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ABSTRACT

This paper studies the impact of cross-country variation in financial market development on firms' financing choices and growth rates using comprehensive firm-level datasets. We document that in less financially developed economies, small firms grow faster and have lower debt to asset ratios than large firms. We then develop a quantitative model where financial frictions drive firm growth and debt financing through the availability of credit and default risk. We parameterize the model to the firms' financial structure in the data and show that financial restrictions can account for the majority of the difference in growth rates between firms of different sizes across countries.

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1 Introduction

Financial restrictions can hinder firms' ability to use inputs efficiently and affect firm growth. Recent theoretical models of firm dynamics predict that limited credit makes inefficient small firms grow faster than large firms.[1] However, evidence for the magnitude of these effects in actual firm-level data is scarce. The central goal of this paper is to use cross-country variation in financial market development to evaluate empirically and quantitatively the impact of financial frictions on firms' financing choices and growth rates with firm-level datasets.

We analyze the relations of firm size with growth and debt financing using comprehensive firm-level data from 22 European countries that vary in financial market development. In our analysis, we focus on the *relative* behavior of firms of different sizes across countries with *varying* financial development, as indicated by the ratio of private credit to GDP and the coverage of credit information for consumers and firms. Consistent with theories of financial frictions, we find that small firms grow disproportionately faster than large firms in less financially developed countries. The growth rate differential across firms' sizes and countries is not only statistically significant but also economically important. We find that a 83 percentage points difference in the ratio of credit to GDP (as found between the United Kingdom and Finland) is associated with a 12 percentage points difference in growth rates between firms with asset shares equal to 0.01 and 0.0001.

We also find that small firms in less financially developed countries finance their assets with disproportionately less debt than large firms. Small firms tend to have higher leverage ratios than large firms on average. But this difference shrinks or even reverses as financial market development worsens. The relation of the debt financing patterns and financial market development is also economically sizable. For example, a 83 percentage points difference in the ratio of credit to GDP is associated with a 5 percentage points difference in leverage ratios across firms with asset shares equal to 0.01 and 0.0001. Importantly, all these findings are robust to controlling for country, industry, or age-specific characteristics.

Our empirical contribution consists of providing a systematic cross-country investigation of the interactions between financial market development, firm size, debt financing, and growth with firm-level datasets that include a large number of small private firms across multiple countries. The analysis focuses on the relative firm growth and financing using an extensive firm database covering many economies with varying financial development. This strategy allows us to identify more sharply the implications of financial frictions because we measure the additional effect that financial market development has on the differential

[1] Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Quadrini (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007), among others.

growth rate and debt financing, after controlling for a large set of fixed effects. For additional robustness of the results, we also include in the investigation measures that control for the two other leading theories for firm dynamics, which are based on selection mechanisms and mean reversion in the accumulation of factors of production. We find that even after introducing these controls, financial considerations continue to play a prominent role in the dynamics of firms.

In the second part of the paper, we develop a quantitative model of heterogeneous firms where default risk interacts with firm growth and debt financing. The model identifies the mechanisms that link firm growth to financial conditions, and allows us to perform a counterfactual exercise as well as a quantitative assessment of the theory. Credit restrictions arise in our model because debt is unenforceable and firms can default. Lenders offer firm-specific debt schedules that compensate for default risk and for a fixed credit cost they incur when issuing debt. We proxy differences in financial market development across economies with differences in the fixed credit cost. A high credit cost induces high default risk, which in turn limits credit. The debt schedules restrict credit disproportionately for small firms in less financially developed economies and make their scale inefficient. These small firms grow faster because they can expand their scale. Hence, in the model small firms in less financially developed economies have less debt financing and higher growth rates, as in the data.

The framework is a dynamic stochastic model where firms use a decreasing returns to scale technology to transform capital into output and face uncertain productivity. They finance capital and dividends with debt and profits and have the option to default on their debt. The restrictions on loans, due to default risk, impact firms' debt financing and capital choices. Increasing debt is useful for financing capital and dividends, but larger loans are costly because of higher default risk. Hence, firms prefer to shrink their capital and become inefficiently small to avoid excessively large loans. Firms can also be small simply because the persistent component of their productivity is low.

The firm-specific loan schedules determine their size and growth. Small unproductive firms have high default risk and thus face restricted loans, especially in less financially developed economies. Restricted credit makes them more likely to be inefficient in scale and hence to grow faster in response to good shocks because they use the additional output to increase their scale to a more efficient level. This implies that small firms grow faster in all economies, and particularly fast in economies with high credit costs and high default risk. Hence, our model matches the first empirical regularity that small firms grow faster than large firms especially in less financially developed economies.

The debt financing patterns across economies are determined by the firm-specific loan schedules and also by the history of shocks. Unproductive small firms face the most restrictive schedules, which tend to lower their equilibrium level of debt. But inefficient small firms have larger loans due, as they have built up debt after a history of bad shocks. These dynamics tend to increase the equilibrium level of debt of small inefficient firms. Hence, small firms can have higher or lower levels of debt than large firms. Nonetheless, as credit costs and default risk increase, the restrictions on loan contracts become so severe for the small unproductive firms that the level of debt of small versus large firms decreases. Thus, our model can match the second empirical regularity that the difference in debt financing of small and large firms decreases in less financially developed economies.

We quantitatively evaluate the model implications in rationalizing the cross-sectional financing and growth patterns jointly. We calibrate our model using the firm-level data of Bulgaria and the United Kingdom as representative countries with weak and strong financial market development. Our calibration strategy consists of choosing the parameters capturing the degree of debt enforceability to match the financing patterns observed in the cross section of firms in each country. The calibrated credit cost for Bulgaria equals 0.08% of output for the average firm and restricts credit such that the average debt to asset ratio equals 0.60. For the UK this cost is zero, which delivers a debt to asset ratio equal to 0.84. We then evaluate the model's predictions on growth rates for firms of different sizes. The model replicates quantitatively the observed patterns of sales growth and firm size in the Bulgarian calibration. For the UK calibration, the model delivers a substantial difference in growth rates between small and large firms, though smaller than the data.

With our calibrated model economy, we perform two counterfactual experiments to quantify how much of the differential growth and debt financing for firms in Bulgaria and the UK is due to the cross-country variation in financial markets versus the productivity structure. In the first experiment, we offer the UK financial market development to firms in the Bulgarian calibration. Consequently, the difference in growth rates between the small and large firms decreases from 0.37 to 0.18 and the difference in leverage ratios increases from -0.21 to 0.09. Better financial market development also increases the output of the small firms by 19%.

In the second experiment, we give the productivity and financial market development parameters of the UK calibration to firms in the Bulgarian calibration. We find that the difference in growth rates between small and large firms further decreases from 0.18 to 0.08, while the difference in leverage ratios increases from 0.09 to 0.11. Nonetheless, differences in productivity structure deliver differentials in growth rates and leverage ratios across firms only because of the presence of financial frictions. Hence, this experiment indicates that lack of enforcement in debt contracts is especially costly in economies with volatile productivity, as in Bulgaria.

These two experiments show that the differential growth and leverage ratios across firms and economies are largely driven by financial factors: both cross-country differentials in financial markets and cross-country productivity differentials in the presence of default risk.

Related Literature

Our empirical findings are novel because we are the first to examine the cross-sectional firm financing and growth patterns simultaneously across countries with a broad coverage of firms. In regard to growth, the cross-sectional firm-level analyses have considered only one country, as in Rossi-Hansberg and Wright (2007) for the United States.[2] In regard to firms' financing patterns, cross-country comparisons have been studied only for large public firms; Rajan and Zingales (1995) examine G7 countries, and Booth et al. (2001) study 10 developing countries. Public firms, however, constitute a small percentage of firms in all countries, which limits the scope of these previous findings.[3]

The theoretical model is related to the literature that studies the implications of financial frictions on firm growth. Our theory is closest to Cooley and Quadrini (2001), who develop a model where financing restrictions arise from limited commitment in debt contracts. They show that these frictions can potentially deliver large differences in the growth rates between small and large firms. In our paper, we use firm-level data to quantify the extent to which financial considerations impact growth rates. We further concentrate on how differences in financial market development can explain firm financing and growth patterns across countries. Our paper is also closely related to Albuquerque and Hopenhayn (2004), who analyze the effects of enforcement problems under a full set of state-contingent assets. In our model, we use incomplete markets to allow firms with a history of bad shocks to decrease their value and to allow precautionary savings to play a role.[4]

Apart from financial frictions, the two leading theoretical explanations for why small firms grow faster are based on selection mechanisms and mean reversion in the accumulation of factors of production. Hopenhayn (1992) and Luttmer (2007), for example, propose theories where the growth of small firms reveals a selection effect: small firms tend to exit with bad shocks, and so they grow faster when they survive after good shocks. Rossi-Hansberg and Wright (2007) develop a model where the mean reversion in the accumulation of industry-specific human capital makes small firms grow faster. We view these theories as complementary to the financial frictions theory. In fact, we find some empirical support for

[2] The cross-country analysis of growth has been restricted to industry-level data, as in Rajan and Zingales (1998).

[3] For example, in the United Kingdom less than 2% of firms in our dataset are public firms.

[4] Quadrini (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007) also study theoretically financial constraints that arise due to informational asymmetries between lenders and entrepreneurs.

these explanations. Nonetheless, theories of firm growth without financial frictions are silent (by construction) regarding the joint financing and growth patterns of firms across countries.

The paper is also related to the literature in corporate finance on the capital structure of firms.[5] Hennessy and Whited (2005) develop a dynamic model of debt financing and show that progressive taxes induce larger firms to use more debt financing. Interestingly, this theory is at odds with the data in the United Kingdom where corporate taxes are progressive, yet the relation between size and leverage is negative. Miao (2005) also studies optimal capital structure of firms in a model with endogenous exit in response to productivity shocks. In his model, firms choose debt only when they enter, yet small firms have higher leverage ratios because their equity value is small. In our model, the firm's debt choice is time varying and the interest rate on debt reflects endogenous default probabilities.

The rest of the paper is organized as follows. Section 2 presents the new empirical findings on firm growth and debt financing across countries with varying financial development. Section 3 introduces and characterizes the model. Section 4 presents the quantitative assessment of the model and counterfactual experiments. Section 5 concludes.

2 Empirical Facts

This section studies empirically the relation of firm size with debt financing and growth across countries. We use the cross-country variation in the development of their financial markets to identify the interaction of financial frictions with debt financing and firm growth. We find that the difference in debt financing and growth across firms of different sizes varies systematically with the country's financial market development. Small firms use disproportionately more debt financing than large firms in more financially developed countries. And small firms grow disproportionately faster than large firms in less financially developed countries.

