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ROLLOVER RISK AND MARKET FREEZES

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### **ABSTRACT**

The crisis of 2007-09 has been characterized by a sudden freeze in the market for short-term, secured borrowing. We present a model that can explain a sudden collapse in the amount that can be borrowed against finitely-lived assets with little credit risk. The borrowing in this model takes the form of a repurchase agreement ("repo") or asset-backed commercial paper that has to be rolled over several times before the underlying assets mature and their true value is revealed. In the event of default, the creditors can seize the collateral. We assume that there is a small cost of liquidating the assets. The debt capacity of the assets (the maximum amount that can be borrowed using the assets as collateral) depends on the information state of the economy. At each date, in general there is either "good news" (the information state improves), "bad news" (the information state gets worse), or "no news" (the information state remains the same). When rollover risk is high, because debt must be rolled over frequently, we show that the debt capacity is lower than the fundamental value of the asset and in extreme cases may be close to zero. This is true even if the fundamental value of the assets is high in all states. Thus, a small change in information, as measured by a change in the fundamental value, can lead to a "market freeze." Interpreted differently, the model explains why discounts in overnight repo borrowing, the so-called "haircuts," rose dramatically during the crisis for asset-backed securities with low credit risk once bad news about the underlying cash flows arrived.

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# 1 Introduction

## 1.1 Motivation

One of the many striking features of the crisis of 2007-09 has been the sudden freeze in the market for the rollover of short-term debt. While the rationing of firms in the unsecured borrowing market is not uncommon and has a long-standing theoretical underpinning (see, for example, the seminal work of Stiglitz and Weiss, 1981), what is puzzling is the almost complete inability of financial institutions to issue (or roll over) short-term debt backed by assets with relatively low credit risk. From a theoretical standpoint, this is puzzling because the ability to pledge assets and provide collateral has been considered one of the most important tools available to firms in order to get around credit rationing (Bester, 1985). From an institutional perspective, the inability to borrow overnight against high-quality assets has been a striking market failure that effectively led to the demise of a substantial part of investment banking in the United States. More broadly, it led to the collapse in the United States, the United Kingdom, and other countries of banks and financial institutions that had relied on the rollover of short-term wholesale debt in the asset-backed commercial paper (ABCP) and overnight secured repo markets.

The first such collapse occurred in the summer of 2007. On July 31, 2007, two Bear Stearns hedge funds based in the Cayman Islands and invested in sub-prime assets filed for bankruptcy. Bear Stearns also blocked investors in a third fund from withdrawing money. In the week to follow, more news of problems with sub-prime assets hit the markets. Finally, on August 7, 2007, BNP Paribas halted withdrawals from three investment funds and suspended calculation of the net asset values because it could not “fairly” value their holdings:<sup>1</sup>

“[T]he complete evaporation of liquidity in certain market segments of the US securitization market has made it impossible to value certain assets fairly regardless of their quality or credit rating. . . Asset-backed securities, mortgage loans, especially sub-prime loans don’t have any buyers. . . Traders are reluctant to bid on securities backed by risky mortgages because they are difficult to sell on. . . The situation is such that it is no longer possible to value fairly the underlying US ABS assets in the three above-mentioned funds.”

This announcement appeared to cause investors in ABCP, primarily money market funds, to shy away from further financing of ABCP structures. These investors could no longer be guaranteed that there was minimal risk in ABCP debt. In particular, many ABCP vehicles had recourse to sponsor banks that set up these vehicles as off-balance-sheet structures but provided them with liquidity and credit enhancements. If ABCP debt could not be rolled

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<sup>1</sup>Source: “BNP Paribas Freezes Funds as Loan Losses Roil Markets” (Bloomberg.com, August 9, 2007).

over, the sponsor banks would have to effectively take assets back onto their balance-sheets. But, given the assets had little liquidity, the banks' ability to raise additional finance – typically rollover debt in the form of financial CP – would be limited too. Money market funds thus faced the risk that the assets underlying ABCP would be liquidated at a loss. This liquidation and rollover risk produced a “freeze” in the ABCP market, raised concerns about counter-party risk amongst banks, and caused LIBOR to shoot upwards. The sub-prime crisis truly took hold as the European Central Bank pumped 95 billion Euros in overnight lending into the market that same day in response to the sudden demand for cash from banks.

The failure of Bear Stearns in mid-March 2008 was the next example of a market freeze.<sup>2</sup> As an intrinsic part of its business, Bear Stearns relied day-to-day on its ability to obtain short-term finance through secured borrowing. At this time, they were reported to be financing \$85 billion of assets on the overnight market (Cohan, 2009). Beginning late Monday, March 10, 2008 and increasingly through that week, rumors spread about liquidity problems at Bear Stearns and eroded investor confidence in the firm. Even though Bear Stearns continued to have high quality collateral to provide as security for borrowing,<sup>3</sup> counterparties became less willing to enter into collateralized funding arrangements with the firm. This resulted in a crisis of confidence late in the week, where counterparties to Bear Stearns were unwilling to make even secured funding available to the firm on customary terms. This unwillingness to fund on a secured basis placed enormous stress on the liquidity of Bear Stearns. On Tuesday, March 11, the holding company liquidity pool declined from \$18.1 billion to \$11.5 billion. On Thursday, March 13, Bear Stearns' liquidity pool fell sharply and continued to fall on Friday. In the end, the market rumors about Bear Stearns' liquidity problems became self-fulfilling and led to the near failure of the firm. Bear Stearns was adequately capitalized at all times during the period from March 10 to March 17, up to and including the time of its agreement to be acquired by J.P. Morgan Chase. Even at the time of its sale, Bear Stearns' capital and its broker dealers' capital exceeded supervisory standards. In particular, the capital ratio of Bear Stearns was well in excess of the 10% level used by the Federal Reserve Board in its well-capitalized standard.

In his analysis of the failure of Bear Stearns, the Federal Reserve Chairman Ben Bernanke observed:<sup>4</sup>

“[U]ntil recently, short-term repos had always been regarded as virtually risk-

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<sup>2</sup>The discussion that follows is based on the Securities and Exchange Commission press release on March 20, 2008 entitled, “Chairman Cox Letter to the Basel Committee in Support of New Guidance on Liquidity Management,” available at: <http://www.sec.gov/news/press/2008/2008-48.htm>

<sup>3</sup>This high quality collateral mainly consisted of highly rated mortgage-backed assets which had low but not inconsequential credit risk by this time in the crisis.

<sup>4</sup>The quotes are based on Ben Bernanke's remarks to the Risk Transfer Mechanisms and Financial Stability Workshop at the Bank for International Settlements, May 29, 2008.

free instruments and thus largely immune to the type of rollover or withdrawal risks associated with short-term unsecured obligations. In March, rapidly unfolding events demonstrated that even repo markets could be severely disrupted when investors believe they might need to sell the underlying collateral in illiquid markets. Such forced asset sales can set up a particularly adverse dynamic, in which further substantial price declines fan investor concerns about counterparty credit risk, which then feed back in the form of intensifying funding pressures. . . In light of the recent experience, and following the recommendations of the President’s Working Group on Financial Markets (2008), the Federal Reserve and other supervisors are reviewing their policies and guidance regarding liquidity risk management to determine what improvements can be made. In particular, future liquidity planning will have to take into account the possibility of a sudden loss of substantial amounts of secured financing.”

In this paper, we are interested in developing a model of this “adverse dynamic.” More specifically, we are interested in explaining the drying up of liquidity in the absence of obvious problems of asymmetric information or fears about the value of collateral. In August 2007, a large number of ABCP conduits could not roll over their paper in spite of not having experienced any defaults. Some of them experienced credit downgrades of some sort, but the sudden erosion of their debt capacity was not explained by these factors. Similar liquidity concerns arose with the freezing of secured repo market for Bear Stearns, which ultimately caused the bank’s collapse.

We study the debt capacity of an asset under the following three conditions:

- (i) the debt is short-term in nature and, hence, needs to be frequently rolled over;
- (ii) in the event of default by the borrower, the underlying assets are sold to buyers who also use short term finance and a (small) liquidation cost is incurred;
- (iii) information arrives slowly relative to the frequency of refinancing the debt.

These are essentially the features alluded to in the preceding discussion of the conditions surrounding the freeze in the market for ABCP and the collapse of Bear Stearns.

Our main objective is to characterize the rollover debt capacity<sup>5</sup> of an asset used as collateral for short-term borrowing. The standard result in efficient markets is that the debt capacity of an asset is equal to its NPV or “fundamental” value. We show in the sequel that this result almost never holds. We characterize the debt capacity as the solution to a dynamic programming problem that can be easily solved by backward induction. Using

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<sup>5</sup>The *rollover debt capacity* is the maximum amount of debt that can be obtained when the debt has to be rolled over each period.

this characterization we show that, *when the maturity of the debt is sufficiently short* (the frequency of rolling over the debt is sufficiently high):

- The debt capacity of an asset is *always* smaller than its fundamental value.
- In fact, the debt capacity in a given state is always less than the terminal value of the asset in that state.
- And, in the worst case, the debt capacity equals the *minimum possible value of the asset*.

We call this last phenomenon a “market freeze.” This remarkable and perhaps counter-intuitive result captures the scenario that Bear Stearns experienced during its failure in March 2008.

The intuition for the market freeze result can be explained as follows. When information arrives slowly relative to the rollover frequency, it is likely that no new information will have arrived by the next time the debt has to be refinanced. The upper bound on the amount of money that can be repaid is the debt capacity at the next rollover date. Since there is a small liquidation cost, issuing debt with face value greater than the next period’s debt capacity is unattractive. So the best the borrower can do is to issue debt with a face value equal to the next period’s debt capacity assuming no new information arrives. But this locks the borrower into a situation in which he is forced to act as if his condition remains the same forever. In fact, his situation is somewhat worse, because there is always the possibility of bad news arriving, which will force him to default and realize the liquidation cost. These two facts, the need to set the face value low and the possibility of default if bad news arrives, together guarantee that the debt capacity is always less than the fundamental value. In fact, the difference between the fundamental value and the debt capacity can be very large, because the fundamental value reflects the substantial probability of good news arriving whereas the debt capacity ignores this upside potential. Thus, a small change in the fundamental value of the assets may be accompanied by a drastic fall in the debt capacity, resulting in a market freeze.

Our results can alternatively be stated in terms of the so-called “haircut” of an asset when it is pledged for secured borrowing or used in a repo transaction. The “haircut” is defined as one minus the ratio of the debt capacity to the “market value.” In the period 2007-2008, price discovery was very difficult. When active transaction prices were not available, banks obtained their “marks” from a variety of sources. In order to operationalize the concept of the “haircut,” we choose the economic or fundamental value as the “mark.” Then we can measure the haircut as one minus the ratio of the debt capacity of an asset to the asset’s expected or fundamental value. Our model shows that, while the haircut is naturally affected by the credit risk of the asset, its primary determinant is the maturity mismatch in

the funding structure of the asset, that is, the maturity of the debt relative to that of the asset. In particular, when rollover frequency of debt is sufficiently high, repo haircuts may be large even though the credit risk and liquidation costs are very small.

Gorton and Metrick (2009) show that during 2007-08, the repo haircuts on a variety of assets rose on average from zero in early 2007 to nearly 50 percent in late 2008. Interestingly, while some of the collateralized debt obligations (CDO) had no secured borrowing capacity at all (100% haircut) during the crisis, equities – which are in principle much riskier assets – had only around a 20% haircut (see Box 1.5 from Chapter 1, Page 42 of IMF, 2008). Our model can explain both these pieces of evidence. First, from early 2007 to late 2008, there was a series of information events that had the effect of reducing the fundamental value of the assets. Our model provides conditions under which the debt capacity may fall much further than the fundamental value, and in the worst case may fall as far as the minimum possible asset value. Second, the model shows that the haircut also depends on how the assets are likely to be financed by potential buyers. A large number of mortgage-backed and other asset-backed securities were funded by almost all potential buyers before and during the crisis by short-term rollover debt, whereas equities are held by relatively long-term investors. This difference in funding structure would be sufficient to generate lower haircuts for equities, even if equities were riskier than asset-backed securities.

The rest of the paper is organized as follows. Section 2 provides an introduction to the model and results in terms of a simple numerical example. In particular, it illustrates our main limit result that a market freeze occurs if the number of rollovers is sufficiently high. Section 3 derives the main limit result for the special case of a model with two states. It also illustrates, in terms of the numerical example, that market freezes can occur even if the debt maturity is not “short.” Section 4 provides a complete characterization of the debt capacity for the general model and extends the limit result to an arbitrary number of states. The proof of the limit result is relegated to Appendix A. Section 5 explains why our results have implications for repo haircuts observed in markets during stress times. Section 6 discusses policy implications regarding ex-post and ex-ante avoidance of market freezes. Section 7 discusses the related literature. Section 8 ends.

## 2 Model and results

In this section, we introduce the essential ideas in terms of a numerical example. For concreteness, consider the case of a bank that wishes to repo an asset. The question we ask is: What is the maximum amount of money that the bank can borrow using the asset as collateral?

Time is represented by the unit interval,  $[0, 1]$ . The asset is purchased at the initial date  $t = 0$ . The asset has a finite life (e.g., mortgages) which we normalize to one unit. To keep

the analysis simple, we assume that the asset has a terminal value at  $t = 1$ , but generates no income at the intermediate dates  $0 \leq t < 1$ . We also assume that the risk-free interest rate is 0 and that all market participants are risk neutral.

For simplicity, let us assume that there are two states of nature, a low state  $L$  and a high state  $H$ . The terminal value of the asset depends on the state of the economy at the terminal date  $t = 1$ . In the high state, the value of the asset is  $v^H$  and in the low state it is  $v^L$ .

