $\eta(s_0)A_tK_{tot}(s)$, in every future state s. Flexible agents either choose to work for themselves, or work for firms and earn wages equal to their productivity. This means that in some states inflexible wages will exceed productivity. Because firms cannot force investors to make payments, firms require capital to credibly commit to the labor contract. They raise this capital by issuing limited liability equity. In states in which wages exceed productivity, the firm uses this capital to make up the shortfall and does not pay dividends. For the moment we restrict attention to states in which the capital in the firm is positive.

Flexible agents purchase the equity by borrowing the required capital from the bank. Firms then redeposit the capital in the bank (ensuring that the supply of deposits equals the demand for loans) and pay instantaneous dividend flows equal to

$$A_t \max(\alpha K_I(s) + C_t r_s - \eta K_{tot}(s), 0),$$

where $A_t C_t$ is the amount of capital owned by the firm at time t and $\eta \equiv \eta(s_0)$. Thus, flexible agents consume

$$A_t \left[(1-\alpha)K_F(s) + \max(\alpha K_I(s) + C_t r_s - \eta K_{tot}(s), 0) - C_t r_s \right], \tag{19}$$

where we assume (and later show) that flexible agents always choose to adjust their bank loans to match the capital firms deposit in the bank. Using (6), when dividends are positive, the term in square brackets in (19) becomes

$$(1-\alpha)K_F(s) + \alpha K_I(s) + C_t r_s - \eta K_{tot}(s) - C_t r_s = (1-\eta)K_{tot}(s), \tag{20}$$

so flexible agents consume their complete market allocation and $dC_t = 0$. Similarly, when dividends are zero we get

$$(1-\alpha)K_F(s) - C_t r_s + \frac{dC_t}{dt}. \tag{21}$$

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[12]In reality, employment contracts that bind workers are not enforceable. However, about half the working population do in fact work for a single employer (see Hall (1982)). As we discuss in the conclusion, this suggests that the lifetime employment contract is nevertheless common, that is, that firms use other means to commit employees to lifetime employment.

Now the stochastic change in firm capital equals the shortfall, that is,

$$dC_t = (\alpha K_I(s) + C_t r_s - \eta K_{tot}(s))dt.$$

Substituting this expression into (21) gives

$$(1 - \alpha)K_F(s) - C_t r_s + (\alpha K_I(s) + C_t r_s - \eta K_{tot}(s)) = (1 - \eta)K_{tot}(s), \tag{22}$$

so the flexible agent consumes his complete markets allocation in every state in which the firm's capital is positive.

Finally, consider the first time that either the value of the firm drops to zero or the firm's capital drops to zero. In such a state the firm can raise additional capital by issuing new equity (either by repurchasing existing equity for zero and issuing new equity to raise capital, or if the equity is not worth zero, issuing new equity at the market price). Hence by always issuing new capital in this state, the firm can ensure that neither its capital, nor its value, ever drops below zero and that it never pays negative dividends. Thus in this equilibrium both agents always consume their complete markets allocation which is Pareto optimal. This implies that flexible agents cannot be better off by following a different borrowing policy, justifying our assumption that they will always choose to borrow the amount firms deposit in the bank. Moreover, since this outcome implies a passive investment strategy for inflexible, as well as flexible, agents, this equilibrium implementation is optimal under the assumption of a small but positive cost of active rebalancing.[13]

The following proposition summarizes these results:

Proposition 3 *The following implementation leads to the complete market Pareto efficient outcome:*

- *Flexible workers either work for themselves or a firm which pays the instantaneous wage* $w_F = A_t K_F(s_t)$.

- *Inflexible workers work for publicly traded firms, which pay instantaneous wages equal to a constant multiple of aggregate production. In aggregate, firms pay the inflexible wage*

$$w_I = \eta A_t K_{tot}(s_t).$$

- *In states in which inflexible productivity plus interest on bank deposits exceeds wages, firms*

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[13]In fact, if in addition one assumes a (small) one-time cost of stock market participation, it is easy to show that this optimal implementation is *unique*, since it minimizes the fraction of the population that participates in the market.

pay dividends equal to

$$A_t[\alpha K_I(s) + C_t r_s - \eta K_{tot}(s)] \tag{23}$$

and retain capital $A_t C_t$ with $dC_t = 0$

- *In states in which inflexible productivity plus interest on bank deposits does not exceed wages, firms pay no dividends and reduce capital to make wage payments*

$$dC_t = (\alpha K_I(s) + C_t r_s - \eta K_{tot}(s))dt \tag{24}$$

- *The flexible workers own all the equity in the stock market. They pay for this equity by borrowing the capital from banks. Firms redeposit the capital in banks. Flexible workers optimally adjust their borrowing to ensure that at all times the supply of deposits equals the demand for loans.*

- *Whenever: (1) the price of the firm drops to zero, the firm raises new capital by repurchasing old equity for nothing and issuing new equity or (2) the amount of capital drops to zero, the firm raises new capital by issuing new equity at the market price.*

There are three important distinguishing characteristics of this solution that reflect reality. First, it features limited capital market participation — only flexible workers participate in capital markets. Indeed, because job mobility precludes flexible workers from sharing risk in labor markets they must participate in capital markets for any risk sharing to take place. Without understanding the importance of the labor market, one might naively look at inflexible workers' wealth and conclude that because this wealth is non-tradable, they would be better off using asset markets to hedge some of this exposure. In equilibrium inflexible workers choose not to further hedge their human capital risk exposure because it is too costly. Flexible workers choose to hold equity, not because of a desire to hedge — they choose to increase the riskiness of their position — but because of the compensation they receive in terms of a high equity risk premium.

