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TAX AVERSION, DEFICITS AND THE TAX RATE-TAX REVENUE RELATIONSHIP

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ABSTRACT

This paper offers a possible explanation for the existence of continual government budget deficits such as experienced in a number of industrialized countries in recent years. Based on the assumption that higher tax rates cause more intensive tax-aversion behavior (tax avoidance and tax evasion), together with the assumption that the time horizon relevant for political decision makers is shorter than that required for complete private sector response to tax rate change, our analysis suggests why there seems to be an inherent bias toward budget deficits. Because of tax aversion an inverse relationship between tax rates and tax revenues may exist at low levels of the tax rate. Consequently determined attempts to eliminate or reduce deficits can become self-defeating, almost certainly so when there is a structural deficit. Our analysis suggests that if an economy is on the downward sloping portion of a stylized Laffer curve political expedience, uncertainty about the shape of the curve, and a common wisdom that tax rate increases reduce deficits can all conspire to keep the budget trapped in deficit. Finally, in the presence of inflation deficit growth may be less if there is indexation of income tax rates to inflation, contrary to conventional wisdom.

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The existence of continual government budget deficits through periods of economic expansion as well as recession has been a matter of increasing concern in a number of industrialized countries in recent years. This paper offers an explanation of this phenomenon based on the assumption that higher tax rates encourage economic agents to engage more intensively in tax evasion and avoidance (a distinction defined below), and on the assumption that the time horizon relevant for political decision makers is shorter than that required for complete private sector response to tax rate change, an assumption recently advanced by Buchanan and Lee (1982). Given these two assumptions our analysis suggests why there seems to be an inherent bias toward budget deficits. Since our analysis indicates that the conditions for the existence of an inverse relationship between tax rates and tax revenues are considerably less restrictive when tax avoidance and evasion behavior are explicitly recognized, it suggests that determined attempts to eliminate or reduce deficits can become self-defeating, and that when there is a structural deficit this almost certainly will be the case. Moreover, if an economy is on the downward sloping portion of a stylized Laffer curve our analysis suggests that political expediency, uncertainty about the shape of the curve, and a common belief that tax rate increases reduce deficits can all combine to keep the budget trapped in deficit. Finally, when ongoing deficits are accompanied by inflation our analysis suggests that, contrary to conventional wisdom, deficit growth may be less if there is indexation of income tax rates to inflation.

In section I we examine how optimizing tax aversion behavior, which encompasses both tax avoidance (the legal use of tax loopholes) and tax evasion (the illegal underreporting of income), and tax aversion cost functions combine to define a tax aversion function. It is then shown how the tax aversion function serves to define the relationship between the legally stipulated tax rate and the effective tax rate. Section II investigates the characteristics of the tax rate-tax revenue relationship implied by a rather standard, and quite familiar, supply-side macro model which explicitly incorporates tax aversion behavior by use of the tax aversion function. Section III derives the Laffer curve and considers the relationship implied by tax aversion behavior between tax revenue collected before and after complete private sector response to tax rate change. Section IV considers the implications of our analysis for the political economy of deficits and budget policy. Section V summarizes our conclusions.

I. The Tax Aversion Function and the Effective Tax Rate

When economic agents engage in tax aversion behavior they incur costs such as those associated with buying the services of attorneys, accountants, or the opportunity cost of the agent's own time invested in such matters. The theoretical analysis of tax evasion originated in the expected utility analysis of Allingham and Sandmo (1972) and subsequently has been extended considerably.[1] Work focusing strictly on tax avoidance within an expected utility framework can be found in Kane and Valentini (1975) and in Kane (1976).

It can be argued however that tax evasion and tax avoidance behavior should be analyzed as joint activities because of the potentially significant degree of substitutability and possible complementarity between them. Taking this point of view, Cross and Shaw (1982) argue that an increase in the penalty for evasion or in the probability of detection of evasion raises the relative rate of return on avoidance activity. Similarly, tax evasion is likely to be stimulated by any reduction of avoidance loopholes by the tax authorities. Cross and Shaw also point out that the costs of avoidance and evasion are likely to be interdependent because certain avoidance (evasion) activities can affect the marginal cost of evasion (avoidance) activity. For instance, the cost of investigating and using perfectly legal loopholes in the tax code for tax avoidance also buys insight into the possibilities for illegal exploitation of such loopholes for tax evasion.

Cross and Shaw's (1982) analysis of tax aversion recognizes the potentially significant degree of substitutability and possible complementarity between tax avoidance and tax evasion, and thus has expected utility depending on both. They represent the potentially interdependent costs of the two activities by a joint cost function. Given a proportional income tax rate τ, assuming decreasing absolute risk aversion and that actual income W is exogenous, expected utility is

$$E(U) \equiv (1-p)U(Y) + pU(Z)$$

where

$$Y \equiv W(1-\tau) + (\theta_1 W+\theta_2 W)\tau-C(\theta_1 W,\theta_2 W)$$

and

$$Z \equiv W(1-\tau)+\tau\theta_1 W+\tau\theta_2 W(1-F)-C(\theta_1 W,\theta_2 W)$$

with $0 < p, \tau, \theta_1, \theta_2 < 1$; $F > 1$; $C_1, C_2, C_{11}, C_{22} > 0$ and $C_{12} < 0$; and where p is the probability of detection, θ_1 and θ_2 are the portions of income avoiding and evading tax respectively, F is the fine imposed if evasion is detected, and $C(\cdot,\cdot)$ is the joint cost function specifying

complementarity between avoidance and evasion activity (i.e., $C_{12} < 0$).[2]
It can be shown (see Cross and Shaw (1982), p. 42) that

$$\frac{\partial \theta_1}{\partial \tau}, \frac{\partial \theta_2}{\partial \tau} \gtrless 0.$$

Hence even in this relatively simple model it is not possible to sign a priori the effects of a tax rate change on tax avoidance and tax evasion activity. However if $C_{12} < 0$ it can be shown (see Cross and Shaw (1982), p. 41) that $\frac{\partial \theta_1}{\partial \tau} > 0$ only if $\frac{\partial \theta_2}{\partial \tau} > 0$ since in that case a decrease (an increase) in the tax rate τ reduces (increases) tax evasion activity which in turn raises (lowers) the marginal cost of tax avoidance and causes a reduction (an increase) in avoidance activity. This result is of particular interest in view of some recent empirical evidence on the effects of tax rates on tax evasion. In a study analyzing over 47,000 individual U.S. tax returns for 1969 Clotfelter (1983) obtains estimates of the elasticity of tax evasion with respect to marginal tax rates that are significantly positive, varying between 0.5 and 3.0 across different occupation and income groups. On the basis of this evidence we will assume in the ensuing analysis that $\frac{\partial \theta_2}{\partial \tau} > 0$ and, since $\frac{\partial \theta_2}{\partial \tau} > 0$ implies $\frac{\partial \theta_1}{\partial \tau} > 0$ in the above theoretical framework, we will also assume that
$\frac{\partial \theta_1}{\partial \tau} > 0$.

Since $\theta_1 + \theta_2$ is the fraction of income subject to tax aversion, the fraction of income not subject to tax aversion can be defined as $\theta \equiv 1-\theta_1-\theta_2$. Given that θ_1 and θ_2 are functions of τ, and the assumption that $\frac{\partial \theta_1}{\partial \tau}, \frac{\partial \theta_2}{\partial \tau} > 0$, then θ is a function of τ, $\theta(\tau)$, and $\frac{\partial \theta}{\partial \tau} \equiv \theta'(\tau) < 0$. We refer to τ as the <u>legally stipulated</u> tax rate on income, assumed to be a proportional tax rate, $0 \leq \tau \leq 1$. Since the function $\theta(\tau)$ characterizes tax aversion behavior, we call it the tax aversion function, a decreasing function of τ such that $0 \leq \theta(\tau) \leq 1$. The product of the legally stipulated tax rate τ and the tax aversion function $\theta(\tau)$ define the effective tax rate $\tau\theta(\tau)$. It can be thought of as the fraction of income the government actually realizes in tax revenue when economic agents engage in the optimal amount of tax aversion associated with a particular level of the legally stipulated tax rate τ.