In what follows, we first describe the firm-level database, Amadeus, which we use for the analysis of firms in Europe. We highlight our findings with two example countries: the United Kingdom and Bulgaria. We then present our main empirical findings regarding the debt financing and growth patterns of firms in 22 European countries.

2.1 Data Description

The data source is Amadeus, which is a comprehensive European database. Amadeus contains financial information of over 7 million private and public firms in 38 European countries

[5] See Harris and Raviv (1991) for a comprehensive review.

covering all sectors in the economy. Nonetheless, the coverage of Amadeus is limited for some countries. Given our aim to document firms' financing and growth patterns for a comprehensive and representative sample of firms, we need to select the countries for which Amadeus contains a sufficiently large number of firms.

We first exclude countries that do not require private firms to report their balance sheets. We next use a simple criterion to select the countries that have a ratio of the number of firms reporting positive assets to PPP-adjusted GDP larger than 20% of the ratio for the United Kingdom in 2005. The dataset for the United Kingdom in AMADEUS is especially attractive because it contains the largest number of firms by far relative to all the other countries. These criteria leave us with 22 countries: Belgium, Bulgaria, Croatia, Czech Republic, Denmark, Estonia, Finland, France, Iceland, Ireland, Italy, Latvia, Lithuania, Malta, the Netherlands, Norway, Portugal, Romania, Russian Federation, Spain, Sweden, and the United Kingdom.[6] In the appendix we show that the datasets for these 22 countries are in fact quite comparable and representative of the universe as reported by the European Commission.

We examine the firms' balance sheet data for these 22 countries in 2004 and 2005. Firm size is measured by the book value of the total assets of the firm. To measure debt financing, we compute the firm's leverage ratio in 2005. Leverage is defined as the broad measure of total liabilities over total assets of the firm. We use this broad definition because it is a more consistent measure across countries and because it provides the largest sample of firms. Firm growth is measured by the net real growth rate of sales from 2004 to 2005, adjusted by the CPI in each country. We exclude firms in the financial and government sectors following Rajan and Zingales (1995). We also clean the data by restricting the sample to firms that report positive assets and non-negative liabilities each year. For the growth statistics, we further restrict the sample to firms that also report positive sales in both 2004 and 2005. Finally, we remove firms with outlier observations of growth and leverage in the top 1st percentile.[7]

The development of financial markets in these 22 countries is measured using two statistics. The first one is the average private credit to GDP ratio over 2000–2004 taken from the *World Development Indicators*. Higher ratios of private credit to GDP indicate better financial development. The second measure is the coverage of credit registries. Credit registries in countries track the loans and defaults of individuals and firms and facilitate lending by banks and financial institutions. The statistic we use is the percentage of adults that are

[6] The threshold of 20% is not important. If we use a threshold of 15%, only Slovak is added to the sample of countries.

[7] The appendix contains more details about the data cleaning procedure.

included in the public and private credit registries in 2005 in each country.[8] Larger credit bureau coverage indicates better financial development because it implies that it is easier for financial intermediaries to make loans when credit information of borrowers is available. Credit bureau coverage is taken from the Doing Business publications of the World Bank.

Table 1 reports descriptive statistics for the firm-level datasets and the two measures of financial markets development for each country. Countries are ordered by their level of private credit to GDP. The table shows the variability of financial development is large across these 22 countries. For example, the private credit to GDP ratio is 143% in the Netherlands and only 18% in Russia; the credit bureau coverage is 100% in Sweden and 0% in Croatia. As expected, these two financial development indices are highly correlated in our sample with a correlation of 0.64.

Table 1: Summary of Firm-Level Datasets and Financial Development

	Firm-Level Datasets					Financial Development	
	Mean Asset	Median Asset	Mean Leverage	Mean Growth	No. Firms	Credit Coverage (%)	Credit to GDP (%)
Denmark	5909	365	0.58	0.16	116726	7.7	147
Netherlands	13791	523	0.92	0.11	147754	68.9	143
United Kingdom	13269	86	0.84	0.11	846910	76.2	143
Portugal	2750	159	0.80	0.12	198162	63.7	138
Iceland	3295	129	0.91	0.59	16528	100	120
Ireland	7588	202	0.91	0.18	86736	100	116
Spain	5023	405	0.75	0.26	526455	42.1	109
Malta	11186	887	0.75	0.33	1749	—	108
Sweden	6496	197	0.62	0.18	192240	100	91
France	5102	215	0.74	0.09	802371	1.8	87
Norway	5020	261	0.78	0.26	144400	100	83
Italy	5247	650	0.81	0.12	528374	59.9	81
Belgium	4000	236	0.74	0.07	290332	55.3	75
Finland	4933	153	0.56	0.16	73556	14.7	60
Croatia	4729	318	0.66	0.04	18942	0.00	48
Czech Republic	3664	168	0.76	0.32	57302	24.9	37
Latvia	3068	576	0.71	0.43	4596	0.6	34
Estonia	585	34	0.42	0.54	50326	12.5	29
Bulgaria	2227	86	0.65	0.53	29731	13.6	22
Lithuania	4273	622	0.61	0.58	6006	4.4	19
Russia	4671	73	0.79	0.63	163628	0.0	18
Romania	307	16	0.98	0.46	419251	1.4	11

The mean and median level of assets for firms in each country are reported for 2005 in

[8] We use data for 2005 because this statistic is not available for many countries before 2005.

terms of current euros in the table. Firm asset levels vary across countries, and they tend to be larger in countries with more developed financial markets. Moreover, the distribution of firms in all countries is highly skewed, as the mean asset levels are much larger than the median asset levels. We also report the average leverage ratio and the average growth rate across all firms in each country. Both mean leverage and mean growth vary substantially across countries. The mean leverage ratio is 0.92 in the Netherlands, but only 0.42 in Estonia; the mean net growth rate is 11% in the Netherlands, but 54% in Estonia. The table also reports the number of firms with positive assets and liabilities in the dataset of each country.

Overall, these aggregate statistics are systematically related to financial market development. First, firms in countries with more developed financial markets tend to have larger leverage ratios. The cross-country correlation of mean leverage and the private credit to GDP ratio is 0.31, and the correlation of mean leverage and the credit bureau coverage is 0.43. Second, firm growth is on average smaller in countries with better financial development. The cross-country correlation of mean growth and the private credit to GDP ratio is -0.58, and the correlation of mean growth and the credit bureau coverage is -0.29. Third, firms in countries with more developed financial markets are larger. The correlation of the mean asset level and private credit to GDP equals 0.65, and the correlation of the mean asset level and credit coverage is 0.44.

2.2 Example: United Kingdom and Bulgaria

To provide a stark illustration of our main empirical findings, we analyze two example countries that differ substantially in their financial market development: the United Kingdom and Bulgaria. Let's first consider the unconditional relation of leverage and firm size in Bulgaria and in the United Kingdom. To this end, we divide firms in each country into 10 quantiles according to their assets and compute their leverage ratios. Figure 1 plots the mean leverage ratio of firms in each quantile in Bulgaria and the UK for the year 2005. The figure illustrates the remarkably distinct pattern of size and leverage across countries. In the UK the leverage-size relation is generally downward sloping: small firms have relatively higher leverage ratios than large firms. In particular, the mean leverage ratio of the smallest firms is above 1 and that of the largest firms is 0.66.[9] In Bulgaria, the difference in leverage ratios between small and large firms shrinks, and in fact the leverage-size relation is generally increasing, ranging from 0.35 for the smallest firms to 0.69 for the largest firms.[10]

[9]When leverage is greater than 1, firms have negative equity. Herranz, Krasa, and Villamil (2008) document that 21% of the small firms in the United States have negative equity in 1998.

[10]In an earlier version of this paper, we documented that in Ecuador, which has a degree of financial development similar to that in Bulgaria, small firms have lower leverage ratios than large firms, as we document here for Bulgaria.

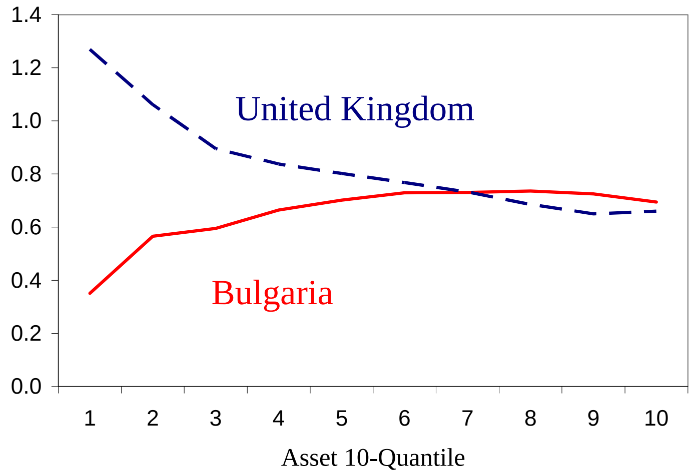

Figure 1: Firm Size and Leverage

The relation between firm size and firm growth is also different across these two countries. To analyze the unconditional relation of growth and size, we again divide firms in each country into 10 quantiles according to their assets in 2004 and compute average sales growth from 2004 to 2005 for each quantile. Figure 2 reports the mean sales growth rate for firms in each asset quantile in Bulgaria and in the UK. The figure illustrates that small firms grow faster than large firms in both countries. The difference in growth rates of small and large firms, however, is bigger in Bulgaria than in the UK. Small British firms in the first asset quantile grow at the rate of 54%, whereas large British firms in the tenth asset quantile grow at the rate close to zero. Small Bulgarian firms, however, grow at the rate of 157%, while large Bulgarian firms grow at about 12%.