The implication, that inflexible workers choose not to participate in markets is consistent with one of the most robust findings in the literature — that wealth and education are positively correlated with stock market participation (see Mankiw and Zeldes (1991)). Clearly, flexible workers are wealthier in our model, but more importantly, if productive flexibility derives from education, then they are likely to be better educated. In fact, Christiansen, Joensen, and Rangvid (2008) show that the degree of *economics* education is casually (positively) related to stock market participation. They interpret this result as evidence that non-participation derives from educational barriers to entry. But their results are also consistent with flexibility. Not all education provides productive flexibility so we would expect to see variation in the type of

education and stock market participation. Their study clearly documents this variation. Finally, note that non-participation in capital markets implies that inflexible workers also do not hold bonds, that is, they choose not to save. This result might help to explain the low savings rate observed in the U.S. — the reason workers choose not to save is that their labor contracts effectively do the saving for them.

A second distinguishing characteristic of our solution is that firm equity can be thought of as an option-like claim on total consumption. We therefore expect the volatility of equity returns to exceed the volatility of total consumption. Because we do not have idiosyncratic risk in our setting, this volatility imparts risk — equity is considerably more risky than total consumption.

That equity can be viewed as an option is well known. However, normally this insight is derived using financial leverage. In our case the firm has no debt, indeed it actually holds cash. In a standard setting this would mean that equity would not have option characteristics, indeed, because of the cash, equity would be *less* risky than the firm's assets. In our setting, it is not financial leverage that gives equity option-like characteristics, but the operating leverage resulting from wage commitments. Notice that this operating leverage is considerably more risky than the typical kind of operating leverage studied in the literature. Typically, firms have the option to shut down — if a firm is losing money on the margin then it can reduce its scale or shut down altogether. However, in our case firms optimally choose to give up this option — they commit to continue to pay wages even when, *ex post*, the value maximizing decision would be to shut down and pay out the remaining capital to equity holders. As we will demonstrate in the next section, by giving up this option, the firm substantially increases the risk of its equity.

Finally, note that asset returns will vary with the business cycle in a highly nonlinear fashion, because of the role of firms as insurers. This means that the unconditional link between real variables and asset returns can look quite weak even though they are instantaneously perfectly correlated. In fact, as we shall see, our results are in line with the findings in Duffee (2005), that consumption and equity returns are only weakly related in bad times, but are highly correlated in good times.

Two important challenges to the neoclassical asset pricing model are its inability to produce a realistic equity risk premium and the seemingly large disparity between consumption and equity volatility. As we have already pointed out, our model has the potential to meet these challenges. Consequently, in the next section, we investigate whether, for realistic parameter values, our model can reproduce the magnitude of the observed empirical anomalies.

5   Parameterization

The objective of this section is to show that the effects we have identified are first order, that is, large enough to potentially explain the empirical facts. We will demonstrate that for a set of realistic parameter values, the model can match the magnitude of the important moments in

the data. Although we view this as an achievement in itself — without the labor market, the model in this paper, essentially a Lucas tree economy, could not come close to matching these moments — deciding whether labor markets really can explain empirical puzzles such as limited market participation will require further empirical investigation.

We assume that flexible workers make up 1/3 of the working population, implying that 2/3 of the population choose not to participate directly in capital markets, in line with the estimates reported in Haliassos and Bertaut (1995), Guiso, Haliassos, Jappelli, and Claessens (2003), Hong, Kubik, and Stein (2004), and Christiansen, Joensen, and Rangvid (2008). Assume that f has the form (shown in Figure 2):

$$f(b) = \frac{(1-b)\left(3\overline{K}\sqrt{K - b\overline{K}} - (1-b)\sqrt{K - b\overline{K}} + 2\overline{K}^2\right)}{\left(\sqrt{K - b\overline{K}} + (1-b)\right)\left(\sqrt{K - b\overline{K}} + \overline{K}\right)}. \tag{25}$$

It is easy to see that f satisfies the required conditions, leading to the total production of flexible agents

$$K_F(s) = 1 + \overline{K}\frac{s^2}{s+1}. \tag{26}$$
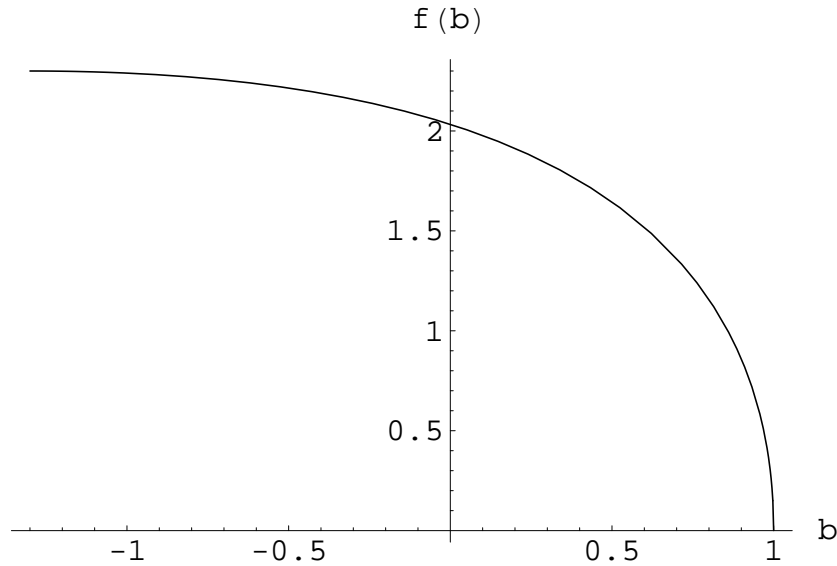