Two examples of tax aversion functions are shown in Figure 1. Given any legally stipulated tax rate such as τ_m, horizontal axis, and the corresponding value of the tax aversion function $\theta(\tau_m)$, vertical axis, the associated effective tax rate $\tau_m\theta(\tau_m)$ is represented by the hatched rectangular area with upper right corner at point m. Point m is the unit elastic point for this particular tax aversion function so that $\tau_m\theta(\tau_m)$ is the maximum effective tax rate; legally stipulated tax rates either higher or lower than τ_m yield lower effective tax rates. For the tax aversion function passing through m' the unit elastic point occurs at m'. Hence the legally stipulated tax rate yielding the maximum effective tax rate for this particular tax aversion function is considerably higher than that for the tax aversion function passing through point m. In general, for tax aversion functions such as those in Figure 1 there is some legally stipulated tax rate, which we may term the critical tax rate, that yields a maximum effective tax rate. At that value of τ

$$\frac{d\tau\theta(\tau)}{d\tau} = \theta(\tau) + \tau\theta'(\tau) = 0, \qquad (1)$$

while at a lower stipulated tax rate $(\theta(\tau) + \tau\theta'(\tau)) > 0$, and at a higher

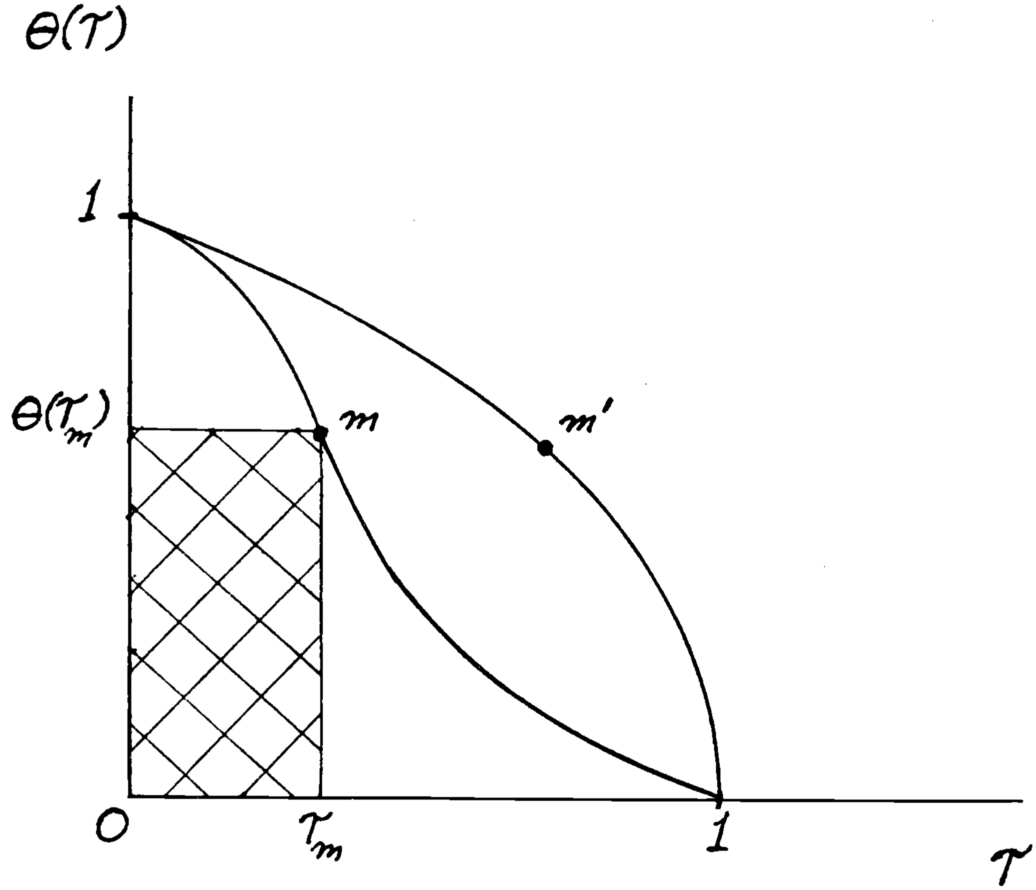

Figure 1

stipulated tax rate $(\theta(\tau) + \tau\theta'(\tau)) < 0$; these facts play an important role in the ensuing analysis. Of course the particular functional form of $\theta(\tau)$ determines the specific value of the legally stipulated tax rate, the critical tax rate, which gives the maximum effective tax rate.[3]

We will see in what follows that the tax aversion function $\theta(\tau)$ plays a critical role in determining the shape of the tax rate-tax revenue relationship, the Laffer curve.[4]

II. Tax Aversion and the Tax Rate-Tax Revenue Relationship

The tax aversion function and the effective tax rate have significant implications for the characteristics of the tax rate-tax revenue relationship implied by a macroeconomic model. This is basically due to the fact that when the stipulated tax rate is raised above the critical tax rate the sign-switching which occurs (where (1) obtains) can determine where the negatively sloped region of the tax rate-tax revenue relationship (the Laffer curve) begins.[5] To illustrate more specifically, we will now incorporate tax aversion behavior (the tax aversion function and the effective tax rate) into a rather standard, and quite familiar, classical-type macroeconomic model and examine the implied tax rate-tax revenue relationship. The supply-side plays a prominent role in this model, a feature often stressed by those concerned that higher tax rates may actually generate lower tax revenues. The basic structure of the model has the virtue, for our purposes, that it is a broadly familiar framework and, as such, it is intended as an illustrative vehicle; the tax aversion function and the effective tax rate in principle could be embedded in any macroeconomic model.[6]

II.A. The Model

In the ensuing analysis of sections III and IV it will be assumed that the relevant time horizon of political decision makers is shorter than that required for complete private sector adjustment of tax aversion activity to any change in the legally stipulated tax rate. A description of how this assumption is incorporated into the analysis is in order before laying out the macroeconomic model. In particular it will be assumed that the private sector's tax aversion behavior adjusts with a lag to any change in the tax rate τ; it takes time for economic agents to adjust their tax aversion activity to the optimal level associated with the new tax rate. Specifically, it is assumed that $\theta(\tau)$ adjusts with a one period lag subsequent to any change in τ. Hence the effective tax rate in any period t is $\tau_t\theta(\tau_{t-1})$. Consider the adjustment in the effective tax rate when the legally stipulated tax rate τ is changed by the amount of $d\tau$ in period t-2 so that $d\tau+\tau_{t-3} = \tau_{t-2} = \tau_{t-1} = \tau_t = \tau$. In period t-2 the effective tax rate is then $\tau\theta(\tau_{t-3})$, in period t-1 it is $\tau\theta(\tau_{t-2}) = \tau\theta(\tau)$; in other words the effective tax rate is affected by the change in τ when that change occurs, the first period, but adjustment of the effective tax rate to the change is not complete until the next period, the second period.

It should be <u>emphasized</u> that in the subsequent analysis the discrete two-period adjustment process described here will serve simply as a <u>stylized</u> way of representing the assumption (advanced by Buchanan and Lee) that the time horizon relevant for political decision makers is shorter than that required for complete private sector response to tax rate change. In reality of course this adjustment process is continuous, but the essence of the time sequence envisioned by the assumption is preserved in our framework. We will not examine the implications of the two-period adjustment process for the Laffer curve and budget deficits until sections III and IV below, but we have described it here and will now incorporate it in the macroeconomic model in this section for completeness.