Our findings for the UK and Bulgaria suggest that the size-growth and size-financing patterns are related to the development of the financial market in each country. To establish these observations for a broad country sample, we start by analyzing the unconditional relations of firm size with growth and leverage for all sample countries. In every country we divide firms into asset quintiles according to their assets, and for every quintile we compute mean growth and mean leverage. Table 2 reports these statistics. We find that across these 22 European countries, small firms have, on average, higher leverage ratios and higher growth rates than large firms. We then look at the unconditional correlations of the difference in growth rates and leverage ratios of firms in the smallest quintile and in the largest quintile with financial development across countries. The correlations of the growth difference with private credit to GDP and the credit coverage equal -0.63 and -0.41, respectively. The correlations of the leverage difference with private credit to GDP and with credit coverage both

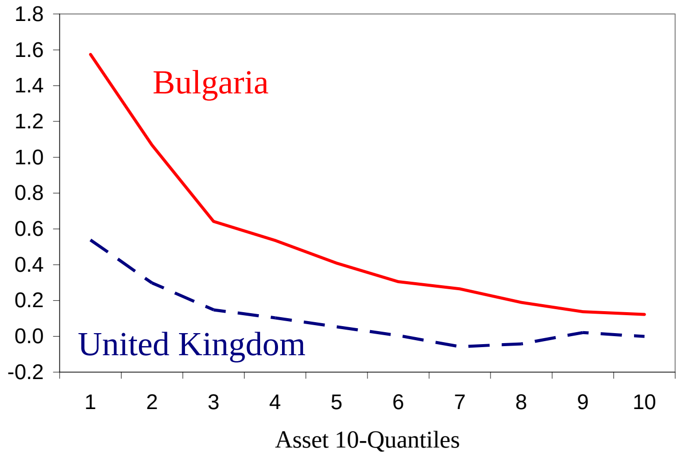

Figure 2: Firm Size and Sales Growth

equal 0.42. These unconditional correlations confirm the unconditional patterns observed in the UK and Bulgaria. In the next subsection, we further examine these relations using a detailed set of controls and for comprehensive firm-level datasets.

2.3 Cross-Country Empirical Findings

Our hypothesis is that in countries with more developed financial markets, small firms have higher leverage ratios and lower growth rates relative to large firms. Therefore, we pool all the countries together and estimate two regressions of the following forms:

$$Leverage_{k,c}(\text{or } Growth_{k,c}) = \beta_0 + \beta_1 \log(Asset\ Share_{k,c}) \quad (1)$$

$$+\beta_2 \log(Asset\ Share_{k,c}) \times Financial\ Development_c + Dummy\ Variables + \nu_{k,c},$$

where c denotes the country, and k the firm. The dependent variable is the firm's leverage ratio for the leverage regressions and the firm's real sales growth rate for the growth regressions. $Asset\ Share_{k,c}$ is the share of the firm k's assets in the total assets of country c. Given the highly skewed firm size distribution, we use the log of firms' asset shares as firm size. $Financial\ Development_c$ corresponds to the two measures of financial development in country c, namely, private credit over GDP and coverage of credit registries. The term *Dummy Variables* corresponds to fixed effects at the country $\times$ industry $\times$ age level. Hence, the regression gives each country $\times$ industry $\times$ age group an independent intercept.

The regression specification controls for country-specific effects, 2-digit industry-specific effects, and 7 age-group-specific effects. Country effects control for any country characteristic,

Table 2: Unconditional Leverage and Growth across Asset Quintiles

	Leverage					Growth				
	Asset Quintiles					Asset Quintiles				
	1	2	3	4	5	1	2	3	4	5
Belgium	0.98	0.69	0.68	0.67	0.66	0.13	0.05	0.05	0.05	0.05
Bulgaria	0.45	0.63	0.72	0.73	0.71	0.73	0.51	0.51	0.49	0.39
Croatia	0.63	0.67	0.68	0.68	0.64	0.01	0.01	0.02	0.05	0.08
Czech Rep	0.89	0.87	0.76	0.69	0.58	0.66	0.36	0.25	0.17	0.17
Denmark	0.54	0.60	0.59	0.58	0.58	0.25	0.13	0.11	0.15	0.17
Estonia	0.15	0.40	0.50	0.53	0.54	0.88	0.56	0.45	0.37	0.43
Finland	0.51	0.58	0.57	0.56	0.56	0.26	0.15	0.12	0.12	0.13
France	0.86	0.75	0.72	0.70	0.66	0.17	0.08	0.06	0.06	0.06
Iceland	1.10	1.00	0.88	0.83	0.74	0.91	0.55	0.36	0.50	0.61
Ireland	1.54	0.95	0.76	0.66	0.63	0.24	0.20	0.17	0.15	0.16
Italy	0.79	0.83	0.83	0.81	0.77	0.16	0.14	0.11	0.09	0.08
Latvia	0.76	0.75	0.71	0.69	0.66	0.78	0.42	0.27	0.33	0.34
Lithuania	0.65	0.64	0.61	0.59	0.58	1.46	0.50	0.36	0.31	0.29
Malta	1.01	0.77	0.72	0.66	0.61	0.57	0.35	0.34	0.13	0.26
Netherlands	1.38	0.95	0.82	0.75	0.69	0.20	0.12	0.07	0.07	0.10
Norway	0.86	0.80	0.78	0.76	0.70	0.44	0.23	0.21	0.20	0.21
Portugal	0.94	0.79	0.77	0.75	0.73	0.24	0.12	0.09	0.08	0.08
Romania	1.32	1.05	0.91	0.81	0.77	0.84	0.57	0.42	0.25	0.18
Russia	0.88	0.85	0.78	0.74	0.69	1.09	0.78	0.59	0.36	0.34
Spain	0.86	0.80	0.75	0.70	0.65	0.45	0.22	0.17	0.19	0.25
Sweden	0.56	0.61	0.62	0.63	0.66	0.28	0.16	0.15	0.15	0.17
UK	1.18	0.87	0.79	0.71	0.66	0.23	0.13	0.05	0.07	0.05
Average	0.89	0.76	0.72	0.69	0.66	0.51	0.30	0.23	0.20	0.21

for instance, business cycles, institutional quality, the legal system, the political system, and many others. Industry effects are at the 2-digit level constructed with NACE codes. They control for any inherent features of industries, including capital intensity, competition structure, liquidity needs, and tradability. The 7 age groups are constructed at 5-year intervals up to 30 years and a final group for firms with age greater than 30 years. Age effects control for any inherent life cycle features of firms, such as market share and technological development.

As discussed in Rajan and Zingales (1998), the use of fixed effects enables us to control for a much wider array of omitted variables. These dummy variables will capture the peculiar features of each age group within each sector of each country, such as the particular technological characteristics or specific tax treatments varying at the country $\times$ industry $\times$ age level. Only additional explanatory variables that vary within each of the industry-country-age groups need be included. These are firm size and the primary variable of interest, the interaction between firm size and financial market development. According to our hypothesis, we must find the coefficient estimate for the interaction between size and financial development to be negative in the leverage regression and to be positive in the growth regression.

Table 3 reports the regression results using the two measures of financial development. The first two columns report the leverage regressions, and the last two columns report the growth regressions. For the regressions using coverage of credit registries, we drop Malta because this statistic is not available for this country. We report the coefficient on firm size and the coefficient on the interaction term between firm size and financial market development in the table. The standard errors of the regression coefficients are reported in parentheses and are robust to heteroskedasticity throughout the paper.

Table 3: Firms' Leverage, Growth, and Financial Development

	Leverage		Growth	
	Private Credit to GDP	Credit Bureau Coverage	Private Credit to GDP	Credit Bureau Coverage
Size (log(firm's asset share))	-0.039***	-0.044***	-0.052***	-0.029***
	(0.0003)	(0.0003)	(0.0009)	(0.0006)
Interaction (credit to GDP × size)	-0.012***		0.030***	
	(0.0003)		(0.0007)	
Interaction (credit bureau coverage × size)		-0.016***		0.007***
		(0.0003)		(0.0006)
Fixed effects	Yes	Yes	Yes	Yes
Country × Industry × Age				
Adjusted R^2	0.06	0.06	0.05	0.05
Number observations	4564461	4563685	2568782	2568559
Number of groups	4773	4662	4550	4486

Let's start with the regression that analyzes the size-leverage relation. The estimated coefficient on the interaction variable is negative as expected and statistically significant at the 1% level under both measures of financial market development. The coefficient estimate on size is also negative and statistically significant under both measures. Thus, smaller firms have on average higher leverage ratios than large firms, other things being equal. Moreover, when private credit to GDP or credit bureau coverage increases, the leverage ratios of small firms relative to large firms increase.

The interaction term is similar to a second derivative. To interpret its magnitude, let's look at the regression with private credit to GDP and compare a small firm with an asset share equal to 0.001% to a large firm with an asset share equal to 1% in Bulgaria and the United Kingdom. The leverage difference between these comparable small and large firms is 6.7 percentage points higher in the UK than in Bulgaria, as private credit to GDP is higher in the UK by 121 percentage points. These numbers are economically significant given that the mean leverage ratio for Bulgaria equals 0.65.

Let's now look at the regressions that analyze the size-growth relation. Size continues to be a significant determinant; smaller firms grow faster overall. The estimated coefficients on the interaction term are positive as expected and statistically significant at the 1% level for both measures of financial development. That is, the growth difference between small and large firms decreases with both private credit to GDP and credit bureau coverage. We can interpret the coefficient on the interaction of private credit to GDP and size as follows. The difference in growth rates of a small firm with an asset share equal to 0.001% relative to a large firm with an asset share equal to 1% is 17 percentage points less in the United Kingdom than in Bulgaria.

2.4 Robustness Tests

In this section we perform robustness on the main results by considering alternative explanations for the negative relation between firm size and growth in addition to financial frictions. One important theoretical explanation of the growth-size relation is the *selection* theory: small firms are more likely to exit under adverse shocks and thus tend to have higher growth rates conditional on survival. If selection differs across countries, one concern is whether our results are robust when we control for such variation. Unfortunately, our dataset does not have precise information on firm exit. Nevertheless, we proxy the degree of selection by the mean growth rate of firms in each country because this theory implies that average firm growth should be higher in countries where selection is more important. Specifically, we add an interaction term between firm size and mean growth to the main regressions. The results are reported in Table 4 .

We find that the coefficient of the interaction between firm size and mean growth is significantly negative as expected by the selection theory. However, even after we control for selection, the coefficients of the interaction between firm size and financial market development continue to be significant and positive as in the main regressions. That is, small firms tend to grow relatively faster than large firms in less financially developed countries.[11]

In a recent work, Rossi-Hansberg and Wright (2007) propose another theory for the relation between firm size and growth based mean reversion in the accumulation of factors.

[11] We also examine the sample attrition issue and find that the size-attrition relation is uncorrelated with financial development. For details, see the appendix.