Figure 2: $f(b)$: Flexible Agents' optimal choice of f for a given choice of b

We assume that flexible workers' limiting productive sensitivity to the state variable, $\overline{K}$, is 2.3 and that inflexible workers all work in an industry with $b = \bar{b} = 0.5$, and $f = \overline{f} = 1.656$.[14]

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[14]Note that here we assume that $f(\bar{b}) = \overline{f}$ implying that a state exists in which all workers work in the same industry.

The total production of inflexible agents is then

$$K_I(s) = 0.5 + 1.656s. \tag{27}$$

These are the two production functions we plotted in Figure 1.

The state process, s, evolves according to (1), with parameter values $\theta = 0.003$, $\sigma = 6\%$ and $\bar{s} = 0.67$. The economy is thus in a recession when $s < 0.67$ and when $s > 0.67$ it is in a boom. With these parameters, the unconditional probability that $0 < s_t < 2$ is 97%, so we focus on this range. The long-term growth rate of the economy is $r = 1.2\%$. This implies an unconditional consumption growth volatility of about 4%, which is in line with what was used in Mehra and Prescott (1985). We use standard values for the preference parameters. We pick a relative risk aversion coefficient of 8.5, within the range Mehra and Prescott (1985, p. 154) consider reasonable, and impatience parameter $\rho = 0.5\%$.

Variable	Symbol	Value
Inflexible constant production	$\bar{b}$	0.5
Flexible Limiting Production Sensitivity	$\overline{K}$	2.3
Impatience Parameter	ρ	0.5%
Risk Aversion	γ	8.5
Long-term Growth Rate	r	1.2%
% of population Inflexible	α	67%
Inflexible variable production	$\bar{f}$	1.656
State Variable Volatility	σ	6%
Initial s	s_0, s^*	1.8
Mean reversion speed	θ	0.003
Long-term mean	$\bar{s}$	0.67
Initial Capital	C_0	0.88

Table 1: **Parameter Values**

We set the initial s equal to s^*. Since this implies that the inflexible workers get their highest possible share of production, it is never optimal for them to quit the firm and so they will never have an incentive to depart from the long-term contract they sign with firms.

The initial capital the firm raises is arbitrary in our model — because we have an effectively complete asset market, the Modigliani-Miller proposition implies that the firm's capital structure is irrelevant. Of course, in a world with frictions the amount of capital raised will affected by a tradeoff between the benefits (e.g., lower transaction costs) and costs (e.g., increased taxes and agency costs). We pick a level of initial capital that ensures that firms almost never need to return to capital markets[15] and to match the price volatility of the market, which we set to 16.6%. This leads to initial capital of $C_0 = 0.88$. Table 1 summarizes these parameter values.

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[15]The expected time to refinancing is over 1,000 years.

We solve for the complete markets equilibrium by solving (18) numerically. Table 2 summarizes this equilibrium. To compute W_i, we set $\delta = K_i(s)$ for each agent type $i \in \{I, F\}$ in (15).[16] We then compute η, the inflexible agents' wealth share, by solving (14) and get 65.0%. In this equilibrium the risk free rate is 4.2% and the firm's expected return is 8.3%, leading to a equity risk premium of 4.2%. The model also delivers realistic second moments. Firm volatility is 16.6% whereas consumption volatility is only 4.3%. More interesting is the unconditional correlation between equity returns and consumption. Because this is a standard neo-classical model, consumption prices assets, so the instantaneous correlation between equity returns and consumption is either 1 or -1 (as we will shortly see, equity values can be decreasing in the state variable). However, the average correlation across all states is only 0.4, that is, the unconditional correlation (what empiricists typically measure) is substantially lower than the instantaneous correlation. The instantaneous equity premium is $r_e - r_s = \rho_{pc}\gamma\sigma_c\sigma_p$. If we use *unconditional* moments to evaluate this expression we get $0.40 \times 8.5 \times 4.3\% \times 16.6\% = 2.4\%$, which is approximately half of the actual unconditional equity premium of 4.1%. This disparity occurs because the unconditional estimate of the correlation is not a good proxy for the actual variable of interest, the instantaneous correlation.