The model is specified by the equations

$$I(r_t) + G_t - \{y_t - C[(y_t - T), (1 - \tau_t\theta(\tau_{t-1}))r_t]\} = 0 \quad (2)$$

$$M[y_t, (1 - \tau_t\theta(\tau_{t-1}))r_t] - \frac{M}{P} = 0 \quad (3)$$

$$y_t - F(N_t) = 0 \quad (4)$$

$$N_t - f(w_t) = 0 \quad (5)$$

$$N_t - N[(1 - \tau_t\theta(\tau_{t-1}))w_t] = 0 \quad (6)$$

$$\tau_t\theta(\tau_{t-1})(w_tN_t + r_tK_t) - T = 0 \quad (7)$$

where y is real output, N is labor hours, w is the real wage, M is the stock of money, p is the price level, r is the interest rate, G is government spending, τ is the legally stipulated tax rate, and t indexes the time period. $\theta(\tau)$ is the tax aversion function already described above, and T is the total tax revenue.[7] Equation (2) gives flow-of-funds (or goods market) equilibrium where investment $I(\cdot)$ is a decreasing function of the interest rate ($I_r < 0$), and consumption $C(\cdot;\cdot)$ is an increasing function of disposable income ($C_y > 0$) and a decreasing function of the after-tax interest rate ($C_r < 0$). Equation (3) gives money market equilibrium where the demand for real money balances $M(\cdot,\cdot)$ is an increasing function of y ($M_y > 0$) and a decreasing function of the after-tax interest rate ($M_r < 0$). Equation (4) gives the economy's production function $F(\cdot)$. Equation (5) gives the demand for labor hours (implied by (4)), a decreasing function $f(\cdot)$ of the real wage ($f_w < 0$). Equation (6) gives the supply of labor hours, an increasing function $N(\cdot)$ of the after-tax real wage ($N_w > 0$). Equation (7) stipulates that total tax revenue equals the sum of the tax revenue from wage income and interest income. Consumption in (1) and real money demand in (2) are functions of the effective after-tax interest rate $(1-\tau_t\theta(\tau_{t-1}))r_t$, while labor supply in (6) is a function of the effective after-tax real wage $(1-\tau_t\theta(\tau_{t-1}))w_t$. Tax revenues collected depend on the effective tax rate in (7), and thus disposable income which enters the consumption function in (2) is a function of the effective tax rate. The supply-side character of the model is evidenced by the fact that equations (4)-(6) jointly determine the output level y, the real wage w, and employment N independently of the rest of the model.[8]

II.B. Tax Rate-Tax Revenue Relationship

The tax rate-tax revenue relationship can be examined by differentiation of (7); allowing for complete adjustment of tax aversion behavior, which requires two periods subsequent to a change in the tax rate $d\tau$, so that

$\tau_{t-2} + d\tau = \tau_{t-1} = \tau_t$ and time subscripts may be ignored,

$$\frac{dT}{d\tau} = (\theta(\tau) + \tau\theta'(\tau))(wN + rK) + \tau\theta(\tau)\ N \frac{dw}{d\tau} + \tau\theta(\tau)\ w \frac{dN}{d\tau}$$

$$+ \quad \tau\theta(\tau)\ K \frac{dr}{d\tau} \lesseqgtr 0 \tag{8}$$

where

$$\frac{d\tau\theta(\tau)}{d\tau} = \theta(\tau) + \tau\theta'(\tau) \lesseqgtr 0 \tag{9}$$

is the change in the effective tax rate due to a change in the legally stipulated tax rate τ. If (9) is negative the change in $\theta(\tau)$ more than offsets the change in τ--for example, an increase in the tax rate τ would be more than offset by the resulting decline in the effective tax rate $\tau\theta(\tau)$. If (9) is positive, the change in τ is only partially offset by the change in $\theta(\tau)$ in the opposite direction.

It can be readily shown (section A of the appendix) that if (9) is positive then in (8) $\frac{dw}{d\tau} > 0$, $\frac{dN}{d\tau} < 0$, while the sign of $\frac{dr}{d\tau}$ is still ambiguous. However suppose $\frac{dT}{d\tau} < 0$, and recognize that equations (4), (5), and (6) determine the equilibrium level of y independently of the rest of the model. Then, assuming (9) is positive, differentiation of (2) and evaluation at equilibrium y implies

$$\frac{dr}{d\tau} = \frac{(1-C_y)\frac{dy}{d\tau} + C_y \frac{dT}{d\tau} + (\theta(\tau) + \tau\theta'(\tau))rC_r}{I_r + (1-\tau\theta(\tau))\ C_r} > 0 \tag{10}$$

Hence if $\frac{dT}{d\tau} < 0$ then it must be true that $\frac{dr}{d\tau} > 0$. ($\frac{dN}{d\tau} < 0$ implies $\frac{dy}{d\tau} < 0$ via (4).) Therefore if a reduction of the tax rate τ is to cause an increase in total revenue T in the case where (9) is positive, such a result can only be due to the dominance of the term containing $\frac{dN}{d\tau}$ in (8).[9]

Consider the case where (9) is negative. In this case it can be shown (section A of the appendix) that $\frac{dw}{d\tau} < 0$ and $\frac{dN}{d\tau} > 0$, just the opposite of the case where $(\theta(\tau) + \tau\theta'(\tau))$ is positive. Thus the terms containing (wN+rK) and $\frac{dw}{d\tau}$ in (8) are negative while the term containing $\frac{dN}{d\tau}$ is positive. The first and third terms in the numerator of (10) are now positive ($\frac{dN}{d\tau} > 0$ implies $\frac{dy}{d\tau} > 0$ via (4)) so that when $\frac{dT}{d\tau} < 0$ it is now possible that $\frac{dr}{d\tau} < 0$; hence the term containing $\frac{dr}{d\tau}$ in (8) can be negative even when $\frac{dT}{d\tau} < 0$. We see therefore that when (9) is negative a reduction in the tax rate can lead to an increase in total tax revenue via the first and second terms in (8), and possibly via the last term.

II.C. The Tax Aversion Function and the Sign-Switching Tax Rate

The tax aversion function $\theta(\tau)$ clearly plays a crucial role in determining the sign of (8) by virtue of the presence of $(\theta(\tau) + \tau\theta'(\tau))$, the change in the effective tax rate $\tau\theta(\tau)$ resulting from a change in the legally stipulated tax rate τ, from (9). Of particular interest, the sign of $(\theta(\tau) + \tau\theta'(\tau))$--hence the sign of (8)--can switch as the size of the effective tax rate changes with any change in τ.[10] Given a particular functional form for $\theta(\tau)$, an important question is at what level of the tax rate τ will $(\theta(\tau) + \tau\theta'(\tau))$ switch sign and thus cause a sign switch in $\frac{dT}{d\tau}$? As the tax rate τ is increased, for example, at what level of τ would total tax revenues cease rising and begin to decline--in other words, at what τ do we get onto the negatively sloped portion of a stylized Laffer curve?

We do not know the specific form of actual real-world tax aversion functions. Those shown in Figure 1 are nothing more than two possible examples; the sign-switch for the tax aversion function passing through point m occurs at a tax rate $\tau = .3$ (as demonstrated in footnote 3), while for the tax aversion function passing through point m' it occurs when $\tau = .66$. These observations would seem of more than academic interest in light of recent estimates of aggregated measures of tax rates. In a recent study Barro and Sahasakul (1983) report that, when weighted by adjusted gross income, the arithmetic average of marginal tax rates from the United States federal individual income tax schedule was 30 percent in 1980. The $\theta(\tau)$ function passing through point m in Figure 1 suggests that the Laffer curve could become negatively sloped at tax rates in this range, bearing in mind of course that this is but a hypothetical example embedded in a simple model. Tobin (1981) cites estimates of the federal marginal rate of personal income tax averaged over all tax brackets which put the average marginal tax rate in the United States at .22 in 1980.

III. Tax Aversion and the Implied Laffer Curve

In order to consider the consequences of tax aversion for budget deficits it is useful to examine the nature of the Laffer curve implied by the two period tax-aversion process. It should be emphasized again that the discrete two-period adjustment process modeled here is simply a stylized way of representing the assumption (advanced by Buchanan and Lee) that the time horizon relevant for political decision makers is shorter than that required for complete private sector response to tax rate change. In reality of course this process is continuous, but the essence of the time sequence envisioned by the assumption is preserved in our framework.

For the purpose of deriving a graphical representation of the two-period tax aversion process and the tax rate-tax revenue relationship, or Laffer curve, the discussion is made more manageable if we ignore the taxation of interest income. Mathematically this simplification sets K equal to zero in the expression for $\frac{dT}{d\tau}$ (given in Appendix A) so that

$$\frac{dT}{d\tau} = -[\theta(\tau) + \tau\theta'(\tau)](\phi - wN) \gtrless 0 \tag{11}$$

where

$$\phi = N_w(N+wf_w)[f_w - (1-\tau\theta(\tau))N_w]^{-1}\tau\theta(\tau)$$

and

$$\phi \gtrless 0 \text{ as } \eta_w \lessgtr -1$$

where

$$\eta_w = \frac{w}{N} f_w$$

is the elasticity of the demand for labor with respect to the real wage w. (11) is the slope of the Laffer curve. The sign of (11) depends on the sign of $[\theta(\tau) + \tau\theta'(\tau)]$, the sign of ϕ, and if $\phi > 0$, the size of ϕ relative to wN; the sign of ϕ depends on whether the equilibrium occurs along the elastic or inelastic region of the labor demand curve.

