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PROTECTION AND THE PRODUCT LINE: MONOPOLY AND PRODUCT QUALITY

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ABSTRACT

There are three points made in this paper. The first is that the question concerning choice of a product line by a monopolist is similar in structure to other adverse selection problems - and can be analyzed in an elementary way by adapting techniques recently developed for such problems. Such an analysis is developed in the first section.

The second is that when a foreign monopolist produces a product line, protection will change the composition of the entire product line. The nature of such effects is studied in the second section and this analysis is greatly simplified by the results of the first section. In line with empirical work on the subject, quotas are shown to raise the average quality of imports, while the effects of tariffs are ambiguous.

The third concerns the possibility of profit shifting protection which is welfare increasing. The welfare consequences of protection are analyzed in the third section, and are shown to depend crucially on the distribution of consumers.

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PROTECTION AND THE PRODUCT LINE: MONOPOLY AND PRODUCT QUALITY

Introduction

This paper deals with the effects of protection on the product line selected by a foreign monopolist, on his pricing policy for the product line, and therefore on national welfare. While trade restrictions tend to raise domestic prices, harm consumers, and thereby lower national welfare, they also transfer profits from the foreign monopolist to the government in the form of revenues collected. This raises national welfare. This profit shifting motive for trade restrictions has aroused considerable interest lately.[1] This paper explores the possibilities for such welfare increasing profit shifting, in the scenario where the foreign producer decides on a pricing policy for the entire product line.

Existing work in the trade literature on the effects of trade restrictions with endogenous quality focuses mainly on the nature of these effects in a competitive world. The specifications of the models are therefore particularly suited to the perfectly competitive paradigm. Unfortunately, they also tend to obscure some significant aspects of firm behavior in an imperfectly competitive world.

Falvey (1979) considers the effect of trade restrictions on the product line. He argues that specific tariffs and quotas raise the quality composition of imports, while ad-valorem tariffs do not. The essence of his argument is that prices are closely related to costs. Specific tariffs, or quotas implemented by the sale of licenses, raise all prices by the same amount. Thus, the relative price of higher quality goods falls. If relative demand depends inversely on relative price, the relative demand for higher quality products rises and this raises the quality composition of imports. Ad-valorem tariffs do not change relative prices and so have no effect on the composition of imports.

While the argument is relatively plausible in the context of a competitive market structure, it is less so when market power on the part of the producer exists. In such cases, the prices of goods of different qualities must be set together to allow the producer to discriminate between consumers. It is therefore essential to study the pricing of an entire product line, as is done in this paper.

The idea that a monopolist might want to produce a product line is captured by allowing the monopolist to differentiate his products. I assume that there is a continuum of products, indexed by their quality, which are "vertically" differentiated. By this I mean that while all consumers agree on the ranking of the products in terms of their quality, they choose to buy different qualities because of their differing preferences over income and the products.

The kind of protection I will consider is a uniform ad-valorem or specific tariff (or a quota implemented by the sale of licenses as discussed in Krishna (1984)) on the entire product line. This is a common feature of trade restrictions as they are usually imposed on broad categories of products since quality is not costlessly observable by the government, as well as for reasons of administrative convenience.

The monopoly pricing problem[2] was first solved by Mussa and Rosen (1978).[3] As the monopolist's problem is that of maximizing profits by choosing a price quality function, the problem is one of maximization in an infinite dimensional space. Even though they assume that all consumers either purchase one unit of some good or none, and a specific utility function, the analysis is nonetheless quite complicated. I present an elementary analysis of their problem for a slightly more general utility function. The technique is based on the work of Myerson (1981) and Baron and Myerson (1982). The simplicity of the technique allows me to consider the effect of parameter changes on the solution and therefore on welfare. In particular, I analyze the effects of a quota and tariff on the profit maximizing choice of the monopolist. In addition, the welfare effects of both a small quota and tariff are calculated. I show that a quota does not affect the quality chosen by each type of consumer, but removes consumers with a low valuation of quality who purchase low quality products, from the market, and raises the price of all qualities by the same amount. Thus, average quality rises. A tariff, on the other hand, lowers the quality chosen by each consumer, thus reducing average quality. In addition, it removes consumers with a low valuation of quality from the market, which tends to raise average quality. The effect on average quality is therefore ambiguous.

In addition, I show that the effect on welfare, of a slightly restrictive quota, or small specific tax on imports, depends on the distribution of consumers and the market served by the monopolist. When consumers, indexed by θ, are distributed as $f(\theta)$, and θ^* is the marginal consumer, who is indifferent between purchasing and not purchasing, then as long as $\frac{f(\theta^*)}{1-F(\theta^*)} > \frac{-f'(\theta^*)}{f(\theta^*)}$, prices rise by less than the specific tariff and welfare increases. The reason is that while the marginal consumer is removed from the market by the imposition of the special tariff, he derives no surplus and this does not affect welfare. However, the government collects more in revenue than consumers pay due to increased prices as the increase in price is less than the specific tariff. This shifts part of the foreign monopolist's profits into the hands of the government, which raises national welfare.

The welfare effects of an ad-valorem tariff are less easy to interpret because the quality chosen by each consumer is affected by the tariff. Although the direct effect on welfare of an increase in t is shown to be beneficial, the tariff always causes a lower quality to be purchased by individuals in the market and decreases the size of the market. These effects are shown to be harmful and the total effect of a tariff on welfare is ambiguous.

Section 1

The Problem

The model I will present here is basically that of Mussa and Rosen (1979). There is a continuum of consumers, indexed by θ . The distribution of θ is given by $f(\theta)$, which is assumed to be continuous and differentiable over $[\theta_0, \theta_1] \subset R^+$, the range of θ . All consumers either purchase one unit of the good or none. Quality is indexed by "q" , with higher values of q denoting higher qualities. There is a constant marginal cost of producing a unit of output of quality q given by $c(q)$, which is assumed to be increasing and strictly convex in q . The consumer of type θ derives utility $U(\theta,q)$ from consuming the good of quality q . I assume that $U(\theta,q) = \theta h(q) + g(q) + n$,[4] where "n" is the consumption of a competitively produced numeraire good. It is assumed that $h,g > 0$, $h',g' > 0$, $h'',g'' < 0$, so that the utility function is non-negative, increasing and concave. Consumers have an endowment, I , of the numeraire good and maximize expected utility, subject to their budget constraint.