Information becomes available continuously according to a Poisson process with parameter  $\alpha > 0$ . That is, the probability that an information event occurs in a short time interval  $[t, t + \tau]$  is  $\alpha\tau$ . When an information event occurs, the state of the economy changes randomly according to a fixed probability transition matrix

$$\mathbf{P} = \begin{bmatrix} p_{LL} & p_{LH} \\ p_{HL} & p_{HH} \end{bmatrix}.$$

We assume that the asset will be financed by debt that has to be rolled over repeatedly. The debt is assumed to have a fixed maturity, denoted by  $0 < \tau < 1$ , so that the debt must be rolled over  $N$  times, where

$$\tau = \frac{1}{N + 1}.$$

The unit interval is divided into intervals of length  $\tau$  by a series of dates denoted by  $t_n$  and defined by

$$t_n = n\tau, \quad n = 0, 1, \dots, N + 1,$$

where  $t_0$  is the date the asset is purchased,  $t_n$  is the date of the  $n$ -th rollover (for  $n = 1, \dots, N$ ), and  $t_{N+1}$  is the final date at which the asset matures and the terminal value is realized. This time-line is illustrated in Figure 1.

— Figure 1 here —

If the bank is forced to default and liquidate the assets, we assume that the assets can be sold for a fraction  $\lambda \in [0, 1]$  of the *maximum amount of finance that could be raised using the asset as collateral*. It is important to note that the recovery rate  $\lambda$  is applied to the maximum debt capacity rather than to the fundamental value of the assets. If the buyer of the assets were a wealthy investor who could buy and hold the assets until maturity, the fundamental value would be the relevant benchmark and the investor might well be willing to pay some fraction of the fundamental value, only demanding a discount to make sure that he does not mistakenly overpay for the assets. What we are assuming here is that the buyer of the assets is another financial institution that must also issue short term debt in order to finance the purchase. Hence, the buyer is constrained by the same forces that determined the debt capacity in the first place. While the debt capacity provides an upper bound on



the purchase price, there is no reason to think that the assets will reach this value. In fact, we assume  $\lambda < 1$  in what follows.<sup>6</sup>

## 2.1 A numerical example

To illustrate the method of calculating debt capacity in the presence of roll over risk, we use the following parameter values: the Poisson parameter is  $\alpha = 10$ , the recovery rate is  $\lambda = 0.90$ , the maturity of the repo is  $\tau = 0.01$ , the values of the asset are  $v^H = 100$  and  $v^L = 50$  in the high and low states, respectively, and the transition probability matrix is

$$\mathbf{P} = \begin{bmatrix} 0.20 & 0.80 \\ 0.01 & 0.99 \end{bmatrix}.$$

The transition matrix gives the probabilities of moving from one state to another conditional on the occurrence of an information event. For example, the probability of switching from state  $L$  to state  $H$  at an information event is  $p_{HL} = 0.80$ , whereas the probability of remaining in state  $L$  is  $p_{LL} = 0.20$ . But the occurrence of an information event is itself random, so the number of information events in a given time interval is random and affects the probability of observing a transition from one state to another. The transition probability matrix for an interval of unit length can be calculated to be<sup>7</sup>

$$\mathbf{P}(1) = \begin{bmatrix} 0.01265 & 0.98735 \\ 0.01234 & 0.98766 \end{bmatrix}.$$

At time 1, the fundamental values are 100 in state  $H$  and 50 in state  $L$  by assumption. So the fundamental values at time 0 can be calculated by using the terminal values and the transition probabilities in the matrix  $\mathbf{P}(1)$ . The fundamental value in state  $H$  at time 0 is

$$V_0^H = 0.98766 \times 100 + 0.01234 \times 50 = 99.383$$

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<sup>6</sup>We could obtain the same results in the limit as the period length gets vanishingly small by assuming that the liquidation costs were constant instead of proportional.

<sup>7</sup>In particular, it is

$$\mathbf{P}(1) = \sum_{k=0}^{\infty} \left\{ \left( \frac{e^{-\alpha} (\alpha)^k}{k!} \right) \begin{bmatrix} p_{LL} & p_{LH} \\ p_{HL} & p_{HH} \end{bmatrix}^k \right\}.$$

For the numerical example, we approximate this as

$$\mathbf{P}(1) = \sum_{k=0}^{200} \left\{ \left( \frac{e^{-10} (10)^k}{k!} \right) \begin{bmatrix} 0.20 & 0.80 \\ 0.01 & 0.99 \end{bmatrix}^k \right\}.$$

since, starting in state  $H$  at time 0, there is a probability 0.98766 of being in state  $H$  and a probability 0.01234 of being in state  $L$  at time 1. Similarly, the fundamental value in state  $L$  at time 0 is

$$V_0^L = 0.98735 \times 100 + 0.01234 \times 50 = 99.367.$$

Note that the fundamental values are nearly identical. Even though the expected number of information events in the unit interval is only 10 (this follows from the assumption  $\alpha = 10$ ), the transition probabilities are close to their ergodic or invariant distribution, which means that the fundamental values are almost independent of the initial state. In spite of this, we shall find that the debt capacity of the asset, defined to be the maximum amount that can be borrowed using the asset as collateral, can be very different in the two states.

Whereas the fundamental value only depends on the state, debt capacity is determined by equilibrium in the repo market and has to be calculated for every one of the dates,  $t_0, \dots, t_{99}$ , at which repo contracts mature. To do this, we first have to calculate the transition probabilities over an interval of length  $\tau = 0.01$ , that is, the length of the period between rollover dates. We find that<sup>8</sup>

$$\mathbf{P}(0.01) = \begin{bmatrix} 0.92315 & 0.07685 \\ 0.00096 & 0.99904 \end{bmatrix}.$$

Notice that the initial state has a much bigger impact on the transition probability. For example, the probability of ending up in state  $H$  after an interval 0.01 has passed is almost 1 if you start in state  $H$  but is close to 0.077 if you start in state  $L$ . This is because the interval is so short that an information event is unlikely to occur before the next rollover date.

Consider now the debt capacities at the last rollover date  $t_{99} = 0.99$ . In what follows, we let  $D$  denote the face value of the debt issued and denote the optimal value of  $D$  at date  $t_n$  in state  $s$  by  $D_n^s$ . It is never optimal to choose  $D > 100$  because this leads to default in both states, with associated liquidation costs, but without any increase in the payoff. For values of  $D$  between 50 and 100 or less than 50, the expected value of the debt is increasing in  $D$  holding constant the probability of default. Then it is clear that the relevant face values of debt ( $D$ ) to consider are 50 and 100. For any other face value we could increase  $D$  without changing the probability of default.

If we set  $D = 50$ , the debt can be paid off at date 1 in both states and the expected value of the payoff is 50. So the market value of the debt with face value 50 is exactly 50.

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<sup>8</sup>We use the approximation

$$\mathbf{P}(0.01) = \sum_{k=0}^{200} \left\{ \left( \frac{e^{-0.1} (0.1)^k}{k!} \right) \begin{bmatrix} 0.20 & 0.80 \\ 0.01 & 0.99 \end{bmatrix}^k \right\}.$$

Now suppose we set  $D = 100$ , there will be default in state  $L$  but not in state  $H$  at time 1. The payoff in state  $H$  will be 100 but the payoff in state  $L$  will be  $(0.9) 50 = 45.0$ , because the recovery rate after default is 0.90. Then the market value of the debt at time  $t_{99}$  will depend on the state at time  $t_{99}$ , because the transition probabilities depend on the state. We can easily calculate the expected payoffs in each state:

$$\begin{aligned} \text{state H} & : 0.99904 \times 100 + 0.00096 \times 0.9 \times 50 = 99.947; \\ \text{state L} & : 0.07685 \times 100 + 0.92315 \times 0.9 \times 50 = 49.226. \end{aligned}$$

For example, if the state is  $H$  at date  $t_{99}$ , then with probability 0.99904 the state is  $H$  at date 1 and the debt pays off 100 and with probability 0.00096 the state is  $L$  at date 1, the asset must be liquidated and the creditors only realize 45.

Comparing the market values of the debt with the two different face values, we can see that the optimal face value will depend on the state. In state  $H$ , the expected value of the debt when  $D = 100$  is  $99.947 > 50$ , so it is optimal to set  $D_{99}^H = 100$ . In state  $L$ , on the other hand, the expected value of the debt with face value  $D = 100$  is only  $49.226 < 50$ , so it is optimal to set the face value  $D_{99}^L = 50$ . Thus, if we use the notation  $B_n^s$  to denote the debt capacity in state  $s$  at date  $t_n$ , we have shown that

$$\begin{aligned} B_{99}^H & = 99.947, \\ B_{99}^L & = 50.000. \end{aligned}$$

Next, consider the debt capacities at date  $t_{98} = 0.98$ . Now, the relevant face values to consider are 50 and 99.9470 (since these are the maximum amounts that can be repaid in each state at date  $t_{99}$  without incurring default and the associated liquidation costs).

If  $D = 50$ , the expected payoff is 50 too, since the debt capacity at date  $t_{99}$  is greater than or equal to 50 in both states and, hence, the debt can always be rolled over. In contrast, if  $D = 99.947$ , the debt cannot be rolled over in state  $L$  at date  $t_{99}$  and the liquidation cost is incurred. Thus, the expected value of the debt depends on the state at date  $t_{98}$ :

$$\begin{aligned} \text{state H} & : 0.99904 \times 99.9470 + 9.6057 \times 10^{-4} \times 0.9 \times 50 = 99.894, \\ \text{state L} & : 7.6846 \times 10^{-2} \times 99.9470 + 0.92315 \times 0.9 \times 50 = 49.222. \end{aligned}$$

Comparing the expected value corresponding to different face values of the debt, we see that the optimal face value is  $D_{98}^H = 99.947$  in state  $H$  and  $D_{98}^L = 50$  in state  $L$ , so that the debt capacities are

$$\begin{aligned} B_{98}^H & = 99.894, \\ B_{98}^L & = 50. \end{aligned}$$

In fact, we did not really need to do the calculation again to realize that  $B_{98}^L = 50$ . The only change from the calculation we did at  $t_{99}$  is that the payoff in state  $H$  has gone down, so the expected payoff from setting  $D = 99.947$  must have gone down too and, *a fortiori*, the optimal face value of the debt must be 50.

It is clear that we can repeat this argument indefinitely. At each date  $t_n$ , the debt capacity in the high state is lower than it was at  $t_{n+1}$  and the debt capacity in the low state is the same as it was at  $t_{n+1}$ . These facts tells us that if it is optimal to set  $D_{n+1}^L = 50$  at  $t_{n+1}$ , then *a fortiori* it will be optimal to set  $D_n^L = 50$  at date  $t_n$ . Thus, the debt capacity is equal to 50 at each date  $t_n$ , including the first date  $t_0 = 0$ .

What is the debt capacity in state  $H$  at  $t_0$ ? The probability of staying in the high state from date 0 to date 1 is  $(0.99904)^{100} = 0.90842$  and the probability of hitting the low state at some point is  $1 - 0.90842 = 0.09158$  so the debt capacity at time 0 is

$$\begin{aligned} B_0^H &= 0.90842 \times 100 + 0.09158 \times 0.9 \times 50 \\ &= 94.9603. \end{aligned}$$

So the fall in debt capacity occasioned by a switch from the high to the low state at time 0 is  $94.963 - 50 = 44.963$  compared to a change in the fundamental value of  $99.383 - 99.367 = 0.016$ . This fall is illustrated sharply in Figure 2 which shows that while fundamental values in states  $H$  and  $L$  will diverge sharply at maturity, they are essentially the same at date 0. Nevertheless, debt capacity in state  $L$  is simply the terminal value in state  $L$ . Thus, a switch to state  $L$  from state  $H$  produces a sudden drop in debt capacity of the asset.

— Figure 2 here —

It is also interesting to compare the roll over debt capacity with the debt capacity if the asset were financed using debt with maturity  $\tau = 1.0$ , so that there is no need to roll over the debt. To be consistent with the way we calculated the roll over debt capacity, we need to allow for liquidation costs at date 1. In each state, it is optimal to choose the face value of the debt to be  $D = 100$ , so that there is always default if the low state occurs at date 1. To calculate the expected value of the debt, we use the transition probabilities from the matrix  $\mathbf{P}(1)$ , just as we did with the calculation of fundamental values. The expected value of the debt in each state is

$$\begin{aligned} B_0^H &= 0.98766 \times 100 + 0.9 \times 0.01234 \times 50 = 99.321. \\ B_0^L &= 0.98735 \times 100 + 0.9 \times 0.01265 \times 50 = 99.304. \end{aligned}$$

As with the fundamental values, the debt capacities with long term debt are much higher than in the roll over case and are also much closer together.

The comparison debt capacities with short ( $\tau = 0.1$ ) and long ( $\tau = 1.0$ ) assumes the rate of interest is the same for short-term and long-term debt. Since the yield curve slopes upwards, this is unrealistic. However, even with a higher interest rate for debt with the longer maturity, the basic point remains valid.

In the rest of the paper, we explore the determinants of debt capacity in a richer model with many states and a broad range of parameters. We model the release of information in terms of the arrival of “news.” The two-state example only allows for no news or bad news in state  $H$  and for no news or good news in state  $L$ . The model presented in Section 4 extends the setup and the results to an arbitrary finite number of information states. At each date, one of three things can happen: either there is “good news” (the information state improves), there is “bad news” (the information state gets worse), or there is “no news.” If the period between roll-over dates is sufficiently short, it is most likely that “no news” will have been released by the time the debt has to be refinanced.