Since the concensus estimate of the elasticity of labor demand in the United States reported by Hammermesh (1976) equals approximately -1/3, we will assume that $-1 < \eta_w < 0$ so that $\phi < 0$ and the sign of (11) depends solely on the sign of $[\theta(\tau) + \tau\theta'(\tau)]$. Given this assumption, the interpretation of (11) and its sign is facilitated by proceeding directly to the derivation of a graphical representation of the tax rate-tax revenue relationship implied by the two-period tax adjustment process and equations (4)-(7), the equations determining y, N, w, and T simultaneously and independently of the rest of the model (assuming interest income is not taxed).

Figure 2, part (a), depicts the labor demand and supply curves specified by equations (5) and (6). The labor demand curve is unit elastic at point f. Suppose the tax rate τ is initially set at zero and that $[\theta(\tau) + \tau\theta'(\tau)] > 0$, which means that the change in τ is only partially offset by the change in $\theta(\tau)$ in the opposite direction. The relevant labor supply curve is $N^s(\tau_0=0)$ which intersects the labor demand curve N^d in its inelastic range at point a to determine the equilibrium real wage and employment level. Tax revenue T is zero, corresponding to point a in part (b) of Figure 2 where tax revenue is measured on the horizontal axis and the legally stipulated tax rate τ is measured on the vertical.

Complete adjustment by the private sector to any tax rate change takes two periods: during the first period labor adjusts its supply to the new legally stipulated tax rate; during the second period labor adjusts its tax aversion behavior to the new tax rate and labor supply is then adjusted to the new effective tax rate associated with the new level of tax aversion activity. Now suppose the tax rate is increased from $\tau_0 = 0$ to $\tau_1 > 0$. During the first period the labor supply curve in part (a) of Figure 2 shifts leftward to $N^s(\tau_1)$ where it intersects the labor demand curve at point b. The real wage paid by employers rises to w_1 and the after-tax real wage received by labor is $(1-\tau_1)w_1$, corresponding to point b in part (b). Since τ initially equaled zero, θ equals 1 during

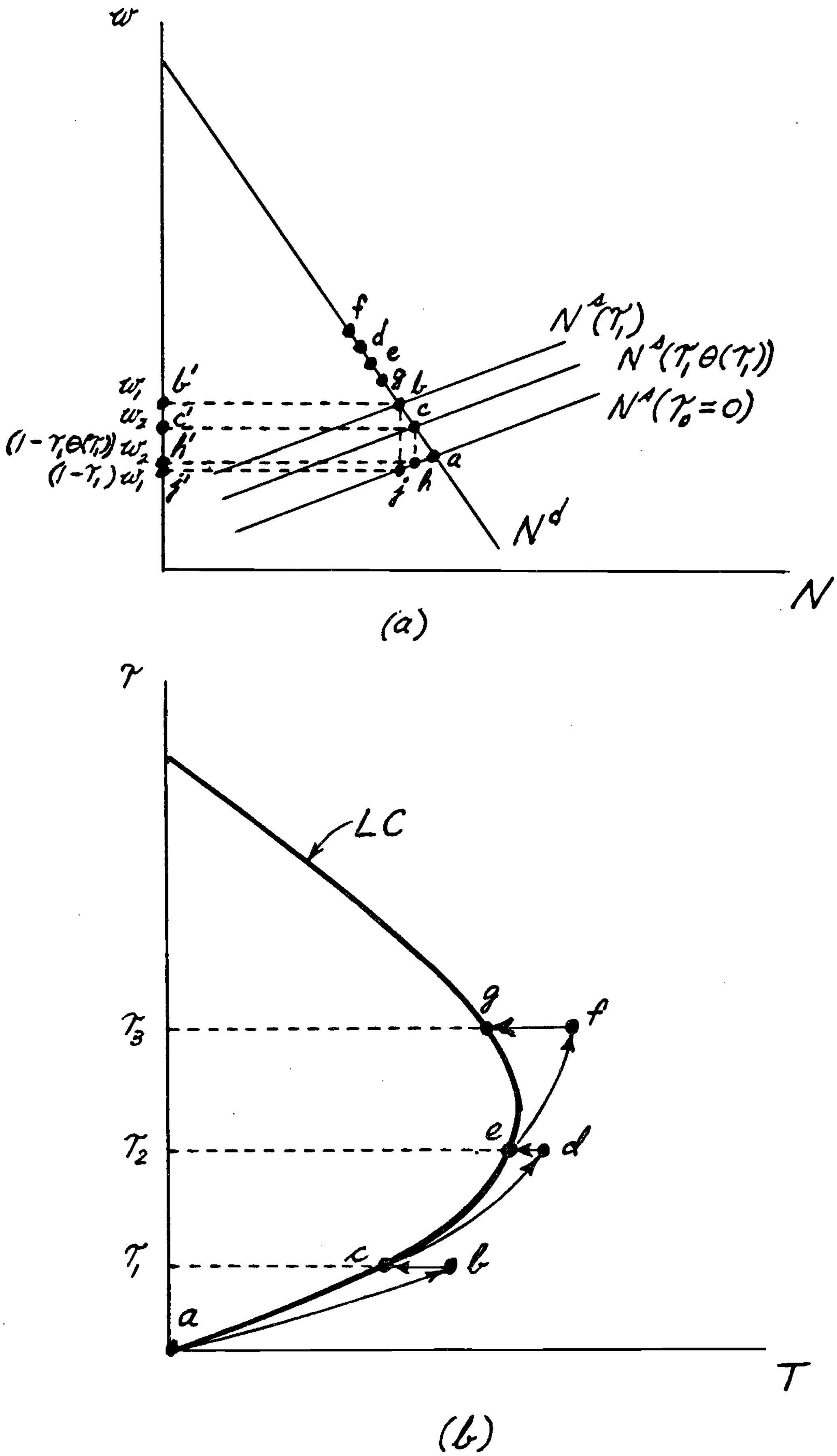
w
f
d
e
g
b
c
a
h
j
w₁ b'
w₂ c'
(1 − τ₁θ(τ₁)) w₂ h'
(1 − τ₁) w₁ j'
N^s(τ₁)
N^s(τ₁, θ(τ₁))
N^s(τ₀ = 0)
N^d
N
(a)
τ
LC
g
f
τ₃
e
d
τ₂
c
b
τ₁
a
T
(b)

Fig. 2

the first period. However during the second period, labor increases its tax aversion activity (from a zero level since the tax rate initially equaled zero) to the optimal level corresponding to the new higher tax rate τ_1. Hence, during the second period θ falls from 1 to $\theta(\tau_1) < 1$ and the effective tax rate equals $\tau_1\theta(\tau_1)$. Since labor now keeps a larger portion of any real wage, i.e., $\tau_1 > \tau_1\theta(\tau_1)$, labor is willing to supply more labor hours at any real wage and therefore the labor supply curve shifts rightward from $N^s(\tau_1)$ to $N^s(\tau_1\theta(\tau_1))$ where it intersects N^d at point c. The second period's rightward shift only partially offsets the first period's leftward shift because $[\theta(\tau) + \tau\theta'(\tau)] > 0$ and hence the change in τ is only partially offset by a change in $\theta(\tau)$ in the opposite direction. The real wage paid by employers falls from w_1 to w_2 and the after-tax real wage received by labor rises from $(1-\tau_1)w_1$ to $(1-\tau_1\theta(\tau_1))w_2$ during the second period. Total tax revenue collected during the second period is given by the area h'hcc'. Total tax revenue has fallen (i.e., area h'hcc' is less than area j'jbb'). This is represented in part (b) by the move from point b to point c.