The monopolist is aware of the distribution of θ , and of the preferences of all types of consumers. However, he is unable to directly identify consumers by their types. Thus, he cannot choose his allocation to consumers on the basis of their "type" unless the allocation offered to each type of consumer is actually his choice among all possible allocations offered. In other words, the problem may be posed as a maximization problem, subject to the usual self selection and individual rationality constraints.

The allocation, $A(\theta)$, assigned to a consumer of type θ is assumed to consist of a triple, $\langle p(\theta), q(\theta), \psi(\theta)\rangle$, where p is the price, q is the quality, and ϕ is the probability of getting the good of the given quality at the given price. It will become apparent that ϕ has only a notational role.

Let $S(\hat{\theta}/\theta)$ denote the surplus of the consumer of type θ with the allocation $A(\hat{\theta})$. By definition

$$s(\hat{\theta}/\theta) = [u(\theta, q(\hat{\theta})) - p(\hat{\theta})]\phi(\hat{\theta}) \quad .$$

The incentive compatibility condition then requires that for all $\theta \in [\theta_0, \theta_1]$, the following condition holds:

$$s(\theta) \equiv s(\theta/\theta) = \max_{\hat{\theta}} s(\hat{\theta}/\theta) \ . \tag{1.1}$$

In addition, individual rationality, requires that:

$$s(\theta) \geq 0 \quad \forall\, \theta \in [\theta_0, \theta_1] \tag{1.2}$$

since consumers cannot be forced to be in the market. Finally, as ϕ is a probability:

$$0 \leq \phi(\theta) \leq 1 \quad \forall\, \theta \in [\theta_0, \theta_1] \ . \tag{1.3}$$

An allocation, $A(\theta) = \langle p(\theta), q(\theta), \phi(\theta)\rangle$, is said to be "feasible" if it satisfies conditions (1.1), (1.2) and (1.3). The monopolist has to find the feasible policy that maximizes his profits. Such a policy is constructed in the next section.

Section 2

The Solution

The solution is derived by using a series of lemmas which allow the profit function of the monopolist to be written in a form where the solution to the problem is apparent by inspection. The solution technique is based on the work of Baron and Myerson (1982). The first lemma defines an equivalent, but more useful way of defining a feasible allocation.

Lemma 2.1: An allocation is feasible, if and only if, it satisfies the following conditions for all $\theta \in [\theta_0, \theta_1]$:

$$0 \leq \phi(\theta) \leq 1 \tag{2.1}$$

$$s(\theta) = s(\theta_0) + \int_{\theta_0}^{\theta} h(q(\tilde{\theta}))\phi(\tilde{\theta})d\tilde{\theta} \tag{2.2}$$

$$h(\theta)\phi(\theta) \geq h(\hat{\theta})\ \phi(\hat{\theta}) \qquad \forall\ \theta \geq \hat{\theta} \tag{2.3}$$

and

$$s(\theta_0) \geq 0\ . \tag{2.4}$$

Proof: First we show that conditions (1.1)-(1.3) imply conditions (2.1)-(2.4). Condition (1.1) requires that:

$$s(\theta/\theta) \geq s(\hat{\theta}/\theta) \qquad \forall\ \hat{\theta} \in [\theta_0,\ \theta_1]\ .$$

Subtracting $s(\hat{\theta}/\hat{\theta})$ from both sides and simplifying yields

$$s(\theta) - s(\hat{\theta}) \geq (\theta - \hat{\theta})\ h(q(\hat{\theta}))\phi(\hat{\theta})\ . \tag{2.5}$$

Performing the same operation with the roles of θ and $\hat{\theta}$ reversed shows that:

(2.6) $$s(\hat{\theta}) - s(\theta) \geq (\hat{\theta} - \theta)h(q(\theta))\phi(\theta)$$

Combining (2.5) and (2.6) gives:

(2.7) $$(\theta - \hat{\theta})h(q(\theta))\phi(\theta) \geq s(\theta) - s(\hat{\theta}) \geq (\theta - \hat{\theta})h(q(\hat{\theta}))\phi(\hat{\theta}) .$$

Equation (2.7) shows that for $\theta \geq \hat{\theta}$ condition (2.3) holds. Since $h(q(\theta))\phi(\theta)$ is a non-decreasing function of θ, it must be continuous almost everywhere in $[\theta_0, \theta_1]$. Thus, if we divide by $(\theta - \hat{\theta})$ and take the limit as $\hat{\theta} \to \theta$ we get:

$$\frac{ds(\theta)}{d\theta} = h(q(\theta))\phi(\theta) .$$

Integrating this equation gives condition (2.2). Condition (2.1) is just condition (1.3) and condition (2.4) is implied by condition (1.2).

Next we show that conditions (2.1)-(2.4) imply conditions (1.1)-(1.3).

As condition (1.2) is implied by conditions (2.2) and (2.4), it only remains to show that conditions (2.2) and (2.3) imply condition (1.1).

Using condition (2.2) gives,

$$s(\theta) - s(\hat{\theta}/\theta) = s(\theta_0) + \int_{\theta_0}^{\theta} h(q(\tilde{\theta}))\phi(\tilde{\theta})\, d\tilde{\theta} - s(\hat{\theta}/\theta) .$$

Adding and subtracting $s(\hat{\theta})$ from the right-hand side and simplifying shows that:

$$s(\theta) - s(\hat{\theta}/\theta) = \int_{\theta_0}^{\theta} h(q(\tilde{\theta}))\phi(\tilde{\theta})d\tilde{\theta} - \int_{\theta_0}^{\hat{\theta}} h(q(\tilde{\theta}))\phi(\tilde{\theta})d\tilde{\theta}$$

$$+ (\hat{\theta} - \theta)h(q(\hat{\theta}))\phi(\hat{\theta}) .$$

This expression is always non-negative for the following reason. If $\hat{\theta} > \theta$

$$s(\theta) - s(\hat{\theta}/\theta) = \int_{\hat{\theta}}^{\theta} h(q(\tilde{\theta}))\phi(\tilde{\theta})d\tilde{\theta} - (\theta - \hat{\theta})h(q(\hat{\theta}))\phi(\hat{\theta}) .$$

By condition (2.3) $h(q(\tilde{\theta}))\phi(\tilde{\theta})$ is non-decreasing, and this expression must be non-negative. If $\hat{\theta} < \theta$,

$$s(\theta) - s(\hat{\theta}/\theta) = - \int_{\theta}^{\hat{\theta}} h(q(\tilde{\theta}))\phi(\tilde{\theta})d\tilde{\theta} + (\hat{\theta} - \theta)h(q(\hat{\theta}))\phi(\hat{\theta}) .$$

Again, by condition (2.3) this expression is non-negative.