Table 4: Robustness on the Growth Regression

	Private Credit to GDP		Credit Bureau Coverage	
Size (log(firm's asset share))	-0.017***	-0.013***	-0.009***	-0.006
	(0.0009)	(0.005)	(0.0005)	(0.004)
Interaction (Financial Development × size)	0.008***	0.011***	0.004***	0.007***
	(0.0007)	(0.001)	(0.0005)	(0.0006)
Interaction (Mean growth × size)	-0.084***	-0.088***	-0.096***	0.007***
	(0.002)	(0.002)	(0.002)	(0.0006)
Interaction (Industry × size)	No	Yes	No	Yes
Fixed effects	Yes	Yes	Yes	Yes
Country × Industry × Age				
Adjusted R^2	0.06	0.06	0.06	0.06
Number observations	2568782	2568782	2638072	2638072
Number of groups				

In their model the growth difference between small and large firms is larger in sectors that use physical capital more intensively. They also document that in the United States, the growth rate of firms declines faster with size in the manufacturing sector than in the service sector. To control for the industry effect on the size-growth relation, we add an additional interaction term between firm size and two-digit industry categories to the main regressions in addition to the interaction between size and mean growth. With this added interaction variable we allow the relation of size with growth to be industry dependent. As shown in Table 4, the main regression results remain almost unchanged for both measures of financial development.

We also conduct similar robustness tests on the leverage regressions. The results are reported in Table 5. The estimated coefficients on the interaction terms of firm size and private credit to GDP have the same sign and the same significance under all of these alternative specifications as in the main regressions. The same is true for the estimated coefficients on the interaction of firm size and credit bureau coverage.

Finally, we add two additional interaction terms: the interactions of firm size with the seven age groups and with the country's GDP per capita. These variables allow for the relation of size with growth to be age dependent and to vary with the log of the country's GDP per capita. We find that our main results are robust to adding these two additional interaction terms. We also conduct robustness checks by using employment as an alternative measure of size and find that the main results are unchanged. All these details are reported

Table 5: Robustness on the Leverage Regression

	Private Credit to GDP		Credit Bureau Coverage	
Size (log(firm's asset share))	-0.030***	-0.037***	-0.043***	-0.045***
	(0.0005)	(0.002)	(0.0003)	(0.002)
Interaction (Financial Development × size)	-0.017***	-0.017***	-0.016***	-0.015***
	(0.0004)	(0.0004)	(0.0003)	(0.0003)
Interaction (Mean growth × size)	-0.023***	-0.026***	-0.005***	0.006***
	(0.0009)	(0.0009)	(0.0007)	(0.0007)
Interaction (Industry × size)	No	Yes	No	Yes
Fixed effects Country × Industry × Age	Yes	Yes	Yes	Yes
Adjusted R^2	0.06	0.06	0.06	0.06
Number observations	4564461	4564461	4563685	4563685

in the appendix.

In summary, we find that small firms use less debt financing and grow disproportionately faster than large firms in countries with worse credit bureau coverage and lower ratios of private credit to GDP. These empirical findings are important for providing a comprehensive picture of the relations of financial market development with financing and growth across firms and across countries.

3 Model Economy

To study theoretically firms' financing choices and dynamics, this section presents a dynamic model of heterogeneous firms that face default risk. The model builds on Cooley and Quadrini (2001) and Albuquerque and Hopenhayn (2004) while incorporating differentiation across economies in the development of their financial markets. In the model, entrepreneurs decide on the level of capital and debt financing for their firms. Credit restrictions arise because debt is unenforceable and firms can default. Lenders offer firm-specific debt schedules that compensate for default risk and for a fixed credit cost they incur when issuing debt. We proxy financial market development with credit cost because large costs induce high default risk, which limits the economy-wide credit.

3.1 Firms

Entrepreneurs in the economy are infinitely lived and have access to a mass one of risky project opportunities, which we refer to as firms. Each entrepreneur owns at most one firm and decides on entry, exit, production, and financing plans to maximize the present value of dividends.

An operating firm starts the period with capital K and debt B_R. It produces output with a stochastic decreasing returns technology with capital as input: $y = zK^{\alpha}$, where $0 < \alpha < 1$ and the productivity of the project z follows a Markov process given by $f(z', z)$. It finances the new capital K' and dividends D with internal funds which consist of the firm's output net of debt repayment $zK^{\alpha} - B_R$ and with external funds by acquiring a new loan B'. We define the leverage of this firm as the ratio of total debt due to capital installed B_R/K if $B_R \geq 0$. If the firm starts with assets $B_R < 0$, the firm has no liabilities due, and thus its leverage ratio is equal to zero.

The timing of decision for an operating firm within the period is as follows. At the beginning of the period, δ fraction of firms exit exogenously. All surviving firms receive their shocks. An entrepreneur with debt B_R, capital K, and shock z decides whether or not to default. If the entrepreneur repays his debt, he chooses a new loan, capital for the following period, and dividends. If the entrepreneur defaults, the firm exits.

We lay out the recursive formulation for the entrepreneur operating a firm. Upon observing the shock realization, the entrepreneur decides whether to default by comparing the default value V^d with the repayment value V^c:

$$V(K, B_R, z) = \max_{d \in \{0,1\}} (1 - d)V^c(K, B_R, z) + dV^d(z), \tag{2}$$

where $V(K, B_R, z)$ denotes the present value of the firm to the entrepreneur. The entrepreneur's default decision can be represented by a binary variable $d(K, B_R, z)$ that equals 1 if default is chosen and 0 if repayment is chosen.

If the entrepreneur chooses to default, his debt is written off, but he loses the project. We assume that with probability θ the entrepreneur can start a new project with the same productivity z. The default value is then given by

$$V^d(z) = \theta V^e(z),$$

where $V^e(z)$ denotes the value of a potential entrant with productivity z.

If the entrepreneur repays his debt, he keeps his project in operation and decides on production and financing. Given the set of loan contracts, the entrepreneur chooses the amount to be received from the creditor this period B' and the amount to be repaid the following period B'_R conditional on not defaulting, capital K', and dividends D to maximize the repayment value:

$$V^c(K, B_R, z) = \max_{\{B', B'_R, K', D\}} D + \beta(1-\delta)EV(K', B'_R, z') \tag{3}$$

subject to a non-negative dividend condition given by

$$D = zK^\alpha - B_R + B' - K' \geq 0, \tag{4}$$

where $\beta < 1$ denotes the discount rate of the entrepreneur. $V^c(K, B_R, z)$ is increasing in K and decreasing in B_R, and $V^d(z)$ is independent of these variables. Thus, default is more attractive for firms with smaller capital and larger debt due.

Optimal debt is determined by trading off costs and benefits of various loans within the set of contracts offered. Debt is beneficial for financing investment. Debt can also be used for dividends, which is attractive when loans are cheap and entrepreneurs discount the future heavily. In addition, debt can be used to relax the non-negative dividend condition when the firm's output is low and the loan due is large. On the other hand, large debt is costly because it can lead firms to default. In particular, a large loan today implies a large repayment the next period that will be costly especially when the productivity shock is low. In this case, income might be so low that the entrepreneur fails to satisfy the non-negative dividend condition, defaults, and loses the project. In anticipation of these possible adverse outcomes, the entrepreneur might have precautionary motives to reduce debt and save.[12]

In our model with limited enforceability of debt contracts, financing decisions interact with firms' investment. In contrast, in an environment where non-contingent contracts are perfectly enforceable and the non-negative dividend condition is relaxed, firms choose capital such that the expected marginal product of capital equals the risk-free rate:

$$E(z)\alpha K_{fb}(z)^{\alpha-1} = (1+r). \tag{5}$$

We refer to this level of capital $K_{fb}(z)$ as first-best capital for a firm with expected productivity equal to $E(z)$.

With enforcement frictions, investment also depends on the set of loan contracts available. In particular, investment is distorted downward. For example, if a firm starts with large debt,

[12] Contrary to Cooley and Quadrini (2001), our model does not impose that debt is used for capital only, which adds a lower and an upper bound on debt. This feature gives more room for the precautionary savings usage and allows a better match of the data where many firms have negative equity.

it might want to borrow a big loan B' to satisfy the non-negative dividend condition and to keep the investment level at the unconstrained optimal. Nonetheless, given that the set of loans is bounded due to possible defaults, such a big loan might not be offered to the entrepreneur. Hence, the entrepreneur might have to reduce the level of investment, making the project inefficiently small.

The problem for a potential entrant is simple in this model. Whenever an idle entrepreneur receives a project opportunity of productivity z, he decides to undertake the project and enter if the expected value of the project is positive. Thus, the value for a potential entrant is given by

$$V^e(z) = \max\{0, V^c(0,0,z)\}.$$

Note that the new entrant starts with no assets and thus the value conditional on entering is exactly equal to the value of the contract $V^c(0,0,z)$ when K and B_R are equal to zero.

3.2 Loan Contracts

In the model, financial frictions are embodied in the schedule of loan contracts that firms face. The schedule of potential loans that firms choose from in turn depends on their default decisions. A loan contract (B', K', B'_R, z) specifies that a firm with productivity z receives B' from the creditors the current period, pays back B'_R the next period conditional on not defaulting, and invests K'.[13] In addition, creditors have to pay a fixed credit cost ξ for every loan they offer. Debt schedules include all contracts that allow creditors to break even in expected value such that:

$$B' + \xi = \frac{B'_R(1-\delta)}{(1+r)} \left(1 - \int d(K', B'_R, z') f(z', z) dz'\right) \text{ for } B' > 0. \tag{6}$$

The left-hand side of equation (6) are the resources creditors spend today. The right-hand side is the expected repayment discounted by the risk-free rate and the death shock.

Default risk determines the availability and the terms of debt contracts. If a firm that invests K' and has productivity z wants a larger transfer B' today, it will need to promise an increasingly larger repayment tomorrow B_R because of higher default risk. Moreover, for every K' and z the schedule contains an upper bound $\bar{B}'$, which is associated with excessively high default risk, that limits the possible firm's borrowings. However, firms can improve the availability and terms of their loans by choosing a higher investment K'. Increasing capital

[13] We generalize the endogenous debt schedules developed in Chatterjee et al. (2007) and Arellano (2008) in their study of unsecure consumer credit and sovereign default by adding an interaction of capital and default risk in the study of firm dynamics.

makes firms less likely to default and hence allows for larger and more favorable loans.[14]

The credit cost ξ affects the availability of debt through its impact on default risk. In particular, a high credit cost increases the risk of default by raising the costs of financing. Higher aggregate default risk in turn limits the economy-wide availability of credit. However, the impact of credit costs ξ on firm-specific default risk also depends on the firm's level of capital and productivity. In general, high ξ increases default risk disproportionately for firms with low capital and productivity.