Variable	Symbol	Value
Risk Free Rate	r_s	4.2%
Firm Expected Return	r_e	8.3%
Firm Volatility	σ_p	16.6%
Consumption Volatility	σ_c	4.3%
Wealth fraction	η, η^*	65.0%
Probability $s < 2$		97%
Unconditional correlation	ρ_{pc}	0.40
Equity premium	$r_e - r_s$	4.2%

Table 2: **Equilibrium Moment Values at s_0**

An important feature of the model is that the value of equity is highly non-linear in the state variable. Note that the value of the firm, $V(s_t, C_t)$, is equal to the amount of cash plus the value of inflexible worker productivity minus the value of the wage commitment:

$$V(s_t, C_t) = \alpha W_I(s_t) + C_t - \eta W_{tot}(s_t), \tag{28}$$

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[16]Notice that $K_i(s)$ is time independent, so (18) degenerates into an ODE, and the price function is also time independent. It is easy to show that the boundary condition at $z = 1$ for the ODE is $Q(1) = 0$. We know of no simple boundary specification at $z = 0$, however, since the boundary is not absorbing for the square root process. In other words, finding the transversality condition that rules out bubbles is nontrivial. Therefore, we solve the PDE and let the terminal date, T, become so large that the solution becomes effectively time independent. This is the correct, no-bubble, solution in the infinite horizon economy.

where $W_{tot}(\cdot)$ is the total value of production. As the top panel of Figure 3 demonstrates, this
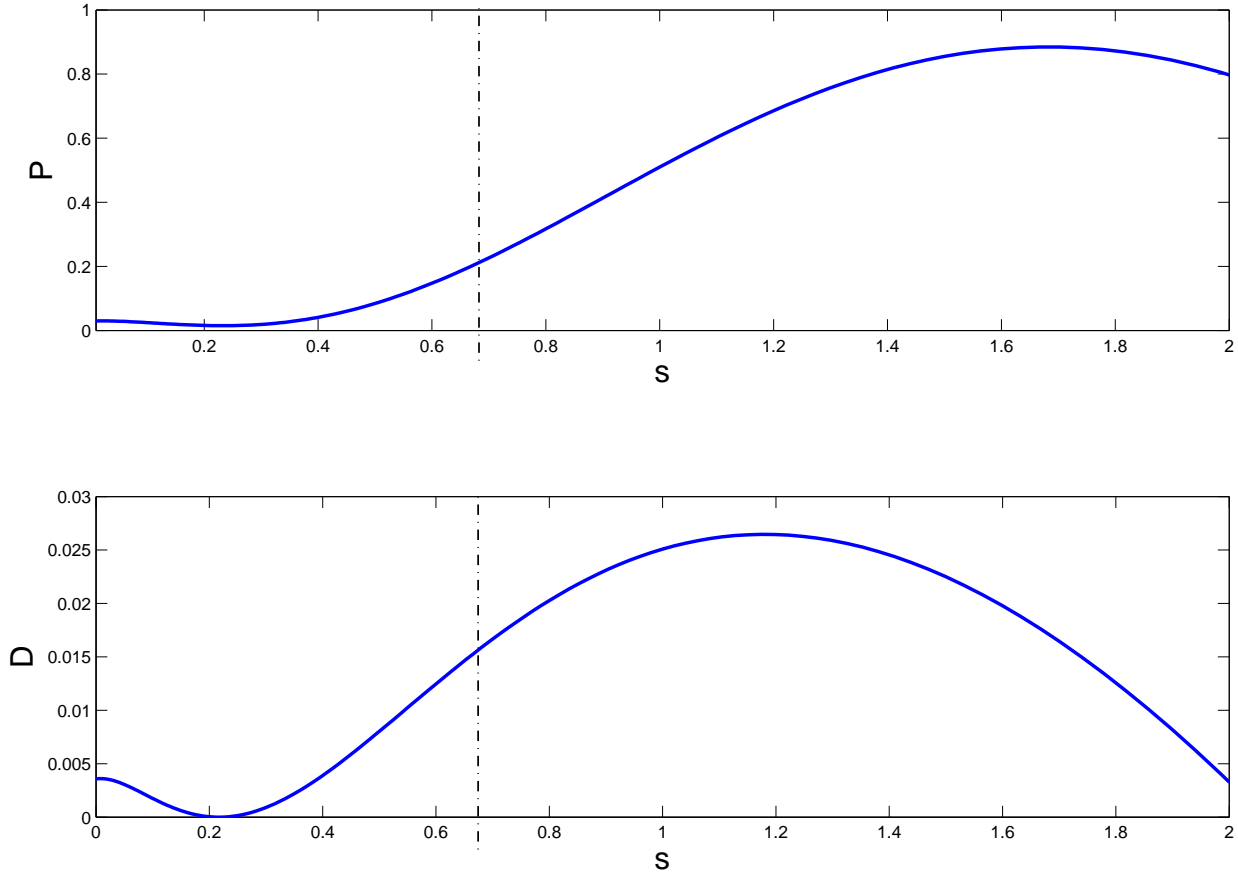

Figure 3: **Value of equity (upper panel) and total dividends (lower panel) as a function of s.** The vertical dashed line marks $\bar{s}$, the average value of the state variable.

price function is highly nonlinear in s. Its option-like qualities are self evident. The function is insensitive to the state variable for low values of s — it is actually slightly decreasing for very low s. It then increases rapidly in a convex fashion for low to high s, after which it reaches a maximum for very high s and becomes decreasing.

To understand the dynamic behavior of equity across the business cycle, note that as the state of the economy worsens, dividend payouts decrease, reflecting that fact that wages exceed productivity. Consequently, the value of the firm drops sharply. To understand what happens when the economy deteriorates further, we must first understand the dynamics of interest rates.