## 2.2 Discussion

The intuition for the market freeze result can be explained in terms of the tradeoff between the costs of default and the face value of the debt. Suppose we are in the low information state at date  $t_n$ . If the period length  $\tau$  is sufficiently short, it is very likely that the information state at the next rollover date  $t_{n+1}$  will be the low state. Choosing a face value of the debt greater than  $B_{n+1}^L$ , the maximum debt capacity in the same state at date  $t_{n+1}$ , will increase the payoff to the creditors if good news arrives at the next date (the state switches to  $H$ ), but it will also lead to default if there is no news (the state remains  $L$ ). Since there is a liquidation cost, issuing debt with face value greater than the debt capacity is always unattractive if the probability of good news is sufficiently small. Then, the best the borrower can do is to issue debt with a face value equal to the debt capacity assuming no new information. But this implies that the debt capacity in the low state is  $v^L$  at every date. In other words, no matter how high the fundamental value is in state  $L$ , the borrower is forced to act as if the asset is only worth  $v^L$  in order to avoid default.

In the remainder of this section, we consider the role of different features of the model in driving the limit result on market freezes.

**Credit risk** If  $v^H = v^L$ , the terminal value of the asset is equal to the fundamental with probability one, so we can set the face value of the debt equal to  $v^H = v^L$  without any risk of default. In this case, the debt capacity must be equal to the fundamental value regardless of any other assumptions. So one necessary assumption is the existence of credit risk, that is, a positive probability that the terminal value of the asset will be less than the initial fundamental value. However, this credit risk can be arbitrarily small, as we illustrated in

the numerical example where, at time 0, the probability that the asset's value is 50 is less than 0.01. We could obtain the same results for smaller values of credit risk at the cost of increasing the number of rollovers.

**Liquidation cost** Secondly, we need a liquidation cost in order to have a market freeze. If the recovery ratio is  $\lambda = 1$ , then regardless of the credit risk, the debt capacity will equal the fundamental value. To see this, simply put the face value of the debt equal to 100 at each date. The market value of the debt will equal the fundamental value of the asset, which must equal the debt capacity. So a necessary condition of the market freeze is  $\lambda < 1$ . The liquidation cost does not need to be large, however. In the numerical example, the loss ratio was 0.1 and it could be set equal to a smaller value at the cost of reducing the maturity of the debt.

In fact, there are two components of the liquidation cost. The first is the finance constraint, which assumes that the buyer of the asset is using short-term finance to purchase the asset. If the buyer of the assets were a wealthy investor who could buy and hold the assets until maturity, the fundamental value would be the relevant benchmark and the investor might well be willing to pay some fraction of the fundamental value, only demanding a discount to make sure that he does not mistakenly overpay for the assets. What we are assuming here is that the buyer of the assets is another financial institution that must also issue short term debt in order to finance the purchase. Hence, the buyer is constrained by the same forces that determined the debt capacity in the first place. The second component could be the result of (a) market power, (b) transaction costs, (c) margin requirement, (d) a bid-ask spread, or some other friction that is not related to the fundamental value of the asset. See Pedersen (2009) for a survey of similar frictions in illiquid markets.

**Short term debt** Among the key assumptions of our model, we take as given the short-term nature of debt and liquidation costs. That investment banks are (or used to be!) funded with rollover debt and that debt capacity can be higher with short-term debt under some circumstances for many underlying assets, are interesting facts in their own right. Indeed, there exist agency-based explanations in the literature (for example, Diamond, 1989, 1991, 2004, Calomiris and Kahn, 1991, and Diamond and Rajan, 2001a, 2001b) for the existence of short-term debt as optimal financing in such settings. In contrast to this literature, Brunnermeier and Oehmke (2009) consider a model where a financial institution is raising debt from multiple creditors and argue that there may be excessive short-term debt in equilibrium as short-term debt issuance dilutes long-term debt values and creates among various creditors a "maturity rat race." Other reasons for the use of short-term debt are the attraction of betting on interest rates if bankers have short-term horizons and choose to shift risk.

Our model presents another counter-example to the claim that short-term debt maximizes debt capacity: debt capacity through short-term borrowing may in fact be arbitrarily small, suggesting that institutions ought to arrange for this possibility by funding themselves also through sufficiently long-term financing. Providing a micro (for example, an agency-theoretic) foundation for debt maturity in a model where the information about the underlying assets’ fundamental value can change as in our model is a fruitful goal for future research, but one that is beyond the scope of this paper.

**Rollover frequency** We have highlighted the role of rollover risk and indeed our main result requires that the rate of refinancing be sufficiently high in order to obtain a market freeze. Figure 3 illustrates the role of rollover frequency on debt capacity in state  $L$  by varying the number of rollovers as  $N = 10, 50$  and  $100$ . Debt capacity with just 10 rollovers is over 90, but falls rapidly to just above 60 with 50 rollovers, and 100 rollovers are sufficient to obtain the limiting result that debt capacity is the terminal value of 50 in state  $L$ .

— Figure 3 here —

We focus on this limiting case of sufficiently many rollovers for two reasons. First, it seems realistic in light of the fact that most ABCP conduits had CP that was rolled over within a week (See Acharya and Schnabl, 2009) and banks were essentially relying on overnight repo transactions for secured borrowing (a market whose failure brought Bear Stearns down). Secondly, this assumption allows us to get a clean result that is easy to understand. But even if the period length is longer than our result requires, so that it is optimal to set the face value greater than the debt capacity, we can still get a market freeze, as we can show with numerical examples in Section 3.

**Information structure** There are two aspects of the information structure that are crucial. First, information is a Poisson process in which the probability of “news” is proportional to the period length. Secondly, because we are assuming the period is short (the rollover frequency is high), information arrives slowly relative to the rollover frequency.

It is important to note that we do not make any special assumptions about the transition probabilities  $\mathbf{P}$ . In particular, we can impose a substantial amount of symmetry if desired. For example, the information state can be a symmetric random walk with reflecting barriers. The only essential property is that the probability of “no news” converges to one as the period length converges to zero. The Poisson process is one way to satisfy this regularity condition, which seems quite natural in this context.

### 3 Debt capacity with two states

In this section we provide a proof for the market freeze result when there two states. We make the same assumptions as for the numerical example but the parameters are otherwise arbitrary. For the time being, we treat the maturity of the commercial paper  $\tau$  and the number of rollovers  $N$  as fixed. Later, we will be interested to see what happens when the maturity of commercial paper  $\tau$  becomes very small and the number of rollovers  $N$  becomes correspondingly large.

There are two states, a “low” state  $L$  and a “high” state  $H$ . Transitions between states occur at the dates  $t_n$  and are governed by a stationary transition probability matrix

$$\mathbf{P}(\tau) = \begin{bmatrix} 1 - q(\tau) & q(\tau) \\ p(\tau) & 1 - p(\tau) \end{bmatrix},$$

where  $p(\tau)$  is the probability of a transition from state  $H$  to state  $L$  and  $q(\tau)$  is the probability of a transition from state  $L$  to state  $H$  when the period length is  $\tau > 0$ . So,  $p(\tau)$  is the probability of “bad news” and  $1 - p(\tau)$  is the probability of “no news” in the high state. Similarly, in the low state, the probability of “good news” is  $q(\tau)$  and the probability of “no news” is  $1 - q(\tau)$ . The one requirement we impose on these probabilities is that the shorter the period length, the more likely it is that no news arrives before the next rollover date:

$$\lim_{\tau \rightarrow 0} p(\tau) = \lim_{\tau \rightarrow 0} q(\tau) = 0.$$

The terminal value of the asset is  $v^H$  if the terminal state is  $H$  and  $v^L$  if the terminal state is  $L$ , where  $0 < v^L < v^H$ .

The debt capacity of the assets can be determined by backward induction. Suppose that the economy is in the low state at date  $t_N$ , which is the last of the rollover dates. Let  $D$  be the face value of the debt issued by the bank. If  $D > v^H$ , the bank will default in both states at date  $t_{N+1}$  and the creditors will receive  $\lambda v^H$  in the high state and  $\lambda v^L$  in the low state.<sup>9</sup> Clearly, the market value of the debt at date  $t_N$  would be greater if the face value were  $D = v^H$ , so it cannot be optimal to choose  $D > v^H$ . Now suppose that the bank issues debt with face value  $D$ , where  $v^L < D < v^H$ . This will lead to default in the low state at date  $t_{N+1}$  and the creditors will receive  $D$  in the high state and  $\lambda v^L$  in the low state. Clearly, this is dominated by choosing a higher value of  $D$ . Thus, either  $D = v^H$  or  $D \leq v^L$ . An exactly similar argument shows that it cannot be optimal to choose  $D < v^L$ , so we are left with only two possibilities, either  $D = v^H$  or  $D = v^L$ . In the first case, the market value of the debt is  $(1 - q(\tau)) \lambda v^L + q(\tau) v^H$  and in the second case it is  $v^L$ . Clearly, for  $\tau$  sufficiently small,

$$(1 - q(\tau)) \lambda v^L + q(\tau) v^H < v^L. \tag{1}$$

---

<sup>9</sup>To simplify the argument, we are assuming that there is a liquidation cost at date  $t_{N+1}$  even though there is no need to sell the asset at that date. None of the results depend on this.



Let  $\tau^* > 0$  denote the critical value such that (1) holds if and only if  $\tau < \tau^*$ . For any  $\tau < \tau^*$  it is strictly optimal to put  $D = v^L$ . Then we have found the debt capacity in the low state at date  $t_N$ , which we denote by  $B_N^L = v^L$ .

Now consider the high state at date  $t_N$ . It is easy to see, as before, that the only candidates for the face value of the debt are  $D = v^H$  and  $D = v^L$ . If the bank issues debt with face value  $v^H$ , there will be default in the low state. The creditors will receive  $\lambda v^L$  in the low state and  $v^H$  in the high state and the market value of the debt at date  $t_N$  will be  $(1 - p(\tau))v^H + p(\tau)\lambda v^L$ . If the bank issues debt with face value  $v^L$ , there will be no default, the creditors will receive  $v^L$  in both states at date  $t_{N+1}$  and the market value of the debt at date  $t_N$  will be  $v^L$ . If the period length  $\tau$  is sufficiently short, we can see that

$$(1 - p(\tau))v^H + p(\tau)\lambda v^L > v^L. \quad (2)$$

Let  $\tau^{**} > 0$  denote the critical value such that (2) is satisfied if and only if  $\tau < \tau^{**}$ . So if  $\tau < \tau^{**}$ , it is strictly optimal to put  $D = v^H$ . Then we have found the debt capacity in the high state at date  $t_N$ , which we denote by  $B_N^H = (1 - p(\tau))v^H + p(\tau)\lambda v^L$ .

Now suppose that we have calculated the debt capacities  $B_n^H$  and  $B_n^L$  for  $n = k+1, \dots, N$ . We show that

**Proposition 1** For  $0 < \tau < \min\{\tau^*, \tau^{**}\}$  and

$$v^H - \lambda v^L > e^\lambda (1 - \lambda) v^L,$$

the debt capacities of the asset in states  $H$  and  $L$  are given respectively by the formulae

$$B_n^H = (1 - p(\tau))^{N-n} v^H + \left[1 - (1 - p(\tau))^{N-n}\right] \lambda v^L, \text{ for } n = k, \dots, N, \quad (3)$$

and

$$B_n^L = v^L, \text{ for } n = k, \dots, N. \quad (4)$$

**The low state** Consider what happens in the low state at date  $t_k$ . If the face value of the debt issued by the bank is  $D$  at date  $t_k$ , then the bank will default in the low state if  $v^L < D < B_{k+1}^H$  and the bank will default in both states if  $D > B_{k+1}^H$ . By our usual argument, the only candidates for the optimal face value are  $D = v^L$  and  $D = B_{k+1}^H$ . If the face value is  $D = v^L$ , the creditors will receive  $v^L$  in both states at date  $t_{k+1}$  and the market value of the debt at date  $t_k$  will be  $v^L$ . On the other hand, if the face value of the debt is  $D = B_{k+1}^H$ , the creditors receive  $B_{k+1}^H$  in the high state and  $\lambda v^L$  in the low state, so the market value of the debt at date  $t_k$  is

$$(1 - q(\tau))\lambda v^L + q(\tau)B_{k+1}^H \leq (1 - q(\tau))\lambda v^L + q(\tau)v^H,$$

since  $B_{k+1}^H \leq v^H$ . But  $\tau < \tau^*$  implies that  $(1 - q(\tau))\lambda v^L + q(\tau)v^H < v^L$ , so the debt capacity is  $B_k^L = v^L$ .

**The high state** Now consider the high state. Again, our two candidates for the face value of the debt are  $D = B_{k+1}^H$  and  $D = v_L$ , which yield market values at date  $t_k$  of  $(1 - p(\tau)) B_{k+1}^H + p(\tau) \lambda v^L$  and  $v_L$ , respectively. From our induction hypothesis,

$$\begin{aligned} & (1 - p(\tau)) B_{k+1}^H + p(\tau) \lambda v^L \\ &= (1 - p(\tau)) \left\{ (1 - p(\tau))^{N-k-1} v^H + \left[ 1 - (1 - p(\tau))^{N-k-1} \right] \lambda v^L \right\} + p(\tau) \lambda v^L \\ &= (1 - p(\tau))^{N-k} (v^H - \lambda v^L) + \lambda v^L. \end{aligned}$$

Then  $D_k^H = B_{k+1}^H$  is strictly optimal if

$$(1 - p(\tau))^{N-k} (v^H - \lambda v^L) + \lambda v^L > v^L$$

or

$$v^H - \lambda v^L > \frac{(1 - \lambda) v^L}{(1 - p(\tau))^{N-k}}. \quad (5)$$

In order for this inequality to be satisfied for all  $n$  it must be satisfied for  $n = 0$ . We can show that  $(1 - p(\tau))^N \geq e^{-\lambda}$ , because  $e^{-\lambda}$  is the probability of no information events in the unit interval. Hence, a sufficient condition for (5) to be satisfied is

$$v^H - \lambda v^L > e^\lambda (1 - \lambda) v^L.$$

By induction, we have proved that the debt capacities are given by the formulae in (3) and (4) for all  $n = 0, \dots, N$ . ■

**Debt capacity with intermediate rollover risk** The qualitative properties of the debt capacities characterized in Proposition 1 are the same as in the numerical example in Section 2.1. In the low state, the debt capacity  $B_n^L$  is constant and equal to the lowest possible terminal value,  $v^L$ . The fundamental value of the asset in the low state  $V_n^L$  is greater than the debt capacity at every date  $t_n$  except at the terminal date, when they are both equal to  $v^L$ . In the high state, the debt capacity  $B_n^H$  is always less than the fundamental value  $V_n^H$ , except at the terminal date when both are equal to  $v^H$ . We call this behavior of the debt capacity a “market freeze” since a switch in the information state from high state to the low state can produce a sudden, sharp drop in debt capacity that is much larger than the drop in fundamental value associated with the switch.