The next lemma derives an expression for profits when the foreign monopolist is faced with a specific and an ad-valorem tariff at the rates m and t , respectively. As a quota implemented by selling import licenses acts like a specific tariff, the effects of a specific tariff may be equated with those of a quota. (See Krishna (1984) for details).

Lemma 2.2: For any feasible outcome function, the profit function of the monopolist faced with an ad-valorem tariff at the rate t and a specific tariff at rate m is given by:

$$(2.8) \qquad \Pi(t,m) = \int_{\theta_0}^{\theta_1} \{[z(\theta)h(q(\theta)) + g(q(\theta))](1-t) - c(q(\theta))$$

$$- m\}\phi(\theta)f(\theta)d\theta - (1-t)s(\theta_0) \ .$$

where $z(\theta) = \theta - \frac{[1-F(\theta)]}{f(\theta)}$.

Proof:

Profits are defined by:

$$\Pi(t,m) = \int_{\theta_0}^{\theta_1} [p(\theta)(1-t) - c(q(\theta)) - m]\phi(\theta)f(\theta)d\theta \ .$$

Using condition (2.2) to substitute for $p(\theta)\phi(\theta)$ gives:

$$(2.9) \quad \Pi(t,m) = \int_{\theta_0}^{\theta_1} \{[h(q(\theta))\theta + g(q(\theta))](1-t) - c(q(\theta)) - m\}\phi(\theta)f(\theta)d\theta \ .$$

$$- s(\theta_0)(1-t) - (1-t) \int_{\theta_0}^{\theta_1} [\int_{\theta_0}^{\theta} h(q(\theta))\phi(\theta)d\theta]f(\theta)d\theta \ .$$

The last term may be simplified by integrating by parts.

$$(2.10) \qquad \int_{\theta_0}^{\theta_1} [\int_{\theta_0}^{\theta} h(q(\theta))\phi(\theta)d\theta]f(\theta)d\theta =$$

$$\{[\int_{\theta_0}^{\theta} h(q(\theta))\phi(\theta)d\theta]F(\theta)\} \Big|_{\theta_0}^{\theta_1} - \int_{\theta_0}^{\theta_1} h(q(\theta))\phi(\theta)F(\theta)d\theta$$

$$= \int_{\theta_0}^{\theta_1} h(q(\theta)\phi(\theta)d\theta - \int_{\theta_0}^{\theta_1} h(q(\theta))\phi(\theta)F(\theta)d\theta \ .$$

$$= \int_{\theta_0}^{\theta_1} (1 - F(\theta))h(q(\theta))\phi(\theta)d\theta$$

Substituting back into equation (2.9) and collecting terms proves Lemma 2.2.

The optimal outcome function for the monopolist when $z(\theta)$ is non-decreasing in θ is now apparent.

Theorem 2.1: Assume that $z(\theta)$ is non-decreasing in θ and that [5]

$$(2.11)\quad z(\theta)h''(q(\theta)) + g''(q(\theta))(1-t) - c''(q(\theta)) < 0 \qquad \forall\, \theta \in [\theta_0, \theta_1] \text{ and } \forall\, q(\cdot)\ .$$

Then the optimal feasible outcome function is given by $\langle q^*(\theta), p^*(\theta), \phi^*(\theta)\rangle$, which are defined as follows. $q^*(\theta)$ is the solution to:

$$(2.12)\quad [z(\theta)h'(q^*(\theta)) + g'(q^*(\theta))](1-t) - c'(q^*(\theta)) = 0\ .$$

$$v(\theta) = [z(\theta)h(q^*(\theta)) + g(q^*(\theta))](1-t) - c(q^*(\theta))\ .$$

θ^* is set at the highest value of θ which has $v(\theta) = 0$, and

$$\phi^*(\theta) = 1 \quad \text{if} \quad \theta > \theta^*$$

$$\phi^*(\theta) = 0 \quad \text{if} \quad \theta \leq \theta^*\ .$$

and

$$p^*(\theta) = u(\theta, q^*(\theta)) - \int_{\theta_0}^{\theta} h(q^*(\tilde\theta))\phi^*(\tilde\theta)d\tilde\theta\ .$$

Proof: Equation (2.8) is maximized by setting $s(\theta_0)$ at its lowest feasible value, zero. The integral in (2.8) is of a simple form and is maximized by setting $q^*(\theta)$ to maximize the terms in curly brackets for each θ, and giving the value 1 to $\phi^*(\theta)$ if the maximized value is positive, and zero otherwise. The assumption that $z'(\theta) > 0$ ensures that $v'(\theta)$ is > 0 as the envelope theorem shows that $v'(\theta) = h'(q^*(\theta))z'(\theta)$. The definition of $\phi^*(\theta)$ ensures that those terms making a positive contribution to the integral, are included with as much weight given to them as possible, and that $\phi^*(\theta)$ is non-decreasing in θ.

As

$$\text{(2.13)} \qquad \frac{dq^*(\theta)}{d\theta} = \frac{h'(q^*(\theta))z'(\theta)(1-t)}{-\{z(\theta)h''(q^*(\theta)) + g''(q^*(\theta))(1-t) - c''(q^*(\theta))\}} ,$$

$q^*(\theta)$ is non-decreasing in θ given the assumptions of the theorem. Thus, $\phi^*(\theta)h(q^*(\theta))$ is non-decreasing in θ, and condition (2.3) is met. As $p^*(\theta)$ is defined using condition (2.2), this condition is automatically satisfied. Thus, the outcome function constructed is both feasible and optimal.

It is instructive to compare the monopolist's choice when he can costlessly identify consumers by their types to the situation we have been studying, where he cannot do so. In the former case, he could charge each type of consumer the entire utility derived from any quality. Thus, as is usual with a perfectly discriminating monopolist, the quality assigned to each type of consumer would be optimal, but the monopolist would appropriate the entire surplus. In this case the monopolist would set $q^M(\theta)$ to maximize $u(\theta,q) - c(q)$, and would serve the consumer of type θ if $u(\theta,q^M(\theta)) - c(q^M(\theta)) > 0$. This is exactly the assignment a planner with perfect and costless information would make, by pricing at marginal cost.