When the economy is in a very bad state, productivity is low but agents propensity to consume does not drop by as much. The reason is that because the state variable is mean reverting, agents understand that the current state is temporary. They therefore anticipate that the economy is likely to improve and because they want to smooth consumption, they have a propensity to borrow to consume. In equilibrium net borrowing is zero, so interest rates have to rise to clear markets, as is evident in Figure 4. Because the firm holds cash, this increase in interest rates generates interest income for the firm. Eventually, the increase in interest rates dominates the decrease in worker productivity, so that dividends begin to increase again, as is evident in the lower panel in Figure 3. The result is that for very low values of the state variable the value of the firm is actually decreasing in s. Note that D and P are both nonnegative within the range of the plots, and therefore there is no need for refinancing within this range.[17]
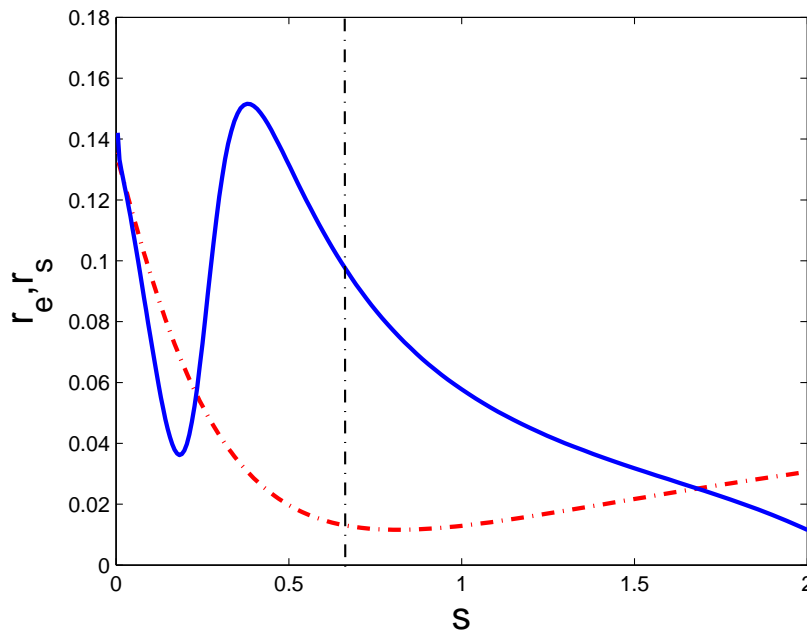

Figure 4: **Expected Return: Expected return on the firm's stock (solid blue line) and the (short) risk free rate (dotted red line).** The vertical dashed line marks $\bar{s}$, the average value of the state variable.

Because of this nonlinearity of the price function and the behavior of interest rates, the equity risk premium is highly variable in the state variable. The instantaneous expected equity

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[17]For $s > 2$, there will be a point at which dividends turn negative, so that refinancing might be needed at some point.

return is

$$r_e dt = \frac{E[dV]}{V} + \frac{D}{V} dt = \left(\frac{\theta(\bar{s} - s)V' + \frac{\sigma^2}{2} s V''}{V} + \frac{D}{V} \right) dt. \tag{29}$$

Using (28), Figure 4 plots this function, together with the risk-free rate. The most striking element in the plot is the difference in behavior of the equity risk premium in expansions and contractions. When $s > \bar{s}$ the equity risk premium is decreasing in the state variable, which makes intuitive sense. Equity is an insurance contract, for high values of the state variable the insurance contract is not very risky. In contractions, the equity risk premium continues to increase as the state deteriorates, reaching a maximum of about 13%. But then, as the value of equity becomes less sensitive to the state of the economy, the risk premium begins to drop and, for low enough values of s, actually becomes negative. At the mean point of state variable, $s = \bar{s} = 0.67$, expected stock returns are about 9.5%, which is close to the unconditional expected return of 8.3%. However, at $\bar{s}$, the equity risk premium is 8.2%, which is substantially higher than the unconditional equity premium of 4.1%.

An important implication of the nonlinearity in the price function is that it weakens the link between consumption and returns. In particular, the correlation between consumption and returns is different in expansions and contractions. The estimated correlation in our calibration conditional on being in a contraction ($s < 0.67$) is 0.23, whereas the estimated correlation conditional on being in an expansion ($s > 0.67$) is 0.81. This disparity is in line with the results in Duffee (2005) that the correlation between stock returns and consumption growth is low (about 0) in bad times, and high (about 0.6) in good times.

It is informative to compare our results with those in Danthine and Donaldson (2002). Although Danthine and Donaldson (2002) also model the effect of labor markets on asset prices, they are unable to match the equity risk premium in a model without frictions.[18] By introducing frictions they are able to match the risk premium, but at the cost of perfect risk sharing. In particular, to get a significant risk premium, their restriction limiting market participation becomes important. Without it, workers would invest in markets and thereby increase risk sharing. Not only would the equilibrium not feature limited stock market participation, but the ability to share additional risk would likely reduce the risk premium. Because we do not impose limited capital market participation, flexible workers must be induced to take on additional risk in equilibrium. That implies that the return on equity (the means by which flexible workers take on this risk) has to be high enough to induce this behavior. This is a key insight in our model — rather than a place to hedge risk and smooth consumption, asset markets are a place where investors are induced to take on extra risk.