We can get similar results even if the period length is not short enough to generate the result stated in Proposition 1. A simple adaptation of the numerical example will illustrate a scenario in which it is optimal to choose a high face value of debt in the low state, with the result that the bank faces a positive probability of default if the economy remains in the low state. Suppose that the value of the asset in the low state is  $v^L = 40$ . All the other

parameters remain the same. Now the loss from default in the low state is less than the gain from a high face value in the high state, so it is optimal for the face value of the debt to be set equal to next period's debt capacity in the high state.

As before, we calculate the debt capacity, beginning with the last rollover date. The last rollover date is  $t_{99}$ . The transition probabilities are given by

$$\mathbf{P}(0.01) = \begin{bmatrix} 0.92315 & 0.07685 \\ 0.00096 & 0.99904 \end{bmatrix}$$

as before. If the face value of the debt is set equal to  $v_H = 100$  in the low state, the market value of the debt issued will be

$$0.07685 \times 100 + 0.92315 \times 0.90 \times 40 = 40.918$$

which is higher than the face value obtained by setting the face value equal to 40. Thus, the optimal face value implies default if the economy remains in the low state. It is still optimal to set the face value of the debt equal to  $D_{99}^H = 100$  in the high state, but the debt capacity is now

$$\begin{aligned} B_{99}^H &= 0.99904 \times 100 + 0.00096 \times 0.90 \times 40 \\ &= 99.939. \end{aligned}$$

To continue the backward induction, we have to solve for the debt capacities in the two states simultaneously. As long as the face value of the debt is set equal to  $B_{n+1}^H$  in both states, the debt capacity satisfies

$$\begin{bmatrix} B_n^H \\ B_n^L \end{bmatrix} = \begin{bmatrix} 0.99904 & 0.90 \times 0.00096 \\ 0.07685 & 0.90 \times 0.92315 \end{bmatrix} \begin{bmatrix} B_{n+1}^H \\ B_{n+1}^L \end{bmatrix}$$

so

$$\begin{aligned} \begin{bmatrix} B_0^H \\ B_0^L \end{bmatrix} &= \begin{bmatrix} 0.99904 & 0.90 \times 0.00096 \\ 0.07685 & 0.90 \times 0.92315 \end{bmatrix}^{99} \begin{bmatrix} 99.939 \\ 40.918 \end{bmatrix} \\ &= \begin{bmatrix} 94.469 \\ 43.058 \end{bmatrix}. \end{aligned}$$

Unfortunately, this procedure does not give us the correct answer. We have assumed that it is optimal to have default in the low state at every rollover date, but this is not necessarily true. Starting at the last rollover date, it can be shown that the debt capacity in state  $L$  rises as we go back in time, reaches a maximum at  $t_{80}$ , and then declines as we move to earlier and earlier dates. The problem is that as the debt capacity rises, the liquidation costs (which are proportional to the debt capacity) also rise and eventually outweigh the upside

potential of a switch to the high state. At the point where the maximum is reached, it is optimal to change the face value of the debt from  $B_{n+1}^H$  to  $B_{n+1}^L$  and avoid default in the low state. Then the debt capacity is given by the formula above for  $n = 80, \dots, 99$  and is given by  $B_n^L = B_{80}^L$  for  $n = 0, \dots, 80$ . We can calculate the value of  $B_{80}^L$  using the formula

$$\begin{aligned} \begin{bmatrix} B_{80}^H \\ B_{80}^L \end{bmatrix} &= \begin{bmatrix} 0.99904 & 0.90 \times 0.00096 \\ 0.07685 & 0.90 \times 0.92315 \end{bmatrix}^{19} \begin{bmatrix} 99.939 \\ 40.918 \end{bmatrix} \\ &= \begin{bmatrix} 98.847 \\ 44.918 \end{bmatrix}. \end{aligned}$$

So the debt capacity at date  $t_0$  is 44.918 rather than 43.058. The gap between the debt capacities in the two states is  $94.469 - 44.918 = 49.551$ , compared to the negligible difference in the fundamental values in the two states. Thus, even if it is optimal to capture the upside potential of a switch to the high state, the debt capacity in the low state does not rise much above the minimum value of the asset, i.e., it is 44.918 rather than 40.

## 4 Debt capacity in the general case

We allow for a finite number of information states or signals, denoted by  $\mathcal{S} = \{s_1, \dots, s_I\}$ . The current information state is public information. Changes in the information state arrive randomly. The timing of the information events is a homogeneous Poisson process with parameter  $\alpha > 0$ . The probability of  $k$  information events, in an interval of length  $\tau$ , is

$$\Pr[K(t + \tau) - K(t) = k] = \frac{e^{-\alpha\tau} (\alpha\tau)^k}{k!},$$

where  $K(t)$  is the number of information events between 0 and  $t$ , and the expected number of information events in an interval of length  $\tau$  is

$$\begin{aligned} \mathbf{E}[K(t + \tau) - K(t)] &= \sum_{k=1}^{\infty} \left( \frac{e^{-\alpha\tau} (\alpha\tau)^k}{k!} k \right) \\ &= \alpha\tau. \end{aligned}$$

Conditional on an information event occurring at date  $t$ , the probability of a transition from state  $s_i$  to state  $s_j$  is denoted by  $p_{ij} \geq 0$ , where  $\sum_{j=1}^I p_{ij} = 1$ . These transition probabilities are described by the  $I \times I$  matrix

$$\mathbf{P} = \begin{bmatrix} p_{11} & \cdots & p_{1I} \\ \vdots & \ddots & \vdots \\ p_{I1} & \cdots & p_{II} \end{bmatrix}.$$

The transition probabilities over an interval of length  $\tau$  depend on the number of information events  $k$ , a random variable, and the transition matrix  $\mathbf{P}$ . The transition matrix for an interval  $\tau$  is denoted by  $\mathbf{P}(\tau)$  and defined by

$$\mathbf{P}(\tau) = \sum_{k=0}^{\infty} \frac{e^{-\alpha\tau} (\alpha\tau)^k}{k!} \mathbf{P}^k.$$

The information state is a stochastic process  $\{S(t)\}$  but for our purposes all that matters is the value of this process at the rollover dates. We let  $S_n$  denote the value of the information state  $S(t_n)$  at the rollover date  $t_n$  and we say that there is “no news” at date  $t_{n+1}$  if  $S_{n+1} = S_n$ . In other words, we regard the current state as the status quo and say that news arrives only if a new information state is observed. Of course, “no news” is also informative and beliefs about the terminal value of the assets will be updated even if the information state remains the same. Again, it is important to note is that, when the period length  $\tau$  is short, the probability of “news” becomes small and the probability of “no news” becomes correspondingly large. In fact,

$$\lim_{\tau \rightarrow 0} \mathbf{P}(\tau) = \mathbf{P}(0) = \mathbf{I}.$$

In that case, the informational content of “no news” is also small.

The terminal value of the assets is a function of the information state at date  $t = 1$ . We denote by  $v_i$  the value of the assets if the terminal state is  $S_{N+1} = s_i$  and assume that the values  $\{v_1, \dots, v_I\}$  satisfy

$$0 < v_1 < \dots < v_I.$$

Let  $V_n^i$  denote the fundamental value of the asset at date  $t_n$  in state  $i$ . Then clearly the values  $\{V_n^i\}$  are defined by putting  $V_{N+1}^i = v_i$  for  $i = 1, \dots, I$  and

$$V_n^i = \sum_{j=1}^I p_{ij} (1 - t_n) v_j, \text{ for } n = 0, \dots, N \text{ and } i = 1, \dots, I,$$

where  $p_{ij} (1 - t_n)$  is, of course, the  $(i, j)$  entry of  $\mathbf{P}(1 - t_n)$  denoting the probability of a transition from state  $i$  at date  $t_n$  to state  $j$  at date  $t_{N+1} = 1$ .

Figure 4 illustrates the fundamental values in a setup with  $I = 11$  states where terminal values  $v_1$  through  $v_{11}$  are equally spaced from 0 to 100, and  $\alpha = 10$ . The transition matrix  $\mathbf{P}$  is described in Appendix B. As in our two-state example, the fundamental values in different states are virtually identical at date 0 though they diverge in steps of 10 at maturity.

— Figure 4 here —

Let  $B_n^i$  denote the equilibrium debt capacity of the assets in state  $s_i$  at date  $t_n$ . By convention, we set  $B_{N+1}^i = v_i$  for all  $i$ .

**Proposition 2** *The equilibrium values of  $\{B_n^i\}$  must satisfy*

$$B_n^i = \max_{k=1, \dots, I} \left\{ \sum_{j=1}^{k-1} p_{ij}(\tau) \lambda B_{n+1}^j + \sum_{j=k}^I p_{ij}(\tau) B_{n+1}^k \right\}$$

for  $i = 1, \dots, I$  and  $n = 0, \dots, N$ .

The result is immediate once we apply the now familiar backward induction argument to show that it is always optimal to set the face value of the debt  $D_n^i$  equal to  $B_{n+1}^j$  for some  $j$ . Although the result amounts to little more than the definition of debt capacity, it is very useful because it allows us to calculate the debt capacities by backward induction.

We use the formula in Proposition 2 to obtain the limiting value of the debt capacity as  $\tau \rightarrow 0$ . An auxiliary assumption is helpful in proving this result: higher information states are assumed to be “better” in the sense that

$$V_{in} < V_{i+1,n}, \text{ for all } i = 1, \dots, I-1 \text{ and } n = 0, \dots, N+1. \quad (6)$$

A sufficient (but, as we show in Appendix A, not necessary) condition for (6) is that  $\{p_{i+1,j}(\tau)\}$  strictly dominates  $\{p_{i,j}(\tau)\}$  in the sense of first-order stochastic dominance. That is, for all  $i = 1, \dots, I-1$ ,

$$\sum_{j=1}^k p_{ij}(\tau) > \sum_{j=1}^k p_{i+1,j}(\tau), \text{ for all } i, k = 1, \dots, I-1. \quad (7)$$

**Proposition 3** *Suppose that (7) is satisfied. Then there exists  $\tau^* > 0$  such that for all  $0 < \tau < \tau^*$ , for any  $n = 0, \dots, N$  and any  $i = 1, \dots, I$ ,*

$$B_n^i = \sum_{j=1}^{i-1} p_{ij}(\tau) \lambda B_{n+1}^j + \sum_{j=i}^I p_{ij}(\tau) B_{n+1}^i.$$

**Proof.** See Appendix A. ■

Several properties follow immediately from Proposition 3 whenever  $0 < \tau < \tau^*$ . We provide these results formally in the form of four corollaries. First, in the lowest state,  $s_1$ , the debt capacity is constant and equal to  $v_1$ , the worst possible terminal value.

**Corollary 4**  $B_n^1 = v_1$  for all  $n$ .

**Proof.** From the formula in Proposition 3,

$$B_n^1 = \sum_{j=1}^I p_{1j}(\tau) B_{n+1}^1 = B_{n+1}^1$$

for  $n = 0, \dots, N$  so claim follows from our convention that  $B_{N+1}^1 = v_1$ . ■

Second, the debt capacity  $B_n^i$  is monotonically non-decreasing in  $n$ , that is, debt capacity increases as the asset matures, holding the state constant. This follows directly from the fact that, if the face of the debt equals  $B_{n+1}^i$ , the debt capacity  $B_n^i$  cannot be greater than  $B_{n+1}^i$ .

**Corollary 5**  $B_n^i \leq B_{n+1}^i$  for any  $i = 1, \dots, I$  and  $n = 0, \dots, N$ .

**Proof.** The inequality follows directly from the formula in Proposition 3:

$$\begin{aligned} B_n^i &= \sum_{j=1}^{i-1} p_{ij}(\tau) \lambda B_{n+1}^j + \sum_{j=i}^I p_{ij}(\tau) B_{n+1}^i \\ &\leq \sum_{j=1}^{i-1} p_{ij}(\tau) B_{n+1}^i + \sum_{j=i}^I p_{ij}(\tau) B_{n+1}^i \\ &= \sum_{j=1}^I p_{ij}(\tau) B_{n+1}^i = B_{n+1}^i, \end{aligned}$$

since  $\lambda B_{n+1}^j \leq B_{n+1}^i$  for  $j = 1, \dots, i-1$ . ■

Third, since  $B_{N+1}^i = v_i$  by convention, the preceding result immediately implies that the debt capacity  $B_n^i$  is less than or equal to  $v_i$ .

**Corollary 6**  $B_n^i \leq v_i$  for all  $i = 1, \dots, I$  and  $n = 0, \dots, N$ .

Finally, we can show that the debt capacity in state  $s_i$  at any date  $t_n$  is less than the fundamental value  $V_n^i$ .