In this case, $q^M(\theta)$ is determined by:

$$\theta h'(q^M(\theta)) + g'(q^M(\theta)) - c'(q^M(\theta)) = 0 . \tag{2.14}$$

Comparing (2.12) to this (with $t = m = 0$) shows that $z(\theta)$ plays the same role in (2.12) as θ does in (2.14). Thus, $\frac{1 - F(\theta)}{f(\theta)} = z(\theta) - \theta$ can be thought of as the implicit cost borne by the monopolist because of his inability to directly identify consumers by their types.

Now we return to the analysis of the profit maximizing outcome function for the monopolist. If $z(\theta)$ is not monotonic, the solution outlined in Theorem 1 does not meet condition (2.3), and is therefore not feasible. In order to derive the optimal policy in this case we need more machinery. We will construct a function $\bar{z}(\theta)$ from $z(\theta)$ which is monotonic and is, in a way, closest to $z(\theta)$, and show that it plays the same role as $z(\theta)$.

Let $\ell(\psi)$ be a function defined over the interval $[0,1]$. ℓ is constructed from z as follows:

$$\ell(\psi) = z(F^{-1}(\psi)) .$$

Thus, if at a given $\theta = \hat{\theta}$, as the value of z is $z(\hat{\theta})$, the same value will be assigned to the number $F(\hat{\theta})$ by the function ℓ . (This is illustrated in Diagram 1.) Define:

$$L(\psi) = \int_0^{\psi} \ell(\tilde{\psi})d\tilde{\psi} .$$

Let $\bar{L}(\psi)$ be the greatest convex function that lies below $L(\psi)$ on the interval $[\theta_0,\theta_1]$. Let $\bar{\ell}(\psi)$ be the slope of $\bar{L}(\psi)$. Now let $\bar{z}(\theta) =$

$\bar{\ell}(F(\theta))$. Notice that the construction of $\bar{z}(\theta)$ ensures that $\bar{z}(\theta)$ is made up of segments of $z(\theta)$ connected by flat portions, and that $\bar{z}(\theta)$ is non-decreasing in in θ, as portrayed in Diagram 1.

The following lemma (due to Myerson (1981)) is of use in deriving the optimal policy.

Lemma 2.3: Define $G(\theta) = L(F(\theta)) - \bar{L}(F(\theta))$. $G(\theta)$ is a continuous non-negative function. $\bar{z}(\theta)$ is a non-decreasing function and is locally constant if $G(\theta) > 0$. If $G(\theta) = 0$, then $z(\theta) = \bar{z}(\theta)$. In addition, if $z(\theta)$ is non-decreasing, $z(\theta) = \bar{z}(\theta)$. Also,

$$\int_{\theta_0}^{\theta_1} A(\theta)z(\theta)f(\theta)d\theta = \int_{\theta_0}^{\theta_1} A(\theta)\bar{z}(\theta)f(\theta)d\theta - \int_{\theta_0}^{\theta_1} G(\theta)dA(\theta)$$

for any monotone function $A(\theta)$.

Proof: Only the last statement need to be proved as the others are apparent by the construction of these functions.

$$\int_{\theta_0}^{\theta_1} A(\theta)\ [z(\theta) - \bar{z}(\theta)]f(\theta)d\theta = \int_{\theta_0}^{\theta_1} A(\theta)[\ell(F(\theta)) - \bar{\ell}(F(\theta))]f(\theta)d\theta$$

$$= \int_{\theta_0}^{\theta_1} A(\theta)d[L(F(\theta)) - \bar{L}(F(\theta))]$$

$$= \int_{\theta_0}^{\theta_1} A(\theta)dG(\theta) .$$

Integrating by parts yields:

$$\int_{\theta_0}^{\theta_1} A(\theta)[z(\theta) - \overline{z}(\theta)]f(\theta)d\theta = A(\theta)G(\theta)\Big|_{\theta_0}^{\theta_1} - \int_{\theta_0}^{\theta_1} G(\theta)dA(\theta)$$

$$= A(\theta_1)G(\theta_1) - A(\theta_0)G(\theta_0) - \int_{\theta_0}^{\theta_1} G(\theta)dA(\theta)$$

$$= - \int_{\theta_0}^{\theta_1} G(\theta)dA(\theta) \ .$$

The last equality follows because $L(\theta_0) = \overline{L}(\theta_0)$ and $L(\theta_1) = \overline{L}(\theta_1)$ as the convex hull of a continuous function always equals the function at the end points of the domain in R.

Now we can use Lemma 2.3 and condition (2.3) to rewrite equation (2.8), the profits for a feasible outcome function, as:

$$(2.15)\quad \Pi(t,m) = \int_{\theta_0}^{\theta_1} \{[\overline{z}(\theta)h(q(\theta)) + g(q(\theta))](1-t) - c(q(\theta)) - m\} f(\theta)\phi(\theta)d\theta - (1-t)\int_{\theta_0}^{\theta_1} G(\theta)d[h(q(\theta))\phi(\theta)] - (1-t)s(\theta_0) \ .$$

This is possible because feasibility implies $h(q(\theta))\phi(\theta)$ is a non-decreasing function so that $h(q(\theta))\phi(\theta)$ can be used in place of $A(\theta)$. The profit maximizing outcome function is now apparent.

<u>Theorem 2.2</u>: The optimal outcome function is derived as in Theorem 2.1 when $z(\theta)$ is replaced by $\overline{z}(\theta)$. Let this be denoted by $\langle p^*(\theta), q^*(\theta), \phi^*(\theta)\rangle$.

<u>Proof</u>: This choice of $q(\theta)$ maximizes the value of the first integral in 2.15 by arguments identical to those of Theorem 2.1. In addition, as $\overline{z}(\theta)$ is

non-decreasing, condition (2.3) is automatically met. The value of the second integral must be non-negative. If $q(\theta)$ is chosen to maximize the first integral, the value of the second integral is zero which is its lowest possible value. (This is ensured as $h[q^*(\theta)]\phi^*(\theta)$ is locally constant whenever $G(\theta)$ is positive and whenever $h[q^*(\theta)]\phi^*(\theta)$ is increasing, $G(\theta)$ is zero.) $s(\theta_0)$ is set at zero, which is its lowest value. This concludes the proof.

Some comparisons of the monopoly outcome and the outcome under competition are worth noting. These results are due to Mussa and Rosen (1978), though the proofs differ. They are provided for the sake of completeness.

Theorem 2.3: (Mussa and Rosen (1978)) A comparison of the monopolist's quality choice, to that under competition shows:

(1) The monopolist assigns a lower quality to each type of consumer served in both markets,

(2) The same quality is assigned in both markets to the consumer of type θ_1,

(3) The monopolist serves a smaller market than under competition.