One can rationalize the expense of ξ as costs lenders pay to obtain information about the entrepreneur's total debt. Knowing this information is necessary for the lender to correctly assess the probability of default of each entrepreneur. We interpret ξ as the economy's ease to acquire credit information, and it controls financial market development of the model economy. The parameter ξ can be naturally linked to the coverage of credit registries across countries as well as the aggregate level of credit. When ξ is low, credit registries in the economy have wide coverage, and it is very easy and cheap to access credit information. When ξ is large, the lender has to spend some resources to screen the entrepreneur and obtain his debt information.[15] As documented in the empirical section, the coverage of credit registries across countries varies widely, and this variable is linked to the ways firms grow and finance their assets. Thus, our model focuses on variation in ξ to capture differences in financial market development across economies.

Entrepreneurs in our model can also save $B' < 0$. We assume that when the entrepreneur saves creditors do not need to pay ξ as default probabilities are zero. Savings contracts satisfy the following condition:

$$B' = \frac{(1-\delta)}{(1+r)} B'_R \text{ for } B' \leq 0. \tag{7}$$

3.3 Equilibrium

Before defining the equilibrium of this economy, we make an assumption on the relation between the risk-free rate and the discount factor of entrepreneurs. The assumption imposes that the rate at which entrepreneurs discount the future is higher than the risk-free rate. This condition can be interpreted as a general equilibrium property of economies with lack

[14] Our model shares this additional benefit of capital of relaxing borrowing constraints with many models of collateral constraints such as Kyotaki and Moore (1997) and Cagetti and De Nardi (2006).

[15] This specification of credit issuance costs is similar to the one used in Livshits, MacGee, and Tertilt (2008). They document that improvements in credit scoring in the United States are important for understanding the rise in bankruptcies and volume of debt.

of enforcement and incomplete markets.[16]

Assumption 1 The risk-free rate r is such that $1/\beta - 1 > r > 0$.

The model delivers an endogenous distribution of firms, denoted by $\Upsilon(K, B_R, z)$, which depends on the decisions of firms to borrow and invest. Whenever existing firms exit either exogenously or endogenously, their z projects become available to potential entrant entrepreneurs such that the mass of projects is always equal to one. New entrants start their operation with zero capital and zero loans. Thus, the measure of entrants $\mu(z)$ is given by the following:

$$\mu(z) = \int [(1-\delta)d(K, B_R, z) + \delta]\, \Upsilon(K, B_R, z)\mathbf{d}(K \times B_R).$$

The evolution of the distribution of firms is given by

$$\Upsilon'(K', B'_R, z') = \int Q((0,0,z),(K', B'_R, z'))\mu(z)\mathbf{d}z +$$

$$(1-\delta)\int (1 - d(K, B_R, z))Q((K, B_R, z),(K', B'_R, z'))\Upsilon(K, B_R, z)\mathbf{d}(K \times B_R \times z), \tag{8}$$

where $Q(\cdot)$ denotes a transition function that maps current states into future states. The distribution of firms the following period includes the set of surviving firms that do not default and do not receive the death shock. It also includes the new firms that enter after project opportunities from the exiting firms become available.

The *recursive equilibrium* for this economy consists of the policy and value functions of firms, the loan contracts offered by creditors, and the distribution of firms such that (i) given the schedule of loan contracts offered, the policy and value functions of firms satisfy their optimization problem; (ii) loan contracts reflect the firm's default probabilities such that with every contract creditors break even in expected value; (iii) the distribution of firms follows (8) and is consistent with the policy functions of firms and shocks given the initial distribution Υ_0.

3.4 Borrowing Limits and Financial Development

Limited enforceability of debt contracts generates endogenous borrowing limits for firms because creditors do not provide loans that will be defaulted on in all future states. These

[16]If $\beta(1+r) = 1$, firms strictly prefer to accumulate assets rather than distribute dividends because of the additional benefits of assets in terms of avoiding firm failure. This would generate an excessive supply of loans that would in turn drive down the risk-free rate.

borrowing constraints play a key role in determining optimal debt. Moreover, borrowing limits vary across firms and with financial market development. In particular, weak financial development limits borrowing relative to assets. And this limitation is more severe for small firms than for large firms.

We provide an analytical characterization of these findings by considering the case when firms are heterogeneous with respect to z yet this productivity is constant over the firm's lifetime. In addition, for simplicity we assume that firms do not face the death shock ($\delta = 0$). We also impose an assumption on credit costs. This assumption guarantees that firms have an incentive to borrow to the limit every period. It also ensures that the borrowing limit is at least as large as the first best level of capital for all firms.

Assumption 2 Credit costs are such that $\xi \leq \left(\frac{\alpha z}{1+r}\right)^{\frac{1}{1-\alpha}} \frac{1-\alpha}{\alpha}, \ \forall z.$

When productivity is certain and constant over time, firms will either repay or default with probability one on any loan. Thus, there is no equilibrium default, as loans that will be defaulted upon with probability one are not offered. Hence, loan contracts available are offered at the risk-free rate. Let $\overline{B}(z)$ denote the borrowing limit of a firm with productivity z and $\overline{B}_R(z) = (1+r)(\overline{B}(z) + \xi)$ denote the associated debt repayment limit.

The assets of the firm are equal to the level of capital $K_{fb}(z)$, which is constant over time at the first best level, as its return is equalized in equilibrium to the constant return on bonds. Also the firm wants to borrow to the limits given $\beta(1+r) < 1$. Hence, the value of a new entrant firm with productivity z is given by

$$V^c(0,0,z) = [\overline{B}(z) - K_{fb}(z)] + \beta[zK_{fb}(z)^\alpha - K_{fb}(z) - r\overline{B}(z) - (1+r)\xi]/(1-\beta).$$

The value of any existing firm with productivity z and debt repayment $\overline{B}_R(z)$ is given by

$$V^c(K_{fb}(z), \overline{B}_R(z), z) = [zK_{fb}(z)^\alpha - K_{fb}(z) - r\overline{B}(z) - (1+r)\xi]/(1-\beta).$$

The borrowing limit for a firm with productivity z is the level of debt that makes the contract value equal to the default value, and is given by

$$V^c(K_{fb}(z), \overline{B}_R(z), z) = \theta V^c(0,0,z).$$

Hence, we derive the debt limit as

$$\overline{B}(z) = \kappa_1 K_{fb}(z) - \kappa_2 \xi, \tag{9}$$

where both κ_1 and κ_2 are positive.[17] More productive and larger firms (bigger z) have looser borrowing limits than small firms, independent of the degree of financial market development. Also, stronger financial market development (lower ξ) increases the loan availability for all firms, independent of productivity.

Furthermore, the maximum loan relative to capital for a firm with productivity z is

$$\frac{\overline{B}(z)}{K_{fb}(z)} = \kappa_1 - \frac{\kappa_2}{K_{fb}(z)}.$$

The relation between debt limits to assets and size is affected by the economy's financial development or easiness to acquire credit information, ξ. When credit information is free ($\xi = 0$), all firms face the same borrowing limits relative to assets. This is because the problem is homogeneous with respect to z. When credit costs are large ($\xi > 0$), small firms are constrained in their borrowing relative to large firms because the credit cost increases default risk disproportionately for them. Moreover, the disadvantage of small firms relative to large firms becomes more pronounced as ξ increases. The following proposition summarizes this finding.

Proposition 1. *In the case without uncertainty, $\delta = 0$, and under assumptions 1 and 2, the relation between debt limits to assets and firm size is decreasing in the degree of financial development:*

$$\frac{d^2\overline{B}(z)/K(z)}{dK(z)d\xi} > 0. \tag{10}$$

Proof. Direct differentiation of equation (10) delivers the result.

Deriving analytical expressions for debt limits in the case with uncertain productivities is difficult due to lack of analytical solutions for the firm's decision rules of debt and investment. All the results regarding borrowing limits, sizes, and financial market development, however, carry through when we solve numerically the model for the more general case with uncertainty.

4 Quantitative Implications of the Model

We now assess quantitatively how default risk shapes firms' financing and growth. We calibrate the model to two representative countries, Bulgaria and the United Kingdom. The

[17] $\kappa_1 = \frac{[(1+r-\alpha)(1-\theta)+\theta(1-\beta)(1+r)]}{\alpha(r(1-\theta\beta)+\theta(1-\beta))}$ and $\kappa_2 = \frac{\alpha(1+r)(1-\theta\beta)}{\alpha(r(1-\theta\beta)+\theta(1-\beta))}$.

important parameters that capture financial frictions are calibrated to obtain the financing patterns observed in the cross section of firms in each country. We then find that default risk can quantitatively account for the relation of firm size with growth found in each country. Improving financial markets in the model reduces the difference in both growth rates and leverage ratios of small versus large firms, which makes the model fully consistent with the empirical evidence. Better financial markets also increase significantly the output of small firms. We also find that lack of contract enforcement is especially detrimental for firms in economies with more volatile productivity shocks.

4.1 Calibration

We calibrate the model twice to match Bulgarian and British data in 2005. The following parameters are chosen independently of the model equilibrium. The interest rate r is set to 4% per annum for Bulgaria and 2% per annum for the UK, which corresponds to the real interest rates in these countries from IFS.[18] The decreasing returns parameter α is chosen to be 0.90, following Atkeson and Kehoe (2005). The probability of re-accessing credit markets after default θ is set to 0.10 following Chatterjee et al. (2007) so that the average number of years that defaulters are excluded from credit markets equals 10 years.

All other parameters are calibrated jointly such that our model produces relevant moments of Bulgarian and British firm datasets. We assume that firms' idiosyncratic productivity consists of a permanent component μ_z and an i.i.d. component ε such that the productivity for firm i equals $z_t^i = \mu_z^i \cdot \varepsilon_t^i$. To make the distribution of firms in our model tractable, we choose a finite number of μ_z and ε_t to parameterize the distribution of productivity. We assume that μ_z can take five values $\mu_z \in \{\mu_z^1, \mu_z^2, \mu_z^3, \mu_z^4, \mu_z^5\}$ and that ε_t can take two values $\{\varepsilon_L, \varepsilon_H\}$. Each μ_z is assumed to have equal mass. Without loss of generality, we assume that transitory shocks have a mean of one, and thus the low shock ε_L and its probability p_L are sufficient to capture the transitory idiosyncratic shock process. We jointly calibrate $\{\mu_z^1, \mu_z^2, \mu_z^3, \mu_z^4, \mu_z^5, \varepsilon_L, p_L, \beta, \xi, \delta\}$ to match the following 10 moments in the data: the median asset levels of five asset quintiles in each country, the average real sales growth rate from 2004 to 2005 of 53% in Bulgaria and 11% in the UK, the average coefficient of variation for sales across firms of 0.40 in Bulgaria and 0.3 in the UK, the mean leverage ratio of 0.65 in Bulgaria and 0.84 in the UK, the leverage ratio of firms in the first asset quintile of 0.45 in Bulgaria and 1.18 in the UK, and the mean age of firms of 10 years across countries in Europe.[19] Table 6 summarizes all the parameter values in the calibration.