**Corollary 7**  $B_n^i \leq V_n^i$  for any  $n = 0, \dots, N$  and  $i = 1, \dots, I$ .

**Proof.** The inequality follows directly from the formula in Proposition 3 for  $n = N+1$  and any  $i$ , so suppose that it holds for  $n, \dots, N$  and any  $i = 1, \dots, I$ . Then the formula in Proposition 3 implies that

$$\begin{aligned} B_n^i &= \sum_{j=1}^{i-1} p_{ij}(\tau) \lambda B_{n+1}^j + \sum_{j=i}^I p_{ij}(\tau) B_{n+1}^i \\ &\leq \sum_{j=1}^{i-1} p_{ij}(\tau) \lambda V_{n+1}^j + \sum_{j=i}^I p_{ij}(\tau) V_{n+1}^i \\ &\leq \sum_{j=1}^I p_{ij}(\tau) V_{n+1}^j = V_n^i, \end{aligned}$$

for any  $i = 1, \dots, I$ , so by induction the claim holds for any  $n = 0, \dots, N$  and any  $i = 1, \dots, I$ .

■

Some of these properties are illustrated in Figures 5a and 5b which show the debt capacities in the 11 states of our numerical example for  $N = 10$  and  $N = 10,000$  rollovers, respectively. For 10 rollovers,  $\tau$  is not sufficiently small and the debt capacity is large even in the worst state. By contrast, with 10,000 rollovers, the debt capacity in the worst case is (essentially) zero; furthermore, as we go from good states to worse states, debt capacity falls roughly by a magnitude of 10 even though the fundamental values (Figure 4) are almost identical in these states.

— Figure 5a and 5b here —

## 5 Application: Repo haircuts

Our results can alternatively be stated in terms of the so-called “haircut” of an asset when it is pledged for secured borrowing or used in a repo transaction. Our measure of the haircut is  $1 - \frac{B}{V}$ , that is, one minus the ratio of the debt capacity to the fundamental value. As we discussed in the introduction, a haircut is usually defined with reference to the market value. Here we assume that banks are using the fundamental or economic value as the appropriate “mark.” While the haircut so defined is affected by credit risk and liquidation costs, the primary determinant is the maturity of the debt, that is, the “maturity mismatch” in the funding structure of the asset. When rollover frequency of debt is very high, large haircuts are compatible with very small liquidation costs and credit risks.

Shin (2008) uses data from Bloomberg to document that typical haircuts on treasuries, corporate bonds, AAA asset-backed securities, AAA residential mortgage-backed securities and AAA jumbo prime mortgages are, respectively, less than 0.5%, 5%, 3%, 2% and 5%; whereas, in March 2008, these haircuts rose to between 0.25% and 3%, 10%, 15%, 20% and 30%, respectively. Brunnermeier and Pedersen (2008) also discuss the widening of haircuts in stress times. Gorton and Metrick (2009) show that during 2007-08, the repo haircuts on a variety of assets rose on average from zero in early 2007 to nearly 50 percent in late 2008. Interestingly, while some of the collateralized debt obligations (CDO) have had no secured borrowing capacity at all during the crisis of 2007-09, equities – which are in principle riskier assets – had a haircut of only around 20% (see Box 1.5 from Chapter 1, Page 42 of IMF (2008)).

Our model can mimic these stylized facts for appropriate parameter values. We have seen how small changes in the fundamental value can lead to much larger changes in the debt capacity. A series of negative announcements, such as were made from early 2007 to late 2008, will raise haircuts dramatically if the roll over risk is assumed to be sufficiently high.



Second, the model shows that repo haircuts should depend on a number of factors, including the maturity of the debt. It appears that a large number of MBS and other ABS were funded before and during this crisis by short-term rollover debt. Equities by contrast were held by relatively long-term investors. This difference in funding structure would be sufficient to generate lower haircuts for equities, even though equities are riskier assets overall than MBS.

— Figures 6 and 7 here —

Figure 6 illustrates how the debt capacity falls dramatically, for high rollover frequencies, as the state moves from good (left part of the x-axis) to bad (right part of the x-axis). The data are taken from the 11-state numerical example introduced earlier. By comparison, Figure 4 shows that the same change in states from good to bad entails only a small decrease in the fundamental value. Figure 7 shows the behavior of the haircut,  $1 - B/V$ , for different values of  $N$ , once again as we vary the state from good to bad. The figure shows strikingly how the haircuts rise as state worsens, but more importantly, that they start approaching 100% in the worst states, when the rollover frequency is sufficiently high.

The implications for repo haircuts represent a novel contribution of our model that may apply to a number of other institutional settings. One candidate is the commercial paper market, accessed primarily by financial institutions (such as Northern Rock, Bear Stearns and Lehman Brothers), but also by highly-rated industrial corporations, where rollover at short maturities is a standard feature. These markets experienced severe stress during the sub-prime crisis and froze (large haircut) for many days at a stretch once the expectations about the quality of mortgage assets became pessimistic, even though, prior to this period, they appeared to be the cheapest form of financing available (near-zero haircut). Another candidate, as we have discussed, is the practice of taking assets off-balance-sheet, putting enough capital and liquidity/credit enhancements to make them “bankruptcy-remote” and AAA-rated, and then borrowing short-term against these assets. Such structures, characterized by a maturity mismatch between assets and liabilities, were prevalent in many forms (“structured investment vehicles” or SIVs, “conduits,” and others) in the period leading up to the crisis (Crouhy, Jarrow and Turnbull, 2007).

Overall, the crisis of 2007-09 illustrates our key assumptions: Assets were funded with short-term rollover debt; once the fundamentals deteriorated and crisis broke out, further short-term financing was difficult to obtain for most institutions; and there have been substantial discounts in the sale of assets in SIVs and conduits and stress or freeze in the overnight repo markets.<sup>10</sup>

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<sup>10</sup>See, for instance, “SIV restructuring: A ray of light for shadow banking,” *Financial Times*, June 18 2008; and “Creditors find little comfort in auction of SIV Portfolio assets,” *Financial Times*, July 18 2008, which both report that net asset values due to asset fire sales have fallen below 50% of paid-in capital. As

## 6 Policy implications

It is tempting to ask the question: Can a regulator do something to unfreeze the market? Note that our model is partial equilibrium in nature and takes as given several features (short-term debt, liquidation costs) that should be endogenous. Any policy conclusions must be tentative and limited to partial equilibrium settings where these assumptions apply. attention to any other related efficiency issues or unintended consequences.

We discuss two possible policy interventions, one that supports liquidation values and another that increases the maturity of the debt issued.

### 6.1 Improving the liquidation value of assets

In our model, debt capacity shrinks with maturity of the debt, assuming that potential buyers are also using very short term debt. One way to raise the debt capacity and lower the haircut would be to insist that longer term debt be used to finance asset purchases. If the maturity of the debt used by potential buyers increases, then the risk of a firesale haircut would fall and so would the loss from fire sales. Why do banks not make these adjustments themselves? It may be that longer term borrowing is more expensive or that banks are short sighted and do not see the liquidity risk. In any case, during a systemic crisis, the willingness to lend at a longer term may disappear because everyone becomes focused on the short term.

A regulator such as the Central Bank or the Treasury can, in principle, directly improve the liquidation value of the affected assets either by lending against the asset as collateral based on its full buy-and-hold value in the event of a market freeze. This argument could provide a rationale for the wide variety of lending facilities created by the Federal Reserve during the crisis of 2007-09 to lend to a large number of borrowers, against a wide variety of collateral, at minimal (if any) haircut.<sup>11</sup> Notice that, according to our model, it is not

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the first article reports: “[W]hen defaults on US subprime mortgages rose last summer, ABCP investors stopped buying [short-term ABCP] notes – creating a funding crisis at SIVs. . . . This situation prompted deep concern about the risk of a looming firesale of assets. The prospect was deemed so alarming that the US Treasury attempted to organize a so-called “super-SIV” last autumn, which was supposed to purchase SIV assets.”

<sup>11</sup>For example, in addition to the traditional tools the Fed uses to implement monetary policy (e.g., Open Market Operations, Discount Window, and Securities Lending program), new programs have been implemented since August 2007: 1) Term Discount Window Program (announced 8/17/2007) - extended the length of discount window loans available to institutions eligible for primary credit from overnight to a maximum of 90 days; 2) Term Auction Facility (TAF) (announced 12/12/2007) - provides funds to primary credit eligible institutions through an auction for a term of 28 days; 3) Single-Tranche OMO (Open Market Operations) Program (announced 3/7/2008) - allows primary dealers to secure funds for a term of 28 days. These operations are intended to augment the single day repurchase agreements (repos) that are typically conducted; 4) Term Securities Lending Facility (TSLF) (announced 3/11/2008) - allows primary dealers to pledge a broader range of collateral than is accepted with the Securities Lending program, and also to borrow

necessary for the Central Bank to lend directly to the market, just that it guarantees to lend, based on the fundamental value of the assets, in the event of a market freeze. Guaranteeing some fraction of the value of loans made by private lenders would have the same effect.

## 6.2 Requiring higher “capital” in asset-backed finance

One implication of our model is that, while short-term rollover debt entails only a small financing cost in good times, its availability can dry up suddenly when fundamentals deteriorate. The excessive reliance on roll-over finance creates a severe maturity mismatch for borrowers and exposes them to low probability but high magnitude funding risk. It may be more prudent for such borrowers to account for such funding risk and complement rollover debt in their capital structure with forms of capital such as long-term debt and equity capital that face lower rollover risk.

Of course, long-term finance may be difficult to raise once a crisis erupts due to information reasons or if the crisis is systematic in nature. Hence, the reduced reliance on rollover debt must be a part of prudential capital structure choice in good times rather than being undertaken during bad times. While financial institutions should have privately recognized the risks of rollover finance, it cannot be ruled out that such risks were not yet fully understood. Going forward, a prudential regulator could play the role of a supervisory watchdog, looking out for excessive reliance on rollover finance, not just in the regulated sector through overnight repos, but also in the shadow banking sector of SIVs and conduits, and encouraging in such cases a greater reliance on long-term capital, through moral suasion or rule-based policies. Since long-term capital may be infeasible beyond a point for opaque, off-balance-sheet structures such as SIVs and conduits, such regulatory push will most likely reduce their incidence in the first place.

## 7 Related literature

At a general level, our result on market freezes can be considered a generalization of the Shleifer and Vishny (1992) result that when potential buyers of assets of a defaulted firm are

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for a longer term — 28 days versus overnight; and, 5) Primary Dealer Credit Facility (PDCF) (announced 3/16/2008) - is an overnight loan facility that provides funds directly to primary dealers in exchange for a range of eligible collateral; 6) Commercial Paper Funding Facility (CPFF) (announced 11/7/2008) - is designed to provide a liquidity backstop to U.S. issuers of commercial paper; 7) Money Market Investor Funding Facility (MMIFF) (announced 11/21/2008) - is aimed to support a private-sector initiative designed to provide liquidity to U.S. money market investors; 8) Term Asset-Backed Securities Loan Facility (TALF) (announced 11/25/2008) - is designed to help market participants meet the credit needs of households and small businesses by supporting the issuance of asset-backed securities (ABS) collateralized by auto loans, student loans, credit card loans etc.

themselves financially constrained, there is a reduction in the ex-ante debt capacity of the industry as a whole. We expand on their insight by considering short-term debt financing of long-term assets with rollovers to be met by new short-term financing or liquidations to other buyers also financed through short-term debt. Our “market freeze” result can be considered as a particularly perverse dynamic arising through the Shleifer and Vishny (1992) channel at each rollover date, that through backward induction, can in the worst case drive short-term debt capacity of an asset to its most pessimistic cash flow.

More specifically, our paper is related to the literature on haircuts, freezes and runs in financial markets. Rosenthal and Wang (1993) use a model where owners occasionally need to sell their assets for exogenous liquidity reasons through auctions with private information. Because of the auction format, sellers may not be able to extract the full value of the asset and this liquidation cost gets built into the market price of the asset, making the market price systematically lower than the fundamental value. In our model, the source of the haircut is not the private information of potential

He and Xiong (2009) consider a model of dynamic bank runs in which bank creditors have supplied debt maturing at differing maturities and each creditor faces the risk at the time of rolling over that fundamentals may deteriorate before remaining debt matures causing a fire sale of assets. In their model, volatility of fundamentals plays a key role in driving the runs even when the average value of fundamentals has not been affected. Our model of freeze or “run” of short-term debt is complementary to theirs, and somewhat different in the sense that *both* average value and uncertainty about fundamentals are held constant in our model, but it is the nature of revelation of uncertainty over time – whether good news arrives early or bad news arrives early – that determines whether there is rollover risk in short-term debt or not.

Huang and Ratnovski (2008) model the behavior of short-term wholesale financiers who prefer to rely on noisy public signals such as market prices and credit ratings, rather than producing costly information about the institutions they lend to. Hence, wholesale financiers run on other institutions based on imprecise public signals, triggering potentially inefficient runs. While their model is about runs in the wholesale market as is ours, their main focus is to challenge the peer-monitoring role of wholesale financiers, whereas our main focus is the role of rollover and liquidation risk in generating such runs.

An alternative modelling device to generate market freezes is to employ the notion of Knightian uncertainty (see Knight, 1921) and agents’ overcautious behavior towards such uncertainty. Gilboa and Schmeidler (1989) build a model where agents become extremely cautious and consider the worst-case among the possible outcomes, that is, agents are uncertainty averse and use maxmin strategies when faced with Knightian uncertainty. Dow and Werlang (1992) apply the framework of Gilboa and Schmeidler (1989) to the optimal portfolio choice problem and show that there is an interval of prices within which uncertainty-averse

agents neither buy nor sell the asset. Routledge and Zin (2004) and Easley and O’Hara (2005, 2008) use Knightian uncertainty and agents that use maxmin strategies to generate widening bid-ask spreads and freeze in financial markets. Caballero and Krishnamurthy (2007) also use the framework of Gilboa and Schmeidler (1989) to develop a model of flight to quality during financial crises: During periods of increased Knightian uncertainty, agents refrain from making risky investments and hoard liquidity, leading to flight to quality and freeze in markets for risky assets.