(Proof in the appendix.)

Having characterized the profit maximizing outcome function, we are in a position to evaluate the effects of specific and ad-valorem tariffs on the quality choice of the monopolist. This is the topic of the next section.

Section 3

Specific and Ad-Valorem Tariffs

The imposition of an ad-valorem tariff, or a quota implemented by the sale of licenses, which acts like a specific tariff, changes the profit maximizing policy of the monopolist. In the previous section, we derived the monopolist's profit function over feasible outcome functions. This had built into it the parameters "t" and "m" which represented a uniform ad-valorem tariff and specific tariff on goods of all qualities.[6] From this we calculated the monopolist's optimal policy.

Consider the profit function, $\Pi(m,t)$ when the specific tariff "m" and ad-valorem tariff "t" is imposed.

$$(3.1)\qquad \Pi(m,t) = \int_{\theta_0}^{\theta_1}\Big[\{h(q(\theta))\bar{z}(\theta) + g(q(\theta))\}(1-t) - c(q(\theta)) - m\Big]f(\theta)\phi(\theta)d\theta$$

$$- s(\theta_0)(1-t) - (1-t)\int_{\theta_0}^{\theta_1} G(\theta)dh(q(\theta))\phi(\theta)\ .$$

Whatever be the level of m or t, the last term is identically equal to zero as $G(\theta)dh(q(\theta))\phi(\theta)$ is equal to zero for each θ in $[\theta_0,\theta_1]$ when the monopolist maximizes profits over the set of feasible policies. The effect of changing m or t on the profit maximizing policy may be calculated by deriving the effect, of changes in m and t, on the value of the optimal feasible $q(\theta)$, $q^*(\theta)$. The effects on price and on customers served, of changes in m and t can be derived from the effects on $q^*(\theta)$, and using the characterization of the optimal policy in terms of $q^*(\theta)$ as provided by Theorem 2.2.

In order to simplify the notation in what follows, I define θ^* as the lowest type of consumer served, and use it to drop the variable $\phi(\theta)$ from the expression previously derived. θ^* is defined by equation (3.2) below.

$$(3.2) \qquad \{[h(q^*(\theta^*))\overline{z}(\theta^*) + g(q^*(\theta^*))](1-t) - c(q^*(\theta^*)) - m\} = 0$$

where $q^*(\theta)$ is the optimal choice of q on the part of the monopolist, and where

$$(3.3) \qquad [h'(q^*(\theta))\overline{z}(\theta) + g'(q^*(\theta))](1-t) - c'(q^*(\theta)) = 0$$

defines $q^*(\theta)$.

An examination of equation (3.3) reveals that $q^*(\theta)$ does <u>not</u> depend on m, only on t. Recognizing this dependence, define $q^*(\theta,t)$ to capture it. Similarly, θ^* depends on both m and t, as shown by equation (3.2) and the properties of $\theta^*(m,t)$ may be derived by using this equation. In addition, $p^*(\theta)$ is defined by:

$$(3.4) \qquad p^*(\theta) = \theta h(q^*(\theta)) + g(q^*(\theta)) - \int_{\theta^*(m,t)}^{\theta} h(q^*(\tilde{\theta}))d\tilde{\theta} .$$

p^* is affected by both m and t. Because m only affects θ^*, and not q^*, the change in price due to a change in m is a constant, independent of θ. This is not the case when t changes, as both q^* and θ^* change in this case. The function $p^*(m,t)$ is implicitly defined by equation (3.4).

A technical problem needs to be taken care of before the effect of parameter changes on the solution can be calculated. If the maximized value of profits gained by serving type θ , when restricted to feasible policies, is zero, or,

$$[h(q^*(\theta))\overline{z}(\theta) + g(q^*(\theta))](1-t) - c(q^*(\theta)) - m = v(\theta) = 0$$

for an interval of θ's , say $(\underline{\theta},\overline{\theta})$, then the monopolist is indifferent between serving the consumers between $\underline{\theta}$ and $\overline{\theta}$. If some consumers in this region are served, he gets a larger market than if none are served, but because $p(\theta)$ is determined by equation (3.4), he has to charge a lower price. In this case a continuum of equilibria exist and it is not possible to estimate the effect of changing parameters, as this depends on which equilibrium is taken as the initial point. I will assume that $\phi(\theta) = 0$ unless $v(\theta)$ is positive. This means that in such cases, $\overline{\theta}$ is taken as the initial point. Thus, for increases in m , $\overline{z}'(\theta^*(m))$ is by definition not equal to zero. We are now in a position to determine the effects of an increase in m , or t .

Theorem 3.1: An increase in m has the following effects:

(1) It leaves the quality assigned to all types of consumers, who remain in the market, unaffected.

(2) It reduces the size of the market by raising θ^* .

(3) It increases price for each quality by a uniform amount.

Proof: The first point is apparent as equation (3.3) defines q^* for every value of θ . Equation (3.2) defines θ^* for each m and t . Implicitly differentiating it gives:

$$\frac{d\theta^*}{dm}(m,t) = \frac{1}{\bar{z}'(\theta^*(m,t))h(q^*(\theta^*(m,t)))} > 0 . \tag{3.5}$$

Note that the first two points imply that average quality must rise, as each $q^*(\theta)$ is unaffected by m, $q^*(\theta)$ is non-decreasing in θ, and as consumers with low θ's are eliminated from the market. Differentiating equation (3.4), which defines $p^*(\theta)$ for each value of m and t gives

$$\frac{dp^*}{dm}(\theta,m,t) = h(q^*(\theta^*(m,t)))\frac{d\theta^*}{dm}(m,t) = \frac{1}{\bar{z}'(\theta^*(m,t))} . \tag{3.6}$$

which is a positive constant independent of θ. It follows that surplus for each type of θ falls as well.

Theorem 3.2: An increase in t has the following effects:

(1) The quality assigned to each type of consumer remaining in the market falls.

(2) The size of the market falls.

(3) The surplus associated with each type of consumer remaining in the market falls.