Suppose the tax rate is raised further, from τ_1 to τ_2, say, where τ_2 is sufficiently high that during the first period after this increase the labor supply curve in part (a) of Figure 1 shifts leftward until it passes through point d, corresponding to an effective tax rate equal $\tau_2\theta(\tau_1)$: total tax revenue is now represented by the rectangle with upper right-most corner at point d in part (a) and corresponds to the point d associated with tax rate τ_2 in part (b). θ equals $\theta(\tau_1)$ during the first period since the tax increase was initiated from a position corresponding to point c in parts (a) and (b). During the second period labor increases its tax aversion activity to the optimal level associated with the higher tax rate τ_2; θ falls from $\theta(\tau_1)$ to $\theta(\tau_2) < \theta(\tau_1)$ and the effective tax rate now equals $\tau_2\theta(\tau_2)$. As a result, labor now keeps a larger proportion of any real wage $(\tau_2\theta(\tau_2) < \tau_2\theta(\tau_1))$ and is therefore willing to supply more labor hours at any real wage. Hence the labor supply curve shifts rightward until it intersects N^d at point e. Once again, total tax revenue declines during the second period; this is represented in part (b) by the move from point d (associated with τ_2) to point e.

To this point we have assumed that $[\theta(\tau) + \tau\theta'(\tau)] > 0$--any change in τ is only partially offset by a change in $\theta(\tau)$ in the opposite direction. But now suppose the tax rate is raised to τ_3 and that this is above that tax rate at which $[\theta(\tau) + \tau\theta'(\tau)]$ switches sign in the manner and for the reasons described in the previous section. Now $[\theta(\tau) + \tau\theta'(\tau)]$ becomes negative, which means that any change in τ is more than offset by a change in $\theta(\tau)$ in the opposite direction.

In Figure 2, part (a), the increase in the tax rate from τ_2 to τ_3 initially shifts the labor supply curve leftward from the position where it passes through point e to a position where it passes through point f. During the first period of the adjustment process tax revenue rises as before, represented by the movement from e to f in part (b) of Figure 2. However during the second period the increase in tax aversion activity in response to the increase in the tax rate now causes a rightward shift in the labor supply curve which more than offsets the initial leftward shift, so that after the adjustment process is complete the labor supply curve passes through point g on N^d which is below the initial point e. The tax revenue decline during the second period of the adjustment process, represented by the move from f to g in part (b), actually results in a lower level of tax revenue at tax rate τ_3 than was realized at the lower rate τ_2 (point g lies to the left of point e in part (b) of Figure 2). Moreover the economy is now on the negatively sloped portion of the Laffer curve.

The heavily drawn curve LC in part (b) is a tax rate-tax revenue relationship, or Laffer curve, which for any given tax rate τ indicates the amount of tax revenue T realized after there has been complete adjustment of tax aversion behavior to the given tax rate. The segments such as ab, cd, or ef, represent tax rate-tax revenue relationships which indicate the amount of tax revenue realized before there has been complete adjustment of tax aversion behavior to any given tax rate.

IV. Implications for the Political Economy of Deficits and Budget Policy

What are the implications of the above analysis for the political economy of government deficits and budget policy? To answer this question we will use the graphical representation of the model of equations (2)-(7) as illustrated in Figure 2. All that needs to be added to Figure 2, part (b), is government spending G which appears in equation (2). In addition to measuring tax revenue T along the horizontal axis, we also can measure government spending G along that axis, as shown in Figure 3. The lynchpin of our analysis of the political economy of budget deficits is the assumption (due to Buchanan and Lee) that the time horizon relevant for political decision makers is shorter than that required for complete private sector response to tax rate change. This assumption is modeled here in stylized fashion by the discrete two-period adjustment process wherein the time horizon relevant to political decision makers only extends over the first period.

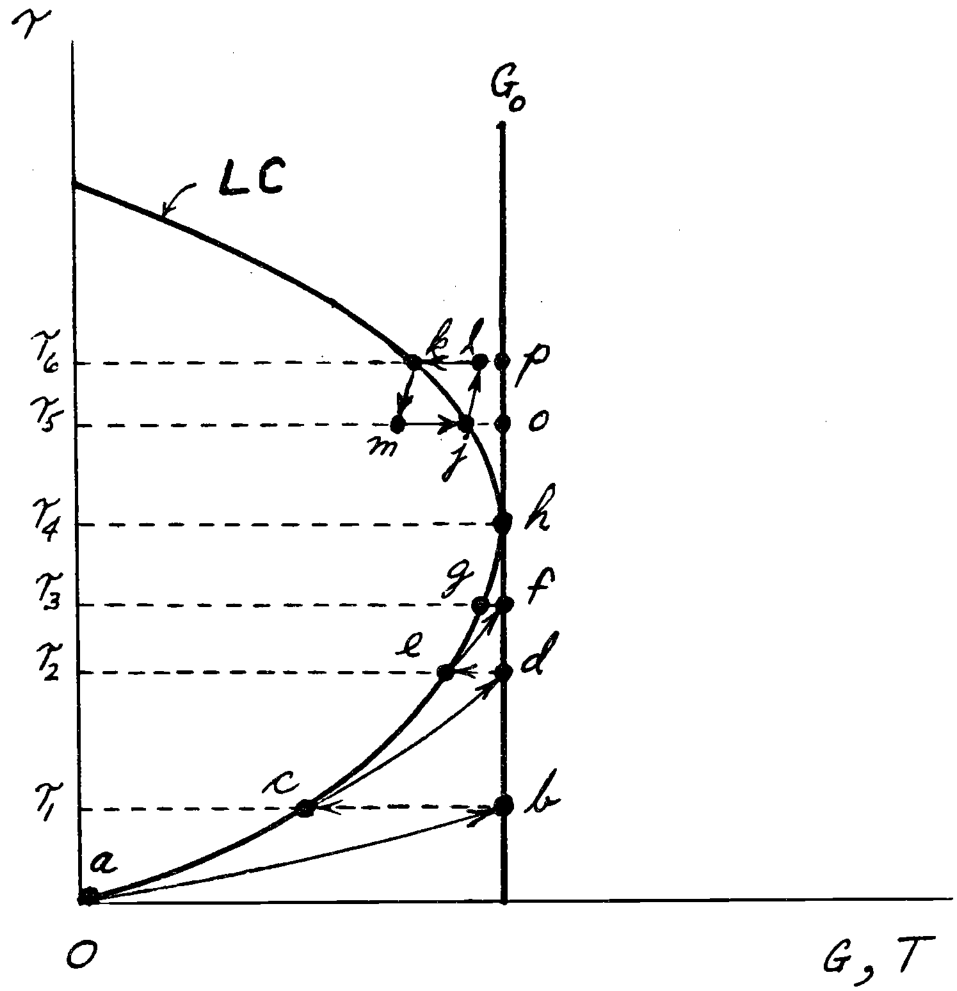

Fig. 3

In the ensuing discussion we also make the following assumptions:

1. Political decision makers desire to balance the budget.
2. Considerations other than budget policy objectives determine the level of government spending (it must be taken as a given when attempting to balance the budget), so that the burden of attaining the balanced budget objective must fall on tax rate changes.
3. Political decision makers will be reluctant to change the tax rate if they fear adverse voter reaction.

IV.A. The Budget Deficit Bias

Suppose that government spending is set at the level indicated by the heavily drawn vertical line G_0 in Figure 3. (It may be regarded as coincidental that G_0 passes through point h; it wouldn't affect the discussion of this section if G_0 were to the left of the position indicated in Figure 3.) Given the assumption that political decision makers have a shorter time horizon than the period of time required for complete private sector response to tax rate change, the Laffer curve LC is not relevant to the actions of political decision makers. Instead, the relevant rate-revenue relationship only reflects private sector response to any tax rate change over the horizon relevant to the political decision maker. This response is only part of the complete response.

Given the assumption that complete adjustment by the private sector to any tax rate change takes two periods, and that the time horizon relevant to political decision makers only extends over the first period, if we start from a zero tax rate position in Figure 3 and increase the tax rate to τ_1, the relevant (one period) rate-revenue relationship is given by ab. The level of realized tax revenue, given by point b, is just sufficient to balance the budget. However when private sector response to the new tax rate is complete (after two periods), realized tax revenue at the tax rate τ_1 will be less, corresponding to point c on LC. Moreover, despite the balanced-budget objective of political decision makers, a budget deficit emerges, equal cb. Once again political decision makers are faced with the need to raise the tax rate in order to achieve a balanced-budget objective. The relevant (one period) rate-revenue relationship is now cd. Political decision makers raise the tax rate to τ_2 and tax revenue increases to the level corresponding to point d (after one period). Again, however, after complete private sector response to the higher tax rate (after two periods), a budget deficit emerges, equal ed. If the process is repeated, eventually a viable balanced budget position is achieved at point h, corresponding to the tax rate τ_4.