As opposed to these models, in our model agents maximize their expected utility and the main source of the market freeze is rollover and liquidation risk which become relevant when the rate of arrival of good news is slower than the rate at which debt is being rolled over. While both types of models can generate market freezes, we believe our model, by emphasizing the rollover and liquidation risk, better captures important features of the recent crisis where the market for rollover debt completely froze while equities — typically considered to be more risky — continued to trade with haircuts of only 20%.

## 8 Conclusion

In this paper, we have attempted to provide a simple information-theoretic model for freezes in the market for secured borrowing against finitely lived assets. The key ingredients of our model were rollover risk, liquidation risk and slow arrival of good news relative to the frequency of debt rollovers. In particular, our model could be interpreted as a micro-foundation for the funding risk arising in capital structures of financial institutions or special purpose vehicles that have extreme maturity mismatch between assets and liabilities. As an important side benefit, the model helps understand the behavior of repo haircuts in stress times.

In future work, it would be interesting to embed an agency-theoretic role for short-term debt, which we assumed as given, and see how the desirability of such rollover finance is affected when information problems can lead to complete freeze in its availability. While we took the pattern of release of information about the underlying asset as either ordained by nature or determined by investors’ expectations, it seems worthwhile to reflect on its deeper foundations, and thereby assess whether a strategic disclosure of information by agents in charge of the asset can alleviate (or aggravate) the problem of freezes to some extent.

## Appendix A: Proofs

We can solve for the equilibrium debt capacities by backward induction. Let  $D$  denote the face value of the debt issued in state  $s_i$  at date  $t_n$ . This debt will pay off  $D$  in state  $s_j$  at date  $t_{n+1}$  if  $D \leq B_{n+1}^j$  and  $\lambda B_{n+1}^j$  otherwise. In other words, the market value of the debt is

given by the formula

$$\sum_{B_{n+1}^j < D} p_{ij}(\tau) \lambda B_{n+1}^j + \sum_{B_{n+1}^j \geq D} p_{ij}(\tau) D$$

and the debt capacity is given by

$$B_n^i = \max_D \left\{ \sum_{B_{n+1}^j < D} p_{ij}(\tau) \lambda B_{n+1}^j + \sum_{B_{n+1}^j \geq D} p_{ij}(\tau) D \right\}.$$

**Proposition 8**  $B_n^i \leq B_n^{i+1}$ , for  $i = 1, \dots, I - 1$  and  $n = 0, \dots, N + 1$ .

**Proof.** The claim is clearly true by definition when  $n = N + 1$ , so suppose it is true for some arbitrary number  $n + 1$ . Then, for any  $D$  and  $i = 1, \dots, I - 1$ , it is clear that

$$\sum_{B_{n+1}^j < D} p_{ij}(\tau) \lambda B_{n+1}^j + \sum_{B_{n+1}^j \geq D} p_{ij}(\tau) D \leq \sum_{B_{n+1}^j < D} p_{i+1,j}(\tau) \lambda B_{n+1}^j + \sum_{B_{n+1}^j \geq D} p_{i+1,j}(\tau) D,$$

because  $\{p_{ij}(\tau)\}$  is first-order stochastically dominated by  $\{p_{i+1,j}(\tau)\}$ . It follows immediately that  $B_n^i \leq B_n^{i+1}$  for  $i = 1, \dots, I - 1$ . The claim in the proposition then follows by induction. ■

Let  $D_n^i$  denote the optimal face value of the debt in state  $i$  at date  $t_n$ . It is clear that the market value of the debt is maximized by setting the face value  $D = B_{n+1}^j$ , for some value of  $j = 1, \dots, I$ . Thus, we can write the equilibrium condition as

$$B_n^i = \max_{k=1, \dots, I} \left\{ \sum_{j=1}^{k-1} p_{ij}(\tau) \lambda B_{n+1}^j + \sum_{j=k}^I p_{ij}(\tau) B_{n+1}^k \right\},$$

for  $i = 1, \dots, I$  and  $n = 0, \dots, N$ .

Now suppose that it is optimal to set  $D_n^i = B_{n+1}^i$  for every  $i = 1, \dots, I$  and  $n = 0, \dots, N$ . The following proposition gives a lower bound to the difference between  $B_n^{i+1}$  and  $B_n^i$ .

**Proposition 9** Suppose that it is optimal to set  $D_n^i = B_{n+1}^i$  for every  $i = 1, \dots, I$  and  $n = 0, \dots, N$ . Then  $B_n^{i+1} \geq B_n^i + e^{-\alpha} (v_{i+1} - v_i)$ , for  $i = 1, \dots, I - 1$  and  $n = 0, \dots, N + 1$ .

**Proof.** Think of  $B_n^i$  as the average of two quantities, one being the debt capacity conditional on the occurrence of at least one information event after date  $n$ , denoted by  $\tilde{B}_n^i$ , and the other being the debt capacity conditional on no information events after  $n$ , denoted by  $\hat{B}_n^i$ . In the first case, the preceding argument suffices to show that  $\tilde{B}_n^{i+1} \geq \tilde{B}_n^i$ . In the second case, we have  $\hat{B}_n^{i+1} = \hat{B}_n^i + (v_{i+1} - v_i)$  since the state remains constant in each case. Since the probability of no information events after date  $t_n$  is  $e^{-\alpha(1-t_n)} \geq e^{-\alpha}$ , it follows that

$$B_n^{i+1} - B_n^i \geq e^{-\alpha} (v_{i+1} - v_i)$$

as claimed. ■

**Proposition 10** For all  $\tau > 0$  sufficiently small,  $D_n^i = B_{n+1}^i$  for all  $i = 1, \dots, I$  and  $n = 0, \dots, N$ .

**Proof.** For a fixed but arbitrary date  $t_n$  and state  $s_i$ , we compare the strategy of setting  $D = B_{n+1}^i$  with the strategy of setting  $D = B_{n+1}^k$ . First suppose  $k > i$  and consider the difference in the expected values of the debt:

$$\begin{aligned}
& \sum_{j=1}^{i-1} p_{ij}(\tau) \lambda B_{n+1}^j + \sum_{j=i}^I p_{ij}(\tau) B_{n+1}^i - \sum_{j=1}^{k-1} p_{ij}(\tau) \lambda B_{n+1}^j - \sum_{j=k}^I p_{ij}(\tau) B_{n+1}^k \\
&= \sum_{j=i}^{k-1} p_{ij}(\tau) (B_{n+1}^i - \lambda B_{n+1}^j) + \sum_{j=k}^I p_{ij}(\tau) (B_{n+1}^i - B_{n+1}^k) \\
&= p_{ii}(\tau) (B_{n+1}^i - \lambda B_{n+1}^i) + \sum_{j=i+1}^{k-1} p_{ij}(\tau) (B_{n+1}^i - \lambda B_{n+1}^j) + \sum_{j=k}^I p_{ij}(\tau) (B_{n+1}^i - B_{n+1}^k) \\
&\geq p_{ii}(\tau) (1 - \lambda) v_1 + \sum_{j=i+1}^{k-1} p_{ij}(\tau) (v_1 - v_I) + \sum_{j=k}^I p_{ij}(\tau) (v_1 - v_I) \\
&= p_{ii}(\tau) (1 - \lambda) v_1 + \sum_{j=i+1}^I p_{ij}(\tau) (v_1 - v_I).
\end{aligned}$$

since  $B_{n+1}^i \geq v_1$ ,  $B_{n+1}^i - \lambda B_{n+1}^j \geq B_{n+1}^i - B_{n+1}^j \geq (v_1 - v_I)$  for  $j = i + 1, \dots, I$  and  $B_{n+1}^i - B_{n+1}^k \geq v_1 - v_I$ . Then it is clear that, for  $\tau$  sufficiently small, the last expression above is positive.

Similarly, for  $k < i$ ,

$$\begin{aligned}
& \sum_{j=1}^{i-1} p_{ij}(\tau) \lambda B_{n+1}^j + \sum_{j=i}^I p_{ij}(\tau) B_{n+1}^i - \sum_{j=1}^{k-1} p_{ij}(\tau) \lambda B_{n+1}^j - \sum_{j=k}^I p_{ij}(\tau) B_{n+1}^k \\
&= \sum_{j=k}^{i-1} p_{ij}(\tau) (\lambda B_{n+1}^j - B_{n+1}^k) + \sum_{j=i}^I p_{ij}(\tau) (B_{n+1}^i - B_{n+1}^k) \\
&\geq \sum_{j=k}^{i-1} p_{ij}(\tau) (\lambda B_{n+1}^j - B_{n+1}^k) + p_{ii}(\tau) (B_{n+1}^i - B_{n+1}^k) \\
&\geq (1 - p_{ii}(\tau)) (\lambda v_1 - v_I) + p_{ii}(\tau) (B_{n+1}^i - B_{n+1}^k).
\end{aligned}$$

since  $\lambda B_{n+1}^j - B_{n+1}^k \geq \lambda v_1 - v_I$ .

At the last roll over date,  $n = N$  and  $B_{N+1}^i - B_{N+1}^k = (v_{i+1} - v_i) > 0$  by definition. Then the last line above is positive for  $\tau$  sufficiently small and this proves that it is optimal to set  $D_N^i = B_{N+1}^i = v_i$ .

Now suppose that we have shown that it is optimal to set  $D_n^i = B_{n+1}^i$  for  $\hat{n} + 1, \dots, N$  and consider the inequalities above for  $n = \hat{n}$ . Then Proposition 9 tells us that  $B_{\hat{n}+1}^i - B_{\hat{n}+1}^k \geq e^{-\alpha}(v_i - v_k) > 0$  and the last line above must be positive for  $\tau$  sufficiently small. This in turn establishes that  $D_{\hat{n}}^i = B_{\hat{n}+1}^i$  and, by induction, we have shown that it is optimal to set  $D_n^i = B_{n+1}^i$  for all this proves that it is optimal to set  $D_N^i = B_{N+1}^i = v_i$  for  $i = 1, \dots, I$  and  $n = 0, \dots, N$ . ■



## Appendix B: Numerical parameters for the example with $I = 11$ states

The terminal values for the 11 states are chosen as  $v_i = 10(i-1)$ , for  $i \in \{1,2,\dots,11\}$ . As with the two-state example, we choose the Poisson rate to be  $\alpha = 10$  and time to maturity as 1. The transition matrix given an information event  $\mathbf{P}$  is given as follows. Also shown below is the unconditional transition matrix  $\mathbf{P}(\tau = 0.01)$ , that is, the transition matrix taking account of the information events when there are 100 rollovers so that length of each rollover period is  $\tau = 0.01$ .

$$\mathbf{P} = \begin{bmatrix} 0.0068283 & 0.0075870 & 0.0085354 & 0.0097548 & 0.011381 & 0.013657 & 0.017071 & 0.022761 & 0.034142 & 0.068283 & 0.80000 \\ 0.0061797 & 0.0068663 & 0.0077246 & 0.0088281 & 0.010299 & 0.012359 & 0.015449 & 0.020599 & 0.030898 & 0.061797 & 0.81900 \\ 0.0055310 & 0.0061455 & 0.0069137 & 0.0079014 & 0.0092183 & 0.011062 & 0.013827 & 0.018437 & 0.027655 & 0.055310 & 0.83800 \\ 0.0048823 & 0.0054247 & 0.0061028 & 0.0069747 & 0.0081371 & 0.0097645 & 0.012206 & 0.016274 & 0.024411 & 0.048823 & 0.85700 \\ 0.0042336 & 0.0047040 & 0.0052920 & 0.0060480 & 0.0070560 & 0.0084671 & 0.010584 & 0.014112 & 0.021168 & 0.042336 & 0.87600 \\ 0.0035849 & 0.0039832 & 0.0044811 & 0.0051213 & 0.0059748 & 0.0071698 & 0.0089622 & 0.011950 & 0.017924 & 0.035849 & 0.89500 \\ 0.0029362 & 0.0032624 & 0.0036702 & 0.0041946 & 0.0048936 & 0.0058724 & 0.0073405 & 0.0097873 & 0.014681 & 0.029362 & 0.91400 \\ 0.0022875 & 0.0025417 & 0.0028594 & 0.0032678 & 0.0038125 & 0.0045750 & 0.0057187 & 0.0076250 & 0.011437 & 0.022875 & 0.93300 \\ 0.0016388 & 0.0018209 & 0.0020485 & 0.0023411 & 0.0027313 & 0.0032776 & 0.0040970 & 0.0054627 & 0.0081940 & 0.016388 & 0.95200 \\ 0.00099011 & 0.0011001 & 0.0012376 & 0.0014144 & 0.0016502 & 0.0019802 & 0.0024753 & 0.0033004 & 0.0049505 & 0.0099011 & 0.97100 \\ 0.00034142 & 0.00037935 & 0.00042677 & 0.00048774 & 0.00056903 & 0.00068283 & 0.00085354 & 0.0011381 & 0.0017071 & 0.0034142 & 0.99000 \end{bmatrix}$$