Because we do not have idiosyncratic risk in our model, an increase in the risk premium must

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[18]They introduce two frictions in their model: (1) Adjustment costs, which provides only a modest increase in the risk premium and (2) changes in the "bargaining power" of workers and investors that effectively prevents perfect risk sharing.

be associated with an increase in volatility. Figure 5 confirms this insight. The volatility of the firm initially explodes in bad states, in line with the empirical evidence cited in the introduction. But then it actually starts decreasing, reflecting the fact that the value of equity becomes less sensitive to the state variable (see Figure 3, top panel), until it actually reaches zero close to 0.2. For even lower s, price is decreasing in s, and so volatility increases. Finally, volatility decreases again as s approaches 0 and as the volatility of ds becomes negligible, once again decreasing the equity volatility. Thus, the volatility of the firm is a nonlinear function of s, with regions of very high volatility in bad states of the world. More importantly, the same is not true of the volatility of consumption growth — it is virtually unaffected by the level of s. Our model actually delivers an almost complete disconnect between consumption volatility and asset volatility, something that is traditionally regarded as perhaps the greatest challenge to the neoclassical asset pricing model. This figure, more than any other, makes clear how the interaction between labor and financial markets can dramatically affect the inferences of the neoclassical asset pricing model. In our model, the consumption CAPM holds by construction. In line with the empirical evidence referenced in the introduction, consumption growth volatility is low and virtually unaffected by the business cycle. Asset volatility, on the other hand, is significantly higher and is very sensitive to the business cycle.

Finally, it makes sense to consider how the equilibrium changes as a function of the initial state. Figure 6 plots the equilibrium share of total consumption of inflexible agents, $\eta(s_0)$. What is clear is that for most values of the state variable, the equilibrium consumption share is insensitive to the initial state. The figure also shows the initial aggregate instantaneous relative productivity, $\frac{K_I}{K_{tot}}$, of inflexible agents. We see that $\frac{K_I}{K_{tot}}$, reaches its maximum, α (i.e., 2/3), at $s_0 \approx 0.7$, whereas $\eta^* \approx 0.64$ occurs at $s_0 = s^* \approx 1.8$. The reason η is maximized to the right of the point where the inflexible agent's relative productivity is maximized is that the value of insurance is greater in the bad states than in the good states. Hence, inflexible agents are willing to pay more for insurance at the point where their production share is maximized than at points to the right, so η continues to increase.

6   Conclusions

Our objective in this paper is to demonstrate the potential importance of explicitly modeling labor markets within the neoclassical asset pricing model. We show that neither the asset market nor the labor market, by themselves, can share risk efficiently. Labor markets are restricted because it is inefficient for some workers to enter long-term contracts, and asset markets are restricted because the (static) asset span is too narrow. But together the two markets can share risk efficiently. By explicitly modeling the interaction between the two markets, we demonstrate the potential to explain some of the most important normative challenges faced by the neoclassical asset pricing model: (1) limited asset market participation, (2) the seemingly high equity
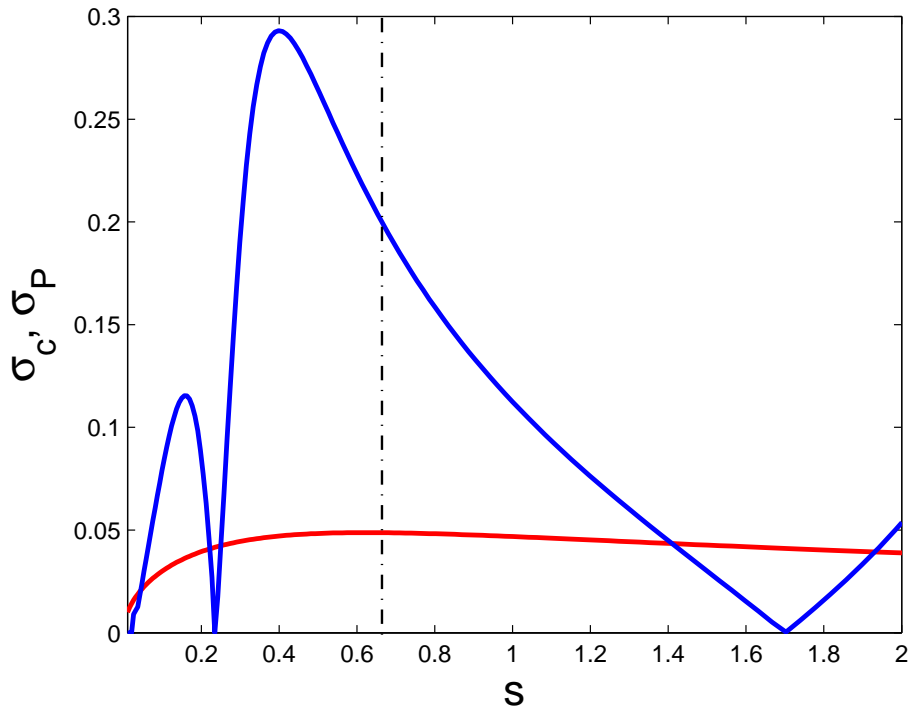
Figure 5: **Volatility: The blue solid curve is firm volatility $\left(\frac{V'}{V}\sigma(s)\right)$, and the red dotted curve is consumption growth volatility $\left(\frac{K'_{tot}}{K_{tot}}\sigma(s)\right)$.** The vertical dashed line marks $\bar{s}$, the average value of the state variable.

risk premium, (3) the very large disparity in the volatility of consumption and the volatility of asset prices, and (4) the time dependent correlation between consumption growth and asset returns.