Proof: The effect on $q^*(\theta)$ of an increase in t is obtained by differentiating equation (3.3). This shows that,

$$\frac{dq^*(\theta,t)}{dt} = \frac{h'(q^*(\theta,t))\bar{z}(\theta) + g'(q^*(\theta,t))}{[\{h''(q^*(\theta,t))\bar{z}(\theta) + g''(q^*(\theta,t))\}(1-t) - c''(q^*(\theta,t))]} < 0 , \tag{3.7}$$

as the numerator is equal to $\frac{c'}{1-t}$ by (3.3) and the denominator is negative by assumption. θ^* is determined by equation (3.2). Differentiating this gives:

$$(3.8) \qquad \frac{d\theta^*(m,t)}{dt} = \frac{h(q^*(\theta^*))\bar{z}(\theta^*) + g(q^*(\theta^*))}{\bar{z}'(\theta^*)(1-t)} > 0 .$$

Using equation (3.4) shows that,

$$(3.9) \qquad \frac{ds(\theta)}{dt} = -h(q^*(\theta^*)) \frac{d\theta^*}{dt} + \int_{\theta^*(m,t)}^{\theta} h'(q^*(\theta)) \frac{dq^*(\theta,t)}{dt} d\theta < 0$$

so surplus must fall. Notice also that the <u>fall</u> in surplus is increasing in θ .

Note that it is <u>not</u> possible to say that the price paid by each <u>type</u> rises, as the quality assigned to each type also falls. However, as surplus falls for each θ , the price assigned to each <u>quality must</u> rise. The effect on average quality of an increase in t is unclear. The increase in t lowers the quality purchased by each consumer remaining in the market. This lowers average quality. However, consumers with low values of θ , who buy low quality products, leave the market, and this raises average quality.

Even though consumer surplus falls as m or t rise, it could be in the interests of national welfare to levy such taxes if value of the gain in revenue was larger than the value of the loss in consumer surplus. The conditions under which this is likely are investigated next.

Section 4

Welfare Effects

In this section I will consider the effect of a small ad-valorem or specific tariff on welfare when the initial situation is that of free trade. Thus, the initial levels of t and m are set at zero.

In the previous section, the assumption made (which is maintained throughout) was that revenue raised by the government is either returned to consumers in a lump sum fashion or that the government acts like a separate consumer with the utility function given by revenue raised. The existence of a numeraire good ensures that lump sum transfers do not affect demand.

Making this assumption, the national welfare function is given by:

$$(4.1) \qquad W(m,t) = \int_{\theta^*(m,t)}^{\theta_1} \{[u(\theta,q(\theta)) - p(\theta)] + R(t,m,\theta)\} f(\theta)d\theta$$

where $R(t,m,\theta)$ is the revenue raised by the government, from the consumer of type θ. Total revenue is given by summing $R(t,m,\theta)$ over the consumers in the market to be:

$$R(m,t) = \int_{\theta^*(m,t)}^{\theta_1} [m + tp^*(\theta)]f(\theta)d\theta$$

and is the utility derived by the government.

I will only consider the effect on welfare of raising t <u>or</u> m from zero. If only m is raised, welfare is given by:

$$W(m) = \int_{\theta^*(m)}^{\theta_1} [s(\theta) + m]f(\theta)d\theta \ .$$

Using condition (1.6), $s(\theta_0) = 0$, and integrating by parts gives:

$$W(m) = \int_{\theta^*(m)}^{\theta_1} \left[\int_{\theta^*(m)}^{\theta} h(q^*(\tilde{\theta}))d\tilde{\theta}\right] f(\theta)d\theta + m[1-F(\theta^*(m))]$$

$$= \int_{\theta^*(m)}^{\theta} h(q^*(\tilde{\theta}))d\tilde{\theta}F(\theta) \Big|_{\theta^*(m)}^{\theta_1} - \int_{\theta^*(m)}^{\theta_1} h(q^*(\theta))F(\theta)d\theta$$

$$+ m[1 - F(\theta^*(m))]$$

Rearranging this gives:

$$(4.2) \qquad W(m) = \int_{\theta^*(m)}^{\theta_1} (1 - F(\theta))h(q^*(\theta))d\theta + m[1 - F(\theta^*(m))]$$

Differentiating the above and using equation (3.5) yields:

$$(4.3) \qquad \frac{dW(m)}{dm}\Big|_{m=0} = [1 - F(\theta^*(m))] \left[1 - h(q^*(\theta^*(m))\frac{d\theta^*(m)}{dm}\right]$$

$$= [1 - F(\theta^*(m))] \left[1 - \frac{1}{\bar{z}'(\theta^*(m))}\right] .$$

When the monopolist was indifferent between serving and not serving a group of consumers, we assumed he did not serve them. This means that for <u>increases</u> in m ,

$$\bar{z}' = z' > 0 \quad \text{and}$$

$$(4.4) \qquad \frac{dW(m)}{dm} = [1 - F(\theta^*(m))] \left[\frac{z'(\theta^*(m)) - 1}{z'(\theta^*(m))}\right]$$

$$= \frac{[1 - F(\theta^*(m))] \; [f(\theta^*)^2 + (1-F(\theta^*))f'(\theta^*)]}{z'(\theta^*(m))f(\theta^*)^2} .$$

Thus, if $\frac{f(\theta^*)}{1-F(\theta^*)} > - \frac{f'(\theta^*)}{f(\theta^*)}$, $\frac{dW(m)}{dm} > 0$.

Equation (4.3) is easy to interpret. It merely states that welfare rises if the loss in welfare due to higher prices falls short of the revenue extracted by the tax. As the magnitude of both these terms is independent of θ , the effect on a given consumer may be blown up by the number of consumers served $(1 - F(\theta^*))$. What is different about this result from the standard results on incidence of a tax, is that the incidence is identified with the distribution of the characteristic θ . The parameter θ is often thought of as being associated with income. The distribution of θ might thus be thought to be related to the Pareto or log-normal distributions. $f'(\theta^*)$ would then be positive if θ^* was relatively low. This interpretation of θ would suggest that using a specific tax on a foreign monopolist who produces a range of qualities, and serves most of the population would be beneficial, while taxing a monopolist serving the upper end of the market would not.