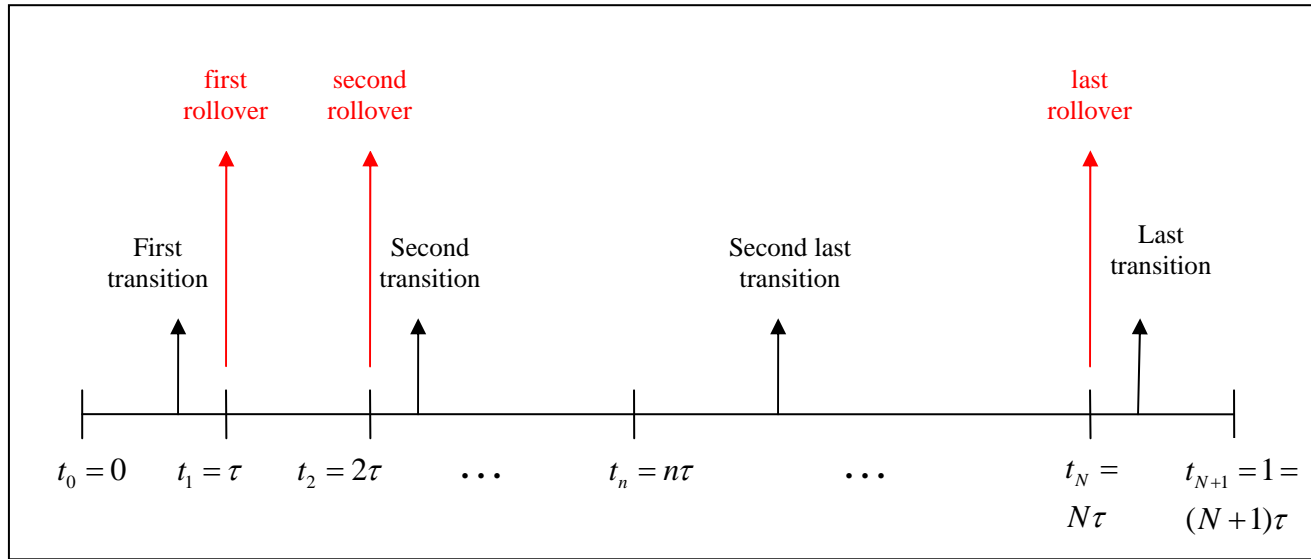
$$\mathbf{P}(\tau = 0.01) = \begin{bmatrix} 0.90546 & 0.00069051 & 0.00077683 & 0.00088780 & 0.0010358 & 0.0012429 & 0.0015537 & 0.0020715 & 0.0031073 & 0.0062146 & 0.076960 \\ 0.00056258 & 0.90546 & 0.00070322 & 0.00080368 & 0.00093763 & 0.0011252 & 0.0014064 & 0.0018753 & 0.0028129 & 0.0056258 & 0.078685 \\ 0.00050369 & 0.00055965 & 0.90547 & 0.00071956 & 0.00083948 & 0.0010074 & 0.0012592 & 0.0016790 & 0.0025184 & 0.0050369 & 0.080410 \\ 0.00044480 & 0.00049422 & 0.00055600 & 0.90547 & 0.00074134 & 0.00088960 & 0.0011120 & 0.0014827 & 0.0022240 & 0.0044480 & 0.082134 \\ 0.00038592 & 0.00042880 & 0.00048239 & 0.00055131 & 0.90548 & 0.00077183 & 0.00096479 & 0.0012864 & 0.0019296 & 0.0038592 & 0.083859 \\ 0.00032703 & 0.00036337 & 0.00040879 & 0.00046718 & 0.00054505 & 0.90549 & 0.00081757 & 0.0010901 & 0.0016351 & 0.0032703 & 0.085584 \\ 0.00026814 & 0.00029794 & 0.00033518 & 0.00038306 & 0.00044690 & 0.00053628 & 0.90551 & 0.00089381 & 0.0013407 & 0.0026814 & 0.087309 \\ 0.00020925 & 0.00023251 & 0.00026157 & 0.00029894 & 0.00034876 & 0.00041851 & 0.00052314 & 0.90553 & 0.0010463 & 0.0020925 & 0.089034 \\ 0.00015037 & 0.00016708 & 0.00018796 & 0.00021481 & 0.00025061 & 0.00030074 & 0.00037592 & 0.00050123 & 0.90559 & 0.0015037 & 0.090758 \\ 9.1481 \times 10^{-5} & 0.00010165 & 0.00011435 & 0.00013069 & 0.00015247 & 0.00018296 & 0.00022870 & 0.00030494 & 0.00045740 & 0.90575 & 0.092483 \\ 3.2594 \times 10^{-5} & 3.6216 \times 10^{-5} & 4.0742 \times 10^{-5} & 4.6563 \times 10^{-5} & 5.4323 \times 10^{-5} & 6.5188 \times 10^{-5} & 8.1485 \times 10^{-5} & 0.00010865 & 0.00016297 & 0.00032594 & 0.99905 \end{bmatrix}$$

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**Figure 1:** Timeline (illustrating  $N+1$  state transitions and  $N$  rollovers).

Figure 2: Fundamental value (V) and debt capacity (B) in high (H) and low (L) states as a function of time (t)

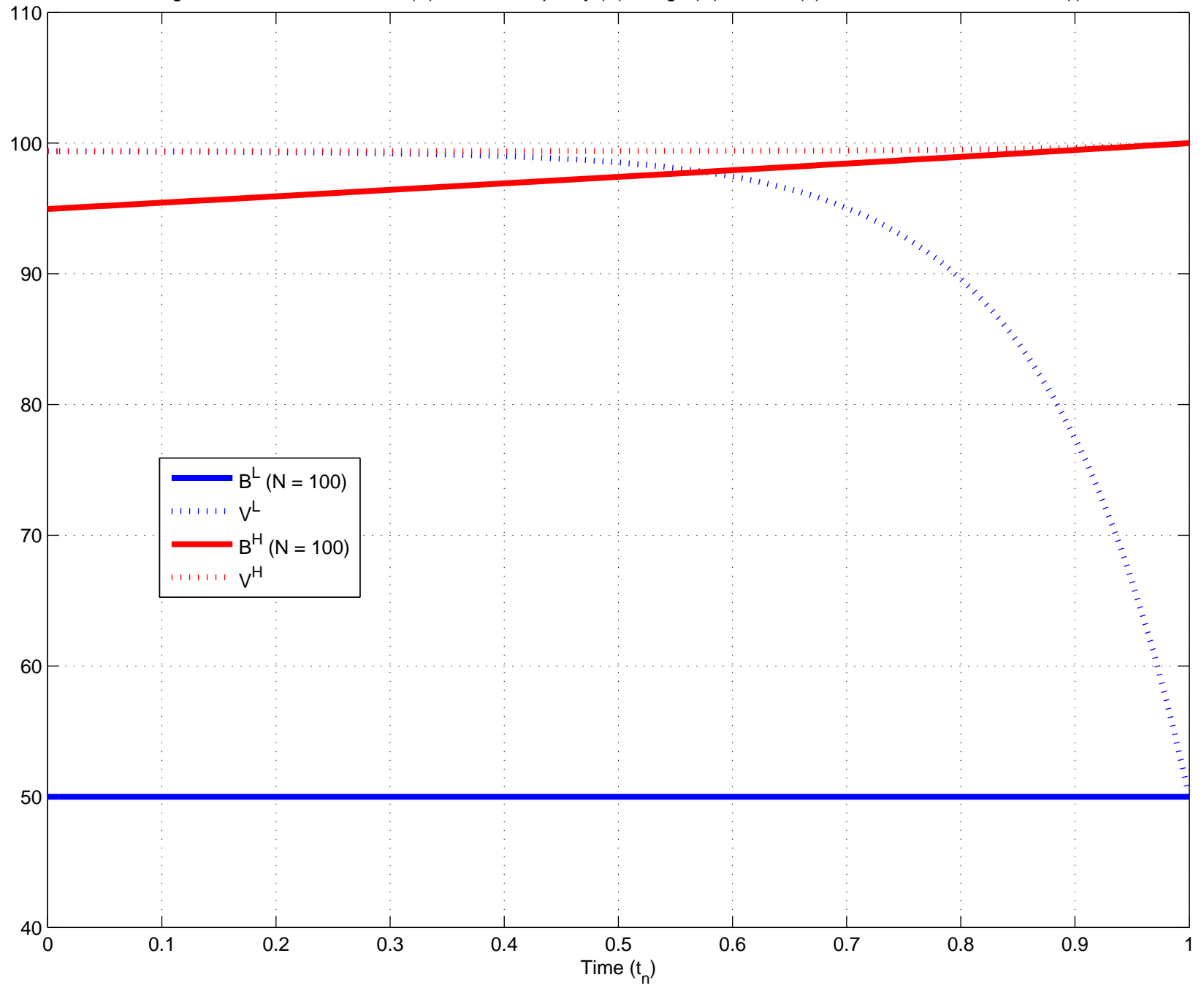


Figure 3: Fundamental value (V) and debt capacity (B) in low (L) state as a function of time (t) for rollover frequencies (N)