[18] The real interest rate is constructed as the difference between the annual nominal lending rate and the inflation rate.

[19] The coefficient of variation for sales is computed from the detrended time series of real sales of each

Table 6: Parameter Values in Benchmark Calibrations

	Parameter	Bulgaria	United Kingdom	Target
Interest rate	r	0.04	0.02	Annual real interest rate
Re-entry prob.	θ	0.10	0.10	Chatterjee et al (2007)
Technology	α	0.90	0.90	Atkeson and Kehoe (2005)
Permanent prod.	$\mu_z^1, ..., \mu_z^5$	1.3, 1.5, 1.6 1.8, 2.1	1.2, 1.4, 1.6, 1.9, 2.3	Median quintile asset
Temporary prod.	$\varepsilon_L, \varepsilon_H$	0.21, 1.13	0.48, 1.08	Mean CV sales
	p_L	0.145	0.13	Mean sales growth rate
Death rate	δ	0.08	0.08	Mean age of firms
Credit cost	ξ	0.03	0.0	Leverage for 1st asset quintile
Discount factor	β	0.94	0.96	Mean leverage

The calibrated ξ parameter for Bulgaria equals 0.03, which corresponds to 0.08% of output for the average firm. The credit costs are higher for the smallest firms and equal 4.3% of the output of firms in the first asset quintile. The calibrated ξ parameter for the UK equals zero.[20]

4.2 Debt Schedules and Dynamics

Before presenting the quantitative results, it is informative to understand how default risk affects firms' debt schedules and how these restrictions on credit impact firms' choices of debt, capital, and dividends.

Let's start by looking at the equilibrium debt schedules (B', K', B'_R, z) that arise due to default risk. The left panel of Figure 3 shows that debt schedules are more lenient if firms choose larger capital levels. The panel plots the possible pairs (B', B'_R) (relative to the first best capital as in eq. 5) for two firms with mean productivity μ_z^1 and capital choice equal to 100% and 80% of the first best level. The slope of the schedule equals $(1-\delta)/(1+r)$ when B_R ranges from 0.02 to about 0.4.[21] For larger B_R values, the firm defaults in the low shock the following period; the slope of the schedule in this range is $(1-p_L)(1-\delta)/(1+r)$. For even larger B_R values, the firm defaults with probability one, and thus these contracts are not offered in equilibrium. Default risk not only increases the effective interest rate paid on loans, but also generates borrowing limits. Importantly, these limits are increasing in the capital choice of the firm. As shown in the figure, the maximum transfer B' that the firm can get is 1.20 if the firm chooses capital equal to the first best or 1 if capital is 20% lower.

firm for 2000–2005.

[20] In the calibration we restrict ξ to be non-negative, and for the UK this constraint is binding.

[21] The schedule doesn't start at the origin due to the fixed cost.

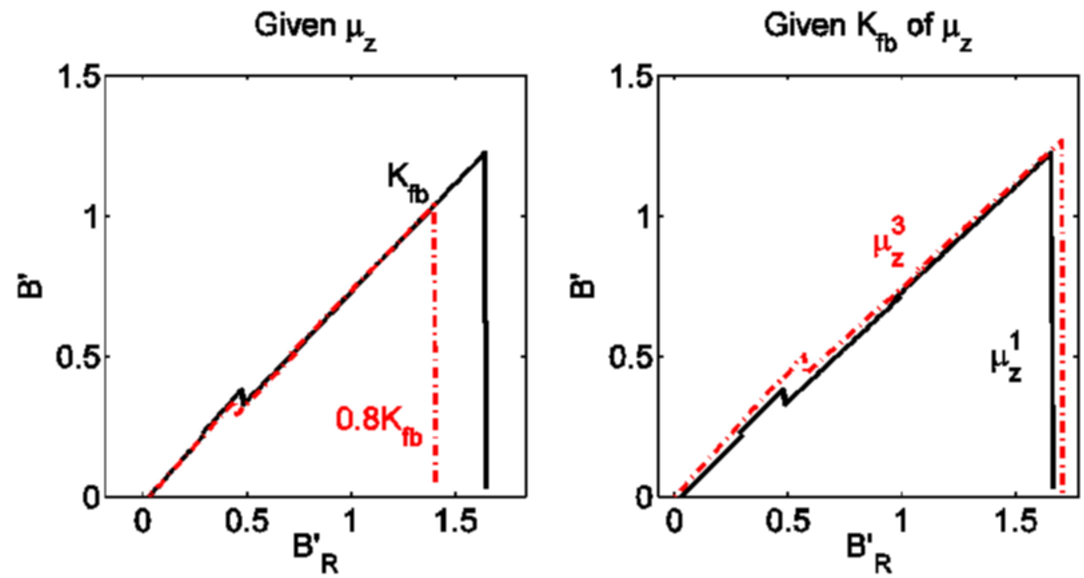

Figure 3: Debt Schedules

Debt schedules are also more lenient for firms with higher productivity. The right panel of Figure 3 plots the possible pairs (B', B'_R) (relative to their corresponding first best capital levels) for two firms with mean productivity μ_z^1 and μ_z^3 when capital K' equals $K_{fb}(\mu_z^i)$. As in Proposition 1 for the deterministic case, firms with higher productivity default for disproportionately higher levels of debt. Hence, they can borrow relatively larger loans at both risk free rates and risky rates.

The limitations on debt contracts affect the way firms respond to shocks. When experiencing sequences of bad shocks, firms tend to reduce their scale to maintain non-negative dividends and increase their debt financing, climbing up their debt schedules. When experiencing a good shock, these inefficient firms expand their scale and reduce their debt, sliding down their debt schedules. These dynamics imply that firms with the same permanent productivity display different sizes that depend on the history of shocks. Across these firms, inefficiently small firms tend to have higher growth rates and higher leverage ratios. To illustrate these dynamics, consider the decision rules for a firm with median permanent productivity μ_z^3 in the Bulgarian calibration shown in Figure 4.

Optimal policies depend on the permanent component of productivity and a single endogenous state variable: *cash on hand*, which equals output minus debt repayment, $\mu_z \varepsilon K^\alpha - B_R$. Cash on hand encodes the information regarding the firm's history of transitory shocks and it is low when firms have a low productivity shock, large debt due, and small capital. In Figure 4 we plot the optimal capital choice, dividends, and debt relative to the

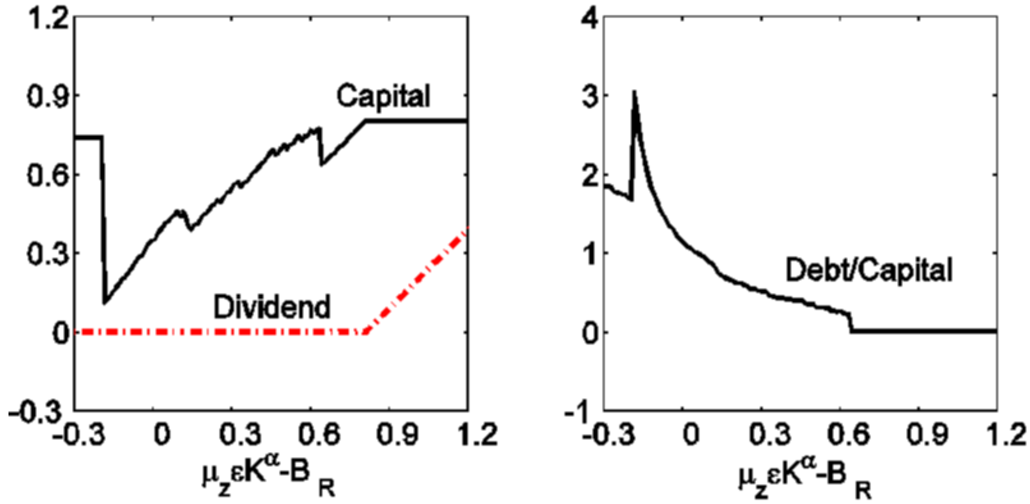

Figure 4: Policy Rules

capital choice as a function of cash on hand. We report capital, dividends, and cash on hand relative to the first best capital for this firm.

With large cash on hand, the firm invests a constrained efficient level, distributes dividends, and holds a low level of debt. The low debt level is due to a precautionary motive, similarly as in standard precautionary savings models (Aiyagari 1994 and Huggett 1993). With uncertainty the firm may not find it optimal to exhaust its borrowing opportunities because large debt increases the likelihood of firm failure. Thus, the firm has incentives to reduce its debt to this low level whenever possible under good transitory shocks.

With intermediate levels of cash on hand, the firm stops paying dividends, and tends to increase debt and decrease investment. Although larger capital choices allow firms to face more lenient debt schedules, large capital choices might also require firms to choose larger loans, such that dividends remain non-negative. The non-monotonicities in the capital choice reflect precisely this trade-off. However, overall we find that as the firm's cash on hand decreases, the choice of capital is lower to prevent debt from increasing too rapidly. All else equal, smaller loans are beneficial to avoid future default, which is costly because the expected value of keeping the project is large.

With low levels of cash on hand, the firm has very large debt to repay and finds it no longer optimal to avoid default. In anticipation of default under the low shock the following period, the firm chooses high debt and adjusts investment to a more appropriate scale for the high shock only.[22]

[22] The jump in investment in the default region of Figure 4 is an artifact of our two discretized transitory shocks.

Firm size depends on its permanent productivity and also its history of transitory shocks. A firm is small either because it has a low level of permanent productivity (*unproductive*) or because it has a sequence of bad shocks (*unlucky*). The different reasons why firms are small have different financing implications. Unproductive small firms tend to have low debt to asset ratios given their restrictive loan schedules. Unlucky small firms, however, tend to have high debt to asset ratios as a result of the bad shocks. But both unlucky and unproductive small firms tend to have inefficient scales because either they are closer to their borrowing limits or have restrictive debt schedules. These small firms have high growth rates when hit with good shocks, as these shocks alleviate their needs of credit and they can expand their size to a more efficient level.

4.3 Main Quantitative Results

Let's now examine the model's main quantitative results. We compute and simulate the model twice: one under the Bulgarian calibration and one under the British calibration. In each simulation, we obtain a model economy with 15,000 firms over 500 periods. The model delivers in the long run a cross-sectional distribution of firms, which we use to compute the model's statistics. For each period, we divide the cross section of firms into five asset quintiles. In the model, firm size equals the assets of the firm: capital K, plus savings B_R if $B_R < 0$. We compute for every asset quintile and for the entire distribution of firms, average sales growth rates, leverage ratios, and median asset levels. The model results are reported in Table 7, together with the data statistics.