Recall that the level of government spending is assumed to be given (determined by considerations other than the balanced budget objective) so that the burden of attaining budget policy objectives must fall on tax rate changes. Given this, and assuming that complete private sector response to the tax rate change takes longer than the time period relevant for political decision makers, if political decision makers are generally reluctant to risk adverse voter reaction to tax rate increases, especially a sequence of such increases, budget policy will have an inherent bias towards deficits. For example, suppose political decision makers raise the tax rate from τ_1 to τ_2 in Figure 3, attaining a balanced budget (after one period) at point d, and suppose they are then evicted from office in large numbers by adverse voter reaction. After complete private sector response (two periods), a deficit reemerges, equal ed, but new political decision makers having witnessed the fate of their predecessors are gun-shy of the further tax rate increases necessary to eliminate the deficit. Finally, note that even if the negatively sloped region of the Laffer curve is not in the relevant tax rate range, the existence of the type lagged private sector response envisioned here is conducive to the existence of a budget deficit bias.

IV.B. Ignorance and Overshooting

Now suppose, quite realistically, that political decision makers don't know the exact configuration of the Laffer curve, and in particular whether they are on the upward or downward sloping portion of the curve. Furthermore, suppose voters feel so strongly about achieving a balanced budget that they are willing to tolerate repeated tax rate increases. If tax rate increases occur along the upward sloping portion of the Laffer curve, a balanced budget objective eventually can be realized. However, given ignorance about the exact configuration of the Laffer curve, it is entirely possible that political decision makers will overshoot the tax rate which is consistent with a viable balanced budget--the rate τ_4 in Figure 3. Once this occurs the tax rate will be in the range of the downward sloping portion of the Laffer curve. Pursuit of a balanced budget objective initiated by tax rate increases in the downward sloping range of the Laffer curve are self-defeating. If the tax rate in Figure 3 is τ_5, for example, an attempt to reduce the associated budget deficit jo by raising the tax rate to τ_6 still would give rise to the deficit lp (after one period). This deficit would increase to kp after complete private sector response (after two periods) and would be larger than the initial deficit jo. It is readily apparent that a repeat of this process would cause even larger deficits.

IV. C. The Deficit Trap

If the economy is on the downward sloping portion of the Laffer curve prospects for reducing the deficit by lowering the tax rate may well run into political impediments, given that political decision makers are uncertain both about the shape of the curve and the economy's location on it.

The potential difficulty arises because of the nature of the two-period adjustment process accompanying a tax rate reduction, in particular the fact that tax revenue <u>falls</u> during the first period after the rate reduction. Suppose the economy is on the downward sloping portion of the Laffer curve at point k on Figure 3 with the tax rate at τ_6, for example, and the associated budget deficit equals kp. Suppose enough political decision makers believe this to be the case, though they can't be certain, that it is possible through the political process to initiate a tax rate reduction from τ_6 to τ_5. Furthermore, suppose that this rate reduction is not large enough to cause $[\theta(\tau) + \tau\theta'(\tau)]$ to switch from a negative to a positive sign, recalling that along the downward sloping portion of the Laffer curve $[\theta(\tau) + \tau\theta'(\tau)] < 0$ so that any change in τ is more than offset by a change in $\theta(\tau)$ in the opposite direction.

During the first period of the two-period adjustment process following the rate reduction from τ_6 to τ_5 the effective tax rate <u>falls</u> from $\tau_6\theta(\tau_6)$ to $\tau_5\theta(\tau_6)$; during the second period economic agents respond to the lower tax rate τ_5 by reducing their tax aversion efforts so that θ <u>rises</u> from $\theta(\tau_6)$ to $\theta(\tau_5)$ and the effective tax rate <u>rises</u> from $\tau_5\theta(\tau_6)$ to $\tau_5\theta(\tau_5)$. In terms of the labor demand and supply curves (such as in Figure 2, part (a)), during the first period of the adjustment process the labor supply curve shifts rightward along the inelastic range of the labor demand curve and tax revenue declines. However during the second period the labor supply schedule shifts back leftwards by more than the amount of the first period's rightward shift, and tax revenue rises by more than the amount of the first period's tax revenue decline. Hence, in Figure 3 the deficit increases from kp to mo in the first period and then declines to jo during the second period.

After the complete two-period adjustment the deficit has been reduced from kp to jo. However given that the relevant time horizon for political decision makers it only the first period, the emergence of an even larger deficit during the first period (from kp to mo) after the tax rate reduction may well shake their faith in the wisdom of their action, especially since they can't be certain that they were on the downward-sloping portion of the Laffer curve in the first place. Conventional wisdom (appropriate for the upward sloping portion of the Laffer curve) might now well prevail, arguing that the tax rate should be increased to reduce the deficit, thus reversing the initial tax rate reduction.

In sum, when the economy is on the downward sloping portion of the Laffer curve, attempts to reduce the deficit by lowering the tax rate can cause an even larger deficit over the politically relevant time horizon. Conventional wisdom may then prevail and cause political decision makers, uncertain about the shape of the Laffer curve, to reverse themselves and raise the tax rate, moving the economy back up along the downward sloping portion of the curve. Thus, once the economy is on the downward sloping portion of the Laffer curve it may become trapped in deficit by a combination of political expediency, uncertainty about the shape of the curve, and a conventional wisdom which holds that increases in tax rates reduce deficits.

IV. D. Consequences of a Structural Deficit

Suppose the level of government spending exceeds the maximum maintainable level of tax revenue, giving rise to what may be termed a structural deficit. This is illustrated in Figure 4 where the level of government spending is represented by the heavily drawn vertical line G_1.

Unlike the situation depicted in Figure 3, it is not possible to achieve a viable balanced budget. Moreover, given ignorance of the true configuration of the Laffer curve, persistent attempts to balance the budget by raising the tax rate will lead inevitably to positions along the downward sloping portion of the long-run Laffer curve. For example, suppose the tax rate is initially set at τ_1. After two periods a deficit equal to cb emerges. Successive attempts to balance the budget by raising tax rates from τ_1 to τ_2 and eventually to τ_4, say, give rise to the path cdefghi and the economy ends up on the ever diverging, self-defeating downward sloping region of the Laffer curve. Indeed it may not even be possible to attain a balanced budget during the first period after a tax rate change--that is, before the onset of tax evading behavior. In that case the vertical line representing government expenditure would be so far to the right in Figure 4 that it would be beyond the reach of segments of the adjustment path given by cd, ef, and gh.

Perhaps the sort of calamity that Figure 4 suggests is highly improbable if political decision makers are reluctant to engage in repeated tax rate increases for fear of adverse voter reaction. Because of this reluctance, political decision makers might simply decide to live with an ongoing deficit, staying with a tax rate such as τ_1 and deficit cb. However, it doesn't take blind pursuit of a balanced budget objective to get onto the downward sloping portion of the Laffer curve. Attempts to simply reduce the size of the deficit could trigger the same result.