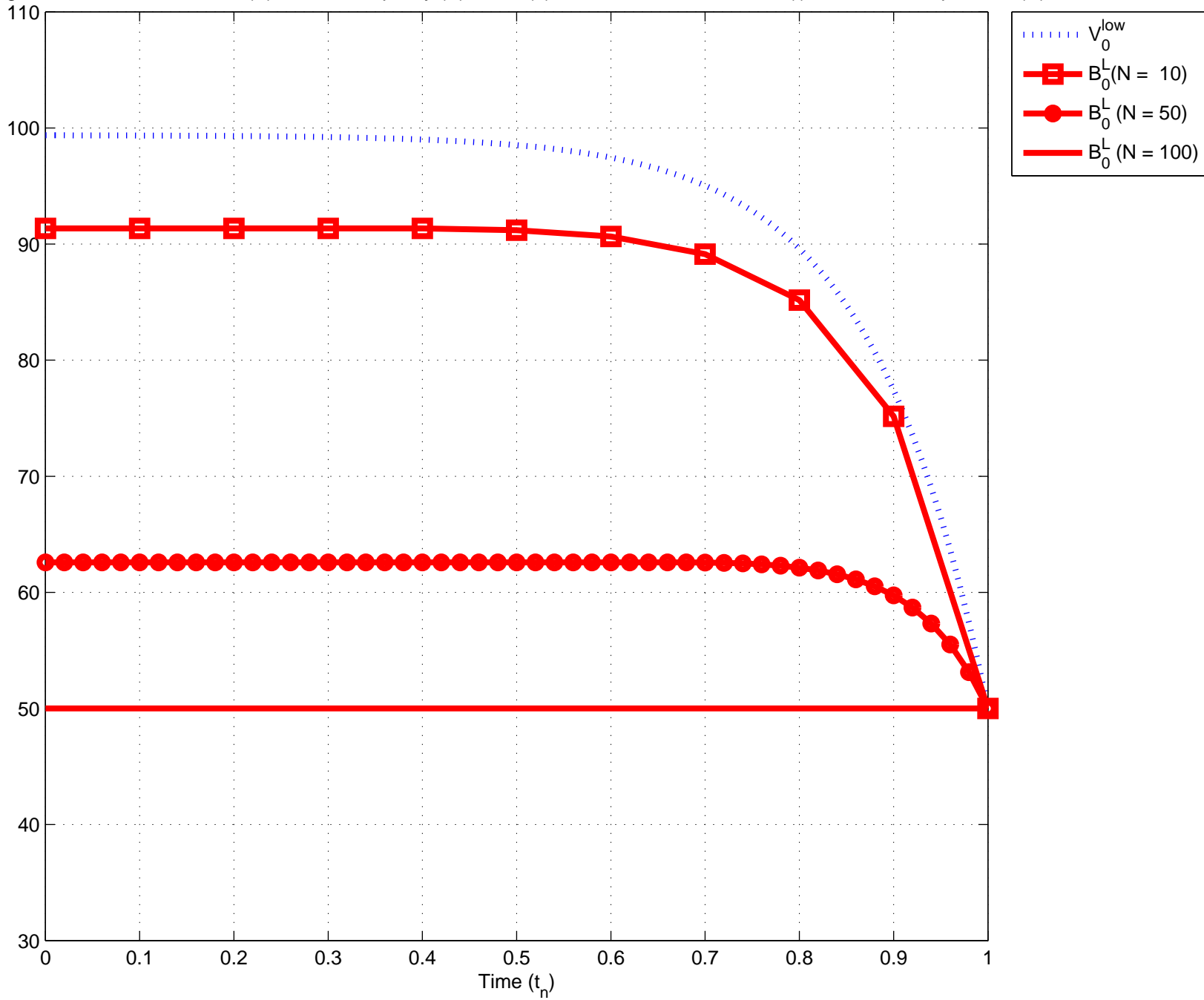


Figure 4: Fundamental value (V) as a function of time

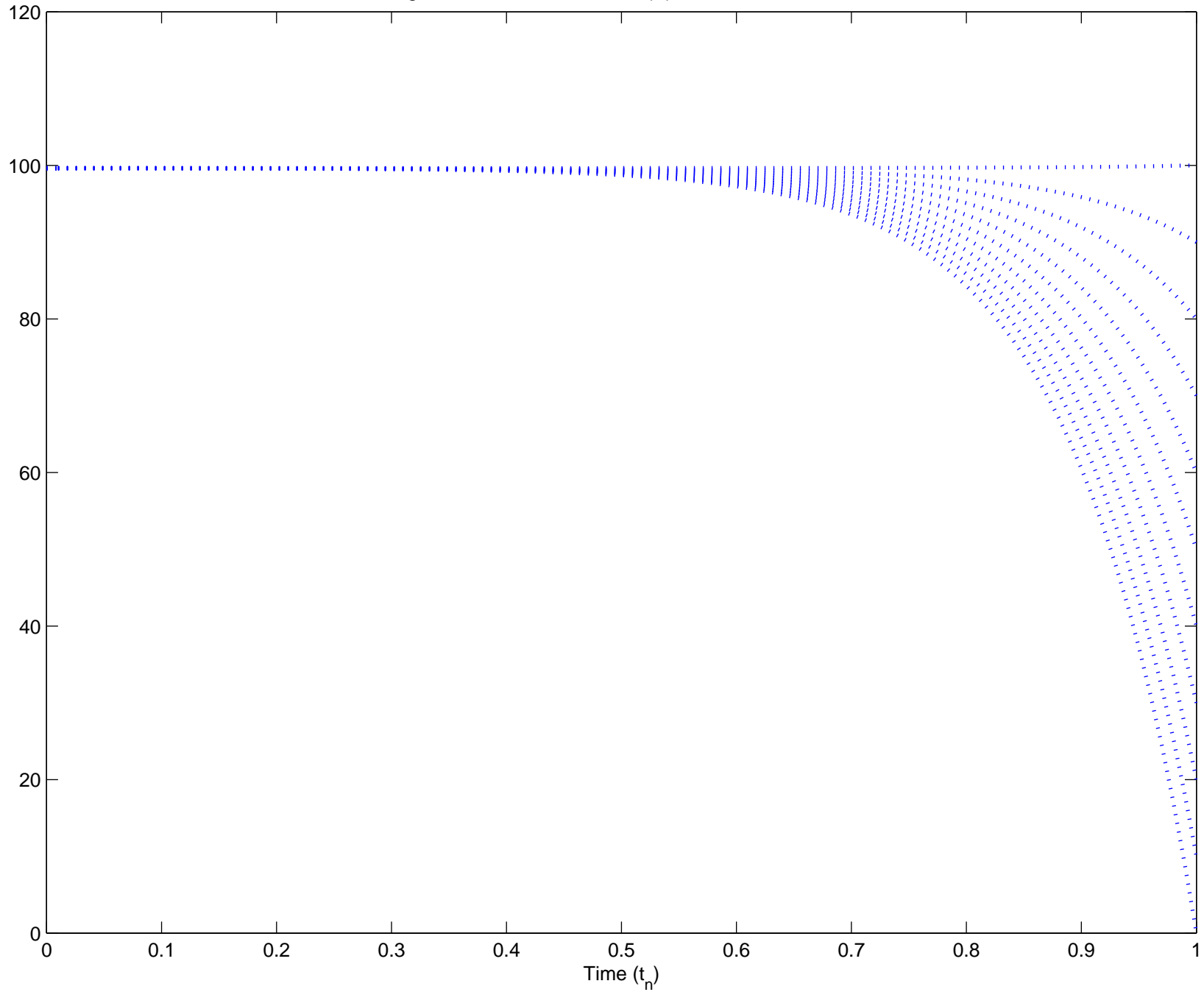




Figure 5a: Debt capacity (B) as a function of time (N = 10)

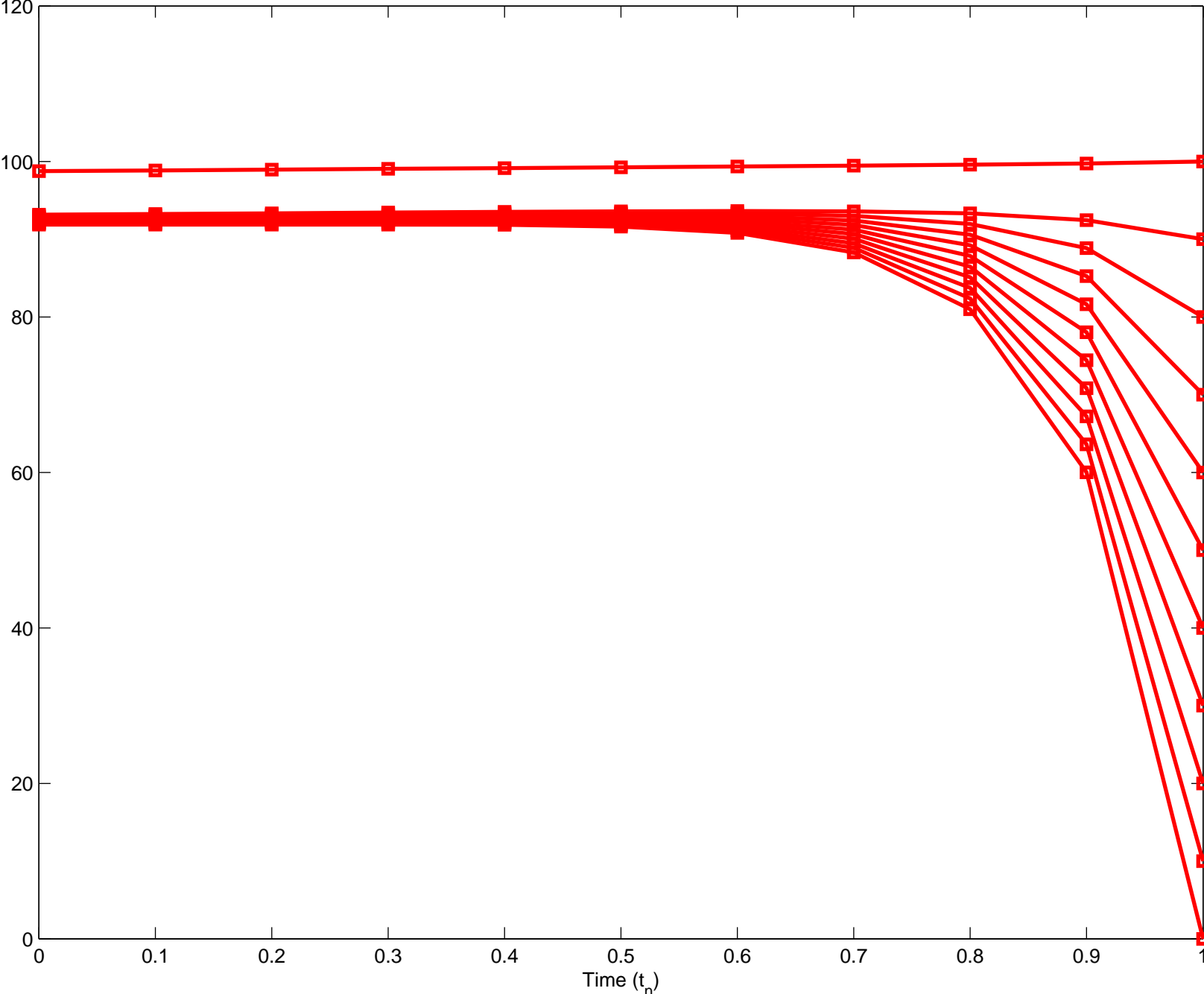


Figure 5b: Debt capacity (B) as a function of time (N = 10,000)

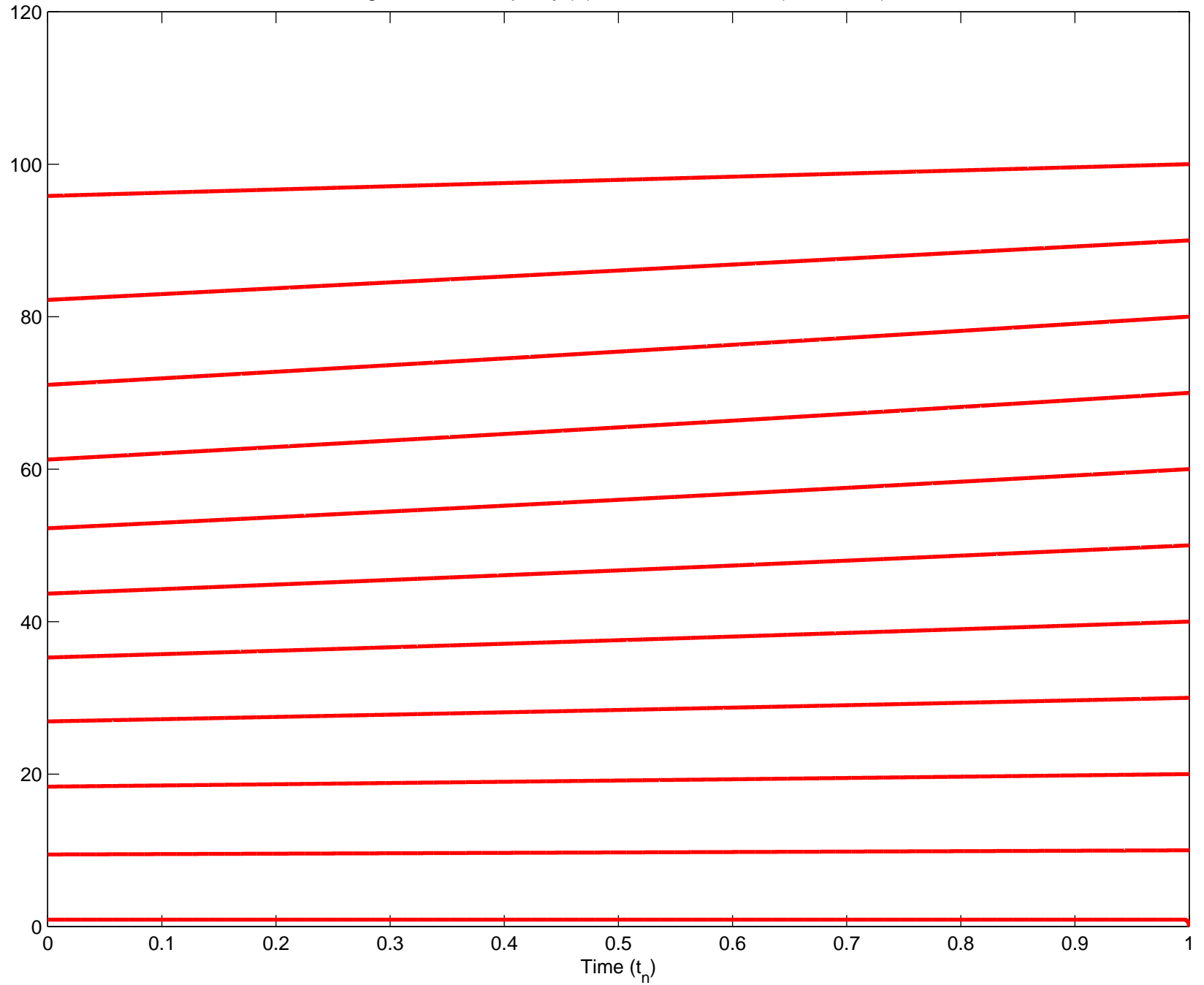


Figure 6: Fundamental value (V) and debt capacity(B) over states

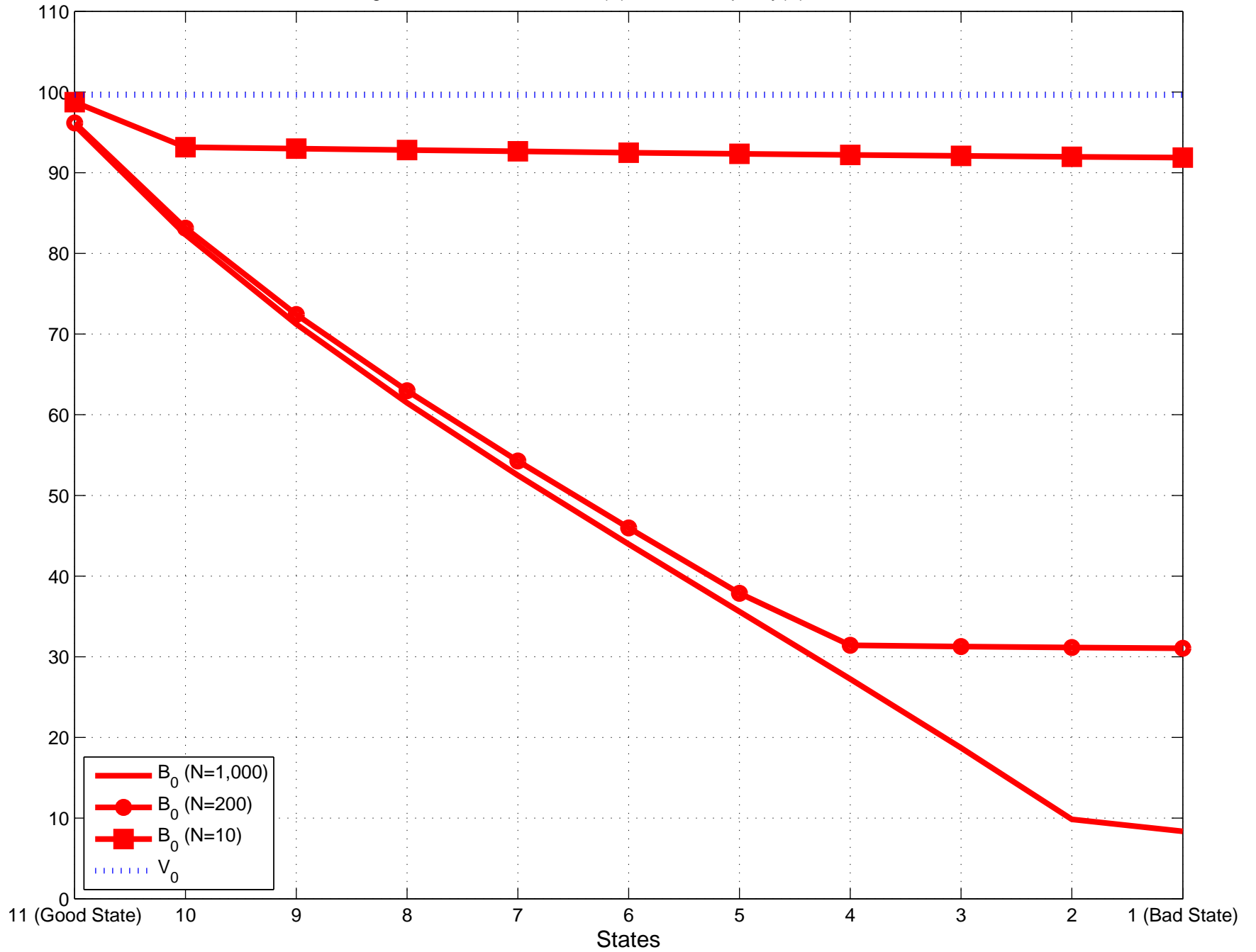


Figure 7: Repo haircuts over states