The table shows that the Bulgarian calibration matches successfully the target moments in the data. In particular, the model reproduces the median asset in each asset quintile, the average sales growth, the mean leverage ratios of the whole sample and of the first asset quintile. The UK calibration matches the median assets, the average leverage and sales growth, but produces a leverage ratio of the first asset quintile lower than that in the data, 0.95 versus 1.18.[23]

We now evaluate the model implications on the size-growth and size-financing patterns. The model under the Bulgarian calibration generates a negative size-growth relation that matches the data well. The growth rates for the smallest and largest firms are 77% and 40%, respectively, in the model, close to 73% and 39%, respectively, in the data. The model also generates an increasing leverage pattern ranging from 0.47 for the smallest firms to 0.68 for the largest firms, similarly as in the data.

The quantitative implications are less successful under the British calibration. The model

[23] This feature is mainly due to the restriction that the fixed credit cost ξ be non-negative.

Table 7: Main Quantitative Model Results

Bulgaria	Data			Model		
Quintile	Assets	Growth	Lev.	Assets	Growth	Lev.
1	1	0.73	0.45	1	0.77	0.47
2	5	0.51	0.63	5	0.56	0.62
3	12	0.51	0.72	13	0.46	0.60
4	38	0.49	0.73	40	0.48	0.62
5	198	0.39	0.71	202	0.40	0.68
Mean	51	0.53	0.65	52	0.53	0.60
U.K.	Data			Model		
Quintile	Assets	Growth	Lev.	Assets	Growth	Lev.
1	1	0.23	1.18	1	0.17	0.95
2	5	0.13	0.87	4	0.09	0.79
3	17	0.05	0.79	14	0.10	0.82
4	66	0.07	0.71	71	0.10	0.82
5	508	0.05	0.66	516	0.08	0.80
Mean	120	0.11	0.84	121	0.11	0.84

delivers a negative size-leverage relation and a negative size-growth relation, but the overall fit is less tight than that under the Bulgarian calibration. Specifically, the growth rate declines from 0.17 for the smallest firms to 0.08 for the largest firms in the model, while the growth rate declines from 0.23 to 0.05 in the data. The leverage ratio for the largest firms is 0.80 in the model, but 0.66 in the data.

The firm-specific debt schedules and the firm's position on its debt schedule drive these results. In the Bulgarian calibration, the overall results are driven by the variation in debt schedules across firms with different permanent productivity as the majority of firms in asset quintile i have permanent productivity μ_z^i. The unproductive small firms in economies with weak financial markets have disproportionately restrictive debt schedules, which induces these firms to have low leverage ratios and high growth rates. The model under the British calibration is homogeneous across permanent productivity as $\xi = 0$, and thus the overall results are driven by the unlucky firms who have climbed up their debt schedules. In this calibration, the majority of firms in asset quintile i also have permanent productivity μ_z^i; however, we find that a larger fraction of firms with higher permanent productivity end up in lower asset quintiles. Firms in this economy have better borrowing opportunities to sustain a longer sequence of bad shocks without defaulting while becoming inefficiently small. For example, there are 98% of firms with permanent productivity μ_z^1 in the first asset quintile in the Bulgarian calibration and 94% of firms with productivity μ_z^1 in this quintile in the British calibration. The small unlucky firms have higher debt to asset ratios and also higher growth rates, and hence the model can match the size-growth and size-leverage patterns of the British economy.[24]

Our model is also consistent with several other empirical predictions. First, the model predicts that firms who default have larger leverage ratios than continuing firms. This implication is consistent with Campbell, Hilscher, and Szilagyi (2006), who find that the leverage ratios of failing public firms in the United States are larger than those of continuing firms. The Bulgarian calibration generates a default rate of 1.8% every period, and a mean leverage ratio of 1.78 for these defaulting firms, compared with 0.58 for continuing firms. The British calibration also delivers larger leverage ratios for defaulting firms than for continuing firms, 1.87 versus 0.83. In this calibration, 1% of firms default every period.

Second, the model predicts that large firms are the ones that distribute dividends, which is consistent with U.S. data as documented in Fazzari et al. (1988). In the Bulgarian calibration, 75% of firms in the first asset quintile do not pay any dividends, compared to 62% for firms in the fifth asset quintile. In the UK calibration, 61% of firms in the first asset quintile do not pay any dividends, compared to 52% for firms in the fifth asset quintile. We also note that firms in economies with better financial markets are more likely to pay dividends. Paying dividends more often is intrinsically related to a lower precautionary motive for firms in economies with larger loan availability.

Our results demonstrate that financial frictions can rationalize quantitatively the growth-size relation observed in both Bulgaria and the UK, though the fit is tighter for Bulgaria than for the UK. These exercises are revealing because we use the financial variables of firms to discipline the extent to which the growth-size relation can be attributed to financial imperfections. By parameterizing the model to mirror the debt financing patterns of firms, we find that the model delivers quantitatively the growth-size relation in the data.

4.4 Counterfactuals

We now use our calibrated model to perform two counterfactual experiments. We want to quantify how much of the differential growth rate and debt financing for firms in Bulgaria and the UK is due to the cross-country variation in financial frictions versus the productivity structure. We find that country-specific financial frictions are most important for the differential leverage ratios and growth rates across firms. Removing financial frictions also increases output especially for the small firms. The country-specific productivity structure also contributes modestly toward the leverage-size and growth-size patterns in an environment with default risk. We find that lack of enforcement in debt contracts is especially

[24] The intrinsic positive comovement of growth rates and leverage ratios present in our model with zero credit costs is similar to the one analyzed by Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), and DeMarzo and Fishman (2007).

damaging for economies with a more volatile productivity structure.

In the first experiment, we lower the fixed credit cost to zero, while keeping the remaining parameters of the Bulgaria calibration unchanged. These results are presented in the second panel of Table 8, where asset levels are normalized to the mean asset level in the first quintile under the Bulgaria benchmark calibration. In this experiment, the size-leverage relation becomes negative, and the size-growth relation becomes flatter. In particular, the difference in growth rates between the smallest and largest firms declines from 37% to 15%, and the difference in leverage ratios increases from -21% to 9%. Also, lowering credit costs increases the mean leverage from 60% to 73%, and decreases the mean growth rate from 53% to 49%. All these implications are fully consistent with our empirical findings.

Table 8: Counterfactual Experiments

	Bulgaria benchmark			UK credit costs			UK credit cost and productivity structure		
Quintile	Assets	Growth	Lev.	Assets	Growth	Lev.	Assets	Growth	Lev.
1	1	0.77	0.47	1.2	0.56	0.77	0.9	0.17	0.92
2	5	0.56	0.62	5.2	0.55	0.79	3.3	0.11	0.83
3	13	0.46	0.60	13	0.51	0.71	13	0.11	0.82
4	40	0.48	0.62	41	0.41	0.68	62	0.11	0.84
5	202	0.40	0.68	211	0.41	0.70	458	0.09	0.81
Mean	52	0.53	0.60	54	0.49	0.73	107	0.12	0.84

Improved financial markets also impact the output of firms. When the fixed credit cost is lowered from 0.03 to zero, the average output rises by 19% for firms in the first asset quintile and by 1.5% for firms in the fifth asset quintile.[25] In our model, the impact of financial market development on aggregate output is modest because aggregate statistics are driven mainly by the large firms. However, one potential additional channel that default risk can affect aggregate output is through its impact on firm entry. Firms start to operate when their value is larger than zero. High fixed credit costs reduce the value of firms such that firms enter only when their productivity is high enough. This results in fewer operating firms and lower output in less financially developed countries.

For the second experiment, we maintain the zero fixed credit cost and change the productivity structure (both permanent and transitory) to the one calibrated for the UK firm dataset. These results are presented in the third panel of Table 8. In this model economy, the difference in growth rates between the smallest and largest firms further decreases from 15% to 8%, and the difference in leverage ratios further increases from 9% to 11%.

[25] All asset levels in Table 8 are normalized to the mean asset level of firms in the first quintile of the Bulgarian benchmark.

Changing the productivity structure has a modest effect on the differences in growth rates and leverage ratios across economies and firms relative to changing financial market development. But in this second experiment, it is the presence of financial frictions that allows productivity differences across economies to change the differential growth rates of small and large firms in the model. If debt contracts were perfectly enforceable, all productivity structures would deliver a flat growth-size relation. We find that economies with higher volatility, as in the second panel of Table 8, are disproportionately affected by the lack of contract enforcement and default risk. Changing the productivity structure alone does have an substantial impact for the mean firm growth, as seen in Table 8.

These two experiments reveal that the differential growth and leverage ratios across firms and economies are mostly driven by financial factors: both cross-country differentials in financial markets and cross-country productivity differentials in the presence of default risk.

5 Conclusion

We have studied both empirically and theoretically the growth and debt financing patterns of firms across countries. Using a broad and comprehensive firm-level database from 22 European countries, we documented that in less financially developed countries — countries with low private credit to GDP ratios or limited credit bureau coverage — small firms grow faster and use less debt financing than large firms. These findings are robust to controlling for age, sector, and country fixed effects. Our empirical analysis provided a new picture of the relation of financial market development with debt financing and growth across firms and countries.

We then developed a quantitative dynamic model of heterogeneous firms where financial development affects firm financing and growth through the availability to credit. By calibrating the degree of financial development to the observed debt financing patterns of firms, we assessed the model implications on firm growth. We found that financial market development is important in explaining quantitatively the difference in growth rates across firms and across countries.

A contribution of the paper is to use micro firm-level data in a quantitative model to study the growth and financing patterns in the cross section of firms of multiple countries. A natural next step is to analyze a time dimension by introducing aggregate fluctuations in the model to study the cyclical features of firm dynamics. Moscarini and Postel-Vinay (2009) document that for the United States, the variance in the firm size distribution is procyclical, and the early phases of booms are mainly driven by the expansion of small firms. Our framework can prove useful in analyzing the impact of financial frictions on the cyclical cross-sectional firm dynamics. More generally, we view our quantitative methodology that combines firm-level data with theory as a useful tool for analyzing the interaction of micro decisions with macro implications.

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Empirical Appendix

In this appendix, we first examine the comparability of the country samples. We then describe in detail the procedure for cleaning the data in the regressions. Finally, we present additional robustness of the main empirical regressions.