Although our model is stylized it captures most of the salient features of both labor and asset markets. The one apparent exception is that in the model it is optimal for some agents to commit to lifetime employment with a single firm. In reality, although firms often commit to employ agents, rarely do agents *explicitly* commit to stay with firms. Nevertheless, there is strong empirical evidence suggesting that in fact a large subset of employees implicitly commit to firms, consistent with the assumptions of our model. Hall (1982) finds that after an initial job search in which employees might work for short periods for different employers, half of all men then work the rest of their lives for a single employer.

There are a number of ways employees may implicitly commit to firms. First, employees might never actually face an incentive to leave, making an explicit commitment unnecessary as is the case in our parameterization in Section 5. To the extent that there are costs to switching jobs, workers will not choose to switch jobs so long as the switching costs exceed the benefits.
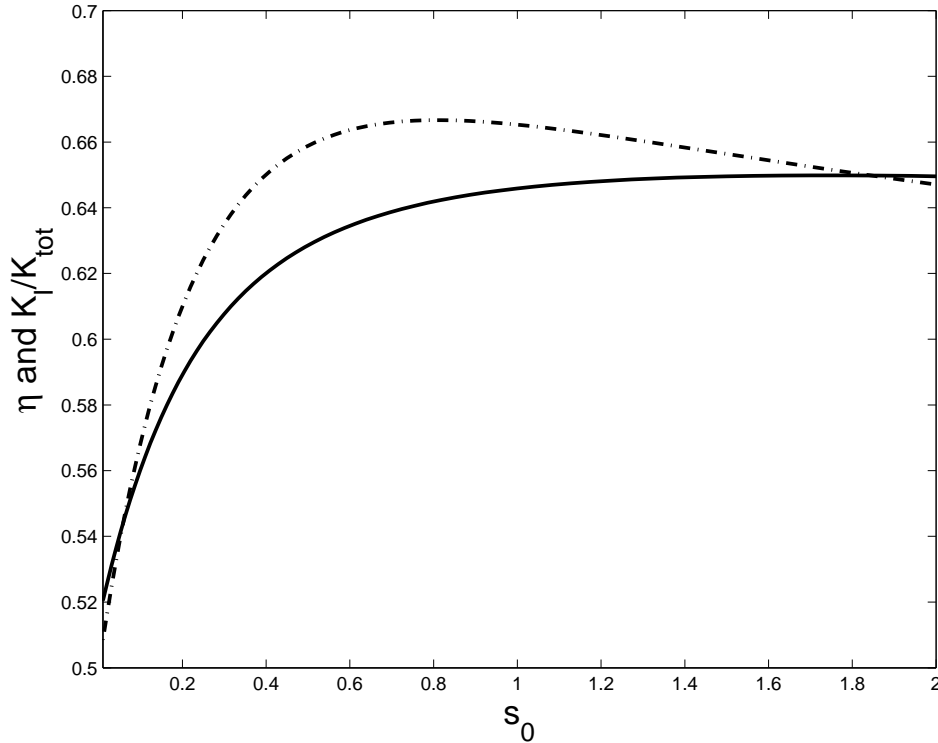
Figure 6: **Sensitivity of the Equilibrium to the Initial State:** The solid curve is the equilibrium share of wealth of the inflexible worker at $t = 0$, η, as a function of the initial s_0. The dashed curve is the inflexible agents aggregate share of total productivity at time 0.

Because $\eta(s)$ is flat around s^* (see Figure 6), only a small cost of switching jobs is required to ensure that agents will not have an incentive to switch jobs for most starting values of s.

Second, agents can increase the costs of switching further and thereby effectively commit to firms by, for example, accepting deferred compensation contracts. Examples include pension funds and stock vesting periods. In addition many union contracts explicitly tie wages to seniority with the firm, making a job switch very costly.

Third, the initial training costs that workers must bear when they start a new job act as a commitment device. Although employers might ultimately compensate workers for these costs through higher long-term wages, workers will have to remain employed to collect this compensation.

Notice that while it is socially suboptimal for inflexible workers to switch jobs, it is optimal for flexible workers to do so. Hence, one might expect to see differences in the costs of switching, because incentives exist to lower the switching costs for flexible workers and increase these costs for inflexible workers. This observation might explain why, for example, head hunters tend to specialize in managerial jobs (where the skill set can be readily transfered across industries).

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Although outside the scope of this paper, our model has potentially interesting implications for the cross-section of asset returns. For simplicity we assumed only one type of firm in our calibration, but, in general, firms with different labor market exposures will have expected returns which will depend on the insurance contract that the firm signs with its workers. Firms that are sensitive to changes in s (i.e., firms with b_i close to 0), will provide more insurance in bad states than firms that are less sensitive (firms with b_i close to 1). Equity holders in these firms will therefore require higher returns. In addition, because of the non-linearity of returns, CAPM betas — either unconditional or conditional — are likely to have little explanatory power in the cross section.

Because inflexible workers only consume their wage income, our model implies that the wages of inflexible workers should have explanatory power in the cross-section. A measure of labor mobility across firms (which should be closely related to our flexibility notion) is constructed in Donangelo (2009), using survey data from the Bureau of Labor Statistics between 1988-2008. Such a measure could potentially be used to disentangle the two sides of the work force. It is also interesting to note that in our model *aggregate* wages, which include wages of both flexible and inflexible workers — i.e., wages of both "insurers" and "insurees" — should be considerably less informative about expected returns, in line with the findings of Fama and Schwert (1977). These observations provide a potentially interesting path for future research.