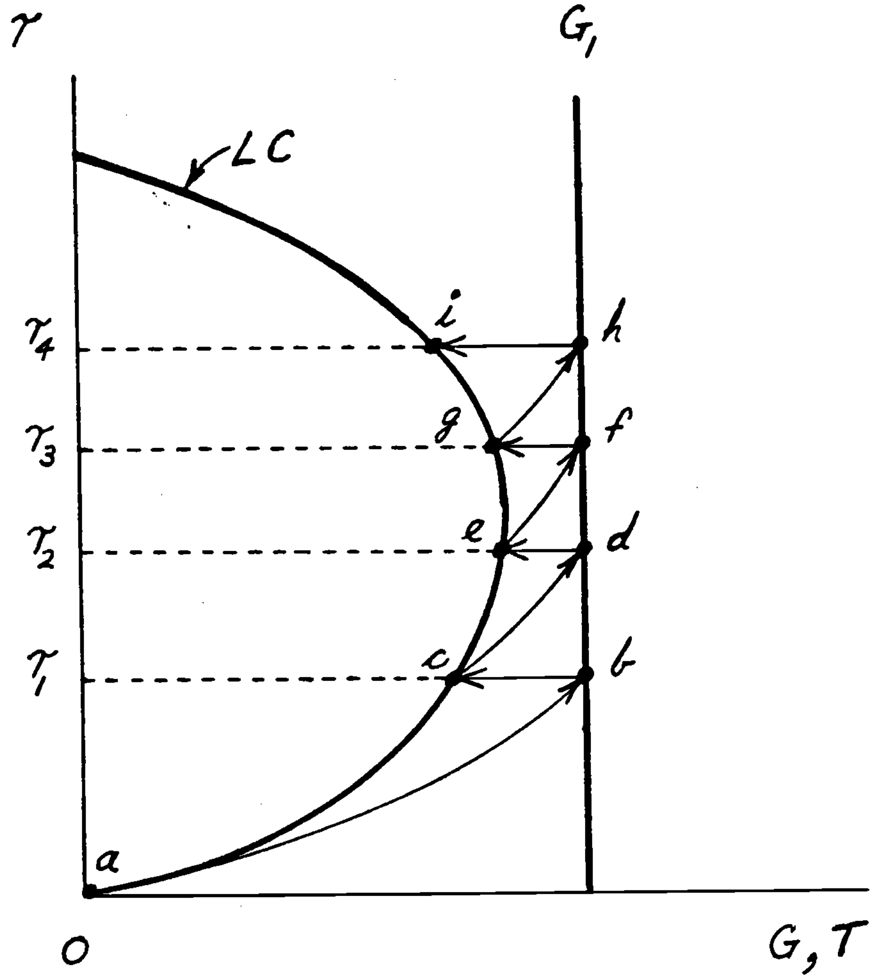

Fig. 4

IV. E. Implications for Inflation and Indexation

We have not explicitly incorporated marginally progressive tax rates and the indexation of tax brackets to inflation in our model, as represented by equations (2)-(7). Nonetheless our analysis does seem to suggest some implications for tax indexation and the effects of inflation on budget deficits. A rough idea of the significance of inflationary bracket creep during the inflationary decade of the 1970s is suggested by the average marginal tax rate estimates of Barro and Sahaskul (1983): weighted by adjusted gross income, the arithmetic average of the marginal tax rates rose from 24 percent in 1970 to 30 percent in 1980. The tax rate referred to in the ensuing discussion is envisioned as such a measure--the weighted average of the different tax rates in a marginally progressive tax structure.

It is commonly argued that indexation of a marginally progressive income tax structure will give rise to larger deficits in the presence of inflation than wouldbe the case without indexation. The argument is predicated on the idea that the tax bracket creep caused by inflation automatically generates more tax revenue; hence indexation would eliminate such automatic tax revenue growth and therefore deficits would become larger. However our line of analysis suggests that <u>just the opposite may be the case</u>. Why? We have noted that the existence of a structural deficit, such as depicted in Figure 4, combined with a reluctance to increase the tax rate for fear of adverse voter reaction, might lead political decision makers to refrain from tax rate increases and simply live with an ongoing deficit--the deficit cb associated with the tax rate τ_1 in Figure 4, for example. However, even absent <u>discretionary</u>

tax rate increases, an ongoing deficit such as cb may be inherently unstable. To the extent that an ongoing deficit is accompanied by inflation, and to the extent that the tax structure is not indexed to inflation, the greater is the likelihood that the tax rate will be automatically increased by bracket creep and the economy moved onto the downward sloping region of the Laffer curve with evergrowing deficits. Hence, contrary to conventional wisdom, indexation of the tax structure might well help curb deficit growth rather than promote it.

V. Conclusions

When the effects of tax aversion behavior are explicitly recognized the conditions for being on the negatively sloped portion of the Laffer curve are considerably less restrictive than when tax aversion behavior is not taken into account. However, even if there were only a positively sloped tax rate-tax revenue relationship, or if a negatively sloped region were not in the relevant tax rate range, the existence of a lagged private sector response to tax rate change that exceeds the relevant time horizon for political decision makers is conducive to the existence of a budget deficit bias. Given the existence of a negatively sloped region of the Laffer curve, especially if it begins at reasonably low tax rates, determined attempts to eliminate or just reduce deficits can become self-defeating, particularly if there is a structural deficit. Moreover, once the economy is on the downward sloping portion of the Laffer curve a combination of political expediency, uncertainty about the shape of the curve, and a common belief that tax rate increases reduce deficits all can conspire to keep the budget trapped in deficit. Finally, given the existence of inflation and a marginally progressive income tax, deficit growth may be less if there is indexation of income tax rates to inflation, contrary to conventional wisdom.

Footnotes

1. See, for example, Srinivasan (1973), Yitzhaki (1974), McCaleb (1976), Weiss (1976), Andersen (1977), Penceval (1979), Christiansen (1980), Issachsen and Strøm (1980), Sandmo (1981), and Cowell (1981).

2. Cross and Shaw focus on the amounts of income avoiding and evading taxes which they designate as A and E respectively, where in our notation $A \equiv \theta_1 W$ and $E \equiv \theta_2 W$. In our analysis θ_1 and θ_2 are used to develop the concept of the effective tax rate.

3. The specific form of the $\theta(\tau)$ function represented by the curve passing through point m in Figure 1 is given by

$$\theta(\tau) = (3\tau+1)(1-\tau)^3. \qquad \text{(i)}$$

The function is decreasing in τ since

$$\frac{\partial\theta(\tau)}{\partial\tau} = -12(1-\tau)^2\tau < 0, \; 0 < \tau < 1$$

and $\frac{\partial\theta(\tau)}{\partial\tau} = 0$ when $\tau = 0$ or 1. It has an inflection point at $\tau = 1/3$ since

$$\frac{\partial^2\theta(\tau)}{\partial\tau^2} = 12(3\tau-1)(1-\tau) \quad \begin{cases} < 0, & 0 \leq \tau < \frac{1}{3} \\ > 0, & \frac{1}{3} < \tau < 1 . \end{cases}$$

For the function given by (i) the effective tax rate $\tau\theta(\tau)$ is maximized when $\tau = .3$, and $(\theta(\tau) + \tau\theta'(\tau)) > 0$ for $\tau < .3$, and < 0 for $\tau > .3$.

4. One might well ask at this point about the empirical significance of tax aversion. Tax avoidance is a legal and common practice as evidenced by the employment of an industry of tax accountants and attorneys, and the widespread use of itemized deductions. Tax evasion, the illegal nonpayment of taxes, is extensive and growing, and its magnitude has been estimated in several recent studies. See for example, Simon and Witte (1982), Witte (1984), and the recent U.S. Internal Revenue Service report (1983), as well as the extensive research in this area cited in each of these documents. Tax evasion as envisioned inthese studies refers to amounts of income taxes individuals and corporations should pay but do not. It refers to income earned from both legal and illegal activity, and encompasses the activities of the "underground" as well as the "above ground" economy. The latest IRS report estimates that $81.5 billion of federal income tax was lost in the U.S. in 1981 due to unreported legal incomes and another $9.0 billion due to unreported incomes earned in illegal endeavors; the total loss of $90.5 billion approximately equaled 22 percent of total federal corporate and personal income taxes actually collected in 1981. Summarizing findings in several countries, Witte reports that in general the Scandinavian countries, West Germany, and the United Kingdom have unrecorded economic activity (therefore untaxable) comparable to that of the U.S. where such activity amounted to approximately 12 percent of national income in 1979; in Italy such activity was estimated to equal 20-25 percent of GNP, while Belgium and France were estimated to lie somewhere between the estimates for the U.S. and Italy.

5. In general this is significant for two reasons. First, the concept of the tax aversion function and its associated effective tax rate can in principle be incorporated in any macroeconomic model. Second, the effective tax rate typically will enter a macro model in several crucial places, as will become apparent in the ensuing analysis.

6. Due largely to the pivotal role played by the tax aversion function and the effective tax rate, the ensuing analysis and its implications are not altered significantly if a more demand-side (Keynesian-type, say) framework is employed (see section B of the appendix).

7. The tax rate τ is assumed to be proportional, as noted earlier.

8. The model represented by equations (2)-(7) is merely intended as an illustrative vehicle for the analysis to follow. It was chosen because of its generally familiar and recognizeable structure.

9. This point has been made by Shaller (1983) in a classical-type model where the effects of tax aversion were not considered.