Comparability of Country Samples

This section analyzes the coverage and comparability of the AMADEUS dataset across countries. The European Commission Report contains information on the distribution of the universe of firms in the business sector for most of the countries in our sample. They report the percentage of enterprises that have 1 to 9 employees, 10–49 employees, 50–250 employees, and over 250 employees. Hence, we compare the fraction of firms for each employment category in our dataset with that in the universe from the report.[26]

Unfortunately, the employment information is not reported for every firm in AMADEUS. The lack of employment data can be a severe problem for some countries. For example, in the Netherlands only 65% of firms in the business sector that report assets and liabilities report employment. Moreover, this lack of employment information is the most severe for small firms. Hence, we impute employment measures for firms that do not report employment in AMADEUS. To do this, we run regressions country by country of log(employment) on log(assets) and log(liabilities). The fit of these regressions is good, with R squares above 0.6 for all countries.[27] We then impute employment for the firms that do not report it using the estimated coefficients and their assets and liabilities.

Table 9 reports the firm distribution in AMADEUS and in the universe for countries for which we have data. The table shows that in our sample, the majority of firms are small with only 1 to 9 employees (micro firms) as in the data. In our sample, on average 70% of firms are micro firms, whereas in the universe of firms, 89% are micro. In our sample, only about 1% of firms have more than 250 employees, which is consistent with the universe where less than 1% of firms fall into this category. Importantly, the coverage in AMADEUS is similar across countries, with most countries having micro firms between 60% and 80%.

[26] For this comparison, we include only firms in sectors that correspond to the business sectors in the European Commission Report.

[27] Introducing additional controls such as firm age and sector dummies changes the fit of the regressions only marginally.

Table 9: Coverage and Comparability of Country Datasets

	AMADEUS Dataset				European Commission – Universe			
	Micro 1-9	Small 10-49	Medium 50-250	Large >250	Micro 1-9	Small 10-49	Medium 50-250	Large >250
Belgium	0.895	0.087	0.014	0.004				
Bulgaria	0.689	0.233	0.062	0.016	0.902	0.08	0.016	0.002
Croatia	0.625	0.264	0.089	0.022				
Czech Rep	0.601	0.269	0.106	0.024	0.953	0.038	0.008	0.001
Denmark	0.765	0.189	0.037	0.009	0.869	0.109	0.019	0.003
Estonia	0.788	0.179	0.029	0.004	0.815	0.151	0.03	0.004
Finland	0.797	0.160	0.033	0.010	0.924	0.061	0.012	0.003
France	0.810	0.156	0.027	0.006	0.923	0.064	0.01	0.003
Iceland	0.911	0.083	0.005	0.001				
Ireland	0.736	0.236	0.025	0.002				
Italy	0.695	0.264	0.034	0.007	0.946	0.048	0.005	0.001
Latvia	0.317	0.408	0.229	0.046	0.831	0.139	0.027	0.003
Lithuania	0.270	0.444	0.250	0.035	0.755	0.197	0.043	0.005
Malta								
Netherlands	0.750	0.198	0.040	0.012	0.89	0.091	0.016	0.003
Norway	0.795	0.182	0.020	0.003				
Portugal	0.773	0.196	0.028	0.004				
Romania	0.875	0.098	0.023	0.005	0.881	0.09	0.023	0.006
Russia	0.314	0.485	0.164	0.037				
Spain	0.727	0.238	0.030	0.006	0.923	0.068	0.008	0.001
Sweden	0.819	0.144	0.030	0.007	0.947	0.043	0.008	0.002
UK	0.755	0.190	0.040	0.015	0.864	0.114	0.018	0.004
Average	0.700	0.224	0.063	0.013	0.887	0.091	0.017	0.003

Data Cleaning Procedure

In this section, we describe the detailed procedures in assembling the cross-country datasets analyzed in the empirical section. In particular, we present step-by-step data cleaning procedures, construction methods of all the variables, and data sources for the country-level statistics.

Firm Data

We download the data from the AMADEUS database compiled by Bureau Van Dijk Electronic Publishing. We delete all firms in the financial and government sectors which correspond to NACE codes 65, 66, 67, and 75. We delete firms that have one or more of the following characteristics: missing total assets, non-positive total assets, missing total liabilities, and negative liabilities.

For the leverage regressions, we generate the leverage variable for each firm by taking the ratio of the firm's total liabilities to total assets. We drop the outlier firms with leverage ratios in the top 1st percentile of the leverage distribution in each country. We generate the *Asset Share* variable by dividing the firm assets by the sum of total assets in its country. We generate the interaction variables by multiplying *log(Asset Share)* by the corresponding variables of interest such as private credit to GDP or credit coverage.

For the growth regressions, we follow these additional steps. We drop the firms with missing, zero, or negative operating revenue (or sales) in 2004 and 2005. We generate the real growth variable as

$$\frac{\text{operating revenue}_{05} * \text{exchange rate depreciation}_{05/04}}{\text{operating revenue}_{04} * \text{CPI inflation}_{05/04}} - 1.$$

We drop outlier firms with growth rates in the top 1st percentile of the growth distribution in each country. For this new clean sample, we generate the *log(Asset Share)* variable and the interaction variables as described above for the leverage regressions.

For both regressions, we construct dummy variables for age groups. Firms are classified into 7 age groups based on the firm age in terms of years: $[0, 5)$, $[5, 10)$, $[10, 15)$, $[15, 20)$, $[20, 25)$, $[25, 30)$, $[30, \infty)$.

Country Data

The country-level statistics are obtained from various data sources. Private credit to GDP from 2000 to 2004 is from the *World Development Indicators* of the World Bank. Credit bureau coverage in 2005 is from *Doing Business 2006* published by the World Bank. Exchange rates, defined as local currency per euro, and CPI inflation from 2004 to 2005 are from the *International Financial Statistics* of the International Monetary Fund.

Robustness of the Main Regressions

This section analyzes the robustness of the cross-country regression results presented in Tables 3, 5, and 4. We first present robustness checks on the sample attrition. We then consider employment as an alternative firm size measure. Finally, we consider adding additional interactions of size with age and GDP per capita. We find that the results of the main regressions are maintained under these alternative specifications.

Sample Attrition

In our analysis we considered the growth rate of firms conditional on being in the sample in both periods 2004 and 2005. Even though sample attrition is actually very small in the growth regressions (1% on average), a potential concern for our results is whether the interaction coefficients on size and financial market development might be capturing a higher sample attrition for small firms in less financially developed countries. However, we find that sample attrition is uncorrelated with both measures of financial market development, and hence our main growth results are not driven by differential size-attrition patterns across countries.

In particular, we compute the sample attrition rate for each asset quartile in each country. We then pool the quartile sample attrition rates for all countries and regress these attrition rates against the asset quartiles and the interaction variables of size (quartile) by financial market development. We find a significant negative coefficient on the size categories and an insignificant coefficient on the interaction between size and financial development. Hence, small firms in the first asset quartile have higher sample attrition on average across all countries, but the differential attrition rates across firms of different sizes is uncorrelated with the country's financial market development.

Employment as Firm Size

Table 10 reports four leverage and growth regressions where firm size is defined by employment. Employment is either the actual number of employees reported by each firm or the imputed employment measure constructed in the section of the comparability of the country samples. The share of employment of a firm equals the ratio of its employment to the total employment in its country. The sample is the same as that in the main regressions, with the exception of the firms that do not report employment and at the same time have zero liabilities. We exclude these firms when imputing their employment.

The results show that using employment as an alternative measure of firm size does not change our main conclusions. Small firms grow faster than large firms and use less debt financing in less financially developed countries. The interaction coefficients in the four regressions have signs as expected and are statistically significant at the 1% level.

Additional Interactions: Size with Age and GDP Per Capita

We now conduct additional robustness tests of the leverage and growth regressions by adding two additional variables: the interactions of firm size with the seven age groups and with the country's GDP per capita. By doing so, we allow the relation of size with growth and leverage

Table 10: Robustness: Employment as Size

	Leverage		Growth	
	Private Credit to GDP	Credit Bureau Coverage	Private Credit to GDP	Credit Bureau Coverage
Size (log(employment share))	-0.015***	-0.019***	-0.056***	-0.033***
	(0.0004)	(0.0003)	(0.001)	(0.001)
Interaction (credit to GDP × size)	-0.012***		0.026***	
	(0.004)		(0.001)	
Interaction (credit bureau coverage × size)		-0.020***		0.0013***
		(0.0003)		(0.0006)
Fixed effects	Yes	Yes	Yes	Yes
Country × Industry × Age				
Adjusted R^2	0.05	0.05	0.05	0.05
Number Observations	4459799	4459799	2553490	2553490

to be age dependent and to vary with the log of the country's GDP per capita. For both the leverage and growth regressions with either measure of financial development, we add these two interaction terms. Table 11 shows that in the leverage regressions, the coefficient on the interaction of firm size and private credit to GDP or credit bureau coverage remains negative and significant when adding the interactions of firm size with age and with the country's per capita GDP. Table 11 shows that in the growth regressions, the coefficient on the interaction of firm size and private credit to GDP remains positive and statistically significant with these two additional interaction variables. The table also shows that the coefficient on the interaction of firm size and credit bureau coverage remains positive and significant when the additional age interaction is introduced. This coefficient, however, becomes insignificant when we add the interaction of firm size and GDP per capita.

Table 11: Robustness on Growth Regression: Industry, Age, and GDP Per Capita Interactions

	Growth		Leverage	
	Private Credit to GDP	Credit Bureau Coverage	Private Credit to GDP	Credit Bureau Coverage
Size (log(firm's asset share))	-0.181***	-0.202***	-0.204***	-0.25***
	(0.011)	(0.011)	(0.005)	(0.005)
Interaction (Financial Development × size)	0.016***	0.005***	-0.019***	-0.020***
	(0.0008)	(0.0006)	(0.0004)	(0.0003)
Interaction (Mean growth ×size)	-0.064***	-0.078***	0.002	0.030***
	(0.003)	(0.003)	(0.001)	(0.001)
Interactions (industry × size)	Yes	Yes	Yes	Yes
Interactions (age group × size)	Yes	Yes	Yes	Yes
Interaction (log(GDP per capita) × size)	0.013***	0.016***	0.016***	0.020***
	(0.001)	(0.001)	(0.0005)	(0.0005)
Fixed effects Country × Industry × Age	Yes	Yes	Yes	Yes
Adjusted R^2	0.06	0.06	0.07	0.07
Number Observations	2568782	2568782	4564461	2568559