The effects of a tariff on welfare are less clear-cut. The welfare function is given by:

$$W(t) = \int_{\theta^*(t)}^{\theta_1} [s(\theta) + tp^*(\theta)]f(\theta)d\theta \tag{4.5}$$

$$= \int_{\theta^*(t)}^{\theta_1} [u(\theta,q^*(\theta,t)) - (1-t)p^*(\theta,t)]f(\theta)d\theta \ .$$

Using equation (3.5) and integrating the second term by parts gives

$$W(t) = \int_{\theta^*(t)}^{\theta_1} \left\{\left[\theta t + (1-t)\frac{(1-F(\theta))}{f(\theta)}\right]h(q^*(\theta,t)) + tg(q^*(\theta,t))\right\} f(\theta)d\theta \ . \tag{4.6}$$

Notice that W depends on t in three ways. First, there is the direct effect of t ; second, there is the effect via changes in $q(\theta,t)$ induced by changes in t ; and last, the effect via $\theta^*(t)$. Differentiating (4.6) gives

$$\left.\frac{dW(t)}{dt}\right|_{t=0} = \int_{\theta^*(t)}^{\theta_1} \{z(\theta)h(q^*(\theta,t)) + g(q^*(\theta,t))\} f(\theta)d\theta$$

$$+ \int_{\theta^*(t)}^{\theta_1} (1-F(\theta))h'(q^*(\theta,t)) \frac{dq^*(\theta,t)}{dt} d\theta$$

$$- [1-F(\theta^*(t))] \, h(q^*(\theta^*(t))) \frac{d\theta^*(t)}{dt} \quad .$$

The expression for $\frac{dq^*(\theta,t)}{dt}$ (which is < 0) and $\frac{d\theta^*(t)}{dt}$ (which is > 0) are given by equations (3.7) and (3.8), respectively. The first term in the expression is the direct effect on welfare via the ad-valorem tariff t . It is positive as $z(\theta)$ can be replaced by $\bar{z}(\theta)$ in the above integral by using Lemma 2.3 and Theorem 2.2. As $v(\theta)$ is non-negative for all θ greater than or equal to $\theta^*(t)$, the first term in the above expression must be positive. The second term in the expression is the effect of welfare via induced changes in $q(\theta,t)$ of an increase in t , and is negative as q falls as it rises. The third term is the effect on welfare via the change in the market served. This is again negative. The overall effect is ambiguous.

Conclusion

Although the effects of trade restrictions with endogenous quality have been previously studied, the specification of the structures analyzed have been particularly suited to the paradigm of perfect competition.

In an imperfectly competitive world, a large number of questions arise which do not have corresponding analogues in a competitive world. In order to study such questions, it is important to develop simple models to capture, possible in isolation, the factors which might be important in answering such questions. This paper is to be viewed as an attempt at doing just this.

This paper analyzed the effect of trade restrictions on the product line. A monopolist who produces a line of products will price them jointly so as to extract the maximum profit from the whole market. He will be limited in how much surplus he can extract, by the fact that he cannot perfectly price discriminate since he cannot identify consumers by their "types".

In this paper I have tried to see whether the analysis of a simple model, capturing this stylized fact, leads to any predictions about the behavior of such a foreign monopolist when faced with a specific or ad-valorem tariff, and what policy prescriptions might be derived for such situations.

However, to the extent that variety is expensive, a producer would try and target a product to groups of consumers as discussed in Krishna 1984. As shown there, protection has different effects on such a model.

Future research might be directed to analyzing such questions in less simple market structures, and in a unified framework where the existence of product specific fixed costs would limit the number of products offered.

Appendix

Theorem 2.3: (Mussa and Rosen (1978)) A comparison of the monopolist's quality choice, to that under competition shows that when $V(\theta,q)$ is concave in q for all θ:

(1) The monopolist assigns a lower quality to each type of consumer served in both markets,

(2) The same quality is assigned in both markets to the consumer of type θ_1,

(3) The monopolist serves a smaller market than under competition.

Proof: (This uses the characterization of the monopoly solution provided in Theorem 2.2, by setting $t = m = 0$.)

(1) The competitive quality assignment is given by $q^c(\theta)$, and the lowest θ served by θ^c. These are defined by

$$\theta h'(q^c(\theta)) + g'(q^c(\theta)) - c'(q^c(\theta)) = 0$$

and

$$\theta^c h(q^c(\theta^c)) + g(q^c(\theta^c)) - c(q^c(\theta^c)) = V(\theta^c) = 0$$

respectively. ($V(\theta) = V(\theta,q^c(\theta))$ is the surplus of the consumer of type θ under competition.) The quality assignments under monopoly are the solutions $q^*(\theta)$, θ^*, to:

$$\bar{z}(\theta)h'(q^*(\theta)) + g'(q^*(\theta)) - c'(q^*(\theta)) = 0$$

and

$$\bar{z}(\theta^*)h(q^*(\theta^*)) + g(q^*(\theta^*)) - c(q^*(\theta^*)) = v(\theta^*) = 0 .$$

$v(\theta)$ is defined in the text. $v(\theta,q) = \bar{Z}(\theta)h(q) + g(q) - c(q)$ and $v(\theta,q^*(\theta)) = v(\theta)$ is its value function. Notice that by definition, $z(\theta) \le \theta$. Also, note that as $\bar{z}(\theta)$ consists of increasing segments of $z(\theta)$, connected by flat portions, $\bar{z}(\theta) \le \theta$. This, combined with the definitions of $q^c(\theta), q^*(\theta)$ together with the assumptions made in Theorems 2.1, 2.2 on the concavity of $v(\theta,q)$ in q for all θ proves (1).

(2) Note that for θ close to θ_1, $z'(\theta)$ must be positive. This means that $L(\psi)$ is convex about $\psi = 1$. Therefore, $\bar{L}'(\psi) > L'(\psi)$ around $\psi = 1$, and so $\bar{z}(\theta) > z(\theta)$ for θ close to θ_1 . But as θ gets close to θ_1 , $z(\theta)$ approaches θ . As $z(\theta) < \bar{z}(\theta) < \theta$, for all θ close to θ_1 , $\bar{z}(\theta)$ must approach θ as well. This, combined with the definition of $q^*(\theta)$, $q^c(\theta)$ proves (2).

(3) If we show:

$$V(\theta^c) = 0 \Rightarrow v(\theta^c) < 0,$$

Then $\theta^c < \theta^*$ and (3) is true.

$$V(\theta^c) = \theta^c h(q^c(\theta^c)) + g(q^c(\theta^c) - c(q^c(\theta^c)) = 0 \; .$$

As $V(\theta^c)$ is the maximized value of $V(\theta^c,q)$

$$\theta^c h(q^*(\theta^c)) + g(q^*(\theta^c)) - c(q^*(\theta^c)) \le 0 \; .$$

As $\bar{z}(\theta^c) \le \theta^c$

$$\bar{z}(\theta^c)h(q^*(\theta^c)) + g(q^*(\theta^*)) - c(q^*(\theta^c)) \le 0 \quad .$$

Hence, $v(\theta^c) \le 0$.