Appendix

Proof of Lemma 1: (a) Follows since f is decreasing and $f(0) = 1$.

(b) Clearly, $\overline{K}s \leq C_F(s) \leq \overline{K}s + 1$ for all s, since the lower bound can be realized by choosing $b(s) = 1$, and the upper bound follows from the constraint that $b \leq 1$. (b) therefore immediately follows.

(c) Follows since $b + f(b)s$ is (weakly) convex as a function of s for each b and the maximum of a set of convex functions is convex. ∎

Proof of Lemma 2: The flexible worker solves $\max_{b \in [0,1]} b + f(b)s$. The first order condition is $f'(b) = -\frac{1}{s}$, and since f' is a continuously differentiable, strictly decreasing, mapping from $[0,1]$ onto $(-\infty, 0]$, the implicit function theorem implies that there is a unique, decreasing, continuously differentiable solution to the first order condition, $b^*(s)$, such that $b^*(0) = 1$ and $\lim_{s \to \infty} b^*(s) = 0$. Since the second order condition is $f''(b)s < 0$, this function indeed yields the maximal strategy, $K_F(s) = b^*(s) + f(b^*(s))s$.

Now, $K'_F(s) = b^{*\prime}(s) + b^{*\prime}(s)f(b^*(s))s + f(b^*(s)) = 0 + f(b^*(s))$, so $K'_F(0) = f(b^*(0)) = f(1) = 0$, and $K'_F(\infty) = f(b^*(\infty)) = f(0) = \overline{K}$. Moreover, $K''_F(s) = b^{*\prime}(s)f'(b^*(s)) = -\frac{b^{*\prime}(s)}{s}$, which is continuous and positive, so K_F is indeed strictly convex and twice continuously differentiable. ∎

Proof of Lemma 3: We have

$$Vol\left(\frac{dK_F}{K_F}\right)^2 = \left(\frac{K'_F}{K_F}\sigma(s)\right)^2 dt,$$

whereas

$$Vol\left(\frac{dK_I}{K_I}\right)^2 = \left(\frac{K'_I}{K_I}\sigma(s)\right)^2 dt = \left(\frac{\overline{f}}{\overline{b} + \overline{f}s}\sigma(s)\right)^2 dt.$$

From Lemma 2, $\frac{K'_F}{K_F}$ converges to 0 for small s, whereas $\frac{\overline{f}}{\overline{b} + \overline{f}s}$ converges to $\frac{\overline{f}}{\overline{b}} > 0$, so for small s, the inflexible worker's productivity indeed has higher volatility.

For large s, we have

$$\frac{dK_I}{K_I} = \frac{\overline{f}}{\overline{b} + \overline{f}s} = \frac{1}{\frac{\overline{b}}{\overline{f}} + s}.$$

Moreover, from Lemma 2 it follows that

$$\frac{dK_F}{K_F} = \frac{f(b^*(s))}{b^*(s) + f(b^*(s))s} = \frac{1}{\frac{b^*(s)}{f(b^*(s))} + s}.$$

The inequality therefore follows if $\frac{b^*(s)}{f(b^*(s))} < \frac{\overline{b}}{\overline{f}}$, but since $b^*(s) \to 0$ for large s (from the proof of Lemma 2) and therefore $f(b^*(s)) \to \overline{K}$, the flexible worker's productivity is indeed riskier for large s. The lemma is proved. ∎

Proof of Proposition 1: From (9), the price at $t = 0$ of a general asset, paying an instantaneous dividend stream, $g(s,t)dt$, where g is a continuous function, is

$$P(s) = K^{\gamma}_{tot} E\left[\int_0^{\infty} e^{-\rho t}\frac{g(s,t)}{K_{tot}(t)^{\gamma}}dt\right] \overset{\text{def}}{=} K^{\gamma}_{tot}Q(s,0), \tag{30}$$

where

$$Q(s,t) \overset{\text{def}}{=} E\left[\int_t^{\infty} e^{-\rho(\tau-t)}\frac{g(s,\tau)}{K_{tot}(\tau)^{\gamma}}d\tau\right].$$

From Feynman-Kac's formula (see, e.g., Karatzas and Shreve (1991)) it follows that $Q \in C^{1,2}(\mathbb{R}_+ \times \mathbb{R}_+)$, and that Q satisfies the PDE

$$Q_t + \mu(s)Q_s + \frac{\sigma(s)^2}{2}Q_{ss} - \rho Q + \frac{g}{K_{tot}^\gamma} = 0. \tag{31}$$

Since K_{tot} is smooth, it follows that P is also smooth and since $Q = \frac{P}{K_{tot}^\gamma}$, it follows that $Q' = \frac{P'}{K_{tot}^\gamma} - \gamma\frac{PK_{tot}'}{K_{tot}^{\gamma+1}}$ and $Q'' = \frac{P''}{K_{tot}^\gamma} - 2\gamma\frac{P'K_{tot}'}{K_{tot}^{\gamma+1}} - \gamma\frac{PK_{tot}''}{K_{tot}^{\gamma+1}} + \gamma(\gamma+1)\frac{P(K_{tot}')^2}{K_{tot}^{\gamma+2}}$. By plugging these expressions into (31), and defining $R(s) = \frac{K_{tot}'}{K_{tot}}$, and $T(s) = \frac{K_{tot}''}{K_{tot}}$, we arrive at (15). $\blacksquare$

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