10. Inspection of the expression for $\frac{dT}{d\tau}$ given in section A of the appendix indicates that every term in the expression is multiplied by $(\theta(\tau) + \tau\theta'(\tau))$. Therefore when $(\theta(\tau) + \tau\theta'(\tau))$ switches sign $\frac{dT}{d\tau}$ will switch sign unless: the terms $(N + wf_w)$ and/or $[(wN + rk)C_y + rC_r]$ happen to switch sign at just the same level of τ where $(\theta(\tau) + \tau\theta'(\tau))$ switches sign and/or the magnitudes of the other terms in the expression become such as to cause a sign switch at just that level of τ. Such coincidence seems remote.

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Appendix

A. Derivation of $\frac{dT}{d\tau}$, $\frac{dw}{d\tau}$, $\frac{dN}{d\tau}$, and $\frac{dr}{d\tau}$ in a Classical-type model

The model is specified by equations (2)-(7) in the text. Recognize that by the structure of the classical model, equations (2) and (4)-(7) may be solved independently of (3), so that by substitution of (4) into (2) the model may be represented more compactly (ignoring the time subscript on τ)

$$\begin{aligned} I(r)+G-\{F(N)-C[F(N)-\tau\theta(\tau)(wN+rK),\ (1-\tau\theta(\tau))r]\} &= 0 \\ N-f(w) &= 0 \\ N-N[(1-\tau\theta(\tau))w] &= 0 \\ \tau\theta(\tau)(wN+rK)-T &= 0 \end{aligned}$$

Totally differentiating this system of equations and using Cramer's rule yields

$$\frac{dT}{d\tau} = \frac{|D|}{|C|} \gtrless 0 \tag{8a}$$

where

$$C = \begin{bmatrix} \overset{(-)}{I_r-\tau\theta(\tau)KC_y+(1-\tau\theta(\tau))C_r}, & \overset{(-)}{-F_N(N)(1-C_y)-\tau\theta(\tau)wC_y}, & \overset{(-)}{-\tau\theta(\tau)NC_y}, & 0 \\ 0 & \overset{(+)}{1} & \overset{(+)}{-f_w} & 0 \\ 0 & \overset{(+)}{1} & \overset{(-)}{-(1-\tau\theta(\tau))N_w} & 0 \\ \overset{(+)}{\tau\theta(\tau)K} & \overset{(+)}{\tau\theta(\tau)w} & \overset{(+)}{\tau\theta(\tau)N} & \overset{(-)}{-1} \end{bmatrix}$$

and where D is obtained from C by replacing the last column of C with d where

$$d = \overset{(?)}{(\theta(\tau)+\tau\theta'(\tau))} \begin{bmatrix} \overset{(+)}{(wN+rK)C_y}+rC_r \\ 0 \\ \overset{(-)}{-N_w} \\ -(wN+rK) \end{bmatrix}$$

Expanding

$$|C| = -[I_r-\tau\theta(\tau)KC_y+(1-\tau\theta(\tau))C_r][f_w-(1-\tau\theta(\tau))N_w] < 0$$

and

$$|D| = (\theta(\tau)+\tau\theta'(\tau))\{[I_r+C_r-\tau\theta(\tau)(KC_y+C_r)]E-\tau\theta(\tau)KF\}$$

where

$$E = N_w(N+wf_w)\tau\theta(\tau)-(wN+rK)(f_w-(1-\tau\theta(\tau))N_w)$$

and

$$F = [(wN+rK)C_y+rC_r](f_w-(1-\tau\theta(\tau))N_w)$$

$$-N_w[\tau\theta(\tau)(N+wf_w)C_y+f_wF_N(N)(1-C_y)]$$

where the signs of E and F and hence $|D|$ are ambiguous. Setting K equal zero in $|C|$ and $|D|$ gives (11) in the text.

Again using Cramer's rule, replacing the third column of C with d, it is readily shown that

$$\frac{dw}{d\tau} = -\frac{(\theta(\tau)+\tau\theta'(\tau))N_w}{[f_w-(1-\tau\theta(\tau)N_w]} \gtrless 0 \text{ as } (\theta(\tau)+\tau\theta'(\tau)) \gtrless 0 .$$

Replacing the second column of C with d, by Cramer's rule

$$\frac{dN}{d\tau} = -\frac{f_w[(\theta(\tau)+\tau\theta'(\tau))N_w]}{[f_w-(1-\tau\theta(\tau))N_w]} \gtrless 0 \text{ as } (\theta(\tau)+\tau\theta'(\tau)) \lessgtr 0 .$$

Replacing the first column of C with d to form a matrix H, by Cramer's rule

$$\frac{dr}{d\tau} = \frac{|H|}{|C|} \gtrless 0$$

where

$$|H| = (\theta(\tau)+\tau\theta'(\tau))\{N_w[f_w(F_N(N)(C_y-1)-\tau\theta(\tau)wC_y)+\tau\theta(\tau)NC_y]$$

$$+ [(wN+rK)C_y+rC_r][f_w-(1-\tau\theta(\tau))N_w]\} \gtrless 0 .$$

B. Tax Revenue-Tax Rate Relationship in a Fixed Wage, Flexible Price Keynesian Model

It is possible to generate a Laffer curve from Keynesian-type models. This is illustrated here for the conventional fixed-wage-flexible-price Keynesian model.

The model is specified quite conventionally as follows (ignoring the time subscript on τ):

$$I(r)+G-\{y-C[(y-T),(1-\tau\theta(\tau))r]\} = 0 \tag{1b}$$

$$M(y,(1-\tau\theta(\tau))r)-M/p = 0 \tag{2b}$$

$$F_N(N)-\overline{W}/p = 0 \tag{3b}$$

$$w-\overline{W}/p = 0 \tag{4b}$$

$$y-F(N) = 0 \tag{5b}$$

$$\tau\theta(\tau)wN+\tau\theta(\tau)rK-T = 0 \tag{6b}$$

where $\overline{W}$ is the fixed money wage and all other variables and functions are as defined previously in the paper. Goods market equilibrium is given by (1b), money market equilibrium by (2b), the labor market is represented by (3b) and (4b), the production function by (5b), and the tax revenue function by (6b). The model may be represented more compactly by substituting (3b), (4b), and (5b) into (1b), (2b), and (6b) to give

$$I(r)+G-\{F(N)-C[F(N)-\tau\theta(\tau)(F_N(N)N+rK),\ (1-\tau\theta(\tau))r]\} = 0$$

$$M(F(N),(1-\tau\theta(\tau))r)-\frac{M}{\overline{W}}F_N(N) = 0$$

$$\tau\theta(\tau)F_N(N)N+\tau\theta(\tau)rK-T = 0$$

Totally differentiating this system of equation and using Cramer's rule yields

$$\frac{dT}{d\tau} = \frac{|B|}{|A|} \tag{7b}$$

where

$$A = \begin{bmatrix} 0 & \overset{(?)}{-(1-C_y)F_N(N)-\tau\theta(\tau)(F_{NN}(N)N+F_N(N))} & \overset{(-)}{I_r-\tau\theta(\tau)KC_y} \\ 0 & \overset{(+)}{M_yF_N(N)-\frac{M}{W}F_{NN}(N)} & \overset{(-)}{(1-\tau\theta(\tau))M_r} \\ \overset{(-)}{-1} & \overset{(?)}{\tau\theta(\tau)(F_{NN}(N)N+F_N(N))} & \overset{(+)}{\tau\theta(\tau)K} \end{bmatrix}$$

and where B is obtained from A by replacing the first column of A with b where

$$b = \overset{(?)}{(\theta(\tau)+\tau\theta'(\tau))} \begin{bmatrix} \overset{(+)}{C_y(F_N(N)+rK)} \\ \overset{(-)}{rM_r} \\ \overset{(-)}{-(wN+rK)} \end{bmatrix}$$

Even if Samuelson's correspondence principle is invoked to establish that $|A| < 0$, it is clear from inspection of the signs of the elements of A and b that the sign of $|B|$, and hence of (7b), is ambiguous. Again using Cramer's rule and assuming $|A| < 0$, if the second column of A is replaced with b to form a matrix C, it follows that (assuming $(\theta(\tau)+\tau\theta'(\tau))>0$)

$$\frac{dN}{d\tau} = \frac{|C|}{|A|} < 0$$

and from (5b) that

$$\frac{dy}{d\tau} < 0 .$$

Hence it is not difficult to see that while increasing the tax rate τ causes output to fall over the whole range of the tax rate $(0\leq\tau\leq 1)$, total tax revenue T could at first rise over some range of τ and then decline over the remainder.