Footnotes

[1] See Brander and Spencer (1982) on this possibility as well as Dixit (1983) for a discussion of recent work on trade policy in oligopolistic markets.

[2] The tendency is for the monopolist to produce lower quality goods, compared to competition so as to extract more surplus from consumers who have greater willingness to pay. Phlips (1983, pp. 215-216) points out that the idea is not a new one:

> "As for the relationship between quality choices and price discrimination, I know of only three references. The first one is a fascinating remark made by Dupuit in his discussion of railroad tariffs for passenger traffic. The following excerpt, taken from Ekelund, introduces the idea of a reduction in quality (of the lower-quality goods) as a market segmentation technique:
>
>> It is not because of the few thousand francs which would have to be spent to put a roof over the third-class carriages or to upholster the third-class seats that some company or other has open carriages with wooden benches... What the company is trying to do is to prevent the passengers who can pay the second-class fare from traveling third-class; it hits the poor, not because it wants to hurt them, but to frighten the rich... And it is again for the same reason that the companies, having proved almost cruel to third-class passengers and mean to second-class one, become lavish in dealing with first-class passengers. Having refused the poor what is necessary, they give the rich what is superfluous. [1979, p. 275]
>
> Dupuit did not work out an analytical solution to the problem, nor did Ekelund."

[3] Later work includes that of Maskin and Riley (1982) who point out the similarities in various adverse selection problems.

[4] A special case of this might be when

$$h(q)\theta + g(q) = (a_1 + b_1 q) + (a_2 + b_2 q)\theta \ ,$$

so that the functions $h(q)$, $g(q)$ are linear. The specification I use is not really much more general as the curvature of $h(q)$ is limited by second order conditions to be fairly small. However, this specification is used as it is notationally more convenient, and is slightly more general. Mussa and Rosen assume that $u(\theta, q) = \theta q$.

[5] If g and h are linear this condition is automatically met. This condition restricts the curvature of $h(q)$, and this is why the specification is not really much more general than that of Mussa and Rosen.

[6] t must be less than one for the producer to want to produce.

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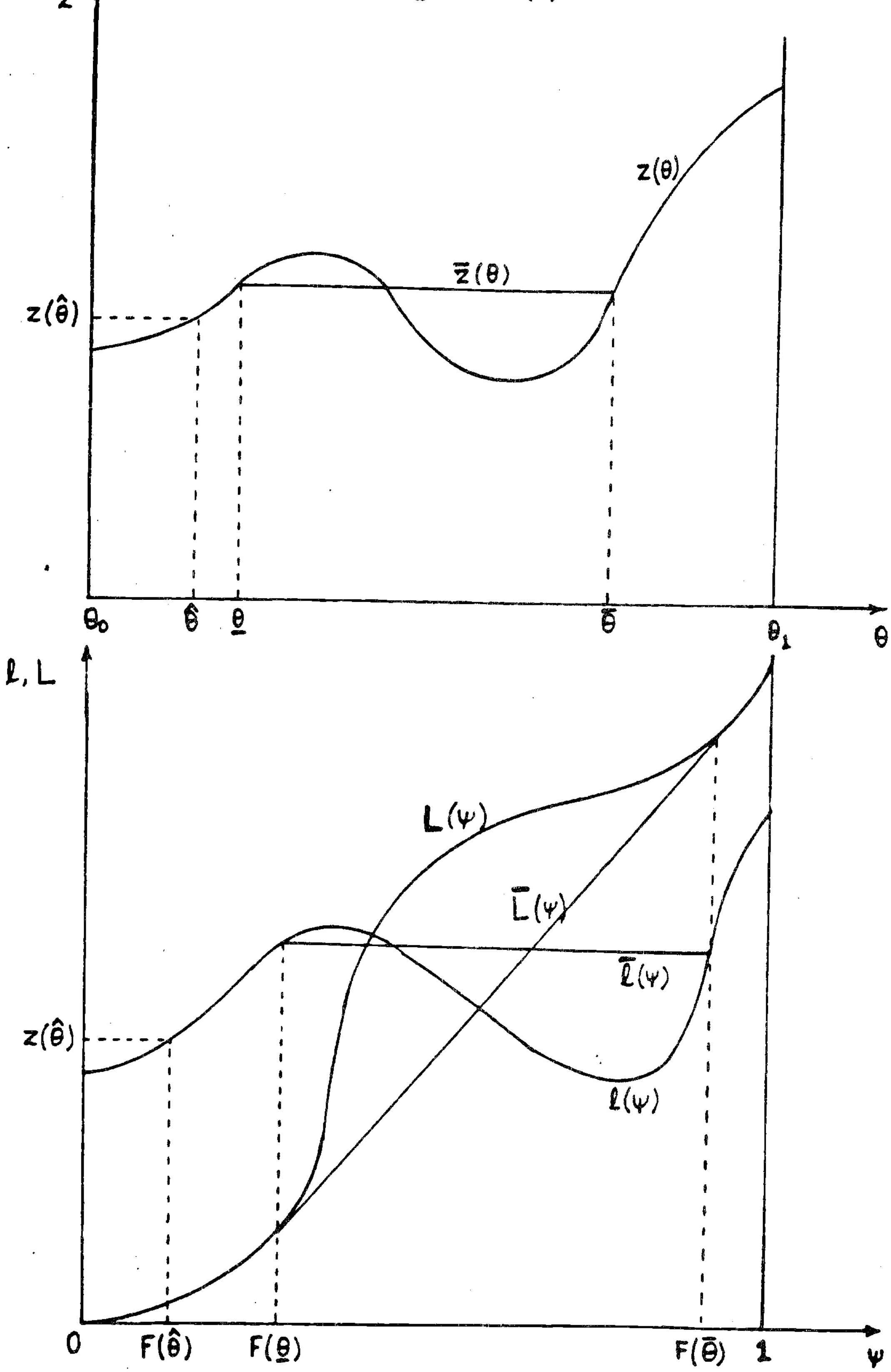
Diagram 2.1(a)


z
z(θ)
$\bar{z}(\theta)$
z($\hat{\theta}$)
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$\underline{\theta}$
$\bar{\theta}$
θ_1
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ℓ, L
L(ψ)
$\bar{L}(\psi)$
$\bar{\ell}(\psi)$
z($\hat{\theta}$)
ℓ(ψ)
0
F($\hat{\theta}$)
F($\underline{\theta}$)
F($\bar{\theta}$)
1
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